

METACOGNITIVE STRATEGIES EMPLOYED DURING MATHEMATICAL  
PROBLEM SOLVING: A COMPARATIVE CASE STUDY OF FIFTH GRADERS  
WHO ARE GIFTED AND HAVE ADHD

by

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Metacognitive Strategies Employed During Mathematical Problem Solving: A  
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## **DEDICATION**

For Papu.

Thank you for always being there, for believing in me, and for encouraging me to do my best.

## **ACKNOWLEDGEMENTS**

A special thank you to my family and friends who have been so patient with me and encouraging during this process. It has been quite a journey. Drs. Suh, Hjalmarson, and Baker were of invaluable help and showed incredible patience. Thank you to the students who participated in this study and shared their “math brains” with me.

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## **ABSTRACT**

### **METACOGNITIVE STRATEGIES EMPLOYED DURING MATHEMATICAL PROBLEM SOLVING: A COMPARATIVE CASE STUDY OF FIFTH GRADERS WHO ARE GIFTED AND HAVE ADHD**

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George Mason University, 2014

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While many studies have examined mathematical problem-solving processes, heuristics, and strategies, few studies have looked at the effect of metacognitive knowledge and beliefs as related to mathematical performance since behavior related to monitoring, assessing, and strategy-selecting is difficult to observe and analyze.

This study looks at the amount of time students spend engaged in various metacognitive-type behaviors when faced with multistep rational number problem-solving challenges, and then compares and contrasts these results across the four categories of students to uncover any patterns. Once discoveries were made as to engagement in metacognitive stages and any changes over time were noted, the researcher created a “teacher-friendly” tool to be used by classroom teachers to help inform teaching and learning of mathematical problem solving.

Overall, self-awareness of the problem-solving process was increased during this study. The participants expressed that they did not do a lot of this type of reflection on their metacognitive processes during a typical school day. All of the students in this study spent an inconsistent amount of time in the verification stage of problem solving. In fact, they did not spend much, if any, time in this phase of problem, yet this is mentioned as an important phase for successful problem solving by Polya, Schoenfeld, and other researchers. These results could suggest that teachers need to help students see the value in “checking their work” and teach them how to do so. More research is needed to understand how behaviors related to control and metacognition are manifested during mathematical problem solving.

## 1. INTRODUCTION

Today's economy requires highly skilled workers who are able to apply mathematical knowledge to solve tasks for which the solution is not known in advance. In fact, students who complete advanced math courses are more likely to succeed in college and get better paying jobs (Cavanagh, 2007). Being successful in mathematics is so important in the 21<sup>st</sup> century; as Schoenfeld (2002) wrote, "to fail children in mathematics, or to let mathematics fail them, is to close off an important means of access to society's resources" (p. 13). He proposed four conditions that are needed in order to assist children in being successful in mathematics: high-quality curriculum; a stable, knowledgeable, and professional teaching community; high-quality assessment aligned with curricular goals; and stability and mechanisms for the evolution of a curricula, assessment, and professional development.

Since solving mathematical problems can be challenging, students may spend more time trying to avoid solving problems than solving them (Kapa, 2001). Metacognitive processes support problem solvers during the solution process and improve their ability to achieve the goal (Fortunato, Hecht, Tittle, & Alvarez, 1991). The more individuals control and monitor the strategies they use, the better their ability to solve a problem (Swanson & Trahan, 1990). In problem situations, good problem solvers are able to identify and modify strategies they have implemented from previous tasks that

are related to the ones they are solving, even if only indirectly related (Schoenfeld, 1985). The use of metacognitive prompts is a way to focus students on the thinking process, which is a key part of problem solving which can be transferred to other kinds of problems (Kapa, 2001) or other domains. Early metacognitive achievements are the foundation for higher order thinking that appears later in life (Kuhn, 2000) and being aware of what one knows is important to being able to extend understandings. An educational and developmental goal in today's society is to enhance metacognitive awareness and metastrategic consistency in how individuals apply, select, and interpret strategies (Kuhn, 2000). By developing these skills in students, they will have the conceptual knowledge and skills to be contributing members of society.

The study of metacognition has the potential to bridge concerns of both educators and researchers interested in how thinking skills are developed (Kuhn & Dean, 2004). There are differences in understandings of metacognition between educators and researchers in terms of what the higher order thinking skills are that students need to effectively participate in our society. While it is relatively easy to test for knowledge of concrete mathematical skills, it is more difficult to test students for problem solving and conceptual understanding. It is the metacognitive processes that are part of problem solving and conceptual understanding that this study explores with fifth-grade students.

The National Research Council's report *Everybody Counts* (1989) and The National Council of Teacher of Mathematics' (NCTM) national standards for curriculum (1989, 1991) discuss the need for mathematics instruction to be adapted to meet the needs of all students while maintaining high expectations for all. NCTM's *Professional*

*Standards for Teaching Mathematics* (1991) states that meaningful mathematical tasks are based on “knowledge of students’ understandings, interests and experiences” and “knowledge of the range of ways that diverse students learn mathematics” (p. 25). Further, the NCTM *Principles and Standards for School Mathematics* (2000), also known as Process Standards, which include problem solving, reasoning and proof, communication, connections, and representation, encourage the use and awareness of metacognitive processes for students of mathematics.

Mathematics instruction in elementary school is often delivered following a bottom-up processing approach (Sternberg, 2003). Under this approach, students are taught concepts and skills they need to know to solve problems, and then practice solving these problems with these skills. The approach focuses on memorizing step-by-step ways of how to do specific skills before applying the skills for problem solving. The primary form of instruction in the bottom-up approach is repetition and memorization (cognitive skills), without regard for understanding the processes—therefore ignoring deeper learning that allows for retention and transfer of knowledge (Sternberg, 2003).

Metacognitive skills are needed for a deeper understanding of the knowledge.

Conversely, a top-down approach to teaching mathematics focuses on strategies of how to approach math problems, using various techniques that can be widely applied (Kilpatrick, 1985). This strategy follows the belief that children learn by doing and by thinking about what they do and follows Polya’s (1957) four phases for problem solving: understanding the problem, devising a plan, carrying out the plan, and looking back.

Gourgey (1998) summarized the ideas of Polya, Sternberg, Kilpatrick, and others:

Cognitive strategies help *build knowledge*, while metacognitive strategies *monitor and improve progress*; that metacognition is important for effective cognition to occur.

Problem solving and critical thinking require a mastery of skills and dispositions that can be generalized across a variety of contexts. These skills include concepts such as interpreting, predicting, analyzing, and evaluating (Abrami et al., 2008). Ennis (1989) classified critical thinking interventions as general, infusion, immersion, and mixed. Many educators are trained in Bloom’s Taxonomy, developed by Benjamin Bloom (1956), which is a ladder of mental steps a learner goes through when developing an understanding of a concept (Table 1). Bloom’s taxonomy is used as a way for teachers to write lesson plans that ensure learners are moving toward information synthesis, reaching the highest possible cognitive level. When using Bloom’s taxonomy, students are given an opportunity to think and teachers are given an opportunity to check for understanding (Fisher & Frey, 2007).

Table 1

*Bloom’s Taxonomy From Highest to Lowest Level of Cognitive Domain*

| Educational Objective Level | Mental Activity                              |
|-----------------------------|----------------------------------------------|
| Evaluation                  | Exercise of Learned Judgment                 |
| Synthesis                   | Create New Relationships                     |
| Analysis                    | Determine Relationships                      |
| Application                 | Use of Generalizations in Specific Instances |
| Comprehension               | Translate, Interpret, and Extrapolate        |
| Knowledge                   | Recall and Recognition                       |

*Note.* Adapted from “Taxonomy of Educational Objectives, Handbook 1: The Cognitive Domain,” by B. S. Bloom, 1956, Copyright 1956 by David McKay Company, Inc.

## Key Concepts

### Metacognition

Metacognition is defined as the deeper understanding of cognitive processes and products (Flavell, 1976). Other terms that have been used to name the process of thinking about one's thinking are *reflective intelligence* and *reflective abstraction*. The use of many terms, along with the challenge of distinguishing the differences between cognition and metacognition, has made metacognition a difficult term to define. Garofalo and Lester (1985) defined cognition as being involved in choosing and planning what to do, and monitoring what is being done. They further define two aspects of metacognition: knowledge of cognition and monitoring of cognition. Knowledge of cognition relates to what someone knows about their cognitive abilities, processes, and resources as related to performing cognitive tasks. Cognition can be further broken down into person, task, and strategies categories. In the person category, metacognitive knowledge includes what one believes about himself or herself and others as cognitive beings. Task knowledge is what one knows about what is required to complete a task and the factors and conditions that make some tasks more difficult than others. Metacognitive knowledge about strategies involves having knowledge of both general and specific strategies and an awareness of their usefulness for approaching and completing tasks. While the use of rote strategy, such as standard algorithms for computation, involves cognition, it does not involve metacognition. Knowledge of cognition is important for the ability to monitor task understanding and strategy use, that is, regulating cognition. This aspect of metacognition relates to the many decisions and strategies one might use while working through a

cognitive task or problem, which could include choosing strategies to help understand the nature of a task or problem, planning how to solve the problem, choosing strategies to solve the problem, monitoring activities while implementing strategies, and evaluating the outcomes of various strategies and plans. When needed, regulation of metacognition also includes revising and abandoning strategies and plans that are not productive (Garofalo & Lester, 1985). These decisions and strategies, as used by fifth graders, were measured over time in this study.

Metacognition is this highest level of cognition (Baum & Owen, p. 97) and involves an awareness and personal control of strategies used to learn and perform. Metacognition refers to the active monitoring, regulation, and orchestration of these processes as related to the concrete goal or objective. There are two aspects to metacognition: knowledge and beliefs about cognitive phenomena and the regulation and control of cognitive actions. These include the learners' awareness of things like planning, monitoring, and evaluating their work toward the solution of a specific math problem (Fortunato et al., 1991). Metacognition is dependent upon the ability to "evaluate or monitor one's own cognitive processes, such as thoughts and memories, so that a reasonable assessment can be made about future performance" (Shimamura, 2000, p. 142). Metacognition is viewed as a cyclical interaction of two systems: the object-level and meta-level systems. Object-level systems are mental representations such as knowledge-based memory. Meta-level systems monitor information processing at the object level and control information processing.

Metacognitive functions can be procedural or declarative. Procedural metacognition is awareness and management of one's thinking, while declarative metacognition involves having a broad understanding of thinking and knowing in general (Kuhn & Dean, 2004). Montague (1992) specified three metacognitive strategies that support monitoring and control while solving mathematical problems at the object level: (a) self-instruction to help the student identify the problem's parts before executing a solution to the problem; (b) a self-question directed by a self-dialogue, or methodical analysis of the problem; (c) self-monitoring that encourages the student to control the performance process.

Metacognition begins early in life, when children first become aware of their minds, but these skills do not typically independently develop at the level needed to be contributing members in our society, including the field of mathematics. Providing challenging tasks that are relevant to students' lives may increase their interest in mathematics, possibly leading to increased achievement (Kramarski, Mevarech, & Arami, 2002). According to Schoenfeld (1992a), the field of education lacks a model that explains the mechanisms of self-monitoring and self-regulation, and there is not a clear understanding of how metacognition works to give individuals their *mathematical point of view*. This study aimed to add to the understanding of the use of metacognitive processes for fifth graders solving single- and multistep problems, and how the use and awareness of these processes varies over time and across types of students.

## **Attention-Deficit/Hyperactivity Disorder (ADHD)**

There is no definitive, objective test that can be administered to determine if a child has Attention-Deficit/Hyperactivity Disorder (ADHD). The organization Children and Adults With Attention-Deficit/Hyperactivity Disorder (CHADD) defines ADHD as “a condition affecting children and adults that is characterized by problems with attention, impulsivity, and over-activity” (n.d., para. 1). It affects 5 to 10% of school-age children and is a lifespan disorder (CHADD). Attention-deficit/hyperactivity disorder is the current medical diagnostic label named in the *Diagnostic and Statistical Manual of Mental Disorders 5* (DSM-5) (2013) by the American Psychiatric Association (APA) for a condition that has been recognized and studied for over a century. Currently, in accordance with the Individuals With Disabilities Education Act (2004), students with a medical diagnosis of ADHD can be served under the label *Other Health Impairment* (OHI), depending upon the level and impact of their ADHD on educational performance. They may also be served under Section 504 of the Rehabilitation Act (1973) in order to access reasonable accommodations outside of the special education system. Students with ADHD who manage the impacts of the disorder with minimal effect on educational performance are not provided specialized support services.

The diagnosis of ADHD is made using test material, behavioral checklists, and parental histories (Fertig, 2009). Students with ADHD often have a high energy level. The key features of ADHD include poor investment and maintenance of effort, deficient modulation of arousal to meet situational demands, a strong inclination to seek immediate reinforcement, and deficient impulse control (Purdie, Hattie, & Carroll, 2002). DuPaul

and Stoner (2003) made three causal connections between ADHD and academic problems: (a) problems with academic skills leading to showing ADHD-related behaviors, (b) behavioral symptoms of ADHD disrupting knowledge acquisition and performance, and (c) both ADHD and learning difficulties caused by one or more third variables like neurological deficits.

According to the DSM-5 (APA, 2013), signs of inattention among students with ADHD include becoming easily distracted by irrelevant sights and sounds; failing to pay attention to details and as a result making careless mistakes; rarely following instructions carefully and completely; and losing or forgetting things like toys, pencils, books, and tools needed for a task. Further, signs of hyperactivity and impulsivity that students with ADHD may experience include feeling restless; fidgeting with their hands or feet or squirming, running, climbing, or leaving a seat in situations where sitting or quiet behavior is expected; blurting out answers before hearing the whole question; and having difficulty waiting their turn in line. DSM-5 defines ADHD as a biologically based condition causing persistent patterns of difficulties resulting in inattention and/or hyperactivity/impulsivity. Six of the symptoms in one of the domains (inattention and hyperactivity/impulsivity) would need to be evident before age 12 for students to be diagnosed as having ADHD (p. 59).

Mathematical reasoning is one category of learning disabilities that focuses on the discrepancy between mathematical achievement and a student's capacity to learn (Lyon, 1996). While it should not be discounted that students with ADHD may also have a mathematical learning disability, the academic underachievement many students with

ADHD have may also be related to the behavioral problems these students have (Scime & Norvilitis, 2006). There have been higher rates of ADHD in children with learning disabilities and higher rates of learning disabilities in children with ADHD, although the reasons why are not completely understood (Faraone et al., 2001).

The symptoms of attention disorders can cause challenges in school and family functioning and in relationships with classmates. These challenges can have negative long-term psychiatric, social, and academic impacts on students (Miranda, Jarque, & Tarrage, 2006). Students with ADHD have problems with impulsivity, sustained attention, and overactivity, which often result in difficulty attending to and following instructions, competing instructional activities, and complying with class rules. These behaviors can keep students from learning new skills and prevent the development of relationships with peers and classmates (Stahr, Cushing, & Lane, 2006). Since children with ADHD may have fewer opportunities to respond to teacher questions during school and may complete less written work than their classmates, they may underachieve academically (DuPaul & Stoner, 2003). However, students with ADHD may be able to focus when receiving frequent positive reinforcement or when they are under strict control. Other problems associated with ADHD include conduct problems, academic underachievement, specific learning disabilities, and peer relationship problems (DuPaul, 2007). Methods of active learning, including the use of mathematical manipulatives, could possibly benefit children with ADHD who often need some special accommodations to help them learn.

## **Giftedness and ADD/ADHD: Twice-Exceptional**

There is no clear definition of twice-exceptional students, but these students have both obvious strengths (gifted characteristics) and evident weaknesses in their academic performance (Stewart, 2007); in this particular study, ADHD impacts student learning. Part of the trouble with identifying students with ADHD as also being gifted is that medical professionals who diagnose ADHD are not trained in giftedness since it is not a medical condition (Baum & Olenchak, 2002).

While awareness of the needs of twice-exceptional students is increasing, many school districts do not provide programs for them. A better understanding of the issues faced by twice-exceptional students is needed in order to better meet their needs (Ruban & Reis, 2005). Children who are bright and children who have ADHD often share many characteristics. However, a student who is bright and has ADHD is often more self-aware, is more likely to perceive himself or herself as inadequate, and may develop narcissism to defend against feelings of low self-esteem (Watkins, 2008). Often these students may test in the gifted range but only perform as an average student in class.

Because intelligence quotient (IQ) tests that measure students' potential do not help identify services that twice-exceptional students may need, McCoach, Kehle, Bray, and Siegle (2004) developed an eight-step system for identifying twice-exceptional students, including behavioral observations, intelligence testing, measures of cognitive processing, and achievement tests. McCoach and colleagues discussed the importance of assessing student functioning in the classroom environment, on curriculum-based

assessments, and through student interviews that address their perceptions and attitudes about academics.

For the purpose of this study, *twice-exceptional* will be used to refer to “students who have or show potential for remarkable gifts and talents in specific areas, but whose deficits and difficulties in learning, paying attention, or meeting social and emotional expectations impede their development” (Baum, Rizza, & Renzulli, 2006). The two students in this study identified as twice-exceptional received accommodations within their classrooms to help them be successful in school. Such accommodations may include things like frequent reminders to maintain focus, restating questions, and reading aloud for written activities. These students often see themselves as less able than they are because they focus on their disability instead of their strengths. A disability may hide their giftedness, and conversely, giftedness may hide a disability. Since disorganization can be a characteristic of these students, tests may indicate giftedness despite average performance in classes. Teachers of twice-exceptional students may find themselves struggling to give these students time to focus on building their strengths because so much time is needed to improve upon weaknesses.

Mendaglio (1995) suggests that combining social and emotional factors that exist for children who are gifted and have ADHD produces a heightened sense of alienation, sensitivity, and overreaction, affecting attitudes toward school, achievement, and self-esteem. The gifted child’s sensitivity coupled with the emotional overreaction, frustration, and irritability of the ADHD child can cause manipulative and egocentric reactions that require skills for emotional development.

Students who are gifted and have ADHD require a wide range of opportunities for success. As noted earlier, traditional ways of identifying gifted students do not cover all of the facets of giftedness and do not allow for the characteristics of ADHD (Leroux, 2000). Maker, Neilson, and Rodgers (1994) proposed combining Gardner's (1983) theory of multiple intelligences, which explains how individuals learn, remember, perform, and understand, and a variety of problem types, to design ways to identify and serve students who are gifted and have ADHD. Vaidya (1994) suggested using portfolio assessment, IQ and achievement tests, and creativity testing to identify gifted/ADHD students.

### **Giftedness**

It is difficult to define giftedness within the education field because most gifted children show a range of abilities in many, but not all, traits. The Jacob Javits Gifted and Talented Students Education Act uses the U. S. Government Title IX definition found in the Elementary and Secondary Education Act and defined a gifted person as someone who "gives evidence of high achievement capability in areas such as intellectual, creative, artistic, or leadership capacity, and who need services and activities not ordinarily provided by the school in order to fully develop those capabilities" (7801 U.S.C. § 22). Gagne (2002) defined gifted individuals as those who have an innate superior ability in one of four domains who develops superior talent through practice and learning. He stated that environmental factors such as parental and educational support, as well as physical and psychological factors contribute to the development, or lack of development, of talent. The Elementary and Secondary Education Act defines gifted and talented students as

Students, children, or youth who give evidence of high achievement capability in areas such as intellectual, creative, artistic, or leadership capacity, or in specific academic fields, and who need services and activities not ordinarily provided by the school in order to fully develop those capabilities. (Title IX, Part A, Definition 22, 2002)

In the particular school district in Virginia where this study took place, students were identified as being gifted through general intellectual aptitude. The Virginia Department of Education (2010), in *Understanding the Virginia Regulations Governing Educational Services for Gifted Students*, defines general intellectual aptitude students as those who

demonstrate or have the potential to demonstrate superior reasoning; persistent intellectual curiosity; advanced use of language; exceptional problem solving; rapid acquisition and mastery of facts, concepts, and principles; and creative and imaginative expression across a broad range of intellectual disciplines beyond their age-level peers. (VR270-01-002 § 22.1-16)

Ridge & Renzulli (1981), pioneers in the field of gifted education, describe gifted behaviors as “an interaction among three basic clusters of human traits—these clusters being above average general or specific abilities, high levels of task commitment, and high levels of creativity” (p. 204). Strip and Hirsch (2000) expanded upon Renzulli’s (1977) definition by including students who both demonstrate and show *potential* to demonstrate an interaction of the three clusters as needing educational opportunities and services that are not typically a part of general education.

There are three common endogenous needs of gifted student that appear across research: uneven or asynchronous development, a tendency toward perfectionism, and a tendency for excessive self-criticism. The most common example of gifted students' exogenous needs surrounds their need for academic engagement in a school environment that is not accepting of students who are very serious about learning (Cross, 1997). As a result, any or all of the following can emerge: (a) the need to feel accepted; (b) the need to affiliate with other gifted students; and (c) the need for recognition of accomplishments.

### **Conceptual Framework**

Many studies have examined mathematical problem-solving processes, heuristics, and strategies; few studies have looked at the effect of metacognitive knowledge and beliefs as related to mathematical performance (Garofalo & Lester, 1985) since behavior related to monitoring, assessing, and strategy selecting is difficult to observe and analyze. This study was based upon Schoenfeld's (2007) conceptual framework that acknowledges that knowledge as well as how individuals use problem-solving strategies, how they use what they already know, and their beliefs, are all important parts of the problem-solving process. Schoenfeld analyzed the amount of time individuals spent in each of six problem-solving stages (read, analyze, explore, plan, implement, and verify) and used a variety of teaching actions to assist students to develop their mathematical problem-solving skills as related to each of the six stages.

Attention disorders can lead to deficits in arithmetic calculation and/or mathematics reasoning. Children with ADHD often have more cognitive and memory

deficits (Burden & Mitchell, 2005) than their peers who do not have ADHD. This could relate to the difficulties many students with ADHD have with memorizing math facts. Deficits in arithmetic calculation skills are more frequently identified than deficits in arithmetic reasoning (Lyon, 1996). Students with ADHD often struggle more than those only with hyperactivity when solving mathematical word problems (Marshall, Schafer, O'Donnell, Elliott, & Handwerk, 1999). Since these students are unable to focus on the most relevant information in word problems, students with attention difficulties may have trouble creating categories for problem solving because they focus more on ideas like objects, tasks, and people and less on the mathematics involved in the problems (Lucangeli & Cabrele, 2006). Those who focus on the structure of problems usually have better conceptual knowledge and procedural skills. Without attending to relevant ideas, students do not always build the conceptual knowledge needed over time to correctly problem solve. Students with ADHD-Predominantly Inattentive Type may be at increased risk for mathematical calculation difficulties, and are typically slower and less accurate than their peers who do not have ADHD. These difficulties may be a result of an overload of working memory that is caused by cognitive effort needed when completing mathematical calculations (Lucangeli & Cabrele, 2006).

### **Purpose of the Study**

The purpose of this case study research was to examine the metacognitive processes of eight fifth-grade students as they solved a series of rational number problems to see if there was an increased awareness in their metacognitive processes over time, and if and how the amount of time spent in various phases of the process changed

over time, using Schoenfeld's categories as a conceptual framework. The research questions were:

1. What types of metacognitive behaviors emerge in fifth-grade students with ADHD, ADHD/gifted, gifted students, and nonlabeled students when they approach rational number problem-solving mathematical tasks?
2. How does the problem-solving debrief session affect the amount of time spent in each of the six metacognitive problem-solving stages and student performance patterns over time?

Two of the students in this study were identified as having ADHD, two students were identified as gifted and having ADHD, two students were labeled gifted, and two had no labels. Through reflection on their metacognitive processes, I hypothesized that increased metacognitive awareness would result in students who are gifted, and that there would will also be some, although less, improvement in those students who are gifted and have ADHD. I further hypothesized that there would be less consistency in regard to metacognitive processes and ability to reflect on these processes for students who have ADHD, as compared to other students in the study. Further, this study examined if creating an awareness of metacognitive processes for any or all groups of these students would assist in the development and/or change of the students' metacognitive processes while problem solving.

The methodology included clinical interviews of the students while solving one-step and multistep mathematical problems and individual viewing of videotaped sessions following the problem-solving sessions, which included creating an awareness of

metacognitive processes used by the students. I met with each student for a total of eight problem-solving sessions.

As a teacher of both Mathematics and Advanced Academics, I have observed many challenges students who are identified as gifted face while solving challenging mathematical problems. Further, with the inclusion of twice-exceptional students in advanced academic classes, informal observations of student struggles suggested varied awareness of problem-solving strategies and abilities to explain metacognitive processes. While working with these two groups of students is my primary interest, including students who were not identified as being gifted but did have ADHD, allowed me to begin to identify any metacognitive processes and/or experiences that were unique to each group as well as those that intersect among two or all three groups of students. Further, by including two students who did not have labels, there was a basis for comparison with fifth-grade students in typical classroom settings who were not receiving any special services in a large urban public school system.

### **Significance of the Study**

In light of lower levels of mathematics achievement in the United States than in other countries, many professional groups (National Council of Supervisors of Mathematics (NCSM) and National Council of Teachers of Mathematics (NCTM)) continue to call for the reform of mathematics education in the United States. Ideas presented include incorporating conceptual understanding as playing a central role, and viewing mathematical knowledge as a system of relationships among mathematical symbols, concepts, operations, activities, and situations (Niemi, 1996).

According to NCTM,

a conceptual approach enables children to acquire clear and stable concepts by constructing meaning in the context of physical situations and allows mathematical abstractions to emerge from empirical experience. A strong conceptual framework also provides anchoring for skill acquisition. Skills can be acquired in ways that make sense to children and in ways that result in more effective learning. A strong emphasis on mathematical concepts and understandings also supports the development of problem solving. (1989, p. 17)

Problem solving has come to be viewed as a process involving the highest faculties: visualization, association, abstraction, comprehension, manipulation, analysis, reasoning, synthesis, generalization, each needing to be “managed” and all needing to be “coordinated” (Garofalo & Lester, 1985). NCTM (2000) stresses the importance of problem solving as a way to help students recognize the *power and usefulness* of mathematics. Problem solving has been an important issue in the mathematics community for several decades.

Mathematics educators need to teach students about metacognition and help them develop an awareness of metacognitive processes involved in problem solving and for proficiency in applying algorithms, yet metacognition has not been systematically studied by mathematics educators. This is likely because problem-solving behaviors are difficult to observe and analyze, and self-reports of task performance may affect cognitive processes while problem solving (Garofalo & Lester, 1985). Past research and cognitive analyses of mathematical performance has been inadequate because it overlooks

metacognitive actions (Garofalo & Lester, 1985). Metacognitive beliefs may help determine success and failure in a variety of activities. Students need both knowledge and awareness, and control of knowledge, to maintain successful cognitive performance.

In 2006 it was estimated that 3 to 5% of school-aged children had attention deficit disorders (Dillon & Osborne, 2006) and that estimate is now closer to 5 to 10% (APA, 2013). Schools need to be prepared to address behaviors related to attention deficit disorders through teaching and classroom management. Math may be the preferred subject area for students with ADHD because there are different types of instruction and content, which could lead to fewer unfocused behaviors (DuPaul et al., 2004). Depending upon how math is taught, it can be difficult for students with ADHD because keeping information in their memories long enough to apply it can be challenging. Short-term memory issues are a result of distractions and frustrations for students when they have not constructed meaning or developed a true understanding of a concept.

The difficulties students with ADHD have with mathematics have not been studied as much as the difficulties these students have with reading. Most of the research with problem solving and students with ADD/ADHD focuses on cognitive processes and mental and graphic representations.

Furthermore, many teachers lack clear guidelines as to how to help develop higher order reasoning and problem-solving skills among their students (Goos, Galbraith, & Renshaw, 2002). Schoenfeld (1992a) worked with college students in groups and monitored and evaluated the groups' progress, but such studies have not provided a lot of insight as to how students think and learn while interacting with their peers, as the studies

have focused more on learning outcomes than learning processes related to mathematical reasoning (Goos et al., 2002).

One of the greatest risks for individuals with ADHD is academic underachievement (DuPaul, Ervin, Hook, & McGoey, 1998). Learning mathematics can be especially difficult for students with ADHD because “you have to hang on to meaningless information until it hooks up with meaning” (Johns, 1998). Keeping meaningless information in working memory long enough to apply it can be very difficult for students with ADHD, and distractions and frustrations contribute to these deficits. These students are also less likely to identify a problem-solving trick, if there is one, for solving a particular type of problem. Routines are important for many students with ADHD when learning mathematics because routine habits are less vulnerable to distraction (Johns, 1998).

### **Definition of Terms**

For the purposes of this study, the following terms will be used as defined below.

*Metacognition:* Metacognition is thinking about one’s thinking; knowledge about how one thinks and his or her ability to monitor that thinking when completing tasks; does not involve the use of rote strategies including things like standard algorithms.

*Attention-Deficit/Hyperactivity Disorder (ADHD) Student:* A student who has this clinical diagnosis by a psychologist, pediatrician, or other doctor; characterized by problems with attention, impulsivity, and/or overactivity; such participants in this study had either an individualized education plan (IEP) or 504 plan outlining necessary accommodations for ADHD to be successful in school.

*Twice-Exceptional Student:* A student who exhibits both the characteristics of a student who is gifted and a student who has ADHD; a student who has been identified as both gifted and having ADHD; and, in this study, was receiving services for each in his or her local school.

*Gifted Student:* A student who has been identified by the local school system as needing, and in this study, is receiving, academic services that provide challenges above and beyond what are provided in the general education classroom as shown by capabilities of high achievement in one or more academic areas.

## **2. REVIEW OF LITERATURE**

In order to inform the study, areas of literature that were reviewed included metacognition and mathematical problem solving, as well as an exploration of the needs of students who are identified as gifted, those identified as having ADHD, and twice-exceptional students.

### **Metacognition and Problem Solving**

While cognitive abilities may vary, there is a definite relationship between cognitive ability and performance in academic settings (Efklides, Papadaki, Papantoniou, & Kiosseoglou, 1997). While academic performance is influenced by ability factors, one can assume that part of the effect of performance anxiety is due to the interaction of anxiety with cognitive abilities. Efklides et al. (1997) found that subjects in mathematics with high cognitive ability were more accurate in difficult and success estimation than those with low ability.

There are three levels of cognitive ability (Efklides et al., 1997):

- Superordinate level: general intelligence; the person's capacity to identify relations in novel situations;
- Middle level: domain-specific abilities that refer to the person's capability to deal with problems in a particular knowledge domain; and

- Subordinate level: task-specific skills that tap into an individual's capability to deal with specific tasks in a domain

Metacognition is knowledge about one's cognition (Efklides et al., 1997) which also consists of three levels:

- General level: knowledge or theories about one's self, the world and the mind;
- Intermediate level: knowledge one has about tasks, strategies, and criteria of knowing; and
- Subordinate level: online, task-specific knowledge and experiences such as feelings.

According to Vygotsky (1978), children's higher mental processes function first on the social level and then on the individual level. In order to find where metacognitive skills begin, children have to be watched in social interactions. Following Vygotsky's ideas, Kuhn and Ho (1980) showed that self-directed activity improved problem solving over time. Theories about the origins of metacognition, then, need to take into account adult-child interaction and problem solving alone before and following formal schooling. It appears that the basics of metacognitive skills develop during preschool years as a result of "adult assistance and solitary persistence at a problem-solving task" (Kontos, 1983). Schoenfeld (2007) talked about the fundamental shift from the emphasis on knowledge (what does the student know?) to a focus on what students know and can do with their knowledge. The idea during this paradigm shift is that being able to use the knowledge one has in appropriate circumstances is essential for mathematical proficiency. Further, Schoenfeld (2007) said that good problem solvers are "flexible and

resourceful” (p. 60). They have many ways to think about problems including many ways to think about problem solving when they get stuck, and are efficient with the knowledge they have. Being able to recite facts and definitions on command is not enough for students to be good mathematical problem solvers. Good problem solvers are willing to approach difficult mathematical challenges and believe they will make progress, without giving up. All of Schoenfeld’s aspects of mathematical proficiency can be learned in school and can help explain why attempts at problem solving are successful or not.

Polya (1957) focused on the problem-solving process, while more recent research is focusing on the attributes of the problem solver that contribute to the process. Polya saw effective problem-solving processes as making use of analogies, making generalizations, restating or reformulating problems, exploiting the solution, exploiting symmetry, and working backwards. Through many examples in *How To Solve It*, Polya showed how the right kind of questioning of a student can lead him or her to consider an easier problem that is related to the more difficult task at hand. Polya defined a four-phase description of problem solving including (a) understanding the problem, (b) planning how to solve the problem, (c) carrying out the plan, and (d) looking back (1957). He saw problem solving as a linear progression from one phase to the next. Polya also provided heuristic strategies which were his rules for making progress on difficult problems; for example, when attempting to understand a problem, students can focus on the unknown, on the data, or draw a diagram. Polya’s problem-solving steps follow the top-down model of mathematics instruction and support the development of discovery and metacognitive skills in students.

Like metacognition, problem solving is a complex term to define, and researchers have varied explanations as to what makes a “good” problem solver. After completing a research synthesis, Suydam (1980) stated that the evidence strongly says that “problem-solving performance is strongly enhanced by teaching students to use a wide variety of strategies or heuristics, both general and specific” (p. 43). Based upon Piagetian tasks, and the high level of mathematical achievement believed to be associated with such tasks, Freyberg (1966) conducted a study with 5- to 7-year-olds that supported his idea that Piagetian task performance predicted increased mathematical achievement two years later. Hiebert and Carpenter (1982) reported that training in conservation did not improve arithmetic achievement over a 4-year period from kindergarten to fourth grade. They claimed that students who spontaneously achieve conservation earlier benefitted from more mathematical instruction, but that training in conservation did not lead to the same benefits. While there are positive correlations found in many studies between Piaget’s logical reasoning abilities and mathematical learning, there are still many questions that need to be answered, mainly, what actions result in these correlations?

In order to teach students to think metacognitively, teachers may have them work in small groups to reason mathematically by answering questions that focus on comprehending the problem, constructing connections, using strategies appropriate for solving the problem, and reflecting on the process and the solution. Positive effects of cooperative-metacognitive instruction have been found on students’ mathematical achievement. These cooperative group sessions have resulted in more positive outcomes than individualized metacognitive instruction, which was more positive than the effects

of cooperative or individual instruction without metacognitive instruction (Kramarski et al., 2002). Teachers should include metacognitive supplements when training students to become proficient in applying mathematical algorithms and heuristics and help students adopt a “metacognitive posture” toward mathematics (Garofalo & Lester, 1985). Before beginning mathematical problem solving, Lester, Garofalo, and Kroll (1989) suggested that teachers read the problem, use whole-class discussions to focus on understanding the problem, and conduct whole-class discussions of possible problem-solving strategies. These teaching moves will show students the importance of reading carefully, put a focus on important vocabulary and data, and may provide ways to solve problems. While problem solving, Lester et al. suggested that teachers watch and question students and provide hints and extensions as needed. These moves will help teachers identify student strengths and weaknesses and move them forward in their thinking. Finally, Lester et al. recommend that following problem-solving sessions teachers show and talk about solutions and relate work to previously solved problems. This will help students be able to show and name different strategies and see how problem-solving strategies can be applied to many different problems.

Flavell (1976) and Miller (1980) found that different task forms are not equal predictors of student competencies in mathematics. Therefore, different tasks that measure the same concept can produce different results (Hiebert & Carpenter, 1982) since different tasks use different types of logical reason abilities. Behr, Wachsmuth, Post, and Lesh (1984) found that targeted interventions were not a part of strategies that students implemented when solving problems. Further, learning rational number concepts

is not a result of formal processes presented in textbooks, but comes from strategies the students themselves have devised (Moseley, 2005). Moseley (2005) found that when one group of fourth-grade students was taught rational number concepts using a single perspective and another was taught with a curriculum that had a multiple-perspective view of rational numbers through problem solving, the first group had more difficulty solving problems. Many of the strategies these students employed were not adequate for the tasks they were asked to complete. Students from the multiple-perspective group were able to make more connections to related concepts which aided in their sense-making than their peers in the single-perspective group. Therefore, Moseley concluded that conceptual diversity of content should be a part of instruction when students are developing their rational number knowledge. A study done with 7- and 8-year-old students (Fennema, 1972) indicated that students can learn direct recall of mathematical ideas with concrete or symbolic representations. When students were expected to transfer knowledge or extend the mathematical principle, students who used the symbolic models performed better than those who used concrete models. When planning mathematics lessons, then, teachers should use symbolic models, which now extend to include virtual models which were not as frequently used at the time Kalyuga's study was conducted (Kalyuga, 2008). When asked to transform graphs of simple linear and quadratic functions using animated vs. static diagrams, Kalyuga's (2008) study's results indicated a significant interaction between levels of the learners' expertise and the instructional formats. Novice learners benefitted more from static diagrams than animated diagrams, while more knowledgeable learners benefitted more from animated rather than static

diagrams. This could be the case because more experienced learners need to integrate dynamic knowledge structures with redundant details displayed in graphics. This may cause learners to pull on other cognitive resources such as working memory and reduce relative learning effects (Kalyuga, 2008).

Mathematical task knowledge includes an individual's beliefs about mathematics as a subject and the nature of mathematical tasks. It also includes an awareness of how problem content, context, structure, and syntax affect the difficulty of the task (Garofalo & Lester, 1985). Garofalo and Lester further extended Polya's work describing four categories of activities that are involved in solving mathematics problems: orientation, organization, execution, and verification. Orientation is the strategic behavior designed to help a student get to an understanding of the problem, including a representation. Organization is the creation of a solution plan while execution is putting this plan into place. Finally, verification is the evaluation of the other three phases and checking of final results. While strategy knowledge includes algorithms, it also includes an awareness of how strategies help in understanding problem-solving statements, organizing information, planning, implementing plans, and checking results. During the problem-solving process, person, task, and strategy all interact. Beliefs, attitudes, and emotions also are thought to influence problem-solving behaviors (Schoenfeld, 1989a), authentic mathematical task planning, and reflecting. Many students have trouble solving authentic tasks (Kramarski et al., 2002); these difficulties for students occur at all stages of the problem-solving process.

Hohn and Frey (2002) devised several sequential phases involved in the processes of problem understanding and solving. The first phase is problem representation which includes problem translation (interpretation of factual information in the problem) and problem integration (knowledge of types of problems is used to create a structure of relationships with the problem). The next phase is solution planning (the learner chooses a solution procedure), followed by solution execution (problem is solved). Finally, in solution monitoring, the problem solver reviews his or her work and looks for errors. According to Hohn and Frey (2002), if one does not effectively engage in these phases, he or she will likely perform poorly on problem-solving tasks. Through implementation of Hohn and Frey's SOLVED intervention, they found that elementary school students can be taught to use simple heuristic strategies and that the result is improved problem-solving skills when compared to a more traditional textbook approach to problem solving. They also found that these students concentrated their work in the solution execution phase and less in the significant reflective phase of problem solving.

Zentall and Ferkis (1993) documented that "when IQ and reading are controlled, math deficits are specific to mathematical concepts and problem types" (p. 6). They demonstrated that students who have difficulties in mathematics in general have differences in cognitive ability and style that interact with instructional contexts and produce problem-solving skill deficits. Further, students who have slow and inaccurate computational skills often have poor mathematical problem-solving skills (Zentall & Ferkis, 1993). Many of the current mathematical texts are designed based upon on the idea of the learner as someone who receives mathematical knowledge rather than one

who participates in the construction of mathematical knowledge, resulting in many negative misconceptions about mathematics, especially when students see math as a set of rules that requires memorization and rote practice to be successful. Conversely, teachers who actively engage students in mathematical problem solving often have students who have better problem-solving abilities. As a result, Zentall and Ferkis (1993) concluded that mathematics instruction for students with ADHD and other learning disabilities should include: (a) mastery learning that builds upon prerequisite skills and understanding instead of spiral learning; (b) learning that involves active construction of meaning; (c) teacher interactions with students to assess and stimulate problem-solving strategies; (d) increased emphasis on assessment and teaching of mathematical concepts; (e) use of strategies for reading comprehension and memory in problem solving, as needed; (f) attentional cues to help students prepare for changes in problem action, operation, and order of operation; and (g) a variety of unique instructional activities that facilitate automatization of basic calculations.

There is some evidence that how well students achieve in mathematics is less related to the use of aids in the classroom, such as manipulatives, and more related to how teachers cover the course content (Raphael & Wahlstrom, 1989). By having students focus on the solution to the problems being solved, which included both planning and executing the steps needed to solve the problem, and then having the students use self-questioning to guide them through their problem-solving process, greater awareness of metacognitive processes was achieved in Raphael and Wahlstrom's study. This cognitive strategy was taught to 12 students every other day for 40 minutes. After 4 months, the

students who received the intervention showed that they were using the instructed strategy and scoring higher than students who had not had the intervention in problem representation and solution. Creating a classroom environment that encourages math talk may increase students' abilities to explain their thinking and communicate their mathematical ideas. For many teachers this requires a shift from a teacher-centered to a more student-centered environment. In such a classroom, learning is more directed by the students and their ideas influence the direction of lessons. The goal of such an environment would include students listening to understand, clarifying each other's ideas, and helping each other understand and correct errors.

Carlson and Bloom (2005) characterized five problem-solving attributes that are necessary for problem-solving success: resources, control, methods, heuristics, and affect. Resources are the conceptual understanding, knowledge, facts, and procedures used during problem solving. Control includes the metacognitive behaviors and global decisions that influence the solution, including the selection and implementation of resources and strategies and behaviors that determine the efficiency with which facts, techniques, and strategies are used. While methods includes general strategies used when problem solving, including construction of new statements and ideas, carrying out computation, and accessing resources, heuristics are more specific procedures and approaches. Finally, affect refers to attitudes, beliefs, emotions, and values/ethics related to problem solving.

Cramer, Behr, Post, & Lesh (2009) suggested that mathematical ideas can be represented in five ways: real-life situations, pictures, verbal symbols, written symbols,

and manipulatives. He found that students learn best by exploring mathematical ideas in a variety of ways and explored the relationships between and among these five in mathematical problem solving.

This current study is based upon the work of Schoenfeld (1983), who developed a scheme for breaking problem solving into episodes of executive decision points to aid in analyzing problem-solving moves. These episodes are reading, analysis, exploration, planning, implementation, and verification. Following the development of these episodes, Schoenfeld (1984) identified three levels of knowledge and behavior which he believed need to be considered if an accurate picture of one's problem-solving abilities is to be obtained: resources, control, and belief systems. Both Schoenfeld (1984) and Lester (1994) claimed that good problems solvers regulate their problem-solving efforts and are able to connect their knowledge. These individuals tend to also have a high level of awareness of their strengths and weaknesses as related to problem solving and tend to focus on the structure and relationships in problems. Good problem solvers are also flexible in their thinking. According to Schoenfeld (1992a), "It's not just what you know; it's how, when, and whether you use it" (p. 355). One aspect of metacognition—reflecting on progress while engaged in problem solving—is important in guaranteeing success in problem solving. Schoenfeld (2007) believed that when people do things that do not help solve problems they may never get to use their knowledge in the right ways. Further, he stated that "effective problem solvers behave differently and students can learn to be much more efficient at monitoring and self-regulation and become more successful problem solvers" (p. 67). Schoenfeld (1989) conducted a study of 230 high

school students to examine the ways individuals' conceptions of mathematics shape the ways they engage in mathematical activities. As a whole, the participating students felt that mathematics was objective and objectively graded and a subject that can be mastered. The students felt that working on mathematics results in good grades, rather than luck, and when these students did badly in math they believed it was their own fault. These highly motivated students reported finding math interesting, believed that learning math would help them think clearly, and they wanted to do well academically. Schoenfeld (1989) concluded that these students have separated school mathematics from abstract mathematics, the latter of which they have not experienced much of and will need more experience with in order to be successful in advanced mathematics.

Similar to Schoenfeld (1989), Chi, Basook, Lewis, Reimann, and Glasser (1989) defined characteristics of students with effective problem-solving skills. They clarified the goal of instruction as being able to enable effective problem solving to facilitate students incorporating the following characteristics through consistent application of real-world situations. Students who are better problem solvers:

1. Attempt to justify each step of a solution to figure out how they could construct a solution to a similar problem.
2. Try to relate specific steps of current problem-solving activity to principles, concepts, and definitions that are related.
3. Spend twice as long on the sample problem-solving exercise than poorer students.

4. Treat sample problem-solving exercises as typical of a class of problems and abstract how each step of the solution would be generalized to a class of problems.
5. Explain what conditions led to choosing what to do for each step.
6. Extract tacit aspects of the meaning from examples.
7. Try to figure out the goals of the expert problem solver that are tacit in the overall problem solution.
8. Notice that they do not understand or fully comprehend either the solution or the rationale.
9. Seek to understand the “why” rather than the answer.
10. Do not show the need to reread information and redo instructional examples; they apply new skills immediately.
11. Formulate a plan to solve the new problem and then go back to retrieve additional information that is relevant beyond what is presented.
12. Verbalize explanations or justifications for solutions.
13. Anticipate how they will be using the information in examples for working problems.
14. Turn principles and concepts into rules for future applications. (p. 175-176)

Passolunghi, Marzocchi, and Fiorillo (2005) operated under that assumption that the ability to inhibit irrelevant information is linked to working memory tasks and solving arithmetic word problems. They found that poor problem solvers recalled fewer target words in working memory tasks, but also made more intrusions. Their study aimed to

determine whether there was a relationship between working memory, inhibitory mechanisms, and the ability to solve mathematical word problems. Thirty fourth graders, 10 with ADHD, 10 with arithmetic learning disabilities, and 10 with arithmetic abilities at or above grade level, participated in the study. Each student received four mathematical word problems during the first session and four additional word problems during the second day. During the second session they also took the digit span task. Researchers asked the students to listen to each of the four problems and remember the problems by focusing on just the relevant information. Information recalled by students was considered to be relevant if it was included in the original version of the word problem. After completing this task, students were given the four problems from each session and asked to solve them. Passolunghi et al. (2005) found that students with ADHD were more impaired when it came to solving problems with irrelevant literal information, and students with arithmetic learning disabilities were more impaired solving problems with irrelevant numerical information. Further, students with ADHD were not able to correctly process relevant literal information and maintain it in working memory to help with problem solving. Students performing at or above grade level were able to recall almost all relevant information and recalled very little irrelevant information, while those with ADHD recalled almost an equal amount of relevant and irrelevant literal information.

Sternberg (2003) identified six metacomponents, or higher order control processes, used for decision making and executive planning: decisions as to what the problem is, selection of lower order components, selection of one or more representations

for information, selection of a strategy for combining lower order components, decision regarding speed–accuracy trade-off, and solution monitoring. The central part of these components in Sternberg’s theory is intellectual functioning. The metacomponents control the activities of the other components, and all of the information from these components needs to be filtered through the metacomponents.

Garofalo and Lester (1985) developed a cognitive–metacognitive framework which addresses key points where metacognitive decisions are likely to influence cognitive actions and is meant to analyze the metacognitive processes involved in mathematical problem solving, including orientation, organization, execution, and verification. Similar to Polya’s (1957) categories, Garofalo and Lester (1985) had more broadly defined the categories. The orientation category includes strategic behaviors used to assess and understand a problem: comprehension strategies, analysis of information and conditions, assessment of familiarity with the task, initial and subsequent representation, and assessment of the level of difficulty and chances of success. The organization category involves planning of behavior and choice of actions, specifically, identifying goals and subgoals, global planning, and local planning to implement the global plan. The execution category involves regulating behavior to conform to plans, including performing local actions, monitoring the progress of local and global plans, and decisions like speed vs. accuracy. Verification includes the evaluation of decisions made and the outcomes of the plans. The verification phase is further broken down into evaluation of orientation and organization, and evaluation of execution.

According to social-cognitive theory, self-regulation, a part of metacognitive processes, is contextually dependent. Zimmerman (2000) believed self-regulation is cyclical since the feedback from prior performance is used to make adjustments during current problem solving. Since personal, behavioral, and environmental factors are always changing, individuals have to monitor these changes and know if any adjustments are required. He described three feedback loops involved in self-monitoring an individual's internal state, behaviors, and environment. Zimmerman stated that self-regulated individuals are more adaptive and can evaluate their task performance appropriately.

Students with strong metacognitive abilities and self-regulation skills are often able to evaluate what they have learned and whether they have to study further to do well on a test. However, higher achieving students may give up easily because ready-made algorithms are not available for solving authentic tasks, and they may have trouble linking what they know about standard tasks to authentic tasks (Kramarski et al., 2002).

Lucangeli and Cornoldi (1997) studied the relationship between a mathematics test and metacognitive tasks with third- and fourth-grade students. Students were given specific metacognitive tasks following each task on their mathematics test. Results showed that metacognitive components were positively associated with high mathematical achievement. Therefore, mathematics educators need to be aware of and attentive to students' metacognition. They need to appreciate metacognitive activity and develop ways to foster it within students. Helping students understand their thinking should be a critical aspect of mathematics curriculum (Tang & Ginsburg, 1999). In an

ideal learning environment for students, teachers adjust their teaching strategies to enhance learning for students based on judgments about success of previous lessons for students. While instructional routines create order, efficiency, and predictability, they may also limit the necessary ongoing change and flexibility for instructional adaptations (Fuchs & Fuchs, 1998), and tension between classroom routines and instructional adaptations can increase as a result. Instructional adaptations include both routine and specialized adaptations. With routine adaptations, the underlying assumption of the general education teacher is that differentiated instruction is needed to address the range of abilities among students in a class. Specialized adaptations go above and beyond routine adaptations and respond to specific student difficulties. Fuchs and Fuchs (1998) found that general education classrooms typically do not differentiate student learning activities, even with the presence of a special education coteacher. By reorganizing classroom formats for 10 teachers as part of their study, they hoped to provide teachers with the information and structure to incorporate adaptations into daily instructional programs. Teachers incorporated 35-minute peer-assisted learning sessions into their weekly math time to help with remediation and review of material, along with curriculum-based measurement that included weekly assessments, student biweekly feedback, and teacher reports. Three of the 10 teachers in the study effected substantial improvement for their students with learning disabilities, therefore suggesting that with rich assessment information, routine structure with adaptations, and consultative support, regular classroom teachers may be able to address the problems of many, but not all, of their students with learning disabilities. While there is not enough information about

when metacognitive strategies should be taught to students, Montague (1992) outlined seven “Say-Ask-Check” metacognitive prompts, similar to Schoenfeld’s six stages of problem solving, that students can use when solving word problems. These metacognitive prompt targets include self-instruction, self-questioning, and self-monitoring for each of the seven cognitive strategies he outlined.

Some students, especially those with learning disabilities, may have problems with mathematical problem solving and those problems requiring higher order thinking skills. These students can struggle to transfer newly learned skills to novel, complex tasks. This may be because they do not understand the new strategy well enough to be able to generalize it to a new situation; they may not have the executive processes needed to implement the strategy; or due to repeated failures in learning situations, the students may not believe in using strategies to lead to successful task completion. They need to be able to recognize and believe in the importance of being strategic and attribute successes to the successful use of strategies (Borkowski, Estrada, Milstead, & Hale, 1989). One way to support the development of metacognition is to have students reflect on and evaluate their activities. Students should be asked “How do you know?” or “What makes you say that?” during classroom discussions. The goal is that these questions will become internalized and will begin to frame much of the students’ independent thinking (Kuhn & Dean, 2004).

Maccini and Hughes (2000) taught students the mnemonic STAR to assist with algebraic problem solving. Students were to search the word problem by reading it carefully, translate the words into an equation as a picture, choose the correct operation to

solve the problem, and represent the problem in the correct format. Next they answered the problem and reviewed their solution by checking their answer. When students were taught the STAR strategy with teacher modeling, guided practice with feedback, and independent practice, Maccini and Hughes (2000) found that all of the students they worked with learned to accurately complete computation and word problems with integers. Their students began with limited generalization abilities to transfer tasks, but 6 weeks after the intervention, still solved the problems with 90% accuracy.

A study of college students with comparable achievement levels (Trainin & Swanson, 2005) showed that students with learning disabilities scored significantly lower than students without learning disabilities in word reading, processing speed, semantic processing, and short-term memory. Students with learning disabilities were found to compensate for their processing deficits by relying on their verbal abilities, learning strategies, and by seeking help. In this study, students used metacognition to compensate for the cognitive processes in which they were deficient. The researchers found that high self-efficacy and high perceived control were positive indicators of metacognition in this study. By redefining learning strategies to fit the demands and content being studied, students achieved more success.

Many lower achieving mathematics students quickly read math problems and therefore do not always comprehend the task and do not realize there may be more than one way to approach solving the problem. Lower achieving students often have trouble reorganizing information and in distinguishing between relevant and irrelevant information. According to Hegarty, Mayer, and Monk (1995),

Whereas good problem solvers construct a model of the given problem on the basis of all the information given in the problem text, poor solvers translate the key words given in the problem text directly into the mathematical operations that the key words usually prime without considering other information given in the text. (p. 29)

There are two main aspects to metacognition according to Garofalo and Frank (1985): knowledge and beliefs about cognitive phenomena, and regulation and control of cognitive actions. Based upon the work of Schoenfeld (1985) and Corno and Mandinach (1983), Fortunato et al. (1991) created 21 metacognitive statements divided into four sections to work with seventh-grade students. The sections focused on interpreting the problem and planning the solution strategy, monitoring the solution processes, evaluating and executing the problem, and specific strategies for solving the problem. Students were asked their opinions before, during, and after solving nonroutine problems. When given routine problems in which students had to apply a familiar algorithm or procedure, some did not have to think to solve the problem and were not challenged. Therefore, they were not able to reflect on how they solved the problem. Student responses to nonroutine problems that have more than one possible approach can be used to develop classroom activities and foster classroom discussions. Through discussion, metacognitive items can foster student awareness of how strategies they used to solve one problem can be used to solve another. Nonroutine problems that promote discussion can also help teachers and students understand how students are thinking about problem solving and encourage teachers and students to explore their awareness of ways to improve the teaching and

learning of problem solving. While approximately half of the students in the study stated they did not use pictures to help them solve problems, many of these students actually drew circles to represent coins or wrote down the value of the coins. The discrepancies between actual and reported behavior indicate that there needs to be clarification of what defines a “problem-solving strategy.” The discrepancies also touch on the importance of checking student responses against other work they have completed (Fortunato et al., 1991).

Results of three studies done by Desoete, Roeyers, and Buysse (2001) indicate three metacognitive components: global metacognition, off-line metacognition, and attribution. They were investigating whether the relationship between metacognition and problem solving could be found in elementary school children, since developmentally, metacognitive knowledge comes before metacognitive skills. Desoete et al. worked with third-grade students divided into two groups: one with average general intelligence and one with specific mathematics learning disabilities. In the nondisability group, students with below-average mathematical problem-solving skills had lower metacognitive scores than their peers with average mathematical problem-solving skills. Children with average mathematical problem-solving skills did worse than their peers with above-average problem-solving skills. In the group with specific math learning disabilities, children with severe math learning disabilities had less developed metacognitive skills than their peers with moderate mathematics learning disabilities. Both of these groups did worse than children with average mathematical problem-solving skills without mathematical learning

disabilities. These results support the idea of using metacognitive assessments to inform instruction differently among differing mathematical ability groups (Desoete et al., 2001).

Kramarski et al. (2002) worked with 91 seventh-grade students within two conditions. One group received cooperative learning with embedded metacognitive instruction, and the other group received cooperative learning with no metacognitive instruction; all students were exposed to the same tasks. The cooperative learning with metacognition students were trained in small groups to formulate and answer self-addressed metacognitive questions including comprehension, connection, strategic, and reflection questions. The cooperative learning with no metacognitive instruction also met in small heterogeneous groups, but did not use metacognitive questions for the 6-week treatment period. Student responses on pre- and posttest tasks were scored on four criteria: referring to all data, organizing information, processing information, and making an offer and justifying it. Findings showed that the cooperative learning with metacognitive instruction group significantly outperformed the no metacognitive instruction group on both authentic and standard tasks.

Carlson and Bloom (2005) studied eight research mathematicians and four Ph.D. candidates using the multidimensional problem-solving framework to investigate, analyze, and explain mathematical behavior. They found that learning to become an effective problem solver requires the development and coordination of many reasoning patterns, types of knowledge, and behaviors; effective management of resources and emotional responses; as well as practice and experience. Each of the participants in their study had a well-connected sense of conceptual knowledge that influenced all phases of

the problem-solving process. Their results further supported the idea that “the ability to access useful knowledge at the right moment during problem solving depends upon the richness and connectedness of the individual’s conceptual knowledge” (p. 70).

The National Council of Teachers of Mathematics (2000) took the position that “writing in mathematics can also help students consolidate their thinking because it requires them to reflect on their work and clarify their thoughts about the ideas” (p. 61). Writing is related to the development of metacognitive behaviors and sustains the development of reasoning, communication, and connections. Zhu and Simon (1987) found that monitoring, self-regulation, and orientation processes occurred more frequently when successful students were involved in problem solving. Pugalee (2001) worked with ninth-grade students to determine whether their written descriptions of problem-solving methods showed evidence of metacognitive behaviors, and what those behaviors were. Students completed six problems, two each categorized as easy, medium, and hard. Pugalee found that a metacognitive framework was present in the writing of these ninth-grade students, including metacognitive behaviors in the orientation, organization, execution, and verification phases of problem solving. These results raise the question of whether writing can function as a vehicle for supporting the metacognitive behaviors necessary for mathematical problem solving.

Kapa (2001) worked with eighth-grade students using a computerized environment which enabled a variety of metacognitive supports during different phases of the problem-solving process. The main findings were that learning environments which provide metacognitive support during mathematical word problem solving in each

of its phases are, “significantly more effective for students with low previous knowledge than learning environments which supply metacognitive support at the conclusion of the process or provide no metacognitive support” (p. 329). Kapa found that metacognitive directing questions asked of students along with the solution process, such as “What are you asked to find?” and “What is given in the problem?”, were more helpful for students who had lower previous knowledge than the correcting/directing feedback such as “Is there another way to solve the problem?” and “Where is the mistake?” given by the computer program at the end of the problem-solving process. Kapa hypothesized this was due to the fact that directive questions given during problem-solving stimulate students’ prior knowledge. By the end of the treatment, students with low prior knowledge had improved their ability to solve word problems correctly, but the high previous knowledge students did not see a change in their problem-solving abilities, possibly due to “the ceiling effect” (p. 331). Kapa also found that students with high previous knowledge spend more time on the problem-solving analysis during the phases of identifying the problem and its representation than students with low previous knowledge. Kapa concluded that learning environments which provide metacognitive support during the process of mathematical problem solving are more effective for students with low previous knowledge than learning environments which provide metacognitive support at the end of the problem-solving process, or provide no support at all.

While some studies have shown that lower achievers benefited from metacognitive instruction more than higher achievers, other studies showed the reverse. One reason for this may be the inconsistency in how different teachers implemented

metacognitive instruction, which may have affected student achievement (Kramarski et al., 2002). Many students are programmed to search for one correct answer while working on mathematical word problems (Kulik & Kulik, 1988), which may cause thinking to be more outcome oriented than process oriented.

Schoenfeld (1992b) presented students with a problem after they took his problem-solving class where they focused on metacognition and he acted as a “coach” while students solved problems in groups. Schoenfeld found that students started to talk about their reasons for problem-solving choices after working on problems for a while. Students often jumped into a solution without thinking much after reading the problem. After working as a group for a few minutes, the students decided to choose another direction to solve the problem. They repeated this action twice. Schoenfeld found that these students had become effective problem solvers because they not only had the knowledge to solve the problems, but they gave themselves the chance to use their knowledge by remaining determined in the problem-solving process and not giving up when the first few strategies were not successful.

Different from this study, since it involves peer interaction, much of the literature that focuses specifically on improving metacognitive strategies uses peer interaction, and has produced conflicting results. Goos and Galbraith (1996) found that student-to-student interaction could either help or hurt metacognitive decision making during problem solving, depending upon the students’ ability to flexibly share metacognitive roles. Stacey (1992) found that among ninth-grade students, a range of strategies were proposed for problem solving, but the correct solution method was often overlooked since students

did not check their work or evaluate the procedures they were using. Artzt and Armour-Thomas (1992) found that small group mathematical problem solving may promote metacognitive behaviors that help students arrive at correct solutions to problems. (p. 166).

Departure from normal teaching becomes an intervention when it: (a) is outside what the teacher intended to do in the course of teaching; (b) requires an outside person to design and evaluate the intervention; (c) involves a formal experimental design that includes provision for evaluating the effects of the intervention; and (d) focuses on independent variables that aim to increase various kinds of performances, usually including academic performance but going beyond content learning itself (Hattie, Biggs, & Purdie, 1996). Cognitive interventions are further defined as those that focus on enhancing a specific task-related skill or strategy. Metacognitive interventions focus on self-management of learning (planning, implementing, and monitoring one's learning efforts) and on conditional knowledge of when, where, why, and how to use certain tactics and strategies in their correct contexts. Affective interventions focus on noncognitive aspects of learning like motivation and self-concept (Hattie et al., 1996). Unistructural interventions are based on one feature or dimension, such as how to do one algorithm. Multistructural interventions involve a range of independent strategies without integrating individual differences of content or context. Relational interventions integrate all components to meet the demands of a particular task and context and are self-regulated and include metacognitive interventions. Finally, extended abstract interventions are used for far transfer (Hattie et al., 1996).

Several studies focused on specific actions teachers can use when working with students with learning disabilities while teaching problem solving. Many of these strategies can be used to develop metacognitive problem-solving skills with all students. Following their study with third graders with mathematics learning disabilities, Fuchs et al. (2008) suggested that teachers need to assess computational and problem-solving skills separately when diagnosing and instructing students who have learning disabilities. When helping students who are having difficulty academically, educators need to prioritize which skills and/or behaviors to target for assessment and intervention. Volpe et al. (2006) suggested considering academic enablers of motivation, engagement, study skills, and interpersonal skills when prioritizing these skills and/or behaviors. Ennis (1989) found that the mixed intervention method, when critical thinking is taught independently within a specific content, has the largest effect on students, while the immersion method, where critical thinking is not explicitly taught, had the smallest effect. It was also concluded that when the teacher had advanced training in teaching critical-thinking skills, interventions had the greatest effect. In contrast to Elbaum and Vaughn (2001), Ennis (1989) found that collaboration among students while developing critical-thinking skills seemed to provide some advantage, but the effect was minor.

Based upon the work of Harwood (1995), which examined group interaction of primary students discussing world issues when the teacher facilitated their conversations and when the teacher was elsewhere, Chiu's study of ninth-grade algebra students (2004) analyzed teacher interventions and how they impacted students' progress when working in small groups. Harwood (1995) had found that when teachers were present, students

were on topic more often, made more correct inferences, and made more justifications for their thinking, than when teachers were not present. Chiu found that teachers initiated most interventions when students were off task or were making little progress in problem solving. In Chiu, after the teacher interventions, students' time on task and problem solving often improved. Interventions which included teacher evaluation of the students' action had the greatest positive effect. Students were more likely to be on task after talking with the teacher than before, and were more likely to see their errors, develop new ideas, and explain their ideas. Further, groups that were more on task usually had better solutions than groups that were not as on task. However, the positive effects of the teacher interventions lasted for about five minutes, but decreased over time; students' on-task behavior declined gradually, and problem-solving effectiveness decreased after the first minute the teacher walked away following the intervention.

In contrast to the findings of Harwood (1995) and Chiu (2004), Meloth and Deering (1999) found that teacher interventions that involved questions were not necessarily helpful, and that high-content teacher interventions often improved student performance. Hattie et al. (1996) found that metacognitive programs taught in context of a particular task were the most successful since the intervention created the effective deployment and monitoring of strategies which resulted in productive cognitive and affective outcomes. They also concluded that interventions for enhanced learning were more likely to have a large effect when the focus was attribution, memory, or structural aids. The lowest effects were associated with motivation and study skills. All of these results had the greatest effect sizes for students with abilities in the middle of their classes

and those who were underachieving. Medium-ability students benefitted most from unistructural and multistructural programs, while underachieving students benefitted from all programs; also, underachieving students and higher-ability students benefitted more from relational programs. While interventions may be effective across age groups, younger students were found to have the most benefits across interventions. Further, self-directed programs were more effective than those directed by teachers.

Bryant et al. (2008) worked with 42 first-grade students who were struggling with mathematics at school. These students received 20-minute intervention sessions (“booster lessons”) 4 days a week for 23 weeks. During the intervention time, students were explicitly taught strategic instructional procedures and content. It was found that, with intervention, these students improved their number sense and performance on arithmetic combinations. While working with fourth graders, Swing, Stoiber, and Peterson (1988) examined the effects of two classroom-based interventions on mathematics achievement. They found that elementary school students did not spontaneously use effective, sophisticated strategies, and therefore focused their interventions on instructing teachers how to teach their students to use metacognitive skills. They found that thinking skills instruction led to better high-level and conceptual achievement for higher ability students than increased learning time. Their results showed that class ability was more important to achievement in the thinking skills classes than in the learning time classes. The lower ability learning time classes performed better than the lower ability thinking skills classes, and the higher ability thinking skills classes scored higher than higher ability learning time groups. The fourth graders in the thinking skills classes reported using more

thinking skills when solving story problems than the learning time students, while the learning time students showed improved engagement in instructional activities. In the thinking skills intervention, teachers learned how to teach their students cognitive strategies including defining and describing, thinking of reasons, comparing, and summarizing. In the learning time intervention, teachers learned how to increase student engagement and learning time. It can be concluded from these studies that improvements in critical thinking skills and dispositions result from being explicitly taught to students by well-trained teachers.

The Mathematics Assessment Project (MAP) out of the University of California at Berkeley (2012) developed a professional development program for teachers to encourage and support educators in using formative assessment with nonroutine problems. The project discusses the role of questioning in mathematics classes and reminds teachers of some of the many possible reasons to ask questions:

- To interest, engage, and challenge;
- To assess prior knowledge and understanding;
- To stimulate recall, in order to create new understanding and meaning;
- To focus thinking on the most important concepts and issues;
- To help students extend their thinking from the factual to the analytical;
- To promote reasoning, problem solving, evaluation, and the formation of hypotheses;
- To promote students' thinking about the way they have learned;
- To help students to see connections. (p. 2)

Further, the MAP lists some of the mistakes more commonly made by teachers which are important to keep in mind while developing a supportive classroom community which encourages problem solving and talking about mathematics:

- Asking too many trivial or irrelevant questions;
- Asking a question and answering it yourself;
- Simplifying the questions when students do not immediately respond;
- Asking questions of only the most able or likeable students;
- Asking several questions at once;
- Asking only closed questions that allow one right/wrong possible answer;
- Asking “guess what is in my head?” questions, where you know the answer you want to hear and you ignore or reject answers that are different;
- Judging every student response with “well done,” “almost there,” or “not quite”;
- Not giving students time to think or discuss before responding; and
- Ignoring incorrect answers and moving on. (p. 2)

### **Summary of Metacognitive Literature**

The review of current and seminal metacognition-related literature revealed generalized information about each of the student groups that was used to explain some of the results of this current study. In summary, giftedness often has a direct correlation to task persistence or perseverance. It is believed that gifted students may be more able to see relationships among ideas and be better able to think through complex ideas than their nongifted peers. These characteristics explain some gifted students’ strength in

mathematical problem solving. Students with ADHD may have low engagements with academic materials and inconsistently complete schoolwork accurately. These characteristics may explain why some students with ADHD often score lower on problem-solving parts of conceptual math assessments. Students who are gifted and have ADHD often face challenges including a lack of persistence, poor problem-solving skills, and an inability to make connections. While these characteristics may lead to difficulty with metacognitive processes while problem solving, other students who are gifted and have ADHD may be able to use their giftedness to compensate for the challenges they face as a result of their ADHD.

### **Students With ADHD**

There has been a quick rise in the reported cases of ADHD in the last decade (Purdie et al., 2002). Males are more likely to be identified as having ADHD than females, and some researchers report that 90% of all children diagnosed with attention disorders are boys (Sagvolden & Archer, 1989). Girls with ADHD show lower levels of hyperactivity than boys with ADHD, and fewer of the girls have conduct disorders and externalizing behaviors. It is important for educators to understand the behavioral manifestations of ADHD because they can have important implications for behavior management and student learning (Dillon & Osborne, 2006). The four executive functions directly affected by response inhibition in attention disorders are working memory, self-regulation of affect/motivation/arousal, internalization of speech, and reconstitution. There are three originating points for attention deficit disorders: the genetic level, the neurological level, and the physiological level (Dillon & Osborne,

2006). Physiological conditions can affect behaviors directly, certain physical attributes may get in the way of learning and/or school performance, or beliefs about the effects of the condition can interact to have greater negative effects on attention or learning.

Trouble regulating behavior can be a result of genetic ADHD and can lead to poor self-regulation.

According to the American Psychiatric Association's criteria (2013), in order for children to be diagnosed with ADHD, they must exhibit at least six inattention or at least six hyperactive-impulsive symptoms before age seven, for at least six months, with consistent academic and/or social impairment. The three subtypes of ADHD are the mostly inattentive type, the mostly hyperactive-impulsive type, and the combined type. In school, students with ADHD are often inattentive and have higher rates of off-task behavior than their non-ADHD peers. These behaviors can lead to disruptive behaviors including talking without permission, leaving the assigned area, bothering other students, and interrupting teacher instruction (DuPaul, 2007). Students with ADHD may have more trouble developing and maintaining positive relationships with peers, teachers, and other school personnel. Their difficulties with inattention and impulsivity can inhibit the development of appropriate social relationships in many ways.

It is estimated that 1 in 20 children in the United States have attention-deficit/hyperactivity disorder (Kercood, Grskovic, Lee, & Emmert, 2007). In Kercood et al.'s study with eight 4<sup>th</sup> and 5<sup>th</sup> graders, visual and auditory mathematical problem solving was used to evaluate the effectiveness of fine motor and physical activity with tactile stimulation. Following studies by Zentall (1993) and Zentall and Ferkis (1993) in

which optimal stimulation theory was explored, Kercood et al. (2007) found that fine motor manipulation of a tactile stimulation object reduced excessive motor movement and increased task completion of students with attention problems. It did not, however, result in more accurate problem solving for students. This leads to the suggestion that teachers may want to provide students with ADHD a tool, such as a small ball they can squeeze in their hands while working, to help them maintain focus during class time.

While in school, students with ADHD may have high rates of disruptive behavior; low rates of engagement with instruction and academic materials; inconsistent completion and accuracy on their schoolwork; poor performance on homework, tests, and long-term assignments; and difficulties getting along with peers and teachers (DuPaul & Weyandt, 2006). Children with ADHD are traditionally academic underachievers and have been shown to score lower on measures of problem-solving ability in conceptual math (Barry, Lyman, & Klinger, 2002). The more severe behavioral problems children with ADHD have, the more negative the impact on their school performance. These students often have trouble with executive functions such as planning, organizing, and maintaining problem-solving sets; cognitive flexibility; and deduction—all of which can lead to difficulties with mathematical problem solving. Planning processes that are needed in solving word problems to organize steps and complete calculations are often lacking in students with ADHD (Lucangeli & Cabrele, 2006). This may be a result of the off-task behaviors children with ADHD present when solving mathematical word problems. Students with ADHD also may have difficulty with mathematical computation. Depending upon the level of adverse effect on educational performance, students with

ADHD may qualify for support services. Students who meet the eligibility criteria for special education under the Individuals With Disabilities Education Act (IDEA, 2004) are provided an Individualized Education Plan (IEP), which is a blueprint for educational services and supports. Those who meet the less rigorous criteria afforded by Section 504 of the Rehabilitation Act of 1973 have their accommodations detailed in a 504 plan. Accommodations for students eligible under Section 504 are determined by what is needed for the student to have an equal opportunity to succeed when compared to their peers who do not have a disability. Some students with ADHD do not require formalized support plans.

There are several treatments that students with ADHD can undergo to change their behaviors. Since central nervous system stimulants do not always lead to positive effects and can have negative side effects, behavioral based and academic interventions can be used, sometimes in conjunction with medications. Behavioral interventions include token reinforcement and response cost systems, both of which have been found to increase on-task behavior and work productivity in classroom settings (DuPaul, 2007). Token economies provide immediate reinforcement or tokens that are contingent upon appropriate behavior. Response cost is the removal of tokens earned based on inappropriate behaviors. When using behavioral interventions, functional assessment data should be used. Students who have ADHD and struggle with the development of core reading and math skills often need academic interventions that directly address their academic deficits. Academic interventions may also be proactive or preventive in terms

of behavior management and may lead to changes in problematic behaviors (DuPaul, 2007).

Performance difficulties in children with ADHD in comparison to their peers without ADHD have been documented in much research (Benedetto-Nasho & Tannock, 1999), likely due to an inability to show automaticity with computational skills at age-appropriate paces, and can lead to an inability to learn more advanced mathematics. DuPaul et al. (2006) evaluated two different consultation-based models for assisting children with ADHD in mathematics and found that there was significant improvement in academic skills resulting from academic interventions through school-based consultations. Students with ADHD tend to work less efficiently than their non-ADHD peers, but they do not benefit significantly when given extended time on speed-based math tasks (Lewandowski, Lovett, Parolin, Gordon, & Coddington, 2007). Barkley (2008) suggested nine management principles for using consequences for students with attention disorders to gain maximum effect:

- Rules must be clear, brief and delivered through more visible and external modes than is necessary for other children.
- Positive and negative consequences must be delivered more immediately than is necessary for other children.
- Consequences and feedback need to be delivered more frequently to build and maintain adherence to rules than is typical for other children.
- Consequences often need to be of greater magnitude to be effective with these children.

- A richer degree of incentives needs to be provided within the setting to reinforce appropriate behavior before punishment can be implemented.
- Reinforcement must be changed frequently to prevent rapid satiation.
- Anticipation is important; children need to know about transitions and rule changes before they happen.
- Children with ADHD need to be held more publicly accountable for their behavior than other children.
- Punishment, like time out from positive reinforcement, will likely be more effective when used with strong positive reinforcement. (p. 7-9)

Other researchers including DuPaul and Stoner (2003) and Scime and Norvilitis (2006) have recommended that teachers approach behavior management using proactive strategies when working with students with attention disorders, as much academic failure and disruptive behavior can be prevented by using effective instructional strategies.

DuPaul and Stoner (2003) suggested the following:

- Rules should be clear, brief, explicitly taught, and frequently reviewed.
- Use eye contact to secure student attention to instruction.
- Remind students of the behavior required before beginning an activity or transitioning to another activity.
- Develop and use nonverbal cues worked out with individual students to redirect them while teaching.
- Circulate around the classroom, monitoring and giving frequent feedback during instruction and independent work periods.

- Maintain a brisk pace of instruction that engages students in frequent responding to maintain student attention.
- Ensure that instruction and assignments are at the appropriate instructional level for students.
- Communicate expectations about the activities for the class or school day with posted schedule and oral review of behavioral expectations. (p. 172-173)

In Scime and Norvilitis (2006), when given three mathematics worksheets to complete, students who had ADHD did not have less accurate results than students who did not have ADHD, although the students with ADHD did complete fewer total problems. Therefore, limiting the amount of problems required for students with ADHD to complete when they can demonstrate mastery of a concept with the completion of fewer problems may be an effective intervention for teachers to use in mathematics classes. During the same study, students were asked to complete a puzzle and children with ADHD were more likely to quit before they completed the puzzle, but persisted for a similar amount of time as children without ADHD. These results suggest that children with ADHD are working harder than their end results may show. Students who have ADHD exhibit behaviors that negatively influence their ability to meet academic, behavioral, and social demands of the school setting. Stahr et al. (2006) conducted a case study with a nine-year-old boy who had off-task behavior that was maintained by attention and escape strategies. The researchers implemented an intervention package with a communication system, a self-monitoring component, and extinction, and also addressed his anxiety and speech and language problems. The intervention included a

self-monitoring checklist where the subject evaluated his own behavior on a 15-minute fixed-interval schedule. The teacher and other students worked to ignore the student and implemented a card system to allow the subject to get the attention of the teacher when he needed it. This intervention increased the subject's on-task behaviors and decreased the subject's anxiety on difficult tasks because he was able to effectively communicate his need for assistance. The self-monitoring helped the subject become aware of his behavior and helped him increase his task engagement.

In 1995, public schools in the United States spent over \$3.2 billion for students with ADHD (Forness, Kavale, Sweeney, & Crenshaw, 1999). Meeting the demands of school is a challenge for students who are not able to self-regulate. These students have difficulty with attention components, an inhibitory component, and strategic and organizational components of self-regulation, and will not likely be successful when learning tasks require attention, inhibition, and active involvement. They will usually fail at tasks that require organizational skills because they have difficulty effectively using higher order processes, and often suffer social rejection from their peers (Miranda et al., 2006).

Teachers may wish to seat a child with ADHD in an area with few distractions, provide an area where the child can move around and release excess energy, or establish a clearly posted system of rules and reward appropriate behavior. Sometimes just keeping a card or a picture on the desk can serve as a visual reminder to use the "right" school behavior, like raising a hand instead of calling out, or staying in a seat instead of wandering around the room. Giving such children extra time on tests can make the

difference between passing and failing, and gives them a more fair chance to show what they have learned. Reviewing instructions or writing assignments on the board, and even listing the books and materials students will need for the task, may make it possible for disorganized, inattentive children to complete their work. In addition to medication and interventions, there are home-school communication programs that involve teachers, parents, and sometimes students collaborating to reinforce appropriate behavior at school with contingencies delivered at home. One such strategy includes daily report cards with targeted behavioral goals. Students with ADHD may also benefit from school-based management interventions such as token reinforcement, academic tutoring, and daily report cards or school-home notes (DuPaul et al., 2006). School-based treatments for attention disorders are effective in the short term for reducing disruptive behaviors and improving on-task behavior and academic performance. Results from a Multimodal Treatment Study of Children with ADHD suggest that the most effective treatment for ADHD is an intervention that includes medication concurrent with parent training, school interventions, and child intervention (Miranda et al., 2006).

There are many strategies for helping students with ADHD in regard to teaching and learning mathematics. Trial and error is often an important part of discovering what works for each child. Some concrete strategies include:

1. Color the processing signs on math tests for students who do not focus well on details and make careless errors due to inattention.
2. Allow calculators and multiplication charts to be used by students.
3. Provide manipulatives and number lines to help students visualize problems.

4. Have students use calculators to check their work.
5. Allow extra time.
6. Have students use graph paper rather than notebook paper to solve computation problems and to help with number alignment.
7. Provide models of sample problems.
8. Teach steps needed for solving problems and list these steps clearly for students to follow.
9. Teach students strategies how to solve word problems.
10. Have students keep index cards with specific math skills, concepts, rules, and algorithms taught. (Rief, 2005, p. 297-290)

Students with ADHD have more academic success when tasks are structured to meet their individual academic levels and when the results of their performance are monitored with frequent and consistent consequences (Mautone, DuPaul, & Jitendra, 2005), specifically resulting in better classroom behavior, attention, and academic performance. When working with peer tutors, students in grades one through five with ADHD showed an increase in on-task behavior (DuPaul et al., 1998). Half of the students showed improvements in academic performance in math or spelling and increased their level of active engagement. It is believed that peer tutoring was so successful because it increased task-related attention and required students to make active responses to the material.

Behavioral symptoms of ADHD are associated with current and later academic underachievement in students. The greater the severity of the symptoms, the greater the

degree of underachievement a student is likely to face (DuPaul et al., 2004). Two effective intervention approaches for managing ADHD in students are pharmacological and behavioral strategies. Interventions based on functions rather than forms of behavior have been more beneficial for students. Emphasizing skill building and prosocial behaviors and deemphasizing punitive intervention strategies that reduce behavior problems increase the likelihood that new behaviors will produce meaningful, lasting change (Stahr et al., 2006).

Students with ADHD may not attend to important stimuli, and as a result, not build the conceptual knowledge they need for certain problem-solving tasks (Kercood, Zentall, & Lee, 2004). Arithmetic performance on the Wide Range Achievement Test decreases over time for students with ADHD (Zentall, 1993). These results may explain why problem solving and computation can be difficult for these students. Students who are unable to comprehend word problems and identify relevant from nonrelevant information struggled more with word problems on this achievement test. When reading differences were eliminated in Zentall's (2003) study of children with ADHD, these students were not found to have greater difficulty than their peers who did not have ADHD. When problems with mathematical problem solving could not be explained by low reading ability, IQ, or lack of exposure, students with ADHD had trouble with math problems with mixed operations or mixed problem-actions (Zentall, 1993).

According to Hooks, Millich, and Lorch (1994), students with ADHD appear to have "more difficulty in maintaining attention over time than their normal counterparts" (p. 69). Students with ADHD have significantly lower rates of academic engagement and

higher rates of off-task behaviors than their peers who do not have ADHD, and efforts are needed to bring perceptions of these students more in line with their abilities (Hooks et al., 1994). Eisenberg and Schneider (2007) noted teachers tend to have more negative perceptions of girls with ADHD than other girls, but the differences for boys are less pronounced than for girls. Boys with ADHD tend to have more negative self-perceptions about mathematical ability than do girls (Eisenberg & Schneider, 2007). Gender differences in students with ADHD and school-based interventions that have been used with students who have ADHD are also noted in the literature. Children with ADHD often fail to complete tasks satisfactorily or show inconsistent academic performance (Scime & Norvilitis, 2006). The minds of these students are often going in many directions and they need to increase self-regulation skills to direct their energy in the appropriate ways.

In DuPaul et al. (2004), teacher perceptions of student academic skills were greater predictors of student success on mathematical achievement tests than a diagnosis of ADHD and reduction of its symptoms. Many of the strategies used in special education are simply good teaching methods. Telling students in advance what they will learn, providing visual aids, and giving written as well as oral instructions are all ways to help students focus and remember the key parts of a lesson. Students with ADHD often need to learn techniques for monitoring and controlling their own attention and behaviors.

Students who have ADHD can benefit from mastery learning that builds on prerequisite skills and understandings instead of spiral learning, like several of the

National Science Foundation-funded curricula including *Everyday Mathematics*, learning that involves active construction of meanings, verbal teacher interactions with students to assess problem-solution strategies, more emphasis on assessment and teaching of mathematical concepts, the use of strategies for reading comprehension and memory in problem solving, attentional cues to help students prepare for changes in problem actions and order of operation, and novel instructional strategies to facilitate the learning of basic calculations (Zentall, 1993). If students are given advanced notice of features of math problems, they will more easily be able to identify those features (Kercood et al., 2004). Additionally, teachers need to be aware that students with ADHD may have more difficulty with strategies involving grouping or sorting, as they have more difficulty forming categories than their peers, and may need more sorting time and direct instruction (Kercood et al., 2004). Further, when teachers have knowledge of what and how children are thinking, it is possible for them to challenge and extend students' thinking and modify and extend activities for students (Maher & Davis, 1990).

Students with attention disorders often have low levels of attention, are inattentive, and are easily distracted (Shimabukuro, Prater, Jenkins, & Edelen-Smith, 1999). Hypothesizing that self-monitoring of academic performance is effective in increasing academic productivity, accuracy, or use of strategies for students with learning disabilities, attentional difficulties, and behavioral disabilities, Shimabukuro et al. (1999) worked with three male middle school students of average intelligence who each had a learning disability and were medically diagnosed as having ADD/ADHD. The researchers looked at academic accuracy, academic productivity, and on-task behavior of

each of the students, the first two monitored by the students and the last monitored by their teacher. It was found that self-monitoring of academic performance resulted in greater positive effects for productivity than for accuracy for both reading comprehension and math for all three students. There were also overall improvements in the accuracy scores in both areas with self-monitoring. Overall, gains in productivity were greater than student gains in accuracy, and productivity gains were greater for reading comprehension and mathematics than for written expression.

Students with attention disorders often have trouble with executive functions such as planning, organizing, and maintaining problem-solving sets, cognitive flexibility, and deduction (Lucangeli & Cabrele, 2006), all of which can lead to difficulties with mathematical problem solving. Experiences in school affect students' perceptions about their academic abilities, social acceptance, popularity, behavior, and self-efficacy, which can affect their school performance. Students who have severe academic difficulties are at risk for having poor self-concept (Elbaum & Vaughn, 2001). Research on self-concept of students with learning disabilities has found that students with learning disabilities have lower academic self-concepts than students without learning disabilities. To address this issue, many studies have been done to see if providing interventions for these students will help their self-concept. Hattie (1992) found from a meta-analysis that about 10% of students who received some intervention showed higher self-concept than those students with learning disabilities who did not receive an intervention. These interventions included peer tutoring, cooperative learning, metacognitive strategy instruction, computer-assisted instruction, and target instruction in specific subject areas.

Hattie (1992) further concluded that there was no difference in the intervention effects whether students were in self-contained or regular education classrooms, received pull-out instruction, or received a combination of services.

As a result of the Multi-Modal Treatment of ADHD study, DuPaul and Weyandt (2006) concluded that a school-based intervention plan involving parents, teachers, and students is critical. They suggested the intervention plan should be balanced in terms of proactive and reactive strategies and should include changes agents to implement interventions. Further, these intervention plans should be based upon assessment data rather than trial and error. School-based interventions need to be evaluated and treatment strategies modified using data-based decision making.

#### **Students Identified as Gifted and Having ADHD: Twice-Exceptional Students**

Children with ADHD can have unusually high creative abilities (Healey & Rucklidge, 2005) and in many cases these creative gifts may mask the ADHD and the ADHD may mask the gifts. Healey and Rucklidge (2005) studied 67 10- to 12-year-olds, some with ADHD and some who did not have ADHD, to see if creative abilities were evenly distributed between the two groups, and if students with ADHD showed more creativity. They found that students who had ADHD had evenly distributed creative abilities, and there were no group differences found between students with ADHD and students without ADHD on their IQ, creativity, generation of ideas, or abstract thinking. This study leads to the suggestion that being highly creative is not a common feature of ADHD and that students who do not have ADHD are as creative as their peers who do have ADHD.

Seeing the difference between behaviors that are sometimes associated with giftedness but also characteristic of ADHD is not easy (Webb & Latimer, 1993). While gifted children typically do not exhibit problems in all situations, children with ADHD typically exhibit the problem behaviors in almost all settings, although the extent of the problem behaviors may fluctuate from setting to setting (Barkley, DuPaul, & McMurray, 1990). A gifted child may have a low perceived ability to stay on task due to boredom, curriculum, mismatched learning style, or other factors. These students will often respond to nonchallenging or slow-moving classes by displaying off-task behaviors. Some of the most common challenges faced by students who are gifted and have ADHD include a lack of persistence, poor problem-solving skills, and an inability to make connections between cause and effect. These students also likely lean toward being perfectionists (Stewart, 2010).

Teachers of twice-exceptional students should place an emphasis on developing academic strengths as well as remediating academic weaknesses (Stewart, 2007). Some barriers these students may face in mathematics include numeric transpositions in mathematical computations, which can be helped using tools like talking calculators and electronic spreadsheet programs. Other barriers to learning for twice-exceptional students include poor organizational skills, poor short- or long-term memory, and sequencing problems (Stewart, 2007). If teachers or parents notice a large difference between a child's school performance in different subjects, it may be a clue that the child has a learning disability. This does not diminish his or her giftedness, but means the student will have to learn ways to compensate for the disability (Strip & Hirsch, 2000).

Children with ADHD who are also gifted may be able to channel their activity constructively and focus their attention in their area(s) of talent. Conversely, giftedness may allow these same children to compensate sufficiently and look average in a traditional classroom setting. While children with ADHD rely on kinesthetic and sensory stimulation, students who are gifted and have ADHD may also need intellectual or creative ways to create stimulation, depending upon their area of talent (Gardner, 1983). Zentall, Hall, and Grskovic (2001) conducted a study with nine boys, three of whom had ADHD, three identified as gifted, and three identified as having ADHD and as being gifted. They found that attention problems were reported twice as frequently for the ADHD and gifted/ADHD groups as for the gifted student group. Students with ADHD failed to get on the right track more frequently, and when on the right track, they consistently got off track. Getting off track included losing interest; looking off task; doing minimal work; and failing to edit, complete assignments, or other routines. It was concluded from teacher and parent observations about organization that students with ADHD may have difficulty organizing a larger amount of physical objects or information in their brains. When asked to solve problems with the researchers, students in the ADHD/gifted group showed their creativity through humor, creating games, putting things or ideas together in novel ways, and other forms of imaginative expression. These same students described a bad day at school as one in which, as one student said, he had to “sit in his seat all day and was asked to do worksheet after worksheet, math problem after math problem” (p. 506). Both groups of students who had ADHD preferred group learning activities that had social stimulation, while participants from the gifted group

preferred to work alone unless it was more efficient to share a project by dividing up various jobs.

There is a demonstrated relationship between performance on tasks of working memory and intelligence, yet the Wechsler subtests of Digit Span, Coding, and Arithmetic are lower in students with ADHD due to the need to tap their working memory (Antshel, 2008). By having less developed working memory skills, IQ scores for students with ADHD may be artificially depressed (Antshel, 2008). Another cognitive construct, executive functions, is associated with intelligence, yet ADHD has been referred to as a disorder of executive functioning. These facts, along with others, including attention span, make it difficult for practitioners to identify students who have ADHD as also being gifted. Antshel (2008) found that when compared to nongifted ADHD peers, students who had ADHD and had a high IQ needed more academic support, more frequently repeated grades, and were rated by their parents as having more functional impairments. Teachers and parents should be aware that for students who have ADHD and a high IQ, academic struggles may be as great as students who are ADHD and have a normal IQ. These students may be less able to manage gifted curricula and can have work production difficulties. Further, as students who are gifted and have ADHD discover the gap between the standard(s) they would like to reach and the actual work they produce, they tend to get frustrated, negative, and alienated from the school system. Parents of these students may notice their children are having trouble socially and have lower self-esteem (Stewart, 2010). Keeping these possible challenges in mind, problem-solving instruction for students who are gifted and have ADHD may need to be

very structured. Providing students with a written step-by-step approach to problems may help them be more successful in their efforts. Additionally, story problems that connect the mathematics to the real world may hold students' interest for longer periods of time, allowing them to be more persistent in their problem solving.

Education programs for students who are twice-exceptional need to include collaboration between the classroom teacher, gifted educator, special educator, parents, and the student. The programs must include strategies that nurture student strengths and interests, foster social and emotional development, enhance student capacity to cope with mixed abilities, identify learning gaps and provide explicit remediation, and support the development of compensatory strategies (Reis & McCoach, 2000; Smutny, 2001).

### **Gifted Students**

In 1988, the Jacob K. Javits Gifted and Talented Students Education Act established a federal program for educating gifted and talented students. This act calls for the identification of gifted and talented students, but does not require that schools provide services for these students. The program was reauthorized by the Improving American's Schools Act of 1994, and in 2002 became part of No Child Left Behind. The Javits Grant is designed to provide funds to provide the services that meet the needs of gifted and talented students, including methods for teaching gifted students, professional development, and model projects and exemplary programs.

Gifted students are those who show exceptional aptitude or competence in one or more areas. This includes those who are exceptionally capable and are able to learn significantly faster than other children their age (NAGC, n.d.). Sometimes interventions

are required to meet the needs of these gifted learners, while other times these students may be low achievers as a result of limited opportunities to learn, physical or learning disabilities, or motivational or emotional problems. Students who are gifted in mathematics have an ability to effectively use facts and perform mathematical tasks, but need opportunities to further develop this talent (Dimitriadis, 2012). Gardner's (1983) Theory of Multiple Intelligences, which connects specific types of giftedness with different centers in the brain, includes *logical mathematical intelligence*. Teachers of students with this type of intelligence must focus on developing positive beliefs, attitudes, and motivation in mathematics. They should focus on higher cognitive levels of mathematics including using higher order thinking skills like reasoning, hypothesizing, communicating, decision making, refining ideas, problem solving, and metacognition (Dimitriadis, 2012). Rather than focusing on programs, this type of instruction is focused on teacher instruction and questioning to develop student higher order thinking skills. Many researchers think task persistence or perseverance has an important correlation to giftedness (Konstantopoulos, Modi, & Hedges, 2001). Renzulli (1977) advocated for the inclusion of *task persistence* as an important component of the definition of talent, while Terman and Oden (1959) included *motivation* as part of their definition.

There are many different ideas and contradictory opinions about how to best meet the needs of gifted learners. In 2010, the National Association for Gifted Children (NAGC) set forth six gifted programming standards. These standards set the stage for differentiating programs and services for gifted students. Each of these standards results in a variety of desired student outcomes as summarized in Table 2.



Table 2

*National Association for Gifted Children Gifted Education Program Standards and Student Outcomes*

| Standard                               | Student Outcomes                                                                                                                                |
|----------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------|
| 1: Learning and Development            | Self-Understanding<br>Awareness of Needs<br>Cognitive and Affective Growth                                                                      |
| 2: Assessment                          | Identification<br>Learning Progress and Outcomes<br>Evaluation of Programming                                                                   |
| 3: Curriculum Planning and Instruction | Curriculum Planning<br>Talent Development<br>Instructional Strategies<br>Culturally Relevant Curriculum<br>Resources                            |
| 4: Learning Environments               | Personal Competence<br>Social Competence<br>Leadership<br>Cultural Competence<br>Communication Competence                                       |
| 5: Programming                         | Variety of Programming<br>Coordinated Services<br>Collaboration<br>Resources<br>Comprehensiveness<br>Policies and Procedures<br>Career Pathways |
| 6: Professional Development            | Talent Development<br>Socio-emotional Development<br>Lifelong Learners<br>Ethics                                                                |

*Note.* From National Association for Gifted Children.(2010). *Pre k-grade 12 gifted programming standards*. Retrieved from [http://nagc.org/uploadedFiles/Information\\_and\\_Resources/Gifted\\_Program\\_Standards/K-12%20programming%20standards.pdf](http://nagc.org/uploadedFiles/Information_and_Resources/Gifted_Program_Standards/K-12%20programming%20standards.pdf)

Goldring (1990) explored many of the questions that have been raised in regard to educating gifted students. The first of these questions is whether teaching gifted children in homogeneous classes would lead to higher achievement. One side of the argument says that heterogeneous classrooms may cause gifted students to be frustrated if not appropriately challenged. Supporting the idea of homogeneous grouping for gifted students is the argument is that gifted students are more stable emotionally, and should therefore be able to adjust well to greater pressures of a special class and may not feel out of place, gaining social acceptance. In contrast to this argument is that homogeneous programs for gifted students can result in a lack of relationships with peers who are not identified as gifted. Goldring (1990) found no difference in general self-concept between gifted students in special classes and those in regular classes; further, the study discovered a negative effect on attitudes toward nongifted students by those in separate gifted classes. When looking at program materials, Goldring found that when in special classes, gifted students who used enrichment materials had higher levels of achievement than those who an accelerated program. Overall, Goldring found that the older students get, the academic advantages for special classes increases, but these students had more negative attitudes toward their peers in regular classrooms and often struggled with peer relationships.

Snyder, Nietfeld, and Lennenbrink-Garcia (2011) examined the effects of metacognition among gifted high school students in the classroom. They found that the gifted students in their study were better able to identify their strengths than their nongifted peers and were better able to judge how well they had performed on an

assessment. The study did not show any significant differences in the two groups of students in terms of how they could regulate their cognition. The gifted students began with greater metacognitive abilities but both groups maintained and improved their abilities during the study. Snyder et al. (2011) concluded that the gifted students in their study were better able to reflect on prior knowledge and monitor their performance by question because they were more careful in their solution planning. The researchers believed these students performed better on assessments due to being more relaxed and less impulsive than their nongifted peers, possibly due to past positive performance. Snyder et al. were surprised to find that there was not a difference between student knowledge of metacognition between the two groups. This could be explained by the fact that many students do not know what they do not know in terms of effective and efficient strategy use.

A study by Span and Overtoom-Corsmit (1986) challenged 28 students, 14 of whom were *highly gifted* and 14 of whom were *averagely gifted*, with seven divergent mathematical tasks. The researchers assessed student work on orientation, execution, evaluation, and accuracy. They found that the highly gifted students solved the problems more quickly and required less support than the averagely gifted students. This same group also solved five out of the seven problems more successfully than the averagely gifted students. The highly gifted students in this study took more time to orient themselves to the task and completed it more thoroughly, while the averagely gifted students started to solve the problems before fully understanding what they needed to do. These students used trial and error most often while problem solving, while the highly

gifted students tried to use other problem-solving strategies. The researchers also found that more than half of the highly gifted students were able to discuss their use of strategies in detail, while less than 8% of the averagely gifted students were able to do so. As a result of this study, Span and Overtoom-Corsmit (1986) recommended four ways instruction can be adjusted to support gifted students:

- Provide difficult problems to be solved on a regular basis.
- Teach students to persist and analyze problems that are difficult.
- Students have to be reminded to use known strategies and knowledge to solve difficult problems.
- Teachers must be experts in the field of mathematics. (p. 289)

Konstantopoulos et al. (2001) found that students who were self-reliant and spent more time on homework and reading for pleasure were more likely to be academically gifted than other students. Further, they claimed that parents with high educational aspirations and high socioeconomic status were important predictors of academic giftedness. Identification and appropriate education of talented students is critical if the United States is to maintain high national standards and be competitive in the international market. Further, entry into high-level professions like medicine, engineering, and mathematics, are dependent upon high levels of ability.

People have two categories of needs, including endogenous needs and exogenous needs (Webb, 1982). Endogenous needs start with the characteristics of an individual, while exogenous needs exist from the students' interaction with the environment. Exogenous needs do not always exist across different environments. When considering

Maslow's (1962) hierarchy of needs—physiological, safety, need for belonging and love, esteem, and self-actualization—gifted students would have to have the first two needs satisfied before being able to focus on the final three needs. This can present a challenge to those working with gifted students since Maslow's hierarchy is designed to focus on a lifetime of growth and development, and exceeds time students spend in school. However, creating an environment that encourages maximizing students' potential must take these needs into account. While it is easy to first focus on academic needs of gifted students, it is important to consider their psychological and social needs and to remember that they are children first (Cross, 1997).

Many gifted students learn how to be students within their school environment by learning what happens when they or other students act in certain ways. Coleman (1985) created a stigma of giftedness paradigm (SGP) with three basic ideas:

- a. Gifted students want to have normal social interactions.
- b. They learn that when others discover their giftedness, they will be treated differently.
- c. They learn they can manage information about themselves that will allow them to be able to maintain a greater amount of social latitude. (p. 37)

Rimm (1997) found that the pressure gifted underachievers feel often comes from internal stress they put on themselves because they are gifted and because adults may have admired them for their academic accomplishments. When these types of things occur in school, social coping becomes a remedy for lack of acceptance. Underachieving can be interpreted as a coping strategy.

For parents and teachers working with gifted students, it is critical to recognize the whole child (Cross, 1997), as these students have the same range of personal characteristics as their nongifted peers. However, these students tend to be more self-sufficient, dominant, and concerned for human rights and equity (Cross, 1997). Gifted students often see relationships among ideas and are better able to think through complex ideas than their nongifted peers (Heid, 1983). Those students who are gifted in mathematics may not demonstrate high mathematical achievement due to a lack of in-depth interest in the subject. Those who do demonstrate high achievement in problem solving are often able to make quick and comprehensive generalizations. They remember similar problems they have solved and think about how they solved those problems, as well as focus on the structure of the problem they are working on, allowing them to switch from one problem-solving method to another relatively easily (Heid, 1983).

Everyone working with students identified as gifted should have a clear understanding of the goals for the children and maintain open communication. Many gifted students will require the opportunity to develop talents outside of a traditional school setting, including mentors; counseling for these students could also be beneficial in meeting their needs. Gifted students should be taught prosocial skills and adaptive coping strategies that do not risk their academic performance, and should be able to spend time with other gifted students. This time allows them to realize they are not alone and that it is acceptable to have serious academic pursuits (Cross, 1997).

VanTassel-Baska (1991) suggested several characteristics teachers need to have in order to be effective with gifted students: (a) eager backing of acceleration options for

able learners, (b) the capability to modify a curriculum, (c) adequate training and competence in the content area, and (d) preparation in organizing and managing classroom activities. Clark (1997) expanded upon these ideas and stated that teachers of gifted students should have an

uncommon ability to empathize with and inspire students; share enthusiasm, a love of learning, and a joy of living; be authentic and humane; be alert, knowledgeable, and informed: tolerate ambiguity; and value intelligence, intuition, diversity, uniqueness, change, growth, and self-actualization. (p. 226)

In order to meet the needs of students who are gifted in mathematics, curricula needs to be broad, in depth, and flexible. Math teachers in particular need to support and nurture a variety of behaviors in an environment that encourages independent thought and study about mathematical ideas (Heid, 1983).

Gavin, Casa, Firmender, and Carroll (2013) at the University of Connecticut developed an elementary mathematics program for gifted students, Project M2 and Project M3, that encourages students to think, discuss, and write as mathematicians. The goal of both of these projects is to get students to think in depth about challenging math concepts, and make their thinking accessible to the whole class through verbal and written communication. The characteristics of their program, based on standards of gifted, early childhood, and mathematics education, include important and advanced mathematics, depth of understanding and complexity, differentiated instruction, mathematical communication, and a nurturing classroom environment. Gavin et al. (2013) have found positive impacts on student achievement for those using their

curriculum, especially on the open-ended assessments, which showed that these students were able to successfully solve the problem-solving tasks and explain and defend their reasoning.

### **3. METHODOLOGY**

The purpose of this study was to extend the current understanding of metacognitive behaviors involved in complex mathematical problem solving in fifth-grade students who are identified as gifted, who have ADHD, who are gifted and have ADHD, and students who have no label. Specifically, following Schoenfeld's model (1985), this research investigated the metacognitive processes related to solving grade-level mathematics problems, and focused on the amount of time students spent engaged in the six stages or episodes of problem solving (read, analyze, explore, plan, implement, verify). This information is used to explain problem-solving processes needed to answer the following research questions:

1. What types of metacognitive behaviors emerge in fifth-grade students with ADHD, ADHD/gifted, gifted students, and nonlabeled students when they approach rational number problem-solving mathematical tasks?
2. How does the problem-solving debrief session effect the amount of time spent in each of the six metacognitive problem-solving stages and student performance patterns over time?

#### **Research Design**

This multiple case study employed qualitative research methods as described by Yin (1984), who suggested that case studies may be more powerful for the purposes of

explanation than explorations—a hopeful end result of this study on a limited scale. A case study is a research tool that is very conducive to a constructivist approach, allowing the questions of “how” and “why” to be answered in depth and detail. For that reason, I chose a case study design to examine the question of how students approach rational number problem-solving tasks in mathematics. Stimulated-recall interviews were used to help students identify metacognitive actions they engaged in while problem solving. Calderhead (1981) discussed using stimulated recall as the use of audio or videotapes of behaviors to help an individual recall his or her thought processes at the time of a behavior. Stimulated recall has been used to describe students’ development of meaning in mathematics classes (Frid & Malone, 1995) and to identify students’ thoughts during mathematical problem solving (Artzt & Armour-Thomas, 1992). In this current study, case study methodology was used to explore ideas about student metacognitive processes and to describe the possible effects of exposure to a debriefing session which caused students to think about and reflect on their own mathematical problem-solving processes.

In order to deepen the understanding of metacognitive processes among the four groups of students, as well as the effectiveness of the sessions, this study included eight cases. Presenting an additional challenge was the fact that there is not one single method that could adequately portray the complexity of students’ thought processes while solving complex problems. Metacognition can only be seen by observers through behaviors exhibited by students or by having students verbally describe the metacognitive processes they used while solving various problems. Self-awareness can also be shown in the

participants' tone of voice and facial expressions (Tang & Ginsburg, 1999). Therefore, a variety of data collection and analysis methods had to be used in this study.

In order to develop an understanding of the occurrences of metacognitive behaviors in problem-solving settings, and be able to draw comparisons and contradictions in behavior, researchers need to be aware of general processes and characteristics of situations that provide the genesis of these behaviors. Both qualitative and case study methodologies worked well for the goals of this study. Krathwohl (1998); Franklin, Allison, and Gorman (1997); and Yin (2003) showed that qualitative methods are effective for developing an understanding of individual's behaviors. Lester (1980) addressed a need for a changing the focus from concentrating only on quantitative information about problem solving to focusing on how problem solvers analyze and control their actions. Further, Lester observed that "the interest in qualitative aspects of problem-solving behavior has been accompanied by recognition of the importance of investigating the interactions among the characteristics of problem solvers, and the processes they use" (p. 314). Merriam (1998) and Gillham (2000) described case study methodology as having the ability to provide researchers with an understanding and description of processes that lead to observable behaviors. Further, Kroll (1988) stated that clinical case study designs are most appropriated for individual task-based interviews. These types of structured, task-based interviews were described by Goldin (2000) as those in which a subject and an interviewer interact with tasks that are introduced in a preplanned way to the subject. In this current study's interviews, subjects interacted with both the problems they were solving and the interviewer. According to

Pugalee (2001), students may use metacognitive behaviors at any stage of problem solving. Therefore, there is not one method that can capture the entire metacognitive process any student uses while engaged in mathematical problem solving. Therefore, this study included semistructured task-based interviews, observations, and students' reflections on their metacognitive processes while solving complex mathematical problems.

The goal of this study was to provide information about the amount of time students spent engaged in various metacognitive-type behaviors when faced with complex problem-solving challenges, and then compare and contrast these results across the four categories of students and to uncover any patterns in the problem-solving context. The goal of this study was not to generalize about the four groups of students. Once discoveries were made as to engagement in metacognitive stages and any patterns over time were noted, I created a "teacher-friendly" tool to be used by classroom teachers to help inform teaching and learning mathematical problem solving.

### **Participants**

Table 3 provides an overview of each participant including their pseudonym, label, age, CogAT and/or NNAT-2 standardized test core, and their average math grades from grades 3-5, when available.

Table 3

*Participant Information*

| Student Name | Label           | Age | CogAT (Quantitative)/<br>NNAT-2 Scores | Average<br>Math<br>Grades                                                |
|--------------|-----------------|-----|----------------------------------------|--------------------------------------------------------------------------|
| Aiden        | ADHD            | 11  | CogAT: 101                             | 3 <sup>rd</sup> Grade:<br>B<br>4 <sup>th</sup> Grade:<br>B<br>Current: B |
| Alex         | ADHD            | 11  | N/A                                    | 3 <sup>rd</sup> Grade:<br>B<br>4 <sup>th</sup> Grade:<br>B<br>Current: C |
| Brian        | Gifted and ADHD | 10  | CogAT: 128<br>NNAT-2: 132              | 3 <sup>rd</sup> Grade:<br>B<br>4 <sup>th</sup> Grade:<br>B<br>Current: B |
| Brett        | Gifted and ADHD | 11  | CogAT: 110                             | 3 <sup>rd</sup> Grade:<br>C<br>4 <sup>th</sup> Grade:<br>B<br>Current: B |
| Gary         | Gifted          | 10  | N/A                                    | 3 <sup>rd</sup> Grade:<br>A<br>4 <sup>th</sup> Grade:<br>A<br>Current: A |
| Greg         | Gifted          | 10  | N/A                                    | 3 <sup>rd</sup> Grade:<br>A<br>4 <sup>th</sup> Grade:<br>A<br>Current: A |
| Nate         | None            | 10  | CogAT: 110<br>NNAT-2: 112              | 3 <sup>rd</sup> Grade:<br>B                                              |

|      |      |    |            |                                                                          |
|------|------|----|------------|--------------------------------------------------------------------------|
|      |      |    |            | 4 <sup>th</sup> Grade:<br>B<br>Current: B                                |
| Neal | None | 11 | CogAT: 115 | 3 <sup>rd</sup> Grade:<br>A<br>4 <sup>th</sup> Grade:<br>B<br>Current: B |

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*Note.* ADHD = Attention-deficit/hyperactivity disorder. CogAT = Cognitive Abilities Test; scores are out of 160 possible. NNAT = Naglieri Nonverbal Ability Test; scores are out of 160 possible. The average score is 100 for each test.

Eight fifth-grade male students participated in the study. Students were suggested by their classroom teachers based upon fitting into the four categories of participants needed for the study. Two of the students were clinically identified as having ADHD (labeled as “other health impairment” in their Individualized Education Plan); two students were labeled as gifted and having ADHD; two students in the study were labeled by the school district as gifted by a school-based multiple criteria identification procedure that included achievement and intelligence test data, teacher recommendations/ratings, and work samples. These students were assessed in the areas of ability to learn, application of knowledge, creative/productive thinking, and motivation to succeed. The remaining two students did not have any labels placed on them by the school system. Fifth-grade students were chosen because they are at a developmentally appropriate age to be expected to explain and justify their thinking on mathematical tasks. It is during fifth grade that students are developing their reasoning and proof skills in preparation for middle school math. Further, given the school that these particular students attended, they

had all had minimal exposure to being taught to explain their mathematical problem-solving processes aloud or to reflect on these processes.

Permissions were granted to me from both the George Mason University Human Subjects Review Board and the school system to complete the study. All students and their parents granted written consent to participate (Appendices A and B). Once consent was given, classroom teachers of each student were notified by me about their students' participation and asked verbally about each student's perceived problem-solving ability and ability to verbalize his or her own thinking in the mathematics classroom.

The ADHD students, Aiden and Alex (pseudonyms are used to protect all students' identities) had both been diagnosed with ADHD by their family physicians, and were currently taking medication for their condition. Both Brian and Brett had been identified as twice-exceptional by the school system. Nate and Neal had no label put on them by either the school system or medical community. Gary and Greg were both identified as needing gifted academic services by the school system. It was merely a nice coincidence that all of the participants in the study ended up being male. Gender would have presented an additional variable to be considered with further studies.

Aiden was an 11-year-old African American male. He had attended the same school since second grade. Aiden's family physician had identified him as having ADHD. He scored a 101 on the quantitative section of the Cognitive Abilities Test (CogAT) given in second grade. Aiden passed the state math test in third grade and missed passing by three points in fourth grade. His report cards reflected an inconsistent pattern of "A," "B," and "C" grades in math beginning in third grade.

Alex was an 11-year-old African American male. He began attending the subject school this year. Alex's family physician had identified him as having ADHD. Alex was tested by a psychologist and had an overall IQ that fell between 115 and 120. He had mostly "Bs" in math on his report cards, with two "Cs" so far the year of this study. Alex had a hard time getting his work finished, according to his classroom teacher, who said, "I think he knows more than he can put down on paper. He doesn't always finish his work and really likes to talk about things other than school."

Brian was a 10-year-old Hispanic male. He began attending this school in fourth grade, but has attended other schools within the same county since kindergarten. Brian's family physician had identified him as having ADHD and the school system had determined he was in need of receiving gifted academic services. Brian scored 128 on the quantitative section of the CogAT test given in second grade. He scored 132 on the Naglieri Nonverbal Ability Test (NNAT-2) given in first grade. Brian passed the state-required math test in both third and fourth grades by just meeting the benchmark score both times. His report cards reflected mostly "Bs" in math since third grade, with the occasional "C" or "B+." Brian was receiving gifted services for math at school.

Brett was an 11-year-old Caucasian male. He began attending the school in the same year of the study, for fifth grade. Brett's family physician had identified him as having ADHD and the school system had determined he was in need of receiving gifted academic services. Brian scored a 129 on the quantitative section of the CogAT test given the fall of the year of this study. There was no record of previous standardized academic testing. Brett's grades in math beginning in second grade were "Bs" and "Cs," with one

“A” in second grade. He was receiving gifted services for both math and language arts at school.

Gary was a 10-year-old African American male. He had attended the same school for fourth and fifth grade. Gary had been identified by a central screening committee as needing gifted academic services, and received these services for both math and language arts at school. He made a perfect score on the fourth grade state math test. Gary’s report cards reflected all “As” in mathematics since third grade. Gary’s teacher said,

He is a doll. A good student in all academic areas. He likes to help other kids in math when he finishes his work and can get them to understand what they are doing when I can’t. Gary likes a challenge.

Greg was a 10-year-old Hispanic male. He had attended the same school since third grade. He had been identified by a central screening committee as needing gifted academic services, and received these services for both math and language arts at school. He scored in the “pass advanced” range on the fourth-grade state math test. Greg’s report cards reflected all “As” in mathematics since third grade, with the exception of one “B” during third quarter of third grade.

Nate was a 10-year-old African American male. He had attended the same school since first grade. Nate scored 110 on the quantitative section of the CogAT. In first grade, Nate scored 112 on the NNAT-2. Both of these scores fall within the average range. Nate passed the state-required math standards test in both third and fourth grade. He has consistently received a grade of “B” on his report cards. Nate’s teacher reported that Nate

worked quickly and did not know how to explain what he was doing when working during math class.

Neal was an 11-year-old Caucasian male who had attended the same school since second grade. Neal scored a 115 on the quantitative section of the CogAT test given in second grade. He passed the state-required math standards test in both third and fourth grade, scoring in the “pass advanced” range in fourth grade. Neal received all “As” on his report cards in third grade, “As” and “Bs” in fourth grade, and had a “B” average in fifth grade at the time of this study.

### **Setting**

This study took place in a large public elementary school located in Northern Virginia where the students had a high mobility rate. Special needs staffing throughout the school supported the academic needs of all students. While a Reading Recovery team offered an early intervention reading program for “at risk” first-grade students, there was not a similar program for mathematics. Using the inclusion model, a team of school-based and local-area specialists coordinated special education services for students with learning, emotional, physical, and speech-related disabilities. The school system relied on a variety of resources for elementary mathematics instruction and did not follow one set curriculum. The school system provided standards, which are often beyond state standards, that teachers were expected to teach each year. Teachers were provided with specific information regarding *what* to teach, and while there was a list of county-approved resources, there was a lot of freedom for teachers to choose *how* to teach these standards.

## **Task Selection**

Tasks used to collect data and observe occurrences of metacognitive behaviors were chosen carefully so that they would be appropriately challenging for fifth-grade students, but would also require mathematical concepts that should be accessible to students of this age. It was expected that these problems would allow students to solve the problems in a variety of ways, eliciting a variety of metacognitive behaviors over an extended period of time in which students would be engaged in problem solving. Hatfield (1978) suggested that problems for research about problem-solving processes should be “non-trivial mathematical problems of the sort [students] might meet in the classroom . . . should emphasize commonly used as well as non-routine settings which utilize appropriate mathematical concepts, principles, and skills either known or readily learned by the subjects” (p. 35).

The use of multistep problems in a study of metacognition is justified in the work of Baker and Cerro (2000), who found that the complexity of problem-solving tasks is important and stimulates the emergence of metacognitive behaviors. Problems used in the study were selected from The Rational Number Project Initial Fraction Ideas (Cramer, Behr, Post, & Lesh, 2009). The Rational Number Project (RNP) is a cooperative research and development project that was funded by the National Science Foundation. The program includes developing meaning for fractions using a part-whole model, constructing informal ordering strategies based on mental representations for fractions, creating meaning for equivalence concretely, and adding and subtracting fractions using concrete models. RNP does not include instruction of formal algorithms (Cramer et al.,

2009). According to the fourth and fifth grade Virginia Standards of Learning, students should be familiar with the concept of equivalent fractions. These problems are open-ended and allow for various strategies to be used as proof of understanding and of the problem solution. Problems 2-8 of this study also included skills that students should know based upon state standards, but required students to use more problem-solving skills to reach a solution. For each of the sessions, several problems were presented, allowing for scaffolding and student choice as needed. Table 4 outlines the concepts and problem types used during each of the eight problem-solving sessions. Appendix C presents the problems used in each session.

Table 4

*Summary of Problem-Solving Session Concepts and Problem Types*

| Session Number | Problem Concepts                                                                                           | Problem Types |
|----------------|------------------------------------------------------------------------------------------------------------|---------------|
| 1              | Comparing unit fractions; the greater the number of parts a unit is divided into, the smaller each part is | Single step   |
| 2              | Model fractions using chips                                                                                | Single step   |
| 3              | Fraction equivalence; reconstruct a unit given the fractional part; model fractions greater than 1         | Multistep     |
| 4              | Fraction circles; fractions greater than 1                                                                 | Multistep     |
| 5              | Fraction circles; fraction names for one-half                                                              | Multistep     |
| 6              | Estimation; adding fractions                                                                               | Multistep     |
| 7              | Estimation; subtracting fractions                                                                          | Multistep     |
| 8              | Reasoning with fractions using addition and subtraction                                                    | Multistep     |

The task chosen for the first meeting with students was relatively simple. This was done deliberately to help students feel comfortable with the process and to help them not to become discouraged with a difficult problem as the first meeting. In order to encourage observable metacognitive behaviors, complex problems included real-world situations. The first two problems the students solved were used to get a baseline of their understandings and abilities and allowed students time to get used to being videotaped. These sessions were not followed by a debriefing session. The remaining six problems

solved by students included the debriefing session with me and a discussion of metacognitive strategies used.

## **Procedures**

### **Initial Meeting**

Prior to beginning the problem-solving sessions, all eight students were brought together and I explained the goals and procedures to them. At this time students were encouraged to ask any questions they had about the study and their participation in it. The only questions were about whether they would be graded on the math they did during the study and who would see the videotapes. Students were briefly introduced to Schoenfeld's Problem Solving Protocol (1985).

### **Problem-Solving Sessions**

Students then met individually with me twice a week for 4 weeks and completed one problem or a series of problems related to one basic problem at each of the sessions. These sessions/interviews were completed in a classroom and students were provided with a supply of sharpened pencils, erasers, and blank paper to use as needed. In order to compensate for potential academic differences, students were given a choice of problems to solve. Each of the problems presented were of the same type and content area as the others that session. The sessions were videotaped, with the camera focused on the paper the student was working on at the time. Based upon the work of Schoenfeld (1982), students were given instructions prior to beginning each problem-solving session that let the participant know that I was interested in everything they were thinking about as they worked through the problem. This included: (a) things the participant tried that did not

work, (b) approaches to the problem the participant thought might work but did not try, and (c) the reasons why the participant chose to try what he did. Students did their problem solving in pen so I could preserve everything they had written. If the participant decided something he had written was incorrect, he was instructed to put a large X through the item(s). Intervention by me during the problem-solving sessions was minimal. If the student was quiet, and appeared to have difficulty expressing his thoughts, I encouraged the student to share his ideas with questions like, “What are you thinking?” or “Tell me why you did that.”

In order to gain further insight into student metacognition, immediately following each of the problem-solving sessions, the students and I viewed the videotape of the session together. I asked questions to prompt students to think about their thinking prior to beginning viewing the video (modified from a study done by Yimer, 2004). The questions were:

1. Please read the problem out loud and explain to me what it is asking you to do.
2. Do you think the problem is clear?
3. Have you ever solved a similar problem?
4. Was the problem difficult?
5. What were your first thoughts as you read the problem?
6. What were you thinking as you got into the problem?

## **Debriefing Sessions**

Following each problem-solving session, the individual students and I viewed the video of the subject solving the problem, looked at work samples, and discussed the problem-solving and metacognitive processes involved. Task-based interviews were used to help the student focus on and identify his thought processes throughout the problem-solving process. The goal of the video-watching session was to obtain information about metacognitive behaviors that students can describe while recalling their thinking and looking at their work. Reflective questions that I asked students following each session were modified from the work of Schoenfeld (1985):

1. What (exactly) are you doing here?
2. Why are you doing it?
3. How does doing this help you?
4. What do you do next?
5. What are you thinking at this stage of problem solving?
6. What do you mean by this?
7. Why are you doing this? What are you thinking as you do it?
8. Can you explain what you are doing here?
9. What do you do when you get stuck?
10. Does what you are doing make sense? Why?
11. How would you explain your solution?
12. How did you know that you had finished solving the problem?

Prior to beginning the study, I hypothesized that with each session following problem solving, all students would become more aware of their metacognitive processes and how they could vary their problem-solving approaches in future problem-solving sessions. According to Scime and Norvilitis (2006), academic performance and frustration need to be assessed in naturalistic ways in order to have implications for classroom practice. Curriculum-based assessments are often more relevant for this purpose. I believe this type of mathematical intervention can be effective for teachers to use with students to help develop both metacognitive and problem-solving skills.

Other probing questions asked after viewing the videotaped sessions included asking students if they had seen the problem or a closely related problem before, if the student had an idea of how to start the problem right away, and if he felt his work was organized or disorganized.

### **Data Collection**

All data collected in this study came primarily from the eight fifth-grade students who solved a series of eight problems. Each student completed two problems in each of the four weeks. Primary data sources included written solutions to problems, videos of students solving problems, field notes while observing problem solving, and audiotapes of conversations between students and me while viewing problem-solving videos. Additional data including standardized test scores and school grades came from school files.

All problem-solving sessions with students were videotaped. The students and I watched each video together immediately following each session. In order to ensure

consistency in implementation and “recording” of the problem-solving sessions, field notes were written after each session and systematically reviewed to monitor and ensure the consistency of the data collection procedures. Time spent in each of Schoenfeld’s problem-solving stages was verified by the myself and two additional elementary school teachers who did not have students participating in the study. Interview sessions while watching the problem-solving videos were audiotaped for transcribing and then for analysis and coding. Along with field notes, these transcriptions were used to assist in analyzing each problem-solving session. Students’ written work, as well as videos, were used to identify individual metacognitive behaviors. As a form of member checking, at the conclusion of the study, students were asked to review their profile written by me. Student feelings of agreement and disagreement, along with other comments and explanations, were noted.

### **Data Analysis**

This was a qualitative multicase study with a focus on observations/videos, interviews/student reflections, and student work samples. Using the grounded theory constant comparative method, interviews and videos were coded comparing all information across the four categories of students (Maykut & Morehouse, 1994). The constant comparative method can be used for theory/model building (Maykut & Morehouse, 1994). To help ensure consistency in recording the problem-solving sessions, field notes were written after each session. Interviews/video reflections were transcribed the same day the sessions were held, when possible, in preparation for analysis. Select portions of the videotapes were transcribed and coded for instances of metacognitive

thinking following Schoenfeld's (1983) methods. Schoenfeld's timeline graph was used to put problem-solving protocols into episodes and executive decision parts for the purpose of analyzing problem-solving actions. These episodes included reading, analysis, exploration, planning, implementation, and verification. Metacognitive decisions that can affect solution attempts often occur at the transition points between these episodes (Schoenfeld, 1983). Schoenfeld's timeline presented a condensed summary of a problem-solving attempt, which represented a summary of the full analysis and pointed to places where important events likely took place (Schoenfeld, 1992b). The analysis stage involves understanding that statement, simplifying the problem, and reformulating the problem. From this point, if the students had a useful formulation of a plan, they may move to the exploratory stage in which they are structuring their argument for problem solving. If students are having some difficulty, they may move to the exploration stage and look at equivalent problems, slightly modified problems, or a broadly modified problem. From here, once there is a schematic solution, students move into the implementation phase that includes step-by-step execution and local verification of the problem. Once they have a tentative solution, students move to the verification stage to check their work.

Schoenfeld (1984) also identified three different levels of knowledge and behavior he felt needed to be considered if problem-solving performance is to be accurately measured that were addressed in this current study through researcher–student interviews during and following the intervention session and case notes. These levels are:

- Resources: knowledge an individual brings to a particular problem

- Control: knowledge that guides the problem solver's selection and implementation of resources
- Belief systems: perceptions about oneself, the environment, the topic, or mathematics that may influence the problem solver's behavior.

Analysis of data was divided into two main phases: coding the data and then looking for patterns across the data. In order to code the data, interviews were transcribed and student statements and work samples were coded and the amount of time spent in each of Schoenfeld's phases—reading, analyzing, exploring, planning, implementing, and verifying—was recorded. Once coded to identify metacognitive episodes, data was analyzed to identify patterns and changes over time for each of the students in each of the coding categories. Data was then further analyzed to identify patterns between and among groups of students and then the four subgroups (ADHD, gifted and ADHD, gifted, and no label). Using this method, metacognitive behaviors identified from one student were compared to other students' metacognitive behaviors while solving the same problem. Patterns were looked for across all of the problems and behaviors were categorized according to their similarities within and across other problems and behaviors of students using Schoenfeld's protocol.

Challenges in coding occurred when student actions did not match their work on paper at the time. There were also times when student comments included more than one phase in Schoenfeld's protocol, requiring me to separate the statement into two parts, resulting in overlapping time in different areas of the protocol. It makes sense that this would happen, especially as the sessions continued and students become more aware of

their metacognitive processes and used this awareness to evaluate their work and thinking. Since not all metacognitive processes could be articulated, it is reasonable to expect that the data collected does not necessarily include *all* metacognitive activity that occurred during each of the problem-solving sessions.

In order to help ensure consistency in coding, two elementary school educators who were not otherwise involved in the study coded a sample of transcripts after being trained by me using coding categories found in Table 5. Each was given written definitions of each of Schoenfeld's stages. Each of the educators and I coded transcripts of four randomly selected interviews from different students independently, which resulted in a 96% agreement among the raters in coding. As a form of member checking, at the conclusion of the study, students were asked to review their profile written by me. Student feelings of agreement and disagreement, along with other comments and explanations, were noted.

Table 5

*Coding Categories*

| Stage of Problem Solving and Explanation                                           | Questions to be Considered When Coding Transcripts to Help Determine Categories                                                                                                                                                                                                                                                                                                                                                                                            |
|------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Reading: When the student begins to read the problem                               | <ul style="list-style-type: none"> <li>• Have all conditions of the problem been noted?</li> <li>• Has the goal been correctly noted?</li> <li>• Is there an assessment of the current status of the problem-solver's knowledge as it relates to the problem-solving task?</li> </ul>                                                                                                                                                                                      |
| Analysis: When the student is attempting to understand the problem                 | <ul style="list-style-type: none"> <li>• What choice of perspective is made? Is the choice made explicitly or by default?</li> <li>• Are the actions driven by the conditions of the problem? (working forward)</li> <li>• Are the actions driven by the goals of the problem? (working backward)</li> <li>• Is there a relationship sought between the conditions and goals of the problem?</li> <li>• Are the actions taken by the problem solver reasonable?</li> </ul> |
| Exploration: Less structured than analysis; may lead to examining related problems | <ul style="list-style-type: none"> <li>• Is problem solving condition driven? Goal driven?</li> <li>• Is the action directed or focused? Is it purposeful?</li> <li>• Is there any monitoring of progress? What are the consequences for the solution of the presence or absence of monitoring?</li> </ul>                                                                                                                                                                 |
| Planning: May be difficult to assess due to possible absence of overt planning     | <ul style="list-style-type: none"> <li>• Is there evidence of planning at all? Is it overt or must the presence of a plan be inferred from the purposefulness of the problem-solver's behavior?</li> <li>• Is the plan relevant to the problem solution? Is it appropriate? Well structured?</li> <li>• Does the subject assess the quality of the plan as to relevance, appropriateness, or structure?</li> </ul>                                                         |
| Implementation: How the plan is put into action                                    | <ul style="list-style-type: none"> <li>• Does implementation follow the plan in a structured way?</li> <li>• Is there an assessment of the implementation at the local or global level?</li> <li>• What are the consequences for the solution of the assessments if they occur, or of their absence if they do not?</li> </ul>                                                                                                                                             |

## Verification

- Does the problem solver review the solution?
  - Is the solution tested in any way? If so, how?
  - Is there an assessment of the solution, either an evaluation of the process, or an assessment of confidence in the result?
-

## **4. RESULTS**

This chapter will present the case studies of the eight participants and the themes and patterns that emerged within each case and across the cases. Information was gathered from interviews with students and school records. All eight of the students were fifth graders attending the same large elementary school.

### **Individual Case Findings**

In order to address research question one, What types of metacognitive behaviors emerge in fifth-grade students with ADHD, ADHD/gifted, gifted students, and students with no label when they approach rational number problem-solving mathematical tasks, a table was created showing the amount of time each of the participants spent in total during each session (Table 6). Sessions were then further broken down to show the amount of time each student spent engaged in each of Schoenfeld's six problem-solving stages throughout the sessions. Qualitative results of each individual student's work are followed by a table (Tables 7-14) listing each session number and whether the student solved the problem from that session correctly or incorrectly.

Table 6

*Total Length of Problem-Solving Sessions in Minutes*

| Session | Nate  | Neal  | Gary  | Greg  | Aiden | Alex  | Brian | Brett |
|---------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1       | 17:01 | 16:05 | 15:40 | 14:24 | 12:29 | 15:35 | 17:08 | 15:28 |
| 2       | 11:48 | 16:35 | 15:22 | 25:52 | 17:51 | 17:02 | 20:38 | 15:05 |
| 3       | 15:24 | 14:28 | 14:59 | 16:27 | 15:09 | 24:26 | 15:19 | 16:32 |
| 4       | 16:38 | 15:52 | 15:37 | 15:02 | 13:24 | 16:26 | 25:28 | 16:59 |
| 5       | 16:46 | 14:56 | 16:34 | 15:53 | 17:41 | 16:57 | 26:32 | 24:52 |
| 6       | 16:52 | 15:46 | 14:65 | 13:39 | 15:52 | 14:29 | 25:46 | 16:24 |
| 7       | 15:32 | 14:52 | 15:46 | 14:28 | 15:02 | 15:29 | 12:54 | 15:56 |
| 8       | 14:47 | 15:28 | 15:12 | 15:36 | 14:58 | 15:52 | 15:26 | 15:51 |

### **Case #1: Aiden**

During the first two problem-solving sessions, Aiden read and then reread each problem twice, and did not spend any time in the verification phase. During sessions 3 and 4, he read and then reread the problems and did spend some time verifying his answers. Sessions 5 and 6 also included time spent verifying his work, with one less reread than the previous week. This could be a positive result of increasing Aiden's metacognitive awareness through viewing and discussing his problem-solving sessions. During session 7, Aiden reread the question twice, the last time after completing the implementation phase and prior to verifying his work. During the final session, Aiden planned for a second and shorter time, implemented his plan, then reread the problem, and completed implementation of his plan. This was a more detailed process than he used during the first session. During this session he did not verify his answer. During each of the problem-solving sessions, Aiden appeared physically relaxed and smiled often. While

viewing himself the first few times Aiden appeared a bit stiffer, but relaxed during the remaining debriefing sessions.

During debriefing after the third session, we had the following dialog.

Researcher: What did you do differently when solving this problem than during our other sessions together?

Aiden: I am remembering to read the problem more than one time to make sure I get it. Wait . . . I think I did that last time, too. I forgot to check my work.

Researcher: Is it helpful for you to check your work?

Aiden: Sometimes. I know I'm supposed to, but sometimes I forget. Sometimes I just don't feel like it. When I know I got a problem right I don't check my work 'cause I know it's right.

Over the course of the intervention sessions it became clear from Aiden's feedback that he only verifies problems when he thinks they are hard. He does not think it is necessary to verify answers for "easy" problems. Despite the fact that Aiden did not solve all of the problems correctly (Table 7), he appears to be confident in his math problem-solving abilities. Aiden reported that his father helps him with his math work and reinforces ideas taught in school. He tends to rely on drawing pictures to solve problems.

During the viewing of the videotapes of problem-solving sessions 2-8, Aiden had no difficulty reading each problem and explaining what the problem was asking him to do. He felt that each of the problems were clear and could not relate any of the problems to another problem he had solved.

Researcher: Can you remember if you have solved another problem like this before?

Aiden: With you? Or ever?

Researcher: Either way.

A: I don't know. I don't think so. I mean . . . we do a lot of story problems in class.

Table 7

*Aiden's Problem-Solving Session Results*

| <u>Session #</u> | <u>Correct/Incorrect</u> |
|------------------|--------------------------|
| 1                | Correct                  |
| 2                | Correct                  |
| 3                | Correct                  |
| 4                | Incorrect                |
| 5                | Correct                  |
| 6                | Correct                  |
| 7                | Correct                  |
| 8                | Incorrect                |

**Case #2: Alex**

Sessions 1 and 2 had Alex spending the more time in the implementation phase of problem solving than any of the other phases. He appeared more nervous during these two sessions than the others, often putting a finger in his mouth, as if to bite his fingernail. With each session Alex's posturing relaxed and he was less stiff. During the first session he took a break from implementing his plan to reread the problem for the second time, and then returned to the implementation phase without doing any additional planning. During Session 2, Alex did spend a few minutes in the planning phase in

between blocks of time he spent implementing his plan. During both of these sessions Alex planned for 2-3 minutes prior to implementation. During Sessions 3 and 4 Alex's problem-solving patterns changed, spending time planning prior to implementation. Again he paused partway through implementation, however, during these sessions it was to verify his work. Alex reported doing this because he thought he was finished with the problem and wanted to check his work. When he realized he was incorrect or there was more for him to do, he reread the problem during Session 3 and then continued to implement his plan. During Session 4 he felt more confident in his work and was able to continue implementing his plan after verifying his work. The third week of the study found Alex more consistent in his problem-solving behaviors. After reading the problem he moved from planning to implementation to rereading the problem. During Session 5 he spent time exploring and rereading prior to planning, while during Session 6 he reread the problem after implementing his plan and then continued to implement his plan briefly. He did not verify his work during this session or during Session 7. Alex was the most unfocused during Session 7, where after planning he began to implement his plan, said he forgot what the problem was, reread the problem, returned to implementation for a bit longer this time, planned for about a minute, and then returned to implementing his plan.

Researcher: Tell me what you are doing and thinking about here.

Alex: Um . . . I'm not sure. I think I started writing and then forgot what I wanted to do so I had to start over again.

Researcher: Do you do that a lot?

Alex: I don't know. Sometimes, I guess. I usually have to read something two times to figure it out, but then I'm good.

This session was in stark contrast to Session 8 in which Alex read the problem "carefully," spent some time analyzing what he needed to do before beginning to plan his actions, and then worked consistently for over five minutes to solve the problem.

Appearing confident with the outcome, Alex spent a few minutes verifying the accuracy of his answer. Alex's written work from each of the sessions was messy and disorganized. He did not appear to have a logical sequence for where he wrote things on paper and could not explain the placement of information on paper. In fact, when asked about this, Alex appeared puzzled about why anyone would ask such a question.

During the viewing of the videotapes of problem-solving sessions 2-8, Alex had a hard time sitting still. His teacher had noted that providing a fidget tool similar to a stress ball often helped him focus in class, so one was provided for him. While this appeared to calm the gross motor movement of his body, Alex's explanations of his work appeared to be disorganized, particularly during sessions 6 and 7—the same sessions when he did not spend any time in verify stage of problem solving. The difficulty focusing was not seen during the problem-solving sessions themselves, with the exception of session 7. Alex struggled to explain his strategies and struggled to verbalize what he was doing as he watched himself on video. He was unable to articulate any changes in how he approached or solved problems throughout the course of the problem-solving sessions. Table 8 shows whether he solved the problems correctly in each session.

Table 8

*Alex's Problem-Solving Session Results*

| <u>Session #</u> | <u>Correct/Incorrect</u> |
|------------------|--------------------------|
| 1                | Correct                  |
| 2                | Correct                  |
| 3                | Correct                  |
| 4                | Correct                  |
| 5                | Correct                  |
| 6                | Incorrect                |
| 7                | Incorrect                |
| 8                | Correct                  |

**Case #3: Brett**

Brett consistently read the problems he was tasked with solving at least twice, except during Session 4. He consistently verified his answers while problem solving, with the exception of Session 6. Sessions 1, 2, and 7 were the only times Brett verified his answers at the conclusion of the problem-solving process. He said this was because those were the times he “just wanted to be sure he was correct.” During the first problem-solving session, Brett spent some time working in each of Schoenfeld’s problem-solving stages. Upon reflection, Brett shared that he was being extra careful to do everything he had learned in school about solving problems since he thought that was what I wanted him to do. During the first session, Brett spent a few minutes in both the analyze and explore stages. He did not explore again until Session 5, for approximately the same amount of time as during Session 1, immediately after reading the problem for the first time; the same was true for Brett during Session 8. Brett sat up straight during each of the sessions and made little eye contact with me. He kept his eyes focused on the paper he was working on throughout much of each session, rigidly responding to any questions he

was asked. His body and tone were more relaxed while watching himself solve the problems on video. During the debriefing after Session 6, he noted,

Researcher: Today you spent some time exploring the problem and problem-solving ideas.

Brett: Yeah. That's good, right?

Researcher: It's just an observation. You don't always do that. Tell me about what you did today and why you did it.

Brett: I don't know. Today it helped me, I guess. I know I'm supposed to slow down and think about what I'm doing in math.

Researcher: Really?

Brett: Yep. My teacher and my mom say that all the time. So I tried to remember to do that today.

Researcher: Was it helpful?

Brett: Yeah. I guess. See, I drew the tower two different ways: up and down, and sideways with the 12 cubes. Then I shaded 2, skipped 1, shaded 2, and skipped 1.

Researcher: You used that strategy for both ways: up and down and sideways. How did that help you solve the problem?

Brett: [Pause] Well, I guess it didn't. It was really the same thing two times. Kind of like a check.

Other than during Sessions 1 and 6, Brett did not spend time analyzing the problems during the sessions. During both of these sessions, analysis occurred after

reading the problem the first time; time spent in this phase was double that of Session 1 in Session 6.

While watching the videos of his problem-solving sessions, Brett often jumped in to say what else he could have done to get to the answer “quicker.” He takes pride in his work and written notes were consistently well organized. While Brett began Session 1 doing what he felt I wanted, even he noticed how his body relaxed during each of the following sessions. At the conclusion viewing the final video, Brett was very reflective about his learning:

I need to spend more time thinking about what I’m going to do before I do it. I think that would make me get better grades in math and make less mistakes. I just always forget to do it. I want to finish my work and the other kids work so fast.

Table 9 shows whether he solved the problems correctly in each session.

Table 9

*Brett’s Problem-Solving Session Results*

| <u>Session #</u> | <u>Correct/Incorrect</u> |
|------------------|--------------------------|
| 1                | Correct                  |
| 2                | Correct                  |
| 3                | Incorrect                |
| 4                | Correct                  |
| 5                | Correct                  |
| 6                | Correct                  |
| 7                | Correct                  |
| 8                | Incorrect                |

#### **Case #4: Brian**

Of all of the students in the study, Brian spent the longest amount of time reading the problems presented the first time. With the exceptions of Sessions 2 and 7, he reread the problem at least once during each of the problem-solving sessions. During the first three sessions, Brian spent less than five minutes in the planning phase of problem solving. Following the first debriefing session and Brian's reflection that he needed to spend more time thinking about what he was going to do before he started doing it, during the fourth session Brian almost doubled the amount of time he spent in the planning phase. This time decreased slightly during Sessions 5 and 6, but was still greater than the amount of time spent planning during Sessions 1-3.

By sessions 7 and 8, Brian's planning time was comparable to the time he spent planning during the first three sessions, yet the time he spent during the implementation phase decreased from around 10 minutes to between 5 and 7 minutes. Brian believed this decrease in time spent planning and implementing his problem-solving plans was due to the fact that he felt more comfortable and confident solving the problems presented than at the beginning of the study. While he was unable to provide specific examples, Brian said he has solved problems in the past that were like the problems he was working on during the study and asked for more problems to do for "practice" at the conclusion of the study. While Brian said he was nervous about being taped doing math, at the conclusion of Session 7 he said,

I guess it's not so bad watching myself. It's kinda weird. Now I can see that I do need to slow down, I guess, just like my teacher told me. I like to finish first, though. The other kids think I'm smart.

Table 10 shows whether he solved the problems correctly in each session.

Table 10

*Brian's Problem-Solving Session Results*

| <u>Session #</u> | <u>Correct/Incorrect</u> |
|------------------|--------------------------|
| 1                | Correct                  |
| 2                | Incorrect                |
| 3                | Correct                  |
| 4                | Correct                  |
| 5                | Correct                  |
| 6                | Correct                  |
| 7                | Incorrect                |
| 8                | Correct                  |

**Case #5: Gary**

Prior to beginning the problem-solving sessions Gary shared that he does not usually check his work. "I do the problems once. I'm careful when I do them and I'm usually right. When I finish my work I turn it in. That's it." Gary appeared confident and comfortable solving problems and having his work recorded.

The first three problem-solving sessions proved Gary to be correct. He read the problems, spent a few moments analyzing the problem in Session 1, and a few minutes exploring strategies during Sessions 2 and 3, he quickly planned and then implemented his plan to solve the problems. Once Gary had finished solving the problems from each of these sessions he looked at me if to say, "Now what?" When viewing himself problem

solving during Session 3, Gary realized he had made an error. He seemed a bit embarrassed by his mistake and quickly asked if he could do the problem again and explained what he would do differently. It is likely this realization contributed to Gary changing his approach to problem solving during Session 4. After reading this problem, Gary spent about five minutes planning his problem-solving strategies. He spent about equal time implementing his strategies, then a few minutes verifying his work. Without saying anything, Gary went back to work and added a few things to his written work on the problem and finished up with a brief verification of his work, mumbling to himself, and again, looking at me and saying, “Ok, I’m done. It’s right now.”

Gary’s verification of his work and strategies continued for Sessions 5 and 6. Viewing himself working during Session 7, Gary commented that he had forgotten to check his work without being prompted, and during Session 8 was sure to take a couple of minutes to verify his use of strategies and his answer. Gary struggled to explain his use of strategies during and after the videotape viewing. He was much more concerned with getting the right answer and being “done” than with the process involved in getting to that answer. Table 11 shows whether he solved the problems correctly in each session.

Table 11

*Gary's Problem-Solving Session Results*

| Session # | Correct/Incorrect |
|-----------|-------------------|
| 1         | Correct           |
| 2         | Correct           |
| 3         | Correct           |
| 4         | Correct           |
| 5         | Correct           |
| 6         | Correct           |
| 7         | Incorrect         |
| 8         | Correct           |

**Case #6: Greg**

Greg approached each problem-solving session with less enthusiasm and confidence than the other students. During the first session he looked at me every few minutes to “check in” and had to be reminded of the purpose of the study and that he would not get any feedback as to whether he was solving the problems correctly or incorrectly during the sessions. Before beginning to plan to solve the first problem, Greg carefully read, analyzed, and reread the problem. He worked meticulously to plan, implement, continue planning, and then continue implementing his problem-solving strategies. Due to his apparent insecurity it was surprising that Greg did not check his work on this problem. The second session appeared a bit less anxiety producing for Greg. When asked how the second session felt different than the first session, Greg said, “I’m not so worried this time since I know that you are just watching and you won’t make me fix any mistakes if I don’t know how to do the work.” He read the problem and moved directly into the planning, implementation, and verification phases of problem solving. Greg then repeated the same pattern of behaviors, fluently moving between problem-

solving phases, once he realized there was an error in his work. As Greg continued he spent approximately half the time planning and double the amount of time implementing his strategy prior to verification. He followed the same pattern of planning, implementing, planning, and then implementation during Session 3. During this session, Greg only verified his work following the second chunk of time spent in the implementation phase. Greg struggled to watch himself on video and had difficulty explaining his problem-solving process.

At the beginning of Session 4 Greg confirmed with me that he would have to watch himself on the video again. When this was confirmed he sighed heavily and said “I look so weird on TV . . . and I don’t always remember what I was doing. I just do it.” Greg hesitantly sat down and began to work after I reassured him that he was just to do his best and if he could stop if he ever became uncomfortable. After reading the problem, Greg immediately started planning his strategy, reread the problem, spent a couple of minutes planning, and then spent an extended amount of time implementing his plan prior to verification. He followed similar patterns during Sessions 5-7 without rereading the problem. During Session 5 he spent 2 minutes analyzing the problem, yet did not analyze again until Session 8. Sessions 6 and 7 found Greg spending some time in the exploration phase of problem solving, with that time doubled in session 6. Greg did not verify his work in Sessions 6 or 8. Table 12 shows that Greg is the only student who solved the problems correctly in every session.

Table 12

*Greg's Problem-Solving Session Results*

| Session # | Correct/Incorrect |
|-----------|-------------------|
| 1         | Correct           |
| 2         | Correct           |
| 3         | Correct           |
| 4         | Correct           |
| 5         | Correct           |
| 6         | Correct           |
| 7         | Correct           |
| 8         | Correct           |

**Case #7: Nate**

Nate did not verify his work during the first two sessions, but did so during each of the following sessions. When asked about this, he said, “I know we will be talking about what I am doing in these problems and my teacher always tells me I have to check my work.” It was only during Sessions 5 and 7 that verification led to further implementation for Nate. Interestingly, after working more on each of these problems, he did not return to reverify his solutions. Nate was not one to spend a lot of time planning his problem-solving strategies, and during Session 2 spent no time visibly planning since he “just knew” how to solve it. With the exception of Session 2, Nate did return to reread the problem he was working on at some time during the problem-solving session “to be sure I was not leaving out anything I need to do.” Nate appeared to be the most lackadaisical of the students in this study in his approach to each of the sessions. He came in, did what he needed to do, and was done. Nate was not interested in extraneous chatting before, after, or during each of his sessions. Table 13 shows whether he solved the problems correctly in each session.

Table 13

*Nate's Problem-Solving Session Results*

| Session # | Correct/Incorrect |
|-----------|-------------------|
| 1         | Correct           |
| 2         | Incorrect         |
| 3         | Correct           |
| 4         | Correct           |
| 5         | Correct           |
| 6         | Correct           |
| 7         | Correct           |
| 8         | Incorrect         |

**Case #8: Neal**

Patterns in Neal's problem-solving approach seemed to vary from week to week. During the first two sessions he was inconsistent other than reading the problems twice and verifying his work at least once. He started planning and then went between implementation and verification during Session 1, while Session 2 found him going between planning and implementation prior to a final verification. Sessions 3 and 4 were more fluid than the first of Neal's problem-solving sessions. He read the problem once, spending more time reading during Session 4, spent a minute in the analysis phase, and then began planning his approach. Neal then spent about a solid five minutes implementing his problem-solving strategies prior to verifying his solutions. Most of this was done silently. When asked to think aloud, Neal struggled to explain his thought processes. While experiencing this metacognitive discomfort, Neal moved around in his seat and would sometimes sway from side to side.

Sessions 5 and 6 found Neal rereading the problems each twice. During both of these sessions he would plan, once following a few minutes in the analysis phase, the

other time after exploration. During Session 5 after planning he started to implement his strategies, reread, and then continued with implementation. During Session 6, after planning he reread, implemented, reread again, implemented strategies, and finally spent a quick minute verifying his work. During Sessions 7 and 8, Neal read the problem and then spent a minute or two in the analysis phase. Session 7 required some time in the explore phase of problem solving and then planning. After a minute or two planning, Neal implemented his strategies, planned again for a minute or two, and then returned to the implementation phase. In Session 7 he then verified his strategy, reread, planned briefly, and finished with implementation, while in Session 8 he moved directly back to reading the problem followed by implementation and then verification before finishing. The final implementation phase during both Sessions 7 and 8 was for the same amount of time (3 minutes). Table 14 shows whether he solved the problems correctly in each session.

Table 14

*Neal's Problem-Solving Session Results*

| <u>Session #</u> | <u>Correct/Incorrect</u> |
|------------------|--------------------------|
| 1                | Correct                  |
| 2                | Correct                  |
| 3                | Correct                  |
| 4                | Correct                  |
| 5                | Correct                  |
| 6                | Correct                  |
| 7                | Correct                  |
| 8                | Incorrect                |

## **Emergent Patterns**

Aiden and Alex were both identified as having ADHD and worked within a general education classroom at school. Inability to maintain control or focus did not appear to be an issue for either student in this one-on-one environment. Alex did struggle to stay still during the viewing of the videotaped sessions, but this concern was remedied a bit by the use of a stress ball. While Aiden read each problem two or three times during each session, after the first session in which he read it three times, Alex was inconsistent, reading the problem anywhere from one to three times. While Aiden said he only checked his work when a problem was hard for him, Alex recognized his mistakes and attempted to rework problems he had not answered correctly.

Brett and Brian were the two students in the study are identified as having ADHD and receiving gifted services (twice-exceptional). Both of these boys were careful when reading the problems they were solving. Brett read each problem at least twice and Brian spent the longest amount of time reading the problems the first time of all of the students in the study. While both boys valued speed as part of their problem-solving process, Brett was more concerned with being accurate while Brian was more concerned with other students' perceptions of his abilities, which he saw as being directly related to the speed in which he finished his work.

Both Gary and Greg were receiving Advanced Academic (gifted) services for mathematics in their school. While Gary did not like to check his work he was embarrassed by a mistake he made and immediately wanted to "fix" it. Greg approached the problem-solving sessions with less confidence than Gary. Greg seemed to get stuck

on the idea that he could not always remember why he chose certain strategies for problem solving and appeared less outwardly confident than Gary. However, while Gary appeared to be outwardly self-confident about his mathematics abilities it became clear during the study that he used his strong verbal abilities and speed to mask his insecurity.

Neither Nate nor Neal had any labels placed on them by the school system. Both were in a general education classroom. Of all of the students in the study, Nate became more aware of his metacognitive processes while problem solving and consciously made changes to his problem-solving approaches as a result. However, verification led to changes and additions to problems in only two sessions. Nate spent little time planning how to solve problems when he “just knew it.” While Nate consistently read and reread the problems he was solving, Neal was inconsistent in this practice, sometimes reading the problem only once, while other times reading it as many as three times. Nate moved more fluently through the problem-solving phases, while Neal started moving between and among the stages beginning with Session 5. While unable to articulate specifics, when asked, Neal said watching the videos of himself helped him see that he needed to do things differently when working on math problems.

### **Self-Awareness of Problem-Solving Process**

Overall, self-awareness of the problem-solving process was increased during this study. The participants expressed that they did not do a lot of this type of reflection on their metacognitive processes during a typical school day. All of the students in this study spent an inconsistent amount of time in the verification stage of problem solving. They did not spend much, if any, time in this phase of problem solving, yet this is mentioned as

an important phase for successful problem solving by Polya, Schoenfeld, and other researchers. These results could suggest that teachers need to help students see the value in “checking their work” and teach them how to do so. The students identified as being in need of gifted services in this study were better able to interpret their answers, but struggled to verbalize their metacognitive processes and were uncomfortable watching themselves complete the work on the videos. This could be a result of the fact that they had high expectations for themselves and had not had enough experience articulating their mathematical problem-solving strategies.

### **Accuracy of Problem-Solving**

While no direct correlation was found between the amount of time spent in various problem solving stages and problem solving accuracy in this study, all students answered the question during the first problem-solving session correctly, while four of the students incorrectly answered the question correctly in problem-solving Session 8. There were no patterns to accuracy or inaccuracy that developed with these students during the problem-solving sessions (Table 15). Problem 1 was chosen to help students become comfortable with the environment. Due to the varying abilities of each of the students, each problem-solving session contained several problems from which students were able to choose. Each of the problems for a session included the same mathematical concept and were the same type of problem.

Table 15

| <i>Incorrect Answers By Session</i> |                                 |
|-------------------------------------|---------------------------------|
| Problem-Solving Session             | Students With Incorrect Answers |
| 1                                   | None                            |
| 2                                   | Brian<br>Nate                   |
| 3                                   | Brett                           |
| 4                                   | Aiden                           |
| 5                                   | None                            |
| 6                                   | Alex                            |
| 7                                   | Alex<br>Brian<br>Gary           |
| 8                                   | Aiden<br>Brett<br>Nate<br>Neal  |

### **Verification Stage**

When looking at the timelines of the final problem-solving session for each student, it is evident that these students did not spend much, if any, time in the verification stage of the problem-solving process, even when the stages of the process were discussed during sessions with me. When students did verify their work, only two students, Brett and Greg, returned to make some changes to their original solution.

Using Schoenfeld's timeline, the amount of time spent in each of the six problem-solving stages was charted for each student. During the final problem-solving session

(see Figure 1), Nate began the session by spending about three minutes reading the problem. He then moved into the analysis stage for 2 minutes and then started planning his strategy. Nate then reread the problem before beginning to implement his strategies, followed by a brief verification of his answer. Appendix D includes the timeline graphs for each of the students during their first and last problem-solving sessions. These sessions were chosen as representative of any changes that occurred in the amount of time spent in the various stages of problem solving over the duration of the study.

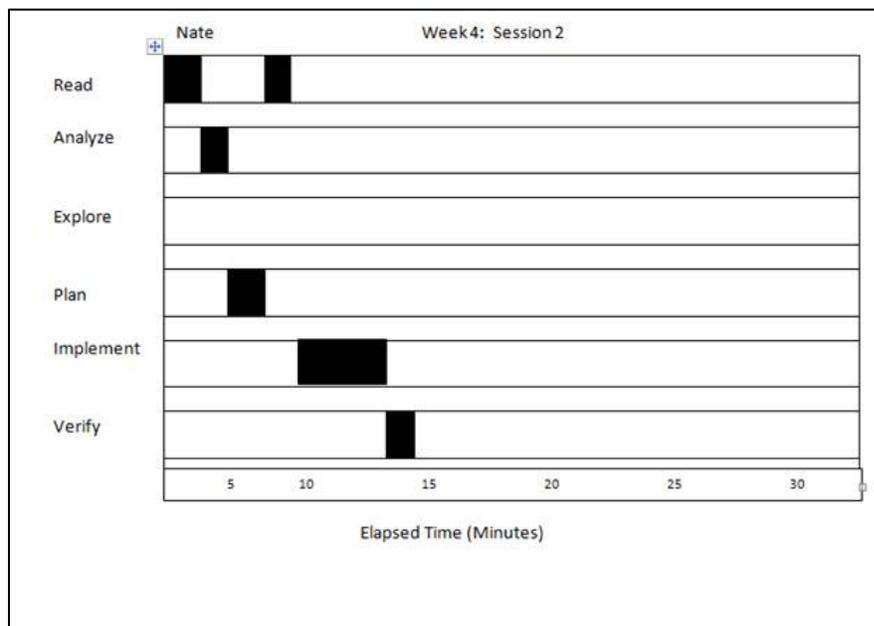


Figure 1. Timeline graph of Nate's final problem-solving session.

### Metacognitive Behaviors

In regard to the first research question, what types of metacognitive behaviors emerge in fifth-grade students with ADHD, ADHD/gifted, gifted students, and

nonlabeled students when they approach rational number problem-solving mathematical tasks, the results are varied. There do not appear to be consistent patterns in these behaviors between or across the particular students in this study.

### **Debriefing**

The second question, how does the problem-solving debrief session effect the amount of time spent in each of the six metacognitive problem-solving stages in subsequent problem-solving sessions, and does it impact student performance patterns over time, yielded inconclusive results. While there were some changes in the amount of time spent in each of the problem-solving stages (Appendix D), it is unknown if this is due to increased metacognitive awareness resulting from the debriefing sessions or from the repeated exposure to the types of problems being solved and limited questioning done during the eight sessions for each student. It would be interesting with further research to see if students who were reminded throughout problem solving to take time to analyze, plan, explore, and verify their work would have more detailed and/or accurate work.

## 5. CONCLUSIONS AND IMPLICATIONS

The primary research question in this study was to examine the metacognitive processes of fifth-grade students while they engaged in rational number problem solving. This was done by analyzing the amount of time each student spent in each of Schoenfeld's six problem-solving stages (read, analyze, explore, plan, implement, and verify) during each of eight problem-solving sessions. The second research question looked to examine if there were changes in patterns that emerged over time when students were encouraged to be reflective about their metacognition. This reflection was done following each problem-solving session and consisted of students being asked about what they were doing while problem solving and why they chose particular actions.

I found that each of the students had difficulty, to varying degrees, articulating their thinking while solving these rational number problems. However, from the perspective of an educator, a lot more was learned about how the students were thinking mathematically than if they had completed the work independently and had not been asked to reflect on their thinking. In order to obtain this kind of knowledge about how students are thinking mathematically, teachers may need to give explicit instruction on how to articulate such thinking and allow students time to practice doing this. While working one-on-one with students and probing them provides additional insight into their

thought processes, classroom teachers are not able to give this kind of one-on-one time to each of their students on a regular basis.

### **Metacognitive Behaviors**

While further research is needed, several big ideas emerged from this study. In regard to research question one, what types of metacognitive behaviors emerge in fifth-grade students with ADHD, ADHD/gifted, gifted students, and nonlabeled students when they approach rational number problem-solving mathematical tasks, I found that regardless of their particular designation, these fifth-grade students struggled to explain their thinking while solving rational number problems; gifted students in this study were the strongest in this area.

Verification of work, as discussed by Polya, Schoenfeld, and others, did not lead students to correct any mistakes—particularly if there was not a clear understanding of the problem to be solved. Students in this study did not spend much (if any) time in this phase of the problem-solving process. Each of the eight students had at least one session in which they did not spend any time in the verification stage of the problem-solving process. Brett and Neal were the only students who verified their solutions during the first session. Brett ended this session by verifying his solution, while Neal verified his solution twice, following implementation of his plan. Aiden did not verify his solution during three of the sessions, but verified his work during five of the sessions as his final step in the problem-solving process. Alex did not verify his work during four of the problem-solving sessions. During two of the sessions he verified his work and then returned to the implementation stage, while two of the sessions ended with time in the

verification stage. Both Brett and Brian had only one session in which they did not spend time in the verification stage of problem solving. Brian ended five of his problem-solving sessions briefly in the verification stage, while Brett did so in three of his sessions. Like Alex, Gary did not verify his work during half of the sessions and Greg did not verify his work during three of the sessions. During the other five sessions, Greg moved from the implementation phase briefly to verification before completing the session. Nate and Neal went from implementation to verification for more than half of their problem-solving sessions.

When working one-on-one, students with ADHD did not have difficulty maintaining focus during this study; however, lack of organization can be evident in their approach to problem solving. Although the amount of “jumping” from stage to stage decreased over the course of the eight sessions, Aiden, followed by Alex, had more movement between problem-solving stages than any of the other students. Each of these students was able to work to find a solution to the problems they were presented with during each of the sessions.

Finally, advanced students in this study were uncomfortable making mistakes and admitting they did not know how to approach a problem. For example, during Session 3, Gary realized he had made an error while reviewing the video and quickly asked if he could fix his mistake. This awareness led to a change in Gary’s approach to problem solving during the next session in which he spent more time in the verification stage of problem solving. Greg was very uncomfortable with the idea of watching himself on

video and often looked to me for reassurance he was doing his work correctly during the sessions.

### **Effect of Debriefing on Metacognition**

In regard to research question two, how does the problem-solving debrief session effect the amount of time spent in each metacognitive problem-solving stage and student performance patterns over time, I found no significant differences in awareness of metacognitive processes in any of the students. While students moved somewhat fluidly through the problem-solving processes, there was not a consistent a pattern to the amount of time spent in any of the six stages. Aiden read each problem two or three times and spent almost no time exploring during any of the sessions. During half of the sessions he read the problem and then moved into the planning phase, determining how he would solve the problem, and during three of these four sessions Aiden then began to implement his plan. Alex spent large amounts of time (sometimes as much as five minutes) implementing problem-solving strategies after reading the problem. After some implementation, with the exception of the last session, he went back to reread the problem before continuing to implement his strategies. Both of the students in this study with ADHD seemed to require reading the problems more than once in order to be successful problem solvers. With the exception of Session 4, Brett also always read the problems two or three times during each session. He moved fluidly between planning and implementation phases during Sessions 1, 5, and 7. Brett ended half of the problem-solving sessions by verifying his answer and ended the other half by finishing the implementation of his plan, without verifying his work. Brian only spent time in

analyzing the problems during the first and last session, and followed a pattern of implementation, rereading, and then returning to implementation during Sessions 1, 3, 4, and 8. Both Gary and Greg spent little time analyzing or exploring during problem solving. When Gary spent time either analyzing (Sessions 1, 6, and 7) or exploring (Sessions 2 and 3), it was only once during the sessions and this never lasted for more than two minutes. Greg spent more time in the implementation phase of problem solving than any of the other students in the study. During Session 6 he spent almost as long exploring the problem as he did implementing his problem-solving strategies, yet during six of the eight sessions he spent no time at all exploring, and only during half of the sessions did he spend any time analyzing his work. Nate spent the most amount of his time problem solving in the implementation stage. He did not verify his work during the first two sessions, and verified as a way of ending sessions during half of his sessions. During the remaining two sessions, Nate verified his work and then returned to further implement his plan. During each of the eight sessions, Neal spent the majority of his time in the implementation phase of problem solving. Three of his eight sessions ended with the implementation phase, while five ended in verification of his solutions.

Each of these eight students had their own ways of approaching problem solving throughout the study, with inconsistent patterns emerging throughout. This lack of consistency in problem-solving strategies, and the variety of patterns that emerged, leads me to the conclusion that reflecting upon problem solving over these eight sessions with these particular students did not increase their metacognitive awareness while problem solving.

## **Implications for Teachers**

Teachers in today's classrooms need to meet the needs of a variety of students and could have students that fall into each of the categories in this study. Therefore, they need to have a large toolbox of resources and strategies to help all students be successful in mathematics. This is not an easy task. Students need to be instructed on *how* to solve problems in order to be able to monitor problem-solving steps and accuracy.

Student responses to mathematical questions reveal information about their level of understanding as well as the different problem-solving approaches they implement. Doing this type of exercise with students prior to teaching a new concept can help to inform teachers of issues that may arise during a lesson—information which can be used to inform instruction. Teachers could summarize student difficulties in the form of questions when teaching the whole class and preplan their questions to help guide whole class student learning. The students in this study felt uncomfortable and had difficulty articulating their strategies, which could suggest they have not had enough experience doing such analysis. Mathematics educators need to be aware of and attentive to students' metacognition. They need to appreciate metacognitive activity and develop ways to foster it within all students.

## **Implications for Teaching and Assessing Metacognitive Processes**

While classroom teachers of mathematics often do not have the time to sit one-on-one with students and have them reflect on their metacognitive processes, a modified version of Schoenfeld's protocol can be used by teachers as follows. By analyzing the amount of time a student spends in each stage of the process, teachers may be able to

assess problem-solving skills and design targeted interventions for those students who need them. These questions could be used by teachers to reflect upon student understanding or could be posed to the students as a reflection on their metacognitive processes while problem solving. Teachers might also allow students to work in small groups to reason mathematically by answering questions that focus on comprehending the problem, constructing connections, using strategies appropriate for solving the problem, and reflecting on the process and the solution. Some students, particularly those with ADHD, will require direct modeling in order to do this effectively.

Table 16 provides questions for teachers to consider, based upon Schoenfeld's problem-solving stages, when assessing metacognition in problem solving.

Table 16

*Teacher Tool to Assess Metacognition Using Schoenfeld's Problem-Solving Stages*

| Schoenfeld's Stage | Questions for Teacher to Consider                                                                                                                                                                                                         |
|--------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Read               | <ul style="list-style-type: none"> <li>• Can the student read the problem?</li> </ul>                                                                                                                                                     |
| Analysis           | <ul style="list-style-type: none"> <li>• Does the student understand the question being asked?</li> <li>• Can the student simplify the problem?</li> <li>• Can the student start to plan how he or she will solve the problem?</li> </ul> |
| Design Plan        | <ul style="list-style-type: none"> <li>• Can the student break down (decompose) the problem into more manageable pieces?</li> </ul>                                                                                                       |
| Implementation     | <ul style="list-style-type: none"> <li>• How does the student put his or her plan into place to solve the problem step by step?</li> </ul>                                                                                                |
| Verification       | <ul style="list-style-type: none"> <li>• Does the solution work in this problem?</li> <li>• Would the problem-solving process used work in other similar problem-solving situations?</li> </ul>                                           |

Exploration

- Can the knowledge gained from solving this problem be applied to other similar or different problems?
- 

### **Implications for Working With Students With ADHD**

Teaching students to recognize patterns in mathematics and mastering mathematical symbols are essential for the success of students with ADHD in mathematics classes. When solving word problems, these students should be encouraged to reread the problem, look for guiding questions in the problems, be provided with real-life examples in word problems, and be allowed to use number lines, graph paper, and other manipulatives in class. Further, teacher perceptions of a student's academic skills can have a tremendous influence on his or her success in mathematics. Given the association between ADHD and academic achievement, education professionals need to be aware of the potential learning difficulties among these students. They must implement effective prevention and intervention strategies to enhance academic functioning. Students must receive academic intervention in mathematics, not just a reliance on reducing ADHD symptoms and disruptive behaviors (DuPaul et al., 2004).

Other suggestions for assisting students with ADHD develop self-monitoring and metacognitive strategies include: provide direct instruction to help student think about their approach in problem solving; help students self-monitor their level of alertness when working, so they maintain attention to task, stay paced, and work problems with accuracy; model how to first read problems and plan a strategy for solving before beginning the work; teach how to work problems carefully and check for accuracy; teach

how to estimate and determine whether an answer is reasonable or not; guide students through the steps of a problem, modeling what questions you ask yourself when solving; and model talking out loud while reasoning/thinking about mathematical problems and encourage students to do the same (Rief, 2005).

### **Implications for Working With Gifted Students**

Renzulli (1975) believed that training teachers appropriately is the best way to ensure high-quality program for students. It follows that this would hold true for teachers of all students: Better trained teachers can better identify student needs. While specially trained teachers are more supportive of gifted students and programming for gifted students (Shore & Kaizer, 1989), it seems this could also hold true for students with any type of special educational need. While differentiation is required for many special populations, when working with gifted students, teachers may have additional challenges (VanTassel-Baska & Stambaugh, 2005) due to the degree of differentiation required, the need to provide learning opportunities beyond grade levels, philosophical barriers toward gifted learners and their needs, and lack of state mandates requiring support services for gifted learners. Teachers need to be aware of these challenges and have a plan to address them so these students do not regress in their mathematical achievement. Teachers can address these concerns by using varied learning approaches such as nonstandardized tests, extended deadlines, providing a variety of modes in which students can produce their work, and providing choice in assignments.

### **Implications for Future Research**

Examining the metacognitive processes of eight fifth graders as they solve mathematical problems is a beginning to the research that needs to be done to start understanding how students who are identified as gifted, who have ADHD, are identified as gifted and having ADHD, and those with no label approach mathematical problem solving. Due to the small size and short duration of this study, these results are not generalizable to larger groups of students. Future studies could explore differences/similarities in larger samples of like students and their awareness of and abilities to explain metacognitive processes.

Future studies could also explore and/or evaluate various “programs” which could be implemented to increase metacognitive awareness while problem solving. While Garofalo and Lester (1985) explained that students with low prior knowledge in mathematics do not routinely analyze information in a problem, monitor their own progress, or evaluate results, more information is still needed to understand how behaviors related to control and metacognition are manifested during problem solving and how they interact with other resources to influence the problem-solving process (Carlson & Bloom, 2005).

More research is needed to determine whether the motivation and effort in mathematics of children with ADHD are being adequately measured when it appears they are not accomplishing as much as their peers without ADHD, since emotional competence and lack of ability of these children to identify and regulate their emotions

may negatively impact students with ADHD when it comes to their performance on mathematical tasks (Scime & Norvilitis, 2006).

If I were to replicate this study, I would conduct it over a longer period of time, perhaps following students for an entire academic year. During this time I would meet with students weekly and do informal classroom observations to see if there were any correlations between their problem-solving processes when working one-on-one and in the classroom environment. It would also be helpful to establish a stronger baseline of student problem-solving abilities prior to beginning the study.

### **Study Limitations**

Limitations of this study include limited generalizability because of the small sample size and limited amount of time during which the study was conducted. Since results have not been replicated in many cases, the findings are not as strong as they could be. As stated by Swing et al. (1988), there is also the possibility of interactions between the metacognitive intervention and students' initial aptitudes and abilities that is not accounted for in this study. The participants in this study were all male, leaving one to wonder if the results would differ if subjects were female. These limitations could be addressed using a larger, mixed-gender group of participants over an extended period of time.

While Schoenfeld's timeline model is helpful for focusing on decision making of the problem solver, he does acknowledge some of the limitations of this model (1992b). The model does not indicate the importance of each decision made. While some problem-solving decisions determine the direction of a solution, others may be choices between

two equal ways of finding a solution. Further, decisions that were tacitly made, and those that were not made but should have been, were also not indicated using this tool.

Schoenfeld further discusses challenges in coding as to whether an episode is one of exploration or analysis, and whether a period of exploration should be coded as one longer exploration or as two shorter explorations. While high inter-rater reliability can be achieved using the timeline model, different groups have been found to come to different consensuses.

### **Conclusions**

This study is just the beginning of analyzing students' metacognitive thinking when solving mathematical problems. It adds to the literature by expanding on how elementary school teachers can use Schoenfeld's protocol to assess metacognition. Further, this study begins the process of evaluating similarities and differences in metacognition across and between students who are identified as having ADHD, being Gifted and having ADHD, Gifted, and those who have none of these labels placed on them by the school system. In particular, adding the postproblem-solving debriefing experience enabled obtaining qualitative information from participants as to why they chose to work the way they did and why they made the problem-solving decisions they did. This type of information can be used by teachers to ensure they are helping all students reach their mathematics potential.

## **APPENDIX A. INFORMED CONSENT FORM**

### **“Metacognitive Strategies Employed During Mathematical Problem Solving: A Comparative Case Study of Fifth Graders Who Are Gifted and Have ADHD”**

#### **INFORMED CONSENT FORM**

##### **RESEARCH PROCEDURES**

This research is being conducted to fulfill the requirements of a Ph.D. at George Mason University. If you agree to allow your child to participate, they will be asked to meet individually with the researcher once a week for eight weeks and complete one math problem at each of the sessions. Problem-solving sessions will be videotaped and reviewed by the participant and researcher in order to reflect upon metacognitive processes involved in problem solving and measure any changes and/or growth over time. Students will be asked to reflect on their problem-solving processes. Each problem-solving session is expected to take from 30-60 minutes. Problem-solving sessions will take place before and/or after school or during student lunch times, if acceptable to the student and classroom teacher. If necessary, additional sessions will be held during the summer at a time and place convenient for you and your child.

##### **RISKS**

There are no foreseeable risks for participating in this research.

##### **BENEFITS**

There are no benefits to you or your child as a participant other than to further research in metacognitive processes in the area of mathematical problem solving.

##### **CONFIDENTIALITY**

The data in this study will be confidential. Your child’s name will not be included on the problems they solve or on other collected data; (1) a code will be placed on the work they complete and other collected data; (2) through the use of an identification key, the researcher will be able to link their work to their identity; and (3) only the researcher will have access to the identification key. Any videotapes will be stored in a locked storage cabinet and destroyed upon submission of the completed dissertation.

**PARTICIPATION**

Your child’s participation is voluntary, and your child may withdraw from the study at any time and for any reason. There will be no academic penalty if your son/daughter chooses not to participate in the study or to withdraw from the study. There are no costs to you or any other party.

**CONTACT**

This research is being conducted by Wendy Schudmak, Advanced Academics Resource Teacher at xxxxxx and Ph.D. candidate at George Mason University. She may be reached at (xxx)xxx-xxxx or xxxxxx@xxx.xxx for questions or to report a research-related problem. The faculty advisor for this project is Dr. Jennifer Suh at George Mason University. She may be reached at (xxx)xxx-xxxx. You may contact the George Mason University Office of Research Subject Protections at xxx-xxx-xxxx if you have questions or comments regarding your rights as a participant in the research.

This research has been reviewed according to George Mason University procedures governing your participation in this research.

**CONSENT**

I have read this form and agree to allow my child to participate in this study.

\_\_\_\_\_  
Name

\_\_\_\_\_  
Date of Signature

**PLEASE CHOOSE ONE OF THE OPTIONS BELOW**

\_\_\_\_\_ I agree to videotaping.

\_\_\_\_\_ I do not agree to videotaping.

## APPENDIX B. LETTER OF ASSENT

### Metacognitive Strategies Employed During Mathematical Problem Solving: A Comparative Case Study of Fifth Graders Who are Gifted and Have ADHD

I, \_\_\_\_\_, agree to participate in Ms. Schudmak's research project for George Mason University. I understand that I will meet with Ms. Schudmak eight times and she will ask me to solve math problems. I will do my best to explain my thinking aloud and Ms. Schudmak will videotape me working. Then we will watch the videos together and talk about my problem-solving/metacognitive process.

I understand that meetings with Ms. Schudmak will take place before and/or after school, or during my lunch time in xxxxxx's classroom. If needed, we will meet during the summer at a time and place that is convenient for me and my family.

I will not be given a grade for participating in the project and none of my grades will be affected by how well I do. If at any time I decide I no longer want to be a part of this project, all I have to do is tell Ms. Schudmak and I will no longer have to participate. I will not have any consequences for choosing to participate or not participate. If I want to contact Ms. Schudmak with any questions about this project or if I decide I no longer want to be a part of this study, I can reach Ms. Schudmak at xxxxxx@xxx.xxx or (xxx)xxx-xxxx.

I have read this form and agree to participate in this study.

\_\_\_\_\_  
Name

\_\_\_\_\_  
Date of Signature

## APPENDIX C. QUESTIONS USED DURING PROBLEM-SOLVING SESSIONS

### Session 1 (From Rational Numbers Project Lesson 6)

1. Mr. Hickman made a large apple pie. His daughter ate  $\frac{1}{2}$  of the pie. His son ate  $\frac{1}{3}$  of the pie. Who ate less? Draw a picture to show your thinking.
2. Spinner A was divided into 6 equal parts shaded green. Spinner B was divided into 10 equal parts with 4 parts shaded green. Which spinner had the larger amount of green? Explain “in your own words” your reasoning.
3. Jessica and Kim shared a large pizza. Jessica ate  $\frac{2}{6}$  of the pizza. Kim ate  $\frac{3}{6}$  of the pizza. Who ate more? Draw a picture to show your thinking.
4. Matthew and Cassandra shared a bag of candy. Matthew ate  $\frac{2}{3}$ . Cassandra ate  $\frac{2}{5}$ . Who ate more? Explain your thinking.
5. Andrew spent  $\frac{1}{2}$  of his allowance on candy. Ellen spent  $\frac{1}{3}$  of her allowance on a movie. Is it possible that Ellen spent more than Andrew? Explain.

### Session 2 (From Rational Numbers Project Lesson 14)

Draw pictures to model each story.

1.  $\frac{2}{3}$  of Mr. Vega’s math class are girls. There are 21 students in the class.
2. You can buy a box of 16 gumdrops for 35 cents. If you share the box with three others, what fraction will each receive? How many gumdrops will each receive?
3. William and his friend shared a small pizza evenly. How much did each eat?
4. Jessica and Jennifer shared their bag of M&Ms with LeAnna. If each received a fair share, how much of the bag did Jessica and Jennifer get together?

### Session 3 (From Rational Numbers Project Lessons 15, 16, and 17)

1. Jess ate  $\frac{2}{3}$  of the peanuts in the bag. There are 7 peanuts left. How many did Jess eat? How many were there in the bag originally?

2. Order fractions from smallest to largest. Explain your thinking.  $\frac{6}{7}$ ,  $\frac{2}{3}$ ,  $\frac{99}{100}$ ,  $\frac{9}{10}$ ,  $\frac{3}{4}$
3. Imagine a tower made of 1-inch cubes. You can't see my tower but I will tell you that 12 cubes would be  $\frac{2}{3}$  the height of my tower. How many cubes in my tower?

Session 4 (From Rational Numbers Project Lesson 17)

1. Brenda ate  $\frac{2}{3}$  of a candy bar for lunch. She finished it after and ate  $\frac{1}{3}$  more of a second candy bar of the same type. How much candy did Brenda eat?
2. Marcia's dad was making pancakes. He added  $\frac{2}{3}$  cup milk to the pancake mix. He decided to make a bigger batch so he poured another  $\frac{2}{3}$  cup of milk in. How much milk did he use?
3. The dress designer needed some yellow ribbon for 3 dresses. He needs  $\frac{2}{3}$  yard for one dress,  $\frac{1}{3}$  for another, and  $\frac{2}{3}$  for the third. Draw a picture to show how many yards of ribbon he bought.

Session 5 (From Rational Numbers Project Lesson 18)

1. Margo and Jose shared a couple of large pizzas. Margo ate  $\frac{5}{8}$  of a pizza. Jose ate  $\frac{6}{16}$  of a pizza. Who ate more? Explain how you know.
2. Imagine that you shared your bag of minidoughnuts with your sister. You ate  $\frac{3}{5}$  off the bag while your sister ate  $\frac{4}{10}$  of the bag. Who ate more? Explain how you know.
3. Chou-Mei ran 2 and  $\frac{7}{8}$  miles. Her sister ran 2 and  $\frac{3}{10}$  miles. Who ran the shorter distance? Explain how you know.

Session 6 (From Rational Numbers Project Lesson 19)

1. Marty divided a candy bar into 12 equal parts. He ate  $\frac{1}{6}$  of the candy bar before lunch. He ate  $\frac{1}{4}$  of the candy bar after lunch. Did he eat more or less than  $\frac{1}{2}$  of the candy bar? Did he eat the whole candy bar? Explain your reasoning.
2. Terri ate  $\frac{5}{6}$  of a small pizza and  $\frac{11}{12}$  of another small pizza. Did she eat more than one whole pizza? Explain your reasoning.
3. Alex used  $\frac{1}{3}$  cup of flour in one recipe and  $\frac{1}{4}$  cup of flour in another recipe. Together did he use more than  $\frac{1}{2}$  cup of flour? Explain your reasoning.

Session 7 (From Rational Numbers Project Lesson 21)

1. A clerk sold three pieces of ribbon. The red piece was  $\frac{1}{3}$  of a yard long. The blue piece was  $\frac{1}{6}$  of a yard long. The green piece was  $\frac{10}{12}$  of a yard long.
  - a. How much longer was the green ribbon than the red ribbon?
  - b. How much longer was the green ribbon than the blue ribbon?
  - c. Are the red ribbon and blue ribbon together greater than, less than, or equal in length to the green ribbon?
  - d. If the red and blue together are greater than the green, how much greater are they? If shorter, how much shorter are they?

Session 8 (From Rational Numbers Project Quiz 6)

1. Ty noticed that there was  $\frac{6}{8}$  of a pizza left over. He ate an amount equal to  $\frac{1}{4}$  of the pizza. How much of a whole pizza was left?
2. Is the answer to this problem greater than or less than one? Explain your thinking?  
 $\frac{4}{5} + \frac{1}{6}$
3. Is this a reasonable answer? Explain your thinking.  
 $\frac{8}{9} - \frac{4}{6} = \frac{4}{3}$

## APPENDIX D. TIMELINE GRAPHS

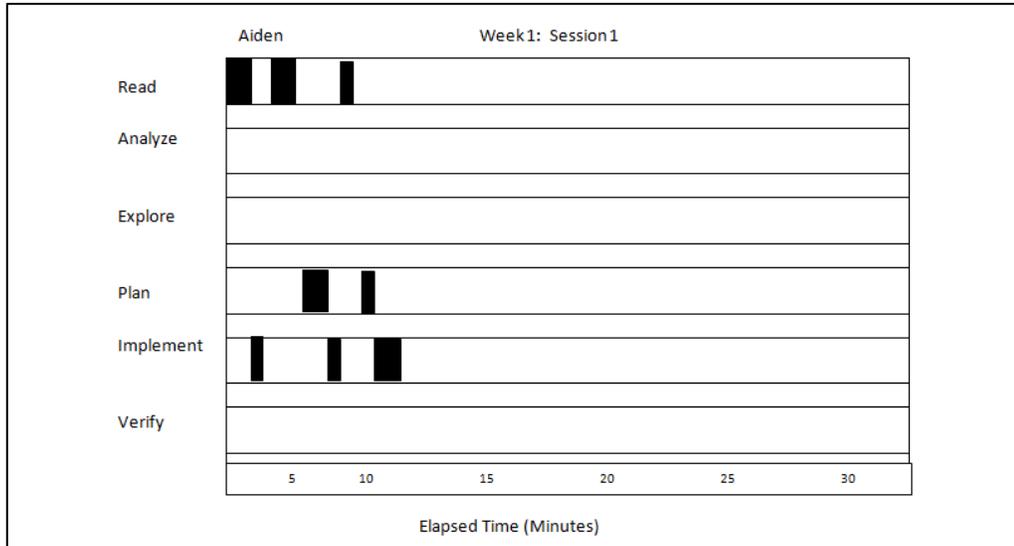


Figure D1. Aiden's problem-solving timeline for the first session.

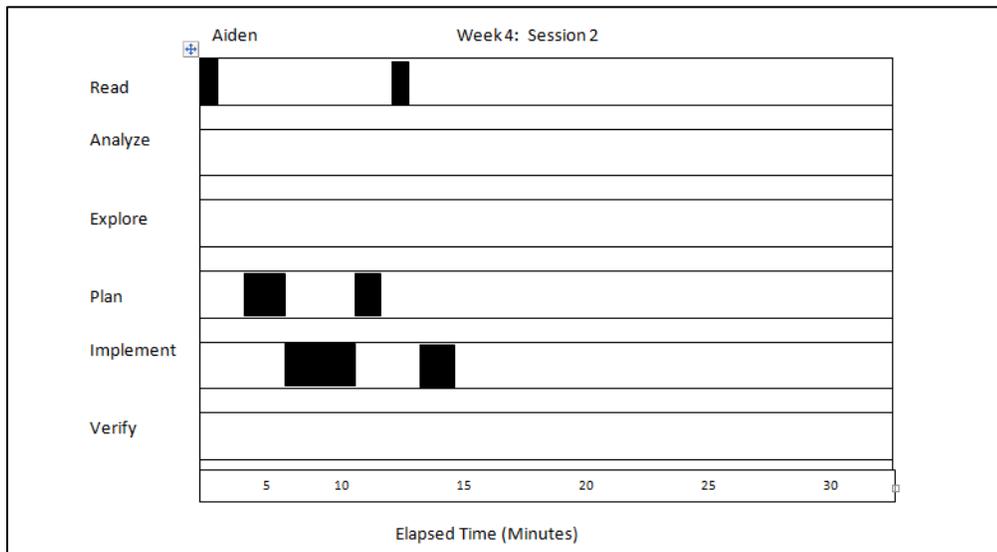


Figure D2. Aiden's problem-solving timeline for the last session.

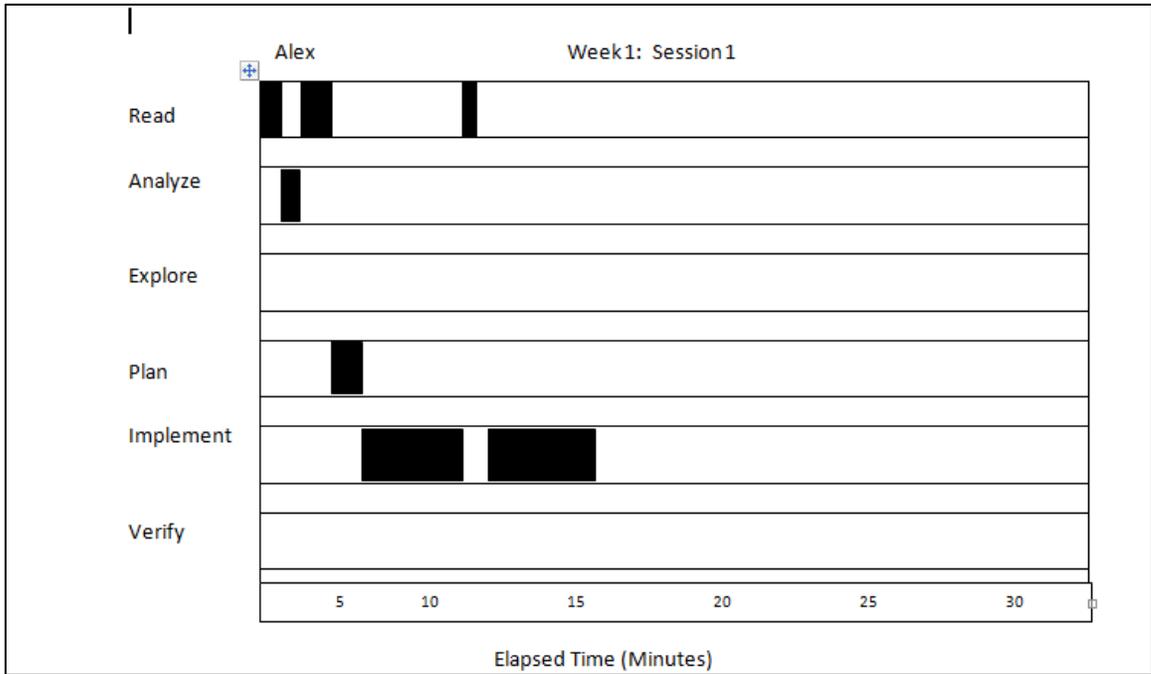


Figure D3. Alex's problem-solving timeline for the first session.

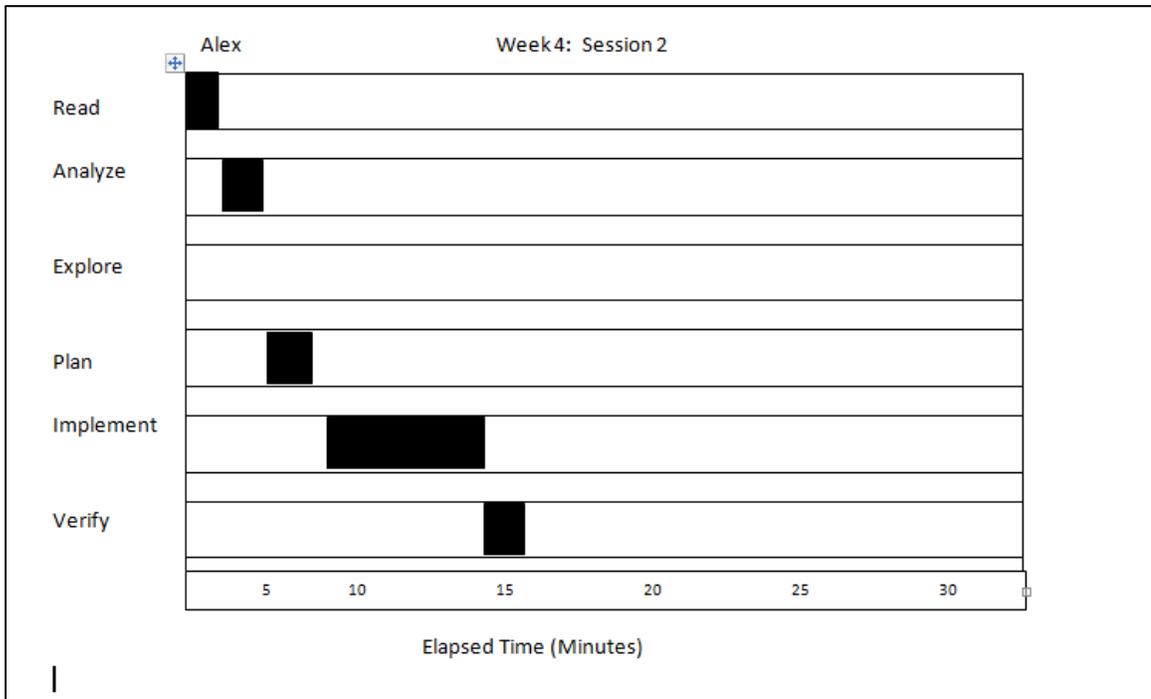


Figure D4. Alex's problem-solving timeline for the last session.

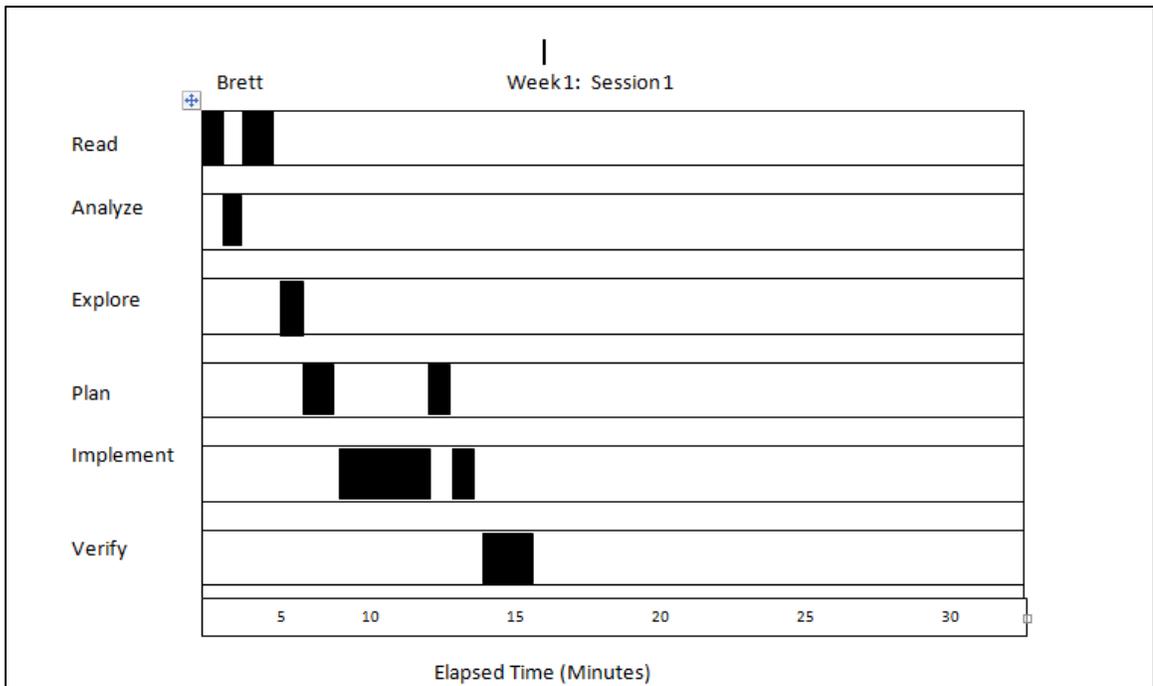


Figure D5. Brett's problem-solving timeline for the first session.

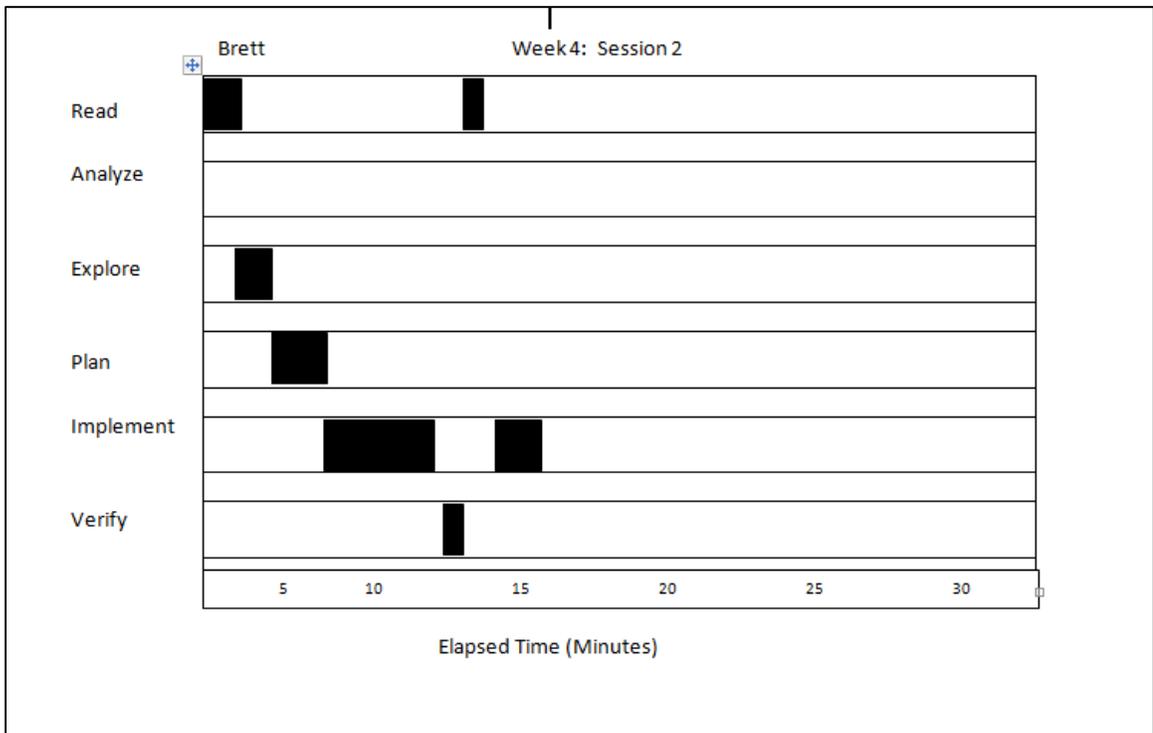


Figure D6. Brett's problem-solving timeline for the last session.

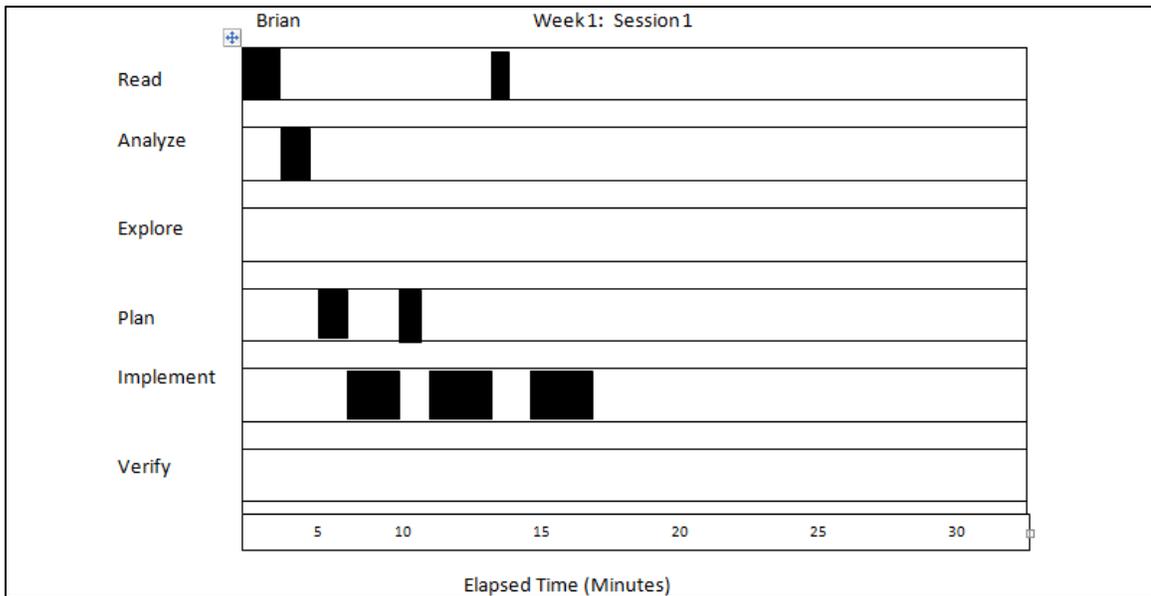


Figure D7. Brian's problem-solving timeline for the first session.

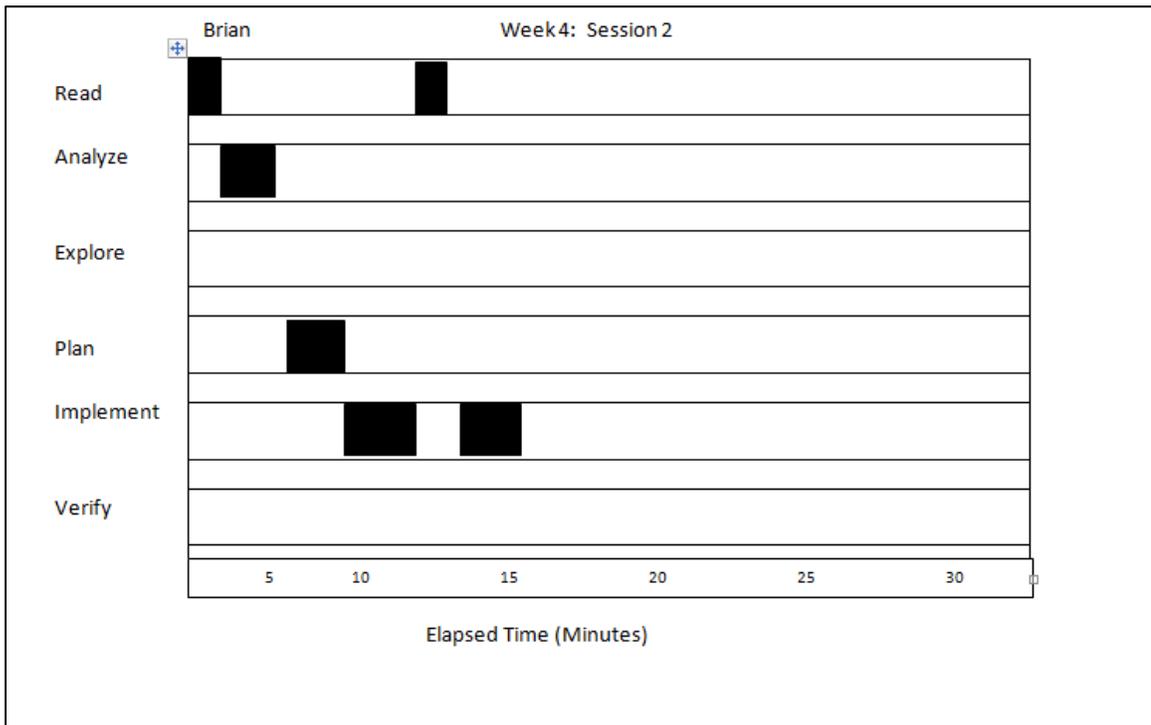


Figure D8. Brian's problem-solving timeline for the last session.

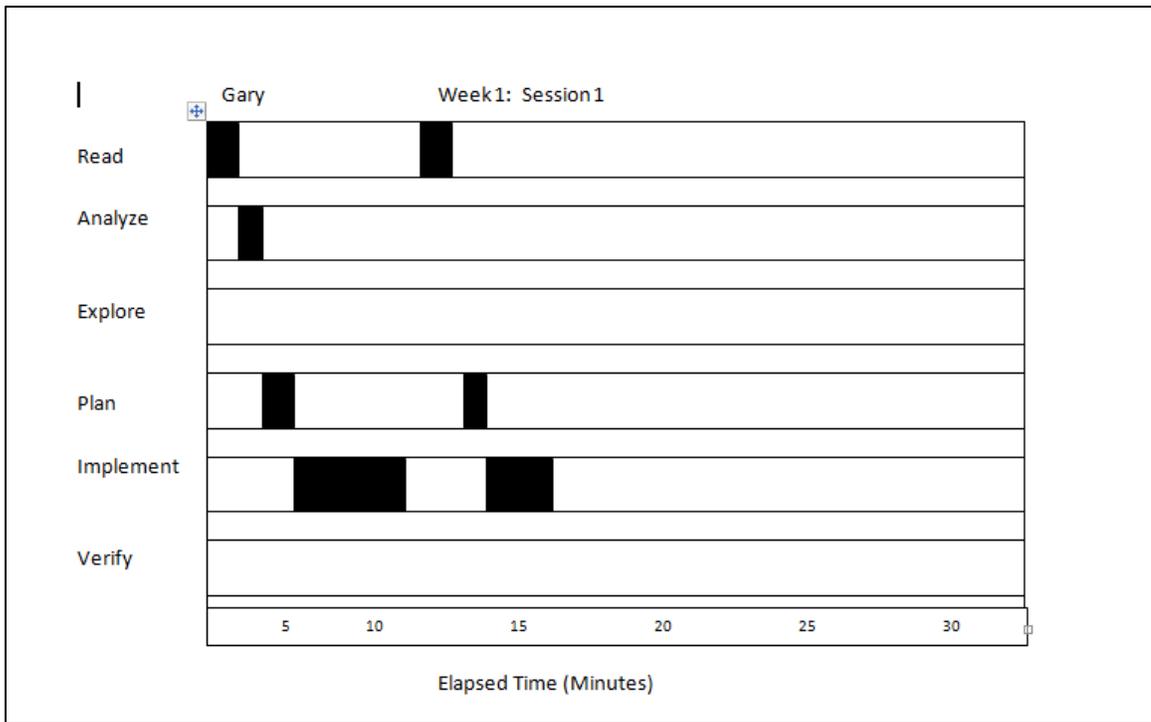


Figure D9. Gary's problem-solving timeline for the first session.

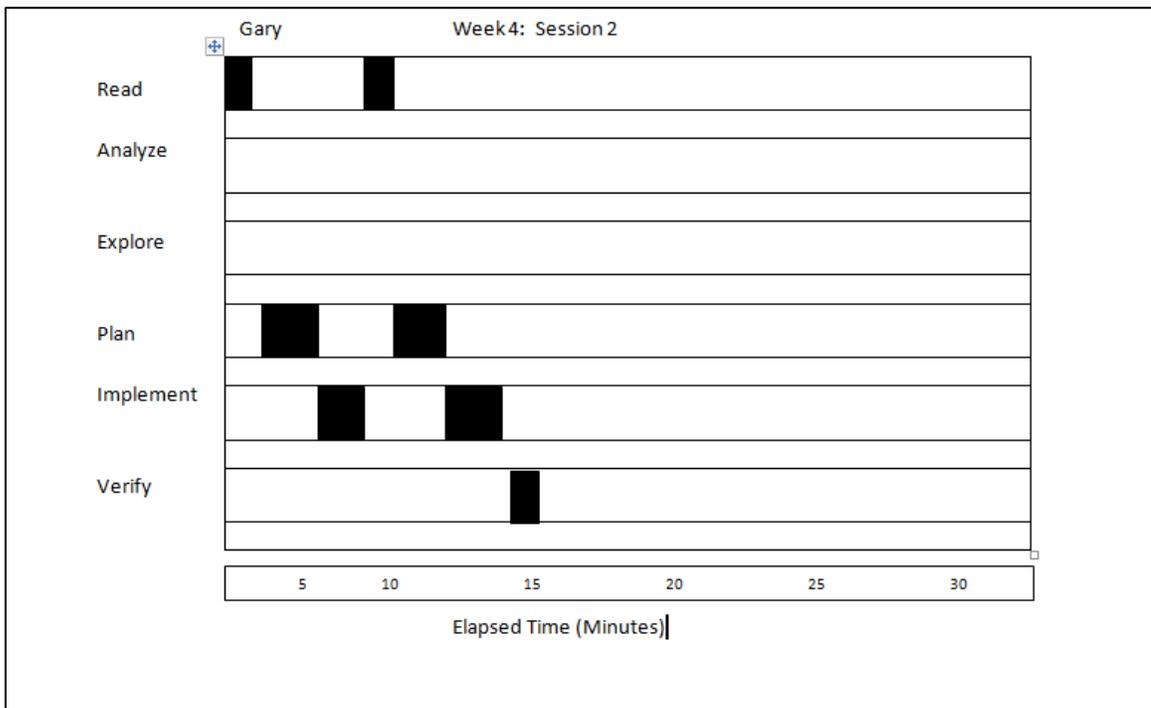


Figure D10. Gary's problem-solving timeline for the last session.

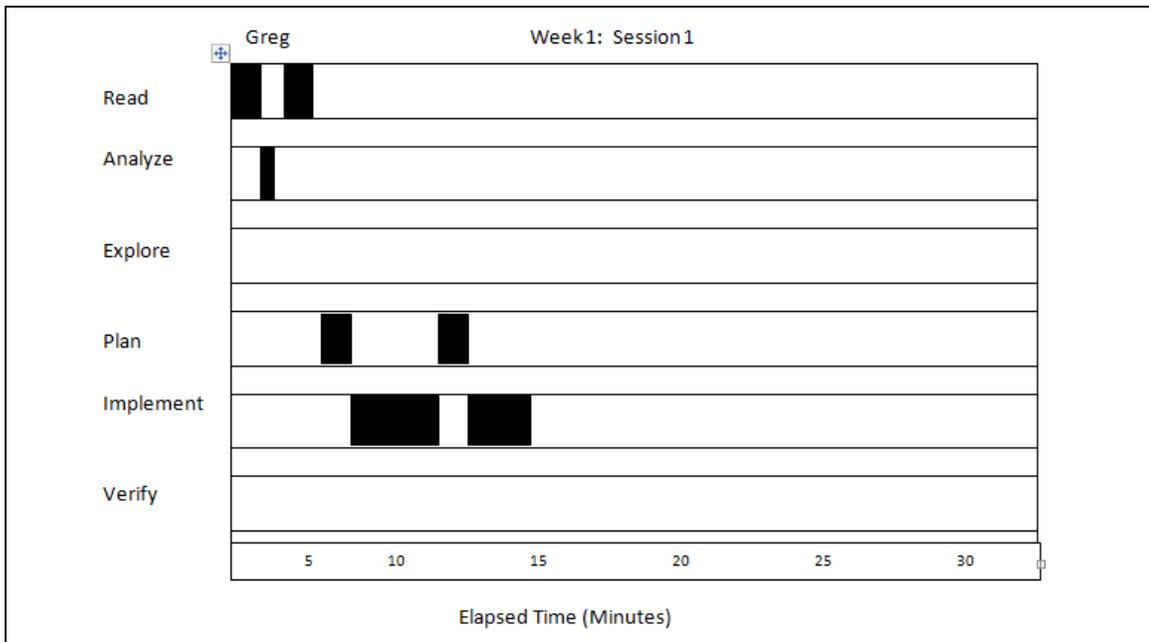


Figure D11. Greg's problem-solving timeline for the first session.

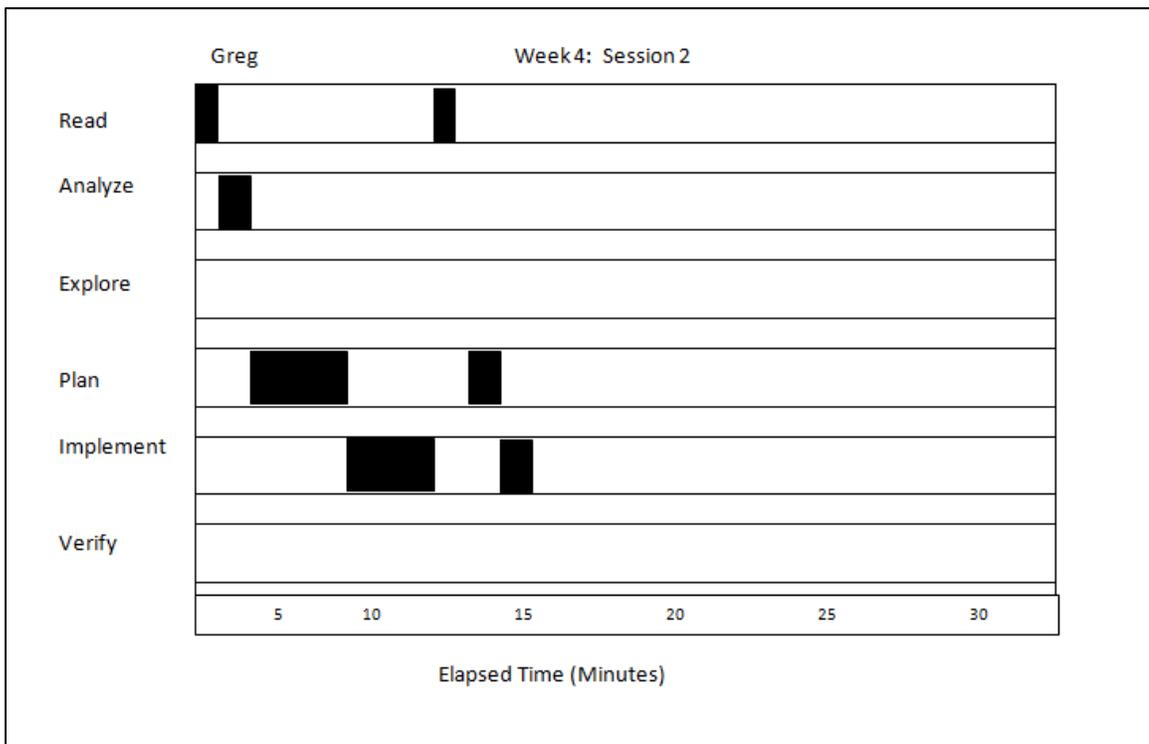


Figure D12. Greg's problem-solving timeline for the last session.

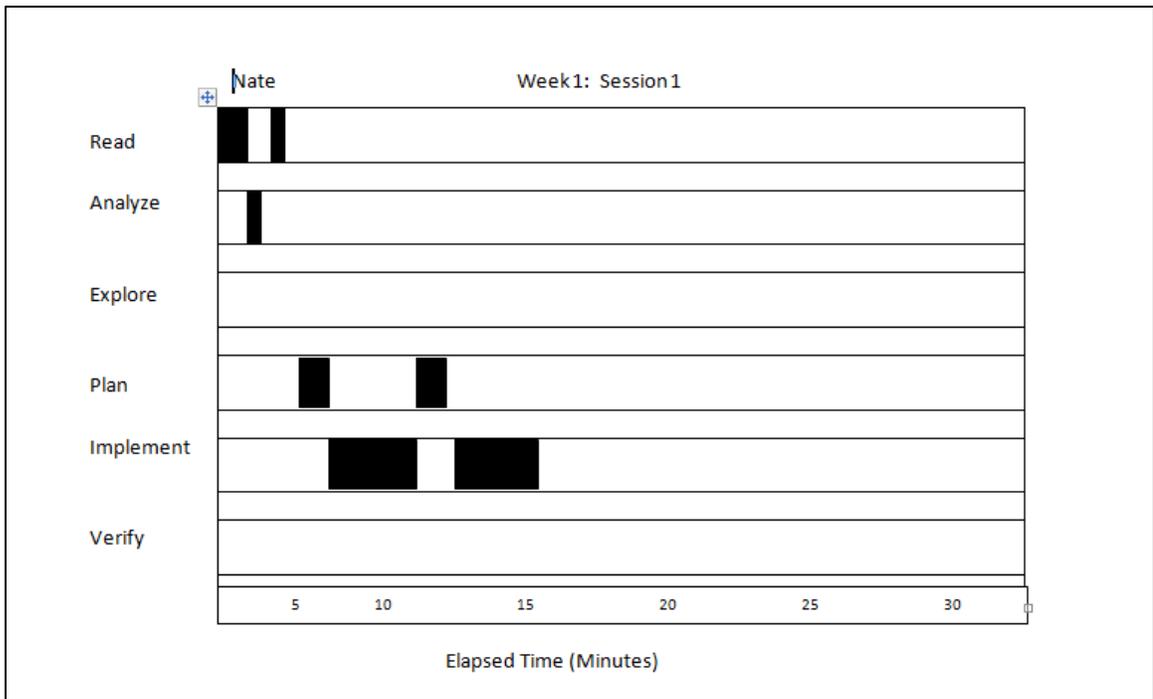


Figure D13. Nate's problem-solving timeline for the first session.

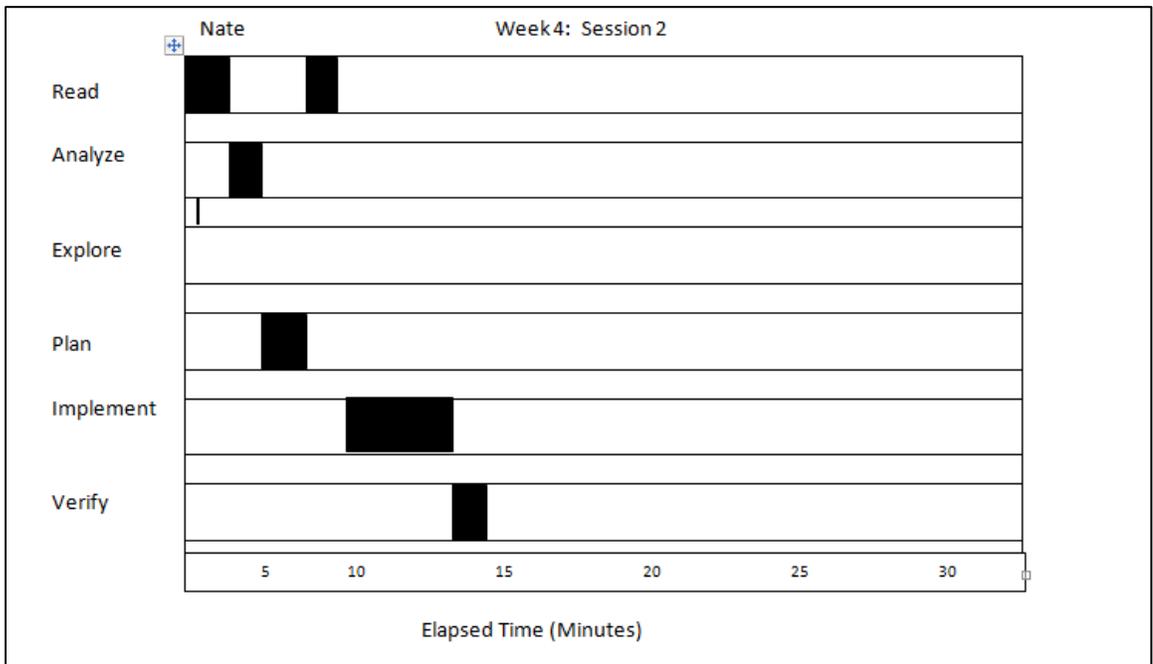


Figure D14. Nate's problem-solving timeline for the last session.

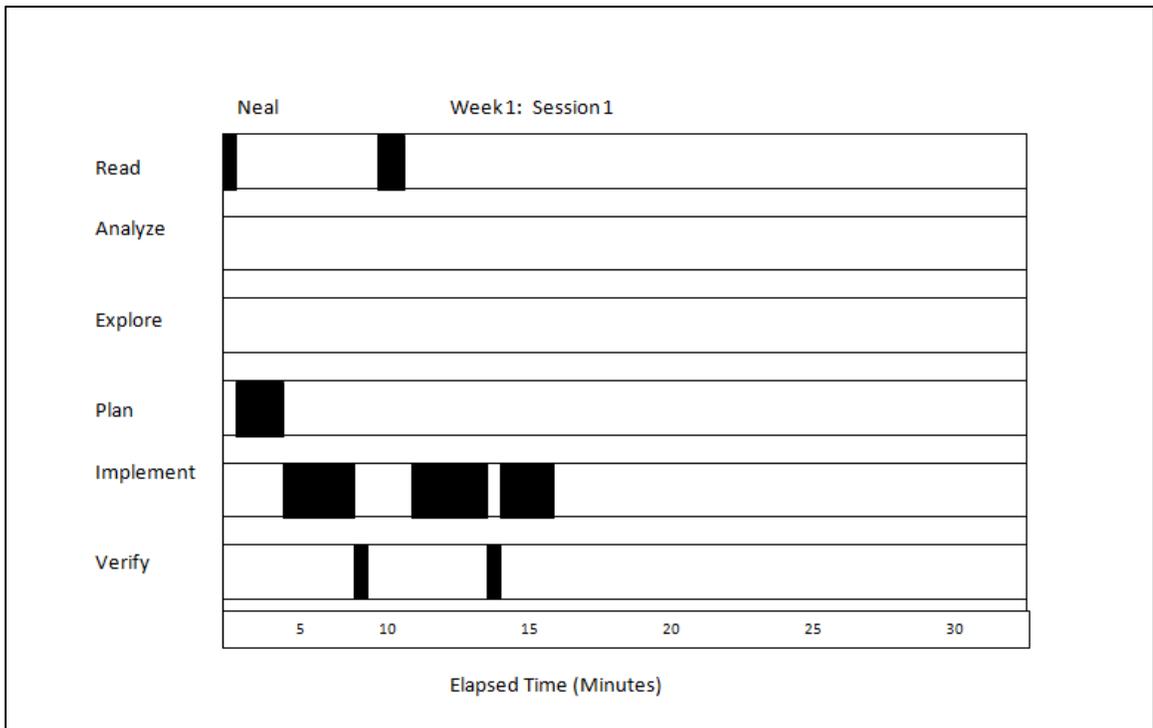


Figure D15. Neal's problem-solving timeline for the first session.

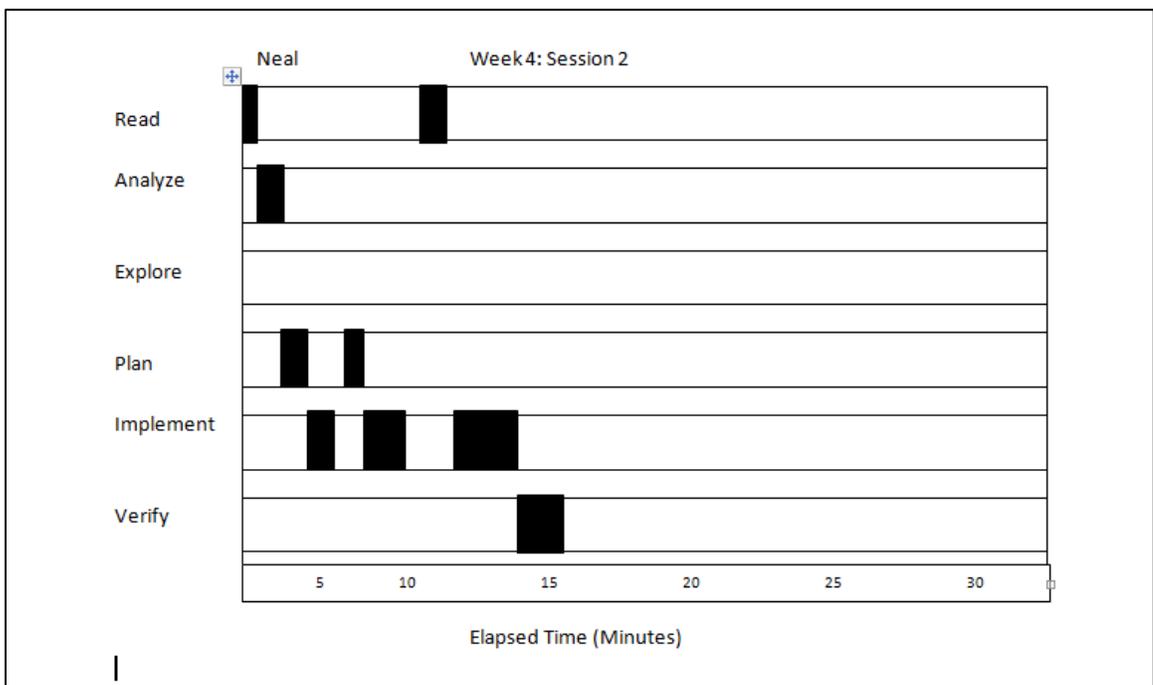


Figure D16. Neal's problem-solving timeline for the last session.

## REFERENCES

- Abrami, P. C., Bernard, R. M., Borokhovski, E., Wade, A., Surkes, M. A., Tamim, R., & Zhang, D. (2008). Instructional interventions affecting critical thinking skills and dispositions: A stage 1 meta-analysis. *Review of Educational Research, 78*, 1102-1134. doi:10.3102/0034654308326084
- American Psychiatric Association. (2013). *Diagnostic and statistical manual of mental disorders* (5<sup>th</sup> ed.). Arlington, VA: American Psychiatric Publishing.
- Antshel, K. M. (2008). Attention-deficit hyperactivity disorder in the context of a high intellectual quotient/giftedness. *Developmental Disabilities, 14*, 293-299.
- Artzt, A. F., & Armour-Thomas, E. (1992). Development of a cognitive-metacognitive framework for protocol analysis of mathematical problem solving in small groups. *Cognition and Instruction, 9*(2), 137-175. doi:10.1207/s1532690xci0902\_3
- Baker, L., & Cerro, L. C. (2000). Assessing metacognition in children and adults. In G. Schraw & J. C. Impara (Eds.), *Issues in the measurement of metacognition* (pp. 99-146). Lincoln, NE: Buros Institute of Mental Measurements.
- Barkley, R. A., DuPaul, G. J., & McMurray, M. B. (1990). Comprehensive evaluation of attention deficit disorder with and without hyperactivity as defined by research criteria. *Journal of Consulting and Clinical Psychology, 58*, 775. doi:10.1037/0022-006X.58.6.775
- Barkley, R. A. (2008). Classroom accommodations for children with ADHD. *The ADHD Report* (pp. 7-10). New York, NY: Guilford Press.
- Barry, T. D., Lyman, R. D., & Klinger, L. G. (2002). Academic underachievement and attention-deficit/hyperactivity disorder: The negative impact of symptom severity on school performance. *Journal of School Psychology, 40*, 259-283. doi:10.1016/S0022-4405(02)00100-0
- Baum, S., & Olenchak, F. R. (2002). The alphabet children: GT, ADHD, and more. *Exceptionality, 10*(2), 77-91. doi:10.1207/S15327035EX1002\_3

- Baum, S. M., & Owen, S. V. (2004). *To be gifted and learning disabled: Strategies for helping bright students with LD, ADHD, and more*. Mansfield Center, CT: Creative Learning Press.
- Baum, S. M., Rizza, M. G., & Renzulli, J. S. (2006). Twice-exceptional adolescents: Who are they? What do they need? In F. Dixon & S. M. Moon (Eds.), *The handbook of secondary gifted education* (pp. 137-164). Waco, TX: Prufrock Press.
- Behr, M., Wachsmuth, I., Post, T., & Lesh, R. (1984). Order and equivalence of rational numbers: A clinical teaching experiment. *Journal for Research in Mathematics Education*, 15, 323-341. doi:10.2307/748423
- Benedetto-Nasho, E., & Tannock, R. (1999). Math computation, error patterns and stimulant effects in children with attention deficit hyperactivity disorder. *Journal of Attention Disorders*, 3(3), 121-134. doi:10.1177/108705479900300301
- Bloom, B. S. (1956). *Taxonomy of educational objectives, handbook 1: The cognitive domain*. New York, NY: David McKay Company Inc.
- Borkowski, J. G., Estrada, M. T., Milstead, M., & Hale, C. A. (1989). General problem-solving skills: Relations between metacognition and strategic processing. *Learning Disability Quarterly*, 12(1), 57-70. doi:10.2307/1510252
- Bryant, D. P., Bryant, B. R., Gersten, R. M., Scammacca, N. N., Funk, C., Winter, A., . . . Pool, C. (2008). The effects of tier 2 intervention on the mathematics performance of first-grade students who are at risk for mathematics difficulties. *Learning Disability Quarterly*, 31(2), 47-63.
- Burden, M. J., & Mitchell, D. B. (2005). Implicit memory development in school-aged children with attention deficit hyperactivity disorder (ADHD): Conceptual priming deficit? *Developmental Neuropsychology*, 28, 779-807. doi:10.1207/s15326942dn2803\_3
- Calderhead, J. (1981). Stimulated recall: A method for research on teaching. *British Journal of Educational Psychology*, 51, 211-217. doi:10.1111/j.2044-8279.1981.tb02474.x
- Carlson, M. P., & Bloom, I. (2005). The cyclic nature of problem solving: An emergent multidimensional problem-solving framework. *Educational Studies in Mathematics*, 58(1), 45-75. doi:10.1007/s10649-005-0808-x
- Cavanagh, S. (2007). What kind of math matters? *Education Week*, 26(40), 21-23.

- Children and Adults With Attention-Deficit/Hyperactivity Disorder. (n.d.). *What Is ADHD or ADD?* National Resource Center on AD/HD. Retrieved from <http://www.help4adhd.org/en/about/what>
- Chi, M. T., Basook, M., Lewis, M. W., Reimann, P., & Glasser, R. (1989). Self-explanations: How students study and use examples in learning to solve problems. *Cognitive Science, 13*(2), 145-182. doi:10.1207/s15516709cog1302\_1
- Chiu, M. M. (2004). Adapting teacher interventions to student needs during cooperative learning: How to improve student problem solving and time on task. *American Educational Research Journal, 41*(2), 365-399.
- Clark, B. (1997). *Growing up gifted: Developing the potential of children at home and at school*. New York, NY: Simon and Schuster.
- Coleman, L. J. (1985). *Schooling the gifted*. Menlo Park, NJ: Addison-Wesley.
- Corno, L., & Mandinach, E. (1983). The role of cognitive engagement in classroom learning and motivation. *Educational Psychology, 19*(2), 88-108. doi:10.1080/00461528309529266
- Cramer, K., Behr, M., Post, T., & Lesh, R. (2009). Rational number project: Initial fraction ideas. Originally published in 1997 as Rational Number Project: Fraction Lessons for the Middle Grades-Level 1, Dubuque, IA: Kendall/Hunt Publishing Co.
- Cross, T. L. (1997). Psychological and social aspects of educating gifted students. *Peabody Journal of Education, 72*, 180-200. doi:10.1080/0161956X.1997.9681873
- Dimitriadis, C. (2012). How are schools in England addressing the needs of mathematically gifted children in primary classrooms? A review of practice. *Gifted Child Quarterly, 56*, 59-76. doi:10.1177/0016986211433200
- Desoete, A., Roeyers, H., & Buysse, A. (2001). Metacognition and mathematical problem solving in grade 3. *Journal of Learning Disabilities, 34*, 435-449. doi:10.1177/002221940103400505
- Dillon, R. F., & Osborne, S. S. (2006). Intelligence and behavior among individuals identified with attention deficit disorders. *Exceptionality, 14*(1), 3-18. doi:10.1207/s15327035ex1401\_2

- DuPaul, G. J. (2007). School-based interventions for students with attention deficit hyperactivity disorder: Current status and future directions. *School Psychology Review, 36*(2), 183-194.
- DuPaul, G. J., Ervin, R. A., Hook, C. L., & McGoey, K. E. (1998). Peer tutoring for children with attention deficit hyperactivity disorder: Effects on classroom behavior and academic performance. *Journal of Applied Behavior Analysis, 31*, 579-592. doi:10.1901/jaba.1998.31-579
- DuPaul, G. J., Jitendra, A. K., Volpe, R. J., Tresco, K. E., Lutz, J. G., Junod, R. E. V., . . . Mannella, M. C. (2006). Consultation-based academic interventions for children with ADHD: Effects on reading and mathematics achievement. *Journal of Abnormal Child Psychology, 34*, 635-648. doi:10.1007/s10802-006-9046-7
- DuPaul, G. J., & Stoner, G. (2003). *ADHD in the schools: Assessment and intervention strategies* (2<sup>nd</sup> ed.). New York, NY: Guilford.
- DuPaul, G. J., & Weyandt, L. L. (2006). School-based interventions for children and adolescents with attention-deficit/hyperactivity disorder: Enhancing academic and behavioral outcomes. *Education and Treatment of Children, 29*, 341-358.
- DuPaul, G. J., Volpe, R. J., Jitendra, A. K., Lutz, J. G., Lorah, K. S., & Gruber, R. (2004). Elementary school students with AD/HD: Predictors of academic achievement. *Journal of School Psychology, 42*, 285-301. doi:10.1016/j.jsp.2004.05.001
- Efklides, A., Papadaki, M., Papantoniou, G., & Kiosseoglou, G. (1997). Effects of cognitive ability and affect on school mathematics performance and feelings of difficulty. *The American Journal of Psychology, 110*(2), 225-258. doi:10.2307/1423716
- Eisenberg, D., & Schneider, H. (2007). Perceptions of academic skills of children diagnosed with ADHD. *Journal of Attention Disorders, 10*(4), 390-397. doi:10.1177/1087054706292105
- Elbaum, B., & Vaughn, S. (2001). School-based interventions to enhance the self-concept of students with learning disabilities: A meta-analysis. *Elementary School Journal, 101*, 303-329. doi:10.1086/499670
- Elementary and Secondary Education Act of 1965, Pub. L. No. 107-110 as amended, Title 1, Part A; 20 U.S.C. 6301-6339, 6571-6578 (2002).
- Ennis, R. H. (1989). Critical thinking and subject specificity. *Educational Researcher, 18*(3), 4-10. doi:10.3102/0013189X018003004

- Faraone, S. V., Biederman, J., Monuteaux, M. C., Doyle, A. E., Seidman, L. J. (2001). A psychometric measure of learning disability predicts educational failure four years later in boys with attention-deficit hyperactivity disorder. *Journal of Attention Disorders, 4*(4), 220-230. doi:10.1177/108705470100400404
- Fennema, E. H. (1972). The relative effectiveness of a symbolic and a concrete model in learning a selected mathematical principle. *Journal for Research in Mathematics Education, 3*, 233-238. doi:10.2307/748490
- Fertig, C. (2009). *Raising a gifted child: A parenting success handbook*. Waco, TX: Prufrock Press.
- Fisher, D., & Frey, N. (2007). *Checking for understanding: Formative assessment techniques for your classroom*. Alexandria, VA: Association for Supervision and Curriculum Development.
- Flavell, J. H. (1976). Metacognitive aspects of problem solving. In L. B. Resnick (Ed.), *The nature of intelligence*, (pp. 231-236). Hillsdale, NJ: Erlbaum.
- Forness, S. R., Kavale, K. A., Sweeney, D. P., & Crenshaw, T. M. (1999). The future of research and practice in behavioral disorders: Psychopharmacology and its school treatment implications. *Behavioral Disorders, 24*, 305-318.
- Fortunato, I., Hecht, D., Tittle, C. K., & Alvarez, L. (1991). Metacognition and problem solving. *The Arithmetic Teacher, 39*(4), 38-40.
- Franklin, R. D., Allison, D. B., & Gorman, B. S. (1997). Introduction. In R. D. Franklin, D. B. Allison, & B. S. Gorman (Eds.), *Design and analysis of single-case research* (pp. 1-12). Mahwah, NJ: Lawrence Erlbaum.
- Freyberg, P. S. (1966). Cognitive development in Piagetian terms in relation to school attainment. *Journal of Educational Psychology, 57*, 164-168. doi:10.1037/h0023411
- Frid, S., & Malone, J. A. (1995). Negotiation of meaning in mathematics classrooms: A study of two year 5 classes. *Mathematics Education Research Journal, 7*(2), 132-147. doi:10.1007/BF03217281
- Fuchs, L. S., & Fuchs, D. (1998). General educators' instructional adaptation for students with learning disabilities. *Learning Disability Quarterly, 21*(1), 23-33. doi:10.2307/1511370

- Fuchs, L. S., Fuchs, D., Powell, S. R., Seethaler, P. M., Cirino, P. T., & Fletcher, J. M. (2008). Intensive intervention for students with mathematics disabilities: Seven principles of effective practice. *Learning Disability Quarterly, 31*(2), 79-92.
- Gagne, F. (2002). Toward a differentiated model of giftedness and talent. In N. Colangelo & G. A. Davis (Eds.), *Handbook of gifted education* (3<sup>rd</sup> ed., pp. 65-80). Boston, MA: Allyn and Bacon.
- Gardner, H. (1983). *Frames of mind: The theory of multiple intelligences*. New York, NY: Basic Books.
- Garofalo, J., & Lester, F., Jr. (1985). Metacognition, cognitive monitoring, and mathematical performance. *Journal for Research in Mathematics Education, 16*(3), 163-176. doi:10.2307/748391
- Gavin, M. K., Casa, T. M., Firmender, J. M., & Carroll, S. R. (2013). The impact of advanced geometry and measurement curriculum units on the mathematics achievement of first-grade students. *Gifted Child Quarterly, 57*(71). doi:10.1177/0016986213479564
- Gillham, B. (2000). *Case study research methods*. New York, NY: Continuum.
- Goldin, G. (2000). A scientific perspective on structures, task-based interviews in mathematics education research. In A. E. Kelly and R. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 517-545). New Jersey: Lawrence Erlbaum.
- Goldring, E. B. (1990). Assessing the status of information on classroom organizational frameworks for gifted students. *The Journal of Educational Research, 83*, 313-326.
- Goos, M., & Galbraith, P. (1996). Do it this way! Metacognitive strategies in collaborative mathematical problem solving. *Educational Studies in Mathematics, 30*, 229-260.
- Goos, M., Galbraith, P., & Renshaw, P. (2002). Socially mediated metacognition: Creating collaborative zones of proximal development in small group problem solving. *Educational Studies in Mathematics, 49*(2), 193-223. doi:10.1023/A:1016209010120
- Gourgey, A. F. (1998). Metacognition in basic skills instruction. *Instructional Science, 26*, 81-96. doi:10.1023/A:1003092414893

- Harwood, D. (1995). The pedagogy of the world studies 8-13 project: The influence of the presence/absence of the teacher upon primary children's collaborative group work. *British Educational Research Journal*, 21, 587-611.  
doi:10.1080/0141192950210504
- Hatfield, L. L. (1978). Heuristical emphases in the instruction of mathematical problem solving. In L. L. Hatfield & D. A. Bradbard (Eds.), *Mathematical problem solving: Papers from a research workshop* (pp. 21-42). Columbus, OH: ERIC Information Analysis Center for Science, Mathematics and Environmental Education.
- Hattie, J., Biggs, J., & Purdie, N. (1996). Effects of learning skills interventions on student learning: A meta-analysis. *Review of Educational Research*, 66(2), 99-136. doi:10.3102/00346543066002099
- Hattie, J. (1992). *Self-concept*. Hillsdale, NJ: Erlbaum.
- Healey, D., & Rucklidge, J. J. (2005). An exploration into the creative abilities of children with ADHD. *Journal of Attention Disorders*, 8(3), 88-95.  
doi:10.1177/1087054705277198
- Hegarty, M., Mayer, R. E., & Monk, C. A. (1995). Comprehension of arithmetic word problems: A comparison of successful and unsuccessful problem solvers. *Journal of Educational Psychology*, 87(1), 18-32. doi:10.1037/0022-0663.87.1.18
- Heid, M. K. (1983). Characteristics and special needs of the gifted student in mathematics. *The Mathematics Teacher*, 76(4), 221-226.
- Hiebert, J., & Carpenter, T. P. (1982). Piagetian tasks as readiness measures in mathematics instruction: A critical review. *Educational Studies in Mathematics*, 13(3), 329-345. doi:10.1007/BF00311247
- Hohn, R. L., & Frey, B. (2002). Heuristic training and performance in elementary mathematical problem solving. *The Journal of Educational Research*, 96(6), 374-380. doi:10.1080/00220670209596612
- Hooks, K., Millich, R., & Lorch, E. P. (1994). Sustained and selective attention in boys with attention deficit hyperactivity disorder. *Journal of Clinical Child Psychology*, 23(1), 69-77. doi:10.1207/s15374424jccp2301\_9
- Individuals With Disabilities Act Amendments (IDEA) of 1997, Pub. L. No. 105-17. (1997).
- Individuals With Disabilities Education Act, 20 U.S.C.§1400 (2004).

Jacob K. Javits Act of 1998, 20 U.S.C. 1001 § 702 (1998).

Jacob K. Javits Gifted and Talented Students Education Program. (2007). Available: <http://www.ed.gov/programs/javits/index.html>.

Johns, D. (1998, June). Helping students with ADHD solve math problems. *Alert: The Official Newsletter of the Association on Higher Education and Disability*, 22(3), 14-17.

Kalyuga, S. (2008). Relative effectiveness of animated and static diagrams: An effect of learner prior knowledge. *Computers in Human Behavior*, 24, 852-861. doi:10.1016/j.chb.2007.02.018

Kapa, E. (2001). A metacognitive support during the process of problem solving in a computerized environment. *Educational Studies in Mathematics*, 47, 317-336. doi:10.1023/A:1015124013119

Kercood, S., Grskovic, J. A., Lee, D. L., & Emmert, S. (2007). The effects of fine motor movement on tactile stimulation on the math problem solving of students with attention problem. *Journal of Behavioral Education*, 16, 303-310. doi:10.1007/s10864-007-9042-1

Kercood, S., Zentall, S. S., & Lee, D. L. (2004). Focusing attention to deep structure in math problems: Effects on elementary education students with and without attentional deficits. *Learning and Individual Differences*, 14, 91-105. doi:10.1016/j.lindif.2003.09.001

Kilpatrick, J. (1985). Doing mathematics without understanding it: A commentary on Higbee and Kunihira. *Educational Psychologist*, 20(2), 65-68. doi:10.1207/s15326985ep2002\_2

Konstantopoulos, S., Modi, M., & Hedges, L. V. (2001). Who are America's gifted? *American Journal of Education*, 109(3), 344-382. doi:10.1086/444275

Kontos, S. (1983). Adult-child interaction and the origins of metacognition. *The Journal of Educational Research*, 77(1), 43-54.

Kramarski, B., Mevarech, Z. R., & Arami, M. (2002). The effects of metacognitive instruction on solving mathematical authentic tasks. *Educational Studies in Mathematics*, 49(2), 225-250. doi:10.1023/A:1016282811724

- Krathwohl, D. R. (1998). *Methods of educational and social science research: An integrated approach*. New York, NY: Addison-Wesley Educational Publisher, Inc.
- Kroll, D. L. (1988). Cooperative mathematical problem-solving and metacognition: A case study of three pairs of women. *Dissertation Abstracts International*, 49(10), 2958. (UMI No. 8902580)
- Kuhn, D. (2000) Metacognitive development. *Current Directions in Psychological Science*, 9(5), 178-181. doi:10.1111/1467-8721.00088
- Kuhn, D., & Dean, D., Jr. (2004). Metacognition: A bridge between cognitive psychology and educational practice. *Theory Into Practice*, 43(4), 268-273. doi:10.1207/s15430421tip4304\_4
- Kuhn, D., & Ho, V. (1980). Self-directed activity and cognitive development. *Journal of Applied Developmental Psychology*, 1(2), 119-133. doi:10.1016/0193-3973(80)90003-9
- Kulik, J. A., & Kulik, C. I. (1988). Timing of feedback and verbal learning. *Review of Educational Research Journal*, 21, 79-97. doi:10.3102/00346543058001079
- Leroux, J. A. (2000). The gifted child with attention deficit disorder: An identification and intervention challenge. *Roeper Review*, 22(3), 171-176. doi:10.1080/02783190009554028
- Lester, F. K. (1980). Research on mathematical problem solving. In R. J. Shumway (Ed.), *Research in mathematics education* (pp. 286-323). Reston, VA: National Council of Teachers of Mathematics.
- Lester, F. K., Garofalo, J., & Kroll, D. L. (1989). Self-confidence, interest, beliefs, and meta-cognition: Key influences on problem-solving behavior. In D. B. McLeod & V. M. Adams (Eds.), *Affect and mathematical problem solving* (pp. 75-88). New York, NY: Springer-Verlag. doi:10.1007/978-1-4612-3614-6\_6
- Lewandowski, L. J., Lovett, B. J. Parolin, R., Gordon, M., & Coddling, R. S. (2007). Extended time accommodations and the mathematics performance of students with and without ADHD. *Journal of Psychoeducational Assessment*, 25(1), 17-27. doi:10.1177/0734282906291961
- Lucangeli, D., & Cabrele, S. (2006). Mathematical difficulties and ADHD. *Exceptionality*, 14(1), 53-62. doi:10.1207/s15327035ex1401\_5

- Lucangeli, D., & Cornoldi, C. (1997). Arithmetic education and learning in Italy. *Journal of Learning Disabilities, 37*(1), 42-49.
- Lyon, G. R. (1996). Learning disabilities. *The Future of Children, 1*(6), 54-76.
- Maccini, P., & Hughes, C. A. (2000). Effects of a problem-solving strategy on the introductory algebra performance of secondary students with learning disabilities. *Learning Disabilities Research and Practice, 15*, 10-21.  
doi:10.1207/SLDRP1501\_2
- Maher, C. A., & Davis, R. B. (1990). Building representations of children's meanings. *Journal for Research in Mathematics Education, 4*, 79-95.
- Maker, C. J., Neilson, A. B., & Rodgers, J. A. (1994). Giftedness, diversity, and problem-solving. *Teaching Exceptional Children, 27*, 4-19.
- Marshall, R. M., Schafer, V. A., O'Donnell, L., Elliott, J., & Handwerk, M. L. (1999). Arithmetic disabilities and ADD subtypes: Implications for DSM-IV. *Journal of Learning Disabilities, 32*(3), 239-247. doi:10.1177/002221949903200305
- Maslow, A. H. (1962). *Toward a psychology of being*. Princeton, NJ: Van Nostrand.
- Mathematics Assessment Project: Assessing 21<sup>st</sup> Century Math. (2012). *Professional Development*. Retrieved from Shell Center for Mathematical Education  
<http://map.mathshell.org/materials/background.php?subpage=pd#wherepd>
- Mautone, J. A., DuPaul, G. J., & Jitendra, A. K. (2005). The effects of computer-assisted instruction on the mathematics performance and classroom behavior of children with ADHD. *Journal of Attention Disorders, 9*(1), 301-312.  
doi:10.1177/1087054705278832
- Maykut, P. S., & Morehouse, R. (1994). *Beginning qualitative research: A philosophic and practical guide*. Washington, DC: Falmer Press.
- McCoach, D. B., Kehle, T. J., Bray, M. A., & Siegle, D. (2004). The identification of gifted students with learning disabilities: Challenging, controversies, and promising practices. In T. M. Newman & R. J. Sternberg (Eds.), *Students with both gifts and learning disabilities* (pp. 31-48). New York, NY: Kluwer.  
doi:10.1007/978-1-4419-9116-4\_3
- Meloth, M. S. & Deering, P. D. (1999). The role of the teacher in promoting cognitive processing during collaborative learning. In A. M. O'Donnell & A. King (Eds.), *Cognitive perspectives on peer learning* (pp. 235-255). Mahwah, NJ: Erlbaum.

- Mendaglio, S. (1995). Children who are gifted/ADHD. *Gifted Child Today*, 18, 37-40.
- Merriam, S. B. (1998). *Qualitative research and case study: Applications in education, revised and expanded from case study research in education*. San Francisco, CA: Jossey-Bass Publishers.
- Miller, S. A. (1980). Assessment of conservation of number: An attempt to enhance ecological validity and diagnostic sensitivity. *Genetic Psychology Monographs* 101, 271-285.
- Miranda, A., Jarque, S., & Tarrage, R. (2006). Interventions in school settings for students with ADHD. *Exceptionality*, 14(1), 35-52.  
doi:10.1207/s15327035ex1401\_4
- Montague, M. (1992). The effects of cognitive and metacognitive strategy instruction on the mathematical problem solving of middle school students with learning disabilities. *Journal of Learning Disabilities*, 25, 230-248.  
doi:10.1177/002221949202500404
- Moseley, B. (2005). Students' early mathematical representation knowledge: The effects of emphasizing single or multiple perspectives of the rational number domain in problem solving. *Educational Studies in Mathematics*, 60(1), 37-69.  
doi:10.1007/s10649-005-5031-2
- National Association for Gifted Children. (n.d.). *What is gifted?* Retrieved from <http://www.nagc.org/>
- National Association for Gifted Children. (2010). *2010 Pre k-grade 12 gifted programming standards*. Retrieved from [http://nagc.org/uploadedFiles/Information\\_and\\_Resources/Gifted\\_Program\\_Standards/K-12%20programming%20standards.pdf](http://nagc.org/uploadedFiles/Information_and_Resources/Gifted_Program_Standards/K-12%20programming%20standards.pdf)
- National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.
- National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: Author.
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
- National Research Council. (1989). *Everybody counts: A report to the nation on the future of mathematics education*. Washington, DC: National Academy Press.

- Niemi, D. (1996). Assessing conceptual understanding in mathematics: Representations, problem solutions, justifications, and explanations. *The Journal of Educational Research*, 89, 351-363. doi:10.1080/00220671.1996.9941339
- Passolunghi, M. C., Marzocchi, G. M., & Fiorillo, F. (2005). Selective effect of inhibition of literal or numerical irrelevant information in children with attention deficit hyperactivity disorder (ADHD or arithmetic learning disorder (ALD)). *Developmental Neuropsychology*, 28(3), 731-753. doi:10.1207/s15326942dn2803\_1
- Polya, G. (1957). *How to solve it*. Princeton, NJ: Princeton University Press.
- Public Law 107-110 (January 8, 2002) *The elementary and secondary education act (The no child left behind act of 2001)*. Retrieved May 30, 2005, from <http://www.ed.gov/policy/elsec/leg/esea02/index.html>.
- Pugalee, D. K. (2001). Writing, mathematics, and metacognition: Looking for connections through students' work in mathematical problem solving. *School Science and Mathematics*, 101(5), 236-245. doi:10.1111/j.1949-8594.2001.tb18026.x
- Purdie, N., Hattie, J., & Carroll, A. (2002). A review of the research on interventions of attention deficit hyperactivity disorder: What works best? *Review of Educational Research*, 72(1), 61-99. doi:10.3102/00346543072001061
- Raphael, D., & Wahlstrom, M. (1989). The influence of instructional aids on mathematics achievement. *Journal for Research in Mathematics Education*, 20(2), 173-190. doi:10.2307/749281
- Rehabilitation Act of 1973 (RA) (29 U.S.C. § 794(a), 34 C.F.R. § 104.4(a))
- Reis, S. M., & McCoach, D. B. (2000). The underachievement of gifted students: What do we know and where do we go? *Gifted Child Quarterly*, 44, 152-170. doi:10.1177/001698620004400302
- Renzulli, J. S. (1975, October). Media Reviews. [Review of the book *Thinking goes to school: Piaget's theory in practice*, by H. G. Furth & H. Wachs]. *Exceptional Children*, 42(2), 105-106.
- Renzulli, J. S. (1977). *The enrichment triad model: A guide for developing defensible programs for the gifted and talented*. Mansfield Center, CT: Creative Learning Press.

- Ridge, H. L. & Renzulli, J. S. (1981). Teaching mathematics to the talented and gifted. In J.G. Vincents (Ed). *The mathematical education of exceptional children and youth* (pp. 91-266). Reston, VA: National Council of Teachers of Mathematics.
- Rief, S. A. (2005). *How to reach and teach children with ADD/ADHD* (2<sup>nd</sup> ed.). San Francisco, CA: Jossey-Bass Teacher.
- Rimm, S. B. (1997). *Smart parenting: How to parent so children will learn*. New York, NY: Crown Publishing.
- Ruban, L. M., & Reis, S. M. (2005). Identification and assessment of gifted students with learning disabilities. *Theory Into Practice*, 44(2), 115-124.  
doi:10.1207/s15430421tip4402\_6
- Sagvolden, T., & Archer, T. (1989). *Attention deficit disorder: Clinical and basic research*. Hillsdale, NJ: Lawrence Erlbaum.
- Schoenfeld, A. H. (1982). Measures of problem-solving performance and of problem-solving instruction. *Journal for Research in Mathematics Education*, 13(1), 31-49. doi:10.2307/748435
- Schoenfeld, A. H. (1983). Episodes and executive decisions in mathematical problem solving. In R. Lesh & M. Landau (Eds.), *Acquisition of mathematics concepts and processes* (pp. 345-395). New York, NY: Academic Press.
- Schoenfeld, A. H. (1984). Beyond the purely cognitive: Belief systems, social cognitions and metacognitions as driving forces in intellectual performance. *Cognitive Science*, 7, 329-363. doi:10.1207/s15516709cog0704\_3
- Schoenfeld, A. H. (1985). *Mathematical problem solving*. Orlando, FL: Academic Press.
- Schoenfeld, A. H. (1989a). Explorations of students' mathematical beliefs and behavior. *Journal for Research in Mathematics Education*, 20(4), 338-355.  
doi:10.2307/749440
- Schoenfeld, A. H. (1989b). Ideas in the air: Speculations on small group learning, environmental and cultural influences on cognition and epistemology. *International Journal of Educational Research*, 13, 71-88. doi:10.1016/0883-0355(89)90017-7
- Schoenfeld, A. H. (1992a). Learning to think mathematically: Problem solving, metacognition and sense making in mathematics. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334-370). New York, NY: Macmillan.

- Schoenfeld, A. H. (1992b). On paradigms and methods: What do you do when the ones you know don't do what you want them to? Issues in the analysis of data in the form of videotapes. *The Journal of the Learning Sciences*, 2(2), 179-214. doi:10.1207/s15327809jls0202\_3
- Schoenfeld, A. H. (2002). Making mathematics work for all children: Issues of standards, testing, and equity. *Educational Researcher*, 31(1), 13-25. doi:10.3102/0013189X031001013
- Schoenfeld, A. H. (2007). What is mathematical proficiency and how can it be assessed? In A. H. Schoenfeld (Ed.), *Assessing Mathematical Proficiency*. (pp. 59-73). New York, NY: Cambridge University Press.
- Scime, M., & Norvilitis, J. M. (2006). Task performance and response to frustration in children with attention deficit hyperactivity disorder. *Psychology in the Schools*, 43(3), 377-386. doi:10.1002/pits.20151
- Shimabukuro, S. M., Prater, M. A., Jenkins, A., & Edelen-Smith, P. (1999). The effects of self-monitoring of academic performance on students with learning disabilities and ADD/ADHD. *Education and Treatment of Children*, 22(4), 397-414.
- Shimamura, A. P. (2000). Review: What is metacognition? The brain knows. *The American Journal of Psychology*, 113(1), 142-146. doi:10.2307/1423465
- Shore, B. M., & Kaizer, C. (1989). The training of teachers for gifted pupils. *Canadian Journal of Education*, 14(1), 74-87. doi:10.2307/1495202
- Smutny, J. F. (2001). *Stand up for your gifted child: How to make the most of kids' strengths at school and at home*. Minneapolis, MN: Free Spirit Publishing.
- Snyder, K. E., Nietfeld, J. L., & Linnenbrink-Garcia, L. (2001). Gifted and metacognition: A short-term longitudinal investigation of metacognitive monitoring in the classroom. *Gifted Child Quarterly*, 55(3), 81-93.
- Span, P., & Overtoom-Corsmit, R. (1986). Information processing by intellectually gifted pupils solving mathematical problems. *Educational Studies in Mathematics*, 17(3), 273-295. doi:10.1007/BF00305074
- Stacey, K. (1992). Mathematical problem solving in groups: Are two heads better than one? *Journal of Mathematical Behavior*, 11, 261-275.
- Stahr, B., Cushing, D., & Lane, K. (2006). Efficacy of a function-based intervention in decreasing off-task behavior exhibited by a student with ADHD. *Journal of*

- Positive Behavior Interventions*, 8(4), 201-211.  
doi:10.1177/10983007060080040301
- Sternberg, R. J. (2003). What is an expert student? *Educational Researcher*, 32, 5-9.  
doi:10.3102/0013189X032008005
- Stewart, W. (2007). Electronic assistive technology for 2e students. *2e Newsletter*.  
Retrieved from <http://www.2eNewsletter.com>
- Stewart, W. (2010). Providing the support a 2e student needs. *2e Newsletter*  
(January/February). Retrieved from <http://www.2eNewsletter.com>
- Strip, C. A., & Hirsch, G. (2000). *Helping gifted children soar*. Scottsdale, AZ: Great Potential Press.
- Suydam, M. (1980). Untangling clues from research on problem solving. In S. Krulik (Ed.), *Problem solving in school mathematics: 1980 Yearbook of the National Council of Teachers of Mathematics*. Reston, VA: The National Council of Teachers of Mathematics.
- Swanson, H. L. & Trahan, M. (1990). Naturalistic memory in learning disabled children. *Learning Disability Quarterly*, 13(2), 82-95. doi:10.2307/1510653
- Swing, S. S., Stoiber, K. C., & Peterson, P. L. (1988). Thinking skills versus learning time: Effects of alternative classroom-based interventions on students' mathematics problem solving. *Cognition and Instruction*, 5(2), 123-191.  
doi:10.1207/s1532690xci0502\_2
- Tang, E. P. & Ginsburg, H. P. (1999). Young children's mathematical reasoning: A psychological view. In L. Stiff & F. R. Curcio (Eds.), *Developing mathematical reasoning in grades K-12: 1999 Yearbook of the National Council of Teachers of Mathematics*. Reston, VA: The National Council of Teachers of Mathematics.
- Terman, L. M., & Oden, M. H. (1959). *Genetic studies of genius: Vol. 5: The gifted group at mid-life*. Stanford, CA: Stanford University Press.
- Trainin, G., & Swanson, H. L. (2005). Cognition, metacognition, and achievement of college students with learning disabilities. *Learning Disability Quarterly*, 28(4), 261-272. doi:10.2307/4126965
- Vaidya, S. R. (1994). Gifted children with learning disabilities: Theoretical implications and instructional challenge. *Education*, 113, 568-573.

- VanTassel-Baska, J. (1991). Serving the disabled gifted through educational collaboration. *Journal for the Education of the Gifted*, 14(3), 246-266.
- VanTassel-Baska, J., & Stambaugh, T. (2005). Challenges and possibilities for serving gifted learners in the regular classroom. *Theory Into Practice*, 44(3), 211-217. doi:10.1207/s15430421tip4403\_5
- Virginia Department of Education. (2009). *Virginia standards of learning: Grade five, mathematics*. Retrieved from [http://www.doe.virginia.gov/testing/sol/standards\\_docs/mathematics/2009/stds\\_math5.pdf](http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/2009/stds_math5.pdf)
- Virginia Department of Education. (2010). *Understanding the Virginia regulations governing educational services for gifted students*. Richmond, VA: Department of Education, Commonwealth of Virginia.
- Volpe, R. J., DuPaul, G. J., DiPerna, J. C., Jitendra, A. K., Lutz, J. G., Tresco, K., & Junod, V. (2006). Attention deficit hyperactivity disorder and scholastic achievement: A model of mediation via academic enablers. *School Psychology Review*, 35(1), 47-61.
- Vygotsky, L. S. (1978). *Mind in society*. Cambridge, MA: Harvard University Press.
- Watkins, C. (2008). Between two worlds: The gifted student with AD/HD. *2e Newsletter*, March/April 2008. Retrieved from <http://www.2eNewsletter.com>
- Webb, N. M. (1982). Peer interaction and learning in cooperative small groups. *Journal of Educational Psychology*, 74, 642-655. doi:10.1037/0022-0663.74.5.642
- Webb, J. T., & Latimer, D. (1993). ADHD and children who are gifted. *Exceptional Children*, 60(2), 183-184.
- Yimer, A. (2004). Metacognitive and cognitive functioning of college students' mathematical problem solving. *Dissertation Abstracts International*, 65(04). (UMI No. 3128290)
- Yin, R. K. (1984). *Case study research: Design and methods: Applied Social Research Methods Series* (Vol. 5). Beverly Hills, CA: Sage Publications.
- Yin, R. K. (2003). *Case study research, design and methods* (3<sup>rd</sup> ed.). Newbury Park, CA: Sage Publications.
- Zentall, S. S. (1993). Research on the educational implications of attention deficit hyperactivity disorder. *Exceptional Children*, 60(2), 143-153.

- Zentall, S. S., & Ferkis, M. A. (1993). Mathematical problem solving for youth with ADHD, with and without learning disabilities. *Learning Disability Quarterly*, *16*(1), 6-18. doi:10.2307/1511156
- Zentall, S. S., Moon, S. M., Hall, A. M., & Grskovic, J. A. (2001). Learning and motivational characteristics of boys with AD/HD and/or giftedness. *Exceptional Children*, *67*(4), 499-519.
- Zhu, X., & Simon, H. A. (1987). Learning mathematics from examples and by doing. *Cognition and Instruction*, *4*, 137-166. doi:10.1207/s1532690xci0403\_1
- Zimmerman, B. J. (2000). Attaining self-regulation: A social cognitive perspective. In M. Boekaerts, P. R. Pintrich, & M. Ziedner (Eds.), *Handbook of self-regulation* (pp. 13-39). San Diego, CA: Academic Press. doi:10.1016/B978-012109890-2/50031-7

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Wendy Schudmak graduated from Benjamin Franklin Senior High School, New Orleans, Louisiana, in 1994. She received her Bachelor of Science from Boston University, Boston, Massachusetts in 1998 and her Master of Arts in Teaching from Simmons College, Boston, Massachusetts in 1999. Wendy has led e-workshops for the National Council of Teachers of Mathematics and presented at a variety of conferences. She was employed as an elementary school teacher in Annapolis, MD, for two years, followed by six years as an elementary school classroom teacher and then teacher of Talented and Gifted students in Alexandria City, VA. Wendy has worked in Fairfax County, VA, as an Advanced Academics Resource Teacher and is currently a Mathematics Resource Teacher/Instructional Leader.