PROPORTIONAL REASONING WORD PROBLEM PERFORMANCE FOR MIDDLE SCHOOL STUDENTS WITH HIGH-INCIDENCE DISABILITIES (HID)

by

Anne Eichorn Brawand
A Dissertation
Submitted to the Graduate Faculty of George Mason University in Partial Fulfillment of The Requirements for the Degree of Doctor of Philosophy Education

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George Mason University
Fairfax, VA
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Summer Semester 2013
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DEDICATION

This dissertation is dedicated to my amazing husband Brian, whose consistent support and love has made this whole adventure possible.
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ABSTRACT

PROPORTIONAL REASONING WORD PROBLEM PERFORMANCE FOR MIDDLE SCHOOL STUDENTS WITH HIGH-INCIDENCE DISABILITIES

Anne Eichorn Brawand, PhD
George Mason University, 2013
Dissertation Director: Dr. Margaret King-Sears

Schema-based instruction (SBI) was used to examine the solving of proportional reasoning word problems for middle school students with high-incidence disabilities (HID). Seventh- and eighth-grade students with HID participated in the study. Students were randomly assigned to one of three groups. A multiple-baseline-across-groups design was utilized, which included baseline and staggered treatment phases. The SBI process consisted of nine lessons for all groups, lasting 35 minutes each, for 2-3 days per week.

The SBI process consisted of three phases. Phase I was baseline when data were gathered about students’ current level of performance solving proportional reasoning word problems. After Phase I, students in the three groups were provided with an overview on schema instruction using proportional reasoning problems that were already solved (worked problems). During the overview, schema instruction was introduced using already worked problems to identify and represent information in story problems.
on a diagram. In Phase II (SBI Process), students needed to enter information from the word problem into a diagram. Students also had to determine missing numbers needed to calculate an answer, write a number sentence, and solve word problems in this phase. In Phase II the students received explicit instruction on how to use the SBI process to solve proportional reasoning word problems. Teaching of the SBI process also incorporated FOPS, a four-step problem-solving strategy. A FOPS checklist was referred to as the following steps were introduced and explained: F- Find out if it is a proportional problem, O-Organize the information in the problem using the diagram, P-Plan to solve the problem, and S- Solve the problem. Finally, Phase III measured whether the students maintained the number of categories completed correctly in the SBI process 4 weeks after the SBI intervention.

The three proportional reasoning types (unit rates, proportions, and scale drawings) were taught separately and had different difficulty levels of computation (novice, intermediate, and advanced). Data were analyzed separately for all groups, both with and without a calculator. The levels of analysis included overall improvement with the SBI process to solve proportional reasoning word problems, as well as across groups.

Performance was measured using a five-category rubric for the Proportional Reasoning Word Problem Probes. The five categories were drawing shapes for the diagram, labeling, knowing the type of problem, writing a number sentence, and having the correct answer. Probes had three problems on the front, which students solved without a calculator, and three problems on the back, for which students could use a calculator.
On the probes without a calculator, all groups increased to an overall level of 64.2%, $SD = 16.0$ (from 3.8% in baseline); with a calculator, the increase for all groups was to a level of 53.9%, $SD = 19.6$, (from 1.6% in baseline). All groups (both with and without a calculator) had a flat trend in baseline and demonstrated an increasing trend at different times per group within Phase II. The overall variability in baseline was low as compared to the levels, and students’ performance in Phase II also demonstrated low variability as compared to the accelerating trends. All groups had a slow immediacy of change when the SBI process was introduced and minimal overlap of data points. In Phase II, the data points were consistently higher than in baseline. Results indicated that all groups of students learned the first three categories of the SBI process but did not learn the last two categories for writing a number sentence and having the correct answer. In Phase III, students also maintained learning for the first three categories with increases for two categories (shapes and labeling) four weeks after instruction.
1. INTRODUCTION

Deficits in mathematics for low-performing students tend to persist and increase over time (Nelson, Benner, Lane, & Smith, 2004). For example, Rao and Mallow (2009) expressed concern that a lack of basic mathematical knowledge is a common obstacle to learning higher-level mathematics for all students, including those with disabilities, and that students with a deficiency in basic mathematical skills may have difficulty learning mathematical computation and higher-order mathematics. The National Council of Teachers of Mathematics (NCTM, 2000) has called for standards that range beyond skills of basic procedural competency, placing heavy value on the teaching of problem solving to students in Grades K through 12. High-stakes standardized tests like the National Assessment of Educational Progress (NAEP) also place a large emphasis on word problems (National Center for Education Statistics, 2009). The current mathematical standards largely affect students with disabilities in the area of problem solving instruction because some students with disabilities have more difficulty recalling information and remembering strategies. The current study examined the functional relation between the schema-based instruction (SBI) process and solving proportional reasoning word problems for middle school students with learning disabilities (LD).
Statement of Problem

The No Child Left Behind Act of 2001 (NCLB) put in place accountability measures including annual assessment of students, and one of the areas of assessment is mathematics. These annual assessments include students with and without LD solving word problems in mathematics at all levels. Results from the NAEP (2011) mathematics assessment indicated that 73% of students had at least a basic knowledge of eighth-grade mathematics in 2011 compared to 52% of students in 1990. The NAEP mathematics assessment is designed to measure what students know and can do across the following five mathematical content areas: number properties and operations, measurement, geometry, data analysis/statistics/probability, and algebra. Although these percentages indicate growth, “basic” performance indicated only a partial mastery of fundamental skills.

Conversely, proficiency in mathematics, such as knowing how to reason and problem-solve, is also necessary because it is critical to functioning adequately in the context of daily life situations (Jitendra, George, Sheetal, & Price, 2010). The National Mathematics Advisory Panel (2008) reported that mathematical problem solving is one of the most important concepts taught in the grade-level curriculum. Targeted instruction in word problems is emphasized as well as a recommendation for explicit instruction when teaching word problems for students with LD. Explicit instruction involves the presentation of new information in small steps, the use of examples and nonexamples, and student practice until mastery is achieved (Miller & Hudson, 2007). Legislation and state testing requirements such as NCLB’s state assessments (2001) and the NAEP
(2011) focus more on a conceptual understanding of mathematical concepts, skills, and relations of numbers than mathematical instruction from memorization of mathematical procedures (Jitendra, 2007). This focus a conceptual understanding has implications for the mathematical problem-solving instruction for students with learning disabilities (LD) due to some of their characteristics.

Many students with learning disabilities (LD) exhibit difficulties in the area of memory and general strategy use, literacy and communication, specific processes and strategies associated with mathematical problems, and low motivation and affect (Bryant & Bryant, 2008). Deficits in memory are connected to the process of problem solving in mathematics. According to a literature review of mathematical interventions at the secondary level, only 23 studies were found that examined middle school interventions in mathematics for students with LD (Maccini, Mulcahy, & Wilson, 2007). Additionally, only seven of the 23 studies were in the domain of problem-solving interventions for students with LD. Focus on mathematical problem-solving instruction for students with LD is critical to providing access to the general education curriculum (Individuals with Disabilities Education Act [IDEA], 1997), preparing for annual assessments (NCLB, 2002), and having the mathematical skills ready to function in the real world (i.e., problem solving and reasoning).

**Background of Problem**

Improving the problem-solving skills of some students with learning disabilities (LD) is challenging due to deficits in attention, memory, background knowledge, vocabulary, language processes, strategy knowledge and use, and visual spatial
processing (Geary, 2003). The next section includes a description of how the characteristics of students with LD affect their mathematical problem-solving skills. Research on math instruction for students with LD will also be discussed.

**Students with Learning Disabilities (LD) and Mathematical Problem Solving**

Characteristics of students with a LD (i.e., problems with memory and language use) contribute to these students’ having difficulty in mathematics. Basic mathematical skills require memory, and problem solving also requires use of language and knowledge of terms. According to IDEA (2004), a specific learning disability is defined as a disorder in one or more of the basic psychological processes involved in understanding or in using language, spoken or written, that may manifest itself in an imperfect ability to listen, think, speak, read, write, spell, or to do mathematical calculations, including conditions such as perceptual disabilities, brain injury, minimal brain dysfunction, dyslexia, and developmental aphasia. (Section 300.7[c][10])

Mathematical problem-solving requires a combination of prerequisite skills that secondary students with LD are likely to lack because the content gets more difficult at that level, terms are less familiar, and teaching is more abstract (Hutchinson, 1993; Xin, Jitendra, & Deatline-Buchman, 2005). Word problems are also more difficult for some students with LD because of the numerous steps and skills necessary to solve them (Jitendra et al., 2002; Krawec, Huang, Montague, Kressler, & Melia de Alba, 2013; Parmar, Cawley, & Frazita, 1996).
Students are commonly taught to solve word problems by following a process or series of steps (Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Fuchs et al., 2004; Hutchinson, 1993; Maccini & Hughes, 2000; Montague, Enders, & Dietz, 2011). For example, in solving a word problem the first step is usually reading the problem and identifying the question. Some students with LD, however, may struggle with comprehending text in the word problem, understanding the meaning of any new terms, and distinguishing relevant from irrelevant information. Any of these issues may prohibit the student from continuing past that first step because they do not understand the problem or the terms or what the problem is asking them to do. The second step usually involves finding the necessary information to solve the problem, and for some students with LD any extra information in a word problem can be distracting. That is, students with LD are not sure where to focus. Moreover, students may not know how to distinguish the relevant from irrelevant information (particularly if they did not understand the first step: understanding what the problem is asking).

A third step in solving word problems has to do with identifying how to solve the problem, which is sometimes referred to as the strategy or the process. That is, students may know which steps to take, but if they do not remember the process or strategy that someone has shown them for how to carry out those steps (the “how”), they are stuck. This step is difficult to recognize for students with LD, especially if they have not mastered or completed the pervious steps accurately. Finally, calculating the final answer in a word problem (the final step) is a challenge for some students with LD if they have not mastered the computational skills required. For example, if the computation is
multiplication with multidigit numbers and regrouping, some students with LD may not know how to accurately calculate these numbers, perhaps even with the use of a calculator. Moreover, although many students with LD do have calculator use as an accommodation on their Individualized Education Program (IEP), their conceptual understanding of the answer may be “going through the motions” when calculators are used (e.g., just entering numbers from the problem into the calculator). Ultimately, there are multiple steps in the process for accurately solving math word problems. Students with LD who are not proficient with the procedure and sequence for each step for solving mathematical words problems require instruction responsive to their learning needs.

General solutions for addressing achievement problems in secondary mathematics include organizing explicit teaching of important concepts, providing numerous examples of new concepts that address the overall range of the concepts, direct teaching of relevant cognitive routines, and systematic teaching of prioritized objectives (Montague & Jitendra, 2007). In 2007, Maccini et al. extended a review completed by Maccini and Hughes (1997) on mathematics interventions for secondary students with LD. The purpose of this extension was to determine the nature and focus of current mathematics interventions that were effective for secondary students with LD. Interventions that produced significant gains for secondary students with a learning disability in mathematics included mnemonic strategy instruction (Manalo, Bunnell, & Stillman, 2000; Test & Ellis, 2005), graduated instructional approach (i.e., concrete-representational-abstract [CRA] sequence; Butler et al., 2003; Cass, Cates, Smith, & Jackson, 2003), cognitive strategy instruction involving planning (Joseph & Hunter,
Although these extensively researched practices in mathematics instruction focused on secondary students with disabilities, Maccini et al. (2007) noted that the interventions targeted mathematics skills that were remedial (i.e., multiplication facts, decimals, integers, and computation skills) versus secondary mathematical concepts (i.e., algebraic concepts, proportional reasoning, operations with fractions). Word-problem solving in math is dependent on mastery of individual mathematical skills, especially for students with disabilities. Proportional reasoning is one skill that has been identified as a foundational topic for further success in understanding higher levels of mathematics (Lamon, 2007; National Mathematics Advisory Panel, 2008).

Proportional reasoning requires an understanding of multiplicative relationships between quantities (Boyer, Levine, & Huttenlocher, 2008). The area of proportional reasoning in mathematics is so important for secondary students with LD to know because research does not focus on this skill for these students. In addition, knowing the foundational skill of proportional reasoning will assist with learning of more complex secondary mathematical content (i.e., algebra and geometry).

**Proportional Reasoning and Students with Learning Disabilities (LD)**

Reasoning with proportions requires an understanding of relationships among numbers that is especially difficult for all adolescents (Boyer et al., 2008). For students with LD, proportional reasoning may be especially difficult because there are multiple steps or operations to perform. For example, in the following proportional reasoning
word problem, students need to align where information goes and what the question is in order to solve the problem: If a school bus holds 50 students, then how many buses are needed to bus 960 students to their school? This problem requires students to find information from word problems where there are missing numbers for ratios or proportions, organize the information in a diagram, set up the proportion with the correct ratios, cross multiply to develop a number sentence (multiplication), and use the opposite operation (divide) to solve for the unknown number.

Sinicrope and Mick (1983) compared the proportional reasoning abilities of students with and without LD. The researchers sought to determine whether students with LD differed in their development of proportional reasoning and whether their disability was in the use of symbols and language (without words) and not in their ability to solve proportional problems. Results indicated that the students with LD were more successful with problem solving ($M = 28.7\%$) than with problems involving the use of symbols and language ($M = 28.3\%$). Conversely, students without LD were more successful with the purely symbolic form of numerical fractions ($M = 70.1\%$) as opposed to problem solving ($M = 55.6\%$). The symbolic form refers to just the numbers, operation signs, and symbols (i.e., $1/4 + 2/4 = ?$). Therefore, researchers concluded that the proportional reasoning skills for students with LD are developmental regarding proportional reasoning skills (similar to students without LD).
Mathematical Problem-Solving Instruction for Students with Learning Disabilities (LD)

Because students with LD have problems with memory and with organizing information, two factors that influence academic learning during instruction for these students are cognitive load and scaffolding (Kalyuga & Sweller, 2004; Lang & Pagliaro, 2007). Both cognitive load and scaffolding relate to the development of problem-solving skills for students with LD because these students may only be able to store a small amount of information at a time (memory) and need scaffolds (gradual release of support during instruction of steps) to remember the process for mathematical problem solving.

**Cognitive load.** In mathematical problem-solving instruction, it is first essential to know how much information someone is capable of working with at one time (cognitive load), particularly for the population of students with disabilities (Kalyuga & Sweller, 2004). How many steps one can remember is dependent on the individual’s proficiency with the task. For example, in a series of driving directions, the first two steps may comprise a “few chunks.” Trying to remember additional steps, however, may be too overwhelming because there may not be enough room in working memory to store the remaining steps of the set of directions (i.e., that person’s cognitive load). Also, the person may not even be familiar with the geographical area in which they are driving, so they are also trying to remember landmarks, which takes up additional space in memory.

Cognitive load is an important factor that impacts academic learning for students with LD. Cognitive load theory is based on the assumption that the processing limitations of working memory may be a major factor that influences the effectiveness of instruction
Working memory, or short-term memory, temporarily stores information, and processing refers to a series of actions or steps taken toward a specific target. If more than a few chunks of information are processed simultaneously, working memory is said to be overloaded (Baddeley, 1986).

Cognitive load to process instruction may also depend on the learner’s level of knowledge (Kalyuga & Sweller, 2004). For example, students with LD who learn new concepts in labeling pictures or diagrams to solve problems may have difficulty adding words or numbers due to the additional information they need to process (Kalyuga & Sweller, 2004; McNamara, Kintsch, Songer, & Kintsch, 1996). Cognitive load is an important factor in academic learning for students with LD due to their speed of processing information as well as their organization skills, and what they can manage about learning is dependent on what they already know and what they can do well.

**Scaffolding.** Scaffolding is an effective teaching method to deal with cognitive load (Baker et al., 2002; Stone, 2002). An example of applying scaffolding to mathematics problem solving would be students’ using a graphic organizer that includes the steps of problem solving and prompts for each step and gradually using fewer prompts. Research supports the effectiveness of scaffolds in helping students to problem solve in mathematics (Rittle-Johnson & Koedinger, 2005; Simons & Klein, 2007). For example, Rittle-Johnson and Koedinger (2005) evaluated scaffolding using story tasks to scaffold for introducing new skills or types of problems. The researchers applied these tasks to solve addition and subtraction problems involving fractions for students after they received computer-based classroom instruction. This procedure can be applied to
mathematics by having students solve already worked problems. Other research studies have demonstrated that students learn more by alternating between studying examples of worked-out problem solutions and solving similar problems on their own than they do when given problems to solve on their own (Cooper & Sweller, 1987; Kalyuga, Chandler, Tuovinen, & Sweller, 2001; Ward & Sweller, 1990). Using already worked problems helps students recognize and understand the key information and mathematical relationships among numbers (Marshall, 1995; Steele, 2005).

When students learn a process, the steps are first demonstrated by the teacher, and teacher support is provided in guided practice. In teacher demonstration, the process can be modeled both verbally through think alouds and visually by referring to a chart of steps. The guided practice is sometimes via verbal cues and sometimes via visual cues so that students receive the supports (i.e., the scaffolds) that they need to move to higher levels of proficiency. The purpose of teaching the steps is so when students need support they can be cued to use the steps and shown again how to use the steps. Scaffolding is similar to a child riding a bike with training wheels for initial success; when the training wheels eventually are removed, the child is able to ride independently. Scaffolding benefits students with LD because they require guided practice followed by a gradual release of the support, such as for processes involving multiple steps. Scaffolds support students’ cognitive processes by decreasing the amount of cognitive load required to perform a task and by making more room in memory for higher-order thinking as teachers gradually release cues (Jonassen & Carr, 2000). An example of applying scaffolding to mathematical problem solving would be when a teacher models how to use
a graphic organizer that includes the steps of problem solving and prompts for each step. Gradually the teacher fades her modeling to include students’ modeling and fades her support for cues as students use the graphic organizer more independently and accurately.

Solving word problems is an area of mathematics that is also affected by students who have difficulty reading text in a word problem (Lang & Pagliaro, 2007). For example, students who have difficulty reading may have difficulty solving word problems because of the decoding and vocabulary skills required to comprehend what the problem is asking them to find. Additionally, students who are not confident in their reading ability may have lower confidence in the subject of mathematics when the task requires them to read independently. The reading ability for middle school students with LD may be even more so affected if their decoding and comprehension skills are multiple levels below their grade level. Students may also have difficulty with the English language and communication aspects of mathematics (Lang & Pagliaro, 2007). For example, Fuchs et al. (2008) concluded that embedding mathematics within a word problem may challenge students who have reading difficulties because many students with LD struggle with reading as well as mathematics.

**Problem-Solving Interventions for Students with Learning Disabilities (LD)**

Once cognitive load and scaffolding have been incorporated into the learning process for mathematical problem solving, teachers need to decide how to approach the content to be delivered. Xin and Jitendra (1999) examined the effectiveness of problem-solving instruction in a meta-analysis of 25 outcome studies. Computer-assisted instruction (CAI) had the largest effect sizes in the group design studies (ES = 1.8, high
according to Cohen’s *d*). Cohen (1988) identified an effect size of .8 as large, .5 as moderate, and .2 as small. Effects for representational techniques and strategy training (i.e., a schema-based intervention) were also found to be significantly higher (*ES* = 1.77; *ES* = .74) than alternate approaches (*ES* = 0.0) for both group design and single-subject studies. Because this review occurred in 1999, however, and technology has greatly evolved since then, a more recent meta-analysis would be necessary to determine the effect size for CAI.

Representation techniques in the meta-analysis by Xin and Jitendra (1999) referred to pictorial, concrete (i.e., manipulatives), and mapping (i.e., recording numbers from the word problem onto a diagram) approaches to problem solving. The category of strategy training referred to any explicit problem-solving procedure or direct instruction (either of the latter may consist of demonstration, guided practice, and independent practice). The most effective interventions both used strategy training and were noted as a schema-based intervention, which involved recognition of schema and mapping problem information on a diagram, and a cognitive intervention, which consisted of self-questioning procedures for how to solve a problem. Three types of interventions that have proven to be effective for students with disabilities will be described in this section, and those interventions include pictorial representation, cognitive instruction, and schema-based instruction (SBI).

**Pictorial representation.** Some math problem-solving interventions rely on different types of representation, including pictorial displays, to organize information (Butler et al., 2003; Maccini & Hughes, 2000). Using the concrete-representational-
abstract (CRA) teaching sequence integrates the use of manipulative devices (concrete), pictorial representations (visual), and numeric display of numbers (abstract) into explicit instruction designed to teach important concepts (Butler et al., 2003; Miller & Hudson, 2007). The explicit teaching sequence begins with an advance organizer including a review of prerequisite knowledge followed by teacher demonstration, guided practice, independent practice, and checks for maintenance.

Students with LD have been successful in applying CRA in the area of math problem-solving. Butler et al. (2003), however, examined how well middle-school students learn when manipulatives (concrete) were not used. Higher mean scores were obtained for the CRA group \((M = 69\%)\) compared to the representational-abstract (RA) group \((M = 63\%)\) on the word problem measure of fractional equivalency \((ES = .17)\), and students in the two treatment groups performed as well as the students in the comparison group of general education students. RA was about as effective as the CRA, which has implications for further examinations in problem solving for middle school students with LD.

Research also supports that high school students with LD can be taught to demonstrate word problems involving integer numbers through pictorial displays or representation of mathematical problems (Maccini & Hughes, 2000). Both concrete and representational methods for displaying numbers are useful for students’ understanding and communicating (i.e., language) of information in word problems (Butler et al, 2003; Maccini & Hughes, 2000; NCTM, 2000). Regarding students with disabilities, varied representations of numbers are particularly useful when students have deficits in the
processing of information. Representing problem information in a concrete or visual display can help students see the relationship among numbers in a word problem. Concrete and visual representation also target receptive language so students have a better understanding what the problem is asking.

**Cognitive instruction.** Cognitive instruction is another category of mathematical problem-solving interventions that has had positive results for this population of students. Cognitive instruction is a strategy for problem solving consisting of modeling self-questioning by thinking aloud, providing prompts, and providing corrective feedback (Hutchinson, 1993). Cognitive instruction is overtly and verbally sharing with the students how teachers are making a decision by bringing in what they are thinking, talking out loud, making mistakes, and/or modeling corrections. Other examples of cognitive instruction may include components such as reading the problem, making a drawing, and performing the operation. Cognitive strategy instruction has demonstrated improvements in verbalization of both problem representation and solution for students both with and without LD (Hutchinson, 1993; Krawec et al., 2013; Montague et al., 2011). The cognitive interventions, however, have been quite lengthy, lasting 4-7 months (i.e., students met with researcher individually for 40 minutes on alternate days over 4-7 months). In some cases, cognitive strategy instruction is an effective way to teach and have students remembers a process but can require significant time for students to learn, although some students need this intensity.

**Schema-based instruction (SBI).** Schema-based instruction (SBI) is another problem-solving strategy that applies both representation and cognitive instruction and
has been successful for elementary and secondary students both with and without disabilities (Jitendra & Star, 2012; Jitendra, Star, Rodriguez, Lindell, & Someki, 2011; Jitendra et al., 2009). SBI focuses on the identification of problem types and considers the mathematical structure. SBI also uses diagrams to solve problems and includes an instructional component with already worked problems. In the SBI approach, each problem type is presented first as a story situation with the answer. This presentation allows students to focus on the identification and representation of the features of the story situation (i.e., entering numbers from the problem into a diagram; Jitendra et al., 2010).

SBI incorporates scaffolding of student learning when the teacher first uses explicit instruction and then gradually reduces support with fading and fewer prompts as the student becomes more proficient with the task being learned (Jitendra & Star, 2011). Applying scaffolding in SBI for mathematics problem solving involves guided practice with teacher support so students are able to independently apply the SBI process to solve problems. Due to the difficulty students with LD may have storing information for a brief period of time, teaching these students to recognize the schema in mathematical problem solving is critical to reducing working memory resources and cognitive load (Jitendra & Star, 2011). Once students know how to read and understand the problem asked, they can organize the word problem information.

The National Mathematics Advisory Panel (2008) advocated SBI as one of the few conceptual approaches that addresses effective problem-solving and reasoning skills that students need time to develop. In the next section, how SBI utilizes diagrams of
problem patterns and structure and has been used successfully to improve the problem-solving skills for students with LD are described (Jitendra et al., 2007).

**Schema-Based Instruction (SBI) in Math**

SBI provides an explicit framework for understanding how to solve word problems in mathematics (Fuchs et al., 2004; Jitendra, 2012; Jitendra et al., 2009; Marshall, 1990; Riley et al., 1983). Marshall explained three steps that lead to the solution of a mathematical word problem. The first step involves processing the problem schemata necessary to represent the situation described in a problem. Processing the problem schemata means knowing what type of problem it is (i.e., proportional) and identifying relevant information. The second step involves the selection of action procedures/operations that correspond with the representation of the problem identified in the first step. The selection means choosing the operation to employ with the relevant information. The third and final step of solving word problems in math comprises a set of rules that can be implemented to arrive at a solution. In the third step, math rules are applied to calculations of the number sentence involving each of the four operations. Providing extensive practice and mastery in each of these components of the SBI process in word problem solving is important for students with disabilities because students need more practice to reach mastery (Bryant & Bryant, 2008; Pressley, 1986).

**Summary**

The next chapter details a review of research on mathematical problem-solving for students with learning disabilities (LD). Research studies involving SBI have demonstrated higher improvement for the SBI group over the control group in
mathematical problem solving for students both with and without disabilities in both the third and seventh grades (Fuchs et al., 2004; Jitendra, 2012). Evidence also supports higher performance for the SBI group when compared to a traditional instruction group for students in the elementary setting with mild disabilities (Jitendra et al., 1998). Regarding specific math skills in word problems, SBI had a greater impact on high-achieving students’ problem-solving scores in the area of fractions and percent (Jitendra & Star, 2012). Researchers have also found that SBI students in general education classes perform higher than the control group in the area of ratios and proportions at the seventh-grade level (Jitendra et al., 2009). However, there has not been research done involving the application of the SBI process to solve proportional reasoning word problems for students with LD at the middle school level. The area of proportional reasoning in mathematics is so important for secondary students with LD to know because research does not focus on this skill for these students. In addition, knowing the foundational skill of proportional reasoning will assist with learning more complex secondary mathematical content (i.e., algebra and geometry). The specific population targeted for the SBI intervention in the current study was seventh-grade middle school students with LD, and the curriculum skills of unit rates, proportions, and scale drawings were the focus (proportional reasoning skills).
Definition of Terms

Cognitive instruction

Cognitive instruction is a strategy for problem solving consisting of modeling self-questioning by thinking aloud, providing prompts, and corrective feedback (Hutchinson, 1993).

Diagram

In mathematics, a diagram refers to a chart or graph that illustrates a procedure or relationship between numbers using shapes.

Explicit teaching

The explicit teaching sequence begins with an advance organizer including a review of prerequisite knowledge followed by teacher demonstration, guided practice, independent practice, and checks for maintenance.

Graduated instructional approach (i.e., CRA)

CRA integrates the use of manipulative devices and pictorial representations into explicit instruction designed to teach important concepts (Butler et al., 2003; Harris, Miller, & Mercer, 1995; Miller & Hudson, 2007).

Graphic organizer

A graphic organizer is a specific type of diagram with shapes connected by lines that usually indicate relationships (i.e., sequence, compare/contrast) that is used to organize information in any content area (i.e., reading, mathematics, science).
If-then diagram

The if-then diagram is used in SBI instruction to map information from proportional word problems. This diagram includes four shapes used in a horizontal and vertical manner to guide students’ placement of numbers from the word problems, and this formation of the numbers results in a proportion.

Mapping of relationships

Mapping of relationships refers to diagramming or placing information from the problem into a graphic organizer.

Mnemonic

A mnemonic refers to a word, sentence, picture device, or technique for improving or strengthening memory (Test & Ellis, 2005).

Organizing/filtering necessary information

Organizing and filtering necessary information refers to the second step of solving mathematical word problems. This step involves extracting the essential facts/numbers and any irrelevant information in order to solve the problem.

Problem solving in mathematics

Problem solving in mathematics is using a process or series of steps to find a mathematical solution to a word problem that may involve one or more specific skills, concepts, and procedures (i.e., algebra, geometry).

Proportional reasoning

Reasoning with proportions requires an understanding of multiplicative relationships between quantities (Boyer et al., 2008). In the current study, specific
proportional reasoning skills that were used include unit rates, proportions, and scale
drawings. For example, if a school bus holds 50 students, then how many buses are
needed to bus 960 students to their school?

**Proportion problem**

A proportion problem is similar to a unit rate problem; however, it does not
include the number “1.” For example, if Carl can read six pages in 14 minutes, how long
will it take him to read a chapter that has 21 pages?

**Scale drawing**

A scale drawing is used to represent something that is too large or too small for an
actual size drawing. For example, a map is usually a scale drawing because the actual
distance on the map is too large for an actual drawing. For scale drawings, the ratio of the
distance on a map is used to represent the actual distance, such as 1 inch = 20 miles.

**Schema**

A schema is a domain-specific long-term memory structure that allows people to
categorize information (Larkin, McDermott, Simon, & Simon, 1980). It can also be
referred to as an organized pattern of thought or behavior or as a mental structure of
preconceived ideas. For example, stereotypes, scripts, academic rubrics, and social roles
are types of schema that people could use to represents some aspect of the world.

**Schema-based instruction (SBI)**

Schema-based instruction (SBI) is a problem-solving strategy that applies both
representation and cognitive instruction and has been successful for both elementary and
secondary students both with and without disabilities. SBI focuses on identifying
problem types, looking beyond the surface similarities, and considering the mathematical structure. SBI also uses diagrams to solve problems and includes a training period with already worked problems. In the SBI approach, each problem type is presented first as a story situation with the answer so students can focus on the identification and representation of the features of the story situation using schematic diagrams (Jitendra et al., 2010).

**Specific learning disability (LD)**

LD is a disorder in one or more of the basic psychological processes involved in understanding or in using language, spoken or written, that may manifest itself in an imperfect ability to listen, think, speak, read, write, spell, or to do mathematical calculations, including conditions such as perceptual disabilities, brain injury, minimal brain dysfunction, dyslexia, and developmental aphasia (IDEA, 2004).

**Word problems**

Word problems are verbal descriptions of problem situations in which one or more questions are asked to obtain the answer by applying mathematical operations to numerical data given in the problem (Verschaffel, Greer, & De Corte, 2000).

**Unit Rate**

A unit rate is a rate expressed as a quantity of one. For example, if one worker packs 25 tomatoes in one box, then the unit rate is 25 tomatoes per one box (i.e., 2 feet per second or 5 miles per hour).
2. LITERATURE REVIEW

This chapter presents a review of research on mathematical problem-solving for students with learning disabilities (LD). First, a description of specific factors that affect academic learning for students with LD is provided. These factors, which also influence problem-solving skills, are cognitive load and scaffolding. Then the rationale for measuring mathematical self-efficacy with problem-solving instruction is described, followed by a review of research on problem solving in mathematics.

The review of research generated two major problem-solving interventions in mathematics for middle school students with LD. Research studies involving those interventions, cognitive instruction and schema-based instruction (SBI), will be described. SBI is used for students with and without high-incidence disabilities (i.e., LD, autism, Other Health Impairments [OHI]) in both elementary and secondary grades. Descriptions for cognitive instruction and the SBI process are also provided in the chapter.

Literature Search Procedures

There were four phases for conducting the literature search. Peer-reviewed journals were selected during all phases of the literature search. In the first phase of the literature search, the database PsycINFO was searched using the following keywords:
math, intervention, problem solving, students with LD, and middle school.” The Academic Search Complete, Education Research Complete, and ERIC databases were used simultaneously (because these three databases could be searched at the same time) in the second phase of the search using the same sequence of keywords as the first phase of the search. In the third phase of the search, the acronym SBI was added as a search term with problem solving (“SBI, problem solving”) in all the previous databases (PsycINFO, Academic Search Complete, Education Research Complete, and ERIC). Finally, Jitendra, the name of an author in many of the studies, was used as a keyword in the fourth phase of the search.

Multiple searches occurred in the first phase of the search because the purpose of this phase was to yield studies that met the following parameters: (a) described a problem-solving intervention in the area of mathematics, (b) involved students with learning disabilities (LD), and (c) were conducted in Grades 6-8. The first search in this phase did not yield any results, so “students with LD” was then omitted from the search terms, leaving “math, intervention, problem solving, middle school.” Only one study emerged that measured problem solving for middle school students, so the search terms were revised to “word problem solving” and “intervention,” resulting in two more studies, for a total of three studies. Due to the low number of studies that included students with LD in Grades 6-8, the search criteria was expanded to include all students in general education in all Grades 1-12 as well as all students with disabilities in these grades. This modification resulted in three additional studies, for a total of six studies.
In the second phase of the search, three additional databases—Academic Search Complete, Education Research Complete, and ERIC—were searched because only six intervention studies in the area of mathematics problem solving were found in the first phase of the search. The same keywords in the same sequence as described in the first phase were used in this second phase as well. So the first search terms entered were “math, intervention, problem solving, students with LD, middle school,” followed by that same set of keywords excluding students with LD: “math, intervention, problem solving, middle school.” One more study emerged with the second set of search terms that measured problem solving for middle school students for a total of seven studies. Next, “word problem solving, intervention” was entered. These last keywords entered did not yield any additional studies.

According to research that emerged from the first and second phases of the search, schema-based instruction (SBI) is a problem-solving intervention used for students with LD in mathematics. Therefore, in the third phase of the search, the acronym SBI was added as a search term only with problem solving—“SBI, problem solving”—to determine if there were additional studies about problem-solving interventions in mathematics at both the elementary and secondary levels for students both with and without disabilities in all databases used (PsycINFO, Academic Search Complete, Education Research Complete, and ERIC). The studies were examined to see if they met the following revised criteria: (a) relevant to problem-solving intervention research in mathematics using SBI and (b) inclusive of elementary and secondary students both with
and without disabilities. This third phase of the search resulted in five more studies to be included in the review, for a total of 12 studies.

Jitendra’s name emerged as author eight times in the 12 problem-solving interventions in mathematics studies found in the first three phases, and so her name was used in the fourth phase of the search to see if new studies originated. Specifically, the search term “Jitendra, Asha” was entered into the four databases (PsycINFO, Academic Search Complete, Education Research Complete, and ERIC) in this fourth phase of the search. Three additional intervention studies met the following criteria: relevant to problem-solving intervention research in mathematics using SBI and inclusive of elementary and secondary students both with and without disabilities. It is possible these three studies did not show up previously because the title and abstract of the studies emphasized the mathematics skill (i.e., ratios, proportions, addition, subtraction) addressed in SBI and not the phrase “problem solving” as searched. In total, 15 research studies met the search criteria and are reviewed in this chapter. A synthesis of these studies is also presented. In the next section, two specific factors that influence instruction in mathematical problem solving learning (cognitive load and scaffolding) are discussed.

Factors That Influence Academic Learning

This section describes two specific factors that influence academic learning for students: cognitive load and scaffolding. The rationale for highlighting these factors is due to their role in the instructional process. First, the role of cognitive load in the instructional process can be to inform the teacher of how much information someone is
capable of working with at one time, which is especially important for students with disabilities (Kalyuga & Sweller, 2004). Second, once a learner’s cognitive load is identified, the role of scaffolding as a teaching method can be to structure the instructional process (Baker et al., 2002; Stone, 2002). Cognitive load and scaffolding, as each pertains to the learning of students with disabilities, are described in the next sections.

**Cognitive Load**

Cognitive load theory is based on the assumption that the processing limitations of working memory may be a major factor that influences the effectiveness of instruction (Sweller et al., 1998). Working memory, or short-term memory, temporarily stores information, and processing refers to a series of actions or steps taken toward a specific target. For some students, if more than a few chunks of information are processed simultaneously, working memory is said to be overloaded (Baddeley, 1986). The number of steps one can remember, however, is dependent on the individual’s proficiency with the task. For example, in a series of driving directions, the first two steps may comprise a “few chunks.” Trying to remember additional steps, however, may be too overwhelming because there may not be enough room in working memory to store the remaining steps of the set of directions. Also, the person may not even be familiar with the area in which they are driving so they are memorizing landmarks, which takes up additional space in memory.

Cognitive load to process instruction may also depend on the learner’s level of knowledge (Kalyuga & Sweller, 2004). For example, students with LD who have to enter
information into diagrams to solve problems may have difficulty adding the words or numbers due to the additional information they need to process (Kalyuga & Sweller, 2004; McNamara et al., 1996). Cognitive load is an important factor in learning mathematical skills for students with LD due to their speed of processing information and their organization skills, and what they can manage about learning is dependent on what they already know and what they can do well. It is essential for teachers to consider cognitive load for students with LD when designing mathematics instruction, especially in the area of problem solving, because instruction can be presented in smaller steps that are more manageable to process.

Scaffolding

Scaffolds are instructional tools that teachers use during instruction by gradually withdrawing supports as the learner becomes more proficient (Baker et al., 2002; Stone, 2002). An example of applying scaffolding to mathematics problem solving would be when a teacher uses a graphic organizer that includes prompts for each step of how to solve an equation and then gradually uses fewer prompts. Scaffolds support students’ cognitive processes by decreasing the amount of cognitive load required to perform the steps in that task because teachers gradually release cues (Jonassen & Carr, 2000). The gradual removal of the prompts is scaffolding. Scaffolding benefits students with LD because during the initial instruction of processes involving multiple steps, they need more cues and prompts; eventually, as students become more proficient, the scaffolds (i.e., cues, supports) can be removed. For example, when teaching students with LD how to solve an addition equation, a teacher might demonstrate an example problem showing
written directions for each step of the problem. Then, as the student becomes proficient with a step, the teacher covers up that part of the example problem, while the remaining steps are visible until mastered. Scaffolding may benefit students with LD in middle school because as they begin to get more proficient, supports are removed, and they are able to apply strategies they could not initially perform on their own (Stone, 1998, 2002).

Research supports the effectiveness of scaffolds in helping students to problem solve in mathematics (Rittle-Johnson & Koedinger, 2005; Simons & Klein, 2007). For example, Rittle-Johnson and Koedinger (2005) evaluated scaffolding using story tasks, or examples, for introducing new mathematical skills or types of problems. The researchers applied these story tasks to solve addition and subtraction problems involving fractions for students after they received computer-based classroom instruction. Seethaler, Fuchs, Fuchs, and Compton (2012) also applied scaffolding by using a support structure referred to as DA, which was defined by the researchers as the degree of scaffolding required to learn unfamiliar mathematics content in that study for problem solving in mathematics. The researchers assessed the value of the DA supports in predicting both calculations and word-problem development for first-grade students. Results indicated that DA may be useful in predicting mathematics development, especially word problems.

Once a learner’s cognitive load is identified, scaffolding as a teaching method structures the instructional process (Baker et al., 2002; Stone, 2002). Scaffolds support students’ cognitive processes by decreasing the amount of cognitive load required to process steps in the context of mathematical problem solving (Jonassen & Carr, 2000). The next section describes students’ self-efficacy for mathematical problem solving.
Students’ Self-Efficacy for Mathematical Problem Solving

Self-efficacy is the degree to which one believes one can perform under certain conditions (Bandura, Barbaranelle, Caprara, & Pastorelli, 1996). According to Bandura (1997), self-efficacy beliefs develop as individuals interpret information from the following four sources: mastery experience, observation of others, social persuasions, and emotional and psychological states. Mastery experience, or past performance, is the most powerful contributor to self-efficacy, especially for students with LD. For example, students with LD most likely experience more failures than successes in academics and may look at difficult tasks and believe they cannot do them. Research provides evidence that secondary students with LD possess lower self-efficacy regarding academic areas such as mathematical problem solving and have lower self-efficacy related to their classroom peers (Hampton & Mason, 2003; Klassen, 2006; Klassen & Lynch, 2007;).

In one study, Klassen and Lynch (2007) examined academic self-efficacy beliefs held by adolescents with LD and found that they (a) had low self-efficacy, (b) were accurate in their calibration of self-efficacy and performance, (c) viewed verbal persuasion as a valued source of self-efficacy, (d) attributed their failures to lack of effort, and (e) were less self-aware of their own learning processes, although they were aware they performed not as well as others. In relation, Hampton and Mason (2003) examined the impact of sources of self-efficacy on self-efficacy beliefs and academic achievement and found that students with LD in Grades 9-12 had accomplished significantly less in the past, had fewer role models, received less positive reinforcement from others, had higher anxiety levels, and had lower self-efficacy for performing classroom tasks.
Additionally, Klassen (2006) used predictions of performance to learn about the writing self-efficacy of early adolescents with and without LD for students in Grades 8 and 9 and found that students with LD had significantly lower levels of self-efficacy and performance in relation to writing than students without LD. Research supports that self-efficacy of students with LD is directly affected by past performance because if these students have not had success in the past with a task (i.e. solving a word problem), they would not believe they could succeed in that area at present.

Other research provides information related to self-efficacy in the area of math and problem solving. Math self-efficacy assesses individuals’ judgements of their capabilities to solve specific math problems, perform math-related tasks, or succeed in math-related courses (Pajares & Miller, 1994). Researchers who have explored the relationship between prior experience and math self-efficacy have found both significant correlations and direct effects (Cooper & Robinson, 1991; Hackett, 1985). For example, Hackett (1985) found that students with high self-efficacy had a strong positive effect on math anxiety as well as on the choice of math-related careers and predicted math anxiety better than prior high school math experience. Regarding the students’ self-assessments, studies indicated that more students overestimate than underestimate their self-efficacy (Pajares & Miller, 1994; Schunk, 1995).

Possessing high self-efficacy may motivate students to continue learning even when their performance is deficient because they are more likely to take on complex tasks if they believe they can do them. Also, high achievers estimate their ability to solve a particular math problem more accurately than low achievers because a direct
relationship exists between learners high in self-efficacy and application of metacognitive strategy to generate successful performance (Braten, Samuelstuen, & Stromso, 2004). Self-efficacy also provides insight about the relationship between the individual beliefs of students with LD in their capability to problem solve and their actual performance in solving math word problems (Pajares & Miller, 1994).

Finally, some researchers have measured self-efficacy as both a pretest and posttest (Feldman et al., 2011; Garcia & Fidalgo, 2008), although many studies only administer a self-efficacy scale one time during implementation of an intervention. Garcia and Fidalgo (2008) measured students’ self-efficacy with writing skills both before and after a writing task and found that students with LD showed a statistically significant higher writing self-efficacy than the typical student group after the intervention implementation. Feldman et al. (2011) examined the effects of accommodations on adolescents’ self-efficacy in both a pretest and a posttest as well. Results indicated that students who received testing accommodations showed significant increases from pretest to posttest on the measure of self-efficacy compared to students who did not receive accommodations. Therefore, due to connections evident between self-efficacy and math performance for students with LD using a pretest-posttest approach, there is more value in assessing self-efficacy both before and after an intervention.

**Review of Research on Problem Solving in Mathematics**

As described in the beginning of this chapter, a methodological search was conducted using five search terms (“math, intervention, problem solving, students with
LD, middle school”) and four databases (PsycINFO, Academic Search Complete, Education Research Complete, and ERIC) that yielded 15 studies. The 15 studies reviewed measured problem-solving in mathematics for both elementary and secondary students with and without LD. Three of the studies focused on cognitive instruction, and the remaining 12 studies focused on schema-based instruction (SBI).

**Cognitive Instruction**

Cognitive instruction is a category of problem-solving interventions that has had positive results for this population of students with LD. Cognitive instruction involves knowing how to solve a problem, including components such as reading the problem, making a drawing, and performing the operation. Hutchinson (1993) designed a cognitive strategy for students in Grades 8-10 with LD to represent and solve three types of algebra word problems. The cognitive strategy consisted of modeling self-questioning by thinking aloud and providing prompts and corrective feedback. Hutchinson employed a modified multiple baseline design with two groups (intervention and control groups). Material used for instruction of the cognitive strategy included task sheets of problems and prompt cards for self-questioning. The self-questioning prompts consisted of self-questions to monitor for how the students represented the information given in problem and carried out the operation(s) selected to solve the problem, such as “Have I read and understood each sentence?” or “Have I written an equation?” Instruction and testing occurred during the regularly scheduled 40-minute resource class, and students’ progress with think-aloud data for representation and solution proportion problems was measured both before and after the intervention. Think-aloud data in this situation referred to the
verbalization of both problem representation and problem solution. Each of the 12 students in the intervention condition met with the instructor on alternate days for one 40-minute session for approximately four months. The control group received regular instruction in their resource classes.

The pretest scores on the think-aloud data for the intervention group increased from 3.5 to 10.7 out of 12 problems correct on the posttest for the representation measure (goal and information given in problem), and from 1.8 to 7.5 out of 8 correct on the solution measure (choice of operation and carrying out operation). Meanwhile, the scores for the control group had a lower increase from pretest to posttest on the representation measure (2.5 to 4.3 out of 12) and a slight decrease in scores on the solution measure (1.6 to .9 out of 8 correct). These findings support the approach to algebra problem-solving instruction in two phases (instruction and representation). Given these results, cognitive instruction has been successful for students with LD in the area of problem solving and should be considered when teachers can incorporate it into the curriculum for a few months of the school year.

Similarly, Montague et al. (2011) studied the effectiveness of a research-based cognitive strategy instructional program, Solve It!, in both inclusive general education and intensive math classes using 24 middle schools (eight in intervention group, 16 in control group). Solve it! is a cognitive strategy designed to improve the problem-solving of middle school students with LD. Eighty of the 719 participants (11% of the total) were identified as LD, and these students were from both general education and the intensive math classes. The skills measured were all four basic operations using whole numbers or
decimals, and the control group received typical classroom instruction. The study utilized a four-level data structure, so repeated measures were nested within classrooms, and classrooms were nested within schools. Overall, students who received Solve It! instruction over the 7 months of the intervention had higher improvement in math problem solving over the school year than students in the control group who received typical classroom instruction. The intervention was equally successful across all ability levels, including students with LD, low-achieving students, and average-achieving students. In some cases, then, when teachers incorporate cognitive instruction into the daily curriculum for an extended period of time they get positive results.

Krawec et al. (2013) also implemented the Solve It! intervention for average-achieving students as well as for students with LD in seventh and eighth grades. Using a pretest-posttest design, the researchers measured knowledge of problem-solving strategies using the Math Problem-Solving Assessment (MPSA), which consisted of 34 items in a structured interview. Across ability levels, results indicated that students who received the Solve It! intervention used significantly more strategies than students in the control group. The average-achieving students improved from 46% on the pretest to 51% on the posttest, which was a greater increase than gained by the control group (from 45% to 42%). Students with LD also had a higher increase from pretest (39%) to posttest (44%) than the control group (37% to 36%). According to the results of these studies, (Krawec et al., 2013; Montague et al., 2011) both students with and without LD demonstrated improvements in problem solving through the cognitive intervention, Solve it!
Schema-Based Instruction (SBI)

Cognitive instruction has had positive effects for middle school students with LD, but teachers can use schema-based instruction (SBI) to teach students in this population as well. SBI is a combination of representation and cognitive thinking, which is likely to be more beneficial for these students because they are applying effective behaviors from each strategy (i.e., thinking aloud and diagramming). The remaining 12 interventions that were part of the review of research for this study involved SBI and are described in this section. The 12 SBI studies are described in one of four categories: SBI versus control group for all students, SBI versus control group for students with LD, single-subject design, and SBI versus another treatment for students with and without LD. Because the studies in this section are reported according to type of design, group studies are separated from single-subject studies. The rationale for this presentation was to allow for focus on characteristics of the single-subject design studies that could later be compared to the current study.

SBI versus control group for all students. Jitendra et al. (2009) evaluated the effectiveness of SBI for seventh-grade students on their performance with word problems involving ratios and proportions. Researchers implemented a pretest-intervention-posttest-retention design. For this study, 148 students were assigned to a treatment or control condition, and the SBI program replaced regular instruction for the SBI treatment condition on the topics of ratio and proportions for 40 minutes daily, 5 days per week, across 10 school days. The intervention was delivered by the general education classroom teachers in their intact math classes. The students in the SBI treatment classes
outperformed students in control classes for all ability levels (high, average, and low) on the posttest problem-solving measure and maintained similar results 4 months later. Based on a ceiling of 18 points, the mean score for students in the SBI treatment classes was a higher increase from pretest \( M = 9 \) to posttest \( M = 13 \) compared to the control group classrooms from pretest \( M = 9.5 \) to posttest \( M = 11.5 \).

Jitendra et al. (2011) examined the effectiveness of SBI for seventh-grade students on their performance with word problems involving ratios/rates, scale drawings, and percent. The researchers implemented a pretest-intervention-posttest design with a retention test. The researcher-designed SBI program replaced regular instruction for the SBI treatment condition on the topics of ratio, proportions, and percent during the regularly scheduled math period for about 50 minutes daily, 5 days per week, across 29 school days. Based on a ceiling of 23 points, the mean score for students in the SBI condition was a higher increase from pretest \( M = 11.2 \) to posttest \( M = 15 \) compared to the control group classrooms from pretest \( M =10.2 \) to posttest \( M = 13 \).

Similarly, Jitendra and Star (2012) evaluated whether schema-based-instruction (SBI) impacted the learning of percent word problems for seventh-grade students. The researchers also explored the extent to which SBI improved high- and low-achieving students’ learning. The intervention occurred during students’ intact math classes for 40 minutes daily, across 9 consecutive classroom periods. Results indicated that SBI had a greater impact on high-achieving students’ problem-solving scores. With a ceiling of 6 points, there was a slightly higher increase in mean scores from pretest \( M = 1.91 \) to posttest \( M = 3.25 \) for the SBI intervention group compared to the control condition at
pretest ($M = 1.95$) and posttest ($M = 2.55$). The low-achieving students improved only slightly by a mean score of .4 points (1.2 to 1.6), and the mean scores of high-achieving students increased from 2.2 to 4.1 points. These studies also provide evidence that SBI is effective for problem-solving instruction in proportional reasoning skills for general education students.

**SBI versus control group for students with LD.** Although SBI has demonstrated improvement in math problem solving for general education students, SBI has also been effective for students with LD (Fuchs et al., 2004; Jitendra et al., 1998). In a study including elementary students with mild disabilities, Jitendra et al. (1998) compared the effects of SBI to traditional instruction in the areas of problem solving and strategy use for one-step word problems. Based on a ceiling of 15 points, the traditional group increased from a score of 7.29 on the pretest to 9.81 on the posttest. Similarly, the SBI group improved from a score of 7.65 on the pretest to 11.48 on the posttest.

Fuchs et al. (2004) also studied the use of schemas to solve word problems for students with LD. The researchers assessed the effects of SBI in promoting mathematical problem-solving while investigating whether schema instruction results in schema development for third graders. Schema development for this study was defined as the students’ ability to categorize novel problems into four problem types: shopping list problems, half problems, buying bags problems, and pictograph problems. Shopping list problems involve information about quantities of items needed, how much each item costs, and total cost. Half problems involved giving half of the quantity away and asking the amount remaining. Buying bags problems give information about how many of the
items were in a bag and ask how many bags would be needed for a certain number of items. In pictograph problems, each picture stands for a certain amount, and the reader uses the pictograph to answer the question.

Fuchs et al. (2004) implemented a pretest-posttest design using three groups of students. The 26 lessons for the SBI intervention were delivered across 14 school weeks. The two groups of students receiving SBI with and without sorting practice improved more than the contrast group on problem-solving measures ($ES = .45$). Sorting practice consisted of categorizing more than one problem-type schema. Results also indicated that both SBI groups developed stronger schemas compared to a control group, and the effect sizes were large: 2.88-3.06 for the SBI group and 2.99-3.99 for the SBI plus sorting group. This development of stronger schemas meant the students improved their ability to formulate a category for a problem that translates to steps for a problem type they would be more likely to recognize how to set up. For example, when a problem states, “If one pencil costs seven cents, then a pack of 12 pencils will cost 84 cents,” the student would learn to recognize that the “if-then” scenario makes it a proportional problem. This research provided support that students with disabilities also benefit from using SBI for problem-solving instruction, including a variety of math skills, in the grade-level curriculum.

**Single-subject design.** The SBI process has been compared to a control group in the studies mentioned previously. Research, however, has also been conducted on SBI using a single-subject research (Jitendra et al., 2002; Jitendra & Hoff, 1996; Jitendra, Hoff, & Beck, 1999). Jitendra and Hoff (1996) applied schema-based instruction to
examine its effect on the performance of three fourth-grade students with LD using addition and subtraction word problems. The researchers utilized an adapted multiple-probe-across-students design over 16 sessions (40 to 45 minutes per session), and the intervention was implemented by one of the researchers in a separate room. Mean scores measured before and after the intervention increased for all participants. The first student improved from 20.3% correct in the baseline condition to 97.6% accuracy after the intervention. The second participant increased from 31.3% in baseline performance to 95% in the intervention phase, and the last student improved from 26.6% to 95.2% correct.

Jitendra et al. (1999) taught sixth- and seventh-grade students with LD to solve addition and subtraction word problems using SBI. The researchers implemented a multiple-baseline-across-students design. Results indicated that students answered 58% of 1-step problems correctly, an increase of 26% over baseline performance. The students were able to identify the correct operation and compute 72% of the 1-step problems, an increase of 17% from baseline. The mean performance in writing the correct operation and answer was 60% for 1-step and 2-step problems, an increase of 24% over baseline performance. Following the second level of treatment, the students performed 86% on 2-step problems. Then when both 1-step and 2-step problems were measured, students successfully completed 86% of the problems, an increase of 54% from baseline performance. SBI has demonstrated repeated effectiveness for elementary students with mild disabilities in problem solving and strategy use of 1-step and 2-step word problems involving addition and subtraction.
Jitendra et al. (2002) also investigated the effects of schema-based strategy instruction on math problem solving for middle school students with LD in solving 1-step multiplication and division word problems. The researchers implemented the SBI intervention utilizing a single-subject design in a room separate from the regular classroom. The intervention consisted of 18 sessions, lasting 35 to 45 minutes each. Strategy use was examined on all completed worksheets and measured by whether students followed SBI and identified the problem schemata by drawing a diagram and developing a plan or by writing a number sentence. The researchers waited for mastery before introducing next problem type. The criteria for mastery were 100% performance for accuracy and strategy use in two consecutive sessions.

All participants reached mastery, and the average improvement in mean scores of strategy use measured before and after the intervention was a 66% increase on drawing diagrams and a 32% increase in writing a number sentence. The first student increased her performance from 44% correct on the word problem test in the baseline condition to 58% after the first type of problem was taught. The second participant improved from 50% in the baseline condition to 58% after the first type of problem was taught, and the third student improved from 37% during baseline to 92%. Finally, the last participant’s score increase from 29% in the baseline phase to 75%. Results after this first level of learning one problem type indicate that the first two participants were not able to generalize the use of schema diagrams to solve the untaught problem type. After the next level of problem types was taught during the intervention, however, all participants were able to discriminate between the two problem types and use the correct diagram with
100% accuracy. The participants also maintained performance up to 10 weeks after the intervention, and they generalized the strategy to additional types of word problems (PND = 82.3%). This study demonstrated evidence that SBI is successful for middle school students with LD in 1-step multiplication and division word problems as well.

**SBI versus another treatment for students with and without LD.** The studies reviewed up to this point highlight the effectiveness of SBI for students both with and without disabilities at both the elementary and secondary levels. SBI has also proven to be more effective than general strategy instruction (GSI; Griffin & Jitendra, 2009; Jitendra et al., 2007; Xin et al., 2005). GSI for the purposes of the studies being described utilizes a four-step problem-solving procedure requiring students to read to understand, develop a plan, solve, and look back. First, Xin et al. (2005) found SBI to be more effective than GSI. The researchers employed a pretest-posttest comparison group design with random assignment of subjects to groups to investigate the effects of SBI and GSI on the proportional problem-solving performance of middle school students with LD. The 12 sessions of the intervention were delivered 3-4 times a week with each session lasting approximately 60 minutes. The posttest effect size comparing the two intervention groups was significant (ES = 1.69), meaning that students in the SBI condition performed significantly higher than students in the GSI condition.

Second, Jitendra et al. (2007) assessed the difference between SBI, a single problem-solving strategy, and GSI, multiple problem-solving strategies. The researchers utilized a pretest-posttest design. Multiple problem-solving strategies in this investigation referred to using objects, acting the word problem out or drawing a diagram, choosing an
operation, writing a number sentence, and using data from a table or graph. A four-step word problem solving procedure was also taught consisting of the following steps: read and understand the problem, plan to solve the problem, solve the problem, and look back or check. Researchers measured the effects on problem-solving, math achievement, and the influence of word problem instruction on computational skills for third-grade students. The 32 lessons of the intervention were delivered during the regularly scheduled math period for 25 minutes daily, 5 days per week, over 9 weeks. SBI was found to be more effective than GSI as determined by posttest scores. The effect sizes comparing the SBI and GSI group were .52 on the immediate posttest, and .69 on a maintenance test administered 6 weeks later.

Griffin and Jitendra (2009) also compared SBI to GSI to investigate the effects of word problem-solving strategy instruction for third-grade students utilizing a between subjects, experimental, pretest-to-posttest-to-delayed-posttest design. The 20 instructional lessons for the intervention were implemented 1 day per week. As for progress on word problem solving during the intervention, a large effect size was found for the SBI condition (ES = .94) when compared to the GSI group. Results, however, also revealed that all students, regardless of whether they received SBI or GSI, improved in their word problem-solving performance (ES = .78) from pretest to posttest. The word problem measure included 16 problems that totaled 32 possible points. The SBI group had a mean score of 16.88 on the pretest and a mean of 22.5 on the posttest, while the GSI group also showed improvement from a mean score of 17.4 on the pretest to 21.7 on the posttest.
Additionally, Jitendra et al. (2013) studied the effects of SBI in comparison to a school-provided standards-based curriculum (SBC) for third-grade students at risk for math difficulty. The researchers utilized a pretest-intervention-posttest-retention test design to measure the skill of additive problems to represent and solve word problems. In addition to the regular 60 minutes of core math instruction from classroom teachers, students received 30 minutes of supplemental math instruction using assigned program (SBI or SBC) five days a week for 12 weeks from a trained tutor outside the classroom. Both groups improved by 25% from pretest to posttest on the word problem-solving measure. The SBI group increased from 51% to 76%, and the SBC group improved from 53% to 78%. Results also indicated that SBI students with higher pretest scores outperformed SBC students with higher pretest scores, whereas SBC students with lower pretest scores outperformed SBI students with lower pretest scores. The majority of these research studies demonstrated that the SBI intervention has had positive results across ability levels and grade levels, even when compared with general problem-solving strategies. The next section provides a synthesis of the 15 research studies that concerned problem-solving intervention in the area of mathematics.

**Synthesis of the Research Studies**

This synthesis presents an overview of the current research on problem-solving for students both with and without disabilities in both elementary and secondary settings. The results of the studies reviewed suggest that strategy instruction effectively influences the problem-solving performance for the following populations: (a) general education students (3 studies), (b) students both with and without disabilities (8 studies), and (c)
students with learning disabilities (4 studies). Improvement in math problem-solving ability was also noted at all grade levels, including elementary (6 studies), middle school (8 studies), and high school (1 study). The research for the problem-solving interventions will be discussed according to the following areas: mastery of learning, duration, instructional features, and mathematical skills addressed.

Mastery of Learning

Cognitive strategy instruction has demonstrated improvements in verbalization of both problem representation and solution for students both with and without LD (Hutchinson, 1993; Montague et al., 2011). SBI has increased strategy use and math proficiency for students with LD in middle school (Jitendra et al., 2002). Mean test scores have increased for students with LD in the third and fourth grades after the SBI intervention (Jitendra & Hoff, 1996). SBI has also demonstrated gains for high-achieving students’ problem-solving scores (Jitendra, 2012).

Three of the 15 studies reviewed found SBI effects to be higher than GSI for students with and without disabilities and at risk for low achievement in math at both the elementary and middle school levels (Griffin & Jitendra, 2009; Jitendra et al., 2007; Xin et al., 2005). Additional studies demonstrated higher improvement for the SBI group over the control group in both the third and seventh grades (Fuchs et al., 2004; Jitendra et al., 2009; Jitendra & Star, 2012). Evidence also supports higher performance for the SBI group when compared to a traditional instruction group for students in the elementary setting with mild disabilities (Jitendra et al., 1998).
**Duration**

The interventions for cognitive strategy instruction were quite lengthy, lasting 4-7 months (Hutchinson, 1993; Krawec et al., 2013; Montague et al., 2011). Students met with researchers individually for 40 minutes on alternate days over this period of time. Researchers who employed the SBI process with elementary students taught an average of 20 lessons that lasted 35 minutes each and were conducted 2-3 days per week during the intervention (Fuchs et al., 2004; Griffin & Jitendra, 2009; Jitendra et al., 2007; Jitendra & Hoff, 1996). In contrast to SBI studies in the elementary grades, research studies involving SBI at the middle school level were taught for an average of 12 lessons lasting approximately 45 minutes each for about four days per week (Jitendra et al., 2002; Jitendra et al., 2009; Jitendra & Star, 2012; Xin et al., 2005). In addition, some of the studies in the elementary grades lasted 9-14 weeks (Fuchs et al., 2004; Jitendra et al., 2007; Jitendra et al., 2013).

**Instructional Features**

Cognitive instruction was an approach to problem solving consisting of modeling self-questioning by thinking aloud, providing prompts, and corrective feedback (Hutchinson, 1993). Cognitive instruction involved knowing how to solve a problem including components such as reading the problem, making a drawing, and performing the operation. The SBI process also incorporated a series of steps to solve a problem (Griffin & Jitendra, 2009; Jitendra et al., 1999; Jitendra et al., 2005; Jitendra et al., 2007; Jitendra & Hoff, 1996). The direct, explicit instruction included in SBI assists students to employ the strategy, step-by-step (Jitendra et al., 2002). The diagrams used in SBI
created a relationship between representation of the problem and the solution because students are required to identify relevant information from the problem necessary to find the answer.

**Mathematical Skills in Word Problems**

Regarding specific mathematical skills in word problems, SBI has been used in the area of fractions and percent (Jitendra, 2012). In another study, SBI was applied to 1-step multiplication and division problems for students with LD in middle school (Jitendra et al., 2002). Addition and subtraction word problems were addressed using SBI for students with LD in the third and fourth grades (Jitendra & Hoff, 1996). Finally, researchers who examined the SBI intervention for general education students at seventh-grade level employed the skill of ratios and proportions (Jitendra et al., 2009).

In some cases, cognitive strategy instruction is an effective way to teach and have students remember a process, but the approach can require significant time for students to learn. Schema-based instruction (SBI), another instructional strategy, is a combination of representation and cognitive thinking and has demonstrated success for both general education students as well as students with disabilities. Studies have been conducted using SBI for students with disabilities at the elementary and middle school levels in the areas of addition and subtraction and for students without disabilities in the area of ratios and proportions. There is limited research, however, on the effectiveness of using the SBI process for middle school students with LD to solve proportional reasoning word problems.
**Schema-Based Instruction (SBI)**

Schema-based instruction (SBI) is a problem-solving strategy that applies representation and cognitive instruction and has been successful for elementary and secondary students both with and without disabilities. The history of SBI will be provided first in this section, followed by an explanation of the SBI procedure and phases of the intervention and the connection between SBI and problem solving.

**History of SBI**

SBI is a strategy that applies pictorial representation to word-problem solving through the use of an individual’s schema. Marshall (1986) defined a schema as a memory structure describing common elements in seemingly different situations. According to Marshall, it is necessary to know which information is considered by the student to be important in solving the problem in order to assess use of appropriate schema knowledge. Schema identification instruction makes semantic relations or meanings of words and symbols within the problem explicit by emphasizing a logical structure to organize the information from the problem (Marshall, 1987).

**Description of SBI**

SBI uses diagrams to solve problems and includes a training period with worked problems. In the SBI approach, each mathematical problem type (i.e., addition, multiplication, proportion) is presented first as a story situation with the answer so students focus on the identification and representation of the features of the story situation using schematic diagrams (Jitendra et al., 2010). SBI focuses on recognizing a problem’s schema, using a diagram based on the schema (schemata diagram), and solving
the problem. The schema of a problem is the way it is organized or the pattern that guides problem representation (Jitendra et al., 2002). Schemata diagrams are diagrams of problem patterns and structures used in SBI that have successfully improved students’ problem-solving skills (Jitendra et al., 2007). Schemata diagrams facilitate translation of the problem in order to arrive at the solution by emphasizing the identification of relationships among information in a problem (see Figure 1). The components of SBI include knowing what the problem is asking and recognizing the schema in order to understand how to arrive at the solution (Jitendra et al., 2002). Figure 1 shows that if the car travels 25 miles on one gallon of gas, then the car is able to travel 75 miles on three gallons of gas. The figure also shows that the ratios of 1/25 in the “if” column and 3/75 in the “then” column are equivalent/proportional.

**Vary.** A car travels 25 miles on a gallon of gas. It can travel 75 miles on 3 gallons of gas.

![Figure 1. Example of schemata diagram](image)

In contrast to schemata diagrams, simple illustrations of word problem content do not highlight the mathematical operation to employ because they may omit the numerical data necessary to solve the problem (refer to left side of Figure 2). Knowledge of the schema in word problems activates relevant patterns that would guide problem representation necessary to solve problems (refer to right side of Figure 2). An advantage of SBI is its focus on identification of problem types, looking beyond the surface similarities and considering the schema. The structure similarity or “schemata” of a problem is the way it is organized or the pattern that guides problem representation. For example, a proportional problem includes an association between two things, as illustrated in the following problem: *Erica uses 7 gallons of gas to drive 14 miles. How many miles can she drive using 1 gallon of gas.* The structure or schema of the problem involves two things being compared, gallons of gas used and miles driven. Once the schema is determined, important information in the problem can be mapped using schemata diagrams.
SBI incorporates scaffolding of student learning by providing explicit instruction (Jitendra & Star, 2012). In scaffolding instruction, the teacher gradually reduces support as the student becomes more proficient with the task being learned. Applying scaffolding in SBI for mathematics problem solving involves teacher demonstration of word problem solving, guided practice with teacher support, and gradual fading of that support to let students independently solve problems. Due to the difficulty students with LD may have storing information for a short period, teaching these students to recognize the schema in math problem solving is critical to reduce working memory resources and cognitive load (Jitendra & Star, 2012). Once students know how to read and understand the problem
asked and can organize the word problem information, they may have a better understanding for what the problem is about. Then the students can use their working memory to focus on solving the problem and reduce cognitive load, as opposed to having to use their working memory to both organize and solve the problem at the same time.

Successful problem solving requires the ability to accurately translate information in a word problem in order to plan how to arrive at the solution. Translating a proportional word problem involves placing a question mark where the missing information belongs in the schemata diagram, then developing a mathematical number sentence that leads to finding the solution. Figure 3 demonstrates how information in a proportional word problem would be translated into a number sentence. In multiplying the cross products, a question mark indicates what information is missing in the equation $42 \times ? = 630 \times 1$.

*The Dale Company packs 42 cans of tomatoes in each crate. How many crates will the company need to pack 630 cans of tomatoes?*

If

```
Cans of tomatoes
42 cans
```

Then

```
630 cans
```

```
42 \times ? = 630 \times 1
```

```
42 \times 15 = 630
```

*Figure 3. Sample of translating information in schematic diagram*
**Explanation of Steps in SBI Process**

In the first step of the SBI approach, each problem schema is presented first as a story situation that contains only known information so students are able to focus on representing information in the problem using schematic diagrams. This step of instruction is an overview of schema instruction (Jitendra, 2007). In this overview, students are taught to identify the problem schema and to represent the story information in schematic diagrams. For example, students may be presented with a story that states *If two baskets hold six flowers, then there are 24 flowers in eight baskets* (see Figure 4). The student would identify the two things being compared and label the two columns (flowers and baskets in Figure 4), and then set up the first ratio of six flowers to two baskets in the box and oval as shown and enter the remaining ratio in the box and oval below. The purpose of this overview is for students to interpret the main concepts in the story situation, discard all unnecessary information, and map the details of the story onto the schema diagram to display the mathematical relationship.
If two baskets hold six flowers, how many flowers will eight baskets hold?

If 6 flowers Then ? flowers

Answer __________ 24 flowers __________________

Figure 4. Sample of story situation and schematic diagram
Adapted from Solving math word problems: Teaching students with learning disabilities using schema-based instruction by A. K. Jitendra, 2007, p. 222.

Jitendra (2007) referred to the second step of SBI instruction as “problem solution.” During the problem-solution step, students learn to solve problems with unknown information. To assist students with following a sequence to solve the problems, FOPS is used as an acronym representing each step. Each letter in FOPS stands for a task, as follows: F- Find the problem type, O-Organize the information in the problem using the diagram, P-Plan to solve the problem, and S- Solve the problem (Jitendra et al., 2009). The goal of acquisition of the SBI process is for students to learn the steps to mastery and show evidence of application of the SBI process in the work they show for each problem. Another prerequisite of SBI is that when the word problems are first introduced, the students need to be able to read and understand them (Jitendra, 2007) because the goal of SBI is for students to learn how to use the process.
Summary

Pictorial representation and cognitive instruction play an essential role in problem solving instruction, especially for students with LD. This review of research indicates that the SBI approach to problem solving incorporates both representation and cognitive instruction. SBI can be implemented by the classroom teacher and infused into daily instruction with grade-level curriculum skills and standards. Studies have been conducted using SBI for students with disabilities at the elementary and middle school levels in the area of addition and subtraction (Jitendra & Hoff, 1996) and for students without disabilities in the area of ratios and proportions (Jitendra et al., 2009). However, there is limited research on the effectiveness of the SBI process for middle school students with LD in the area of proportional reasoning. The area of proportional reasoning in mathematics is so important for secondary students with LD to know because this foundational skill of proportional reasoning will assist with learning of more complex secondary mathematical content (i.e., algebra and geometry).

Purpose and Research Questions

The purpose of the current study was to examine the functional relation between the SBI process and solving proportional reasoning word problems for middle school students with LD. The word problems studied involved proportional reasoning skills. The components of schema-based instruction (SBI) include recognizing the schema and then using schemata diagrams to arrive at the solution; these components support problem-solving instruction for students with LD (Jitendra et al., 2002). The functional relation of SBI was examined by whether students with LD increased the number of categories
completed correctly for the SBI process to solve proportional reasoning word problems. Implementing SBI for the identification of math problem schema was anticipated to allow students to represent the problem accurately and to calculate that information to get an accurate solution. Teaching the SBI process also incorporated FOPS, a 4-step problem-solving strategy developed by Jitendra et al. (2011). A FOPS checklist was referred to as the following steps were introduced and explained: F- Find out if it is a proportional problem, O-Organize the information in the problem using the diagram, P-Plan to solve the problem, and S- Solve the problem.

Self-efficacy of participants was also measured before and after the intervention. Questions on self-efficacy were included in a preassessment and social validity measure given after the intervention. Self-efficacy beliefs help students maintain the effort and perseverance needed to compensate for their low academic ability or to maximize the high ability they already possess (Usher & Pajares, 2006). Reporting the self-efficacy measure of students’ beliefs about their abilities gave more clarity to the improvement of participants in the area of problem-solving skills by providing insight about the relationship between students’ individual beliefs in their capability to problem solve and their actual performance in solving math word problems. Moreover, many studies have not measured self-efficacy in math problem solving (or other content areas) both before and after the intervention.

The specific population targeted for the SBI intervention in the current study was seventh-grade middle school students with LD; the curriculum skills of unit rates,
proportions, and scale drawings were the focus. A multiple-baseline design across groups of participants was employed. Specific research questions were:

1) Does the number of categories completed correctly in the SBI process increase after the process is taught, using proportional reasoning mathematical word problems with and without the use of calculators, for middle school students with LD?
   1a) Does the number of categories completed correctly to solve proportional reasoning mathematical word problems increase without calculators?
   1b) Does the percentage of accurate answers increase after the SBI process is taught without calculators?
   1c) Does the number of categories completed correctly to solve proportional reasoning mathematical word problems increase with calculators?
   1d) Does the percentage of accurate answers increase after the SBI process is taught with calculators?

2) Is the number of categories completed correctly in the SBI process maintained 4 weeks after instruction with and without the use of calculators?
   2a) Do students maintain the number of categories completed correctly to solve proportional reasoning mathematical word problems 4 weeks after instruction without calculators?
   2b) Is the percentage of accurate answers maintained 4 weeks the SBI process is taught without calculators?
2c) Do students maintain the number of categories completed correctly to solve proportional reasoning mathematical word problems 4 weeks after instruction with calculators?

2d) Is the percentage of accurate answers maintained 4 weeks after the SBI process is taught with calculators?

3) Do middle school students with LD increase self-efficacy for solving mathematical word problems after the SBI process is taught?
III. METHODS

The schema-based instruction (SBI) process was used to examine the solving of proportional reasoning word problems for middle school students with learning disabilities (LD; disability category of LD later becomes high-incidence disabilities, HID; refer to Participant section). SBI utilizes diagrams of problem patterns and structure, and this intervention has been successfully used to improve students’ word problem performance in mathematics (Fuchs et al., 2004; Jitendra et al., 2002; Jitendra et al., 2007; Jitendra et al., 2011; Jitendra & Hoff, 1996; Jitendra & Star, 2012).

There were three phases of this intervention. Phase I was a baseline period when data were gathered from students about performance on word problems, which involved proportional reasoning skills. Three types of proportional reasoning problems were on the probes: unit rates, proportions, and scale drawings. After baseline data were gathered in Phase I, students received a 1-day overview about the SBI process. During the overview, the researcher showed students the SBI process using completed problems (i.e., already worked problems). The purpose of the overview was to expose students to the vocabulary of the proportional reasoning problems and to how numbers were placed on a diagram. Although students actively participated during the overview (i.e., by answering questions), they were not expected to master any of the SBI content at that point. Using
already worked problems helped students become familiar with the key information and mathematical relationships among numbers (Marshall, 1995; Steele, 2005). On the SBI overview day, no probe data were gathered.

In Phase II (SBI process), students were taught the SBI process via the explicit instruction process for solving word problems that had missing information. Explicit instruction consisted of demonstration, guided practice with scaffolding, and independent practice. During Phase II, students were taught and practiced entering information from the word problem into diagram, determining missing numbers needed to calculate an answer, writing a number sentence, and solving word problems. Data were collected prior to each session in Phase II before any instruction occurred. Students completed the same type of probe that they completed during baseline.

Finally, during Phase III, maintenance was measured. These data were collected four weeks after the SBI process instruction ended. The same type or probe was used throughout the study to determine whether the students were able to solve the proportional reasoning word problems.

The three proportional reasoning types were taught separately in this research for two reasons. First, these types are taught separately in the grade-level mathematics curriculum. Second, students with learning disabilities benefit from having information introduced one skill at a time. Each of the three proportional reasoning types had different stages of difficulty, or scaffolds of instruction, for computation. These scaffolds of instruction are described in more detail in this chapter.
This chapter provides information regarding the research design, the setting, recruitment of and eligibility procedures for identification of participants, materials used, and an explanation of procedures. The total number of participants was nine, which comprised three groups of three students, and the intervention was taught by the researcher. Materials for the SBI intervention included a student workbook (see Appendix A) that contained key terms related to SBI, a visual of the “if-then” diagram (see Appendix B), and a checklist of problem-solving steps (see Appendix C). The “if-then” diagram was a graphic organizer used to display word-problem information. The “if-then” diagram included four shapes used in a horizontal and vertical manner to guide students’ placement of numbers from the word problems, and this formation of the numbers resulted in a proportion.

The methods used for this SBI intervention were employed to address the following research questions (disability category of LD in research questions later becomes HID; refer to Participant section):

1) Does the number of categories completed correctly in the SBI process increase after the process is taught, using proportional reasoning mathematical word problems with and without the use of calculators, for middle school students with LD?

1a) Does the number of categories completed correctly to solve proportional reasoning mathematical word problems increase without calculators?

1b) Does the percentage of accurate answers increase after the SBI process is taught without calculators?
1c) Does the number of categories completed correctly to solve proportional reasoning mathematical word problems increase with calculators?

1d) Does the percentage of accurate answers increase after the SBI process is taught with calculators?

2) Is the number of categories completed correctly in the SBI process maintained 4 weeks after instruction with and without the use of calculators?

2a) Do students maintain the number of categories completed correctly to solve proportional reasoning mathematical word problems 4 weeks after instruction without calculators?

2b) Is the percentage of accurate answers maintained 4 weeks the SBI process is taught without calculators?

2c) Do students maintain the number of categories completed correctly to solve proportional reasoning mathematical word problems 4 weeks after instruction with calculators?

2d) Is the percentage of accurate answers maintained 4 weeks after the SBI process is taught with calculators?

3) Do middle school students with LD increase self-efficacy for solving mathematical word problems after the SBI process is taught?

**Research Design**

Single-subject designs, such as multiple-baseline-across-groups, seek to establish a functional relation between the targeted behavior and the intervention over repeated instruction and manipulation of the independent variable (Gast, 2010). A multiple-
baseline-across-groups design was utilized for the current study, which included baseline and staggered treatment phases. The multiple baseline design permitted evaluation and demonstration of intrasubject direct replication, which increased the external validity of the findings because the performance of each of three groups had the potential to demonstrate an effect (Gast, 2010). This design also provided a practical means to teach a nonreversible academic skill, which was mathematical word problem performance for the current intervention. The multiple-baseline design repeatedly monitored progress over time. The start of the intervention was staggered across groups, and the order in which each group was introduced to intervention was determined randomly.

According to Kratochwill et al. (2010), single-subject design studies must meet the following criteria in order to meet design standards:

1. Systematic manipulation of the independent variable, which means the researcher determines when and how the independent variable conditions change
2. Interobserver agreement in each phase, which means each variable must be measured systematically over time by more than one assessor for at least 20% of the data points in each condition
3. Three attempts to demonstrate an effect at three different points in time or with three different phase repetitions (i.e., multiple baseline designs with at least three baseline conditions)
4. At least three data points per phase are required to qualify as an attempt to demonstrate an effect
The current study met these four different criteria in order to meet design standards set by Kratochwill (2010).

Participants

There were four waves of selection criteria for students to participate in the research. The first wave was specific to teacher recommendations, the second was attendance information, the third was parent permission, and the fourth wave was specific to an initial assessment on multiple computations. Before the intervention took place, parents’ informed consent was obtained to gather data for student ability levels, and then an initial assessment on multiple computations was administered to students.

Participant Selection Criteria

Twelve students were targeted for the intervention to comprise at least three groups of three students in each group. Seventh-grade students with learning disabilities (LD) were the targeted population for this SBI intervention involving proportional reasoning problems. To increase the number of participants, the final sample of students comprised both seventh- and eighth-grade students. Three of the students in the sample were identified as having other high-incidence disability (HID) categories than LD (two with other health impairments and one with autism and an LD in mathematics). In the case of this study, nine students were included as participants for the intervention.

After school-system research permission was acquired, both phone and email contact was made with the school administrator. At that time, the researcher provided the school administrator with an overview of the research (such as found on the Parent Informed Consent and Student Assent documents; see Appendices D and F), including
the approximate amount of time the students would be investing to participate in the research. The researcher also informed the administrator that official approval had been obtained to conduct the research in their school system.

In the first wave of selection criteria, each student needed to meet the following requirements in the order listed to be considered for the intervention:

1) Student must have had an Individualized Education Program (IEP) indicating a primary or secondary disability of LD, with mathematical problem-solving skills as a goal on the IEP. As noted earlier in this chapter, this criterion was later changed to include students with other high-incidence disabilities (HID; i.e., other health impairments and autism) in order to acquire a sufficient number of participants. The student may or may not have had a calculator accommodation noted on their IEP.

2) Student must have been enrolled in the seventh grade for the 2012-13 school year and must have been 12 or 13 years old to participate. As noted earlier in this chapter, this criterion was later changed to include eighth-grade students (ages 14 and 15) in order to acquire a sufficient number of participants.

3) A recommendation from the previous year’s mathematics and/or special education teacher indicating the student was having difficulty solving word problems involving proportional reasoning was also required.

Waves of participant selection. After 14 eligible students were identified from the first wave of selection criteria, the researcher proceeded with the second wave and obtained attendance information from school office staff for the designated students. A
student with a consistent attendance record during the previous school year could not have been absent more than 3 days per quarter or 12 days during that school year. The eligibility of the designated students was used as a predictor of students whose previous attendance could minimize the possibility of attrition during the study due to attendance issues. From the 13 out of 14 students who had a consistent attendance record, the third wave of student selection began. Parental informed consent to participate in the research (see Appendix D) was acquired through written permission forms sent home with 13 students. A script (see Appendix E) was provided for the teachers to use when distributing the permission forms. Then from a total of 12 students who obtained parental informed consent, each student’s willingness to participate in the research was then collected as indicated by his/her signature on a Student Assent form (see Appendix F).

Although 12 participants received the intervention, if a student had more than three absences during the study, his or her data did not apply for the intervention. The researcher conducted make-up lessons for students who missed one or two lessons. For the current study, data from nine students were used. The remaining students continued to receive the SBI intervention, but their scores were not calculated as part of the study. Table 1 displays eligibility criteria used to determine which students were eligible for the intervention.
Table 1

*Student Eligibility Criteria*

<table>
<thead>
<tr>
<th>Student</th>
<th>Grade</th>
<th>Age</th>
<th>Disability</th>
<th>IEP Goal</th>
<th>Calc.</th>
<th>MSCA Score</th>
<th>Parent Consent</th>
<th>Student Consent</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>7</td>
<td>13</td>
<td>Autism</td>
<td>√</td>
<td>√</td>
<td>66% (6/10 skills)</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>A2</td>
<td>7</td>
<td>13</td>
<td>LD</td>
<td>√</td>
<td>√</td>
<td>50% (3/10 skills)</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>A3</td>
<td>7</td>
<td>14</td>
<td>OHI</td>
<td>√</td>
<td>√</td>
<td>60% (6/10 skills)</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>B1</td>
<td>8</td>
<td>14</td>
<td>LD</td>
<td>√</td>
<td>√</td>
<td>57% (5/10 skills)</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>B2</td>
<td>7</td>
<td>13</td>
<td>OHI</td>
<td>√</td>
<td>√</td>
<td>63% (7/10 skills)</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>B3</td>
<td>8</td>
<td>15</td>
<td>LD</td>
<td>√</td>
<td>√</td>
<td>53% (6/10 skills)</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>C1</td>
<td>8</td>
<td>13</td>
<td>LD</td>
<td>√</td>
<td>√</td>
<td>77% (8/10 skills)</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>C2</td>
<td>8</td>
<td>13</td>
<td>LD</td>
<td>√</td>
<td>√</td>
<td>53% (5/10 skills)</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>C3</td>
<td>7</td>
<td>14</td>
<td>LD</td>
<td>√</td>
<td>√</td>
<td>57% (6/10 skills)</td>
<td>√</td>
<td>√</td>
</tr>
</tbody>
</table>

*Note.* “PSSA” = Pennsylvania System of School Assessment and the Reading level was measured by the Science Research Associates (SRA) Reading Test. Below B refers to “below basic.” For ethnicity: “C” = Caucasian, “H” = Hispanic, and “M” = mixed race.

**Multiple Skills Computation Assessment (MSCA).** Criteria for student selection in the fourth wave involved using the results of a Multiple Skills Computation Assessment (MSCA) created by the researcher using an online mathematical worksheet generator (Intervention Central, n.d.). The results of the MSCA were used to determine whether a student proceeded further with the intervention by demonstrating proficiency in at least three of the 10 skills assessed. Twelve of the 13 students met this criterion and were selected to receive the intervention (although only data for nine students who were in consistent attendance to receive the intervention were used). The MSCA was a 30-item customized assessment (see Appendix G) and consisted of 10 skills: basic facts in addition, subtraction, multiplication, and division; addition and subtraction problems with
and without regrouping up to 1000; multiplication of two-digit numbers; and division by two-digit numbers. There were three computations included for each of the 10 skills.

There were two reasons for including multiple skills on the MSCA. First, the researcher could form intervention groups among similarly skilled students. Second, the researcher could identify students’ computational skill level proficiency and use that information to develop appropriately challenging word problems for each group. Parents were to be informed in writing if their child did not meet criteria for the intervention and was not eligible to participate. However, this stipulation was not applicable in the current study because all of the students who returned the consent forms met all eligibility criteria for the intervention. Individual results were displayed on a matrix (see Appendix H) of three columns including the name of the particular skill, recording of a slash (/) in the first column if the skill was demonstrated on all of those type of problems on the MSCA, and writing of a zero (0) in the second column if there were errors on some problems for a particular skill. For example, for a student who only had basic addition problems correct, a “/” was entered in the second column after “basic addition,” and a “0” in the third column for the remaining skills.

The MSCA employed two versions consisting of problems that have the same operations and types of numbers. Students were directed to respond with a calculator on one version and without a calculator on the other version. Some students with disabilities who had an IEP with mathematical objectives have a calculator accommodation, so allowing all students to use a calculator on one version of the MSCA adhered to that need. In addition, the researcher provided students with an extra sheet of paper to work
out problems to provide the “needs extra space” accommodation or in case they just
needed more room.

**Student Characteristics**

The categories of demographic data acquired for each student are identified in
Table 2. These demographic categories include gender, grade, primary disability,
ethnicity, proficiency level on standardized state mathematics assessment, and reading
level. The first three students listed are from Group A (A1= first student in Group A), the
next three students are from Group B, and the final three from Group C respectively.
Although there was a specific procedure for creating groups (see Procedures section),
both students in Group C who had scores available for the PSSA (Pennsylvania System
of School Assessment) test ended up having below basic performance on that measure.
Further descriptions of each student participant include: academic and behavior
characteristics, additional school services, report card grade in mathematics, total years in
special education, and any instructional adaptations.
Table 2

*Participant Demographics*

<table>
<thead>
<tr>
<th>Student</th>
<th>Gender</th>
<th>Age</th>
<th>Grade</th>
<th>Ethnicity</th>
<th>Primary Disability</th>
<th>PSSA score</th>
<th>Reading level</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>F</td>
<td>13</td>
<td>7</td>
<td>C</td>
<td>Autism</td>
<td>Basic</td>
<td>4&lt;sup&gt;th&lt;/sup&gt;</td>
</tr>
<tr>
<td>A2</td>
<td>F</td>
<td>13</td>
<td>7</td>
<td>C</td>
<td>LD</td>
<td>Basic</td>
<td>5&lt;sup&gt;th&lt;/sup&gt;</td>
</tr>
<tr>
<td>A3</td>
<td>M</td>
<td>14</td>
<td>7</td>
<td>C</td>
<td>OHI</td>
<td>Basic</td>
<td>6&lt;sup&gt;th&lt;/sup&gt;</td>
</tr>
<tr>
<td>B1</td>
<td>F</td>
<td>14</td>
<td>8</td>
<td>M</td>
<td>LD</td>
<td>Basic</td>
<td>7&lt;sup&gt;th&lt;/sup&gt;</td>
</tr>
<tr>
<td>B2</td>
<td>M</td>
<td>13</td>
<td>7</td>
<td>M</td>
<td>OHI</td>
<td>Basic</td>
<td>7&lt;sup&gt;th&lt;/sup&gt;</td>
</tr>
<tr>
<td>B3</td>
<td>M</td>
<td>15</td>
<td>8</td>
<td>C</td>
<td>LD</td>
<td>Basic</td>
<td>7&lt;sup&gt;th&lt;/sup&gt;</td>
</tr>
<tr>
<td>C1</td>
<td>M</td>
<td>13</td>
<td>8</td>
<td>C</td>
<td>LD</td>
<td>Basic</td>
<td>7&lt;sup&gt;th&lt;/sup&gt;</td>
</tr>
<tr>
<td>C2</td>
<td>M</td>
<td>13</td>
<td>8</td>
<td>C</td>
<td>LD</td>
<td>Basic</td>
<td>6&lt;sup&gt;th&lt;/sup&gt;</td>
</tr>
<tr>
<td>C3</td>
<td>M</td>
<td>14</td>
<td>7</td>
<td>H</td>
<td>LD</td>
<td>Basic</td>
<td>4&lt;sup&gt;th&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

*Note.* “PSSA” = Pennsylvania System of School Assessment and the Reading level was measured by the Science Research Associates (SRA) Reading Test. Below B refers to “below basic.” For ethnicity: “C” = Caucasian, “H” = Hispanic, and “M” = mixed race.

**Group A: Student A1.** Student A1 was a 13-year-old seventh grader in a junior-senior high school. She was diagnosed as a student with autism approximately eight years prior to the study and had a secondary disability of LD in mathematics and a speech and language impairment. Her IEP indicated that she received speech and occupational therapy services and special education resource support in mathematics. Student A1 was proficient with basic mathematical facts. She had difficulty, however, with regrouping in addition and subtraction problems and 2-or-more digit multiplication and division problems. Student A1 also received a calculator accommodation. As far as behavior in mathematics, the classroom teacher reported that Student A1 demonstrated excellent work habits, cooperated with peers, and was respectful to authority. She participated
during class and benefitted from structure and reminders/visuals in the learning process, and her report card grade in mathematics with a modified curriculum was an “A.”

**Group A: Student A2.** Student A2 was also a 13-year-old seventh grader in the same junior-senior high school. She was identified as a student with a specific learning disability (LD) and had been receiving special education services almost 2 years prior to the study. Student A2 knew basic mathematical addition, subtraction, and multiplication facts. She had difficulty with division, regrouping in addition and subtraction problems, and multiplication and division problems with two or more digits. Student A2 also had a calculator accommodation on her IEP. In mathematics class, the classroom teacher reported that Student A2 was hesitant to participate in class at times. She would complete assignments, though not always accurately, and did not ask questions for clarification. Student A2 was also respectful to authority. Student A2 performed at the basic level (below average) on the most recent PSSA test in mathematics, and her report card grade in mathematics with modifications was a “B.” In addition, Student A2 read on a fifth-grade level as measured by the SRA Reading Test.

**Group A: Student A3.** Student A3 was a 14-year-old seventh grader in the same school. He had been receiving special education services as a student with other health impairments (OHI) due to an auditory processing disorder about 2 years prior to the study. Student A3 was proficient with basic mathematical facts involving all operations. He had difficulty, however, with multiplication and division problems of 2-or-more digit numbers. Student A3 also had a calculator accommodation in mathematics on his IEP. When motivated, Student A3 completed assignments and participated in mathematics
class; however, his work habits were inconsistent. According to the classroom teacher, he
was positive toward adults and peers, and his report card grade in mathematics with
resource support was an “A.”

**Group B: Student B1.** Student B1 was a 13-year-old seventh grader in the same
school. He was diagnosed as a student with OHI 5 years prior to the study due to a
diagnosis of Attention Deficit Hyperactivity Disorder (ADHD). Student B1 received
special education resource support in mathematics and physical therapy services.
Students B1 knew basic mathematical addition, subtraction, and multiplication facts;
however, he had difficulty with division and multiplication and division problems with
two or more digits. Student B1 also had a calculator accommodation on his IEP. In
mathematics class, the classroom teacher noted that Student B1 liked to get work
completed but may have rushed through it and not have been as accurate. Regarding
overall school behavior, he respected adults and peers in the classroom, and his report
card grade in mathematics with a modified curriculum was an “A.”

**Group B: Student B2.** Student B2 was a 14-year-old eighth grader in the same
school. She had been receiving special education services as a student with LD for 5
years prior to the study. Student B2 was proficient with basic mathematical facts
involving all operations and regrouping in addition and subtraction problems. She was
not as accurate with multiplication and division problems of 2-or-more digit numbers.
Student B2 also had a calculator accommodation on her IEP. In mathematics class, the
classroom teacher reported that Student B2 followed directions, cooperated with peers,
and respected others. Overall, she had good work habits, enjoyed learning, and did well
in school. Student B2’s report card grade in mathematics with resource support was a “B.”

**Group B: Student B3.** Student B3 was a 15-year-old seventh grader in the same school. He was also identified as a student with a LD and had been receiving special education services almost 7 years prior to the study. Student B3 was proficient with basic mathematical facts, but he had difficulty with regrouping in addition and subtraction problems and 2-or-more digit multiplication and division problems. Student B3 also had a calculator accommodation on his IEP. According to the classroom teacher, Student B3 participated orally in mathematics class yet was behind on written assignments. He was also cooperative and respectful to authority, and his report card grade in mathematics with modifications was a “B.”

**Group C: Student C1.** Student C1 was a 13-year-old eighth grader in the same school. He was identified as a student with LD and transferred to the school a year prior to the study. His records only included the most recent evaluation, so it was not known how long he had been receiving special education services. Student C1 was proficient with basic mathematical facts involving all operations and regrouping in addition and subtraction problems. He was not as accurate with multiplication and division problems of 2-or-more digit numbers. Student C1 also had use of a calculator for application problems in mathematics on his IEP. The classroom teacher in mathematics noted that Student C1 had excellent potential when focused but was easily distracted by peers. He benefitted from redirection to the task and displayed satisfactory social skills most of the time. At times, Student C1 had verbal disagreements with peers that affect learning in
some situations, yet he responded to teacher intervention well. Student C1’s current report card grade in mathematics with resource support was an “A.”

**Group C: Student C2.** Student C2 was also a 13-year-old eighth grader in the same school. He was diagnosed as a student with a LD almost 9 years prior to the study and also had speech language impairment. He received speech language services in addition to special education resource support in mathematics. Student C2 knew basic mathematical addition, subtraction, and multiplication facts. He had difficulty with division, regrouping in addition and subtraction problems, and multiplication and division problems with two or more digits. Student C2 also had a calculator accommodation on his IEP. Regarding mathematics class, the classroom teacher noted that Student C2 took time to process class material, was hesitant to ask questions, and did well working one-on-one with an adult. In relation to overall behavior, he was positive to classmates and teachers. Student C2’s report card grade in mathematics with a modified curriculum was an “A.”

**Group C: Student C3.** Student C3 was a 14-year-old seventh grader in the same school. He had been receiving special education services as a student with a LD for approximately eight years prior to the study. Student C3 was proficient with basic mathematical addition, subtraction, and multiplication facts, though he had difficulty with division, regrouping in addition and subtraction problems, and multiplication and division problems with two or more digits. Student C3 also had a calculator accommodation on his IEP. According to the classroom teacher, Student C3 had satisfactory work habits in mathematics most of the time but became distracted by peer behavior when the
opportunity presented itself. He was respectful to authority and bothered by students who were not cooperative. Student C3’s report card grade in mathematics with resource support was an “A.”

Setting

This study was conducted in a public secondary school in the suburbs of the northeastern region of the United States. The school served 959 students in Grades 7-12, and approximately half of the students were eligible for free or reduced price lunch programs. The intervention was implemented by the researcher in an unoccupied classroom of the school building during the students’ mathematics class times. The groups of three students were seated in desks, which were arranged in rows facing the front of the classroom during the intervention lessons. The researcher delivered instruction from the front of the classroom. Materials were placed on a teacher desk on the side of the room near the door. If the researcher was working with one student to clarify any details, there was ample room to walk between the rows of desks. A digital clock was also placed at the front of the room for students to view and for the purpose of recording the starting and ending times for data collection. The perimeter of the classroom was mostly shelves and filing cabinets holding materials from a previous teacher who had been reassigned to another building. The door was kept closed during the lessons to avoid distraction, and the students were not able to see into the hallway.

Independent Variable

The independent variable in the current study was teaching of the SBI process. The SBI process included scaffolding of student learning by providing explicit instruction
in which the teacher gradually reduced support with fading and fewer prompts as the students became more proficient with the process for solving of proportional reasoning word problems (Jitendra & Star, 2011). SBI assists with cognitive load in mathematical problem-solving instruction because the SBI process is presented in smaller steps that are more manageable to process. Due to the difficulty students with LD may have storing information for a short period, teaching these students the process of SBI in mathematical problem solving is critical to reducing cognitive load (Jitendra & Star, 2012).

There were three scaffolds of instruction in Phase II (SBI Process) in the study so that students began learning how to solve proportional word problems with numbers that were at incremental stages of difficulty for the students. Teaching the SBI process for proportional reasoning word problems incorporated a four-step problem-solving strategy known as FOPS and developed by Jitendra (2007). The three scaffolds of instruction for the lessons and the FOPS process will be described in this section.

**Scaffolds of Instruction**

The three scaffolds of instruction were novice, intermediate, and advanced. The word problems in the novice level used numbers at the third- and fourth-grade levels (i.e., multiplication and division of 1- and 2-digit numbers without regrouping), intermediate problems had numbers from the fifth- and sixth-grade levels (i.e., multiplication and division of 1- and 2-digit numbers with regrouping), and the advanced level consisted of word problems including numbers from the seventh- and eighth-grade levels (i.e., multiplication and division with 2- and 3 digits with regrouping). The novice numbers were determined from the numbers the students were proficient with on the Multiple
Skills Computation Assessment (MSCA), which informed the researcher of types of numbers to use for the word problems that were tailored to the skill level of each intervention group. It would not have been appropriate to go from novice numbers to grade-level numbers because there needed to be a gradual shift in degree of difficulty during instruction, so an intermediate level of numbers was also included in the intervention. These three levels of word problems were labeled for each lesson in the teacher script and student workbook. Examples of proportion problems at each level of difficulty can be found in the student workbook (see Appendix A).

FOPS Process

FOPS is a four-step problem-solving strategy developed by Jitendra (2007) that includes the following steps: F- Find out if it is a proportional problem, O-Organize the information in the problem using the diagram, P- Plan to solve the problem, and S- Solve the problem. The steps of the FOPS strategy were demonstrated using a checklist to teach the SBI process for the intervention. The FOPS checklist referring to the schema, or “if-then” diagrams, was briefly overviewed before explicit instruction began in each lesson.

Materials

There were two categories of materials used in this research. The first category was student materials, and the second category was researcher materials. The student materials described in this section include a student workbook, an “if-then” diagram, a FOPS checklist of mathematical problem-solving steps, and calculators. The researcher materials consisted of laminated charts of the “if-then” diagram and FOPS problem-solving strategy checklist, chart paper, a teacher script, and a digital clock.
**Student Materials**

Students were provided with a student workbook that contained key terms related to SBI instruction (see Appendix A), a visual of the “if-then” diagram (see Appendix B), and a FOPS checklist of mathematical problem-solving steps (see Appendix C). The “if-then” diagram was a graphic organizer used to display proportional reasoning word problem information. This diagram included four shapes used in a horizontal and vertical manner to organize numbers from the word problems, and this formation of the numbers resulted in ratios, which then formed proportions.

Calculators were also provided for students to use when completing specific assessments (refer to MSCA and proportional reasoning word problem probes). Although calculators were allowed for a portion of assessments during the intervention, calculators were not used during instruction because the intervention focused on students’ processing of information from the proportional reasoning problems. That is, multiple steps were taught for how to set up and solve proportional reasoning word problems. The proportional reasoning word problem probes (PRWPP) were scored using a rubric that was divided into five categories that will be described later in the chapter. Although accurate calculations are important, the rubric also focused on organizing information from a proportional reasoning word problem.

**Researcher Materials**

Laminated charts of both the “if-then” diagram (see Appendix C) and the FOPS problem-solving strategy checklist (see Appendix D) were used during demonstration and guided practice of the word problems. The “if-then” diagram was a graphic organizer
used to record information from proportional reasoning word problems. Chart paper was used to display the word problems from the student workbook. A digital clock was also displayed so students could record their start time and end time for completing the PRWPP. A teacher script was used for the intervention, which detailed what the researcher was to say and do when implementing the three phases of SBI (Phase I-baseline, Phase II- SBI process, and Phase III- maintenance; refer to Appendix I for the teacher script) and for the overview of the SBI process with worked problems provided between Phase I and Phase II.

**Dependent Measures**

Dependent measures used during the study to assess students’ word problem performance are described in this section. There were two dependent measures: proportional reasoning word problem probes (PRWPP) and the Mathematical Problem-Solving Self-Efficacy Scale (MPSES). The PRWPP were used to assess student progress during baseline (Phase I), the teaching of the SBI process (Phase II), and maintenance (Phase III). During Phase II, students completed the PRWPP at the beginning of each session, prior to any instruction. The MPSES was a pre and postmeasure that elicited how students perceived their self-efficacy in the area of mathematical problem solving.

**Proportional Reasoning Word Problem Probes (PRWPP)**

The proportional reasoning word problem probes (PRWPP; see Appendix J) consisted of six word problems. The proportional reasoning types were unit rates, proportions, and scale drawings (examples given later in this section). There were two of
each type of problem on the PRWPP; three problems were on the front, and three were on the back. Calculators were permitted for the last three problems on the back side of the PRWPP. Content validity of the PRWPP was also assessed (described in detail later in this section). The PRWPP were used to assess student progress during baseline (Phase I), teaching of the SBI process (Phase II), and maintenance (Phase III). The PRWPP were administered before each lesson in Phase II to measure accuracy with the SBI process for proportional reasoning word problems. The computations on the PRWPP were at the seventh- and eighth-grade levels (i.e., multiplication and division with two and three digits with regrouping).

The wording of the word problems was adapted from resources used in middle school mathematics classrooms, including Glencoe Online, Big Ideas Math, and Solving Math Word Problems (Jitendra, 2007). Word problems were randomly assigned to 22 probes according to type, and probes were randomly assigned to lessons. Ten minutes were allotted for students to complete the six problems, and each student was responsible for recording his or her start and end times.

**Types of proportion problems.** Examples of each type of proportion problem are as follows:

- If 24 school buses are needed to bus 960 students to their school, how many students does one bus hold? (unit rate)
- Carl can read 6 pages in 14 minutes. How long will it take him to read a chapter that has 21 pages? (proportion)
• The scale on a blueprint is 1 cm for every 2 m. If the width of an arch is to be 5 m, then how wide should it be drawn on the blueprint? (scale drawing)

In the lessons for all the unit rate word problems, the answer was finding the quantity for one of something. The math problem asked how many students are on one bus in the following example: *If 24 school buses are needed to bus 960 students to their school, how many students does one bus hold?* The proportion problems were similar to unit rate problems except they did not include the quantity of one of something. An example of a proportion problem is as follows: *Carl can read 6 pages in 14 minutes; how long will it take him to read a chapter that has 21 pages?* The scale drawing problems were recognized by a scale described by metric units such as inches or feet, such as, *The scale on a blueprint is 1 cm for every 2 m. If the width of an arch is to be 5 m, then how wide should it be drawn on the blueprint?*

**Validation of content.** Two experts in the area of mathematics were selected to validate the contents used to assess progress with performance on solving grade-level word problems. The criterion for selecting these experts was that they were certified to teach mathematics in middle school (Grades 6-8). The experts chosen were responsible for checking that the word problems on the PRWPP were of similar degrees of difficulty and that the word problems include two problems for each type (unit rate, proportions, and scale drawings). These experts in the area of mathematics also verified that grade-level numbers were used on the PRWPP. An example of a problem that used grade-level
numbers is as follows: If Mr. Hall’s car will run 21 miles on each gallon of gas, then how many gallons will be needed to drive 84 miles?

**Schema-Based Instruction (SBI) process.** Data were separated according to the five rubric categories (refer to section on scoring of PRWPP) of the PRWPP to determine where within the SBI process the students were learning or not learning. The rubric for PRWPP with weighted points yielded a raw score of 16 for each word problem. This raw score did not necessarily mean the students had the correct answer; it meant they accumulated points, as indicated on the graphs of results. The researcher calculated each category of the SBI process for problems completed with a calculator and without a calculator. The percentages for application per category were first calculated per individual student. The individual student data were then averaged with the rest of the students according to each group. Finally, the group data were averaged to provide an overall percentage for application of the FOPS process per category.

**Mathematical Problem-Solving Self-Efficacy Scale (MPSSES)**

The Mathematical Problem-Solving Self-Efficacy Scale (MPSSES) was administered to the students before and after the SBI intervention to elicit how students perceived their competence in mathematical problem solving. Students responded to the following statements using a 4-point Likert scale (*strongly disagree*-1, *disagree*- 2, *agree*-3, *strongly agree*- 4):

1) When I start to solve a mathematical problem, I usually feel I cannot find a solution.

2) I do not feel sure about myself when solving word problems in mathematics.
3) I have difficulty solving mathematical word problems.

The three questions on the MPSSSES were located on the Multiple Skills Computation Assessment (MSCA, which was used with the students only at the beginning of the study to determine their computational level) before the intervention, and the same three questions were also included on the Social Validity Scale after the intervention.

Analysis of Dependent Measures

Rubrics detailing how the PRWPP and the MPSSSES were scored are described in this section. PRWPP measured performance on solving word problems in both the baseline and treatment phases, showing the average performance of each group of students. Self-efficacy was measured before and after the SBI intervention using the MPSSSES to determine how students perceived their competence in the area of solving mathematical word problems.

How proportional reasoning word problem probes (PRWPP) were scored.

The PRWPP measured progress with the SBI process and mathematical problem-solving accuracy. The following rubric described the five categories that were measured on the PRWPP:

1) Drew an accurate number of shapes for the “if-then” diagram ___ / 4 points

2) Labeled words/phrases correctly on the “if-then” diagram ___ / 6 points

3) Inserted the words “if” and ”then” into diagram to indicate type of problem ___ / 2 points

4) Translated diagram from proportion into number sentence
(2-full credit, 1-partial, 0-none) ___/ 2 points

5) Provided correct digits in answer (2-full credit, 1-partial, 0-none) ___/ 2 points

The total amount of points per problem was 16, which meant 96 total possible points on each PRWPP (e.g., 16 x 3 problems without the calculator + 16 x 3 problems with the calculator). The rationale for the higher weight for the first two categories on the rubric (Categories 1 and 2) was that the “if-then” diagram of the four shapes, in which problem information was displayed and labeled, was the more difficult to figure out because the student had to organize information from a proportional reasoning word problem. The labeling of words and phrases needed to be correct for the association between two things to be accurately organized into the proportion that needs to be solved. Because it was critical for the information to be organized, the weight of those points was 2 and 3 times higher than the weight of the last three categories (Categories 3, 4, and 5) on the rubric. If the first two steps were not done correctly, the final answer would most likely be inaccurate.

Regarding Category 2 on the rubric, there was partial credit given if part of the phrase was labeled correctly (i.e., “red” for “red balls”). For Category 4 on the rubric, “translate” had to do with changing the proportion into number sentence by cross-multiplying diagonal numbers (i.e., from 10/1 = 50/5 to 10 x 5 = 50 x 1). A check box for students was also included on the proportional reasoning word problem probe to indicate whether students chose to use a calculator on the last three designated problems that
allowed the choice of using a calculator. The researcher monitored to know when
students used calculators for the last three word problems.

**How Schema-Based Instruction (SBI) Process was scored.** The rubric with
weighted points yielded a raw score of 16 for each word problem, and the final correct
answer was a portion of those points. The rubric scoring meant that the students
accumulated points, as indicated on the graphs of results (provided in Chapter 4). The
categories on the rubrics were separated to determine which steps the students completed
correctly in the SBI process (shapes, labeling, type, number sentence, and/or correct
digits). The percentage was calculated for each category by adding the points earned per
category for each of the three problems and then dividing by the total possible points for
each category for the three problems (see Table 3). The steps in FOPS aligned to the
rubric categories (see Table 3). This breakdown for each category on the rubric for the
PRWPP is for one proportional reasoning word problem (16 points). The scores for
rubric categories per PRWPP, however, are equivalent to 48 points (i.e., 3 problems
without a calculator). For example, the first category was worth 4 points, so the
denominator of the percentage when-calculated was 12 (total amount of points possible
for three problems). The same procedure was followed for the three problems on the back
side of the PRWPP (with calculator).

For each probe administered per designated phase (Phases I, II, and III) for each
student, the three problems on the front side (without calculator) were averaged together
to get a percentage for each category according to the rubric. The percentages of strategy
application per category were first calculated per individual student both with and
without a calculator. The individual student data were then averaged with the rest of the students according to each group. Finally, the group data were combined to provide an overall percentage of strategy application per category both with and without a calculator.

Table 3

**Content of Rubric for PRWPP**

<table>
<thead>
<tr>
<th>Rubric Category Number</th>
<th>Category for SBI Process</th>
<th>Raw Number of Points Per Category Per Problem</th>
<th>Highest Category Percentage of Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>shapes</td>
<td>4 points</td>
<td>25%</td>
</tr>
<tr>
<td>2</td>
<td>labeling</td>
<td>6 points</td>
<td>37.5%</td>
</tr>
<tr>
<td>3</td>
<td>“if-then”</td>
<td>2 points</td>
<td>12.5%</td>
</tr>
<tr>
<td>4</td>
<td>number sentence</td>
<td>2 points</td>
<td>12.5%</td>
</tr>
<tr>
<td>5</td>
<td>correct answer</td>
<td>2 points</td>
<td>12.5%</td>
</tr>
<tr>
<td>TOTAL</td>
<td></td>
<td>16 points</td>
<td>100%</td>
</tr>
</tbody>
</table>

**Procedures**

The schema-based instruction (SBI) intervention consisted of at least 15 scripted lessons lasting 35 minutes each. There were three phases of the study (Phase I- baseline, Phase II- SBI process, and Phase III- maintenance). After baseline data were gathered in Phase I, students received a 1-day overview about the SBI process. During the overview, the researcher showed students the SBI process using completed problems (i.e., already worked problems). The purpose of the overview was to expose students to the vocabulary of the proportional reasoning problems and to how numbers were placed on a diagram.

In Phase II (SBI process), the students were taught the SBI process via the explicit instruction process for solving word problems that had missing information. During
Phase II students were taught to and practiced entering information from the word problem into a diagram, determining missing numbers needed to calculate an answer, writing a number sentence, and solving word problems. The teaching of the SBI process (Phase II) lasted of 9 days for all groups. Finally, Phase III (maintenance) was 1 day, and this day did not include an instructional piece. Table 4 provides an example SBI intervention schedule for Group A, and the schedule similar for Groups B and C.

Table 4

Sample SBI Intervention Schedule for Group A

<table>
<thead>
<tr>
<th>Group</th>
<th>Phase</th>
<th>Problem Type</th>
<th>Scaffold</th>
<th>Duration</th>
<th>Lesson(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>I - Baseline</td>
<td>All</td>
<td></td>
<td>5 days</td>
<td>1-5</td>
</tr>
<tr>
<td></td>
<td>Overview of SBI Process with Worked Problems</td>
<td>All</td>
<td></td>
<td>1 day</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>II - SBI Process</td>
<td>Unit Rate</td>
<td>Novice</td>
<td>1 day</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Intermediate</td>
<td>1 day</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Advanced</td>
<td>1 day</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Proportion</td>
<td>Novice</td>
<td>1 day</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Intermediate</td>
<td>1 day</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Advanced</td>
<td>1 day</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Scale Drawing</td>
<td>Novice</td>
<td>1 day</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Intermediate</td>
<td>1 day</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Advanced</td>
<td>1 day</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td></td>
<td>III - Maintenance</td>
<td>All</td>
<td></td>
<td>1 day</td>
<td>16</td>
</tr>
</tbody>
</table>

Multiple Skills Computation Assessment (MSCA)

The Multiple Skills Computation Assessment (MSCA) was administered prior to the start of the intervention. The MSCA (see Appendix G) was a 30-item customized
assessment developed by the researcher and consisted of 10 mathematical skills: basic facts in addition, subtraction, multiplication, and division; addition and subtraction problems with and without regrouping up to 1000; multiplication of two-digit numbers; and division by two-digit numbers. There were three computations included for each of the 10 skills. There were two reasons for including multiple skills on the MSCA. First, the researcher could form intervention groups among similarly skilled students. Second, the researcher could identify students’ computational skill level proficiency and use that information to develop appropriately challenging word problems for each group for both scaffolding and cognitive load purposes.

**Student Group Selection**

Students were divided into three groups to receive the SBI intervention. The students in each group had similar learning histories and performed at the same levels for the initial assessment (Gast, 2010). There were two ways the students were grouped. First, the students were grouped according to mathematics class times. The students were selected from two mathematics classes, one of all seventh-grade students and the other of seventh- and eighth-grade students combined. The researcher accessed the mathematics class times from a staff member at the school and planned lessons accordingly.

The second way that students were grouped was by their ability level. The second mathematics class the students were pulled from comprised two of the three total groups. Accordingly, the students from the second mathematics class were matched by ability level to determine student placement into each of the groups, with three students in each group. Significant gaps in mathematical ability were indicated by deficits in different
skills on the Mathematical Skills Computation Assessment (MSCA) version taken without a calculator. In this case the grouping of students was according to needs in different skill areas. For example, in the second mathematics class, three of the students had difficulty with basic division and multiplying two or more numbers, so these three students comprised one group. The other set of three students in the second mathematics class had difficulty subtracting with regrouping and dividing two or more numbers, and so they formed another group. Because there were two groups of students in the second mathematics class, they rotated days they met with the researcher (every other day).

**Lesson Plan Framework per Proportional Reasoning Type**

Each of the lessons in the overview of the SBI process and in Phase II included the three instructional components for explicit instruction: demonstration, guided practice, and independent practice. Explicit instruction is the demonstration of new information in small steps, guided practice with examples, and student independent practice until mastery is achieved (Miller & Hudson, 2007). Additionally, each of the three types of proportional reasoning word problems (unit rate, proportion, scale drawing) were taught for three lessons according to the three scaffolds of instruction levels (novice, intermediate, advanced) for each group. For example, Lesson 1 of the intervention was on unit rates at the novice level, and Lesson 2 continued with unit rates at the intermediate level (refer to Table 4). Each day’s lesson plan for all types and levels included teacher demonstration followed by guided practice and independent practice.

**Demonstration.** In each day’s demonstration component of the lesson, the teacher modeled and explained the FOPS strategy by showing how to identify the
important information and how to record the information in the “if-then” diagram using two example word problems.

**Guided practice.** Guided practice began that same day with teacher guidance through two word problems. The teacher stayed in guided practice longer if a student’s practice of the level and type indicated he or she still did not know how to accurately use FOPS or solve problem accurately. This extension of guided practice was determined by whether the student scored at least 80% out of 16 points (at least 14/16) on the third. If 80% on the third word problem in guided practice was not attained, the student did a fourth problem on his/her own. The score on this fourth problem needed to be 80-100% before moving to independent practice.

**Independent practice.** Proficiency continued to be monitored for the three independent practice problems using the same criteria as in guided practice of 80-100% (at least 14 out of 16 points). The score needed to be 80-100% on the third independent practice problem before moving to the next day’s lesson.

**Phase I: Baseline**

The multiple-baseline-across-groups design, which was employed for the current study, included baseline and staggered treatment phases. The length of the baseline phase (Phase I) for each group varied according to the staggered start of the intervention for the multiple-baseline-across-groups design. Phase 1 required at least five data points, which was in agreement with single-subject design standards (Kratochwill et al., 2010), to gather sufficient data from students on word problem performance. The order in which each group was introduced to intervention was determined randomly. Whatever number
was chosen for each designated group was the first day of the intervention, which was also when the overview of schema instruction with worked problems began for each group. The group that was selected to begin the intervention first was designated as Group A, continuing with Group B for the next, and Group C for the last group. Group A was in baseline for 5 days, Group B had 9 days of baseline, and Group C was in baseline for 11 days before the intervention began.

The baseline phase consisted of students’ completion of PRWPP with six advanced word problems that included two problems of each type of proportional reasoning (unit rates, proportions, and scale drawings). An advanced word problem was one written using grade-level numbers (i.e., multiplication and division with two and three digits with regrouping). Each probe was on one 8½-by-11 inch sheet of paper. On the front were the first three problems, and on the back directions indicated that the students could use a calculator for the last three problems. Once students’ papers were turned over and they began Problems 4, 5, and 6, students could not go back to Problems 1, 2, and 3. Permitting calculator use for a portion of the problems showed what students knew with and without a calculator and allowed more focus on the process of solving word problems. The researcher distributed PRWPP to students and directed them to record the start time in the designated area on the probe. A digital clock was also in view of the students so they could record accurate starting and ending times. Although recording of the start and end times allowed measure of fluency, the intent was to acquire these data for reporting versus analysis purposes.
The teacher provided the following directions when administering the PRWPP:

This is a proportional reasoning word problem probe including 6 word problems with ratios, proportions, and scale drawings. First, complete #s 1, 2, and 3 on the front side. Then raise your hand to let me know you are ready to do Numbers 4, 5, and 6 on the back. You can use a calculator when completing those problems. You cannot return to Numbers 1, 2, and 3 once you have the calculator, so be sure to check your work before asking for the calculator. There is a box on the back to check whether you did use a calculator. When you have completed all six word problems, raise your hand to let me know you are finished.

The teacher did not provide prompts or cues for student(s). When the students completed all six problems, the teacher directed them to record the end time on the PRWPP, and then the teacher collected their work. At least five administrations of the PRWPP occurred for each group.

**Overview of Schema-Based Instruction (SBI) Process**

As recommended by Jitendra et al. (2010), the overview of the SBI process with worked problems was when the researcher showed students the SBI process using completed problems (i.e., already worked problems). Using already worked problems helps students recognize and understand the key information and mathematical relationships among numbers (Marshall, 1995; Steele, 2005). The purpose of the overview was to expose students to the vocabulary of the proportional reasoning problems and to how numbers were placed on a diagram. The overview of the SBI process using worked problems utilized teacher-led demonstration and modeling to
identify and underline sentences in story situations (Jitendra et al., 2002). This overview also employed teacher modeling to find information in the problem (i.e., numbers used to set up a proportion) and then record that information onto a diagram.

To introduce the overview of the SBI process (with worked problems), the researcher defined proportional reasoning as an area of mathematics that dealt with a form of measurement and included the following types: unit rates, proportions, and scale drawings. An example of each type of proportional reasoning problem was explained to the students through examples given in the student workbook (see Appendix A). Next, the researcher introduced key vocabulary for proportional problems also found in the student workbook. The key terms and word problems for demonstration and guided practice from the student workbook were displayed on chart paper in the classroom. Before introducing the FOPS process, the researcher referred students to the FOPS checklist in the student workbook. The checklist was referred to as the steps in the FOPS process were introduced, and each step was explained separately.

The FOPS strategy (application of the FOPS process) was demonstrated with word problems that did not have missing information so students were able to receive instruction on the strategy and could then focus on where the information belonged in the “if-then” diagram. The researcher described the problems as usually having an “if-then” kind of statement that tells about an association between two things. Students were introduced to the meaning of “association” during review of key vocabulary (definitions also in student workbook), as a relationship that compares the values of two or more amounts. In an example, the “if-then” statement for the problem, *If 10 boxes fit in 1 truck,*
then 50 boxes will fit in 5 trucks, indicated that the two things with an association were boxes and trucks. The researcher also modeled how to cross off the steps of FOPS as they were implemented on the FOPS problem-solving strategy checklist. Although the “if-then” statement may not have been directly stated in the problem, the researcher modeled how an “if-then” relationship could be constructed from a description of an association between two things and a question that asks about an unknown number. Underlining and circling information in the already worked word problems was also modeled on the chart paper. The boxes for each step (F, O, P, and S) were checked off as they were implemented on a laminated chart displayed in the room throughout the intervention.

**F- Find out if it is a proportional problem.** The researcher explained that in the “F” step of the FOPS strategy, students would read and retell the story to verify that the problem was proportional. The researcher then told the students the way that they retell the story was to describe in their own words or paraphrase. Paraphrase means to give a summary of information. Whether or not the problem was a proportional was justified by whether the problem told about the association between two things. The researcher shared a problem example to demonstrate recording of a word problem into the “if-then” diagram while modeling a think aloud. In the problem, *If there are six seats in each rowboat, then how many seats will be in five rowboats?*, “if” tells about an association between a rowboat and the number of seats in it (six), and “then” statement tells about an association or ratio between rowboats (five) and seats. So, this problem was proportional because it told about the association between the number of rowboats and the number of seats in rowboats.
**O- Organize the information in the problem using the diagram.** The second step of the FOPS strategy, “O,” involved organizing the information using the “if-then” diagram. In the “O” step, the researcher first modeled how to underline the two things that formed a specific ratio and to write those names in the “if-then” diagram. The researcher explained that that the students were to also underline the two things. Then the researcher modeled how to write numbers and write the labels, or names, in the “if-then” diagram (referring to “if-then” diagram in student workbook). After the researcher wrote the numbers for the second ratio (“then” statement in diagram), the researcher checked off the second box under the “O” step.

**P- Plan to solve the problem.** The researcher then told how the third step of the FOPS strategy (“P”) was used in the next phase (Phase II) of the intervention in which students plan an answer by translating information from the diagram and writing a number sentence.

**S- Solve the problem.** The researcher also explained and modeled how the fourth step of the FOPS strategy (“S”) would be applied to solve the problem and checked for accuracy and to see if their answer made sense in the next phase of the SBI process.

**Phase II: SBI process**

The lesson plan framework per proportional type for the SBI intervention began with daily PRWPP. The FOPS checklist referring to the schema, or “if-then” diagrams, was briefly overviewed next (before explicit instruction began). Then demonstration of the FOPS steps in the process of solving word problems at each designated type and level
took place, followed by guided and independent practice. This section includes specific procedures for data collection and instruction in Phase II.

**Data collection in Phase II.** Data were collected daily before each lesson in Phase II (SBI process) using the PRWPP (refer to baseline procedures for administration of PRWPP). Students were allotted 10 minutes to complete PRWPP and were then given their student workbook for the instructional component when they submitted the probe to the researcher. According to the directions for administering the PRWPP (same as baseline procedures), the students were directed to record the start and end times on their probes by accessing a clock in the front of the classroom. Calculators used for the back side of the PRWPP (last three word problems) were given to the students by the researcher when they reached that point on the probe. For transition into the lesson, the students remained in the same seats and were not directed to another location.

**Instruction in Phase II.** For the review of the FOPS strategy, the researcher directed students to refer to the FOPS problem-solving strategy in their student workbooks to recite/review steps. The researcher then prompted students to list and explain the steps with questions such as, *What is Step 1? What do you do in Step 1? What do we do in Step 2? What are the directions for Step 3? Finally, what is Step 4?* The researcher then demonstrated two word problems at the designated type/level. The demonstration of two problems of one type/level was followed by guided practice of at least three problems of the same type/level, and then independent practice of at least two problems of that type/level occurred.
**F- Find out if it is a proportional problem.** In the demonstration component of each lesson in Phase II (SBI process), the researcher explained that the FOPS problem-solving strategy would be applied beginning with the “F” step. The directions for this step included verifying that the word problem was proportional. The researcher modeled a think aloud to find out if it was a proportional problem by reading the problem aloud and retelling it in her own words. When the problem was retold, the researcher asked what students knew and what they were asked to find. The researcher then summarized that the story told about an association, or relationship, between two things. The researcher went on to explain that since there was unknown information, that was the missing the information they were asked to find. The researcher continued to model the “F” step by asking if the problem was proportional by determining whether there was an “if-then” statement, and whether the “if-then” statement told about an association between two things. The researcher also explained that although an “if” and “then” were not directly stated in the problem, there was an association between two things described.

**O- Organize the information in the problem using the diagram.** For the “O” step (second step of the FOPS strategy), the researcher continued to follow the FOPS checklist and crossed off the boxes as each was completed. The researcher demonstrated the “O” step by underlining two things that formed a specific ratio and wrote those two things’ names, or labels, in the “if-then” diagram. First, the researcher asked what the problem talks about. For example, in the problem, *The Dale Company packs 42 cans of tomatoes in each crate. How many crates will the company need to pack 630 cans of tomatoes?* The problem talked about “cans of tomatoes” and “crates.” So those terms were underlined in
the problem. Then “cans of tomatoes” was written to label the first column on the “if-then” diagram, and “crates” was written to identify the second column in the diagram. The researcher then directed students to check off the first box for the second step of the FOPS strategy.

Next, the researcher read each sentence of the problem to circle numbers for each ratio in the proportion and wrote numbers and their labels in the “if-then” diagram. The researcher also clarified that in the first sentence “each” meant one of something, for purposes of recording accurate numbers in the diagram. The students were taught to figure out what must be solved by first identifying what information needed to go in ovals as opposed to boxes on the “if-then” diagram. For “if” in the diagram, the number “42” was circled in the problem. Then “42 cans” was written in the box for the “cans of tomatoes” column, and “each” was circled in the problem and “1 crate” was written in the oval for the “crates” column. The researcher continued to demonstrate the problem-solving steps by reading the next sentence to circle numbers for each association and by writing numbers and labels in the “if-then” diagram.

In the next sentence of the problem, the number of cans was 630, so “630” was circled in the problem and “8 cans” was written in the box (first column) because it told about the number of “cans of tomatoes.” The problems in this SBI process phase included missing information to solve for, unlike the word problems used in the overview of the SBI process using already worked problems. The researcher described how students could show where the missing information was by placing a question mark on the diagram for what must be solved after they found the question sentence. To number and
label the oval for the crates column, the number of crates was unknown, so a “?” was written in the oval for the crates column. Then the second box under the “O” step (second step of the FOPS strategy) was crossed off.

**P- Plan to solve the problem.** In planning to solve the problem, the “P” step of FOPS, the researcher showed how to translate the information from the diagram into a mathematical number sentence. Translate means change into another form. The researcher directed students to look at the “if-then” diagram for the problem and explained that the number of crates that can pack 630 cans of tomatoes needed to be found. This step was planning to solve the problem, so information from the diagram needed to be translated into a mathematical sentence. The researcher demonstrated how to cross multiply to set up a mathematical equation from the proportion. The ratio of 42 cans/630 cans = 1 crate/? crates, so that plan was used to solve the problem, and Step 3 was crossed off on the checklist. Then the researcher showed how to multiply cross products to set up a mathematical sentence from the proportion. The researcher explained that students need to solve the unknown “?” by multiplying cross products to get a number sentence: 42 x ? = 630 x 1. The researcher told students that the “=” sign indicated that the left side of the number sentence was equal to the right side of the number sentence. Next, the researcher explained that they can get the “?” by itself by doing the opposite operation; however, whatever happened on the left side needed to happen on the right because both sides were equal.

**S- Solve the problem.** In the “S” step of the FOPS strategy, the researcher then showed how to perform the opposite operation to solve: 42/42 canceled out and was
equal to 1 on the left side and 630 ÷ 42 was left over on the right side. After applying the cross-multiplication method, the researcher described how to check accuracy of the answer by multiplying the cross products again using all the numbers. Finally, the researcher repeated that the unknown number of crates when she and the students solved 630 ÷ 42 was 15. The researcher then explained that they could check their answer by multiplying cross products: 42 x 15 = 630. The calculation was accurate, and Step 4 (the “S” step) could be crossed off on the checklist.

After the demonstration of three proportional reasoning problems, guided practice began. The guided practice component of each lesson consisted of researcher guidance through three new proportional reasoning problems. If a student’s practice of the level and type on the third problem indicated that he or she still did not know how to accurately use FOPS or solve problems accurately, the guided practice component was extended. The researcher checked student work to see if the student scored at least 80% out of 16 points (at least 14/16) following the rubric for PRWPP on the third word problem, which meant the extension of guided practice would not be necessary. Proficiency continued to be monitored for additional guided practice problems that may have been given using the rubric for PRWPP. Finally, the two problems in the independent practice part of the lessons were also scored for proficiency using the rubric for PRWPP before the next day’s lesson, when another type/level of proportional reasoning problem was presented.
Phase III: Maintenance

Four weeks after the conclusion of the SBI intervention for each group of students, a proportional reasoning word problem probe containing six word problems (two of each type) was administered. The purpose of this phase was to determine maintenance of whether the students were able to apply the SBI process to solve the same types of proportional reasoning problems (unit rates, proportions, and scale drawings) from baseline and each of the phases 4 weeks later. The same rubric for PRWPP used in the baseline and problem-solution phases was utilized for scoring in the maintenance phase. All problems were evaluated on the SBI process using the rubric of five categories for PRWPP (shapes, labeling, “if-then,” number sentence, correct answer). Each problem was 16 points for a total of 96 possible points on each probe (48 points without a calculator and 48 points with a calculator). These raw data were then converted to a percentage (refer to Analysis of Dependent Measures section).

The data were also separated by the five categories on the rubrics for the maintenance probe to determine the number of categories students completed correctly in the SBI process. The percentages for application of the FOPS strategy process per category were first calculated per individual student with and without a calculator. The individual student data for maintenance were then averaged with the rest of the students according to each group. Finally, the maintenance group data were combined to provide an overall percentage of strategy application per category with and without a calculator. It was important to measure maintenance because it helped determine whether the experimental effect was durable over time (Gast, 2010).
Reliability

Procedural Reliability

Procedural reliability during the intervention measured how true the researcher was to administering the PRWPP. Three staff members at the school were assigned to complete procedural reliability checklists (one designated for each of the three groups of students). The designated staff members were chosen according to referrals from the special education department chair as well as staff availability during the time of the intervention. The first staff member designated to complete procedural reliability checklists was certified in special education (K-12) and had been in the classroom 5 years prior to the study. The second staff member was also certified in special education (Grades K-12) and had been in the classroom 10 years prior to the study. Finally, the third staff member was certified in special education (Grades K-12) and elementary education (Grades K-8). The third designated staff member had been teaching mathematics to special education students in Grades 7-12 for 6 years prior to the study.

The three staff members were trained to a minimum of 80% criterion by the researcher so they learned all the components to be evaluated on the procedural reliability checklist (the staff members reached 100% in this training). Thirty percent of these data collection sessions for each group of students in both the baseline (Phase I) SBI process (Phase II), and maintenance (Phase III) were observed using the procedural reliability measure. The designated staff members were instructed to mark each step completed or not completed by the researcher on the procedural reliability checklists (see Appendix L). For example, the checklist for the baseline phase included the following six steps:
1) Distributed the PRWPP to participant(s).

2) Directed student(s) to record the start time on the data collection sheet.

3) Explained to students that they would complete six word problems involving unit rates, proportions, and scale drawings.

4) Directed students to solve the first three problems on front side and raise hand when completed to receive a calculator to do the three remaining problems on the back. Reminded students to check whether he or she did use a calculator in the box on the back side.

5) Did not prompt student(s) to produce the correct responses.

6) Directed student(s) to record the end time on the PRWPP.

The procedural reliability for the baseline (Phase I) and SBI process (Phase II) was calculated by dividing the number of steps completed by the six planned steps in each phase. All six steps in each phase were seen within one session. The calculations resulted in 100% reliability across all sessions and phases measured.

**Fidelity of Treatment**

Fidelity of treatment measured how true the researcher was to following the SBI intervention scripts. The same three staff members (one for each of the three groups of students) who completed the procedural reliability checklists were also recruited to complete fidelity of treatment checklists. The staff members were trained to the minimum of 80% criterion by the researcher using a mock lesson so they learned all the components to be evaluated. Although the minimum for training of staff members for fidelity was 80%, each staff member reached 100% criterion in the training. Thirty
percent of the lessons presented to each group of students in the problem-solution phase were observed using the fidelity of treatment measures. The staff members were instructed to mark each step completed or not completed by the researcher on the fidelity of treatment checklist (see Appendix K) during the designated lessons.

The fidelity of treatment checklist included the following eight steps:

1) Followed teacher script for PRWPP demonstration, guided practice, and independent practice of SBI lessons
2) Referred to visuals and problems in student workbook during lesson
3) Modeled think-alouds and defined unfamiliar terms in word problems
4) Modeled how to cross off the steps of FOPS on laminated poster as they were implemented
5) Displayed key terms and word problems from the student workbook on chart paper
6) Demonstrated how to add problem information to the “if-then” diagram poster during lesson
7) Modeled underlining and circling of information in the word problems on chart paper for demonstration and guided practice word problems
8) Stayed in guided practice longer if a student’s practice of the skill indicated he or she still did not know how to apply the SBI process to 80% mastery (each student’s workbook was reviewed by the researcher to determine this mastery)
The fidelity of treatment was calculated by dividing the number of steps completed by the eight planned steps. All eight steps were seen within one session. The calculations resulted in 100% fidelity across all sessions measured.

**Interobserver Agreement for Scoring of the PRWPP**

There were two scorers who scored the students’ work. One was the primary scorer, and the other person was the secondary scorer. The researcher calculated the scores as the primary scorer. The secondary scorer checked scoring on the PRWPP for 30% of the student work in each phase for each student. The secondary scorer was a staff member at the school who was certified in special education (Grades K-12) and elementary education (Grades K-8). This staff member had been teaching mathematics to special education students in Grades 7-12 for 6 years prior to the study.

All problems were evaluated on the SBI process using the rubric of five categories for PRWPP (shapes, labeling, “if-then,” number sentence, correct answer). Each problem was 16 points for a total of 96 possible points on each probe (48 points without a calculator and 48 points with a calculator). The secondary scorer was trained to a criterion of 90% using sample PRWPP completed by the researcher for practice. The interrater reliability before the actual scoring happened was 90-100% in training. Photocopies of the students’ work for 30% of the lessons were provided for the secondary scorer to evaluate according to the rubric. The use of permanent products as a recording sheet allowed for the calculations of interobserver agreement to be done in a setting separate from the intervention after the lessons took place. The formula used to calculate interobserver agreement (IOA) was as follows (Kennedy, 2005):
Agreements \times 100 = \text{Percent of Agreement}

Agreements + Disagreement

The IOA calculations resulted in 100% agreement across all sessions and phases measured.

**Validity**

**Threats to Validity**

To control for internal validity threats, information about outside events were obtained by inquiring about school functions such as school-wide assemblies or field trips prior to the start of the intervention, and the dates of lessons were planned accordingly. Three groups of at least three students in each group were targeted in order to have enough students in case group attrition occurred. To demonstrate experimental control, the researcher implemented the intervention according to the teacher script and calculated IOA measured by designated staff. The researcher conducted the study in a separate location in order to control for treatment diffusion, which means that other students involved in the intervention were not exposed to SBI lessons unless it was their group’s time. If a student had more than three absences, he or she was not included in the intervention. However, he or she continued to receive the intervention but did not have their scores calculated as part of the study. This situation applied to five students (one student in Group A, two students in Group B, and two students in Group C). The researcher also kept lessons motivating by using real-life word problems to combat attrition.
The external validity was addressed by replicating the intervention across three groups under the same experimental conditions. A detailed log, which was used in analysis of data and description of results, was kept on description of the students, settings, conditions, and procedures. This log assisted in understanding how similar the conditions were and the extent to which the groups differed.

**Social Validity**

Social validity was measured after the intervention ended using student responses to the following statements on a 4-point Likert scale ranging from *strongly disagree* to *strongly agree* (*strongly disagree*-1, *disagree*-2, *agree*-3, *strongly agree*-4; see Appendix M):

- I enjoyed learning how to solve mathematical problems using the SBI process.
- I liked showing my progress with solving word problems on the daily Proportional Reasoning Word Problem Probes (PRWPP).
- My mathematical word problem performance increased as a result of this problem-solving intervention.
- This new skill development of solving proportional reasoning word problems will assist my learning of more complex mathematical word problems.
- I would like to continue the FOPS problem-solving strategy for additional skills in mathematics.
- I plan to use the FOPS problem-solving strategy on my own for word problems involving other skills in mathematics.
The MPSSES measured students’ individual beliefs regarding their ability to problem solve in mathematics, and this belief was assessed before and after the intervention. The social validity measure was different from the Mathematical Problem-Solving Self-Efficacy Scale (MPSSES) in that it assessed student satisfaction with the SBI intervention.

**Data Analysis**

Repeated measurements of mathematical word problem performance were calculated using a rubric, showing the average performance of each group of students to compare the effects of the intervention across groups (Gast, 2010). Graphs displayed acquisition of the SBI process using a rubric for proportional reasoning word problems, and separate graphs were created for percentage of accuracy with the SBI process both with and without the calculator (48 points and 48 points). Each group of students’ mean scores were also compared with adjacent phases to assess whether manipulation of the SBI process was associated with an effect, or predicted change, in student performance. The information from all phases of the study was examined to determine whether there was an effect demonstrated three different times. Two data analyses techniques used to determine effectiveness of the intervention, visual analysis and percent of nonoverlapping data (PND), are described in this section. Overall percentages of strategy application per category were also calculated as well as across groups and individual students both with and without a calculator. Then the results were separated to explain overall percentages of strategy application per category both with and without a calculator. Detailed results using these data analysis techniques are presented in the next chapter.
Visual Analysis

Visual inspection was made among the groups of students’ data for baseline and treatment (Phase II-SBI process) data points only. The visual analysis of data was interpreted through six components: level, trend, variability, immediacy, overlap, and consistency (Kratochwill et al., 2010). Descriptions of each component of interpreting data through visual analysis are as follows:

- The level is the mean score of data within a phase. For this study, the level is expressed in means (averages) of data for percentages of accuracy with the SBI process. The levels reported in treatment for the current study were the mean of the last three data points due to the increasing trend (Kennedy, 2005).
- The trend is the slope of the best-fitting line for data within a phase.
- Variability of the data refers to the range of data about the best-fitting straight line.
- Overlap refers to the percentage of data from the intervention phase that enters the range of data from the previous phase.
- Immediacy is the magnitude of change between the last three data points in one phase and the first three data points in the next phase.
- Consistency is the extent to which data patterns were similar in similar phases.

These six components of visual analysis were used to determine the performance of each of the three groups.
Percent of Non-Overlapping Data (PND)

The nonregression effect size of percent of nonoverlapping data (PND) was reported for each group’s data to support visual analysis and to assess the degree to which data in adjacent phases overlaps. PND scores were represented by the proportion of overlapping data displayed between treatment and baseline phases (Scruggs, Mastropieri, & Casto, 1987). The PND was incorporated into the visual inspection of data points as a primary indicator of effectiveness. PND scores were calculated by dividing the number of data points in the treatment phase (Phase II-SBI process) that did not overlap with the highest baseline data point by the total number of data points in the treatment phase. This calculation was then converted to a percentage by multiplying by 100. By determining the PND between each baseline and problem-solution phase for all three groups both with and without use of a calculator, it was determined whether the SBI intervention was effective. Scruggs, Mastropieri, Cook, and Escobar (1986) have suggested that a PND higher than 90% indicates high effectiveness, 70-90% represents fair effectiveness, 50-70% indicates questionable effectiveness, and a PND of less than 50% suggests that the intervention was ineffective.

Disaggregated Data for Schema-Based Instruction (SBI) Process

Data were disaggregated according to the five categories on the rubric for the PRWPP (shapes, labeling, type, number sentence, and/or correct digits) to describe overall percentages of SBI process completed accurately per category both with and without a calculator. This separation of data determined the number of categories students were able to complete correctly in the SBI process. Each category was converted
to a percentage for each student according to the weight on the PRWPP rubric (see Table 3). The percentages for application of the SBI process per category were first calculated per individual student both with and without a calculator (refer to Analysis of Dependent Measures in this chapter). The individual student data were then averaged with the rest of the students according to each group. Finally, the group data were combined to provide an overall percentage of strategy application per category both with and without a calculator. Overall percentages for application of the SBI process per category were reported first, followed by group data, then individual performance.
4. RESULTS

This chapter presents the results of a multiple-baseline-across-groups design. The current study examined the functional relation between the SBI process and solving proportional reasoning word problems for middle school students with high-incidence disabilities (HID). Schema instruction utilized teacher-led demonstration and modeling to identify and underline sentences in story situations and indicate necessary information in the problem, such as the numbers used to set up a proportion, and then record this information onto appropriate diagrams (Jitendra et al., 2002). As explained in Chapter 3, three groups of three students in each group participated in all three phases of the study.

Phase I was a baseline period when probes with specific types of proportional reasoning problems were used to gather data from students about performance on word problems, which involved proportional reasoning skills. Proportional reasoning consisted of three types: unit rates, proportions, and scale drawings. A 1-day overview of the SBI process was provided after Phase I for students in the three groups using proportional reasoning problems that were already solved (worked problems). Using already worked problems helps students recognize and understand the key information and mathematical relationships among numbers (Marshall, 1995; Steele, 2005). During Phase II, the students were taught the SBI process through an explicit instruction sequence of
demonstration, guided practice, and independent practice. Students were taught how to use the FOPS strategy during Phase II as well. As part of the FOPS strategy, students were taught how to use information from the proportional reasoning word problems to set up a diagram and write a number sentence to solve word problems. Cognitive load and scaffolding were among the instructional factors incorporated into the SBI instruction. Prior to each Phase II session, students completed probes similar to those used during baseline. As such, data were gathered before instructional sessions throughout Phase II. Finally, in Phase III, students’ maintenance of the SBI process was measured 4 weeks after the last intervention session for each group.

The proportional reasoning word problem performance for all three groups of students is described in relationship to the following research questions (RQ):

1) Does the number of categories completed correctly in the SBI process increase after the process is taught, using proportional reasoning mathematical word problems with and without the use of calculators, for middle school students with HID?

1a) Does the number of categories completed correctly to solve proportional reasoning mathematical word problems increase without calculators?

1b) Does the percentage of accurate answers increase after the SBI process is taught without calculators?

1c) Does the number of categories completed correctly to solve proportional reasoning mathematical word problems increase with calculators?
1d) Does the percentage of accurate answers increase after the SBI process is taught with calculators?

2) Is the number of categories completed correctly in the SBI process maintained 4 weeks after instruction with and without the use of calculators?

2a) Do students maintain the number of categories completed correctly to solve proportional reasoning mathematical word problems 4 weeks after instruction without calculators?

2b) Is the percentage of accurate answers maintained 4 weeks the SBI process is taught without calculators?

2c) Do students maintain the number of categories completed correctly to solve proportional reasoning mathematical word problems 4 weeks after instruction with calculators?

2d) Is the percentage of accurate answers maintained 4 weeks after the SBI process is taught with calculators?

3) Do middle school students with HID increase self-efficacy for solving mathematical word problems after the SBI process is taught?

First, overall performance on solving proportional word problems both with and without a calculator is reported for both baseline (Phase I) and the teaching of the SBI Process (Phase II). Next, a description of each group’s percentage of accuracy with the SBI process as well as a table indicating individual student performance is provided. Then the data were disaggregated to determine the number of categories completed
correctly in the SBI process (both with and without a calculator). The disaggregated results are reported across groups and by individual student.

**Research Design**

A multiple-baseline-across-groups design was utilized, which included baseline and staggered treatment phases. Three groups of three students in each group participated in all three phases of the study. The multiple baseline design permitted evaluation and demonstration of intrasubject direct replication, which increased the external validity of the findings because the performance of each of three groups had the potential to demonstrate an effect (Gast, 2010).

According to Kratochwill et al. (2010), single-subject evidence standards require four criteria in order to be considered strong evidence:

1) In baseline, there must be documentation of a research problem and a predictable pattern.

2) There must be documentation of a predictable pattern in each phase of the analysis.

3) The data must have documentation of a predicted change in the dependent variable when the independent variable is manipulated, demonstrating an effect.

4) There must be three demonstrations of basic effect at different points in time to illustrate experimental control. There also needs to be no demonstration of intervention failure at any point throughout the study.

This study meets these evidence standards proposed by Kratochwill et al. (2010).
The two data analyses techniques used to determine effectiveness of the intervention were visual analysis and percent of nonoverlapping data (PND). The visual analysis of data was interpreted through six components: level, trend, variability, immediacy, overlap, and consistency (Kratochwill et al., 2010). In the current study, the level was expressed in means (averages) of data for percentage of accuracy with the SBI process. The levels reported in treatment were the mean of the last three data points due to the increasing trend (Kennedy, 2005). Analysis of data determined whether baseline data were stable (i.e., a flat trend) and when an accelerating trend was evident in treatment. Variability in baseline was compared to the level (i.e., low) and to the accelerating trend in treatment. Degree of overlap (how many data points) was also observed between treatment and baseline data, and speed of change (after number of data points) was identified between baseline and treatment. Finally, consistency of data in treatment was observed according to how often the data were higher than in baseline, and the data in the maintenance (Phase III) were compared to how similar the data were in treatment.

The PND was incorporated into the visual inspection of data points as another way data were analyzed to indicate of effectiveness. PND scores were represented by the proportion of overlapping data between baseline and treatment (Scruggs et al., 1987). PND scores were calculated by determining the percent of data points in the treatment phase that did not overlap with the highest baseline data point. By determining the PND between each baseline and treatment phase for all three groups both with and without use of a calculator, it was determined whether the SBI intervention was effective. Scruggs et
al. (1986) suggested that a PND higher than 90% indicates high effectiveness, 70-90% represents fair outcomes, 50-70% indicates questionable effectiveness, and a PND of less that 50% suggests that the intervention was ineffective.

**Dependent Measures**

There were two dependent measures used: proportional reasoning word problem probes (PRWPP) and a Mathematical Problem-Solving Self-Efficacy Scale (MPSSES). PRWPP measured performance on solving word problems in the baseline and the treatment phases, showing the average performance of each group of students. According to the directions for the PRWPP, calculators were not used for the first three word problems and were used for last three word problems. The researcher monitored to know when students used calculators for the last three word problems. Separate graphs were created for calculator use to compare group performance both with and without a calculator. Self-efficacy was also measured before and after the SBI intervention using the MPSSES to determine how students perceived their competence in the area of solving mathematical word problems. Details for how the PRWPP and the MPSSES were scored are also described in this section.

**Proportional Reasoning Word Problem Probes (PRWPP)**

The PRWPP were used to acquire data about students’ accuracy with the SBI process during baseline and SBI process phases (see Appendix J). During Phase II for the SBI process, data were gathered at the beginning of each session prior to instruction. The probes were also used to gather data about students’ maintenance of accuracy with the SBI process 4 weeks after the SBI process was taught. Although the SBI lessons during
the intervention included different difficulty levels of computations (including students’ grade level computations), the PRWPP only included numbers from the seventh-grade level (i.e., multiplication and division with two and three digits with regrouping) because this level of problems were taught in the curriculum. The following rubric described the five categories measured on the PRWPP:

1) Drew an accurate number of shapes for the “if-then” diagram ___ / 4 points

2) Labeled words/phrases correctly on the “if-then” diagram ___ / 6 points

3) Inserted the words “if” and ”then” into diagram to indicate type of problem ___ / 2 points

4) Translated diagram from proportion into number sentence (2-full credit, 1-partial, 0-none) ___ / 2 points

5) Provided correct digits in answer (2-full credit, 1-partial, 0-none) ___ / 2 points

The total amount of points that could be earned per problem was 16. There were three problems (one per type of proportional reasoning problem) on the front side of the probe; students could not use calculators for these problems, which equaled 48 points. On the back side of the probe, there were similar problems for which students could use calculators, which also equaled 48 points. In total, there were 96 raw score points possible for each PRWPP. Points were converted to percentages derived from the raw score on the rubric, and the percentages were used on the graphs.
The “if-then” diagram included four shapes used in a horizontal and vertical manner to guide students’ placement of numbers from the word problems, and this formation of the numbers resulted in a proportion. A check box for students was also included on the PRWPP to indicate whether they chose to use a calculator on the last three designated problems that permitted calculator use. These data allowed the researcher to know which students used calculators for the last three proportional reasoning word problems. Even when the calculator was permitted, all groups chose to use the calculator only 19-30% of the time (Group A-, 29%; Group B, 19%; and Group C, 30%). Regardless of whether students used the calculator, students’ data for the problems when students were permitted to use the calculator were plotted on the corresponding “with calculator” graph to indicate the students had the opportunity to use the calculator.

The SBI process was related to improvement in three of the five categories of the PRWPP rubric for solving proportional reasoning mathematical word problems (RQ 1: shapes, labeling, “if-then”). The students increased the number of steps completed correctly to solve proportional reasoning mathematical word problems (RQ 1a and 1c). However, the percentage of accurate answers did not increase (RQ 1b and 1d). In maintenance (Phase III), the students remembered three categories of the SBI process to solve proportional reasoning mathematical word problems (RQ 2, 2a, and 2c). The students’ answers in maintenance, however, were not accurate (RQ 2b and 2d).

Data were analyzed separately for all groups, both with and without a calculator. The levels of analysis included the accuracy of the use of the SBI for all groups combined
as well as across groups and by individual students. Throughout this section, data for the six components of interpreting visual analysis are provided first according to phases. Next, percent of nonoverlapping data (PND) is reported for Phase II. Finally, maintenance of the SBI process (Phase III) is described.

**Accuracy with SBI Process for All Groups Without a Calculator**

Each of the six components of interpreting data through visual analysis (level, trend, variability, immediacy, overlap, and consistency) was employed for all groups without a calculator. The overall level of Phase I, the baseline phase, for all three groups on solving of proportional reasoning word problems without use of a calculator was 3.8% ($SD = 3.6$). Although performance changed per group in a staggered fashion via the multiple-baseline-across-groups design, when the SBI process was taught (Phase II) there was a 60.4% increase for solving word problems for all groups of students. The overall level of Phase II (SBI process) for all groups was 64.2% ($SD = 16.0$).

In baseline, the data for all three groups were stable and demonstrated a flat trend. The changes in trend for all three groups for Phase II (SBI process) were in the upward direction. All three groups had low variability in baseline, and the problem-solution phases for all groups also demonstrated low variability from the accelerating trends. All groups had a slow immediacy of change (after 1-2 data points) between baseline and problem-solution phase. In relation to overlap, Group A had slight overlap (2 data points), while the overlap in Groups B and C was minimal (1 data point). Regarding consistency, or the extent to which data patterns were similar in similar phases, data points in Phase II (SBI process) were regularly higher than in baseline (Phase I). Also
related to consistency, the data in the maintenance (Phase III) were the same as in the SBI process phase for all groups.

Percent of nonoverlapping data (PND) and maintenance were also calculated for all groups. The overall PND, as measured by calculating the percent of data points in the treatment phase that did not overlap with the highest baseline data, for Groups A, B, and C was 85%. This percentage indicated that the SBI intervention was fairly effective for these groups of students when the calculator was not used (Scruggs et al., 1986).

**Group A**

Group A’s baseline (Phase I) level without a calculator was 2.7% ($SD = 3.3$, range of 0-5.7%; refer to Figure 5). When the SBI process was taught in Phase II (SBI process), Group A’s level was 56.7%, $SD = 18.8$ (range of 0-62.7%) for solving proportional reasoning math word problems, which was a 54% increase from the level in baseline. The data in baseline were stable and demonstrated a flat trend. An accelerating upward trend was evident after the second data point. Variability for Group A in baseline was low, and the SBI process phase had low variability when compared to the accelerating trend.

There was no immediacy of change (improvement was after 2 data points) between baseline and Phase II for Group A, and there was minimal overlap of two data points. Regarding consistency, the data in Phase II were regularly higher than baseline, and the data in maintenance (Phase III) were the same as in Phase II. Additionally, the PND for Group A was 78% in Phase II. This figure indicated that the SBI process was fairly effective for this group of students without the use of a calculator (Scruggs et al., 1986).
Figure 5. Percentage of accuracy with SBI process without calculator across groups
Group B

Group B’s baseline performance (Phase I) resulted in a level of $M = 3.4\%$ ($SD = 2.2$, range of 0-6.7%), and Group B’s word problem performance increased to a level of 59.1\% ($SD = 12.6$, range of 5.3-61.7%), when the SBI process was taught in Phase II. The data in baseline were stable and demonstrated a flat trend. The change of trend for Group B was accelerating in the upward direction. The variability in baseline was low, and the SBI process phase also had low variability as compared to the upward trend.

There was a slow immediacy of change between the baseline and Phase II (SBI process) for Group B, and then an abrupt change from 3\% to 28\% for the second data point in Phase II. There was also minimal overlap of one data point. Regarding consistency, the data were regularly higher than in baseline throughout Phase II and Phase III (maintenance). Additionally, the PND for Group B was 78\% in Phase II. This percentage indicated that the SBI process was effective for this group of students without the use of a calculator (Scruggs et al., 1986).

Group C

Group C had a baseline level of $M = 5.5\%$ ($SD = 5.2$, range of 1-10.3\%) for accuracy with the SBI process in solving proportional reasoning word problems without a calculator (Phase I). When the SBI process was taught in Phase II, the level increased to $M = 76.9\%, SD = 16.6$ (range of 11.3-83.3\%). The data in baseline were stable and demonstrated a flat trend, and the change in trend for the SBI process phase was
accelerating upward. The variability of data in baseline was low, and the variability in Phase II was also low compared to the accelerating trend.

There was also a slow immediacy of change between baseline and the SBI process phase (improvement was after the first data point) for Group C, and there was minimal overlap of one data point. In relation to consistency, the data in Phases II (SBI process) and III (maintenance) were regularly higher than baseline. Finally, the percent of nonoverlapping data (PND) for Group C was 89% in Phase II. This figure indicated that the SBI process was effective for students in that group as well (Scruggs et al., 1986).

**Summary of Individuals**

Individual proportional reasoning word problem performance without a calculator was analyzed. Levels, standard deviations (SD), and percent of nonoverlapping data (PND) were calculated for each student (see Table 5). The first three students listed are from Group A (A1= first student in Group A), the next three students are from Group B, and the final three from Group C respectively. The individual students’ performance with a calculator was 0-11.7% in baseline (Phase I). When the SBI intervention was introduced (Phase II), the amount of improvement ranged from 33% for one individual student to an increase of 82.6% for another student. All nine students increased accuracy with the SBI process to solve proportional reasoning word problems by at least 33% with a calculator. Also, the PND for the SBI process was in the 78-100% range for seven of the nine students (78% of total number of students). In maintenance (Phase III), the accuracy with the SBI process in Phase II ranged from a decrease of 22.8% to an increase of 25%.
Table 5

Levels and PND of Individual Student Performance without a Calculator

<table>
<thead>
<tr>
<th>Student</th>
<th>Baseline M (SD)</th>
<th>Treatment M (SD)</th>
<th>PND</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.0% (0.0)</td>
<td>69.2% (10.3)</td>
<td>78%</td>
</tr>
<tr>
<td>A2</td>
<td>2.0% (2.2)</td>
<td>35.0% (25.2)</td>
<td>44%</td>
</tr>
<tr>
<td>A3</td>
<td>6.4% (6.5)</td>
<td>65.8% (3.1)</td>
<td>44%</td>
</tr>
<tr>
<td>B1</td>
<td>0.9% (2.7)</td>
<td>53.5% (5.6)</td>
<td>89%</td>
</tr>
<tr>
<td>B2</td>
<td>3.6% (4.2)</td>
<td>73.5% (2.6)</td>
<td>78%</td>
</tr>
<tr>
<td>B3</td>
<td>3.6% (3.1)</td>
<td>50.2% (6.0)</td>
<td>78%</td>
</tr>
<tr>
<td>C1</td>
<td>11.7% (7.5)</td>
<td>94.3% (9.8)</td>
<td>78%</td>
</tr>
<tr>
<td>C2</td>
<td>1.0% (2.4)</td>
<td>61.2% (7.0)</td>
<td>89%</td>
</tr>
<tr>
<td>C3</td>
<td>4.8% (4.8)</td>
<td>75.3% (4.9)</td>
<td>100%</td>
</tr>
</tbody>
</table>

Accuracy with SBI Process for All Groups With a Calculator

Each of the six components of interpreting data through visual analysis (level, trend, variability, immediacy, overlap, and consistency) was employed for all groups with a calculator. The level of the baseline phase (Phase I) for all three groups in accuracy with the SBI process to solve proportional reasoning word problems with a calculator was $M = 1.6\%$ ($SD = 1.4$). Although performance changed per group in a staggered fashion via the multiple-baseline-across-groups design, when the SBI process was taught (Phase II-SBI process) there was an increase in level to $M = 53.9\%$ ($SD = 19.6$), which was a 52.3% increase in level from baseline. All groups demonstrated an accelerating upward trend at some point within phases. All three groups also had low variability in baseline and low variability in the SBI process phase when compared to the accelerating upward trend.
All groups demonstrated a slow immediacy of change during Phase II when learning the SBI process, with changes occurring after 1-5 data points (Group A had slowest immediacy of change after 5 data points). In addition, Group A had more overlap (5 data points) than Groups B and C (1 data point). Regarding consistency, the data in the SBI process (Phase II) and maintenance phases (Phase III) were regularly higher than baseline (Phase I) for all groups. Finally, the PND data for accuracy with the SBI process to solve proportional reasoning word problems and maintenance were calculated for all three groups in Phase II. Group A, B, and C’s data resulted in an overall PND of 70%. This percentage translates to the SBI intervention having questionable effectiveness for these particular groups of students when a calculator was permitted (Scruggs et al., 1986).

**Group A**

Group A’s baseline (Phase I) level for accuracy with the SBI process with a calculator was $M = 0.3\%$ ($SD = 0.5$; see Figure 6). The level increased to $M = 37.8\%$ ($SD = 35.5$) when the SBI process was taught in Phase II. The data in baseline were stable and demonstrated a flat trend. In addition, there was a zero-celerating trend, followed by a rapid upward change in trend. Baseline variability was low, and Phase II for Group A had lower variability than the other groups as compared to the accelerating upward trend.

There was no immediacy of change between the baseline and problem-solution phases (improvement only occurred after 6 data points) for Group A. A much higher overlap of data for Group A compared to the other groups was also evident (5 data points for Group A, compared to only 1 data point for Groups B and C). In relation to consistency, the data were not regularly higher than in baseline throughout the SBI
process phase. The data were regularly higher, however, than in baseline for Phase III (maintenance). The PND for Group A was 44%. This result indicated that the SBI process was not effective for this group of students without the use of a calculator (Scruggs et al., 1986).

**Group B**

Group B’s baseline (Phase I) level was $M = 0.9\%$ ($SD = 0.9$). When the SBI process was taught in Phase II (SBI process), Group B’s level increased to $M = 54.1\%$ ($SD = 8.4$, range of 3-57.7%). The data in baseline were stable and demonstrated a flat trend. The changes in trend for Group B were accelerating in the upward direction. Variability for Group B in baseline was low, and the problem-solution phase also had low variability when compared to the accelerating trend.

There was also a slow immediacy of change between the baseline and treatment phases (improvement after 1 data point) for Group B. Minimal overlap of one data point is also evident. Regarding consistency, the data in the treatment and maintenance phases were consistently higher than baseline. The PND for Group B was 89% in Phase II, which indicated that the SBI process was effective for this group of students with the use of a calculator (Scruggs et al., 1986).
Figure 6. Percentage of accuracy with SBI process with calculator across groups
**Group C**

Group C’s baseline level (Phase I) was $M = 3.7\%$ ($SD = 3.0$). When the SBI process was taught in Phase II (SBI process), Group B’s level increased to $M = 69.8\%$, $SD = 14.9$ (range of 1-77.6%), demonstrating a 73.9% increase over the baseline level. The data in baseline were stable and demonstrated a flat trend. The changes in trend for Group C were accelerating in the upward direction. Variability for Group C in baseline was low, and the SBI process phase also had low variability when compared to the accelerating trend.

There was also a slow immediacy of change between the baseline and treatment phases (improvement after 1 data point) for Group C. Minimal overlap of 1 data point is also evident for Group C. Regarding consistency, the data in the treatment and maintenance phases were consistently higher than baseline. Finally, the PND for Group C was 89% in Phase II. This figure indicated that the SBI process was effective for this group of students with the use of a calculator (Scruggs et al., 1986).

**Summary of Individuals**

Individual proportional reasoning word problem performance for students with a calculator was analyzed. Levels, standard deviations (SD), and percent of nonoverlapping data (PND) were calculated for each student (see Table 6). The individual students’ performance without a calculator was between 0.0-6.6% in baseline (Phase I). When the SBI process was introduced (Phase II), the amount of improvement ranged from a decrease of 0.1% for one student to an increase of 76.4% for another student. Overall, eight out of the nine students (89% of students) increased accuracy with the SBI process.
to solve proportional reasoning word problems by at least 42.8%. The PND for the SBI process was in the 67-100% range for six of the nine students (67% of total number of students). In maintenance (Phase III), the degree of improvement from Phase II ranged from a decrease of 70.5% for one student to an increase of 68.1% for another student.

Table 6

*Levels and PND of Individual Student Performance with a Calculator*

<table>
<thead>
<tr>
<th>Student</th>
<th>Baseline M (SD)</th>
<th>Treatment M (SD)</th>
<th>PND</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.0% (0.0)</td>
<td>42.8% (29.2)</td>
<td>33%</td>
</tr>
<tr>
<td>A2</td>
<td>1.0% (1.79)</td>
<td>0.9% (0.0)</td>
<td>0%</td>
</tr>
<tr>
<td>A3</td>
<td>0.0% (0.0)</td>
<td>70.5% (8.7)</td>
<td>44%</td>
</tr>
<tr>
<td>B1</td>
<td>0.0% (0.0)</td>
<td>49.5% (1.7)</td>
<td>100%</td>
</tr>
<tr>
<td>B2</td>
<td>1.8% (2.9)</td>
<td>63.8% (6.1)</td>
<td>78%</td>
</tr>
<tr>
<td>B3</td>
<td>0.4% (1.3)</td>
<td>49.0% (6.1)</td>
<td>89%</td>
</tr>
<tr>
<td>C1</td>
<td>6.6% (5.0)</td>
<td>83.8% (25.9)</td>
<td>67%</td>
</tr>
<tr>
<td>C2</td>
<td>1.0% (2.4)</td>
<td>54.2% (5.9)</td>
<td>89%</td>
</tr>
<tr>
<td>C3</td>
<td>4.0% (4.0)</td>
<td>71.3% (8.4)</td>
<td>89%</td>
</tr>
</tbody>
</table>

**Disaggregated Data for Schema-Based Instruction (SBI) Process**

Application of the SBI process to solve proportional reasoning word problems without a calculator was measured for all groups (Groups A, B and C). The rubric of categories for the PRWPP with weighted points yielded a raw score of 16 for each word problem. This raw score did not necessarily mean the students had the correct answer; it meant they accumulated points as indicated on the graphs of results. Data were separated to determine the number of categories completed correctly in the SBI process to solve
proportional reasoning word problems. Each category was converted to a percentage for each student according to the weight on the PRWPP rubric (as presented previously in Table 3). The breakdown for each category on the rubric for the PRWPP is for one proportional reasoning word problem (16 points). The scores for rubric categories per PRWPP on the results tables included in the next chapter are equivalent to 48 points (i.e., 3 problems without a calculator).

Table 3
*Content of Rubric for PRWPP*

<table>
<thead>
<tr>
<th>Rubric Category Number</th>
<th>Category for SBI Process</th>
<th>Raw Number of Points Per Category Per Problem</th>
<th>Highest Category Percentage of Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>shapes</td>
<td>4 points</td>
<td>25%</td>
</tr>
<tr>
<td>2</td>
<td>labeling</td>
<td>6 points</td>
<td>37.5%</td>
</tr>
<tr>
<td>3</td>
<td>“if-then”</td>
<td>2 points</td>
<td>12.5%</td>
</tr>
<tr>
<td>4</td>
<td>number sentence</td>
<td>2 points</td>
<td>12.5%</td>
</tr>
<tr>
<td>5</td>
<td>correct answer</td>
<td>2 points</td>
<td>12.5%</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td></td>
<td><strong>16 points</strong></td>
<td><strong>100%</strong></td>
</tr>
</tbody>
</table>

The results in this section are reported for two areas of the SBI process, acquisition and maintenance. Data were analyzed separately for all groups, both with and without a calculator. The data will first be described for all groups overall, then across groups, and finally by individual students. The results will be reported according to the five rubric categories for the PRWPP. In this chapter, the categories will be referred to as Category 1, Category 2, Category 3, Category 4, and Category 5.
All Groups without a Calculator

Application of the SBI process to solve proportional reasoning word problems without a calculator was measured for all groups (Groups A, B and C). Individual student data are available in Appendix O. The overall baseline (Phase I) level for acquisition of the SBI process in all five rubric categories was 3.8% (SD = 1.6, range of 2.7-5.5%). The highest means in Phase II (SBI process) were 23.7% in Category 1 (shapes) and 25.6% in Category 2 (labeling). The lowest means in Phase II were 2.2% in Category 4 (number sentence) and 1.7% in Category 5 (correct answer).

When students completed maintenance (Phase III) probes 4 weeks after instruction ended, Category 1 increased to 25.0% (highest mean that could be obtained in that category), and Category 2 decreased only slightly from 25.6% to 24.1%. Category 3 also decreased slightly from 11.0% in Phase II to 9.9% in Phase III. The last two categories decreased from 2.2% to 1.4% (Category 4), and from 1.7% to 0.7% (Category 5).

Group A. For the students in Group A, the level for all five categories in baseline (Phase I) was 2.7% (SD = 3.3, range of 0.0-6.4%), and 56.7% in Phase II. Group A had a mean of 21.2% in Category 1 (shapes) in Phase II and a mean of 23.1% in Category 2 (labeling; see Table 7). The lowest means in Phase II were $M = 1.1\%$ in Category 4 (number sentence) and $M = 1.3\%$ in Category 5 (correct answer).

When Group A students, completed maintenance (Phase III) probes 4 weeks after instruction ended, Category 1 increased from 21.2% to 25.0%. Category 2 also had a slight increase from 23.1% to 24.1%. Category 3 increased from 9.9% in Phase II to
12.5% in Phase III (highest mean that could be obtained in that category). The last two categories decreased from 1.1% to 0% (Category 4) and from 1.3% to 0% (Category 5).

Table 7
Percentage of Accuracy with SBI Process without Calculator across Groups

<table>
<thead>
<tr>
<th>Group</th>
<th>Rubric Category</th>
<th>Baseline</th>
<th>Treatment</th>
<th>Maintenance</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1 (out of 25%)</td>
<td>0.0%</td>
<td>21.2%</td>
<td>25.0%</td>
</tr>
<tr>
<td></td>
<td>2 (out of 37.5%)</td>
<td>0.0%</td>
<td>23.1%</td>
<td>20.3%</td>
</tr>
<tr>
<td></td>
<td>3 (out of 12.5%)</td>
<td>0.0%</td>
<td>9.9%</td>
<td>12.5%</td>
</tr>
<tr>
<td></td>
<td>4 (out of 12.5%)</td>
<td>1.1%</td>
<td>1.1%</td>
<td>0.0%</td>
</tr>
<tr>
<td></td>
<td>5 (out of 12.5%)</td>
<td>1.6%</td>
<td>1.3%</td>
<td>0.0%</td>
</tr>
<tr>
<td></td>
<td>TOTAL (out of 100%)</td>
<td>2.7%</td>
<td>56.7%</td>
<td>57.8%</td>
</tr>
<tr>
<td>B</td>
<td>1 (out of 25%)</td>
<td>0.0%</td>
<td>25.0%</td>
<td>25.0%</td>
</tr>
<tr>
<td></td>
<td>2 (out of 37.5%)</td>
<td>0.0%</td>
<td>23.4%</td>
<td>26.8%</td>
</tr>
<tr>
<td></td>
<td>3 (out of 12.5%)</td>
<td>0.0%</td>
<td>10.6%</td>
<td>11.0%</td>
</tr>
<tr>
<td></td>
<td>4 (out of 12.5%)</td>
<td>1.1%</td>
<td>0.0%</td>
<td>1.3%</td>
</tr>
<tr>
<td></td>
<td>5 (out of 12.5%)</td>
<td>2.3%</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td></td>
<td>TOTAL (out of 100%)</td>
<td>3.4%</td>
<td>59.1%</td>
<td>64.2%</td>
</tr>
<tr>
<td>C</td>
<td>1 (out of 25%)</td>
<td>0.0%</td>
<td>25.0%</td>
<td>25.0%</td>
</tr>
<tr>
<td></td>
<td>2 (out of 37.5%)</td>
<td>0.0%</td>
<td>30.3%</td>
<td>25.0%</td>
</tr>
<tr>
<td></td>
<td>3 (out of 12.5%)</td>
<td>0.0%</td>
<td>12.5%</td>
<td>6.3%</td>
</tr>
<tr>
<td></td>
<td>4 (out of 12.5%)</td>
<td>2.2%</td>
<td>5.4%</td>
<td>3.0%</td>
</tr>
<tr>
<td></td>
<td>5 (out of 12.5%)</td>
<td>3.3%</td>
<td>3.7%</td>
<td>2.0%</td>
</tr>
<tr>
<td></td>
<td>TOTAL (out of 100%)</td>
<td>5.5%</td>
<td>76.9%</td>
<td>61.3%</td>
</tr>
</tbody>
</table>
**Group B.** For the students in Group B, the level for all five categories in baseline (Phase I) was 3.4% ($SD = 2.2$, range of 0.9 to 4.4%), and a level of 59.1% in Phase II. Group B’s highest means in Phase II were 25.0% out of 25.0% in Category 1 (shapes) and 23.4% in Category 2 (labeling). The lowest means in Phase II were 0.0% and 0.0% in both Category 4 (number sentence) and Category 5 (correct answer).

When Group B students completed maintenance (Phase III) probes 4 weeks after instruction ended, Category 1 stayed at 25.0%, and Category 2 increased from 23.4% in Phase II to 26.8% in maintenance. Category 3 increased from 10.6% to 11.0%. The last two categories had the lowest degrees of improvement: Category 4 increased from 0% to 1.3%, and Category 5 stayed at 0.0%.

**Group C.** For the students in Group C, the level for all five categories in baseline (Phase I) was 5.5% ($SD = 5.2$, range of 1.0% to 11.0%) and 76.9% in Phase II. Group C’s highest means in Phase II were 25.0% out of 25% in Category 1 (shapes), 30.3% in Category 2 (labeling), and 12.5% out of 12.5% in Category 3 ("if-then"). The lowest means in Phase II were 5.4% in Category 4 (number sentence) and 3.7% in Category 5 (correct answer).

When Group C students completed maintenance (Phase III) probes 4 weeks after instruction ended, Category 1 remained at 25.0%, the highest mean that could be obtained in that category. Category 2 slightly decreased from 30% to 25.0%. Category 3 decreased from 12.5% in Phase II to 6.3% in maintenance. The last two categories decreased from 5.4% to 3.0% (Category 4) and from 3.7% to 2.0% (Category 5).
All Groups with a Calculator

Application of the SBI process to solve proportional reasoning word problems without a calculator was measured for all groups (Groups A, B and C). Individual student data are available in Appendix O. The overall baseline (Phase I) level for accuracy with the SBI process in all five rubric categories was 1.6% ($SD = 1.4$, range of 0.3 to 3.7%). The highest means in Phase II were 20.7% in Category 1 (shapes) and 20.3% in Category 2 (labeling). The lowest means in Phase II were 1.8% Category 4 (number sentence) and 1.3% in Category 5 (correct answer).

When students completed maintenance (Phase III) probes 4 weeks after instruction ended, Category 1 increased from 20.7% to 22.2%. Category 2 slightly decreased from a level of 20.3% to 21.7%. Category 3 also decreased from 9.9% in Phase II to 8.4% in Phase III. The last two categories improved the least, remaining at 1.8% for Category 4 and increasing from 1.3% to 2.2% in Category 5.

Group A. For the students in Group A, the level for all five categories in baseline (Phase I) was 0.3% (range of 0.0-0.8%), and 37.8% in Phase II. Group A’s highest means in Phase II were 12.9% in Category 1 (shapes) and 16.4% in Category 2 (labeling; see Table 8). The lowest means in Phase II were 1.1% in Category 4 (number sentence) and 0.9% in Category 5 (correct answer).
Table 8

*Percentage of Accuracy with SBI Process with Calculator across Groups*

<table>
<thead>
<tr>
<th>Student</th>
<th>Rubric Category</th>
<th>Baseline</th>
<th>Treatment</th>
<th>Maintenance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group A</td>
<td>1 (out of 25%)</td>
<td>0.0%</td>
<td>12.9%</td>
<td>16.7%</td>
</tr>
<tr>
<td></td>
<td>2 (out of 37.5%)</td>
<td>0.0%</td>
<td>16.4%</td>
<td>15.7%</td>
</tr>
<tr>
<td></td>
<td>3 (out of 12.5%)</td>
<td>0.0%</td>
<td>6.4%</td>
<td>8.3%</td>
</tr>
<tr>
<td></td>
<td>4 (out of 12.5%)</td>
<td>0.0%</td>
<td>1.1%</td>
<td>0.0%</td>
</tr>
<tr>
<td></td>
<td>5 (out of 12.5%)</td>
<td>0.3%</td>
<td>0.9%</td>
<td>0.0%</td>
</tr>
<tr>
<td>TOTAL (out of 100%)</td>
<td>0.3%</td>
<td>37.8%</td>
<td>40.7%</td>
<td></td>
</tr>
</tbody>
</table>

| Group B | 1 (out of 25%)  | 0.0%     | 25.0%     | 25.0%       |
|         | 2 (out of 37.5%)| 0.0%     | 18.0%     | 22.3%       |
|         | 3 (out of 12.5%)| 0.0%     | 11.1%     | 9.7%        |
|         | 4 (out of 12.5%)| 0.1%     | 0.0%      | 2.0%        |
|         | 5 (out of 12.5%)| 0.7%     | 0.0%      | 2.7%        |
| TOTAL (out of 100%) | 0.9%     | 54.1%    | 61.7%     |

| Group C | 1 (out of 25%)  | 0.0%     | 24.1%     | 25.0%       |
|         | 2 (out of 37.5%)| 0.0%     | 26.4%     | 27.0%       |
|         | 3 (out of 12.5%)| 0.0%     | 12.0%     | 7.5%        |
|         | 4 (out of 12.5%)| 0.6%     | 4.3%      | 3.5%        |
|         | 5 (out of 12.5%)| 3.1%     | 2.9%      | 4.0%        |
| TOTAL (out of 100%) | 3.7%     | 69.8%    | 67.0%     |

When Group A students completed maintenance (Phase III) probes 4 weeks after instruction ended, Category 1 increased from 12.9% to 16.7%. Category 2 had a slight decrease from 16.4% to 15.7%. Category 3 increased from 6.4% in Phase II to 8.3% in Phase III. The last two categories decreased or remained the same from 1.1% to 0.0% (Category 4) and from 0.9% to 0.0% (Category 5).

**Group B.** For the students in Group B, the level for all five categories in baseline (Phase I) was 0.9% (range of 0.0 to 1.8%) and 54.1% in Phase II. Group B’s highest
means in Phase II were 25.0% in Category 1 (shapes), 18.0% in Category 2 (labeling), and 11.1% in Category 3. The lowest means in Phase II were 0% in both Category 4 (number sentence) and Category 5 (correct answer).

When Group B students completed maintenance (Phase III) probes 4 weeks after instruction ended, Category 1 remained at 25%. Category 2 increased from 18.0% to 22.3%. Category 3 decreased from 11.1% in the SBI process phase to 9.7% in maintenance. The last two categories had the lowest increases, with Category 4 increasing from 0.0% to 2.0% and Category 5 from 0.0% to 2.7%.

**Group C.** For the students in Group C, the level for all five categories in baseline (Phase I) was 3.7% (range of 0.7 to 6.6%) and 69.8% in Phase II. Group C’s highest means in Phase II were 24.1% in Category 1 (shapes) and 26.4% in Category 2 (labeling). The lowest means in Phase II were 4.3% Category 4 (number sentence) and 2.9% in Category 5 (correct answer).

When Group C students completed maintenance (Phase III) probes 4 weeks after instruction ended, Category 1 increased from 24.1% to 25.0%. Category 2 slightly increased from 26.4 to 27.0%, and Category 3 decreased from 12.0% in Phase II to 7.3% in maintenance. The last two categories changed minimally, decreasing from 4.3% to 3.5% in Category 4 and increasing from 2.9% to 4.0% in Category 5.

**Maintenance**

In Phase III of maintenance without a calculator, the overall percentage of accuracy with the SBI process for all three groups was \( M = 61.1\% \) \((SD =18.3)\) when tested 4 weeks following instruction. The mean for Group A was \( M = 57.8\% \) \((SD =5.7)\) in
maintenance. Group B performed at $M = 64.2\%$ ($SD = 16.2$), and Group C had a mean of $M = 61.3\%$ ($SD = 32.9$). It should also be noted that only two of the three students were present to complete maintenance PRWPP in Group B.

With a calculator in Phase III of maintenance, the overall percentage of accuracy with the SBI process for all three groups was $M = 56.4$ ($SD = 30.4$) when tested 4 weeks following instruction. The mean for Group A was $M = 40.7\%$ ($SD = 36.0$) in maintenance. Group B performed at $M = 61.7\%$ ($SD = 22.5$), and Group C had a mean of $M = 61.1\%$ ($SD = 18.3$) in maintenance.

**Self-Efficacy**

The students increased self-efficacy to some extent for solving mathematical word problems after SBI was taught (RQ 3). The Mathematical Problem-Solving Self-Efficacy Scale (MPSSES) was administered both before and after the intervention to measure students’ individual beliefs regarding their ability to problem solve in mathematics. On the MPSSES, students responded to three statements related to self-efficacy using a 4-point Likert scale (*strongly disagree* - 1, *disagree* - 2, *agree* - 3, *strongly agree* - 4). The total number of students who were in agreement with each statement self-efficacy was reported.

Prior to the SBI intervention, three of the nine students surveyed were in agreement that they usually feel they cannot manage a solution when they start to solve a mathematical problem, and four of the students agreed that they did not feel sure of themselves when solving word problems in mathematics. In addition, three students also agreed that they have difficulty solving word problems. After the SBI intervention was
administered, individual student ratings on the self-efficacy indicated that one more student felt he or she could not manage a solution when starting to solve a mathematical problem for a total of four students who indicated that they have difficulty solving word problems. In addition, three more students were now more sure of themselves when solving word problems in mathematical for a total of seven students. Finally, the same number of students (3 students) who were in agreement they had difficulty solving word problems before the SBI intervention continued to agree with this concept after the treatment. Two of the three of these students were the same, which had reported continued difficulty solving word problems after the SBI intervention.

**Social Validity**

Social validity was also measured after the intervention using student responses to statements on a 4-point Likert scale from *strongly disagree* to *strongly agree* (*strongly disagree*-1, *disagree*- 2, *agree-* 3, *strongly agree*- 4). For calculation of feedback on social validity, the *agree* and *strongly agree* responses were combined under the category of *agree*. Similarly, the *disagree* and the *strongly disagree* categories were grouped together under *disagree*. According to the feedback from students on the social validity scales, eight of the nine students indicated that they enjoyed learning how to solve word problems using SBI, and seven of the students liked showing their progress on the daily PRWPP. All students were in agreement that their proportional reasoning word problem performance increased as a result of the SBI process learned (9 out of 9 total students). Seven of the students felt that this new skill development would assist in their learning of other mathematical concepts. According to feedback given, six students indicated they
would like to continue use of the SBI process for other skills in mathematics. Finally, six of the students reported that they planned to use this process on their own for word problems involving other skills.

**Summary**

Analyses of the results of this study suggest that all of the students improved accuracy with the SBI process, as measured by accuracy for each of five categories on a rubric, to solve mathematical word problems both with and without use of a calculator. Students improved their word-problem performance by 60.4% without a calculator and by an increase of 52.3% when a calculator was permitted. All three of the groups also demonstrated maintenance of learning some categories of the SBI process 4 weeks after the intervention, both with and without use of a calculator. Disaggregation of the data indicated that the highest percentages in Phase II (SBI process) were 23.7% in Category 1 (shapes) and 25.6% in Category 2 (labeling). The lowest percentages in Phase II were 2.2% Category 4 (number sentence) and 1.7% in Category 5 (correct answer).

The measure of self-efficacy indicated that after receiving the SBI process, four students (the original three plus one more student) felt they could manage a solution when they start to solve a mathematical problem; three more of the students (total of seven students) were more sure of themselves when solving word problems in mathematics; and three students were in agreement that they continued to have difficulty solving word problems after the SBI intervention was completed. In addition, all groups of students reported that they enjoyed learning how to solve word problems using SBI. The next chapter provides a discussion of major findings from the study.
5. DISCUSSION

This chapter includes discussion of the major findings and their implications for both researchers and practitioners that derived from the single-subject research study on schema-based instruction (SBI) process across three groups of students with high-incidence disabilities (HID). First, the findings are summarized related to each research question (RQ). Next, the discussion of results will also refer to five rubric categories (shapes, labeling, “if-then,” number sentence, and correct answer) for the proportional reasoning word problem probes (PRWPP). Limitations and recommendations for future research are also addressed in the chapter.

Summary of Findings

This section summarizes findings according to the research questions. The SBI process was related to improvement in only three categories for solving proportional reasoning mathematical word problems (RQ 1). Results were summarized for the number of categories completed correctly to solve proportional reasoning mathematical word problems (RQ 1a and 1c), and the percentage of accurate answers did not increase (1b and 1d). The SBI process was also related to learning only three categories for maintenance (RQ 2). Results were provided to demonstrate whether students maintained the number of categories completed correctly to solve proportional reasoning
mathematical word problems (RQ 2a and 2c), and the percentage of accurate answers was not maintained (RQ 2b and 2d). The students also increased self-efficacy to some extent for solving mathematical word problems after the SBI process was taught (RQ 3).

**Acquisition of the Schema-Based Instruction (SBI) Process**

Visual analysis of the multiple baseline data provided answers to parts of the first two research questions concerning the SBI process (RQ 1, 1a, 1c, 2, 2a, and 2c). The disaggregation of data by rubric categories provided answers to the remaining parts of the first two research questions (RQ 1, 1b, 1d, 2, 2b, and 2d). Overall findings indicated that students were able to learn three categories (shapes, labeling, and “if-then”) of the SBI process, although not to the point of arriving at the correct answer (RQ1a, 1b, 1c, and 1d). The number of categories students completed correctly in the SBI process to solve proportional reasoning mathematical word problems with and without calculators was assessed using the PRWPP (RQ 1a and 1c). The visual analysis of data was interpreted through six components: level, trend, variability, immediacy, overlap, and consistency (Kratochwill et al., 2010). Percent of nonoverlapping data (PND) was another data analysis technique used to determine effectiveness of the intervention (Scruggs et al., 1987).

The SBI intervention was focused on the “process” so the researcher was able to distinguish that the students were able to learn the SBI process (FOPS strategy) for solving proportional reasoning word problems. All groups without a calculator had a 60.4% increase from baseline, and all groups with a calculator increased by 52.3% from baseline. Although the students accumulated high points on the PRWPP during
implementation, the initial cumulative scoring of all five rubric categories combined (shapes, labeling, “if-then,” number sentence, and correct answer) did not reflect that students were not getting correct answers (RQ 1b and 1d). Plausible explanations for these findings are described after the summary of results.

The overall baseline (Phase I) mean for acquisition of the SBI process in all five rubric categories was 3.8%, $SD = 3.6$ (range of 2.7 to 5.5%) without a calculator and 1.6%, $SD = 1.4$ (range of 0.3 to 3.7%) with a calculator. The highest means in Phase II (SBI process) were 23.7 and 20.7% (without calculator and with calculator) in Category 1 (shapes) and 25.6 and 20.3% in Category 2 (labeling) of the PRWPP rubric. In Category 3, students performed at 11.0% without a calculator and 9.9% with a calculator (out of a ceiling of 12.5%). The lowest means were 2.2% and 1.8% in Category 4 (number sentence) and 1.7% and 1.3% in Category 5 (correct answer).

Results indicated that mastery per rubric category needed to occur for students to move to the next steps and finally to arrive at the correct answer (RQ 1b). The initial breakdown of the SBI process was observed in Category 2 (labeling). Without a calculator, students’ accuracy with the SBI process was 25.6% (out of a ceiling of 37.5%) in Phase II for Category 2 (from 0% in baseline). Similarly, students performed at 20.3% (from 0% in baseline) with a calculator. In labeling the “if-then” diagram (Category 2 of the PRWPP rubric), students had to rely on language use and mechanics to properly label the columns and then place the given numbers in the appropriate columns. Another factor to consider was the deficits in memory skills characteristic to this population of students (Geary, 2003; Pressley, 1986). Many students with LD exhibit difficulties in the areas of
memory and general strategy use, literacy and communication, and specific processes and strategies associated with mathematical problems (Bryant & Bryant, 2008). The strength of the SBI process was demonstrated for the students in the current study as they overcame deficits in memory to show improvement in three of the five rubric categories. The specific categories the students remembered most of the time were Category 1 (shapes) and Category 2 (labeling).

The overall PND, as measured by calculating the percent of data points in the treatment phase that did not overlap with the highest baseline data, for all three groups without a calculator was 85%, indicating fair effectiveness (Scruggs et al., 1986). The data for all three groups with a calculator resulted in an overall PND of 70%, indicating fair effectiveness as well. The PND data indicating fair effectiveness, however, is confined to the groups’ effectiveness as measured by the rubric’s first three categories (shapes, labeling, “if-then”); the data do not indicate students’ mastery of the fourth and fifth categories (number sentence and accurate answer) of the SBI process.

**Accuracy of Solving Proportional Reasoning Word Problems**

Students had the most difficulty with translating the proportion into a number sentence (Category 4 of PRWPP rubric) and arriving at the correct answer (Category 5). The mean of Category 4 (number sentence) was 2.2% without a calculator and 1.8% with a calculator. Category 5 (correct answer) also had low mean of 1.7 % without a calculator and 1.3% with a calculator. This difficulty was most likely caused by errors in labeling of the diagram in Category 2. During the intervention, the researcher attributed the incorrect answers (Category 5) on the PRWPP to students’ not retaining knowledge of how to
solve a proportion (the skill). As such, the researcher continued instruction with more guided practice until each student reached the 80% mastery. Additionally, the students not having knowledge of the prealgebra skills could have been a reason why they did not increase performance in Categories 4 and 5. The academic skill of cross multiplying in order to develop a number sentence (Category 4) was measured by Category 5 (correct answer). The prerequisite skill of cross multiplying to solve proportions needed to be pretaught with time to master that skill within instruction, or this skill could have been a required pre-requisite computational skill.

When the students were learning the SBI process during instruction, they were getting the answers correct (RQ 1b). Analysis by rubric categories on the PRWPP indicated that the students were mastering previous steps of the SBI process. The students learned the first three categories (shapes, labeling, and “if-then”) of the SBI process, although not to the point of getting the correct answers (Category 5) for the proportional reasoning word problems.

**Mastery of learning.** Guided and independent practice lessons during the SBI process included instructional feedback and gave students opportunities to practice the process of solving word problems by applying the process both with the teacher and independently. The rule when monitoring performance on one guided practice problem before moving to independent practice, however, was only to 80% criteria. This practice resulted in students having the incorrect answer even though they may have been improving in the SBI process.
The researcher continued with the intervention when Categories 4 and 5 (number sentence and correct answer) were not being mastered. The researcher’s focus was on progression in the overall SBI process when scoring the students’ PRWPP daily, according to overall performance on the five categories combined (not separated). Although the researcher monitored for correct answers during instruction and observed during the daily instructional lessons on which students obtained 100% during instruction, the students did not generalize that knowledge to PRWPP when they were assessed independently. It was observed by the researcher that many of the students were eager to set up the shapes for the “if-then” diagram during the instructional lessons, which demonstrated development of proficiency in Category 1 (shapes). Another observation by the researcher was that the previously worked problems from guided practice were on display during independent practice. In hindsight, the researcher wondered if the students accessed the charts that were on display during instruction to get correct answers, but the students completed the PRWPP without any materials (including the charts) to refer to.

**Group A.** Of the three students in Group A, two participated more often during guided practice than the third. All students, however, applied the SBI process to solve the third proportional reasoning word problems to at least 80% accuracy (100% accuracy most of the time). During independent practice, all of the students obtained 100% accuracy by at least the second attempt (i.e., the second problem during independent practice). As such, the researcher observed that the students were able to complete the
SBI process to the point of arriving at the correct answers during the instructional lessons.

**Group B.** All of the students in Group B participated in applying the SBI process during guided practice to at least the 80% criteria, and they each reached 100% accuracy by the second attempt. During independent practice, two of the students reached 100% criterion on the first attempt most of the time. The third student independently performed to 80% on the third guided practice problem and 100% on the second attempt. As such, the researcher observed that the students in Group B were able to complete the SBI process to the point of arriving at the correct answers during the instructional lessons.

**Group C.** For the students in Group C, one of the three students was initially resistant to “show all the work” for the problems during instruction and would have preferred to just write the answer. The researcher emphasized the importance of each step of FOPS and being able to use that process for more difficult numbers later on. That student began drawing the diagrams and applying the other categories and reached 100% criterion during instruction. The other two students reached 100% in guided practice and performed at 100% in independent practice by the second attempt. One reason these students were usually incorrect the first time was due to errors in solving the proportion and multiplying incorrect numbers (related to Category 4, writing the number sentence). The researcher observed that the all students in Group C were able to complete the SBI process to the point of arriving at the correct answers during the instructional lessons.

For all three groups, the students were getting correct answers during the instructional lessons, and yet they did not perform similarly on the PRWPP. One reason
may be that the PRWPP were considered “cold probes,” and students were administered this measure before instruction began the next session. Although during instruction, the students performed better and with accurate answers, this result may have been because they had practice with gradual removal of cues and prompts (i.e., scaffolding) before completing the problems on their own. In addition, the lessons were not on consecutive days for most groups. For students with learning disabilities (LD), remembering the steps of the SBI process and how to arrive at the correct answer over a few days may have been difficult, contributing to lower performance on the PRWPP. Overall, students did improve in three categories of the SBI process (shapes, labeling, “if-then”), although not to the point of having accurate answers. To reach 100% accuracy on the PRWPP and remember over time and accurately complete all five categories of the SBI process may have been achieved if there had been more sessions.

**Rubric.** One change that could have been made during instruction of the SBI process is use of the PRWPP rubric categories as a diagnostic tool to identify with which categories students were having problems and then tailor the next session’s instruction to focus on those categories. Additionally, although 80% mastery during guided practice seems high, it was not high enough for students to learn and remember the SBI process with grade-level numbers in the proportional reasoning problems. As such, the current method of determining mastery before independent practice should be replaced with an alternative method in classroom instruction as well as in future research.

One alternative would be to monitor for 100% on one or more guided practice problems to ensure each of the five rubric categories is mastered. Another solution would
be for students to have five problems to do for guided practice, and they must get four out of five problems correct before proceeding to independent practice. Ultimately, though, the students’ performance on the PRWPP should reach 100% on at least one occasion, if not more, to indicate mastery of the SBI process. Correspondingly, with a requirement that students achieve 100% on at least one occasion on the PRWPP to demonstrate mastery, the quantity of instructional sessions would also be increased.

**Maintenance**

The maintenance data provide evidence that students with HID can remember three categories of the SBI process 4 weeks later for some groups (RQ 2, 2a, 2b, 2c, and 2d). When all three groups of students completed maintenance (Phase III) PRWPP 4 weeks after instruction ended, Category 1 (shapes) increased to 25% out of a ceiling of 25% (from 21.7%) without a calculator and increased to 22.2% (from 20.7%) with a calculator. Category 2 (labeling) decreased to 24.1% (out of a ceiling of 37.5%) from 25.6% without a calculator and to 20.2% from 20.3% with a calculator. Category 3 (“if-then”) decreased to 9.9% out of a ceiling of 12.5% (from 11%) without a calculator and to 8.4% (from 9.9%) with a calculator. Category 4 in the maintenance data decreased from 2.2% and 1.4% (out of a ceiling of 12.5%) without a calculator and remained at 1.8% with a calculator. In Category 5 the students decreased from 1.8% to 1.2% (out of a ceiling of 12.5%, without a calculator) and increased to 2.2% from 1.3% (with a calculator). The students’ performance in maintenance highlights the strength of the SBI process because strategy instruction is effective for those first three rubric categories.
Self-Efficacy

The Mathematical Problem-Solving Self-Efficacy Scale (MPSSES) measured students’ individual beliefs regarding their ability to problem solve in mathematics, and this sense of self-efficacy was assessed both before and after the intervention. Prior to the SBI intervention, three of the students surveyed were in agreement that they usually feel they could not manage a solution when they start to solve a mathematical problem, which increased to four students after the SBI process was taught. Four students agreed that they did not feel sure of themselves when solving word problems in mathematics before the intervention. After the SBI intervention three more students were surer of themselves when solving word problems in mathematics for a total of seven students. Finally, the same number of students (3 students) that were in agreement they had difficulty solving word problems before the SBI intervention continued to agree with this concept after the treatment (two of three of these students were the same).

Although students were more certain of themselves when solving word problems, over half of the students still admitted to having difficulty. Student awareness that they were not getting the SBI process to mastery for all five categories may have attributed to why their scores on the MPSSES did not change or were not very high. Students likely knew they were learning well three of the SBI process, but they were also likely aware they were still not getting the correct answer.

Social Validity

Results of how satisfied the students were with the SBI intervention on the social validity scale may be attributed to positive attitudes of the students toward learning to
solve mathematical problems. Nevertheless, only six of the nine students would like to continue use of the strategy for other mathematical skills, according to the feedback given. It would have been beneficial to conduct interviews with students to ask why they would not like to continue using the SBI process for other skills in mathematics. Finally, six of the students reported that they plan to use this problem strategy on their own for word problems involving other skills. For the students who do not plan to use this strategy for other mathematical skills, it is possible that they were not learning the SBI process.

**Description of Major Findings Compared to Previous Research**

The results of the current research are discussed in this section and compared to results acquired by other researchers according to accuracy with the SBI process using proportional reasoning mathematical word problems, with and without the use of calculators (RQ 1). The steps of the FOPS strategy were used to teach the SBI process for the intervention: F- Find out if it is a proportional problem, O-Organize the information in the problem using the diagram, P-Plan to solve the problem, and S- Solve the problem. Accuracy of the students’ answers was also discussed (RQ 1b and 1d) as well as the extent to which students maintained the number of categories completed correctly in the SBI process 4 weeks after instruction (RQ 2a, 2b, 2c, and 2d). Finally, students’ responses about their self-efficacy for solving mathematical word problems after the SBI was taught were provided (RQ 3).
Duration of SBI Instruction

Regarding the duration of the intervention (across how many weeks, number of lessons, how many times per week, and length of each lesson), the current study included nine instructional lessons two to three times a week, lasting 35 minutes each over a course of 2-3 weeks (including baseline). Winter break affected the scheduling of the intervention for two of the groups (Groups B and C). The students continued to demonstrate knowledge of three categories of the SBI process, however, upon their return to school.

Other researchers who employed the SBI process with elementary students taught an average of 20 lessons for the intervention (Fuchs et al., 2004; Griffin & Jitendra, 2009; Jitendra et al., 2007; Jitendra & Hoff, 1996). However, research studies involving SBI at the middle school level were taught for an average of 12 lessons, lasting approximately 45 minutes each, for about 4 days per week (Jitendra et al., 2002; Jitendra et al., 2009; Jitendra & Star, 2012; Xin et al., 2005). The researcher considered this information to determine the number of lessons as a starting point for the current study. The researcher also factored into account that SBI procedures were modified in the current study to focus on only skill (proportional reasoning), while previous studies at the middle school level included multiple skills (i.e., incorporating percent problems as well as proportional). In the case of this study, the students may have mastered all five categories of the SBI process (including the last two categories: writing the number sentence and calculating the accurate answer) if there were more than nine instructional lessons and if attention was given to accuracy for each of the five rubric categories. Factors such as disability
type, number of skills being learned, and grade level also need to be considered when planning a timeline for the SBI intervention. For example, more time may be needed for mastery for some students with LD.

The current intervention was not administered on a daily basis, which could have affected the retention of skills for students with HID, resulting in the slight variability of data in the treatment phase. All three groups of students received SBI lessons with proportional reasoning word problem two to three times a week for the majority of the intervention. Other researchers, such as Xin et al. (2005), conducted the SBI intervention for middle-school students with LD (or for students at risk in academics) three to four times a week for 12 lessons. Jitendra et al. (2009) implemented the SBI intervention on consecutive days across 10 lessons and had a lower degree of improvement (only a 22% increase) than students in the current study of nine lessons. However, in other researchers’ studies, the students got the correct answer.

Phase II (SBI process) for all groups (both with and without a calculator) demonstrated a slow immediacy of change. This delay was to be expected since students needed to learn the SBI process first. It is characteristic for students with LD to need more time for instruction to learn and master the processes that require memory, use of language, and knowledge of terms (Bryant & Bryant, 2008; Geary, 2003). The fact that the students did remember three categories of the SBI process in the current study suggests that students with LD may have been able to learn and master the entire SBI process with more time.
**Calculator Use**

There were two of each type of problem (unit rates, proportions, and scale drawings) on the PRWPP. Three problems were on the front, and three were on the back. Calculators were permitted on for the last three problems on the back side of the PRWPP. Previous researchers have not noted use of calculators during teaching of the SBI process. Students in the current study increased in the SBI process by a higher percentage (60.4%) without the use of a calculator than when a calculator was permitted (52.3% increase). These results indicate that students can accurately learn the parts (as measured by the rubric categories) of the SBI process before there is a need for a calculator. Additionally, when students in each group could use calculators, all groups chose to use the calculator 19-30% of the time (Group A, 29% of the time; Group B, 19%; and Group C, 30%).

These percentages are low. Eight of the nine students had a calculator accommodation on their Individualized Education Plan (IEP), and the remaining student was permitted use of a calculator for mathematical application problems according to his IEP. According to progress students made in the first three categories of the SBI process, it was questionable whether students always needed a calculator. That is, providing calculators for students’ use does not take the place of students’ need to learn the reasoning for solving word problems. In this study, if students did not know how to set up the problem (Category 4), then the calculator was not beneficial.

**Instructional Features of SBI Process**

All groups, both with and without a calculator, demonstrated an accelerating trend within Phase II (SBI process). Some reasons for the accelerating trend may be that the
small group instruction allowed more focus, and students had motivation to learn the strategy. The study was based on carefully designed instruction and only focused on proportional reasoning skills, or one problem type, which may have influenced the students’ performance. The small 1:3 teacher-to-student ratio allowed more attention to the participants and possibly more focus on the part of the students.

The instructional features of SBI also contributed students’ learning of the SBI process. According to Hutchinson (1993), explicit instruction has been proven to aid students with understanding the structure of the problem so that students could effectively master problem-solving skills. The explicit instruction in the current study assisted students in learning the SBI process step by step. In addition, the diagrams created a representation of the problem when students entered problem information into the diagram. These diagrams were a representation of shapes in which to enter numbers, as opposed to representations that are pictures.

Results of the current study are not comparable to previous research as measured by accurate answers acquired when SBI was used because the current study’s rubric (SBI process) included five categories, and students demonstrated mastery only in the first three categories, not in the last two. Groups A, B, and C had a high degree of improvement in accuracy with the SBI process (60.4% increase without calculator and 52.3% increase with calculator). As previously stated, in other research studies students obtained the correct answer when applying the SBI process. Students in a study involving general education students at the same grade level improved by 22%, and the students did get the correct answer (Jitendra et al., 2009). In another SBI study, Jitendra et al. (2011)
examined the effectiveness of SBI for seventh-grade general education students, and results indicated a 16% increase in the SBI group also with getting the correct answer.

**Maintenance of Learning**

Some students with disabilities have more difficulty recalling information and remembering strategies due to deficits in cognitive load. For these students to maintain or exceed progress in certain categories of the SBI process after 4 weeks as reported in the current study is quite an accomplishment. It was not expected that students with HID would maintain learning and actually improve in three categories of the SBI process (shapes, labeling, and “if-then”) 4 weeks after instruction because of deficits in memory, attention, background knowledge, vocabulary, language processes, strategy knowledge and use, and visual spatial processing (Geary, 2003; Pressley, 1986). Jitendra and Star (2011) have emphasized the importance of teaching these students to recognize the schema in mathematical problem solving to reduce working memory resources and cognitive load.

Jitendra et al. (2007) assessed the difference between SBI, a single problem-solving strategy, and GSI, multiple problem-solving strategies. Those researchers also obtained higher maintenance scores than on the posttest immediately following treatment ($ES = .52$ on the immediate posttest and $ES = .69$ on a maintenance test 6 weeks later). The interval of 4 weeks between the intervention and maintenance session is the timeframe recommended between the intervention and maintenance session (Xin & Jitendra, 1999). However, seventh-grade general education students maintained their proportional word problem performance 4 months later in one study conducted without
students with disabilities that included students getting the correct answer (Jitendra et al., 2009).

**Practical Implications**

The findings for this study revealed a number implications for teachers who want to have their students learn the SBI process. One implication for these findings related to rubrics used for teaching the SBI process is that instruction for teaching a mathematical “process” needs to be carefully monitored so that students are mastering each step of the process necessary before arriving at the correct answer. The rubric categories in the current study included two categories (shapes and number sentence) measured in previous research. Jitendra et al. (2002) examined strategy use by whether students followed SBI and identified the problem schemata by drawing a diagram and developing a plan or writing a number sentence. However, the separation of data done in the current study via the five categories was a more detailed assessment to determine number of categories completed accurately for mastery of the SBI process.

Students with HID may experience more success if their teachers focus on teaching for mastery and measure for mastery with carefully designed rubrics. In a diagnostic capacity, teachers could use rubrics such as the one used in this research to alert them to what category students need more instruction in and what categories they mastered. Then teachers could pinpoint the location of the breakdown before students are able to advance to the next steps necessary to obtain the correct answer. Teachers need to be assessing all components of a process or strategy in the classroom.
FOPS as an acronym is a beneficial tool for students to remember the steps to solve proportional reasoning word problems and the sequence of the steps. Each step of the acronym had corresponding actions for students to do. Some students who learn FOPS and the corresponding actions, including when and how to use FOPS, master the strategy. That is, the acronym alone, FOPS, evolves to a strategy when students know when to use it, how to use it, and how to self-correct. The implications for teachers is that they need to give sufficient attention to teach the corresponding actions for each letter in the acronym so that students learn the strategy. The students in this study learned the “F” and “O” steps but were still acquiring mastery of the “P” and “S” steps with grade-level computations.

During instruction, one of the things that helped the students learn the strategy is that they had materials to refer to (i.e., visual of FOPS steps and “if-then” diagram). The teachers used the student materials and other charts or worked examples in the classroom that students could refer to during instruction. It is typical for teachers to have materials displayed in the classroom that relate to the content students are learning. In hindsight, the researcher wonders if the students accessed these materials as their cues and prompts in the independent practice component because the researcher observed students obtaining high scores (i.e., 100%) during the instructional lessons. One implication for teachers may be to have a “challenge activity” during the guided practice and independent practice. In this challenge activity, students would be able to see how much they could do on their own before they needed to refer to materials in the classroom.
Ultimately the students’ attainment of proficiency is demonstrated by a cold probe (assessment separate from instructional lesson/access to materials).

It would also be beneficial for both teachers and students to review student work samples or scored assessments together to clarify student understanding of FOPS and inform the teacher of student needs. Knowing how to solve word problems to the point of getting the right answer is what students need to do in life and on high-stakes tests. This review could also be incorporated into the classroom when assessing students’ mastery for different skills. Another approach to targeting the source of misunderstanding would be to do error analysis. Error analysis in a process such as SBI would assist in locating the source of the breakdown before students continue to practice without mastery of initial steps of the process. The PRWPP rubric could be used for that error analysis.

Another implication for instruction is that teachers of students with IEPs who note calculators as accommodations need to be focused on the process students use before students turn to the calculators. Focus on numbers and calculations in mathematics can hinder progression with solving math word problem, just as spelling deficits in the writing process may delay quality of the writing. It may be possible that if students are not distracted by calculation deficits, they can focus more on the process. Calculators themselves may not help students determine accurate answers if the students do not also comprehend the word problem and know what to do and how to solve the problem.

**Limitations**

There were six limitations to this study. A major limitation relevant to procedures is the first to be discussed. Limitations according to duration of the intervention,
participant characteristics (number of students and setting), and materials (researcher-created probes and scales) are described next.

**Limitations to Procedures**

A major limitation of this research is reliance on 80% versus 100% mastery criteria during guided practice of the SBI process instruction with each individual and each group. The criteria should have been 100%, and guided practice instruction should have extended until the students mastered all five categories of the rubric, culminating in the correct answer. The levels of performance during the intervention were graphed by total percentage of accuracy with the SBI process by group (all five categories together). The researcher did not wait for students each individual in the group to master Categories 4 and 5 (number sentence and correct answer) because the students were showing progression in the overall SBI process according the predesignated 80% criteria for guided practice.

During instruction, the students were arriving at the correct answer in guided practice after multiple attempts. The error in the procedures was to have 80% on the third guided practice problem as a mastery criteria. If the students only scored 80%, they were not getting the correct answer. Another limitation to the current study is that data were not collected on the amount and type of teacher feedback given during guided and independent practice.

**Limitations to Duration**

The current study was implemented for each of the three groups through nine instructional lessons. The lessons lasted 35 minutes each and were conducted two to three
times a week over the course of 2-3 weeks (including baseline). Students with disabilities may need an intervention over a longer timespan for adjusting to the structure and having time for mastery of information to be learned.

**Limitations to Participant Characteristics**

Regarding the participants, only nine students participated in the study in small-group settings. Therefore, even though groups of students showed improvement in the SBI process for those nine students, generalizability to more students in larger classes is not possible. Another limitation of this study also related to the generalization of the findings. Results are not generalizable to students beyond those included in the study. Due to all nine participants being identified with a disability, the findings from this study may not have similar results for average students in mathematics or for students in the same or other education disability categories, including other students with LD, OHI, and autism.

**Limitations to Materials**

One final limitation of the study is the researcher-created probe (PRWPP) used to measure progress with the SBI intervention. Data collected with nonstandardized instruments is more susceptible to systematic errors (Mertens, 1998). However, the researcher probe was validated by two experts in the area of mathematics. In addition, the Mathematical Problem-Solving Self-Efficacy Scale only included items related to mathematical problem-solving in general, whereas the intervention itself dealt with proportional reasoning mathematical word problems.
Recommendations for Future Research

Recommendations for future research are described in this section pertaining to populations of students, grade-levels, other settings (e.g., cotaught classrooms), and mathematics curriculum. There is limited research on the SBI intervention in the area of proportional reasoning skills for students with HID in the middle school setting. Future research should address this population because they may benefit from learning the SBI process to solve proportional reasoning word problems. High-school students with HID may also benefit from instruction in the SBI process to solve these types of word problems. Providing scaffolds or schema diagrams help these students to organize information in word problem, reduce cognitive load, and teach key information in the problem (Xin et al., 2005).

Based on the current research, a recommendation for procedures is to monitor error patterns. An example of this would be to measure student progress with the five rubric categories used in this study (shapes, labeling, “if-then”, number sentence, and correct answer) which can provide diagnostic instructional information for teachers. Moreover, SBI process mastery per category should be 100% (as opposed to 80% criterion during guided practice as used in the current study).

Other recommendations for future research involve replication of the current study in additional settings, such as a cotaught classroom, incorporating the changes that have been identified (i.e., increase the 80% criteria to 100% in guided practice) from the current study. Research could also be conducted to teach the SBI process with other grade-level mathematical skills in the middle-school mathematics curriculum (e.g.,
fractional word problems and algebraic concepts). Finally, more comparative studies can be done to analyze the effect of additional interventions to improve the process of solving word problems compared to the SBI intervention.

Another recommendation for future research would be to collect data on the amount and type of teacher feedback given during guided and independent practice. In the current study, the students did require teacher feedback in guided and independent practice. An approach also related to feedback in future research of the SBI process would be to focus on mastery of the SBI process without teacher feedback and to probe or assess students after mastery.

Regarding prerequisite skills, the current study revealed that prealgebraic skills to solve proportional reasoning need to be pretaught or have time built into the lessons for mastery of these skills within the SBI process. A recommendation for future research would be to require prealgebraic skills as a prerequisite or measure these skills on the Multiple Skills Computation Assessment (MSCA). Finally, the self-efficacy measure should also capture the teaching process by replacing the word “solve” with “process” when applicable. In the Mathematical Problem Solving Self-Efficacy Scale for the current study, the behavior of solving problems, not the process, was emphasized.

**Summary**

Previous researchers found evidence that the ability to improve mathematical problem solving performance for both elementary and middle school students is possible (Butler et al., 2003; Fuchs et al., 2004; Hutchinson, 1993; Maccini & Hughes, 2000; Montague et al., 2011). The results from this study show that SBI was also an effective
intervention to improve accuracy with the SBI process in solving proportional reasoning word problems for middle school with HID. It is imperative that the teaching of SBI or any other mathematical process be carefully monitored so that students are mastering each step of the process necessary before arriving at the correct answer. In addition, instructional time needs to be flexible for students with disabilities to allow time for mastery of the overall process.
APPENDICES
APPENDIX A

Student Workbook

If

Then

Workbook Contents:
Key terms
FOPS checklist
If-Then diagram
Student practice problems
**Key terms**

- **Ratio:** compares the values of 2 or more amounts

  ![Diagram of triangle to circle ratio]

  The ratio of circles to triangles is \( \frac{2}{3} \) (can also be written as 2 to 3 or 2:3)

  The ratio of triangles to circles is \( \frac{3}{2} \)

- **Rate:** ratio that compares amounts measured in different units

  \[ \frac{12}{4} \text{ hotdogs} \] OR \[ \frac{130}{2} \text{ miles} \]

- **Unit:** the part being measured (dollars, hotdogs, miles, hours)

- **Unit rate:** rate expressed as a quantity of 1

  \[ \frac{2 \text{ feet}}{1 \text{ second}} \] OR \[ \frac{5 \text{ miles}}{1 \text{ hour}} \]

  The word "per" is always a clue that you are dealing with a unit rate.

  **Application** → If you have a rate such as 120 students for every 3 buses, and want to find the unit rate, write a ratio equal to the original rate with 1 as the bottom term.

  \[ \frac{120 \text{ students}}{3 \text{ buses}} = \frac{? \text{ students}}{1 \text{ bus}} \]
- **Association**: A relationship that compares the values of 2 or more amounts/also another word for “ratio” (see definition above)

- **Equivalent**: means EQUAL or “same as"

  4 dogs + 6 dogs = 10 dogs

- **Proportion**: 2 ratios that are equal

  \[
  \frac{20 \text{ boxes}}{2 \text{ trucks}} = \frac{40 \text{ boxes}}{4 \text{ trucks}}
  \]

- **Cross products**: diagonal units in ratios

  \[
  \frac{10}{1} = \frac{50}{5}
  \]

- **Translate**: to change proportion into number sentence

  \[
  10 \times 5 = 50 \times 1
  \]

  5050 yes, forms a proportion!

- **Equation**: A number sentence that compares two amounts, with an equal sign

  \[
  10x = 50
  \]
If-then statement:
- If 10 boxes fit in 1 truck, then 50 boxes will fit in 5 trucks
- If there are 6 seats in each rowboat, then how many seats will be in 5 rowboats?

Scale Drawing: used to represent something that is too large or too small for an actual size drawing.
- A map is an example of a scale drawing b/c the scale on a map is a ratio of the distance on a map to the actual distance

Retell: describe in own words (paraphrase)

Graphic organizer: a diagram used to organize information
Map: to diagram, or place information from problem into a graphic organizer
If-then diagram (p.6): graphic organizer used to map information from proportional word problems
Label: Identify or describe information

FOPS strategy (p.5): problem-solving strategy
FOPS Problem-Solving Strategy

1) F- Find out if it is a proportional problem.
   - Read and retell story.
   - Is there an association between 2 things?
   - Is an “if-then” statement included?

2) O- Organize the information (using the “if-then” diagram)
   - Underline 2 things that form a specific ratio.
   - Circle the numbers in the problem that form ratios and label in diagram.
   - Write a “?” for what must be solved (find question sentence)

3) P- Plan to solve the problem
   - Translate information from diagram into number sentence

4) S- Solve the problem
   - Is answer complete, accurate, and does it make sense?
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### Types of Proportional Reasoning Skills in Word Problems

<table>
<thead>
<tr>
<th>Types</th>
<th>Example</th>
</tr>
</thead>
</table>
| **Unit Rate**        | If a school bus holds 50 students, then how many buses are needed to bus 960 students to their school?  
  *
  In the lessons for all the unit rate word problems, "1" will always be one of the numbers used ("a" school bus = 1 school bus).* |
| **Proportion**       | Carl can read 6 pages in 14 minutes, how long will it take him to read a chapter that has 21 pages?  
  *
  The proportion problems are similar to unit rate problems; however, they will not include the number "1."
  |
| **Scale Drawing**    | The scale on a blueprint is 1 cm for every 2 m. If the width of an arch is to be 5 m, then how wide should it be drawn on the blueprint?  
  *
  The scale drawing problems can be recognized including a scale in the problem described in metric units such as inches or feet.* |
Examples of Proportion Problems at each Level of Difficulty

- **Novice**
  - If 1 cup of flour is used to make 5 dumplings, how many cups of flour are needed to make 10 dumplings?

- **Intermediate**
  - Gina used 9 apples to make 12 apple pies. If she has only 3 apples, how many apple pies can she make?

- **Advanced**
  - If Mr. Hall’s car will run 21 miles on each gallon of gas, then how many gallons will be needed to drive 84 miles?
Demonstration 1: If one pencil costs 7 cents, then a pack of 12 pencils will cost 84 cents.

\[
\begin{align*}
1 \text{ pencil} & = 12 \text{ pencils} \\
7 \text{ cents} & = 84 \text{ cents}
\end{align*}
\]

Demonstration 2: If there are 6 seats in each rowboat, then 5 rowboats will have 30 seats.

<table>
<thead>
<tr>
<th>Rowboats</th>
<th>Seats</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 rowboat</td>
<td>6 seats</td>
</tr>
<tr>
<td>5 rowboats</td>
<td>30 seats</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\frac{1}{5} & = \frac{6}{30} \\
1 \times 30 & = 5 \times 6 \\
30 & = 30
\end{align*}
\]

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Guided Practice

1) If a school bus holds 40 students, then 24 buses are needed to bus 960 students to their school.

2) Kelly and Ryan are responsible for making drinks for the party. They used 6 lemons to make 2 quarts of lemonade. To make 8 quarts of lemonade they need 24 lemons.
Independent Practice

1) If 1 circus ticket costs $4, then $24 can buy 6 circus tickets.

If

Then

2) Each skating board has 4 wheels, 9 skating board will have 36 wheels.

If

Then
Demonstration (Unit rate/Novice)

1) The Dale Company packs 4 cans of tomatoes in each box. How many boxes will the company need to pack 8 cans of tomatoes?

If

\[
\begin{align*}
\text{Cans of tomatoes} & \quad \text{Crates} \\
4 \text{ cans} & \quad 1 \text{ box} \\
8 \text{ cans} & \quad ?
\end{align*}
\]

Then

\[
4 \times ? = 8 \times 1 \\
4 \times 3 = 8
\]

2) Sam used 3 bags of candy bars to make 1 ice cream cake. To make 6 ice cream cakes, how many bags of candy bars would he need?

If

Then
Guided Practice

1) Carissa gets paid $6. per hour to babysit, how much would she get paid for 2 hours?

2) If 1 cup of flour is used to make 6 dumplings, how many cups of flour are needed to make 30 dumplings?

3) The cost of 2 bags of candy is 1 dollar. If you have 6 dollars, how many bags of candy can you buy?

Independent Practice

1) Erica uses 7 gallons of gas to drive 14 miles. How many miles can she drive using 1 gallon of gas?

2) Brian can eat 3 hotdogs in 1 minute, how many hotdogs can he eat in 6 minutes?
**Demonstration** (Unit Rate/Intermediate)
1) A total of 48 slices of cake are ready to be served to students sitting at many tables. If 8 slices of cake will be served at each table, how many tables can be fully served?

2) There are 4 chocolate bunnies in each candy box. If you bought 12 boxes, how many chocolate bunnies would you have?

**Guided Practice**

1) If one pack of socks costs $5., how much will 7 packs of socks cost?
2) If 2 baked cookies are $1., how much money will the bakery make for 10 cookies?

3) Bob reads 60 pages of a book in 1 hour. How long should it take him to read 180 pages?

**Independent Practice**

1) Callie gets paid $6. per hour to babysit, how much would she get paid for 20 hours?

2) In a classroom, each row can only seat 7 students. How many students would be seated in 12 rows?
Demonstration (Unit Rate/Advanced)
1) All of the seats at a movie theater were taken by a total of 384 people. If there are 16 rows of seats in the theater, how many seats are there in each row?

2) Each notebook has 70 pages in it. If you bought 5 notebooks, how many pages would you have in all?

Guided Practice
1) If Mr. Hall’s car will run 21 miles on each gallon of gas, then how many gallons will be needed to drive 84 miles?
2) A store packs 42 cans of tomatoes in each crate. How many crates will the store need to pack 630 cans of tomatoes?

3) Eric uses 7 gallons of gas to drive 196 miles. How many miles can he drive using 1 gallon of gas?

**Independent Practice**

1) Only 4 buses are available for a school field trip. If each bus will sit 26 students, how many students in all can be seated in the buses for the field trip?

2) Michelle used 27 apples to make 9 apple pies. If she has only 9 apples, how many apple pies can she make?
**Demonstration** (Proportion/Novice)

1) The cost of 3 chocolate bars is 2 dollars. If you have 6 dollars, how many chocolate bars can you buy?

2) Ben can eat 4 hotdogs in 2 minutes, how many hotdogs can he eat in 6 minutes?

**Guided Practice**

1) If 2 blueberry bagels are $4., how much money will the shop make for 20 bagels?

2) Maria used 6 strawberries to make 3 cups of fruit cocktail. If she needs to make 12 cups of fruit
cocktail for a party, how many strawberries does she need?

3) Gina used 6 apples to make 2 apple pies. If she has only 3 apples, how many apple pies can she make?

**Independent Practice**
1) Carl can read 3 pages in 6 minutes. At this rate, how long will it take him to read a chapter that has 12 pages?

2) Jenny spent 2 hours making 12 Christmas ornaments. How many hours will it take her to make 36 ornaments?
Demonstration (Proportion/Intermediate)

1) In Kylie’s school, 2 out of every 5 seventh graders went to a movie theater over the weekend. If there are 80 students in the seventh grade, how many of them saw a movie?

2) Ms. Bayer made 48 peanut butter cookies using 6 eggs, if she only has 2 eggs, how many peanut butter cookies can she make?

Guided Practice

1) Jill spent 3 hours making 12 Christmas ornaments. How many hours will it take her to make 36 ornaments?
2) Carl can read 6 pages in 14 minutes. At this rate, how long will it take him to read a chapter that has 21 pages?

3) Jill used 9 apples to make 6 apple pies. If she has only 3 apples, how many apple pies can she make?

**Independent Practice**

1) Ben can eat 7 hotdogs in 2 minutes, how many hotdogs can he eat in 6 minutes?

2) If 4 blueberry bagels are $4., how much money will the shop make for 120 bagels?
Demonstration (Proportion/Advanced)
1) If 3 eggs are needed to make 18 cupcakes, and you plan to make 126 cupcakes for a party, how many eggs are needed?

2) If 60 paper rolls are used to cover 12 textbooks, how many paper rolls will be needed to cover 10 books?

Guided Practice
1) If Ray can save $48. In 5 months, how many months will it take Ray to save $144.?
2) Ms. Bayer made 48 peanut butter cookies using 6 eggs, if she only has 2 eggs, how many peanut butter cookies can she make?

3) Only 10 buses are available for a school field trip. If 3 buses will sit 75 students, how many students in all can be seated in the buses for the field trip?

**Independent Practice**

1) A farmer has 444 eggs to pack in cartons. If each egg carton holds 12 eggs, how many cartons will the farmer need?

2) A car used 8 gallons of gas to travel 120 miles. At that rate, how many gallons will be used to travel 360 miles?
Demonstration (Scale Drawing/Novice)

➢ Scale drawing: used to represent something that is too large or too small for an actual size drawing.
   
   A map is an example of a scale drawing b/c the scale on a map is a ratio of the distance on a map to the actual distance

1) If 1 cm on a map represent 3 miles on the road, how many miles will 6 cm represent?

2) The scale of a blueprint is 1in = 2ft. If the actual width of a porch = 6ft, what is the width on a blue print drawing?
**Guided Practice**

1) In the drawing of a car, the scale is 2 inches for each foot. If the drawing is 10 inches long, how long is the actual car?

2) The scale on a blueprint is 1 cm for every 2 m. If the width of an arch is to be 5 m, then how wide should it be drawn on the blueprint?

3) On a scale drawing, 1 in = 2 ft. What would the length of a car on a scale drawing be that that is 6 feet wide?

**Independent Practice**

1) If the scale is 2 cm = 1 m, what would the length be in a drawing of desk that is 4 m long?

2) On a scale drawing, 1 in = 2 ft. Find the length of a car that is 5 feet wide
Demonstration (Scale Drawing/Intermediate)

1) The Burkes are building a deck in their back yard. They want it to be 10 feet wide. If the scale on the blueprint is 1in = 3 feet, what should the dimension for width be in the scale drawing?

2) If the scale is 2cm = 1m, what would the length be in a drawing of desk that is 5m long?

3) On a scale drawing, 1in. = 3ft. What would the length of a car on a scale drawing be that that is 24 feet wide?
Guided Practice

1) If 5in = 1 mi on a map, what is the distance of 2 miles on the map?

2) The scale of a blueprint is 1in = 4ft. If the actual width of a porch = 16ft, what is the width on a blueprint drawing?

Independent Practice

1) If 3 cm on a map represent 12 miles on the road, how many miles will 5 cm represent?

2) In the drawing of a car, the scale is 4 inches for every 2 feet. If the drawing is 12 inches long, how long is the actual car?
Demonstration (Scale Drawing/Advanced)

1) A.J. likes cars a lot. He draws his favorite car on a scale drawing. In the drawing, the scale is 2 inches for each foot. If the drawing is 2 inches long, how long is the actual car?

2) The scale on a blueprint is 3 cm for every 5 m. If the width of an arch is to be 15 m, then how wide should it be drawn on the blueprint?

Guided Practice

1) Tim and Megan are planning a hiking trip. On their map, their route is 9 cm long. If 1 cm = 3 km, what is the actual length of their hike?

2) On a scale drawing, 3 in = 15 ft. What would the length of a car on a scale drawing be that that is 25 feet wide?
3) If 6 cm on a map represent 72 miles on the road, how many miles will 12 cm represent?

**Independent Practice**

1) Adam made a scale drawing of his bedroom. In the drawing, 16 cm represent 4 m. If the windows in the drawing are 48 cm apart, how many meters apart are the actual windows?

2) The Burkes are building a deck in their back yard. They want it to be 15 feet long. If the scale on the blueprint is 1 inch = 3 feet, what should the dimension for width be in the scale drawing?
APPENDIX B

If-Then Diagram

If

Then

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FOPS Problem Solving Checklist

1) F- Find out if it is a proportional problem.
   □ Read and retell story.
   □ Is there an association between 2 things?
   □ Is an "if-then" statement included?

2) O- Organize the information (using the "if-then" diagram)
   □ Underline 2 things that form a specific ratio.
   □ Circle the numbers in the problem that form ratios and label in diagram.
   □ Write a "?" for what must be solved (find question sentence)

3) P- Plan to solve the problem
   □ Translate information from diagram into number sentence

4) S- Solve the problem
   □ Is answer complete, accurate, and does it make sense?
APPENDIX D

George Mason University

EFFECTS OF THE SCHEMA BASED INSTRUCTION ON MATH PROBLEM SOLVING

PARENT INFORMED CONSENT FORM

RESEARCH PROCEDURES
I would like to determine the effects of schema-based instruction (SBI) on math problem solving for students with learning disabilities (LD). SBI utilizes diagrams of problem patterns and structure to organize word problem information. If you agree to have your child participate, he/she will be practicing word problems through a problem-solving intervention on a daily basis for approximately three to four weeks. The schema-based instruction (SBI) intervention consists of 15 scripted lessons, and one lesson will be presented per day. There are three levels of instructional lessons so the word problems are at incremental stages of difficulty for the students. Each session will take no longer than 35 minutes. The research will take place during the child's math class or lunch period, based on schedule and preference. At the end of the study, your child will be questions to measure their degree of satisfaction with the math problem solving intervention.

If you agree for your child to participate in this research, you are agreeing to allow the researchers to use your child’s performance as part of the research.

1. To acquire information that allows comparisons, your child will be asked to complete an assessment on multiple skills so the researcher knows what students’ performance levels are prior to instruction.

2. Information such as age, gender, report card grades, and achievement scores (from teachers and/or school files, as applicable) may be gathered so that the researcher can better describe the students who participate in the research. The researcher will replace names with alphabetical codes so that all research participants’ information becomes anonymous.

3. To acquire feedback from students who participate, the researcher will ask them to rate aspects of the technique on a written scale of nine questions on how they
liked the intervention. This activity will occur during the class period in the classroom or another appropriate space of the school. The activity will last approximately 10 minutes.

RISKS
There are no foreseeable risks for your child when participating in this research.

BENEFITS
There are no benefits to your child as a participant other than to further research in math problem solving.

CONFIDENTIALITY
The data in this study will be confidential. Your child’s name will not be included on any collected data. A code will be placed on the collected data through the use of an identification key so the researcher will be able to link your child’s information to his/her identity. Only the researcher will have access to the identification key.

PARTICIPATION
Your child’s participation is voluntary, and your child may withdraw from the study at any time and for any reason. If you decide not to have your child participate or if your child withdraws from the study, there is no penalty or loss of benefits to which you are otherwise entitled. There is no cost to your child or any other party.

CONTACT
Anne Brawand, a doctoral candidate from George Mason University is conducting this research. If you have any questions you can email her at aeichorn@masonlive.gmu.edu. You may contact her supervisor, Dr. Peggy King-Sears, at 703-993-3916 (mkingsea@gmu.edu) for questions or to report a research-related problem. You may contact the George Mason University Office of Research Integrity & Assurance at 703-993-4121 if you have questions or comments regarding your rights as a participant in the research.

This research has been reviewed according to George Mason University procedures governing your participation in this research.
CONSENT
You received two copies of this consent form. Please sign below to indicate whether you agree to have your child participate, or choose to decline participation. Keep one copy for your records and return the other.

______________________________________________________________________________
(Student’s Name)

I have read this form and agree for my child to participate in this research.

_______________________________________            Date_______________
(Parent Signature)

I have read this form and choose to decline participation for my child at this time.

_______________________________________            Date_______________
(Parent Signature)

Version date: 09-19-12
Teachers Sending Parent Informed Consent Home – Script to Describe for PARENTS – September 2012

Parents: I, (name of teacher), have been asked to have my students participate in a research study conducted by a doctoral candidate from George Mason University, Anne Brawand. The research is about a math problem-solving intervention, and the researcher would also like your child to participate by learning a strategy for problem solving. Participation is voluntary, and your child’s identity will be anonymous because a code will be assigned to his/her data.

In order for your child to participate, your permission is needed. Attached is a Parent Informed Consent. If you are OK for your child to participate, indicate your permission by signing this form. If you would prefer that your child not participate, please sign on the second line on the Parent Informed Consent to indicate that you decline participation. Either way, the Parent Informed Consent should be returned to name the location for that school by a date within one to two weeks of the day distributing the Parent Informed Consent.

If you have any questions for the researcher, Anne Brawand’s name, email, and phone # are on the Parent Informed Consent. This is all the information I have about the research. I (the teacher) am interested in participating in the research. Before your child can participate, the researcher needs your permission, one way or the other. To indicate your decision, please sign this form after indicating your decision regarding your child’s participation. Then return the form to name the place and date.
Script to Read to STUDENTS – September 2012

Students: I, (name of teacher), have been asked to have my students participate in a research study conducted by a doctoral candidate from George Mason University, Anne Brawand. The research is about a math problem-solving intervention, and the researcher would also like you to participate by learning a strategy for problem solving. Participation is voluntary, and your child’s identity will be anonymous because a letter will be assigned to his/her data.

In order for you to participate, your parent or guardian must sign this letter, which the researchers call a Parent Informed Consent, and indicate whether they give permission by signing this form. Either way, the Parent Informed Consent should be returned to name the location for that school/ also written on board by a date within one to two weeks of the day distributing the Parent Informed Consent.

If you have any questions for the researcher, Anne Brawand’s name, email, and phone # are on the Parent Informed Consent. This is all the information I have about the research. I (the teacher) am interested in participating in the research. Before you can participate, your parents need to sign this form after they have indicated whether or not you can participate, then you return the form to name the place and date.
APPENDIX F

George Mason University
EFFECTS OF SCHEMA-BASED INSTRUCTION ON MATH PROBLEM-SOLVING

STUDENT ASSENT FORM

RESEARCH PROCEDURES
I would like to study if a problem-solving strategy will improve your math performance. I will work with you daily to see if you are able to answer math word problems using the strategy learned. If you agree to learn the math problem-solving strategy, we will work together daily for about 3-4 weeks. We will meet each time for no more than 35 minutes. You will learn how to solve word problems with proportions. You will answer some questions about how you liked the problem-solving strategy at the end.

BENEFITS AND RISKS
There is no benefit to you and nothing bad will happen to you if you do or do not participate.

CONFIDENTIALITY
Your name will not be included on any document. A code will be placed on your work so I will be able to link your information to your identity. Only I will have access to the identification key.

PARTICIPATION
You do not have to participate if you do not want to, and you may stop at any time.

CONTACT
My name is Anne Brawand. I’m a student at George Mason University. You can call or email me if you have questions about this study. My phone number is 410-279-4723, and my email is aeichorn@masonlive.gmu.edu. You may contact my supervisor, Dr. Peggy King-Sears, at 703-993-3916 (mkingsea@gmu.edu) for questions or to report a research-related problem.

The George Mason University Office of Research Subject Protections knows all about...
our research. You can call them at 703-993-4121 if you have any questions about being a part of this research.

I have read this form or someone read it to me and I agree to participate in the study.

___________________________________________  Date_______________
( Student Signature)  
Version date: 09-16-12
APPENDIX G

Multiple-Skills Computation Assessment A: Student Copy

Student: ____________________ Date: ____________________

\[
\begin{array}{c|c|c}
2 & +5 & 3 \\
\hline
\end{array}
\]

\[
\begin{array}{c|c|c}
1 & \div 1 & 2 \\
\hline
\end{array}
\]

\[
\begin{array}{c|c|c}
1 & \times 2 & 1 \\
\hline
\end{array}
\]

\[
\begin{array}{c|c|c}
9 & - 7 & 6 \div 6 \\
\hline
\end{array}
\]

http://www.interventioncentral.org/
<table>
<thead>
<tr>
<th>5 [\text{+ 2}]</th>
<th>4 [\times 1]\</th>
</tr>
</thead>
<tbody>
<tr>
<td>55 [\text{+ 39}]</td>
<td>78 [\text{+ 5}]</td>
</tr>
<tr>
<td>87 [-28]</td>
<td>5943 [-1531]</td>
</tr>
<tr>
<td>46 [\text{+ 58}]</td>
<td>7 [\text{+ 27}]</td>
</tr>
<tr>
<td>25 [-18]</td>
<td>3663 [-2531]</td>
</tr>
</tbody>
</table>
\[
\begin{align*}
76 + 8 &= 84 \\
37 + 45 &= 82 \\
641 \div 35 &= 18.31 \quad & 131 \times 12 \\
81 \times 21 &= 1701 \\
9207 \div 27 &= 341 \quad & 212 \times 21 \\
584 \div 30 &= 19.47 \\
\end{align*}
\]
http://www.interventioncentral.org/

Please rate the following statements on the lines provided using the scale below:

1 – Strongly disagree; 2 – disagree; 3 – agree; 4 – strongly agree

_____ When I start to solve a mathematical problem, I usually feel I cannot manage a solution.

_____ I do not feel sure about myself when problem-solving in mathematics.

_____ I have difficulty solving word problems.
APPENDIX H

MSCA Individual Results

<table>
<thead>
<tr>
<th>Skill</th>
<th>Proficient</th>
<th>Area of need</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic addition</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Basic subtraction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Basic multiplication</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Basic division</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Addition with regrouping</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Addition without regrouping</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subtraction with regrouping</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subtraction without regrouping</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multiplication of 2-digit numbers</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Division of 2-digit numbers</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Number of skills proficient \( \frac{\text{__________}}{10} \)

*Students needs to demonstrate proficiency in at least 3 of the 10 skills assessed to participate.*
APPENDIX I

SBI Lesson Plan  Baseline Lessons #1-5

**Note:** First 5 lessons have same format, 35 minutes each lesson. Some participants will complete more than 5 lessons of baseline condition in order to acquire baseline information.

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<table>
<thead>
<tr>
<th>Lesson Component</th>
<th>Activity</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportional Reasoning Word Problem Probes (PRWPP) administered</td>
<td>T distributes the Proportional Reasoning Word Problem Probes (PRWPP) to participant(s), and directs student(s) to record the start time on the data collection sheet. T explains: <em>This is a Proportional Reasoning Word Problem Probe including 6 word problems with ratios, proportions, and scale drawings. First, complete #s 1, 2, and 3 on the front side. Then raise your hand to let me know you are ready to do #s 4, 5, and 6 on the back. You can use a calculator when completing those problems. You cannot return to #s 1, 2, and 3 once you have the calculator, so be sure to check your work before asking for the calculator. There is a box on the back to check whether you did use a calculator. When you have completed all six word problems, raise your hand to let me know you are finished.</em> The T does not provide prompts or cues for student(s). After completed, T directs student (s) to</td>
<td>10 minutes</td>
</tr>
</tbody>
</table>
**record the end time on the PRWPP, and T collects their work.**

| ****Administration of Multiple Skill Computation Pretest (Version A, calculator cannot be used. Version B, calculator can be used) (only during Days 1 and 2 of Baseline condition) | Classroom T will administer the 2 versions of assessments (Version A without calculator, and Version B with the calculator) before baseline phase; however, the researcher may give it to students to complete when necessary. 

(3 Questions on self-efficacy will also be included on this pre-assessment) | **25 minutes** |

**SBI Lesson Plan**

**Overview of SBI Process**

**Lesson #6**

*(may take longer than 1 lesson)*

<table>
<thead>
<tr>
<th><strong>Lesson Component</strong></th>
<th><strong>Activity</strong></th>
<th><strong>Time</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportional Reasoning Word Problem Probes (PRWPP) administered</td>
<td>T distributes the Proportional Reasoning Word Problem Probes (PRWPP) to participant(s) following the procedures in baseline lessons 1-5. When student raises hand to indicate they have completed the PRWPP, the T will collect and then distribute Student Workbook to be used for SBI instruction.</td>
<td><strong>10 minutes</strong></td>
</tr>
</tbody>
</table>
Introduce FOPS strategy

After students have received workbooks, T says:

*I distributed a Student Workbook that we will be using for the next few weeks. Write your name at the top of the cover page. I will be collecting these at the end of our lessons so they are ready for us to use the next time.*

T will explain: *FOPS is a four-step problem-solving strategy we will learn to solve math word problems. The kind of word problems we will be working on are proportional reasoning problems. Proportional reasoning is an area of math dealing with a form of measurement and includes the types of unit rates, proportions, and scale drawings. An example of a problem for each type can be found on p.7 in the Student Workbook.*

Teacher reads through examples with students:

In the lessons for all the unit rate word problems, the answer will be finding the quantity for “1” of something. The problem asks how many students are on “1” bus in the following example: *If 24 school buses are needed to bus 960 students to their school, how many students does one bus hold?*

The proportion problems will be similar to unit rate problems except they will not include the quantity of “1” of something. An example of a proportion problem is: Carl can read 6 pages in 14 minutes, how long will it take him to read a chapter that has 21 pages?

The scale drawing problems will be recognized by a scale described by metric
units in inches or feet, such as: The scale on a blueprint is 1cm for every 2m. If the width of an arch is to be 5m, then how wide should it be drawn on the blueprint?

Note: The teacher will use laminated posters of both the “if-then” diagram and FOPS problem-solving strategy checklist during demonstration and guided practice of the word problems. (The teacher will model how to cross off the steps of FOPS as they are implemented.)

*The key terms and word problems for demonstration and guided practice from the student workbook will be displayed on chart paper in the classroom.
*Underlining and circling of information in the word problems will also be modeled on the chart paper.

T introduces key vocabulary for proportional problems: First, I want to be sure you understand the meaning of the terms we will be using for the word problems. Turn to the list of key terms on p. 2 in your Student Workbook. The first term is ratio, and the definition in your book is “compares 2 or more amounts.” Look at the example at the top of p. 2. There are 3 triangles and 2 circles, so the ratio of circles to triangles is 2/3 which can also be written two other ways shown (2 to 3, or 2:3).

T asks: Then what is the ratio of triangle to circles? (3/2)
The next term is rate, which is defined as a “ratio that compares amounts measured in different units.” Examples of a rate as
shown are: It costs 12 dollars for 4 hotdogs, and someone travels 130 miles in 2 hours. The units are the parts being measured; such as dollars & hotdogs.

T asks: What are the other units shown? (miles, and hours)

The next term is unit rate. A unit rate is a rate expressed as a quantity of 1. For example, finding the cost of 1 hotdog, or the distance traveled in 1 hour.

Look at the examples given in the Student Workbook:

2 feet per second and 5 miles per hour

So if you see the little word "per" in the word problem, that is also a clue that you are dealing with a unit rate.

To apply these terms to find a unit rate in a word problem you can set a second ratio, or rate, equal to the original rate with 1 as the bottom number, or denominator, of the second term.

So, if I have a rate such as: 3 buses are used for 120 students, (T refers students to the application at the bottom of p.2) and I want to find the unit rate. First, I would write a ratio (or rate) equal to the original rate with “1” as the bottom number (denominator) of the second term.

For example: \[
\frac{120 \text{ students}}{3 \text{ buses}} = \frac{\# \text{ of students}}{1 \text{ bus}}
\]

Let’s move on to the next term in your Student Workbook. What is an association, looking at the top of p. 3?

Yes, for the purposes of our task, an association is a relationship that
compares the values of 2 or more amounts.
What other term sounds like it has this definition? Yes, a ratio- so an association and a ratio mean the same thing.

The next term in your Student Workbook on p.3. is “equivalent,” meaning equal or “same as.” A way to remember the meaning of this word is from the same beginning letters (E-q-u) as “equal.”

Now we move on to the next term, “proportion,” also on p.3. A proportion refers to 2 ratios that are equal. So given that ratios and associations are the same thing, a proportion also refers to 2 associations that are equal.

In the example given on p.3:

\[
\begin{align*}
20 \text{ boxes} & = 40 \\
\text{boxes} & \\
2 \text{ truck} & = 4 \\
\text{trucks} &
\end{align*}
\]

20 boxes require 2 trucks, and 40 boxes require 4 trucks.

Cross products is our next term on p. 3. To determine if the ratios are equivalent and form a proportion, we can multiply the cross products.

Cross products refer to the diagonal units in ratios of a proportion. Look at the example on p.3:

\[
\begin{align*}
10 & = 50 \\
1 & 5
\end{align*}
\]

For 2 ratios to form a proportion, the cross products should be equal.

When we multiply the cross products, we first change the proportion into a
number sentence. We can multiply these diagonal units in ratios to create the number sentence. This will also determine if the ratios are equal, or find a missing number in a proportion. In the example given on p.3, \(10 \times 5 = 50 \times 1\), and \(50 = 50\). Yes, it is a proportion!

This process describes our next term, **translate**.

So **translate** means changing the proportion into a number sentence.

Where else have you heard the word **translate**? Yes, if someone speaks a different language they may need someone to translate their words to English. So we can apply that to remember that when deciding if a proportion is equal or when solving for a missing number in a proportion- we can change or translate it into a number sentence. **Translating** into a number sentence involves multiplying the **cross products** or diagonal amounts and setting the sides equal. If the 2 sides are equal, it is a proportion. If there is a missing number, we need to perform the opposite operation to solve and find the missing number.

Look back at the proportion example on p.3:

\[
\begin{align*}
20 \text{ boxes} & \overset{?}{=} 40 \text{ boxes} \\
2 \text{ truck} & \overset{?}{=} 4 \text{ trucks}
\end{align*}
\]

\[
20 \times 4 = 40 \times 2 \text{ (number sentence)} \\
80 = 80, \text{ so it is a proportion with equal ratios}
\]
If one of those numbers was missing, for example:

\[
\begin{align*}
20 \text{ boxes} & = 40 \text{ boxes} \\
2 \text{ truck} & \quad ? \text{ trucks}
\end{align*}
\]

We could translate the proportion into a number sentence and use “x” for the missing number. So:

\[10 \times x = 1 \times 50.\]

A number sentence with an equal sign is called an equation (p.3). We could remember the meaning of equation by using the first 3 letters as a clue as well (E-Q-U). The way we solve for a missing number in an equation is to do the opposite operation.

So in the example the operation is multiplication and that means I must divide to solve it. In \(10x = 50\), I divide both sides by the number that is next to the unknown letter (10) to get the letter by itself, or alone. On the left the “10’s” cancel each other out and we are left with \(50 ÷ 10\) on the right.

Just to remind you, we will be working with proportional reasoning word problems.

We decide if a word problem is “proportional” by deciding whether the story tells about a ratio between 2 things and if there is an “if-then” statement. Remember another word for ratio is an association between two things. So proportional problems usually have an “if-then” kind of statement (refer to top of p.4) that tells about 2 ratios or associations between 2 things.

For example, the “if-then” statement for the problem, “If 10 boxes fit in 1 truck, then 50 boxes will fit in 5 trucks,”
indicates that the two things there is an **association** between are boxes and trucks. Although the “if-then” statement may not be directly stated in the problem, an “if-then” relationship can be constructed from a description of an association between two things and a question that asks about an unknown number.

In another example on p. 4, If there are 6 seats in each rowboat, then how many seats will be in 5 rowboats?, “if” tells about an association between a rowboat and the number of seats in it (6), and the “then” statement tells about an association or ratio between rowboats (5) and seats. So this is a proportional problem because it tells about the **association** between the number of rowboats and the number of seats in rowboats.

Now that we are familiar with the some terms for proportional problems, we will learn the FOPS problem-solving strategy I mentioned before.

T will direct students: Turn to the FOPS strategy on p. 5 to refer to the checklist as the steps are introduced.

When you learn to use FOPS to solve math word problems, you learn 4 steps. The 4 steps are:

1) **F** means to find the problem type
2) **O** means to organize the information in the problem using the diagram
3) **P** means to plan to solve the problem
4) **S** means to solve the problem
In the 1st step ("F"), there are 2 boxes, which means we will do 2 things in the F-step. First, we will read and retell the story to verify it is a proportional problem. **Retell** means to describe in our own words or paraphrase (T direct students to view as a key term on p. 3).

The second part of the F-step is to determine if there is an **association** between 2 things and whether an “if-then” statement included. An if-then relationship can be an association between two things, or a question that asks about an unknown number.

The 2nd step, “O” involves organizing the information into the **if-then** diagram found on p. 6 in your Student Workbook. T monitors that students have turned to p. 6.

OK now let’s look at the FOPS checklist on p. 5. In the “O” step we will:

- Underline 2 things that form a specific ratio
- Circle the numbers in the problem that form ratios and **label** in diagram
- Write a “?” for what must be solved (find question sentence)

What does it mean to **label**? Let’s see if the term “label” is defined on p. 4, (waits for students to get to p. 3). Ok yes it means to identify or describe information. That makes sense because I know of things at home that have labels to tell what they are, like a can of vegetables.

Next, in the 3rd step of “P” - we will plan our answer by **translating** information.
from the diagram into a number sentence. Remember **translate** means changing the proportion into a number sentence by multiplying the **cross products**.

Finally, the last step of “S” has us checking if our answer is accurate and makes sense. If the cross products are equal when we multiply them, our answer should be accurate and make sense. To be a complete answer, we need to include the units as well as the number. For example, 5 **trucks**, not just the number “5.”

T tells the students: We will be applying this FOPS strategy when we begin to solve word problems but today’s lesson involves story problems that already have the F, P, and S completed. So there is not missing information to solve for. Our goal for this activity will be to organize the problem information, which is the “O” step in FOPS.

**Demonstration**

T explains: *In the first phase of this problem-solving strategy, we will initially learn to interpret the main concepts in the story situation, and then map, or diagram, the details of the problem onto the if-then diagram.*

What exactly does it mean to “map” something? Let’s look at our key terms on p. 4. **Map** means to diagram, or place information from a problem into the graphic organizer. The if-then diagram is an example of a graphic organizer (T directs students to locate if-then diagram on p. 6), and that’s what we will be using to solve proportional word problems. In reading class you might use another type
of graphic organizer called a story map.

T demonstrates a story problem example: *Turn to the story problem “Demonstration 1” on p. 9 of your Student Workbook.*

The problem states: *If one pencil costs 7 cents, then a pack of 12 pencils will cost 84 cents. So in this story the 2 things that form a “ratio” are pencils and cents, and 1 pencil costs 7 cents. The “if-then” statement in the story is: If one pencil costs 7 cents, then a pack of 12 pencils will cost 84 cents. So we need to understand that the ratio (1 pencil/7 cents) formed in the “if” statement and the ratio formed in the “then” statement (12 pencils/84 cents) are equivalent or “proportional.”*

This completes Step 1 of the FOPs strategy. Now, look at “Demonstration 2” on p. 9:

*If there are 6 seats in each rowboat, then 5 rowboats will have 30 seats.*

T demonstrates how to map story word problem on a graphic organizer using the following think aloud:

*Now I am ready for Step 1. When I look on my checklist for the “F” step, I see that I need to find out if it is a proportional problem. The first direction is to read and retell the story in my own words so I will read the story: “If there are 6 seats in each rowboat, then 5 rowboats will have 30 seats.”*

*Next, I will retell in my own words to help me understand it and ask myself what I know: The story tells about an association or ratio between rowboats and the number of seats in rowboats. If each rowboat has 6 seats in it, then 5
rowboats will have 30 seats. Alright, now I can check off the 1st box under the “F” step on my FOPS problem-solving checklist.

Then I will ask myself it is a proportional problem by deciding whether the story tells about an association between 2 things and if there is an “if-then” statement continuing the second component of the of “F” step. In this story “if” tells about a ratio between a rowboat and the number of seats in it (6), and the “then” statement tells about an association or ratio between rowboats (5) and seats (30). So this is a proportional problem because it tells about the association between the number of rowboats and the number of seats in rowboats, and I can check off the second box for the “F” step.

Next, I am ready for the “O” step, to organize information- first I need to underline the 2 things that form a specific ratio and write their names in the if-then diagram (T refers to diagram in Student Workbook on p.6 and “if-then” diagram on laminated poster in the classroom). Let’s see rowboats and seats in rowboats are the 2 things that form a specific ratio because in the “F” step, we decided that the story tells about the association or ratio between the number of rowboats and the number of seats in rowboats.

So, I will underline rowboats and seats in the “if-then” statement and write “rowboats” and “seats in rowboats” as labels for the 2 columns in the if-then
diagram. I remember that label means to identify, or describe, which is what I am doing in the problem. So I can check of the 1st box under the “O” step.
(T assures students that they may write either “rowboats” or “seats” in the first column but it’s important to place the numbers for rowboats in the same column and seats in the same column).

Next, I need to read the story to circle numbers for each ratio, and write numbers and labels in the if-then diagram. The “if” sentence states, “If there are 6 seats in each rowboat- so I will now write the numbers and labels for the 1st ratio in my if-then diagram. I know that “each” in the sentence means “one,” so I will circle “each” and write “1 rowboat” in the box for the rowboats column, and circle “6,” and write “6 seats” in the oval for the seats column. Ovals are the egg shapes in the second column of the diagram.

The 2nd part of the sentence is the “then” statement: “then 5 rowboats will have 30 seats” which indicates that there are 30 seats in 5 rowboats. Now I can write the numbers for the 2nd ratio (refer to “then” statement in diagram), I will circle “5” and write “5 rowboats” in the box for the rowboats column, and circle “30” and write “30 seats” in the oval for the seats column.

(T explains that because the 1st column is labeled rowboats, 5 rowboats goes in same column).

Now I can check off the 2nd box under step 2.

When I look at the if-then diagram I can read if 1 rowboat has 6 seats in it, then 5
rowboats have 30 seats in them. It also shows that the ratios of 1/6 in the “if” column, and 5/30 in the “then” column are equal/proportional.

<table>
<thead>
<tr>
<th>Guided Practice</th>
<th>T guides students through 2 additional practice problems on p.8 of the Student Workbook:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3) If a school bus holds 40 students, then 24 buses are needed to bus 960 students to their school.</td>
</tr>
<tr>
<td></td>
<td>4) Kelly and Ryan are responsible for making drinks for the party. They used 6 lemons to make 2 quarts of lemonade. To make 8 quarts of lemonade they need 24 lemons.</td>
</tr>
</tbody>
</table>

T asks for volunteer to read 1st word problem, and directs students to follow along. T tells students to refer to their FOPS problem-solving checklist and asks: What’s the 1st step? After a student responds, T models: To **find out if it is a proportional problem**, I will read the story and retell it in my own words. If a school bus holds 40 students, then 24 buses are needed to bus 960 students to their school. Ok now I have read the story, what is the next step? T acknowledges correct answer: Yes, I will **retell the story in my own words** to help me understand it and ask myself what I know in the story. Alright, the story tells about an association or ratio between buses and students.

Let’s continue retelling the story. If 1 bus holds 40, students, then 24 buses will hold 960 students. Ok I read the story and told it in my own words. Now I can check off the first box under the “F” step.
T asks: What is the next step? T responds to correct answer: How do I know if it is a **proportional problem**? I need to ask whether the story tells about a ratio or rate between 2 things, so I am checking for an “if-then” statement. In this example, the “if” statement tells about a ratio between 1 bus and the number of students (40) it can hold. Since there is only 1 bus it’s a “unit” ratio. A unit ratio has a bottom number of “1.” The “then” statement also tells about this association, so 24 buses will hold 960 students. Now I can check off the 2nd box under The “F” step because this IS a proportional story that tells about an association between buses and the number of students.

Next, I am ready for the “O” step: **organize the information in the story.** T directs students to 1st box under the “O” step: First, we underline 2 things that form a specific ratio, and write their names in the diagram. T asks: What are the 2 things that form a specific ratio in the story? T repeats correct answer: Yes, the story tells about an association between buses and students. Let’s underline buses and students in the “if-then” statement and write buses and students as labels for the 2 columns in the vary diagram. Now we can check off the 1st box under the “O” step on the checklist. We need to read the story again to circle numbers for each ratio and write numbers and labels in the diagram. The first part of the sentence, the “if” statement says, “If a school bus holds 40 students.” Let’s write the numbers and
labels for the first ratio. Alright how many school buses in the first ratio (a school bus = 1)? Let’s circle “a” and write “1” bus in the box for the buses column. Let’s also circle “40,” and write 40 students in the oval for the “students” column. The second part of the sentence is the “then” part that says, “then 24 school buses are needed to bus 960 students to their school.

Now we are ready to write the numbers for this second ratio in the “if-then” diagram. We will circle “24” and write 24 buses in the box for the “buses” column, and circle “960” and write 960 students in the oval for the “students” column. Now we can check off the 2nd box under step 2 because we:

✓ Underlined 2 things in the story that form an association and wrote their names in the 2 columns in the diagram

✓ Circled numbers and wrote numbers and labels in the diagram.

T also points out that the diagram also shows that the ratio 1 to 40 presented in the “if” statement and the ratio 24 to 960 presented in the “then” statement are equivalent or proportional.

T repeats procedure for q#2 above:

<table>
<thead>
<tr>
<th>Independent Practice</th>
<th>Students complete mapping, or diagramming, of 2 story problems independently and T will orally check for accuracy as students volunteer answers:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3) If 1 circus ticket costs $4, then $24 can buy 6 circus tickets.</td>
</tr>
<tr>
<td></td>
<td>4) Each skating board has 4 wheels, 9 skating boards will have 36 wheels.</td>
</tr>
</tbody>
</table>
Lesson #7 (Unit Rate/Novice)

<table>
<thead>
<tr>
<th>Lesson Component</th>
<th>Activity</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportional Reasoning Word Problem Probe</td>
<td>T distributes the Proportional Reasoning Word Problem Probes (PRWPP) to students.</td>
<td>10 minutes</td>
</tr>
<tr>
<td>FOPS strategy (introduced in schema instruction of worked problems phase)</td>
<td>T explains: *Today we will use the if-then diagram we have already learned about to solve proportional word problems. The word problems we will be working are unit rates, proportions, and scale drawings. *Turn to p.7 in your Student Workbook to review examples of word problems using these skills. *T reviews examples of unit rate, proportion, and scale drawing word problems and explains how to recognize each skill type according to table on p. 7 of Student Workbook: Proportional Reasoning Skills in Word Problems.</td>
<td>2 minutes</td>
</tr>
</tbody>
</table>

**TOTAL time: 35 minutes**
<table>
<thead>
<tr>
<th><strong>Proportion</strong></th>
<th>Carl can read 6 pages in 14 minutes, how long will it take him to read a chapter that has 21 pages? *The proportion problems are similar to unit rate problems; however, they will not include the number “1.”</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Scale Drawing</strong></td>
<td>The scale on a blueprint is 1 cm for every 2 m. If the width of an arch is to be 5 m, then how wide should it be drawn on the blueprint? *The scale drawing problems can be recognized including a scale in the problem described in metric units such as inches or feet.</td>
</tr>
</tbody>
</table>

The T will then direct students to refer to the FOPS problem-solving strategy on p.5 of the Student Workbook to recite/revie steps.

T will ask: Let’s review the FOPS steps. What is Step 1? What do you do in Step 1? What’s happens in Step 2? What are the directions for Step 3? Finally, what is Step 4?
<table>
<thead>
<tr>
<th>Demonstration</th>
<th>Teacher demonstration of 2 word problems at new skill/level on p.12 of Student Workbook:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1) The Dale Company packs 4 cans of tomatoes in each box. How many boxes will the company need to pack 8 cans of tomatoes?</td>
</tr>
<tr>
<td></td>
<td>2) Sam used 3 bags of candy bars to make 1 ice cream cake. To make 6 ice cream cakes, how many bags of candy bars would he need?</td>
</tr>
<tr>
<td></td>
<td>T explains that we will use the FOPS problem-solving checklist beginning with step 1: To find out if it is a proportional problem, I will read the problem and retell it in my own words. Follow along as I read aloud the first problem and model how to retell in my own words.</td>
</tr>
<tr>
<td></td>
<td>T reads aloud 1st problem. Now, when I retell the problem in my own words I will ask myself what I know and what I am asked to find out. So this story tells about an association or ratio between the number of cans of tomatoes and the number of crates. I don’t know how many crates will be needed to pack 8 cans of tomatoes, so that is what I’m asked to find out.</td>
</tr>
<tr>
<td></td>
<td>We can check off the 1st box under the “F” step, and next I will ask myself if the problem is a proportional problem, by asking whether there is an “if-then” statement and if it tells about the rate or ratio between 2 things.</td>
</tr>
</tbody>
</table>
|               | Although an “if-then” statement is NOT DIRECTLY stated in the problem, there is an association described between the number of cans of tomatoes and number

---

| 10 minutes |  |
of crates (4 cans of tomatoes in each crate in the first sentence), and a question that asks about the ratio between 8 cans of tomatoes and an unknown number. So let’s check off the 2nd box under the “F” step.

In the “O” step we need to first underline 2 things that form a specific ratio and write their names, or labels, in the diagram.

T asks: So what does the problem talk about? (cans of tomatoes and crates) OK so we can underline cans of tomatoes and crates in the problem and write cans of tomatoes for the 1st column, and crates for the 2nd column in the diagram. T directs students to check off the 1st box for the “O” step. Next, read each sentence to circle numbers for each ratio and write numbers and labels in the diagram.

T asks: How many does “each” stand for in the first sentence? (one)
T asks: So what numbers do you circle and write for “If” in the diagram? (4) So we write 4 cans in the box for the cans of tomatoes column, and circle “each” and write “1 crate” in the oval for the “crates” column.
T asks: From the next sentence, do we know the number of cans? (8) So do we write “8 cans” in the box or oval? (box-because it tells about the number of cans of tomatoes)
OK- so circle 8 and write “8 cans” in the box. Now, what number and label do we put in the oval for the crates column? Do we know the number of crates? (no) So since we do not know how many crates are needed to pack 8 tomatoes, we will
write a “?” in the oval for the crates column. Now we can check off the 2nd box under the “O” step.

T directs students to look at the if-then diagram for problem #1 on p.12 and explains: We need to find out the number of crates that can pack 8 cans of tomatoes, so what must we solve for? (# of crates to pack 8 cans of tomatoes). Now let’s move onto step 3: to plan to solve the problem, we need to translate the information from the diagram into a math sentence. What does it mean to translate? (change into another form) T demonstrates how to cross multiply to set-up math equation from proportion. The ratio of 4 cans/8 cans = 1 crate/? crates, so we will use this “plan” to solve the problem and we can cross off step 3 on our checklist.

Finally, we need to solve the unknown “?” by multiplying cross products to get a number sentence: 4 x ? = 8 x 1. We could use “x” for the missing number. So: 4 x x = 8 x 1. The left side of the number sentence is equal to the right side of the number sentence, so whatever we do on the left side needs to happen on the right side. In 4x = 8, I divide both sides by the number that is next to the unknown letter (4) to get the letter (x) by itself, or alone. On the left the “4’s” cancel each other out and we are left with 8 ÷ 4 on the right.

T asks: So what is the unknown number of crates when we solve 8 ÷ 4? (2) We can check our answer by multiplying cross products: 4 x 2 = 8, so our calculation is accurate and we can cross off the “S” step!
<table>
<thead>
<tr>
<th>Guided Practice</th>
<th>Teacher guides students through 3 practice problems:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1. Carissa gets paid $6. per hour to babysit, how much would she get paid for 2 hours?</td>
</tr>
<tr>
<td></td>
<td>2. If 1 cup of flour is used to make 6 dumplings, how many cups of flour are needed to make 30 dumplings?</td>
</tr>
<tr>
<td></td>
<td>3. The cost of 2 bags of candy is 1 dollar. If you have 6 dollars, how many bags of candy can you buy?</td>
</tr>
</tbody>
</table>

**NOTE:** For all lessons, the T will stay in guided practice longer if a student’s practice of the skill indicates he/she still does not know how to accurately use FOPS / solve problem accurately. This will be determined by whether the student performs at least 80% of 16 points (at least 14/16) on the third guided practice word problem. Then student will do a fourth problem on
his/her own, and the score needs to be at least 80% before moving to independent practice. The fourth problem is #1 under independent practice for each lesson in the Student Workbook. Proficiency will continue to be monitored for independent practice problems using the same criteria of at least 80% (at least 14 out of 16 points).

| Independent Practice | Students complete 2 problems independently if 80% mastery on guided practice problems:
1. Erica uses 7 gallons of gas to drive 14 miles. How many miles can she drive using 1 gallon of gas?
2. Brian can eat 3 hotdogs in 1 minute, how many hotdogs can he eat in 6 minutes? | 5 minutes |

SBI Lesson Plan
Lesson #8 (Unit Rate/Intermediate)

<table>
<thead>
<tr>
<th>Lesson Component</th>
<th>Activity</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportional Reasoning Word Problem Probe</td>
<td>T distributes the Proportional Reasoning Word Problem Probe (PRWPP) sheets to students following the procedures in baseline lessons 1-5.</td>
<td>10 minutes</td>
</tr>
<tr>
<td>FOPS strategy (introduced in schema)</td>
<td>The T will direct students to refer to the FOPS problem-solving strategy on p.4 of the Student Workbook to recite/review</td>
<td>2 minutes</td>
</tr>
</tbody>
</table>

TOTAL time: 35 minutes
### Instruction of Worked Problems Phase

T will ask: *Let’s review the FOPS steps.*

- **What is the “F” step?** What is the 1\textsuperscript{st} thing we do in the “F” step?
- **What do you do in the “O” step?** What else do we do in the “O” step?
- **What are the directions for the “P” step?**
- **Finally, what is involved in the “S” step?**

### Demonstration

Teacher demonstration of 2 word problems at new skill/level using procedure from Lesson 7:

<table>
<thead>
<tr>
<th>Number</th>
<th>Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>3)</td>
<td>A total of 48 slices of cake are ready to be served to students sitting at many tables. If 8 slices of cake will be served at each table, how many tables can be fully served?</td>
</tr>
<tr>
<td>4)</td>
<td>There are 4 chocolate bunnies in each candy box. If you bought 12 boxes, how many chocolate bunnies would you have?</td>
</tr>
</tbody>
</table>

### Guided Practice

Teacher guides students through 3 practice problems:

<table>
<thead>
<tr>
<th>Number</th>
<th>Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>4)</td>
<td>If one pack of socks costs $5., how much will 7 packs of socks cost?</td>
</tr>
<tr>
<td>5)</td>
<td>If 2 baked cookies are $1., how much money will the bakery make for 10 cookies?</td>
</tr>
<tr>
<td>6)</td>
<td>Bob reads 60 pages of a book in 1 hour. How long should it take him to read 180 pages?</td>
</tr>
</tbody>
</table>

### Independent Practice

Students complete 2 problems independently if 80% mastery on guided practice problems:

<table>
<thead>
<tr>
<th>Number</th>
<th>Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>3)</td>
<td>Callie gets paid $6. per hour to babysit, how much would she get</td>
</tr>
</tbody>
</table>
In a classroom, each row can only seat 7 students. How many students would be seated in 12 rows?

<table>
<thead>
<tr>
<th>Lesson Component</th>
<th>Activity</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportional Reasoning Word Problem Probes (PRWPP)</td>
<td>T distributes the Proportional Reasoning Word Problem Probes (PRWPP) to students following the procedures in baseline lessons 1-5.</td>
<td>10 minutes</td>
</tr>
</tbody>
</table>
| FOPS strategy (introduced in schema instruction of worked problems phase) | The T will direct students to refer to the FOPS problem-solving strategy on p.4 of the Student Workbook to recite/review steps. T will ask: *Let’s review the FOPS steps.*

*What is the “F” step? What is the 1st thing we do in the “F” step?*

*What do you we do in the “O” step? What else do we do in the “O” step?*

*What are the directions for the “P” step? Finally, what is involved in the “S” step?* | 2 minutes |
| Demonstration                                         | Teacher demonstration of 2 word problems at new skill/level using procedure from Lesson 7: 3) All of the seats at a movie theater were taken by a total of 384 people. If there are 16 rows of seats in the theater, how many | 10 minutes |

TOTAL time: 35 minutes
seats are there in each row?
4) Each notebook has 70 pages in it. If you bought 5 notebooks, how many pages would you have in all?

**Guided Practice**
Teacher guides students through 3 practice problems:
4) If Mr. Hall’s car will run 21 miles on each gallon of gas, then how many gallons will be needed to drive 84 miles?
5) A store packs 42 cans of tomatoes in each crate. How many crates does the store need to pack 630 cans of tomatoes?
6) Eric uses 7 gallons of gas to drive 196 miles. How many miles can he drive using 1 gallon of gas?

**Independent Practice**
Students complete 2 problems independently if 80% mastery on guided practice problems:
1) Only 4 buses are available for a school field trip. If each bus will sit 26 students, how many students in all can be seated in the buses for the field trip?
2) Michelle used 27 apples to make 9 apple pies. If she has only 9 apples, how many apple pies can she make?

**TOTAL time:** 35 minutes
### Lesson Component

<table>
<thead>
<tr>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Proportional Reasoning Word Problem Probes (PRWPP)</strong></td>
</tr>
<tr>
<td><strong>FOPS strategy (introduced in schema instruction of worked problems phase)</strong></td>
</tr>
<tr>
<td><strong>Demonstration</strong></td>
</tr>
<tr>
<td><strong>Guided Practice</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 minutes</td>
</tr>
<tr>
<td>2 minutes</td>
</tr>
<tr>
<td>10 minutes</td>
</tr>
<tr>
<td>8 minutes</td>
</tr>
</tbody>
</table>
how many strawberries does she need?
3) Gina used 6 apples to make 2 apple pies. If she has only 3 apples, how many apple pies can she make?

Independent Practice

Students complete 2 problems independently if 80% mastery on guided practice problems:

1) Carl can read 3 pages in 6 minutes. At this rate, how long will it take him to read a chapter that has 12 pages?
2) Jenny spent 2 hours making 12 Christmas ornaments. How many hours will it take her to make 36 ornaments?

TOTAL time: 35 minutes

SBI Lesson Plan

Lesson #11 (Proportion/Intermediate)

<table>
<thead>
<tr>
<th>Lesson Component</th>
<th>Activity</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportional Reasoning Word Problem Probes (PRWPP)</td>
<td>T distributes the Proportional Reasoning Word Problem Probes (PRWPP) to students following the procedures in baseline lessons 1-5.</td>
<td>10 minutes</td>
</tr>
<tr>
<td>FOPS strategy / introduced in schema instruction of worked problems</td>
<td>The T will direct students to refer to the FOPS problem-solving strategy on p.4 of the Student Workbook to recite/review steps.</td>
<td>2 minutes</td>
</tr>
</tbody>
</table>
T will ask: Let’s review the FOPS steps.

What is the “F” step? What is the 1st thing we do in the “F” step?
What do you we do in the “O” step? What else do we do in the “O” step?
What are the directions for the “P” step?
Finally, what is involved in the “S” step?

<table>
<thead>
<tr>
<th>Demonstration</th>
<th>Teacher demonstration of 2 word problems at new skill/level using procedure from Lesson 7:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4) In Kylie's school, 2 out of every 5 seventh graders went to a movie theater over the weekend. If there are 80 students in the seventh grade, how many of them saw a movie?</td>
</tr>
<tr>
<td></td>
<td>5) Ms. Bayer made 48 peanut butter cookies using 6 eggs, if she only has 2 eggs, how many peanut butter cookies can she make?</td>
</tr>
<tr>
<td></td>
<td>10 minutes</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Guided Practice</th>
<th>Teacher guides students through 3 practice problems:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3) Jill spent 3 hours making 12 Christmas ornaments. How many hours will it take her to make 36 ornaments?</td>
</tr>
<tr>
<td></td>
<td>4) Carl can read 6 pages in 14 minutes. At this rate, how long will it take him to read a chapter that has 21 pages?</td>
</tr>
<tr>
<td></td>
<td>5) Jill used 9 apples to make 6 apple pies. If she has only 3 apples, how many apple pies can she make?</td>
</tr>
<tr>
<td></td>
<td>8 minutes</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Independent Practice</th>
<th>Students complete 2 problems independently if 80% mastery on guided practice problems:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5 minutes</td>
</tr>
</tbody>
</table>
3) Ben can eat 7 hotdogs in 2 minutes, how many hotdogs can he eat in 6 minutes?
4) If 4 blueberry bagels are $4., how much money will the shop make for 120 bagels?

<table>
<thead>
<tr>
<th><strong>Lesson Component</strong></th>
<th><strong>Activity</strong></th>
<th><strong>Time</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportional Reasoning Word Problem Probes (PRWPP)</td>
<td>T distributes the Proportional Reasoning Word Problem Probes (PRWPP) to students following the procedures in baseline lessons 1-5.</td>
<td>10 minutes</td>
</tr>
<tr>
<td>FOPS strategy <em>(introduced in schema instruction of worked problems phase)</em></td>
<td>The T will direct students to refer to the FOPS problem-solving strategy on p.4 of the Student Workbook to recite/review steps. T will ask: <em>Let’s review the FOPS steps.</em> What is the “F” step? What is the 1st thing we do in the “F” step? What do you we do in the “O” step? What else do we do in the “O” step? What are the directions for the “P” step? Finally, what is involved in the “S” step?</td>
<td>2 minutes</td>
</tr>
<tr>
<td>Demonstration</td>
<td>Teacher demonstration of 2 word problems at new skill/level using procedure from Lesson 7:</td>
<td>10 minutes</td>
</tr>
</tbody>
</table>

TOTAL time: 35 minutes
1) If 3 eggs are needed to make 18 cupcakes, and you plan to make 126 cupcakes for a party, how many eggs are needed?
2) If 60 paper rolls are used to cover 12 textbooks, how many paper rolls will be needed to cover 10 books?

### Guided Practice

Teacher guides students through 3 practice problems:

4) If Ray can save $48. In 5 months, how many months will it take Ray to save $144.
5) Ms. Bayer made 48 peanut butter cookies using 6 eggs, if she only has 2 eggs, how many peanut butter cookies can she make?
6) Only 10 buses are available for a school field trip. If 3 buses will sit 75 students, how many students in all can be seated in the buses for the field trip?

### Independent Practice

Students complete 2 problems independently if 80% mastery on guided practice problems:

3) A farmer has 444 eggs to pack in cartons. If each egg carton holds 12 eggs, how many cartons will the farmer need?
4) A car used 8 gallons of gas to travel 120 miles. At that rate, how many gallons will be used to travel 360 miles?

**TOTAL time:**
<table>
<thead>
<tr>
<th>Lesson Component</th>
<th>Activity</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportional Reasoning Word Problem Probes (PRWPP)</td>
<td>T distributes the Proportional Reasoning Word Problem Probes (PRWPP) to students following the procedures in baseline lessons 1-5.</td>
<td>10 minutes</td>
</tr>
<tr>
<td>FOPS strategy (introduced in schema instruction of worked problems phase)</td>
<td>The T will direct students to refer to the FOPS problem-solving strategy on p.4 of the Student Workbook to recite/review steps. T will ask: Let’s review the FOPS steps. What is the “F” step? What is the 1st thing we do in the “F” step? What do you we do in the “O” step? What else do we do in the “O” step? What are the directions for the “P” step? Finally, what is involved in the “S” step?</td>
<td>2 minutes</td>
</tr>
<tr>
<td>Demonstration</td>
<td>Teacher demonstration of 2 word problems at new skill/level using procedure from Lesson 7: 4) If 1 cm on a map represent 3 miles on the road, how many miles will 6 cm represent? 5) The scale of a blueprint is 1in = 2ft. If the actual width of a porch = 6ft, what is the width on a blue print drawing?</td>
<td>10 minutes</td>
</tr>
<tr>
<td>Guided Practice</td>
<td>Teacher guides students through 3</td>
<td>8 minutes</td>
</tr>
<tr>
<td>Independent Practice</td>
<td>Students complete 2 problems independently if 80% mastery on guided practice problems:</td>
<td></td>
</tr>
<tr>
<td>----------------------</td>
<td>------------------------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1) If the scale is 2cm = 1 m, what would the length be in a drawing of desk that is 4m long?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2) On a scale drawing, 1in = 2ft. Find the length of a car that is 5 feet wide.</td>
<td></td>
</tr>
</tbody>
</table>

**SBI Lesson Plan**

**SBI Process**

**Lesson #14 (Scale Drawing/Intermediate)**

<table>
<thead>
<tr>
<th>Lesson Component</th>
<th>Activity</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportional Reasoning Word Problem Probes (PRWPP)</td>
<td>T distributes the Proportional Reasoning Word Problem Probes (PRWPP) to students following the procedures in baseline lessons 1-5.</td>
<td>10 minutes</td>
</tr>
<tr>
<td>Activity</td>
<td>Description</td>
<td>Duration</td>
</tr>
<tr>
<td>-----------------------------------</td>
<td>----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
<td>-----------</td>
</tr>
</tbody>
</table>
| FOPS strategy                     | The T will direct students to refer to the FOPS problem-solving strategy on p.4 of the Student Workbook to recite/
review steps. T will ask: Let’s review the FOPS steps. What is the “F” step? What is the 1st thing we do in the “F” step? What do you do in the “O” step? What else do we do in the “O” step? What are the directions for the “P” step? Finally, what is involved in the “S” step? | 2 minutes |
| Demonstration                     | Teacher demonstration of 2 word problems at new skill/level using procedure from Lesson 7: 4) The Burkes are building a deck in their back yard. They want it to be 10 feet wide. If the scale on the blueprint is 1in = 3ft, what should the dimension for width be in the scale drawing? 5) If the scale is 2cm = 1 m, what would the length be in a drawing of desk that is 5m long? | 10 minutes|
| Guided Practice                   | Teacher guides students through 3 practice problems: 1) If 5in = 1 mi on a map, what is the distance of 2 miles on the map? 2) The scale of a blueprint is 1in = 4ft. If the actual width of a porch = 16ft, what is the width on a blueprint drawing? 3) On a scale drawing, 1in = 3ft. What would the length of a car on a scale drawing be that is 24 feet wide? | 8 minutes |
| Independent Practice              | Students complete 2 problems independently if 80% mastery on guided practice problems:                                                                                                                                              | 5 minutes |
1) If 3 cm on a map represent 12 miles on the road, how many miles will 5 cm represent?
2) In the drawing of a car, the scale is 4 inches for every 2 feet. If the drawing is 12 inches long, how long is the actual car?

<table>
<thead>
<tr>
<th>Lesson Component</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Proportional Reasoning Word Problem Probes (PRWPP)</td>
<td>T distributes the Proportional Reasoning Word Problem Probes (PRWPP) to students following the procedures in baseline lessons 1-5.</td>
<td>10 minutes</td>
</tr>
<tr>
<td>FOPS strategy (introduced in schema instruction of worked problems phase)</td>
<td>The T will direct students to refer to the FOPS problem-solving strategy on p.4 of the Student Workbook to recite/review steps. T will ask: Let’s review the FOPS steps. What is the “F” step? What is the 1st thing we do in the “F” step? What do you we do in the “O” step? What else do we do in the “O” step? What are the directions for the “P” step? Finally, what is involved in the “S” step?</td>
<td>2 minutes</td>
</tr>
<tr>
<td>Demonstration</td>
<td>Teacher demonstration of 2 word problems at new skill/level using</td>
<td>10 minutes</td>
</tr>
</tbody>
</table>
procedure from Lesson 7:

3) A.J. likes cars a lot. He draws his favorite car on a scale drawing. In the drawing, the scale is 2 inches for each foot. If the drawing is 26 inches long, how long is the actual car?

4) The scale on a blueprint is 3 cm for every 5 m. If the width of an arch is to be 15 m, then how wide should it be drawn on the blueprint?

<table>
<thead>
<tr>
<th>Guided Practice</th>
<th>Teacher guides students through 3 practice problems:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Tim and Megan are planning a hiking trip. On their map, their route is 9 cm long. If 1 cm = 3 km, what is the actual length of their hike?</td>
<td></td>
</tr>
<tr>
<td>2) On a scale drawing, 3 in = 15 ft. What would the length of a car on a scale drawing be that is 25 feet wide.</td>
<td></td>
</tr>
<tr>
<td>3) If 6 cm on a map represent 72 miles on the road, how many miles will 12 cm represent?</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Independent Practice</th>
<th>Students complete 2 problems independently if 80% mastery on guided practice problems:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Adam made a scale drawing of his bedroom. In the drawing, 16 cm represent 4 m. If the windows in the drawing are 48 cm apart, how many meters apart are the actual windows?</td>
<td></td>
</tr>
<tr>
<td>2) The Burkes are building a deck in their back yard. They want it to be 15 feet long. If the scale on the</td>
<td></td>
</tr>
<tr>
<td>blueprint is 1 inch = 3 feet, what should the dimension for width be in the scale drawing?</td>
<td></td>
</tr>
<tr>
<td>TOTAL time: 35 minutes</td>
<td></td>
</tr>
</tbody>
</table>
Name __________________ Date ____________________

Baseline A: NO calculator

1. Each notebook has 60 pages in it. If you bought 4 notebooks, how many pages would you have in all?

2. A car used 9 gallons of gas to travel 108 miles. At that rate, how many gallons will be used to travel 360 miles?

3. Adam made a scale drawing of his bedroom. In the drawing, 9 cm represent 3 m. If the windows in the drawing are 6 cm apart, how many meters apart are the actual windows?

_____/16      ____/16      ____/16
4. Only 5 buses are available for a school field trip. If 2 buses will sit 50 students, how many students in all can be seated in the buses for the field trip?

\[
\text{\underline{\hspace{2cm}}} /16
\]

5. If 4 eggs are needed to make 24 cupcakes, and you plan to make 144 cupcakes for a party, how many eggs are needed?

\[
\text{\underline{\hspace{2cm}}} /16
\]

6. The scale on a blueprint is 4 cm for every 6 m. If the width of an arch is to be 24 m, then how wide should it be drawn on the blueprint?

\[
\text{\underline{\hspace{2cm}}} /16
\]

**Calculator used:**

<table>
<thead>
<tr>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
</table>

**Baseline A: Calculator Permitted**
APPENDIX K

FIDELITY OF TREATMENT CHECKLIST

Observer: __________________________  Condition: 

Student: ___________________________  Date: 

Score: _______ /8 points = _______%

Definitions

**Word Problem Probe**- 6 word problems administered before each lesson

**Teacher Script**- detailed description of procedure for each lesson with examples

**Student Workbook**- student booklet that includes problems and examples from script

**FOPS**- 4-step problem-solving strategy checklist

“if-then”**diagram**- graphic organizer used to display problem information

Note: Mark each step completed or not completed by the researcher. The fidelity of treatment will be calculated by dividing the number of steps completed by the number of steps planned.

**Yes**

1. Follows teacher script for Proportional Reasoning Word Problem Probes (PRWPP), Demonstration, Guided Practice, and Independent Practice of lessons. □ □

2. Refers to visuals and problems in Student Workbook during lesson. □ □


4. Models how to cross off the steps of FOPS as they are implemented on
laminated poster.

5. Displays key terms and word problems from the student workbook on chart paper.

6. Demonstrates how to add problem information to the “if-then” diagram poster during lesson.

7. Models underlining and circling of information in the word problems on chart paper for demonstration and guided practice word problems.

8. Stays in guided practice longer if a student’s practice of the skill indicates he/she still does not know how to apply the SBI process to 80% mastery (each student’s Student Workbook was reviewed by the researcher to determine this).

Notes:________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

________________________
________________________

250
### PROCEDURAL RELIABILITY CHECKLIST: BASELINE

**Observer:** __________________________  **Condition:** __________________________

**Student:** __________________________  **Date:** ______________________________

Note: Mark each step completed or not completed by the researcher. The fidelity of treatment will be calculated by dividing the number of steps completed by the number of steps planned.

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Distributes the Proportional Reasoning Word Problem Probes (PRWPP) to participant(s).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>Directs student(s) to record the start time on the data collection sheet.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>Explains to students that they will complete 6 word problems involving unit rates, proportions, and scale drawings.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>Directs students to solve the first 3 problems on front side and raise hand when completed to receive a calculator to do the 3 remaining problems on the back. Reminds students to check whether he/she did use a calculator in the box on the back side.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>Does not prompt student(s) to produce the correct responses.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>Directs student(s) to record the end time on the Proportional Reasoning Word Problem Probes (PRWPP).</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
PROCEDURAL RELIABILITY CHECKLIST: TREATMENT

Observer: __________________________ Condition: __________________________
Student: ___________________________ Date: ______________________________

Note: Mark each step completed or not completed by the researcher. The fidelity of
treatment will be calculated by dividing the number of steps completed by the number of
steps planned.

1. Administers the **Proportional Reasoning Word Problem Probes** (PRWPP) to participant(s) by directing them to complete #s 1, 2, and 3 on the front side, and to raise hand when ready for #s 4, 5, and 6 on the back. Informs students they may use a calculator when completing the last 3 problems on the back, and cannot return to #s 1, 2, and 3 once they have the calculator.

   Yes ☐ No ☐

2. Distributes student workbook to be used for SBI instruction, and directs students to refer to the **FOPS problem-solving strategy** in the Student Workbook to recite/review steps. Queries students on what is done in each step of the strategy.

   Yes ☐ No ☐

3. **Demonstrates** 2 word problems at new skill/level using the FOPS strategy, referring to visuals and problems in student workbook, modeling think alouds, and defining unfamiliar terms in word problems.

   Yes ☐ No ☐
4. **Guides** student(s) through 3 practice problems. Stays in guided practice longer if a student’s performance on the 3rd problem indicates he/she still does not know how to apply the SBI process to 80% mastery (each student’s Student Workbook was reviewed by the researcher to determine this) □   □

5. **Demonstrates** 2 word problems at new skill/level using the FOPS strategy, referring to visuals and problems in student workbook, modeling think alouds, and defining unfamiliar terms in word problems. □   □

6. Directs students to complete 2 problems **independently** if 80% or higher on guided practice problems. □   □

Notes:

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________
APPENDIX M (Social Validity Scale)

Math Problem-Solving Intervention

Rate the following statements on the lines provided using the scale below:

1 – Strongly disagree; 2 – disagree; 3 – agree; 4 – strongly agree

1) I enjoyed learning how to solve mathematical problems using the FOPS strategy. 

2) I liked showing my progress with solving word problems on the daily Proportional Reasoning Word Problem Probes (PRWPP). 

3) My mathematical word problem performance increased as a result of this problem-solving intervention. 

4) This new skill development of solving proportional reasoning word problems will assist my learning of more complex mathematical word problems. 

5) I would like to continue the FOPS problem-solving strategy for additional skills in mathematics. 

6) I plan to use the FOPS problem-solving strategy on my own for word problems involving other skills in mathematics. 

7) When I start to solve a mathematical problem, I usually feel I cannot not
find a solution.

8) I do not feel sure about myself when solving word problems in mathematics.

9) I have difficulty solving mathematical word problems.
### APPENDIX N

#### Overall Percentage of SBI Process without Calculator

<table>
<thead>
<tr>
<th>Group</th>
<th>Rubric Category</th>
<th>Baseline</th>
<th>Treatment</th>
<th>Maintenance</th>
</tr>
</thead>
<tbody>
<tr>
<td>All groups</td>
<td>1 (out of 25%)</td>
<td>0.0%</td>
<td>23.7%</td>
<td>25.0%</td>
</tr>
<tr>
<td></td>
<td>2 (out of 37.5%)</td>
<td>0.0%</td>
<td>25.6%</td>
<td>24.1%</td>
</tr>
<tr>
<td></td>
<td>3 (out of 12.5%)</td>
<td>0.0%</td>
<td>11.0%</td>
<td>9.9%</td>
</tr>
<tr>
<td></td>
<td>4 (out of 12.5%)</td>
<td>1.5%</td>
<td>2.2%</td>
<td>1.4%</td>
</tr>
<tr>
<td></td>
<td>5 (out of 12.5%)</td>
<td>2.4%</td>
<td>1.7%</td>
<td>0.7%</td>
</tr>
<tr>
<td>TOTAL (out of 100%)</td>
<td>3.8%</td>
<td>64.2%</td>
<td>61.1%</td>
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</tr>
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#### Overall Percentage of SBI Process with Calculator

<table>
<thead>
<tr>
<th>Group</th>
<th>Rubric Category</th>
<th>Baseline</th>
<th>Treatment</th>
<th>Maintenance</th>
</tr>
</thead>
<tbody>
<tr>
<td>All groups</td>
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<td>0.0%</td>
<td>20.7%</td>
<td>22.2%</td>
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<tr>
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<td>2 (out of 37.5%)</td>
<td>0.0%</td>
<td>20.3%</td>
<td>21.7%</td>
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<td>3 (out of 12.5%)</td>
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<td>9.9%</td>
<td>8.4%</td>
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<tr>
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<td>4 (out of 12.5%)</td>
<td>0.3%</td>
<td>1.8%</td>
<td>1.8%</td>
</tr>
<tr>
<td></td>
<td>5 (out of 12.5%)</td>
<td>1.3%</td>
<td>1.3%</td>
<td>2.2%</td>
</tr>
<tr>
<td>TOTAL (out of 100%)</td>
<td>1.6%</td>
<td>53.9%</td>
<td>56.4%</td>
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# APPENDIX O

### Individual Percentage of SBI Process Without Calculator

<table>
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<th>Student</th>
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<th>Baseline %</th>
<th>Treatment %</th>
<th>Maintenance %</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>1 (out of 25%)</td>
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<td></td>
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<td>12.5</td>
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<tr>
<td></td>
<td>4 (out of 12.5%)</td>
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<td>3.3</td>
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</tr>
<tr>
<td></td>
<td>5 (out of 12.5%)</td>
<td>0.0</td>
<td>0.0</td>
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</tr>
<tr>
<td></td>
<td><strong>TOTAL (out of 100%)</strong></td>
<td><strong>0.0</strong></td>
<td><strong>69.2%</strong></td>
<td><strong>62.5%</strong></td>
</tr>
</tbody>
</table>

| A2      | 1 (out of 25%)  | 0.0        | 13.7        | 25.0          |
|         | 2 (out of 37.5%)| 0.0        | 16.0        | 22.0          |
|         | 3 (out of 12.5%)| 0.0        | 4.7         | 12.5          |
|         | 4 (out of 12.5%)| 0.0        | 0.0         | 0.0           |
|         | 5 (out of 12.5%)| 1.6        | 0.7         | 0.0           |
|         | **TOTAL (out of 100%)** | **1.6%** | **35.0%** | **59.5%** |

| A3      | 1 (out of 25%)  | 0.0        | 25.0        | 25.0          |
|         | 2 (out of 37.5%)| 0.0        | 25.0        | 14.0          |
|         | 3 (out of 12.5%)| 0.0        | 12.5        | 12.5          |
|         | 4 (out of 12.5%)| 3.2        | 0.0         | 0.0           |
|         | 5 (out of 12.5%)| 3.2        | 3.3         | 0.0           |
|         | **TOTAL (out of 100%)** | **6.4%** | **65.8%** | **51.5%** |

### Individual Percentages of SBI Process with Calculator

<table>
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<th>Student</th>
<th>Rubric Category</th>
<th>Baseline %</th>
<th>Treatment %</th>
<th>Maintenance %</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>1 (out of 25%)</td>
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<td>13.7</td>
<td>25.0</td>
</tr>
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</tr>
<tr>
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<td>0.0</td>
<td>0.0</td>
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<tr>
<td></td>
<td><strong>TOTAL (out of 100%)</strong></td>
<td><strong>0.0%</strong></td>
<td><strong>42.8%</strong></td>
<td><strong>53.5%</strong></td>
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## Individual Percentage of SBI Process without Calculator

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<th>Rubric Category</th>
<th>Baseline</th>
<th>Treatment</th>
<th>Maintenance</th>
</tr>
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<td>25.0%</td>
</tr>
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<td>TOTAL (out of 100%)</td>
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<td>74.5%</td>
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## Individual Percentages of SBI Process with Calculator

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<th>Treatment</th>
<th>Maintenance</th>
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258
<table>
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<th>Treatment (out of 100%)</th>
<th>Maintenance (out of 100%)</th>
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<td>51.5%</td>
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Individual Percentage of SBI Process without Calculator

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<th>Treatment (out of 100%)</th>
<th>Maintenance (out of 100%)</th>
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<td>8.3%</td>
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<tr>
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<td>4.0%</td>
<td>0.0%</td>
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<td></td>
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<tr>
<td></td>
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<td>0.7%</td>
<td>61.2%</td>
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<td>33.8%</td>
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<tr>
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<td>3 (out of 12.5%)</td>
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<td>4 (out of 12.5%)</td>
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<td>4.8%</td>
<td>75.3%</td>
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Individual Percentages of SBI Process with Calculator

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<td>3.6%</td>
<td>71.3%</td>
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REFERENCES


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Anne Eichorn Brawand has 12 years of experience teaching both general and special education at the elementary and middle school levels in Prince George’s County Public Schools, Maryland. She completed her Bachelor’s degree in Elementary Education at King’s College in Wilkes-Barre, PA, and obtained her Master’s degree in Mild to Moderate Disabilities from Johns Hopkins University in Baltimore, MD. Anne completed her doctoral studies in Special Education and Special Education Leadership at George Mason University in Fairfax, VA, in August 2013. She is currently employed as an Assistant Professor of Special Education at Kutztown University in Kutztown, PA. Anne’s areas of focus at the secondary level for students with disabilities include math intervention research, coteaching, teacher preparation, and self-efficacy.