A STUDY OF THE INITIATION PROCESS OF CORONAL MASS EJECTIONS
AND THE TOOL FOR THEIR AUTO-DETECTION

by

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Dedication

To my wife Les, parents Ana and Oscar, grandmother Olga, and family, who always encourage and support me.
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Abstract

A STUDY OF THE INITIATION PROCESS OF CORONAL MASS EJECTIONS AND THE TOOL FOR THEIR AUTO-DETECTION

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Coronal mass ejections (CMEs) are the most energetic and important solar activity. They are often associated with other solar phenomena such as flares and filament/prominence eruptions. Despite the significant improvement of CME study in the past decade, our understanding of the initiation process of CMEs remains elusive. In order to solve this issue, an approach that combines theoretical modelling and empirical analysis is needed. This thesis is a combination of three studies, two of which investigate the initiation process of CMEs, and the other is the development of a tool to automatically detect CMEs.

First, I investigate the stability of the well-known eruptive flux rope model in the context of the torus instability. In the flux rope model, the pre-eruptive CME structure is a helical flux rope with two footpoints anchored to the solar surface. The torus instability is dependent on the balance between two opposing magnetic forces, the outward Lorentz self-force (also called curvature hoop force) and the restoring Lorentz force of the ambient magnetic fields. Previously, the condition of stability derived for the torus instability assumed that the pre-eruptive structure was a semicircular loop above the photosphere without anchored footpoints. I extend these results to partial torus flux ropes of any circularity with anchored footpoints and discovered that there is a dependence of the critical index on the fractional
number of the partial torus, defined by the ratio between the arc length of the partial torus above the photosphere and the circumference of a circular torus of equal radius. I coin this result the partial torus instability (PTI). The result is more general than has been previously derived and extends to loops of any arc above the photosphere. It will be demonstrated that these results can help us understand the confinement, growth, and eventual eruption of a flux rope CME.

Second, I use observations of eruptive prominences associated with CMEs to examine the behaviour of their initiation and compare these observations to theoretical models. Since theoretical models specify the pre-existence of a flux rope, the observational challenge is the interpretation of the flux rope in solar images. A good proxy for flux ropes is prominences, because of its obvious elongated helical structure above the magnetic polarity line. I compare the prominence kinematics and the associated extrapolated magnetic fields. This observational study yields two key conclusions. The first is that there is a dependence of the ejecta’s kinematics on how the ambient magnetic field decays. The second is that the critical decay index, theorized to be where the flux rope transitions from a stable to unstable configuration, is dependent on the geometry of the loop. This second result is in qualitative agreement with the theorized PTI.

Finally, I develop a tool to automatically detect CME events in coronagraph images. Because of the large amount of data collected over the years, searching for candidate events to study can be daunting. In order to facilitate the search of CME event candidates, an algorithm was developed to automatically detect and characterize CMEs seen in coronagraph images. With this tool, one need not scroll through the large number of images, and only focus on particular subsets. The auto-detection reduces human bias of CME characterization. Such automated detection algorithms can have other applications, such as space weather alerts in near-real time.

In summary, this thesis has improved our understanding of the initiation process of CMEs by taking both theoretical and observational studies. Future work includes investigating a larger number of events to give a better statistical characterization of the results found in the observational study. Furthermore, modification to the theoretical model of
the PTI, for example by including a repulsive force due to induced photospheric currents, can improve the quantitative agreement with observations. The complete knowledge of the initiation of CMEs is important because it can help us to predict when such an event may occur. Such a prediction can aid in mitigating severe space weather effects at the Earth.
Chapter 1: Introduction

In this thesis the major theme will be the study of the initiation of eruptions that lead to CMEs. This is done through theory and observation. Theoretical work extends the concept of the torus instability to loops of varying height/shallowness. The observational study focuses on comparing the kinematics of prominences and extrapolated coronal magnetic fields in the context of the theory proposed. Finally, an automatic method for the detection of CMEs within the FOV of coronagraphs is developed to aid scientists in event exploration and characterization of CMEs.

1.1 Motivation

Ever since prominences were observed to erupt from the Sun, over a century ago (Pettit, 1940, 1944), solar physicists have searched for the underlying physics involved. The fundamental question as to what drives and causes these eruptions is still not fully understood, but we have come a long way. It is now known that magnetic energy dominates the low corona where the precursor structure of a CME originates. This structure is thought to be either a magnetic loop arcade or a flux rope that can support mass against gravity. What drives this structure to eruption and the physical nature of the eruption are highly debated topics. For example, many concepts have been proposed to explain the eruption, such a converging or shearing of the structure’s footpoints (e.g. Mikic et al., 1988; Mikić & Linker, 1994; Linker & Mikic, 1994; Amari et al., 1996), flux injection (Chen, 1989, 1990; Cargill et al., 1994; Chen, 1996; Krall et al., 2000), instability/loss of equilibrium (e.g. Forbes & Isenberg, 1991; Forbes & Priest, 1995; Isenberg et al., 1993; Démoulin & Aulanier, 2010), or resistive process involving magnetic reconnection (e.g. Antiochos et al., 1999; Forbes, 2000;
Lin & Forbes, 2000). Complete agreement by the scientific community of the eruption process is not reached, and much work is needed toward this goal.

Aside from studying CMEs for their scientific insight, the understanding of the physics of these events have immediate real world application. Complete knowledge of their formation and initiation is needed to help aid in the prediction of potential space weather effects. Figure 1.1 is NASA’s illustrative depiction of the Sun-Earth connection that drives space weather. Space weather encompasses a wide range of phenomena with the most predominant being geomagnetic storms, solar energetic particle events (SEPs), and elevated X-ray fluxes. Global fluctuations of the Earth’s magnetic field have been studied since the mid-1800’s, then just referred to as storms and now as geomagnetic storms, which are marked by a decrease in the $Dst$ (Disturbance Storm Time) index (Gonzalez et al., 1994; Zhang et al., 2007). Space weather effects on humans and technological systems include induced currents in power lines causing power system interference or blackouts, elevated radiation doses to aircraft crew members especially for polar flying aircrafts, and damage to the electronics on-board spacecraft and satellites. See appendix A for a brief introduction on these and
other space weather effects.

1.2 CME Phenomenon

One of the most energetic and impressive manifestations of solar activity is the coronal mass ejection (CME). These events originate from the build up and release of solar magnetic energy. They have been measured to release a large quantity of mass, on the order of $10^{14}$ to $10^{17}$ g, into the heliosphere, with speeds measuring from 100 km/s to over 3000 km/s, and magnetic energy on the order of $10^{30}$ to $10^{32}$ ergs (Howard et al., 1985; Hundhausen, 1987, 1999; Gosling, 1990; Webb et al., 1994; Hudson et al., 2006). These events are known to be correlated with other solar phenomena such as prominences eruptions, solar flares, solar energetic particle events, and radio bursts, not as separate events but connected by the same energy release (e.g. Harrison, 1995; Shibata, 1996; Forbes, 2000; Lin, 2004; Schwenn et al., 2006, and references therein). The first white light observations of the CME would occur during the 1970’s and must have been a most beautiful and spectacular finding. Measurements of metric type II radio burst had previously shown that material does in fact propagate outward through the corona (Wild et al., 1963), the white light observations would confirm what was seen in the radio. In addition, it validated the theories and ideas of such authors as Lindemann (1919), Chapman & Ferraro (1931), Morrison (1954, 1956), and Gold (1962) who had predicted the existence of a transient ejection of mass from the Sun, that if encountered the Earth would result in a geomagnetic storm. The direct correlation between CMEs and shocks (Gosling et al., 1975), and CMEs and magnetic clouds (Burlaga et al., 1982) lead to the deduction that CMEs are the drivers of geomagnetic storms at the Earth. Furthermore, with the discovery of halo CMEs, called so because of its circular appearance around the occulter of the coronagraph, it was then observationally verified that CMEs can have a trajectory directed towards the Earth (Howard et al., 1982). It is now recognized that CMEs are the major drivers of space weather events at the Earth.

The first observation of a CME was reported by Tousey (1973) of a coronal transient that
Figure 1.2: Coronal mass ejection photographed by the coronagraph onboard Skylab taken on August 10, 1973. From A to D the times of observation were taken at 1332, 1343, 1424, and 1448 UT. (From: Gosling (1974))
occurred on 14 December 1971 travelling at over 1000 km/s. This CME was observed with the coronagraph onboard the NASA Orbiting Solar Observatory 7 (OSO-7) and had a field of view (FOV) from 3 to 10 solar radii (Koomen et al., 1975). This was the first coronagraph to be placed into orbit to allow continuous observation of the corona from space. The weakness of this instrument was that it only took 4 full images of the corona per day (Howard, 2006). During the life time of OSO-7, from 1971 to 1973, approximately 30 CMEs were observed. The next significant instrument to be flown was the High Altitude Observatory’s white light coronagraph on the NASA Skylab mission (MacQueen et al., 1974). Figure 1.2 shows a sample time series of an event that occurred on August 1973 (Gosling, 1974). This figure shows the corona and the evolution of the CME, the first evidence of the CME is seen as over exposed loops just above the occulting disk in panel A. In subsequent frames, once the CME leaves the FOV, the legs of the loop appear to become rays projecting almost radially, the interpretation of this by Gosling (1974) is that the CME remains magnetically connected to the Sun. This interpretation offered had previously been proposed to describe gas ejected into interplanetary space by the Sun. Gold (1962) offers a description of such an event and explores the idea that an expelled plasma cloud drags out solar lines of force and remains magnetically connected to the Sun, an idea also proposed by other authors of the time (e.g. Cocconi et al., 1958; Piddington, 1958). Figure 1.3 shows a cartoon of the description that Gold (1962) offers of the propagating plasma cloud. Observationally, a magnetic cloud is characterized by its in situ signatures. It is identified as having an enhanced magnetic field with rotation in direction and have a plasma β that is small. The low plasma β indicates that magnetic pressures dominate over thermal pressures in its expansion. Preceding the magnetic cloud a shock region is typically observed and its strength dependent on the speed of the cloud. When magnetic clouds were first observed their connection to the Sun was not clear, though it was suggested by Burlaga et al. (1981); Klein & Burlaga (1982) that they might be related to CME events. The first definite correlation between a magnetic cloud and CME was made by Burlaga et al. (1982) using in situ observations from the Helios 1 spacecraft and the NRL coronagraph onboard the P78-1 satellite.
1.3 Models of the Onset and Acceleration of a CME

The basic observational features of an erupting CME are depicted graphically in figure 1.4. This picture is a culmination of many years of observations amongst the associations of different solar phenomena and the theoretical insight of many authors. Many reviews on the theoretical understanding of the initiation of CMEs have been written in the last decade (see e.g. Forbes, 2000; Klimchuk, 2001; Lin et al., 2003; Forbes et al., 2006; Mikić & Lee, 2006; Mittal & Narain, 2010, and references therein). This section will attempt to summarize the key points of these authors. What the "initiation" of a CME refers to here is the physical process by which some initial structure on the Sun subsequently evolves and results in the expulsion of mass and energy. This subsequent mass and energy is manifested in flares, prominence eruptions, and the CMEs which are seen in the FOV of coronagraphs. These observed features are labeled in Figure 1.4, which is a cartoon that illustrates how they are spatially interrelated. Flares are labelled as X-ray loops, whose footpoints map
Figure 1.4: Cartoon showing the various observational features associated with a CME eruption. (From: Forbes (2000))
to the surface labelled as Hα ribbons and are features observed on the solar surface. The prominence is also labelled, this is a feature that typically can be observed possibly for days before it may erupt. Finally, the plasma pileup and cavity are thought to be the bright front and dark cavity observed in the FOV of coronagraphs and the prominence is typically interpreted as being the bright core within the dark cavity.

CMEs originate from the solar surface as large scale structures that transition from a state of rest to a very dynamic one releasing a large quantity of mass and energy into the heliosphere. Where this energy comes from and how it is released has been a highly debated subject over the last few decades. When CMEs were first observed it was not clear where the energy was derived to drive such a large quantity of mass to be accelerated and ejected from the Sun, though it was quickly realized early on that magnetic forces should dominate in the initiation of the CME (e.g. Gold & Hoyle, 1960; Anzer, 1978; Mouschovias & Poland, 1978; Canfield et al., 1980; Webb et al., 1980). The energy that is released could be stored in the corona prior to eruption. Analysis of the energetics involved with the eruption indicate that the magnetic energy stored in the corona exceeds kinetic, thermal, and gravitation energy densities by 2 to 3 orders of magnitude combined (Forbes, 2000). This leads to the condition in the corona that currents associated with the magnetic energy must be force-free or confined to current sheets since other counter balancing forces such as gas pressure or gravity are small compared to the $J \times B$ Lorentz force. The magnetic structure in the corona before the onset and release of the energy of the CME is modelled as either force-free magnetic loop arcades or a flux rope. In these configurations the currents are flowing parallel to the fields. These initially closed filed lines, through destabilization, open to infinity with a vertical current sheet separating the oppositely directed field lines. This idea is also supported by the interpretations of observations of CMEs through the FOV of coronagraphs that seem to open the coronal fields after an eruption (Gosling, 1974; Hundhausen, 1987). This process releases the stored magnetic energy of the associated coronal current that had built up. Models of this type are known as "storage and release" and it is expected that the energy must decrease once the field opens (Low, 1981; Sturrock
et al., 1984). In an idealized case of simply connected open magnetic field lines it has been found that this open state in fact has greater magnetic energy than the force-free closed state (Aly, 1991; Sturrock, 1991). This is called the Aly & Sturrock condition. At first glance this seemingly contradicts the premise of the storage and release models. Those models predict that closed magnetic field regions open releasing energy, and hence, the open field state should have less magnetic free energy. This is the challenge to the storage and release models, which have found a way to bypass this by employing such mechanisms as magnetic reconnection (e.g. Mikić & Linker, 1994), non force-free process in which thermal or gravity play a role (e.g. Low, 1981, 1999), or only partially opening of the field lines (e.g. Wolfson & Low, 1992; Antiochos et al., 1999) in which cases the the Aly & Sturrock condition is no longer valid.

Resistive MHD models and the ideal-resistive hybrid models both of which can be considered storage and release type models require that resistive MHD be invoked for reconnection to occur to dissipate the stored magnetic energy (e.g. see Forbes, 2000; Lin et al., 2003, for a review). The purely resistive models typically increase magnetic energy of the system by the shearing of and/or convergence of footpoints which leads to the formation of a sheared arcade or flux rope. With further stressing of the system reconnection dissipates the magnetic energy and the structure is released opening the field lines (e.g. Mikic et al., 1988; Inhester et al., 1992; Amari et al., 1996; Antiochos et al., 1999; Mikić et al., 1999; Amari et al., 2003a,b). The ideal-resistive hybrid models predict that an ideal MHD instability occurs when a quasi-static evolution of a system leads to a sudden loss of MHD equilibrium (e.g. Low, 1981; Forbes & Isenberg, 1991; Isenberg et al., 1993) or instability (e.g. Klimchuk & Sturrock, 1989; Wu et al., 1994; Török & Kliem, 2004; Kliem & Török, 2006). The ideal loss of equilibrium has also been termed catastrophic in nature (Forbes & Isenberg, 1991). The resistive part of these models is included because the ideal loss of equilibrium is not sufficient to account for the total energy that is dissipated (e.g. Forbes & Isenberg, 1991; Isenberg et al., 1993; Lin & Forbes, 2000).

In addition to the storage and release models an alternative class of models have been
identified as "driven" models (e.g. see Klimchuk, 2001, for a review). The earliest of the driven models are the so called "thermal blast" models which predict that a CME is driven outward by the thermal energy of a flare (e.g. Dryer, 1982; Wu, 1982). But it is now known that this thermal energy is not sufficient to account for the energy required to drive a CME through eruption (see e.g. Forbes, 2000). Another concept of the driven class of models are ones that are initiated by the dynamo of the Sun. It is hypothesised that convective zone motions may produce a field align current along a loop in the corona. This current is of such magnitude that it can drive the loop during the eruption (e.g Sen & White, 1972; Heyvaerts, 1974; Kan et al., 1983). The field-align currents will cause the shear component of the magnetic field to be amplified causing an increase in magnetic pressure producing an outward inflation of the loops (e.g. Klimchuk, 1990). If this inflation occurs rapidly enough there may result an eruption and an ensuing CME (Klimchuk, 2001), though, this idea has been objected to because such currents should cause upward and horizontal surface motions, an observational feature which has not been detected (Lin et al., 2003). Finally, another class of model that may be thought of as analogues to the dynamo driven model is that of flux injection (e.g. Chen, 1989, 1996; Krall et al., 2000; Chen et al., 2006). Physically, flux injection refers to the gradual or sudden increase in the poloidal flux passing through the loop and the photosphere. In ideal MHD it is assumed that this flux is conserved and determines how the toroidal current along the loop will evolve. During the flux injection process this flux is allowed to increase and since the current is dependent on the flux it will increase. It should be noted that it has previously been pointed out by Chen & Krall (2003) that it may be the case that the current may decrease even though the flux in being increased. This typically occurs some time after the loop becomes greater than semi-circular. In these types of models it is typically presupposed that a stable flux rope has formed on the solar surface. The driving flux injection is a parameter that must be specified. The source of this flux has been interpreted as originating from the photosphere (cf. Chen, 1989), or chromosphere (cf. Forbes, 2000), or from coronal fields possibly as a result of reconnection (cf. Vršnak, 2008). This scenario could be interpreted in two ways, either the flux rope is
driven through eruption by the current (e.g. Chen, 1996; Krall et al., 2000), or the flux rope is driven through stable quasi-static states until it reaches a sudden loss in equilibrium and subsequently erupts (e.g. Chen et al., 2006; Olmedo & Zhang, 2010). The latter scenario will be rigorously described in chapter 2.

1.4 Topics Covered in this Thesis

1.4.1 Loss of Equilibrium

In the solar corona the forces that dominate the initiation of a CME are of magnetic origin. Several forces have been identified between which stability of coronal structures can be established. Essentially there is an upward and a downward force including gravity. A good observational feature to the pre-eruptive structure of CMEs are filaments. These structures are long lived features on the Sun and erupting prominences are highly associated with CMEs. Filaments are observed to lie above polarity inversion lines (Martres et al., 1966). The scale height of prominences indicates that plasma pressure alone cannot support the observed mass against gravity and that magnetic fields support this mass. The configuration of the magnetic field includes a magnetic dip where the mass is supported against gravity and several magnetic topologies have been identified (see review by Démoulin, 1998). One such topology that can support prominences are twisted flux ropes, where the mass will lie in the bottom part of the helical field lines. The magnetic field of the flux rope may be modelled as from a current channel parallel to the polarity inversion line. During the time scale of an eruption no appreciable changes to the normal photospheric magnetic field are observed. Mathematically, this is modelled with an image current in order to satisfy this condition (e.g. Kuperus & Raadu, 1974). Equivalently this is thought of as the induction of photospheric currents that lead to a net repulsive outward force of the filament, modelled as a current channel. The Lorentz force between this current channel and the potential magnetic field of the dipole depends on the direction of the current. The current can be aligned such that this Lorentz force is downward to balance the upward repulsive force (Kuperus & Raadu,
1974; van Tend & Kuperus, 1978; Forbes & Isenberg, 1991). This topology is known as the inverse polarity configuration (e.g. Leroy et al., 1983). Alternatively, the current could be aligned such that the Lorentz force were directed upwards (Kippenhahn & Schlüter, 1957). In this case, the configuration is known as normal polarity. The net upward Lorentz and repulsive forces are balanced by the downward gravitational force. The inverse polarity is now thought to be the dominant configuration since most measurements of the magnetic fields in prominences indicate this (Démoulin, 1998) and because magnetic energy dominates in the corona, even over gravity (Forbes, 2000).

Another force of magnetic origin to consider is that of the ”hoop” Lorentz self force which result from the interaction of the toroidal loop current with its self fields. Stability is achieved when an external magnetic field is applied perpendicular to the loop axis. This type of configuration was first studied for toroidal plasma confinement in tokamaks (Shafranov, 1966; Bateman, 1978), then placed into the context of solar eruptions (Chen, 1989; Titov & Démoulin, 1999; Kliem & Török, 2006; Isenberg & Forbes, 2007). Just like the straight current channel case, this configuration places the loop axis over the polarity inversion line. Because the flux rope is curved it is assumed that it has two footpoints that connect towards the photosphere at opposite polarities completing the circuit below the photosphere. The Lorentz self force is outwards away from the center of curvature. The Lorentz force between the loop current channel and the potential magnetic field of the dipole perpendicular to the inversion line, like the straight current configuration, can be outward or downwards depending on the direction of the current. For stability, the downward solution is taken, in this way the outward self Lorentz force is balanced (e.g. Cargill et al., 1994; Chen, 1996; Titov & Démoulin, 1999).

Three magnetic forces have just been described, a repulsive force due to induced photospheric currents, a Lorentz self force arising from the curvature, and an external Lorentz force between the current channel and ambient magnetic field. In some models only the external Lorentz and the repulsive force are considered (e.g. Kuperus & Raadu, 1974; van Tend & Kuperus, 1978; Forbes & Isenberg, 1991), and in others the external Lorentz and
the curvature force are considered (e.g. Anzer, 1978; Chen, 1989; Titov & Démoulin, 1999; Kliem & Török, 2006). A model which includes all three force components has recently been proposed Isenberg & Forbes (e.g. 2007). It has been argued that the repulsive force and the Lorentz self force may be equivalent when a circular current channel is half way embedded in the photosphere such that the embedded half acts as the image current (Démoulin & Aulanier, 2010), though in the embedded circular current channel model it has been pointed out that the footpoints at the photospheric boundary would not remain stationary during dynamic evolution (Chen, 2007).

In the model of Chen (1989, 1996) the Lorentz self force, also called the Lorentz curvature force, is considered as the radially outwards driving force. The Lorentz force between the current channel and the external magnetic field is such that it is directed downwards. An equilibrium can be achieved between these two forces. The dependence of the external magnetic field in this model is specified as a function of height into the corona. The dynamical stability comes down to how these two forces compete with each other with increasing height. If the outward force cannot be restored at a rate equal to or faster than its decay with height by the downward external Lorentz force, then the flux rope is unstable. Along these lines it has been formulated that the location of stability is most dependent on how the external magnetic field strength decays (Cargill et al., 1994). In the early analysis by Cargill et al. (1994) it was found that for a shallow low lying loop with a large radius of curvature an external magnetic field with increasing height was needed, though it was not made clear at what curvatures or how “low” the low lying loop needed to be to require a field that is increasing with height. In a new analysis, by Olmedo & Zhang (2010) (chapter 2 in this thesis), we calculate the exact dependence on the external magnetic field required for stability. This analysis also further clarifies the “magnetic energy release” scenario proposed by Krall et al. (2000) (a model which is an extension of Chen (1989); Chen & Garren (1993); Chen (1996)) that states that at some point the external magnetic field falls off very rapidly such that the outward force can no longer be restored. However, it was not explicitly shown how fast the external magnetic field needs to decay. We have calculated
the explicit expression of the required decay of the external magnetic field for stability and is discussed in detail in chapter 2. In this way, the exact height where the loss of stability occurs can be calculated. This contribution significantly improves our theoretical understanding of how a flux rope CME configuration will experience loss of equilibrium and erupt. Future work will further develop the model presented by including all three magnetic forces discussed. This study only considered stability between the outward curvature force and the external Lorentz force. By including the repulsive force due to induced photospheric currents a more complete picture of the initiation should arise.

1.4.2 Study of Eruptive Prominences and Their Related Magnetic Fields

Based on the ideas presented in the previous section, that the instability depends on the competing magnetic forces, an observational approach is taken to interpret the loss of equilibrium. Theoretical models predict that a critical height exists where stability is no longer possible. Three such scenarios have been proposed depending on the geometry. These geometries include, straight current channel (van Tend & Kuperus, 1978), semi-circular torus (Kliem & Török, 2006), and partial torus (Olmedo & Zhang, 2010). All three predict different conditions for the onset of eruption. In this thesis the equilibrium between the magnetic forces of a partial torus is for the first time studied in detail (in chapter 2). We find that the condition for stability depends on the arc angle of the partial torus above the photosphere. The next step in this research is to observationally explore the nature of the theory, and empirically infer evidence to support the theory presented. We are encouraged by previous studies that have investigated magnetic fields in the corona with a connection to the eruptive nature of CMEs (e.g. Filippov & Den, 2001; Wang & Zhang, 2007; Liu, 2008). They have found that the ambient magnetic field, extrapolated from the photospheric surface, will decay faster with height for eruptive events that have resulted in a CME. The intention of our study is to examine the connections between models of the initiation of CMEs (as discussed in the previous section) and coronal magnetic fields, in an observational context of events of did result in CMEs.
In the model of Kuperus & Raadu (1974) a straight current channel is embedded in the corona over a polarity inversion line of a dipole. The Lorentz force of the potential magnetic field with the current channel balances the repulsive force of the induced photospheric currents. This force balance can be written in the following way, including gravity, (van Tend & Kuperus, 1978)

\[ F = \frac{I^2}{c^2 h} - \frac{I}{c} B(h) - mg, \]  

(1.1)

where the first term is the repulsive force of the image current, the second term is the Lorentz force, and the third term is gravity. Stability analysis finds that for this configuration to be in equilibrium the external magnetic field must decay radially slower than the repulsive force. This condition can be expressed as \( n_{ct} = -d\ln B/d\ln h = 1 \), where \( n_{ct} \) is the critical decay index of the magnetic field, where if the field were to decay any faster the configuration would not be in equilibrium. Another way to think of this is to express this as a critical height, which is the height where \( n_{ct} = 1 \). In this model, the exact nature of the external magnetic field is not specified only that it is of external origin, and that it is the vectorial component that lies above and is orthogonal to the polarity inversion line. This condition is the same for all three geometries (straight, torus, and partial torus). Observationally, magnetic field in the corona can be calculated through extrapolating from photospheric observations. The model just described above was used by Filippov et al. (2006) to empirically explain the stability of prominences on the solar surface. A leap between observations and theory is made by making the assumption that \( B \) in equation 1.1 is the extrapolated magnetic field. Interpretation is required to make the comparison between the filament and the required flux rope configuration. It is known that a possible magnetic configuration that can support prominence material is a flux rope, then a possible interpretation is that the top part of the prominence represents the apex of the flux rope current channel. The observational explanation by Filippov & Den (2001); Filippov et al. (2006) is that a critical height exist where the decay index of the external magnetic field corresponds exactly to \( n = 1 \).

A limitation to the study of Filippov & Den (2001), and Filippov et al. (2006) is that
Figure 1.5: Position of the STEREO satellites on March 6, 2010. Image courtesy NASA.
the measurement of the height of the filament is estimated using only one vantage point (from the Earth) and suffers from projection effects. In my study I overcome this problem by making use of multi-instrument observations spatially separated. I use observations from the STEREO satellites, SOHO, and SDO. Combining the observations from the STEREO satellites with SOHO and SDO, features on the solar surface can be tracked from three vantage points. With simple geometric triangulation the exact three dimensional trajectory of an erupting prominence can be measured. One event studied occurred on March 6, 2010. Figure 1.5 shows the position of the STEREO satellites on this date, which were separated by 137 degrees at that time. The position of the SOHO and SDO satellites are approximated to be taken at the Earth. In this way the exact evolutionary track can be compared with the extrapolated magnetic fields. An ideal event for study is one where a prominence is seen by both the STEREO satellites and where this prominence is seen as a filament near the disk center from the vantage point of the Earth. In this way, the line of sight magnetic field measurement is best observed.

A new approach of interpretation is explored in this work. The new calculation of the dependence of the critical parameter that determines stability of a partial torus flux rope (Olmedo & Zhang, 2010) is used as an effort to understand the observations. Chapter 3 of this thesis presents preliminary results of a new ongoing observational study to try to validate the ideas of the presented theory. Going beyond the straight current channel model, another model to consider is the torus loop half embedded in the photosphere. In this model a critical decay index of \( n_{ct} = 1.5 \) is predicted instead of \( n_{ct} = 1 \), as in the straight current model. Simulations of a semi-circular line-tied flux rope have shown that the critical decay index can vary. For example Fan & Gibson (2007) found that the critical index can be close to 2, and Török & Kliem (2007) found that \( n \lesssim 1.5 \). So the generally accepted value for this decay index is between 1 and 2. The theoretical study presented in chapter 2 of the stability of a partial torus flux rope indicate that the critical index is a function of the geometry of the loop and that it can be outside of the range between 1 and 2. The fundamental result of the theoretical study is that there is a functional dependence
of the critical index on the geometry of the loop, more specifically on the shallowness of the loop. This function monotonically increases and plateaus to a maximum value as the radius of curvature becomes larger. The fundamental result of the observational study is that a more shallow loop has a smaller critical index versus a more circular loop. A quantitative comparison between this result and the theory presented in chapter 2 show a discrepancy in the exact numerical dependence. We argue that the observations qualitatively capture the essence of the theory presented by producing a similar trend. Another aspect explored is a result of the simulations of Török & Kliem (2007) that show that the kinematics of an erupting flux rope are affected by the slope of the decay index of the external magnetic field at the height of the onset. A prominence is used as a proxy of a flux rope and its kinematics are modelled by an exponentially dependent acceleration function characterised by its time constant. It is empirically found that the steeper the slope of the decay index with height the faster the eruption will proceed. What is meant by "faster" is that the exponential time decay constant is smaller meaning that the acceleration increases at a faster rate.

1.4.3 Automatic Detection and Characterization of CMEs

Since the launch of the Solar and Heliospheric Observatory (SOHO) spacecraft in 1995, CME observations have mainly been made with the LASCO instrument onboard. The identification and cataloguing of LASCO CMEs is an important task which provides the basic knowledge for further scientific studies. Currently two prominent catalogues exist that catalogue CMEs observed with the LASCO coronagraphs: the Naval Research Lab (NRL) catalogue (currently online at http://lasco-www.nrl.navy.mil/index.php?p=content/cme_list under the "Preliminary List" section of this page) and the Coordinated Data Analysis Workshop Data Center (CDAW) catalogue (currently online at http://cdaw.gsfc.nasa.gov/CME_list/). The NRL catalogue is compiled by LASCO observers who look through the sequence of LASCO coronagraph images and report on events that have taken place on a daily basis. This is a preliminary catalogue and provides information on CME time and approximate position as well as a brief description of the event. The CDAW catalogue
on the other hand provides some measurements of CME properties including: the position angle, the angular width, and the height of the CME in each individual image. This catalogue combines the height measurements in the time sequence to determine the velocity and acceleration, which are also provided in the catalogue (Yashiro et al., 2004). These measurements are made by dedicated human operators who look at and then choose the CME height and position on coronagraph images displayed one by one on a computer screen. This human-based process is rather time-consuming and the events provided and parameters measured are subject to human bias.

The STEREO SECCHI instruments observing the corona and the heliosphere have been produced valuable observations of the propagation of transients from the Sun to the Earth. Onboard there two coronagraphs (COR1 & COR2) and two instruments to observe the heliosphere (HI1 & HI2). It should be recalled that the STEREO mission is composed of two satellites, therefore any catalogue of events observed with instruments onboard will have two points of view for the same event depending on the trajectory of the event. A catalogue for CME events is compiled for the COR1 instrument by the NASA Goddard Space Flight Center COR1 team (currently online at http://cor1.gsfc.nasa.gov/catalog/). This catalogue is compiled by human operators that look at both images from the STEREO A & B satellites and provide a description of the event similar to the NRL catalogue of LASCO CMEs. In addition they also provide movies of the events observed. A catalogue is compiled for transient events observed with the HI1 instrument by the HI1 team at the Rutherford Appleton Laboratory, UK (currently online at http://www.sstd.rl.ac.uk/stereo/HIEventList.html). This catalogue provides height measurements (unit of elongation angle), and an estimated trajectory angle with which the speed is calculated, and a time of arrival to 1 AU is predicted.

To date, several automated CME detection schemes have been described in the literature. The first of its kind, the Computer Aided CME Tracking (CACTus) software package was introduced in 2002 (Berghmans et al., 2002; Robbrecht & Berghmans, 2004). It implements the image processing technique of the Hough transform, which finds lines within
a 2D image. CACTus utilizes this transform to identify CMEs as a bright streak in 2D time height images composed along a specific position angle from a series of coronagraph images. Currently CACTus has compiled an online catalogue that spans from 1996 to the present day using LASCO C2 and C3, and STEREO SECCHI observations (currently online at http://sidc.oma.be/cactus/). This system operates in near-real time, meaning that detections of CMEs can be found online on a daily basis. Another method proposed is the Automatic Recognition of Transient Events and Marseille Inventory from Synoptic maps (ARTEMIS) that utilizes LASCO C2 synoptic maps and looks for signatures of CMEs (Boursier et al., 2005; Boursier et al., 2009). This method detects CMEs as vertical streaks within the maps and determines their velocity by comparing synoptic maps constructed at different distances from the Sun. The result of ARTEMIS are placed online and a complete catalogue using LASCO C2 observations has been compiled (currently online at http://www.oamp.fr/lasco/). Another interesting method proposed by Liewer et al. (2005) that has not received much attention is a scheme that tracks arc-like features in coronagraph image pairs for use by the STEREO SECCHI coronagraph instruments to preferentially downlink data containing CMEs. A method proposed by Qu et al. (2006) use image-segmentation techniques to find CMEs in running-difference and running-ratio images. Once the image is segmented machine-learning techniques are implemented to further classify CMEs into different categories. Finally, the Solar Eruptive Event Detection System (SEEDS) algorithm we developed (Olmedo et al., 2008), fully described in chapter 4, detects CMEs by using image segmentation techniques to identify them in coronagraph running difference images and then subsequently tracks them in subsequent frames with a time-dependent causal filter. A complete catalogue of the algorithm applied to LASCO C2 data is found online (currently online at http://spaceweather.gmu.edu/seeds/). On this site a near-real time module has been implemented to continuously make detections as data becomes available to us from the instrument operators. The near-real time module includes detections made with LASCO C2 and STEREO A & B COR2 beacon mode data. A complete catalogue of STEREO A & B COR2 data is forthcoming. It should be emphasized
here the importance of automated detection in a general sense. As the technologies of
detectors and communication advance, the capabilities for observing at higher resolutions
and higher cadences yield data sets that are tremendous in volume. And as more space
missions are flown with advanced capabilities, the data acquisition rate begins to overcome
the rate at which human operators can analyse and interpret the data. An automated
method to analyse data is a valuable tool, not only for the present ongoing missions but,
more importantly, for future missions.

Automatic detection of CMEs in coronagraph images can aid scientist to reduce the size
of the date sets and extract valuable information that can be used for further examination.
I give here an example on how SEEDS has helped supplement this thesis. With the use
of the derived catalogue created by the automated method, CMEs within the FOV of the
LASCO coronagraph images were identified. The output is a list of events, where each event
is defined by its measured parameters. These include its starting and ending time, position
angle, angular width, and height. Other parameters can be derived from these, such as
velocity and acceleration. A criteria for the type of event required for the observational
study of chapter 3 is a filament eruption that resulted in a CME. To begin the search I look
for events in the SEEDS CME catalogue that have an intermediate angular width, say > 45
degrees. Upon finding appropriate events, I then look for the solar surface source region
and if a filament is identified then that event can become a candidate for further study.
There are a number of data sets which could be looked in where filaments are seen, such
as Hα, or extreme ultraviolet. The SOHO and STEREO 304Å images are very accessible,
and in this wavelength typically filaments/prominences can clearly be seen. An additional
criteria for the observational study of chapter 3 is that the prominence eruption be observed
by multiple vantage points. I limit the search to events that occur during the STEREO era
for the satellites separation angle of > 90 degrees. That is, events that occur after about
February 2009. With these criteria, the 6 March 2010 event was identified as a candidate
event for further study. On this date the STEREO satellites were separated by about 137
degrees. Figure 3.6 top panels shows the precursor filament (left) and subsequent flare
after eruption (right). In the SOHO EIT 304Å images a data gap prevented tracking this filament during the eruption. But these images do constrain the heliographic coordinate of the filament location. Figure 1.7 is the height-time plot for this event found with the SEEDS algorithm using LASCO C2 data. The line through the data points is a linear best fit with velocity 445 km/s. Using a linear and second order extrapolation to the solar surface an onset time of 7:15 UT and 7:00 UT respectively are predicted. As it turns out, the onset time of this event is between 5:26 UT and 6:03 UT, so the prediction made by the SEEDS detection is off by at most 1.5 hours. Determination of the exact onset time requires detailed analysis of the prominence kinematics. Further detail on this event and associated extrapolated magnetic fields will be explored in chapter 3. In addition, a second event, the 1 August 2010 event, is studied. But because of a data gap in SOHO, we were not able to use this method to find this event. Other methods to search for events may include randomly scrolling through the solar images (which can be daunting), or by word of mouth within the community.
Figure 1.6: The 6 March 2010 prominence and CME eruption event. The top panels show the prominence eruption in two subsequent frames in SOHO EIT 304Å images, where the arrow points to the filament before eruption. After eruption (top right panel) a solar flare is seen. The bottom panels are the detection made by the SEEDS algorithm of the associated CME using LASCO C2 data.
Figure 1.7: Height-time plot made by SEEDS for the 6 March 2010 event. The line through the data points is a linear best best fit with velocity 445 km/s.
1.5 Organization of Thesis

The first part of this thesis describes the theoretical and observational work related to the initiation and onset of CMEs. The last part is the description of the automatic detection method developed as a tool for data reduction/exploration, and characterization of CMEs. Chapter 2 is on the partial torus instability (PTI), that theoretically studies the equilibrium between the Lorentz self force and the restoring Lorentz force between the ambient field and the current channel. Chapter 3 observationally explores the concepts of the theoretical study of chapter 2. This is done by comparing the kinematics of prominences and associate extrapolated magnetic fields. Chapter 4 describes in detail the automatic detection algorithm to identify CMEs. Finally, chapter 5 concludes the thesis.
Chapter 2: Stability Analysis of a Partial Torus Flux Rope

2.1 Introduction

In this chapter, I study the onset mechanism of coronal mass ejections (CMEs) and propose that the onset is caused by the partial torus instability of a pre-existing flux rope magnetic structure prior the eruption. CMEs have origins in the magnetic fields of the Sun. The magnetic structure of a CME is now generally accepted to be a flux rope. Whether or not the flux rope is pre-existing in the corona prior to an eruption is still in debate; though mounting evidence is pointing to its existence before the eruption initiates (e.g. Rust, 2003; Gibson & Fan, 2006; Green & Kliem, 2009; Tripathi et al., 2009; Yeates & Mackay, 2009). Many theoretical studies have therefore presupposed the existence of a flux rope in the corona anchored to the photosphere in order to do stability and kinematical studies of CMEs (e.g. Chen, 1989; Lin et al., 2002; Roussev et al., 2003; Fan & Gibson, 2007; Isenberg & Forbes, 2007; Török & Kliem, 2007). Torus instability (TI) has been investigated before. This type of instability arises when considering the stability of a plasma ring with toroidal current. When Lorentz forces dominate, stability of such a plasma ring can be established with an external magnetic field that is perpendicular to the axis of the torus (Shafranov, 1966). However, if the external field decreases rapidly enough in the direction of the major radius $R$, then any outward perturbation will cause the inward Lorentz force to decrease faster than the outward Lorentz force resulting in the expansion of the ring (Bateman, 1978). This kind of instability is recognized as a driver for the eruption of Coronal Mass Ejections (CMEs), where a flux rope is modeled as a current-carrying toroidal loop (e.g. Chen, 1989; Cargill et al., 1994; Titov & Démoulin, 1999; Krall et al., 2000; Lin et al., 2002; Kliem & Török, 2006; Fan & Gibson, 2007; Isenberg & Forbes, 2007; Török & Kliem, 2007). The external magnetic field can be quantified by a decay index $n$, and it has been shown
that there exist a critical value $n_{ct}$. If $n > n_{ct}$, the flux rope will be unstable to major radius perturbation and will rapidly erupt (Kliem & Török, 2006; Fan & Gibson, 2007; Török & Kliem, 2007). Studies of extrapolated magnetic fields overlying active regions that have produced either an erupting or confined event have shown a clear distinction between the two, observationally indicating the existence of a $n_{ct}$ (Wang & Zhang, 2007; Liu, 2008). Furthermore, laboratory experiments where solar like conditions were reproduced have also shown that the external field plays a major role in the eruption of a flux rope like structure (Hansen & Bellan, 2001).

Almost all earlier works on torus instability have assumed the critical index to be a constant and are based on the geometry of full torus. Theoretical calculations of stability done by Bateman (1978) and Kliem & Török (2006) considered a current-carrying toroidal plasma ring. To achieve stable equilibrium, a spatially decaying external magnetic field that follows a particular functional form was used. This form assumed that the external field satisfy $n = -\frac{Rd\ln B_{ex}}{dR} = \text{const}$ (the choice of this form will become clear later on in this chapter). A value for $n_{ct}$ was found by Bateman (1978) to be $3/2$. Kliem & Török (2006) found that $n_{ct}$ could be written as a function of the aspect ratio, which is defined as the ratio between the major and minor radius of the torus. However, since the aspect ratio term only appears in their equation within an inverse logarithmic term, their expression is only weakly dependent on aspect ratio. In the limit of the very large aspect ratio, it will recover $n_{ct} = 3/2$. It is also interesting to note the analysis work done by Hansen & Bellan (2001), who considered the the stability problem of a flux rope when describing their experimental results. They only considered a constant external field (since that was how their experiment was set up) and calculated the required field strength needed for the flux rope to be in equilibrium. A constant external field would yield a decay index of $n = 0$, which is less than $n_{ct}$, so one would expect the flux rope to be in a stable equilibrium. But this equilibrium can only occur if the magnitude of the external field has an exact value. This corresponds to a value such that the $J \times B_{ex}$ force between the current in the flux rope and the external field balance the outward Lorentz force of the loop. Therefore, if the magnitude
is different than what is required, no equilibrium can be established. Hence, there are two factors for the stability of the flux rope modeled as a torus: (1) the balance between the internal and external magnetic field strengths (so that the Lorentz forces balance), and (2) the rate of decay of the external magnetic field strength at the location of the equilibrium, which determines whether the flux rope is in a stable equilibrium or if a perturbation will cause it to expand. It may also be possible to achieve equilibrium in a condition where no external magnetic field is present. In this case thermal gradients between internal and external pressures balance the outward Lorentz force Xue & Chen (1983); though such a case may not be realistic in the Sun’s corona.

Different from previous works, we re-examine the torus instability through considering a partial current-carrying torus loop. As a result, the instability analysis is generalized. We consider that the flux rope only exists above the photosphere with two footpoints anchored to the photosphere. Our model also assumes the footpoint separation is fixed. The fixed footpoint separation leads to several effects, most notably the scaling law of acceleration, which states that there is a relationship between the height at which the flux rope reaches a maximum acceleration and the footpoint separation (Chen & Krall, 2003; Chen et al., 2006), and the instability of low-lying loops (Cargill et al., 1994), which will be discussed as part of the theory derived herein. Likewise, it is presented here that footpoint separation also plays a role in the value of \( n_{ct} \) for varying values of the ratio \( Z/S_0 \), that is the ratio between the apex height \( Z \) and half footpoint separation \( S_0 = S_f/2 \). This dependence arises due to the geometrical assumption made for the evolution of the model. In the limit where \( Z/S_0 \) goes to infinity, or the circular torus limit, \( n_{ct} \) as derived by Bateman (1978), and Kliem & Török (2006) is recovered. The derived expressions in this thesis are therefore more general. Because of the dependence of \( n_{ct} \) on the geometrical shape, we coin this the partial torus instability (PTI).
2.2 Equilibrium Analysis

The forces used here were first derived by Shafranov (1966) for the equilibrium of a current-carrying circular loop. One of the earliest adaptations of this type of model, as applied to solar coronal loops, was by Anzer (1978) and Xue & Chen (1983). This type of model is distinguished from other models because it contains a term in its force balance equation that has the form $\ln(8R/a)$, which arises from certain approximations when deriving the inductance of the current loop. The derived forces of this model were later adopted by Chen (1989) to describe the dynamics of an evolving solar loop, now known as a flux rope. The modifications made to the model include a fixed footpoint separation anchored to the solar surface and the assumption of local curvature. In this way one can model the 3D structure of the loop by following the apex height. The forces considered in the model are the Lorentz force, thermal pressure gradients, gravity, and aerodynamic drag. See appendix B for a detailed description of this model, including an illustration relating the various terms. For analytical simplification, we will only consider the Lorentz forces following analysis. In other words, it is assumed that in the regime of validity the plasma has a very low beta value, such as in the lower corona, and that Lorentz forces dominate over all other forces. The equations of motion with this assumption are written as force per unit length; for the position of the apex $Z$

$$M \frac{d^2Z}{dt^2} = \frac{I^2_t}{c^2 R} \left[ \left( \ln \left( \frac{8R}{a} \right) - 2 + \frac{\xi}{2} \right) - \frac{1}{2} \frac{B^2_t - B^2_{zt}}{B^2_p} + 2 \frac{R}{a} \frac{B_s}{B_p} + 1 \right] \quad (2.1)$$

and for the minor radius $a$

$$M \frac{d^2a}{dt^2} = \frac{I^2_t}{c^2 a} \left[ \frac{B^2_t - B^2_{zt}}{B^2_p} - 1 \right]. \quad (2.2)$$

In this section each of the variables will be later described. A major distinction must be made between equation (2.1) and the equation of motion used in the analysis of Kliem &
Török (2006). That is, that this equation of motion follows the position of the apex height \( Z \) above the photosphere, whereas the equation of Kliem & Török (2006) follows the major radius \( R \) and does not take into account the effect of the anchored footpoints. Because of the assumed fixed footpoint separation in the model considered here, the major radius is defined as a function of \( Z \):

\[
R = \frac{Z^2 + S_0^2}{2Z}
\]  

(2.3)

This model does not strictly follow the definition of a line-tied flux-rope, though it does contain aspects which are required. These include footpoints that are fixed and embedded in the solar surface, and as will be discussed further, conservation of magnetic flux enclosed between the loop axis and the surface. A feature not included in this model but required of line-tying is that the normal component of the magnetic field remain constant at the surface. This model does not include any prescription of the field at the surface, therefore this condition cannot be met. A more rigorous derivation of the force equation can be found in the work of Isenberg & Forbes (2007), where the line-tying of the magnetic field on the surface is incorporated and its effects on the flux rope are modelled. A more stringent analysis of stability should consider the forces on all points along the flux rope. It must be emphasized that in the analysis presented here only the forces at the apex are considered, and although it may be the case that the sum of forces at the apex are positive (radially outward) the total net force on all of the points may not be. Finally, an aspect not explicitly described by this model is any current sheet that may form below the flux rope. Nevertheless it has been found that for a strictly line-tied flux rope, the formation of a current sheet may only contribute a second order correction to the forces acting on the flux rope (Isenberg & Forbes, 2007).

In equation (2.1), the first term enclosed by parenthesis is related to the self-inductance \( L_p \), which arises due to the toroidal current \( I_t \) and is approximated with the assumption...
that $R >> a$

$$L_p = \frac{4\pi \Theta R}{c^2} \left[ \ln \left( \frac{8R}{a} \right) - 2 + \frac{\xi_i}{2} \right], \quad (2.4)$$

where $\xi_i$ is the internal inductance, which depends on toroidal current distribution across the minor radius, and

$$\Theta = \begin{cases} 
1 - \theta / \pi, & Z \geq S_0 \\
\theta / \pi, & Z < S_0 
\end{cases}, \quad (2.5)$$

with $\theta = \sin^{-1}(S_0/R)$. $\Theta$ is the fractional number of the partial torus and defined by the ratio between the arc length of the torus above the photosphere and the circumference of a circular torus of equal radius. The arc length of the partial torus is $2\pi \Theta R$. Equation (2.4) makes the assumption that the minor radius is uniform along the flux rope from the apex to the footpoints and evolves following equation (2.2). Other studies have imposed that the minor radius remain constant at the footpoints and model the evolution of the minor radius only at the apex (Chen & Garren, 1993, 1994; Chen, 1996; Krall et al., 2000). In this case the tapering of the minor radius from the apex to the footpoints must be modeled. Chen & Garren (1993, 1994) proposed that the minor radius from the apex to the footpoints vary exponentially, and Krall et al. (2000) have proposed a linear dependence, though the choice of not modelling the tapering of the minor radius will not significantly affect the results herein, a further discussion on the tapering is presented in section 2.4. We will show that by including this effect, the results of this analysis can differ by at most approximately 10%.

Because of the approximations made in deriving $L_p$, its accuracy compared to its integral definition will remain to a few percent to $Z_0/S_0 \gtrsim 0.5$, where $Z_0$ is the initial height, and for some parameter choices this accuracy persist to $Z_0/S_0 \approx 0.4$ (Cargill et al., 1994). Therefore the subsequent analysis shall be limited to $Z_0/S_0 > 0.5$ in order to stay on the conservative side. It should also be noted that the comparison of equation (2.4) in the circular torus limit ($\Theta = 1$) with a numerical solution of the integral form of $L_p$ shows that this equation
remains accurate to within approximately 10% for aspect ratio $R/a \gtrsim 10$. The deviation appears to exponentially rise for smaller aspect ratios (Zic et al., 2007).

There are four magnetic field components to take into consideration: $B_t$ is the toroidal and $B_p$ is the poloidal component of the internal field, $B_{et}$ is the toroidal and $B_s$ is the poloidal component of the overlying external field. Here $B_{et}$ is taken to be zero but could potentially be prescribed as some function, $B_s$ is a prescribed function of $Z$, and $B_p$ is described as a function of the toroidal current $I_t$ and minor radius

$$B_p = \frac{2I_t}{ca}$$  \hspace{1cm} (2.6)

The final aspect of the model is the definition of how the toroidal current evolves. This is an important aspect in the model that couples the poloidal flux due to the toroidal current and an external flux $\Phi_s$ due to the magnetic field $B_s$ that is bound between the photosphere and the loop. The total poloidal flux $\Phi_p$ is given by

$$\Phi_p = cI_tL_p + \Phi_s = cI_tL_p + \int B_s \cdot dA.$$  \hspace{1cm} (2.7)

This integral can only be solved for the simplest functional forms for $B_s$, else it will need to be solved numerically. An example that is analytically solved, assuming that $B_s$ varies linearly, will be given in the following section. The reader is here referred to figure B.1 in appendix B to get a pictorial/geometric perspective of the relationship between the variables that describe this model.

The stability of the flux rope can be studied by examining small height perturbations ($\delta Z$) about an equilibrium state. Through linearizing of equation (2.1), the evolution of a small perturbation satisfies an equation of the following form

$$\frac{d^2\delta Z}{dt^2} = \Gamma^2 \delta Z$$  \hspace{1cm} (2.8)
where by evaluating $\Gamma^2$, the behaviour of the perturbation will be either oscillatory or exponentially divergent depending on the sign of $\Gamma^2$. Cargill et al. (1994), as an extension of Chen (1989) with $B_s \neq 0$, derived an expression of $\Gamma^2$ for the full equations of motion given by Chen (1989) that include magnetic, thermal pressure, and gravity forces. Through this analysis they identified the instability of low-lying loops and were able to predict the oscillation frequency of the major axis of the flux rope for given initial conditions. The condition for stability of a flux rope was found to be most dependent on the rate of decay of the overlying field. The following will extend this work into the domain of an erupting flux rope and consider different scenarios for the configuration of the overlying field. It is also assumed that $\Phi_p$ remains constant to any perturbation, an assumption which satisfies the line-tying condition (Lin et al., 2002) and determines how the toroidal current will evolve, though, by prescribing $\Phi_p$ as a dynamical function of time the flux rope can be driven. The analysis begins by defining a variable

$$\hat{L} = \frac{c^2 L_p}{4\pi \Theta R} = \left( \ln \left( \frac{8R}{a} \right) - 2 + \frac{\xi_i}{2} \right),$$

(2.9)

which is equivalent to the variable $c$ in Kliem & Török (2006) written in MKS units. Three assumptions can be made about the evolution of the flux rope that lead to slightly different stability criteria. (1) The major and minor radius expand independently (i.e. no assumption made about self-similarity). (2) The system expands quasi self-similar by assuming that $R/a$ varies only by a small amount and that $\hat{L} \approx \text{const}$, which would be exactly constant if the system were self-similar $[1]$. Treating $\hat{L}$ as a constant is justified because this variable depends on $R/a$ logarithmically and should only contribute little error, a fact that will become clear shortly. (3) The flux rope expands exactly self-similarly with $R/a = \text{const} [2]$, and hence $\hat{L} = \text{const}$.

$[1]$This approximation assumes that $d\hat{L}/dZ = 0$ but not that $d(R/a)/dZ = 0$.

$[2]$When the system is self-similar $R^{-1}dR/dZ = a^{-1}da/dZ$. 
The linearization of equation (2.1) is written in the following way

\[
\frac{d^2 \delta Z}{dt^2} = \left\{ \frac{I^2}{c^2 RM} \left[ \frac{d\hat{L}}{dZ} + \frac{\Delta R}{R \, dZ} - \frac{\Delta da}{a \, dZ} + \frac{\Delta dB_s}{B_s \, dZ} - \frac{\Delta dB_p}{B_p \, dZ} \right] \right\} \delta Z
\]  

(2.10)

where \( \Delta = (2RB_s)/(aB_p) \), \( M = \pi a^2 \bar{n} m_i \) is the mass per unit length, and \( \bar{n} \) is the average total density. The term within the curly brackets is \( \Gamma^2 \) of equation (2.8). The decay index \( n \) of the external magnetic field is identified in the fourth term within the brackets and defined as

\[
n \equiv - \frac{Z \, dB_s}{B_s \, dZ}.
\]

(2.11)

The solutions of equation (2.8) are either oscillatory and stable, when \( \Gamma^2 < 0 \), or exponentially divergent indicating instability when \( \Gamma^2 > 0 \).

In equation (2.10) there are four differential terms. \( dR/dZ \) is straightforward to derive from equation (B.2), \( dB_s/dZ \) comes from the external field and is part of the definition of \( n \). To find \( dB_p/dZ \) consider the equilibrium of the minor radius, where it is found that \( B_p = B_t \), and \( B_t \) can be written as a function of initial values derived from the conservation of toroidal flux \( \Phi_t = \pi a^2 B_t \)

\[
B_p = B_t = B_{t0} \frac{a^2}{a_0^2},
\]

(2.12)

from here it is straightforward to find \( dB_p/dZ \). Using the linearized form of equations (2.6), (2.7), and (2.12) leads to

\[
\frac{1}{a} \frac{da}{dZ} = \frac{1}{R} \frac{dR}{dZ} + \left[ \frac{1}{\Theta} \frac{d\Theta}{dZ} + \frac{1}{cL_p I_t} \frac{d\Phi_s}{dZ} \right] \left( 1 + \frac{1}{L} \right)^{-1},
\]

(2.13)

where \( d\Theta/dZ \) is derived from equation (2.5), and \( d\Phi_s/dZ \) comes from the external magnetic
Figure 2.1: Solutions of $\Gamma$ versus the fractional number $\Theta$ with various values of decaying index $n$. Lines A to F increase from $n = -0.25$ to $n = 2.25$ in steps of 0.5 respectively. Positive values of $\Gamma$ correspond to the exponential growth rate of the instability, and negative values to the oscillation frequency. In units of minutes$^{-1}$. 
field profile. The final expression for $\Gamma^2$ is found to be

$$\Gamma^2 = \frac{I_t^2}{c^2 R M Z} \Delta \left[ 2Z \frac{dR}{dZ} + \left( Z \frac{d\Theta}{dZ} + \frac{Z}{c L_p I_t} \frac{d\Phi_s}{dZ} \right) \frac{1 - \frac{1}{\Delta}}{1 + \frac{1}{\hat{L}}} - n \right]. \quad (2.14)$$

Finally, $n_{ct}$ is found by setting $\Gamma^2 = 0$. In an equilibrium state the equation of motions yield the following conditions: $B_t = B_p$ for the minor radius and

$$-\Delta = \hat{L} + 1/2 = (\ln(8R/a_a) - 3/2 + \xi/2). \quad (2.15)$$

Plugging this equation for $\Delta$ into $(1 - 1/\Delta)/(1 + 1/\hat{L})$ yields a form that approaches one as the aspect ratio increases.

Solutions of $\Gamma$, equation (2.14), for different constant value of $n$ are plotted in figure 2.1. This is calculated with $\bar{n} = 3.62 \times 10^8$ cm$^{-3}$, $d\Phi_s/dZ = 0$, and $(1 - 1/\Delta)/(1 + 1/\hat{L}) = 1$, which is an approximation that introduces little error. The value of $\bar{n}$ was chosen arbitrarily, other choices would essentially stretch the y-axis and will not affect the position where any one particular line crosses from positive to negative. The units in this plot are such that positive values correspond to exponential growth, and negative values to the oscillation frequency. In this calculation and calculations hereafter an aspect ratio of 10 is assumed. The values of $n$ for each corresponding line A to F are from -0.25 to 2.25 in intervals of 0.5 respectively. For a particular value of $n$, the solution of $\Gamma$ goes from positive (unstable) to negative (stable), indicating that smaller values of $\Theta$ and $Z_0/S_0$ are unstable to major radius perturbation. This fact was coined as the instability of low-lying loops (Cargill et al., 1994). Therefore, for a particular value of $n$, a lower lying loop is unstable, while a higher lying loop would be stable. It should be emphasized that the stability at any value of $Z_0/S_0$ is solely dependent on $n$ whether or not the loop is low-lying. The instability of low-lying loops is thus the core of the partial torus instability theory presented in this chapter. Figure 2.1 is a kind of extension of figure 2d of Cargill et al. (1994). In that work $n = -\delta$, 36
$d\Phi_s/dZ \neq 0$ with $B_s$ assumed to vary linearly as $\delta Z$ where $\delta$ is the slope, and thermal and gravitational forces are included in the calculation. Even though thermal and gravitational forces were considered, the stability of the loop was found to be most dependent on $n$, which is associated to the Lorentz force, and only weakly on the former, therefore justifying the low beta limit approximation made in the present analysis.

### 2.3 Result of Partial Torus Instability (PTI)

By setting $\Gamma^2 = 0$, the boundary between stability and instability can be identified and is assumed to occur at some critical height $Z = Z_{ct}$. Since $n$ is a function of $Z$, the critical index is hence defined as

$$n_{ct} \equiv n(Z_{ct}).$$  \hspace{1cm} (2.16)

By rearranging the equation $\Gamma^2(Z = Z_{ct}, n = n_{ct}) = 0$ such that $n_{ct}$ is on one side the formal rule for the PTI is established. This definition says that if $n$ of the external magnetic field as calculated by equation 3.1 is greater (less) than $n_{ct}$ the loop is unstable (stable) to major radius perturbations at $Z_{ct}$. In case (1), the most general case where the major and minor radius expand independently, $n_{ct}$ reduces to

$$n(Z_{ct}) = n_{ct} = \frac{2Z_{ct}}{R} \frac{dR}{dZ} + \left( \frac{Z_{ct}}{\Theta} \frac{d\Theta}{dZ} + \frac{Z_{ct}}{cL_p I_t} \frac{d\Phi_s}{dZ} \right) \frac{1 - \frac{1}{\Delta}}{1 + \frac{1}{\Delta}}.$$  \hspace{1cm} (2.17)

In case (2) where quasi self-similar expansion is assumed, $n_{ct}$ reduces to

$$n(Z_{ct}) = n_{ct} = \frac{2Z_{ct}}{R} \frac{dR}{dZ} + \frac{Z_{ct}}{\Theta} \frac{d\Theta}{dZ} + \frac{Z_{ct}}{cL_p I_t} \frac{d\Phi_s}{dZ},$$  \hspace{1cm} (2.18)

where $\Phi_s$ is given by the integral on the right hand side of equation (2.7) and is a function of $B_s$. The only difference between these two equations is the factor on the right hand side of equation (2.17). This term approaches one for increasing $R/a$ and equation (2.15) for $\Delta$. 37
For all calculations of $n_{ct}$ herein, the quasi-self similar expression will be assumed. Because of the way that this model is set up $B_s$ must be given as a function of $Z$ and other variables $x_1, x_2, ...$ that characterize it, where $Z = 0$ defines the photospheric boundary. Plugging $B_s(Z, x_1, x_2, ...)$ into equation (3.1) results in $n$ as a function of $Z$ and other variables. At the critical height $Z = Z_{ct}$, the equation $n_{ct} = n(Z_{ct}, x_1, x_2, ...)$ will define a critical surface.

In equations (2.17) and (2.18), it is seen that $n_{ct}$ is found on both sides, since $d\Phi_s/dZ$ is a function of $n_{ct}$ at $Z = Z_{ct}$. This leads to a condition that makes it not easily possible to analytically solve for $n_{ct}$ except for the simplest functional forms of $B_s$.

In the circular torus limit where $Z_0/S_0$ goes to infinity, assuming quasi-self similar expansion, and a power law external field ($B_s \propto R^{-n}$), equation (2.18) reduces to

$$n_{ct} = \frac{3}{2} - \frac{1}{4L}$$

reproducing Kliem & Török (2006), and in the limit of a very large aspect ratio $n_{ct} = 3/2$. In this expression of $n_{ct}$ the external field profile is assumed a function of $R$, and may not necessarily be appropriate for the flux rope model examined here because this model assumes that the center of curvature is moving radial upwards from below to above the photospheric surface. This would imply that the origin of the external field would be moving radial upwards with the motion of the flux rope. Moreover, the loop’s radius of curvature, modelled by equation (B.2), must first decrease until the loop is semicircular then begin to increase leading to $dR/dZ = 0$ when $Z/S_0 = 1$ (Chen & Krall, 2003).

A more appropriate external field profile could be one that is a function of $Z$. The local magnetic field at the apex is approximated as a linear function of $Z$

$$B_s = B_{ct} \left(1 - \delta\frac{Z - Z_{ct}}{Z_{ct}}\right),$$

where $B_s(Z = Z_{ct}) = B_{ct}$. Equation (2.20) is the equation defined by Cargill et al. (1994)
with a minus sign instead of a plus sign, and a redefinition of the variables from initial condition to critical. Plugging equation (2.20) into equation (3.1) yields \( n \) as a function of \( Z, Z_{ct}, \) and \( \delta, \) and at \( B_s(Z = Z_{ct}), n_{ct} = \delta. \) With equation (2.20) it is possible to calculate an analytical expression for \( n_{ct}. \)

The linear case for an external magnetic field, as expressed by equation (2.20), may be the only functional form for \( B_s, \) which when used in deriving a criterion for stability with equation (2.17) or (2.18) will yield an analytical solution. The difficulty in the calculation is in finding \( d\Phi_s/dZ, \) and other functional forms for \( B_s, \) such as a power law \( B_s \propto Z^{-n}, \) yield more complex solutions that are better solved numerically. Deriving an expression for a critical criterion is straight forward once \( d\Phi_s/dZ \) is known. Using equation (2.20) for \( B_s, \) \( \Phi_s \) can be integrated and yields the following result

\[
\Phi_s = B_{ct} \left[ \varphi \left( 1 - \delta \left( \frac{R \cos(\pi(1 - \Theta))}{Z_{ct}} - 1 \right) \right) - \frac{2\delta S_0^3}{3 Z_{ct}} \right],
\]

(2.21a)

where

\[
\varphi = R^2 \left( \pi \Theta + \frac{S_0}{R} \cos(\pi(1 - \Theta)) \right).
\]

(2.21b)

Deriving \( d\Phi_s/dZ \) is straight forward and plugging this into equation (2.18) and rearranging for \( \delta, \)

\[
\delta = n_{ct} = \left( \frac{2Z_{ct}}{R} \frac{dR}{dZ} + \frac{Z_{ct}}{\Theta} \frac{d\Theta}{dZ} - \left( \frac{1}{2} + \frac{1}{4L} \right) \frac{Z_{ct}}{2\pi R^2} \frac{d\varphi}{dZ} \right) \left( 1 - \left( \frac{1}{2} + \frac{1}{4L} \right) \frac{\psi Z_{ct}}{2\pi R^2} \right)^{-1},
\]

(2.22a)

where

\[
\psi = \frac{d\varphi}{dZ} \left( \frac{R \cos(\pi(1 - \Theta))}{Z_{ct}} - 1 \right) + \varphi \left( \frac{dR \cos(\pi(1 - \Theta))}{dZ} + \frac{R \frac{d\Theta}{dZ} \sin(\pi(1 - \Theta))}{Z_{ct}} \right)
\]

(2.22b)

and \( \delta \) is evaluated at \( Z = Z_{ct}. \) although not explicitly written, this expression is in fact a
function of the ratio $Z_{ct}/S_0$ and not independent of either. It is also found that this function
is independent of $B_{ct}$. The term $(1/2+1/4\hat{L})$ comes from the equilibrium of equations (2.1),
and (2.2).

$$\frac{1}{2} + \frac{1}{4\hat{L}} = \frac{1}{2} + \frac{\pi \Theta R}{c^2 L_p} = \frac{-2\pi \Theta R^2 [B_s(Z)]}{c L_p I_t} \tag{2.23}$$

The dash line in figure 2.2 shows the behaviour of $n_{ct}$, given by equation (2.22a), as a
function of $\Theta$ evaluated at $Z = Z_{ct}$. Vertical dotted lines indicate the position of $Z_{ct}/S_0$,
and as $Z_{ct}/S_0 \to \infty$, $\Theta \to 1$. This plot is essentially a prediction of this flux rope model
that tells us what the decay index of the external field is at the apex of an unstable flux
rope.

All of this discussion thus far has assumed that $d\Phi_s/dZ \neq 0$. On the other hand, if it is
assumed that $d\Phi_s/dZ = 0$, then $n_{ct}$ becomes independent of the functional form of $B_s$. The
difference between the two assumptions becomes apparent when considering a perturbation
of the flux rope from $Z \to Z + dZ$. Consider the convective derivative of $\Phi_s$ that describes,
in the Lagrangian frame, the effective change in external flux that the flux rope sees as it
propagates

$$\frac{d\Phi_s}{dt} = \frac{\partial \Phi_s}{\partial t} + V \cdot \nabla \Phi_s, \tag{2.24}$$

where $V = dZ/dt$ is the velocity at the apex of the expanding flux rope, $\partial \Phi_s/\partial t = 0$ since
it has been assumed that the external field $B_s$ is independent of time, and $\nabla \Phi_s = d\Phi_s/dZ$.
The convective derivative therefore reduces to $d\Phi_s/dt = V d\Phi_s/dZ$. In ideal MHD $\Phi_s$ is a
conserved quantity and hence $d\Phi_s/dt = V d\Phi_s/dZ = 0$. The physical interpretation is that
as the flux rope is expanding it is pushing the overlying field aside. On the other hand if
$d\Phi_s/dZ \neq 0$ ideal MHD is broken and in this case it could be interpreted as the external
flux converted to internal flux, e.g., through magnetic reconnection. This can be understood
with equation (2.7) where the term $c I_t L_p$ is considered the internal flux and $\Phi_s$ the external
flux. Ideal MHD requires that $\Phi_p$ be conserved during the perturbation ($d\Phi_p/dt = 0$)
Therefore if $\Phi_s$ is not conserved then the rate of flux conversion (or flux injection) is

$$\frac{d(cI_tL_p)}{dt} = -\frac{d\Phi_s}{dt} = -V \frac{d\Phi_s}{dZ},$$  \hspace{1cm} (2.25)$$

and will determine how $I_t$ will evolve. The rate of flux conversion, RHS term, will be dependent on the sign of $B_s$. If it is assumed that $B_p > 0$, then it is necessary for $B_s < 0$ so that Lorentz force arising from $I_t \times B_s$ be downwardly directed opposing the upward directed Lorentz force of the loop (Cargill et al., 1994). Hence, if $B_s < 0$ then the flux conversion term will be positive.
Figure 2.2 shows the solution of $n_{ct}$ when assuming $d\Phi_s/dZ = 0$ in equation (2.18) (solid line). The dash-dot line is the solution of $n_{ct}$ when the self-similar approximation is made, previously referred to as case (3). In this limit $n_{ct}$ becomes independent of the $d\Theta/dZ$ and $d\Phi_s/dZ$ terms, and reduces to

$$n_{ct} = \frac{2Z_{ct} dR}{R dZ}$$  \hspace{1cm} (2.26)

This function flips sign at $Z_{ct}/S_0 = 1$ because of the fact that at this value $dR/dZ = 0$. This means that an external field is required to be increasing in magnitude at the apex with increasing height for a flux rope that is $Z/S_0 < 1$. This zero crossing is not unique to this case but is also seen in the solutions to the other cases discussed with the only difference being that the zero crossing occurs at some smaller values of $Z_{ct}/S_0$. For flux ropes in the regime where $n_{ct} < 0$ an overlying field that is increasing with height is necessary for stability. This was the scenario considered by Chen (1996) that specified an external field that first increases with height then decreases, except that here the exact value for the index of the overlying field required for stability can be calculated. Therefore, the only requirement for stability is that the index of the overlying field be less than the critical index, $n < n_{ct}$, at the position of the apex.

### 2.4 Discussion on the Minor Radius Tapering

In this section, I discuss the effect of the tapering of the minor radius from the apex to the footpoints. The model studied specifies that the minor radius at the footpoints remains fixed, but that the minor radius at the apex evolves with equation (2.2). A model as to how the minor radius tapers from the footpoints to the apex is specified. This can be modelled as an exponential relationship between the footpoint and apex, with a functional form (Chen & Garren, 1993, 1994)

$$a(\theta) = a_f e^{s(\theta - \theta_f)},$$  \hspace{1cm} (2.27)
where $a_a$ is the minor radius at the apex (defined just as $a$ above), $a_f$ is minor radius at the footpoints and is assumed fixed, $s = \ln(a_a/a_f)/(\pi - \theta_f)$, and $\pi - \theta_f$ is the angle subtending from one footpoint to the apex. Or, a linear relationship with a functional form, (Krall et al., 2000)

$$a(\theta) = m(\theta - \theta_f) + a_f, \quad (2.28)$$

where $m = (a_a - a_f)/(\pi - \theta_f)$. Using these expressions the inductance is found, $L = \frac{1}{\pi^2} \int J \cdot A dV$ (Jackson, 1962), where $A$ is the vector potential of a ring with current density $J$ and total current $I$. The volume integral is separated in half because of symmetry, integrated between $\theta_f$ and $\pi$, and multiplied by 2. The inductance $L_p$ presented above in equation (2.4) was calculated in this way except without the assumption that the minor radius tapers from footpoint to apex. In that assumption the minor radius at the footpoints will evolve in the same way as the apex. In the tapering assumption, the minor radius at the footpoints remains fixed. Using the expressions for the minor radius tapering the inductance is found.

With the expression of the exponential case the inductance can be written as

$$L_e = \frac{4\pi R}{c^2} \left( \frac{1}{2} \log \left( \frac{8R}{a_a} \right) + \log \left( \frac{8R}{a_f} \right) \right) - \frac{\xi_i}{2} = \frac{4\pi R}{c^2} \left( \log \left( \frac{8R}{a_a} \right) - 2 + \frac{\xi_i}{2} + \frac{1}{2} \log \nu \right), \quad (2.29)$$

where $\nu = a_a/a_f$. For the linear case the inductance is written as

$$L_l = \frac{4\pi R}{c^2} \left( \log \left( \frac{8R}{a_a} \right) - 2 + \frac{\xi_i}{2} + \frac{1}{2} \log \nu + \left\{ 1 - \frac{(\nu + 1)}{2(\nu - 1)} \log \nu \right\} \right), \quad (2.30)$$

In either approximation, as the flux rope evolves in interplanetary space $\nu$ becomes large, i.e. $a_a >> a_f$, and the inductance scales as $L \propto R \ln(8R/a_f)$ (Chen & Krall, 2003). This scaling applies to both $L_e$, and $L_l$. This is unlike $L_p$ (equation (2.4)) which scales more like $\propto R$ since the aspect ratio $R/a_a \sim \text{const}$ during the flux rope’s evolution (Chen et al., 1997, 2000; Krall et al., 2001). On the other hand, during the initiation phase the $\ln(R/a_a)$
and \( \ln(R/a_f) \) terms are comparable to each other and it is not till \( \nu \) becomes large that \( \ln(R/a_f) \) dominates. During the initiation at \( t = 0 \) it is assumed that \( \nu = 1 \), or \( a_a = a_f \).

Following the steps outlined above for deriving the critical index, using these expressions for the inductance, a general form for \( n_{ct} \) can be written as follows. Note that the \( \hat{L} \) term in equation (2.10) is always defined as \( (\ln(8R/a) - 2 + \xi/2) \), as in the major radial force, equation (2.1). Whereas the inductance in equation (2.7) is redefined according to the tapering model, either \( L = L_e \), or \( L = L_l \), and that equation determines how \( I_t \) evolves.

\[
n_{ct} = \alpha \frac{Z}{\hat{L}_e} \frac{dR}{dZ} + \beta \left( \frac{Z_{ct}}{\Theta} \frac{d\Theta}{dZ} + \frac{Z_{ct}}{cL_I} \frac{d\Phi_s}{dZ} \right), \tag{2.31a}
\]

where

\[
\alpha = 1 + \frac{1}{\Delta} + \beta(1 + \frac{1}{\hat{L}}). \tag{2.31b}
\]

The different expressions for the inductance determine \( L, \hat{L}, \alpha, \) and \( \beta \). In the case of the inductance \( L_p \) that does not assume footpoint tapering, equation (2.4), \( \beta = (1 - 1/\Delta)/(1 + 1/\hat{L}) \). Plugging this into equation (2.31a), the equation for the critical index with \( L_p \), equation (2.17), is recovered. As it turns out, if the quasi self-similar approximation is made, then regardless of the tapering the expression for the critical index always reduces to equation (2.18), where \( \alpha = 2 \), and \( \beta = 1 \). Recall that \( \hat{L} = c^2L/(4\pi \Theta R) \). As defined in equation (2.9), \( L = L_p \), which can be extended to include \( L = L_e \) or \( L = L_l \). In either case the quasi self-similar approximation is defined such that \( d\hat{L}/dZ = 0 \).

In the exponential case the \( \beta \) term is,

\[
\beta_e = \frac{1 - 1/\Delta}{1 + 1/(2L_e)}, \tag{2.32}
\]
Figure 2.3: Critical index including the effect of minor radius tapering, compared to not including the effect. The dot-dash lines (orange, green, blue) are the solution to $n_{ct}$ that includes the effects of tapering. In this solution $\nu = 1$. The aspect ratio $R/a$ is varied where the blue line is 2.5, green is 10, and orange is 100. The solid line (black) is the solution of $n_{ct}$ assuming quasi self-similar with $d\Phi_s/dZ = 0$, equation (2.18), and is independent of the aspect ratio. The dash line (red) is the exact solution of $n_{ct}$ not assuming anything about self-similarity with $d\Phi_s/dZ = 0$, case (1) as discussed in the text, equation (2.17), and aspect ratio of 10.

where $\hat{L}_e = c^2L_e/(4\pi\Theta R)$. And in the linear case the $\beta$ term is

\[
\beta_l = \frac{1 - 1/\Delta}{1 + 1/\hat{L}_l \left( \frac{1}{2} - \frac{\nu\ln\nu}{(\nu-1)^2} + \frac{\nu+1}{2(\nu-1)} \right)}, \tag{2.33}
\]

where $\hat{L}_l = c^2L_l/(4\pi\Theta R)$. Note that the $\hat{L}$ term in equation 2.31b is replaced by $\hat{L}_e$ or $\hat{L}_l$ depending on the model. In this model the initial state of the flux rope assumes that $\nu = 1$ and in this limit the equations of inductance reduce to equation (2.4), $L_e = L_l = L_p$. The
limit of the linear solution $\beta_l$ as $\nu$ approaches 1 is equal to the exponential solution $\beta_e$.

Figure 2.3 compares the critical index as calculated with and without modelling the tapering. The dot-dash lines (orange, green, blue) are the solutions with $\nu = 1$ and the $d\Phi_s/dZ = 0$ assumption. The aspect ratio $R/a$ is varied where the blue line is 2.5, green is 10, and orange is 100. The solid line (black) is the solution of $n_{ct}$ assuming the quasi self-similar approximation with $d\Phi_s/dZ = 0$, equation (2.18). With the assumption that $d\Phi_s/dZ = 0$ the quasi self-similar solution becomes independent of the aspect ratio. The dash line (red) is the exact solution of $n_{ct}$ not assuming anything about self-similarity with $d\Phi_s/dZ = 0$, case (1) as discussed in the text, equation (2.17), and aspect ratio of 10. As can be seen, the difference between the exact solution and the quasi self-similar solution is small. Thus, validating the approximation that $(1 - 1/\Delta)/(1 + 1/\hat{L}) = 1$. In fact these two curves essentially overlap each other except for smaller values of $\Theta$ where the deviation is a few percent. The difference between those solutions and the solution that includes the tapering is much greater. The deviation increases to $\sim 10\%$ as $\Theta$ becomes large for an aspect ratio of 10 (green dot-dash line). If the aspect ratio were to be made larger this deviation decreases (orange dot-dash line), and if made smaller the deviation becomes larger (blue dot-dash line). As can be seen from figure 2.3, as the aspect ratio increases the solution of the critical index that includes the tapering effects approaches the quasi self-similar solution (solid black line).

2.5 Model Run

The model is solved numerically to demonstrate the theory presented here. The forces included in the numerical calculation are Lorentz, thermal pressure gradient, and gravity, but the drag force is omitted because it would only contribute a negligible amount during the initiation phase modelled. See Appendix B for a complete description of thermal pressure gradient, gravity, and the ambient parameters such as temperature and density. One example model run is performed to illustrate the analysis presented, essentially that the loop
will remain in equilibrium until it reaches a critical height where the index of the overlying field becomes greater than \( n_{ct} \). A low-lying flux rope is assumed with footpoint separation of \( S_0 = 3 \times 10^9 \) cm, initial height of \( Z_0 = 2.25 \times 10^9 \) cm, aspect ratio of 10, and mass of \( 1 \times 10^{15} \) g. \( d\Phi_s/dZ = 0 \) was assumed, and a value of \( \xi_i = 1.2 \) was used as was derived by Chen (1996). In this example a critical height of \( Z_{ct} = 1.75S_0 \) is chosen corresponding to a critical index of \( n_{ct} = 1.42 \), an overlying field then needs to be chosen such that its index is less than \( n_{ct} \) when the apex height of the loop \( Z < Z_{ct} \). A slightly modified version of the field proposed by Chen (1996) is used, the modified form is given by

\[
B_s = B_{s0}\text{sech}^2[(Z - Z_0)/h],
\]

(2.34)

where \( B_{s0} = B_s(Z_0) \), and \( h \) is a scale length chosen such that at \( Z = Z_{ct} \) the index of this profile, found with equation (3.1), is equal to \( n_{ct} \) as predicted by the theory. This profile has the property that at \( Z = Z_0 \) the index of the overlying field is equal to 0 and increases as \( Z \) increases, indicating that \( B_s \) decreases in magnitude at an increasing rate as \( Z \) increases. For the proposed example \( B_{s0} = -20 \) G was specified, and \( h = 1.46S_0 \) when \( n_{ct}(Z_{ct} = 1.75) = 1.42 \).

In order to perturb the system from an initial equilibrium state, poloidal flux injection is assumed as the driver. The reader is referred to chapter B equation (B.30) for the functional form of the flux injection profile. This function is specified by three basic components: 1. a ramp up with time parameter of \( \tau_1 \) that begins at time \( t1 \) and ends at time \( t2 \), 2. a plateau from time \( t2 \) to time \( t3 \), and 3. a ramp down with time parameter \( \tau_2 \) starting at time \( t3 \).

The origin of poloidal flux injection from the sub-photosphere is of much debate, though it has recently been proposed that it may be related to reconnection below the flux rope (Vršnak, 2008) and that it has some physical connection with the CME-flare phenomena (Chen & Kunkel, 2010). In this paper we do not explicitly assume the physical origin of the injected flux except that it serves as a specified driver to trigger the instability studied. For the example model run calculated here a flux injection profile with the following parameters
is assumed: \( \tau_1 = 30 \text{ min} \), \( \tau_2 = 60 \text{ min} \), \( t_2 - t_1 = 70 \text{ min} \), and \( t_3 - t_2 = 30 \text{ min} \). The solution to the equation of motion of the apex with this flux injection profile is shown in Figures 2.4, and 2.5. The maximum flux injection rate is varied from \( 3.9 \times 10^{-5} \Phi_{p0} \text{ s}^{-1} \) to \( 5.8 \times 10^{-5} \Phi_{p0} \text{ s}^{-1} \), where \( \Phi_{p0} = 8 \times 10^{20} \text{ Mx} \) is the initial poloidal flux. In Figure 2.4, it can be seen that, as flux is injected, the height of the apex increases. Further, as the maximum rate of injection is increased, the apex will reach new equilibrium at higher heights. However, new equilibrium can not be established infinitely. Instead, what is found is that once the loop exceeds the critical height, loss of equilibrium occurs. Figure 2.4c shows the resulting acceleration profile, it is seen that the acceleration is sudden, in this particular run the peak occurred at approximately 1000 m/s\(^2\). The loss of equilibrium can be further understood in figure 2.5: (a) shows the magnitude of the overlying magnetic field at the apex. (b) The index \( n \) of the overlying field calculated with equation (3.1) (solid line), where at \( Z_0 \), \( n = 0 \) and \( n_{ct} = 0.19 \), hence \( n < n_{ct} \) and the flux rope is in an equilibrium stable to major radius perturbation. The dot-dash line is the function of \( n_{ct} \), given by equation (2.18) with \( d\Phi_s/dZ = 0 \). It is seen that for \( Z < 1.75S_0 \), \( n < n_{ct} \) and the flux rope is stable. Figure 2.5c shows the velocity of the rising flux rope which corresponds to the height-time plot in figure 2.4b. Once the apex reaches and exceeds the critical height, then \( n > n_{ct} \), and the flux rope becomes unstable and subsequently erupts. This must be the instability that Krall et al. (2000) implicitly refer to when considering the "magnetic energy release" scenario with a prescribed ambient field that "falls off so rapidly that the flux rope can no longer be held in place and erupts". Although not seen in the figures, oscillations of very small amplitude occur with a frequency given by \( \Gamma \). As the flux rope approaches the critical height the frequency decreases. These types of oscillations, as pertaining to this model, were studied by Chen (1989), assuming that \( B_s = 0 \), and by Cargill et al. (1994) and Chen & Schuck (2007), assuming that \( B_s \neq 0 \), who found that the oscillations are damped by the drag force that couples the flux rope with the ambient medium. Since the drag force was omitted in the example model run, no damping of the oscillations occurs.
Figure 2.4: A model run to demonstrate the partial torus instability. (a) The flux injection profiles of four different cases. (b) Corresponding Height-Time plots. It is seen that, once the flux rope apex exceeds the critical height $Z_{ct} = 1.75S_0$, it suddenly erupts. (c) The acceleration profile for the case of instability.
Figure 2.5: A model run to demonstrate the partial torus instability. (a) Height dependence of the external magnitude of external field. (b) Decay index $n$ of the external field. (c) Apex velocity plotted against apex height. The velocity corresponds to the height-time plot in figure 2.4. The dot-dash line in (b) is the curve of $n_{ct}$, equation (2.18), with $d\Phi_s/dZ = 0$. 

\[ Z_0 \quad Z_{ct} \]

\[ \text{Magnetic Field } |G| \]

\[ \text{Index } n \]

\[ \text{Velocity (Km/s)} \]

\[ \text{Apex Height } Z/S_0 \]

Stable

Eruptive

with $d\Phi_s/dZ = 0$. 

50
2.6 Conclusion and Discussion

In this section, the PTI was proposed and investigated for a flux rope model. We found the critical decay index $n_{ct}$ as a function of $Z/S_0$. This is a novel finding because it shows that $n_{ct}$ is a function of both apex height and footpoint separation and not a single value. The inclusion of $S_0$ in this model introduces a scale length that does not exist in the full torus limit. A model run was made to demonstrate the theory and shown that as the rate of flux injection is increased, the loop can rise to new equilibria, and if it were to rise above a critical height, it will subsequently erupt due to the PTI, or loss of equilibrium between the competing Lorentz forces (outward hoop force versus the downward Lorenz force).

Several assumptions have gone into the derivations presented herein. These include: (1) the assumption of low beta, where magnetic forces dominate over all others, (2) the geometrical assumption for the dependence of the radius of curvature with height $R(Z)$, (3) the large aspect ratio limit that is used in the derivation of the equations of motion, and (4) the assumption of local curvature at the apex. The model presented here also assumes a geometrical line-tying condition as opposed to a physical condition. The geometrical condition states that the footpoints of the loop should remain fixed during the evolution of the flux rope. This is satisfied with the geometrical function $R(Z)$ by holding $S_0$ fixed. The physical line-tying condition is motivated by the fact that, during an eruption, the radial observations of magnetic fields appear to remain fixed. To satisfy this boundary condition, a mirror current is mathematically imposed (e.g. Isenberg & Forbes, 2007). This assures that the radial field lines will remain fixed during any evolution. In addition the physical line-tying also includes fixed footpoints as a condition. Inclusions of the physical line-tying into the model will be a future work. The tapering of the minor radius was also investigated. It was found that, if the aspect ratio is small, then the deviation by including the tapering into the calculation of the critical index is large, $\gtrsim 10\%$, as compared to the quasi self-similar solution. As the aspect ratio increases, this deviation becomes small and the solution approaches the quasi self-similar solution.

In the model run, poloidal flux injection was specified as the dynamical driver of flux
rope till the critical height was reached. Other mechanisms to drive the flux rope could include footpoint twisting, or hot plasma injection (Krall et al., 2000), though the "magnetic energy release" scenario of Krall et al. (2000) seems to be universal because, as was just shown in this chapter, a flux rope remains in a stable equilibrium until the critical height is reached. This is the place where the field "drops off quickly", as stated in the "magnetic energy release" scenario. There exists a critical index where the outward Lorentz force will dominate over the restraining Lorentz force between the flux rope current and the ambient field. The profile of the critical index derived could be thought of as the rate of which the outward Lorentz force is changing. If the rate of change of the restoring force is faster than the outward hoop force, then the flux rope will be unstable to radial perturbation.

Observations of decay index $n$ from extrapolated magnetic fields that overly active regions have shown that a clear distinction between eruptive and confined events exist. In one particular study it was found that no eruptive event had a decay index smaller than 1.74 and no confined event had a decay index that exceeded 1.71 (Liu, 2008). A similar result was obtained by Wang & Zhang (2007), who used the ratio of the coronal fluxes above and below a certain height in the corona as a measurement of the significance of the magnetic fields overlying an active region on producing an eruptive or confined event. The question that one must ask is, does the Sun have some property that would lead it to have the same critical index for all events? and what is the significance of that number? Because, in the context of PTI, $n_{ct}$ is a function of $Z/S_0$ and is calculated with some assumed external field profile. Therefore, one might not expect there to be just one value for $n_{ct}$ in the solar environment as any reasonable value may seem equally possible. A hint to a possible answer to these questions may be found with the scaling law of acceleration (Chen & Krall, 2003; Chen et al., 2006). The scaling law of acceleration states that there exist two critical heights that bound where the acceleration of an eruptive event peaks. The first critical height is defined as $Z_s = S_0$ and the second is $Z_m \approx 3S_0$, and $Z_{max}$ is where the acceleration of the centroid of the apex of the flux rope is maximum. The scaling law then states that $Z_s < Z_{max} < Z_m$. The onset of the acceleration would then have begun at
some point less than $Z_{\text{max}}$, and possibly even less than $Z_\star$. This law, theoretically derived and demonstrated empirically, gives an upper and a lower bound for $n_{ct}$ between $\lesssim 0.5$ and $\sim 2$ when $Z/S_0 \lesssim 1$ and $Z/S_0 \sim 3$ respectively.

An observational test of the PTI theory presented here could potentially be made if the index of the overlying field were observed at the height of the onset of eruption as well as the footpoint separation of the flux rope (This will be the subject of chapter 3, however further investigation is needed as only preliminary results are shown). If this prediction is correct, then this type of observation should yield a plot that would monotonically increase between $Z_\star$ and $Z_m$ and look something like figure 2.2 with data points clustered between $Z/S_0 = [1,3]$ and $n_{ct} = [0.5,2]$. To take things even further, such an observation could be a probe of coronal fields and give us an idea of the functional form of the fields overlying active regions that have produced eruptive events.
Chapter 3: Study of Eruptive Prominences and Their Related Magnetic Fields

3.1 Introduction

The flux rope theory for the eruption of coronal mass ejections (CMEs) states that (1) the initial pre-eruptive configuration of a CME is a flux rope, and (2) it is the Lorentz forces that dominate over all other forces in the dynamics of the eruption. The second statement stems from the fact that in the low corona the dominant form of energy is magnetic, which exceeds gravity, thermal, and kinetic by several order of magnitude (Forbes, 2000). The relationship to prominences is that they are thought to be supported by the flux rope magnetic fields (e.g. Démoulin, 1998). The earliest of these models is that of Kuperus & Raadu (1974). A flux rope is modelled as an infinite straight current channel above the photosphere that is embedded above the polarity inversion line (PIL) of a dipole boundary. This part of the model represents the fact that prominences are always observed above the PIL of photospheric magnetic fields (Kiepenheuer, 1953; Martres et al., 1966). This boundary can give rise to two conditions of either open or closed magnetic field lines. For the present discussion of this chapter the closed condition is assumed. In this case the potential magnetic field loops that are closed between the dipole represent an overlying loop arcade. Two magnetic forces are considered, one is directed outward and is the repulsion of the current channel by induced photospheric currents. Mathematically this is described by an image current to satisfy the boundary condition that the radial magnetic field remain approximately constant during an eruption. The other force is a Lorentz force between the current channel and presumed external potential field arcade. These magnetic fields are orthogonal to the PIL and depending on the sign of the current channel can either be directed downwards or upwards away from the photospheric boundary. The current channel
alignment that is typically taken is the one that gives a solution where this Lorentz force is directed downwards (e.g. Kuperus & Raadu, 1974; Chen, 1996; Forbes & Isenberg, 1991; Olmedo & Zhang, 2010). This configuration is referred to as the inverse-polarity model. The two magnetic forces balance each other and there exists a critical height (or critical current) that determines the threshold for stability. If this critical value is exceeded the outward force will dominate and lead to an eruption.

Another way of quantifying this scenario is by the recognition that it is the overlying magnetic fields that determine stability since they provide the restoring downward force. On the Sun these magnetic fields decrease with increasing height with inverse power index

\[ n = -\frac{d\ln B}{d\ln Z} \]  

(3.1)

increasing from zero (close to the surface) to three (dipole field) (van Tend & Kuperus, 1978). In addition to the straight line configuration, curved flux rope configurations have also been studied, in this case the loss of equilibrium has been referred to as the torus instability (Kliem & Török, 2006). In the curved case, just like the straight line case loss of equilibrium occurs when a critical value is exceeded. It has recently been recognized that both the curved, and straight line case are two limits of more general current paths with the same underlying physics (Démooulín & Aulanier, 2010). The curved case has in addition a force that is dependent on the curvature of the loop, it is physically described as the Lorentz self force, or ’hoop force’ (Chen, 1989; Garren & Chen, 1994; Chen, 1996). The index by which the magnetic field decays has a critical value above which the flux rope will no longer be in equilibrium and the repulsive outward force will dominate. For the straight line case the critical value is \( n_{ct} = 1 \) (van Tend & Kuperus, 1978), and for the circular case it is \( n_{ct} = 1.5 \) (Bateman, 1978). It should be noted that the circular case assumes that half of the circle is above the surface and the lower half is considered as the image current (Démooulín & Aulanier, 2010). The value of \( n_{ct} = 1.5 \) is therefore only valid for a semicircular loop whose footpoints are not fixed. For the circular curved case an index
between 1 and 2 is typically an accepted value depending on differing theoretical details, like fixed footpoint separation or magnetic line-tying (Kliem & Török, 2006; Török & Kliem, 2007; Fan & Gibson, 2007; Isenberg & Forbes, 2007; Liu, 2008; Olmedo & Zhang, 2010). The model of Olmedo & Zhang (2010) predicts that $n_{ct}$ can be out side of the range from 1 to 2 and is dependent on the loops arc. A flux rope is therefore in a stable equilibrium to perturbation if the overlying field decays with an index that has a value smaller than the critical value.

The question that can be asked is; what can observations of the magnetic fields tell us about the index necessary for eruption? In its initial conception by Kuperus & Raadu
(1974), and van Tend & Kuperus (1978) the overlying fields are theorized to be potential fields with a current channel embedded within. In the study by Filippov & Den (2001), and Filippov et al. (2006) coronal potential fields are extrapolated and compared with observed heights of prominences on the limb $h_p$. According to the straight line flux rope case the critical index for loss of equilibrium is $n_{ct} = 1$. Since it is know that filaments lie above neutral lines, the extrapolated magnetic field above the neutral line was calculated. The height where the index of this field was equal to 1 is the critical height $h_c$. Combining the heights of the prominences and the critical height an interesting relationship was found. Filippov & Den (2001) found that the closer the height of the prominence to the critical height the more likely it will erupt. This relationship is shown in Figure 3.1, where the open circles represent filaments that were seen to disappear, i.e. erupt, in Hα observations. The closed circles are filaments that were seen to cross the solar disk without erupting. It is seen that the prominences with heights closest to the critical height subsequently erupted. These results seem to be consistent with the straight line flux rope models of Kuperus & Raadu (1974), and van Tend & Kuperus (1978), but not consistent with the circular models that predict a critical index of $n_{ct} = 1.5$ (Kliem & Török, 2006). Their results might also be in conflict with those of Liu (2008) who found observationally that the critical index separating eruptive and confined events was closer to 1.7, though that particular study did not take into account the height of the pre-eruptive structure and also averaged the value of the critical index over the height range from 42 to 142 Mm, which may have lead to a larger value. In another study it was shown that the magnetic flux radially above the PIL should decrease faster for eruptive versus confined events (Wang & Zhang, 2007). In both cases an X class flare was observed, where eruptive refers to events that resulted in a CME and confined to those that did not result in a CME. Those results are consistent with Filippov & Den (2001), and Filippov et al. (2006) who conclude that if the decay of the magnetic field is fast ($n \geq 1$ in their study) the prominence may erupt and result in a CME, and if the decay is slower no eruption nor CME is observed. A recent reanalysis of the model of Chen (1989, 1990), and Cargill et al. (1994) has shown that the critical index should
be a function of the arc angle of the flux rope above the photosphere with a characteristic curve that monotonically increases (see chapter 2 for details, and Figure 2.2) (Olmedo & Zhang, 2010). Furthermore, in the curved flux rope case, simulations have indicated that the eruption should occur when the flux rope has reached an approximately semi-circular shape above the photosphere (Fan & Gibson, 2007; Török & Kliem, 2007).

A detailed look at the simulation result of Török & Kliem (2007) show that, for varying the decay rate of the magnetic field with height, the steeper the slope of $n$ the larger the acceleration peak. Not only is the peak acceleration larger but the resultant velocity is also larger. This indicates that the kinematics are directly affected by the slope of the index $dn/dh$. Figure 3.2 shows the results of the simulation of Török & Kliem (2007). The shaded region in the top panel indicated the initial height of the flux rope. With the exception of the green line that has $n > 2$, from red, to blue, to black, it is clearly seen that the slope of $n$ increases in the range $n < 2$. The value of $n = 2$ is of significance because it is an upper limit predicted by analytical models for the stability of a flux rope (Kliem & Török, 2006; Olmedo & Zhang, 2010). Based on the details of these models, if the index of the external magnetic field is greater than two, $n > 2$, the outward Lorentz forces will dominate the restoring external Lorentz force. During dynamical evolution, the outward force drops more slowly than the restoring force, thus producing a net outward force. Since $n$ is the decay of the external magnetic field, the downward restoring force, which is dependent on this field, should decrease at least as fast as or proportional to $n$. This means that the decay of the outward Lorentz force should be smaller than $n$ to produce a net outward force.

In this chapter the idea that the slope of the index affects the kinematics of an eruption will be observationally explored. We present, for the first time, the observations that clearly show that there is a relationship between how the ambient magnetic field decays radially and the kinematics of an eruption. It has previously been suggested that the ideal observation of this system is to co-temporally observe the prominence from the limb and magnetic fields from the disk (Démoülin, 1998; Filippov et al., 2006). Such observations are now possible with the advent of the Solar Terrestrial Relations Observatory (STEREO, Kaiser et al.,
Figure 3.2: Simulation by Török & Kliem (2007) showing the effect of the slope of the index ($dn/dh$) on the CME kinematics. The shaded region in the top panel indicates the initial height of the flux rope. (From: Török & Kliem (2007))
2008), in combination with disk observations from the Earth. Disk observations at Earth’s orbit from space include the Solar and Heliospheric Observatory (SOHO, Domingo et al., 1995), and the Solar Dynamic Observatory (SDO Schwer et al., 2002). In the previous study of Filippov & Den (2001), and Filippov et al. (2006) observations only from the Earth were used to measure the height of the pre-eruptive structure and the magnetic fields. Once a prominence moved onto the disk the resultant filament height was tracked by a proxy based on it morphological evolution. The magnetic fields were calculated as the filament approached the central meridian and tracked for a number of days. As discussed above, in their study a critical index of $n = 1$ was assumed and the critical height was calculated based on this value. In the study presented in this chapter we do not make this assumption. We use STEREO observations to measure the height of observed prominences in conjunction with disk measurements of line-of-sight magnetic fields taken with the Michelson Doppler Imager (MDI Scherrer et al., 1995) onboard SOHO. Based on the exponential fitting model of Gallagher et al. (2003), we decomposed the observed height time plots into a pre-onset linear part characterized by constant velocity, and the post-onset part characterized by an exponential dependence. The onset time is defined at the point where the exponential component becomes larger than the linear part. In this way an approximate onset height can be determined. Based on the ideas of the torus instability (Kliem & Török, 2006; Olmedo & Zhang, 2010), this onset height will be considered the critical height. And at this critical height the observed index is calculated and said to be the critical index $n_{ct}$. Our recent analysis (Olmedo & Zhang, 2010) has shown that there is a dependence on the critical index with the geometrical shape of the flux rope loop. The result shows that the index monotonically increases as the flux rope loop goes from being shallow to more circular. The observational results presented in this chapter will show this dependence. Although a qualitative comparison to our previous results could not be made the essence is captured by showing the geometrical dependence to the critical index. This will be further discussed later.

This chapter is organized by first presenting the observations of the magnetic fields
and the kinematics of the events studied. Section 3.2 describes how the magnetic fields are extrapolated into the corona. Section 3.3 describes how the kinematics are fit to an exponential function. Two events were chosen for this study, one took place on 6 March 2010, and the other on 1 August 2010, the kinematic and magnetic field observations are shown in section 3.4. Section 4.3 compares the kinematics and extrapolated magnetic fields. It is in this section that the kinematics and slope of the index, and the geometrical dependence to the critical index are discussed. Finally section 3.6 concludes the study.

3.2 The Magnetic field Observations and Extrapolation

3.2.1 Potential Field Source Surface model (PFSS)

The Potential Field Source Surface (PFSS) (Schatten et al., 1969; Altschuler & Newkirk, 1969; Hoeksema et al., 1982; Wang & Sheeley, 1992; Zhao & Hoeksema, 1995) model is a method to extrapolate coronal magnetic fields to the first order. The key element in this model is the assumption of a current free region in the lower corona, since it is known that in this region the plasma beta is less than one and dominated by magnetic forces. This is expressed as $\nabla \times B = 0$. This assumption holds to about 2.5 solar radii at which point the magnetic field lines are carried out by the solar wind and completely open. The dragging of the solar wind induces currents and the current free assumption is no longer valid. A source surface boundary is assumed at 2.5 solar radii to simulate this by imposing that at this height all magnetic field lines are radial. The lower boundary is taken to be the photosphere. Solving for the field is essentially a boundary value problem where one boundary comes from observations of photospheric magnetic fields and the other is the source surface, chosen to be 2.5 solar radii.

The magnetic field in the limit of this model is expressed as a potential field $B = -\nabla \phi$. From Maxwell’s equation $\nabla \cdot B = 0$ we get the Laplace condition $\nabla^2 \phi = 0$. The general
solution to this problem is written as

\[
\phi(r, \theta, \varphi) = R \sum_{l=1}^{N} \sum_{m=0}^{l} \left[ \left( \frac{R_{\odot}}{r} \right)^{l+1} P_{l}^{m}(\theta) (g_l^m \cos m\varphi + h_l^m \sin m\varphi) \right], \tag{3.2}
\]

where \( P_{l}^{m}(\theta) \) are given by the associated Legendre polynomials and \( g \) and \( h \) are coefficients to be solved for. The solution of the magnetic field follows from this general solution as

\[
B_r = -\frac{\partial \phi}{\partial r} = \sum_{l=1}^{N} \sum_{m=0}^{l} (l + 1) \left( \frac{R_{\odot}}{r} \right)^{l+2} (g_l^m \cos m\varphi + h_l^m \sin m\varphi) P_{l}^{m}(\theta)
\]

\[
B_{\theta} = -\frac{1}{r} \frac{\partial \phi}{\partial \theta} = -\sum_{l=1}^{N} \sum_{m=0}^{l} \left( \frac{R_{\odot}}{r} \right)^{l+2} (g_l^m \cos m\varphi + h_l^m \sin m\varphi) \frac{dP_{l}^{m}(\theta)}{d\theta}
\]

\[
B_{\varphi} = -\frac{1}{rsinz} \frac{\partial \phi}{\partial \varphi} = \frac{1}{rsinz} \sum_{l=1}^{N} \sum_{m=0}^{l} m \left( \frac{R_{\odot}}{r} \right)^{l+2}
\]

The only remaining step to getting the solution is to solve for the \( g \) and \( h \) coefficients, which can be solved for with the Neumann boundary condition.

A drawback to this model is that current-carrying fields are not modelled. Such fields can arise near neutral lines and emerging active regions. The model is best for studying the larger scale configuration of the fields and gives a good approximation to these fields. Larger scale here refers to a scale the size of an active region. Therefore, this model is an appropriate tool for this study.

### 3.2.2 Preparation of the Synoptic Map

In order to better model the magnetic fields around the time of the eruption a 'synoptic frame' (Zhao et al., 1997) was used as input to the PFSS model code. The synoptic frame takes a traditional Carrington synoptic map (available from Stanford) and the region of the date and time of interest is replaced by data values using MDI magnetograms. To improve the signal to noise ratio several (6 to 8 images) MDI magnetograms are averaged together. Since the cadence of this data is 96 minutes 6 to 8 images corresponds to less than 12 hours prior to the event. Doing this is justified with the assumption that the large scale magnetic
Figure 3.3: Procedure for making a synoptic frame. Top left is an MDI magnetogram taken on March 6, 2010 8:00 UT. The arrow points to the dipole region from where the filament studied erupted. The magnetogram is transformed to Carrington coordinates (top right). The square region (red) is cut out and inserted into Carrington synoptic map rotation 2094 (bottom).
fields of interest have not significantly changed during the time period for which the data was averaged. This technique is demonstrated in Figure 3.3. The top panels shows the transformation of an MDI magnetogram to Carrington coordinates. The example given here is for March 6, 2010. This event is fully described in section 3.4.2. Six MDI images are used to create the transformed image (top right). A prominence eruption occurred on this date and the arrow in the top left overlaid on the MDI magnetogram indicates the location where the event erupted from. This event will be further discussed in the following sections. Clearly this event originated from a dipole region. A square region (red) that is 80° by 80° is cut out and inserted into Carrington synoptic map rotation 2094 (bottom). The advantage to using a synoptic frame is that it better approximates the magnetic fields at the time of the event in study.

With the synoptic frame as input to the PFSS model the global magnetic fields are calculated. Using the March 6, 2010 event as an example, resultant magnetic field lines calculated are shown in Figure 3.4. The green lines correspond to closed field lines, and the red (negative polarity) and blue (positive polarity) to open field lines.

### 3.3 Measurement and Fitting of the Kinematics

Height-time measurements of prominence eruptions have been made for a long time (e.g. McMath & Pettit, 1938; Pettit, 1940). Being that prominences are seen to erupt from the solar limb, projection effects are usually taken to be small. A more precise way to measure the height would be to use multiple vantage points of view, therefore giving an additional constrain to the observational measurement. Simple geometrical triangulation between the vantage points of the same object can be used to give an estimate of the 3D position in space. With the use of STEREO observations it is possible to determine detailed 3D trajectories. In this study, simple geometrical triangulation is used to determine the height of prominences above the solar surface. The data to make the measurement of the prominences was obtained from STEREO EUVI 304Å observations. We justify using simple
Figure 3.4: Extrapolated magnetic field lines for 6 March 2010. The green lines correspond to closed field lines, and the red (negative polarity) and blue (positive polarity) to open field lines.
geometrical triangulation because prominences are typically narrow coherent features, which in a still frame appear almost as ridged objects. On the other hand, simple geometrical triangulation can not readily be used for CMEs observed in the high corona, on the order of several solar radii. This is because CMEs are diffuse cloud like objects and features seen between the two vantage points between the STEREO spacecraft’s are ambiguous. Other techniques of geometrical triangulation (e.g. de Koning et al., 2009; Liu et al., 2010) and forward modelling (e.g. Thernisien et al., 2009; Wood et al., 2009; Poomvises et al., 2010) have been implemented to track CMEs from a few solar radii up to 1 AU.

Simple geometrical triangulation refers to the identification of the same feature within the two views of STEREO. In this way the 3D position of this feature can be calculated. This is done by first identifying the feature in one image. A line that connects the spacecraft to that feature is in turn projected into the field of view (FOV) of the other spacecraft. One then simply picks the point along the line that corresponds to the feature being tracked. The SCCMEASURE idl routine part of the Solar SoftWare (SSW) package was used to make the measurements. The uncertainty of the measurements are estimated as 10Mm within the FOV of the STEREO EUVI instrument. This error mostly comes from the difficulty in identifying the same feature in both A and B images. Because an erupting prominence is not a static feature and evolves on a time scale of minutes, it is required that observations be made simultaneously between the two STEREO instruments. This is not a problem as capturing images simultaneously is the normal operation of these instruments.

Fitting of time height profiles has been done ever since the heights of prominences have been measured. Early authors considered piece wise linear fits to these profiles (McMath & Pettit, 1938). More recent authors consider not only piece wise linear fits but also second order polynomials, power law, and exponential fits (Alexander et al., 2002; Gallagher et al., 2003; Schrijver et al., 2008; Robbrecht et al., 2009). One such functional form, proposed by Gallagher et al. (2003), assumes only that the acceleration is a function of time $a(t)$. 
Integrating twice yield the following equation,

\[ h(t) = c_1 + c_2 t + \int_0^t \int_0^t a(t) dt^2, \quad (3.4) \]

where \( c_{1,2} \) are constants depending on the initial conditions at \( t = 0 \). If \( a(t) \) is assumed constant then this equation reduces to the well known second order polynomial kinematic equation. Gallagher et al. (2003) has proposed that in the low corona, or the early rise phase of eruptive events, the acceleration can be modelled as an exponential

\[ a(t) = a_0 \exp(t/\tau). \quad (3.5) \]

Solving for \( c_{1,2} \) assuming the initial conditions that \( h(t = 0) = h_0, \) \( dh(t = 0)/dt = V_0, \)
\( dV(t = 0)/dt = a_0 \) yields the following kinematic equation,

\[ h = h_0 + (V_0 - a_0 \tau) t + a_0 \tau^2 \left( \exp \left( \frac{t}{\tau} \right) - 1 \right). \quad (3.6) \]

This equation satisfies the following differential equation,

\[ \frac{d^3 h}{dt^3} = \frac{1}{\tau} \frac{d^2 h}{dt^2}, \quad (3.7) \]

where \( d^3 h/dt^3 \) is the derivative of the acceleration, also called the jerk \( j = da/dt \). The exponential time constant can be found as \( \tau = a/j \). In this study, equation (3.6), will be used to fit the measurements. It will also be assumed that \( (V_0 - a_0 \tau) \) is small compared with the exponential term as \( t \) increases. This is an assumption that will not only be confirmed a posteriori but is also in agreement with Gallagher et al. (2003). In their work the term \( (V_0 - a_0 \tau) = v_0 \), and their fit to the April 21, 2002 event yielded a fit such that \( v_0 = 0 \). With this assumption the ratio between velocity and the acceleration is equal to the exponential
time constant $\tau = V/a$.

3.4 Description and Analysis of Events

3.4.1 Curve Fitting and Onset Time/Height Determination

With the approximation in equation (3.6) that $(V_0 - a_0\tau)$ is small, the exponential time constant $\tau$ can be approximated as $\tau = V/a$. By plotting $V$ vs. $a$, $\tau$ can be determined as the slope of the corresponding curve. Here we have chosen to plot $V$ as the abscissa and $a$ as the ordinate. The slope in this case is $1/\tau$. Figure 3.5 shows the fitting of the slope for the two events studied: 6 March and 1 August, 2010. The difference between them is clearly seen. Because the acceleration is modelled as an exponential, $\tau$ determines how rapidly it increases. The 6 March event has value of $\tau$ that is smaller than the 1 August event. This shows that the 6 March event accelerated more rapidly than the 1 August event. A detailed account of these events will be discussed in the following sections.

After having found $\tau$, $V_0$ and $a_0$ can be calculated. This is done using the equation for the velocity, written as

$$\frac{dh}{dt} = V = V_0 + a_0\tau \left( \exp \left( \frac{t}{\tau} \right) - 1 \right).$$

(3.8)

$V_0$ and $a_0$ are simply determined by linearly fitting this equation as $y = mx + b$ where $y = V$, $m = a_0$, $b = V_0$, and $x = \tau(\exp(t/\tau) - 1)$. For the fit, $V$ comes from the observations for each time $t$. The observed velocity is numerically calculated from the time height observations using three point Lagrangian interpolation. The final term in equation (3.6) to determine is $h_0$. Equation (3.6) is written using the observed data points as

$$h_0 = h_{\text{obs}}(t) - \left[ (V_0 - a_0\tau)t + a_0\tau^2 \left( \exp \left( \frac{t}{\tau} \right) - 1 \right) \right],$$

(3.9)
Figure 3.5: Empirical determination of $\tau$ as the slope of the equation $\tau = V/a$. Here we have chosen to plot $V$ as the abscissa and $a$ as the ordinate. The slope in this case is $1/\tau$. 
where $h_{\text{obs}}(t)$ is the observed height at time $t$. The mean of this equation over all of the data points gives $h_0$ and the standard deviation the uncertainty.

Once the parameters have been fit, the onset time and height are found. We define the onset time when the exponential part of the velocity fit equation becomes larger than the linear part. The linear part is defined as the constant $V_0$, and the exponential part is $a_0 \tau (\exp(t/\tau) - 1)$. The onset time is found by setting these two equal to each other and rearranging for $t$

$$t_{\text{onset}} = \ln \left( \frac{V_0}{a_0 \tau} + 1 \right).$$

Finally, by plugging this value into equation (3.6) the onset height is found. An estimate to an upper bound of the onset time is found by replacing $V_0$ in equation (3.10) by one sigma in variation $V_0 + V_\sigma$, where $V_\sigma$ is determined by the fit of equation (3.8). In order to compare the linear and exponential parts of the function both parts are plotted together. The linear part is plotted as $h = h_0 + V_0 t$, and the exponential part is equation (3.6). These plots will be shown in the following sections that describe the events studied.

### 3.4.2 Event: 6 March 2010

This eruptive prominence event was observed between 02:00 UT and 08:00 UT on 6 March 2010. The event originated from a decaying active region, which had been labelled NOAA AR 11045 in the previous solar rotation, approximately one month before. In association with this event, a GOES X-ray flare was observed. This flare was only partially observed because the GOES 14 satellite was in orbit night for part of the flare. It was during the rise phase of the flare that the satellite did not collect data. The data gap extends from 6:45 UT to 7:30 UT. But from the data which is available a peak is observed at about 8:00 UT making this flare at least a B5 class. The solar source of the flare is identified as originating from the filament eruption site.

For the measurement the height of the filament is taken to be the highest observable point above the solar surface. Using both STEREO EUVI 304Å A and B observations, the
Figure 3.6: Plots of the kinematics for the 6 March 2010 event. Top panel is the height-time profile. The vertical lines indicate the onset time and its upper bound. The horizontal line is linear part of the fit equation assuming constant velocity. The middle panel is the velocity, and the bottom is the acceleration. The solid line gives the fit to the data using equation (3.6).
3D trajectory of the filament was triangulated. On the date of this event the separation angle between the two satellites was about 137 degrees. Figure 3.9 shows the measurement, the top panel is the height, middle the velocity, and bottom the acceleration. The velocity and acceleration are numerically calculated using three point Lagrangian interpolation. The data is fit using the method described in section 3.4.1. The fit parameters are found to be: 
\[ \tau = 1205 \pm 154 \, \text{s}, \quad h_0 = 44 \pm 4 \, \text{Mm}, \quad V_0 = 0.4 \pm 2 \, \text{km/s}, \quad \text{and} \quad a_0 = 1.5 \times 10^{-5} \pm 9 \times 10^{-7} \, \text{m/s}^2. \]

The onset time found with equation (3.10) was 5:26 UT at onset height of 49.6 Mm. The upper bound for the onset time was found to be 6:03 UT at onset height of 53 Mm. The onset time and its upper bound are plotted as vertical lines in the top panel of figure 3.6. The horizontal line is the linear part of the fit to the pre-onset regime with constant velocity of \( V_0 = 0.4 \, \text{km/s} \). It is clearly seen that post-onset the exponential part of the fit dominates the kinematics. The footpoint separation was measured to be \( S_f = 0.24 \pm 0.03 \) solar radii using a SOHO EIT 304 Å observation at 4:06 UT.

The magnetic fields for this event were extrapolated with the PFSS model. The input synoptic frame used seven MDI magnetogram images averaged together spanning from 5 March 22:27 UT to 6 March 8:00 UT. In the theoretical model the magnetic field component of interest is the one orthogonal to the PIL. Using the extrapolated magnetic fields the PIL is tracked into the corona. Figure 3.7 top panel shows the photospheric magnetic field measurements overlaid with the outline of the PIL with height. The PIL increasing with height morphs from the top downward. The color code corresponds to different traces of the PIL with height. The bottom panel shows the index of the magnetic field as calculated with equation (3.1). The colours of the bottom panel correspond to the top panel so the index at different locations along the PIL are being shown. It should be noted that the magnetic field used in the calculation is a mixture of both the poloidal and toroidal components. This assumption has previously been made by Liu (2008) when calculating the magnetic field index. A 3D rendering of the PIL with height, seen as a 2D projection in figure 3.7, is shown in figure 3.8, where the color code corresponds to the index \( n \).
Figure 3.7: The photospheric magnetic field and the calculated index for the 6 March 2010 event. The top panel shows the photospheric magnetic field overlaid with the PIL with height. The PIL increasing with height morphs from right to left. The bottom panel is the index of the magnetic field calculated with equation (3.1). The color code corresponds to different traces with height along the PIL and the '⋄' correspond to sampled heights.
Figure 3.8: 3D rendering of the PIL with height for the 6 March 2010 event. The color code corresponds to the calculated index.
3.4.3 Event: 1 August 2010

On August 1, 2010 several filament eruption and flare events are seen to take place. The first event to be seen is an observed GOES C-class flare and subsequent CME eruption. This flare event was interesting because the pre-eruptive structure was observed to be a sigmoid and at the onset of the eruption this structure was seen to transform to a flux rope (Liu et al., 2010). Occurring contemporaneously with the flare, a crown filament eruption close to the central meridian (as seen from Earth) occurred. This is the event which we analyse here and focus on the initiation mechanisms. Upon erupting it is observed to begin to writhe once the filament height reached 1.5 solar radii (Cheng et al., 2010).

The filament under investigation was observed between 31 July 20:00 UT to 1 August 10:00 UT, 2010. The height of the filament is taken to be the highest observable point above the solar surface. Using both STEREO EUVI 304Å A and B observations, the 3D trajectory of the filament was triangulated. On this date of this event the separation angle between the two satellites was about 150 degrees. Figure 3.9 shows the measurement, the top panel is the height, middle the velocity, and bottom the acceleration. The velocity and acceleration are numerically calculated using three point Lagrangian interpolation. The data is fit using the method described in section 3.4.1. The fit parameters are found to be: \( \tau = 4819 \pm 2332 \text{ s}, \ h_0 = 84 \pm 4 \text{ Mm}, \ V_0 = 0.4 \pm 1.4 \text{ km/s}, \) and \( a_0 = 9 \times 10^{-4} \pm 7 \times 10^{-5} \text{ m/s}^2. \)

The onset time found with equation (3.10) was 2:12 UT with onset height of 94.4 Mm. The upper bound for the onset time was found to be 4:15 UT at onset height of 104.4 Mm. The onset time and its upper bound are plotted as vertical lines in the top panel of figure 3.9. The horizontal line is the linear part of the fit to the pre-onset regime with constant velocity of \( V_0 = 0.4 \text{ km/s}. \) It is clearly seen that post onset the exponential part of the fit dominates the kinematics. The footpoint separation was measured to be \( S_f = 0.8 \pm 0.2 \) solar radii and was measured using an SDO AIA 304 Å observation at 1:19 UT.

The magnetic fields for this event were extrapolated with the PFSS model. The input synoptic frame used seven MDI magnetogram images averaged together spanning from 31
Figure 3.9: Plots of the kinematics for the 1 August 2010 event. Top panel is the height-time profile. The vertical lines indicate the onset time and its upper bound. The horizontal line is linear part of the fit equation assuming constant velocity. The middle panel is the velocity, and the bottom is the acceleration. The solid line gives the fit to the data using equation (3.6).
Figure 3.10: The photospheric magnetic field and the calculated index for the 1 August 2010 event. The top panel shows the photospheric magnetic field overlaid with the PIL with height. The PIL increasing with height morphs from the top downward. The bottom panel is the index of the magnetic field calculated with equation (3.1). The color code corresponds to different traces with height along the PIL and the ‘⋄’ correspond to sampled heights.
Figure 3.11: 3D rendering of the PIL with height for the 1 August 2010 event. The color code corresponds to the calculated index.
July 14:27 UT to 1 August 0:03 UT. There was an MDI data gap between 0:03 UT and 16:00 UT on 1 August 2010, therefore there were no magnetic field measurements during and after the onset time of the eruption. It is assumed that small magnetic field variations that occur on the time scale of hours do not significantly affect the global magnetic field configuration, which we are interested in. For that reason the available data is taken to be a good representation of the magnetic fields during and after the onset time. In the theoretical model the magnetic field component of interest is the one orthogonal to the PIL. Using the extrapolated magnetic fields the PIL is tracked into the corona. Figure 3.10 top panel shows the photospheric magnetic field measurements overlaid with the outline of the PIL with height. The PIL increasing with height morphs from the top downward. The color code corresponds to different traces of the PIL with height. The bottom panel shows the index of the magnetic field as calculated with equation (3.1). The colours of the bottom panel correspond to the top panel so the index at different locations along the PIL are being shown. A 3D rendering of the PIL with height, seen as a 2D projection in figure 3.10, is shown in figure 3.11, where the color code corresponds to the index \( n \).

### 3.5 Discussion Of The Magnetic fields And The Prominence Kinematics

Combining the kinematic and magnetic field information we deduce a relationship which is quantitatively in agreement with theory. The first aspect we discuss is the relationship between the slope of the index and the resultant kinematical behaviour of the filament. The slope of the index between \( n = 1 \) and \( n = 1.5 \) is calculated. This range is chosen because it was found, as presented in the previous section, that the corresponding index at the onset height was smaller than \( n \lesssim 1.5 \) for both events. Figure 3.12 shows the magnetic field index for the two events studied plotted together. It is clearly seen that the 6 March event rises more quickly than the 1 August event. The slope of in the range \( n = [1, 1.5] \) for the 6 March event was found to be \( 0.021 \pm 0.003 \text{ Mm}^{-1} \), and for the 1 August event \( 0.008 \pm 0.003 \)
Figure 3.12: The magnetic field index plotted with height, with the slope calculated between $n = 1$ and $n = 1.5$. The horizontal dash lines are visual guides at $n = [1, 1.5, 2]$. 
Mm$^{-1}$. Based on the simulations of Török & Kliem (2007) it was found that the slope of the index directly affects the kinematics of the ejecta. The kinematics of the onset are quantified via the exponential time constant $\tau$. Plotting these two pieces of information together shows that there may exist a relationship between the slope of the index of the magnetic fields and the kinematics. Figure 3.13 shows $\tau$ plotted against the slope of $n$, $dn/dh$. It is seen that the larger the value of $\tau$ the smaller the slope. Considering that we only have two data points, we come to this conclusion with some reservation. A larger statistical sample of event are needed to make a more sound conclusion. In any event these results are compelling enough to justify the conclusion.
The partial torus instability theory of Olmedo & Zhang (2010) shows that the critical index $n_{ct}$ for stability is a function of the geometry of the loop above the photosphere. It was found that the critical index increases monotonically and asymptotically approaches a maximum value as a function of the loops arc. A lower lying loop is found to have a smaller critical index, whereas a more circular loop has a larger value. Theoretical models predict that the onset of an eruption occurs when the index of the ambient magnetic field exceeds the critical index (Kliem & Török, 2006; Olmedo & Zhang, 2010). The resultant kinematical evolution is subsequently found to have an exponential dependence with time.

We relate the theoretical onset with the onset time found using the method outlined in the previous sections. We next define that the corresponding index at the height of the filament at the time of onset as the critical index $n_{ct}$. The geometry of the loop is quantified as the quotient between the onset height and the half footpoint separation $S_0 = S_f/2$, $h_{onset}/S_0$. This quotient is also used by theoretical models to describe the geometry of the loop with the assumption that the footpoints remain fixed (Chen, 1989; Cargill et al., 1994; Olmedo & Zhang, 2010). The critical index theorized by Olmedo & Zhang (2010) is a function of this quotient. If $h_{onset}/S_0$ is equal to 1 then the loop is considered to be semi-circular, and a low lying, more shallow, loop would have a value < 1. The values found for the events studied for the onset height, footpoint separation, and corresponding index at the onset height $n_{ct} = n(h_{onset})$ are plotted together. This plot is shown in figure 3.14, where the abscissa is $h_{onset}/S_0$, and the ordinate is $n_{ct}$. It is seen that the 6 March event has a larger value of $h_{onset}/S_0$ and a larger value of $n_{ct}$. The 1 August event has a smaller value of $h_{onset}/S_0$ and a smaller value of $n_{ct}$. This shows that $n_{ct}$ increases for larger values of $h_{onset}/S_0$.

The outcome shown in figure 3.14 reproduces the essence of the results of Olmedo & Zhang (2010), namely that as the quotient $h_{onset}/S_0$ increases, $n_{ct}$ increases. A direct quantitative comparison finds that the empirical values are much larger than the theoretical ones derived by Olmedo & Zhang (2010). A possibility to this discrepancy could be the fact that the model of Olmedo & Zhang (2010) does not include the repulsive force of
Figure 3.14: Empirical determination of the critical index as a function of $h_{\text{onset}}/S_0$. It is seen that $n_{ct}$ increases for larger values of $h_{\text{onset}}/S_0$. 
induced photospheric currents. Mathematically this is included as an image current below the surface. Other theoretical models do include this image current force (e.g. Kuperus & Raadu, 1974; Forbes & Isenberg, 1991; Isenberg & Forbes, 2007; Démoulin & Aulanier, 2010). Instead, the model of Olmedo & Zhang (2010) implicitly assume that the current that closes the loop below the surface is a circular continuation of the radius of the loop above the surface. This is not the same as in the image current formulation. To explain the results of figure 3.14 we take the fact that the straight line current loop model predicts a critical index of 1, and a circular loop model, without fixed footpoints, predicts a critical index of 1.5. This value may change when making the assumption of fixed footpoints. Recall that in the circular model only half of the circular loop is thought to be above the surface, the resultant loop is therefore semi-circular above the photosphere (Démoulin & Aulanier, 2010). If $h_{\text{onset}}/S_0 << 1$ then the loop would resemble a very long low lying loop that could be thought of as being resemblant of the straight line current model. We presume that the geometry of the loop can vary between the straight line and circular current model. With this idea the critical index may be in the range between 1 and 1.5 for intermediate loops that are smaller than semi-circular $h_{\text{onset}}/S_0 \leq 1$. This anecdotal evidence may help explain the results of figure 3.14. For a lower lying loop the critical index should approach 1, and for a loop that is more semi-circular the index should approach 1.5. The exact nature of the functional dependence of the critical index is unknown at this time and will be a topic of future work.

3.6 Conclusion

Two events were investigated and compared with each other. The events studied occurred on 6 March 2010, and 1 August 2010. It was empirically shown that there is a relationship between the magnetic field index and the kinematics of the initiation of erupting filaments. Furthermore, these observations are in agreement with theory of the onset of eruptions. These theories specify that a flux rope has formed in the solar corona and is in equilibrium
between opposing magnetic forces. The decay index $n$ quantifies how the ambient magnetic field decays, and hence tells us something about how the downward restoring Lorentz force decays. Observationally the top of the filament is used as a proxy of the flux rope apex. It was shown that there exist a relationship between the kinematics of the eruption, quantified by the exponential time constant $\tau$, and the slope of the magnetic field index at the onset. In concurrence with theory it was empirically shown that a lower lying loop has a smaller critical index, whereas a more circular loop a larger critical index. The observational outcome was not found to be quantitatively in agreement with Olmedo & Zhang (2010), instead the fact that there is a dependence of $n_{ct}$ with the geometry of the loop was captured. Future work includes making observations of a larger sample of events to present a stronger statistical argument to the results presented. Also updating the model of Olmedo & Zhang (2010) to including the image current force may reduce the discrepancy between the empirical and theoretical result.
4.1 Introduction

The previous chapters address the physical mechanism responsible for the eruption of CMEs. The focus of this chapter is the problem of automatically detecting CMEs in white light coronagraph data, which enables further statistical study of CMEs. In addition to detecting CMEs, it is also valuable to extract their kinematical properties for complete description of the event and for statistical studies. Several algorithms have been proposed in the literature to try to tackle this problem, these will be reviewed in the following section. A major hurdle for detection of CMEs is the lack of a detailed definition of what one would look like in a coronagraph. As such, the best definition of what classifies a CME is a brightness enhancement that is moving radially away from the Sun (e.g. Gosling, 1974; Hundhausen et al., 1984; Yashiro et al., 2004). This definition is ambiguous in nature and many authors have tried to classify CMEs based on their morphological appearance (e.g. Howard et al., 1985; Burkepile & St. Cyr, 1993). Furthermore, an equivalent set of observations by multiple observers will yield slightly different numbers of CMEs identified, and the measured quantities such as angular width, and speed may be different. As is discussed by Yashiro et al. (2004), who compared the number of CMEs they observed with the LASCO instrument between 1996 and 1998 with those observed by St. Cyr et al. (2000), it seems that this difference is a result of biases on the part of the observer. For example, small features that showed radial motion along streamers were not counted by St. Cyr et al. (2000) because they faded very quickly but were counted by Yashiro et al. (2004). The difference in disagreement between these two observers turns out to be approximately 7%. A major motivating factor for the development of an automated system to identify CMEs is
essentially to remove this human bias. The bias is then shifted towards how the algorithm is tuned, i.e. internal threshold that may make up the algorithm, but the advantage is that the detections remain consistent for a given data set. The goal for the algorithm is then to detect CMEs without any human bias and to identify as many unambiguously as possible such that a human operator may look at the results and be in agreement with them.

The art of detecting and cataloguing CMEs is that of considerable difficult and subjectiveness on the part of observers. Spending time looking at coronagraph data one quickly realizes the difficulty that lies in detecting and making measurements of CMEs. For clarity, the difference between these two terms is that "detection" refers to the identification of a CME within the field of view (FOV) of a coronagraph, and "measurement" refers to deduce physical/kinematical properties of a CME such as its angular extent, radial height, mass, speed, etc. The issue with making measurements of CMEs has to do with the fact that they are optically thin, diffuse, cloud-like features with no definite boundary. The light that is observed in coronagraphs arises from the Thompson scattering of photospheric photons by electrons of the CME mass. The intensity observed is an integration of the scattered light along the line of sight.

On many occasions a CME is observed to have three-part structure consisting of an outer bright rim, followed by a relatively dark cavity, and finally a trailing bright core (Illing & Hundhausen, 1983, 1985). Typically, the outer bright rim will appear as a relatively featureless loop, which is where the description of "loop-like" originates (Illing & Hundhausen, 1985), though this loop in many cases may not have a perfectly round shape and instead may by deformed. The bright core shows intricate structure of twisted or helical shaped magnetic fields. This material in most cases originates from a corresponding eruptive prominence (House et al., 1981; Hundhausen, 1999; Simnett, 2000). Figure 4.1 shows an observation of a CME exhibiting the three-part structure just described.

In the compiled catalogue of CMEs observed by the SSM mission, Burkepile & St. Cyr (1993) provides several descriptions of the morphological shapes seen within the FOV. Two
of these classifications are the "loop/cavity" and "core" morphologies which can best describe the three part structured CME. The "loop/cavity" is described as having a bright frontal loop and trailing faint cavity. The "core" description extends the loop/cavity morphology by also including a bright core within the dark cavity trailing the front and is almost exclusively associated with the loop/cavity morphology. Their "core" description is therefore the best classification of the three part structured CME. Of the CMEs seen by the SMM mission, which spanned almost a complete solar cycle, Burkepile & St. Cyr (1993) reports that 32.1% are of type loop/cavity and that 16.5% are of the core type. This indicates that a large majority of transient CME events have morphologies other than the three part structure and that more events are seen without the bright core than there are with. The issue of how to identify a CME has been a long standing problem, even with the earliest of observers. Gosling (1974) reports on transient events, now known as CMEs (I will use "CME" and "transient" interchangeably), as observed with the High Altitude Observatory’s white light coronagraph experiment aboard Skylab. In that work, Gosling
defines that the "term transient is used to describe changes in the corona easily discernible on a time scale of tens of minutes." And are "events that obviously represent the ejection of material from the Sun." In this early definition the radial motion must be an implied property of the transient. So, it seems that the definition of what is a CME as observed within a coronagraph has for the most part gone unchanged since the first observations of the phenomena (the first reported CME was by Tousey (1973)). A more refined classical description of what defines a CME in the coronagraph FOV is given by Hundhausen et al. (1984) "to be an observable change in coronal structure that (1) occurs on a time-scale between a few minutes and several hours and (2) involves the appearance [and outward motion] of a new, discrete, bright, white-light feature in the coronagraph field of view". The statement in the brackets is an update to this definition to explicitly state that the feature should be moving radially outwards (cf. St. Cyr & Burkepile, 1990; Schwenn, 1996). This definition makes no assumption about the underlying physical nature of the phenomena nor does it imply any geometrical shape. Based on these observations the interpretation of the geometrical shape of the CME is quite rather ambiguous. For example, CMEs have been described as having a loop or rope like shape, or being a shell or bubble of dense plasma (Crifo et al., 1983; Hundhausen, 1999). Either geometry seems to be able to reproduce what is seen in the coronagraph image. Physical models have favoured the loop over the shell interpretation and many authors have proposed that this loop can be modelled as current carrying "flux-rope" (e.g. Mouschovias & Poland, 1978; Anzer, 1978; Yeh, 1982; Xue & Chen, 1983; Chen, 1989; Titov & Démoulin, 1999).

4.2 Review of Existing Algorithms

Coronagraph data can be thought of as a three dimensional data set with two spatial, and one temporal dimension $t$. Measurements of CMEs could be made with any combination of these. The two spatial dimensions correspond to the projected radial height $r$ above the solar disk, and the position angle $\theta$ taken counterclockwise from the solar north. To date no
detection algorithm takes advantage of all three dimensions simultaneously, instead, every proposed algorithm will apply some transformation to two dimensions simultaneously then merge the third and get a complete picture of the CME. Olmedo et al. (2008) have developed a Solar Eruptive Event Detection System (SEEDS) algorithm to detect CMEs using the two spatial dimension \([r, \theta]\), then use the temporal dimension to track it as it moves radially outward. The detection algorithm developed by Qu et al. (2006) also utilizes the two spatial dimensions to detect the CME and the temporal dimension to track. The Computer Aided CME Tracking (CACTus) algorithm developed by Berghmans et al. (2002), and Robbrecht & Berghmans (2004) utilize the radial and time dimensions \([r, t]\) by making slices for every position angle and detect the CME within these slices. They then combine with the position angle to get the full angular width. Finally the Automatic Recognition of Transient Events and Marseille Inventory from Synoptic maps (ARTEMIS) algorithm developed by Boursier et al. (2005); Boursier et al. (2009) utilizes the position angle and time dimensions \([\theta, t]\) by making synoptic maps for given radial height and detect CMEs within the synoptic maps. They then combine multiple radial heights to derive kinematical motions of the CME.

Automated methods such as CACTus and SEEDS have a tendency to report more than twice as many CMEs as are identified in catalogues made by a human observer. This major difference is primarily due to a sensitive balance between internal thresholds of the algorithm and the goal of trying to include every CME detected by a human operator. The excess CMEs detected are primarily CME outflow, blob like features, streamer disturbance, or noise. These kinds of small features have been reported and studied (e.g. Howard et al., 1985; Burkepile & St. Cyr, 1993; Wang et al., 2000; Bemporad et al., 2005). It is important to note that because of the diffused, cloud-like nature of CMEs, any detection, be it by human or computer algorithm, will have some subjectivity. For example, a CME, as detected by a human or computer algorithm, may be detected differently by another human or computer algorithm, such that the CME may be seen as two CMEs, have different angular width and velocity, or different starting and ending times (Qu et al., 2006). Although a computer algorithm attempts to reduce human subjectivity, the algorithm itself is a heuristic process
that is a set of rules and internal thresholds. Through experimentation with the rules and thresholds it may be possible to create a catalogue, although it may not contain 100% of the CMEs that emerge from the Sun, which is at least consistent with a given set of rules. Currently no automated system is capable of detecting 100% of all CMEs reported in the CDAW catalogue and the events missed tend to be weak in nature. Furthermore, the measured parameters, radial height, angular width, speed, etc. may be different between the various automated methods and CDAW.

4.3 Comparison of automated catalogues and CDAW

The most prominent and widely referenced CME catalogue is the one produced by the Coordinated Data Analysis Workshop (CDAW) Data Center and is compiled using the LASCO C2 and C3 coronagraphs. The CDAW catalogue provides some measurements of CME properties, including the position angle, the angular width, and the height of the CME in each individual image. This catalogue combines the height measurements in the time sequence to determine the projected velocity and acceleration, which are also provided in the catalogue (Yashiro et al., 2004). These measurements are made by dedicated human operators who look at and then choose the CME height and position on coronagraph images displayed one by one on a computer screen. Although there is human bias and possibly a few missed events it is commonly used by the community and considered as ground truth. This is an important assumption especially for comparing and validating catalogues built upon automated methods.

There are two major issues when making comparisons between a catalogue made by human observers such as the CDAW and a catalogue made by automated methods. The first issue is related to bias and the second is related to the apparent over detection of CMEs by automated methods. An advantage to using automated methods is the fact that, although still biased by internal thresholds, the bias is consistent throughout the detection process. On the other hand, human bias may change with time. This is evident in the CDAW catalogue where an increase in narrow CMEs in the period after 2004 was seen.
Figure 4.2: Comparison between the CDAW and automated methods, CME detection rates. The top panel shows the detection rate between CDAW and CACTus and is compared with the average sunspot number (From: Robbrecht et al. (2009)). Middle panel shows the detection rate between CDAW and SEEDS. The bottom panel shows the detection rate between CDAW and ARTEMIS (From: Boursier et al. (2009))
Although this may be corrected by revisiting the data and identifying missed events, as was done by Gopalswamy et al. (2009). Comparing the CME detection rates between CDAW and automated catalogues (using LASCO data) and sunspot number one finds that the general trend of automated catalogues better follows the sunspot number, where as manual CDAW catalogue does not. This trend is shown in figure 4.2 top panel. This could be due to the many outflow features that accompany CMEs. These features are typically ignored by a human operators but are reported by automated methods, this is probably the reason why automated methods may report more events. This leads to the second issue which is related to the high detection rates. The trend is that at least twice as many detections are made with automated methods than made with manual methods. An interesting feature is seen during solar minimum of 2007, it appears that the number of detections reported by CDAW exceed automated methods. This could be due to a sensitivity issue related to thresholds imposed by automated methods. Narrow CMEs (angular width < 30°) are an important topic of discussion when it comes to automated detection. Based on a comparison study between the CDAW and CACTus catalogues made by Yashiro et al. (2008), narrow CMEs comprīse 60% of the entire CACTus catalogue most of which are true events (46%) and many that are false (35%). In some cases a wider angle (> 30°) CME may be split into smaller pieces, each piece considered as a narrow CME, upon visual inspection based on a sample of 100 narrow CMEs Yashiro et al. (2008) found that 7% were a piece of a wider CME. They also estimated that CACTus list about 1000-2300 more narrow CMEs than the CDAW catalogue. Comparing the detection rates of SEEDS, CACTus, and ARTEMIS all follow the basic trend (Boursier et al., 2009).

### 4.4 Online Catalogue

The output of our detection system is available on-line at [http://spaceweather.gmu.edu/seeds/](http://spaceweather.gmu.edu/seeds/). On the site one will find a calendar that contains links to all of the data that have been processed. This encompass LASCO C2 data spanning from 1996 to the present day.
**Latest Detection Movie:**

Click here to see the latest detection movie: 20100818

**SEEDS Monthly Catalog Version 1.0:**

![LASCO C2 CMEs](image)

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(*) An asterisk denotes months in which only quicklook data is available

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**Figure 4.3:** Screen shot of the LASCO C2 SEEDS catalogue.
Height-time plots with linear and second-order fits are available as image files. Also provided are Java script movies of each detection that contains the running-difference images, with the projected leading edge, as well as a second panel that contains the direct coronagraph image for comparison. The leading edge referred here, which is created automatically to outline the CME (see figure 4.5.5), will be discussed in section 4.5.4. Figure 4.3 shows a screen capture of the current version of the website as seen on August 18, 2010. One can tell the date of the capture because the ”Latest Detection Movie” says ”2010/08/18”. This latest detection movie is a result of the near-real-time module. This module continually queries for new LASCO C2 data several times a day and upon downloading new images processes them by applying our detection algorithm. By clicking on the ”Near-Real-Time” link, processed data for the last 21 days is found. This includes LASCO C2 detections as well as detections made with the STEREO COR2 instrument in BEACON mode. We hope that this website will be a valuable asset to the scientific community to aid in the identification and characterization of CME events.

4.5 Solar Eruptive Event Detection System (SEEDS)

In this section the Solar Eruptive Event Detection System (SEEDS) algorithm developed to detect CMEs is described. The first few sections will describe the pre-processing necessary to prepare and ”clean” the image. This is followed by the steps taken to make an initial detection and track a CME in a time sequence. Finally, the algorithm is validated by comparing its output to the CDAW catalogue.

4.5.1 Pre-Processing

The input to the detection system is LASCO C2 ”LEVEL 0.5” images. The pre-processing module involves several steps. First, the input 1024 × 1024 pixel image is normalized by the exposure time. It is then passed through a noise filter to suppress sharp noise features such as stars and cosmic rays. Then a mask is made to indicate areas of missing blocks in the telemetry. This mask is useful because the missing data blocks will cause anomalous
false signals in the difference image and thus the false detection. For the same reason, the planets seen in the images are also masked to avoid any false signals in CME detection. Finally the image is transformed to a polar coordinate system and then put through a running-difference filter.

The telemetry of the SOHO spacecraft is such that data are sent in packets. On occasion, packets will not be transmitted properly and hence create missing blocks in the coronagraph images. These data blocks within the images are represented with zero pixel value. A bivalence mask is created, of equal size as the input image, such that the area of missing data blocks is represented with zero value and everything else with value one.

An important step in the pre-processing procedure is the application of a noise filter to the input images. There are two general purposes for this filter; the first is to remove such features such as cosmic rays and random noise. The second is to remove background stars, and correct for planets and comets. The basis to this filter is the procedure developed by Llebaria et al. (1998) which discriminates point-like objects in astronomical images using surface curvature. There are three primary steps to this procedure. In the first step the log of the image is taken and the surface curvature, which parameterizes how the brightness gradient changes within the 2D spatial coordinates of the image, is calculated. The surface curvature is parameterized by two principle curvature coefficients $k_1$ and $k_2$ which characterize the curvature on the surface for each pixel location. Within the 2D $[k_1,k_2]$ space, different areas represent spike-like features, such as stars and planets, or elongated features, such as cosmic rays. This leads to the second step in the procedure: Finding the location of valid and invalid pixels and making a mask of ones and zeros, equal in size to the input image. Spike like and elongated features, found in the input image within the 2D $[k_1,k_2]$ space, are represented with zeros in the mask while the rest are represented with ones. The third and final step is to correct the invalid pixels such that they are representative of the background. This is done with the aid of the mask created and by a local, pyramidal interpolation using a non-linear, multiresolution method, where the end result is an image that is devoid of background stars, cosmic rays, and random noise.
For the removal of features such as planets and comets, which tend to be larger than background stars and cosmic rays, a two step process is followed, where the end product is a mask that identifies the feature. First, a copy of the original input image is made and a median filter followed by a smoothing filter is applied to it, this essentially smoothes out the small features (cosmic rays and background stars) and leaves the larger ones. The next step involves again calculating the surface curvature, finding the spike like and elongated features in the 2D \([k_1,k_2]\) space, and creating a mask of ones and zeros. This new mask will contain the location of the larger features (planets, comets, etc.) and the invalid pixels will be represented with 0’s. In order to fully cover the larger features, morphological dilation is applied to this mask. Morphological dilation is an image processing technique where a binary images is used to dilate or grow objects in size based on a smaller structuring element (see appendix C for the mathematical formulation of morphological operators). Hence, the area covered by the invalid pixels is slightly increased.

To make image processing efficient, the input images, which are in \([x,y]\) Cartesian coordinates, are transformed into a \([\theta,r]\) polar coordinate system, because the features of interest are intrinsically in polar coordinates due to the spherical structure of the Sun. This kind of transformation has been used in other CME-detection algorithms (Robbrecht & Berghmans, 2004; Qu et al., 2006). The transformation takes the \([x,y]\) field of view (FOV) of the LASCO C2 image, and starting from the North of the Sun going counter clockwise, transforms each angle onto a \([\theta,r]\) FOV so that each column in the resulting 360 × 360 image corresponds to one degree in angle. The radial FOV in this image corresponds to 360 discrete points between 2.2 to 6.2 solar radii.

To enhance moving features, a running-difference sequence of the processed images is made using the following equation

\[
 u_i = (n_i - n_{i-1}(\bar{n}_i/\bar{n}_{i-1})) \frac{\alpha}{\Delta t} 
\]  

(4.1)

where \(u_i\) is the running-difference image, \(\bar{n}\) is the mean of the pixels in the entire FOV of
Figure 4.4: An image of the CME that occurred on 12 September 2002 and its polar representation. Panel (a) shows the input LASCO C2 image processed with the exposure time normalization and the noise filtering; the white arrow points to the CME in the FOV. Panel (b) shows the polar representation of the running-difference image made from the image in panel (a). The bright region seen in panel (b) is the corresponding CME seen in panel (a).

The image \( n \), \( \Delta t \) the time difference between the images (in minutes), \( \alpha \) a constant, and the subscript \( i \) denotes the current image and \( i - 1 \) denotes the previous image. The constant \( \alpha \) is set to approximately the smallest possible \( \Delta t \) time difference between any pair of images, typically equal to a value between five and ten minutes. This normalized difference ensures that the mean of the new image \( (u_i) \) will be effectively zero. After performing the differencing, the masks of missing blocks and planets are applied. The masks are first polar transformed, then the difference image is multiplied by them such that the missing block areas as well as the planets become zero value pixels. Figure 4.4 shows the pre-processing of an input LASCO C2 image. In this figure, panel (a) shows the input LASCO C2 input image processed with the exposure-time normalization and the noise filter, panel (b), shows the corresponding polar representation of the running-difference image made from the image in panel (a).
Figure 4.5: The intensity profile along the angular axis showing the 1D projection of the CME image. Only positive pixels along the radial axis are used. This profile effectively indicates the angular positions of a CME when it is present.

4.5.2 Initial Detection

The initial CME detection is made on the polar-transformed running-difference sequence images. First, the 2D images are projected to 1D along the angular axis such that the values of the radial pixels in each degree column in the image are summed; this effectively measures the total brightness of signals along one particular angular degree. Observationally, a CME is best observed in running-difference images. This type of image essentially removes static or quasi-static background features such as coronal streamers and enhances features that change at faster time scales. A CME in a running-difference image will appear as a bright leading edge enhancement (positive pixel values) followed by a dark area deficient in brightness (negative pixel values), and the background will appear gray indicating zero change pixels. Since the detection concerns the bright leading edge, only the positive pixels are counted when making the 1D projection. Hence, the 2D enhancement is seen as a peak in the 1D intensity profile. Also, by excluding the negative enhancement, the positive enhancement becomes more outstanding in the projection profile. The following is
a mathematical formulation of the projection

\[ p_\theta = \frac{1}{c_\theta} \sum_r u(r, \theta) \quad u(r, \theta) > 0 \quad (4.2) \]

where \( c_\theta \) is the number of positive pixels for each given angle \( \theta \). An example of the projection can be seen in Figure 4.5.

To make the detection, a threshold (\( T_1 \)) is chosen to identify the peaks in the 1D projection based on the standard deviation \( \sigma_p \) and mean \( \bar{p}_\theta \) of the projection \( p_\theta \).

\[ T_1 = N_1 \sigma_p + \bar{p}_\theta \quad (4.3) \]

Here \( N_1 \) is a number chosen to set the threshold level. This value remains constant throughout the detection and is determined through experimental methods. The identified peak angles above the threshold are called the core angles. These core angles correspond to the central and the brightest part of a CME.

\( N_1 \) (thus the threshold \( T_1 \)) is a critical number in the performance of the detection algorithm. It is often chosen to be between two and four. The lower the number, the more sensitive the detection, and the more false detections. On the other hand, the higher the number, the lower the sensitivity of the detection, which results in more missed events. Currently we are still experimenting to find an optimal number. An important issue that arises at this point that is not fully addressed in our detection algorithm is the emergence of multiple CMEs at the same time, especially if there were a bright and a faint CME.

Presently, because the threshold \( T_1 \) is calculated with the standard deviation and mean of the whole projection \( p_\theta \), what will happen is that the signal of the faint CME will be overpowered by the bright CME and not detected. This is not a big problem if the brightness of multiple emerging CMEs are close in magnitude, they will in general all be detected. We have been experimenting with a few other methods to try to overcome the issue of bright and faint CMEs simultaneously emerging but have not yet successfully come
to a conclusion as to which may be the best. For example we could take the log of the projection, which is similar to what Llebaria et al. (1998) has done for discriminating point-like sources, where using the log scale is justified because the range of curvatures become narrower and improves the detection of faint signals. Another option could be to use local thresholds, the 360 points of the projection could be split into several sections, compute a threshold for each, then piece all of the detections together to make a final determination of the core angles. A promising solution proposed by Qu et al. (2006) is to not only use running-difference images but to also use running-ratio images. This has the advantage that weak CMEs in the ratio images will appear closer in brightness to the bright CMEs, but has the disadvantage that noise will also be enhanced, causing overflow errors. To overcome this, Qu et al. (2006) have proposed that only the pixels greater than the median in the reference image \( n_i \) be used to make the ratio image. Future investigation is needed in order to find the best method to overcome this issue.

Figure 4.6a shows, with two vertical lines, the core angles found. These angles only yield a small fraction of the CMEs total angular width. To find the full angular width of the CME region “growing” is applied. Region growing is the procedure used to expand a small region found in a multi-dimensional space into a larger region. In our case region-growing takes two inputs: the maximum and minimum values for the core region to be grown to. This means that the CME angles are widened to include all of the pixels that are within the maximum and the minimum range, hence the starting and ending position angles (PAs) of the full-sized CME are determined, figure 4.6b. For our application, we have chosen the maximum input value to be the maximum value in the core-angle range and the minimum is set to a value similar to a reduced \( T1, T2 \), where instead of the mean and standard deviation of the whole projection, the mean and standard deviation of only the values outside of the core-angle range are used. In many cases, especially with limb CMEs, it is observed that the CME will deflect the streamer structure of the Sun. This type of disturbance has the appearance of a wispy-like structure and is caused by the CME pushing the material of the streamer away from it as it expands during its radial propagation.
Figure 4.6: Determination of CME angular position, angular size, and heights. Panel (a) shows the CME core (or brightest) angels within the two vertical lines. Panel (b) indicates the full CME angles after applying the region growing method. Panel (c) shows the CME heights at the half-max-lead, maximum, and the half-max-follow indicated by the three horizontal lines respectively; the white box at the bottom corresponds to the trailing box, where exponential suppression is applied. The right side of panel (c) shows the intensity profile of the 1D projection along the radial direction.
the region-growing step just discussed, it is difficult to distinguish between what is and what is not part of the CME structure in the 1D profile, therefore on some occasions the streamer deflection will become part of the detected angular width of the CME. Another disadvantage to be discussed is the emergence of two CMEs at approximately the same time and angular position such that the full angular widths of both CMEs overlap. After the region-growing step, since the full angular widths overlap, what will happen is that the algorithm will mistake the multiple CMEs as one CME and attempt to track it as though it were a single CME. Currently, no solution to this problem has been found and further investigation is needed.

4.5.3 Tracking

To identify a newly-emerging CME, the CME must be seen to move outward in at least two running-difference images. This condition is also set by Yashiro et al. (2004) to define a newly-emerging CME. This condition is very useful because very often it is found that when only one detection is found with SEEDS it is most likely a noise feature and not a feature that is evolving with time. Currently SEEDS can not distinguish between a noise feature or an object with CME-like features in a single detection. Also, the actual number of CMEs that are found with such a high velocity as to only be seen in one LASCO C2 frame is very small.

In order to determine the presence of a newly-emerging CME we look at running-difference images $i$ and $i + 1$, assuming that a detection has been seen in image $i$. If the detection in image $i$ overlaps with a detection in image $i + 1$ then it is assumed that image $i$ is the initial detection of a newly emerging CME; where overlap here refers to the overlapping of PAs detected between the two images. Once the PAs of the initial detection are established, the leading edge and height can be found.

Determination of the leading edge begins with performing another projection along the radial direction. This projection is made within the starting and ending PAs for each CME seen in the FOV of the image. To do this projection, the pixels within the PAs are averaged
along the theta component such to be left with a 1D array of 360 points corresponding to the 360 discrete points in the radial FOV of the image. This is written as follows, where $w$ is the angular width of the CME detected (equal to the end PA minus the start PA).

$$ P_r = \frac{1}{w} \sum_{\theta=PA_{\text{end}}}^{\theta=PA_{\text{start}}} u(r, \theta) $$

(4.4)

The peak of this array is found and is defined as the height of CME maximum brightness, max-height, see Figure 4.6c. The two half-maximum points that are radially higher and lower than the max-height point are calculated and are called the half-max-lead for the point that is higher and half-max-follow for the point that is lower. At present, the half-max-lead is thought of as the leading edge of the CME and is the point that is used for making calculations of velocity and acceleration. With the two half-max points and the starting and ending PAs we define a region of interest, which we call the CME box. Figure 4.6c shows a detection where the two half maxima’s and the max-height of the radial projection are shown.

At this point, the initial detection of the CME has been established; what follows is the tracking of its path in subsequent running-difference images. The running-difference image that follows the image with the initial detection now becomes the subject in focus and becomes the current image. Starting and ending PAs have already been determined for this image through the initial detection module. They are compared with the starting and ending PAs of the previous image and are combined such that the starting PA is the minimum of the two starting PA and the ending PA is the maximum of the two ending PAs. The next thing to determine is the leading edge of the CME. Radial projection is again performed, as was previously done, but for this projection the lower limit becomes the half-max-follow that was formerly determined in the previous image instead of the lower radial boundary of the FOV of the image. The max-height within this projection is found, as well as the half-max-lead and half-max-follow. Two criteria are applied at this point to the position of the max-height and half-max-lead. First, the max-height is not allowed
to be below the max-height of the previous detection. Second, the distance between the max-height and half-max-lead is to be greater than or equal to the distance between the max-height and half-max-lead of the previous detection. The second criteria is used to avoid the max-height or half-max-lead from being found below the previous detection. This assures that the leading-edge detection in a sequence of images always increases in height with time and never backtracks, as that would not be realistic in the tracking of a CME, which is a radially outward moving object. This is not to say that backtracking material, known as inflow, doesn’t exist and can be seen in LASCO C2 coronagraphs up to 5.5 solar radii (Sheeley & Wang, 2002). This project only focuses on outward-moving CMEs. The criteria also assumes that the CME is a radially-expanding object where it is thought that the distance between the max-height and half-max-lead should increase with time.

To continue tracking the CME, the steps just described in the previous paragraph are applied to successive images in chronological order. Where the current running-difference image becomes the previous running-difference image and the next running-difference image becomes the current running-difference image and so forth. These steps continue until the half-max-lead height is found to be outside of the FOV of the image or until no detection is found in the current image due to the CME becoming too diffused. Finding the half-max-lead height outside of the FOV is possible because of the assumption that the distance between the max-height and half-max-lead height in the current detection must be greater than the distance in the previous detection. In the case that a temporal gap is found, considered to be more than about two hours, the algorithm stops all tracking, outputs the data for any CME tracked and starts over at the time after the gap.

Following a major CME, very often there exist a period of continuous outflow of small blob-like outflow features, which could be detected as other CMEs. To suppress this effect we first define a trailing box, see Figure 4.6c (white box). Within the trailing box, an exponential function is applied. The following equation shows the function used to suppress CME outflow features.

\[ p_0^* = p_0(1 - e^{-(T - T_0)/T_c}) \]  \hspace{1cm} (4.5)
Figure 4.7: The CME leading edge. Panel (a) shows the segmented CME within the segmentation area. The segmentation area is contained within the vertical lines and the horizontal lines excluding the half-max-lead line which is shown only for reference purposes. Panel (b) shows the pixels “∗” that define the leading edge of the CME along the position angles.

Where \( p_0 \) is the pixel values within the trailing area, \( p_0^* \) indicates the exponentially-suppressed pixel values, \( T_0 \) is the onset time of the CME, and \( T_c \) a time constant. This function is applied for several hours after the onset of the CME at \( T_0 \), an optimal time was found to be between five and six hours. Further investigation into the \( T_c \) parameter is needed to find an optimal value; with current experimentation we find a value of in the range between three and five. The value of \( T_c \), which determines the rate of the exponential suppression, is highly correlated with the velocity with which the CME leaves the FOV as well as the extent, or the radial length, of the CME.

4.5.4 Leading edge Determination and Visualization

In this section we describe the method used to find an approximate leading-edge shape of the CME. This is done for visualization purposes which helps guide the eye to show how the CME is evolving; it could also be used to measure velocity. The biggest difficulty in this task is the fact that CMEs tend to appear as diffuse clouds that have no clear boundary. In our application we implement a simple segmentation technique based on a threshold to
find the approximate shape of the leading edge.

There are three main steps in this calculation. We begin with the CME detected within the CME box. The first step is to extend the box and set a new search area for the leading edge. The new upper boundary is set to a height that is half of the height of the CME box (half the distance between the half maximums) above the half-max-lead position. Within this new region, a CME area is segmented by scrolling through each position angle within the box and setting all the pixels that are half maximum of the maximum within each angle to value one and the rest to value zero (Figure 4.7a). Mathematical morphology operators opening and closing, based on set-theory, as used in image processing (e.g. Qu et al., 2004, 2006), are applied to the segmented image to remove noise and connect disjoint pieces. The pixels on the upper border of the area are chosen as the leading edge outline of the CME; these leading-edge points are linearly smoothed and projected back to the coronagraph image (Figure 4.7b).

4.5.5 Example Case

As an example case, the event that occurred on 12 September 2002 21:12 UT is shown in Figure 4.8. We show the CME evolution in four difference images with the leading-edge over plotted with a black dotted line. Figure 4.9 shows the time-height profile for this example CME measured with the automatic detection as compared with the CDAW catalogue entry from human measurement. The “+” line represents the height-time plot from CDAW but using only C2 observations, the “△” is the highest peak found in the leading-edge-segmentation as found with SEEDS, and the “∗” is the half-max-lead. We calculate a velocity using the leading-edge-segmentation and half-max-lead and find values of 543.4 km/s and 474.2 km/s respectively. For the C2 measurements in the CDAW catalogue we found a velocity of 484.9 km/s. The angular width found by SEEDS was 105° and for CDAW 123°. We can see that the SEEDS automatic measurements for this event are very similar to the manual measurements in terms of measuring the angular width and velocity of the CME. The height, on the other hand, seems to fall slightly short of the manual measurement. This
Figure 4.8: The image sequence showing the automatic tracking of the event on 12 September 2002 21:12 UT. The black dotted lines indicate the detected leading edges of the CME overlaid on the CME images.
discrepancy shows the difference between a human and computer choosing the position of the height for the CME. But both capture the fact that the CME is propagating in a radial direction and they find a velocity that is approximately the same. It can also be noticed that the leading-edge-segmentation and the half-max-lead heights are diverging with increasing time. This discrepancy may be due to the expansion of the CME and could possibly be a measure of how the CME expands with time; this finding merits further investigation outside of the scope of this current study.

4.5.6 Results

The output of the SEEDS system that quantifies CMEs includes the PA, angular width, and height, which are further used to calculate the velocity and acceleration as a function of time. There are many other measurements that are made with this system besides the basic CME parameters. As mentioned before, the parameters that make up the CME box include the PAs and the half-max-lead and half-max-follow. Five more parameters that are associated with the area contained within the CME box are the mean, standard deviation, maximum, and minimum brightness values within the CME box. The other parameter comes from taking the difference between the detection box as seen in the current difference image and that in the previous difference image, and then taking the mean of the pixels within this difference. We call this last parameter the mean box difference which effectively indicates the brightness variation of a given CME. Some of these proposed measurements may not have obvious scientific value but they have been discussed in order to show that our technique is flexible and has the capability to extract many measurements. We believe that many of these measurements may be useful as CME signatures and for the possible classification of events. This type of classification, for example, has been explored by Qu et al. (2006), where they used a list of measurements and classified CMEs into three categories: strong, medium and weak.
Figure 4.9: The comparative Height-Time plots of the CME on 12 September 2002 21:12 UT. The measurements of the manual-based CDAW, the automated SEEDS at the highest leading-edge-segmentation position, and at the half-max-lead are denoted with “+”, “△”, and “∗” symbols, respectively. The straight lines show the linear fit to the height-time measurement, yielding the CME velocity.
Figure 4.10: Comparison of CME parameters measured in the automated SEEDS system and the manual CDAW measurements. Panels (a), (b), and (c) show the width, latitude, and velocity, respectively. The straight line in the panels is a diagonal line with slope of one.
4.5.7 Validation and Comparison

The SEEDS CME detection system was tested using data for 2002 and compared with the CDAW catalogue for the same period. Statistics were compiled on latitude (converted from central PA), angular width, and velocity. The data were run at monthly intervals to simplify the comparison between the two catalogues. The monthly text version of the CDAW catalogue for 2002 was used. Because of the large number of events, a manual comparison between the catalogues would be difficult, so an automated method was implemented to compare our results with those of the CDAW catalogue. A criterion was established to compare entries between the two catalogues. It involved looking at the starting and ending PAs as well as the initial time of detection. Because starting and ending PAs are not available on the CDAW catalogue they must be calculated. This was accomplished by subtracting one half the angular width to the central PA, to find the starting PA, and adding one half the angular width to the central PA, to find the ending PA. In order to do the comparison we first look at the time of the initial detection of the entry in the SEEDS catalogue and find all detections in the CDAW catalogue that fall within a range of one and a half hours either before or after the time of detection giving us a three hour window. The reason for this window has to do with the nature of the SEEDS scheme in that it may sometimes detect an event either several frames early or several frames late. So we stipulate that a three-hour window is adequate for making our comparison. We next look at the PAs and find the unique entry in the CDAW catalogue within the time window that overlaps the SEEDSs detection. If no entry exists in the CDAW catalogue then it is thought of as either a new CME detection that had gone unreported in the CDAW catalogue or an erroneous detection that may have been caused by a large streamer disturbance or noise that had not been removed properly by the detection processes.

This analysis yields the following results. For 2002, SEEDS finds a total of 4045 events, the CDAW catalogue lists 1389 CMEs. However, this number of CDAW catalogue CMEs was limited to those seen with C2 measurements and that had a quality index of greater than or equal to one, Where quality index is a visual measure of how well defined the
CME leading edge is. This value ranges between zero and five, zero being ill-defined and five begin excellent. It is found that 1306 SEEDS events corresponded to events in the CDAW catalogue. On occasion SEEDS will not detect a CME as whole, for example a very wide angle or halo CME may be detected in several pieces therefore SEEDS will report that multiple CMEs have been detected when in fact the pieces belong to a single event. Another example of when this occurs is if a CME in the middle of tracking is not detected in one frame but is then re-detected in the next frame and then re-tracked. Further study is needed to associate these two events temporally such that SEEDS will only report one event. We found that, of the 1306 SEEDS events that correspond to events in the CDAW catalogue, 281 were disjoint events that should have corresponded to a single event, leaving 1025 events that mapped from SEEDS to the CDAW catalogue. This leaves us with a true positive rate of approximately 74% assuming the CDAW catalogue is ground truth. On the other hand SEEDS detects over twice as many events as listed in the CDAW catalogue, which is not unusual since the automated system picks up many outflow, narrow, or small events that are ignored by human observers. Upon inspection of the CMEs that were missed it is concluded that they tend to be ones that are weak or faint. This is not uncommon with automated CME detection, for example, Qu et al. (2006) reported that in their case study time period, using their developed CME detection algorithm, all of the CMEs missed were weak events. They also made a comparison with the CACTus catalogue, using the same case study time period, and found that the CMEs missed by CACTus were also weak events.

Figure 4.10 shows scatter plots of the basic parameters for the associated CMEs between SEEDS and CDAW. Figure 4.10a shows the comparison between angular widths which indicate that the widths from SEEDS are in general narrower than CDAW measurements, especially with increasing angular extent. This indicates to us that perhaps the region-growing maximum and minimum thresholds need to be further investigated since the region-growing step in the detection is what determines the full angular width of the CME. Figure 4.10b shows that latitude measurements of SEEDS very closely follow CDAW
measurements. Finally, Figure 4.10c shows the velocity comparison, which shows a broad scatter. This type of scatter indicates that much work is needed on how CMEs are tracked and how the position of the leading edge of the event is determined. This scatter could also be due to events that were detected as disjoint events. For example, one detection may be part of the core of the CME, expanding in the radial direction, and another disjoint detection, of the same CME, is expanding more in the latitudinal direction; therefore it would appear to have a slower velocity.

Histograms of the SEEDS detections of angular width, latitude, and velocity can be seen in Figure 4.11. The mean and median of the angular width for normal CMEs within the distribution were found to be $40^\circ$ and $34^\circ$ respectively. Where normal CMEs are the ones that are found to have an angular width greater than $20^\circ$ and less than $120^\circ$ (Yashiro et al., 2004). The mean and median of velocity distribution were found to be $292 \text{ km/s}$ and $233 \text{ km/s}$, respectively. The critical latitude angles were calculated for the latitude distribution and were found to be $-68^\circ$ and $51^\circ$. The critical latitudes represent the latitudinal positions on the Sun where 80% of all CME emerge in a given time period (Yashiro et al., 2004).

Comparing the statistical properties of SEEDS, for 2002, with those reported by the CDAW catalogue we find similar results. For the mean and median of the angular width for normal CMEs they find $53^\circ$ and $49^\circ$ respectively, the mean and median for the velocity distribution, $521 \text{ km/s}$ and $468 \text{ km/s}$, respectively, and the critical latitude angles they found were $-59^\circ$ and $51^\circ$. The largest discrepancies in the comparison of statistical results are the mean and median of the velocity. The discrepancies could be attributed to the fact that SEEDS has detected many anomalous small objects with much slower velocities. At this point it is important to note the sources of error in the histograms of the CME parameters. As previously stated, many small anomalous detections are made, these anomalous features may include things like streamer deflections or streamers that are bright enough that there signal can be seen in the 1D projection, $p_\theta$. Other objects may include blob-like features that may or may not be associated with either pre or post CME outflow but that are brighter than the background. Finally, especially during times when no CME is erupting,
Figure 4.11: Histogram plots of CME parameters from the SEEDS method for all LASCO C2 observations in the year 2002. From top to bottom the three panels are for CME width, latitude (equivalently position angle), and velocity, respectively. The two vertical lines in panel (b) show the critical latitude that bound the area where 80% of the CME are to be found.
areas of increased brightness may be observed in the 1D projection, $p_\theta$. These bright regions typically may not be associated with any type of solar feature, but if such a region in the same angular position is seen in multiple frames, the algorithm will output the erroneous detection. Because of these anomalous features that may be detected, future statistical studies using the output of SEEDS must be handled with extreme care.

4.6 Conclusion

An automated detection algorithm was developed and implemented to detect, track, and characterize CME events. The algorithm uses two spatial dimension $[r, \theta]$, then combines the temporal dimension to track the CME as it moves radially outward. This technique, compared with other automated methods, has the advantage of making initial detection using only one running difference image. This advantage makes real time detection possible. To track the CME, three running difference images are utilized. This is distinct from other methods that require an array (or cube) of images to be stacked together over a period of time. Using an automatic method to compare the detections spanning the year 2002, SEEDS successfully detected $\sim 74\%$ of the events listed in the CDAW catalogue. An online catalogue of LASCO C2 data has been compiled using the outlined method of detection. This catalogue spans from 1996 to the present day. Automated methods are useful tools that aid in extracting relevant scientific information from large data sets. This supports scientist in their quest to understand the underlying physical nature of the phenomena captured by the data sets.
Chapter 5: Summary and Future Plan

This thesis has yielded new and important result to further our understanding of the initiation of CMEs. I have also described an algorithm to automatically detect and characterize CMEs. Three topics were covered. The first topic is related to the initiation criteria that characterizes the loss of equilibrium between the magnetic forces in the corona. The analysis presented considers the equilibrium of a partial torus flux rope anchored to the photosphere. The second topic makes an effort to understand the initiation of CMEs through observations in the context of this theory. The third topic is the description of a proposed algorithm to automatically detect and characterize CMEs in coronagraph images. The main result of each topic are summarised in the following.

(1) In this study, I model the flux rope as a current carrying partial torus loop with its two footpoints anchored in the photosphere, and investigate its stability in the context of the torus instability (TI). Three forces that dominate the initiation of CMEs have been identified in the literature: (1) The repulsive force of the photospheric induced currents, (2) the Lorentz self (or "hoop") force arising from the loops curvature, and (3) the external Lorentz force. Both the repulsive and self forces are directed radially outwards from the Sun. These are in turn restored by the external Lorentz force between the current channel of the flux rope and the ambient magnetic fields. This configuration of the current channel with respect to the ambient magnetic field is known as the inverse polarity model. In the theoretical study presented, only the Lorentz self force and the restoring force were considered. The rate of change of the restoring force is constrained by the radial decay of the external magnetic fields, characterized by its decay index. Now, if the decay of the external magnetic fields is such that it is faster than some critical value, then, the outward Lorentz self force will dominate and the net force will be radially outwards. This study
reveals that the critical index is a function of the fractional number of the partial torus, defined by the ratio between the arc length of the partial torus above the photosphere and the circumference of a circular torus of equal radius. We refer to this finding as the partial torus instability (PTI).

Part of the future work will be to investigate the effect of inclusion of the repulsive force into the theoretical model. Here I will briefly discuss the general ideas of that work. In a previous study by Garren & Chen (1994), current loops of various geometries were investigated. It was assumed that the loop was partially embedded and was divided into two parts, above and below the photosphere. The Lorentz self force depends on the geometry of the loop of these two parts. It was found that regardless of how shallow the loop was above the surface, as long as the part below the surface was larger, a circular loop of equal radius to the part above the surface gave a good approximation to the Lorentz self force. In a particular example they show that if the loop above and below are both shallow such that the combined loop has an elliptical geometry then the circular approximation is not valid. This latter example is exactly what is required when considering the repulsive force, because mathematically this force is calculated by an image current below the photosphere. This will assure that the normal magnet field component does not change. In this case the loop below will be a mirror of the loop above. I propose to use the fundamental results of Garren & Chen (1994) to develop a new model of the initiation of CMEs that will incorporate this repulsive force. Furthermore, this new model will be compatible with the eruptive flux rope (EFR) model of Chen (1989, 1996), because as the loop expands and becomes large the repulsive force becomes negligible. When the loop expands far from the Sun its dynamics are dominated by the Lorentz self force and solar wind dragging. The EFR model has been shown to reproduce the kinematic behaviour of CMEs from the Sun to the Earth (e.g. Chen & Kunkel, 2010; Kunkel & Chen, 2010). The modification I propose will affect the dynamics only when the CME is near the solar surface and will shed new light on the issue of the initiation of CMEs.
(2) This observational study compared kinematics of prominences to their associated extrapolated magnetic fields. Two distinct events were examined that occurred on the 6 March 2010, and 1 August 2010. Both events were similar in that a prominence was observed to erupt and produce a CME. I used STEREO observations of the prominence to triangulate the 3D position of the apex height, and SOHO and SDO observations were used to measure the footpoint separation. A method for identifying the onset time and onset height was proposed. This is done by fitting the kinematics to an equation that assumes that the acceleration is a function of time. If this function is constant then the fit equation reduces to the well known second order kinematic equation. The function for the acceleration was assumed to be exponential, characterized by the decay constant $\tau$. To find the onset time, this equation is separated into a linear and exponential component. A formula was derived for the onset time that is defined as the time when the exponential part becomes equal to the linear part. It is at this point that the exponential part dominates the kinematic. By knowing the onset time the onset height is readily calculated. Next, I extrapolate, using the PFSS model, the magnetic fields of the polarity inversion line over which the filament lies. The decay index of these magnetic fields is calculated. I compare this result with those of the fit of the kinematics. It was found that (1) there is a link between the slope of the decay index of the magnetic fields and the decay constant $\tau$, and (2) there is a relationship between the decay index of the magnetic fields and the geometry of the loop at the time of onset, parametrized as the $h_{\text{onset}}/S_0$, the apex height at the time of onset over the half footpoint separation. The decay index at the time of onset is considered the critical index as defined by the theory. These results support the theory presented in (1) above in a qualitative way in that the general trend is reproduced. Namely that the critical decay index of the magnetic fields increases with increasing $h_{\text{onset}}/S_0$. Future work includes measurements of a larger sample of events to achieve a better statistical conclusion. Also, the future work outlined above for the modification of theoretical model can improve the quantitative agreement with observations.
Finally, I have developed a tool to automatically detect and characterize CMEs in coronagraph images. Coronagraph data is three dimensional, having two spatial, radial $r$ and angular $\theta$, and one temporal dimension. To date no detection algorithm takes advantage of all three dimensions simultaneously. Instead, every proposed algorithm will apply some transformation to two dimensions simultaneously then merge the third and get a complete picture of the CME. The solar eruptive detection system (SEEDS) algorithm, described in chapter 4, uses the two spatial dimension $[r, \theta]$, then, the temporal dimension to track it as it moves radially outward with a time-dependent causal filter. The algorithm characterizes CME events with their starting and ending times, position angle, angular width, and height. With these variables others can be deduced, such as velocity and acceleration. A full online catalogue of events was developed using LASCO C2 coronagraph data. Additionally, a near real-time module was developed to make detections of CMEs as soon as the data becomes available to us from the instrument operators. This module is currently working with LASCO C2, and STEREO beacon mode data sets. Typically, at most, there may be a two to three hour delay for the availability of the data.

Detecting CMEs in this dimensional space is motivated by the fact that a human operator would look at sequences of images and identify a CME frame by frame. The major challenge is the fact that CMEs appear as diffuse cloud like structures with no definite boundary. For this reason, different observers will identify CMEs slightly differently from each other. By observer, I am referring to either a human or computer. The computer can reduce human bias, though introduce a systematic bias. The reduction of human bias is advantageous. Take the CDAW catalogue for example. This catalogue has been compiled over the last decade by several operators and the visual criteria for each is slightly different from each other. This has lead to a detection rate that does not follow the trend of the sunspot cycle. As it is known that CMEs should, like other solar activity, follow this cycle. On the other hand because the systematic bias of the computer is consistent throughout, the detection rate produced by these methods much more closely follows the sunspot cycle. The use of an automatic CME detection system is helpful to humans because it can aid
in reducing the large amount of data and extract relevant information by characterizing CMEs. This can be further used by scientist in aiding them to search for events to study in more detail. Another important application is for space weather alerts. Since an automatic real-time system works constantly, whenever an event of substantial size is observed an alert could be sent out to the community.

The developed SEEDS algorithm is not perfect and future work is needed to improve the quality of the detections. On occasion the algorithm may miss events or split an event either spatially or temporally. Further research is need to improve the detection system.
Appendix A: Space Weather Effects on the Inhabitants of Earth

Solar activity can affect the livelihood of the inhabitants of Earth. In particular, the greatest effects to humans are to their technologically advanced systems that rely heavily on electronics, which are susceptible to the electromagnetic environment of the Earth and space. In addition, solar activity can endanger human life or health because of the high levels of radiation produced. This is particularly true for astronauts, who have the highest risk since they are not protected by the Earth’s atmosphere, and aircraft crews and passengers. The term "Space Weather" has been coined to describe the solar-terrestrial effects on humans and technological systems. The following quote by the committee for space weather, office of the federal coordinator for meteorological services and supporting research, best defines this term:

"Space weather refers to the conditions on the Sun and in the solar wind, magnetosphere, ionosphere, and thermosphere that can influence the performance and reliability of space-borne and ground-based technological systems and can endanger human life or health." (OFDM, 2000)

In the following sections a brief description of some of the effects of space weather will be discussed. The most notable effects to technological systems arise from induced currents and energetic particles. Although space weather effects are not physically felt like other catastrophic forces on the Earth such as hurricanes, volcanoes, or earthquakes, they can potentially be just as destructive, in their own way, to the way of life of humans. The more that humans rely on technological systems in the future, the more space weather effects will become perceptible. The most tremendous of these effects is the potential of space weather to bring down power grids over large areas of the Earth (as occurred in March 1989 in Quebec, Canada). This is dangerous to the livelihood of humans as we have become so dependent on electricity. Electricity is used to drive such services as water, oil and
gas, communications, transportation, emergency services, etc. all of which we depend on to survive. Understanding the causes and origins of space weather is vital to mitigating harmful effects. Considering that space weather is driven by activity on the Sun, it is important to study and understand the underlying physics of solar origin that drives it.

A.1 Geomagnetically Induced Currents

In the magnetosphere and ionosphere, the natural occurring currents can be disturbed during a geomagnetic storm. This disturbance will cause variations in the geomagnetic field and be observed as a geomagnetic storm on the Earth’s surface. The currents of the magnetosphere and ionosphere can in turn induce currents within the Earth and a geoelectric field. This geoelectric field can produce geomagnetically induced currents (GIC) through technological conductors that are laid out, and span, over a large area of the Earth. These systems include power transmission lines, oil and gas pipelines, telecommunication cables, and railway equipment. Any of these technological systems are vulnerable to the effects of GIC each of which is affected in a different way. In addition to these effects it is interesting to note that the geomagnetic field at the Earth’s surface can also affect humans, in particular the heart and cardiovascular systems (Breus et al., 2008).

The earliest effects of the GIC were observed in telegraph systems of the mid 1800’s (Barlow, 1849; Varley, 1873). The most intense of these was during the 1859 storm (Boteler, 2006), where telegraph systems world-wide were brought to a stand-still. It was reported that in some cases telegraph operators received electric shocks and that in other cases they were able to transmit signals even with the batteries disconnected. Although we may not have understood it at the time the commencement of this event came from the Sun, it was the first time humans experienced such a global event with such an origin.

One of the more dramatic effects that can occur is the loss of electrical power over large regions due to induced currents. In power transmission lines the problem arises when transformers, separated geographically on the Earth, have a potential difference between their grounds (see Boteler et al., 1998; Molinski, 2002, and references therein). This potential
difference will cause a DC current along the transmission lines and can saturate the transformers. Because of the high variability in the magnetosphere, ionosphere, and geoelectric field, the GIC will change polarity. The GIC is therefore considered to be a quasi-DC current with a very low frequency (1 Hz or less). The largest event in modern times occurred on 13 March 1989 causing a near collapse of the Hydro-Québec power system, Canada (Allen et al., 1989; Kappenman & Albertson, 1990). This affected 6 million residents of Québec, leaving them without power for 9 hours. The solar counterpart of this event began on 10 March 1989 with a gigantic solar flare and an eruption of material headed straight for the Earth. On 13 March 1989 spectacular auroras erupted that could be seen as far south as Florida and Texas and of course the disruptions, the Hydro-Québec power system being the most dramatic. Other disruptions also occurred in communication systems, and orbiting satellites.

Metallic oil and gas pipelines buried in the ground can experience corrosion and may be damaged due to induced currents (Camitz et al., 1997; Pirjola et al., 2000). The problem is not due to the GIC flowing along the pipeline but to the the pipe-to-soil voltage difference. Metallic pipes must be kept at a negative voltage with respect to Earth in order to prevent the chemical reactions that lead to rust and other corrosions. Keeping metallic pipes to a negative voltage can be achieved with a cathodic protection system. This protection system works by employing another metal to act as the anode with respect to the pipe hence assuring that the pipes remain the cathode. During increased levels of GIC the pipe-to-soil voltage can exceed the cathodic protection potential and cause an increase in the amount of corrosion. Over time, and after repeated events, the pipes can become permanently damaged and fail.

To railways, GIC can disturb the signaling and train control systems. The GIC can quickly over run susceptible equipment and cause a failure. The first reported case occurred in 1921 in which the entire signal and switching system of the New York Central Railroad failed, in addition to this a fire resulted in the control tower (Ptitsyna et al., 2008). Space weather effects on the railroad system are still for the most part unclear. In the recent study
of Ptitsyna et al. (2008), they compile a list of anomalous events (unstable functioning and failures) that occurred in the East-Siberian Railway system and state that it is unknown what type of anomaly is related to GIC. Though, these anomalies often times lead to train delay and can significantly hamper railway traffic. After a statistical analysis between the anomalous events and geomagnetic activity it was found that during intense geomagnetic storms the daily duration of anomalies increased by a factor of 3.

A.2 Solar Radiation and High Energy Particles

High energy ions and electrons in space can affect the operation of sensitive electronic equipment onboard satellites. In addition these particles can be hazardous to humans operating at high altitudes above the Earth and in orbit. These high energy particles can originate from outside the solar system, known as galactic cosmic rays (GCRs), or from Sun activity. These particles can have energies up to \( \sim 1\text{GeV} \), with corresponding velocities close to the speed of light. During a solar event, solar energetic particles (SEPs) can be produced. These can originate from the energization at a solar flare site on the Sun’s surface or from propagating shock waves driven by CMEs. SEPs, upon arrival to the Earth, can penetrate the magnetosphere and become trapped in the radiation belts. This of course can pose problems to any operation in the vicinity of that region. It is therefore important to understand the fundamental physics that underlie these types of events in order to better predict there occurrence.

Both ions and electrons can cause faults and damage to spaceborne electronics. When an ion interacts with electronic equipment it can be the case that it will deposit enough charge to change the state of the system and cause a fault, such an event is known as single-event upsets (SEUs) (Robinson, 1989). These types of events can cause the satellite to operate in unexpected and undesirable ways. It may also be the case that a major SEU will cause the entire satellite to fail. It is therefore important that engineers take this risk into account when designing space systems. A notable effect that ions have are the interruption to imaging systems in space. For example, the CCD (charged-couple device)
detector used onboard the SOHO spacecraft are degraded by contamination of heavy ions from cosmic/solar particle radiation (Brekke et al., 2004). Ions can also cause immediate and major loss of seeing by contamination of what looks like noise when ions interfere with the CCD detector. Such an event may be caused by a solar flare and the energization of ions to relativistic velocities that reach the detector tens of minutes after the flare. Figure A.1 is an example of the effects of these types of events on the extreme ultraviolet imaging telescope (EIT) onboard SOHO. These two images were taken one hour apart with the flare occurring some time in between. In some cases events are so overwhelming that not even the outline of the solar disk can be made out. These types of events do not permanently damage the CCD as the image is recovered once the event is over.

The Sun produces enormous amounts of electromagnetic radiation that span from radio waves to gamma-rays. During increased levels of radiation, in particular during solar flares, or SEP and geomagnetic storms, the ionosphere can experience an enhanced amount of ionization that may cause errors and failures in communication systems. High frequency (HF) and very high frequencies (VHF) communications, and global positioning systems

Figure A.1: Flare effects on spaceborne sensors. Images taken by the EIT instrument onboard SOHO show dramatic degradation due to a flare. The images are only separated in time by one hour.
(GPS) can be affected by such changes in the ionosphere (Cannon et al., 2004). HF and VHF communication systems are used to communicate over long distances by making use of the ionosphere to reflect radio signals. The ionosphere essentially has two regions of importance to HF and VHF operators that are most affected by space weather. The lower ionosphere can attenuate signals, whereas the higher ionosphere is the layer used to reflect signals. During an event such as a solar flare, increased amount of ionization can occur in the ionosphere from the extreme ultra-violet and X-ray radiation produced. During these time of increased ionization, the attenuation of the lower atmosphere can be enhanced and the reflective properties of the upper ionosphere augmented. The attenuation is of most concern because this can prevent operators from using this type of communication over long distances since the signals will not be able to penetrate the lower ionosphere. It should be noted that the threat of radiation for solar flares is only of concern to the day-side of the Earth since the night-side is protected. Another concern are SEPs that travel along the magnetic field lines to the polar regions. These particles can cause increased amounts of ionization and hence an increased amount of attenuation to signals in the lower ionosphere. These types of events are called polar cap absorption (PCA) events for obvious reasons. PCA events are of most concern to humans operating in the polar region, in particular to airline flights travelling over the north pole (Fisher & Jones, 2007), not only because of the possible loss of communication from lower ionosphere attenuation to HF and VHF signals, but because of increasing levels of high energy particles at high altitudes from GCR sources and during SEP events. These particles are a concern to humans at the altitudes that planes fly because of increased doses of radiation received. Although there seems to be a controversy as to what the risk is in developing cancer as a result of cosmic radiation accumulated over a flying career (Fisher & Jones, 2007), the stance that every radiation exposure will have an effect is usually taken.
Appendix B: Flux Rope Model

The flux rope model described in this section has seen much debate over the last two decades since its conception by James Chen. It describes the initiation and propagation of CME by modelling macroscale forces that would act upon a magnetic toroidal loop (Chen, 1989; Garren & Chen, 1994; Chen, 1996; Krall et al., 2000). The macroscale forces that are involved in this process include gravity or buoyancy, Lorenz, thermal pressure, and drag

\[ \rho \frac{dV}{dt} = J \times B - \nabla P - \rho g - \frac{1}{2} c d \rho V^2. \]  

This model has typically been classified as a "driven" or "injection" type (e.g. Forbes, 2000; Klimchuk, 2001; Lin et al., 2003). This classification is referring to the driving mechanism of the model which is the dynamic increase in the poloidal flux \( \Phi_p \). This increase will cause the loop to rise. Furthermore, increasing the flux causes the magnetic flux between the loop and the photospheric surface to increase. Hence why this model is refereed to as "injection" type. When first proposed it was thought that this flux injection originated from the photosphere (Chen, 1989) it is now thought that its origins may lie in the corona (Vršnak, 2008) and may be related to the flare energy release (Chen & Kunkel, 2010).

Recently, physical aspects of this model have been interpreted as being resemblant of the torus instability (Krall et al., 2000; Mittal & Narain, 2010; Olmedo & Zhang, 2010), which is the loss in equilibrium when the ambient magnetic field drops quicker than the outward Lorentz force of the loop.

This loop model consists of two basic current system, one flowing along the torus in the toroidal direction and the second flowing in the poloidal direction that circles the torus (see figure B.1). An external magnetic field is prescribed and influences the equilibrium of the flux rope by exerting a force opposite in the direction to the internal Lorenz self-force. Although an equilibrium condition can be achieved without an external magnetic field (Xue & Chen, 1983), the external field is thought to originate from overlying coronal...
Figure B.1: Flux rope topology of the model current loop. The subscripts 't' and 'p' refer to toroidal and poloidal directions. See text for details on the physical variable depicted. (From: Chen (1996))
Figure B.2: Loop geometry depicting the formulaic evolution of the apex. (From: Anzer (1978))
loops whose foot points may be anchored in an active region. Four assumptions are made in the derivation of this model (1) The major radius is much larger than the minor radius \( R >> a \). (2) A local curvature assumption is made at the apex. (3) The footpoints are fixed. (4) The major radius is prescribed as a function of height and foot point separation \( S_f = 2S_0 \), where \( S_0 \) is the half foot point separation

\[
R = \frac{Z^2 + S_0^2}{2Z}, \tag{B.2}
\]

\( S_f \) is the foot point separation and \( Z \) the apex height of the flux rope. This kind of evolutionary model for the radius of curvature at the apex had previously been proposed by Anzer (1978). Figure B.2 illustrates this equation. In the notation of this figure \( r \) corresponds to \( R \) of equation B.2, \( R \) of this figure is related to \( Z \) by \( R = Z + R_\odot \), and \( r_0 \) corresponds to \( S_0 \). Equation B.2 has the property that it must first decrease until the loop is semicircular then begin to increase leading to \( dR/dZ = 0 \) when \( Z/S_0 = 1 \) (Chen & Krall, 2003).

The mathematical derivation of this model begins with considering a torus of major radius \( R \) and minor radius \( a \). Two current systems are applied, one considering the current flowing along the tube \( J_t \) (toroidal), and the second following in a solenoid fashion around the tube \( J_p \) (poloidal). An external magnetic field \( B_s \) is applied perpendicular to the plane of the torus. Also since this system describes a magnetized plasma, thermal pressure must also be considered. Following Shafranov (1966) the derivation of the forces on the system begins with finding the total energy

\[
Q = Q_t^B + Q_p^B + Q^P \tag{B.3}
\]

where \( Q_t^B \) is the magnetic energy due to the toroidal current system, \( Q_p^B \) is the magnetic energy due to the poloidal current system, and \( Q_P \) is the thermal energy. The forces on the
two axis of this system, $R$ vector and $a$ vector, are calculated from this energy

$$F_a = \frac{dQ}{da},$$  \hfill (B.4)

and

$$F_R = \frac{dQ}{dR}.$$  \hfill (B.5)

There are essentially five forces acting on the system, $J \times B$ Lorentz self-force, $J \times B_s$ Lorentz force from the external magnetic field, gravity, drag, and pressure force. In order to make the derivation analytically possible a major assumption is made that $R \gg a$. The Lorentz self-force is written as $\vec{J} \times \vec{B} = J_t \times B_p + J_p \times B_t$ and is a vector with components in the major and minor radial directions. The Lorentz self-forces in the major radial direction arising from the two current components written as follows,

$$F_s = \frac{I_t^2}{c^2 R} \left[ \ln \left( \frac{8R}{a} \right) - \frac{1}{2} \frac{B_t^2}{B_p^2} - 1 + \frac{\xi_i}{2} \right]$$  \hfill (B.6)

This describes the Lorentz self-force, also known as the hoop force, where $B_t$ is the toroidal magnetic field component, $B_p$ is the poloidal magnetic field on the surface of the torus system, $\xi_i$ is the internal inductance per unit length, and $c$ the speed of light. The following describes the Lorentz force between the toroidal current and the external magnetic field $I_t \times B_s$,

$$F_x = \frac{2 I_t^2}{c^2 R} \left( \frac{R}{a} \right) \frac{B_s}{B_p}$$  \hfill (B.7)

An alternative way to calculate these forces would be to calculate $J \times B$ directly and is mathematically equivalent to energy approach taken here (Isenberg & Forbes, 2007). Finally the thermal pressure gradient force between the flux rope and the ambient environment is
written as,

\[ F_p = \frac{I_t^2}{2c^2R} \{ 8\pi \bar{p} - p_a \frac{B_t^2}{B_p^2} \}, \]  

(B.8)

where \( \bar{p} \) is the average pressure inside the loop, and \( p_a \) is the ambient coronal pressure. A parameter \( \beta_p \) is defined to simplify the equation, where \( \beta_p = \left[ 8\pi (\bar{p} - p_a) / B_p^2 \right] \). The remaining forces, gravity \( F_g \), and drag \( F_d \), are explicitly specified, and will be described later. Summing all of the forces in the major and radial direction yields the following equation of the force per unit length at the apex \( F_R = F_s + F_x + F_p + F_g + F_d \)

\[ F_R = M \frac{dV}{dt} = \frac{I_t^2}{c^2R} \left[ \ln \left( \frac{8R}{a} \right) + \frac{1}{2} \beta_p - \frac{1}{2} \frac{B_t^2 - B_\text{et}^2}{B_p^2} + 2 \frac{R}{a} \frac{B_t}{B_p} - 1 + \frac{\xi_i}{2} \right] + F_g + F_d \]  

(B.9)

where \( V \) is the propagation velocity at the apex. This force is responsible for the propagation of the CME.

The forces in the minor radial direction are expressed as follows:

\[ F_s^a = \frac{I_t^2}{c^2a} \left( \frac{B_t^2}{B_p^2} - 1 \right) \]  

(B.10)

is the Lorenz self-force in the minor radial direction, also known as the pinch force.

\[ F_p^a = \frac{I_t^2}{c^2a} \beta_p \]  

(B.11)

is the pressure force in the minor radial direction. The final force in the minor radial direction is a drag force term \( F_{dl} \). This term takes into account pressures that are due to laminar/turbulent flows around the CME, it is thought that this drag term is smaller than the drag term in \( F_R \). The functional for \( F_{dl} \) has been suggested to be similar to \( F_d \) (Chen, 1996). Putting all of these terms gives us the force equation in the minor radial direction.
\[ F_a = F_s^a + F_p^a + F_{dl} \]

\[ F_a = M \frac{dw}{dt} = \frac{I_t^2}{c^2 a} \left( \frac{B_t^2}{B_p^2} - 1 + \beta_p \right) + F_{dl} \]  \hspace{1cm} (B.12)

where \( w \) is the expansion rate. This equation describes the expansion in the minor radius.

The next topics of discussion are the gravitational, and drag forces. The gravitational force per unit length is given as

\[ F_g = \pi a^2 m_i g(Z)(n_a - \bar{n}) \]  \hspace{1cm} (B.13)

where \( m_i \) is the ion mass, \( n_a \) is the ambient solar wind density, and \( \bar{n}_T = \bar{n}_c + \bar{n}_p \) is the total density of the loop where subscript \( c \) refers to cavity and \( p \) to prominence. The total gravitational acceleration is given by

\[ g(z) = \frac{g_s}{(1 + Z/R_\odot)^2} \]  \hspace{1cm} (B.14)

where \( g_s = 2.74 \times 10^4 \text{cm/s}^2 \). It has been estimated that the temperature for quiescent prominences is less than the temperature of the cavity \( T_p \ll T_c \) by at least two orders of magnitude (Chen, 1996). The pressure is calculated as \( \bar{p} = 2 \bar{n}_c k \bar{T}_c \), where \( k \) is the Boltzmann constant, and \( \bar{T}_c \) is the averaged cavity temperature. Within the loop the equation of state is given by

\[ \frac{d}{dt} \left( \frac{\bar{p}}{\bar{\rho}^\gamma} \right) = 0, \]  \hspace{1cm} (B.15)

where \( \bar{p} = \bar{n}_c m_i \) is the average mass density within the loop, and \( \gamma \) is the adiabatic index \( (1 \leq \gamma \leq 5/3) \). Chen (1996) uses a value of 1.2, which is an assertion that the parallel thermal conductivity is high. The equation for the pressure becomes \( \bar{p} = \text{const} \bar{\rho}^\gamma \), the
constant is found with the initial equilibrium values and is equal to

\[ \text{const} = \frac{2kT_c}{n_c^{\gamma-1} m_i}, \]  

(B.16)

The explicit equation of the drag force \( F_d \) is given by

\[ F_d = 2c_d a \rho_a \left| V_{sw} - (V + 2w) \right| \left| V_{sw} - (V + 2w) \right|, \]  

(B.17)

where \( c_d \) is the drag coefficient. The reason for the \((V + 2w)\) term is because it has been found that the effective radius is not \( a \) but in fact \( 2a \), a characteristic determined empirically stemming from many model-data comparisons (Krall et al., 2006; Chen & Kunkel, 2010). The effective leading edge is \( Z(t)+2a(t) \), the velocity is therefore \( V(t)+2a(t) \). The minor radius drag force term \( F_{dl} \) will have the same functional form as equation B.17, except that it will have its own drag coefficient \( c_{dl} \). It is thought that this drag coefficient is smaller than that of the major radial direction \((c_{dl} < c_d)\). This is because as the minor radius is expanding equally in all directions flow is incoming only along one direction and flows around the minor radius. So \( c_{dl} \) is a kind of average due to laminar/turbulent flow pressures around the minor radius.

The toroidal current is related to the poloidal flux by

\[ I_t = \frac{\Phi_p}{cL_p}, \]  

(B.18)

where \( \Phi_p \) is the poloidal magnetic flux, and \( L_p \) is the inductance. This is the expression that is used to dynamically evolve \( I_t(t) \). It is known that \( L_p \) for a torus is given by (see e.g. Shafranov, 1966; Miyamoto, 1976)

\[ L_p = \frac{4\pi R}{c^2} \left\{ \ln \left( \frac{8R}{a} \right) - 2 + \frac{\xi_i}{2} \right\}, \]  

(B.19)
which is derived from the fundamental equation for inductance

\[ L = \frac{1}{I^2} \int J \cdot A \, dV, \quad (B.20) \]

where \( J \) is the current density, \( A \) the magnetic vector potential, and the integral is over the volume \( dV \). For the torus case the azimuthal angle integration is taken over \( 2\pi \). In the flux rope case this no longer can hold since only a partial torus is considered, therefore the azimuthal integration must be taken to be then angular extent of the flux rope. Secondly, when the flux rope is taken out of equilibrium the minor radial expansion is asymmetric between the footpoints and the apex. It is assumed that the minor radius at the footpoints \( a_f \) remains fixed or constant. Equation B.12 therefore models the evolution of the minor radius at the apex \( a_\alpha \). The tapering between the apex and the footpoints must be modelled.

Two such models have been proposed, an exponential, and a linear. In the exponential case the minor radius takes the form \( a(\theta) = a_f \exp[s(\theta - \theta_f)] \) (Chen & Garren, 1993; Garren & Chen, 1994; Chen, 1996), where

\[ \theta = \sin^{-1} \left( \frac{S_0}{R} \right), \quad (B.21) \]

and \( \theta_f \) is the angle subtending from one footpoint to the apex. With this expression for \( a \), \( L_p \) is found to be

\[ L_p = \frac{4\pi \Theta R}{c^2} \left[ \frac{1}{2} \ln \left( \frac{8R}{a_f} \right) + \ln \left( \frac{8R}{a_\alpha} \right) - 2 + \frac{\xi_i}{2} \right], \quad (B.22) \]

where \( \Theta \) is the fractional angular extent of the flux rope above the photosphere, and is given by

\[ \Theta = \begin{cases} 
1 - \theta/\pi, & Z \geq S_0 \\
\theta/\pi, & Z < S_0 
\end{cases} \quad (B.23) \]
In the linear case the minor radius takes on the form $a(\theta) = m(\theta - \theta_f) + a_f$ (Krall et al., 2000). Deriving $L_p$ using this expression for $a$ gives

$$L_p = \frac{4\pi \Theta R}{c^2} \left[ \ln(8R) - 1 + \frac{\xi_i}{2} - \frac{[a_a \ln(a_a) - a_f \ln(a_f)]}{(a_a - a_f)} \right]$$

(B.24)

The initial equilibrium of the flux rope is defined by setting the forces $F_R$ and $F_a$ equal to zero. This condition leads to the following expression for the poloidal magnetic field

$$B_{p0} = B_{s0} \pm \left[ B_{s0}^2 - \frac{(a_0^2/R_0^2)\Lambda_0}{2} \left( 8\pi(\bar{\rho} - p_a) + 4\pi R_0 m_t g_0 (n_a - \bar{n}_T) \right) \right]^{1/2}$$

(B.25)

where the 0 subscript refers to the initial values, $g_0 = g(Z_0)$, $B_{s0} = B_s(Z_0)$, and

$$\Lambda_0 = \ln \left( \frac{8R_0}{a_0} - 1.5 + \frac{\xi_i}{2} \right)$$

(B.26)

With equation B.25 the initial equilibrium toroidal current is calculated by

$$I_t = \frac{ca B_{p0}}{2}$$

(B.27)

and comes from the surface current model that does not take into account the poloidal field component within the loop. The toroidal magnetic field can be calculated from the minor radius equilibrium condition ($F_a = 0$), and $B_{p0}$

$$B_t = \sqrt{B_{p0}^2 - 8\pi(\bar{\rho} - p_a)}$$

(B.28)
The $B_t$ is related to the toroidal flux $\Phi_t$ as,

$$B_t = \frac{\Phi_t}{\pi a^2}. \quad (B.29)$$

Since $\Phi_t$ is assumed to be conserved, $B_t$ can be written as a function of $a_a$ and $a_f$ as,

$$B_t = B_{t0}a_f^2/A_a^2, \text{ where } B_{t0} = B_t(Z_0).$$

In order for the flux rope to go out of equilibrium a poloidal flux is injected into the system. This flux could be thought of as coming up from the photosphere or have coronal origin. In the model, a functional form for the rate of flux injection ($d\Phi_p/dt$) is specified, it basically has three parts, 1. a ramp up, 2. a plateau, and 3. a ramp down and has the effect of increasing the total poloidal flux. Hence, during the time period of injection the poloidal flux is not conserved. Depending on time constants and maximum rate of injection for each of the parts the dynamics of the resulting evolution will change. The following is the functional form of the flux injection profile as proposed by Chen & Kunkel (2010)

$$\frac{d\Phi_p(t)}{dt} = \begin{cases} 
Q_0 \equiv \left(\frac{d\Phi_p}{dt}\right)_{t=0}, & 0 \leq t \leq t_1 \\
Q_0 + Q_1\left[\text{sech}^2\left(\frac{t-t_1}{\tau_1}\right) - \text{sech}^2\left(\frac{t_1-t_2}{\tau_1}\right)\right], & t_1 < t < t_2 \\
Q_0 + Q_1 \left[1 - \text{sech}^2\left(\frac{t_1-t_2}{\tau_1}\right)\right] \equiv \left(\frac{\Phi_p}{dt}\right)_{\text{max}}, & t_2 \leq t \leq t_3 \\
\left(\frac{\Phi_p}{dt}\right)_{t=t_3}\text{sech}^2\left(\frac{t-t_2}{\tau_2}\right), & t_3 < t
\end{cases} \quad (B.30)$$

where $\tau_{1,2,3}$, and $Q_{0,1}$ are constants. Typically $Q_0$ is taken to be zero, but could be a small term that could drive the system for $t \leq t_1$.

In the early stages of the eruption it is thought that some or all of the prominence material falls back to the sun. This is modelled and parametrized by the density term of
the prominence material $\bar{n}_p$ with a time dependence

$$
\bar{n}_p = \begin{cases} 
\bar{n}_{p0}, & t_1 \leq t \leq t^* \\
\bar{n}_{p0} \left[ \text{sech}^2 \left( \frac{t-t^*}{\tau_p} \right) + \delta_p \right], & t > t^* 
\end{cases} 
$$

(B.31)

where $\bar{n}_{p0}$ is the initial prominence density, $t^*$ is the time delay before prominence material begins to fall, $\delta_p$ is the percentage of material that does not fall back and continues to propagate with the flux rope. Finally $\tau_p$ is the time scale of the free fall time at approximately 2 solar radii ($\tau_p \simeq 150\text{min}$).

### B.1 The Ambient Variables

The next important step is to model ambient coronal properties such as the solar wind speed, coronal density, and temperature. The following is the model coronal density profile

$$
n_a(Z) = 4(3R_s^{-12} + R_s^{-4}) \times 10^8 + 3.5 \times 10^5 R_s^{-2} 
$$

(B.32)

where $R_s \equiv Z + R_\odot$. The solar wind speed is modelled as a hyperbolic tangent curve,

$$
V_a = V_{a0} \frac{\tanh \left( \frac{Z-Z_0}{R_\odot} - \sigma_1/\sigma_2 \right) - \tanh \left( -\sigma_1/\sigma_2 \right)}{1 - \tanh \left( -\sigma_1/\sigma_2 \right)}, 
$$

(B.33)

where $\sigma_{1,2}$ are constants, and $V_{a0}$ is the asymptotic solar wind speed far from the surface (> $40R_\odot$). We have found $\sigma_1 = 8$, and $\sigma_2 = 20$ to be appropriate values. The temperature follows a two-part curve, where the temperature is taken to be constant bellow $6R_\odot$, and above this it follows $T_a(Z) = T_0 R_s^{-\alpha}$, where $\alpha$ is taken to be 0.9 within 1AU. The final aspect of the model is ambient coronal magnetic field. Chen (1996) has proposed that a
magnetic field profile that first increase, then decreases with magnitude,

\[
B_s(Z) = \begin{cases} 
B_{s0}\text{sech}^2((Z - Z_*)/h_1), & Z < Z_* \\
B_{s0}\text{sech}^2((Z - Z_*)/h_2), & Z > Z_* 
\end{cases}
\]

(B.34)

where \( Z_* \) and \( h_{1,2} \) are scale length of the fields. This magnetic field is perpendicular to the toroidal magnetic field \( B_t \).

### B.2 4th Order Runge-Kutta

To solve the equations of motion of the flux rope model described a Runge-Kutta 4th order method was implemented. Runge-Kutta is an iterative method to numerically find solutions for ordinary differential equations. Take \( f(t, y) \) to be a a set of ordinary differential equations to solve, where \( t \) is time, and \( y \) is the variable/s for which were are solving. The following is the Runge-Kutta algorithm, where \( h \) is the time step:

\[
\begin{align*}
k_1 &= hf(t_i, y_i) \\
k_2 &= hf(t_i + h/2, y_i + hk_1/2) \\
k_3 &= hf(t_i + h/2, y_i + hk_2/2) \\
k_4 &= hf(t_i + h, y_i + hk_3) \\
y_{i+1} &= y_i + \frac{h}{6} (k_1 + 2k_2 + 2k_3 + k_4) \\
t_{i+1} &= t_i + h
\end{align*}
\]

(B.35)

In order to initialize the algorithm the value \( y_0 \) at step \( i = 0 \) must be specified. The equations of motion described in this appendix are written as a set of first order ODE’s and solved with the 4th order Runge-Kutta algorithm:
\[
\begin{align*}
M \frac{dV}{dt} &= \frac{I^2}{c^2 R} \left[ \ln \left( \frac{sR}{a} \right) + \frac{1}{2} \beta_p - \frac{B^2}{2 B_{pa}} + 2 \left( \frac{R}{a} \right) \frac{B_z}{B_{pa}} - 1 + \xi \right] + F_g + F_d \\
\frac{da}{dt} &= w \\
M \frac{dw}{dt} &= \frac{I^2}{c^2 a} \left( \frac{B^2}{B_{pa}^2} - 1 + \beta_p \right) + F_{dl}
\end{align*}
\]
Appendix C: Morphological Operators

The morphological operators are based on a set-theory approach to digital signal and image analysis based on shapes. These operators can be extended to any dimension and are typically used to reduce a signal or image by removing any irrelevant information (such as noise). With these operators one can extract useful information from an image that can be used to describe and represent the shape of an object of interest. The two basic operators upon which other are built are erosion and dilation. Define an input signal or image \( A \) and structuring element \( B \) of arbitrary shape for which \( A \) will be compared with. In the most simple case the input \( A \) will be binary with '1' representing foreground pixels, and '0' representing background pixels. \( A \) and \( B \) are in Euclidean space, \( X \in \mathbb{R}^n \), where \( n = 1 \) is a 1D array and \( n = 2 \) a 2D image. Dilation is denoted as \( A \oplus B \) and is an operator that will essentially expand an image \( A \), it is defined as

\[
A \oplus B = \{ x | (\hat{B})_x \cap A \neq \emptyset \} \quad (C.1)
\]

where \( \hat{B} \) is the reflection of set \( B \)

\[
\hat{B} = \{ w | w = -b, b \in B \} \quad (C.2)
\]

Erosion is denoted as \( A \ominus B \) and is an operator that will essentially shrink an image \( A \), it is defined as

\[
A \ominus B = \{ x | (B)_x \subset A \} \quad (C.3)
\]

Combining these two operators, two others can be defined as opening and closing. The opening operator is denoted as \( A \circ B \) and is the erosion of \( A \) by \( B \) followed by the dilation with \( B \). This operator can be used for removing irrelevant pixels in \( X \) whose dimensions are smaller than the structure \( B \) by not satisfying the erosion condition, such as sparse noise pixels. Its effect will also smooth the image and break down or eliminate narrow gaps.
between objects within an image. The opening operator is defined as

$$A \circ B = (A \ominus B) \oplus B$$  \hfill (C.4)

The closing operator is denoted as $A \bullet B$ and is the dilation of A by B followed by the erosion with B. This operator will smooth an image and can be used to fuse narrow gaps between objects, fill small holes, and fill gaps in contours. The closing operator is defined as

$$A \bullet B = (A \oplus B) \ominus B$$  \hfill (C.5)
Appendix D: Scientific Acknowledgements

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