DESIGN AND TESTING OF AN AUCTION FOR NON-CONVEX COST ENVIRONMENTS

by

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Master of Arts
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Dedication

This is dedicated to Candice, Jack, Ava, Sam, and Noah whom I love very much.

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I would like to thank Dave Porter, Bart Wilson, Dan Houser, Steve Rassenti, and all the people at ICES who helped me out in the lab even though I was clueless at times.

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Abstract

DESIGN AND TESTING OF AN AUCTION FOR NON-CONVEX COST ENVIRONMENTS

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George Mason University, 2008

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Van Boening and Wilcox ran experiments finding that the ordinarily robust Double Auction produced inefficient results in an environment characterized by a small number of sellers with non-convex cost structures (i.e. large avoidable fixed costs, zero incremental costs, and production quantity limits). Advances in computation provide opportunities for new ways to transact multilaterally, which may facilitate efficient production in such environments. In the context of electricity markets, novel competitive institutions have evolved that execute multilateral trading in environments with non-convexities. My experiments use an institution called a Quasi-Uniform Price Auction ("QUPA"), which is modeled after mechanisms that are currently used in electricity auctions. When tested in environments with non-convex cost structures, the QUPA is in some cases more efficient than the Double Auction and the Smart Market, which is another computationally intensive multilateral trading institution. These results suggest

that further experimental research on QUPAs would provide useful lessons for the future design of electricity auctions.

I. Introduction and Motivation

The Double Auction ("DA") has performed well in experiments under a variety of circumstances including markets with small numbers of buyers and sellers. However, Van Boening and Wilcox (1996) ran experiments finding that the DA performed poorly in an environment characterized by a small number of sellers with large avoidable fixed costs, low incremental costs, and capacity constraints. They reported low efficiency, erratic prices, and lack of improvement with experience. Van Boening and Wilcox[1] and several others[2] have done additional work but have not found a competitive institution that consistently results in efficient outcomes in such an environment.

The novel aspect of Van Boening and Wilcox's (1996) experiments was that they used an environment with sellers unlike the examples that are generally used to illustrate concepts in classical microeconomics textbooks. The following figure illustrates the difference between the typical example of a seller's cost function (shown on the left) and the type of cost function that was by Van Boening and Wilcox (shown on the right).

Firm A, the firm shown in the left panel of Figure 1, has a marginal cost curve that steadily increases from a cost of 3/unit and intersects the average total cost curve at a cost of 7/unit. In the short-run, Firm A will respond to changes in the equilibrium price with small changes in output, and if the price falls below 3/unit, Firm A will produce zero

[1] See Van Boening and Wilcox (2005a) and Van Boening and Wilcox (2005b)
[2] See Durham et al (1996)

units. In the long-run, Firm A will exit the market unless it believes future prices are likely to be above 7/unit.

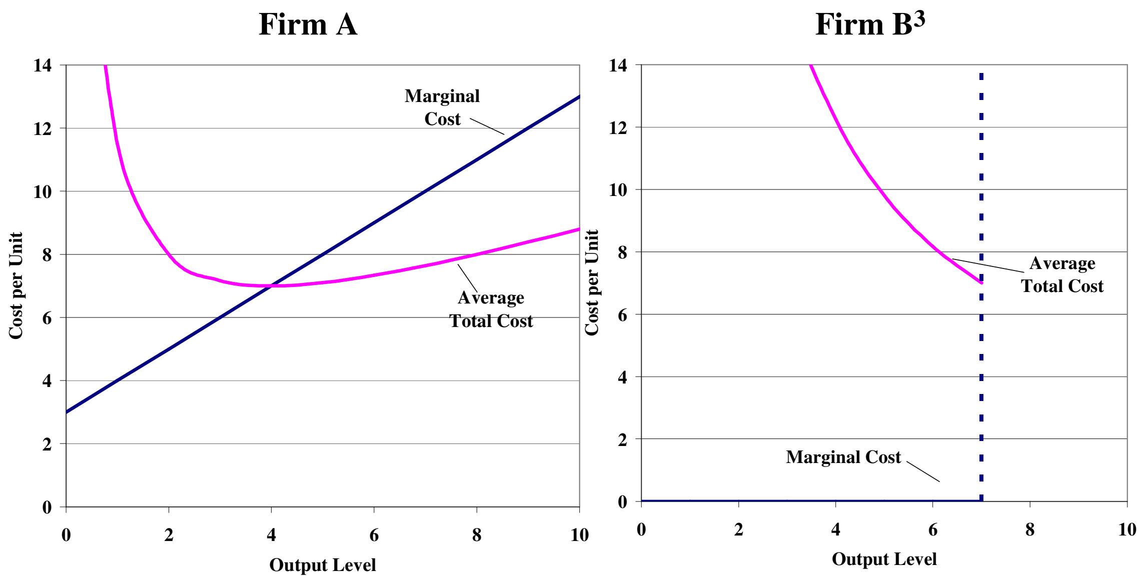

Figure 1: Example Cost Functions for Two Suppliers

Firm B, the firm shown in the right panel of Figure 1, has a marginal cost of 0/unit, but a substantial fixed cost. In the short-run, Firm B will sell every unit up to its output limitation unless the price falls to zero. In the long-run, Firm B will exit the market unless it believes future prices are likely to be above 7/unit, which is the same threshold used by Firm A. If other firms in this market are similar to Firm B, variations in factors such as the level of demand will have a large impact on the equilibrium price making it difficult for Firm B to predict when it will be profitable to stay in the market.

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[3] Firm B has a feasible range of output that is continuous, although Van Boening and Wilcox's (1996) sellers have a feasible range that is discrete.

The term “long-run” is used, but in some industries, this can be a matter of months, days, or even hours.

Commenting on the results of experiments using several different institutions in such non-convex environments, Van Boening and Wilcox (2005b) suggest that such environments may be poorly suited to bilateral contracting institutions such as the DA and auction institutions that do not address difficult coordination problems. Archibald, Van Boening, and Wilcox raise the possibility that collusive arrangements such as mergers, monopolies, and certain regulatory arrangements may bring about more efficient results than traditional competitive institutions. Van Boening and Wilcox (2005b) predict the greatest potential for efficient outcomes in such environments is with multilateral contracting institutions that address combined value problems.

Advances in computation have led to the emergence of competitive institutions that have the potential to execute complex multilateral trading in difficult environments with non-convexities. One recent example is the electric power industry. Previously, this was considered to be a natural monopoly industry because of the complex interaction between production, consumption, and the operation of the transmission system. It was thought that these elements of the market needed to be owned and operated by a single closely-regulated entity. But advances in computation have allowed for the divestiture of the old monopolies into multiple buyers and sellers who compete in auction-style markets. Hence, there are real world institutions that have been used to address the problems that arise from non-convex cost structures.

The objective of my research is to identify a competitive institution that facilitates efficient trading in an environment characterized by a small number of sellers with large avoidable fixed costs, low incremental costs, and capacity constraints. Specifically, I use experiments to assess the performance of a Quasi-Uniform Price Auction ("QUPA"), which is a multilateral trading institution that is modeled on existing wholesale electricity auctions. The results of these experiments are analyzed and compared with outcomes from previous experiments using other institutions with non-convex cost structures.

This paper is divided into the following sections:

1) Introduction – The introduction explains the motivation for this project.

2) Survey of Experimental Literature – This discusses experimental economics literature related to environments with non-convex cost structures.

3) Wholesale Electricity Auctions – This discusses mechanisms that have been used to address the problems that arise from non-convex cost structures in wholesale electricity markets.

4) Environment and Institution Design – This describes the environments and the QUPA institution that are used in my experiments.

5) Incentives in the QUPA – This section discusses the incentives of buyers and sellers when participating in the QUPA. This section also includes a Nash Equilibrium analysis of the institution and two environments used in my experiments.

6) Experiment Design – This is describes the procedures that were used in my experiments.

7) Empirical Results – This analyzes the data collected in these experiments. The first part evaluates the efficiency of the QUPA institution and compares it to

several other institutions. This section also evaluates the transaction prices and the strategies that were used by buyers and sellers.

8) Conclusions – This summarizes my conclusions and provides suggestions for further research.

II. Review of Experimental Literature

This section of the paper surveys the experimental literature on environments that have a small number of sellers with non-convex cost structures. Section II.A discusses Van Boening and Wilcox's (1996) experiments, which found the standard Double Auction ("DA") did not perform efficiently in an environment that had a small number of suppliers with non-convex cost structures. Section II.B summarizes subsequent work by Van Boening and Wilcox (2005a) to identify variations on the standard DA that might perform efficiently in the same environment. Section II.C discusses several experimental papers that use sealed-offer auctions in similar environments.

A. The Double Auction in Environments with Non-Convexities

Since the emergence of experimental economics research, the DA has been robust, performing efficiently under a wide variety of conditions. Van Boening and Wilcox (1996) note that the DA has performed well in particularly challenging conditions, including environments with high concentration.[4] Van Boening and Wilcox evaluate the performance of the DA in an environment that has a small number of sellers with non-convex cost structures. Their first motivation is to determine whether such "ill-behaved" structures are inherently incompatible with competitive institutions, which would help explain the emergence of non-competitive institutions (e.g., mergers, regulated industries,

[4] See Plott, Charles R. (1989).

natural monopoly industries). Their second motivation is to determine whether the DA's uniform-pricing tendency is too strong to allow core surplus divisions when no Competitive Equilibrium exists for a particular environment.

In this paper, Van Boening and Wilcox report that the DA performs poorly in an environment that has a small number of sellers with non-convex cost structures. They observe low efficiency, price fluctuations that are unusual in a DA, and a lack of improvement with experienced subjects. Furthermore, there is some evidence that efficiency is even lower for avoidable cost environments that have no Competitive Equilibria. Finally, the paper points to several areas for future experimental research related to environments with ill-behaved structures.

1. Environment

As mentioned, the novel aspect of this paper was that it used a distinctive environment, which was characterized by non-convex cost structures for sellers. In each period, the sellers had zero incremental costs and non-zero avoidable fixed costs, which differ from fixed costs because they can be avoided in a particular round by producing nothing in that round. The sellers also had production quantity limits that restricted the quantity that they could produce in each round. The sellers also were not able to inventory unsold units, forcing them to sell all of their production in each round. The following figure summarizes the cost structure used in their experiment.

	Supplier:			
	#1	#2	#3	#4
Avoidable Cost	**960**	**750**	**540**	**420**
Marginal Cost	**0**	**0**	**0**	**0**
Quantity Limit	**8**	**5**	**3**	**2**
AC / Quantity Limit	**120**	**150**	**180**	**210**

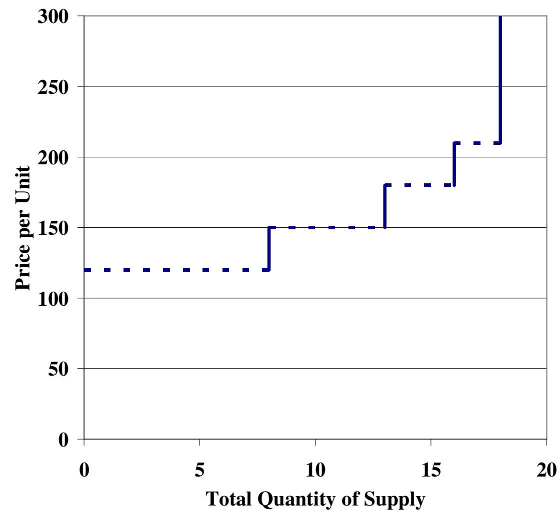

Figure 2: Supplier Costs Used in Several Experiments by Van Boening and Wilcox

The left-panel of Figure 2 reports the avoidable fixed costs and production quantity limits for the four suppliers. The last row shows the minimum average cost, which occurs when the supplier produces a number of units equal to its quantity limit. The costs were chosen such that the smallest supplier has the smallest avoidable fixed cost and the largest minimum average cost. As the size of the supplier increases, the avoidable fixed cost increases but at a rate that leads to decreasing minimum average cost.

The right-panel of Figure 2 shows the market supply curve, which is derived by ranking the suppliers according to minimum average cost. The horizontal portions of the market supply curve are shown with a dotted line since those price-quantity pairs are not actually achievable due to the non-convex costs of the suppliers.

The environment included four identical buyers. The buyers had a value of 250 per unit or 264 per unit depending on the session. The buyers desired a quantity of three units or four units depending on the treatment.

2. Treatments

The paper reported five treatments:

1. Inexperienced AC3: avoidable cost structure and each buyer desires three units.
2. Inexperienced AC4: avoidable cost structure and each buyer desires four units.
3. Experienced AC3: avoidable cost structure and each buyer desires three units. Subjects participated in a prior inexperienced session.
4. Experienced AC4: avoidable cost structure and each buyer desires four units. Subjects participated in a prior inexperienced session.
5. MC4: marginal cost structure and each buyer desires four units.

These five treatments enable the authors to make at least three comparisons. First and most importantly, the paper compares the results of the four AC treatments, which have avoidable cost structures, to MC4, the baseline treatment with marginal cost structures, to determine whether avoidable cost structures undermine efficiency in the DA. Second, the inexperienced and experienced treatments allow for the authors to assess whether efficiency changes as the subjects gain experience with the avoidable cost structure. Third, the AC3 and AC4 treatments are compared to determine whether efficiency is sensitive to the existence of a Competitive Equilibrium ("CE").

Although core allocations exist for both the AC3 and AC4 treatments, Van Boening and Wilcox show that a CE exists for the AC4 treatments but a CE *does not* exist for the AC3 treatments. Intuitively, the CE exists in the AC4 treatments because the total market demand is 16 units and the three suppliers with the lowest minimum average costs collectively produce 16 units when producing at their average cost minimizing levels. However, the CE does not exist in the AC3 treatments because the total market demand of 12 units does not equal the collective production of any group of suppliers that are producing at their average cost minimizing quantities.

In the introduction, Van Boening and Wilcox discuss how the DA institution is likely to interact with an environment where there is no CE (which is the key feature of the AC3 treatments). In such environments, the authors hypothesize that an institution must support non-uniform pricing in order to produce efficient results. Although the DA does not impose uniform clearing prices, there is a strong tendency for the DA to produce uniform clearing prices. Hence, the authors expect that the AC3 treatments will produce either uniform clearing prices or efficient results, but not both.

3. Results and Discussion

The paper reports that the AC treatments performed far less efficiently than the marginal cost baseline. 81 percent of the trading periods in the marginal cost baseline achieved at least 90 percent efficiency, whereas only 39 percent of the trading periods in the AC treatments achieved the at least 90 percent efficiency. Furthermore, just 5 percent of the trading periods in the marginal cost baseline exhibited efficiency lower than 70 percent, whereas 26 percent of the trading periods in the AC treatments exhibited efficiency lower than 70 percent.

Both the experienced and inexperienced AC treatments exhibited poor overall efficiency, and the authors find no statistically significant effect from experience. However, they note that efficiency tends to improve within each session.

The authors find weak statistical evidence for a positive effect from the existence of a CE. The mean efficiency was 84 percent for the AC4 treatments and only 77 percent for the AC3 treatments. Using a rank-sum test, the authors find a statistically significant difference between the AC4 and AC3 treatments (p-value = 0.06). However, they conjecture that this effect would probably not hold up if the test controlled for serial correlation.

The pattern of inefficient market outcomes is explained in some detail. The authors observe that if too many sellers entered the market early in the trading period, prices dropped as the sellers with zero incremental costs realized there was an excess of supply. As a result, some sellers that engaged in trading actually earned negative profits on average. This was the case for Supplier #3 in the AC4 treatments and Suppliers #2 and #3 in the AC3 treatments. It might have been difficult for sellers to predict whether it would be profitable to incur the fixed cost, but once this cost was sunk, sellers might have no choice but to sell off their remaining units at very low prices.

Based on the poor performance of the DA in the AC treatments, the paper identifies four classes of institutions that might achieve better results. First, the paper proposes modifying the standard DA to facilitate block transactions, which might be more compatible with the non-convex cost structures. Van Boening and Wilcox ran experiments on several DA variations, and these are discussed in the second part of this section. Second, the paper suggests that an institution with two-part pricing (i.e. a price for entry and a price for each unit of production) might perform efficiently. Subsequent research in this area is discussed in Section II.C. Third, the paper speculates that a uniform price auction might improve efficiency in the AC4 environment, although it is likely to exacerbate problems in the AC3 environment. Fourth, the paper returns to the explanation that some environments are inherently incompatible with competitive institutions, and in the real world, such environments are likely to spawn cooperative arrangements such as mergers of competing firms or regulation when the industry is considered to be a natural monopoly.

B. Variations on the Standard Double Auction

Following the poor performance of the standard DA in an environment with avoidable cost structures, Van Boening and Wilcox (2005a) ran experiments with several variations on the standard DA. They modified the standard rules of the DA to facilitate block transactions, which were expected to be more compatible with avoidable cost structures. Ultimately, the results of the modified DA experiments were not more efficient than the results of the standard DA experiments.

The standard DA is here referred to as the Multi-Unit DA ("MUDA"), because buyers and sellers were able to specify bids and asks of one, two, or three units. Van Boening and Wilcox ran experiments with two variations on the standard DA: the Bundled-Unit DA ("BUDA") and the Restricted Bundled-Unit DA ("RBUDA"). The BUDA is like the MUDA except that buyers and sellers must accept the entire bundle of one, two, or three units in a particular bid or ask. Hence, if a seller submits an ask of two units, a buyer in the MUDA could accept one or two units whereas a buyer in the BUDA would only have the option of accepting both units. The RBUDA is like the BUDA except that single-unit bids and asks are not allowed.

The paper reports the results of experiments with experienced subjects in the environment that has no CE (i.e., same as the experienced AC3 treatment). Efficiency of at least 90 percent was achieved in 54 percent of trading periods with the MUDA, 55 percent of trading periods with the BUDA, and 57 percent of trading periods with the RBUDA. Efficiency of at least 70 percent was achieved in 83 percent of trading periods with the MUDA, 87 percent of trading periods with the BUDA, and 85 percent of trading periods with the RBUDA. Overall, the alternate DA institutions did not achieve substantially better results than the standard DA. The paper also reports the results of experiments using an institution called the "Smart Market," which is discussed in detail in Section II.C.

Although the BUDA, RBUDA, and Smart Market institutions were designed specifically to address environments with avoidable cost structures, these experiments did not achieve typical levels of efficiency. The authors conclude that environments with avoidable cost structures pose significant problems for competitive institutions. They go on to summarize results from experiments with avoidable cost environments in three cooperative institutions, suggesting that they might be more promising than competitive institutions for such environments. [5] In subsequent research, Van Boening and Wilcox (2005b) performed additional experiments using the BUDA and the RBUDA in an

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[5] See Archibald, G., M. Van Boening, and N. Wilcox (2002)

avoidable cost environment with only three suppliers. They report inefficient results, concluding that multilateral trading institutions such as a Combined Value Call Markets have potential to address the inefficiencies that arise in bilateral institutions based on the DA.

C. Sealed-Offer Auctions with Two-Part Pricing

This part of the section summarizes experimental research on three competitive institutions that were specifically designed to work with avoidable fixed cost environments.

All three institutions are sealed-offer auctions that allow sellers to submit two-part offers, which include a fixed part and an incremental part with a quantity limit. In these institutions, offers are accepted subject to the constraint that the fixed part must be accepted before any increments. However, the three designs use different rules for selecting and paying the winning offers. Experiments on all three institutions used fully revealing robot buyers. The key differences among the three institutions are as follows:

Table 1: Key Features of Three Sealed-Offer Auctions

<table>
<tr><th>Institution</th><th>Offer Selection</th><th>Payments</th></tr>
<tr><td>Smart Market</td><td>Bids and offers that maximize surplus</td><td>Fixed fee based on fixed part of offer (called a “vendor fee”)
Incremental fee equals the incremental offer price times the number of units accepted</td></tr>
<tr><td>Offer Cost Minimization (“OCM”)</td><td>Offers that minimize cost while satisfying the quantity of demand (This is equivalent to maximizing surplus)</td><td rowspan="2">Fixed fee based on fixed part of offer (called a “start-up fee”)
Incremental fee equals the clearing price times the number of units accepted
Clearing price is equal to the highest accepted incremental part of offer</td></tr>
<tr><td>Payment Cost Minimization (“PCM”)</td><td>Offers that minimize buyer payments while satisfying the quantity of demand</td></tr>
</table>

There were several other differences among the experiments on the three institutions. Durham et al (1996) ran experiments using the Smart Market institution in environments that were identical to the environments in Van Boening and Wilcox (1996) except that buyers were played by fully revealing robots rather than subjects. Baltadounis (2007a and 2007b) ran experiments using the OCM and PCM institutions in an environment that was like a simplified electricity market. Hence, demand was cyclical, mimicking the daily pattern of electricity usage, rather than constant across trading periods and suppliers had moderate avoidable fixed costs with non-zero incremental costs and quantity limits.

For each of the three institutions, the authors conclude that the two-part offer increased the capabilities of sellers to raise their offers to extract more surplus. The

Smart Market institution exhibited higher prices for buyers and lower efficiencies than in the Marginal Cost Baseline that was used by Van Boening and Wilcox. The OCM and PCM institutions exhibited much higher prices for buyers than the Simple-Offer institution.

1. Experiments with Smart Market

Durham et al (1996) summarized the results of experiments using the Smart Market institution, which was designed to address the problem put forward by Van Boening and Wilcox (1996). For this reason, Durham et al (1996) experiments used two environments identical to the AC3 and AC4 environments by Van Boening and Wilcox (1996) with one caveat: the Smart Market experiments used fully revealing robot buyers.

The Smart Market institution allows sellers to submit a two-part offer comprising a vendor fee and an incremental fee. Winning offers are selected by an integer program, which has the objective of maximizing the surplus between the buyers' bids and the sellers' offers, subject to the constraint that a seller's vendor fee must be accepted before any of its incremental units. Winning offers are paid the vendor fee plus the accepted portions of their incremental offer.

After each round, subjects were given several pieces of information about the other three sellers. For each of the other three sellers, subjects were told the vendor fee that was offered, the incremental fee that was offered, and the quantity sold. Hence, after each round, each seller knew the range of offers and whether they were accepted.

Two features of the design were added to help subjects cope with the complexity of the institution. First, the user interface included a "calculator" that each seller could use to calculate the profit he/she would earn from selling at a given vendor fee and incremental fee. Second, subjects were advised that they could protect themselves from selling at a loss by submitting a vendor fee equal to their avoidable fixed cost. Hence, it is interesting that approximately half of the 2,972 offers submitted during the experienced rounds included vendor fees that were well below their avoidable fixed costs. It is also note-worthy that in each of the ten experienced sessions, virtually all of the sellers adopted the same strategy regarding the size of the vendor fee relative to their avoidable fixed cost. In other words, in most sessions, all four suppliers submitted a large vendor fee and a smaller incremental fee, or all four suppliers submitted a small vendor fee and a large incremental fee, but there were very few sessions where some used one component while the others used the other component. This suggests that most subjects did not give much thought to this but simply copied what other subjects did.

The Durham et al compared the results of the Smart Market experiments to the results of Van Boening and Wilcox's (1996) experiments. The Smart Market achieved higher overall efficiency than the standard DA but also experienced a similar frequency of periods with very low efficiency (e.g., lower than 70 percent). The Durham et al reported frequent efficiency "Roller Coasters" where several 100 percent efficiency trading periods might be followed by periods with very low efficiency. As a result, the Smart Market did not achieve levels of efficiency that were as high as the Marginal Cost Baseline (i.e., Treatment MC4) that was used Van Boening and Wilcox.

However, the Durham et al note that comparisons between the Smart Market and DA are obscured by the use of robot buyers in the former. They speculate that the use of robot buyers reduced efficiency in the Smart Market, explaining that the lack of strategic bidding allowed prices to rise above the competitive range, which led to entry by inefficient sellers. Consistent with this explanation, prices were significantly higher in the Smart Market experiments than in the DA experiments.

Durham et al conclude that the single-shot sealed-offer format of the Smart Market is a drawback when compared with the continuous format of the DA. In a single trading period, the Smart Market provides each subject with four price-quantity messages about the other sellers from the previous period, whereas the DA provides each subject with 48 or 64 messages. Hence, in the Smart Market, when an efficient seller mistakenly submits an overly high offer, it may severely affect efficiency, whereas in the DA, such a seller can revise his offers downward if they are initially too high, thereby limiting the effect on efficiency.

For future research, Durham et al propose changing the Smart Market institution to reduce the strategic possibilities of sellers. Ideas include dropping the incremental fee altogether or replacing the vendor fee with a minimum purchase quantity. They believe such changes would reduce the complexity of the institution and lead to more efficient allocations.

2. Experiments with OCM and PCM Institutions

Baltadounis (2006, 2007a, and 2007b) indicates that theoretical evaluations of the PCM and OCM institutions have generally assumed that sellers will reveal their costs. Under this dubious assumption, the PCM institution will, by definition, always result in lower costs for consumers and the OCM institution will, by definition, always result in lower production costs, and therefore, greater social welfare. However, if sellers can act strategically, it is not obvious which institution will perform better in lowering consumer costs and/or lowering production costs, and it is not known how market power will affect performance.

This paper reports the results of experiments on the PCM and OCM institutions under conditions with and without market power. The experiments included a total of four treatments: two PCM treatments and two OCM treatments. For each institution, the experiment included one treatment with market power and one treatment without market power.

The environment was identical across the four treatments except that the ownership shares were reallocated in the market power treatments to create market power. Consistent with the electricity market context, the sellers submitted offers for the next trading day which comprised four trading periods: an overnight low demand period, a morning moderate demand period, an afternoon high demand period, and an evening moderate demand period. Demand was a fixed quantity, although small amounts of demand were "interruptible" at high prices.

In addition to private information about their costs, transactions, and profits, subjects were given several pieces of public information that were likely important to the outcomes. Subjects were told the criteria for selecting offers in their respective institution. Subjects were not told the precise manner in which the clearing price was determined, although they were told that they might be paid more than their incremental offer component and that all sellers were paid the same price. Subjects were given complete information about the cost characteristics and ownership of each plant, and they were told the number of units sold by all sellers in each period.

Baltadounis (2007a) finds that the sellers did not generally reveal their costs, and in particular, used the start-up fee (fixed component of the offer) in a strategic manner. Baltadounis notes that the evening was the most competitive period of the day, explaining that this was because plants were already started in earlier periods, and thus, the start-up fee did not play a significant role. The extent of strategic behavior made it difficult to detect a treatment effect from the institution. In conclusion, Baltadounis determines that the ability to submit a start-up fee encourages strategic behavior and ultimately reduces overall efficiency. This leads to the inference that a simpler institution might produce more efficient outcomes than the OCM and PCM institutions.

3. Comparison of OCM and PCM with a Simple-Offer Institution

Comparing the performance of the OCM and PCM institutions, Baltadounis finds that sellers generally raise their offers above their costs, a conclusion that greatly undermines the theoretical arguments for using the OCM or PCM institution. This leads him to ask whether the institutions that allow sellers to submit two-part offers can perform better than a uniform price auction with simple offers.

This paper reports the results of experiments comparing the PCM institution, the OCM institution, and a simple-offer institution under conditions without market power. Accordingly, the experiments included three treatments: a PCM treatment, an OCM treatment, and a simple-offer. All three treatments used the same environment as the previous paper.

Baltadounis finds that the three institutions yielded similar levels of efficiency with the PCM institution performing marginally better than the OCM and simple-offer institutions. This is notable given that the PCM institution is the only one that does not even attempt to minimize the overall cost of offers. This results from the tendency for sellers to not reveal their true costs in the OCM and PCM institutions. Baltadounis also finds that the simple-offer institution leads to lower costs for consumers, leading to the conclusion that the two-part offers better enable sellers to extract surplus by raising their offers above their true costs.

Although the OCM and PCM institutions did not perform better than the simple-offer institution, the simple-offer institution also did not achieve the levels of efficiency that are theoretically possible in an institution with two-part offers. Hence, if an institution with two-part offers gave sellers incentives to reveal their true costs, it would perform better than the OCM, PCM, and simple-offer institutions in environments with significant avoidable fixed costs.

4. Information in Sealed-Offer Institutions

Experimental subjects, who are mostly drawn from American universities, can be expected to have extensive experience with posted-offer markets. However, most subjects have had little or no experience with sealed-offer auction institutions, and virtually none have had experience with ones that use complex two-part offers. Therefore, when designing the institution, it is especially important to consider what information the subjects receive during the experiment and how the subjects are likely to apply the information when deciding what to offer in subsequent rounds.

There are several elements of the design of sealed-offer auctions that affect incentives but may be difficult for subjects to understand when they participate. First, when offers have multiple components, the criteria for selecting winning offers may be overly complex for subjects. It is intuitive for most subjects that lower-priced offers will be accepted before higher-priced offers, but it is easier for subjects to see how this works when offers are for incremental units only.

Second, depending on the rules of the institution, subjects may find it difficult to understand the method of determining payments to sellers with accepted offers. In the Smart Market institution where sellers are paid according to their offer, subjects probably had no difficulty understanding the payment method. However, in the OCM and PCM institutions, subjects may have had more difficulty understanding how payments were determined.

Third, subjects might find it difficult to grasp basic strategies that can be used in sealed-offer institutions. For example, even though subjects in the Smart Market experiments were told that they could protect themselves from losses by submitting a vendor fee equal to their fixed cost, a large share chose to submit a vendor fee well below their fixed cost. Moreover, in each of the ten experienced sessions, virtually all of the sellers adopted the same strategy regarding the size of the vendor fee relative to their avoidable fixed cost, suggesting that most subjects did not give much thought to this but simply copied what they observed other subjects doing.

Fourth, price discovery may occur slowly in a sealed-offer auction. In contrast, the DA provides subjects with a lot of information about the concentration of bids and asks, which quickly leads to an understanding of the price range where supply and demand intersect. A sealed-offer auction is a relative black box that simply outputs a final answer, so it may take several trading periods for subjects to understand the likely range of transaction prices. In a sealed-offer auction with multi-part offers and payments, it will be even harder for subjects to see the market converge toward equilibrium.

Price discovery is important because it helps subjects understand how the market will respond to a change in strategy. The DA allows subjects to change strategies within a single trading period so that efficient allocations are possible even if subjects initially use poor strategies. In a sealed-offer institution, many rounds of inefficient allocations may occur before subjects refine their strategies. This discovery process is likely to take a long time since other subjects are simultaneously changing their strategies.

The next section examines the sealed-offer auctions that are used in electricity markets. Like the Smart Market, OCM, and PCM institutions, most electricity auctions are very complex. However, the repeated nature of electricity auctions (most regions conduct between 30,000 and 120,000 spot auction rounds annually!) provides market participants with a lot feedback on their strategies. This increases the challenges for experimenters that try to replicate such institutions in the laboratory.

III. Wholesale Electricity Auctions

The environment examined by Wilcox and Van Boening (1996) is highly stylized but still applicable to the real world. Manufacturers must periodically decide whether to incur the cost necessary to keep a plant in operation. Airlines must decide whether to schedule a flight, which is costly, although it costs very little to add passengers once the flight has been scheduled. Electricity generators must decide whether to turn their plants on and off based on forecasts of future prices. In the context of wholesale electricity markets, several mechanisms have been used to reduce the inefficiencies that can arise from avoidable cost structures. The Quasi-Uniform Price Auction ("QUPA") that is used in this paper is based on mechanisms that have used in wholesale electricity markets to deal with the problems posed by non-convex cost structures.

A. Background on Electricity Auctions

1. Uniform Price Auctions for Electricity

Electricity markets are different from other commodity markets primarily because electricity cannot be stored efficiently on a large scale. In most commodity markets, producers fill inventories while periodically making sales and deliveries out of that stock. However, electricity must be produced at the same moment it is being consumed, so supply and demand must always be kept in nearly perfect balance. Since end-users do not schedule their consumption in advance, considerable effort goes into forecasting demand and ensuring sufficient supply is available to meet it. Unlike other commodity markets where buyers have time to reject offers from one producer and accept those of another producer, electricity markets require that all transfers occur instantaneously in a coordinated fashion.

Under the regulated system that existed for decades, a single vertically integrated "natural" monopoly would handle all of the necessary coordination from the power plant to the individual customer. It was not difficult for customers to choose the low bid supplier in real-time because there was only one supplier.

Conversely, under deregulation, many buyers and sellers are allowed to participate and the necessary coordination is accomplished by a single market operator, commonly referred to as the Independent System Operator ("ISO"). Most ISOs coordinate supply and demand by running a sealed-offer single-shot uniform price auction every few minutes. This type of auction has been popular for at least two reasons. First, the single shot format allows the ISO to speed up the process of accepting and rejecting offers so that the ISO can constantly adjust production schedules to match demand.

Second, the uniform price auction format is also used because of its tendency to facilitate production by the lowest-cost suppliers. In a uniform price auction, all suppliers are paid a market clearing price that is based on the highest-priced accepted offer or the lowest-priced unaccepted offer.[6] Under highly competitive market conditions, there is a low probability that a particular supplier's offer will affect the market clearing price, so suppliers usually have an incentive to reveal their true marginal cost (i.e., offer at marginal cost). To submit profit-maximizing offers in a uniform price auction under competitive conditions, suppliers only need to know their own costs and do not need any additional information about their rapidly changing environment. To the extent that suppliers provide the ISO with offers that are consistent with their marginal costs, the ISO will be able to update the dispatch of supply and demand resources efficiently by running a new auction every few minutes.

2. Multi-Part Offers in Electricity Auctions

The uniform price auction mechanism generally does a good job of scheduling production from the suppliers with the lowest incremental costs, because such costs are incurred in the timeframe of the spot auction, which runs every five or fifteen minutes. However, some suppliers incur significant costs from starting-up their generators. Depending on the physical characteristics of the generation facility, the decision to start-up may need to be made hours or days in advance. But since the timing of these start-up decisions are out of sync with the spot auction, the spot auction mechanism cannot ensure that they are coordinated efficiently. If there is no other mechanism for coordinating these start-up decisions, then individual suppliers must forecast price levels based on predicted supply and demand conditions to determine whether and when starting units and keeping them on-line will be profitable. Suppliers risk either (i) starting generators

[6] This distinction is not important in electricity auctions, where tens of thousands of units are transacted in each round.

that will lose money or (ii) not starting generators when it would have been profitable to do so. Van Boening and Wilcox (1996) show the inefficiencies that can arise from errors in prediction since there were cases when sellers frequently entered the market only to find prices plummet while sellers who stayed out sometimes missed opportunities to earn profits.

When multiple sellers must decide whether it will be profitable to start, it results in a coordination problem, which can be likened to the Entry Game. If too many suppliers enter, the price will drop to unprofitable levels, while if too few suppliers enter, the price will rise to very high levels. For this reason, there is great potential benefit from an institution that coordinates efficient decisions. Such an institution might also reduce the risk to individual sellers, which would enhance efficiency to the extent that sellers are risk averse.

Some ISOs have tried to solve the problem of coordinating start-ups and shutdowns by allowing suppliers to submit multi-part offers that include a start-up cost, minimum running cost, and an incremental cost. The various components of the offer are used by the ISO to determine an offer-cost-minimizing production quantity for every seller in the market. Depending on the particular rules in each region, the determination of whether a unit is started may or may not be folded into a forward auction market. Since start-up costs and minimum running costs are not incremental, they cannot set the clearing price directly. As a consequence, it is possible for a generator to be started when the clearing price is not sufficiently high for it to recoup its start-up cost, minimum running cost, and incremental cost. Such generators are given side payments to "make them whole," which are equal to their multi-part offer minus the revenue they receive from the clearing price.[7]

Several issues can arise when sellers are allowed to submit multi-part offers. First, although make-whole payments ensure that sellers in the auction do not lose money by revealing their true costs, these side payments can give suppliers pay-as-bid incentives. Pay-as-bid incentives encourage suppliers to raise their offers above their true costs, and may undermine the efficiency benefits of a uniform price auction, namely that suppliers have the incentive to offer at cost.

Second, uniform-price auctions set clearing prices that are based on the incremental offer price of the marginal supplier, and thus, clearing prices do not reflect the non-incremental components of sellers' multi-part offers. This is particularly problematic when the marginal supplier of energy to the market has a production cost function primarily made up of non-incremental costs. In this regard, quick start combustion turbine generators ("CTs") pose significant challenges for electricity markets. Many CTs are either on or off, having limited ability to vary their output between zero and the maximum level. Such CTs do not have incremental costs and their minimum running cost *is* their cost of producing at maximum output. Some CTs can start and reach full output in as little as five minutes and have modest start-up costs, enabling them to participate in the spot auction even when they are off-line (i.e. turned-off). The following

[7] For example, suppose a generator submits the following offer: start-up offer = $1000, minimum running offer = $750/hour, incremental offer = $50/MWh, minimum output level = 10 MW, and maximum output level = 20 MW. Further suppose that the ISO schedules the generator to produce 20 MW for 3 hours at a clearing price of $70/MWh. This would result in $4200 (= 20 MW * 3 hours * $70/MWh) of revenue from the clearing price and $4750 (= $1000 + 3 hours * $750/hour + 10 MW * 3 hours * $50/hour) of accepted offer components. To make up the difference, the ISO would pay the generator $550 (= $4750 minus $4200).

two examples illustrate the difficult nature of setting prices when the multi-part offers of off-line CTs are accepted in the spot auction.

Suppose that in the spot auction for the previous five-minute period, the clearing price was set by the marginal offer of \$100/MWh, and that in the spot auction for the current five-minute period, the ISO needs an additional 10 MW of supply to satisfy demand. Further suppose that the ISO has two alternatives: (i) accept 10 MW from a seller that has available incremental supply at \$200/MWh for a total cost of \$2000/hour or (ii) accept 10 MW from an off-line CT that has a maximum output level of 10 MW and a running cost offer of \$1800/hour. The second alternative is clearly more attractive since the total cost of starting the CT is lower by \$200/hour. In this case, it seems intuitively reasonable to set the clearing price anywhere between \$180/MWh, which is the average cost of the CT's offer, and \$200/MWh, which is the lowest-priced offer that was not accepted. Moreover, any price between \$180 and \$200 would constitute a Competitive Equilibrium Price. However, the next example shows that it can be impossible to set a price that clears the market.

Suppose the ISO needed only 9 MW of additional supply. Now, the ISO has two alternatives: (i) accept 9 MW from a seller that has available incremental supply at \$200/MWh for a total cost of \$1800/hour or (ii) accept 10 MW from an off-line CT that has a maximum output level of 10 MW and a running cost offer of \$1800/hour and reduce by 1 MW the quantity purchased from the seller that has an incremental cost of \$100/MWh for a total net cost of \$1700/hour. Although the second alternative is still more attractive to the ISO, there is no longer a Competitive Equilibrium Price that clears the market. Any price greater than \$100/MWh will make the seller of \$100/MWh electricity unhappy about having its quantity reduced, while anything lower than \$180/MWh will make the CT earn less than its running cost offer of \$1800/hour. In such cases, most ISO-run markets use the marginal incremental offer to set the clearing price, which, in this particular case, would result in a \$100/MWh clearing price and a make whole payment to the owner of the CT.

The lack of a price that clears the market in the example provides a practical illustration of one of the key issues identified Van Boening and Wilcox (1996), which is that there is frequently no Competitive Equilibrium price that clears a market with non-convex cost structures. They further suggest that if an institution is to produce efficient allocations in an environment with no competitive equilibrium, it must support non-uniform pricing. The use of make whole payments is a form of non-uniform pricing since it allows some sellers to be paid more than others for providing the same quantity. However, this particular mechanism gives the owner of the CT pay-as-bid incentives rather than uniform-price auction incentives.

In conclusion, uniform price auctions have become the most common mechanism for clearing electricity spot markets, because they facilitate rapid changes in the production schedules of sellers and because they tend to elicit cost-revealing offers from sellers. However, the non-convex cost structures of most generating facilities are not well-suited for the requirement that in uniform price auctions offers must be incremental and monotonically non-decreasing. In response, most ISOs have allowed sellers to submit multi-part offers that have non-incremental components, but multi-part offers present significant difficulties for the determination of efficient prices, because, as Van Boening and Wilcox (1996) demonstrated, a competitive equilibrium price may not exist in an environment with non-convex cost structures. The next part of this section discusses mechanisms that have been used and/or proposed to set clearing prices that better reflect the non-incremental costs of marginal suppliers.

B. New Pricing Mechanisms for Auctions with Non-Convexities

Uniform price auctions are used in wholesale electricity markets to induce buyers and sellers to reveal their costs. However, coordination problems arise when suppliers that have non-convex cost structures are unable to predict when entry will be profitable. In ISO-run markets, multi-part offers have evolved as a way to deal with the non-convex cost structures of electricity suppliers, but the use of multi-part offers also has negative consequences. Clearing prices may not represent the costs of generators near the margin such as in the example of the 10 MW CT in Section III.A.2, and thus, clearing prices may not give efficient incentives to participants in the auction or prospective entrants. Large amounts of "make whole payments" may be required, which can lead the putative *uniform* price auction to function more like a discriminatory price auction in some situations.

Henceforth, this paper refers to the class of putative uniform price auctions that allow non-convex offers and the resulting make whole payments as *Quasi-Uniform* Price Auctions. These institutions are quasi-uniform price auctions in the sense that they determine quasi-clearing prices that clear most of the buyers and sellers in the market while make whole payments are used when a supplier sells at a quasi-clearing price that is not sufficiently high for the supplier to recoup its offer-cost. In most existing quasi-UPAs for electricity, the quasi-clearing price is set by the marginal incremental offer as discussed in Section II.A.2. Henceforth, this paper refers to this type of auction as the S*tandard Quasi-UPA*, because it is the de facto method of pricing by ISOs that run putative UPAs that allow non-convex offers.

This part of the section describes several mechanisms that have been used to better reflect non-convex offers in quasi-clearing prices. First, several ISOs in the U.S. have devised special pricing rules to "set the clearing price" when the ISO procures supply or reduces demand outside the spot auction in order to satisfy the mandated reliability requirements. Second, the New York ISO has devised a more generalized mechanism for allowing a CT to "set the clearing price" when it is the marginal source of supply. Third, several other novel institutions have been proposed to set quasi-clearing prices that better reflect non-convex cost components of electricity supply. The question that this paper seeks to answer is whether quasi-UPAs can achieve greater efficiency than the institutions that were used to test the environment used by Van Boening and Wilcox (1996).

1. Special Pricing Rules for Out-of-Market Adjustments by the ISO

When the wholesale market is close to a shortage of supply, the ISO may be obliged to act outside the spot auction to increase supply or decrease demand in order to satisfy mandated reliability requirements. Consequently, quasi-clearing prices in the spot auction sometimes clear at moderate levels that do not reflect the costs of the actions taken by the ISO. Such prices lead to significant "make whole payments" and do not provide efficient incentives for entry by other suppliers. To address these issues, several ISOs have devised special pricing rules that allow generators with non-convex cost structures and buyers with non-convex bids to “set the clearing price” under limited circumstances.

First, the ISO that operates the wholesale market in most of Texas, which is known as ERCOT, purchases a product called Non-Spinning Reserves[8] in a day-ahead auction. Whenever ERCOT forecasts that the offers available to the Balancing Energy Auction[9] are not sufficient to meet forecasted demand at any point in the subsequent hour, the ISO deploys Non-Spinning Reserves to make up the difference. In the Balancing Energy Auction, the Non-Spinning Reserves that was deployed is treated as a price-taker, thereby displacing the most expensive offers. Hence, the deployment of Non-Spinning Reserves shifts the market from a condition of extreme scarcity to a condition of excess supply that produces a moderate clearing price. As a result, suppliers that do not sell Non-Spinning Reserves are less likely to offer into the Balancing Energy Auction when they anticipate the market will be in shortage conditions. To increase the availability of supplies under tight market conditions, ERCOT implemented a special pricing rule that re-calculates the clearing price during Non-Spinning Reserve deployments by excluding the price-taker offers of the Non-Spinning Reserves.

Second, the ISO-New England allows prospective importers to submit offers before each hour. Since the ISO runs its spot auction close to every five minutes, and the

[8] In the ERCOT market, Non-Spinning Reserves is generating capacity that is paid to be available for deployment within 30 minutes.

[9] The ERCOT Balancing Energy Auction is run every 15 minutes to make up the difference between supply that is scheduled through the forward bilateral market and forecasted demand.

ISO must decide whether to accept import offers before the results of the spot auction are known, the ISO schedules imports based on a forecast of spot prices. If the offer price of the import is greater than the eventual clearing prices, the ISO gives the importer a Make Whole Payment. Like Non-Spinning Reserves in the ERCOT market, high priced imports can shift the spot auction in New England from a shortage of supply to a condition of adequate supply and moderate clearing prices. As a result, suppliers complained that the clearing price did not reflect the cost of the supplies that were accepted to meet demand. In 2002, the ISO implemented, on a temporary basis, a special pricing rule whereby import offer prices could set the clearing price whenever the market would have experienced a shortage if the import had not been accepted.[10]

Third, when the New York ISO anticipates at least two hours in advance that it will experience a shortage of reserves, it can curtail "Demand Response" resources. Demand Response resources are electricity consumers that indicate a willingness to curtail their consumption at a price of $500/MWh. They must be curtailed for at least four hours, so the reduced demand can reduce clearing prices well below the strike price of $500/MWh for a substantial portion of the duration of the curtailment. As a result, suppliers complain that the clearing prices do not reflect the cost at which consumers were curtailed. In 2003, the ISO implemented a special pricing rule whereby Demand Response resources could set the clearing price at $500/MWh whenever the market would have experienced a shortage if the Demand Response had not been curtailed.[11]

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[10] See Patton, David B. (2002)

[11] See New York Independent System Operator, Inc. FERC Electric Tariff, Attachment B.

One drawback of all three special pricing rules is that there will inevitably be sellers who submit offers that are not accepted in the auction even though they are priced below the market clearing price. Such sellers are like the $100/MWh supplier in Section III.A.2 who was not fully accepted to make room for the 10 MW CT. This raises the concern that the $100/MWh supplier would no longer have an incentive to reveal its costs, but instead, would have an incentive to reduce its offer price below the next supplier's offer.

The three special pricing rules have been used to address the disjunction between the offer prices of resources with non-convex characteristics that are deployed under shortage conditions and the resulting uniform auction clearing prices. The scope of these special pricing rules is limited to relatively infrequent circumstances where the ISO circumvents the spot auction to add supply or reduce demand. The remainder of this part of this section describes mechanisms that are designed for a wider set of conditions when a Competitive Equilibrium Price does not exist due to supply offers with non-convexities.

2. New York ISO Approach to Pricing Non-Convex Offers

The New York ISO has developed a generalized mechanism for letting certain generators with non-convex cost structures set quasi-clearing prices.[12] The mechanism allows CTs to set price when shutting down the CT would require the ISO to schedule more expensive supply to replace it. Such cases imply that the offer of the CT would be accepted if it were convex. But if shutting down the unit would enable the ISO to schedule less expensive supply, the CT does not set price. In such cases, the CT is

[12] See New York Independent System Operator, Inc. FERC Electric Tariff, Attachment B.

typically running due to operating restrictions that limit the ability of the CT to shut on and off in the timeframe of the spot auction.

The following example in Figure 3 illustrates how the mechanism would work in a load pocket[13] where the only source of supply besides imports is a group of CTs. Suppose the transmission lines into the area can carry up to 300 MW of imports at a cost of $50/MWh for the first 100 MW, $55/MWh for the second 100 MW, and $60/MWh for the third 100 MW. Suppose power consumption is 500 MW and four 50 MW generators are running with average running costs of $80/MWh, $90/MWh, $100/MWh, and $110/MWh. The following figure shows two supply curves for the load pocket. The first illustrates how a standard quasi-UPA would treat it, while the second illustrates how the NYISO mechanism would work.

The pink line represents the supply curve in the standard quasi-UPA. In the standard quasi-UPA, when a CT is running and cannot be shut down in the timeframe of the spot auction, the CT is effectively a price taker. As a price taker, the CT displaces supply that can be adjusted in the spot auction. Hence, the first 200 MW of demand are satisfied by the four 50 MW CTs before any imports are needed. The last 300 MW of demand are satisfied by the 300 MW of imports which have an incremental cost of $60/MWh at the intersection of supply and demand. If the quasi-clearing price is set to $60/MWh, make whole payments will be needed to compensate the CTs, which have offers ranging from $80 to $110/MWh, and the quasi-clearing price inside the load pocket will be the same as the quasi-clearing price outside of it.

[13] A load pocket is an area where the quantity of demand typically exceeds the amount of power that had be imported to the area and where the cost of supply within the load pocket exceeds the cost of the imports.

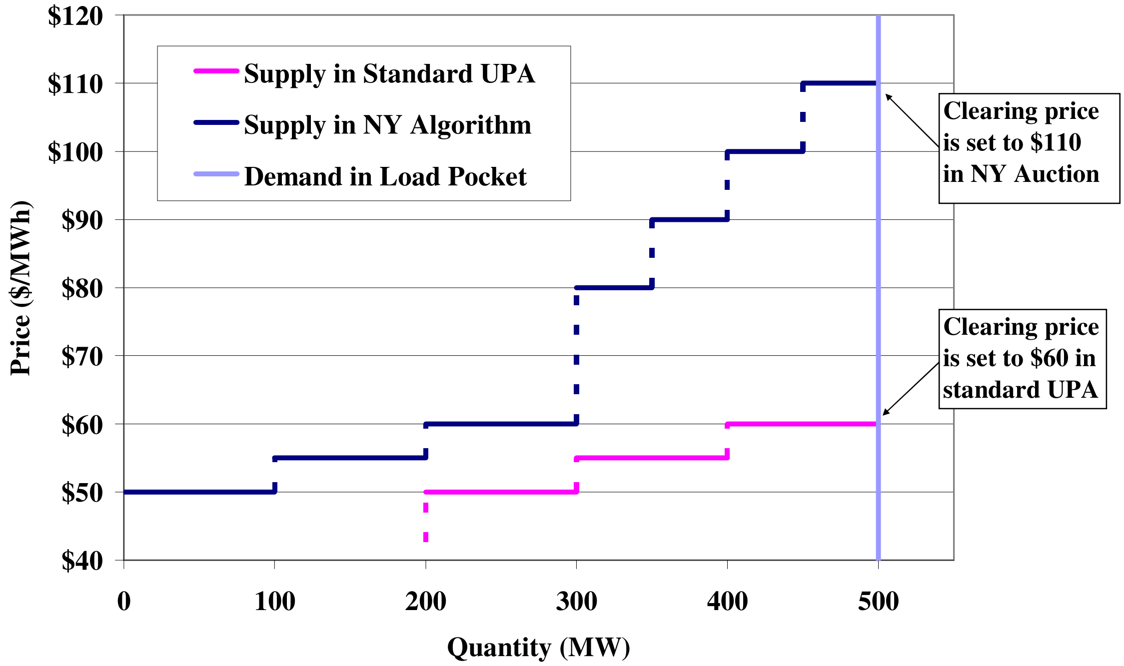

Figure 3: Illustration of Pricing in Quasi-Uniform Price Auctions

The dark blue line represents the supply curve in the New York ISO's pricing mechanism, which treats the output of the CTs as if it were flexible in the timeframe of the spot auction. Each of the CTs is treated as flexible from 0 MW to 50 MW with an incremental offer equal to its average running cost offer. Using this logic, the first 300 MW of demand are satisfied by imports and the last 200 MW of demand are satisfied by the four 50 MW CTs. The quasi-clearing price is set by the intersection of supply and demand at $110/MWh. In this stylized example, the New York ISO's pricing mechanism sets a quasi-clearing price that is representative of the costs of the marginal source of supply, and thereby, avoids the need for make whole payments. Furthermore, there are no suppliers that submitted offers priced below the clearing price that were not accepted. Hence, this is a special case where the quasi-clearing price actually clears the market.

However, the example becomes more complicated if demand is reduced to 425 MW. In the standard quasi-UPA, this would lead to a 75 MW reduction in imports since the output of the CTs cannot be reduced in the timeframe of the spot auction. The price would continue to be set at $60/MWh, which is the marginal cost of imports. The New York ISO's physical deployment of resources would be consistent with the standard quasi-UPA, while the New York ISO's pricing algorithm would reduce the "quantity" of the $110/MWh CT from 50 MW to 0 MW and the "quantity" of the $100/MWh CT from 50 MW to 25 MW. Thus, the quasi-clearing price would now be set to $100/MWh. Since the mechanism for determining physical schedules is different from the mechanism that produces quasi-clearing prices, make whole payments are still necessary in the New York ISO's auction. The following table summarizes the settlement under this system.

Table 2: Payoffs in a Quasi-Uniform Price Auctions

Resource	**Physical Quantity**	**Payment at Clearing Price**	**As-Offered Cost**	**Make Whole Payment**	**Lost Oppor-tunity**
Imports @ $50	100 MW	$10,000	$5,000	$0	$0
Imports @ $55	100 MW	$10,000	$5,500	$0	$0
Imports @ $60	25 MW	$2,500	$1,500	$0	$3,000
CT @ $80	50 MW	$5,000	$4,000	$0	$0
CT @ $90	50 MW	$5,000	$4,500	$0	$0
CT @ $100	50 MW	$5,000	$5,000	$0	$0
CT @ $110	50 MW	$5,000	$5,500	$500	$0

The physical quantity of sales from each resource is taken from the standard quasi-UPA while the quasi-clearing price of $100/MWh is taken from the pricing algorithm. The As-Offered Cost is the minimum payment that each resource is guaranteed to receive under the rules of the auction. The $110/MWh CT receives a $500 make whole payment to make up the difference between its As-Offered Cost and the payment it receives based on the quasi-clearing price. The Lost Opportunity is the value of foregone sales for units that are instructed to produce less than the level that would be most profitable. The table shows a Lost Opportunity for the $60 importer of $3,000, which is equal to the 75 MW of $60/MWh imports that would have been profitable at a quasi-clearing price of $100/MWh. The rules of the NYISO's spot auction do not compensate the owners of such resources for the Lost Opportunity under these circumstances.

The example with 425 MW of demand illustrates how the pricing rules affect incentives in an auction. In the standard quasi-UPA, the quasi-clearing price is set by the $60 import, resulting in make whole payments ranging from $20/MW to the $80 CT to $50/MW to the $110 CT. Each CT receives a different payment for the same quantity of production, leading the standard quasi-UPA to function less like a UPA and more like a Discriminatory Price Auction ("DPA"). In the NYISO's quasi-UPA, the quasi-clearing price is set by the $100 CT, resulting in a make whole payment to just one of the CT owners. The NYISO's quasi-UPA generates transaction prices that are more uniform than the standard quasi-UPA in situations like the example above.

UPAs and DPAs are compared in greater detail in Section III.C, but a key feature of DPAs is that sellers have greater incentives to raise their offers above their true cost, because there is a significant chance that raising their offer will result in a larger payment.

The incentives of the importers are also affected by the choice of pricing rules. In the example with 425 MW of demand, the $60 importer is reduced from 100 MW to 25 MW. When the quasi-clearing price is $60, the importer is indifferent between selling into the load pocket and its next alternative. But when the quasi-clearing price is $100, the importer must forego profits of $40/MW. If the importer anticipates being reduced when the quasi-clearing price is above its offer, the importer could reduce its offer price to $54. At this offer price, the importer would be scheduled at 100 MW while the $55 importer would be reduced to 25 MW.

In conclusion, the NYISO pricing algorithm is designed to address an issue that undermines efficiency in the standard quasi-UPA for electricity, which is that areas containing generators that have non-convex cost structures frequently exhibit quasi-clearing prices that are lower than the costs of meeting demand in the area. However, the NYISO's algorithm also leads to situations when suppliers are instructed to operate below the level that would be most profitable for them. The use of quasi-UPAs in environments with the non-convex cost structures leads to incentives that are too complex for analytical solution methods and game theoretic approaches. Hence, experimental methods provide an excellent way to evaluate the efficiency of quasi-UPAs.

3. Other Novel Institutions for Pricing Non-Convex Offers

In recent years, several groups have approached the problem of how to set prices in auctions that allow participants to submit non-convex bids and offers when a competitive equilibrium price does not exist. This part of the section discusses three novel institutions that were motivated by the difficult nature of setting efficient prices in electricity auctions. Unlike the novel institution that is used in the New York ISO's electricity auctions, the following three institutions have not been used in any real world setting.

First, O'Neill, Sotkiewicz, Hobbs, Rothkopf, and Stewart propose a pricing mechanism for a market where suppliers submit two part offers with a start-up cost and an incremental cost.[14] The first step of the mechanism solves a Mixed-Integer Programming ("MIP") problem in order to determine the offer-cost minimizing set of start-ups and production levels. The second step converts the MIP to a Linear Programming ("LP") problem by bounding each integer variable at the optimal value from the MIP problem plus or minus epsilon. The dual of this problem produces a lagrange multiplier for each commodity and a lagrange multiplier for each start-up decision. When the lagrange multiplier on a start-up decision is positive in their formulation, it denotes that the commodity price is not sufficiently high for the seller to recoup its startup offer-cost and incremental offer-cost and the lagrange multiplier will be equal to the difference between the total offer-cost and commodity revenue. When the lagrange multiplier on a start-up decision is negative in their formulation, the commodity

[14] See O'Neill et al

price is higher than necessary for the seller to recoup its startup offer-cost and incremental offer-cost, leading the payment to the seller to be reduced by the magnitude of the lagrange multiplier for their start-up decision. Whether the lagrange multiplier of the start-up decision is positive or negative, the seller receives the same payment which is also equal to the total offer-cost, leading the proposed mechanism to function like a DPA. For this reason, sellers can be expected to have weaker incentives to reveal their true costs than in the standard quasi-UPA.

Second, Hogan and Ring present their novel institution[15] as a superior approach to the difficult environment that was addressed by O'Neill et al. Hogan and Ring say that while O'Neill et al's institution produces prices that do not require make whole payments, the prices are highly discriminatory, and thus, are unlikely to lead sellers to reveal their costs. Hogan and Ring's institution is called the "Uplift Minimizing Model" because it sets the quasi-clearing price at a level that minimizes uplift. In this case, uplift is defined as the sum of (i) all make whole payments to suppliers that sell when the quasi-clearing price is not sufficiently high for them to recoup their offers and (ii) all make whole payments to suppliers that do not sell when it would be profitable for them to do so at the quasi-clearing price.[16] The first step of the mechanism solves a MIP problem in order to determine the offer-cost minimizing set of start-ups and production levels. The second step solves an LP in order to determine the quasi-clearing price that minimizes uplift holding constant the start-up decisions and production levels determined in the first

[15] See Hogan, William W. and Brendan R. Ring (2003)

[16] Relating this definition of uplift to the example in Section III.C.2, component (i) is equivalent to the column called *Make Whole Payment* in Table 2 and component (ii) is equivalent to the column called *Lost Opportunity* in Table 2.

step. By minimizing the use of make whole payments, Hogan and Ring argue that their quasi-UPA will provide better incentives for suppliers to reveal their true costs.

Third, Gribik, Hogan, and Pope propose a novel institution called the *Convex Hull Model*.[17] The Convex Hull Model describes an approach that can be used to solve for the uplift-minimizing quasi-clearing price in complex systems such as large-scale wholesale electricity auctions. This is an improvement on Hogan and Ring's model, which stated the objective function without proposing a practical method to solve it. Gribik et al solve for the uplift minimizing quasi-clearing prices by forming the dual problem of the primal of the convex hull of the MIP problem. The convex hull of the MIP problem is the smallest convex region that envelops the feasible region of the MIP problem. In other words, the Convex Hull Model convexifies the primal problem and uses it to formulate a dual problem. The lagrange multipliers of the dual problem are the uplift-minimizing set of quasi-clearing prices.

Both Hogan and Ring and Gribik et al comment on the New York ISO's quasi-UPA, which was already being used when both papers were published. They indicate that the uplift-minimizing quasi-clearing prices will be the same as the quasi-clearing prices that are determined by the New York ISO's auction if generators considered for start-up have a constant incremental cost rather than an increasing incremental cost.[18] These conditions hold for the example in Table 2, so the Uplift Minimizing Model would set the same quasi-clearing price as New York ISO's auction, but it would do this by

[17] See Gribik, Paul R., William W. Hogan, and Susan L. Pope (2007)

[18] This was originally shown in Ring, Brendan J. (1995).

minimizing the sum of the quantities in the columns called *Make Whole Payment* and *Lost Opportunity*.

In the environment used by Van Boening and Wilcox (1996), it would be possible for the Uplift-Minimizing Model and the New York ISO's quasi-UPA to result in different quasi-clearing prices, because the sellers by Van Boening and Wilcox (1996) are able to produce any integer amount between one unit and their maximum production level, and this enable them to submit increasing incremental offers.

4. Conclusions Regarding Quasi-Uniform Price Auctions

In this section, we have identified several different institutions that can be called *quasi-uniform price auctions*, which have been proposed or have evolved in the context of electricity markets. This class of auction allows participants to submit convex and/or non-convex bids and offers, which inherently require some degree of non-linear pricing. However, the non-linear pricing of quasi-UPAs differs from the non-linear pricing of DPAs, because quasi-UPAs are designed to set quasi-clearing prices that are the primary basis for payments to suppliers by consumers.

All quasi-UPAs calculate the set of production quantities that minimizes the sum of accepted offers. Quasi-UPAs differ in how the quasi-clearing price is determined. The following quasi-UPAs were discussed in this section:

- Standard Quasi-UPA – The quasi-clearing price is set by the marginal incremental offer.
- Standard Quasi-UPA with Special Pricing Rules – The quasi-clearing price is usually set by the marginal incremental offer, but under limited circumstances when the operator secures supply or curtails demand outside of the spot market to

maintain reliability, the quasi-clearing price is set to a level that is consistent with the amortized cost of the operator's action.

- New York ISO Approach – The quasi-clearing price is determined by re-running the auction treating the non-convex offers of certain CTs as if they were flexible between zero and their maximum output level.
- Uplift Minimizing Model – The quasi-clearing price is set to the level that minimizes the sum of (i) all make whole payments to suppliers that sell when the quasi-clearing price is not sufficiently high for them to recoup their offer and (ii) all make whole payments to suppliers that do not sell when it would be profitable for them to do so at the quasi-clearing price.

The special characteristics of quasi-UPAs hold promise for addressing the problem posed by Van Boening and Wilcox (1996). Quasi-UPAs allow participants to submit non-convex offers and produce non-linear prices when strictly uniform prices do not exist. These two characteristics were specifically discussed by Van Boening and Wilcox (1996) as necessary for producing efficient outcomes.

C. Discriminatory versus Uniform Pricing in Sealed-Offer Auctions

Quasi-UPAs have characteristics of both discrimatory-price auctions and uniform-price auctions. They function as UPAs to the extent that the quasi-clearing price is the sole basis for payments from buyers and to sellers. They function as DPAs to the extent that make whole payments are used to make up the difference between the seller's offer and the revenue it earns from the quasi-clearing price. Although limited attention has been given to the incentives that participants have in quasi-UPAs, research on behavior in DPAs and UPAs can provide some intuition about how participants are likely to behave in quasi-UPAs. This part of the section also discusses the incentive effects that arise from the various mechanisms for allocating the costs of make whole payments in DPAs and UPAs. Lastly, this part of the section compares each of the sealed-offer institutions discussed in Sections II and III and places them on the continuum from most-discriminatory to most-uniform in their pricing methods.

1. Discussion of DPA and UPAs

This part of the section evaluates the incentives of participants in DPAs and UPAs in the unique context of electricity markets. A review of the optimal strategies of sellers in DPAs and UPAs suggests that UPAs are likely to be more efficient in markets with significant uncertainty about the equilibrium price where supply equals demand. Experimental evidence indicates that UPAs produce prices that are closer to the competitive equilibrium level than DPAs when there is not significant market power. It is notable that nearly all existing regional electricity markets use some form of UPA or quasi-UPA to clear spot transactions,[19] reflecting a widespread belief that DPAs are less efficient than UPAs for spot transactions of electricity. However, the market for England and Wales is a notable exception, having adopted a DPA in 2001 after dissatisfaction with the UPA that was used from 1990 to 2001.

In DPAs, sellers maximize their profit by submitting the highest possible offer price that will be accepted because they are paid their offer price for units that are accepted. Sellers gather information about supply and demand to predict the price levels at which electricity will trade in the auction. Sellers that can accurately predict the

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[19] In the U.S., the list includes the markets in the following regions: California, New England, New York, Texas, the region operated by the Midwest ISO, the region operated by the PJM ISO, and the region operated by the Southwest Power Pool. Outside the U.S., the list includes, but is not limited to, Ontario, Australia, New Zealand, Singapore, and Norway.

transaction prices in the auction will be able to submit the highest possible offer prices that will be accepted. If a seller submits an offer price that is lower than the profit-maximizing level, the seller will be able to sell but will earn less than the optimal profit. If the seller is among the lowest cost set of producers, this is not inefficient, it simply means the seller will realize a smaller share of the surplus. However, if a seller submits an offer price that is higher than the profit-maximizing level, the seller will have its offer rejected, which is inefficient if the seller would have been among the lowest cost set of producers. Due to the unique physical characteristics of electricity and the lack of price-responsive demand, electricity spot prices tend to be extremely volatile. Hence, it is inevitable that electricity producers will make errors in predicting the rapidly changing value of electricity, leading to inefficient production schedules if the electricity is sold through a DPA.[20]

In UPAs, the clearing price is set by the marginal offer price, which is either the highest accepted or lowest unaccepted offer price.[21] So a seller's profit-maximizing strategy depends on the ability of the seller to influence the marginal offer price. There are two ways that a seller can influence the marginal offer price. First, if the seller is able to predict that its own offer will be on the margin, the seller will be able to raise the clearing price to just below the price of the next lowest priced offer of another seller. While this is a theoretical possibility, the complexity of electricity markets makes it difficult for sellers to predict such opportunities. Second, if the seller has market power,

[20] See Peter Cramton and Steven Stoft (2007)

[21] In single-unit uniform price auctions, the distinction has a substantial impact on incentives. But in electricity auctions, where tens of thousands of units are being transacted, the highest accepted offer price and the lowest unaccepted offer price are usually equivalent.

the seller may be able to profit by withholding supply, leading a higher priced offer to be on the margin. If the foregone profits from reduced sales are smaller than the increased profits from sales at the higher clearing price, withholding will be profitable. If the seller concludes that it cannot profitably influence the marginal offer price, the seller will have an incentive to reveal its true costs in its offer, because this will lead the seller to sell when the clearing price is greater than its costs and to not sell when the clearing price is lower than its costs. When participants reveal their costs, the UPA coordinates an efficient allocation between buyers and sellers. Coordination is particularly important in electricity markets because clearing prices are volatile and there is no opportunity for negotiating, submitting follow-up offers, or re-selling in secondary markets.

The experimental evidence presented by Rassenti, Smith, and Wilson (2003) is consistent with the basic theory outlined above. In an environment with five sellers and limited market power, they found that the UPA produces much lower prices than the DPA. This could only occur if prices in the DPA were substantially above the cost of the marginal seller. The authors attribute this result to tacit collusion among the sellers in the DPA. In an environment with unilateral market power, the authors report prices above the competitive equilibrium level in both institutions. In this case, the prices in the DPA were not statistically different from prices in the UPA.

While most regional electricity markets use UPAs to clear spot transactions, the British switched from a UPA to a DPA in 2001. One of the chief reasons for the change was the notion that UPAs are more susceptible to the exercise of market power than DPAs. Another reason was that buyers had grown dissatisfied with uplift charges, which had grown large under the UPA. Uplift refers to charges that are necessary to recoup operating costs that are incurred by the ISO but not reflected in the price of electricity. Make whole payments were responsible for a large share of the uplift charges in the British market when it used a UPA. [22]

2. Uplift Charges Arising from Make Whole Payments

Up to this point, I have discussed how make whole payments can lead sellers to have incentives that are comparable to the incentives of a seller in a DPA. However, it is also important, when designing an institution, to consider how efficiency will be affected by the allocation of uplift charges that result from make whole payments. This part of the section considers the incentive effects of allocating uplift and identifies mechanisms that can be used to allocate it more efficiently.

Uplift charges are costs that are socialized because the auction does not provide a mechanism for allocating them efficiently. There are at least two strategies for allocating these costs more efficiently. The first strategy is to charge the uplift to the class of participants who are most directly responsible for the costs. For example, if make whole payments are incurred to ensure reliable electric service to a particular region, the resulting uplift charges could be assessed to the end-users in that region.

The second and best strategy for allocating uplift costs efficiently is for the ISO to define a product that meets the operational requirements that necessitate make whole payments in the first place. Such products may be procured through the auction, allowing sellers to compete to provide the product and buyers to benefit from reducing their

[22] For a discussion of the factors that led to the change, see Staropoli (2003).

consumption of it. Examples that have been developed by ISOs include pricing of transmission losses, transmission congestion management, contingency reserves, and reactive power. A large portion of the uplift that accrued in the British market resulted because the auction set a single Commodity Price for the entire market. The Commodity Price did not reflect the value of minimizing losses or reducing congestion, nor did it differentiate by location the price of contingency reserves or reactive power production. [23] There is substantial evidence from several regions that most uplift results from make whole payments that arise from providing these services rather than make whole payments that arise from non-convex offers.[24]

To make the electricity market as efficient as possible, uplift should be charged in a manner that minimizes inefficient incentives. Usually, uplift is charged to at least one of the following three groups: end-users, generators, or financial transactions. End-users dislike paying uplift charges, partly because they are difficult to hedge. End-users that purchase power in long-term contracts to insulate themselves from price fluctuations may be surprised if uplift charges are larger than expected. However, there is little evidence that end-users change their consumption patterns due to uplift charges, which is to be expected since electricity end-users are generally not responsive to prices either.

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[23] See Green (1998).

[24] For example, the Texas wholesale market does not have non-convex offers, and hence, does not make related whole payments but still experience substantial uplift charges. For more on this, see Potomac Economics (2007) Likewise, the New England market experiences substantial uplift charges, but a very small portion is unrelated to generators that are not providing a separate service such as reserves or local congestion management. For more on this, see Patton and LeeVanSchaick (2007). Electricity markets are developing better ways to price such reliability products. For a discussion of this subject, see Kirsch and Morey (2006).

Generators are more likely to change their behavior in order to avoid uplift charges. Electricity generation is a low margin business for most generators in most periods, so uplift charges may eat into variable operating profits but generally will not eliminate them. Usually, generators will incorporate expected uplift charges in slightly higher offer prices, leading to elevated clearing prices. In some cases, generators will change their behavior in order to reduce their exposure to these charges.

Entities that engage in financial transactions through the ISO-run auctions are most likely to change their behavior as a result of uplift charges. This is because many financial transactions are made to arbitrage price differences that may be smaller than the typical uplift charge. So it seems that uplift charges might have a substantial effect on the behavior of financial transacters. This would be efficient if uplift charges were caused by the activity of financial transacters, but otherwise it is inefficient.

3. Institutions on a Continuum from Discriminatory to Uniform

This part of the section reviews each of the sealed-offer auction institutions that have been discussed thus far. The Smart Market, OCM, and PCM institutions are discussed in Section II while the UPAs, DPAs, and several quasi-UPAs are discussed in this section. Each of these institutions, which enable sellers to submit non-convex offers, can be classified on a continuum with pure discriminatory pricing at one end and pure uniform pricing at the other end. This is useful because it provides some intuition about how incentives are likely to vary according to the rules for determining transaction prices.

The following six institutions are shown in order from most-discriminatory to most-uniform.

- Smart Market – This institution is a pure DPA because winning are offers are paid their offer prices. It differs from other DPAs because sellers can submit two-part offers.
- OCM and PCM – These are hybrids between DPAs and UPAs, because winning offers receive two-part compensation: a start-up payment equal to their start-up offer price and a per-unit payment that is based on a uniform clearing price, which is set by the highest accepted incremental offer. The OCM and PCM differ in the mechanism for selecting winning offers but they set the same prices for a given set of accepted offers.
- Quasi-UPA: Standard – This is also a hybrid between a DPA and a UPA, because winning offers may receive a make whole payment in addition the per-unit payment based on a uniform clearing price. Quasi-UPAs are more similar to UPAs than are the OCM and PCM, because sellers only receive a make whole payment if the margin on the per-unit sales is not sufficient for a seller to recoup its start-up offer while the start-up payment is always made in the OCM and PCM.
- Quasi-UPA: Standard with Special Rules – Under limited circumstances, special rules set the uniform clearing price higher than the marginal incremental offer. The special rules are intended to reduce the use of make whole payments.
- Quasi-UPA: NY Method – This routinely allows the uniform clearing price to be set higher than the marginal incremental offer by allowing amortized non-convex offers to set the clearing price.
- Quasi-UPA: Minimum Uplift Pricing – This routinely allows the uniform clearing price to be set higher than the marginal incremental offer. The clearing price is a choice variable for the objective of minimizing inconsistencies between the actual quantity allocation and the quantity that would be optimal at the clearing price.

While the Smart Market is clearly a DPA, the other five institutions combine characteristics of UPAs and DPAs. In a UPA, incentives depend on the ability of the seller to influence the transaction price. For this reason, a seller in the OCM or PCM will have a strong incentive to raise its start-up offer above its actual start-up cost, because if the offer is accepted, it will lead directly to a larger payment. In any of the quasi-UPAs, the seller will have less of an incentive to raise its start-up offer since it will not necessarily receive a make whole payment. Among the quasi-UPAs, some have rules that are designed to reduce the use of make whole payments, making them more likely to induce sellers to truly reveal their costs.

IV. Environment and Institution Design

The objective of this paper is to identify a competitive institution that performs more efficiently than the DA in an environment with non-convex cost structures. Section II discusses experiments that have been performed in such environments using several variations on the DA as well as several sealed-offer auction institutions. The authors of these studies reported poor overall results, characterized by low efficiency and/or supra-competitive prices. Section III surveys several sealed-offer auction mechanisms that have been developed for use in electricity markets and that are here referred to as *Quasi-Uniform Price Auctions*. Quasi-UPAs have the potential to coordinate efficient trading in environments that have non-convex cost structures. This section constructs a quasi-UPA to be used in experiments on the same environment that presented challenges for Van Boening and Wilcox (1996) and several other experimenters.

Although Section III identifies some of the differences among the quasi-UPA designs that have been proposed or implemented, all of the quasi-UPA designs share the following key features: (i) participants are allowed to submit non-convex bids and offers, (ii) quasi-clearing prices are the primary basis for settling transactions between buyers and sellers, and (iii) make whole payments are made so that no seller receives less than the value of the accepted portions of its offer and no buyer pays more than the value of the accepted portions of its bid.

When choosing the specific rules of the quasi-UPA design that will be used in these experiments, it is important to consider two key findings from Section III. First, a quasi-UPA with strong uniform-pricing tendencies is more likely to give participants the incentive to reveal their costs/values. The incentive to reveal is particularly important in markets where efficient coordination is often difficult for participants to achieve in a decentralized institution. Second, to the extent that make whole payments are necessary, the resulting charges should be allocated to a group that will not have its incentives altered in an inefficient manner.

The institution described in this section is based on the New York ISO approach, which has strong uniform-pricing tendencies. Section III identified the New York ISO approach as having stronger uniform-pricing characteristics than the Standard Quasi-UPA or any of the Quasi-UPAs with special pricing rules. The Minimum Uplift Pricing Model likely has the stronger uniform-pricing characteristics, since it formulates an optimization problem whereby the use of make whole payments is explicitly minimized, using the quasi-clearing price as a decision variable. However, my experiments use the New York ISO approach for several reasons. First, my experiments were designed and performed before the Minimum Uplift Pricing Model was sufficiently developed for use in electricity markets. Second, experimental testing of the New York ISO approach has more potential for real world application since it is already being used to clear a multi-billion dollar market. The institution that is described in this section and analyzed in subsequent sections is hereafter referred to as the *QUPA* institution, while the class of auctions that has the characteristics outlined above is still referred to as *Quasi-UPAs*.

A. Environment

To enable comparisons with the experimental results of Wilcox and Van Boening, my experiments use two environments. To enable comparisons with the Durham et al, my experiments also use their modified versions of the Wilcox and Van Boening environments. Durham et al used the same sets of sellers, buyers, costs, and values; however, their buyers are fully revealing robots while Wilcox and Van Boening used human subjects.

The environment has four sellers and four buyers. The sellers have the following costs:

Table 3: Supplier Costs Used in Several Experiments by Van Boening and Wilcox

	Supplier:			
	#1	**#2**	**#3**	**#4**
Avoidable Cost	**960**	**750**	**540**	**420**
Marginal Cost	**0**	**0**	**0**	**0**
Quantity Limit	**8**	**5**	**3**	**2**
AC / Quantity Limit	**120**	**150**	**180**	**210**

The Fixed Cost is incurred in each auction round where the seller sells one or more units. The Per Unit Cost is zero for all four sellers. The Quantity Limit prevents the seller from selling more than a particular number of units in each auction round. The Fixed Cost / Q Limit is the minimum possible per unit cost for each seller.

The buyers have values of 250 per unit. The number of units desired by each buyer depends on the treatment. The treatments are as follows:

- D3 – Each buyer desires three units in each round.
- D4 – Each buyer desires four units in each round.
- R3 – Each robot buyer desires three units in each round.
- R4 – Each robot buyer desires four units in each round.

B. Rules of the QUPA Institution

This section gives a detailed description of the QUPA institution, which is used in my experiments. The QUPA is modeled on the NY approach, but it is not identical due to differences between the actual New York market and the more stylized environment that was developed by Van Boening and Wilcox (1996).[25]

1. Bid/Offer Parameters

The bid and offer parameters used in the QUPA are intended to fit with the environment. In the environment used by Van Boening and Wilcox (1996), each seller has an avoidable fixed cost that is incurred in each round where the seller sells one or more units. Each seller has a maximum production level and can produce any integer quantity between zero units and the maximum production level.

In the QUPA, the sellers have the following offer parameters: (i) A Fixed Offer Component, which is the minimum compensation that the seller must receive in order to sell one or more units in a particular round, and (ii) Incremental Offer Components, which are the minimum compensation that the seller must receive for each individual unit in addition to the Fixed Component. A separate price may be defined for each increment,

[25] In the New York market, CTs are guaranteed to run for at least one hour, which typically includes 12 auction rounds, and CTs offer a start-up cost and a single inflexible operating point. The stylized environment has no dependence of one trading period on another and sellers can sell any integer from one unit to the maximum production level.

but they are accepted from lowest to highest. The Appendix shows a sample of the user interface where sellers enter their offers.

If the seller's offer is accepted, the seller will receive an amount that is greater than or equal to the sum of the fixed component submitted and the incremental components that were accepted. For instance, a seller with a maximum production level of three units could offer a fixed component of $10 and three units priced at $2, $3, and $4. If one of the units is sold, it will be the one with the lowest offer price and the seller will receive at least $12 (= fixed charge of $10 + the unit offered at $2). If all three units are accepted, the seller will receive at least $19 (= fixed charge of $10 + the units offered at $2, $3, and $4).

In the environment used by Van Boening and Wilcox (1996), each buyer has a value for a limited number of units. In the QUPA institution, the buyers submit: Incremental Bid Components, which are the maximum the buyer is willing to pay for each individual unit. A separate price may be defined for each increment. The Appendix shows a sample of the user interface where buyers enter their bids.

If the buyer's bid is accepted, the amount the buyer pays will be less than or equal to the accepted bid increments. For instance, a buyer with a demand for three units could submit a bid of one unit for $40, one unit for $30, and one unit for $20. If all three units were accepted, the buyer would pay no more than $90 (= $40 + $30 + $20). If two units were accepted, the buyer would pay no more than $70 (= $40 + $30).

The following figure shows an example set of offers and bids for an environment with three sellers and two buyers. The sellers each have a maximum production level of five units and the buyers both have a demand for up to four units. This example is used throughout this section to illustrate how the QUPA works.

Supplier 1	Unit1	Unit2	Unit3	Unit4	Unit5
Fixed Offer	120				
Increment Offers	20	30	40	75	140

Supplier 2	Unit1	Unit2	Unit3	Unit4	Unit5
Fixed Offer	220				
Increment Offers	0	0	50	80	85

Supplier 3	Unit1	Unit2	Unit3	Unit4	Unit5
Fixed Offer	500				
Increment Offers	0	0	0	0	0

	Unit1	Unit2	Unit3	Unit4
Buyer 1 Increment Bids	200	180	160	140
Buyer 2 Increment Bids	210	170	130	60

Figure 4: Example Offers and Bids for Three Sellers and Two Buyers

2. Step One – Determine Transaction Quantities

Step One of the auction determines the quantities bought by each buyer and sold by each seller. This is determined by running an integer program that maximizes the surplus based on the submitted bids and offers.[26] The objective function is:

$$\text{Surplus} = \sum_i \sum_k (b_{ik} * x_{ik}) - \sum_j (f_j * y_j + \sum_k \{c_{jk} * z_{jk}\})$$

Subject to the following constraints:

[26] Note, this is identical to the formulation used in the Smart Market except that the Smart Market does not allow an individual offer price or bid price for each increment. In the Smart Market, a buyer or seller must submit the same offer price or bid price for each unit.

$\sum_i \sum_k x_{ik} = \sum_j \sum_k z_{jk}$ (demand = supply)

$\sum_k z_{jk} <= y_j * m_j$ (production for j is less than cap)

$\sum_k x_{ik} <= u_i$ (production for i is less than cap)

$x_{ik}, y_j, z_{jk} \in \{0,1\}$

Where the indices are as follows:

i is a particular buyer,

j is a particular seller,

k is a particular increment for a particular buyer or seller.

Where the buyers and sellers submit the following:

b_{ik} is the bid price for increment k of buyer i,

c_{jk} is the offer price for increment k of seller j,

f_j is the fixed offer price for seller j,

m_j is the number of increments offered by seller j, and

u_i is the number of increments bid by buyer i.

The integer program chooses the optimal values for:

x_{ik} is 1 if increment k of buyer i is accepted, and is 0 if it is not accepted,

z_{jk} is 1 if increment k of seller j is accepted, and is 0 if it is not accepted,

and

y_j is 1 if the fixed component of seller j is accepted, and is 0 if it is not accepted.

Because more than one allocation can result in the same surplus, the algorithm applies three tie-breakers in the following order. First, if two or more bid increments are submitted with the same bid price or if two or more offer increments are submitted with the same offer price, the auction accepts them in random order. Second, if two or more unique sets of values for the vector f produce the same surplus, the auction chooses the set of values for the vector f that has the largest number of units bought and sold. If there is still a tie between two or more unique sets of values for the vector f, the auction selects one of them randomly. Occasionally, the tie-breakers affect outcomes in the experiments, but they are critically important in the Nash Equilibrium analysis in Section V.

For the example shown in the exhibit above, Step One would find the surplus maximizing allocation: seller 1 produces four units, seller 2 produces three units, seller 3 produces zero units, buyer 1 buys four units, and buyer 2 buys three units. The accepted units are shown highlighted in the following exhibit:

Supplier 1	**Unit1**	**Unit2**	**Unit3**	**Unit4**	**Unit5**
Fixed Offer	**120**				
Increment Offers	**20**	**30**	**40**	**75**	**140**

Supplier 2	**Unit1**	**Unit2**	**Unit3**	**Unit4**	**Unit5**
Fixed Offer	**220**				
Increment Offers	**0**	**0**	**50**	**80**	**85**

Supplier 3	**Unit1**	**Unit2**	**Unit3**	**Unit4**	**Unit5**
Fixed Offer	**500**				
Increment Offers	**0**	**0**	**0**	**0**	**0**

	Unit1	**Unit2**	**Unit3**	**Unit4**
Buyer 1 Increment Bids	**200**	**180**	**160**	**140**
Buyer 2 Increment Bids	**210**	**170**	**130**	**60**

Figure 5: Example of Step One in the QUPA

Note that the last accepted unit was Unit4 of Supplier 1, which was slightly less expensive than Unit4 of Supplier 2. Although Supplier 3 offered incremental units at 0, the high fixed offer component made these too expensive. The total surplus from this allocation is 635, which is the difference between the shaded bids totaling 1190 and the shaded offers totaling 555.

3. Step Two – Determine the Quasi-Clearing Price

Step Two of the auction determines the Quasi-Clearing Price. The portions of each offer that were accepted in Step One are linearized in the following fashion. For seller j:

$$\text{AvgTotalOffer}_j(n) = (f_j \div n + \textstyle\sum_k^n \{c_{jk}\})$$

$$\text{MinAvgTotalOffer}_j = \text{Min of } \{\text{AvgTotalOffer}_j(1 \text{ to } m_j)\}$$

$$\text{LinearOffer}_j(k) = \text{Max of } \{\text{MinAvgTotalOffer}_j, c_{jk}\}$$

The following figure illustrates the calculation of AvgTotalOffer, MinAvgTotalOffer, and LinearOffer:

Supplier 1	**Unit1**	**Unit2**	**Unit3**	**Unit4**	**Unit5**
Fixed Offer	**120**				
Increment Offers	**20**	**30**	**40**	**75**	**140**
AvgTotalOffer	**140**	**85**	**70**	**71.25**	**85**
MinAvgTotalOffer			**70**		
LinearizedOffer	**70**	**70**	**70**	**75**	

Supplier 2	**Unit1**	**Unit2**	**Unit3**	**Unit4**	**Unit5**
Fixed Offer	**220**				
Increment Offers	**0**	**0**	**50**	**80**	**85**
AvgTotalOffer	**220**	**110**	**90**	**88**	**87**
MinAvgTotalOffer					**87**
LinearizedOffer	**87**	**87**	**87**		

Figure 6: Example of Step Two in the QUPA

For each supplier, the AvgTotalOffer generally decreases as the number of units increases due to the large size of the Fixed Offer Component. The MinAvgTotalOffer for Supplier 1 is based on producing three units while the MinAvgTotalOffer for Supplier 2 is based on producing five units. The LinearOffer, which is only calculated for accepted units, is the higher of the increment offer component and the MinAvgTotalOffer.

Once the linearized offers have been calculated, they are ranked and the highest one sets the quasi-clearing price. The following exhibit shows the ranking of the linearized offers as well as the ranking of the incremental offers of Supplier 1 and Supplier 2. It also shows demand.

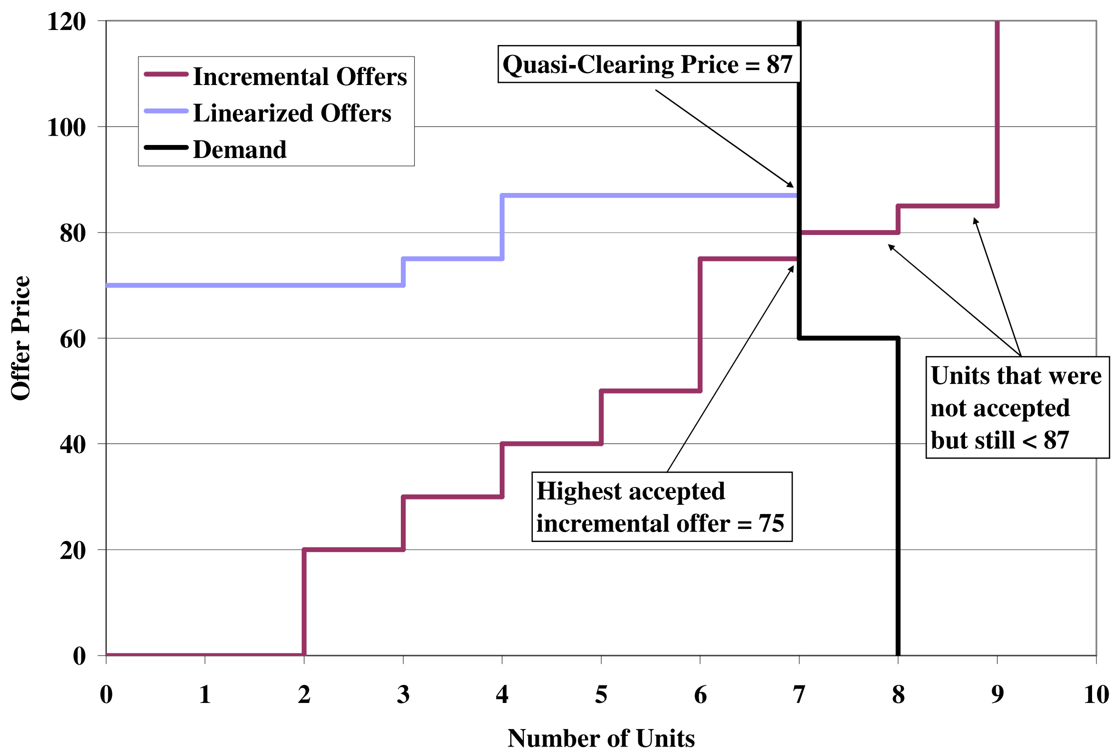

Figure 7: Comparison of the QUPA and the Standard Quasi-UPA

The QUPA institution sets the Quasi-Clearing Price based on the linearized offer of 87 for Supplier 2. The figure shows two out-of-equilibrium units that were offered at less the Quasi-Clearing Price, 80 and 85, but were not accepted. These two units would not be out-of-equilibrium if the Quasi-Clearing Price was determined using the Standard Quasi-UPA institution, which would set the Quasi-Clearing Price using the highest accepted incremental offer of 75 or the lowest non-accepted incremental offer of 80. In this example, there are no out-of-equilibrium units of demand, but it is possible for a bid to be accepted when it is priced below the Quasi-Clearing Price.

4. Step Three – Determine Make Whole Payments and Charges

Make whole payments are necessary in the QUPA because otherwise some buyers would be forced to pay more at the quasi-clearing price than the accepted portions of their bids and some sellers would be paid less at the quasi-clearing price than the accepted portions of their offers.

When allocating the uplift charges resulting from make whole payments, it is important to regard the principle stated at the beginning of this section, which is that uplift should be allocated in a manner that minimizes its effect on incentives. For this reason, I chose to allocate the cost of make whole payments on a per-unit basis to accepted bids (subject to the constraint that the sum of the Quasi-Clearing Price and allocation of make whole payments cannot exceed the sum of the accepted bids).[27] This method of allocation distributes the charges as uniformly as possible, which is consistent

[27] If this constraint prevents the allocation of a portion of the uplift charge, this portion will be reallocated to the remaining accepted bids. The reallocation will be subject to the same constraint, which may result in another reallocation.

with the overall purpose of the quasi-UPA institution, so that buyers will not have an incentive to inefficiently reduce their bids to minimize the charges. The exception is if a buyer is subject to the constraint mentioned above that total payments cannot exceed the sum of accepted bids, although I do not expect this exception to play a significant role in the incentives of buyers. These incentives are discussed in greater detail in Section V.

I chose to allocate the uplift charges from make whole payments to buyers instead of sellers for two reasons. First, in the environment used in these experiments, I expect buyers to receive more of the surplus than sellers, so the uplift will be smaller to buyers and therefore less significant. Second, most of the uplift charges will come from payments to sellers, and since these sellers cannot both receive make whole payments and pay the resulting uplift, there might not be any sellers left to pay the uplift charges in some cases.

Section III mentions quasi-UPA designs that give lost opportunity payments to offers that are not accepted when priced lower than the quasi-clearing price. These payments seem fairer to sellers, which has merit in the one-sided environment of wholesale electricity markets where most buyers are not price-sensitive. However, since it is not clear that they substantially improve the incentives of sellers to reveal their true costs and they would substantially increase the total amount of uplift, I have chosen not to include them in the QUPA.

The following discussion uses the previous example to illustrate the determination of make whole payments and the resulting uplift charges. The following payments are made according to the Quasi-Clearing Price of 87 per unit. Buyer 1 pays 348 for 4 units, even though his accepted bids sum to 680. Buyer 2 pays 261 for 3 units, even though his accepted bids sum to 510. Seller 1 receives 348 for 4 units, even though his accepted offers sum to 285. Seller 2 receives 261 for 3 units, even though his accepted offers sum to 270.

In this example, the following make whole payments are given to each buyer and seller: Buyer 1 receives 0 because the accepted portions of its bid sum to 680, which exceeds its payment of 348. Buyer 2 receives 0 because the accepted portions of its bid sum to 510, which exceeds its payment of 261. Seller 1 receives 0 because the accepted portions of its offer sum to 285, which is less than its payment of 348. Seller 2 receives 9 because the accepted portions of its offer sum to 270, which exceeds its payment of 261 by 9. Note that under this formulation, the buyers would not have received make whole payments if one of their accepted bid increments had been priced below 87. A buyer would have only received a make whole payment if the *average* accepted bid increment was priced below 87.

In this example, the uplift is allocated to the buyers in proportion to the number of units purchased. Buyer 1 pays 4/7 of 9 (≈ 5.1). Buyer 2 pays 3/7 of 9 (≈ 3.9).

A final summary of payments is as follows. Both buyers pay an average uplift charge of 1.3 per unit, resulting in a total charge of 88.3 per unit. Seller 1 receives a payment of 87 per unit based solely on the quasi-clearing price. Seller 2 receives a total payment of 90 per unit.

C. Information in the QUPA Institution

This section discusses the information that is revealed to subjects through the QUPA. In the QUPA, each round consists of (i) a brief period for subjects to submit bids and offers, (ii) a brief pause while the auction software runs, and (iii) a brief period for subjects to review the results of the auction round.

The following information is provided by the institution after each round: [28] (i) the number of units bought or sold by the individual buyer or seller, (ii) the total number of units transacted by all buyers and sellers, (iii) the levels of the accepted bid or offer components and their sum,[29] (iv) the total transaction payment charged to the buyer or paid to the seller, (v) the subject's average transaction price equaling the total transaction payment divided by the number of units transacted by the subject, (vi) the minimum and maximum transaction prices of the other buyers and sellers, (vii) the value of the units bought or the cost of the sold, (viii) the subject's profit from the round, and (ix) the subject's cumulative profits from the group of 12 rounds. This information for the three previous rounds is shown on the screen in summary format while subjects are deciding on their offers.[30]

There are several pieces of information that are not given to subjects by the institution. Subjects are never given a breakdown of transaction prices according to the quasi-clearing price, the make whole payment, and the uplift charge. Subjects are never told about the bids and offers that are submitted by other buyers and sellers. No

[28] The Appendix shows samples of the screens that are shown to buyers and sellers after each round.
[29] If a seller offers three units and only two are accepted, the results will indicate that the fixed offer component and the two lowest-priced incremental components were accepted.
[30] The Appendix shows samples of the screens that buyers and sellers use to enter their bids and offers.

information is given to them besides what they see on the screen, and buyers and sellers never communicate directly with one another.

It is important to highlight how features of the QUPA compare with other institutions. First, the QUPA is a single-shot sealed-offer auction, so subjects receive no feedback from the institution during a particular round. This feature of the QUPA is consistent with the other single-shot sealed-offer institutions discussed in Sections II and III. In contrast, the DA gives subjects information from early trades as well as bids and offers that are not accepted, enabling them to refine their strategies during the round. Hence, a disadvantage of the QUPA is that if subjects make strategic errors, it will take multiple rounds for them to adjust accordingly. The flipside of this disadvantage is that it facilitates the rapid pace of transactions that is necessary in electricity markets.

Second, the QUPA provides subjects with less non-private information than most of the other institutions. The Smart Market authors point out that in each round of the Smart Market, subjects receive three messages—the offers of each of the other three sellers in the previous round. Conversely, their conservative estimate is that in each round of the DA, subjects receive four messages for each unit transacted, totaling 48 to 64 messages. The QUPA provides subjects with just two pieces of non-private information after each round: the number of units transacted and the range of transaction prices. In this regard, the QUPA is similar to most electricity spot markets; however, most electricity markets release additional information about offer prices several months afterward.

V. Analyses of Incentives in the QUPA

This section analyzes incentives to predict how subjects will behave in the QUPA and to assess how this will affect the overall efficiency of the institution. These analyses help inform the design of experiment, which is described in Section VI. The findings from this section also provide a framework for analyzing empirical outcomes in Section VII.

Section V uses two approaches to analyze incentives in the QUPA. Section V.A evaluates incentives qualitatively, drawing on the earlier discussion of electricity auctions to characterize several strategies that subjects are likely to use to devise their bids and offers in the QUPA. Section V.B uses Nash Equilibrium analyses to predict outcomes in my experiments. Separate analyses are performed for the D3 environment and the D4 environment. The Smart Market authors also performed a Nash Equilibrium analysis, but it is not applicable to the QUPA, because this type of analysis depends on the environment as well as the rules of the institution.

A. Qualitative Evaluation

The institution and environment are the primary determinants of the incentives of subjects in my experiments. Four buyers each submit three or four separate bid components. Four sellers have non-convex cost structures and each submit between three and nine separate offer components. In all, there are a total of 34 to 38 separate bid and offer parameters that go into each auction round. While the buyers are symmetric, the sellers each have different production costs. The institution reveals a limited amount of information to subjects about the actions of other subjects and how transaction prices and quantities are determined. Under such conditions, it is difficult for formal analytical models to accurately predict behavior. Thus, the following qualitative evaluation of incentives provides additional insight about how subjects are likely to respond to the institution and environment.

Drawing on the earlier discussion of electricity auctions, Section V.A characterizes several strategies that subjects might use to devise their bids and offers in the QUPA. The QUPA was designed to give the incentives of a Uniform Price Auction while allowing sellers to submit non-convex cost offers. However, the unique features of the QUPA that facilitate sales by suppliers with non-convex cost structures may also undermine Uniform Price Auction incentives. Considering this mixture of incentives, the following discussion outlines the range of potential strategies that subjects might use in the experiment.

The incentive properties of the QUPA institution are similar to those of a uniform price auction in certain respects. A key strength of uniform price auctions is that they give buyers and sellers the incentive to reveal their private values under certain circumstances. Buyers and sellers are likely to reveal their private values when they believe that their offer will not significantly affect the clearing price. When buyers and sellers reveal, it leads to highly efficient allocations and competitive prices. Inducing buyers and sellers to reveal would be particularly beneficial in an environment with non-convex cost structures where efficient coordination can be difficult to achieve. However, when sellers (buyers) believe that their offer (bid) is likely to affect the clearing price, they may raise (lower) their offer (bid) prices if they believe that the gains from a higher (lower) clearing price are likely to offset the foregone profits from selling (buying) a smaller quantity. So buyers and sellers are likely to reveal in a uniform-price auction if the environment does not have substantial market power.

The unique features of the QUPA that are designed to facilitate sales from suppliers with non-convex cost structures could also affect incentives in several ways. First, some sellers may have discriminatory price (pay-as-bid) auction incentives rather than uniform price auction incentives. Sellers that anticipate receiving make whole payments or having the linearized offer that directly determines the clearing price have an incentive to raise their offer prices as much as possible without having their offer rejected. For example, Supplier #2 from Figure 4, Figure 5, and Figure 6 in Section IV could have raised the quasi-clearing price considerably before feeling competitive pressure from Supplier #1, Supplier #3, or the bids of the buyers. The environment used in my experiments has only four sellers, at least one of whom will be paid-as-bid in each auction round, so pay-as-bid incentives are likely to be a significant factor at least for the sellers that are close to the margin.

Second, even though bids never directly set the quasi-clearing price in the QUPA, buyers may still have pay-as-bid auction incentives. Buyers are guaranteed to pay no more than the sum of the bid prices of their accepted bid increments, so it is possible for buyers to reduce their allocation of the uplift costs arising from make whole payments or to receive make whole payments by reducing their bid prices below the quasi-clearing price. These two possibilities can be demonstrated using the example in Section IV where the buyers paid the quasi-clearing price of 87/unit plus the uplift allocation of 1.3/unit. Suppose that the lowest accepted bid increment, which was priced at 130, was bid by a buyer with just one unit of demand. The buyer could avoid the allocation of uplift by bidding 87/unit, because the procedure for allocating uplift does not allow the sum of the quasi-clearing price and uplift allocation to exceed the accepted components of the buyer's bid. The buyer could receive a make whole payment by bidding 75/unit. The bid would be accepted because the last accepted increment of supply was offered at 75/unit, and the buyer would receive a make whole payment of 12/unit for the difference between the quasi-clearing price and the buyer's bid.

Third, sellers may have incentives to offer below cost and buyers may have incentives to bid above their values due to inconsistencies between the quasi-clearing price and the marginal incremental bids and offers.[31] In the example in Section IV, the quasi-clearing price was 87 while increments of supply that were offered by Seller #2 at 80 and 85 were not accepted. Under certain circumstances, this inconsistency would give Seller #2 the incentive to offer one of these increments at 74 to undercut the highest accepted increment, which was priced at 75. In this particular case, Seller #2 set the quasi-clearing price and received a make whole payment, and thus, had pay-as-bid incentives. However, if the circumstances were different and Seller #2 had not set the

[31] This includes the range between the highest accepted offer or unaccepted bid and the lowest accepted bid or unaccepted offer. In the example from Section IV, this range would go from 75, the highest accepted offer, to 80, the lowest unaccepted offer.

quasi-clearing price or received a make whole payment, Seller #2 would have benefit from offering its fourth unit at 74 instead of 80. Similar inconsistencies may arise which give buyers the incentive to bid above their value.

Based on these factors, buyers and sellers are likely to pursue one of four strategies at a given time:

- *Revealing Strategy* – Buyers and sellers that perceive little ability to profitably affect transaction prices may bid or offer at levels that reveal their values or costs.
- *Aggressive Strategy* – Buyers and sellers will set their bids and offers in order to influence the transaction prices if they anticipate the benefits of affecting the price will outweigh the profits from foregone purchases and sales.
- *Quantity Maximizing Strategy* – Buyers will raise their bids above their values and sellers will lower their offers below their costs in order to increase the probability of transacting all of their units if they anticipate being harmed by inconsistencies between the quasi-clearing price and the selection of bids and offers. In many cases, this strategy results in the same payoffs as the Revealing Strategy, but for buyers and sellers near the margin, it can result in a larger quantity of profitable sales. However, this must be balanced against the possibility of buying or selling at a loss.
- *Conservative Strategy* – Subjects may have difficulty interpreting the sometimes contradictory incentives of the QUPA. If so, they may fall back on a cautious strategy that compromises between the Aggressive Strategy, which may lead to lost profits from foregone transactions, and the Revealing Strategy, which might not exert sufficient competitive pressure on the quasi-clearing price.

The extent to which various incentives affect buyers and sellers depends on their assessments of the environment and institution.

Uncertainty makes it more difficult for buyers and sellers to optimize their bids and offers to take advantage of opportunities created by the institutional rules. Thus, it is useful to examine how uncertainty is likely to affect the attractiveness of the four strategies listed above. The Revealing Strategy can be executed with no information about what other subjects will do or what the quasi-clearing price will be, so it may be attractive to a subject if changes in the quasi-clearing price from round to round seem unresponsive to changes in his bid or offer. The Aggressive Strategy requires some intuition about how the market will respond to changes in the offer or bid. So uncertainty is likely to discourage this strategy. The Quantity Maximizing Strategy is easy to execute, but a lot of information is necessary to determine precisely when this strategy would be profitable, so uncertainty may discourage this strategy as well. Like the Revealing Strategy, the Conservative Strategy can be executed with little information about what other subjects are doing. It will be attractive to sellers (buyers) that do not have a clear understanding of the factors that determine transaction prices but do have an intuitive sense that revealing their costs (values) would allow buyers (sellers) to push transaction prices down (up). So uncertainty may encourage subjects to adopt the Conservative Strategy.

The QUPA institution is likely to produce efficient results if the incentives of buyers and sellers to reveal their private values are generally stronger than the incentives to take use other strategies. The information that each subject has during the experiment, and therefore, their preference for one strategy over the other, partly depends on the design of the experiment. It is important to carefully design instructions and messages that allow subjects to perceive when efficient behavior is also profitable. However, there is also a need to design the experiment in a manner that makes it applicable to the real world. So, the instructions could be filled with "coaching tips" that encourage subjects to adopt the Revealing Strategy rather than the Aggressive Strategy, but if the QUPA were used in the real world, subjects would presumably be exposed to other profitable strategies as well. The experimental protocols are discussed in Section VI, but they specifically avoid such prompting.

B. Analysis of Nash Equilibria

In Van Boening and Wilcox (1996), one of the primary treatment variations was between the AC4 environment, where there exist core allocations that are also Competitive Equilibria, and the AC3 environment, which was specifically designed to support core allocations but no Competitive Equilibria. The existence of a non-empty core and CE is not dependent on the institution, only the environment, so these findings are applicable to the Smart Market and QUPA experiments. The Smart Market authors perform a Nash Equilibrium analysis, which is dependent on both the environment and the institutional rules, finding that Nash Equilibria exist in the AC3 and AC4 environments. The experiments run by Van Boening and Wilcox (1996) provide weak evidence that the DA is more efficient in the AC4 environment than in the AC3 environment. In contrast, the authors of the Smart Market found weak evidence for the opposite effect, that the no-CE environment was more efficient. The Smart Market authors concluded that the experimental outcomes were generally inconsistent with the Nash Equilibria, noting that the sellers usually pushed prices far above the levels predicted by the Nash Equilibrium analysis. In this section, I attempt a Nash Equilibrium analysis of the QUPA institution with the D3 and D4 environment that were used in my experiments.

The QUPA is a sealed-bid auction institution where each subject has multiple parameters that may be used to influence the outcome. In each round, eight subjects enter a total of 34 parameters in the D3 treatment and 38 parameters in the D4 and treatment, making a comprehensive Nash Equilibrium analysis of the QUPA very difficult. Furthermore, the non-convexity of the environment creates additional difficulties. So the scope of the analysis in this section is limited to showing whether there exists Nash Equilibria that are 100 percent efficient allocations and where all bid increments are priced at the same level.

In the D4 treatment, the only 100 percent efficient allocation is where Seller #1 sells 8 units, Seller #2 sells 5 units, Seller #3 sells 3 units, and Seller #4 sells 0 units. In the D3 treatment, the 100 percent efficient allocations are where Sellers #1 and #2 sell a combined 12 units. This happens in two ways: (i) if Seller #1 sells 8 units and Seller #2 sells 4 units or (ii) if Seller #1 sells 7 units and Seller #2 sells 5 units.

Various abbreviations are used in this section. Seller #X has a true fixed cost (C_X), a fixed offer component (F_X), incremental offer components (I_{X1} to I_{Xm}), and a maximum production capability (m_X). The short-hand T is used such that $T_X = F_X + \sum(I_{X1}$ to $I_{Xm})$ where $\sum(I_{X1}$ to $I_{Xm})$ refers to the summation of increment offers of Seller #X from element 1 to element m_X. The linearized offer (L_{X1} to L_{Xm}) is equal to the higher of the incremental offer components or the minimum average offer cost (M_X). When the incremental components of multiple sellers are referred to as a group, the increments are numbered from lowest to highest (I_{g1} to I_{gj} where j is the sum of m for the sellers). Buyer Y has a true value (V_Y), incremental bids (B_{Y1} to B_{Yq}), and a maximum desired quantity (q). When Buyers A, B, C, and D are referred to as a group, the bids are numbered from highest to lowest (B_1 to B_{12} for the D3 environment or B_1 to B_{16} for the D4 environment).

The following assumptions are used throughout this section. First, continuity is assumed even though the actual QUPA institution rounds or truncates after the first decimal place. Second, negative offers and bids are not allowed in the QUPA, so they are not considered in this section. Third, this section assumes that buyers submit bids that are no higher than their true value even though this was not a limitation in the actual QUPA experiments.

1. D3 Treatment

Three cases are evaluated in this section: Case I where Sellers #1 and #2 use fixed component strategies (i.e. $T_1 = F_1$ and $T_2 = F_2$), Case II where Seller #1 uses an incremental strategy and Seller #2 uses a fixed component strategy, and Case III where Sellers #2 uses an incremental strategy and Seller #1 uses a fixed component strategy.

Case I: Sellers #1 and #2 use fixed component strategies (i.e. $T_1 = F_1$ and $T_2 = F_2$)

Case I examines whether there are Nash Equilibria that are 100 percent efficient and where Sellers #1 and #2 submit incremental offer components that sum to zero. Any allocation where Sellers #1 and #2 sell a combined 12 units will result in one of the 13 increments not being accepted. Since the QUPA breaks a tie between two or more increments by accepting them in random order, there is a $p_A = 5/13$ that Seller #1 sells 8 units and Seller #2 sells 4 units and $p_B = 8/13$ that Seller #1 sells 7 units and Seller #2 sells 5 units. Each seller receives the higher of: (i) the number of units sold times the quasi-clearing price and (ii) the seller's accepted offer. The quasi-clearing price and the expected profit functions of Sellers #1 and #2 are as follows:

$$P = \max(M_1, M_2) = \max(F_1/8, F_2/5)$$
$$\prod_1 = p_A * \max(8*P, F_1) + p_B * \max(7*P, F_1) - C_1$$
$$\prod_2 = p_A * \max(4*P, F_2) + p_B * \max(5*P, F_2) - C_2$$

In order for Sellers #1 and #2 to sell a combined 12 units and for Sellers #3 and #4 to each sell 0 units, several conditions must hold.

Base Conditions

Seller #1 must provide more surplus by selling 7 units than any other combination that includes Seller #3, Seller #4, both, or neither. This results in the following constraints:

- $\sum(B_6 \text{ to } B_{12}) - F_1 >= \sum(B_6 \text{ to } B_8) - T_3 \rightarrow$ $F_1 <= \sum(B_9 \text{ to } B_{12}) + T_3$
- $\sum(B_6 \text{ to } B_{12}) - F_1 >= \sum(B_6 \text{ to } B_7) - T_4 \rightarrow$ $F_1 <= \sum(B_8 \text{ to } B_{12}) + T_4$
- $\sum(B_6 \text{ to } B_{12}) - F_1 >= \sum(B_6 \text{ to } B_{10}) - T_3 - T_4 \rightarrow$ $F_1 <= \sum(B_{11} \text{ to } B_{12}) + T_3 + T_4$
- $\sum(B_6 \text{ to } B_{12}) - F_1 >= 0 \rightarrow$ $F_1 <= \sum(B_{11} \text{ to } B_{12})$

Seller #2 must provide more surplus by selling 4 units than any other combination that includes Seller #3, Seller #4, both, or neither. This results in the following constraints:

- $\sum(B_9 \text{ to } B_{12}) - F_2 >= \sum(B_9 \text{ to } B_{11}) - T_3 \rightarrow$ $F_2 <= B_{12} + T_3$
- $\sum(B_9 \text{ to } B_{12}) - F_2 >= \sum(B_9 \text{ to } B_{10}) - T_4 \rightarrow$ $F_2 <= \sum(B_{11} \text{ to } B_{12}) + T_4$
- $\sum(B_9 \text{ to } B_{12}) - F_2 >= \sum(B_9 \text{ to } B_{12}) - T_3 - T_4 + I_{g5} \rightarrow$ $F_2 < T_3 + T_4 - I_{g5}$
- $\sum(B_9 \text{ to } B_{12}) - F_2 >= 0 \rightarrow$ $F_2 <= \sum(B_9 \text{ to } B_{12})$

Seller #3 will submit an offer with a fixed component exceeding his true cost OR submit an offer where one increment exceeds one-third of his true cost if all three increments would be accepted OR submit an offer as low as zero if all three increments would be accepted and the quasi-clearing price would exceed one-third of his true cost. Hence, Seller #3's offer must satisfy one of the following three conditions:

- $F_3 > C_3$
- $I_{33} > C_3 / 3$ AND I_{33} is accepted
- $T_3 >= 0$ AND I_{33} is accepted AND $P > C_3 / 3$

Seller #4 will submit an offer with a fixed component exceeding his true cost OR submit an offer where one increment exceeds one-half of his true cost if both increments would be accepted OR submit an offer summing to zero if both increments would be accepted and the quasi-clearing price would exceed one-half of his true cost. Hence, Seller #4's offer must satisfy one of the following three conditions:

- $F_4 > C_4$
- $I_{42} > C_4 / 2$ AND I_{42} is accepted
- $T_4 >= 0$ AND I_{42} is accepted AND $P > C_4 / 2$

These are not the only relevant constraints, but they are referred to repeatedly in this section. Given the symmetry of buyers in this environment, we look for Nash Equilibria where $B_i = B$ for all i.

B <= 180

When B <= 180 there is not sufficient surplus for Sellers #3 or #4 to earn a profit, who have minimum average costs of $C_3/3 = 180$ and $C_4/2 = 210$, so we suppose that Seller #3 offers $F_3 = C_3 + e$ and Seller #4 offers $F_4 = C_4 + e$. Suppose that Sellers #1 and

#2 submit the highest possible offers that satisfy the Base Conditions. Thus, Seller #1 offers F_1 = 7*B and Seller #2 offers F_2 = 4*B. This allows us to calculate the expected profits for Seller #2:

$$
\begin{aligned}
\prod_2 &= p_A * \max(4*P, F_2) + p_B * \max(5*P, F_2) - C_2 \\
&= p_A * \max(4*\max(F_1/8, F_2/5), F_2) + p_B * \max(5*\max(F_1/8, F_2/5), F_2) - C_2 \\
&= p_A * \max(4*F_1/8, F_2) + p_B * \max(5*F_1/8, F_2) - C_2 \\
&= 5/13 * \max(F_1/2, F_2) + 8/13 * \max(5/8*F_1, F_2) - 750 \\
&= 5/13 * \max(7/2*B, 4*B) + 8/13 * \max(5*7/8*B, 4*B) - 750 \\
&= 5/13 * 4*B + 8/13 * 5*7*B/8 - 750
\end{aligned}
$$

Notice that $\prod_2$ is an increasing function of B, so it is possible to solve for the minimum value of B that allows Seller #2 to earn a positive expected profit. It is where:

$$
\begin{aligned}
&B = 750 * 13 * (5*4 + 5*7) = 177.27 \\
&F1 = 1240.91 = (155.11 * 8 \text{ units}) \\
&F2 = 709.09 = (141.82 * 5 \text{ units})
\end{aligned}
$$

It is interesting to note that this potential solution has Seller #1 submitting a higher minimum average cost than Seller #2. Furthermore, Seller #2 submits an offer that is lower than his true cost, because he expects the quasi-clearing price to be set by Seller #1. Since the profit function $\prod_2$ does not depend on F_2, so F_2 can be any value that will be accepted in step one of the QUPA. This includes $0 <= F_2 <= 709.09$. Thus, we have candidate class of Nash Equilibria for $177.27 < B <= 180$ where:

- $T_1 = F_1 = 7/8*B$,
- $105/16*B - 625 <= T_2 = F_2 <= 4*B$ (which ranges from $538.35 <= F_2 <= 709.09$ for B = 177.27 to $566.25 <= F_2 <= 720$ for B = 180. The reason for the lower bound at 105/16*B – 625 is explained below),
- $T_3 = 540 + e$,

- $T_4 = 420 + e$, and
- $P = T_1 = 7/8*B$.

Seller #2 will not deviate. For all F_2 that satisfy the inequality above, $\prod_2 = 5/13 * 4*B + 8/13 * 5*7*B/8 - 750 > 0$ and $\prod_2$ is invariant with F_2. Increasing F_2 above the range violates the Base Conditions so that Seller #2 sells 0 units. Decreasing F_2 below $105/16*B - 625$ does not change the payoff, but it does lead to a deviation by the buyers, which is explained below, and hence, it is outside the set of Nash Equilibria. Any reallocation from F_2 to I_{2i} for a given T_2 will lead to $p_A = 1$ and $p_B = 0$, leading Seller #2 to sell just 4 units, but since Seller #2 must earn at least 187.5/unit if he sells just 4 units and this is not possible when $B <= 180$, this reduces $\prod_2$.

Seller #3 will not deviate. Any increase would still result in a sale of 0 units. Unless $B > 180$, there is no $T_3 <= 540$ that satisfies the Base Conditions.

Seller #4 will not deviate. Any increase would still result in a sale of 0 units. Unless $B > 210$, there is no $T_4 <= 420$ that satisfies the Base Conditions.

The buyers will not deviate. Each buyer earns $\prod_B = 3 * (V - P) = 3 * (V - T_1/8) = 3 * (250 - 7/8*B)$. If Buyer A raises the bid for one, two, or three increments, the solution does not change since the Base Conditions indicate that only the 6^{th} to 12^{th} highest bid increments have any effect on the allocation. If Buyer A lowers the bid of one increment leads to a violation of the Base Conditions, resulting in Seller #1 selling zero units, Seller #2 selling 5 units, and $P = F_2/5$ and giving Buyer A a less than 100 percent probability of having accepted the two increments bid at B. However, Buyer A can increase the probability to 100 percent by raising the two increments such that $B_{A1} >$

B and $B_{A2} > B$ while causing Seller #1 to sell zero units by bidding $B_{A3} < B$. As a result Buyer A has $\prod_{BA}' = 2 * (V - P) = 2 * (V - F_2/5)$. But $\prod_{BA}'$ is not greater than $\prod_B$ iff: $3 * (250 - 7/8*B) >= 2 * (250 - F_2/5) \rightarrow F_2 >= -625 + 105/16*B$.

Seller #1 will not reduce his offer since doing so would reduce P which reduces $\prod_1 = (p_A*8 + p_B*7) * P - C_1 = (p_A + 7) * P - C_1$. Any increase in F_1 will violate the Base Conditions. However, Seller #1 can switch to an incremental strategy, which reduces pA = 0, but results in a higher quasi-clearing price. Specifically, Seller #1 can reallocate $F_1 = 7*B$ to $F_1 = 0$ and $I_{11} = I_{12} = I_{13} = I_{14} = I_{15} = I_{16} = I_{17} = B$, and since it will not be accepted anyway: $I_{18} = B$. This type of outcome is examined more closely under Case II.

Since Seller #1 decides to use the incremental strategy, we find no Nash Equilibria for cases where $B_i = B$ for all i, $B <= 180$, Seller #1 and #2 sell a combined 12 units, $T_1 = F_1$, and $T_2 = F_2$.

180 < B <= 210

When $180 < B <= 210$, we must consider the possibility that Seller #3 changes strategy. In particular, there is sufficient surplus so that Seller #3 could offer $F_3 = 0$, $I_{31} = 0$, $I_{32} = 0$, and $I_{33} = C_3/3 + e$, have all three units accepted and set P to at least $C_3/3 + e$. Seller #1 must now offer $F_1 <= \sum(B_9 \text{ to } B_{12}) + I_{33}$ or be replaced by Seller #3. Seller #2 must now offer $F_2 <= B_{12} + I_{33}$ or be replaced by Seller #3. There is no solution that satisfies these conditions because even for the largest B (=210), $F_1 <= 4*210 + 180 + e = 1020 + e$ and $F_2 <= 210 + 180 + e = 390 + e$, resulting in $P = 127.5$ and a negative profit for Seller #2. Hence, there are no Nash Equilibria for cases where $B_i = B$ for all i, $180 < B <= 210$, Seller #1 and #2 sell a combined 12 units, $T_1 = F_1$, and $T_2 = F_2$.

210 < B <= 240

For cases where 210 < B <= 250, we must consider the possibility that Seller #4 also changes strategy. In particular, there is sufficient surplus so that Seller #4 could offer $F_4 = 0$, $I_{41} = 0$, $I_{42} = C_4/2 + e$, have both units accepted and set P to at least $C_4/2 + e$. So we must now consider four possible scenarios: (i) Sellers #3 and #4 both use the incremental strategy, (ii) Seller #3 uses the incremental strategy while Seller #4 uses the fixed component strategy, (iii) vice versa, or (iv) Sellers #3 and #4 both use the fixed component strategy.

For scenario (i): to satisfy the Base Conditions, Seller #1 must offer $F_1 <= 2*B + I_{33} + I_{42}$ but Sellers #3 and #4 will not use the incremental strategy if it would result in Seller #1 selling and Seller #2 being left out, because either I_{33} or I_{42} would not be accepted and this would lead one of them to earn negative profit. However, Seller #2 will be left out if $F_1 <= 2*B + F_2$, and since Seller #1 always has the ability and incentive to satisfy this inequality, scenario (i) cannot result in a Nash Equilibrium.

For scenario (ii): to satisfy the Base Conditions, Seller #1 must offer $F_1 <= 2*B + I_{33} + F_4$ but Seller #3 will not use the incremental strategy if it would result in Seller #1 selling and Seller #2 being left out, because then I_{33} would not be accepted and Seller #3 would earn a negative profit. However, Seller #2 will be left out if $F_1 <= 2*B + F_2$, and since Seller #1 will always have the ability and incentive to satisfy this inequality, scenario (ii) cannot result in a Nash Equilibrium.

For scenario (iii): to satisfy the Base Conditions, Seller #1 must offer $F_1 <= 2*B + F_3 + I_{42}$ but Seller #4 will not use the incremental strategy if it would result in Seller #1 selling and Seller #2 being left out, because then I_{42} would not be accepted and Seller #4 would earn a negative profit. However, Seller #2 will be left out if $F_1 <= 2*B + F_2$, and since Seller #1 will always have the ability and incentive to satisfy this inequality, scenario (iii) cannot result in a Nash Equilibrium.

For scenario (iv): to satisfy the Base Conditions, Seller #1 must offer $F_1 <= 2*B + F_3 + F_4$ and Seller #2 must offer $F_2 <= B + F_3$. In scenario (iv), the expected profits for Seller #2 are:

$$
\begin{aligned}
\prod{}_2 &= p_A * \max(4*P, F_2) + p_B * \max(5*P, F_2) - C_2 \\
&= p_A * \max(4*\max(F_1/8, F_2/5), F_2) + p_B * \max(5*\max(F_1/8, F_2/5), F_2) - C_2 \\
&= p_A * \max(4*F_1/8, F_2) + p_B * \max(5/8*F_1, F_2) - C_2 \\
&= p_A * \max(1/2*(2*B+F_3+F_4), F_2) + p_B * \max(5/8*(2*B+F_3+F_4), F_2) - C_2 \\
&= 5/13 * (B+540) + 8/13 * (5*B/4 + 600) - 750 \\
&= 15/13*B - 2250/13
\end{aligned}
$$

$\prod_2$, which is an increasing function of B, must be non-negative. $\prod_2$ is non-negative for $B >= 150$. So Seller #2 offers low enough to stay in the market, resulting in the quasi-clearing price: $P = \max(F_1/8, F_2/5)$, and since $F_1/8 > F_2/5$ for all $210 < B <= 240$, $P = F_1/8 = (2*B + F_3 + F_4)/8 = 120 + B/4$. P is always less than 180, making Seller #3 unprofitable at the quasi-clearing price. This would be a Nash Equilibrium except Seller #3 has the incentive to deviate by switching to the incremental offer strategy, causing Seller #2 to no longer sell. Thus, scenario (iv) eventually devolves into scenario (ii) as follows. To protect against this change by Seller #3, the Base Conditions indicate that Seller #1 must offer $F_1 <= 2*B + I_{33} + F_4$ and Seller #2 must offer $F_2 <= B + I_{33}$. Even for the highest value of B where $210 < B <= 240$, $F_1 <= 1080$ and $F_2 <= 420$, which would result in $P = \max(T_1/8, T_2/5) = 135$. Since this would result in negative profit for Seller #2, there are no Nash Equilibria for cases where $B_i = B$ for all i, $210 < B <= 240$, Seller #1 and #2 sell a combined 12 units, $T_1 = F_1$, and $T_2 = F_2$.

240 < B <= 250

For cases where $240 < B <= 250$, we must consider that Sellers #3 and #4 could use the incremental or fixed strategies. We must consider four possible scenarios: (i) Sellers #3 and #4 both use the incremental strategy, (ii) Seller #3 uses the incremental strategy while Seller #4 uses the fixed component strategy, (iii) vice versa, or (iv) Sellers #3 and #4 both use the fixed component strategy. For the same reasons that applied previously, Scenarios (i), (ii), and (iii) cannot result in Nash Equilibria, so we consider Scenario (iv). For scenario (iv) to satisfy the Base Conditions, Seller #1 must offer $F_1 <= 2*B + F_3 + F_4$ and Seller #2 must offer $F_2 <= B + F_3$. Once again, the expected profits for Seller #2 are:

$$\begin{aligned} \prod\nolimits_2 &= p_A * \max(4*P, T_2) + p_B * \max(5*P, T_2) - C_2 \\ &= 5/13 * (B+540) + 8/13 * (5*B/4 + 600) - 750 \\ &= 15/13*B - 2250/13 \end{aligned}$$

$\prod_2$ is greater than zero for all $240 < B <= 250$, so Seller #2 offers low enough to stay in the market. However, Seller #3 has the incentive to deviate by switching to the incremental offer strategy, causing Seller #2 to no longer sell. To protect against this change by Seller #3, the Base Conditions indicate that Seller #1 must offer $T_1 <= 2*B + I_{33} + F_4$ and Seller #2 must offer $T_2 <= B + I_{33}$. For $B = 250$, $F_1 <= 1100$ and $F_2 <= 430$, which would result in $P = \max(T_1/8, T_2/5) = 137.5$. Since this would result in negative profit for Seller #2, there are no Nash Equilibria for cases where $B_i = B$ for all i, $240 < B <= 250$, Seller #1 and #2 sell a combined 12 units, $T_1 = F_1$, and $T_2 = F_2$.

Case II: Seller #1 uses incremental strategy and #2 uses fixed component strategy

Case II examines whether there are Nash Equilibria that are 100 percent efficient where Seller #2 continues to use a fixed component strategy (i.e. $T_2 = F_2$), but Seller #1 does not. In Case II, Seller #1 sells 7 units and Seller #2 sells 5 units, because Seller #1 always submits an increment that is priced higher than the highest increment offered by Seller #2. Each seller receives the higher of: (i) the number of units sold times the quasi-clearing price and (ii) the sum of the seller's accepted offer components. Under the conditions above, the quasi-clearing price and the expected profit functions of Sellers #1 and #2 are now as follows:

$$\prod_1 = \max(7*P, T_1 – I_{18}) – C_1$$
$$\prod_2 = \max(5*P, F_2) – C_2$$

In Case II, $P = \max(M_1, M_2, I_{17})$. But now $M_1 = \min((F_1+I_{11}), (F_1+\sum(I_{11} \text{ to } I_{12}))/2, (F_1+\sum(I_{11} \text{ to } I_{13}))/3, (F_1+\sum(I_{11} \text{ to } I_{14}))/4, (F_1+\sum(I_{11} \text{ to } I_{15}))/5, (F_1+\sum(I_{11} \text{ to } I_{16}))/6, (F_1+\sum(I_{11} \text{ to } I_{17}))/7, (F_1+\sum(I_{11} \text{ to } I_{18}))/8)$. If $I_{17} <= M_1$, then $M_1 = \min((F_1+\sum(I_{11} \text{ to } I_{17}))/7, (F_1+\sum(I_{11} \text{ to } I_{18}))/8) = \min((T_1-I_{18})/7, T_1/8)$. And of course: $M_2 = F_2/5$. So we can write:

$$P = \max(\min((T_1-I_{18})/7, T_1/8), F_2/5, I_{17})$$

In order for Seller #1 to sell 7 units, Seller #2 to sell 5 units, and Sellers #3 and #4 to each sell 0 units, the following set of conditions must hold.

Base Conditions

Seller #1 must provide more surplus by selling 7 units than any other combination that includes Seller #3, Seller #4, both, or neither. This results in the following constraints:

- $\sum(B_{10} \text{ to } B_{12}) - \sum(I_{15} \text{ to } I_{17}) > \sum(B_{10} \text{ to } B_{12}) - T_3 \rightarrow$ $\sum(I_{15} \text{ to } I_{17}) < T_3$
- $\sum(B_{11} \text{ to } B_{12}) - \sum(I_{16} \text{ to } I_{17}) > \sum(B_{11} \text{ to } B_{12}) - T_4 \rightarrow$ $\sum(I_{16} \text{ to } I_{17}) < T_4$
- $\sum(B_8 \text{ to } B_{12}) - \sum(I_{13} \text{ to } I_{17}) > \sum(B_8 \text{ to } B_{12}) - T_3 - T_4 \rightarrow$ $\sum(I_{13} \text{ to } I_{17}) < T_3 + T_4$
- $B_{5+i} - I_{1i} >= 0$ for i from 1 to 7

Seller #2 must provide more surplus by selling 5 units than any other combination that includes Seller #1's eighth unit, Seller #3, Seller #4, or combination of them. This results in the following constraints:

- $\sum(B_8 \text{ to } B_{12}) - F_2 >= \sum(B_8 \text{ to } B_{10}) - T_3 \rightarrow$ $F_2 <= \sum(B_{11} \text{ to } B_{12}) + T_3$
- $\sum(B_8 \text{ to } B_{12}) - F_2 >= \sum(B_8 \text{ to } B_{11}) - T_3 - I_{18} \rightarrow$ $F_2 <= B_{12} + T_3 + I_{18}$
- $\sum(B_8 \text{ to } B_{12}) - F_2 >= \sum(B_8 \text{ to } B_9) - T_4 \rightarrow$ $F_2 <= \sum(B_{10} \text{ to } B_{12}) + T_4$
- $\sum(B_8 \text{ to } B_{12}) - F_2 >= \sum(B_8 \text{ to } B_{10}) - T_4 - I_{18} \rightarrow$ $F_2 <= \sum(B_{11} \text{ to } B_{12}) + T_4 + I_{18}$
- $\sum(B_8 \text{ to } B_{12}) - F_2 > \sum(B_8 \text{ to } B_{12}) - T_3 - T_4 \rightarrow$ $F_2 < T_3 + T_4$
- $\sum(B_8 \text{ to } B_{12}) - F_2 >= B_8 - I_{18} \rightarrow$ $F_2 <= \sum(B_9 \text{ to } B_{12}) + I_{18}$
- $\sum(B_8 \text{ to } B_{12}) - F_2 >= 0 \rightarrow$ $F_2 <= \sum(B_8 \text{ to } B_{12})$

Seller #3 will submit an offer with a fixed component exceeding his true cost OR submit an offer where one increment exceeds one-third of his true cost if all three increments would be accepted OR submit an offer as low as zero if all three increments would be accepted and the quasi-clearing price would exceed one-third of his true cost. Hence, Seller #3's offer must satisfy one of the following three conditions:

- $F_3 > C_3$
- $I_{33} > C_3 / 3$ AND I_{33} is accepted

- T_3 >= 0 AND I_{33} is accepted AND P > C_3 / 3

Seller #4 will submit an offer with a fixed component exceeding his true cost OR submit an offer where one increment exceeds one-half of his true cost if both increments would be accepted OR submit an offer summing to zero if both increments would be accepted and the quasi-clearing price would exceed one-half of his true cost. Hence, Seller #4's offer must satisfy one of the following three conditions:

- $F_4 > C_4$
- $I_{42} > C_4 / 2$ AND I_{42} is accepted
- $T_4 >= 0$ AND I_{42} is accepted AND $P > C_4 / 2$

These are not the only relevant constraints, but they are used throughout this section. Given the symmetry of buyers in this environment, we look for Nash Equilibria where Bi = B for all i.

B <= 180

For cases where B <= 180, suppose Seller #3 offers $T_3 = F_3 = C_3 + e$ and Seller #4 offers $T_4 = F_4 = C_4 + e$. Suppose Seller #1 submits the maximum fully incremental offer under the Base Conditions: $F_1 = 0$ and $I_{1i} = B$ for all i from 1 to 7 and an arbitrarily high I_{18}. In this case, Seller #2 submits the maximum offer under the Base Conditions: $T_2 = F_2 = \min(5*B, 4*B+I_{18}) = 5*B$. In this case, the quasi-clearing price is:

$$
\begin{aligned}
P &= \max(\min((T_1-I_{18})/7, T_1/8), F_2/5, I_{17}) \\
&= \max((7*B)/7, (5*B)/5, B) \\
&= B
\end{aligned}
$$

In this case, P would be the same based on either the Seller #1's offer or Seller #2's offer. The expected profits for Seller #1 are:

$$\prod_1 = \max(7*P, T_1–I_{18}) – C_1$$
$$= 7*B – 960, \text{ which is positive where } B > 137.14$$

The expected profits for Seller #2 are:

$$\prod_2 = \max(5*P, T_2) – C_2$$
$$= \max(5*B, 4*B) – C_2$$
$$= 5*B – 750, \text{ which is positive where } B > 150$$

Nash Equilibria can only exist for values of B that allow sufficient surplus for Seller #2 to earn 750 on 5 units. So, there may be a class of Nash Equilibria where $150 < B <= 180$ and:

- $F_1 = 0$ and $I_{1i} = B$ for all i from 1 to 7 and $I_{18} > B$
- $T_2 = 5*B$
- $T_3 = 540 + e$,
- $T_4 = 420 + e$, and
- $P = B$.

In this case, Seller #1 has an incentive to deviate in multiple ways. Although he is able to sell 7 units at $P = B$, he could increase the number of units by submitting $F_1 = I_{1i} = 0$ for all i from 1 to 7 and $I_{18} = B – e$. The result would be that Seller #2 sells 0 units and Seller #1 sells 8 units at $P = B – e$. Seller #1 will deviate in this manner whenever 8 units * I_{18}' > 7 units * B $\rightarrow$ $I_{18}' > 7/8*B$. The Base Conditions tell us that Seller #2 must reduce his offer in order to protect himself against this possibility by offering $F_2 <= 4*B + I_{18}' = 39/8*B$. Alternatively, Seller #1 could switch to a fixed component strategy that would enable him to sell 8 units and Seller #2 to sell 0 units. Since he will set the quasi-clearing price, he will submit the highest $T_1 = F_1$ that satisfies $8*B – F_1 >= 5*B – F_2 \rightarrow F_1 <= 3*B + F_2$. This will be profitable whenever $F_1 >$ 7 units * B. Combining the inequalities: $7*B < 3*B + F_2 \rightarrow 4*B < F_2$. So Seller #2 must reduce his offer to $F_2 <= 4*B$ to protect against this strategy. So the new candidate class of Nash Equilibria where $150 < B <= 180$ and:

- $F_1 = 0$ and $I_{1i} = B$ for all i from 1 to 7 and $I_{18} > B$
- $T_2 <= 4*B$
- $T_3 = 540 + e$,
- $T_4 = 420 + e$, and
- $P = B$.

Now Seller #1 has no incentive to deviate, because Seller #2 offers in a range that prevents Seller #1 from profiting by changing his offer to sell more units. Any attempt to increase P by raising F1 or I_{11} to I_{16} will violate the Base Conditions. Seller #1 can reduce the offer prices of I_{11} to I_{16} to zero and I_{18} to B with no effect on the quasi-clearing price. Many reallocations of from the I_{11} to I_{16} increments to F_1 are also possible without affecting profits. Thus, we can broaden the set of conditions above to: $T_1 – I_{17} – I_{18} <= 6*B$, $I_{17} = B$, and $I_{18} > B$.

Seller #2 will not deviate. Increasing T_2 to $T_2 > 4*B$ does not increase $\prod_1$, but it would encourage Seller #1 to deviate. Decreasing T_2 will not affect profits.

Seller #3 will not deviate. Any increase would still result in a sale of 0 units. Unless $B > 180$, there is no $T_3 <= 540$ that satisfies the Base Conditions.

Seller #4 will not deviate. Any increase would still result in a sale of 0 units. Unless $B > 210$, there is no $T_4 <= 420$ that satisfies the Base Conditions.

The buyers have an incentive to deviate under certain circumstances. Each buyer earns $\prod_B = 3 * (V – P) = 3 * (V – B) = 3 * (250 – B)$. If Buyer A raises the bid for one, two, or three increments, the solution does not change since only the 6^{th} to 12^{th} highest bid increments have any effect on the allocation. But if Buyer A:

- Lowers the bid of one increment and it leads to a violation of the Base Conditions, it will result in Seller #1 selling 6 units and Seller #2 selling 5 units. This will be profitable if: $3 * (250 – B) < 2 * (250 – P') \rightarrow P' < 3/2*B – 125$ where $P' = \max(M_1, M_2, I_{16}) = \max((T_1–I_{17}–I_{18})/6, F_2/5, I_{16})$.
- Lowers the bid of more than one increment, there are two cases:
    - If $B_{A2} < I_{16}$, it will result in Sellers #1 and #2 selling 5 units apiece and Buyer A buying 1 unit. This will be profitable for Buyer A if: $3 * (250 – B) < 1 * (250 – B_{A1}) \rightarrow B_{A1} < 3*B – 500$ which ranges from -50 to 40 depending on the value of B. So, this would be a good strategy if B_{A1} would be accepted when very low, which would happen if $I_{15} <= B_{A1} < 3*B – 500$ and $F_1 + I_{11} + I_{12} + I_{13} + I_{14} + I_{15} <= 4*B + B_{A1} < 7*B – 500$.
    - If $B_{A2} >= I_{16}$, it will result in Seller #1 selling 6 units and Seller #2 selling 5 units and Buyer A buying 2 units. This will be profitable for Buyer A if: $3 * (250 – B) < (2*250 – B_{A1} – B_{A2}) \rightarrow B_{A1} + B_{A2} < 3*B – 250$. This allocation would occur if $B_{A1} >= I_{15}$ and $B_{A2} >= I_{16}$ and $F_1 + I_{11} + I_{12} + I_{13} + I_{14} + I_{15} + I_{16} <= 4*B + B_{A1} + B_{A2}$, and it would be profitable if $I_{15} + I_{16} < 3*B – 250$ and $F_1 + I_{11} + I_{12} + I_{13} + I_{14} + I_{15} + I_{16} < 7*B – 250$.

So, deviation by the buyers will occur if any of the following conditions are violated:

- $\max((T_1–I_{17}–I_{18})/6, F_2/5, I_{16}) >= 3/2*B – 125$, OR
- $I_{15} >= 3*B – 500$ and $F_1 + I_{11} + I_{12} + I_{13} + I_{14} + I_{15} >= 7*B – 500$, OR
- $I_{15} + I_{16} >= 3*B – 250$ and $F_1 + I_{11} + I_{12} + I_{13} + I_{14} + I_{15} + I_{16} >= 7*B – 250$.

The threat of deviations by one of the buyers limit how far down Sellers #1 and #2 can reduce their offers. So we conclude that Nash Equilibria exist where $150 < B <= 180$ and:

- $T_1 - I_{17} - I_{18} <= 6*B$, $I_{17} = B$, and $I_{18} > B$
- $T_2 <= 4*B$
- $\max((T_1 – I_{17} – I_{18})/6, F_2/5, I_{16}) >= 3/2*B - 125$,
- $I_{15} >= 3*B - 500$ and $F_1 + I_{11} + I_{12} + I_{13} + I_{14} + I_{15} >= 7*B - 500$,
- $I_{15} + I_{16} >= 3*B - 250$ and $F_1 + I_{11} + I_{12} + I_{13} + I_{14} + I_{15} + I_{16} >= 7*B - 250$,
- $T_3 = 540 + e$,
- $T_4 = 420 + e$, and
- $P = B$.

180 < B <= 250

For cases where 180 < B <= 250, we must consider that Seller #3 could use an incremental strategy and re-evaluate the Nash Equilibrium conditions from above:

- $T_1 - I_{17} - I_{18} <= 6*B$, $I_{17} = B$, and $I_{18} > B$
- $T_2 <= 4*B$
- $\max((T_1 – I_{17} – I_{18})/6, F_2/5, I_{16}) >= 3/2*B - 125$,
- $I_{15} >= 3*B - 500$ and $F_1 + I_{11} + I_{12} + I_{13} + I_{14} + I_{15} >= 7*B - 500$,
- $I_{15} + I_{16} >= 3*B - 250$ and $F_1 + I_{11} + I_{12} + I_{13} + I_{14} + I_{15} + I_{16} >= 7*B - 250$,
- $T_3 = 540 + e$,
- $T_4 = 420 + e$, and
- $P = B$.

Since $I_{15} + I_{16} > 0$ and $I_{17} = B$, Seller #3 could profitably enter by submitting $T_3 = I_{33} = C_3/3 + e < B$, displacing three of Seller #1's units. To protect against this possibility, Seller #1 must offer $I_{17} <= C_3/3 = 180$, but this violates the Nash Equilibrium conditions since Seller #1 can increase the quasi-clearing price by raising I_{17} above this level.

Case III: Seller #2 uses incremental strategy and #1 uses fixed component strategy

Case II examines whether there are Nash Equilibria that are 100 percent efficient where Seller #1 uses a fixed component strategy (i.e. $T_1 = F_1$), but Seller #2 does not. In Case III, Seller #1 sells 8 units and Seller #2 sells 4 units, because Seller #2 always submits an increment that is priced higher than the highest increment offered by Seller #1. Each seller receives the higher of: (i) the number of units sold times the quasi-clearing price and (ii) the sum of the seller's accepted offer components. Under the conditions above, the quasi-clearing price and the expected profit functions of Sellers #1 and #2 are now as follows:

$$\prod_1 = \max(8*P, F_1) – C_1$$
$$\prod_2 = \max(4*P, T_2–I_{25}) – C_2$$

In Case III, $P = \max(M_1, M_2, I_{25})$. But now $M_2 = \min((F_2+I_{21}), (F_2+\sum(I_{21} \text{ to } I_{22}))/2, (F_2+\sum(I_{21} \text{ to } I_{23}))/3, (F_2+\sum(I_{21} \text{ to } I_{24}))/4, (F_2+\sum(I_{21} \text{ to } I_{25}))/5)$. If $I_{24} <= M_2$, then $M_2 = \min((F_2+\sum(I_{21} \text{ to } I_{24}))/4, (F_2+\sum(I_{21} \text{ to } I_{25}))/5) = \min((T_2-I_{25})/4, T_2/5)$. $M_1 = F_1/8$. So we can write:

$$P = \max(\min((T_2-I_{25})/4, T_2/5), F_1/8, I_{24})$$

In order for Seller #1 to sell 8 units, Seller #2 to sell 4 units, and Sellers #3 and #4 to each sell 0 units, the following set of conditions must hold.

Base Conditions

Seller #1 must provide more surplus by selling 8 units than any other combination that includes Seller #2's fifth unit, Seller #3, Seller #4, or combination of them. This results in the following constraints:

- $\sum(B_5 \text{ to } B_{12}) – F_1 >= \sum(B_5 \text{ to } B_7) – T_3 \rightarrow \quad F_1 <= \sum(B_8 \text{ to } B_{12}) + T_3$

- $\sum(B_5 \text{ to } B_{12}) - F_1 >= \sum(B_5 \text{ to } B_8) - T_3 - I_{25} \rightarrow$ $F_1 <= \sum(B_9 \text{ to } B_{12}) + T_3 + I_{25}$
- $\sum(B_5 \text{ to } B_{12}) - F_1 >= \sum(B_5 \text{ to } B_6) - T_4 \rightarrow$ $F_1 <= \sum(B_7 \text{ to } B_{12}) + T_4$
- $\sum(B_5 \text{ to } B_{12}) - F_1 >= \sum(B_5 \text{ to } B_7) - T_4 - I_{25} \rightarrow$ $F_1 <= \sum(B_8 \text{ to } B_{12}) + T_4 + I_{25}$
- $\sum(B_5 \text{ to } B_{12}) - F_1 >= \sum(B_5 \text{ to } B_9) - T_3 - T_4 \rightarrow$ $F_1 <= \sum(B_{10} \text{ to } B_{12}) + T_3 + T_4$
- $\sum(B_5 \text{ to } B_{12}) - F_1 >= \sum(B_5 \text{ to } B_{10}) - T_3 - T_4 - I_{25} \rightarrow$ $F_1 <= \sum(B_{11} \text{ to } B_{12}) + T_3 + T_4 + I_{25}$
- $\sum(B_5 \text{ to } B_{12}) - F_1 >= B_5 - I_{18} \rightarrow$ $F_1 <= \sum(B_6 \text{ to } B_{12}) + I_{18}$
- $\sum(B_5 \text{ to } B_{12}) - F_1 >= 0 \rightarrow$ $F_1 <= \sum(B_5 \text{ to } B_{12})$

Seller #2 must provide more surplus by selling 4 units than any other combination that includes Seller #3, Seller #4, both, or neither. This results in the following constraints:

- $\sum(B_{10} \text{ to } B_{12}) - \sum(I_{22} \text{ to } I_{24}) > \sum(B_{10} \text{ to } B_{12}) - T_3 \rightarrow$ $\sum(I_{22} \text{ to } I_{24}) < T_3$
- $\sum(B_{11} \text{ to } B_{12}) - \sum(I_{23} \text{ to } I_{24}) > \sum(B_{11} \text{ to } B_{12}) - T_4 \rightarrow$ $\sum(I_{23} \text{ to } I_{24}) < T_4$
- $\sum(B_9 \text{ to } B_{12}) - T_2 > \sum(B_9 \text{ to } B_{12}) - T_3 - T_4 \rightarrow$ $T2 < T_3 + T_4$
- $B_{8+i} - I_{2i} >= 0$ for i from 1 to 4

Seller #3 will submit an offer with a fixed component exceeding his true cost OR submit an offer where one increment exceeds one-third of his true cost if all three increments would be accepted OR submit an offer as low as zero if all three increments would be accepted and the quasi-clearing price would exceed one-third of his true cost. Hence, Seller #3's offer must satisfy one of the following three conditions:

- $F_3 > C_3$
- $I_{33} > C_3 / 3$ AND I_{33} is accepted
- $T_3 >= 0$ AND I_{33} is accepted AND $P > C_3 / 3$

Seller #4 will submit an offer with a fixed component exceeding his true cost OR submit an offer where one increment exceeds one-half of his true cost if both increments would be accepted OR submit an offer summing to zero if both increments would be accepted and the quasi-clearing price would exceed one-half of his true cost. Hence, Seller #4's offer must satisfy one of the following three conditions:

- $F_4 > C_4$
- $I_{42} > C_4 / 2$ AND I_{42} is accepted
- $T_4 >= 0$ AND I_{42} is accepted AND $P > C_4 / 2$

These are not the only relevant constraints, but they are used throughout this section. Given the symmetry of buyers in this environment, we look for Nash Equilibria where $B_i = B$ for all i.

B <= 180

For cases where B <= 180, we suppose Seller #3 offers $T_3 = F_3 = C_3 + e$ and Seller #4 offers $T_4 = F_4 = C_4 + e$. Suppose Seller #2 submits the maximum fully incremental offer under the Base Conditions: $F_2 = 0$ and $I_{2i} = B$ for all i from 1 to 4 and an arbitrarily high I_{25}. In this case, Seller #1 submits the maximum offer under the Base Conditions: $T_1 = F_1 = \min(8*B, 7*B+I_{25}) = 8*B$. The quasi-clearing price is:

$$
\begin{aligned}
P &= \max(\min((T_2-I_{25})/4, T_2/5), F_1/8, I_{24}) \\
&= \max((4*B)/4, (8*B)/8, B) \\
&= B
\end{aligned}
$$

In this case, P would be the same based on either the Seller #1's offer or Seller #2's offer. The expected profits for Seller #1 are:

$$
\begin{aligned}
\prod\nolimits_1 &= \max(8*P, F_1) - C_1 \\
&= 8*B - 960, \text{ which is positive where } B > 120
\end{aligned}
$$

The expected profits for Seller #2 are:

$$
\prod\nolimits_2 = \max(4*P, T_1-I_{25}) - C_2
$$

$= \max(4*B, 4*B) - C_2$

$= 4*B - 750$, which is positive where $B > 187.5$

So if Seller #2 sells 4 units, he does not earn a profit until the buyers make sufficient surplus available where $4*B > 750$.

180 < B <= 250

For cases where $180 < B <= 250$, we must consider that Seller #3 could use an incremental strategy. To protect against this possibility, Seller #2 will need to submit $I_{24} < I_{33} = 180 + e$, but this will not result in a quasi-clearing price that allows Seller #2 to earn a profit. So Seller #2 must submit an offer such that $(F_2+I_{21}+I_{22}+I_{23}+I_{24})/4 = M_2 > 187.5$ in order to set the quasi-clearing price at a profitable level. But to be accepted, Seller #2 must offer $F_2+I_{21}+I_{22}+I_{23}+I_{24} <= I_{33} + B$, and these are mutually exclusive when $180 < B <=250$, so Seller #2 cannot sell just 4 units, earn a profit, and prevent entry by Seller #3. So, Seller #2 will try to increase the expected number of units sold by switching to a fixed component strategy, which devolves into Case I: $T_1 = F_1$ and $T_2 = F_2$.

2. D4 Treatment

The only allocation that results in 100 percent efficiency is when Seller #1 sells 8 units, Seller #2 sells 5 units, and Seller #3 sells 3 units. For this to occur, several conditions must hold.

Base Conditions

Seller #1 provides more surplus by selling 8 units in addition to the 8 units sold by Sellers #2 and #3 than any alternative allocation. This results in the following constraints: $- \sum(I_{22} \text{ to } I_{24})$

- $\sum(B_9 \text{ to } B_{16}) - T_1 >= 0 \rightarrow$ $T_1 <= \sum(B_9 \text{ to } B_{16})$
- $\sum(B_9 \text{ to } B_{16}) - T_1 >= \sum(B_9 \text{ to } B_{10}) - T4 \rightarrow$ $T_1 <= \sum(B_{11} \text{ to } B_{16}) + T_4$
- $B_{16} - I_{18} >= 0 \rightarrow$ $I_{18} <= B_{16}$

Seller #2 provides more surplus by selling 5 units in addition to the 11 units sold by Sellers #1 and #3 than any alternative allocation. This results in the following constraints:

- $\sum(B_{12} \text{ to } B_{16}) - T_2 >= 0 \rightarrow$ $T_2 <= \sum(B_{12} \text{ to } B_{16})$
- $\sum(B_{12} \text{ to } B_{16}) - T_2 >= \sum(B_{12} \text{ to } B_{13}) - T4 \rightarrow$ $T_2 <= \sum(B_{14} \text{ to } B_{16}) + T_4$
- $B_{16} - I_{25} >= 0 \rightarrow$ $I_{25} <= B_{16}$

Seller #3 provides more surplus by selling 3 units in addition to the 13 units sold by Sellers #1 and #2 than any alternative allocation. This results in the following constraints:

- $\sum(B_{14} \text{ to } B_{16}) - T_3 >= 0 \rightarrow$ $T_2 <= \sum(B_{14} \text{ to } B_{16})$
- $\sum(B_{14} \text{ to } B_{16}) - T_3 >= \sum(B_{14} \text{ to } B_{15}) - T4 \rightarrow$ $T_2 <= B_{16} + T_4$
- $B_{16} - I_{33} >= 0 \rightarrow$ $I_{25} <= B_{16}$

Seller #4 will submit an offer with a fixed component exceeding his true cost OR submit an offer where one increment exceeds one-half of his true cost if both increments would be accepted OR submit an offer summing to zero if both increments would be accepted and the quasi-clearing price would exceed one-half of his true cost. Hence, Seller #4's offer must satisfy one of the following three conditions:

- $F_4 > C_4$
- $I_{42} > C_4 / 2$ AND I_{42} is accepted
- $T_4 >= 0$ AND I_{42} is accepted AND $P > C_4 / 2$

B <= 210

In this case, the buyers do not provide sufficient surplus for Seller #4 to earn a profit, so we can suppose that $T_4 = F_4 = C_4 + e = 420 + e$. We also start by supposing that Sellers #1, #2, and #3 submit the highest offers that will be accepted: $T_1 = 8*B$, $T_2 = 5*B$, and $T_3 = 3*B$ where $I_{18} <= B$, $I_{25} <= B$, and $I_{33} <= B$. The quasi-clearing price and the expected profit functions of the sellers are:

- $P = \max(M_1, M_2, M_3, I_{18}, I_{25}, I_{33}) = \max(T_1/8, T_2/5, T_3/3, I_{18}, I_{25}, I_{33}) = B$
- $\prod_1 = 8*P - C_1$
- $\prod_2 = 5*P - C_2$
- $\prod_3 = 3*P - C_3$

In cases where $B < 180$, the surplus allowed by the buyers are not sufficient for Seller #3 to sell and earn a non-negative profit. Therefore, there are no 100 percent Nash Equilibria where $B < 180$.

In cases where $180 <= B <= 210$, this is a Nash Equilibrium if no one has an incentive to deviate in a way that reduces the profits of others.

Sellers #1, #2, and #3 do not raise their offers because doing so would violate the Base Conditions. Each may lower his offer all the way to zero as long one of the other sellers sets the quasi-clearing price $P = B$. The sellers do not raise their offers, because doing so would reduce their sales without increasing transaction prices above B.

The buyers have incentives to deviate under certain circumstances. Each buyer earns $\prod_B = 4 * (250 - P)$. A buyer might try to reduce his bids to drive down his transaction price for a smaller number of units. If the sellers use fixed offer strategies such that $T_1 = F_1$, $T_2 = F_2$, and $T_3 = F_3$, the buyers would need to earn $\prod_B' = 3*250 - Pymt > 4*(250 - P) \rightarrow Pymt < 4*B - 250$. Suppose $T_3 = 3*B$ and $T_1 = T_2 = 0$, Buyer A could lower one bid increment $B_{A4} < B$ and raise the other $B_{A1} >= B_{A2} >= B_{A3} > B$. Doing so would cause Seller #3 to be rejected, leading to $M_1 = 0$, $M_2 = 0$, $P = 0$, and $Pymt = 0$. So, if $T_3 = 3*B$, Seller #1 or #2 would have to bid high enough to prevent the buyers from deviating. The threshold that one of them must exceed is $Pymt/3 = 4/3*B - 250/3$. This same principle of the profitability of deviation by the buyers is true if Sellers #1 or #2 are setting the quasi-clearing price. So this results in a set of Nash Equilibria where $180 < B <= 210$ and the sellers use fixed component strategies:

- $T_1 = F_1 = 8*B$ and $0 <= T_2 = F_2 <= 5*B$ and $4/3*B - 250/3 <= T_3 = F_3 <= 3*B$
- $T_1 = F_1 = 8*B$ and $0 <= T_3 = F_3 <= 3*B$ and $4/3*B - 250/3 <= T_2 = F_2 <= 5*B$
- $T_2 = F_2 = 5*B$ and $0 <= T_1 = F_1 <= 8*B$ and $4/3*B - 250/3 <= T_3 = F_3 <= 3*B$
- $T_2 = F_2 = 5*B$ and $0 <= T_3 = F_3 <= 3*B$ and $4/3*B - 250/3 <= T_1 = F_1 <= 8*B$
- $T_3 = F_3 = 3*B$ and $0 <= T_1 = F_1 <= 8*B$ and $4/3*B - 250/3 <= T_2 = F_2 <= 5*B$
- $T_3 = F_3 = 3*B$ and $0 <= T_2 = F_2 <= 5*B$ and $4/3*B - 250/3 <= T_1 = F_1 <= 8*B$

There are also many Nash Equilibria if Sellers #1, #2, and #3 do not use pure fixed offer strategies. For example, in the first set of conditions, if Seller #1 changes from offering $T_1 = F_1 = 8*B$ to offering $F_1 = 0$ and $I_{1i} = B$ for i from 1 to 8. In this case, any reduction by the buyers would reduce quantity without changing transaction prices. A wide range of such reallocations are also Nash Equilibria, but I make no attempt to identify a comprehensive list.

210 < B <= 250

In this case, there is sufficient surplus for potential entry by Seller #4. Start by supposing that Sellers #1, #2, and #3 use fixed offer strategies. If Seller #4 expects one of the other sellers to set $P > 210$, he would be willing to offer $T_4 = 0$. Also, if Seller #4 expects to have both units accepted, he might also offer $T_4 = I_{42} = 210 + e$. If Seller #4 offers $I_{42} = 210 + e$, the other Sellers will be subject to the following Base Conditions:

- $T_1 = F_1 <= 6B + I_{42}$ → $M_1 = ¾*B + 26.25$ → $M_1(B=210) = 183.75$
- $T_2 = F_2 <= 3B + I_{42}$ → $M_2 = 3/5*B + 42$ → $M_2(B=210) = 168$
- $T_3 = F_3 <= B + I_{42}$ → $M_3 = 1/3*B + 70$ → $M_3(B=210) = 140$

So, we see that in order to prevent Seller #4 from entering using the incremental offer strategy $I_{42} = 210 + e$, the other Sellers must reduce their offers significantly. If they offer subject to these constraints, Seller #4 will not have an incentive to use the zero offer strategy $T_4 = 0$. Note, M_1, M_2, and M_3 are increasing in B, and when B = 245:

- $M_1(B=245) = 210$
- $M_2(B=245) = 189$
- $M_3(B=245) = 151.66$

So now we must revise the set of potential Nash Equilibria to cases where $210 < B < 245$. They include the following conditions:

- $T_1 = F_1 = 6B + I_{42}$
- $T_2 = F_2 = 3B + I_{42}$
- $T_3 = F_3 = B + I_{42}$
- $T_4 = F_4 = 420 + e$
- $P = T_1/8$

Seller #1 has an incentive to deviate. He has the profit function $\prod_1 = \max(8*P, T_1) - C_1 = 8*\max(T_1/8, T_2/5, T_3/3) - C_1 = T_1 - C_1$. Increasing T_1 leads to a higher quasi-clearing price, allowing Seller #4 to switch to an incremental and displace Seller #1. So, this is not a Nash Equilibrium. The reason is that, on the margin, the sellers will always have an incentive to push prices above 210 using incremental offers, which will cause Seller #4 to enter.

If Sellers #1, #2, and #3 do not all use fixed component strategies, Nash Equilibria are possible. For example, if $I_{1i} = I_{2i} = I_{3i} = 210$, Seller #4 will not use an incremental offer strategy, because doing so would result in Seller #4 selling 1 unit at a negative profit. Seller #4 could offer $T_4 = F_4 = 420 + e$, but this would give Sellers #1, #2, and #3 the incentive to raise their incremental offer prices to raise the quasi-clearing price above 210. But this would allow Seller #4 to profit from switching to an incremental offer strategy. However, if Seller #4 offers $F_4 = 0$ and $I_{41} = I_{42} = 210 + e$, Sellers #1, #2, and #3 will have no incentive to deviate from $I_{1i} = I_{2i} = I_{3i} = 210$, because any attempt to do so would result in an unprofitable loss of sales. Moreover, any attempt by the buyers to reduce their bids would result in foregone purchase and no reduction in transaction prices. Thus, the following is a Nash Equilibrium:

- $F_1 = 0$ and $I_{1i} = 210$ for i from 1 to 8,
- $F_2 = 0$ and $I_{2i} = 210$ for i from 1 to 5,
- $F_3 = 0$ and $I_{3i} = 210$ for i from 1 to 3,
- $F_4 = 0$ and $I_{4i} = 210 + e$ for i from 1 to 2,
- $P = 210$

Additionally, there are many adjustments that would also be Nash Equilibria. For example, if I_{11} is reduced to 0, it will have no impact on payoffs and no one will have an incentive to deviate. As long as at least 4 units are offered at 210 by Seller #1, #2, and #3, the buyers will be unable to profit by dropping their bids, and Seller #4 will not profit from changing his offer.

C. Conclusions

Two approaches are used in this section to predict how buyers and sellers might act in the D3 and D4 treatments: (i) a qualitative approach, which includes two case studies of the D4 treatment with example outcomes, and (ii) a Nash Equilibrium approach is used to analyze the D3 and D4 treatments.

Qualitative analyses in Section V.A provide some indication that buyers and sellers will tend to use a Conservative Strategy, a middle-of-the-road strategy where buyers bid and sellers offer somewhere in between the Revealing Strategy and the Aggressive Strategy. The analysis was only performed for the D4 treatment.

The Nash Equilibrium analyses have very different results in the D3 and D4 treatments. In the D3 environment, the analysis in this section finds that 100 percent Nash Equilibria arise when Aggressive Strategies are used by the four buyers and Seller #1 and the Quantity-Maximizing Strategy (i.e., offering below cost) is used by Seller #2. In the D4 environment, the analysis in this section finds that 100 percent Nash Equilibria are consistent with the Competitive Equilibrium price range. Nash Equilibria arise under relatively unrestrictive conditions, including where the buyers and sellers are using any of the four strategies listed at the beginning of this section.

VI. Experiment Protocols

When designing the experiments with the QUPA institution, the primary consideration was to make the results comparable to the results of experiments using the Double Auction and Smart Market institutions. The QUPA experiments use environments identical to those in previous experiments on the Double Auction and Smart Market to enable direct comparisons of the results. Hence, the experiments on the QUPA institutions use the following treatments:

- D3 – Each buyer desires three units in each round.
- D4 – Each buyer desires four units in each round.
- R3 – Each robot buyer desires three units in each round.
- R4 – Each robot buyer desires four units in each round.

The results from these four treatments were compared to the Double Auction and Smart Market results. The D3 and D4 treatments were compared to the AC3 and AC4 treatments in Van Boening and Wilcox (1996). The R3 and R4 treatments were compared to the AC3 and AC4 treatments[32] in the Smart Market experiments.

Van Boening and Wilcox (1996) used the MC4 treatment as a baseline for comparisons with the AC3 and AC4 treatments, because the Double Auction has consistently performed efficiently in such environments and they were looking at whether the avoidable cost structure would undermine its efficiency. I also used the MC4

[32] The authors actually refer to the treatments as “Design (b)” and “Design (a).”

treatment as a baseline for comparison with the D3 and D4 treatments, because I am looking at whether the institutional rules of the QUPA that facilitate non-convex offers compensate for the coordination problems that arise from avoidable cost structures.

Four experimental sessions were conducted for each treatment, for a total of 16 experimental sessions. Each session consisted of a set of 10 practice rounds,[33] followed by four groups of 12 rounds per group. Between each group of 12 rounds, the sellers rotated values, so that by the end of the experiment, each subject that played sellers had experienced 12 rounds in each role. The subject in seat A played the sellers in the following order: Seller #3 → #4 → #2 → #1. The subject in seat B played the sellers in the following order: Seller #4 → #3 → #1 → #2. The subject in seat C played the sellers in the following order: Seller #1 → #2 → #4 → #3. The subject in seat D played the sellers in the following order: Seller #2 → #1 → #3 → #4.

This structure was chosen for several reasons. First, the sessions of Van Boening and Wilcox (1996) were mostly in groups of 12 rounds. I chose the same number, anticipating that experience would have a significant effect within each group of 12 rounds. Second, I did not want Seller #4 to get bored, and as a consequence, do something irrational. I reduced this potential by limiting each subject to just 12 rounds as Seller #4. Third, the rotation of values allowed me to compensate subjects that played sellers solely with their earnings from the auction, rather than having different endowments or different exchange rates. Fourth, given the importance that a single

[33] One D4 session had on six practice rounds.

player can have on the efficiency of results, I expected this mixed structure to limit the impact of such fixed effects.

To the extent that clearing prices converge during each group of 12 rounds, players may assume that clearing prices will converge to the same level in the subsequent group of 12 rounds. To mitigate this tendency, the induced values and exchange rates of each player were multiplied by a different factor in each group of 12 rounds. The factors were as follows: 50 percent in the first group, 20 percent in the second group, 80 percent in the third group, and 60 percent in the fourth group.

Several decisions were made to ensure that rewards would be salient in the experiment. First, subjects' were compensated strictly from their earnings, rather than an initial endowment. Second, although the Smart Market experiments and some of the Double Auction sessions treated the seller's avoidable fixed cost as a lost opportunity, I chose to treat it as a normal cost. The reason is that the avoidable fixed cost per unit was large relative to the average transaction price, which would allow subjects to earn relatively high earnings by just sitting and doing nothing. Furthermore, since the QUPA provides subjects with a full-proof way of not losing money, by submitting fixed offer components that are greater than or equal to their avoidable fixed cost, I did not expect that negative earnings would be a significant problem.[34]

In the instructions,[35] subjects were told the number of buyers or sellers in the experiment. However, in the D3 and D4 treatments, the sellers were not told the number

[34] As it turns out, 3 of the 96 subjects ended the experiment with negative earnings, so they were paid only their show-up fee.
[35] A copy of the instructions is shown in the Appendix.

of buyers and the buyers were not told the number of sellers. In the robot sessions, the sellers were told that the buyers were robots. Subjects were told relatively little about the allocation and pricing mechanisms of the auction due to time and cost constraints. They were told that lower offers and higher bids are more likely to be accepted. Sellers were told that they would be paid at least as much as the accepted offer components. Buyers were told that they would pay no more than the accepted bid components.

Subjects were George Mason University undergraduate students with no prior experience in complex-offer sealed offer auctions.[36] Subjects received a $7 show-up fee plus average earnings of $31.82. The minimum earnings were $0 for 3 subjects. The maximum earnings were $87.

[36] One subject was mistakenly allowed to participate in the experiment after participating in a previous session. The results of the session were not particularly notable.

VII. Empirical Results

This section analyzes the data collected in my experiments in order to address three issues. First, the principal issue is whether the QUPA has the potential to produce efficient results in markets with non-convex cost structures. If it does not, the only reason to do further research on this type of auction would be to demonstrate to policy makers that it should not be used in electricity markets. But if the QUPA does produce efficient results, then additional experimental research could contribute to future refinements of electricity market designs and illuminate other product markets where the QUPA could work efficiently. Section VII.A evaluates efficiency in the QUPA and finds promising results that should motivate future research.

Second, this section investigates under what conditions the QUPA is likely to produce prices that are competitive. The degree to which an institution produces competitive prices is particularly important in electricity markets for several reasons: (i) electricity demand generally does not respond to changes in the wholesale price, (ii) many local geographic markets exist with a small number of sellers, and (iii) regulators in many countries are expected to take a proactive approach to addressing potential market power. Experimental research on the QUPA could inform the development of competition policy as it is applied in electricity markets. Section VII.B finds that treatment variations have significant effects on transaction prices in the QUPA.

The third part of this section examines the bids and offers submitted by buyers and sellers in the QUPA to determine how they respond to their incentives. Section V identified several different strategies that might be used by buyers and sellers in the QUPA. However, the QUPA institution is relatively complex, making it difficult to predict how subjects will respond to their incentives. Since the QUPA shows potential for facilitating efficient market outcomes, examination of bid and offer patterns may help experimental researchers design future experiments. For this reason, Section VII.C provides a summary of bids and offers.

The following is an overview of the issues addressed in this section:

A. Efficiency
   1. Experience effects
   2. Treatment effects
   3. Institutional comparison
B. Transaction Prices
   1. Treatment effects
   2. Discussion of Prices and Efficiency
   3. Discriminatory Pricing
C. Examination of Bids and Offers
   1. Bidding Patterns
   2. Offer Patterns

A. Efficiency

The primary problem identified by the experiments of Van Boening and Wilcox (1996) and Durham et al was that the Double Auction and the Smart Market produced relatively inefficient results in an environment with non-convex cost structures. The

QUPA was designed in an attempt to find a competitive institution that produces efficient outcomes in such an environment. This section evaluates the efficiency of the QUPA and analyzes how efficiency is affected by the experience-level of subjects and treatment variations. The treatments seek to determine whether efficiency is affected by the existence of a Competitive Equilibrium and the price-sensitivity of buyers. This section also compares efficiency in the QUPA to efficiency in the Double Auction and the Smart Market.

1. Experience Effects

The QUPA is unfamiliar to most subjects, so their behavior may change as they get a better feel for the rules and incentives. Most institutions that are useful in the real world must perform well after repeated exchange. Hence, it is more important that an institution performs well when subjects are experienced than when they are inexperienced. This section examines the effects of experience on the efficiency of production and exchange in the QUPA. The conclusions from this part of the section are used to classify auction rounds as experienced in the analysis of treatment effects in Part 2 of this section and in the comparison of the QUPA to other institutions in Part 3 of this section.

The following figures analyze experience-effects separately for each treatment, because it is possible that the effects of experience may differ by treatment. For instance, the use of fully revealing robot buyers might speed up the learning process by making the market simpler for subjects to understand. In the treatments with robot buyers, R3 and R4, each subject competes against three other subjects, while in the treatments with human buyers, D3 and D4, each subject competes against seven other subjects. As a result, subjects in D3 and D4 face many more sources of variation in outcomes which may slow the pace at which the market moves toward equilibrium. Likewise, the existence of a Competitive Equilibrium, which is the case for R4 and D4 but not for R3 and D3, may help the market converge toward a consistent outcome and assist subjects in identifying their preferred strategies.

Figure 8 shows the efficiency of each round in the experiments on the QUPA. The figure shows four line graphs: one for each treatment. Each line graph shows four lines: one for each experimental session. Rounds are shown in four groups of 12 on the x-axis, corresponding to the four groups of rounds during the each session. Efficiency is

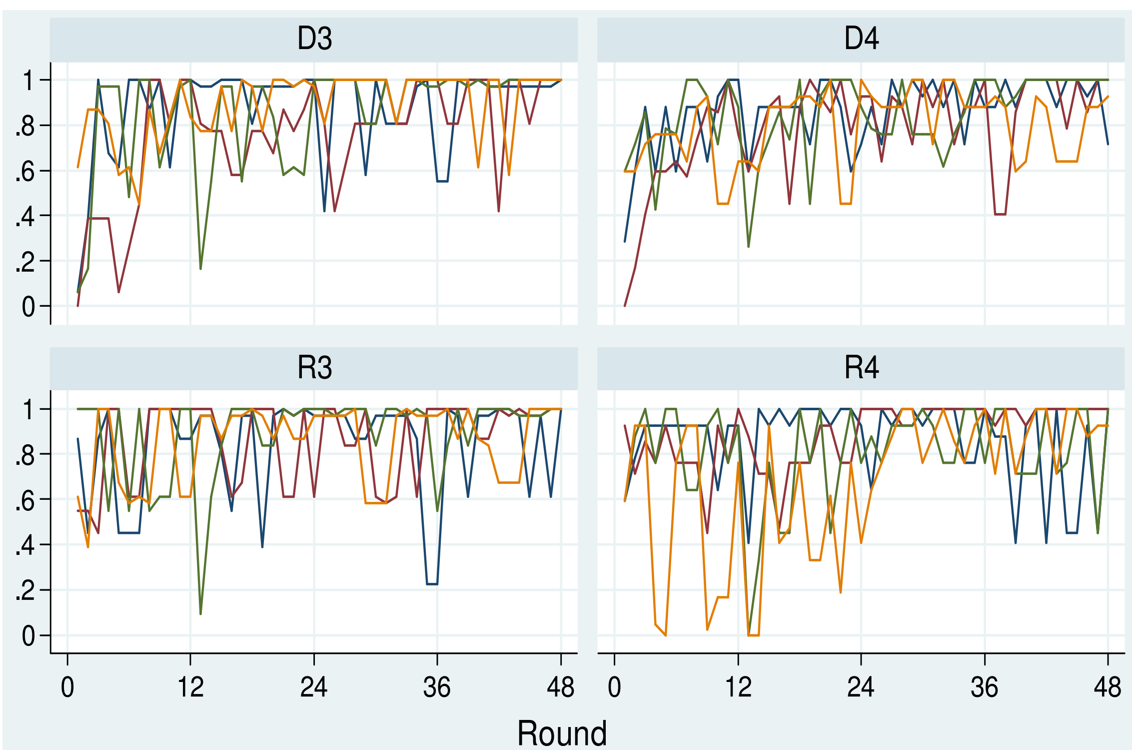

Figure 8: Trend in Efficiency of the QUPA as Experience is Gained

shown on the y-axis.

Figure 8 exhibits an upward trend in efficiency as the subjects gained experience. Efficiency increased considerably in both of the first two groups of 12 rounds, although it is less clear that efficiency improved in the third and fourth groups of rounds.

The following figure shows how the effects of experience vary according to the treatment. It reports the mean efficiency for each treatment for each group of 12 rounds.

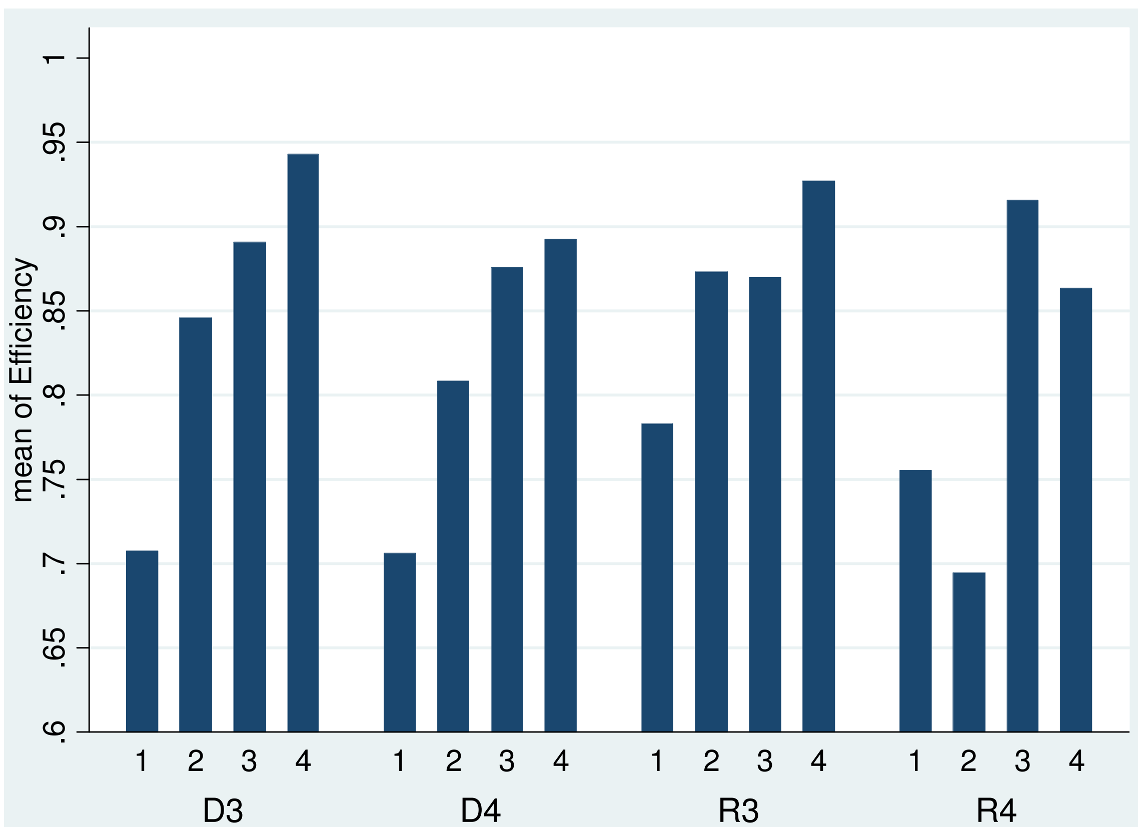

Figure 9: Mean Efficiency as Experience is Gained, by Treatment by Group

The figure shows that the upward trend in efficiency is broadly applicable to all four treatments, although the pattern appears less consistent when we examine individual treatments. D3 improves consistently from Group 1 to Group 4. D4 improves from Group 1 to Group 3 but does not improve significantly in Group4. R3 improves considerably from Group 1 to Group 2 and from Group 3 to Group 4 but not between Group 2 and Group 3. R4 exhibits the least intuitive trend, falling after Group 1 and Group 3 but increasing dramatically from Group 2 to Group 3. The apparent lack of consistency across the four treatments may be due to the reduced sample sizes from splitting the 16 sessions of data shown in the previous figure into four groups.

The data shown in Figure 9 is based on a total of 768 auction rounds (= 4 treatments x 4 sessions per treatment x 4 groups per session x 12 rounds per group). Because of the likelihood that the rounds within each group of 12 rounds are not independent, the mean efficiency from each group of 12 rounds is treated as a single observation for statistical testing purposes. As a result, the data used for statistical testing included 64 observations (= 4 treatments x 4 sessions per treatment x 4 observations per session).

The Mann-Whitney rank sum test is used to determine whether the effects of experience are statistically significant. In particular, I test the null hypotheses for each pairwise comparison that the two samples are drawn from the same population. Each possible pairwise comparison is shown in a different row of the table. The columns report the p-values for each treatment. P-values below 5 percent are shaded in the table below.

Table 4: Experience Effects on Mean Efficiency, by Treatment

Experience Level		Treatment and p-value			
Sample 1	**Sample 2**	**R3**	**R4**	**D3**	**D4**
Group 2	Group 1	.021	.564	.149	.083
Group 3	Group 2	.885	.149	.564	.083
	Group 1	.043	.021	.021	.021
Group 4	Group 3	.083	.468	.564	.773
	Group 2	.083	.248	.248	.248
	Group 1	.021	.387	.021	.043

For all four treatments, the mean efficiency increased after Group 1 and the rank sum test rejected the null hypothesis at the 5 percent level of significance that Group 1 was taken from the same population as one or more of the subsequent groups. This is notable given that each sample reported in Table 4 contains just 4 observations. For each treatment, the mean efficiency of Group 4 is greater than the mean efficiencies of the earlier groups, except for the R4 treatment, where the mean efficiency falls from 91.6 percent in Group 3 to 86.4 percent in Group 4. Examination of the individual session data shows that there was one R4 session where the mean efficiency was 91.8 percent for Group 3 and 73.1 percent for Group 4. Excluding this session, the mean efficiency was 91.5 percent for Group 3 and 90.8 percent for Group 4. So it seems that the decline in efficiency from Group 3 to Group 4 for the R4 treatment was primarily due to a single session rather than a consistent pattern.

The results provide support for the conclusion that efficiency tends to improve as subjects gain experience. This is not surprising since the subjects had no prior experience in the QUPA or any other complex sealed-offer auction. However, based on these results and without running additional experiments with the same subjects, it is impossible to know whether additional experience would lead efficiency to increase further, decrease, or remain constant. It is possible that if subjects were given additional experience, they might learn profitable strategies that undermine the efficiency of the institution. Given the unfamiliarity of the QUPA institution to most subjects, the results shown above also suggest that changes in instructions would have a significant effect on the amount of experience required for efficiency to reach a plateau. More detailed instructions could speed up the learning process or make subjects aware of strategies that would otherwise take a long time to discover.

While the available data does not tell us how the QUPA would perform if subjects were *fully* experienced, it indicates that experience is a significant factor, suggesting that the remainder of this section should focus on the experienced rounds. The results above suggest that in D3, D4, and R3, efficiency reached its peak in the final group of rounds, so I treat Group 4 as experienced for these three treatments. In R4, efficiency reaches its peak in the third group of rounds and then declines due to one session that appears to be an outlier. For this reason, I treat Groups 3 and 4 as experienced for the R4 treatment.

2. Treatment Effects

These experiments used four treatments in order to evaluate two aspects of behavior in the market. First, like Wilcox and Van Boening, I am seeking to determine whether the existence of a Competitive Equilibrium affects efficiency. I compare D3 to D4 (and R3 to R4) to determine whether the ability of the QUPA to produce discriminatory prices results in efficient outcomes regardless of whether there exists a Competitive Equilibrium. Second, I want to determine how the price-sensitivity of buyers affects efficiency. Durham et al conjectured that the lack of competitive pressure from buyers in the Smart Market allowed the sellers to push prices far higher than the competitive range. They further speculate that the lack of competitive pressure may have allowed the inefficient seller, S4, to sell more frequently, thereby reducing efficiency.

The following histograms summarize efficiency by round in each treatment for experienced subjects. The mean efficiency is also shown for each treatment.

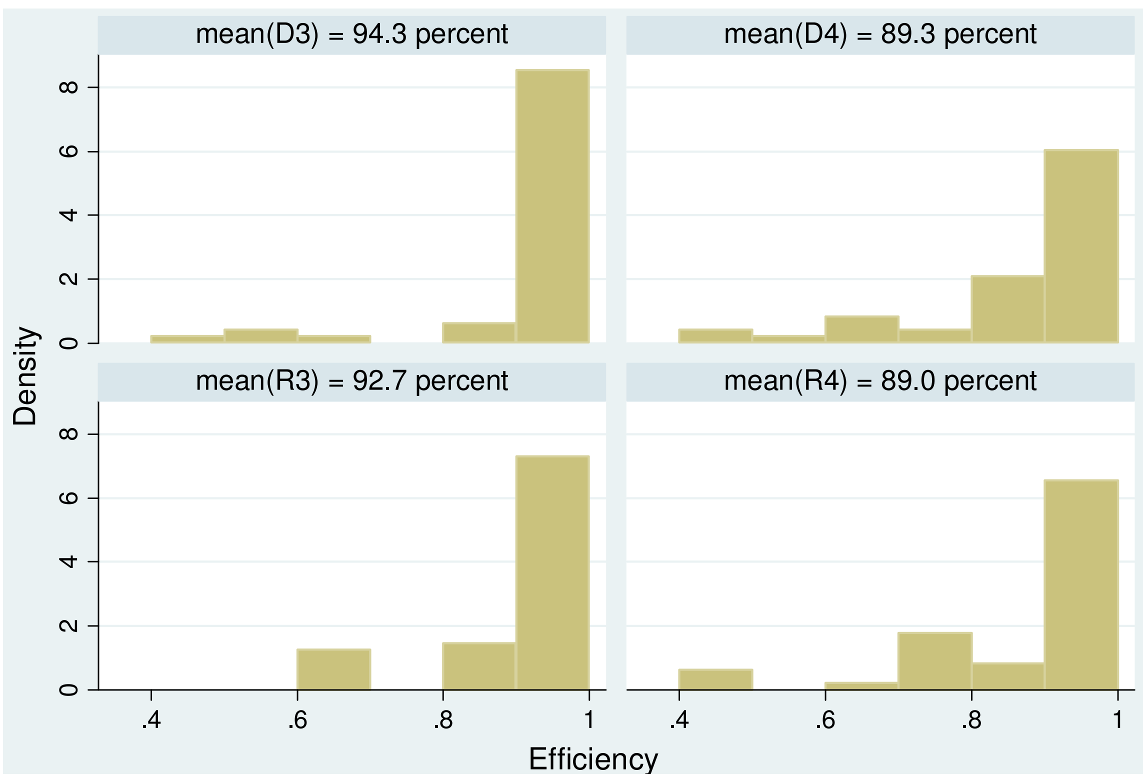

Figure 10: Distribution of Efficiency in Experienced Rounds, by Treatment

The figure suggests that D3 exhibited higher efficiency than its counterpart with a Competitive Equilibrium, D4, and to lesser extent its counterpart with robot buyers, R3. There is also some indication that R3 performed better than R4. The mean efficiency of D4 and R4 were very similar. These results are inconsistent with the hypothesis that the existence of a Competitive Equilibrium improves efficiency. In fact, if anything, these results suggest the opposite. These results provide some indication that the use of robot buyers reduces efficiency, although any effect from robot buyers is smaller in magnitude than the effect from a Competitive Equilibrium.

The Mann-Whitney rank sum method is used to test whether the effects of experience are statistically significant. Three pairwise comparisons are possible between the efficiency from one sample where no Competitive Equilibrium exists and a second sample where a Competitive Equilibrium does exist. Two of these are one-to-one comparisons between treatments: D3 versus D4 and R3 versus R4. Then D3 and R3 are pooled together and compared to D4 and R4 pooled together. Three pairwise comparisons are possible between the efficiency from one sample that uses human buyers and a second sample that uses robot buyers. Two of these are one-to-one comparisons between treatments and one of these pools the data from D3 with D4 and from R3 with R4. For each pairwise comparison, I test the null hypotheses that the two samples are drawn from the same population. The results are summarized in the following table:

Table 5: Treatment Effects on Efficiency in Experienced Rounds

Category	Comparison	Mean (Sample 1)	Mean (Sample 2)	Difference	Rank sum p-value
Compare: non-CE versus CE	D3 v D4	94.3%	89.3%	5.1%	.387
	R3 v R4	92.7%	89.0%	3.8%	.610
	(D3 & R3) v (D4 & R4)	93.5%	89.1%	4.5%	.190
Compare: Robot Buyers versus Human Buyers	D3 v R3	94.3%	92.7%	1.6%	.564
	D4 v R4	89.3%	89.0%	0.3%	.865
	(D3 & D4) v (R3 & R4)	91.8%	90.2%	1.6%	.537

The mean efficiencies reported in the table above suggest that there may be a negative treatment effect on efficiency from the existence of a Competitive Equilibrium and a positive treatment effect from the use of human buyers. However, the Wilcoxon rank sum tests reported in the table above do not provide evidence of treatment effects on efficiency based on any conventional level of significance, although the power of the tests is limited by the small number of observations for each treatment.

Although the results in the table above are not statistically significant, the mean efficiencies of treatments R3 and D3 are larger than the mean efficiencies of their counterparts with Competitive Equilibria. Although this might be interpreted as suggesting that the existence of a Competitive Equilibrium undermines efficiency, there is another reasonable interpretation. Specifically, the marginal efficient seller in the D3 and R3 treatments faces more competitive pressure from the next least expensive seller than the marginal efficient seller in the D4 and R4 treatments. In D4 and R4, Seller #3 has a strong incentive to push the price up to the cost of the inefficient seller, Seller #4. If Seller #3 offers too aggressively and is not accepted, the entry of Seller #4 reduces efficiency from 100 percent to 92.6 percent.[37] In D3 and R3, Sellers #2 and #3 are more evenly matched for competing against one another, so if Seller #2 is too aggressive and allows Seller #3 to enter, efficiency is only reduced to 96.9 percent.[38]

3. Institutional Comparison

The experiments using the QUPA institution were motivated by the problem identified by Van Boening and Wilcox (1996). They reported that the DA did not achieve efficient results in avoidable cost environments, while their baseline (a comparable environment, except that sellers had marginal rather than avoidable fixed costs) did. Van Boening and Wilcox (2005a) also found that additional variations on the DA also did not perform as well as the marginal cost baseline. Durham et al designed a novel institution called the Smart Market to facilitate more efficient production and exchange in the same avoidable fixed cost environment used by Van Boening and Wilcox (1996, 2005a). The Smart Market was tested using robot buyers and produced more efficient results than the DA experiments but still did not perform as efficiently as the

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[37] 100 percent efficiency is achieved when Seller #1 produces 8 units at a cost of 960, Seller #2 produces 5 units at a cost of 750, and Seller #3 produces 3 units at a cost of 540 for a total of 16 units. Since the buyers have a value of 250 per unit, the surplus from this allocation is 1750. If Seller #3 is replaced by Seller #4 producing 2 units at a cost of 420, the surplus is reduced to 1620 (= 1750 – 250 + (540 – 420)).

[38] 100 percent efficiency is achieved when Seller #1 produces 7 or 8 units at a cost of 960 and Seller #2 produces 4 or 5 units at a cost of 750 for a total of 12 units. Since the buyers have a value of 250 per unit, the surplus from this allocation is 1290. If Seller #2 is replaced by Seller #3 producing 3 units at a cost of 540, the surplus is reduced to 1250 (= 1290 – 250 + (750 – 540)).

marginal cost baseline. In this section, I compare the results from the QUPA institution to the results from the experiments using the DA and the Smart Market.

The following figure reports the mean efficiency for each institution and environment. Experience was shown to be significant for the experiments using the QUPA institution, so the figure shows results from only experienced QUPA rounds. Likewise, Durham et al report that experience had a substantial positive effect on efficiency, so the figure shows results from only experienced Smart Market rounds. Van Boening and Wilcox found no statistically significant effect from experience when they used an avoidable cost environment and they did not run experienced sessions for the marginal cost baseline, so the figures below report inexperienced and experienced sessions for the DA sessions.

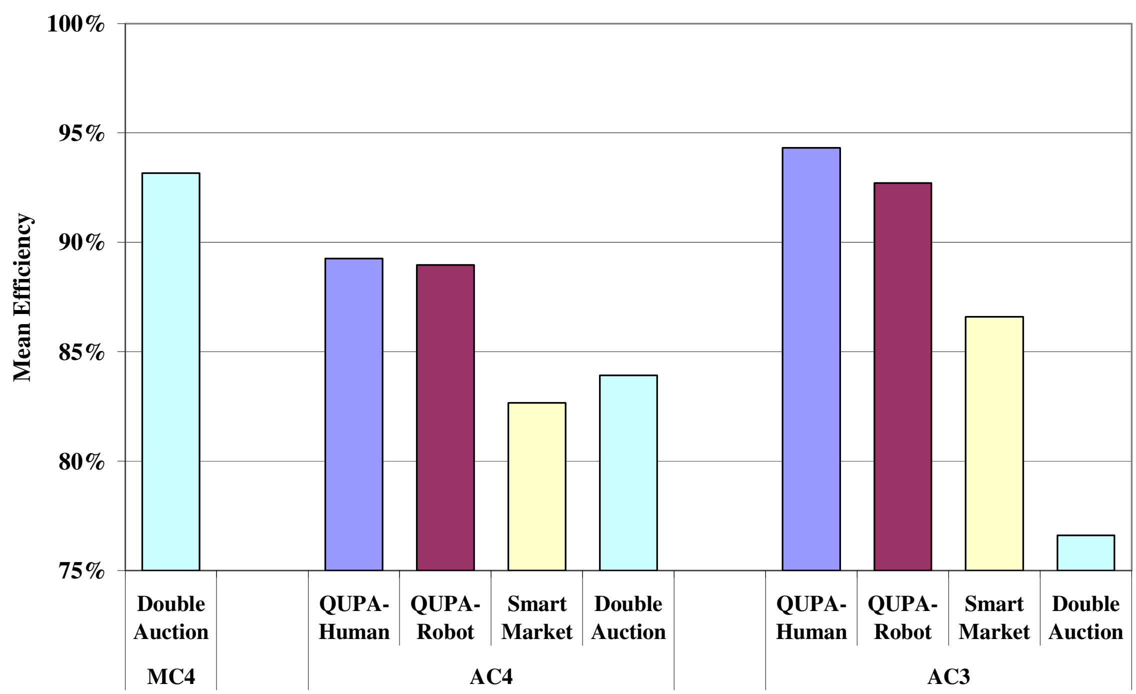

Figure 11: Mean Efficiency by Institution and Environment

Figure 11 shows the mean efficiency for each environment and institution. The QUPA in the AC3 environment with human buyers had the highest mean efficiency, followed closely by the DA in the MC4 environment and the QUPA in the AC3 environment with robot buyers. It is notable that for the QUPA sessions and Smart Market sessions, the AC3 environment, where no Competitive Equilibrium exists, exhibited better efficiency than the AC4 environment. Conversely, for the DA, the AC4 environment exhibited higher efficiency than the AC3 environment. The QUPA in the AC3 environment with human buyers exhibited levels of efficiency comparable to the DA in the MC4 environment, while the QUPA in the AC4 environment with human buyers was somewhat lower. Within each environment, the QUPA sessions achieved higher levels of efficiency than the Smart Market and the DA. The following analyses test whether these samples are drawn from the same population distribution.

The following table reports the mean efficiencies of each treatment from the QUPA, the Smart Market, and the DA. The table also reports the rank sum test comparisons between the QUPA and the DA and the Smart Market. The QUPA sessions that used robot buyers are compared to the Smart Market, and the QUPA sessions that used human buyers are compared to the DA. The AC3 and AC4 treatments are compared to the corresponding treatments and then the AC3 and AC4 treatments are also pooled and compared as a group to the corresponding group. In each case, I test the null hypothesis that the two samples are drawn from the same population. The first three rows show the comparisons between the QUPA sessions with robot buyers and the Smart Market sessions. The last three rows show the comparisons between the QUPA sessions with human buyers and the DA sessions. P-values below 5 percent are shaded in the table below.

Table 6: Comparisons of Mean Efficiency between Institutions

Category	Comparison	Mean (Sample 1)	Mean (Sample 2)	Difference	Rank sum p-value
Compare: QUPA-Robot versus Smart Market	AC3 v AC3	92.7%	86.6%	6.1%	.142
	AC4 v AC4	89.0%	82.7%	6.3%	.107
	(AC3 & AC4) v (AC3 & AC4)	90.2%	84.6%	5.6%	.048
Compare: QUPA-Human versus Double Auction	AC3 v AC3	94.3%	77.0%	17.3%	.019
	AC4 v AC4	89.3%	83.8%	5.3%	.201
	(AC3 & AC4) v (AC3 & AC4)	91.8%	80.4%	11.4%	.006

For each comparison reported in Table 6, the QUPA exhibited a higher mean efficiency than the other institution. The rank sum tests indicate the efficiency in the QUPA is distributed differently from the efficiency in the Smart Market when the AC3 and AC4 treatments are pooled together based on the 5 percent level of significance. The rank sum tests indicate the efficiency in the QUPA is distributed differently from the efficiency in the DA for the AC3 environment and when the AC3 and AC4 treatments are pooled together.   The statistical significance of these results is notable given the small number of observations.

The following table reports the results of comparisons of the mean efficiency between the QUPA using human buyers and the DA sessions that used the MC4 environment. The mean efficiencies of the samples are in the same general range for the QUPA sessions with human buyers in an avoidable cost environment and the Double Auction sessions in a marginal cost environment. No statistical comparisons are performed since only two sessions were run with the Double Auction in a marginal cost environment.

Table 7: Mean Efficiency of QUPA with Human Buyers and Marginal Cost Baseline

Comparison	Mean (Sample 1)	Mean (Sample 2)	Difference
QUPA-H3 v DA-MC4	94.3%	93.2%	1.1%
QUPA-H4 v DA-MC4	89.3%	93.2%	-3.8%
QUPA-(H3 & H4) v DA-MC4	91.8%	93.2%	-1.4%

The results of the QUPA experiments are promising. The QUPA results are more efficient than the Smart Market and Double Auction institutions when placed in comparable environments. Moreover, the QUPA results are comparable to the efficiency of trading in Double Auction experiments that use a comparable marginal cost environment, providing some evidence that the special features of the QUPA substantially address the difficulties posed by "ill-behaved" cost structures.

B. Transaction Prices

This section examines transaction prices in the QUPA. Transaction prices are primarily based on the quasi-clearing price, but are also affected by side payments that enable the QUPA to do discriminatory pricing. Prices provide an indication of the competitiveness of outcomes in each treatment. Prices are discussed with reference to the analyses in Section V.

1. Treatment Effects on Quasi-Clearing Price

Section V predicts that treatment variations will have strong effects on transaction prices. The D4 treatment is expected to produce higher prices than the D3 treatment, because of higher demand levels can be expected to raise prices when cost of supply is fixed and increasing. In the D4 treatment, the Competitive Equilibrium and Nash Equilibrium predictions are that prices will range from 180 to 210, which is between the minimum average costs of Seller #3 and #4. Essentially, this predicts that one of Sellers #1, #2, or #3 will be on the margin and will push prices up until he feels the competitive pressure from Seller #4. In the D3 treatment, it is expected that the competition at the margin will range from 150 to 180, which is between the minimum average costs of Seller #2 and #3. It is expected that the use of robot buyers will remove competitive pressure on the sellers, resulting in higher prices.

The following figure shows the quasi-clearing prices in each auction round. One line graph is shown for each treatment and one line is shown for each session. The labels on the x-axes indicate the round number. Quasi-clearing prices above 250 are shown at 250 and those below 110 are shown at 110.

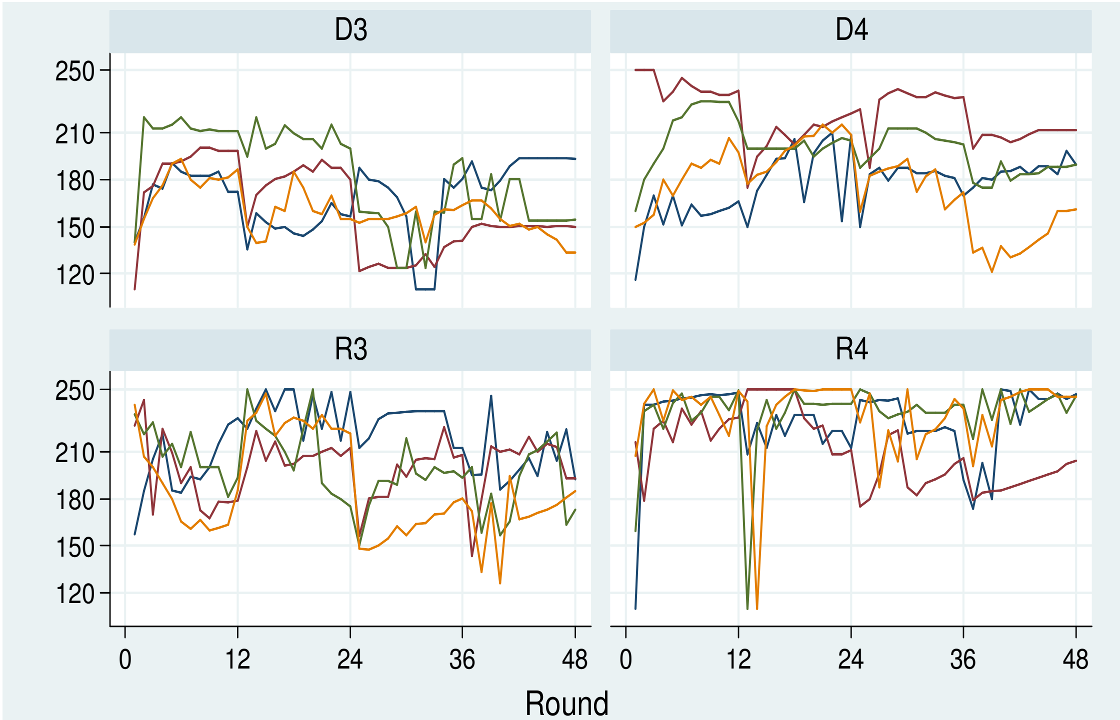

Figure 12: Distribution of Transaction Prices in the QUPA, by Treatment

The Mann-Whitney rank sum method is used to test whether the effects of experience are statistically significant. Three pairwise comparisons are possible between the efficiency from one sample where no Competitive Equilibrium exists and a second sample where a Competitive Equilibrium does exist. Two of these are one-to-one comparisons between treatments: D3 versus D4 and R3 versus R4. Then D3 and R3 are pooled together and compared to D4 and R4 pooled together. Three pairwise comparisons are possible between the efficiency from one sample that uses human buyers and a second sample that uses robot buyers. Two of these are one-to-one comparisons between treatments and one of these pools the data from D3 with D4 and from R3 with

R4. For each pairwise comparison, I test the null hypotheses that the two samples are drawn from the same population. The results are summarized in the following table:

Table 8: Treatment Effects on Quasi-Clearing Price in Experienced Rounds

Category	**Comparison**	**Mean (Sample 1)**	**Mean (Sample 2)**	**Difference**	**Rank sum p-value**
Compare: non-CE versus CE	D4 v D3	179.8	162.6	17.2	.564
	R4 v R3	223.5	190.6	33.0	.042
	(D4 & R4) v (D3 & R3)	209.0	176.6	32.4	.031
Compare: Robot Buyers versus Human Buyers	R3 v D3	190.6	162.6	28.0	.083
	R4 v D4	223.5	179.8	43.7	.017
	(R3 & R4) v (D3 & D4)	212.5	171.2	41.4	.003

The means reported in the table above are consistent with our predictions. The means of D4 and R4 are higher than the means of D3 and R3 by 17.2 and 32.9, respectively. Furthermore, the means of R3 and R4 are higher than the means of D3 and D4 by 28.0 and 43.7, respectively. In the R3 treatment, sellers were able to push up prices considerably but not above the competitive on average. Conversely, in the R4 treatment, sellers were able to push prices well above the competitive range of 180 to 210. Moreover, the rank sum tests indicate that the null hypothesis is rejected for each case except between D4 and D3 and between R3 and D3.

A close review of the quasi-clearing prices also provides insight about the competitive dynamic in each treatment.

- In the D3 treatment, the quasi-clearing price ranged from 150 to 210 in 90 percent of the rounds, indicating most of the competition was between Seller #2 and Seller #3. This finding is confirmed by a review of the quantities sold in each round: Seller #1 sold in 98 percent of the rounds, Seller #2 sold in 77 percent of the rounds, Seller #3 sold in 25 percent of the rounds, and Seller #4 sold in 0 percent of the rounds.
- In the R3 treatment, the quasi-clearing price ranged from 150 to 210 in 71 percent of the rounds, indicating most of the competition was between Seller #2 and #3 but that Seller #4 was involved more frequently than in the D3 treatment. This finding is confirmed by a review of the quantities sold in each round: Seller #1 sold in 94 percent of the rounds, Seller #2 sold in 63 percent of the rounds, Seller #3 sold in 35 percent of the rounds, and Seller #4 sold in 27 percent of the rounds.
- In the D4 treatment, the quasi-clearing price ranged from 180 to 210 in 54 percent of the rounds, indicating much of the competition was outside the price range of highly efficient outcomes. The quasi-clearing price was greater than 210 in 10 percent of rounds, between 150 and 180 in 17 percent of rounds, and between 120 and 150 in 19 percent of rounds. A detailed review of the quantities sold in each round indicates Seller #1 sold in 96 percent of the rounds, Seller #2 sold in 88 percent of the rounds, Seller #3 sold in 60 percent of the rounds, and Seller #4 sold in 21 percent of the rounds. It is perplexing that Seller #4 sold in 21 percent of the rounds even though the quasi-clearing was at a profitable level for him in just 10 percent of the rounds.
- In the R4 treatment, 69 percent of the rounds cleared above 210, indicating that the competitive pressure from Seller #4 was not sufficient to prevent the other sellers from pushing prices up to the values of the robot buyers. A detailed review of the quantities sold in each round indicates Seller #1 sold in 94 percent

of the rounds, Seller #2 sold in 88 percent of the rounds, Seller #3 sold in 75 percent of the rounds, and Seller #4 sold in 32 percent of the rounds.

2. Quasi-Clearing Prices and Efficiency

The following figure examines the relationship between efficiency and the quasi-clearing price in the QUPA. The scatter plot shows the mean quasi-clearing price and the mean efficiency in each session of each treatment. Each of the 16 sessions is labeled according to the treatment and session number. The labels on the x-axis line up with the minimum average costs of the sellers, which range from 120 to 210, and the values of the buyers, which are always 250.

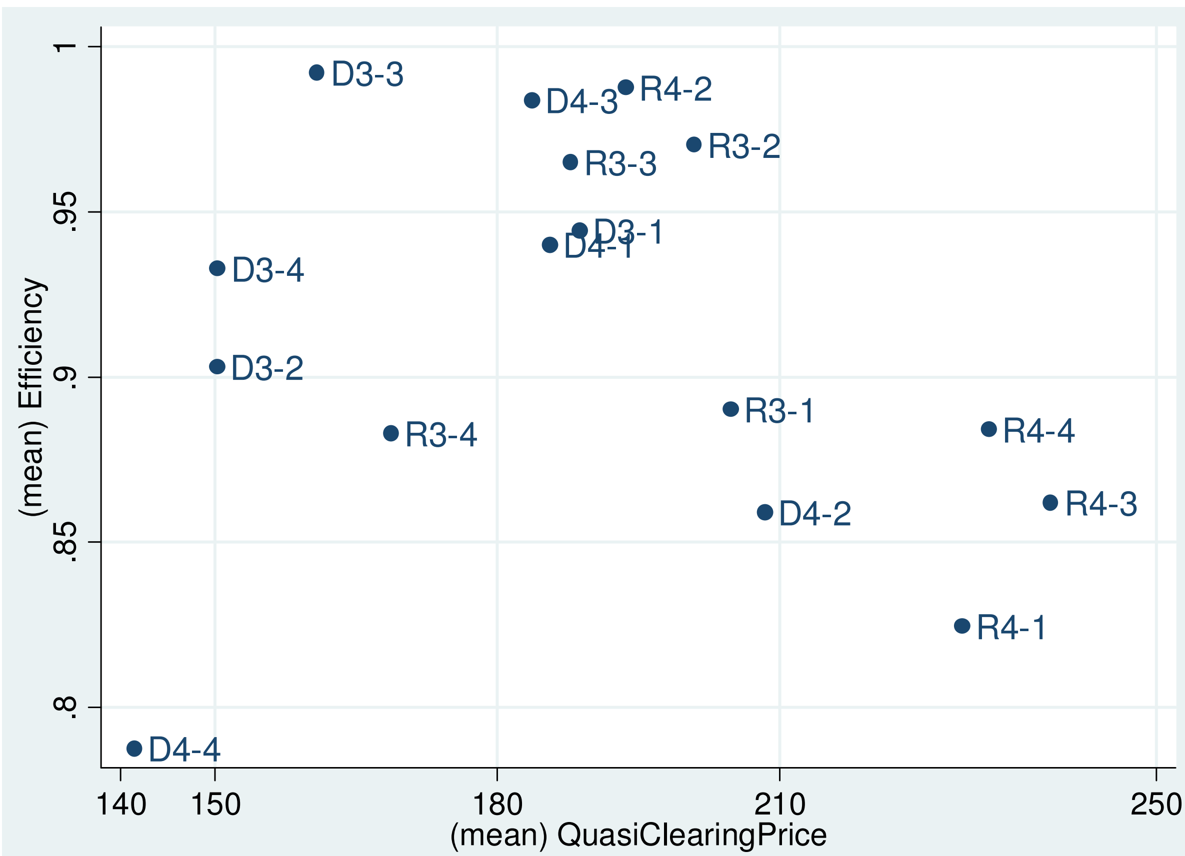

Figure 13: Mean Efficiency, by Session

The scatter plot highlights sessions that were outliers within each treatment.

- Session D4–4 exhibits the lowest efficiency and the lowest average quasi-clearing price of any session. Upon closer examination of this session, this unusual result was driven by Seller #4 who, with a minimum average cost of 210, sold unprofitably at transaction prices ranging from 130 to 161 in 58 percent of the rounds. It may have been that this subject was confused about the incentives of the institution, and that this confusion reduced efficiency by preventing Sellers #2 and #3 from being active in the auction.
- Session R3–4 exhibits lower efficiency and a lower average quasi-clearing price than the other R3 sessions. Similar to Session D4–4, this result is driven by Seller #4 making unprofitable sales at transaction prices ranging from 126 to 194 in 50 percent of the rounds. It is likely that this subject was also confused about the incentives of the institution. Session R3–1 also had relatively low efficiency, but this was the result of Seller #4 making profitable sales after the other sellers had pushed up the quasi-clearing price.
- Session R4–2 exhibits very high efficiency and a much lower average quasi-clearing price than the other R4 sessions. In R4–2, the quasi-clearing price was in the CE range of 180 to 210 in 83 percent of the rounds, suggesting that the sellers were overly conservative.

If we ignore sessions R3–4 and D4–4, the outliers that were affected by subject errors, a pattern emerges in the figure above relating efficiency and quasi-clearing prices. It seems that the efficiency was greater than 90 percent in the sessions where the quasi-clearing price was lower than 201, and the efficiency averaged between 80 and 90 percent in the sessions where the quasi-clearing price was higher than 201. This indicates that the QUPA is likely to produce efficient outcomes in environments without significant market power, but if sellers are able to push prices outside the competitive range, high-cost sellers will enter, undermining efficiency.

3. Discriminatory Pricing

The QUPA was designed to allow limited discriminatory pricing, which theory predicts is necessary to for efficiency when no-CE exists. So we expect a substantial number of rounds with discriminatory pricing in treatments D3 and R3 and little or no discriminatory pricing in treatments D4 and R4. The following figure characterizes the extent to which transaction prices were discriminatory. The figure shows a histogram for each treatment showing how the differences between the maximum transaction price and the minimum transaction price are distributed. The bars of the histogram group observations every 2.5, so the left most bar includes 0 to 2.5, the second bar includes 2.5 to 5, etc.

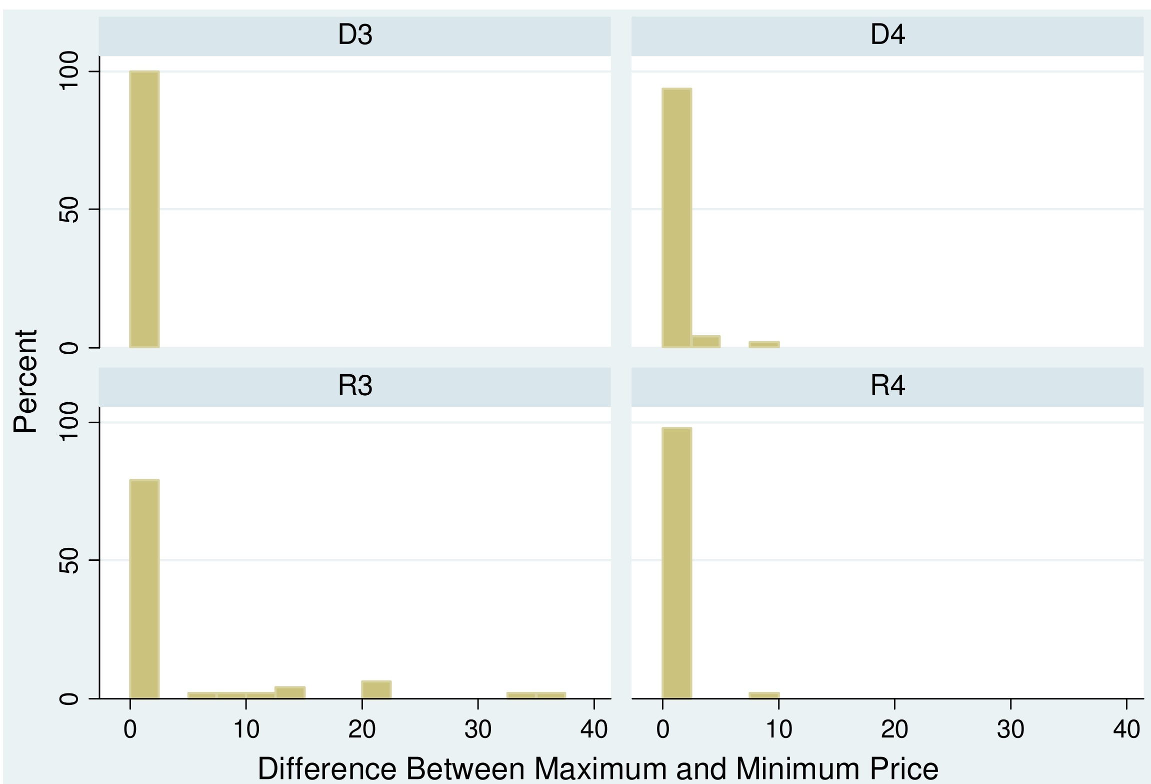

Figure 14: Spread between the High and Low Transaction Prices in Each Round

As expected the D4 and R4 treatments had relatively few round with a significant difference between the maximum transaction price and the minimum transaction price. Surprisingly, the D3 treatment never experienced a difference between the minimum and maximum price exceeding 2.5 per unit, although the difference was between 0 and 1 per unit in 15 percent of the rounds.

In contrast, the R3 treatment exhibited a difference ranging from 5 to 37 per unit in 21 percent of the rounds. 90 percent of these rounds were highly efficient with Sellers #1 and #2 combining to sell 12 units. In cases where Seller #1 sold 7 units and Seller #2 sold 5 units, Seller #1 received a make whole payment, and in cases where Seller #1 sold 8 units and Seller #2 sold 4 units, Seller #2 received a make whole payment. In these rounds, transaction prices were relatively high, ranging from 190 to 245, indicating that the sellers had been offering aggressively. The assessment of bids and offers in Sections VII.C and VII.D shed additional light on the reasons for these outcomes.

C. Bids and Offers

1. Buyers' Bidding Patterns

This section closely examines the bidding patterns of buyers in the D3 and D4 treatments. There is no need to analyze the R3 and R4 treatments, which used robot buyers rather than human subjects. The object is to characterize whether the bidding patterns are consistent with any of the strategies outlined in Section V. Moreover, I try to determine how buyers change their behavior with experience or with changes in the treatment.

Section V.A outlined four strategies that buyers might use in the QUPA.

- *The Revealing Strategy*, which is where the buyer bids at or near his value, would be used by buyers that assume they have little or no effect on the quasi-clearing price, and thus, want to maximize the chance of buying at a profitable price.
- *The Aggressive Strategy* is where the buyer bids near or below where he expects the quasi-clearing price in hopes that this will reduce the price at which the buyer purchases. Such buyers would expect that their strategy would result in some foregone profitable purchases, but that these would be offset by the gains from lower transaction prices.
- *The Conservative Strategy* which is where the buyer bids somewhere between the other two strategies, is used by buyers that want to put some downward pressure on transaction prices without foregoing a substantial amount of profitable purchases.
- *The Quantity Maximizing Strategy* is where buyers raise their bids above their values in order to increase the probability of transacting all of their units if they anticipate being harmed by inconsistencies between the quasi-clearing price and the selection of bids and offers.[39]

To examine which of these strategies were used, the two figures in this section give a detailed view of how human subjects bid. The first figure illustrates how bidding evolved during the D3 and D4 treatments, which comprised eight experimental sessions. A separate graph is shown for each treatment. The x-axis shows the 48 rounds per session, broken into four groups of 12 rounds. The y-axis is shown with labels and grid

[39] This strategy may have been employed, but since there were virtually no occasions where it could have resulted in a different outcome from a simple Revealing strategy, the Quantity-Maximizing Bids are shown at 250, a Revealing Bid.

lines at the minimum average costs of the four sellers (e.g. Seller #1 at 120, Seller #2 at 150, etc) and the per unit value of the buyers.

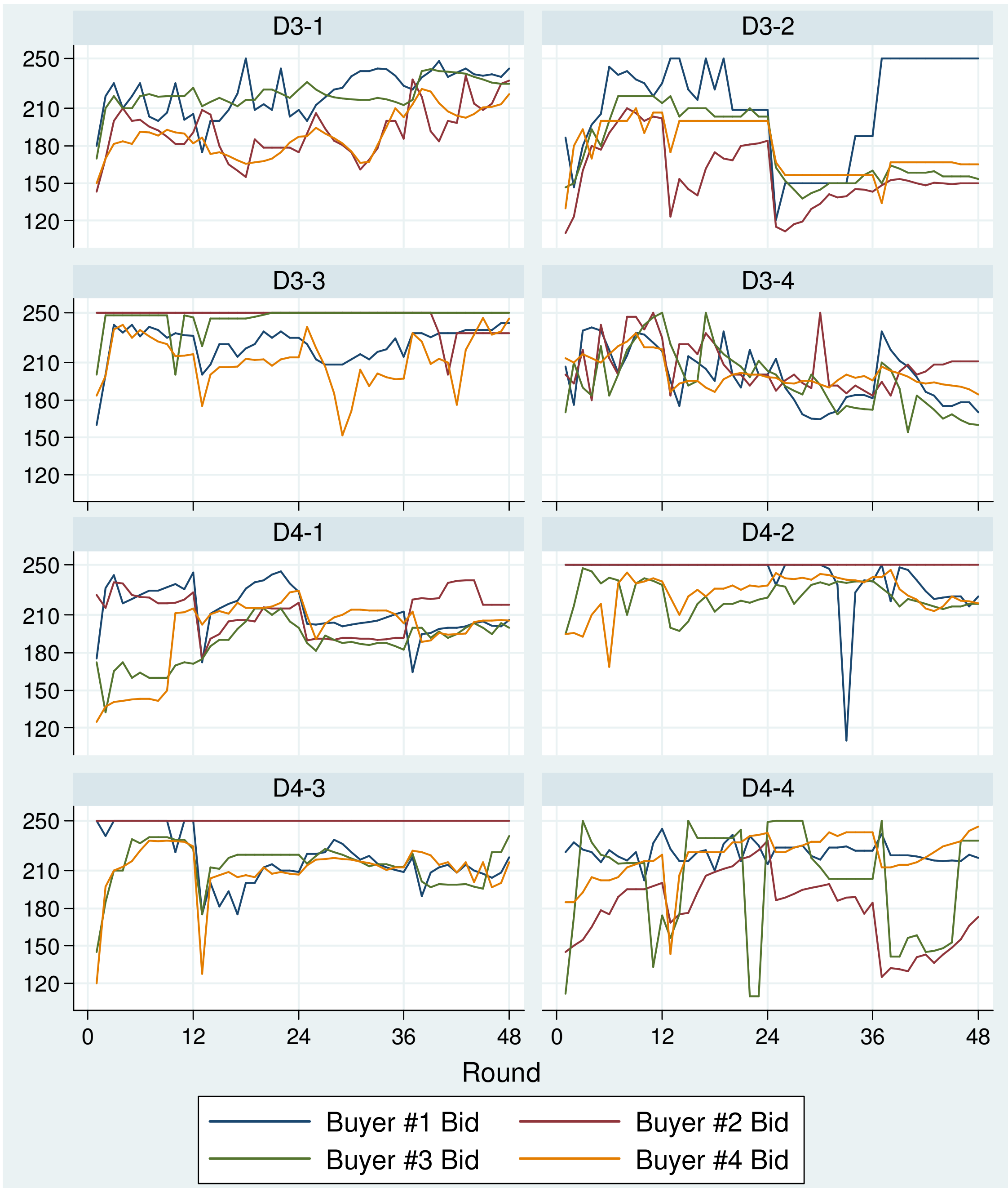

Figure 15: Trend in Bid Prices as Experienced is Gained, by Treatment

Several interesting patterns are evident from the figure above. At the beginning of the first group of 12 rounds, the buyers began by bidding in a wide dispersion from their value down to less than 50 percent of the value. The dispersion quickly narrowed as buyers received feedback from the institution. The second group of 12 rounds was similar, although the dispersion of bids was narrower at the beginning and took fewer rounds to reach a steady level.

One conclusion can be drawn from the figure above. Bid prices in both treatments are widely dispersed between levels consistent with Aggressive bidding and levels indicative of a Revealing Strategy. Hence, it may be inferred either that most buyers adopted a strategy of bidding a broad range of prices or that they adopted a variety of different strategies. The following figure examines bids from the last group of 12 rounds in greater detail to determine the range of different strategies.

The following figure summarizes the distribution of bid prices and quasi-clearing prices in the fourth group of 12 rounds, omitting the first round. In the previous section, it was determined that discriminatory pricing was very limited in the D3 and D4 treatments, so the quasi-clearing prices provide a very accurate picture of the transaction prices that buyers experienced. The figure shows eight histograms: one for each session in the D3 and D4 treatments. Each histogram is labeled with the treatment, the session number, and the fraction of the quantity of the buyers' demand that was satisfied. The green bars show the distribution of quasi-clearing prices while the tan bars show the distribution of bids. The x-axis is shown with labels and gridlines at the minimum average costs of the four sellers (e.g. Seller #1 at 120, Seller #2 at 150, etc.) and the per

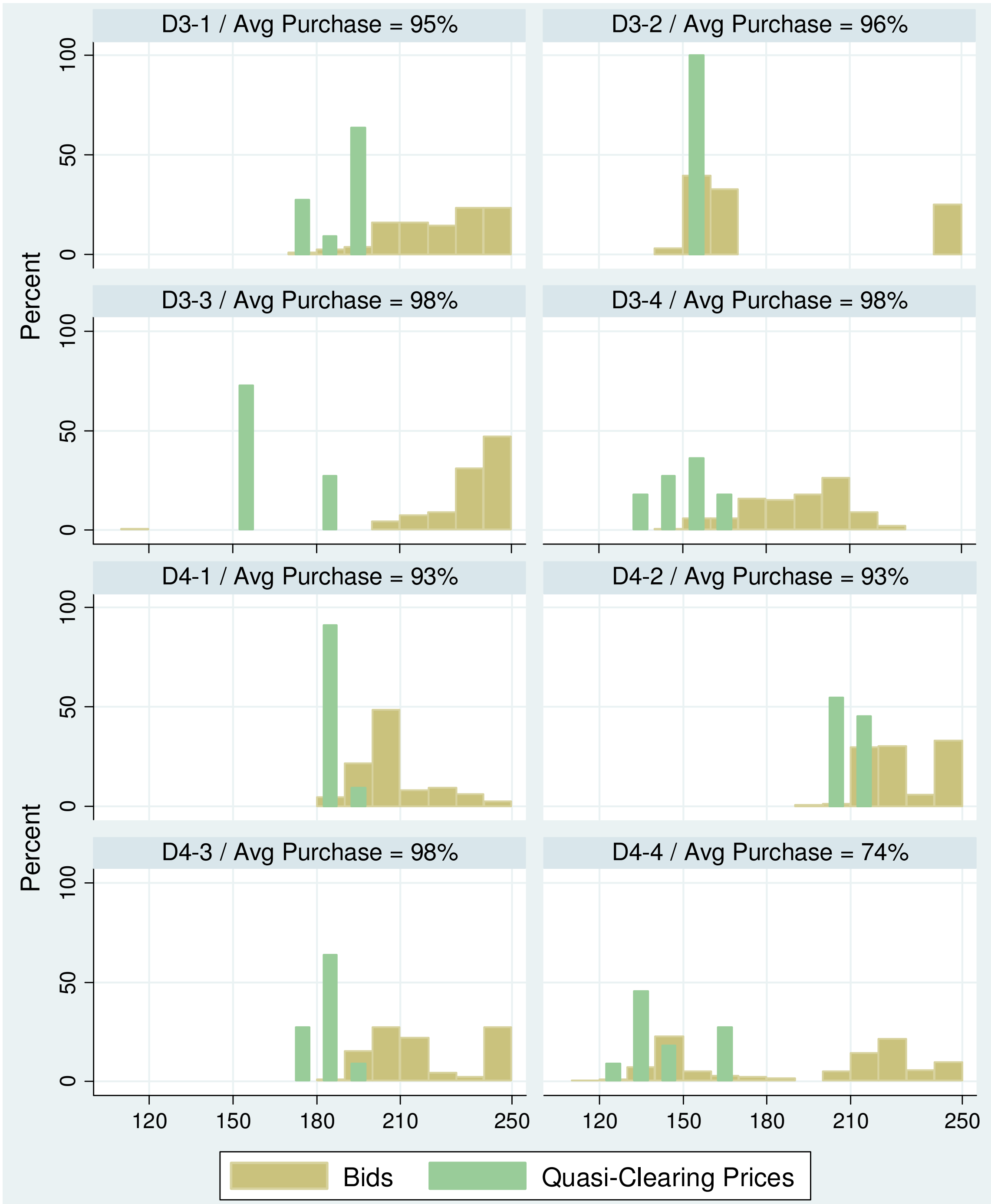

Figure 16: Histograms of Bids and Quasi-Clearing Prices, by Session

unit value of the buyers. The histogram groups the data in bins of ten (e.g. 110 to 120, 120 to 130, etc.). Bids below 110 are shown in the bin from 110 to 120, and bids above 250 are shown in the bin from 240 to 250.

The histograms above illustrate that the buyers used a wide range of strategies. In each session, the four buyers employed different strategies. In each treatment, the pattern of bidding varied widely from session to session. For example, session D3-3 shows four buyers that used Revealing strategies or strategies at the revealing end of Conservative such that an overwhelming share of bids exceeded the quasi-clearing price by 50 or more. In contrast, session D3-2 shows a very different picture: one buyer uses a Revealing in every round while the other three buyers bid in a range that is Aggressive or at the aggressive end of Conservative. The lack of convergence toward a single strategy suggests that additional experience or information might have additional effects on the strategies of buyers. Furthermore, the strategies chosen by each buyer might depend heavily on the strategies of other buyers and sellers in the experiment. More experiments are necessary to determine whether *fully* experienced buyers would eventually converge to some equilibrium strategy.

The quasi-clearing prices provide additional evidence that each session reached a different equilibrium. In sessions D3-2 and D3-4, the quasi-clearing price was consistently near 150, while in D3-1, the quasi-clearing price bounced between 170 and 200, and in D3-3, the quasi-clearing price alternated between 150 and 180. These quasi-clearing prices indicate that the buyers frequently applied sufficient pressure to keep the Seller #2 from pushing the price up to the minimum average cost of Seller #3. In sessions D4-1 and D4-3, the quasi-clearing price was consistently near 180, while in D4-2, the quasi-clearing price was near 210, and in D4-4, the quasi-clearing price was usually below 150. Without running additional experiments, it is impossible to say whether the quasi-clearing prices would have converged toward a more consistent level across sessions.

Based on a cursory review, it appears that a substantial portion of the buyers are using the Aggressive Strategy; however, a closer examination suggests that some of these buyers are using the Conservative Strategy. For example, 75 percent of the bids in the D3-2 session are very close to the range of quasi-clearing prices, suggesting that three of the buyers in the session were Aggressive. However, the label at the top of the histogram reveals that 96 percent of the buyers' demand was satisfied, which seems too high if three buyers were Aggressive. Closer review of the data indicates that three of the four buyers in the session purchased 100 percent of their demand and one of the four purchased 83 percent. Hence, one of the low-bidders was Aggressive, while the other two low-bidders are more accurately described as Conservative, bidding low enough to prevent the sellers from pushing up prices, but not low enough to forego profitable purchases.

The histogram for session D4-4 provides additional insight about how the session went awry. As the offers of Seller #4, who has a minimum average cost of 210, descended into a very unprofitable range of prices, two buyers bid Aggressively. If these two buyers had used Conservative or Revealing Strategies, it is likely that Seller #2 or #3 would have sold units, thereby setting the quasi-clearing price at a level much higher than the offers of Seller #4. Hence, even though Seller #4 used a sub-optimal strategy, possibly out of confusion, Aggressive bidding on the part of two buyers was still necessary to seriously undermine the efficiency of the session.

2. Sellers' Offer Patterns

This section closely examines the offer patterns of sellers in the four treatments. The purpose is to examine whether the offer patterns are consistent with the strategies outlined in Section V. This section also attempts to characterize the effects of experience and treatment variations on the behavior of sellers.

Section V.A outlined four strategies that sellers might use in the QUPA.

- *The Revealing Strategy*, which is where the seller offers at or near his cost, would be used by sellers that assume they have little or no effect on the quasi-clearing price, and thus, want to maximize the chance of selling at a profitable price.
- *The Aggressive Strategy* is where the seller offers near or below where he expects the quasi-clearing price in hopes that this will increase the price at which he sells. Such sellers would expect that their strategy might result in some foregone profitable purchases, but that these would be offset by the gains from higher transaction prices.
- *The Conservative Strategy*, which is where the seller offers somewhere between the other two strategies, is used by sellers that do not expect to directly influence transaction prices, but want to limit the extent to which the transaction prices might fall.
- *The Quantity Maximizing Strategy* is where sellers lower their offers below their costs in order to increase the probability of transacting all of their units if they anticipate being harmed by inconsistencies between the quasi-clearing price and the selection of bids and offers.

There are reasons to expect sellers to be more likely than buyers to use an Aggressive Strategy. The price-setting mechanism in the QUPA ensures that one or more sellers will be paid an amount that is directly determined by their offer. In comparison, the buyers' bids never set the quasi-clearing price, and it is relatively unlikely that their bids will lead them to receive a make whole payment or reduce their allocation of make whole payments.

The following two figures show the offers of all four sellers in all 48 rounds of all 16 sessions. The two figures comprise 16 line graphs: one for each session. The 48 rounds of each session, which are indicated on the x-axes, are divided into four groups of 12 rounds. The y-axes indicate the level of the average total offer, which is the sum of the fixed component and the incremental components divided by the number of units offered, and the quasi-clearing price. The y-axes are labeled with the minimum average costs of the four sellers and the buyers' values. Offers below 100 are shown at 100, and offers above 250 are shown at 250. The first figure shows the results of the D3 and R3 treatments, while the second figure shows the results of the D4 and R4 treatments.

Several general observations can be made from the two figures. For all four sellers, the offers vary significantly throughout most of the sessions of each treatment, indicating that the subjects are testing the unfamiliar institution with different strategies. In some of the sessions, such as D3-2, the variability seemed to settle down as the subjects gained experienced. However, in sessions like R3-3, the sellers' offers remained volatile throughout the 48 rounds. This is more support for the conclusion that if subjects were given additional experience, it might lead to additional effects on outcomes in the QUPA.

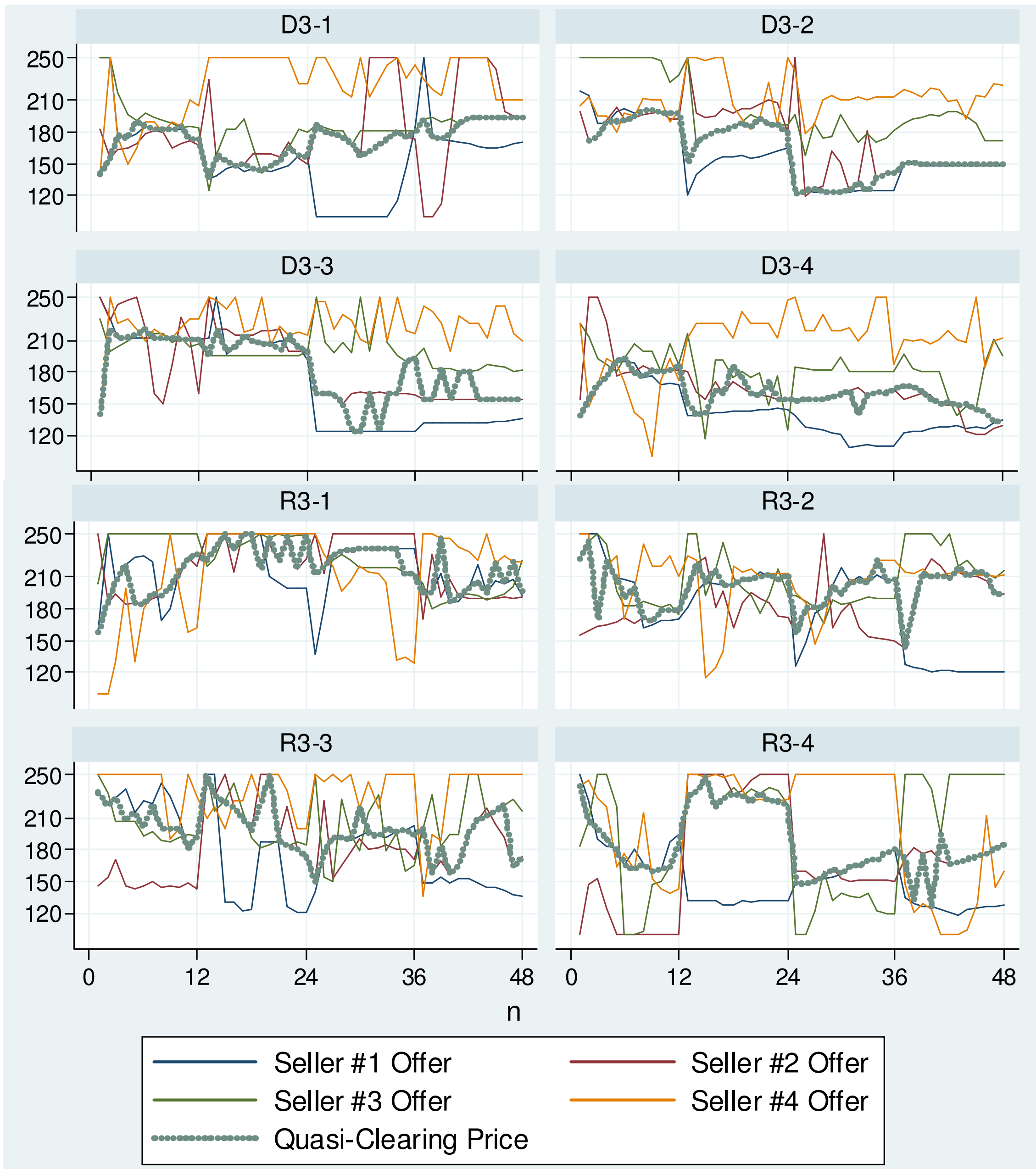

Figure 17: Offers in D3 and R3 Treatments

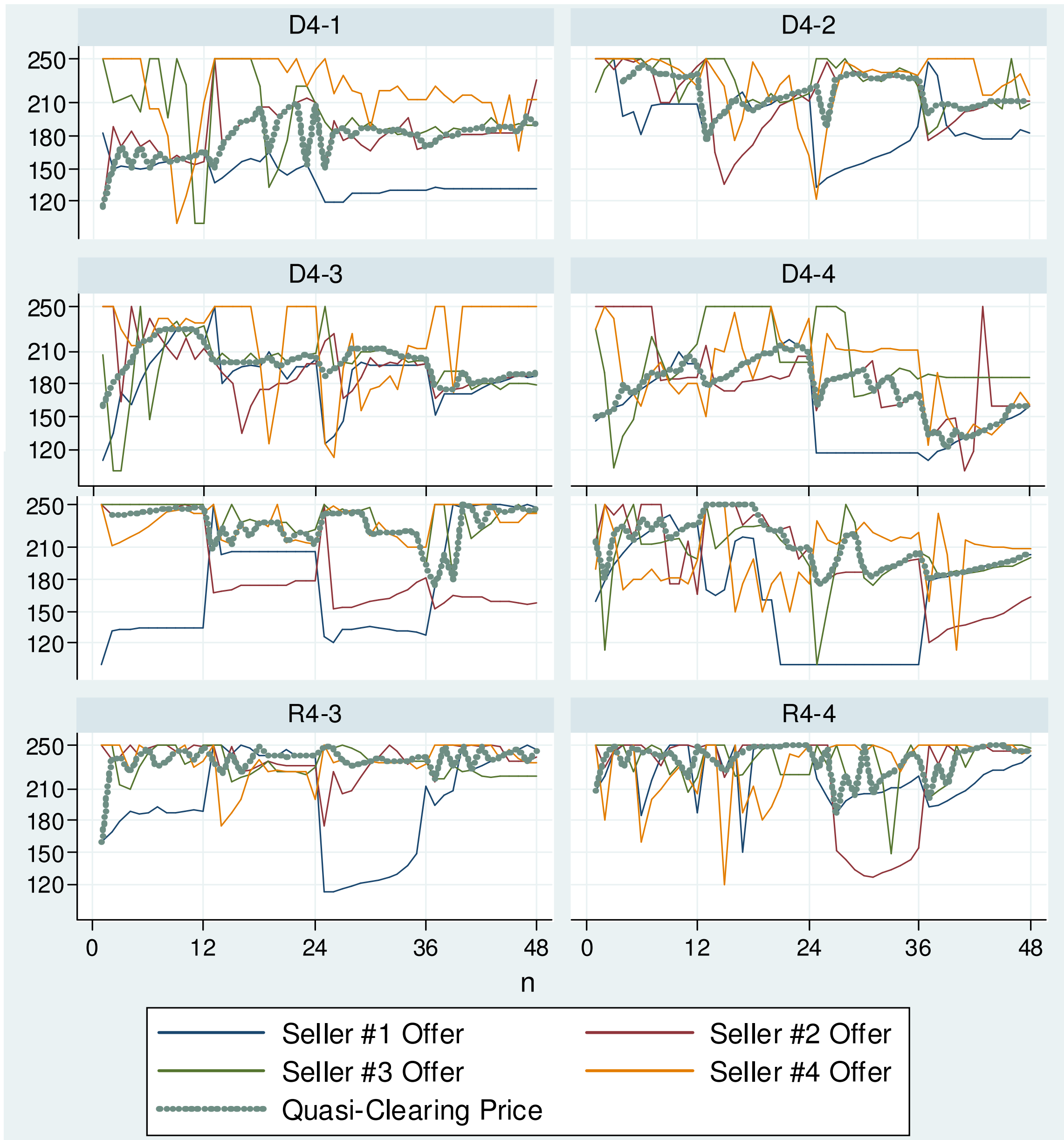

Figure 18: Offers in D4 and R4 Treatments

It was expected that sellers would not generally offer below cost, because it seemed unlikely that this would ever be more profitable than a Revealing Strategy. The results of many sessions, such as D3-3, are consistent with this expectation. However, other sessions were like D3-4, where Sellers #2 and #3 both offered below cost for a significant portion of the final group of 12 rounds. These could be examples of the Quantity-Maximizing Strategy. However, the institution is mentally taxing, requiring subjects to think about how to offer a bundle of units, so arithmetic errors may have been responsible for some instances of below-cost offers. In other cases, the below-cost offers were likely due to confusion. The strangest example of below-cost offering was in session R3-4 when Seller #4 offered well below cost during all 48 rounds. This cannot be written-off to simple confusion by an individual subject, because all four subjects had the opportunity to "play" Seller #4 for 12 rounds. In session R3-4, Seller #4 profited at the beginning of the first group of 12 rounds, consistently profited in the second group of 12 rounds, but lost substantial earning in the last two groups of 12 rounds.

The figures show many examples of subjects using all three of the strategies outlined earlier. This is also true in the final group of 12 rounds after the subjects gained a substantial amount of experience. In the R4 treatment, the Aggressive Strategy appears most prevalent, which is consistent with it producing much higher quasi-clearing prices than the other treatments. The other three treatments exhibit a mix of different strategies. For instance, in the final 12 rounds of D4-3, Sellers #1 and #2 use Aggressive strategies while Seller #3 reveals. In contrast, the final 12 rounds of D4-1 show Sellers #1 and #3 Revealing while Seller #2 is offering Aggressively. Each of the four sellers can be seen using a wide range of strategies in the final 12 rounds of the different sessions.

One aspect of the learning process is where subjects test various strategies. Presumably, this gives them a feel for the institution before they gravitate toward a single strategy. Large changes in the total offer from round-to-round may be an indication that sellers are testing strategies, while smaller changes can be interpreted as fine-tuning.

VIII. Conclusions

Experimental economics researchers have identified a class of problems that are difficult to solve using traditional bilateral contracting mechanisms such as the Double Auction. Similar coordination problems have arisen in the context of electricity markets, which are in a relatively early stage of reform. Advances in computation provide opportunities for new ways to transacted multilaterally. This paper reports the results of experiments using one such computationally intensive auction mechanism called the QUPA, which is modeled on mechanisms that have been used in electricity auctions.

The results of the QUPA experiments are promising. The QUPA sometimes results in greater efficiency than the Smart Market and Double Auction institutions when placed in comparable environments. Moreover, the QUPA exhibits efficiency that is comparable to what is observed in the Double Auction experiments that use a comparable marginal cost environment. These results suggest that further research on the class of institutions called *Quasi-Uniform Price Auctions* would be worthwhile.

The treatment variations showed no evidence that the treatment variations affected efficiency; however, there were significant treatment effects from on the quasi-clearing price. When fully revealing robot buyers are replaced with human buyers, the quasi-clearing price declined considerably. Sellers raised their offers considerably above their true cost when demand increased and when the buyers were robots. This result is consistent with the findings of Baltaduonis (2007a)

Future research could explore several areas. First, it would be useful to determine whether and how buyers' and sellers' strategies converge if given more extensive experience, and whether this improves or degrades the efficiency of the institution. Second, given the widespread use of the *Standard Quasi-Uniform Price Auctions* in electricity markets, research comparing the alternative rule sets has the potential to improve the efficiency of markets that coordinate many billions of dollars of consumption and production annually.

Appendix

The appendix contains:

- Three example instructions screens; one is general, one is for buyers, and one is for sellers.
- Two example screens reporting results from the previous round; one for a buyer and one for a seller.
- Two example user-input screens; one for a buyer and one for a seller.
- A copy of the experimental procedures.

General Instructions:

In this experiment, you are either a buyer or a seller. In each round, buyers submit bids to buy and sellers submit offers to sell. After each round, a computer will accept the set of bids and offers that provides the maximum possible difference between the sum of accepted bids and the sum of accepted offers.

If you are a buyer, you will submit a bid that indicates the maximum dollar amount you are willing to pay for each unit. If your bid is accepted, the amount you pay will be less than or equal to the bid you submitted. For instance, suppose a buyer has an accepted bid of 1 unit for 40, 1 unit for 30, and 1 unit for 20, the buyer will pay *at most* 90 for the 3 units. Increasing your bid increases the probability that it will be accepted.

If you are a seller, your offer includes two parts: a fixed charge indicating the minimum amount you must receive if you sell any units; and a per unit charge indicating the minimum additional amount you must receive for each corresponding unit. If your offer is accepted, you will receive an amount that is greater than or equal to the sum of the fixed charge you submitted and the per unit charges of the units that were accepted. For instance, suppose a seller offers a fixed charge of 10 and 3 units priced at 2, 3, and 4. If 1 of the units is sold, it will be the one with the lowest offer price and the seller will receive *at least* 12 (= fixed charge of 10 + the unit offered at 2). If all 3 units are accepted, the seller will receive *at least* 19 (= fixed charge of 10 + the units offered at 2, 3, and 4). Reducing either your fixed charge or per unit charge increases the probability that your offer will be accepted.

After each auction round, you will be notified of the following:
-The number of units bought or sold by you,
-The total number of units transacted by all buyers and sellers,
-The average price you paid or received for the units you transacted,
-The minimum and maximum transaction prices of buyers and sellers,
-The profits earned by you

Figure: General Instructions for All Buyers and Sellers

Individual Instructions:

You are a buyer. You receive a value of 250 for each unit you buy. You can buy up to a certain number of units in each round. You can submit a different bid price for each unit.

For example, if you purchase 2 units at 170 per unit, your earnings will be (250 - 170) + (250 - 170) = 160.

There are three other buyers in this market besides you.

In this experiment, your earnings are calculated in e-dollars. After the experiment, you will trade your e-dollars for U.S. dollars. Now you will have several practice rounds before the experiment begins.

Figure: Individual Instructions for a Buyer

The final sentence of the Individual Instructions for a Buyer was replaced with information about the exchange rate.

Individual Instructions:

You are a seller. In each round, you will incur a cost of 550 if you sell one or more units. That is, if you sell no units, your cost will be 0. If you sell 1 unit, your cost will be 550. If you sell 2 units, your cost will be 550. The maximum number of units you can sell in the market in each period is 5 units.

For example, if you sell 4 units for 150 per unit, you will receive 4 * 150 = 600 for net earnings of 600 - 550 = 50.

There are three other sellers in this market besides you.

In this experiment, your earnings are calculated in e-dollars. After the experiment, you will trade your e-dollars for U.S. dollars. Until further notice, the e-dollars you earn will be exchanged at a rate of 0 e-dollars for $1.

Figure: Individual Instructions for a Seller

In the R3 and R4 treatments, the following text replaced the third paragraph in the Individual Instructions for a Seller:

> "You can protect yourself against a loss by submitting an offer with a fixed charge that is greater than or equal to your cost. For example, suppose you submit an offer with a fixed charge equal to 550. If it is

accepted, you will be paid at least 550. If it is not accepted, you will be paid zero, but your cost will also be zero.

There are a total of four buyers and four sellers in this market. The buyers are robots programmed to bid the same amount in each round."

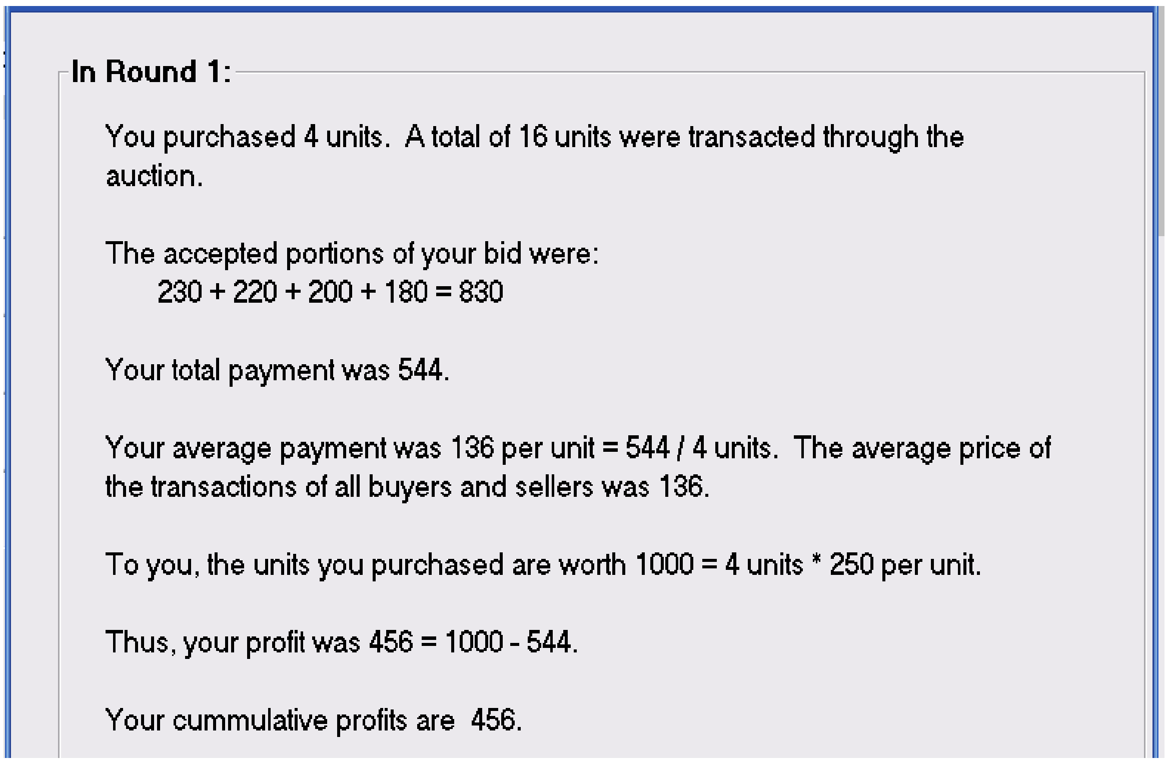

Figure: Round 1 Results that are Reported to a Buyer

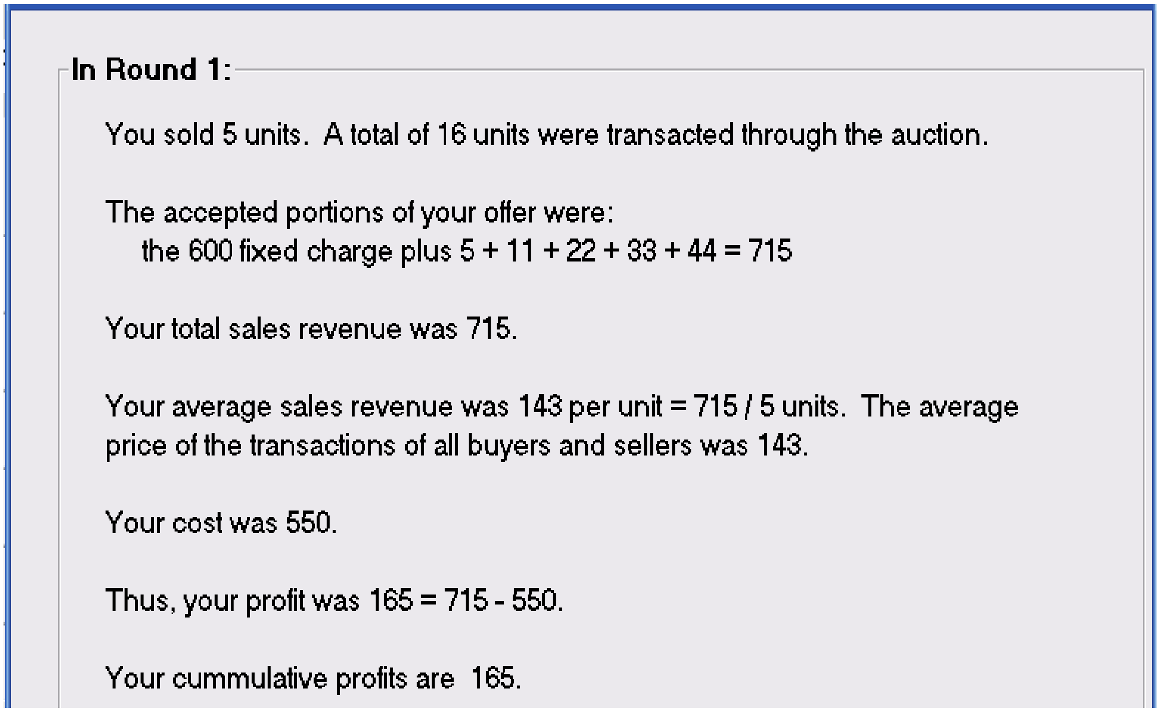

Figure: Round 1 Results that are Reported to a Seller

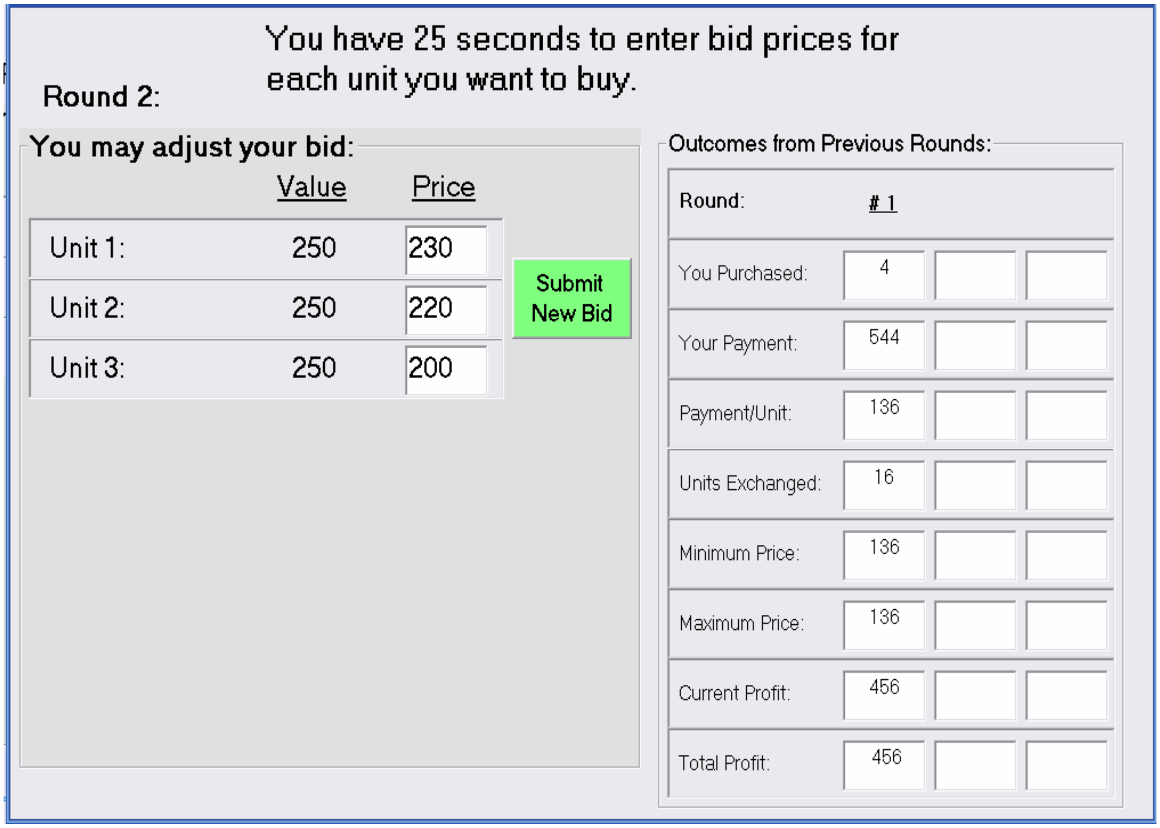

Figure: Round 2 User-Input Window for a Buyer

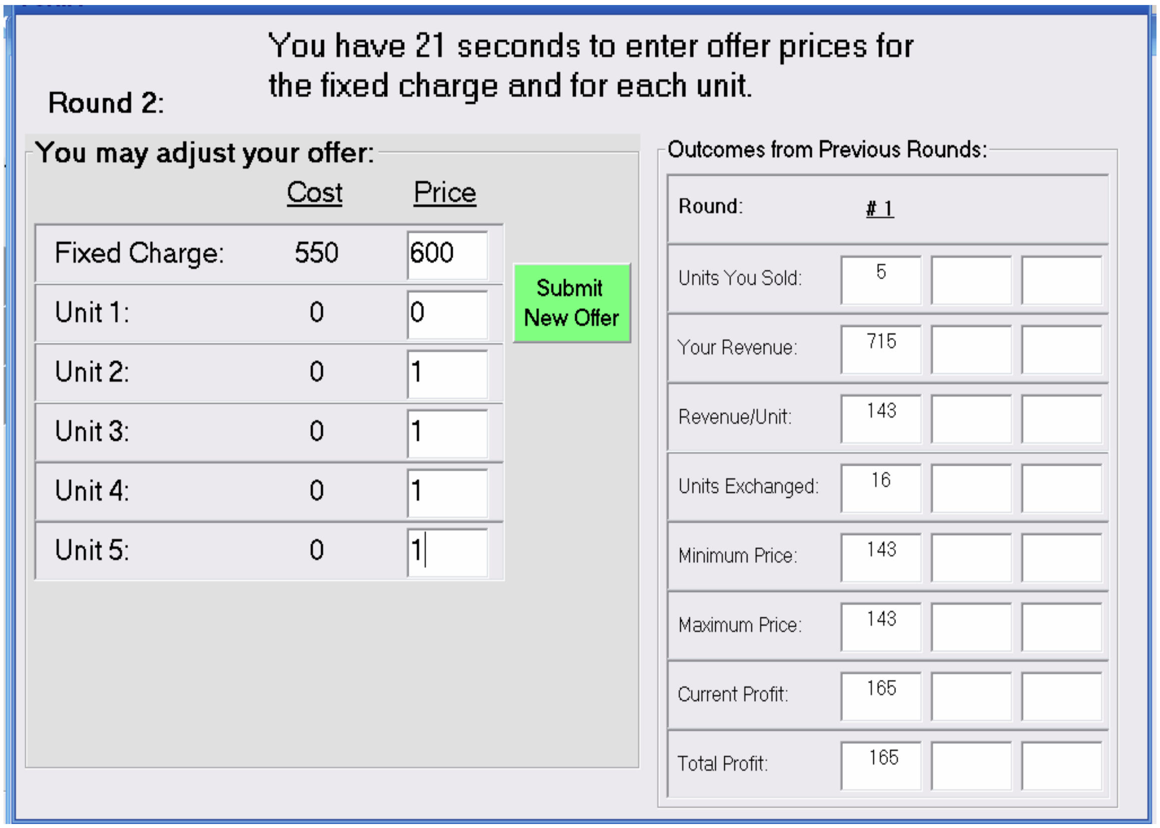

Figure: Round 2 User-Input Window for a Seller

Experimental Procedures

After the subjects sign consent forms, say, "You may now come in and take a seat at one of the computer terminals that is turned-on." While they are getting seated, say, "Go ahead and type-in the code that you see on the post-it note that you received." Subjects are prompted by the computer to type-in their unique code. Once you confirm that each subject entered his code, say "During the session, if you have a question, please raise your hand and I will come to you. Now we will move on to the instructions."

The General Instructions are put on the screen for subjects to see for five minutes. Read aloud the General Instructions. Afterward, say, "You can continue reading while I hand out three things: a pencil, a piece of scratch paper, and a copy of the page of instructions that is on your screen."

The Individual Instructions are put on the screen for subjects to see for three minutes. Say, “Now we will go to the second page of instructions, which you should read silently. You will have several minutes before we go on to the practice rounds.”

Say, “Does anyone have a question before we go on to the practice rounds?”

Subjects are shown the user-input screen for Practice Round 1. Say, “Go ahead and fill in your bid or your offer in the space provided. Please hit the submit button when you are finished.” Subjects have unlimited time to enter bids in practice rounds. If anyone has not hit submit after four minutes, ask, “Does anyone have a question before we go on?” Once all eight subjects click Submit, the server will run the Round 1 auction.

Subjects are shown the results screen for Practice Round 1. Say, “Now review the results of the first round. When everyone is ready, we can go on to the next practice round.” After two minutes, ask “Is everyone ready to go on to the next practice round?”

These steps are repeated for Practice Rounds 2 to 10. If it is Round 2, say, “When we get to the actual experiment, you will have 40 seconds to enter your bid or offer, but during the practice, you have as much time as you need.” In Round 2, when the results appear on their screens, say, “Now please review the results. When we get to the actual experiment, you will have about 25 seconds to review the results, but during the practice we will wait a minute and a half.”

The General Instructions and Individual Instructions are repeated.

Rounds 1 to 12 occur in the same way as the practice rounds, although they no longer have unlimited time after Round 1. They will have 45 seconds to review the results of the first round before moving to the second round. In Rounds 2 to 12, they have 40 seconds to fill in their bid or offer. They will have 27 seconds to review the results of rounds 2 to 12.

The Individual Instructions are repeated.

Rounds 13 to 24 occur in the same way as Rounds 1 to 12.

The Individual Instructions are repeated.

Rounds 25 to 36 occur in the same way as Rounds 1 to 12.

The Individual Instructions are repeated.

Rounds 37 to 48 occur in the same way as Rounds 1 to 12.

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Curriculum Vitae

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