THE RELATION BETWEEN PATTERNING, EXECUTIVE FUNCTION, AND MATHEMATICS

by

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A Dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy at George Mason University

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Master of Arts
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DEDICATION

This dissertation is dedicated to Virginia Louise, Paige Victoria, and Kelly Jean.

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ABSTRACT

THE RELATION BETWEEN PATTERNING, EXECUTIVE FUNCTION, AND MATHEMATICS

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Patterning, or the ability to understand patterns, is a skill commonly taught to young children as part of school mathematics curricula. While a number of studies have demonstrated that patterning is beneficial for young children acquiring mathematical skills, little research exists that examines the cognitive components of the skill. It seems likely that some aspects of executive function, such as cognitive flexibility, inhibition, and working memory, may underlie patterning abilities in children. The primary objective of the present study was to examine the relationship between patterning and executive function skills for first-grade children. In addition, the relations between patterning, executive function, mathematics, and reading were examined. The results showed that patterning was significantly related to cognitive flexibility and working memory, but not significantly related to inhibition. Patterning, cognitive flexibility, and working memory were significantly related to mathematical skills. Only patterning and working memory were significantly related to reading. In sum, these findings suggest that cognitive flexibility and working memory may be two of the processes underlying patterning ability, and also provide further confirmation of the close relationship between patterning and mathematics.

Keywords: first-grade, education, mathematics, patterning, executive function

THE RELATION BETWEEN PATTERNING, EXECUTIVE FUNCTION, AND MATHEMATICS

Patterning, or the ability to identify and manipulate patterns, is a cognitive skill that is considered beneficial for young children both in the United States and around the world (Lee, Ng, Bull, Pe, & Ho, 2011; Mulligan & Mitchelmore, 2009; Papic, 2007; Waters, 2004). Hence, patterning is commonly taught to young children as part of school mathematics curricula, and patterning instruction is included as a mathematics component of the Common Core State Standards (National Governors Association Center for Best Practices, 2010) for children beginning in kindergarten. In the initial years of schooling, the first patterns taught in the classroom are typically basic repeating patterns (Liljedahl, 2004; Rittle-Johnson, Fyfe, McLean, & McEldoon, 2013; Threlfall, 1999). Examples of such repeating patterns are abababab, abbabbabb, or abcabcabc. Patterning is usually taught in preschools and primary grades in the United States and abroad (Papic, Mulligan, & Mitchelmore, 2011; Warren & Cooper, 2006). Manuals are available that describe how to teach patterning skills (Burton, 1982; Ducolon, 2000; Jarboe & Sadler, 2003). Despite the widespread assumption of an inherent benefit of teaching patterning to young children, the causal mechanism by which improving patterning improves performance in other areas, such as mathematics, is still largely unknown.

Empirical Studies of Patterning Instruction

Herman (1973) examined the effects of teaching simple patterns on mathematics achievement of kindergarten students in her dissertation. Following the patterning intervention, results on the Numbers Subtest of the Metropolitan Readiness Test (Hildreth, Griffiths, & McGauvran, 1969) revealed that children in the intervention group coming from English-speaking homes did significantly better on measures of numeracy than did the control group. Meanwhile, children in the intervention group who came from Spanish-speaking homes did not attain higher scores than the control group. However, a problem with this dissertation was that the control group came from kindergarteners in a different school, and so it may have been that a cohort effect was responsible for the differences found (i.e., children may have had different patterning skills at the outset of the research).

Over three decades later, Hendricks, Trueblood, and Pasnak (2006) worked with first graders who were identified by their teachers as having difficulty with their schoolwork. Children were randomly assigned to be taught either a large variety of patterns or academic material recommended by their teachers. Children in the patterning group were taught 480 patterns throughout the academic year, which ranged from simple alternating patterns to more complex multidimensional patterns. The patterning and control instruction sessions were matched in timing and duration. At the end of the year, the children who were given patterning instruction were able to identify and interpret patterns significantly better than were the children who did not receive patterning instruction. In addition, after the intervention and control groups were statistically equated on IQ, children who had made gains in their understanding of patterning also made significantly greater gains in mathematics and language scales of the Diagnostic Achievement Battery-2 than did children in the control group. Thus, the results of this study suggest that patterning instruction may not only improve mathematics skills, but also reading skills.

In 2005, Papic and Mulligan sought to answer the following two questions: is there a connection between children's patterning abilities and their development of pre-algebraic skills, and can a patterning intervention program result in long-term benefits for children's development in mathematics. The researchers conducted their investigation in two Sydney, Australia preschools, one of which received the intervention and one of which acted as the control group. The results of the individual task-assessment interviews suggested, after the patterning intervention, children in the intervention school were better at tasks for all patterning categories designed by the researchers. Observations were reportedly made by teachers and parents in the classroom about how the patterning instructions were translating to the children's mathematical development throughout the course of the study. When a follow-up study was conducted a year later, the children in the intervention group still demonstrated a better understanding of patterning than did the control group, again as measured by the task-assessment interviews (Papic & Mulligan, 2007). However, no statistical analyses were conducted, and children in the control preschool may have been inferior to those in the intervention preschool in regards to these skills prior to the intervention. Thus, the results of this study are not generalizable.

A study with a similar design was conducted again in Australia by Papic, Mulligan, and Mitchelmore (2011). Researchers examined the impacts of a patterning intervention in two preschools, one of which received the patterning intervention while the other school served as the control group. In the intervention school, researchers implemented a six-month patterning intervention that emphasized repeating and spatial patterns. At the end of the intervention, as well as one year later, children from the intervention school performed better on patterning tasks than did children from the control school. Compared to children in the control school, children in the intervention school showed a greater understanding of units of repeat and spatial structure, and they were also better able to explain and extend growing patterns. However, this also was not an adequately controlled study, and again the results may be invalid.

Also in Australia, Warren and Cooper (2007) studied third-grade children's understanding of visual growth patterns. In this study, patterning lessons were taught in a two lesson sequence to children in two third-grade classrooms. The two lessons included copying and continuing simple patterns, using language to describe patterns, and predicting and thinking about patterns. Pre and posttest scores seemed to demonstrate a growth in the children’s ability to understand, describe, and predict patterns. Warren and Cooper stated that they hoped the study would help to convey children’s ability to understand complex patterns in the early elementary school years, and that it would also emphasize the importance of incorporating patterning into elementary school classroom curricula. Nonetheless, again, this study does not allow for generalization; there was no control group assigned and no statistical analyses were conducted.

In 2013, Kidd, Gadzichowski, Gallington, Boyer, and Pasnak examined the effectiveness of patterning instruction with first graders. In this study, children who scored lower than their peers in patterning abilities in pretest assessments were randomly assigned to one-on-one, 15-minute lessons, three times a week on patterning, reading, mathematics, or social studies. Children in the patterning group were taught patterns, including single and double alternations, symmetrical patterns, progressive patterns, rotating patterns, and random repeating patterns. Children in the other groups were given lessons based on the State Standards of Learning, which parallel the Common Core State Standards (National Governors Association Center for Best Practices, 2010).

Scores on the posttests given to the children after six and half months of lessons showed that the children who received patterning instruction substantially outperformed the children in other instruction groups – mathematics, reading, or social studies – on an assessment of patterning abilities and on the Woodcock-Johnson III (WJ-III): 18A - Quantitative Concepts - Concepts (WJ-18A), which measures understanding of mathematical symbols, counting, and terms like greater and lesser (Woodcock, McGrew, & Mather, 2001). Children in the patterning instruction group scored significantly higher on the WJ-III: 18B - Quantitative Concepts - Number Series (WJ-18B) than did children in any other group, and children in the mathematics instruction group scored higher on this scale than did children who received reading or social studies instruction. The results of this study suggest instructing young children in patterning may not only improve children's patterning abilities, but also improve their understanding of mathematical concepts. However, there were no significant differences between any of the groups on the WJ-10 - Applied Problems, which measures mathematics skills, nor on the WJ scales 1, 2, or 9, which measure different aspects of reading.

The possibility of using patterning instruction to improve both reading and mathematics outcomes was also assessed by Kidd et al. (2014). As in Kidd et al.'s (2013) study, first-grade children who scored the lowest in their classroom on a pretest patterning assessment were randomly assigned to either a patterning instruction group or to groups receiving instruction in reading, mathematics, or social studies. The patterning, reading, mathematics, and social studies instruction were the same as those used by Kidd et al. (2013), but the achievement tests administered were different. The reading achievement measures administered in this study were the Gray Oral Reading Test-4 (GORT; Wiederholt & Bryant, 2001), the Test of Word Reading Efficiency (TOWRE; Torgesen, Wagner, & Rashotte, 1999), and the Test of Early Reading Ability-3 (TERA; Reid, Hresko, & Hammill, 2001). The mathematics achievement measures administered were the WJ-18A and WJ-18B, as well as the Key Math 3 test (Connolly, 2007).

At the end of the intervention, which took place over the course of the academic year, children in the patterning group scored significantly better in the assessment of the understanding of patterns than did children in any other group. The children in the patterning group and reading group scored significantly higher than did the children in the mathematics group or social studies group on the TERA Meaning and the TOWRE Word measures. In regards to the mathematics assessments, the patterning group and mathematics group scored significantly higher on the WJ-18A and WJ-18B than did children in the reading or social studies groups. Notably, the patterning group also scored significantly higher than all other groups on a number of the Key Math 3 subscales, which included numeration, addition, algebra, measurement, foundations, computation, data problems, and applied problems. Thus, the results of this study provide compelling evidence to suggest instruction in patterning can yield positive results, not only in regards to the understanding of patterns, but also in mathematics and reading.

Theoretical Speculations

The causal mechanism by which improving patterning improves performance in other areas, such as mathematics, is still largely unknown. However, theorists have proposed numerous ideas just as to why and how patterning might work to help facilitate other positive outcomes. As stated by Clements and Sarama (2007c), patterns may encourage higher order thinking that can help with more advanced concepts, as "identifying patterns helps bring order, cohesion, and predictability to seemingly unorganized situations and allows one to recognize relationships and make generalizations beyond the information directly available" (p. 507). Hendricks et al. (2006) proposed "development from forming and understanding simple sequences to forming and interpreting complex sequences presumably corresponds to development from intuitive, perceptually driven thought to more logical thought" (p. 80).

The inclusion of patterning in today's classroom curricula is largely based on a consensus of educators who believe children's understanding of patterns is of great educational value in kindergarten through eighth grade, and beyond (National Council of Teachers of Mathematics, 2000). According to McGarvey (2012), "recognizing patterns is seen as critical to algebraic thinking because it develops students' ability to express generalities by recognizing commonalities, articulating rules and relationships, and eventually representing those relationships using symbols" (p. 312). The ability to detect the structure in patterns has been described by researchers as a demonstration of "pre-algebraic thinking" in children (Mulligan & Mitchelmore, 2009).

Patterning and Executive Function

While the reasoning about the cognitive underpinnings involving patterning has been sophisticated, there is still little empirical evidence as to what cognitive mechanisms are involved in understanding patterns. However, in a recent study by Bock et al. (2015), researchers examined the relationships between patterning and three components involved in executive functions. The components of executive function Bock et al. examined were cognitive flexibility, inhibition, and working memory. Cognitive flexibility refers to the ability to shift between mental tasks, inhibition refers to the ability to stop a relatively automatic response, and working memory involves actively manipulating information (Duan, Wei, Wang, & Shi, 2010). These three components of executive function have been found to be separate and distinct, although also correlated with one another (Miyake et al., 2000).

The results of Bock et al. (2015) suggest only cognitive flexibility is related to patterning skills. However, researchers have yet to try to replicate these findings. One limiting factor in Bock et al.'s study was that the patterning task included a somewhat limited amount of patterns. The children in this study were asked to complete a set of 18 patterns, which included a line of numbers, letters, or shapes that either increased or decreased in value, position of the alphabet, or size, or were symmetrical. All patterns were presented horizontally, and the missing item was always the last in the sequence. Bock et al. also examined the relation between patterning and reading, although no relation was found, and the relation between patterning and mathematics was not assessed. Thus, in the present study, the goal was to look at the relationship between patterning and the components of executive function when a larger amount and a greater variety of patterns were examined. The other goals included examining the relation between patterning and mathematics, as well as between patterning and reading.

Hypotheses

The hypotheses for the present study included that patterning would be significantly related to each of the three components of executive function examined – cognitive flexibility, working memory, and inhibition. All of the measures of executive function – cognitive flexibility, working memory, and inhibition – were also expected to be correlated with one another. Patterning was expected to be significantly related to all three measures of mathematics. Cognitive flexibility, working memory, and inhibition were also expected to be significantly related to the three measures of mathematics. Finally, patterning was expected to be significantly related to the reading measure. Cognitive flexibility, working memory, and inhibition were also expected to be significantly related to the reading measure.

METHODS

Participants

Parental consent was obtained for first-grade children in two different elementary schools in a public school system of a metropolitan area in the mid-Atlantic region. Many of the children in this system were from immigrant families, and many lived in subsidized housing. Approximately 70 percent of children attending these two elementary schools qualified for free or reduced lunch. All first-grade classrooms in each of the two schools were included (11 classrooms total), except for two classrooms reserved for children not yet proficient in English. The children selected for testing were among those identified by their teachers as performing about average academically as compared to their peers. After three children chose not to participate in the testing, 45 girls and 29 boys remained. Of these 74 children, 27 (approximately 37%) were African American, 19 (26%) were Hispanic/Latino, 12 (16%) were Middle Eastern, 12 (16%) were Caucasian, and 4 (5%) were of two or more races or of an unspecified ethnicity. The mean age of the children at the beginning of testing was 7 years, 2 months, *SD* = 3.42 months.

Measures

Eight different assessments were administered to the children, which included a measure of patterning skills, three measures of mathematical skills, one measure of reading ability, and three measures of different elements of executive function (cognitive flexibility, working memory, and inhibition).

Patterning Assessment. A researcher-designed patterning measure was administered to examine children's ability to recognize a pattern and to then fill in a missing item in the sequence (Gadzichowski, 2012). The patterning assessment had 48 pattern problems. Due to its length, the assessment was administered during two separate testing periods, with 24 pattern problems per session. The patterns included those of numbers, letters, shapes, pictures, and clock faces that either increased or decreased in value, position of the alphabet, or size, and also included symmetrical and rotating patterns. The missing item was presented equally often in the beginning, middle, or end of the sequences. Half of the patterns were presented horizontally, half vertically, and each pattern problem consisted of four items and one blank space. Children were asked to examine the pattern and to then choose from four possible options shown below or next to the sequence in order to fill in the blank. The patterns were presented one at a time, and each child was allowed as much time as needed to choose an answer. Scores of the total number of patterning problems correct (out of 48) were then used in the analyses.

Woodcock-Johnson Tests of Cognitive Abilities III. The Woodcock-Johnson III (WJ-III) is a set of tests for assessing general intellectual ability and academic achievement (Woodcock, McGrew, & Mather, 2001). The particular subtests of the WJ-III used in the present study were: 10 - Applied Problems; 18A - Quantitative Concepts - Concepts; and 18B - Quantitative Concepts - Number Series.

Subtest 10 consists of oral mathematics word problems and measures a student's ability to analyze and solve mathematics problems. Subtests 18A and 18B assess a student's understanding of mathematical concepts, symbols and vocabulary, and the ability to understand number patterns. For all three subtests, the questions start out fairly simple and progress to become increasingly difficult. Each subtest was discontinued when a child consecutively made six incorrect responses for WJ-10, four for WJ-18A, and three for WJ-18B. The total number of correct answers for each subtest was then calculated for analyses. The WJ-III is a very highly regarded and widely used test, with a reliability coefficient of .84 for seven year olds (McGrew & Woodcock, 2001). The WJ-III is also reported to have convergent validity coefficients of .68 - .70 with the Wechsler Individual Achievement Test (Wechsler, 1974) and .62 - .66 with the Kaufman Test of Educational Achievement (Kaufman & Kaufman, 1985).

Test of Word Reading Efficiency, Second Edition. The Sight Word Efficiency (SWE) subtest of the Test of Word Reading Efficiency, Second Edition (TOWRE) assesses print-based word reading skills and provides a quick and reliable way to measure the efficiency of sight word recognition (Torgesen et al., 1999). While the majority of academic assessments in the present study examined children's mathematical skills, this subtest provided a very efficient way to also look at the relationship of reading to patterning and executive function. The SWE measures the number of words, printed in vertical lists, an individual can accurately read within 45 seconds. The subtest can be completed in approximately five minutes, including the time for directions and practice items. The TOWRE meets high standards for both reliability and validity, with reliability coefficients ranging from .90 - .99 and validity coefficients ranging from .77 - .96.

Multiple Classification Card Sorting Test. The Multiple Classification Card Sorting Test (MCCST; Cartwright, 2002) was administered in order to measure the cognitive flexibility component of executive function. The researcher first used a set of 12 training cards to demonstrate to the child how to complete this activity. After this demonstration, the child was then given 12 cards and was instructed to sort them based on two different dimensions concurrently (e.g., by color, such as brown and yellow, and by object type, such as tools and instruments) into four piles within a matrix (e.g., brown tools, brown instruments, yellow tools, yellow instruments), as the researcher had demonstrated. After the child sorted the cards in each set, the researcher asked the child to explain why he or she had sorted the cards this way. Children were then scored on the accuracy of their sorting, the explanation they gave for their sorting (whether they justified sorting by both color and object), and the time it took them to sort the card set. If the child sorted the cards incorrectly, the researcher would demonstrate to the child the way the cards could be sorted by both object and color. After sorting the first pile, each child was given three more sets of cards, one at a time, and instructed to again sort them by color and object. Each card set was made up of two new colors (e.g., purple and blue), and of two new objects (e.g., insects and dogs), but the instructions remained the same. A flexibility composite score was calculated for each child by adding the sorting score and justification score together and then dividing by the sorting time. Reliability for this measure is reportedly high, with a Cronbach's alpha of .86 (Cartwright, Marshall, Dandy, & Isaac, 2010).

Wechsler Intelligence Scale for Children Revised: Memory for Digit Span. The Wechsler Intelligence Scale for Children - Revised (WISC-R; Wechsler, 1974) - Memory for Digit Span - was administered in order to measure the children's working memory capabilities as a component of executive function. The Memory for Digit Span component of the WISC-R is a measure of short term memory and working memory. There are two components to the Memory for Digit Span assessment: Digit Span Forwards and Digit Span Backwards. Digit Span Forwards assesses short-term auditory memory, while Digit Span Backwards assesses working memory ability. For the Digit Span Forwards portion of the assessment, each child was instructed to repeat a series of numbers read aloud by the researcher. The Digit Span Forwards contained 14 sets of numbers, ranging from three numbers to nine numbers, with two trials each. The length of the sequence of numbers increased for each set as the child responded correctly. Testing was discontinued when the child incorrectly answered both trials of the number set.

Next, for the Digit Span Backwards portion of the assessment, the child was instructed to listen to a sequence of numbers read aloud by the researcher and to then repeat the numbers backwards. Digit Span Backwards contains 12 sets of two to eight numbers, also with two trials each of increasing difficulty levels. Like Digit Span Forwards, testing was discontinued when the child was incorrect on both trials of the number set. Each correct response was worth one point, and scores were calculated for Digit Span Forwards and Digit Span Backwards each independently for each child. Only the Digit Span Backwards scores were entered in the analyses, as this is the component of the assessment that measures working memory.

Stroop Color-Word Test. The Stroop Color-Word Test (SCWT) was administered in order to examine inhibition abilities as a component of executive function (Stroop, 1935). The SCWT uses word reading as an automatic process, where individuals are more inclined to read a word even when instructed to name the color of the ink the word is written in instead. The SCWT requires participants to inhibit the dominant response of reading in order to make the required color-naming response. Each child was first shown a list of words where the colors and words matched (e.g., the word red was written in red ink, the word blue in blue ink, etc.) and was then timed on how quickly he or she could read the words. Next, each child shown a list of words where the colors and words did not match (e.g., the word red was written in blue ink, the word blue in red ink, etc.). Again, the child was timed on how long it took he or she to read the list of words. The number of errors children made for each set was recorded, and a z score was calculated by dividing the time it took a child to read the matching set of words by the time it took to read the non-matching words.

Procedures

After obtaining approval from an internal review board, the children were assessed individually in nine separate sessions. Each testing session lasted approximately 5-15 minutes, depending on the assessment being given and the performance of the child. Assessments were administered to one child at a time in a quiet location in the classroom or hallway. Each child was given instructions at the beginning of every activity, and a child could choose to not participate or to stop participating at any time. The order in which children were given the assessments was counterbalanced across the classrooms.

RESULTS

Descriptive statistics were conducted on the variables of interest (see Table 1). All variables were normally distributed except for the WJ-18A, which had a borderline significant positive kurtosis value.

Bivariate Correlations

Pearson correlation coefficients were calculated between all dependent variables.

Patterning. Patterning was found to be significantly related to two of the executive function measures – cognitive flexibility and working memory – but not to inhibition. Patterning was also related to all three mathematics measures (WJ-10, WJ-18A, & WJ-18B) and the reading measure (TOWRE) (see Table 2). Cohen (1992) defined the effect size of correlation coefficients of .10, .30, and .50 as small, medium, and large, respectively. Thus, the correlation between patterning and cognitive flexibility is considered to be a medium effect size, and the correlation between patterning and working memory is considered to be a small effect size. The correlations between patterning and each of the mathematics measures are large effect sizes, and the correlation between patterning and the reading measure is a medium effect size.

Executive Function Measures. None of the executive function measures were related to each other (see Table 2). Cognitive flexibility was significantly correlated with the WJ-10 and WJ-18A (effect sizes were small and medium, respectively), but not to the WJ-18B or the TOWRE. In addition to patterning, working memory was significantly correlated with all three mathematics measures and the TOWRE. All effect sizes were medium, except with the WJ-18B, which was small. Inhibition did not correlate with any of the achievement measures.

Achievement Measures. All of the intercorrelations between the mathematics and reading achievement measures were significant (see Table 2). All effect sizes were large or medium.

Regression Analyses

Patterning. A regression analysis showed that, when patterning scores were predicted from the three executive function measures, only cognitive flexibility was a significant predictor (see Table 3). The relationship with working memory was borderline, but not significant. This indicates part of the bivariate relation between working memory and patterning was due to a small, but not statistically significant, relation between working memory and cognitive flexibility, probably general intelligence (*g*).

Achievement Measures. For the achievement variables, when linear regression analyses were run with working memory, inhibition, cognitive flexibility, and patterning as predictor variables, patterning was a significant predictor for all achievement measures (see Tables 4, 5, 6, & 7). Working memory was a significant predictor for the WJ-10 and WJ-18A mathematics measures, as well as the TOWRE reading measure. There was no relation between cognitive flexibility and any achievement variables.

When only working memory and patterning were entered as predictor variables, patterning was again a significant predictor for all achievement measures, and working memory was again predictive of all achievement measures, except for the WJ-18B. Hence, this suggests the bivariate correlation between working memory and WJ-18B was due to the former's relation to patterning, but working memory predicted the other achievement measures independently.

Further exploration with regression analyses also showed, if only patterning and cognitive flexibility were entered as predictors, there was no relation between cognitive flexibility and any achievement measure. Only patterning remained predictive of the achievement measures. This indicates the bivariate correlation between cognitive flexibility and the WJ-10 and WJ-18A scores was due to cognitive flexibility's relation to patterning.

When working memory and cognitive flexibility were used as predictors, both predicted WJ-10 and WJ-18A scores, but only working memory predicted the WJ-18B and TOWRE scores. This is identical to the relationships shown by the bivariate correlations. Hence, the correlations between cognitive flexibility and WJ-10 and WJ-18A were not due to the relation between working memory and cognitive flexibility. This is a second indication that the bivariate correlation between cognitive flexibility and the WJ-10 and WJ-18A scores was due to the relation between patterning and cognitive flexibility.

DISCUSSION

In sum, the findings of the present study were that patterning was related to cognitive flexibility and working memory, but not to inhibition. In addition, although there were bivariate correlations between patterning, executive function, and achievement measures, only patterning independently predicted all achievement measures. In contrast, the findings of Bock et al. (2015) were that patterning was significantly related to cognitive flexibility, but not to working memory or inhibition. Furthermore, while the results of Bock et al.'s study did not show a significant relationship between patterning and the three GORT-4 reading scales, the results of the present study indicated patterning was significantly related to the TOWRE reading measure.

In addition to the present study assessing mathematics, perhaps the most notable difference between the present study and that of Bock et al. (2015) was the patterning assessment used. As mentioned previously, in the study of Bock et al., children were asked to complete 18 patterns, each of which consisted of a line of numbers, letters, or shapes that either increased or decreased in value, position of the alphabet, or size, or were symmetrical. All of the patterns were presented horizontally and had the last item in the pattern missing. The patterning assessment used in the present study consisted of 48 pattern problems (Gadzichowski, 2012).

The different patterns consisted of numbers, letters, shapes, pictures, and clock faces that either increased or decreased in value, position of the alphabet, or size, and also included symmetrical and rotating patterns. The missing item was presented equally often in the beginning, middle, or end of the sequences, and patterns were presented either horizontally or vertically.

The complex patterns of the patterning assessment of the present study may be part of the reason a relation was found between reading and patterning here, but not in the study of Bock et al. (2015). The briefer, simpler nature of the pattern measure Bock et al. used may have made it a less sensitive measure. Another difference between the two studies was the demands of the reading measures used. For the TOWRE, which was used in the present study, children are asked to read as quickly as possible, and no concern for errors is voiced. For the GORT reading measures used by Bock et al. (2015), care and accuracy is required. Hence, the conflicting demands of the reading measures used in the two studies may also have contributed to the difference in the relation between patterning and reading that was observed.

The greater variety of patterns and the greater number of possible positions for the missing item in the present study may also account for the disappearance of the effect of cognitive flexibility on the WJ-10 and WJ-18A in the regression analyses. To score well on the patterning measure administered, children had to consider the orientation of the pattern, whether the pattern was increasing, decreasing, symmetrical, rotating, or a clock face series when choosing from among the alternatives offered, as well as the possible positions for the missing item when considering the alternatives. The constantly changing demands of the patterning task would demand cognitive flexibility. Hence, the patterning measure would mask the effect of cognitive flexibility and, being more direct, would be primary.

Despite predictions made in both the current study and that of Bock et al. (2015), neither of the results in these studies suggested a relation between patterning and inhibition. The assessment used to examine inhibition used in Bock et al.'s study was the day-night inhibition test, which is an assessment similar to the Stroop task intended for children as young as 3.5 years old. In the present study, the Stroop task was administered. Prior researchers have found all inhibitory tasks make demands that go beyond requiring only inhibitory ability, thus limiting the ability of tasks such as the day-night inhibition task or Stroop task to solely assess inhibition (Simpson and Riggs, 2005). As pointed out by Simpson and Riggs (2005), for the day-night task, the child is instructed to say 'day' to a picture of a moon and 'night' to a picture of a sun. If a child forgets the rules of the task, however, then he or she is likely to make an incorrect response to the pictures, which does not necessarily reflect a lack of inhibitory skills.

The finding in the present study that working memory is related to achievement measures of mathematics and reading is also consistent with the findings of prior studies. According to Christopher et al. (2012), working memory and general processing speed, but not inhibition, were unique predictors of both word reading and comprehension for children ages 8 through 16 years old. In addition, according to McClelland et al. (2007), behavior regulation (which includes working memory) significantly predicted pre-kindergarteners' fall and spring emergent literacy, vocabulary, and mathematics skills on the Woodcock-Johnson Tests of Achievement.

In the present study, results showed a significant relation between patterning and performance on the WJ-10. This finding contrasts with the results of Kidd et al. (2013), who found instruction in patterning did not improve WJ-10 scores. However, the results of the present study are consistent with those of Kidd et al. (2013), in that the present study showed a significant relation between patterning and the WJ-18A, as well as between patterning and the WJ-18B. Kidd et al. (2013) also found that instruction in patterning was significantly related to higher scores on the WJ-18A and WJ-18B.

The results of the present study are consistent with those of Herman (1973) and Hendricks et al. (2006), which also showed significant relations between instruction in patterning and academic achievement measures. Herman (1973) found children from English-speaking homes who were instructed in patterning scored significantly higher on the Numbers Subtest of the Metropolitan Readiness Test, as compared to the control group. In Hendricks et al. (2006), researchers found that children who were taught patterning scored significantly higher than the control group on the mathematics and language scales of the Diagnostic Achievement Battery-2, after the two groups were statistically equated on IQ. The current study is consistent with these findings, in that the results suggest not only a relationship between patterning and mathematics, but also between patterning and reading. The present study shows this relation exists whether or not some children receive instruction in patterning.

Summary and Conclusions

The findings of this study support the suggestions by educators about the relation between patterning and early mathematics skills (Clements & Sarama, 2007a, 2007b, 2007c; McGarvey, 2012; Mulligan & Mitchelmore, 2009). Children's understanding of complex patterns was in fact more highly related to mathematical skills than were measures of executive function, indicating patterning makes a unique contribution to progress in both mathematics concepts and mathematics achievement. Children's understanding of complex patterns was also related to reading achievement. This result confirms the prediction of Sarama and Clements (2004), and the empirical results of Kidd et al. (2014), which suggest a relation between patterning and reading. It implies better understanding of patterns provides a broad foundation for academic achievement.

The present study shows understanding complex patterns is related to working memory and to cognitive flexibility, but not to inhibition. However, the regression analyses show understanding such patterns makes a contribution to mathematics and reading achievement above and beyond that made by cognitive flexibility, and, on one mathematics concepts measure, beyond what is made by working memory. Hence, it appears teaching patterns is likely to be a worthwhile enterprise, particularly if the patterns taught are more complex than the alternations that are normally taught in preschools and kindergartens, as it should contribute to progress in both mathematics and reading. This prediction must be tempered, however, by the observations that different achievement and concepts measures may yield different results, as may sample differences and other extraneous variables.

Limitations and Future Directions

While the present study did provide a number of unique contributions to the field of patterning, there were some limitations. As noted, in contrary to predictions made, inhibition was not found to be related to any other measures. This may have been a result of using the Stroop task to measure inhibition, as it is not typically used with children as young as those in the present study (Stroop, 1935). In future studies with this age group, it would likely be beneficial to use an alternative inhibition measure, other than the Stroop task, used in the present study, and the day-night task, used by Bock et al. (2015), which appeared to be too simple for first-graders. An additional limitation was that only one reading measure was administered in the present study. Future researchers should utilize more reading measures that include a larger scope of the many processes involved in reading for first graders. Likewise, administering a larger variety of mathematics assessments, in addition to the three used in the present study, could yield more generalizable results.

Children who were included in the present study were those identified by their teachers as performing about average academically. In many of the prior studies where researchers implemented patterning interventions, the children included were those performing the lowest in their classrooms either on academic material or patterning ability (Hendricks et al., 2006; Kidd et al., 2013; Kidd et al., 2014). Future studies examining patterning and executive function may also want to include such children who are performing more poorly than their peers, or perhaps those children who are performing at more advanced levels. Also, average levels of performance in first-grade classrooms in some schools are likely different than in other schools, which could also impact results. Future studies examining patterning and executive function could not only look at the correlations between abilities, but also implement a patterning intervention. Researchers could then also look at the dose effect of teaching patterns (i.e., how many patterning lessons are needed to see growth in children's academic achievement). As can be seen here, there is still much more to be discovered in the field of patterning.

APPENDICES

Table 1

Descriptive Statistics for Variables

	Mean	*SD*	Min	Max	Skew	Kurtosis
Patterning	18.57	9.25	5.00	41.00	1.02	-0.08
Cognitive flexibility	0.08	0.04	.00	0.17	0.07	-0.48
Inhibition	0.42	0.20	0.16	0.97	1.12	0.60
Working Memory	3.16	1.31	.00	7.00	-0.31	0.92
WJ-10	26.82	3.67	19.00	34.00	-0.25	-0.56
WJ-18A	15.28	1.88	9.00	19.00	-0.98	2.06
WJ-18B	9.93	2.08	4.00	14.00	-0.25	-0.36
TOWRE	45.00	15.41	9.00	75.00	-0.47	-0.69

Table 2

Correlations among Variables

	P	CF	I	WM	10	18A	18B	T
Patterning	-	.41**	.17	.23*	.54**	.54**	.52**	.33*
Cognitive Flex		-	-.03	.01	.29*	.34**	.20	.15
Inhibition			-	-.12	-.12	-.07	-.04	-.05
Working Mem				-	.37**	.40**	.27*	.40**
WJ-10					-	.67**	.46**	.59**
WJ-18A						-	.54**	.49**
WJ-18B							-	.33**
TOWRE								-

Note: *p < .05, **p < .01.

Table 3

Hierarchical Linear Regression Predicting Patterning from Working Memory, Inhibition, and Cognitive Flexibility

Predictors	R^2	Adj R^2	*β*	*t*
Model 1	.05	.04		
Working Memory			1.65	1.90
Model 2	.08	.05		
Working Memory			1.54	1.76
Inhibition			-7.72	-1.33
Model 3	.24	.21		
Working Memory			1.55	1.94
Inhibition			-7.12	-1.34
Cognitive Flexibility			94.61**	3.63**

Note: *p < .05, **p < .01.

Table 4

Hierarchical Linear Regression Predicting WJ-10 Scores from Working Memory, Inhibition, Cognitive Flexibility, and Patterning

Predictors	R^2	Adj R^2	*β*	*t*
Model 1	.23	.19		
Working Memory			1.00**	3.24**
Inhibition			-1.25	-0.61
Cognitive Flexibility			25.19*	2.50*
Model 2	.40	.36		
Working Memory			0.72*	2.54*
Inhibition			0.06	0.03
Cognitive Flexibility			7.91	0.80
Patterning			0.18**	4.17**

Note: *p < .05, **p < .01.

Table 5

Hierarchical Linear Regression Predicting WJ-18A Scores from Working Memory, Inhibition, Cognitive Flexibility, and Patterning

Predictors	R^2	Adj R^2	*β*	*t*
Model 1	.26	.22		
Working Memory			0.49**	3.42**
Inhibition			-0.26	-0.27
Cognitive Flexibility			14.18**	3.03**
Model 2	.40	.36		
Working Memory			0.37**	2.74**
Inhibition			0.31	0.35
Cognitive Flexibility			6.67	1.43
Patterning			0.08**	3.81**

Note: *p < .05, **p < .01.

Table 6

Hierarchical Linear Regression Predicting WJ-18B Scores from Working Memory, Inhibition, Cognitive Flexibility, and Patterning

Predictors	R^2	Adj R^2	β	*t*
Model 1	.12	.06		
Working Memory			0.42*	2.17*
Inhibition			-0.10	-0.08
Cognitive Flexibility			10.04	1.60
Model 2	.29	.24		
Working Memory			0.25	1.40
Inhibition			0.67	0.58
Cognitive Flexibility			-0.22	-0.04
Patterning			0.11**	3.90**

Note: *$p < .05$, **$p < .01$.

Table 7

Hierarchical Linear Regression Predicting TOWRE Scores from Working Memory, Inhibition, Cognitive Flexibility, and Patterning

Predictors	R^2	Adj R^2	β	*t*
Model 1	.20	.16		
Working Memory			4.81**	3.62**
Inhibition			0.22	0.02
Cognitive Flexibility			53.90	1.24
Model 2	.26	.21		
Working Memory			4.11**	3.09**
Inhibition			3.44	0.40
Cognitive Flexibility			11.03	0.24
Patterning			0.45*	2.20*

Note: *$p < .05$, **$p < .01$.

REFERENCES

Bock, A., Cartwright, K. B., Gonzalez, C., O'Brien, S., Robinson, M. F., Schmerold, K., Shriver, A., & Pasnak, R. (2015, March). *The role of cognitive flexibility in patterning.* Paper presented at Eastern Psychological Association, Philadelphia, PA.

Burton, G.M. (1982). Patterning: Powerful play. *School Science and Mathematics, 82,* 39-44. doi:10.1111/j.1949-8594.1982.tb17161

Cartwright, K. B. (2002). Cognitive development and reading: The relation of reading-specific multiple classification skill to reading comprehension in elementary school children. *Journal of Educational Psychology, 94,* 56-63. doi:10.1037/0022-0663.94.1.56

Cartwright, K. B., Marshall, T. R., Dandy, K., & Isaac, M. C. (2010). The development of graphophonological-semantic cognitive flexibility and its contribution to reading comprehension in beginning readers. *Journal of Cognition and Development, 11,* 61–85. doi: 10.1080/15248370903453584

Christopher, M. E., Miyake, A., Keenan, J. M., Pennington, B., DeFries, J. C., Wadsworth, S. J., Willcutt, E., & Olson, R. K. (2012). Predicting word reading and comprehension with executive function and speed measures across development: a latent variable analysis. *Journal of Experimental Psychology: General*, *141*(3), 470. doi: 10.1037/a0027375

Clements, D. H. & Sarama, J. (2007a). Curriculum, mathematics. In R. S. New & M. Cochran (Eds.), *Early childhood education: An international encyclopedia* (Vol. 1, pp. 193-198). Westport, CN: Praeger. doi: 10.4236/ce.2012.35092

Clements, D. H. & Sarama, J. (2007b). Early childhood mathematics learning. In F. K. Lester, Jr. (Ed.), *Second handbook on mathematics teaching and learning* (pp. 461-555). Charlotte, NC: Information Age.

Clements, D. H. & Sarama, J. (2007c). Mathematics. In R. S. New & M. Cochran (Eds.), *Early childhood education: An international encyclopedia* (Vol. 2, pp. 502-509). Westport, CN: Praeger. doi:10.4236/ce.2012.35092

Cohen, J. (1992). A power primer. *Psychological Bulletin*, *112*, 155-159. doi: 10.1037/0033-2909.112.1.155

Connolly, A. J. (2007). *KeyMath 3: Diagnostic Assessment*. San Antonio, TX: Pearson.

Duan, X., Wei, S., Wang, G., & Shi, J. (2010). The relationship between executive functions and intelligence on 11- to 12-year-old children. *Psychological Test and Assessment Modeling, 52*, 419-431. Retrieved from http://search.proquest.com/docview/1400191031?accountid=14541

Ducolon, C. K. (2000). Quality literature as a springboard to problem solving. *Teaching Children Mathematics, 6*, 442-446. Retrieved from http://search.proquest.com/docview/214138326?accountid=14541

Gadzichowski, K. M. (2012). Examining patterning abilities in first grade children: A comparison of dimension, orientation, number of items skipped and position of the missing item. *Psychology*, *3(12)*, 1177. doi: 10.4236/psyc.2012.312A174

Hendricks, C., Trueblood, L., & Pasnak, R. (2006). Effects of teaching patterning to 1st-graders. *Journal of Research in Childhood Education*, *21*(1), 79–89. doi:10.1080/02568540609594580

Herman, M. L. (1973). Patterning before mathematics in kindergarten*. Dissertation Abstracts International*, *33,* 060.

Hildreth, G. H., Griffiths, N. L., & McGauvran, M. E. (1969). *Metropolitan Readiness Tests*. New York: Harcourt, Brace and World.

Jarboe, T. & Sadler, S. (2003). *It's as easy as 123: Patterns and activities for a creative, balanced math program*. Peterborough, NJ: Crystal Springs.

Kaufman, A. S. & Kaufman, N. L. (1985). *K-TEA, Kaufman Test of Educational Achievement: Comprehensive Form, Easel-kit*. American Guidance Service.

Kidd, J. K., Carlson, A. G., Gadzichowski, K. M., Boyer, C. E., Gallington, D. A., & Pasnak, R. (2013). Effects of patterning instruction on the academic achievement of 1st-grade children. *Journal of Research in Childhood Education*, *27*(2), 224–238. doi:10.1080/02568543.2013.766664

Kidd, J. K., Pasnak, R., Gadzichowski, K. M., Gallington, D. A., McKnight, P., Boyer, C. E., & Carlson, A. (2014). Instructing first-grade children on patterning improves reading and mathematics. *Early Education and Development*, *25*(1), 134–151. doi:10.1080/10409289.2013.794448

Lee, K., Ng, S., Bull, R., Pe, M., & Ho, R. (2011). Are patterns important? An investigation of the relationships between proficiencies in patterns, computation, executive functioning, and algebraic word problems. *Journal of Educational Psychology, 103*(2), 269. doi:10.1037/a0023068

Liljedahl, P. (2004). Repeating pattern or number pattern: The distinction is blurred. *Focus on Learning Problems in Mathematics*, 26(3), 24-42. Retrieved from http://www.peterliljedahl.com/wp-content/uploads/FLPM-Pattern-is-Blurred-2004.pdf

McClelland, M. M., Cameron, C. E., Connor, C. M., Farris, C. L., Jewkes, A. M., & Morrison, F. J. (2007). Links between behavioral regulation and preschoolers' literacy, vocabulary, and math skills. *Developmental psychology*,*43*(4), 947. doi:10.1037/0012-1649.43.4.947

McGarvey, L. M. (2012). What is a Pattern? Criteria used by teachers and young children. *Mathematical Thinking and Learning*, *14*(4), 310–337. doi:10.1080/10986065.2012.717380

McGrew, K. S., & Woodcock, R. W. (2001). *Woodcock-Johnson III technical manual*. Itasca, IL: Riverside Publishing.

Miyake, A., Friedman, N. P., Emerson, M. J., Witzki, A. H., Howerter, A., & Wager, T. (2000). The unity and diversity of executive functions and their contributions to complete “frontal lobe” tasks: A latent variable analysis. *Cognitive Psychology, 41,* 49-100. doi:10.1006/cogp.1999.0734

Mulligan, J. & Mitchelmore, M. (2009). Awareness of pattern and structure in early mathematical development. *Mathematics Education Research Journal, 21,* 33-49. Retrieved from http://link.springer.com/article/10.1007/BF03217544#page-1

National Council of Teachers of Mathematics (Ed.). (2000). *Principles and standards for school mathematics* (Vol. 1). National Council of Teachers.

National Governors Association Center for Best Practices (2010*) Common core state standards*. Council of Chief State School Officers, Washington D.C.

Papic, M. (2007). Promoting repeating patterns with young children - More than just alternating colours! *Australian Primary Mathematics Classroom, 12*(3), 8-13. Retrieved from http://search.informit.com/au/documentSummary; dn135529953955178;res=IELHSS

Papic, M. & Mulligan, J. (2005). Preschoolers' mathematical patterning. *The Proceedings of the 28th Mathematical Education Research Group of Australasia Conference,* 609-616. Melbourne, Australia. Retrieved from researchonline.mq.edu.au

Papic, M. & Mulligan, J. (2007). The growth of early math patterning: An intervention study. *Math: Essential Research Essential Practice, 2,* 591-600. Retrieved from http://www.researchgate.net/publication/255616732

Papic, M., Mulligan, J., & Mitchelmore, M. (2011). Assessing the Development of Preschoolers' Mathematical Patterning. *Journal for Research in Mathematics Education, 14(3),* 237-269. Retrieved from http://www.jstor.org/stable/10.5951/jresematheduc.42.3.0237

Reid, D. K., Hresko, W. P., & Hammill, D. D. (2001). *Test of early reading ability*. Austin, TX: Pro-ed.

Rittle-Johnson, B., Fyfe, E. R., McLean, L. E., & McEldoon, K. L. (2013). Emerging understanding of patterning in 4-year-olds. *Journal of Cognition & Development*, *14*(3), 376–396. doi: 10.1080/15248372.2012.689897

Sarama, J. & Clements, D. H. (2004). Building blocks for early childhood mathematics. *Early Childhood Research Quarterly*, *19*(1), 181-189. doi: 10.1016/j.ecresq.2004.01.014

Simpson, A. & Riggs, K. J. (2005) Inhibitory and working memory demands of the day-night task in children. *British Journal of Developmental Psychology, 23* (3), 471-486. doi:10.1348/026151005X28712

Stroop, J. R. (1935). Studies of interference in serial verbal reactions. *Journal of Experimental Psychology*, *18*(6), 643.

Threlfall, J. (1999). Repeating patterns in the early primary years. *Pattern in the teaching and learning of mathematics*, 18-30.

Torgesen, J. K., Wagner, R. K., & Rashotte, C. A. (1999). *Test of word reading efficiency*. Austin, TX: Pro-Ed.

Warren, E. & Cooper, T. (2006). Using repeating patterns to explore functional thinking. *Australian Primary Mathematics Classroom, 11*(1), 9-14. Retrieved fromhttp://search.informit.com.au/documentsummary;dn=170932599345758;res=IELHSS

Waters, J. L. (2004). A study of mathematical patterning in early childhood settings. *Mathematics Education Research Group of Australasia*, 321-328. Retrieved from http://www.merga.net.au/

Wechsler, D. (1974). *WISC-R, Wechsler intelligence scale for children, revised*. New York: Psychological Corporation.

Wiederholt, J. L. & Bryant, B. R. (2001). *Gray Oral Reading Tests: GORT-4*. Austin, TX: Pro-Ed.

Woodcock, R. W., McGrew, K. S., & Mather, N. (2001). Woodcock-Johnson III. Itasca, IL: Riverside Publishing.

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