Essays on Banking and Capital:  
An Agent-Based Investigation

A dissertation submitted in partial fulfillment of the requirements for the degree of  
Doctor of Philosophy at George Mason University

By

Pedro P. Romero  
Master of Arts  
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Director: Dr. Richard E. Wagner  
Department of Economics

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Fairfax, VA
Dedication

A mi muy querido hermano Marlon, por ser siempre fuente de inspiración, lealtad y tenacidad. Sin su ejemplo no hubiera terminado esta empresa.
Acknowledgments

I would have not being able to come here without the initial mentorship of Dora and Enrique Ampuero. They encouraged me to pursue graduate studies with their example and love for freedom. I am also thankful to Dr. Franklin López for all his intellectual support and friendship.

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ESSAYS ON BANKING AND CAPITAL: AN AGENT-BASED INVESTIGATION

Pedro P. Romero, PhD.

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Dissertation Director: Dr. Richard E. Wagner

How do institutional arrangements in banking affect the occurrence of crises?
The first two chapters present an endeavor in using new modeling techniques to
answer this question. Even if the results are not widely accepted, the way in
which the problem is tackled here is offered for consideration and debate. Is
capital the result of an evolving process that takes advantages of entrepreneurial
networks? The last chapter put forth a model wherein firms develop economic
ties with one another. By doing so, a market network unfolds along time as an
spontaneous process.

In the first chapter, I explore the occurrence of bank runs by developing a
sensitivity analysis to the model in Diamond and Dybvig (1983). I implement an
agent-based economic model to analyze different modifications and extensions
to the original. In 36 experiments based on three different versions of the one-
bank model the frequency of bank runs dropped from 42% to 17%. This was due to changes in the payoffs structure and social network effects whereby depositors go to the bank if at least three of their proximate neighbors went previously.

What is the role of interbank markets and central banks in coping with banking crises? In experiments using an agent-based framework with multiple banks and an interbank market. I found that when banks cannot interact, then runs in isolated banks occur with a higher frequency than when banks have equal market shares. That is, there are no runs escalating to systemic panics. In contrast, if one bank has a market share twice as big as the rest, runs spread. The presence of a central bank may unexpectedly increase the occurrence of bank runs. Institutional complexity helps to reduce the frequency of bank runs. Hence, decentralized institutional structures perform better than centralized ones.

The objective in this chapter is to implement a parsimonious agent-based computational model of economic networks whereby agents make strategic decisions based upon profits and information generated through their immediate social network. In this model firms are represented by nodes and the links between each pair of them are the result of a mutually advantageous economic decision. Therefore, links are two-sided or undirected. The economic decision is based on two elements, namely: a myopic profit motive and local information channeled through collaborating firms. Here I endogenize the formation and
deletion of links. Furthermore the number of firms (nodes) in the network at each time by allowing firms (nodes) to enter and exit the market. Centrality measures are reported together with firms’ profits. The evolution of the network yields higher connectivity and profits when the (positive) externality is high and the rule to exit the market more strict. The higher the network connectivity, the higher the overall profits of firms.
1. Bank Runs, Banking Contracts, and Social Networks

1. Introduction

The model of Diamond and Dybvig (1983) is perhaps the modern canonical statement of the claim that money won’t manage itself because a regime of free banking is subject to contagious bank runs and failures, wherein insolvency in one bank can spread to other banks that initially were solvent. Deposit insurance and various forms of regulation might serve as means of restraining such runs. Diamond-Dybvig (hereafter) is austerely simple, involving, among other things, a single bank that neither makes loans nor allows checking accounts. The point of this paper is not to challenge Diamond-Dybvig, but rather is to explore how computational modeling might be brought to bear on the relationship between agents’ environment and bank runs.

In this paper, I implement an agent-based computational model to analyze different modifications and extensions to the original model. By doing so, I can use discrete agents that have individual properties and follow decision rules according to the economic environment in which they are interacting. Macroeconomic as well as microeconomic causes of banking crises have been
discussed in the literature. Gorton and Winton (2002) present a thorough survey in this regard. My focus in this paper will be on the microeconomic causes of banking crises.

The relationship between business cycles and banking panics\(^1\) in comparing the National Banking Era and the Great Depression points out a small difference regarding when panics are leading or lagging the cycle. During the former most of the six\(^2\) panic occurrences identified in Calomiris and Gorton (1991) happened during the downturn and near to the peak\(^3\). In contrast, the first wave of bank panics during the Great Depression occurred after thirteen months of the turning point in the cycle (Duckenfield et al. 2006 Vols. 2-3; Friedman and Schwartz 1963). Also, Mishkin and White (2003) study the major stock market crashes in the twentieth century in the U.S. and report 15 episodes in which stock market crashes precede financial instability or distress; including banking panics (although without a clear pattern as a leading, coincident or lagged indicator of the business cycle in the U.S.). During the post Second World War period up to 2008; there had been 11 recessions of smaller magnitude (in terms of lost GDP) compared to the previous period according to the National Bureau

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1 Here I follow standard definitions to distinguish a ‘bank run’ as a localized liquidity crisis in one bank when the withdrawal rate is so large that it cannot be served; a ‘banking panic’ whereby several banks face generalized withdrawals that compromise their liquidity; and a ‘bank failure’ in which case a bank or more suspend payments and/or exit the market. See Selgin (1988), Calomiris and Gorton (1991), and Leijonhufvud (1998) for a broader taxonomy of economic crises.

2 Wicker (2000:xii) reports five by excluding the panic in 1896 due to its localized nature to Chicago and Minneapolis-St. Paul without propagating to the whole country.

3 The panic in 1873 anticipated the peak of the business cycle for one month.
of Economic Research (NBER); but only until the 1973 recession did bank failures occur. Also later in the 1980s during the savings and loan crisis and now in the U.S.-led financial crisis in the developed world.

Calomiris and Gorton (1991) test the operational hypotheses stemming from the two competing models that in the early 1980s tried to answer the following question, namely: “How can bank debt contracts be optimal if such contracts lead to banking panics?” 1991:107. The models were: a) random deposit withdrawals, and b) the asymmetric information. The seminal paper for the first strand of models is Diamond-Dybvig. While for the second one its origin is more diverse: Chari and Jagannathan (1988), Gorton and Mullineaux (1987), Diamond (1984), and Jacklin and Bhattacharya (1988) are the most relevant papers.

Random deposit withdrawals models focus on the liability side of the balance sheet of banks, i.e. deposits. There banks’ main role is to provide ‘liquidity’ that contributes to the smoothing pattern of individual consumption. On a pure theoretical basis this model requires two mechanisms to assure the occurrence of a bank run. These are a sequential-service constraint (Wallace

4 Another way in which this question can be posed that is closer to the current research on ‘emergent’ or bottom-up organizational and institutional processes is: How do banks spontaneously evolve in markets to provide liquidity and related financial services?
1988) and a lack of a secondary market for trading assets and bank liabilities (Jacklin 1987).

In the asymmetric information case the asset side of the balance sheet of banks is analyzed, but without any effect from the liability side. Here bank runs happen as a rational response by depositors that neither have full information about the quality of the loans of the banks nor lower transaction costs to monitor that aspect for every loan. Thus, a bank exists to monitor the quality of the loans of a pool of savers to borrowers. Bank runs occur when those savers or depositors are not sure about which banks are solvent.

In both of those cases ‘outside’ equity is omitted (Dewatripont and Tirole 1993). Dowd (1993) includes ‘inside’ equity provided by a bank owner, allowing him to conclude that with this modification bank runs are less likely. On the other hand, in this literature banks’ liabilities do not play any role as a medium of exchange, whether as inside money or outside money. Gorton and Pennacchi (1990) elaborate a model that can be deemed as an approximation of the ‘credit theory of money’ spelled out by Schumpeter (1939). Under a setting similar to that of Diamond-Dybvig, they derive how banks overcome the asymmetry of information between informed and uninformed traders by creating or offering a riskless security that can be used as a medium of exchange\textsuperscript{5}. It remains an open \footnote{Marimon, Nicolini and Teles (2003) present a model where inside money providers’ competition creates incentives that promote efficiency for the government’s supply of outside money.}
avenue for research the modeling of both sides of the balance sheet of banks and the effect for asset and liability management.

Calomiris and Gorton (1991) implement their empirical test with data from the National Banking Era in the U.S.. They proceed by distinguishing three opposing predictions yielded by the two models. First off, the random withdrawals model differs from the asymmetric information model over the source of shocks triggering the panic. In the former case an idiosyncratic shift in the money-demand is the cause of the panic, so unusual increases in withdrawals in the pre-panic periods should be observed. On the contrary, in the latter case the shocks might be falling stock prices, real-state prices, or those occurred in whatever assets mostly held in banks’ portfolio.

Secondly, bank failures or liquidations will come from regionally concentrated demand shocks channeled through the banking network, according to the random withdrawals model. For the second type of models high incidences of bank failures will more likely happen in regions that suffer from negative asset shocks. Lastly, both types of models differ in their predictions regarding the management of the crises. The random withdrawals model predicts that a discount window like the one provided by the Fed during the Great Depression should be a sufficient deterrent for banking panics. While the asymmetric

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6 For a fruitful reading in this line, see Diamond and Rajan (2001).
information model predicts that interbank transfers or some similar sort of collective action by banks will help to internalize and quickly solve bank runs to avoid turning into systemic panics.

All in all, Calomiris and Gorton (1991) develop an exhaustive statistical test for the three stages just described without implementing an explicit econometric test in either case (Gorton 1988, develops an econometric model to test the implications of the first set of predictions supporting the asymmetric information model). Their findings reject the predictions of the random withdrawal model during the National Banking Era and the Great Depression in favor of the predictions of the asymmetric information model. The historical work by Wicker (2000: 139-147) also supports these results but he remarks that some of the data used by Calomiris and Gorton may be incomplete and more work needs to be done to fill that gap.

Despite this, in this paper, I will not get rid of Diamond-Dybvig altogether. Rather I will relax some of their assumptions. By doing so, I will show how even in this tradition of models bank runs are less likely.

The rest of the paper is organized as follows. Section 2 presents a more specific review of the related literature. Section 3 presents the model I purport to
tackle these issues. Section 4 presents the results. Last section contains the summary and possible extensions to the model.

2. An Overview of Diamond and Dybvig and Related Models

In the Diamond-Dybvig model they assume a continuum of depositors with two types of them: impatient and patient ones. The model has three periods and the agents’ types are ‘discovered’—actually, randomly selected from a uniform distribution—in the second period. The agents interact in a coordination game with no mixed strategies. In the first period each depositor makes a deposit in the bank that has a linear production function with constant returns to scale. Also, the bank neither lends money nor owns equity. On the third period the bank’s investment matures providing positive returns. There is no uncertainty either.

The sequence of actions proceeds as follows: if only impatient agents withdraw during the second period then a bank run does not occur and the population coordinates on the Pareto superior equilibrium. Conversely, if patient agents imitate the behavior of impatient agents on the second period instead of waiting to withdraw in the third and last period, a bank run will occur and the population will coordinate on the second Nash equilibrium that is not Pareto optimal.
The following are some relevant extensions to this model to analyze the robustness of its conclusions. Making agents heterogeneous regarding their preferences and discounting rates instead of having only two types of agents’ populations that is tantamount to having only two agents. Increasing the number of banks (see below Temzelides 1997, for such an extension). Adding owner’s capital or equity to the bank’s balance sheet (Dowd 1993). Also including the lending activity of the banks (see Diamond 1984).

In the Temzelides (1997) model the original setup of Diamond-Dybvig is extended to a repeated game environment. Thus, the author is able to analyze the evolution of agents’ learning during the game and it is claimed that this reinforces the reduction in the likelihood of bank runs. This model also incorporates a case of multiple isolated banks, randomizes strategies of patient agents, bank size becomes a control variable, random matching between depositors and banks, banks are subject to demand shocks, there is uncertainty in payoffs due to the random matching process, and furthermore, introduces a small world network for agent(s)-bank(s) interaction for an alternative matching process.

Agents’ learning in the simple repeated game version allows them to coordinate longer on the Pareto superior equilibrium than when the game is played only for one-shot. Moreover, if the bank’s size increases, then the
population of agents coordinates mostly on the inferior Pareto equilibrium. On the other hand, under the local interaction rule, i.e. small world network, financial contagion is more feasible among banks.

In my first approximation to model bank runs within an agent-based computational model, I add heterogeneity across depositors regarding their preferences and discount rates. Initially, there is only one bank that is investing part of its idle funds in bonds that can be turned into cash by selling them in the secondary market. I also analyze how network topology can affect the feasibility of bank runs incorporating neighborhoods.

3. Implementation in an Agent-Based Computational Framework

This version of the model only includes a bank à la Diamond-Dybvig. A model with several banks will be introduced in Romero (2009). A key feature of this computational model involves the specification of the operating rules for any individual bank. The monetary base is all the wealth deposited by agents in the banks that can be withdrawn at any time.

A bank has a multiplicity of discrete depositors, most of whom at any moment will have positive balances on deposit. While a bank will want to keep reserves to maintain liquidity against claims for redemption by depositors, it will
be able to lend out some of its reserves. By doing this, however, also comes a risk of illiquidity that is not present when the bank provides only bailment.

To avoid complexities regarding financial firms and labor markets, I assume all firms are sole proprietorships. Up to now only individuals and a single bank populate the model. There is an initial (uniform) distribution of money among the individuals who in turn entrust their money to a bank. This model is spatial in character and I will specify the details of this below.

For each individual, receipts and expenditures are both subject to some random variation. Any bank will lend based on some myopic forecast on current experience regarding the behavior of its reserves. From this point of departure there are several experiments that can be performed. The first would be to assume that all depositors are subject to the same random variation. This would be a world of homogeneity and would map relatively directly into closed forms of modeling based on averages and representative agents. The challenge and opportunity for computational modeling would involve the presence of heterogeneity, and along several dimensions\(^7\). At any rate, here I seek to answer: under what circumstances a local bank run may or may not happen?

\(^7\) See Axtell (2000) where he points out three different reasons to use agent-based instead of equation-based models. My claim is that this model falls in his third category wherein writing down the equations does not shed light on the problem.
Next, I describe the details of the computational replication (implemented in Netlogo 4.0.2) and introductory modifications to the Diamond-Dybvig model (see Appendix A for pseudo-code, and Appendix B for a screenshot). In this initial version there is only one bank located in the center of the grid and 441 depositors\(^8\). There are two types of depositors: impatient and patient ones.

During the initialization of the model the depositors make their unitary deposits in the bank, then their types are randomly assigned out of an uniform probability of being an impatient agent. The parameterization of the computational model is summarized in Table 1, which is a base scenario that I will explore.

<table>
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<tr>
<th>Model Attribute</th>
<th>Value</th>
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<tbody>
<tr>
<td>Depositors, D</td>
<td>441</td>
</tr>
<tr>
<td>Banks, B</td>
<td>1</td>
</tr>
<tr>
<td>Initial Deposit per agent</td>
<td>1</td>
</tr>
<tr>
<td>Agent Type</td>
<td>p (impatient/deposited) = 0.5</td>
</tr>
<tr>
<td></td>
<td>p (patient/deposited) = 0.5</td>
</tr>
<tr>
<td>Withdrawals</td>
<td>Impatient-type = 0</td>
</tr>
<tr>
<td></td>
<td>Patient-type = 0</td>
</tr>
<tr>
<td>( r_1 )</td>
<td>1.2</td>
</tr>
<tr>
<td>( R )</td>
<td>2</td>
</tr>
<tr>
<td>Initial Bank’s Deposits</td>
<td>Sum initial deposits by all agents</td>
</tr>
<tr>
<td># Agents Withdrawing</td>
<td>n-served 0</td>
</tr>
</tbody>
</table>

\(^8\) Technically, the grid is a torus with 21 by 21 patches that does wrap either vertical or horizontally. Every patch is an agent (depositor), so that this is where the number 441 comes from. Because I am not incorporating any rule for agents’ movement or mutation this is enough for my analysis.
In the running stage (go procedure) impatient agents will start to withdraw a random proportion of the sum of their initial deposit plus a return. But this will be carried out sequentially in order, agent after agent. This allows me to introduce the 'sequential service constraint' of the original model.

Let the payoff for impatient agents withdrawing before those who are patient be:

\[
V_1(f_j, r_1) = \begin{cases} 
1.2 & \text{if } f_j < \frac{1}{r_1} \\
0 & \text{if } f_j \geq \frac{1}{r_1} 
\end{cases}
\]  

(1)

and the return for patient agents be:

\[
V_2(f, r_1) = R \left( \frac{1 - r_1 f}{1 - f} \right)
\]

(2)

where \(f_j\) is the number of bank's depositors being served at time \(t\) as a fraction of the total number of initial depositors and \(r_1\) is the gross return for those withdrawing before the bank's investment has matured otherwise their return is \(R\). The following relationship is established: \(1 \leq r_1 < R\). Finally, \(f\) is the fraction of impatient agents in relation to the total number of agents. Equations (1) and (2) are slightly different from equations (2) and (3) in Diamond-Dybvig model.

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9 Since Netlogo 4.0 exists the possibility of asking agents in an orderly fashion throughout the grid one by one.
There are also two regimes for the rates of return, which are: 'fixed' and 'random'. The first follows the Diamond-Dybvig assumptions regarding homogeneity across agents. The latter allows me to analyze heterogeneity across agents. In Table 1, the case for \( r_1 \) fixed (1.2) and equal across agents is presented. Also, \( R \) here is set equal to 2. For the second case the rates of returns are randomly drawn from a uniform distribution: \( r_1 (1, 1.2) \) and \( R (1.2, 2) \).

Patient agents have a higher fitness or payoff from which they can consume (withdraw). That is \( V_2 > V_1 \), given the sequential service constraint, the availability of funds in the bank, and the rate of withdrawals. Below I describe the model's agents and their features.

*Depositors*: each depositor has information about its deposits, amount withdrew, payoff or fitness, returns for withdrawing at an early or later date, and an account to register how much is left in the bank.

*Bank(s)*: register their initial deposits, the amounts withdrew by depositors at every time-step during the simulation, how many agents have been served, and their final balance. The bank invests according to the following condition:

\[
I_t(b_{t-1}, R) = \begin{cases} 
  R & \text{if } f_j \leq f_{\text{imp}} \\
  0 & \text{if } f_j > f_{\text{imp}} 
\end{cases}
\]  

(3)
where \( I_t \) is the bank's investment per period; that takes its previous positive balance \( b_{t-1} \) to be invested at the rate of return \( R \)—which is the same gross rate of return that agents will receive for being patients. This will happen so long as the number of agents withdrawing before the investment matures \( f_j \) is less than or equal to the total number of impatient agents \( f_{\text{imp}} \). Finally, if the bank goes bankrupt the simulation stops.

**Main interaction rule:** There are two rates of return to determine depositors’ payoffs and accounts. Two different regimes for agents’ consumption can also be chosen. Firstly, agents consume altogether their respective payoffs, \( V_1 \) and \( V_2 \), every time they withdraw. In the second case each agent withdraws a (random) proportion \( w_j (0, 1) \) from their payoffs.

Once all the agents are initialized the depositors or bank’s customers have to decide whether to withdraw at every period in the simulation. Impatient agents withdraw first, and then patient agents have to decide whether to withdraw. The decision of withdrawing now or later also depends upon the following relationship taken from Diamond-Dybvig, that is, that the proportion of customers being served with respect to the total number of customers may or not be less than the inverse of the return for withdrawing earlier, as explained in equation (1).
The bank balances its account and keeps serving its customers until it has run out of money. Every customer can withdraw from the bank only after it has served the previous customer. This is not a concurrent procedure.

4. Results

In Figure 1, I present a computational model based on the conditions exposed above. In this figure there are four panels. The blue line tracks the change in final balances or net deposits in the bank. The black line records total withdrawals from both types of agents. The green line depicts only the total withdrawals from agents (impatient or patient) withdrawing in earlier periods. Finally the brown line depicts those withdrawals from those agents (only patient ones) who wait. In panel a) after two time-steps the bank ran out of savings or liquidity to serve its clients. There were only 336 agents who could be served during this experiment. The remaining 105 could not even get their initial deposits back. This first result obtained with homogenous consumption and rates of return and with $r_1 > 1$, is the same as in Diamond-Dybvig.
Figure 1. Experiments with Original Diamond-Dybvig.

In panel b) the bank does not run out of funds. The variables achieve a stationary equilibrium whereby total withdrawals hover over 300 value units. It is important to observe that the only modification in this experiment from the previous one is that the consumption schedule per agent is variable or heterogeneous across population. In panel c) again with constant consumption schedule per agent but with heterogeneous rates of returns after five time-steps the bank ran out of assets and only could serve to 347 agent depositors. This is 127 more than the total number of impatient agents, i.e. 58% greater. Thus, patient agents withdrawing earlier than they were supposed to do it bring about...
the bank run. There were 94 depositors who were unable to withdraw after the bank went bankrupt. Lastly, in panel d) with variable consumption schedule and heterogeneous rates of returns across agents a bank run does not occur. The bank’s balance and total withdrawals hover over 200. Another stationary equilibrium is again achieved.

In Figure 2, I present a modified version of the previous computational model whereby the depositor-bank contract is modified to have a different payoff structure. In this case, the payoffs for each period are given by:
where \( f_j \) is the number of bank’s customers being served at time \( t \) and \( c_1 \) is the optimal consumption for those agents withdrawing at period one, otherwise they consume \( c_2 \) in period two. The latter is equal to the second expression in the payoff function for \( V_2 \). The following relationships hold: \( c_1 < c_2, c_1 \geq 1, \) and \( R > 1 \).

Finally, \( f \) is the total number of impatient agents. Besides these changes the other characteristics of the agents and the rest of the simulation environment remains the same as before. These payoffs are simpler than those in equations (1) and (2) and yield different results as I will report on Table 2.

The plots in Figure 2 show the results of the same four experiments I implemented previously. In panel a) after three time-steps the bank did run out of funds. The bank served 365 clients, that is 76 of them were unable to get any funds back. In panel b) there is not a bank run, after fifty time-steps. In this case the consumption schedule per agent was heterogeneous across agents. In panel c) with heterogeneous rates of return and the same consumption pattern for all agents a bank run does not occur after fifty time-steps. The bank’s balance

\[
V_1(f_j, r_1) = \begin{cases} 
1 & \text{if } f_j > f \\
\frac{c_1}{1} & \text{if } f_j \leq f
\end{cases}
\]

\[
V_2(f_j, R) = \begin{cases} 
R & \text{if } f_j > f \\
R\left(1 - \frac{c_1 f}{1 - f}\right) & \text{if } f_j \leq f
\end{cases}
\]
declined to 5 value units after three time-steps, but then recovered to fluctuate around 45 value units. Note that in this case earlier withdrawals are always higher than later ones. Finally, in panel d) with rates of return and consumption patterns heterogeneous across agents a bank run does not occur after fifty time-steps. The bank remains liquid with about 200 units in available funds.

Figure 3. Experiments with Diamond-Dybvig and Neighborhoods.

In Figure 3 another modification was added to the previous setting. This time I inserted a social network component to the model. Impatient agents

\[ I \text{ implemented a Moore neighborhood with a radius of one that has, at most, eight neighbors for the agent at the center.} \]
make their decision to withdraw first and then patient agents ask to three of their eight neighbors—whether patient or impatient—if they have already withdrawn any funds from the bank in order for them to start to withdraw. Again, I experimented with these four variations as in both previous cases. In this model only two bank runs occur. The one depicted in Figure 3 is the first case with fixed interest rates and constant consumption across agents (see panel a). Also, total withdrawals are higher than the bank’s final balance in panel c), in which consumption patterns are constant for the agents. The reverse is true when this is changed to a heterogeneous regime across agents, panels b) and d). The second bank run is described next.

<table>
<thead>
<tr>
<th>p(impatient/deposited)</th>
<th>Scenario a</th>
<th>Scenario b</th>
<th>Scenario c</th>
<th>Scenario d</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>Run</td>
<td>no run</td>
<td>Run</td>
<td>no run</td>
</tr>
<tr>
<td>0.5</td>
<td>Run</td>
<td>no run</td>
<td>Run</td>
<td>no run</td>
</tr>
<tr>
<td>0.75</td>
<td>Run</td>
<td>no run</td>
<td>no run</td>
<td>no run</td>
</tr>
<tr>
<td>Model 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>Run</td>
<td>no run</td>
<td>no run</td>
<td>no run</td>
</tr>
<tr>
<td>0.5</td>
<td>Run</td>
<td>no run</td>
<td>no run</td>
<td>no run</td>
</tr>
<tr>
<td>0.75</td>
<td>no run</td>
<td>no run</td>
<td>Run</td>
<td>no run</td>
</tr>
<tr>
<td>Model 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>Run</td>
<td>no run</td>
<td>no run</td>
<td>no run</td>
</tr>
<tr>
<td>0.5</td>
<td>Run</td>
<td>no run</td>
<td>no run</td>
<td>no run</td>
</tr>
<tr>
<td>0.75</td>
<td>no run</td>
<td>no run</td>
<td>no run</td>
<td>no run</td>
</tr>
</tbody>
</table>

In Table 2, I show the overall results not just for the four cases or scenarios that I already described for each of the three different models, but also for two additional variations when the probability of being an impatient agent is 20
0.2 and 0.75. Thus, I ran 12 experiments per each model or 36 experiments overall. In model 1 there were five occurrences of bank runs (42%); in model 2 only three occurrences (25%); and in model 3 only two of these occurrences (17%). Hence, the simple modifications that I have added to the first model reduced the frequency of bank runs. Especially when I modified the topology of the 'artificial' world by adding Moore neighborhoods this frequency dropped by 60% compared with the first model.

5. Conclusions

I have implemented in these simulations changes that allow me to explore the dynamics of the Diamond-Dybvig model within not just a repeated version but also within a population of heterogeneous agents. The aggregate dynamics that I analyzed was the result of the individual or agent-based interacting behavior subject to the rules of the simulated environment. Firstly, I developed a discrete framework for agents and time rather than keeping the unrealistic continuity assumption. Secondly, I introduced the important effects of social networks and provided a better rationale for social interaction.

To the underlying question, what are the causes of localized bank runs? The model presented hitherto does not provide anything that can be considered as a definite answer. Its contribution lies in taking the negative equilibrium
reported in Diamond-Dybvig and showing how by relaxing and modifying their original model bank runs are less probable. Summarizing, in this paper bank runs don’t happen at all when there is heterogeneity in the agents’ consumption schedules. Even if the interest rates where fixed across agents in any of the three models presented here. On the other hand, when consumption is homogenous across population the results are more mixed. By looking the scenarios a) and c) in Table 2, runs are still present in the first two models even if interest rates are heterogeneous. But its frequency is decreasing, of course. Although in the third model this is not true anymore, since the frequency drops to zero when there is heterogeneity in interest rates. Between this and the second model there is no difference in the frequency of bank runs in scenario a). The same happens if we compare the results between figures 2 and 3. As I mentioned before, the most relevant difference in all these models is that introducing social networks among depositors as a variable to make withdrawal decisions seriously reduced the frequency of bank runs overall.

I explore a multibank setting that builds on the one-bank developed here in Romero (2009). In that context I will study the presence of interbank markets, the effects of size on a particular bank, and the policies of a central bank. Within this framework, nonetheless, a further endogeneization of agent types, more relevance to the asset side of the bank, among others are part of my own research agenda.
References


Schumpeter J. (1939) *Business Cycles*.


2. Banking Crises and Institutional Arrangements

1. Introduction

When do banks emerge? Whenever credit and monetary transactions within firms dominate transactions within markets (Coase 1937). In other words, whenever it is cheaper to develop contracts within an organization that engages in credit and exchange, instead of contracting individually on a very short-time basis. Banks bring advantages of specialization and economies of scale to credit, exchange, and transfer activities (de Roover 1974, Crouzet 2001). These characteristics are what distinguish them from other firms. The activities of credit and exchange have evolved a great deal since the twelfth century when Genoese and Venetian bankers were inventing the financial instruments and techniques that are still in use today. Monetary and insurance services are byproducts of this evolution.

Diamond and Rajan (2001) make the case that banks are special because they provide liquidity—not just to other entrepreneurs by financing their projects, but to the bank’s own creditors or depositors. Banks create liquidity on both sides of the balance sheet at the cost of a run prone financial structure. This banking
contract would serve to solve the commitment problem between the depositors and the banker; that is, providing to the latter with funds at a lower cost subject to the feasibility of a run. However, in a thorough review of the theoretical and empirical literature on financial intermediation, Gorton and Winton (2002) claim that the industrial organization of banking usually includes elements of instability, but that banks per se do not\textsuperscript{11}. The goal of this paper is to understand whether certain institutional arrangements are more prone to generate banking crises. Specifically, it focuses on the role of interbank markets and central banks in coping with banking crises.

In a study of historical experience with bank regulation in the United States and international comparisons, Calomiris (1993) observed, “The central lesson of these studies is that instability is associated with some historical examples of banking that had common characteristics; it is not an intrinsic problem of banking per se.” p. 3 He concludes that instability arises from the organization of the banking industry, not the nature of the banking contract itself. Probably, the difference between Calomiris’s empirical results and the results Diamond finds in several theoretical papers, (Diamond and Dybvig 1983, Diamond 1984, Diamond and Rajan 2001) is that Diamond reduces the actors in his models to a ‘representative’ bank or a ‘continuum’ of agents that behave as banks.

\textsuperscript{11} The liquidity that a bank à la Diamond creates is inside money, not ‘fiat money’ that is usually considered to be outside money (Selgin and White 1996: 85-6, and Mises 1980: 278-338).
Here I present a model of a multibank system where banks and depositors are represented by discrete agents within an object-oriented computational framework (see Epstein and Axtell 1996, Weiss 2000), instead of by a representative agent or by a continuum of agents. The objective is to explore the effects due to the presence or lack thereof cooperative arrangements among banks on banking panics. In the first extension of the model, competitive banks are not isolated; rather, they operate within webs of associations and cooperative relationships, as well as creating multi-branch structures. Since branch banking and cooperative associations such as clearinghouses accomplish much the same task regarding the maintenance of liquidity, my model works with an association from within an environment of otherwise independent banks. The rules of association generally map into risk-sharing insurance arrangements. This computational model should generate less insolvency in the presence of such clearinghouse arrangements.¹²

A next extension is to include central banking in the model’s environment. How do things differ when a central bank exists? The central bank must be described by a different rule of operation than what pertained to

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¹² One feature of such arrangements was usually controls placed on individual bank portfolios as a condition for belonging to the association. Such controls relieved some of the moral hazard that would have otherwise resulted.
clearinghouses\textsuperscript{13}. It is also necessary to pay attention to the central bank’s budget constraint.

Laeven and Valencia (2008: 24-5) found that there were 124 systemic banking crises between 1970 and 2007 among 101 developed and developing countries. The fiscal costs of these crises were as high as 55.1\% of GDP, but averaged 13.3\%, while output losses ranged from nil to 98\% of GDP. If the US savings and loans is excluded together with the 2007 onset of the recent crisis in UK and US, then there were 121 systemic banking crises in 99 countries.

Low volatility of inflation and output in most developed countries save Japan between 1984 and 2006 led economists to term the period as ‘the Great Moderation’ (Bernanke 2004). It seemed as if banking crises and deep recessions in advanced economies were things of the past. But financial instability has occurred even in times of low price volatility and booming output, not just in the US during the Great Depression but also in other countries and times—e.g. Korea and Japan in the late 1980s and 1990s. Borio (2006) presents a compelling case for prudential policies even during these tranquil times\textsuperscript{14}.

\textsuperscript{13} Gorton and Mullineaux (1987) describe how private commercial-bank clearinghouses worked originally in New York.

\textsuperscript{14} It is not the first time that this has happened, though. Bronfenbrenner (1969) collects a series of papers from renowned economists where the title of the book reflects what was their view at that time after almost two decades of stability: \textit{Is the Business Cycle Obsolete}? Although their answer was not an absolute negative, Bronfenbrenner (1969: vii) reported “that greater reliance by ‘politicians’ on economic “technocrats,” particularly on econometric macroeconomists, might soon render the cycle obsolete.” A similar optimism was around in 1997 according to Fuhrer and Schuh (1998) just before the East Asian crisis.
In Romero (2009) I presented a one-bank model with multiple discrete agents as depositors. That model had three different versions but all of them were based on the canonical model of Diamond and Dybvig (1983). The most important of the three versions was the last one, which included *social networks* were included in the decision-making processes of depositors. Moving from the original banking contract à la Diamond and Dybvig to the version with depositor networks, the frequency of bank runs dropped from 42 percent to 17 percent, that is by 60 percent.

Here I will build on the modified banking contract used in the second and third versions in Romero (2009). In addition, I will introduce a multiple-bank setting, each bank having a distinctive clientele and constraints. Table 3 displays other models in the literature that deal with the specifics of an interbank market, a central bank, or financial contagion. Except for Temzelides (1997), all of those models make use of a continuum of agents. The model presented here adds to this literature a model wherein both banks and depositors are discrete and can have heterogeneous attributes and decision rules.
Table 3: Selected multi-bank models

<table>
<thead>
<tr>
<th>paper</th>
<th>Interbank Market</th>
<th>Central Bank</th>
<th>Continuum (c)/Discrete (d)</th>
<th>Contagion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Allen and Gale (2000)</td>
<td>X</td>
<td>X</td>
<td>C</td>
<td>X</td>
</tr>
<tr>
<td>Bhattacharya and Gale (1987)</td>
<td>X</td>
<td></td>
<td>C</td>
<td></td>
</tr>
<tr>
<td>Champ et. al. (1996)</td>
<td></td>
<td></td>
<td>C</td>
<td></td>
</tr>
<tr>
<td>Rochet and Tirole (1996)</td>
<td>X</td>
<td>X</td>
<td>C</td>
<td>X</td>
</tr>
<tr>
<td>Smith (1984)</td>
<td></td>
<td>X</td>
<td>C</td>
<td></td>
</tr>
<tr>
<td>Smith (1991)</td>
<td></td>
<td>X</td>
<td>C</td>
<td></td>
</tr>
<tr>
<td>Temzelides (1997)</td>
<td>X</td>
<td></td>
<td>D</td>
<td>X</td>
</tr>
<tr>
<td>This paper</td>
<td>X</td>
<td>X</td>
<td>D</td>
<td>X</td>
</tr>
</tbody>
</table>

The paper is organized as follows. The next section presents a multibank model wherein banks can be isolated or participate within an interbank market. Section 3 adds a central bank as another agent to the environment and studies the interaction of the interbank market with some aspects of monetary policy by a central authority. Section 4 discusses some issues related to the implications of the results presented here and the methodology of the paper. The last section concludes.

2. Multibank Model

There are n depositors and m banks. Each depositor keeps track of her initial deposits, amount withdrawn, payoffs or fitness, returns for withdrawing at
an early or later date, and amount left in her bank account. The payoffs for a depositor are given below and they are the same as the ones presented in the second version of the model in Romero (2009). Let the payoff for impatient agents be:

\[
V_1(f_j, r_1) = \begin{cases} 
1 & \text{if } f_j > f \\
 c_1 & \text{if } f_j \leq f 
\end{cases}
\]  

(1)

and for patient ones be:

\[
V_2(f_j, R) = \begin{cases} 
R & \text{if } f_j > f \\
R \left( \frac{1 - c_1 f}{1 - f} \right) & \text{if } f_j \leq f 
\end{cases}
\]  

(2)

where \( f_j \) is the number of depositors being served at time \( t \), and \( c_1 \) is the efficient consumption allocation for those withdrawing at the early period. Otherwise they will consume \( c_2 \) in the next period, which is equal to second expression in the payoff for \( V_2 \). The total number of impatient depositors is \( f \). Finally, the following relationships hold: \( c_1 < c_2, c_1 \geq 1, \) and \( R > 1 \).

The payoffs are the same as those in Diamond and Dybvig (1983: 415). They argued that a proportional tax levy on the wealth held at the beginning of period 1 can be used to finance a deposit insurance scheme (stated in their second Proposition). Deposit insurance generates incentives for patient depositors to wait until their bank’s investment matures no matter what other
depositors do. This result should hold even if the fraction of impatient depositors is stochastic. Nonetheless, I showed in Romero (2009) that even with these payoffs bank runs occurred in three of the 12 experiments run with the model.

After explaining the attributes for each bank agent, I will describe a slight modification to this banking contract (see Appendices A and B for pseudo-code and a model screenshot).

In the model, banks register their initial deposits, the amounts withdrawn by their depositors at every period during the simulation, how many depositors have been served, depositors’ final balances, and bank’s outstanding balance. A bank again will invest so long as it has a positive balance after serving the depositors who decided to withdraw at that period, and so long as the queue size \( f_j \) is less than or the same as the number of impatient depositors \( f_{\text{imp}} \) in the total population. Thus, this process is given by:

\[
I_t(b_{t-1}, R) = \begin{cases} 
R & \text{if } f_j \leq f_{\text{imp}} \\
0 & \text{if } f_j > f_{\text{imp}} 
\end{cases}
\]  

where \( I_t \) is the bank’s investment per period and \( b_{t-1} \) is the bank’s previous positive balance, which earns a return of \( R \)—which is the same as the gross rate of return that patient depositors will receive when the investment matures. If the bank goes bankrupt and depositors cannot be served the simulation stops.
The model has four banks. Each bank has no more than ten customers. Thus, there is a banking market with four banks and forty depositors. These numbers are large enough to illustrate what happens with multiple agents, yet small enough that one can readily examine each agent’s behavior. Again, impatient agents withdraw first, and then patient depositors have to decide whether to withdraw, since they are the ‘strategic’ agents. In this version of the model, the payoff structure was modified according to equations (1) and (2). The decisions whether to withdraw depend simply on the size of the queue, and the payoff for consuming earlier is always lower than the payoff from waiting. The extension is merely a modification of the rule under which patient agents make their decisions whether to withdraw based not just on the size of the queue, but also on whether the interest rate the bank pays on deposits exceeds the depositor’s ‘subjective’ interest rate.

My aim here is to answer the following questions: Under what conditions can a liquidity crisis in a given bank spread or be contagious to others? How fast does this occur? To make this operational the model contains an interbank market that allows banks that lack sufficient funds to pay all customers in the withdrawal queue to borrow money from any other bank that has a positive balance. After serving its customers the bank will be required to repay the loan with interest. If the bank is unable to repay its debt and/or to serve its customers,
it goes bankrupt. Customers stop withdrawing from the bank if they have consumed all of their savings from it.

Table 4: Multibank Model

<table>
<thead>
<tr>
<th></th>
<th>No Interbank Market</th>
<th>Interbank Market</th>
<th>One Big Bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>#Patient</td>
<td>16</td>
<td>16</td>
<td>14</td>
</tr>
<tr>
<td>#Impatient</td>
<td>24</td>
<td>24</td>
<td>11</td>
</tr>
<tr>
<td>Run</td>
<td>Yes</td>
<td>None</td>
<td>Only big one</td>
</tr>
<tr>
<td>Time-step/period</td>
<td>6</td>
<td>25</td>
<td>4</td>
</tr>
</tbody>
</table>

Note: Constant consumption and heterogeneous interest rates across customers and banks and with \( p(\text{impatient/deposited}) = 0.5 \).

[Referential runs]

Table 4 shows results for three different experiments run within this version of the model. In the baseline scenario of no interbank market, each bank is isolated from the other banks and, in turn, their customers. In the second period, two of the four banks cannot keep serving their clients. In the third period, another bank ‘fails,’ and by the sixth period, the last bank also stop serving its clients. Thus, there is an overall bankruptcy of; i.e. a banking panic; the system that takes place gradually. This banking panic, though, is due neither to a contagion effect brought about by customers sharing information, nor from a localized bank run spreading to the whole system.

The second case (the second column in Table 4) contains a basic ‘interbank market’ to explore how such an institutional environment can facilitate or discourage financial contagion. The interest rate in the interbank lending
market is arbitrarily set at 0.01%, and each bank cannot borrow more than 10 percent of the funds owned by the lending bank. In the simulation, bank runs did not occur in any of the banks. This result was surprising, since I expected that ending the isolation of banks and their customers would result in contagion due to a bank’s financial fragility spreading to other banks. Each bank determines its own interest rate policy and decides whether to borrow from a more liquid bank. The decision whether to borrow depends on how many impatient versus patient agents each bank has in its queue and what are the depositors’ particular ‘subjective’ interest rates expected from trading with the bank.

In the third and last case I present an extension of the second case. Like the second case, it contains an interbank market, but it reduces the number of customers from the initial 40 to 25. Then, I allocate the customers arbitrarily to make sure that only one of them will get 10 customers and the rest only 5 per bank. By doing so, I get an interbank market with one of them twice as big in customers and liabilities (deposits) than the rest. This resulted in another unexpected result, which is a bank run at period 4 only for the bigger bank while the smaller banks were able to serve all of their customers. One interesting aspect of the extension is that before running out of liquidity the bigger bank lent money to another smaller bank that could serve its customers.
3. A New Agent as a Central Bank

Now let us add a central bank to the previous multibank model and its interbank market. The characteristics of the central bank are the following: (a) it controls the monetary base of the economy; (b) it collects the reserves from the commercial banks; (c) it establishes the legal reserve ratio; (d) it determines its policy for a discount rate; and (e) it can lend money to any of the commercial banks. Its balance is the sum of the monetary base plus the total reserves deposited by the commercial banks.

This extension of the model allow us to analyze the interaction between two important institutional features of financial systems in many countries today; a central bank and an interbank lending market. The central bank has three instruments for implementing its policies: altering the quantity of the monetary base; changing the minimum legal reserve ratio for commercial banks; or changing its discount rate below or above the fixed interbank market rate of 0.01% assumed in the previous version of the model. I develop experiments based on the different policy alternatives for the central bank and the probability of depositors being impatient. The results for the frequency of bank runs are reported in Table 5.
Table 5: Effects of adding a Central Bank

<table>
<thead>
<tr>
<th>Panel (a): Reserve ratio 2%</th>
<th>p(impatient)</th>
<th>0.5</th>
<th>0.2</th>
<th>0.75</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MB</td>
<td>MB</td>
<td>MB</td>
<td>MB</td>
</tr>
<tr>
<td>CB-rate %</td>
<td>5</td>
<td>8</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>0.008</td>
<td>25%</td>
<td>0</td>
<td>0.008</td>
<td>50%</td>
</tr>
<tr>
<td>0.012</td>
<td>25%</td>
<td>0</td>
<td>0.012</td>
<td>50%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel (b): Reserve ratio 30%</th>
<th>p(impatient)</th>
<th>0.5</th>
<th>0.2</th>
<th>0.75</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MB</td>
<td>MB</td>
<td>MB</td>
<td>MB</td>
</tr>
<tr>
<td>CB-rate %</td>
<td>5</td>
<td>8</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>0.008</td>
<td>25%</td>
<td>0</td>
<td>0.008</td>
<td>0</td>
</tr>
<tr>
<td>0.012</td>
<td>25%</td>
<td>0</td>
<td>0.012</td>
<td>0</td>
</tr>
</tbody>
</table>

Notes: Percentages are proportions of bank runs. Runs have up to 2800 periods. The payoff structure is the same as in the multi-bank model. MB = monetary base. CB = central bank. [Referential runs]

The monetary base can be either 5 units or 8 units in a period; the central bank’s interest rate can be either 0.008% (below the interbank market rate) or 0.012% (above the interbank market rate); the probability of a depositor being impatient can be 0.5, 0.2, or 0.75; and the reserve ratio is fixed across banks at 2%. The results is $2 \times 2 \times 3 = 12$ experiments, shown as the gray boxes in panel (a) in Table 5. The monetary supply with a central bank present in the model is given by adding the monetary base (5 or 8 units), and the total deposits of the banking system at any period (initially set at 40 units).
Note that with a similar reserve ratio of 2% for all the commercial banks and independently of what is the probability of a depositor being impatient, or what is the central bank’s interest rate; whenever the monetary base increases from 5 to 8 units there are no bank runs at all. Thus, the central bank fulfills its role of lender of last resort.

When the monetary base is only 5 units there are bank runs but in no more than 50% of the banks. If the probability of a depositor being impatient is 0.5 or 0.75 the proportion of bank runs is the same; i.e. 25%. Why is that the proportion of bank runs does not increase when there are probably more impatient depositors in the population? Because the payoffs per depositor actually goes down, since each depositor may be withdrawing earlier and more frequently but the average withdrawal per depositor is lower. In contrast, when that probability goes down to 0.2 unexpectedly the proportion of bank runs increases up to 50% of the banking system. Precisely because of an increase in the average withdrawal per depositor now that there are probably more patient depositors in the population. It is also important to notice that the central bank’s interest rate does not play any role in affecting these results.

Panel (b) in Table 5 shows the results for the same 12 experiments presented in panel (a). The difference is that the reserve ratio now is fixed at 30% for all the banks. Also, in this case whenever the monetary base is increased
from 5 to 8 units, bank runs do not occur. This result holds across the three different probabilities for being an impatient depositor (0.2, 0.5, and 0.75), and for the two different levels of the central bank’s interest rate (either below 0.01% or above it).

A main difference in panel (b) with respect to panel (a) is that no bank runs occur when the probability of being impatient is 0.2 or 0.75 for any amount of monetary base; i.e. 5 or 8 units. Thus, when the reserve ratio increases from 2% to 30% the proportion of bank runs decreases to nil for those values of the probability of being impatient, or any value of the monetary base or the central bank’s interest rate.

The results do not change, though, when the probability of being an impatient depositor is 0.5 in either panel. That is to say there is still a 25% of bank runs in the banking system when the monetary base is only 5 units.

Last but not least, why do bank runs still occur when I have a central bank and an interbank market working together? First, each commercial bank balances its accounts by deducting reserves deposited in the central bank. Secondly, each can borrow no more than 10 percent of the outstanding balance of the central bank at every period. Loans from the central bank and the interbank market are scheduled to pay in the next period plus any interest out of
any remainder in banks' balances. The main difference with the previous model, which has only an interbank market and where no runs occurred, is that in this version the reserves are centralized in the central bank and are no longer at the disposal of each of the banks competing in the interbank market for funds. Banks incur debt first by borrowing in the interbank market, then they proceed to ask to the central bank for any further loans. However, if any bank still has no required funds from any other bank and does not have money to keep serving its depositors it can get the funds from the central bank anyway. Since liquidity problems also arise in a sequential fashion in the banks, the central bank who now centralizes the reserves of the system can provide funds to one bank at a time. Hence, the central bank is also subject to a sequential service constraint.

4. General Implications

I have implemented agents within a microeconomic environment and studied their statistical aggregate patterns. To some extent these patterns are ‘emergent’ in the sense of Epstein and Axtell (1996) because they were not imposed upon the agents’ behavior. The patterns ‘grow up’ from the microeconomic structure in which the agents are embedded. Because the models also include interaction between depositors and banks (and in the third
In each of my models agents’ interaction occur within a set of rules based on economic behavior. The rules were part of the design of the environments for each model. Can the rules themselves also be the result of an emergent process? On one hand, this can be a question answered by evolutionary computation or a more stylized agent-based model such as Axtell (1999). There Axtell shows how firms are ‘emergent’ organizations after individual workers join or leave a firm. On the other hand, one can provide a rationale for that process from an evolutionary economic point of view. I take the latter approach here.

In the model of multiple banks I experimented with a version in which there was neither an interbank market nor a central bank. The isolated banks did not pool reserves when liquidity was scarce. Their behavior was like that of a primitive unit-banking system. A clearinghouse association is an organization that purports to overcome the lack of pooled reserves for a banking system. The clearinghouse and the appearance of an interbank market for loans explains the evolution towards a more integrated system that allocates reserves throughout all banks by portfolio adjustments.
How could these institutional solutions emerge? In the version of the model where banks were isolated, every time that there was a big increase in demand for withdrawals individual banks suffered important reserve losses that led to banking runs. Some banks failed while others did not until later. Banks with excess reserves could not increase profits by lending to other banks with lack of liquidity. It was as if an opportunity for increasing business was not being exploited. Here lies the economic origin of the interbank market. The development of more institutionalized forms to cope with liquidity risks is rather the result of a trial and error process. After the banking industry suffers massive losses or panics, a group of bankers may decide to establish clearinghouse associations to reduce the transaction costs of check clearing and transfer of net balances, and, more importantly to pool reserves to improve liquidity across the banking industry.

This gives place to the distinction between members and non-members of these types of associations or private clubs that provide public goods to members. This is important for naturally test under what scheme banks may reduce the overall risk of panics. Due to a unitary banking industry all the network externalities that a branch-banking industry may offer under clearinghouses will be absent. At a localized level member banks will be covered even in a unitary system by the pooling of reserves with all the other local banks also participating of this type of associations.
In the models, I have not yet incorporated relevant industry characteristics such as branch banking. Calomiris (1992) and Ramírez (2003) present evidence for the pre-Great Depression period comparing branching regulations across the U.S. and in Virginia (which allowed branching) versus West Virginia (which did not). Their results show that banks in states that allowed branching were more resilient to agricultural or seasonal crises than banks in states that did not allow branching. An evolutionary account of banking institutions should make room for an explanation of the different industrial architectures that may flourish within different rules, and other set of institutions belonging to property rights and monetary arrangements. I leave such extensions for future work.

But even more resilient industrial architectures may not eliminate the risk of failure. Tussing (1967) presents a compelling case that fewer resources will be wasted if banks were treated like any other commercial firms when they fail. His claim is another way to argue that if bankers know that they will be bailed out during economic crises, they will have incentives for them to mis-allocate their resources.

Central banks have been established for varied reasons. The Bank of England was explicitly founded for purely fiscal reasons (White 1999: 81-3), while the Federal Reserve System was the result of a prolonged public discussion in
which fiscal concerns were minor. The main argument for establishing the Federal Reserve was not the frequent banking panics of the preceding system, but what was considered its ultimate cause namely the inelastic money supply (Wicker 2005: 22-41).

Some economists consider a fiat-money monetary system headed by a central bank a suboptimal solution compared to a classical gold standard or a competitive private provision of money (Hayek 1978, Mundell 1999, Klein 1974). In this vein, it is interesting how recent historical research on the origins of the Fed (Wicker op. cit.) notes that the original proposals for monetary and banking reform in the U.S did not include at all the existence of a central bank. It was during the travels of the members of the Monetary Commission, organized by Senator Nelson Aldrich between 1908 and 1910, that the idea of establishing a central bank was adopted. Since the other leading economic countries of the time, such as England and France, had central banks, it seems that imitative behavior can also lock us into a standard not necessarily Pareto optimal.

15 At least between 1894 up to its foundation in 1913 there were debates in which bankers from New York, Chicago, the American Bank Association, also merchants from several Chambers of Commerce throughout the states, academicians, and politicians participated in (Wicker 2005).
5. Concluding Remarks

I have increased the number of banks and gradually added institutional complexity to the baseline model of Romero (2009). The agents are very simple in that they do not have sophisticated cognitive capabilities or full information, but they interact dynamically within a microeconomic environment, yielding ‘emergent’ aggregate results à la Epstein and Axtell.

In most of the cases introduced here, except in the interbank market case or when the monetary base was always 8 units when a central bank was present, bank runs persisted. The models as they stand here are still very stylized, yielding mostly qualitative results. An important step forward is to empirically validate their main implications.
References


3. The Evolution of Economic Networks

“A town or city lies at the centre of a number of interlocking catchment areas: there is the circle from which it obtains supplies; the circle in which its currency, weights and measures are used; the circle from which its craftsmen and new bourgeois come; the circle of credit (the widest one); the circle of its sales and the circle of its purchases; and the successive circles through which news reaching or leaving the town travels. Like the merchant’s shop or warehouse, the town occupies an economic area assigned by its situation, its wealth and long-term context.” [emphasis added] Fernand Braudel 1979:188.

1. Introduction

If we replace the word ‘network’ by the word ‘circle’ in the quote above we would realize that those networks evolved out of the initiative of a small group of entrepreneurs. Then, others followed those leaders. By this fashion the reach of the networks was gradually expanding out during the early industrialization period. The picture that is captured by Braudel's words shows probably more the result; or a snapshot at a moment of time; of that process.

The way in which these networks overlapped at each moment of time was not the object of choice of those entrepreneurs. As a matter of fact, each network configuration that could have contributed to the development of societies was not necessarily taken into account in the original plans that motivated those forerunners. Yet, it was due to the particularities of each network that a city
during the first wave of modern industrialization got access to innovations and discoveries. If we think of networks and their relationship to economic development, we would probably have a clearer way of understanding how the ‘invisible hand’ metaphorically used by Adam Smith was actually working.

Networks seen from this perspective can represent how the coordination of economic activities was carried out through different geographical locations (Orsborn and Klein 2007). Here I would propose that these entrepreneurial networks may be seen as ‘coordination structures’ that added value to different economic activities. The particular configuration of these networks at a moment of time can be considered as an unintended consequence of the competitive production process. The general objective of this paper is to understand the evolution of these networks.

I propose an agent-based economic model of formation and evolution of networks whereby agents make strategic decisions based on economic variables and information generated through their immediate social network. It is important to bear in mind that the particularities of a network in a static snapshot may not be an equilibrium situation but rather one of disequilibrium. Then, a simulated environment will help to appreciate this better than other conventional tools.
In the next section, I review the literature that motivated this study. Section 3 presents the research questions. Section 4 introduces concepts that will be used in the rest of the paper. Section 5 presents my strategy in modeling this evolving network. Section 6 shows the reference simulations of the model. Section 7 reports the results of experiments after manipulating key parameters in the model. The last section summarizes the main findings so far.

2. Literature Review

Two strands of literature one theoretical and the other empirical dealing with social networks yield different conclusions concerning the evolution of cooperation within a society of individuals that interact based upon strategic and self-interested behavior. The more abstract and game-theoretical models of social networks sustain that in coordination games in social networks multiple equilibria do emerge as stochastically stable states. This is in contrast to previous results, such as Young (1998), as I will explain in the subsequent section. In line with this, also the study of multiplayer prisoners’ dilemma games leads to the results that cooperation emerges on a sparse matrix rather than on close-knit networks. The second more empirical strand is based upon studies of actual economic sectors yielding as results that particularly in high dense networks (not
sparser) underlies the productivity and economic growth of certain localized industries.

Before I describe the details of these two strands of literature a few definitions are in order. I will follow more closely a graph theoretical approach in doing this (see Beineke and R. J. Wilson 1997). A network is a set of nodes wherein any pair of nodes is connected, at least, by a link. A fixed network is a set with a given and finite number of nodes, and a fixed configuration of links among the nodes. A dynamic (endogenous) network is a set with a given and finite number of nodes, but a variable configuration of links among the nodes. This variable configuration is usually the result of an endogenous formation process for links. An evolving network refers to a variable (even stochastic) number of nodes and link configurations among them (Romero 2006, Cowan et. al. 2006).

2.1 Game-theoretical Approach to Social Networks

Jackson and Watts (2002) study fixed and endogenously formed networks whereby players are playing coordination games with their neighbors. Each player only interacts with those other players whom are directly linked to it, and each link is formed after mutual consent. Also, there are costs of forming links. This results in games with only two pure Nash equilibria where the payoffs matrix
is specified in such a way to model the Pareto equilibrium also as the risk-dominant strategy.

In the fixed network case they analyze three variations: a lattice or complete graph, a circle graph, and a star shaped graph. Stochasticity is added when agents choose their strategies. Here their main contribution is their result for a star shaped network that is in stark contrast to the conventional result; e.g. Young (1998). The latter claims that for any fixed network players always converge to the risk-dominant strategy. On the contrary, Jackson and Watts claim that the two equilibria may be chosen, thus all players may be playing the risk-dominant equilibrium or playing the efficient but not risk-dominant equilibrium in other periods.

In the case in which agents choose not just their strategies, but also whom to play with, their main result is that there is multiple equilibria and players may coordinate even in those equilibria that are neither risk-dominant nor efficient. By manipulating the cost structure of the game, they even go farther to claim that even for fixed networks they may exist multiple stochastically stable states. Thus concluding that the conventional results; e.g. the risk-dominant solution as the unique stochastically stable state; are sensitive to the particularities of agents’ behaviors and interaction technologies.
Hanaki et. al. (2007) address the emergence of cooperation where individuals' behavior and interaction structures are evolving. In this setting there is a \textit{dynamics on the network} generated by the rules that govern individual behavior, moreover a \textit{dynamics of the network} that is generated by the rules governing social behavior. The rules of individual behavior are based on each agent playing a prisoners' dilemma game with each of its surrounding or local neighbors. However, each agent can choose either to cooperate or defect with its whole neighborhood; i.e. it cannot play a different strategy against any other agent within its neighborhood. Because this is a simulated environment the population of players are actually playing a multiplayer prisoners' dilemma with a changing subset of other agents that at each period may be part of its neighborhood. Moreover, each player can imitate the most successful strategy--measured by its payoff-- of the last period by one of its neighbors. Also they can break or create a link with another agent to modify their neighborhood. There is not an exogenous upper limit for the number of neighbors that a given agent can have during the simulation.

Concerning the interaction dynamics this is determined by the marginal increase or decrease in benefits per player from either breaking or creating a link with other player. One important element in this model is the incorporation of triadic relationships through which an agent can find a new partner. Nonetheless, an agent can decide to create a link with an agent randomly drawn from the
population at large. There are costs in both cases; that is, for breaking and creating a link. Moreover, there is an additional procedure to decide whether to trust a new partner. Here two different settings are implemented, namely a full and a zero information case about the history of plays by the new partner in previous periods.

Their main result is the following: “cooperation can persist in sparse, dynamic networks of effectively unlimited size, and in fact tends to fare better in large networks than in small ones.” pp. 1049. They emphasize how assortative matching of partners reinforces cooperation (as in Tullock 1980). But also how allowing defectors to be selected by highly trusting cooperators expands this cooperation. During the report of their results they also acknowledged that a “higher average proportion of cooperating players does not necessarily mean that the population average payoff is higher.” pp. 1004. This point is relegated to a footnote where they mention that despite this result there still exist a positive correlation (0.48) between the average proportion of cooperating players and the population average payoff. Yet from the main text one of their main results is that a high-cost regime for agents’ interaction is what determines both the sparseness of the network and its greater level of cooperation achieved in relation to when there is a lower cost of interaction.
2.2 Empirical Social Network Analysis

The main particularity of economic networks at the producer level is that they change from period to period. The firms representing the nodes may have changed. The networks of raw material providers/retailers and clients may change from period to period, too. Nonetheless, there is a core or nucleus of clients to whom the seller frequently sells and a core of providers from whom usually it buys. These are their permanent clients and providers. Furthermore those not so permanent clients and providers can be called casual ones. This is network complementarity between embedded and `arm's length ties'.

Castilla (2003) and Castilla et. al. (2000) focus on a static network where only embeddedness is studied and thus highlighted as the main driving force of the creation of capital in Sillicon Valley. Next, I explain two examples that were elaborated by Castilla et. al. (2000) and Uzzi (1999). The first work is about Silicon Valley and how the development of that region is generated through the networks of venture capitalists, educators, engineers, lawyers, trade groups, and so on. Regarding the conformation of technological firms a special focus is given to employees and referees, managerial, and information networks that are generated and transmitted through different links or channels among firms. The second one is about bank-borrowers networks in the Chicago area.
Castilla et. al. call attention to the fact that: “Extensive labor mobility creates rapidly shifting and permeable firm and institutional boundaries and dense personal networks across the technical and professional population. The ability of Silicon Valley to restructure itself when conditions change through rapid and frequent reshuffling of organizational and institutional boundaries and members (…"recombining" process) is one of the factors that underlie the dominance of Silicon Valley…” pp. 220. Their analysis show how the creation of capital in Silicon Valley is benefited and fostered by the positive externalities created due to the high degree of density and the openly competitive environment among different networks related to a given firm. An intense competition and high mobility of resources allows for a fast rate of learning of adaptation to the new conditions of the market. One important characteristic that they pointed out is the fact that much of the know-how or informal knowledge produced by this interaction among technological firms remains local.

Using techniques of social network analysis with data collected by journalists they are able to trace--since 1947 up to 1986-- the evolution of the network of firms, managers, educators and others. This was what contributed to the beginnings of projects such as Intel and the like. Those individuals or firms with a high degree of centrality (connected to a lot of others) and those that play the role of ‘crucial linkage’ to reach others are discovered. Hence, entrepreneurial spirit, willingness to support innovative ideas, but specially
networks externalities are the key elements identified by them lying at the great development of Silicon Valley.

A visual representation is in the network from which initial public offers (IPOs) (data from 1999) are originated in Silicon Valley. Figure 4 shows three different kinds of organizations that interact and collaborate to give birth to a new enterprise. These are: investments banks, law firms, and accounting firms. Furthermore, the issuing firm is not portrayed. There is a link between any two firms (from the same or different industry) whenever both are involved in the same IPO. The length of the line also conveys relevant information, namely it is inversely proportional to the number of co-participations. Thus is a proxy for the strength of the link. The more co-participations, the stronger the relationship (i.e. the shorter the link).

The main result is that a particular kind of network; defined by centrality and degree of connectivity; determines particular outcomes. That is, different types of relationships that may exist among the actors of any network. In a posterior work by Castilla (2003), he compares the degree of connectivity or density of the network of venture capital firms in Sillicon Valley to the one in Route 128 (Massachusetts). He found that the higher number of projects and amounts of money invested in California are a consequence of the higher
connectivity among firms through different industrial sectors and within each of them.

On the other hand, Uzzi (1999) carried out an analysis of the effects of social embeddedness of networks in corporate financial dealings. An important contribution of this paper is the triangulation between social network analysis, statistics, and original data collected through field research. The sample included 2400 small or medium size companies and eleven medium size (less than 500 employees) banks in the Chicago area. His focus is upon the credit networks or
the bank-borrower links and their effects on the amount and cost of loans obtained. The first pair of hypotheses is: a) if bonds or social attachments created (and the longer this relation exists) among managers and bankers increase the probability of getting a loan; and b) if given this, the cost will be lower. Data from the fieldwork pointed out that bankers and managers do care about how to establish a social relationship with one another beyond the cold numbers. Because to get to know each other gives them information that is not easily found in figures and increases the degree of trust in their relationships.

The other pair of subsidiary hypotheses tested by Uzzi is: the likelihood to get financing increases if a firm has access to a mix of embedded and arm's length ties. In other words, if a mix of bonding and bridging social relationships in different networks is important to arbitrage opportunities and reduce search costs. The other hypothesis, then, is if costs of financing are lower when a firm has access to these two kinds of social networks. Another way to put this is that if a firm only has been focused on cultivating only one kind of these networks' links (bonding or bridging) it will be less successful getting loans and reducing the costs per loan.

An important concept explored by Uzzi is related to this mix of bonding and bridging networks what he referred as to 'network complementarity.' In his own words: “Networks high in complementarity produce premium outcomes
because the features of different ties reinforce one another's advantages while mitigating their disadvantages.” pp. 491.

The econometric tests yielded these results: the social network bonding links did not affect the probability of a given firm to get loans, but it does affect the price or interest rate of the loan. The latter is in agreement with field data. In regards to the tests about network complementarity these pointed out that these kinds of combined network links do produce optimal benefits relative to networks only of one type or another for a firm.

3. Where All This Lead Us? Research Questions

To what extent the game-theoretical results of network games (Goyal et. al. 2007) explain the empirical evidence of actual social networks in the market? What it is reported in field research on social networks may be just one type of equilibrium explained by the models. But here my purpose is toward building a rationale of: how social networks contribute to the development of commercial ties? In a more general vein: How do firms coordinate to produce technology through networks; i.e. economic networks?

An economic or entrepreneurial network is formed by a profit motive and social links. In this model nodes represent firms and the link between each pair of
them is the result of a mutually advantageous economic decision. The environment is an industrial sector where firms interact locally but contribute to a global evolving network of technological innovation (Cowan et. al. 2006). Links in this case are not one-sided or directed but two-sided or undirected.

Moreover, the temporal dimension of the process will be studied by how long it takes to the network of firms to evolve a network structure (topology). This will serve us to answer the following questions: how do the model’s parameters and rules of interaction affect the network evolution? Under what networks profits may be greater? Are certain network topologies more prone to generate coordination among firms? and, in general: Can this be a part of that intangible capital that accounts for endogenous economic growth through knowledge?

4. The Environment

The agents are firms that will interact within an industry. A firm may cooperate or not with another firm. There will be direct relationships that will be established pairwise, and indirect ones that are a consequence of the former kind of relationships. That is to say, each firm only focus on the relationships that establishes directly with other firms. This pairing of firms can be understood as a contract to collaborate whether in the funding of a new enterprise with innovative
ideas or contributing with knowledge to a particular investment project. This keeps some similarity to what happens in places such as the Silicon Valley, but I keep the model rather general. There is neither a market demand nor a production technology. I have relied on Wasserman and Faust (1994), Scott (1991), and Goyal (2007) to write the next two subsections.

4.1 Definitions

Firms are represented by a set of nodes $N = \{1, \ldots, n\}$ where $n \geq 2$ and a finite number. Their pairwise relationships are links or edges denoted by $g_{ij} \in \{0, 1\}$ for nodes $i$ and $j$. Where $g_{ij}$ takes value 0 when the two nodes are not connected and 1 otherwise. Here I will consider only undirected links, which in this context means that both nodes mutually accept to establish and maintain the link. Let $G_t$ be the network formed by a set nodes and its links at a time $t$. There is a set of networks representing each of the $G$ networks along time.

$$\Gamma = \int_0^T G_t \, dt$$

is a set of networks representing each of the $G$ networks along time.

A neighborhood of agent $i$ is the set of all its neighbors with whom is directly connected represented by $N_i(g) = \{j \mid g_{ij} = 1\}$. The degree of node $i$ is the number of direct neighbors $d_i(g) = |N_i(g)|$ in a given network $G$. The first order neighborhood of node $i$ is $N_i$. The second order is $N_i \cup \{N_z \mid z \in N_i\}$. Other higher order neighborhoods can be defined in a similar manner. Let $\bar{d}$ be the
maximum degree for a given network. The degree distribution of the network is denoted by $P$, and the frequency of nodes with degree $d$ is $P(d)$.

The following are relevant type of networks. The complete network, $g^c$, is the one where every node has the same degree and this is equal to $n - 1$. The empty network, $g^e$, which is not connected or is of degree zero. A connected network is where there is a path between any two nodes even though is not a complete network.

Other important concept is a walk, which is a sequence of nodes whereby two nodes are linked. Here a node or a link may be included more than once in the walk. The length of the walk is the number of links it crosses or the number of nodes involved minus one. A trail is a walk in which all crossed links are distinct. In turn, a trail in which every node is distinct is a path. The length of the path is the number of links that involves. There is a shortest path between nodes $i$ and $j$; called its geodesic distance in network $G$ which is measured by its length and denoted by $t_{ij}$. For every node $i$ in network $G$ there may exist a set of shortest paths to every other node $j$. Whenever there is no path between any two nodes in $G$ then their geodesic distance is $t_{ij} = \infty$. 

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4.2 Measuring a Network

When a network $G$ is connected its average distance between nodes or path length is

$$L = \frac{\sum_{i \neq j}^n t_{ij}}{n(n - 1)}$$

This is useful to know how close is an agent (firm) to another one and how easy or fast information or knowledge can be transferred in a network.

The centrality of an agent in a network refers to its prominence; i.e. how relevant or critical is the presence or absence of this agent in the network. This is measured by

$$C_d = \frac{d_i(g)}{n - 1}$$

A related measure is the degree centralization of a network $G$. If there is a node $i'$ with the highest degree centrality $C_{d_i'}$ then

$$C_d(g) = \frac{\sum_{i=1}^n [C_{d_i} - C_{d_{i'}}]}{n - 2}$$

The density of a network $G$ measures the proportion of potential links present in it. It is expressed as a ratio of actual links to the maximum possible ones. This is
Another measure that will be introduced is the clustering coefficient. This captures the overlapped links that exist among the neighbors of agent \( i \) or what proportions of its neighbors are also neighbors. This is defined for any node \( i \) as

\[
\Delta = \frac{\sum_{i} d_i}{n(n - 1)}
\]

Finally, the total clustering coefficient is the sum of all individual clustering coefficients. That is, \( C = \sum_{i=1}^{n} C_i \). I will use these concepts and measures in different sections along the paper.

5. The Evolving Network

At the beginning, independently of any value of the parameters and exit treatments, there will be only one firm in the market. Then, firms make their appearance in the environment one by one per period. ‘New’ firms arrive and propose to form a link or economic relationship with ‘old’ firms. The latter should decide whether to accept such a proposal. Those firms that are unsuccessful; i.e. the ones that held negative profits for several periods of time; leave the industry. Therefore, a dynamic process is recreated in which firms enter and leave the
market affecting the economic relationships that have also been formed dynamically (see Appendices A and B for pseudo-code and a model screenshot).

Every firm contributes to the technological innovations throughout the network. But every firm arrives in the deterministic fashion I explained previously. Thus at this stage the model does not include elements of stochasticity. The particular topology of the overall network changes every time period. Because some firms are entering while others are leaving the market. Firms are also risk neutral.

The flow of innovations in this industry is the result of not just each firm contribution, but more importantly of the connectivity of the network that all firms form. I will draw on Jackson and Wolinsky (1996) formalization of the ‘connections model’ from now on. Let $w_j$ be the market valuation of firm j’s potential contribution to firm i’s innovative endeavors. Then, the accumulated value after a period t for firm i (which value $w_i$ is of itself) interacting with every other firm j in the network $G_t$ is

$$V_i(G_t) = w_i + \sum_{j \neq i} \delta_{ij} W_j$$  \hspace{1cm} (6)

16 ‘Innovations’ within this literature have also been interpreted as ‘knowledge’ within each firm. A link is formed whenever this ‘knowledge’ is purposefully shared or diffused throughout the network; e.g. Cowan and Jonard (2006). I am avoiding this usage since I consider knowledge a more abstract category of thought than information, for instance. See Polanyi (1974: 69 -260) and Hayek (1937, and 1945) for further distinctions about knowledge and the relevance of its tacitness. So in my case innovation is the same as ‘new’ information.
where $\delta \in (0, 1)$ is a parameter that represents the transferability factor or how firm i gets access to the innovations of firm j via intermediate links and other firms in the network. This is expressed by $d_i(g_t)$ that is the degree of node i for a network g at t. Thus, the connection with node j is indirect via the local neighborhood of node i. The positive externality deteriorates the farther is firm j from i. There are costs; denoted by $c_{ij}$; of forming links between any two firms. Therefore, profits for firm i per unit of time are given by

$$\pi_i(G_t) = w_i + \sum_{j \neq i} \delta^{d_i(g_t)}w_j - \sum_{j \in G} c_{ij}$$  \hspace{1cm} (7)

The dynamics of the network is given here at two levels. Firstly, as I mentioned before there is not a fixed number of firms during the simulated time. As a consequence links between firms cannot be fixed either. Both, the number of firms and their links are permitted to evolve during the simulation. By doing this, the state variables of the firms are also altered every period during the experiments.

When a new firm arrives to this industry it proposes to form a link with an incumbent firm. When there are more than two firms the incumbent firm is randomly chosen from the new firm’s neighborhood. Next, I define a myopic pairwise dynamics in which:
i) A new link is created as long as both firms do not get worse off by establishing this relationship, and at least one of them is strictly better off (Jackson and Watts, 2002).

ii) An old link is severed if at least one of the firms who formed it exits the market due to accumulated negative profits for m successive periods. Otherwise, it is maintained.

The first point is standard in the study of endogenous network formation when there is a finite set of nodes during a simulation. While the links appear and disappear from the network at each period yielding certain network topologies. However, I should emphasize that in this model; in contrast to previous approaches; nodes also appear and disappear from the environment (or rather interface). The number of nodes and links are endogenous at each period. This is the main contribution of this paper.

Before to proceed I should, also, point out that in this model I incorporate the notion of *ex ante* and *ex post* gains for the firms forming or removing links. Thus, I assumed that at the beginning of each period firms get to know the market valuation of others, which are represented by the $w$s. But the realized profits are given by (7) at the end of each period. This is also a different approach from the one implemented in Carayol and Roux (2005) and König(2008) in which case ‘knowledge’ is represented by ‘innovations’ that arrive every period.
according to a known probabilistic distribution. Leaving unspecified the distinction I introduce here regarding ex-ante versus ex-post profits.

The second point, on the other hand, it is the result of merging two processes. The first one is a firm exiting the market due to consecutive negative profits (e.g. four quarters), and the second one is the deletion of the link(s) or economic relationship(s) between that firm and their directly connected firm partners (i.e. neighboring nodes). Here lies another innovation of the paper whereby the evaporation of nodes is paired with the deletion of their direct links.

6. Simulation Results

Figure 5 depicts how firms and their linkages evolve through time for a typical run. Periods should be assumed larger than a day. It could be months and even quarters. The particular length of the period will only make sense when an empirical validation against actual data is carried out. The externality parameter (δ) is set at 0.95. I, also, here only present an exit rule (treatment) for firms leaving the market. This rule states that a given firm with negative profits or unconnected from any other firm for more than 4 consecutive periods will exit the market.
Below, I show snapshots of the evolution of the network at different time periods.

<table>
<thead>
<tr>
<th>Time</th>
<th>Average Degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) t= 40</td>
<td>Ave degree = 1.18</td>
</tr>
<tr>
<td>b) t = 200</td>
<td>Ave degree = 2.89</td>
</tr>
<tr>
<td>c) t = 500</td>
<td>Ave degree = 3.18</td>
</tr>
<tr>
<td>d) t= 1000</td>
<td>Ave degree = 3.2</td>
</tr>
</tbody>
</table>

Figure 5. Evolution of Firms Network. Degree mean values across firms.

At the bottom of each panel in Figure 5 the average degree is reported. Note how this measure increases between panel a) and b) and from then to c). Between c) and d) is more or less the same. I won’t make strong statistical claims at this moment (see more in section below). I just wanted to point out the monotonic increase of the average degree of the network.

In figures from 6 through 9 the evolution of some key aggregate variables is presented. Figure 6 depicts two variables namely the number of ‘surviving’ firms and the ‘surviving’ links. It should be reminded that in this model firms enter the market at a constant rate of 1 per unit of time. In addition the exit rule previously mentioned implies that several firms may exit the market by the end of each period. Note that the number of firms and links increase approximately pari passu up to 200 periods. After that firms grow at a decreasing rate whereas links
grow increasingly faster until they start to fluctuate after 700 periods around a value of 140.

Figure 6. Temporal Dynamics

Figure 7 shows the evolution of average profits and average degree of the network of firms at each time step. It is not surprise that the correlation between both variables is 0.86. This stems from the profit equation per firm as specified in (7). This, of course, does not mean that any firm with the highest degree due to its higher number of related firms will always have the highest profit. Because in the profit equation (7) every firm also faces costs per each related firm that it keeps. In addition these costs vary from period to period; and there are no fixed costs, since all costs are variable.
Figure 8 presents the average fraction of firms in the giant component along the simulation. Note that after 200 periods this fraction reaches 80% of the whole population of ‘surviving’ firms. This population at any time step may include the firms belonging to the giant component, firms that are part of other component(s), and temporarily unconnected firms. This fraction fluctuates between 80% and 90% after 300 periods.
Figure 9 presents the degree distribution of the firms network after 50 simulations each one measured at $t = 1000$. This result shows a power law relationship when the distribution is measured in logs. The power coefficient is close to 1 and statistically significant at 95% level of confidence. I will point out that here this power law relationship is found in spite of the absence of a ‘preferential attachment’ mechanism during the network formation. Next section presents a more formal analysis of robustness of the results.
7. Experiments

What is the effect of the externality parameter ($\delta$) on the network degree and firms’ profits? Since average degree and profits are positively correlated the effect of increasing (decreasing) the externality parameter should be the same for both. Recall that the externality parameter is measuring how fast the rate of knowledge is spread over the network. The higher it is the faster knowledge is being transferred throughout. I run experiments for $\delta = \{0.05, 0.5, 0.95\}$ and two exit rules. A firm will leave the market if for more than 4 or 12 consecutive periods has been having negative profits or has been unconnected from others; i.e. its degree is zero. This adds up to six experiments.
Figure 10. Network Degree vs Externality Parameter. $\delta = \{0.05, 0.5, 0.95\}$. 50 runs per experiment each one for $t = 1000$.

Figure 10 reports the average degree of firms after 50 runs. The results of the six experiments (3 $\delta$ values times 2 exit rules) are showed from left to right. The externality parameter takes the values of $\{0.05, 0.5, 0.95\}$ per each exit treatment. Exit treatments are also read from left to right. The first exit treatment (I) refers to the same rule applied to the results showed in section 6. While the second exit treatment (II) just increases that value to 12 consecutive periods but it works in the same fashion as mentioned in the previous paragraph. Thus, in the figure I report the ordered pair values of $\{\delta, \text{exit rule}\} = \{(0.05, \text{I}); (0.5, \text{I}); (0.95, \text{I}); (0.05, \text{II}); (0.5, \text{II}); (0.95, \text{II})\}$. Note that within each exit treatment the
average network degree monotonically increases with the value of the externality parameter as expected. Then, I proceed to test if the means across exit treatments per each value of $\delta$ are statistically equal or not with an unpaired difference means test with unequal variances. That is, I compare the means difference between the pairs (0.05, I) and (0.05, II); and so on (I implement two-sided and one-sided tests). So that there will be three means difference tests. The results reject the null hypothesis of equal means (i.e. a zero value for the difference of means) at the 5% level of significance. For instance, it can be claimed that the average network degree when $\delta = 0.95$ and the firms are interacting under exit rule (I) is statistically different (and higher) than the average network degree when $\delta = 0.95$ but firms are interacting under exit rule (II).

Next, Figure 11 reports the values of average profits after 50 runs again. The number of total experiments is the same as for the average network degree case. Also, the ordered pair values {$\delta$, exit rule} are the same as before. It is also observed that within each exit treatment average profits monotonically increase with the value of the externality parameter. A means difference test was also applied to determine whether these values are statistically different across exit treatments exactly as I did for the average network degree case. The results of the three means difference test ordered as before yielded also a rejection of the null hypothesis in each case. Again, I can claim that the average profits when $\delta = 0.95$ and the firms are interacting under exit rule (I) is statistically different (and
higher) than the average profits when $\delta = 0.95$ but firms are interacting under exit rule (II). The higher the network degree or connectivity, and the most competitive the market, the higher the overall profits of firms.

![Average Profits vs Externality Parameter](image)

**Figure 11.** Average Profits vs Externality Parameter. $\delta = \{0.05, 0.5, 0.95\}$. 50 runs per experiment each with $t = 1000$.

8. Concluding Remarks

Castilla (2003: 125) found that the average degree of the overall network of venture capitalists in Silicon Valley is 2.8 while the same network statistic for Route 128 (Massachusetts) is 1.5. This is one of the findings on which he based
his conclusion that the greater frequency of cooperation in Silicon Valley is what explains its greater economic success. The empirical average degree reported there is pretty much the same as the one reported here in Figure 10, i.e. 2.78 for the first exit treatment and $\delta = 0.95$. Whereas the second exit treatment and the same externality value yield an average degree of 2.01.

My goal here is not to make a precise quantitative calibration of the model. Rather at this stage a qualitative calibration is what I have in mind (Axtell and Epstein 1994). But, even as it stands the model may shed light on the empirical differences between Silicon Valley and Route 128 venture capitalists' networks, for instance. As a matter of fact, here was also found that the higher the network connectivity the greater the profits; or the economic efficiency loosely defined (Romero 2006). The model, of course, cannot yet portrait the trade-off or complementarity between bonding and bridging links reported by Uzzi (1999). Nonetheless, the model at this stage is more able to explain a type of regional development due to factors within a hub like the one in Silicon Valley.

Goyal (2007: 20-4) summarizes the features of empirical networks across several domains. Including corporate web site, coauthors, sexual contacts and R&D networks. He concludes: "[social and economic networks] have low average degree relative to the total number of nodes, the distribution of degrees is unequal, clustering is high, and the average distance between nodes is
small." (pp. 23-4) The model presented here, also yields a low average degree relative to nodes, see Figure 3. An unequal distribution of degrees was reported in Figure 6. I have reported here neither the clustering coefficient nor the average path length but I would not be surprised if, in fact, it mimics the general pattern reported by Goyal.

These results were yielded by the model and closely match the ones from empirical networks. Yet I did not follow in building the model more traditional approaches: such as preferential attachment mechanisms, random networks, or small world networks. I based my model more on simple economic grounds and local information. Thus, providing a more credible microeconomic behavior of the agents.

Thomas and Griffin (1996) and Lin and Shaw (1998) present complementary works on supply chain networks and how coordination through top management techniques have become less and less of practical use in multinational process of production. Knowledge and practices are so much distributed throughout the supply chain network that no one can manage it only relying on global information. In general, these supply chain networks are comprised by: raw material providers, manufacturers, assemblers, warehouses, and retailers. In turn, these networks can be subdivided into three types of categories, namely: buyer-vendor networks, production-distribution networks,
and inventory-distribution networks. The model presented here falls into a production network category but it can be extended to include distribution and the demand aspects of an artificial industrial environment.

Industries ranging from auto, computer hardware, airlines’ services, is where most of the case studies are found. The general point in all of them is that coordination in these industries that show vast ‘economies of scope’ have recently tended to spread their production processes as moving from vertically integrated towards more flatter networks. This has resulted from the search for coping with uncertainty and adaptation to a more competitive environment. By doing so, leaders in these industries have been able to discover opportunities not known or existent before. This kind of coordinative processes that go beyond a particular firm or even region can account for an important part of economic growth not included in more traditional models.
References


Appendix A. Pseudo-code

OBJECT bank;
   initial-deposits;
   agents-served;
   final-balance;
   FUNCTION initialize;
   FUNCTION rates-of-return;
   FUNCTION investment;
   FUNCTION balance-sheet;
   FUNCTION liquidation.

Pseudo-code block 1: Bank object

OBJECT agent;
   deposits;
   bank-accounts;
   withdrawals-per-period;
   payoffs;
   FUNCTION initialize;
   FUNCTION get-type;
   FUNCTION rates-of-return;
   FUNCTION make-decision-to-withdraw;
   FUNCTION update-accounts;
   FUNCTION compute-payoffs;
   FUNCTION stop.

Pseudo-code block 2: Agent (Depositor) object
OBJECT central bank;

set-monetary-base;
set-reserve-ratio;
set-central-bank-rate;
banks-served;
FUNCTION initialize;
FUNCTION make-loans;
FUNCTION central-bank-balance;
FUNCTION recover-loans;
FUNCTION revise-policies.

Pseudo-code block 3: Central Bank object

PROGRAM bank-model;
initialize depositors;
initialize banks;
initialize central bank;
repeat:
    select 1 depositor at random;
    for each depositor selected:
        discover type;
        impatient agents withdraw first;
        patient agents withdraw after a period and depending on queue size and subjective discount rates:
            withdraw lower or higher return;
        compute payoffs;
    for each bank:
        determine queue;
        determine whether to invest idle funds if any;
        pay any loans if previously requested;
        compute bank balance;
        determine if loans will be necessary next period;
        declare bankruptcy if unable to serve depositors;
    until user terminates.

Pseudo-code block 4: Overall banking model
OBJECT firm;
    market value (if unconnected);
    linking costs;
    profits;
    degree;
    lifetime;
FUNCTION initialize;
FUNCTION propose an economic venture(link);
FUNCTION find path lengths to all linked firms;
FUNCTION compute payoffs;
FUNCTION exit if unconnected and negative payoffs for t periods.

Pseudo-code block 5: Firm object.

PROGRAM market_network;
    initialize firms;
    repeat:
        a new firm arrives;
        for each new firm:
            select at random an incumbent firm;
            propose a link if:
                mutually advantageous
                and at least one firm is strictly better off;
                otherwise stay unconnected;
            determine local network;
            compute payoffs;
        for network;
            select unconnected and unprofitable firms to exit market;
            do layout;
            clustering-coefficient;
            any components;
            do plotting;
        until user terminates.

Pseudo-code block 6: Overall network model.
Appendix B. Screen Shots of the Models

B1: One-bank model
B2: Free-banks model

B3: Central bank model
B4: Economic network model
Pedro P. Romero was born in Quito, Ecuador on February 5, 1977. He earned a Bachelor of Science in Economics with two minors in Finance and Business Administration from the Escuela Superior Politécnica del Litoral in 2003. In 2007 he completed a Master of Arts in Economics at George Mason University. Mr. Romero’s research covers topics such as: constitutional uncertainty in Latin America, banking theory, economic networks, and agent-based economics. In 1999 he worked for a banking organization, Solbanco, as a Treasury’s assistant. Exactly during the financial meltdown in Ecuador. In 2007 he published a constitutional study and reform for Ecuador that circulated in the 2007-8 constitutional assembly. He has conducted public policy research in Ecuador at the Instituto Ecuatoriano de Economía Política. He has taught courses on Economic Development of Latin America, Macroeconomics, Microeconomics, and History of Economic Thought.