

A CASE STUDY OF THE ADOPTION OF AN INNOVATIVE MATHEMATICAL  
TEACHING PRACTICE (USING ANCHORING CONTEXTUALIZED PROBLEMS)  
BY A SMALL GROUP OF ALGEBRA II TEACHERS: A DIFFUSION OF  
INNOVATION ANALYSIS

by

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A Case study of the Adoption of an Innovative Mathematical Teaching Practice (Using  
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## **DEDICATION**

I dedicate this dissertation to my loving wife Carrie for all her support throughout my studies. She was a constant presence who gave me time and space to complete this task as well as a driving force that kept me going when I would waiver in my commitment to finish. I look forward to the years ahead when neither of us is in school.

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## LIST OF ABBREVIATIONS

Anchored Contextualizing Problem.....	ACP
United States .....	US
National Council of Teachers of Mathematics .....	NCTM
Models and Modeling .....	MM

## **ABSTRACT**

**A CASE STUDY OF THE ADOPTION OF AN INNOVATIVE MATHEMATICAL TEACHING PRACTICE (USING ANCHORING CONTEXTUALIZED PROBLEMS) BY A SMALL GROUP OF ALGEBRA II TEACHERS: A DIFFUSION OF INNOVATION ANALYSIS**

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This study explores how cognitively demanding tasks administered prior to units of study (Anchored Contextualizing Problems – ACPs) impact the concerns and perspectives of high school, Algebra II teachers towards Standards-based approaches to teaching mathematics. Participants were observed prior to the study in order to gauge their teaching styles as traditional or reform-oriented. After receiving the first ACP, the participants completed a survey to determine their thoughts, opinions, and interest in the ACP. A professional development (professional development) on the first ACP was conducted and the survey was administered again. The participants were individually interviewed to more individually gauge their perspectives on ACPs and Standards-based approaches. After the participants administered the first ACP to their students, they reconvened as a group to discuss their thoughts and opinions on the administration and to

learn about the second ACP. Prior to administering the second ACP to their students, the participants completed the survey a final time. The participants all met again after the administration of the second ACP for a final group discussion. Finally, a second round of individual interviews was conducted to gauge changes in the participants' concerns and perspectives towards ACPs. Results indicate that the participants adopted the ACPs for the following reasons: (a) They were directly related to the skills covered in the units of study for which the ACPs were created; (b) they served as referencing tools when introducing a topic in a unit of study; and (c) they helped fulfill the participants' desires to incorporate deeper meaning, reflective of NCTM's Process Standards (2000), into their teaching practices. This study will also relate Rogers' (2003) theory on diffusion of innovations to the methods used in this study as a means of introducing ways for diffusing other educational innovations to educators.

## CHAPTER I: INTRODUCTION

Human society has reached a point where future technology and innovation now require more than a select, few individuals to lead the way in our advancement as a species. All branches of science have splintered off into hundreds, if not thousands, of various faculties that each require thousands of innovative individuals to help keep pace with the rate of growth. Compound this with the need for an even larger number of skilled laborers to assist in the dissemination of these technological innovations, and we find ourselves in need of an exponentially, growing number of mathematically literate people with the ability to reason, think conceptually, contextualize situations, and “think outside the box.”

From a purely economic standpoint, there is evidence to suggest that advancement of mathematical literacy in students can have an impact on a country’s Gross Domestic Product (GDP). The Organization for Economic Co-Operation and Development (OECD) created the Program for International Student Assessment (PISA), which assesses students in 70 countries on their problem-solving abilities to apply mathematics in real-world contexts. According to OECD’s website<sup>1</sup>, students of the United States (US) ranked 32nd among the 65 participating countries on the exam. Hanushek and Woessmann (2010) designed a model that relates the scores of these mathematics exams

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<sup>1</sup> <http://www.oecd.org/pisa/keyfindings/pisa-2012-results-overview.pdf>

to future growth of the GDP. Their correlation suggests that a 25-point average rise in scores on the PISA exam for an individual country can account for an increase of 115 trillion USD over the lifetime of the generation born in 2010. These findings seem to indirectly support that learning mathematics from a contextualized, problem-based approach anchored in real-world experiences is not only important, but also fiscally advantageous for today's societies.

In order to meet the growing demand for innovative technology, our educational systems must rise to the challenge of graduating a larger number of mathematically literate students. To do this, educators must foster reasoning, thinking, and the ability to contextualize problem situations with their students. Educators, unfortunately, are not achieving these goals at the rate necessary to meet society's demands.

In 1989, as a means of addressing the gap mentioned above, the National Council of Teachers of Mathematics (NCTM) developed standards for mathematics that outline what students should know and be able to do by graduation from high school: (a) that they learn to value mathematics; (b) they become confident in their ability to do mathematics; (c) they become mathematical problem solvers; (d) they communicate mathematically; and (e) they reason mathematically. In 1991, NCTM published the *Standards for Teaching Mathematics Professional Standards*, which called for a less dominate role of the use of computational algorithms, manipulation of expressions, and paper-and-pencil drill, and instead called for school curricula to incorporate more rigorous exploration of geometry, measurement, statistics, probability, algebra, and functions. More specifically, they stated that students should encounter and use more

mathematics within the context of genuine problems and situations. In 2001, NCTM released the *Principles and Standards for School Mathematics*, which further extended the standards to incorporate six, core principles that all mathematics curricula should address: (a) equity; (b) curriculum; (c) teaching; (d) learning; (e) assessment; and (f) technology. These principles are not meant to be specific mathematics content or processes, but guides for curricular frameworks, instruction, and lesson planning. The Common Core State Standards Initiative (2009) has elaborated on these mathematical principles and stated that students should learn to do the following: (a) Make sense of problems and persevere in solving them; (b) reason abstractly and quantitatively; (c) construct viable arguments and critique the reasoning of others; (d) model with mathematics; (e) use appropriate tools strategically; (f) attend to precision; (g) look for and make use of structure; (h) look for and express regularity in repeated reasoning.

Ultimately, these publications outline what is intended for a *Standards*-based approach to learning mathematics and how administrators, curriculum writers, teachers, and students should approach their efforts to develop, foster, teach, and learn mathematics. Above all else, a Standards-based approach to learning mathematics is a framework that should guide instruction such that *all* students are able to problem solving, reason and prove, communicate, make connections, and represent mathematical concepts in a way meaningful to themselves (NCTM refers to these abilities as *Process Standards*<sup>2</sup>). In this dissertation, I work from the assumption that a Standard-based

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<sup>2</sup> The Common Core State Standards in the previous paragraph were partially inspired by NCTM's Process Standards.



approach to learning mathematics is necessary to increase students' level of mathematical ability commensurate with the rising demands of our advancing society.

### **Background of the Problem**

For decades, the US has been trying to advance our students' mathematics and science abilities to generate a large pool of knowledgeable citizens capable of tackling needed advances within a technological society. Beginning with the Manhattan Project, Sputnik, and the ensuing Space Race, the U.S. government has taken significant steps at increasing the number of students pursuing Science, Technology, Engineering, and Mathematics (STEM) fields. Additionally, the country has attempted to change what students know about mathematics, how they know it, and the rate at which they learn it (Tomlinson, 1987).

### **PISA**

The OECD is a consortium of 70 countries that created the Programme for International Student Assessment (PISA) with the intention of comparing the progress of students in the countries making up 90% of the world's economy. The PISA assessment covers reading, mathematics, and science, and measures students' abilities in these content areas at seven levels, with *Below Level 1* being the lowest possible score and *Level 6* the highest. The assessment is given to 15-year-old students in each country and/or participating educational system, and it geared towards students' ability problem solve in real world context (PISA, 2009).

At the higher end of the measurement scale in mathematics (Levels 5 & 6), the questions are typically unfamiliar and require a degree of reflection and creativity. Such

problems require that students interpret complex and unfamiliar data and construct mathematical models of real-world situations. In the middle of this scale (Levels 3 & 4), questions require students to interpret situations, restate them, and assign mathematical language to them. Examples include using scale distances on a map, spatial reasoning in understanding geometric figures, and speed and time calculations. At the lower end of the scale (Levels 1 & 2), the questions are simpler and require a limited amount of interpretation of a situation and the mathematical concepts describing them.

Of the 65 educational systems taking part in the 2009 assessment, the US ranked 32<sup>nd</sup> with an average score of 487. In the 2009 assessment, a Level 5 score was between 633 and 708; and the 2009 average OECD score was 496. Approximately 14.7% of U.S. students reached a Level 5 or higher, compared with Singapore's 36% and China-Shanghai's 50% (PISA, 2009).

### **What It Means**

While comparing the success rate of U.S. students on assessments measuring their ability to problem-solve to those of other nations like Singapore is informative, it only highlights the need to increase the mathematical literacy of students in the US. Salzman and Lowell (2008) point out that the actual number of high-performing students is most important in the global economy. Comparing the PISA results between the large, U.S. student population (14.7%) to the small, Singapore population (36%) still yields a larger number of capable U.S. students.

Though this fact paints a more positive picture for the U.S.'s economic and innovational future, it does not negate the fact that the US can do better for our students.

In order to increase mathematical literacy, the US must study how mathematics is being taught and identify changes that will have a positive impact on the problem-solving abilities of the nation's students. It is, therefore, imperative that we approach mathematics education in a manner that fosters reasoning and problem solving – from a Standards-based approach.

A promising approach to teaching mathematics with a Standards-based approach is through a models and modeling (MM) perspective on learning mathematics. This perspective views teaching in a way that many find backwards – that instruction should *begin* with “big” problems instead of the skills and procedures supposedly needed to solve such a big problem at the end of a unit of study. These big problems, specifically model-eliciting activities (MEAs), are designed to encourage students to deeply consider the problem, devise their own model, share/express their model with other students, test it, and revise it as needed. Ultimately, MM problems foster NCTM's Process Standards, and aim to make students problem-solvers instead of robotic fact-spouters.

There is a significant body of evidence that suggests modeling activities are quite successful for most students (Lesh & Doerr, 2003; Lesh & Zawojewski, 2007; Schorr & Koellner-Clark, 2003; Schorr & Lesh, 2003; Zawojewski & Lesh, 2003), yet few teachers know of or use this approach. It is disappointing that most U.S. mathematics teachers have some level of awareness of NCTM's Standards-based approach to learning, but according to Hiebert (2005), little has changed in the way mathematics is taught in the US since its first publication in 1989.

At the heart of this dilemma are teachers' beliefs about how students learn to think mathematically. Their beliefs span from a complete disapproval of any NCTM principle to total adherence to, and application of, its principles (Remillard, 2006; Senk & Thompson, 2003). Between these contrasting stances exist a multitude of reasons that hinders educators from adopting a Standards-based approach to learning mathematics (Hoyle, 1992; Raymond, 1997; Skott, 2001; Sztajn, 2006).<sup>3</sup> There is research that shows a positive impact on teachers' beliefs about Standards-based approaches to learning after undergoing training in professional development programs (Ambrose, 2004; Barlow, 2012; Fennema et al., 1996; Grant, 1998; Guskey, 2002; Raymond, 1997; Staub & Stern, 2002), but the vast majority of the studies are done on willing teachers who have already adopted this approach to teaching in one form or another. Further, most of the studies have been conducted on K-8 teachers and not high school teachers. If a Standards-based approach is what is needed to advance students to a level of mathematical ability needed for today's society, how then are teachers' beliefs influenced such that they are willing to make even a small change – commensurate with a Standards-based approach – in the way they teach?

### **Purpose of the Study**

Though there are a significant number of studies related to teachers' beliefs and Standards-based approaches to learning mathematics, few, if any, focus on or include unwilling, skeptical, or unaware teachers (Hoyle, 1992; Raymond, 1997; Remillard, 2006; Senk & Thompson, 2003; Skott, 2001; Sztajn, 2006). Many of the studies

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<sup>3</sup> A detailed description of teachers' beliefs, as well as models and modeling, will be given in chapter II.

mentioned above demonstrate significant growth in teachers' beliefs and perspectives towards teaching mathematics in alignment with the goals of NCTM. There is a clear line of reasoning established that teachers can substantially develop their teaching practices towards a more Standards-based approach after they have re-aligned their beliefs about mathematics with those approaches. What must be more thoroughly considered is how or when do teachers re-align their beliefs, or is it even possible for certain teachers to do so.

The purpose of this study was for participant teachers to use a single, Standards-based approach similar to MEAs, called Anchored Contextualizing Problems (ACPs), in two units of study in high school Algebra II classes. I designed a survey to gauge teachers' concerns, opinions and perspectives on the use of ACPs. The largest source of data came from interviews conducted with each participant. The questions asked in the interviews sought to answer the study's research questions as thoroughly as possible. Using a mixed methods approach grounded in a diffusion of innovations (Rogers, 2003) framework for analysis, I sought to answer the following questions:

1. How do high school, Algebra II teachers' concerns and perspectives about Standards-based approaches to teaching problem solving change after implementing Anchoring Contextualizing Problems in two units of study?
2. What do teachers learn about their students' mathematical capabilities when using a Standards-based tool such as ACPs?
3. What are factors that promote adoption, or the lack thereof, for innovations such as ACPs?

## Definitions

There are frequently used terms in this dissertation that are found in several literature sources: *Standards-based approaches*, *problem solving*, *Anchored Contextualizing Problems*<sup>4</sup>, and *beliefs*. I offer the following definitions of these terms in order to align their meaning with those of the readers’.

### Standards-Based Approach to Teaching and Learning Mathematics

The word, standards, can evoke a number of definitions. To some, it may be a series of skills and concepts a student learns in a particular mathematics course. To others, it may be a guideline for teachers to follow to prepare students for the next level of mathematics. According to NCTM (1989), “...it should be understood that the standards are value judgments based on a broad, coherent vision of schooling derived from several factors: societal goals, student goals, research on teaching and learning, and professional experience (p. 4).”

Some believe that understanding mathematics is being able to perform arithmetic and algebraic procedures and memorize geometric principles. NCTM’s *Curriculum and Evaluation Standards* (p. 7) state that *knowing* mathematics means *doing* mathematics. Specifically, “A person gathers, discovers, or creates knowledge in the course of some activity having purpose.” This active process differs from mastering concepts and procedures.

In this study, a Standards-based approach reflects the ideas of NCTM as they pertain to mathematics as *doing*. Through the process of doing mathematics, students

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<sup>4</sup> This term is not one that a reader would find in broader literature sources, but it is referenced throughout this dissertation and necessary for the reader to know.

acquire a depth of knowledge in mathematics corresponding to NCTM's Process Standards: (a) Reason about; (b) Communicate with; (c) Make connections to; and (d) Represent mathematical concepts in a way meaningful to them. Further, in a Standards-based approach, students learn the skills and procedures necessary to perform calculations in the context of meaningful mathematics rather than students learning meaningful mathematics by first learning a series of skills and procedures. I acknowledge that a Standards-based approach also encompasses more specific guidelines for fostering the Process Standards (i.e., the use of manipulatives as a bridge for students to understand abstract concepts or the use of classroom discourse to create a classroom atmosphere of reasoning and reflection); however, since this study will focus primarily on one approach, I will not expand the definition to include these other facets.

### **Problem Solving**

According to Davidson (2012), the PISA 2012 definition of problem solving states "problem-solving competency is an individual's capacity to engage in cognitive processing to understand and resolve problem situations where a method of solution is not immediately obvious. It includes the willingness to engage with such situations in order to achieve one's potential as a constructive and reflective citizen (p. 5)." In several studies (Capobianco, 2003; Elstein et al, 1978; Gainsburg, 2003; Magajna & Monaghan, 2003; MSEB, 1998), researchers found that members of career fields deeply based in mathematics seldom use mathematics in the procedural manner they were taught in high school and college. Rather, they created systems relevant to a given situation and incorporated mathematics taught in schools as appropriate. Parker (1998) suggested these

fields require a broader understanding of mathematical systems rather than the procedural processes typical of school mathematics. Lesh and Zawojewski (2007) suggested that in light of this situational approach to doing mathematics, a definition of problem solving should incorporate characteristics of what it means to think mathematically:

A task, or goal-oriented activity, becomes a problem (or problematic) when the “problem solver” (which may be a collaborating group of specialists) needs to develop a more productive way of thinking about the given situation. (p. 782)

With this in mind, *problem solving* in this study will reflect both the definitions given by Lesh and Zawojewski as well as PISA. In the simplest form, problem solving will mean learning mathematics by doing it. More specifically, it will mean that it is a process of interpreting a situation mathematically and acquiring the knowledge that mathematics is a study of structure (Lesh & Zawojewski, 2007).

### **Anchored Contextualizing Problems**

Anchoring Contextualizing Problems (ACPs) are a modified version of model-eliciting activities (Lesh & Lehrer, 2003)<sup>5</sup>. The purpose of ACPs is to frame learning around students’ personal understanding of a concept. They are general in nature, but specific to a topic of learning – in this study, quadratic and polynomial functions.

Students receive the problems at the beginning of the units of study and are given at least an hour to generate their own ideas and responses to the problems. Teachers use this time to gain insight to student understanding of the problem and consider how they will frame learning of the skills and concepts of the related unit of study around the problem. Since

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<sup>5</sup> See the section on Models and Modeling in the Literature Review for a description of model-eliciting activities.



the ACPs are given at the beginning of units of study and meant to encompass the majority of the skills and concepts taught, they are considered to be an anchor to subsequent topics taught in the unit. See Appendices 1 and 2 for the “Pig Pen” and “Getting the Most Out of Your Luggage” ACPs.

The goal of ACPs is to create an impetus for students to think deeply about a given situation and develop their own means of interpreting and solving a problem. They are meant to encompass the above definition of problem solving in that students must construct their own meaning of a situation and devise an individual method for finding an answer to a problem for which there is not a clear solution. The goals of ACPs extend beyond developing problem-solving abilities with students and also help create context to better understand specific algebraic skills and procedures taught in a unit of study. In addition to helping students become more reflective and constructive thinkers, ACPs also act as references for later learning and development of skills.

### **Beliefs**

In education, the word belief is used in a variety of ways and it is often undefined by those who study it in relation to mathematics (Parajes, 2003). Further, it is often interchanged with words such as understanding, knowledge, attitudes, values, and perspectives. Philipp (2007) defined beliefs as, “Psychologically held understanding, premises, or propositions about the world that are thought to be true. They are more cognitive than attitudes and more difficult to change. They can be held with varying degrees of convictions unlike knowledge” (p. 259). Thompson (1992) viewed beliefs as dynamic, mental structures that were susceptible to change in light of *experience*.

In this study, I will work from the premise that teachers' beliefs vary significantly and, as Thompson suggested, are susceptible to change through experience. Further, I will incorporate these definitions into Simon and Tzur's (1999) view of perspective – that beliefs, knowledge, and methods of teaching are all components of teachers' perspectives and influence how a teacher approaches planning and teaching a given topic. According to Simon and Tzur, these components should not be viewed separately from one another, and cannot be assessed without gathering rich data, which in the case of this study will be collected primarily from an interview process. Ultimately, beliefs will be viewed in this study as a major component of teachers' malleable perspectives.

## CHAPTER II: LITERATURE REVIEW

Chapter II is comprised of three main objectives. The first purpose of this chapter is to establish a firm understanding of what problem solving means within the context of a Standards-based approach to learning mathematics. Specifically, I will outline the research that led to development of models and modeling perspectives on teaching mathematics in order to frame my dissertation within the appropriate context for understanding Anchored Contextualizing Problems (ACPs). In addition to a review of the literature on the concept of models and modeling perspective, I will discuss specific studies that incorporated a models and modeling approach to programs aimed at teacher development.

This chapter's second purpose is to tie teachers' beliefs about Standards-based approaches to teaching mathematics and models and modeling. Ultimately, its goal is to ascertain the concerns and perspectives of high school teachers when implementing ACPs in their own classrooms – the primary purpose of this dissertation. By outlining the research on teachers' general beliefs and then honing it towards specific studies of teachers' beliefs related to MM, I will establish the following: (a) there is little research pertaining to MM or mathematics teachers' beliefs at the high school level; and (b) that most, if not all, of the research on MM is based on interested individuals who have

already “bought into” the practice and not related to skeptics or those uninformed about the concept.

The final purpose of this chapter is to set out the factors relevant in the dissemination of practices such as ACPs. Specifically, I will list a set of criteria individuals follow when adopting an innovation such as a technology, a social trend, or a methodology used in one’s profession. Using the principles found in the book *Diffusion of Innovation* (Rogers, 2003), I will set out patterns among humans in adopting innovations and identify how these patterns relate to ACPs. I will also outline the standard time frame in which people adopt innovations, the factors that impact people’s decisions to adopt them, and methods used in encouraging “laggards” to adopt them.

### **Problem Solving**

The premise of this section is that problem-solving abilities are of integral importance in mathematics education (Elstein, Shulman, & Sprafka, 1978; Greeno, 1998; Lesh & Doerr, 2003; Lesh & Yoon, 2004; Lester, Garafalo, & Kroll, 1989; Lester & Kehle, 2003 Schoenfeld, 1992) and that problem solving should not be taught as an independent task. Instead, it should be taught within the context of learning mathematics, and, conversely, mathematics should be taught in the context of problem solving (Lesh & Zawojewski, 2007). The following is a discussion of this conceptual framework as it pertains to the development of the MM perspective on learning mathematics. The purpose of this section is to identify MM and model-eliciting activities as specific methods for training students in problem solving. My primary goal is not to show

evidence of the benefit of general problem-solving skills, but to connect MM and model-eliciting activities to ACPs as methods for teaching problem solving.

### **A Linear Approach to Problem Solving**

In 1957, Polya wrote his book, *How to Solve It*, where he outlined a step-by-step process for solving word problems – namely, draw a picture, work backwards, look for examples of similar problems, etc. For more than two decades, researchers tried to use this heuristic as a foundation for research in developing problem-solving abilities with students; however, there was little to no evidence to support that the approach led to an increase in students’ abilities to solve challenging word problems (Lester & Kehle, 2003). Krutetskii (1976) saw Polya’s heuristic successfully implemented by gifted students when solving challenging mathematics problems, but the process was not explicitly taught to them, nor did they have a circuitous set of rules and procedures to follow at each step of the process. These students were naturally successful at problem solving, and their methods were more “organic.”

As far back as 1978, Elstein and colleagues found that the learning of problem solving in medical education was embedded in, and linked to, the content and context of the situation (See also studies of problem solving in everyday settings by Carraher, Carraher, & Schliemann, 1985; Carraher & Schliemann, 2002; Gainsburg, 2003b; Hall, 1999a, 1999b; Lave, 1988; Lave & Wenger, 1991; Saxe, 1988a, 1988b, 1991). They found that problem solving is not a linear process, but a back-and-forth, iterative cycle that cannot simply be broken down into a set of instructions. While similar overall to Polya’s heuristic, Elstein and others determined that the heuristic itself was not a means of

teaching problem solving. At this point, it was not any clearer how to develop a “natural process” in students for whom problem solving did not come easily.

In 1992, Schoenfeld (1992) also concluded that any attempts to teach problem solving as Polya suggested had failed. This led to a call for a new approach to learning problem solving that was not linear but organic – an approach that was embedded in everyday mathematical learning where content is learned through problem solving and problem solving is learned while doing mathematics. Zawojewski and Lesh (2007) later determined that a system was needed for interpreting problem situations where students are encouraged to think with and about a situation to guide them to a model representative of a given problem.

### **Changing Views on the Development of Problem-Solving Abilities**

To better describe the evolution of the history and varying perspectives of teaching, Lesh and Doerr (2003) gave three analogies: (a) the industrial; (b) electronic; and (c) biotechnical. Each analogy gives greater insight into how problem solving has been approached problem solving in the past decades in relationship to the research on Polya’s heuristic. They also show how each leads to a greater understanding of what needs to be done in response to Schoenfeld’s (1992) call for a new teaching approach to problem solving.

**Industrial analogy.** In the industrial analogy, the ability to solve problems is viewed like a machine that creates a product. While each gear and cog does its part in a hierarchal fashion to create a final product, the machine (system) is viewed as no more than sum of its parts. The interplay of the cogs and gears are not considered when trying

to understand how the machine works. In a similar fashion, Polya's (1957) heuristic for solving problems - *draw a picture, work backwards, look for a similar problem, or identify the givens and goals* - is a step-by-step method that is predicated on the notion that achieving one step will naturally lead to the second, third, and fourth step and ultimately lead to a solution to a problem.

Polya's heuristic was determined to be a descriptive (Schoenfeld, 1992) account of expert<sup>6</sup> problem solvers rather than a prescription for novices to solve problems. None of the steps account for how they are related or if they cycle back on one another to bring about a better understanding of the problem itself. Furthermore, the steps do not account for individual nuances of a problem-solving situation that changes from problem to problem.

Problem solving is not linear enough to simply follow a few steps to get to an end goal or to simply view as a machine or system. Polya's heuristic is not, however, a poor account of problem-solving ability. It outlines the general steps that problem solvers use to get to a solution, but it does not explain the dynamic and frequently messy process that really occurs in the mind of a problem solver. Lesh and Zawojewski (2007), suggest that Polya's heuristic be used as a reflective tool for students to analyze the process they used to solve a problem and thus become a means of developing metacognitive abilities for reflection.

**Electronic analogy.** The electronic analogy reflects the fact that Polya's steps were lacking a significant number of processes that problem solvers may (or may not)

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<sup>6</sup> Experts here will be defined simply as people with at least ten years of experience in post-secondary mathematics.

work through when trying to find a solution to a given situation or problem. For years researchers developed an increased number of routes a problem solver could take on his or her way to a solution using “if-then” scenarios similar to a computer model. Like a circuit consisting of hundreds, if not thousands, of recursive pathways leading to a specific function, the electronic analogy likens problem solving as a unidirectional process that builds on itself to make a finished product. It creates a complex flow chart that attempts to explain the process, but it does not create a means of teaching problem solving.

According to Lesh and Doerr (2003), this analogy has three flaws: (a) it is unidirectional and thus does not account for the possibility that problem solvers revert back to “old programming” to help approach new problems differently; (b) it is contingent on the ability of the problem solver to store hundreds, even thousands, of permutations of routes to a single solution; and (c) it is digital and not reflective of how the human mind works. Even if this process is taught extremely well, it is not likely to teach students how to be good problem solvers since it requires too many distinctions and connections to fit any mode of education (Lesh, Yoon, Zawojewski, 2007). Lesh and Doerr’s criticism of this analogy gave rise to a more organic one that embodies the “messier” aspects of human problem solving.

**Biotechnical analogy.** What is needed is a model that reflects the fact that the humans do not approach problems solving in pre-mathematized (all the necessary components are already in place), extremely efficient manners. The mistakes, poor lines of logic, and outright foolish theories that humans have developed over the millennia are



essential in understanding present, working theories. The previous models are based on a starting point that assumes humans do not make mistakes or have naïve understandings of a problem situation.

The biotechnical model is a symbol of a complex and diverse ecosystem that can be ‘fuzzy’ and difficult to understand but is also dynamic and continually adapting to its environment. In the same manner, problem solving is viewed as an often messy but continually adapting scenario where students create a rich environment that describes problem solving as an increasingly evolving system-as-a-whole rather than Polya’s step-by-step process. At the foundation of the biotechnical analogy is the concept of mathematization – a student’s attempt at understanding the “big picture” of mathematical systems. The following section will describe this concept and then discuss the concept of models and modeling.

### **Models and Modeling Perspective**

Over the past 30 years, researchers have conducted extensive studies of students’ production of models (representations of a problem situation that describe a student’s current understanding) and the study of the process (or modeling) that students go through to solve complex problems. The research spans a range of settings and includes several related topics ranging from Constructivism, Piagetian Conceptual Systems as they relate to mathematization, Vygotsky’s Zone of Proximal Development as it relates to the acquisition of problem-solving abilities in a group setting, etc. (Piaget, 1964; Piaget, 1966; Vygotsky, 1978). The scope of this research is far too wide for this chapter and as such, this section will be limited to: (a) an explanation of model eliciting problems

development; (b) how mathematics is “seen” when working with model-eliciting problems; and (c) the research and professional development involved in the these specific areas of MM.

It is necessary to first distinguish the difference between “models” and “modeling.” A model is a system that includes, but is not limited to, representations or tools for understanding a representation (i.e., guess-and-check tables, algorithms, etc.), and descriptions of a process that a student develops when trying to understand a problem situation. The model may be vague, incomplete, or completely inaccurate; however, the model belongs to the student (or the group in which the student is working) and is meaningful to him or her. MM argues that a student’s learning takes place when he or she is given the opportunity to express his or her model to classmates, test it on the problem to see if it works, and *revise* the model if it does not work completely. It is the modeling process of *expressing, testing, and revising* where students are able to learn problem-solving skills (Lesh and Zawojewski, 2007). There is no way to teach problem solving through direct instruction of mathematical skills – students learn by doing.

### **Model-Eliciting Problems**

The development of a student’s ability to solve complex problems requires the skills to generalize a given situation – to mathematize. Rather than adopting a teacher’s or curriculum’s way of thinking, students must come up with their own way of approaching a problem in order to generalize and understand it (Lesh, Yoon, Zawojewski, 2007). Students are given the opportunity to mathematize when they are

presented with model-eliciting problems and/or activities that require them to create tools to help them solve a given problem.

**Creating model-eliciting problems.** In order to develop problems that encourage students to create their own models, it is necessary to establish criteria for producing simulations that are model-eliciting. The following are six principles (Lesh & Doerr, 2003, pp. 43-44) that guide the construction of such simulations and a brief explanation of each one:

1. *The Personal Meaningfulness Principle* – Is the question worth solving? Is it truly a real-life situation? Do students find it interesting? Will they be able to give their own ideas on the problem or will they be expected to follow the teacher’s line of reasoning?
2. *The Model Construction Principle* – Is the problem focused on underlying patterns? Will the students need to create a *model* to represent their understanding of a system-as-a-whole? Will their model require refinement and adjustment?
3. *The Self-Evaluation Principle* – Are the criteria clear to students for assessing the usefulness of their responses in addition to alternative responses? Will the students know how to test whether their models yield correct answers? Do the students know to whom they are presenting their responses?
4. *The Model-Externalization Principle* – Will the students’ responses require them to document their thinking? Or document their models? Will they be

required to state their goals, mistakes, possible solution paths such that an instructor/researcher is able to gain insight to the students' thinking?

5. *The Simple Prototype Principle* – Is the problem sufficiently simple yet requiring the need of a significant model? Will the model created by the students be a useful template for similar problem types? Will the problem-solving experience have the ability to make sense of similar situation?
6. *The Model Generalization Principle* – Will the model be sharable, reusable, and transformable? Can the model created for a particular problem-solving situation be general enough to use in other situations?

These principles are the guiding factors in creating problems that “focus on abilities related to mathematical interpretation and conceptualization (description, explanation, communication) at least as much as computation or deduction” (Lesh, Yoon, & Zawojewski, 2007, p. 335). The principles create environments where students are encouraged to develop means of “seeing” mathematics at deeper levels and as a system-as-a-whole.

**How mathematics is “seen” when working with model-eliciting problems.**

Another way of defining mathematization is a student's attempt at understanding the “big picture” of mathematical systems. In other words, are students able to see the forest for the trees? The purpose of model-eliciting problems is to get students to think about the way they view and understand an overall situation before trying to conduct computations or deductions related to it. Lesh, Yoon, and Zawojewski (2007) gave five types of mathematical abilities needed for seeing the big picture: (a) within-concept systems; (b)

between-concept systems; (c) interacting representational systems; (d) basic skills whose meanings depend on systemic understandings; and (e) higher-order ideas or abilities whose meanings depend on systemic understandings. In order to convey how model-eliciting situations encourage students to mathematize and ultimately acquire the five abilities list above, it is necessary to juxtapose the process of mathematizing with traditional instructional methods used to teach the mathematical abilities directly. The following is a brief synthesis of each of the mathematical abilities and how it compares to traditional approaches.

*Within-concept systems* are the structural or relational tools used to organize learning, much as a map is used to describe the locations of individual venues in a city. In a traditional setting, teachers lay this map out for the students and guide them through it. Though this is helpful, the students do not picture the actual venues on the map in the same manner as the teacher because they did not construct the map themselves. In model-eliciting problems, students make their own system and express, test, and revise it until they have their *own* map to describe the nature of the system. The map may be crude and lacking in detail, but through the process of expressing, testing, and revising their work, students can create a powerful resource for understanding the big picture maps.

*Between-concept systems* are the understanding that most problem situations require knowledge from several different areas. For instance, different chapters of a mathematics book, concepts in science, and even concepts from the humanities can be incorporated into a problem situation to aid the student in finding a solution. In a traditional class, material is presented in a very linear fashion similar to highly organized

notes (i.e. Roman numerals), and this makes sense because of the cumulative nature of mathematics. Unfortunately, this may also pigeon-hole students in singular train of thought that keeps them from thinking outside of the box (Lesh, Yoon, & Zawojewski, 2007). The value of model-eliciting problems is that they are based on an *apply-first-teach-later* methodology that does not front-load students with material that may send them down a singular path. Instead, students are encouraged to gather resources from several sources.

The *representational systems* utilize the ability to move back-and-forth and between different media (i.e. functions, tables, graphs, drawings, etc.) to represent a concept. In model-eliciting problems, students create their own media and understand when to use it, when it is not useful, and when to get rid of it all together. Traditional methods typically present students with a form of media that is all-encompassing that attempts to include every facet of a problem in one form. Much like the issue of pigeon-holing in the *between-concept systems*, representational systems often lock students into one mode of viewing a concept and as a result, students do not develop their own ability to create representational media let alone learn to work between several forms.

Finally, *higher order conceptual abilities based on holistic systems* rely on an ability to see a system for more than the sum of its parts. In traditional classes, mathematics is taught prescriptively, skill by skill, in the hope that those skills will be tied together to form a grander ability in problem solving. Skill training in mathematics is important, but it does not develop problem-solving abilities any more than learning how to dribble a ball teaches a person to play basketball. In model-eliciting situations, students

think *with* models and not just *about* them; overarching conceptual systems drive students' reasoning in problem solving not just the application of skills.

**Summary.** MM is a method of learning mathematics that requires students to generalize their understanding of a problem, create systems that describe their understanding, and develop tools to help them find solutions. Under the MM framework, the use of real life problems helps elicit these processes and lead to mathematization. The processes themselves foster mathematical abilities that create problem solvers capable of translating their abilities into various aspects of their lives. Research has proven that at-risk and/or below average students, and students who have performed poorly on standardized exams are able to successfully learn complex mathematics in an MM setting far beyond anything their instructors would have “dared” considered teaching them (Lesh et al., 2000; Lesh and Doerr, 2000; Lesh, Hoover, & Kelly, 1993). It is the contention of MM that such problem-solving abilities should be one of the primary goals of mathematics education (Lesh & Zawojewski, year). However, educators must become students themselves in order to learn how to foster an MM environment in their classrooms. The following is a summary of key research of professional development (professional development) activities for MM.

### **Research Related to Models and Modeling**

The purpose of this section is to discuss significant studies related to teacher training on the use of MM in order to identify studies similar to mine and determine specific aspect of my work that are not examined in other research. Specifically, I will show that most, if not all, of the work done on MEAs - as they pertain to professional

development for teachers – has included more willing teachers who have already agreed to a particular MEA’s use or to at least go along with their colleagues. Further, I will show that much of the work has been done on a k-8 level and not with high school teachers.

**Teacher interpretation of students’ work on MEA.** Schorr and Lesh (2003) conducted an intervention aimed at helping elementary school teachers consider and then implement more meaningful forms of mathematics instruction and assessment with their students. The intervention was a series of monthly workshops for teachers over the course of three years. Teachers worked on MEAs in groups and then presented their findings to the entire workshop. They then used the MEAs in their classrooms and recorded the lessons for analysis. The teachers were encouraged to recognize and analyze their students’ thinking while working on the MEAs and report on their findings in following workshops. In the workshops that followed each implementation of a classroom MEA, excerpts of selected video recordings of the implementation of the MEAs in teachers’ classrooms were analyzed as a group.

The data were gathered using video, transcripts, student’ written work, students’ reflections of their work, and teachers’ reflection of their own work on MEAs as well as the overall implementation of the MEAs in their classroom. Data were also collected from the researchers who observed, recorded, and took field notes on both the workshops and the classroom implementations of the MEAs. The researchers also took field notes on the teachers’ analysis of student work and how it evolved over the course of the study.



Over the course of three years, teachers met monthly. Initially, their observations on student thinking and reasoning focused on concerns of how well students worked together while doing MEAs. As the study progressed, the teachers' observations evolved to concentrate on the students' mathematical ideas and quality of work during the MEAs, specifically:

- Identification of the problem and understanding of the task
- Choosing and using tools appropriately
- Creativity in selecting and using strategies for solving the problem
- Organization of problem data
- Quality of the product (how it looks visually, organization, completeness)
- Persuasiveness of the justification for all parts of the results
- Mathematical perseverance – staying with the task
- Ability to monitor and assess the product, refining it appropriately, continuing through solution cycles as necessary
- Working within a group to analyze the problem, share insights, questioning both the process and the results
- Roles assumed with the group
- Intellectual characteristics shown – reflection, confidence, curiosity, etc.
- Evidence of knowledge and skill with particular mathematical concepts
- Differences in strategies chosen
- Questioning that the solution “makes sense” – that it in fact appropriately answers the question posed and would be satisfactory to the client

One of the teachers created a self-assessment tool for her students to help them reflect on their own work and compare their work to the ideas of others.

The process allowed teachers to evolve their approaches to teaching problem solving through meaningful mathematics. They went from looking for surface level information during problem-solving sessions to deep, meaningful, and thought-revealing information. They changed their views on how to help students reflect on and assess their own work, and they learned to better assess and understand student work. Since the MEAs were not out of context of student learning or the standards of their grade levels, teachers were not taken away, or out of, their necessary instructional practices. Instead, the MEAs were embedded within the appropriate units of study and teachers were allowed to develop their abilities to foster problem solving without worrying about coverage of material.

**Concept maps of MEA to skills learned.** Clark and Lesh (2003) examined a multi-tiered professional development where middle-school teachers went through several iterations of concept maps related to individual MEAs. The purpose of the study was to determine the degree to which teachers refine their understanding and construction of unit lessons around MEAs. Specifically, how the teachers' psychological, historical, and instructional knowledge evolves while developing, revising, and refining the concept maps. The researchers gathered data for this qualitative study using transcripts of discussions among the participants during workshops.

Over the course of the study, the researchers found that the teachers' refined the concept maps extensively and in turn refined their own understanding their students'

reasoning while working on the MEAs. Teachers also refined their understanding of the mathematics being taught. The most growth documented occurred with teachers who were able to identify the interrelationships between skills and concepts covered in a given MEA.

The researchers concluded that multi-tiered professional development allows teachers to grow without the constraints of traditional approaches where the content of the professional development was often separate from what was taught or being done in the classroom. It takes into account what the teachers are doing in their classrooms; that their knowledge is multidimensional, pluralistic, variable and contextual. Using MEAs for teachers allows them to approach enhancement of their teaching from a multi-faceted foundation that allows them to refine, revise, or reject perspective on teaching after implementing the MEAs in their classrooms and then discussing their decisions with other colleagues

**Problem-solving cycle.** Koellner-Clark et al. (2007) created a study where teachers took part in a series of professional developments focused on assisting teachers to support student mathematical reasoning by having the teachers delve into the problems given to the students. Ten middle school teachers took part in the study and attended three workshops. In the first workshop, the teachers worked on an MEA as their students would, they then came back together as a group, discussed their results and processes for finding solutions to the given MEAs, and later had their students work on the MEA. In the second workshop, the teachers reported back on their experiences teaching with the MEA. In the third workshop, they re-examined the process of teaching with MEAs and

discussed how to extend their own understanding of both the content and pedagogy using MEAs.

The researchers gathered data for this qualitative study using video recordings of the group discussions and the teachers working on the MEAs in small groups. Additionally, researchers took field notes of professional development and interviewed each of the teachers. The interviews sought to gain the teachers' insight on their goals, intentions, and reflections of the use of MEAs in their classrooms. The researchers used vignettes to analyze and summarize their findings.

The research indicated that there was significant growth in the teachers' understanding of the mathematical content related to the MEA and their ability to teach the content. The researchers concluded that the workshops, which were focused on assisting teachers to support student mathematical reasoning, also increased the teachers' content and pedagogical knowledge. The study also found that this professional development approach was also applicable to foster greater content and pedagogical knowledge when teaching within the framework of NCTM's standards-based reform movement<sup>7</sup>, which is discussed in chapter I.

**Teachers' spoken ideas on MM and their actual practices.** Schorr and Koellner-Clark (2003) found that some teachers incorporate aspects of Standards-based approaches to learning (the use of manipulatives, groups, etc.) into their own traditional ways of teaching but do not truly evolve their teaching to the level of NCTM's reform movement. In a 16-week, multi-tiered program, Schorr and Koellner-Clark designed a

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<sup>7</sup> This movement aims to help teachers adopt a Standards-based approach to learning.

study where teachers shared their ideas and experiences after implementing problem-solving activities in their classrooms. In the qualitative study, 12 middle school teachers took part in a reform-oriented professional development. The teachers were interested in establishing a Standards-based approach in their classrooms, and the professional development was created to help the teachers disseminate their ideas and experiences after implementing problem-solving activities in their own classrooms.

The intervention was a video-analysis of the implementation of an MEA in a teacher's classroom and the ensuing discussion of the lesson among the teachers in the professional development. Using transcripts of interviews with each teacher and the transcripts of group discussions, the researchers sought to determine the participants' responses to the following questions about a specific teacher's implementation of an MEA in his classroom:

- What are Roger's (pseudonym) current approaches to the teaching and learning of mathematics?
- What reform-oriented practices is Roger attending to and what are the ways in which he implements them?
- How does Roger promote changes in students' mathematical knowledge?
- What should the research consider to provoke Roger to consider new instructional practices (if appropriate)? (Schorr & Koellner-Clark, 2003, p. 203)

The researchers found that Roger's implementation of the MEA was not aligned with the theory of MM. Roger scaffolded the MEA into piecemeal questions and led his students through the entire problem without allowing them to toil with it and create their

own models. Through the process of analyzing Roger's teaching and implementation of an MEA with the other 11 participants, a cognitive dissonance occurred where Roger realized his shortcomings by expressing, testing, and revising his interpretations of how to teach mathematics from a problem -solving approach.

The researchers concluded that providing an impetus for teachers to develop a new, larger view of their teaching practices allows them to better understand what it means to teach mathematics from a Standards-based approach. The reflections led to progressively better understanding of teaching in a reform-oriented manner. Specifically, all of the participants began to reflect more deeply about their students' thinking and what was necessary to foster such thinking.

**Summary.** Aside from the fact that these studies do not focus on high school mathematics classes, they also do not address the issue of teachers who simply do not agree with or believe in the value of Standards-based approaches to learning mathematics. The participants in each of the studies were willing participants and thus individuals who were already open to the idea of teaching differently. The studies sought to improve teachers' ability to teach mathematics with the Standards-based approach of MM, but they did not address issues with teachers' interpretations of what teaching in this manner truly meant, nor did they seek to discover the perspectives of teachers who may be skeptical or unaware about Standards-based teaching practices.

In my study, I seek to gain insight to teachers' concerns and perspectives about using MEA-like items called Anchoring Contextualizing Problems (ACPs) while they implement ACPs in their own classrooms. My study will mirror some of the aspects of

the professional developments used in the studies above but focus on determining the concerns and perspectives of the teachers implementing ACPs in their classrooms. Before beginning this analysis, it is necessary to explore the literature related to teachers' beliefs and teacher change.

### **Teachers' Beliefs and Change**

The purpose of this section is to juxtapose mathematics teachers' beliefs with actual practices in order to establish a foundation from which to better understand contradictions between the two. This foundation helps establish a link between an NCTM reform approach and learning mathematics as well as the change agents used to alter mathematics teachers' beliefs about these approaches. Ultimately, I will use each of these concepts to introduce my study on ACPs.

#### **Beliefs**

Thompson (1992) stated that researchers should consider the discipline of mathematics, the relationship between what a teacher thinks about mathematics and how s/he actually teaches the subject. The definitions of mathematics posed by NCTM, TIMMS, and PISA are generally accepted as accurate reflections of mathematics and, therefore, teachers' beliefs and application of these definitions must be thoroughly understood. As stated in chapter I, Philipp (2007) defined beliefs as, "Psychologically held understanding, premises, or propositions about the world that are thought to be true. They are more cognitive than attitudes and more difficult to change. They can be held with varying degrees of convictions unlike knowledge" (p. 259). It is important then to

explore how teachers' beliefs about mathematics impact current application and ability to embrace new approaches to teaching.

### **Beliefs and Practice**

Thompson (1992) viewed beliefs as dynamic mental structures that are susceptible to change in light of *experience*. Further, beliefs are different than knowledge, as beliefs are malleable due to varying degrees of commitment one has to them. If people hold an opinion on a matter as factual knowledge, then it is unlikely that a meaningful discussion can be had with others that do not hold the same opinion. Even those whose beliefs align closely with another's knowledge may have difficulty maintaining a meaningful conversation due to differing beliefs.

Differentiating between what one believes and what one accepts as knowledge is a first step in determining if a person is willing to try something new and give it a sincere chance to be considered. In the case of a Standards-based approach to learning, there are several matters to consider when determining a person's beliefs about mathematics. For instances, are individual teaching practices truly indicative of a teacher's beliefs, are there outside forces impacting teaching practices that cause a teacher to not adhere to individual beliefs despite a desire to do so, or are there legitimate and logical reasons that may cause a teacher to counter her beliefs in light of extenuating circumstances?

External pressures are a major factor in education that can impact a teacher's actions. When a teacher adjusts his or her actions due to external pressures, it may cause an outside observer to view the actions as a reflection of the teacher's beliefs. Philipp (2007) called for a more balanced approach to understanding teachers' beliefs and the



contradictions that occur when observing them in action. The following studies support this suggested approach and give insight to the perspectives of teachers' belief systems and actual practices.

Raymond (1997) investigated the relationship between a beginning teacher's beliefs, her mathematics teaching practices, and the inconsistencies between them. Five fourth grade teachers took part in a series of interviews, observations, and questionnaires that sought to determine their individual approaches to teaching: traditional, primarily traditional, even mix of traditional and nontraditional, primarily nontraditional, and nontraditional. Raymond found a significant difference in one participant's (Joanna) responses to the interview questions and her actual practices. Joanna viewed her teaching practices as what she wanted to do more so than what she said she should do. Her practices were more aligned with her beliefs about mathematics content (traditional) and less aligned with her beliefs on mathematics instructions (primarily nontraditional). Raymond concluded that this inconsistency was related to the effects of external variables such as time-constraints, scarcity of resources, concerns over standardized tests, and student behavior.

Hoyle (1992) reviewed previous studies like Raymond's and reconsidered how to view the inconsistencies in teachers' belief statements and actual practices. By considering the circumstances and constraints under which teachers had to work, the inconsistencies became "irrelevant," and new variables could be considered in such settings and may produce different pictures of success. For instance, a future study could compare the accomplishment of teachers' goals with their individual beliefs.

Skott (2001) agreed with Hoyle on teachers' inconsistencies but gave an alternative view that teachers' observed inconsistencies with their beliefs and practices may actually be in line with belief statements. A case study on a student teacher, (Christopher) showed him adhering to his constructivist beliefs in one lesson and being seemingly "traditional" in another. When interviewed and asked about these inconsistencies, Christopher did not view them as such. Instead he stated that he reconciled the needs of the students to feel confident in their findings by reviewing a problem that was explored in the previous class by using a traditional approach. Essentially, Christopher took some time to summarize students' work into a cohesive and traditional method, but he did so from his constructivist belief stance.

These findings are aligned with Hoyle's in that beliefs are situated; however, Skott argued that situations do not necessarily change the teacher. In Christopher's case, his belief did not change between the two lessons, nor did he succumb to pressures of a typical classroom situation where students wanted a traditional approach. Instead, Skott concluded that Christopher did not change his beliefs but recognized the importance of students' process of learning via a constructivist approach as well as the need to solidify students' understanding of their product via a direct instructional approach.

Similarly, Sztajn (2003) found that inconsistencies in teachers' beliefs with their practices were due to students' needs. When comparing two teachers with similar beliefs, Sztajn found that their practices were quite different from one another. Instead of viewing one teacher better than the other because of one's adherence to individual beliefs, Sztajn sought reasons to explain the differences in practices. In doing so, Sztajn found that the

students in each of the teachers' classes had very different needs. In the teacher's class who adhered to her beliefs more firmly, the students came from middle- to upper-middle class families and tended to behave well and complete assignments, whereas the students in teacher's class who adhered less to her beliefs came from less affluent homes and were less attentive. Sztajn concluded that socio-economics was a factor in the implementation of the teachers' beliefs. One teacher did not have to be as concerned with students behaving appropriately or doing their homework, so it was easier to teach from a Standards-based approach. The other teacher needed to attend to more general classroom issues (i.e., discipline, organization, fostering habits of mind, etc.) that are essential for creating a foundation for learning. Much of her focus was centered on these needs and thus took up much of the time she would have used for fostering mathematical thinking instead of just procedural practice.

Zakaria and Maat (2012) also sought to gather information about inconsistencies researchers have found in teachers' beliefs and their actual practices. A total of 51 teachers from seven secondary schools were grouped according to their experiences. They were given a set of questionnaires that gathered responses about the teachers' individual mathematics beliefs and mathematics practices. The mathematics beliefs were delineated along the following dimensions: the nature of mathematics, mathematics teaching, and mathematics learning. Zakaria and Maat found that there was no difference between the less-experienced and more-experienced teachers in regards to mathematics beliefs; however, he did find a moderately significant correlation between the teachers'

mathematics beliefs and teaching practices. He concluded that the establishment of good mathematics beliefs would lead teachers to positive and effective teaching practices.

Philipp (2007) suggested that inconsistencies between a teacher's beliefs and practices do not exist and research in the area of beliefs should adopt this stance. Based on the findings above that obstacles such as discipline and resources hinder teachers from practicing what they believe makes Philipp's suggestion a feasible one. In this section, I sought to establish a baseline of reasons for inconsistencies in teachers' beliefs and actual practices. I will utilize this baseline to highlight the need to change teachers' beliefs about Standards-based approaches to teaching mathematics *before* trying to change their practices.

### **Changing Beliefs**

Pajares (2003) suggested that to change teachers' behaviors (practices), one must change teachers' beliefs. In this section, I will explain how different approaches to training teachers on Standards-based approaches impact their beliefs. The purpose in doing so is two-fold: (a) to establish a need for research on teachers who are unaware of – or “buy-in” to – Standards-based approaches to learning mathematics; and (b) to establish the need for tracking changes in teachers' opinions and concerns about a Standards-based approach as they implement components of it.

Guskey (2002) stated that significant change in teachers' beliefs can only happen after they see changes in student learning outcomes. Further, von Glasersfeld (1993) said “If one succeeds in getting teachers to make a serious effort to apply some of the constructivist methodologies, even if they don't believe in it, they become enthralled after

five or six weeks” (p. 29). He observed that changes in teachers’ beliefs were followed by changes in instruction. The following studies have determined methods capable of changing teachers’ beliefs.

Grant, Hiebert, and Wearne (1998) found that telling teachers about a Standards-based approach to learning was not sufficient for bringing about change in their teaching practices. Instead, they sought to determine what would happen if teachers observed actual Standards-based instruction. Nine elementary school teachers in mathematics classrooms took part in a 12-week observation of Standards-based instruction. The teachers were interviewed and placed on a continuum of beliefs about teacher-student responsibility for learning mathematics. The continuum ranged from a skill/teacher’s responsibility approach to teaching mathematics to a Standards-based, process/student responsibility approach. The teachers then observed the classes and reported back in exit interviews.

The researchers placed four of the nine teachers at the skills/teachers responsibility end of the beliefs spectrum, as they focused on specific aspects of the observed lessons (i.e., manipulatives) and failed to explain their use. Instead of seeing individual components as a means of promoting a link between mathematical meaning and the math being learned, the nine teachers saw these components as the only means of learning the material. Three teachers were placed in the middle of the beliefs spectrum, as they merged the Standards-based practices into their own teaching practices and viewed their role as one that removes obstacles hindering students’ understanding. Instead of allowing students to struggle and find their own ways to a solution, the teachers

essentially identified a “Standards-based” method with teaching the students how to get directly to the solution. Two teachers were placed at the process/student responsibility end of the beliefs spectrum, as they were able to aptly identify that the purpose of the Standards-based instruction (i.e., the use of manipulatives) was to get students to describe their thinking process rather than just solve problems. The researchers concluded that there is evidence for the position that the beliefs teachers hold filter what they see and, consequently, what they internalize.

Borko, Mayfield, Marion, Flexer, and Cumbo (1997) had similar findings with 14 3<sup>rd</sup> grade teachers. Many of the teachers in their study either ignored or inappropriately assimilated the Standard-based approach into their existing practices. The researchers concluded that beliefs served as filters through which ideas were perceived, and teachers needed to be challenged to reflect upon their beliefs. Though these authors concluded that little change to the teachers’ core beliefs occurred during observations of Standards-based instruction, Barlow (2012) found contrary evidence. An elementary school teacher observed a mathematics teacher educator for one year teaching from a Standards-based approach, which resulted in “the establishment of strongly, standards-oriented beliefs about mathematics teaching and learning (p. 3).” It must be noted that the educator taught in teacher’s classroom for one year, the teacher was clearly open to the idea, and the teacher observed the entire year and saw results.

### **Reflection and teachers’ beliefs about students’ mathematical thinking**

The purpose of this section is to establish a link between the impact of changing teachers’ beliefs about students’ role in constructing their own knowledge with positive

changes in the teachers' instructional practices commensurate with a Standards-based approach. The value of the findings in this section will be two-fold: (a) it will establish additional advantages for teaching from a Standards-based approach that will be utilized in the professional development portion of my study; and (b) it will further establish a need for research in the area of teachers' buy-in to Standards-based approaches.

**Cognitively guided instruction.** Fennema, Carpenter, Franke, Levi, Jacobs, and Empson (1996) used a professional development about Cognitively Guided Instruction (CGI), where teachers' beliefs about students' mathematical thinking changed in relation to their instructional practices and vice-versa. They also studied the growth of students' learning in relation to their teachers' instructional changes. Twenty-one primary-grade teachers took part in the study over a four-year period. The intervention used was instruction and professional development in Cognitively Guided Instruction. CGI is an approach to teaching mathematics rather than a curriculum. The core purpose of CGI is for teachers to listen to their students' mathematical thinking and arrange learning around their thought processes.

To describe the patterns of change teachers underwent while using CGI, Fennema et al adapted Hall's (1975) work on measuring the level of use of instructional strategies. Fennema also examined Schifter and Fosnot's (1993) work on developing levels of teachers' decision making based on constructivist perspectives to create their own Mathematics Belief Scales and subscales, which included: (a) the Role of the Learner; (b) the Relationship Between Skills and Understanding; (c) the Sequencing of Topics; and (d) the Role of the Teacher. Each scale is scored from 1 – 4A/B, with 1 being the lowest

level of teachers' beliefs that students can solve problems without help (1 indicates that teachers do not believe they can); level 2 is a teacher struggling with the belief that students can solve problems without help; level 3 is the belief that students can do so in a limited fashion, but using student thinking to guide instruction should be limited; level 4-A is the belief that students can solve problems in specific domains and their thinking should guide teacher/student interaction; and in level 4-B teachers believe that students can solve problems across all domains, and their thinking should guide instruction as well as curriculum design. Data on teachers' instruction were collected via observation, field notes, and interviews.

With regards to teachers changing their beliefs about CGI, the following results were found: (a) 18 out of 21 increased their beliefs about CGI; (b) 18 out of 21 increased their levels of instruction with CGI; and (c) 17 out of 21 did both. In the beginning, 2 teachers held level 4 beliefs and 2 teachers instructed at level 4. By the end of the study, 11 out of 21 had level 4 beliefs and 7 teachers had level 4 instructional practices. Of the 17 teachers who increased their ratings on both beliefs and instruction, 6 changed their beliefs then their instruction, 5 changed instruction then beliefs, and 6 did so simultaneously. Students' increased achievement in problem solving was directly related to teachers' changes in instructional practices. No changes were found in students' computational ability.

Ninety percent of the teachers increased their beliefs and instructional practices to a level 3 or higher – that their students can problem solve on their own. It was not clear why some teachers changed while others did not, nor was it clear why there was variance



in the degree some teachers changed over others. The researchers concluded that the teachers changed for two reasons: (a) they learned the researched-based model for CGI; and (b) they implemented the model in their classrooms. They found that developing an understanding of children's mathematical thinking can be a basis for transitioning teachers to a Standards-based approach to teaching mathematics.

In a similar area of research, Staub and Stern (2002) assessed teachers' pedagogical beliefs and placed them on a scale like Fennema et al. In a longitudinal study with 496 students spread over 27 classrooms in German elementary schools, the researchers assessed students' performance on word problems and arithmetic. They assessed the teachers' pedagogical content beliefs using a questionnaire similar to Fennema et al. Students of teachers whose beliefs aligned more with cognitive constructivist orientation (closer to a 4-A or 4-B level in Fennema's study on Standards-based approach) had greater achievement gains in word problems than those with a direct transmission view (1 – 2 on Fennema's scale) of teaching mathematics. Further, students of teachers with the direct transmission view did not fare better than the other students on computational skills. The researchers concluded that students of teachers whose views are more aligned with a cognitive constructivist orientation had greater gains in problem-solving ability than teachers with a more direct transmission approach.

**Pre-service teachers.** In a study on pre-service teachers' beliefs about teaching mathematics, Ambrose (2004) suggested that teacher trainers should work with pre-service teachers' pre-existing assumptions about teaching mathematics instead of simply tearing down said beliefs. Through a program called Children's Mathematical Thinking

Experience (CMTE), 15 pre-service teachers worked with children in an elementary school while they were enrolled in their first mathematics for teachers course. Their goals for the course were to make sense of “non-standard methods” students devise when reasoning about the procedures on numbers, to understand the different ways operations of numbers can be interpreted, and to be able to represent concepts in several forms.

Using surveys, interviews, written teachers’ work, and field notes, the researchers were able to align the pre-service teachers’ beliefs about teaching mathematics along a continuum of teaching as telling to teaching as doing<sup>8</sup>. Through teaching CMTE, the pre-service teachers recognized that teaching mathematics is far more challenging than they had previously believed, they found the constructivist approaches in CMTE helpful and interesting, and challenged their own views on teaching mathematics. Ambrose concluded that teachers do not let go of old beliefs while forming new ones, but they do become more cognizant of the shortfalls of the old beliefs. She also concluded that pre-service teachers’ failure of teaching a mathematical topic (e.g. fractions) within the purview of their pre-existing beliefs might act as an impetus for making different teaching decisions in the future that are more aligned with CMTE process of teaching mathematics. Ultimately, she concluded that change in beliefs is an incremental process that occurs over a significant period of time, and that this idea should be considered when training pre-service teachers.

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<sup>8</sup> Teaching as telling describes teachers who consistently show students what to do and have them repeat their processes. Teaching as doing describes teachers who spend more time having students work out a problem on their own or in groups.

**Teachers' beliefs and students' algebra development.** In one of the few secondary level studies about teachers' mathematical beliefs, Nathan and Koedinger (2000b) found a contradiction in teachers' beliefs regarding the kinds of mathematics problems students are able solve more easily than others. Specifically, secondary mathematics teachers expected that symbolic problems would be easier for their students to solve than word problems. This *symbol-precedence* view of mathematical development assumes that arithmetic skills develop before algebraic skills and symbolic problem solving develops before verbal reasoning. This view is counter to the NCTM Process Standards, and Nathan and Koedinger conjectured that this contradiction mediates teachers' judgment about instruction and students' mathematical development. Using 105 elementary, middle, and high school teachers, the researchers presented them with a list of mathematics problems and had them rank them according to the difficulty they believe students would have with each. Additionally, the teachers were given a 47-item survey that sought to determine their beliefs about how students learn mathematics and to place them on a continuum that put them in or out of agreement with NCTM's Process Standards.

The researchers found that the high school teachers believed that verbal problems would be more difficult for their students to solve than symbolic problems more so than the elementary and middle school teachers. The researchers also found that the high school teachers' predictions about their students' ability were the least accurate of all the teachers in the study. Further, the researchers found that the high school teachers' responses to questions about reform-oriented views of teaching mathematics were the

least agreeable of all the participants in the study. The researchers concluded that high school teachers might have a “blind spot” about their students’ abstract ability to manipulate symbols and understand them in a meaningful context. Finally, they postulated that the textbooks and curricula perpetuate this misconception.

Rutherford’s (2012) dissertation explored the process teachers undergo while incorporating problem solving into their teaching practices. Using a professional learning community (PLC) consisting of herself and two sixth grade teachers, Rutherford sought to understand how teachers’ beliefs, knowledge, and instructional practices were affected when exploring the use of problem-solving activities in their classrooms. Rutherford interviewed and observed the participants teaching to gain an understanding of their beliefs and practices regarding mathematics teaching. She then oversaw five PLC’s where the participants created and refined problem-solving activities, and then she observed the teachers incorporating the activities into their lessons. After the implementation of the activities, Rutherford interviewed the teachers again to seek further information about their beliefs, knowledge, and instructional practices. Rutherford found that the PLC process of creating the problems, incorporating them into their lessons, and then discussing them again in their PLC’s moved the participants closer to a Standards-based approach to learning.

The purpose of this section was to identify issues affecting teachers’ beliefs about Standards-based approaches to mathematics. Further, it set out to identify change agents used to positively alter mathematics teachers’ beliefs about such an approach to teaching. There is clear evidence that traditionally-oriented teachers can be influenced to change

their practices towards a more Standards-based approach. Getting teachers to try a single Standards-based approach or even observe them practiced by other teachers can have a major impact on their opinions and beliefs towards them.

### **Diffusion of Innovations**

In this section, I outline Roger's diffusion principles of innovation. Roger defines innovation as "...an idea, practice or object that is perceived as new by an individual or other unit of adoption (*loc* 876)." This theory identifies some factors that may play a role in teachers' decision-making process for adopting an educational practice. Additionally, tying Hiebert's (2005) account of the slow rate of change in mathematics educational practices to the factors that impact adoption of innovations will give insight to effective practices for diffusing Standards-based approaches to teaching mathematics.

The general idea behind Roger's *Diffusion of Innovations* theory is that people adopt an innovation (i.e., a particular technology, idea, or practice) at varying rates. Some do so very quickly and other only after a long period of time and then only after seeing the benefit of the innovation. In this theory, "diffusion is the process in which an innovation is communicated through certain channels over time among the members of a social system" (Rogers, *loc* 768). Adoption consists of the stages individuals go through after first hearing about the innovation to the time they adopt it. The following is a description of these stages of adoption and the process of diffusion.

#### **Process**

There are four elements in diffusion: (a) innovation; (b) communication channels; (c) time; and (d) social systems. As stated above, innovations are ideas or objects

perceived as new by a person or group. Communication channels are the methods for getting a message from one individual to another. Time is the period a person takes to move through the decision making process for adopting an innovation, whereas the rate of adoption is the speed members of a social system adopt an innovation. A social system is a “set of interrelated units that are engaged in joint problem solving to accomplish a common goal” (*loc* 1103).

According to Rogers, the decision to adopt an innovation can occur in three ways. One, it can be adopted by an individual who is distinct from those in his or her social system. In the case of adopting new technologies, early adopters are typically leaders in a social system who frequently have greater incomes. The second type of decision is that made collectively by a group. The third type is that made by a few leaders for an entire social system. Before a decision is made to adopt or reject an innovation, individuals within a social system go through a five-stage process: (a) knowledge; (b) persuasion; (c) decision; (d) implementation; and (e) confirmation. Over a period of time, an innovation is transmitted to members of a social system through various means of communication, and each individual moves through the adoption stages at varying rates.

**Stages.** Rogers’s stages were generally based on the findings of Ryan and Gross (1943) in a study on the adoption of a corn seed that had greater crop yields. They found that a typical farmer took a fairly long time to adopt the seed as their own. After learning about the seed through various communication channels (i.e., other farmers, researchers, etc.), they sought more information from other farmers. Further, they had to see the crop yields of others who had used the seed to believe it worked. Even after gaining evidence

of the benefits of the new seed, some farmers still took more time to make the decision to use it. Ryan and Gross's findings led to a stage theory that Rogers refined over the course of his career. These five stages are outlined below as they are in Rogers's text (*loc* 3678):

1. **Knowledge** occurs when an individual (or other decision-making unit) is exposed to an innovation's existence and gains an understanding of how it functions
2. **Persuasion** occurs when an individual (or other decision-making unit) forms a favorable or an unfavorable attitude towards the innovation
3. **Decision** takes place when an individual (or other decision-making unit) engages in activities that lead to a choice to adopt or reject the innovation
4. **Implementation** occurs when an individual (or other decision-making unit) puts a new idea into use
5. **Confirmation** takes place when an individual seeks reinforcement of an innovation-decision already made, but he or she may reverse this previous decision if exposed to conflicting messages about the innovation

### **Rate of Adoption**

The rate at which an innovation is adopted is measured by the time it takes for a percentage of people in a community to adopt the innovation. Each community consists of five categories of adopters that are either quick or slow to adopt a given innovation. For example, if a new phone that allowed for holographic communication were invented, people within a standard town in the United States (a social system) would adopt the

technology at varying rates. There would be people who love technology and have enough money to afford such a gadget who would buy it straight away. There would be enthusiasts who were slightly skeptical about the value and quality of the technology and would adopt after the enthusiast. If the technology panned out and enough of the second group adopted it, then many others in the system who trust the second group would follow suit and create a critical mass that would create a snowball effect for others to do the same. There would be individuals who were still skeptical of the technology and/or not be able to afford it that would still hold out, and there would be those who were very traditional who like things “the way they were” who may never adopt until the only phone that was available were those with the holographic function.

**Adoption categories.** Rogers established specific categories for the types of adopters exemplified above: (a) Innovators; (b) Early Adopters; (c) Early Majority; (d) Late Majority; and (d) Laggards. The following is a brief description of each of these adopters. The Innovators, or the “venturesome” (*loc* 5694), are adopters who are almost obsessed with the acquisition of new ideas and practices. They are generally cosmopolitan and well educated. They usually have resources that allow them to purchase items when they are new and more expensive, or the resources allow them to take risk with new ideas or practices that may result in expensive failures or setbacks. They tend to have communication ties that span a far greater distance than those they have with their community. These ties give them knowledge of new innovations that they would not receive in their community. They may not be well known or respected in their



social system, but they play an integral part in bringing new innovations into the system and introducing it to the larger community.

The Early Adopters are the locally connected individuals in a social system that are well-respected. Innovators seek these people out to disseminate their ideas, and members within the social system look to them for advice on whether or not to adopt an innovation. They are typically well educated, financially well off, and opinion leaders. They maintain a trusted and respected position in a social system by adopting an innovation judiciously and then evaluating it subjectively. Their adoption of an innovation is essentially a “stamp of approval” (*loc 5717*) for others in the community.

The Early Majority, or “deliberate” (*loc 5717*) are those who adopt an innovation right before the average member of a social system does. They generally interact frequently with their peers but are not considered leaders. They make up nearly one-third of the adopters and play an important role of completing the diffusion process, as they are the critical mass of adopters that create a snowball effect for diffusing an innovation.

The late majority, or “skeptics” (*loc 5728*), are those who adopt only after a majority has done so and frequently because they must (i.e., adopting a new corn seed because the alternative becomes scarce due to the new one). Their resources and finances are typically scarce, so it is necessary for them to have their concerns and skepticism about an innovation allayed prior to adoption. In the case of teachers, the adoption of a pedagogical practice would not impact their finances directly; however, their resource of time could be greatly impacted.

The Laggards, or “traditional” (*loc* 5728), are members of a social system who look to the past for ways to approach situations in the present. Their financial situations are typically quite precarious, so adoption of an innovation must come with the knowledge that it will not fail. In the case of a pedagogical tool, a Laggard would need to know that their time would not be wasted and the value of the innovation paid the dividends they valued most.

**Factors influencing adoption.** Rogers identified five factors that affect individuals in their varying stages of adopting an innovation: (a) relative advantage; (b) compatibility; (c) complexity or simplicity; (d) trialability; and (e) observability. These factors can have positive or negative impacts on an individual’s decision to adopt or reject an innovation. A brief description of each is given below:

1. Relative advantage has to do with how an innovation is better than a previous, presently used innovation.
2. Compatibility deals with how well an innovation meshes with an individual’s life.
3. Complexity or simplicity deals with the perception an individual has towards the ease of implementing an innovation. If an individual perceives it as too difficult to implement, then adoption is not likely.
4. Trialability is the opportunity a person has to test an innovation out with little to no risk of financial or resource loss.

5. Observability is how visible an innovation is to others. An innovation's visibility will drive communication among members of a social system and more readily diffuse a negative or positive reaction to it.

### **Social Systems**

As stated earlier, “a social system is defined as a set of interrelated units that are engaged in joint problem solving to accomplish a common goal” (*loc* 1101). These systems can be entire villages or towns debating the implementation of a new water filtration system, a group of doctors questioning the validity of a new drug, or mathematics teachers determining if an approach to teaching problem solving is worthwhile. Within these systems, the most innovative member is often perceived as deviant from the social system and thus accorded little credibility (Rogers, *loc* 1159). However, there are individuals who are considered opinion leaders and their influence significantly impacts the diffusion of an innovation.

Opinion leaders are not necessarily formal leaders of the social system, but they are respected for their technical competence, social accessibility, and conformity to the system's norms. These characteristics afford them an informal authority in their social system, which results in the trust of other members. They reflect the system's structure and move towards innovation if that is the tendency of the system, but they can just as likely shun an innovation. Opinion leaders are more open to external communication than other members of a system and thus reflect a more cosmopolitan tie to the world. Further, they are central to the interpersonal communication channels of a social system and play an integral role in disseminating information and ideas.

Opinion leaders are extremely important to the success or failure of the diffusion of an innovation. The respect these leaders have and the trust they foster put them in a unique position to influence change. Identifying these people and convincing them of the value of an innovation is an integral component to any diffusion process.

### **Next Steps**

As demonstrated in the previous sections, several studies, mostly at the elementary and middle school levels, have indicated that teaching mathematics from a Standards-based approach is beneficial to students in that it develops a deeper understanding of mathematics than do traditional approaches. As discussed above, MM is a venerable and effective Standards-based practice proven to aid most students, but it is an approach many teachers do not know about or are not convinced works. The evidence that many high school teachers have a symbolic precedence (Nathan & Koedinger, 2000a) about teaching mathematics may hinder them from seeing the benefit of a Standards-based approach, which requires them to begin lessons with big, conceptual problems rather than wait to the end of a skill-laden series of lessons that may culminate into the ability to solve such a problem.

Pajares' (2003) statement about changing people's actions by first changing their beliefs is essential to persuading educators to change from practices they believe are sufficient to better practices about which they are not certain. The following study works from von Glasersfeld's (1993) premise that "If one succeeds in getting teachers to make a serious effort to apply some of the constructivist methodologies, even if they don't believe in it, they become enthralled after five or six weeks." My study focuses on high

school teachers' concerns and perspectives regarding a single Standards-based approach to learning mathematics similar to model-eliciting activities – Anchored Contextualizing Problems.

### **CHAPTER III: METHODOLOGY**

This chapter outlines both the research study and its design. In the first section, I describe the participants, data sources, research location, and the researcher's role. In the second section, I discuss the procedures for implementing ACPs in the participants' classes, analysis of the techniques used to collect and interpret data, validity, and limitation.

The following is the definition of ACPs as given in the Introduction of this dissertation: Anchoring Contextualizing Problems (ACPs) are a modified version of model-eliciting activities (Lesh & Lehrer, 2003)<sup>1</sup>. Their purpose is to frame learning around students' personal understanding of a concept. They are general in nature but specific to a topic of learning – in this study, quadratic and polynomial functions. Students receive the problems at the beginning of the units of study and are given at least an hour to generate their own ideas and responses to the problems. Teachers use this time to gain insight to students understanding of the problem and to consider how they will frame learning of the skills and concepts of the related unit of study around it. Since the ACPs are given at the beginning of units of study and meant to encompass the majority of the skills and concepts taught - They are considered to be anchored to everything taught in the unit. See Appendices 1 & 2 for the “Pigpen” and “Getting the Most Out of Your Luggage” ACPs.

The goal of ACPs is to create an impetus for students to think deeply about a given situation and develop their own means of interpreting and solving a problem. They are meant to encompass the above definition of problem solving in that students must construct their own meaning of a situation and devise a method for finding an answer to a problem for which there is not a clear solution. The goals of ACPs extend beyond developing problem solving abilities with students to also include the creation of a context from which to understand specific algebraic skills and procedures taught in a unit of study. In addition to helping students become more reflective and constructive thinkers, ACPs also act as references for later learning and development of skills.

### **The Research Study**

#### **Research Questions**

This research study will seek to answer the following questions:

1. How do high school, Algebra II teachers' concerns and perspectives about Standards-based approaches to teaching problem solving in mathematics change after implementing Anchoring Contextualizing Problems in two units of study?
2. What do teachers learn about their students' mathematical capabilities when using a Standards-based tool such as ACPs?
3. What are factors that promote adoption, or the lack thereof, for innovations such as ACPs?

## **Participants**

In this study, ten Algebra II teachers were asked to participate; however, due to various reasons on the teachers' part, only five took part. The teachers represented in this study took part all of the professional developments and incorporated at least one of the ACPs in their classrooms. One teacher did not administer the second ACP due to time limitations. She attended professional development on it, but she was two weeks behind the other teachers in covering the content of her course and was unable to use the class time to administer the ACP to her students.

## **Data Sources**

Both qualitative and quantitative data were collected to gather as much information from teachers on their concerns and perspectives on using ACPs. The survey of Beliefs and Concerns Regarding Anchored Conceptualizing Problems (designed by me), which incorporates the theories of the Concerns Based Adoption Model (George, Hall, and Stieglebauer's, 2006) and Rogers' (2003) principles of Diffusion of Innovations, was administered three times to the teachers over the course of the study (See Appendix 1). An additional questionnaire was given to the participants after each administration of the ACPs (See Appendix 2). The qualitative components of the study consisted of analyses of interview transcripts conducted with each participant at varying stages of the study.



Table 1

*Research Questions, Data that will be collected, and Data Analyses*

<b>Research Questions</b>	<b>Data</b>	<b>Data Analyses</b>
<p>How do high school, Algebra II teachers' concerns and perspective about Standards-based approaches to teaching problem solving change after implementing Anchoring Contextualizing Problems in two units of study?</p>	<ol style="list-style-type: none"> <li>1. Survey of Beliefs and Concerns Regarding Anchored Conceptualizing Problems</li> <li>2. Transcripts from interviews</li> <li>3. Field notes taken during professional development</li> <li>4. Teacher reflections on their administration of the ACPs</li> </ol>	<ol style="list-style-type: none"> <li>1. Quantitative analyses by averaging deltas</li> <li>2. Qualitative analyses to understand teachers' concerns and perspectives towards ACPs</li> <li>3. Analyses for tailoring questions to individual participants in interview process</li> <li>4. Data gathered from short questionnaires to tailor the second professional development and final interviews around participants' specific concerns and perspectives about their implementation of the ACPs.</li> </ol>
<p>What do teachers learn about their students' mathematical capabilities when using a Standards-based tool such as ACPs?</p>	<ol style="list-style-type: none"> <li>1. Survey of Beliefs and Concerns Regarding Anchored Conceptualizing Problems</li> <li>2. Transcripts from final interview</li> </ol>	<ol style="list-style-type: none"> <li>1. Quantitative analyses by averaging deltas to determine change in participants' concerns and perspectives towards ACPs</li> <li>2. Qualitative analyses to find commonalities among participants who either learned something new about Standards-based approaches due to the use of ACPs or do not learn anything new.</li> </ol>

<p>What are factors that promote adoption, or the lack thereof, of innovations such as ACPs?</p>	<p>1. Transcripts from interviews</p>	<p>1. Qualitative analyses to find evidence of adoption and the factors that led to the adoption of ACPs</p>
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**Research Location**

The research site is located in a mid-size, economically diverse school in Northern Virginia. It is a grades 9 – 12 campus with approximately 2,500 students. The ethnic demographics of the school are as follows: (a) approximately 35% African American; (b) 5% Asian/Pacific Islander and American Indian; (c) 35% Hispanic; (d) 20% White; and (e) 5% Unspecified. Fifty-five percent of the student body is eligible for free or reduced lunch, approximately 20% are classified as having Limited English Proficiency (LEP), and 10% of the student population is categorized as Special Education (SPED).

The school has approximately 200 licensed staff with 25 mathematics teachers and 10 administrators, with one of the administrators overseeing the mathematics department as her primary duty. The school offers the following mathematics courses: Algebra I, Geometry, Algebra II, Pre-Calculus with Trigonometry, Statistics and Probability, Discrete Mathematics, Intro to Computer Science, Advanced Placement Computer Science, Advanced Placement Statistics, Advanced Placement Calculus AB and BC, Dual-Enrolment Calculus II, and Dual-Enrolment Differential Equations. Sixty

percent of the students tested on the 2011 – 2012 Virginia Mathematics *Standards of Learning* (SOL) exam passed.

### **Researcher's Role**

As the researcher, I had several roles to fulfill in this study. First, it was my duty to facilitate a professional development on ACPs where I trained teachers on how to use two ACPs and when to teach them in a unit of study. Further, it was my responsibility to outline the procedures of my study to the participants, so they were aware of what they were taking part in as well as what I expected of them and myself for the duration of the study. Second, I was responsible for keeping track of the participants' progress while implementing the ACPs in their classes and helping them with any questions they had. Third, I gathered and reported on the following data: (a) the participants concerns and perspectives towards ACPs via the survey mentioned earlier (see Appendix 1); (b) commonalities of participants' changing (or unchanging) concerns and perspectives towards ACPs as they implemented them in two units of study; and (c) analyses of the participants' reflections on using ACPs.

### **Research Design**

I used a mixed-methods design where both qualitative and quantitative data were collected in order to gain insight as to how teachers' concerns and perspectives towards ACPs changed (or did not) after implementing them in two units of study in an Algebra II course. The design implemented aspects of Koellner-Clark's study on *Problem-Solving Cycles* (2007) and Rutherford's dissertation on *Exploring Teachers' Process of Change in Incorporating Problem Solving into the Mathematics Classroom* (2012). I then utilized

discussion and reflection of teaching problem solving during professional developments similar to Koellner-Clark's and Rutherford's studies; however, this study focused on high school teachers, not elementary or middle school teachers, and changing concerns and perspectives on a Standards-based approach to learning. Further, the study explored how the professional developments and the innovation (ACPs) itself affected teachers' adoption of the innovation to teach problem solving in their own classroom.

For the research questions to be properly addressed, I first trained the participants on how to use the ACPs prior to their implementation. Further, the participants were extensively interviewed to gain insight of their understanding, thoughts, and opinions of the use of ACPs in mathematics. The following sections describe this data in greater depth and how they were used to answer the research questions.

### **Procedures and Data Analyses**

In this section, I outline the entire study and describe the data sources I used to collect information about the concerns and perspectives of teachers implementing ACPs. Starting with participant selection, I will describe the study as it occurred chronologically and integrate how I used my data sources. The ACPs in this study covered two consecutive units in the participants' curriculum. These units are typically covered between the months of October and January. With the preliminary work needed for the study, the data collection began in September of 2013 and ended in late January of 2014.

### **Participants**

The ACPs used in this study are specific to an Algebra II course and thus require participants who teach the subject. I chose to implement the ACPs at this level of

mathematics because Algebra II is considered a high school class. With the acceleration of Algebra I (and even Geometry) in middle school, using the ACPs at those levels of mathematics could generalize my study to both middle and high school. Since there is a shortage of data at the high school level for teachers' perspectives and concerns towards Standards-based approaches to teaching mathematics, I sought to conduct a solely high school study. Additionally, I believe that Standards-based approaches such as ACPs can be challenging to implement if students have not already learned elementary and middle school mathematics with them, and thus applying ACPs at a later stage such as Algebra II can be quite difficult. If Hiebert (2005) is correct about little changing in the way mathematics has been taught since the inception of the 1989 NCTM Standards, then it is likely that many students enter Algebra II without having learned mathematics in a way that have prepared them to use ACPs. Such a reality makes teaching a Standards-based approach to mathematics this "late in the game" all the more challenging, and the concerns of the participants in my study may be all the greater. These greater challenges may add more depth and richness to the study.

To assure a sufficient number of participants, I cast a wide net and asked all of the Algebra II teachers at the research site to take part in the study. Typically, the research site has up to a dozen Algebra II teachers. I have been a teacher at the research site for six years and have established a rapport with the administration and the teachers in the mathematics department, so attaining permission from administration and participation from the teachers was not a concern. After discussing my plans with both the head principal and the principal in charge of curriculum, I asked each teacher individually to

participate. I contacted teachers via email and briefly explained the study to them, and then I followed up in person to determine if they would take part in the study (see Appendix 3 for the email). The five teachers who agreed to participate received a consent form (See Appendix 4) that outlined the study thoroughly and explicitly stated that they were not required to take part in the study and may drop out at any time.

This “purposeful selection” as Maxwell (2005) coined the phrase, allowed me to gain information that cannot be “gotten” (p. 88) using other forms of participant selection. Other methods would not have allowed me to gather data on the concerns and perspectives of all the kinds of teachers I hope to interview – teachers that may have unique and relevant perspectives to offer. Specifically, the random selection process typical in quantitative research would have hindered my ability as a researcher to gather pertinent data because it would not have assured that I would get the cross-section of Algebra II teachers I was seeking (i.e., varying ages, experiences, educations, etc.). If I had conducted this study across the world and with a sufficiently large enough sample of Algebra II teachers, then it is likely I could have randomly acquired the cross-section I seek; however, this would have been a monumental feat requiring a huge amount of funding.

In order to assess the participants’ perspectives on teaching mathematics (i.e., traditional, Standards-based, those who are open to Standards-based, etc.), I did one of two things prior to the professional development to draw inferences on the way each of them teach mathematics. I either observed a participant teaching a lesson on absolute value equations, or I had the participant describe to me their lesson plan for teaching the

topic. It would have been preferable to have had the participants do both, as doing so would have given me greater insight to any discrepancy between their described practices and their actual practices. According to Maxwell (2005, p. 94):

While interviewing is often an efficient and valid way of understanding someone's perspective, observation can enable you to draw inferences about this perspective that you couldn't obtain by relying exclusively on interview data. This is particularly important for getting at tacit understanding and 'theory-in-use,' as well as aspects of the participants' perspective that they are reluctant to directly state in interviews.

Due to participant scheduling conflicts, doing both was not a possibility. After observing and categorizing each of the teachers, I performed a member check during the interviewing process to verify with each teacher whether my assessment aligned with their perspective on teaching (Maxwell, 2005).

### **Professional Development**

The first ACP, *Building a Pigpen*, was used for the unit on quadratic functions (See Appendix 5 to view the entire problem). The problem requires students to create an algorithm for calculating the area of a rectangular pigpen given 300 feet of fencing. This ACP has the potential of helping students develop the following skills and concepts related to quadratic functions: (a) the behavior of quadratic functions; (b) gathering and organizing data in tables and graphs; (c) the ability to construct a function through the process of guessing and checking solutions; (d) the direction a parabola opens; (e) the relationship corresponding points have with the line of symmetry and vertex of a

parabola; (f) the importance of the zeros of quadratic functions; (g) the domain and range of functions; and (h) how the values on a graph of a function, such as the highest point, relate to a given situation.

This professional development session took place in mid-September 2013. Prior to the professional development, the ACP was sent to each of the participants to familiarize them with the problem. Further, the survey (see below for details) was sent to each of the participants in order to gain a general insight of their concerns and perspectives towards using the ACP. I opened the professional development session by placing the participants into groups of two and three and asking them to solve the Pigpen problem as if they were a student without any knowledge of the Algebra used to find information about quadratic functions. The teachers were not allowed to use any sort of symbol to represent an unknown value. I walked around the classroom asking the teachers questions and giving advice as I would my students. This process gave the participants an idea of how they can facilitate the ACP in their own classrooms as well as convey the limitations they were going to have in giving students too much information.

After the participants completed the ACP, I facilitated a discussion on the process they underwent to find the solutions. Additionally, I asked them to brainstorm a list of skills and concepts they believe the question encompasses so they knew when to reference the ACP during their lessons. I then asked the participants to complete another survey in order to determine if there were any changes in the concerns and perspectives towards the ACP prior the professional development and when they believe it should be administered (their responses were addressed in the first interview cycle). Finally, I



discussed how the ACP would be the first thing they do in the unit and used this opportunity to explain my study.

### **Data source – First and Second Administration of the Survey of Beliefs and Concerns Regarding Anchored Conceptualizing Problems**

Shortly after sending the first ACP to the participants, I administered the survey of Beliefs and Concerns Regarding Anchored Conceptualizing Problems to all of the participants. After the professional development, I administered the survey again in order to see if the professional development had any impact on the participants concerns and perspectives towards the ACP. The survey was a 21-item questionnaire that gauged educators' concerns about ACPs in the following six areas: (a) teacher self-efficacy; (b) teachers' beliefs about students' ability to work with ACPs; (c) teachers' beliefs about ACPs; (d) organizational concerns; (e) adopter categories for Roger's Diffusion Theory; and (f) teachers' understanding of Standards-based approaches to learning mathematics.

The survey was used in two ways: First, as a means of staging individual interviews with teachers (see next section); and secondly, quantifying changes in participants' concerns and perspectives towards using ACPs throughout the study. In the interviewing process, the participants' scores on the survey were used to determine where each of them scaled regarding their concerns and perspectives on ACPs. With this information, I was able to tailor interviews to each individual in the study and more thoroughly acquire qualitative data from each participant. Subsequent administrations of the survey allowed me to examine quantitative data to determine if participants' concerns and perspectives changed during various stages of the study. I will address how this data was calculated and used in a later section.

## **First Round of Interviews**

After the professional development, I set up and conducted interviews with each participant prior to their implementation of the Pigpen ACP. I performed the member check mentioned above to confirm the participants' perspectives on teaching mathematics, and I used the individual responses to the survey to ask each participant about their concerns and perspectives about ACPs (For a list of questions that were asked in the first interview, please see Appendix 6). It must be noted that the questions asked of the participants were not limited to those listed in Appendix 6, as the participants' responses to the survey as well as information gained during the professional development gave rise to follow-up questions or refinement of those already listed. Like the survey, the interview questions were organic in that they changed over the course of the study to fit the individual needs of the participants. After the interviews were completed and transcribed, I began combing through each of the transcripts looking for commonalities. I also began categorizing and coding the commonalities for comparison to later interviews and analyses of the entire study's data.

## **Implementation of the First ACP**

The first ACP was administered in mid-October 2013. The day each participant administers the first ACP, they received a short questionnaire in both hard-copy and electronic format. The questionnaire sought to determine how they felt their lessons went and how beneficial the ACP would be when teaching the skills and concepts in the unit of study related to it. I used this information to better tailor the second professional development to the overall needs of the participants. The teachers were also asked to

collect a sample of students' work for discussion in the following professional development. Not all of the participants returned these questionnaires to me even after several requests; however, I was able to acquire the data I was looking for through the group reflection discussed in the following section.

### **Group Reflection and Second professional development**

The participants completed the quadratics unit associated with the Pigpen ACP in mid-November 2013. Once this was done, I scheduled the second professional development session where we reflected on the implementation of the first ACP. Data collected from the questionnaires given to the participants after administering the ACP as well as student work guided the discussion (like the questionnaires, not all participants contributed student work). The reflection process was recorded for referencing specific statements made by participants in the final set of interviews. After, I administered the second ACP, *Maximizing Your Luggage* (See Appendix 7).

The second ACP is based on a cubic polynomial and has many similarities to the Pigpen ACP. Specifically, the process of gathering and organizing data are similar, and that the goal is to find a maximum value – in this case volume. This ACP has the potential of helping students develop the following skills and concepts: (a) gathering and organizing data in tables and graphs; (b) creating functions from guess and check tables; (c) differentiating between symmetry of quadratic functions and the possible skewed nature of polynomial functions; (d) how the domain and range of general, polynomial functions are limited by the realities of real-world situations; (e) how zeros of polynomial

functions can be helpful in describing the behavior of the functions; and (f) using graphing utilities to aid students in understanding the nature of polynomials in general.

The participants were then administered an expanded survey, which included one additional question that reflected the participants' needs as determined by their notes and comments from the previous administration of the survey. I analyzed each of the two surveys that the participants completed for changes in their concerns and perspectives towards the use of ACPs so I could better tailor the subsequent interviews for each of the participants.

### **Third Administration of the Survey**

Prior to the participants administering the second ACP, I sent each of them the survey again. This was the third and final time the survey was given to the participants. I wanted to see if there were any general changes in participants' concerns and perspectives when presented with a new ACP. For instance, would the participants feel more confident about administering a new ACP after having administered one already? Would they have more or fewer concerns about their students' ability to do the ACPs?

I compared the data gathered on the final survey with the first and second in order to gauge any overall changes in the participants' concerns and perspectives about ACPs. To calculate any significant changes, I created six spreadsheets that were organized by the categories listed earlier. I calculated changes in the Likert-scores from the first survey to the second, the second to the third, and the first to the third. I then calculated the average change for all the participants in each category (See Appendix 8). This data

yielded a few interesting results that aided me in customizing the final interviews for each of the participants.

### **Implementation of the Second ACP**

Using the same process as the Pigpen ACP, participants administered the second ACP to their students, collected a cross-section of student work, and completed the questionnaire. The questionnaire was again sent to the participants to complete after they administered the ACP. New information gathered from this instrument aided me in creating specific questions to drive discussion during the final meeting with the participants.

### **Final Group Reflection**

This was the last time the participants came together as a group and discussed their general impressions of ACPs. The opportunity to reflect on the process and discuss with one another their thoughts on what was and was not useful acted as a stimulus for thoughtful and forthright discussion in the individual interviews that followed. I recorded the discussion and took notes of comments that were relevant to the study. These notes were useful in creating “organizational categories” (Maxwell, 2005, p. 97) for preliminary coding of the participant interviews.

### **Final Interviews**

The final interviews provided the largest source of data for this study (For a list of questions I asked the participants, see Appendix 9). Focusing first on the study’s core questions, I initially searched for data that indicated change in any of the participants’ concerns and perspectives about the use of ACPs. I also knew that teachers whose

concerns and perspectives did not change would also be very important to the review of the data. Specifically, teachers who started off with negative views of ACPs and continued to maintain the same concerns and perspectives reflective of said views would have given insight to what Hiebert (2005) stated about the slow pace of change with regard to the Standards. While those who started off with positive views and maintained them had valuable advice and opinions about how to better design professional developments towards the needs of teachers asked to implement Standards-based approaches to teaching mathematics.

The participants in this study were essentially early adopters (Rogers 2003) and were all quite positive about ACPs early on in the study. They did not provide any negative views towards ACPs during the study. However, their reasons for adopting ACPs as a personal, pedagogical practice went straight to the heart of the third research question: What are factors that promote adoption, or the lack thereof, of innovations such as ACPs?

**Coding.** The interviews were recorded and transcribed for analysis. Using the coding in the first round of interviews, I searched for commonalities between the two interviews and reported on all changes. Additionally, I combed the transcripts for commonalities new to this stage in the study, coded them, and reported on them accordingly. Using Maxwell's (2005, p. 97) categorizing analysis, I created organizational, substantive, and theoretical categories for the following coding process.

1. I started with general organizational categories based on anticipated statements teachers made (i.e., issues with time, students' prior knowledge, etc.).

2. While interviewing each of the teachers, I noted specific comments that stood out and began creating substantive categories that described specific statements made by the participants.
3. I read each interview transcript multiples times and noted commonalities I found among all the participants and used them to hone my substantive categories.
4. I then ran a search<sup>9</sup> on each of the transcripts using the common words and/or comments (data from substantive category) that I noted in the interviews to verify any widespread relationship among the participants and created theoretical categories that reflect my interpretations of the data.
5. I next contacted each of the participants and “member checked” (Maxwell, 2005) how I interpreted their statements that related to each of the theoretical categories.
6. Finally, I had a second researcher do a separate coding, so I could compare my work to hers to determine if my findings and developed theories were valid.

According to Maxwell (2005), organizational categories are broad areas or issues that can be anticipated by the researcher, while substantive categories are descriptions of participants’ concepts and beliefs. The substantive category is used to identify specific statements made by participants, and it is not for general interpretations made by the researcher. Data collected in substantive categories however “can be used in developing a more general theory of what is going on” (p. 97). Theoretical categories, however, are

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<sup>9</sup> Instead of using a program like NVivo or Atlas, I simply used the “Find” function in Microsoft Word to search the number of occurrences of a particular word and/or comment I determined pertinent in the interviewing process and the first reading of the interview transcripts.

more general and abstract and can be created from prior theories postulated by the researcher or theories developed through the coding process.

In this study, I focused on creating theoretical categories as I coded the first and second interviews. Although I suspected there would be categories related to teachers changing their views of Standards-based approaches (i.e., never knew what Standards-based approaches really were, never thought students could do this kind of work, etc.), other interesting topics arose about the reasons why they adopted the use of ACPs.



Table 2

*Timeline of Interview, Professional Developments, and Interviews*

Date	Data Collection
August/September 2013	<ol style="list-style-type: none"> <li>1. Sent first ACP to Participants</li> <li>2. Administered survey, took notes, and expanded/honed based on Participants' concerns and perspectives</li> <li>3. Observed Participants teaching a lesson (or discussed lesson plans) to assess their teaching practices (i.e., traditional, Standards-based, etc.)</li> <li>4. Professional Development – Introduced the Pigpen ACP and study to the Participants</li> <li>5. Administered expanded survey, took notes, and expanded/honed based on Participants' concerns and perspectives</li> </ol>
October/November	<ol style="list-style-type: none"> <li>1. First round of Interviews</li> <li>2. Began coding Interviews</li> <li>3. Participants administered the Pigpen ACP</li> <li>4. Participants collected student work for discussion and reflection in next professional development</li> <li>5. Administered reflection/questionnaire</li> </ol>
November	<ol style="list-style-type: none"> <li>1. Second professional development – Participants reflected on implementation of the first ACP and were trained on the second ACP; the lesson was recorded so notes on key conversations could be made</li> <li>2. Administered expanded survey, took notes, and expanded/honed based on Participants' concerns and perspectives</li> </ol>
December/January	<ol style="list-style-type: none"> <li>1. Participants administered the Luggage ACP</li> <li>2. Participants collected student work for discussion and reflection in next professional development</li> <li>3. Administered reflection/questionnaire</li> </ol>
January/February 2014	<ol style="list-style-type: none"> <li>1. Participants met for the last time to reflect on the Luggage ACP – the discussion was recorded so notes on key conversations could be made</li> <li>2. Final survey administered</li> <li>3. Sought additional questions for the survey from Participants</li> <li>4. Conducted final interviews</li> <li>5. Transcribed interviews and began coding</li> </ol>
February – May	<ol style="list-style-type: none"> <li>1. Determined Results and Report</li> <li>2. Drew Conclusions</li> <li>3. Formatted and Edited</li> </ol>
June	<ol style="list-style-type: none"> <li>1. Defend Dissertation</li> </ol>

## **Validity**

My first concern with validity is my interpretations of the results of the interviews with the teachers. I was concerned that my bias towards MM and the use of ACPs in the classroom would act as rose-colored glasses as I combed through the transcripts of the interviews. Specifically, I was concerned that I would find what I wanted to find in the transcripts rather than the intended meaning of statements made by the teachers. As stated in the coding sub-section, I had a second researcher also code my transcripts and then compare my analyses with hers. She also coded the interviews similarly and made similar findings as mine. By doing so, I reduced the possibility of misinterpretation and more likely confirmed my findings.

My second concern about validity was that the survey was specifically used to guide the interviews and not to draw conclusions. I tried to design the survey so changes in responses to survey question by the participants could not be viewed in a positive or negative manner. Instead, I wanted the survey to capture any shift and serve as a catalyst for gaining greater insight about participant's concerns and perspectives about ACPs in the interviewing process. As the researcher, I was particularly careful not to make inferences or judgments about the participants based on their survey responses.

## **Limitations**

There are several possible limitations in this study due to the number of participants. Though beginning with five participants was a relatively large number for such a qualitative study, I was concerned that some of the participants would withdraw for some of the following reasons. Once they learned the specifics of the study they may

not have approved or desire to be a part of it. They may not been able to commit the time necessary to take part in the professional development and interviewing process. Finally, they may not have been able to make it to one or both of the scheduled professional developments, thus omitting a major component of the study. My concerns were allayed, as all five participants stayed in the study and took part in both of the professional developments. However, due to time constraints, one participant did not administer the second ACP to her class.

Another concern is that this study took place in my school, and I know the participants quite well and have a good rapport with them. As such, they may have been inclined to give responses that they believed I wanted to hear rather than their true opinions, concerns, and perspectives about ACPs. I went to great lengths to emphasize to the participants that I desired their candid responses, which would also be most beneficial to the study. From a researcher's perspective, this may be considered a limitation in that bias may be an issue; however, within the framework of the professional development, my presence and enthusiasm for ACPs turned out to be a primary feature for the participants' adoption of the innovation.<sup>10</sup>

Finally, the implementation of a Standards-based approach to teaching mathematics such as ACPs may require a greater amount experience with the innovation then this study offers. Specifically, participants may need to administer more ACPs in more units of study then just the two given. Administering ACPs over the course of one,

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<sup>10</sup> This factor is discussed in the proceeding chapters.

two, or even three years to allow teachers more time to see their impact may be needed to gain insight to teachers' concerns and perspectives about ACPs.

### **Significance**

As stated earlier, Hiebert (2005) found that little has changed in the way mathematics is taught since the inception of NCTM's Standards; however, many teachers are aware, to some degree or another, of Standards-based approaches to learning mathematics. Developing a framework for understanding teachers' concerns and perspectives about such an approach can allow administrators, educators, and researchers to gain greater insight to related professional developments and implementing a given Standards-based approach to learning mathematics. Understanding these concerns may allow for a smoother, more teacher-centered means of easing educators from traditional methods of teaching mathematics to a more Standards-based model.

## **CHAPTER IV: RESULTS**

In this chapter, I will discuss my three-question study and outline pertinent information that arose during the study. First, I will introduce the participants and briefly examine their experiences as teachers. Second, I will lay out the chronology of the study to set the foundation to aid in answering the study's three questions. Finally, I will thoroughly discuss the findings of the study.

### **Participants**

In this section, I introduce the participants of the study and discuss their general approaches to teaching and their beliefs about incorporating Standards-based approaches in their classrooms to foster problem solving. The participants' beliefs and approaches to learning were gathered using one or more of the following methods: (a) Watching them teach a lesson on absolute value equation; (b) discussing their process in designing their lesson for teaching absolute value equations; and/or (c) collecting information gleaned from the interviews and group reflections. Participants' specific beliefs about teaching towards a Standards-based approach will not be covered in this section, only their general opinion about incorporating such an approach in their teaching.

This section will also address the degree to which each participant *appears* to be a traditional or reform-oriented teacher.<sup>11</sup> It should be noted that a Standards-based approach to teaching does not negate the need for rote examination of procedures<sup>12</sup> for solving math problems so often associated with traditional methods of teaching. On the contrary, the investigation of these procedures can promote all of the NCTM Process Standards if done so in an appropriate manner. In my observation of the participants' lessons, I looked for the number of ways they approached teaching absolute value equations and made an assessment of their traditional versus Standards-based leanings. If I felt they leaned towards a more Standards-based approach it was because they did not simply approach a problem analytically or just do a great job promoting communication among their students. Instead, they *also* included a connection to graphical representations of the solutions to the problems in addition to using analytical methods.

When I observed each of the participants or discussed their lessons with them, I looked for evidence of traditional and Standards-based approaches. I created a simple, two-column instrument with one column titled "Traditional" and the other column titled "Standards-based." When observing the participants, I listed practices indicative of each approach under the appropriate column. For example, if a participant spent a lot of time working through examples, I placed this evidence under the "Traditional" column. If a participant fostered discourse with his/her students regarding multiple approaches to

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<sup>11</sup> As a reminder, a traditional approach to teaching mathematics is generally summarized in this study as a teacher showing students examples of problems and having them work them out with little or no emphasis on application or connection to other mathematical topics. In this study, reform-oriented teachers use a Standards-based approach to promote the NCTM Process Standards - problem solving, connections, reasoning and proof, communication, and representation.

<sup>12</sup> Rote examination of procedures is a series of examples that go through particular types of problems step-by-step. This process does not tie the procedures to any other representation (i.e., graph, table, etc.)

solving an example, then I placed this evidence in the “Standards-based” column. I anticipated that each participant would have a combination of practices and thus have evidence of said practices in each column. The amount of evidence in each column helped me to assess which way each the participants “leaned” in regards to traditional or Standards-based practices.

Of the five participants in this study, I found one to lean towards a Standards-based approach, two lean only slightly towards a traditional approach, and the other two to lean more significantly towards a traditional approach. Interestingly, the participant I found to lean towards a Standards-based approach thought she was far more traditional and one of the participants who leaned slightly towards traditional disagreed with my assessment. I later determined that our different opinions dealt mostly with how we defined traditional and Standards-based approaches to teaching mathematics. These differences will be discussed in the individual discussions of each participant below.

### **Ken**

Ken is a veteran teacher who has been at the study site for more than 20 years. His classroom is set up in rows, but the students are close enough to one another to work on problems together and have discussions on topics Ken asks them to have. When I observed his class, he was teaching absolute value equations analytically by using a method called the cover-up method where students are given an absolute value equation and the portion inside the absolute value is hidden from sight.

$$\begin{array}{l} |x + 2| = 7 \\ | \quad | = 7 \end{array}$$

Since the absolute value of a number asks how far a number is from zero, in this case 7, there are two solutions. For this problem, it would be -7 and +7. However, since the inside part of the absolute value was an algebraic expression, the student must set the expression inside the absolute value sign equal to two different numbers,  $x + 2 = -7$  and  $x + 2 = 7$ , thus yielding two solutions,  $x = -9$  and  $x = 5$ .

Through several examples, Ken was quite successful in conveying the meaning of absolute value to his students. He had the students practice additional problems after their discussion and walked around checking for understanding on the students' part. He aptly promoted discourse among his students about what absolute value meant; however, he did not use multiple representations to make further connections to the solutions of any of the problems he and his students discussed. I considered him a traditional-leaning teacher because he spent a greater amount of time doing examples than he did discussing their meaning, and he did not tie the solutions to any of the problems to graphical or tabular representations. When I performed a member check with him, he did not agree with my assessment. Interestingly, his reasons for doing so were due to a difference in our definition of traditional teaching methods.

For Ken, traditional teaching practices have a rather negative connotation because of his personal experiences as a student. He was educated in the 1970's and 80's by teachers who lectured, wrote problems on the chalkboard, and expected students to quietly write the problems down and figure them out on their own. Ken does not believe he teaches this way, nor do I associate his practices with such methodologies. Despite the



semantics of the situation, Ken and I did not come to an agreement on his approach to teaching.

With regards to his beliefs about Standards-based teaching, he was adamant about the need to go deeper into mathematical concepts with students. When he heard the word ‘standards,’ he first thought of state standards that dictate the skills students must learn. Though Ken did not know the specific Process Standards of NCTM, he stated in his first interview the importance of acknowledging that state standards are a bare minimum and that students must go far deeper than learning simple skills. He referred to the need to make connections and discuss them with his students. He made it clear that he believed that mathematics should be taught from a Standards-based approach even though he did not know the exact terminology.

Though my assessment of Ken’s teaching practices did not completely align with a Standards-based approach, it is clear that his beliefs do and that his practices are geared towards this end. He addressed the looming fact that teachers are on a deadline to cover a huge number of skills in preparation for a standardized test that does not assess meaningful skills like problem solving, connections, communications, etc., and that this reality often holds him back from facilitating more meaningful learning in his classroom.

### **Mike**

Mike has been at the study site for nine years. When I observed him, he had already covered absolute value equations and was teaching absolute value inequalities. He was doing so analytically by using the cover up method as Ken did with absolute value equations, but he had students test their solutions on a number line. This method

allows students to see a graphical representation of values that yield true or false solutions to an inequality. For example,  $|x + 2| > 7$  yields two points on a number line that breaks it into three segments.

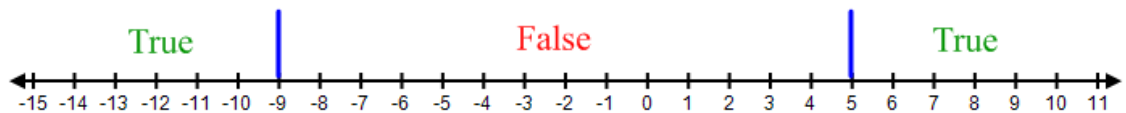


Figure 1. Absolute Values and Number Lines

The points are the same as the solutions to the equation mentioned in Ken's lesson on absolute value equations, but the segments represent sets of solutions rather than just the points representing them. Mike navigated his students through this process and helped them make connections to previous mathematics similar to this kind of problem.

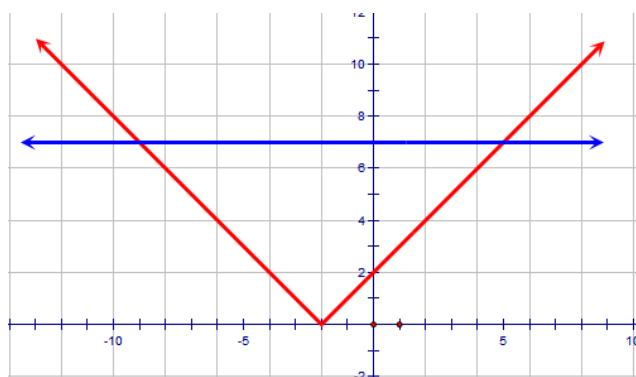
When I asked Mike about his methods for teaching absolute value equations, he stated that it was similar to Ken's. Based on other conversations with Mike, I found that many of his approaches are quite analytical and do not relate graphical methods to his analytical approach as readily as the Process Standards would suggest. I assessed his approaches to teaching mathematics as leaning slightly towards traditional, and when I asked him about his opinion on my assessment he agreed. In our initial interview, I needed to clarify what a Standards-based approach to teaching mathematics is as well as NCTM's emphasis on problem solving. Mike had a general idea of what a Standards-

based approach is, and he agreed that problem solving was integral to mathematics. Despite his belief in the importance of training students to be problem solvers, he conceded that this was an area of difficulty for him.

### **Shareese**

Shareese is a highly structured mathematics teacher with 15-years experience teaching middle and high school mathematics. She scaffolds much of the mathematics she teaches her students by creating worksheets and notes that outlines everything the students need to know. Essentially, she does much of the organization for her students and does not leave them to figure a problem out on their own.

I was not able to observe Shareese teach, so I sat down with her for 30-minutes and discussed how she created a lesson for absolute value equations. I had seen her materials in the past and made the inaccurate assumption that she leaned very strongly towards a traditional approach to teaching. Though she scaffolds her lessons significantly, I found that she not only teaches the subject as Ken and Mike do, but she also incorporates an object-oriented approach to the topic. An object-oriented approach to functions and equations is a multi-representational method of teaching students to tie analytical solutions to their graphical representation. When Shareese teaches the equation  $|x + 2| = 7$ , she demonstrates to her students that the equation can be represented using two different graphs on the same coordinate axes.



**Figure 2. Absolute Values and Graphs**

The v-shaped graph is the absolute value side (left of the equal sign) of the equation, and the horizontal line is the right side of the equation. The  $x$ -coordinates of the points where these two graphs intersect are the solutions to the equation,  $x = -9$  and  $x = 5$ . The fact that an absolute value creates a  $v$  and a constant ( $7$ ) creates a straight-line that can cross the  $v$  at two places gives students a visual representation that reminds them to seek out two answers when solving absolute value equations analytically.<sup>13</sup>

Upon determining Shareese’s multi-representational approach to teaching equations, I assessed that she leaned more towards a Standards-based approach to teaching. Despite her highly structured scaffolding that removes much of the work students must do in order to be organized, Shareese fosters the Process Standards in her classroom quite well. Surprisingly, when I asked her what kind of teacher she thought she was she stated that she was traditional.

In contrast to Ken, she matriculated in the late 1980’s and 90’s and did not experience the one-way lecture where students sat quietly writing down solutions to

<sup>13</sup> A horizontal line can also touch the vertex of the  $v$  tangentially, yielding only one solution, or not at all, yielding no solutions.

problems. She attended classes where students interacted with their teachers and worked on problems together. To Shareese, this is what it meant to be traditional. She did not consider her multi-representational approach to teaching as anything special and thus thought she was not doing anything differently than the way she was taught.

Shareese stated in her first interview that she viewed reform-oriented approaches to teaching mathematics as Discovery Learning where students are given materials and a problem and told to figure something out. This perception of a reform-oriented approach is in significant contrast to her highly structured, scaffolded approach, so she considered herself quite traditional. When considering Ken's view of traditional teaching juxtaposed to Shareese's it all becomes quite relative. Although not completely relevant to the goals of this study, I found that semantics plays a major role in a teacher's self-assessment of his/her approaches to teaching.

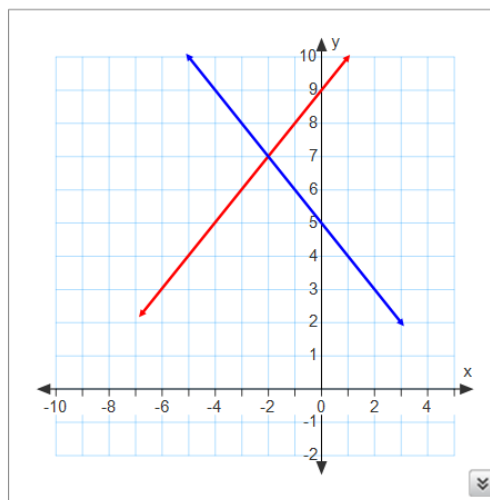
With regards to Shareese's knowledge of Standards-based approaches to learning, she stated that she did not know them specifically but then said, "I think their take (NCTM) is more of a problem solving, reasoning based, using formulas kind of like science formulating, hypotheses way about to solve problems. Like not too much of a teacher led process." She clearly had a grasp about what Standards-based approaches are, but she then conceded that she scaffolds her lessons more than she believes NCTM would suggest doing so stating, "I am probably helping them more than I should."

When referring to scaffolding, she commented that the "kind of students we have" require more guidance. She seemed to indicate that her students would have more difficulty learning in a "student centered" format and thus needed more help. Her beliefs

about teaching mathematics seemed to indicate a certain level of skepticism towards a Standards-based approach to learning. While my overall assessment was that Shareese leans more towards a Standards-based approach than she thinks, it appears that Shareese does not fully understand what the Process Standard's goals truly are for teachers.

### Hartley

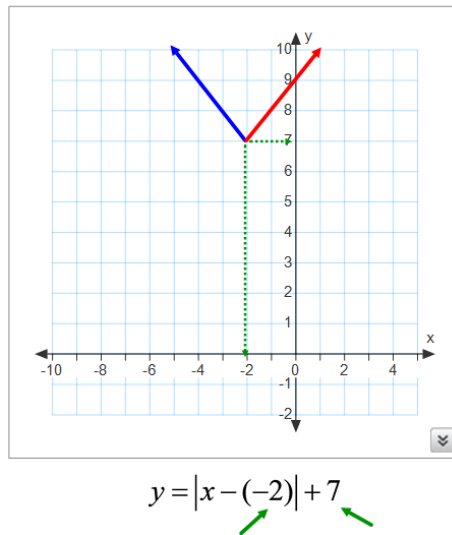
Hartley is a second year teacher at the study site. When I observed her, she was teaching students how to graph absolute value functions using a combination of analytical and functional approaches. The analytical method draws on students' prior knowledge that absolute value functions yield two solutions and in the case of graphs, two linear functions.



$$y = |x - (-2)| + 7 \quad \rightarrow \quad y = \begin{cases} x + 2 + 7 & \text{for } x > -2 \\ -x - 2 + 7 & \text{for } x \leq -2 \end{cases}$$

Figure 3. Absolute Values as Lines

The functional approach looks at absolute value functions as the *v*-shaped object mentioned previously and moves them on their vertices about a Cartesian-coordinate plane based on values inside and outside of the absolute value symbols in the function.



**Figure 4. Transformational Form of Absolute Values**

Hartley spent the majority of time on the analytical approach and tested points on each of the lines to determine which were “true” and which were “false.” This method helps determine which portion of each line to use, left or right of the vertex (point of intersection of the two lines in this case). It is a very analytical approach that ties the meaning of absolute value functions to their *v*-shape. It also can segue to the functional approach of teaching absolute value functions as objects that transform (dilate, translate, etc.) according to the features of their function. The functional method is a more global

and generalized view of the topic and focuses less on procedure. This format can be easier to follow than the analytical approach.

In a Standards-based approach to this topic, a teacher would start with the global perspective of viewing absolute values functions as transformable objects then move to the piece-wise approach with which Hartley began. However, the fact that Hartley taught it from both approaches demonstrated a desire to develop a multi-representational perspective for learning mathematics. The order of approach almost becomes inconsequential when considering her beliefs about teaching mathematics. Though she leaned more heavily towards the analytical approach she had a clear desire to teach both methods and create connections between them.

When I interviewed her about her methods, she believed that she was quite traditional. When asked about what she thought traditional meant in regards to this specific lesson she stated, “It was, this is what you do and this how you do it. There was no, here’s a question and let’s see if you can figure it out.” She was referencing the fact that the lesson was not contextualized; she did not begin with a word problem that could be modeled with an absolute value function. When I asked her if her approach was the only way she believed the topic could be taught from a Standards-based approach, she said no, but she could not come up with examples.

Hartley believes that her approaches to teaching mathematics are more traditional than Standards-based, and I agree based on the evidence I collected while observing her. However, I do not interpret her practices as contradicting her beliefs that teaching mathematics should be more Standards-based, nor would I say that she is solely a



traditional teacher. Many teachers state that they believe mathematics should be taught using Standards-based approaches, but they do not demonstrate such practices when observed (Raymond, 1997). In Hartley's case, she demonstrated a desire to teach a more multi-representational approach that emphasized connections between graphical and analytical methods even though her lesson leaned heavily towards the analytical. Further, the efforts she makes to create novel lessons and to seek better ways to teach a given topic suggests she is striving to become a Standards-based teacher. This evidence suggested that she leans slightly towards a Standards-based approach to teaching mathematics.

### **Nicole**

Nicole has more than ten years' experience teaching high school mathematics. I had observed Nicole teach prior to my study and noticed that she was quite traditional in her approach to teaching mathematics. Her lesson was centered on doing examples with little discussion about the meaning behind the mathematics. Students followed her examples and replicated her procedures on problems given to them on worksheets. When I sat down with her for the study and discussed the constructions of her lesson plan for absolute value equations, I saw much of the same approach as I did when I observed her earlier. When she described her teaching process, she explained how she demonstrates breaking the absolute value equations into two separate linear equations to be solved separately. She briefly mentioned discussing with her students how absolute value problems can have two different answers and thus the need for two different linear equations. Much of our conversation dealt with the procedural aspects of solving these

kinds of equations. Nicole made no mention of connecting the solutions to these equations with graphs, nor did she discuss methods such as Mike's for checking solutions.

When I performed a member check with Nicole, she agreed with my assessment that her approach to teaching absolute value equations was traditional stating, "Oh yeah.... Definitely." Another indicator of her traditional approach to teaching was that she viewed learning procedures as a means of learning to think logically. She believes that learning material like the quadratic formula in a procedural manner made the students smarter because they would later be able to follow other rules in life in a step-by-step manner. For her, much of the value of learning mathematics is learning to follow procedures properly.

### **The Study**

In this section, I outline the study and begin developing common themes found during the observation process, data gathering, and the first and second professional developments. I will discuss the key portions of the study and highlight important elements of participants' comments, concerns, and perspectives towards ACPs. Since the teacher observations are discussed above, I will move straight to the first professional development.

#### **First Professional Development**

The goal of the first professional development was to train the participants on using The Pig Pen problem<sup>14</sup> and ultimately gauge their beliefs on where in the learning

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<sup>14</sup> Recall that the Pig Pen problem is an activity that is modeled by a quadratic equation.

process such a problem should be given to students. Specifically, should it be given at the beginning, middle, or end of their units of study on quadratic functions? Before making this determination, I wanted them to work on the ACP in teams as their students would. They were instructed to approach the problem as if they had little to no knowledge of quadratics. They were given calculators and told only to use arithmetic to make their calculations. Also, I placed a major emphasis on organizing all of their work in a table.

The participants engaged in the activity well and seemed to enjoy acting like their students. Ken was very good playing the role of a student; asking several questions. They divided into two groups and worked together to create organizational charts of their work. While they were working on the problem, I walked between the two groups asking questions about their reasoning for a particular approach to finding dimensions or area of a pigpen, highlighting good practices by an individual to the entire group, and making suggestions for directing their progress. I told them before the activity began that I would be doing this as a means of showing them how they might administer the ACP to their own classrooms.

After the participants completed their ACP, I modeled how they should focus their classes to the work of a single group within their class and discuss that group's process while also asking other students to make comparisons to their own work. I next asked the participants to point out the characteristics of the activity that related directly to skills taught in the quadratics unit (e.g., repeated areas for different dimensions relate to corresponding points, the maximum area is the vertex of a parabola, the dimensions that give no area are the zeros of the functions, etc.). Interestingly, each participant saw a

characteristic that one or more of the other participants did not see. By the time we were finished, the participants created a significant list of how the activity related to the quadratics unit and they emphasized how much they valued these characteristics.

Finally, I listed all of the participants' names on the board and asked each one of them when they thought this lesson should be taught. Ken thought it should be done sometime during the unit or after it was finished. Mike thought it should be done at the end of the unit, prior to or after the unit exam. Shareese believed it should be done at the end of the unit. Hartley felt it should be shown to the students at the beginning of the unit, referenced throughout, but given at the end. Nicole felt it should be given at the end of the unit. At the conclusion of the professional development, I asked them to administer the ACP prior to beginning their units on quadratic functions with their students. I discuss the participants changing views on this matter in the next section.

### **First and Second Administration of the Survey**

Prior to and after the first professional development, I administered the survey to each of the participants. After I received both surveys, I combed over each survey to see if there were any significant changes in the Likert-scores for any or all of the participants. Aside from the first six questions dealing with the participants' beliefs about being able to properly administer the ACP and their students' ability to do them, there were not any significant changes among Likert-scores for any individual; therefore, no group-wide changes were identified. With regard to the participants' beliefs about their ability to administer the ACP, four of the five participants had concerns prior to the professional development, and those participants' concerns lessened after the professional

development. The fifth question asked the participants about any concerns they had about their students being able to do the problems. Prior to the professional development, all of the participants' Likert-scores indicated that they had some concern about their students' ability to the ACPs. After the professional development, the Likert-scores indicated that their concerns had lessened.

To measure the change in the participants' responses to each item, I calculated the change in the Likert-scores for each item for each participant. I then calculated the change and averaged them for all of the participants. I did this for each of the six categories listed in the description of the survey in the Methodology chapter. To reference this spreadsheet, see Appendix 9. As stated above, the only significant change occurred in the average for the first four questions dealing with the participants' beliefs about their ability to administer the ACP. On the first administration of the survey, three of the five participants had Likert-scores of three and/or four to questions asking them about their ability to administer the first ACP. High Likert-scores indicated that their concerns to properly administer the ACP were significant, and low Likert-scores indicated that they had little to no concerns. On average, the participants' concerns about being able to properly administer the ACP dropped (average change was -1.1).

Aside from this change, the survey did not yield any significant results for the participants as a group. There were questions dealing with Standards-based approaches to learning as viewed by the NCTM, but I did not address these in the first professional development, so it was not possible for any changes to occur on these survey items immediately after the professional development. I thought that there would be skepticism

about the value of the ACPs prior to the first professional development and participants would show this on the survey. Thus I anticipated that there would be positive growth on the Likert-scores for these items, but the participants valued them highly prior to and after the professional development.

### **First Round of Interviews**

In the first round of interviews conducted before the teachers administered the first ACP to their classes, a series of commonalities among the participants arose. I used these commonalities to create substantive and theoretical categories that I later explored with the participants in the group reflections. This process allowed me to hone in on specific themes common to all participants and pertinent to the study's research questions.

When I read through the interview transcripts, I annotated key findings in each one and cross-referenced them to determine the commonalities mentioned above. I created three categories for organizing my data based on Maxwell's (2005) format: (a) General; (b), substantive; and (c) theoretical. The general category consisted of concerns, opinions, and belief by the participants that I anticipated ahead of the study. Specifically, the belief that a Standards-based approach to learning was teaching to a series of skills laid out by a governing body (i.e. a state government), that the participants' students would "pushback" against using ACPs, the students would not be prepared, or the ACPs would take too much time to administer in the classroom. The substantive categories I found were based on participants' misconceptions of Standards-based approaches to learning mathematics and their reasons for tentatively adopting the ACPs. Specifically,

most of them would not have considered using ACPs at the beginning of a unit of study, but they saw the value of doing so after seeing the skills and concepts introduced during the professional development that would segue nicely to their later lessons on those skills.

As I compared the transcripts of the first interview and created the substantive categories, I saw some common themes emerge. For instance, despite the participants' initial thoughts that Standards-based learning meant teaching to a series of skills laid out by a governing body, most of them believed that there should be more to them; that they should promote something greater than procedural knowledge. Many even spoke of problem solving and communication without knowing the principles of the NCTM's Process Standards. Also, it was not just a lack of time that kept the teachers from promoting something more than procedural knowledge, but a lack of knowledge about innovations created to help teachers do so. Many of them wanted something more, but did not know what to do, how to do it, or had the time to research it themselves. The ACPs were a possible solution to this issue. Finally, a factor that promoted greater acceptance of ACPs was the various skills they began developing in preparation for the participants' lessons.

The following sections summarize data collected in each of the participants' first interview that substantiate the creation of my substantive and developing, theoretical categories. It must be noted that not every participants' statements fit neatly into each of my categories. The categories are general and were used to hone my questions in later interviews with the participants as a group and as individuals. The descriptions will not

be a narrative of each interview, but rather a “talking points” format where key information is presented.

**Ken.** As with all the participants, I questioned Ken’s thoughts and opinions about Standards-based approaches to teaching mathematics. The first thing he stated was that they are set for us by “...another entity, in our case I believe the state.” He quickly added that he sees them as the minimum standard and minimum competency and that there should be more to them stating, “I’m not saying that, all standards are necessarily the minimum, but I’m saying that can be the place where the debate occurs that, I mean if indeed these are, minimum standards, is that really what we want to be shooting for? Don’t we want to be shooting, well beyond that?” He said he saw them as a work in progress that have been evolving for the last 15-years to a higher level of rigor; however, he could not see how they could lead to a more diverse problem type that dealt with solving real-world problems. His belief is that they should.

He spoke of being more acutely aware of the need for promoting problem solving and reasoning because of his experiences with the College Board and teaching Advanced Placement Calculus. He understood that students were expected to think and reason beyond what was taught in state standards and fostering this needed to happen earlier than Calculus. He then mentioned that by working in the 1990’s and early 2000’s, when discussions of more problem-based learning began occurring, he became more cognizant of the need to concentrate more on the word problems given in textbooks. This was the extent of examples he gave to describe how he promoted problem solving.



When I asked him for his opinions about the first ACP promoting problem solving and reasoning, he agreed that it did promote these skills. He saw it as a reversed approach to the traditional model of teachings skills first and then applying those skills to a real-world problem. When pressed about his comfort level with giving the ACP prior to teaching the lesson, he admitted that it was not something he would do on his own. He said that students needed certain skills to solve such problems, and he was concerned about students being able to do the problem without those skills.

I then asked him what skills taught in a unit on quadratic functions he would be able to reference throughout his lessons after giving the ACP at the beginning of the unit. He stated that he would need to go over the problem again, but as he began discussing it more and more he stated several skills that related directly to the ACP. After several minutes of talking through his thoughts (I said and asked very little), he brought up his stated opinion during the professional development about when to give the ACP. Despite his apprehensions, the professional development and the discussion we were having in the interview led him to see a benefit in giving it at the beginning of the unit. His open pondering of the skills that could be referenced throughout the unit of study seemed to highlight the benefit of giving the ACP earlier than later. He still had reservations about students' lack of necessary skills for the problem, but they seemed to be partially allayed by our discussion.

As I've thought about that; giving it at the beginning, the discussion that we just had about the things that naturally emerge from that, and the added benefit that you're starting right off with this thing that we talked about for twenty

minutes already in terms of the NCTM and their desire to have problem solving be, you know the major focus that we have here, it would seem that giving it at the beginning makes more sense. Again, as long as we work our way back down to get that basic, you know foundation.

After, he still discussed the need to develop foundational skills, so the students would be able to do these problems. In later discussions with Ken, I determined that his concerns with this issue were not foundational skills for quadratics but far more rudimentary skills dealing with middle school mathematics. He wanted to make sure that students would not get caught up making mistakes graphing points or doing minor calculation errors that would ultimately make the ACP difficult to follow and thus understand.

In Ken's first interview, I was able to see the process that he was going through to understand the value of having his students do the ACP. Further, I began understanding that participants realizing the value of giving the ACP at the beginning of a unit of study was a process that took time and thought. Having a new approach and innovation thrown at their feet and told it was great was not enough for them to understand its merits.

**Mike.** After discussing some of the topics specific to the general categories (time, pushback from students for having to do so much work, etc.), Mike and I discussed his views about Standards-based learning. Like Ken, he saw them as a series of mandated skills given by a governing body. He mentioned that prior to there being any mandated standards, teachers just did what they wanted, and so a need for researched, guided standards came about from various outside groups. He threw out an example of such a

governing body, “Like for math maybe, Math Teachers of America or something...” He then mentioned the evolution of these standards to textbooks and state standards such as the Virginia Standards of Learning.

When I prompted Mike towards the NCTM standards, he remembered learning about their general standards, which provide the topics of mathematics students should learn at varying ages (e.g., 6 – 8 Statistics, 3 – 5 Numeracy, etc.), so he had a grasp of some of the work NCTM has done. At this point, I spoke about the Process Standards and how they frame learning around the larger goal of fostering problem solving, communication, reasoning, etc. in students. He pointed out that the standardized exams for which he must prepare his students do not assess these goals; so fostering them in his class is a practice that is easily forgotten.

Mike, like Ken, also teaches Advanced Placement (AP) Calculus, and he has learned that preparing students for a standardized exam is different than preparing students for the Advanced Placement exam. He spoke about the questions on standardized exams seeking right or wrong answers and not concerned with the process students go through to get those answers, whereas the AP exam has portions that do take the process into account when scoring the exams. This fact impacts the way he teaches Calculus differently than Algebra II (a course assessed by the state using a standardized exam) because of the higher expectations for problem solving, communication, and reasoning on the AP exam.

His statements on AP Calculus led me to ask him about the frequency with which he fosters a Standards-based approach to learning in his Algebra II class, and he said that

it was less than 50%. When I asked why, he admitted that he did not know how to due to a lack of experience and examples of activities that fostered the goals of a Standards-based approach. Interestingly, he made reference to doing problems like The Pig Pen ACP and other activities that fostered reasoning, problem solving, communication, and the other Process Standards in his classes, so I was led to believe he taught more towards a Standards-based approach than he thought.

Mike saw value in the ACPs promoting a Standards-based approach to learning mathematics. He liked the fact that his Algebra II students could do problems his Pre-Calculus students were doing without using their algebraic methods. Further, he was enthusiastic about the value the ACPs could have in supporting him teach skills and concepts in his quadratic unit stating,

Cause I could go back, and I could say, hey, you know this looks like, one of those problems from before or, do you remember that we had a parabola, you didn't know it, but it was called a parabola. It went upside down. And so, what type of leading coefficient does it have, I think it's something that we can always go back to, and I think it would be, probably good to even just hang one of those on the wall somewhere, so I could go back and say. For example look at this. It went through zero does that make sense? If you have no width to your rectangle, you're not gonna have any, area whatever.

Yeah, zeros, uh, min max, um, increasing and decreasing intervals, uh so on and so forth, but I think it's good to go back, and that's my thought, I'm hoping I learn a whole bunch of other stuff, but right now that's what I see myself

doing. Is just referring back to it, going back to it, um. Gosh it would be nice to go [back]at the end of this quadratics unit, write the equation, write it in vertex form standards form - Increasing and decreasing - I know we've already done it before, but, just for the kids to do it again.

When I asked Mike about his opinion on giving the ACP prior to the unit of study (At the professional development, he believed it should be given at the end of the unit), he saw value in doing so stating, "I'm thinking that's a good idea now, because of what I can do. Although... Again I think there's, more than one right answer for this. I think, maybe middle of the road could be okay as well." Though he was not completely convinced that the ACP had to be given prior to the unit of study on quadratics, he saw major value in working through the problem and then discussing its merits.

Mike's interview demonstrated how the professional development, and our subsequent discussion on how it pertained to NCTM's Process Standards, led him to a better understanding of what it meant to teach from a Standards-based approach. Like Ken, Mike saw the value in the ACP for the topics it covered and the opportunity it created to reference something meaningful when teaching a given skill in his unit on quadratics. Interestingly, Mike was aware of the need to foster problem solving and reasoning with his students but felt he did not have the knowledge or resources to help him do so to the extent he would like. Mike's interview played a significant role in the creation of my questions for the group reflections and second round of interviews.

**Shareese.** When I interviewed Shareese, I went straight to the question of what she thinks NCTM considers a Standards-based approach to teaching. She said, "I think

their take is more of a problem solving and reasoning based [approach].” She spoke of it being like science where students would be formulating hypotheses and ways to solve problems. She also said that it was student centered. She believed that Standards-based approaches required students to have more autonomy than she gave her own students stating, “ But I’m kind of used to the students we have, and, you know, creating lessons that are more scaffolded for them, and probably helping them more than I should.”

As stated earlier, Shareese sees herself as a traditional teacher but could lean towards a Standards-based approach more than she believes she does. In her interviews, she indicated that her propensity to scaffold lesson was due to the “kind of students we have,” and this fact made her more traditional in her approaches to teaching. Based on non-recorded conversations with Shareese, “the kind of students we have” are those who vary greatly in ability and preparation for a given class. As such, her scaffolding is a result of her need to have all of her students guided to the same results as quickly and efficiently as possible.

When examining her teaching methods, she does scaffold her lessons significantly, and this led me to believe that she would shy away from using an ACP, especially at the beginning of a unit. However, Shareese saw value in The Pig Pen ACP and pointed out that she liked the discourse that it generated during the professional development. That this discourse could lead to the “aha!” moments she loves seeing. She still had reservations about allowing students to go down the wrong path or students in a group not chiming in when another member was leading them down the wrong path; however, she generally liked the ACP and saw value in them.

Like Ken and Mike, Shareese also liked how the ACP would create a reference for teaching the skills associated with quadratics. She liked that it was “broken down into steps” such that her students could follow the problem enough. She also conceded that she would want to “facilitate just a little bit, maybe clarify” what her students should be doing during the ACP but not to the point that she would be doing it for them. She liked the problem and saw its value but clearly wanted to scaffold it and guide her students more than she was supposed to.

When Shareese and I discussed the NCTM Process Standards and their relationship to the ACP, she saw the communication and problem-solving standards most prominently in the activity. She emphasized her interest in the discourse the ACP would foster in her classroom and also foresaw the struggles students might go through which could foster problem solving. Shareese agreed the ACP should be given at the beginning of the unit, due to its referencing qualities. Like Mike, she wanted to create a poster from one of the students and use it to reference certain qualities of The Pig Pen throughout the unit on quadratics. She seemed to have accepted the use of this ACP into her future lessons. Further, she was rather interested in using more problems like the ACP throughout the unit and see how much the students can do as the unit progresses. Specifically, she suggested helping the students with the ACP and then giving them a similar problem halfway through the unit and see what they could do with it given no help.

Shareese reiterated the value Ken and Mike placed on ACPs as a referencing tool for teaching skills and concepts related to quadratics. At this point, I was able to identify

this as a significant theme that was emerging with the participants. Further, I saw how it was likely teachers would modify the ACPs to fit their comfort levels, as Shareese indicated a desire to scaffold the problem and help her students more than was suggested.

**Hartley.** The interview with Hartley was rather straightforward. She had the general concerns I stated earlier about time and students' preparation; however, she was rather optimistic about trying the problem out with her students. She had done a problem like it in an AFDA class the year before, and she had given a very scaffolded version of it to her students at the beginning of their unit on quadratics.<sup>15</sup> She said she was looking forward to seeing what her present students will do with a less scaffolded problem.

When she was asked about a Standards-based approach to teaching mathematics, the only thing she could think of was a series of skills laid out for teachers by a governing body. When I spoke with her about the ACP as a Standards-based approach, she saw it as "student-centered," whereas "...the Standards-based approach is more like the way I would think of it as rote teaching." Like the other participants, the word 'standards' simply invokes what governing bodies set for educators to teach; however, it is clear that they all know there is more to teaching than a series of skills.

Hartley spoke of a desire to instill problem-solving abilities in her students any chance she received, but due to the time needed to complete the curriculum laid out by the district and the state she found it difficult. She saw the ACP as an opportunity to incorporate problem solving in her lessons while still preparing them to acquire all the necessary skills needed to pass the state's standardized exam. She liked the fact that

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<sup>15</sup> It should be noted that I played a role in this version of the ACP being given at the beginning of her AFDA unit on quadratics.



students had to generate their own numbers and test them in order to get a series of values representative of the area of a pigpen.

When I asked Hartley about the value the ACP would have in relationship to NCTM's Process Standards she believed it would help students make connections from the skills they would learn to something tangible like maximizing the area of a pigpen. Like the other participants, she liked that the ACP would introduce the skills taught in the quadratics unit ahead of time and thus give her something to reference. When I asked her if she would have given the ACP at the beginning of the unit without any prompting by me, she laughingly said no. She said that she is presently in a mode where she is thinking about the end of the unit assessment and only teaching the skills necessary for that assessment. There is an assessment, a transfer task similar to the ACP, at the end of the unit, which requires problem solving and reasoning, and she seemed to only see teaching a series of skills as a means of preparing the students for this task. After the professional development on the ACP, she saw how the process of struggling through the ACP would help prepare her students to think through the transfer task more reasonably.

At this point in the study, it was becoming clear that the teachers knew the basic concepts of Standards-based teaching, but the title itself led them to believe I was asking about state standards. Even a relatively new teacher like Hartley was searching for ways to promote more thinking and problem solving with her students. The professional development on ACPs and the ensuing conversations with the participants helped me clarify what an approach to teaching towards the NCTM Process Standards could look like.

**Nicole.** The interview with Nicole was rather interesting, as she had a different perspective on what a Standards-based approach meant. Like the other participants, she thought of state standards when I asked her about Standards-based approaches to learning, and, like the other participants, she wished they were more than a series of skills. Nicole further stated that she wished they would “develop a higher level of thinking, inquiry, and logic.” When I inquired about her ideas on higher-level thinking and logic she had a different approach than I would have thought. She wanted more of an opportunity to look at procedural problems differently. She wanted to incorporate geometry in her Algebra II class and spend more time discussing how to solve a problem analytically. She said this process develops analytical thinking, perseverance, and patience.

Nicole’s desire to dive deeply into an analytical approach to promote discourse in her class made sense. She said she wanted to “twist” problems and discuss what the changes meant. She stated that she wanted to make connections, but I was not sure how her approach to encouraging higher-level thinking would do that, and with further questioning I was not able to ascertain how. Like the other participants, she liked how the ACP could create a reference for her when teaching the skills in the quadratics unit and she again referenced how it fosters patience and perseverance.

While Nicole did not seem to completely grasp the goals of the ACP, she saw the importance of her students generating their own values as a lesson in patience and perseverance rather than an opportunity to “feel” out the problem and test values so it would make more sense. She seemed to have aligned the ACP with her more procedural

approach to teaching mathematics and made it a lesson in fostering the positive habits of perseverance and patience. At this point in the study with Nicole, I was not sure how she would use the ACP, if at all.

### **Initial Findings**

At this point in the interviewing process, I realized that the participants had at least a basic knowledge of the NCTM Process Standards. They may not have known the exact words and definitions given by NCTM, but they clearly desired to teach more than a series of skills laid out by the state. All of them stated that they want to do more, whether it be to promote problem solving, critical thinking, logic, or inquiry-based learning; they all knew there was more to teaching mathematics than a series of skills. The process of interviewing and talking about the ACPs seemed to clarify what it meant to teach problem solving for the participants.

During the first round of interviews, I sought to identify and understand the teachers' concerns and perspectives about Standards-based approaches to learning. The most common concern initially focused on whether the teachers understood what should be done during the study, but gradually shifted to concern over how they could foster problem solving, critical thinking, etc. in their classes. Many of them stated that they did not know how or what to teach in their classes to promote these more global skills, but seemed positive about the ACPs as tools to do so. Through a single professional development and a conversation about ACPs, the participants seemed to be developing an idea for promoting NCTM's Process Standards in their own classes while still adhering to the requirements of the state's standards.

What I found most intriguing was the general enthusiasm the participants had for the ACPs. Though they had not yet given them to their students, the participants seemed to be open to their use, and even hopeful of the benefits the ACPs would offer their students. This enthusiasm added another component to my study that in part led to the creation of the study's third research question – what are factors that promote adoption, or the lack thereof, of innovations such as ACPs? The other reasons I included this question stemmed from the group reflections.

### **Initial Coding and Validation**

As set out earlier in this chapter, I anticipated that the participants would have general concerns about time constraints and pushback by students who wanted more initial guidance for solving the ACP. And as expected, many of the participants shared these general concerns during the initial stages of the study. During the first round of interviews, however, other key topics arose that led to the creation of my substantive categories of data collected. The first substantive category was the participants' general desire to teach beyond the skills laid out in state standards and to promote more global skills like those of NCTM's Process Standards; that they were aware of their lack of experience and knowledge about Standards-based approaches to teaching mathematics. The second substantive category was the value the teachers placed on the ACPs once they saw the skills it covered. My final substantive category was the general enthusiasm and possible adoption of the ACPs by the participants.

These categories helped me form the questions I would ask in the following two group reflections, which in turn helped me develop my theoretical categories to aid me in

answering the study's research questions. In order to ensure that I appropriately categorized data from the interviews and the analysis was not biased to fit my goals, I asked a colleague to validate my findings. I sent her two of the five transcripts (without my annotations) to review. Further, I asked her to make her own annotations and observations to share with me. The goal was to see if any of her observations aligned with mine.

When my colleague returned the transcripts, she had the following general comments: (a) The participants did not have a complete understanding of Standards-based learning; (b) they lacked confidence and experience teaching from a Standards-based approach; and (c) they did not know how to blend Standards-based approaches with direct instructions. Her findings spoke primarily to my first substantive category dealing with teachers' desire to do more in the classes to promote a Standards-based approach to teaching mathematics. Though she did not mention anything about teacher adoption of ACPs, I still felt that it was a topic I needed to pursue, so I did not eliminate it from the questions designed for the two group reflections.

### **First Group Reflection and Second Professional Development**

Much of the first group reflection was spent discussing what the participants thought worked during their administration of the ACP and what they thought they could improve. The participants also widely discussed the value of referencing the ACP throughout the quadratics unit and its benefit to linking skills to a specific context. One participant touched on the value "this kind of teaching has" for fostering problem solving, but it was not a topic that was discussed as widely as the referencing the overall benefits.

Finally, the participants discussed their openness to the use of ACPs and some of the factors for doing so.

The first part of the reflection was spent discussing some of the logistic problems teachers encountered when administering the ACP (i.e., did not allot enough time, a lock down drill occurred, etc.) and then it shifted into methods each teacher used to administer it. For instance, Shareese broke the ACP into parts because of her proclivity to scaffold such problems for her students and because her Algebra II class was a shorter, daily class instead of the lengthy one-and-a-half hour-long block classes the other participants had. Ken gave his students the ACP and allowed them to approach it any way they wanted. Ken also mentioned his initial concerns about administering the ACP at the beginning of the unit. He said that even though he saw benefit in giving it later in the unit, administering it in the beginning now made more sense.

The next discussion dealt with what the participants would do in preparation for the ACP if they administered it again the following year. Mike said that he would have given more time to problems in the previous chapter that required students to scale axes on Cartesian coordinate planes.<sup>16</sup> He and other participants emphasized the issue their students had with not allotting enough space on their graphs when plotting points representing the dimensions and area of the pigpen. Ken discussed how absolute value functions taught in the previous unit could serve as a foundation for teaching many of the helpful practices for understanding quadratics (i.e., symmetry, corresponding points, vertices, etc.).

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<sup>16</sup> Scaling refers to the need to properly “number” the x- and y-axes such that the required domain/range of values needed for a given situation can fit on the graph.

What I found most interesting about these discussions was the participants were noting connections to other topics in Algebra II that would help their students work on the ACP more easily. Also, they were realizing the benefit of their students making the same connections to other topics. They were viewing the ACPs as connections to other areas of mathematics and not just quadratics, and it seemed as if they saw ACPs as a way of thinking about their unit planning in a broader context than just skills dealing with one unit of study.

When I brought up the referencing quality of the ACP, there were some contradictory findings even though during the interviews all the participants had believed the ACP would be a valuable referencing tool. When I asked each participant how many times they referenced the ACP throughout the unit, two said they did so daily, two said two or three times total, and Nicole said a “couple of times,” but she did not think the students were paying attention to her. I found this surprising, as all of the participants indicated that the ACPs referencing quality was one of its strongest characteristics. Ken and Mike were the two participants who said they referenced it only two or three times, with Ken specifically saying he did so only in the beginning of the unit.

The final discussion during the group reflection focused on the participants’ adoption of ACPs as part of their personal pedagogical practice. When I asked them if they would be administering The Pig Pen ACP next year with a new group of students, they all responded affirmatively. Even with Ken, Mike, and Shareese having only referenced the ACP two or three times throughout the unit, they were still very positive about using it again. Ken even said, “If you look at the highest functioning school

systems in the world, this is how they create their lesson plans.” Ken later went on to say that he would “definitely” be giving the ACP again.

After the group reflection, the participants and I went over the next ACP that centered on maximizing the volume of a suitcase. The teachers discussed it among themselves, but did not wish to work on it together as they did with the first ACP. They stated that they felt they understood the problem and were confident they could implement it with their students.

The lack of referencing by three of the participants seemed to indicate that they did not value the referencing quality of the ACP as much as I had earlier thought even though all of the participants made mention to its value during this first group reflection. However, their continued enthusiasm for the ACP indicated that the overall value they placed on it was quite positive. Further, participants’ concerns with the implementation of the ACP before the unit of study seemed to be allayed after they had the opportunity to work on it with their students. All this led to participants’ full, or closely full, adoption of ACPs and their continued use in future school years. What remained to be determined were initial factors that promoted their adoption of ACPs. Specifically, were there reasons the participants were willing to use their limited class time to administer the ACPs beyond helping a colleague, me, do a study for his dissertation?

### **Third Administration of the Survey**

After the participants read through the second ACP, I administered the survey a third and final time. My procedures for organizing my findings were the same as they were for the first two administrations of the survey. I organized the participants’ results in



a spreadsheet and calculated changes from one administration to the next. In this case, I compared the third survey administration to the first and second administrations. Next, I calculated the average changes between the two administrations for each category.

There were only slight changes in each category from the second to the third survey administrations. However, there were moderate changes in three of the six categories from the first to the third. Regarding the category on participants' beliefs about being able to properly administer the second ACP, there was a drop in the average Likert-score (average change was -0.9375). This was not a major surprise, as the participants had not only administered the first ACP to their classes already, but they had also worked through the second ACP.

Another change occurred in the category dealing with participants' beliefs about their students being able to do the ACP. After the first professional development, there was little change (average change was -0.22) in participants' concerns about their students' ability to do the first ACP. However, participants' concerns about their students' ability to do the second ACP were less than they were prior to the first ACP (average change from first to second survey administration was -0.857). This evidence indicated that the participants' concerns were alleviated due to their experience with the first ACP. They also saw that their students were able to do the first ACP and thus were less concerned regarding the second ACP. In the second group reflection, this finding was substantiated by most of the participants.

The other change in participants' Likert-scores had to do with the organizational concerns category. These questions dealt with time issues, specifically that one class

period would not be sufficient to do the problem and/or the time needed to administer the ACP would utilize too much time in the school year otherwise needed to complete all the required material. After the first professional development, these concerns did not change significantly (average change was -0.333). However, after the second professional development the concerns lessened more significantly (average change from first to third survey was -0.833). Again, after administering the first ACP and seeing the students complete it, the participants were more comfortable with the time requirements. Further, according to statements made by participants in the second group reflection and second round of interviews, the value they placed on the ACPs to aid them in covering the necessary skills for a given unit of study helped alleviate concerns regarding time.

### **Second Group Reflection**

After the participants administered the second ACP and finished their units on polynomials, I conducted the final group reflection. The discussion started with the participants' impressions of the ACP and its general success. The majority of the group reflection was spent on the reasons the participants found the overall use of ACPs valuable and their reasons for adopting its use. It must be noted that Nicole did not administer the second ACP because she was two weeks behind the other teachers and did not have time to do so.

The four participants who administered the second ACP found it much easier to implement than the first. Hartley said her students found it easier because they had gained experience from doing the first ACP. Knowing what was expected of them (i.e., choosing their own values, organizing their findings in tables, etc.) was no longer an issue because

the students had learned to work through these types of problems already during the school year. Ken and Mike said that issues with basic prerequisite knowledge (i.e., scaling a graph, plotting points, etc.) had also been learned, so the pitfalls of the first ACP were not issues in the second. Shareese's experience with the first ACP made it easier to scaffold it to her own preferences. She did not lay out a series of steps for the students to solve the problem as with the first ACP, but she drew a picture and set up a graph for them to organize their data. Overall, all the participants found the second ACP easier because of their previous experience with the first.

There was a brief discussion again on the value of referencing the ACP throughout a unit of study. Specifically, Hartley mentioned doing an activity graphing polynomials where the ACP acted as a good reference for her students to better understand that specific topic. Ken mentioned the need to understand the ACPs extremely well, so teachers knew every opportunity available to them to reference the ACP. Despite the fact that three of the participants in the first group reflection said they only referenced the first ACP a couple of times, there was still a significant value placed on ACP referencing qualities. Mike again mentioned his desire to make a poster of the students' work so it could "stare at the kids" throughout the unit (he stated that he had done so in the first group reflection and his interview).

The remainder of the discussion centered on the participants' adoption of ACPs. First, I asked the participants about their initial opinions on when the ACPs should be given. In the first professional development, all of the participants responded that they would administer the ACP either in the middle or the end of a unit. By the time the study

came to a close, all of them agreed that administering the ACPs at the beginning of the study was best; however Ken still saw value in administering them a little later in a unit. Second, I pointed out the common response in the first round of interviews that the participants would not have administered an ACP in the beginning of a unit without having been prompted to do so. Ken stated that he simply did not have the time to construct problems that were both contextualized and laden with worthwhile skills specific to a unit of study. He really emphasized the importance of making ACPs valuable to teachers' instruction as they teach specific topics; that if they did not have this value they would not be worth administering. Mike interjected at this point to say that their value was far greater than the data gathered administering school district-wide benchmark exams. This represented a shift in perception about the relative advantage from a diffusion of innovations perspective (Rogers, 2003). Hartley mentioned that the time requirements for the ACPs kept her from considering such an innovation, but the process of doing so for this study made the value so clear that she found their time expense worthwhile. From a diffusion of innovations perspective, the innovation was compatible with her needs and interests as a teacher

The conversation later turned to the reasons why the participants were initially willing to take part in the study and administer the ACPs beyond trying to help a colleague with his dissertation. Specifically, I asked the participants to compare the professional development for ACPs to other professional developments in which they had taken part. Shareese emphasized that this professional development was specific to mathematics and did not generalize a topic like reading or writing to every subject, as did

other professional developments she was required to attend. She acknowledged that she found reading and writing in all subjects important, but the means of training teachers to help students on these topics was not helpful. She mentioned that she did not think the leaders of other professional developments knew how to incorporate their area of expertise into mathematics. Nicole followed that the facilitator, me, was someone she knew, respected and trusted. She stated that she believed the innovation I was offering was something worthwhile because she knew I would not waste their time. Hartley later added that I was still in the classroom and had not forgotten what it was like having to administer an innovation someone else had handed me. The fact that I was “one of them” made it easy for her take the initial step of using ACPs in her own classroom. As stated in the limitation section chapter III, the research aspect of my placement in the study could have been a bias concern from a research perspective; however, from the perspective of the professional development and the adoption of the ACPs, my presence was a positive feature.

The conversation mostly concentrated on the value of the ACP as it pertains to mathematics education. Mike pointed out that it was not “bologna” unlike most other professional developments in which he had taken part. He said next to his National Board Certification and another extensive professional development in which he had taken part, this professional development “kicked all other professional developments butt.” He mentioned the waxing and waning of superintendents that bring innovations to a school that are quickly thrown out once a new superintendent is hired and how teachers know that the innovations will not last, so there is no reason for them to fully buy into them. He

stated, “This isn’t going away; these types of problems in math.” He said that these types of problems are in several levels of mathematics, and that you will see them if you go to a different school, a different state, or a different country. Ken added that the use of ACPs “speaks directly to our coursework.”

The reflection ended with a short discussion on what the participants believed would help extend the use of ACPs to other members of the faculty. Ken stated the common planning that is available specifically to the faculty at the study’s site made it easy. Shareese said that the initial adoption of ACPs of the participants in this study would help sell it to other faculty members; that the facilitator who is one of them played a role in convincing her to take part, and now her and the other participants’ acceptance of the innovation will convince others.

When the second group reflection was finished, I listened to its recording and expanded the notes I had already taken. I then went back and listened to the first group reflection and did the same thing, and I also read through all the transcripts of the first set of individual teacher interviews. This helped me begin identifying the key factors that influenced the participants’ understanding of Standards-based approaches to learning as well as their willingness to implement Standards-based innovations such as ACPs. First, I realized all of the participants knew there should be more to teaching mathematics than just a series of skills laid out in state standards. They all stated a desire to foster at least one of the NCTM Process Standards in their classrooms and knew teaching a series of skills was not sufficient for doing so. Second, the participants found significant value in the referencing qualities of the ACPs and changed their opinions on the timing of the

administration of the ACPs because of this. They saw how they fostered many of the process standards, but also liked the fact that it made teaching necessary skills easier. Finally, factors beyond the pedagogical value of ACPs influenced the participants' initial willingness to teach ACPs as well as their continued desire to do so in the future. The fact that ACPs are specific to mathematics and the content was presented to them by someone they trusted played a major role in their acceptance of them.

### **Final Interviews**

The three factors listed above helped me create a final set of questions to ask each participant in the final data collection process of this study. In the final interviews with each teacher, I honed in on how these factors addressed each of the study's research questions. I grouped the second interview questions according to the major themes I found in the first set of interviews and the first and second group reflections. Further, I created a series of questions for the final set of interviews that specifically targeted each of the research questions posed in this study. The second interview questions can be reviewed in Appendix 8. The following is a breakdown of each interview. A short narrative will introduce the participant and his/her general views of the study before providing their responses to each question.

**Ken.** As mentioned earlier, Ken is a veteran teacher with nearly 30-years of experience. He frequently talks about mathematics pedagogy and the present state of education in comparison to the past as well as to other country's pedagogical approaches. He may not have been able to quote the NCTM process standards when I asked him about them in the second interview, but he definitely had an intuitive grasp of them.

For Ken, doing the ACPs with other teachers and then administering them to his class did not generate a major epiphany on what it means to teach mathematics. He always desired to strive towards these standards and feels that he has been successful at fostering *reasoning* in most of his lessons and *problem solving* in many of them. With problem solving, his main concern has been to not teach it for the sake of teaching it. He feels that by doing so, educators and textbook publishers create contrived problems that are simply “rubbish.” After some lengthy discussion, he clarified that he essentially saw problem solving as an overarching skill to teach throughout mathematics.

A major concern that Ken also discussed was the pendulum swinging that he has seen so much of in his career. For example, he sees educators either leaning heavily towards teaching problem solving and not concentrating sufficiently, or at all, on foundational skills or conversely leaning heavily on rote, skill practice and little on problem solving. He stated that he has always tried to find a good median for both, so he promotes reasoning and thinking as often as possible while still having his students consistently work on their skills. What he learned most through the process of this study is that he should constantly remind himself of the importance of teaching towards the NCTM Process Standards and have them “at the forefront and center of his instruction” while not forgetting the importance of skills being a major component of actually being able to solve problems.

In regards to his reasons for adopting the ACPs, he stated that they “spoke directly to me as an Algebra II teacher.” He said that they were tailor-made for use in an Algebra II class by a “respected math teacher... who has the same goals towards the



same specific discipline that I have.” In relation to other professional developments, he said that ACPs were more relevant to his goals and not just generalized practices given by people outside of his field. He said, “When somebody is speaking to you and you know that it is directed directly towards your discipline and your area of interest and study, I feel like you’re going to be more inclined to be able to initiate that change.”

**Mike.** Mike does not have the experience that Ken does, but he does have the same ambition Ken has to extend his teaching abilities. He has demonstrated this by earning his National Board Certification, taking a leadership role in a training program at the study site’s district offices, and also serving as the department chair at the study site. Like Ken, he also teaches AP Calculus and frequently ponders how he can use the expectations in the AP course guidelines in his other classes. He had a rather general grasp of the NCTM Process Standards prior to the study without knowing each one, but it was clear that he was actively seeking to use some them in his own practices.

When asked about any concerns or perspective towards Standards-based approaches to teaching, Mike stated that he always felt the state’s standardized exam focused on skills more than problem solving or communication. He did not have concerns with Standards-based approaches; he has concerns that the “high-stakes tests” do not assess the NCTM Process Standards that a Standards-based approach encompasses. As stated earlier, Mike had a general idea of the Process Standards even though he could not quote them. At the time of his second interview, he had been teaching AP Calculus for three years and learned the importance of problem solving and communication in mathematics. The AP test requires students to work through problems using more than

rote procedures, and they are frequently expected to justify their responses. Mike had become all the more aware that the state's standardized exam for his Algebra II class did not focus on this by comparison and thus he did not believe he prepared his students for the rigor of an AP course initially. He said, "...we try to design some of the unit tests and our own end of course tests to have a little bit, but the high-stakes test does not include some of these things we've been talking about (Process Standards)." Mike did not have any concerns about Standards-based approaches to learning. His concern was that the educational model in which he works does not foster this approach.

Regarding when to administer his students the ACP, Mike still waffled between the beginning and middle of the unit. However, he really zeroed in on his idea of using a poster of the ACP to reference it throughout the unit of study. In order to do this, he would have to administer the ACPs at the beginning of each unit. A primary tenet of ACPs as a Standards-based approach to learning is giving students rich tasks early on so they can learn skills through problem solving and not the other way around. Mike indicated that he was open to teaching the ACPs at the beginning of units in the future, but how much value he placed on doing so was not completely clear.

Mike stated several things that he learned about Standards-based approaches while taking part in the professional development on ACPs and administering them in his class. First, he learned to see a Standards-based approach to teaching as more than a set of skills but a way to teach skills within the context of rich problems like ACPs stating, "It's putting those word problems into naked number problems and mixing the two

together, and trying to communicate those results.” Second, he learned that pursuing a Standards-based approach is about the journey not the destination.

Again, it’s not just finding the answer or a naked number problem, it’s this huge word problem. You’re like, wow, what’s this. You break it down, drawing pictures, communicating, talking with someone else, making a table of values, and you’re really thinking about different ways to do it. The nice part about this is the process of doing it, not the goal. It’s okay if you didn’t come up with the most efficient shape, but how did you get that? Length times width times height, or whatever it is. Hopefully, the kids can get out of it the best way to do it. There are other ways and that’s okay as well. In life, there’s always not enough information and you’re making choices based on a certain amount of information. You try to pick the best one. There’s always more than one approach, just like cell phone plans and all this.

A third thing he stated he learned through using ACPs was that skills can be learned and/or reinforced through problem solving, so reviewing facts prior to an activity such as an ACP are not always necessary. A specific example he cited was how students adjusted the scaling of the graphs they used for the two ACPs. Instead of explicitly telling the students what intervals to use, the students figured it out through trial and error and working with one another.

As stated earlier, Mike was aware of the NCTM Process Standards in a general manner. When I asked him how the ACPs affected his knowledge of them he said that he was now more aware of them when designing his lessons and his assessments. When I

asked him how they affected his teaching he said that it helped him see how there is more than one way to teach a topic. Also, he said they allowed him to give the students more control and to make his lessons less teacher-centered. He stated, “More students are building it, making meaning, instead of me giving the meaning.”

When I asked Mike about his adoption of ACPs, he said that they were something he really wanted. He mentioned how that he was trying to create similar problems for his Calculus class because it was required on the AP exam. Despite the fact that such skills are not focused on in the state’s standardized exam for Algebra II, he still wanted questions like this to foster the deeper thinking and reasoning he desired to foster with his students. His reasons for adopting ACPs were that they were specifically relevant to what he was doing. Like Ken, he wanted school mandated professional developments to be specific to mathematics. He did not want a topic generally taught across all faculties, as he would rather “sit and talk about a math problem and how to do it or what’s the best way or technique to do something as opposed to some of these other professional developments that we’ve done before.” Ultimately, Mike wanted more problems like these ACPs and accepted their use with the caveat that they be vetted by he and other teachers to ensure they were pertinent to the topics he taught.

**Shareese.** Like all of the participants in the study, Shareese initially thought of state standards of learning when asked about a Standards-based approach to teaching mathematics. However, like all of the participants, she thought there should be more than just a list of skills to teach students. For Shareese, the extension of that list included some of the Process Standards.

When I asked Shareese about any concerns she had regarding the Process Standards, she said that she thought she did a good job promoting some of them. Specifically, she believed she promoted the *connections* and *communication* standards well, but she felt she was weakest in promoting *problem solving* and *reasoning and proof* in her class. She felt that her students determined her ability to do so stating, "...my students kind of dictate whether I do that or not with their level of ability." Further, she believed problem solving was something that was promoted using certain kinds of word problems, and these problems were hard to create and/or find.

After using the ACPs Shareese saw how she was able to promote problem-solving, reasoning, and proof in her own classroom. She appreciated having the problems she found so difficult to create and/or find given to her by a colleague she "knew and trusted." Further, she saw how Process Standards she was confident promoting were further exercised in the ACPs and that students were "forced" to communicate and make connections.

Shareese originally believed problems such as ACPs were best given at the end of the unit after students had learned all the necessary algebraic skills to solve them quickly and efficiently. After using the ACPs, she completely changed her opinion and saw the value in teaching them at the beginning of a unit of study. Like Mike, she really appreciated how the ACPs facilitated review of basic skills such as scaling a graph. She learned that students were able to come up with a lot of rich data on their own with some facilitation on her part rather than significant amount of scaffolding. That being said, she still scaffolded the ACPs more than the other participants, but after the second ACP was

administered, she conceded that she would give the students less help on these in the future.

Shareese altered each of her mastery objectives for both units of study (quadratics and polynomials) to focus more on problem solving stating, “I’ve been writing my own essential questions and my own mastery objectives based on the fact that we started with problem solving and we’re trying to make connections throughout the unit.” Further, she felt comfortable not focusing on the state’s standards when doing the ACPs. She said,

With these ACPs, I’m not thinking about SOLs (state’s standards) at all. I’m thinking about whether my kids are getting it and making connections. To me, that’s what math is about. It’s not about SOLs. If the kids are making connections, then the SOL’s make no difference, because they already know them. Period.

She indicated that she learned that the Standards-based approach can encompass the necessary skills expected by state standards.

Shareese fully adopted the use of ACPs stating, “I’m a total buy-in for this.” She said she had them in the front of her quadratics and polynomial binders ready for next year. Like the other participants, she appreciated that they came from someone she “knew and trusted.” Further, she liked the fact they encompassed the big ideas of the unit as well as addressed the little things like scaling a graph. When I asked her about a comment she made in the last group reflection on giving testimonials of the professional development on ACPs to other teachers, she said the following:

I think it would be something as easy as showing student samples of work or, if one of us was willing, to videotape ourselves actually doing it in the classroom to

see how it goes along. Then, I guess we could keep a log. After I taught this, I referenced the ACP on this day. After I taught this, I referenced it again. The kids were constantly making connections. It wasn't like, okay, one and done. I think it comes down to, one, them seeing that you're passionate about it and two, you're able to show them that it actually works in the classroom.

Of the five participants, Shareese most significantly accepted the use of ACPs. Her comments above led me to believe she was what Rogers (2003) called an early adopter. She is respected among the small community of mathematics teachers at the study site and will play a major role in diffusing the ACP innovation to others. I will speak more on this in the final chapter.

**Hartley.** Hartley is openly honest about her lack of experience with and knowledge about teaching mathematics from a Standards-based approach. She has only been teaching for two years and knows she has a lot to learn about teaching mathematics. Her candor is both refreshing and hopeful, as she makes it abundantly clear that she is open to learning as much as possible and seeking the help and advice of others.

When I asked her what she knew of the NCTM Process Standards, she admitted to not knowing what they were explicitly and that prior to the study she had not fostered them much in her classroom. She did say she expected students to reason and prove their statements regularly by requiring them to justify and explain each step in their work. She also said she made connections as frequently as possible by explaining to students where the math they learn is used in everyday life. She indicated that she had an idea that she

needed to teach the students more than just a set of skills. Further, she stated that she needed more time and experience to refine her skills as a teacher.

Early in the study when I asked Hartley about her concern regarding teaching from a Standards-based approach, she mentioned that she did not even know what it meant. She was unsure of what she was supposed to do in a Standards-based approach, so she focused on the state standards yet felt that she should be doing more. Through the process of administering the ACPs and the aid she received through the professional development clarified, she learned what the Process Standards were and what it meant to teach from a Standards-based approach.

When I asked her if she felt the ACPs changed her perspectives on teaching towards the NCTM Process Standards, she said that she is much more comfortable with letting students toil with a problem stating, "...I let them sit for longer than I would have. Normally I would've been like, alright, you've got a minute, and seen what they came up with. Now, I'm letting them have more time to actually process it." She said that prior to the ACPs she was concerned that this took up too much time, but learned that it actually saved time in the long run. Further, the ACPs helped her believe that students could do more than she thought they could if she simply expected them to do more challenging problems on their own. She said,

The first ACP was pretty difficult. I was ready to jump in and just do it with them. After seeing the result of it and talking to everybody else, I was much more laid back for the second one. I actually sat down and said I wasn't going to answer any questions for a few minutes. They kind of knew what I expected from



them and I knew that they had to do it on their own. If I did it for them, they weren't going to learn. They weren't going to learn why we're spending this time doing it. Every moment we have in that class is valuable, and they know it. I tell them every day that we have a lot to do today and everything we do has value. Why did it have value? They knew that we were doing it.

Like the other participants, Hartley also mentioned the value of the ACPs as a means of referencing, but she had another perspective on it. She said that her students no longer asked her why they were learning a particular topic in a unit on quadratics or polynomials. She said the ACP made it clear why they were doing something. Further, she stated that the exploration that occurred while doing the ACPs is what created recall for the students, specifically,

Again, if they were just looking at that by itself, they didn't make all the connections. With a little bit of rote learning later on, when we were talking about increasing, decreasing, and relative maximum and minimum, we just kind of threw up some pictures and did a whole bunch at a time, they were able to make that connection back. Originally doing the exploring was what they remembered. The rote learning just enforced what they kind of already knew.

She later said that the ACPs gave students a "hold on what they are doing." Also, she explained how the professional development and the administration made the NCTM Process Standards "more apparent" to her and easier for her to convey to her students.

Further, she stated that the professional development on ACPs gives her more of an idea of where she should be going in her professional growth.

Hartley stated in the first professional development that she would not give ACPs at the beginning of a unit of study. When I asked her about her opinion after administering them twice at the beginning of two units, she stated that she would “most certainly” do so now. She also said that she would use the two ACPs again in the future; however, when I asked her if she had adopted the use of ACPs as her own, she said no. She said, “It’s not my own until I can create things myself and be more part of the creative process.” For Hartley, adoption was not simply using someone else’s material but being part of improving on and expanding the use of ACPs. She had adopted their use but did not consider it her own because she did not make them or have a part in doing so.

In the second group reflection, Hartley was one of the participants who mentioned the trust she had in me that helped her take a leap of faith and do these ACPs that she would not have otherwise done on her own. When I asked her about this in her final interview, she said that I was someone she knew and trusted. When I told her that I had done the ACPs in my own classes, she knew it could be done. She later added, “Not only because I know you, but because I respect and like you.” I mention this not for self-promotion but to lay the groundwork for a claim I will make in the final chapter.

Hartley’s positive attitude and deep desire to improve upon her craft makes her another strong, early adopter. Like Shareese, she is respected within our small mathematics teaching community, and her ability to get students to become problem-solvers despite her minimal experience will be a very positive testimonial for this

Standards-based approach to teaching mathematics. Further, her desire to be a part of the creative process may expand the use of ACPs to more units of study in Algebra II and beyond.

**Nicole.** Nicole's second interview differed from the other participants because she did not administer the second ACP on polynomials although she was very enthusiastic about using ACPs overall. She stated that she was two weeks behind the other teachers and did not have the time to administer it. She was also open about having lost faith in her students' abilities to solve challenging problems due to their lack of basic skills, but she believed that teaching should encompass more than the skills she, herself, focused on in her class.

After I reiterated to Nicole what the NCTM Process Standards were and how a Standards-based approach to teaching encompasses them, I asked about her concerns on teaching towards such a model. She said she lacked the resources to foster the Process Standards because she did not know where to look for questions that encompassed the approach. She also stated that time was a major factor in classes where a standardized test would be given by the state at the end of the year. She spoke about spending more time on asking the "why" questions in classes where one of these tests were not given.

When I asked her about the ACPs, she said that these were the kind of questions she could not find. She said they encompassed each of the Process Standards and covered what she wanted her students to learn. However, she was still concerned about the time they took to administer in a class with a state end-of-course exam.

I then started asking her questions about what she learned about ACPs. She said that prior to the study, she would have felt ACPs should be given at the end of a unit. Specifically, she said, “ Prior to the ACPs, I would have said to stick with the old, teach the tasks (skills) and then go ahead and apply it in a word problem.” She later said that she doubted students with poor basic skills could have done such problems prior to administering the ACPs; however, after doing so, the faith she had lost was “brought back.” She no longer thought that getting students to do such problems was as difficult as she thought.

Like the other participants, she appreciated all the skills the ACPs encompassed and the ability to reference them. Additionally, she found that the ACPs made it easier to give her students other word problems. She spoke frequently of a Johnny Appleseed problem she used to give to her students. It had to do with parabolic motion and the use of quadratics to model that motion. Over the years where she had lost her “faith” in students’ ability to solve such problems, she found it more and more difficult to administer it and ultimately gave up doing so. After doing the ACP, she found that her students were more capable of solving it.

With regards to her adoption of ACPs, Nicole said she would use them again. She stated,

I can’t do it every day, but once again, if you use them to see problems like this, they’ll be more and more comfortable with them and eventually they won’t be scared to do a problem like that. The more I have them, the better it is for my students in developing all the process standards.

Since she had run out of time and was not able to administer the second ACP, I am not sure she will fully implement this Standards-based approach in future years, as it is likely time will always be an issue.

Like many of the other participants, Nicole's willingness to try the ACPs was based on the trust she had in me. She said, "I know you are going to give me resources that will not be a waste of my time." When I asked her what she wanted in other professional developments, she said she wanted them to be geared not just to mathematics but specific levels such as Algebra II or Algebra Functions and Data Analysis (A bridge course to Algebra II). Like other participants, she did not want general professional developments that are geared towards all faculties.

Overall, Nicole shared in the general enthusiasm for ACPs as the other participants did, but the fact that she did not administer the second ACP due to time suggested that time could get in the way of administering ACPs in the future. When offered another ACP during the year the study was conducted, she was enthusiastic about using it in her class, but at the time of this writing she had not yet done so. It made me wonder how the pressures of time and the state's end-of-the year exam would influence the enthusiasm of other participants in the future.

**Validation of interviews.** In order to confirm my findings, I contacted each of the participants via email and sent them a bulleted list of what I extracted from the final interviews. I asked them to read through my findings and determine if they agreed. Further, I asked them to make any clarifications as they saw fit, so I could get the most accurate data of their thoughts, concerns, and perspectives about Standards-based

approaches to learning. For the most part, the participants agreed with what I stated in the emails. In circumstances where clarification was needed, the participants replied to my email with annotations outlining corrections. The information above reflects this process and any clarifications.

### **Results for Research Questions**

In this section, I pool all the data gathered throughout the study in order to answer its research questions. Each question is introduced by listing common themes found throughout the study that are pertinent to those questions. I will briefly review the instruments I used to gather the data that led to those themes and then report my findings. For each theme, I will incorporate data collected from each participant in order to set forth the most rounded perspective possible.

#### **Research Question One**

In the first part of my study, I sought to determine if teachers' concerns and perspectives towards Standards-based approaches to learning changed after implementing a method indicative of such approaches. Specifically, I asked:

*How do high school, Algebra II teachers' concerns and perspectives about Standards-based approaches to teaching problem solving change after implementing Anchoring Contextualizing Problems in two units of study?*

To answer this question, I first had to determine what, if any, concerns and perspectives the participants had towards such an approach. I did so by gathering data using four instruments: (a) The survey of Beliefs; (b) transcripts from two interviews; (c) data

collected during professional developments and two group reflections; and (d) information gleaned from teacher reflections after administering the two ACPs.

The most valuable data was gathered from the transcripts of the two interviews with each participant. The survey gave some data, and discussions in the professional developments revealed integral information that played a significant role in focusing my questions for the interviews. The participants' reflections were the most challenging data source, as it was difficult to retrieve them after the participants administered the ACPs. Further, when the participants did return them they yielded little new information beyond what was already gathered from the other data sources.

Overall, the instruments helped me determine themes that were generally common to all of the participants. The participants had three concerns:

- They were not confident that they would be able to implement ACPs correctly.
- They were not sure their students had sufficient prior knowledge to do the ACPs.
- They were worried about students having enough time to complete the ACP, and they were concerned that it would set them back in their course timeline.

Further, the data presented commonalities among the participants' perspectives on Standards-based approaches to teaching mathematics. For instance, all of the participants assumed I was speaking of state standards for covering a series of skills when I asked them to describe a Standards-based approach to teaching mathematics. More interestingly, most of them interjected (without prompting) that they believed the standards should be more than a series of skills. When I asked them about what the standards should be, all but one of them cited one or more of NCTM's Process Standards:

(a) Problem solving; (b) Reasoning and Proof; (c) Communication; (d) Connections; and (e) Representations. It was clear that they all wanted to do more than teach skills and facts to their students.

In general, the participants concerns were allayed after taking part in the professional development and implementing the ACPs in their classrooms. The process of doing the ACPs themselves and administering them to their classes allowed them to see the feasibility and value of this Standards-based practice. Further, discussions about the ACPs in interviews and professional developments expanded the participants' perspectives on Standards-based approaches to teaching mathematics from a very general, implicit understanding of them to a more specific understanding based on the NCTM Process Standards. The following section is an analysis of these findings.

**Evidence of change in participants' concerns.** In this section I will set out evidence that the participants' general concerns listed above were allayed after taking part in the professional developments and administering the ACPs. I will do so by listing evidence found in the surveys and use it as a foundation for discussing key statements made by the participants in both the professional developments and individual interviews. This evidence will later be used to draw conclusions in chapter V.

As stated earlier, the survey yielded some data regarding the changes in the concerns and perspectives of the participants towards Standards-based approaches to teaching mathematics. It was intended to give information that would hone the interview questions to individual participants, so I was able to elicit useful data from them directly. There were some changes in Likert-scores from one administration to the next, but these



changes were only common to the majority of the participants for a few of the 21 questions. Specifically, the first four questions that dealt with participants' concerns towards administering the ACPs and the fifth and sixth questions dealing with the participants' beliefs about students' ability to work with ACPs.

The first four questions asked participants about their concerns implementing the ACPs properly or whether they could solve them on their own or without using Algebra (a necessary pre-requisite, as the students did not know how to solve them with Algebra at the time they were presented with them). Except for Ken, all of the participants had some level of concerns in this area prior to the first professional development. Giving the survey after the first professional development yielded an average drop in Likert-scores of more than one point. This indicated that the participants working through the ACP and discussing it with their colleagues lessened their concerns pertaining to their ability to do and administer the ACP. Hartley's concerns did not diminish as reflected by her responses to the survey.

When the participants had administered the ACP to their students and completed the quadratics unit, I gave them the second ACP and the survey again. Again, Ken had no concerns about his ability to do the second ACP or to administer it properly, but three of the four participants who had those concerns prior to the first ACP also did not have any concerns with the second ACP. Hartley maintained a relatively high concern in this area; however, during the second group reflection that took place after the participants administered the second ACP, Hartley was quite enthusiastic about how well it went. She, Shareese, and Mike all mentioned that the administration of the first ACP paved the

path for a smoother administration of the second. Despite Hartley's concerns about administering the ACPs, her success in doing so indicated that she was far more comfortable with them by the end of the study.

The fifth and sixth questions dealing with participants' beliefs about their students' abilities to do the ACPs had some minor changes as well. Question five asked teachers whether they believed their students had enough prior knowledge to solve the ACPs. A higher Likert-score indicated that they believed their students did not have enough prior knowledge and lower Likert-score indicated that they believed their students did. There were drops in these scores from the first survey to the second and third administrations of the survey indicating that the participants concerns about their students' prior knowledge lessened. The sixth question asked participants if they believed their students could do the ACP at all. A Likert-score of five indicated they believed their students could not do the ACP, and a Likert-score of 1 meant they believed their students could do it. The scores on the first survey were low and thus indicated that the all of the participants believed their students could do the ACP. The second and third administrations of the survey showed little to no change.

The topic of prior knowledge was frequently discussed in the group reflections and individual interviews. For instance, Ken originally stated in the first professional development that ACPs should be given in the middle or at the end of a unit because the material learned in the unit would help his students do the ACP better. When I discussed this with him in his first interview, he seemed to indicate that there was Algebra the students needed to know in order to solve the ACP; a misconception that we debunked

during the first professional development. After some discussion however, it came to light that he really meant there were pre-requisite skills with which students frequently struggle that would make it harder for his students to do the ACP. Specifically, his students have historically had trouble scaling the axes of a graph that represents a specific scenario. In the case of The Pig Pen ACP, this would be how high a number to place on the y-axis to represent the area of the pigpen and how large a number to place on the x-axis to represent the width or length of the pigpen as well as which interval to use to count up to those values.

In the first group reflection after the participants administered the first ACP, Ken was very quick to state that he understood the value of administering the ACP prior to beginning a unit of study. His concerns about students' prior knowledge (one of the very few concerns indicated on the survey) were reduced after he administered the ACP. He stated that the process of doing the ACP helped the kids learn how to scale the graphs properly. Further, Shareese and Mike stated that their students also had to redo their graphs, sometimes more than once, because of mistakes with scaling; however, their students learned to do so properly through doing The Pig Pen ACP. Even though it took up part of the class period to do the ACP, the participants' students were still able to complete the ACP.

***Time.*** With regard to the participants' concerns about having enough time to administer the ACPs and the time the ACPs would take away from their course timeline, the survey indicated little change. The Likert-scores were rather mixed from participant to participant, so the average drop in concerns on these issues was only two-thirds of a

point. However, there was evidence in the final group reflection?? as well as both sets of interviews that the participants' concerns about time were allayed because the ACPs made it easier to teach the skills in the quadratics and polynomial units. Specifically, many of the participants discussed how the referencing quality of the ACPs anchored instruction of specific skills to something their students had already explored in the ACP; so teaching the skills went more smoothly and quickly.

**Evidence of change in participants' perspectives about standards-based approaches to teaching.** Gauging the participants' change in perspective towards Standards-based approaches to teaching was a more challenging task, as the meaning of the term "Standard-based" was confusing to the participants. All of them mentioned the state standards when I asked them what they thought the term meant, and all of them interjected that they thought such standards should ask more of students and promote grander mathematical practices. Nicole felt that the standards should promote logical thinking, and Shareese thought they should promote connections between topics. Ken, Mike, and Hartley all stated that they should promote problem solving.

All of them had an idea of what a Standards-based approach to teaching mathematics should be; they simply could not articulate them as NCTM's Process Standards. Mike went so far as to mention NCTM subject strands (e.g., Geometry, Statistics, Algebra, etc.) he had learned while in university, but he did not know what the Process Standards were. For the participants, a Standards-based approach to teaching mathematics was implicitly understood. All of them knew that teaching mathematics was not simply a series of skills, and they all tried to encompass at least one of the Process

Standards in their everyday teaching. Hartley stated that she never learned them directly, but the “process of learning to teach naturally encompassed process standards.” Nicole mentioned in the first interview that she always tries to get her students to see how everything is connected. Shareese stated in her first interview how she wanted students to see the connections between topics, and in our discussion about her teaching style, she made it very clear that she worked to get students to make connections to multiple representations of functions. Mike and Hartley had actually used a problem similar to The Pig Pen ACP in previous years. Hartley introduced a unit in a lower level mathematics class with such a problem, and Mike had used his rendition of the problem in previous Algebra II classes but not at the beginning of the unit. Finally, Ken was adamant about promoting discourse and communication in his lessons, and he was equally adamant about fostering reasoning while working through problems in class.

The process of implementing ACPs in two units of study did not necessarily change the participants’ perspectives and/or opinions towards Standards-based approaches. All of the participants had positive views of Standards-based approaches, once defined, prior to the study and worked towards implementing them in one manner or the other in their own classrooms. The likely reason they immediately mentioned state standards when asked about Standards-based approaches is that state uses the term “standards” as well, and teachers are constantly reminded of the need to cover the skills set forth by these standards.

In the second interview, Shareese stated that she had thought the “NCTM approach” meant more of a discovery-learning model where students are given a problem

and some materials and left to their own devices to figure it out. She did not like this approach and likely considered herself a traditional teacher because she did not teach this way. Her perspective towards NCTM's Standards-based model shifted when she learned that it is a model that encompasses approaches such as ACPs for promoting connections and multiple representations. The other participants did not share Shareese's initial view and thus did not demonstrate any major shift in perspectives towards Standards-based approaches to teaching mathematics.

Once it was established that Standards-based approaches to teaching mathematics meant learning mathematics by doing meaningful tasks that promote the Process Standards, all of the participants' saw themselves as teachers who promoted this model. Their views towards such an approach did not significantly change, only their understanding of what I meant by a Standards-based approach. It was significant, however, that the participants learned about Standards-based approaches to teaching mathematics while administering the ACPs in their own classrooms.

### **Research Question Two**

In the second question of my study, I sought to determine what teachers learn about their student's mathematical capabilities when using Standards-based tools, such as ACPs. The instruments I used to answer this question were the *Survey of Beliefs and Concerns Regarding Anchored Conceptualizing Problems* and the transcripts from the interviews conducted during the study. The questions in the survey did not yield useful data, as the participants' Likert-scores on these items had little to no change in value;

however, the interviews offered a wealth of information. The following is a list the common themes that emerged during the interviewing process.

- Participants learned that their beliefs about teaching mathematics beyond the skills set out in state standards were aligned with NCTM Process Standards.
- They learned that problems like the ACPs promote the Process Standards.
- They learned that their students are able to do these problems on their own with less scaffolding and help than they previously believed possible.
- They learned that students could do ACPs at the beginning of a unit, and basic skills, such as scaling a graph, could be reviewed and/or learned while doing ACPs.

Each theme is not specific to every participant, but the themes are linked to three or more participants. In the following sections, I will provide examples of these themes and discuss their relevance.

**Participants' beliefs aligned with NCTM process standards.** As stated earlier, the participants had an implicit understanding of what it means to teach towards a Standard-based approach. They were not able to list every standard explicitly, but their desires to teach beyond a series of skills indicated that they understood what was necessary to promote the Process Standards in their classrooms. The interviewing process, professional developments, or group reflections did not foster an “aha” moment where participants realized they knew the Process Standards all along. Instead, the questions asked in the interviews elicited their views on teaching mathematics, which all

had something to do with the Process Standards. When I showed them the Process Standards, they realized how their beliefs already aligned with them to some extent.

Ken stated that he always knew what the Process Standards were, but the process of doing ACPs reminded him of their importance. Shareese also said she knew what they were prior to and during the study. Mike said that he did not know what they were prior to the study, but his desire to promote problem solving indicated his aligned beliefs with the Process Standards. Nicole stated, "I did not know what NCTM Process Standards were prior to your study. I realized that Standards-based approaches are the basis of my teaching. The ACPs made my students think logically and make conjectures about the given topic."

**ACPs promote the process standards.** During the second interview, I showed the Process Standards to each of the participants, and I asked them which of the standards they believed the ACPs encompassed. All of the participants, except Mike, believed the ACPs encompassed all of the Process Standards. Mike was unsure about the reasoning and proof standard stating, "I would have to look over the problem some more," in reference to determining if the standard was covered by the ACP.

Throughout the study, the participants spoke of time constraints and lack of resources as being the primary reason why they did not do problems like ACPs in their classes. Specifically, they did not have the time to research and locate problems like ACPs to incorporate into their units of study. As previously established, all of the participants try to incorporate one or more of the Process Standards into their daily lessons; however, when I asked them in the second interview if there were any standards



with which they struggle using, all of them said yes. For example, Ken said he was good with communication and reasoning, but problem solving was a challenge. Shareese said she was good with problem solving, but she had trouble with reasoning and proof.

Many of the participants saw ACPs as a means of encompassing all of the Process Standards as well as many of the skill-based standards given by the state. Ken said that the ACPs reminded him of the importance to teach towards the Process Standards. More specifically, he stated,

I did know what the Process Standards were prior to the study. However, what I think is one of the great strengths of the ACPs is that if we followed the instructions for implementation as laid out in the Professional Development sessions, the ACPs almost put the classroom on auto pilot in terms of following the process standards. All 5 elements were present and requiring of attention, and I think that as a specific class were to become more familiar with this type of problem solving and approach to learning, each standard would be highlighted and accentuated even more and more. Success on these problems required elements from all 5 standards, and thus both teacher and student were made more acutely aware of these as the ACP progressed and more ACPs are undertaken.

All of the participants were enthusiastic about using ACPs with future classes. They appreciated that someone had found them questions that were pertinent to their specific subject matter and also encompassed the Process Standards.

**Students can do ACPs with less help from teachers.** A common belief of the participants was their students would need a lot of help and scaffolding to solve the

ACPs. They believed their students did not know enough to do the ACPs or would not take the initiative to start them if they did not do some of the work for them. Shareese took both ACPs and broke them into parts so the problems were easier for her students. She said “our kind of students” are not prepared for doing work like this and need the extra help. Hartley stated in the second interview that statements by other teachers about the lack of preparation of “our kind of students” have kept her from giving her own students cognitively demanding tasks such as ACPs.

After the participants administered both ACPs, most of them believed their students were more capable of doing them than they originally thought (Ken believed his students could from the beginning of the study). For instance, in both the second group reflection and the second interview, Shareese said she should not have scaffolded the problems so much. It was not clear whether she determined this during the administration of the ACPs or in the conversations with other participants, but she indicated that she believed her students needed less help than she had given them. She said that she would scaffold less in future administrations of the ACPs indicating her belief that students could do more on their own changed.

When I asked Hartley in the second interview about what she learned by using ACPs, she said she no longer had an issue with letting her students struggle with a problem. Instead of giving her students a problem and immediately showing them how to do it, she began asking them to figure it out for themselves. When she taught topics like graphing quadratics, she found that the students’ toiling with the ACP prior to teaching such skills made it easier for them to understand them. Likewise, she found when

students had to work through new kinds of problems on their own it helped them learn the material more easily.

Mike said he learned that giving his students more control to solve the ACPs allowed them to develop meaning in their mathematics instead of him telling them the meaning. He said that the ACPs promote problem- solving and they help “kids make meaning out of naked math problems.” For Mike, it was not a matter of learning his students could do the ACPs without his help. Instead, it was learning that his students’ laboring through the ACPs was the benefit, and that doing so promoted problem- solving and greater understanding of general mathematical skills.

All of the participants expressed significant concerns about what the students did not know prior to the administration of the ACPs. When Shareese made the reference about “our kind of students,” she further referenced in the second interview teachers’ perceptions of students who do not think for themselves and expect a teacher to tell them what to do. She also mentioned students who made it through various grade levels without learning the mathematics they were supposed to know for the next level. Because of her experience with “these kinds of kids,” it was difficult for her to believe they could do problems like ACPs without being told what to do or how to do them. The realization that students could do the ACPs with less help than all the participants believed also played a role in when the teachers believed the ACPs should be administered.

**ACPs can be done at the beginning of a unit.** In the first administration of the survey, the participants were asked when they believe the ACPs should be given to their students. All of them said it should be given at the end or the middle of a unit. Even after

the first professional development, most of the participants believed that it should be given sometime after the unit began or at the end of the unit. Of course, the participants administered the ACPs at the beginning of the two units for which they were designed, but only because I asked them to do so. After the first ACP was administered to the participants' students, most of the participants had begun shifting their opinions towards administering the ACPs at the beginning of a unit or at least near the beginning. As stated earlier, Ken mentioned in the first group reflection that he originally thought the ACPs should be given at the middle or end of the unit, but had seen the value of giving them at the beginning. At the first group reflection, Mike still spoke of administering them shortly after the beginning of a unit, but also saw value in doing them at the beginning.

During the final group reflection, after the second ACP had been administered, all of the participants agreed that the ACPs should be given to their students prior to beginning the lesson. All of the participants stated that they would use the ACPs the following school year, and they all stated that they would do so at the beginning of each unit for which the ACPs were designed. What I found to be a key reason for this shift was that most of the participants witnessed how pre-requisite skills they believed their students needed to review *prior* to the ACP could actually be done *during* the ACP. They saw how the ACP created a context for the students to understand the importance of proper scaling, and this made it easier for the participants to revisit the skill within a problem-solving context.

**Summary.** For most of the participants, the professional developments on, and the administration of, the ACPs did not foster pedagogical epiphanies. Instead, these

activities either reminded the participants of the importance of incorporating the Process Standards into their unit planning, or guided them to knowledge they had been seeking throughout their careers. All of the participants had always sought to incorporate some element of the Process Standards into their teaching, but they lacked the time and resources to locate problems that encompassed all of them. When they were presented with problems that encompassed both the state standards and the Process Standards, they realized the ACPs' potential for meeting their needs and learned a lot about their teaching practices and their students' abilities to do challenging problems.

### **Research Question Three**

As stated earlier, all of the participants fully adopted the use of ACPs. Every participant said that they will incorporate the ACPs into future classes, and each participant said they would administer them at the beginning of the units of study for which they were designed.<sup>17</sup> I did not expect all of the participants to adopt the innovation as their own, and as their enthusiasm became clearer over the course of the study, I became more curious about the reasons for their enthusiasm. With question three, I wanted to know what factors promoted, or extinguished, participants' acceptance of the ACPs. There were three significant factors that promoted adoption of the ACPs common among the participants:

- The referencing qualities of the ACPs and the role they played in helping the participants teach specific skills in a problem-solving context.

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<sup>17</sup> It should be noted again that Mike reserved his opinion about giving the ACP after the unit began; however, he repeatedly mentioned his desire to create a poster the students could reference throughout the unit.

- The relevance the ACPs had to the participants' specific needs as Algebra II teachers.
- The trust and respect they had for the person bringing the innovation.

The data sources I used to acquire this information were the questionnaires the participants filled out after each administration of the ACPs, information gathered from the second group reflection, and the transcripts from the second round of interviews. As was the case for question two, the questionnaires did not yield significant data because not all the participants turned them in or gave information that differed from other data sources. The initial evidence of widespread adoption of ACPs occurred in the second group reflection. During my conversation with all of the participants, it was becoming clear that they all liked how the ACPs aided them in teaching, so I started asking them directly about the factors that fostered their acceptance of the ACPs. I took note of individual statements and then brought them up in the proceeding, final interviews.

What was most notable was how the participants' reasons for adopting the ACPs were reflective of Rogers' (2003) factors for influencing adoption: (a) relative advantage has to do with how an innovation is better than a previous, presently used innovation; (b) compatibility deals with how well an innovation meshes with an individual's life; (c) complexity or simplicity deals with the perception an individual has towards the ease of implementing an innovation. If an individual perceives it as too difficult to implement, then adoption is not likely; (d) trialability is the opportunity a person has to test an innovation out with little to no risk of financial or resource loss; and (e) observability is how visible an innovation is to others. An innovation's visibility will drive

communication among members of a social system and more readily diffuse a negative or positive reaction to it. Specifically, the participants appreciated the relative advantage of the ACPs over other innovations given to them by their administration, and they liked how the ACPs were compatible with their specific topics of instruction. The following sections outline the common themes for the enthusiasm I found among the participants and ties specific comments made by the participants to Rogers' factors for influencing adoption of an innovation. Relative advantage has to do with how an innovation is better than a previous, presently used innovation.

In order to help guide the reader through the proceeding sections, I found it worthwhile to organize a table that relates my findings to Rogers' adoption factors. The following table lists a series of words or phrases I used for coding participants' statements about ACPs to those factors. Further, I include a column of quotations or references to statements made by the participants in the final round of interviews that are exemplars of them. Though I connect data from the interviews to each of Rogers' adoption factors, not all of them were common to each of the participants. The most prevalent commonalities were specific to Rogers' relative advantage and compatibility factors, and these are discussed most thoroughly in proceeding sections.

Table 3

*Analysis of Participants' Adoption of ACPs*

My Codes	Rogers' Codes	Exemplars from Data
<b>Better professional development</b>	<b>Relative Advantage</b>	Mike's statement about the professional development on ACPs kicking all other professional developments' "butt."
<b>Speaks to Subject Matter</b>	<b>Compatibility</b>	Kens statement about how the ACPs "spoke directly to him as an Algebra II teacher."
<b>Facilitator can do it, so can I</b>	<b>Complexity or Simplicity</b>	Ken: "When somebody is speaking to you and you know that it is directed directly towards your discipline and your area of interest and study, I feel like you're going to be more inclined to be able to initiate that change."  Hartley's statement about feeling comfortable using the ACPs at the beginning of a unit after finding out that I had done so successfully with my own classes.
<b>Students can do more than teachers think</b>	<b>Trialability</b>	The participants' realization that their students could do the ACPs without review of past material or instruction on new topics. The opportunity to try an ACP out in a single day made this possible.
<b>Testimonial</b>	<b>Observability</b>	Shareese's desire to share ACPs with other teachers and speak about their value; to show them how useful they are.

**Referencing.** In both group reflections, as well as the second round of interviews, all of the participants discussed the value of ACPs when teaching a new skill during a unit of study. For instance, when the participants' taught their students how to graph a quadratic function, they were able to reference specific parts of the Pigpen ACP to aid



them in teaching the topic. Specifically, the Pigpen ACP led students to create a table of values that had a column representing the area of the pigpen based on certain dimensions. The area values repeated themselves for different dimensions because of the nature of rectangles – for instance, a 30 by 120 pigpen has the same area as a 90 by 40 pigpen. The graph of a parabola has repeated points called *corresponding points*, and the participants stated their appreciation for the ability to reference the Pigpen ACP when teaching how to graph a quadratic function.

The referencing quality of ACP played a significant role in shifting the participants' opinion of when to administer the ACP, but it was also one of the key factors to their overall adoption of ACPs. The ACPs made it easier for them to place the skills they were teaching their students into a context with which the students were familiar and this added value strengthened their opinions towards ACPs. Mike repeatedly mentioned his desire to create a poster of the ACPs that could be referenced throughout the units for which the ACPs were created. Nicole said, "The apparent understanding of the topic was shown by my students over and over again throughout the chapter of study when the students were recalling the ACP activities and connecting the new ideas." Shareese and Hartley both mentioned in the second group reflection how they appreciated being able to introduce a topic by reminding the students about its connection to the ACP.

From the beginning of the study, all of the participants believed in incorporating at least one of the Process Standards into each of their lessons. Whether it was fostering logic and reasoning when getting their students to understand the procedures of a particular problem or creating an environment of discourse and communication that

stimulated students' interest in understanding a concept, all of the participants wanted to teach their students more than a series of skills. However, all of them were subject to the realities of teaching; that there are state-mandated skills that must be covered by the end of the year. The participants ultimately found that ACPs met both needs – the ability to teach to the Process and state standards at the same time. Their ability to introduce the majority of the skills in a unit of study by using an activity that encompassed all of the Process Standards such that they were able to reference it throughout played a significant role in their adoption of the innovation. This is reflective of Rogers' compatibility factor in that ACPs are an innovation that meshes well with an individual's life. In the case of the participants, ACPs meshed well with their pre-existing lessons and met the needs of the state's standards.

**Specific to Algebra II.** When I began asking the participants about their adoption to ACPs, all participants felt the ACPs were successful because they were tailor-made to a specific topic. These views led to the discussion of the topic of professional development in education. Every year, the participants had to attend some form of professional development at the study's site, but none of them believed the professional developments were worthwhile. All of them conceded that the general idea of the professional developments were good (e.g., reading in the content area, fostering discourse, etc.), but they all had complaints about the generality of the professional developments. Shareese mentioned that she believed that promoting reading in the content area was important, but she questioned whether the facilitators of the professional development knew what it meant to do so in a mathematics class. She said the

professional development was given to the entire school faculty and thus generalized beyond the point of value to her and her specific subject area.

The common opinion of the participants toward the professional development on ACPs was that it was beneficial because it spoke not just directly to their subject area but specifically to Algebra II. Ken stated, "...it was able to speak directly to me as an Algebra II teacher, for in the case of these ACPs, they were tailor-made for use in an Algebra II class." Both Shareese and Mike made specific reference in their second interview that one of their primary reasons for adoption of the ACPs was the relevance they had to their subject area. Mike also stated in the second group reflections that the professional developments on the ACPs were not "bologna." He said that this kind of professional development is not going to fade away, as ACPs are problems that are relevant to him and his students, to students in different schools, and to students in other countries. Shareese said that if the ACPs were not relevant to her or other teachers, then the professional development would have been "dead in the water."

The relevance of the ACPs to the participants' specific needs was a significant factor that led to their overall acceptance. The fact that the professional development was not a generalized approach to teaching but an actual activity that addressed the needs of the participants as Algebra II teachers was a major factor in the participants' adoption of the innovation and reflects Rogers (2003) relative advantage factor.<sup>18</sup> Additionally, the importance of the person diffusing the innovation also played a major role in the participants' initial willingness to even take part in the study.

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<sup>18</sup> Recall that the relative advantage factor has to do with an innovation being better than a previous one.

**A trusted and respected colleague presented the innovation.** During the second group reflection, the discussion moved from the relevance the professional development had to the participants' specific needs to their initial reasons for taking part in, and staying with, the study. I acknowledged the fact that the participants were helping me, their colleague, complete his dissertation, but I wanted to know if there were other factors for staying with the study as they had the option of dropping out at any time. Further, I wanted to know if those possible factors would be of importance in conducting other professional developments.

Similar to the second theme, the conversation initially dealt with the participants' dislike for the mandated professional developments they had to do every year and then led to their lack of relevance discussed in the previous section. When I asked the participants if there was anything beyond the relevance of ACPs and their willingness to help a colleague, a common theme of trust and respect emerged. Nicole specifically stated,

I think this development is easier to take on because we trust you. I think as a colleague and another math teacher, as a person I respect and trust, I will say that I really did not have to be persuaded... I know that you will not give me something that will be a waste of time.

Mike stated his appreciation that the professional development was on a voluntary basis, and Hartley added that I, the facilitator of the professional development, was "one of them," and had not forgotten what it was like to be in the classroom. These statements are reflective of Rogers (2003) descriptions of early adopters/opinion leaders being trusted members of a community that are respected for their technical competence and

conformity to the system's norms (*loc* 1159). Being a member of the mathematics community at the study site made it easy for me to understand the needs of the participants and gain their trust.

In the second round of interviews, I brought this topic up with the participants individually, and each of them agreed that being "one of them" was a major factor in their overall adoption of the ACPs. Mike said that his positive opinions on the value of the study's professional developments versus those of some other professional developments were different because the study facilitator was a person in his field who is knowledgeable about the innovation being presented as well as the content area for which he is "pushing" the innovation. Hartley said it was easier for her to accept the use of ACPs because she knew the facilitator of the study's professional developments personally and respected and liked me.

The statements above make it clear that a personal tie to the facilitator of a professional development is a significant factor in buying in to an innovation. Since it is not possible for a person diffusing an innovation to know everyone s/he hopes to adopt the innovation (at least not a pool of adopters as big as all mathematics teachers), then a cascading effect needs to occur. Near the end of the second group reflection, Shareese said, "I think you can get more people to buy in at this point because you have people like us who can do testimonials for you." Shareese did not only decide to use ACPs in the future, she decided to share them with other teachers. Shareese's willingness to share the ACPs indicates that she is an early adopter, a trusted member of a community of Algebra II teachers that will diffuse the ACP as a useful innovation. Further, her statement is

indicative of Rogers' observability factor where her future use of the ACPs will likely be visible to other mathematics teachers, and her standing in the community will positively impact the opinions of those teachers towards ACPs.

I realized at this moment that a "ground up" approach to diffusing this Standards-based approach to teaching mathematics could be a positive and effective method. Instead of a top-down approach where unknown, or little known, facilitators tell teachers what to do, teachers can be given the option of participating in professional developments they find most relevant. If they find it relevant, then their adoption and enthusiasm can be the primary, diffusing factor in spreading the innovation.

### **Summary**

As demonstrated throughout the study, the common reasons among the participants for adopting the ACPs were the referencing qualities, relevance to the participants' specific subject matter, and that someone they knew and trusted delivered the innovation. There was a strong alignment between Rogers' elements of the diffusion of innovations model and the factors relevant for the teachers' adoption of the ACPs. Taking these factors into consideration along with the knowledge teachers gain, while taking part in professional developments like those in this study, can have significant implications for promoting Standards-based approaches to teaching mathematics. The following chapter draws on the evidence gathered during this study to draw conclusions about the following issues: (a) Teachers concerns and perspectives towards Standards-based approaches to teaching mathematics; (b) what teachers learn about Standards-based approaches when using ACPs; and (c) what factors encourage teachers to adopt

innovations aligned with Standards-based approaches. I will tie these conclusions to the literature in chapter II and point out differences that need further research. Further, I will make suggestions for future research on the matter as well as make suggestions for implementing professional development from a ground-up approach rather than a top-down approach.

## CHAPTER V: DISCUSSION

Promoting NCTM's Process Standards in mathematics is integral to fostering problem solving and helping students develop into critical thinkers willing to engage in constructive and reflective activities. For decades, NCTM has sought to increase the use of Standards-based approaches to teaching mathematics across the US, and there is a wide body of evidence linking such practices with verified positive effects for students' learning and problem-solving ability (Elstein, Lesh & Doerr, 2003; Greeno, 1998; Lesh et al., 2000; Lesh & Doerr, 2000; Lesh, Hoover, & Kelly, 1993; Lesh & Yoon, 2004; Lester, Garafalo, & Kroll, 1989; Lester & Kehle, 2003; Schoenfeld, 1992). However, according to Hiebert (2005), few changes in teaching practices have occurred since NCTM's first publication on the subject in 1989. Despite teachers' knowledge of Standard-based approaches, Hiebert suggests that they are not implementing them to the extent necessary to change how students learn and do mathematics.

In this study, I presented high school Algebra II teachers a Standards-based approach to teaching mathematics through the use of ACPs and sought to answer the following questions:

1. How do high school, Algebra II teachers' concerns and perspectives about Standards-based approaches to teaching mathematics change after implementing ACPs in two units of study?



2. What do teachers learn about their students' mathematical capabilities when using a Standards-based tool such as ACPs?
3. What are factors that promote adoption, or the lack thereof, for innovations such as ACPs?

Through my analyses of the study's results, I came to following general conclusions: (a) teachers' concerns about their ability or their students' ability to do cognitively demanding problems are alleviated when teachers do the problems themselves in professional developments and administer the problems to their students; (b) teachers learn that their students can grasp or think through new mathematical concepts without previous instruction on a specific topic; and (c) teachers' adoptions of educational innovations are strongly affected by the individuals and the methods used to introduce them. Each of these conclusions is more nuanced than the general statements above, and I will provide deeper analyses below.

As mentioned previously, one factor that separates my study from the majority of other studies on Standard-based approaches to teaching is that I conducted the study solely with high school mathematics teachers. There has been a dearth of research on this topic with high school mathematics teachers, and through my study I sought to begin filling this gap. The perspectives and opinions of this under-represented demographic have the potential of supplying a wealth of information to the field of research in mathematics education, and I seek here to make a small contribution.

The following sections set out the underlying analyses of the conclusions listed above in relation to the research cited in the literature review of this study. I will compare

and contrast my findings to those of other studies to establish my contribution to the body of research on Standards-based approaches to learning. This will, in turn, allow me to suggest questions for further research and make suggestions for future, educational practices.

### **Conclusion 1: Changes in Concerns and Perspective**

As discussed in the results section for question one, the participants' concerns about their ability to do the ACPs without Algebra were allayed after they took part in the professional developments for the ACPs. Further, the concerns the participants had about their students' ability to do the ACPs prior to a particular unit of study significantly lessened after the participants administered the problems to their students. The participants' experience with ACPs played a significant role in changing the beliefs of the participants. The following describes the conclusions I drew from these factors.

#### **Perspective Shift Requires Several Factors**

The most significant result for question one was that the participants all shifted their perspectives on *when* the ACPs should be administered. The primary tenet of the ACPs as they relate to Standards-based practices is that they should foster NCTM's Process Standards, and this is not done by having students solve a problem after they been shown how to do it. If students know how to use a formula to solve a familiar problem, then much of the reasoning, communication, connections, and multiple representation of working through a problem like ACPs is lost.

All of the participants originally thought the ACPs should be given in the middle or at the end of each unit so they could teach the necessary Algebra that could be used to

find the ACPs' solutions prior to the administration of the ACP. Giving the ACPs at the beginning of a unit required a significant shift in the participants' perspectives on teaching such problems. Even after their personal concerns were allayed after doing the ACPs as a group in the first professional development, the participants still believed that the ACP should be given later in units of study due to concerns about their students' abilities to do such problems. When they saw their students struggle through the ACPs, make corrections, and help one another figure them out, the participants began to realize their students could, in fact, do such problems with little to no Algebra prior to a unit of study. Further, participants' concerns about students' necessary prior knowledge (i.e., scaling a graph, graphing points, performing the order of operations, etc.) were allayed when they saw the students review and/or learn such skills while doing the ACPs.

When the participants reconvened after each administration of the ACPs, they discussed their successes and mistakes and learned from one another. Ken interjected early in the study how he saw the benefit of doing the ACPs at the beginning of the unit. Shareese, after listening to the stories of other participants, conceded that she should not have scaffolded the ACPs as much as she did. Like Roger watching recordings of his lessons in Schorr and Koellner-Clark's study (2003) and realizing he was making a Standards-based lesson too traditional, Shareese also came to this conclusion by having discussions with fellow teachers. The process of discussion throughout the administration of the ACPs helped the participants clarify the need to administer the ACPs prior to units of study rather than the end.

## **Beliefs**

The research on teachers' beliefs and methods for shifting their beliefs resonate clearly in this study (Glaserfeld, 1993; Guskey, 2003; Hoyle, 1992; Koellner-Clark et al, 2007; Pajares, 2003; Raymond, 1997; Schorr & Koellner-Clark, 2003; Skott, 2003; Sztajn, 2003; Thompson, 1992). As Phillip (2003) suggests, inconsistencies of teachers' stated beliefs and actual practices do not exist. Instead, a series of factors (i.e., time, lack of knowledge of practices, tools, etc.) influences how teachers are able to carry out their beliefs about teaching mathematics. In this study, all of the teachers believed in the importance of the Process Standards to guide their instruction<sup>19</sup>, but they did not have the time and resources to foster the standards to the degree they wanted. This gap explains any misalignment in the participants' beliefs and their actual practices. All they needed was the opportunity to implement their desired goals, and all of them were ready and willing to foster a Standards-based learning environment in their classrooms beyond what they were already achieving. The ACPs, the related professional development, the administration of the ACPs, and the discussion of each implementation were combined factors in helping them reach those goals, and the confluence of these factors were essential in the success of the ACPs.

### **One Approach is Not Sufficient**

Having a discussion with the participants about the value of ACPs would not have been sufficient for shifting their perspectives towards administering the ACPs prior to each units of study (e.g., Grant, Hiebert, & Wearne, 1998). Simply allaying the participants' concerns about their ability to do the ACPs without using Algebra in

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<sup>19</sup> Recall that they wished to include at least one of them into their regular teaching practices even though they were unable to cite all of them or knew they were specific to the NCTM Process Standards.

professional developments would not have been sufficient either. Further, having the participants administer the ACPs in their classes at the beginning of each unit without professional development or a discussion of the administration would not have led to a successful shift in their perspectives. Similar to Koellner-Clark's et al (2007) findings in their middle school study on *Problem-solving Cycles*, my professional development, the participants' administration of the ACP, and the group reflections all played integral roles in helping shift the participants' beliefs about when a Standards-based tool should be given to students to better foster the NCTM Process Standards.

Both Glasersfeld's (1993) supposition about teachers becoming engrossed in constructivist methods of teaching<sup>20</sup> after using such methods for only a few weeks and Guskey's (2002) statement that teachers' beliefs can only be changed after they see changes in student learning outcomes, strongly support of my conclusion that shifting teachers' perspectives towards Standards-based practices cannot be accomplished with a singular approach. Simply handing teachers tools that are considered Standards-based such as ACPs is not sufficient without professional development, and professional development is not sufficient if teachers cannot see the tools successfully implemented in their classrooms. Further, if such tools are not properly implemented in teachers' classrooms, then corrections cannot be made without thorough reflection. In order to truly foster a shift in teachers' concerns and perspectives about Standards-based approaches to learning a multi-pronged approach must be taken. The combination of proper tools (e.g.,

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<sup>20</sup> Constructivism and Standards-based teaching practices are not one and the same, but they work in tandem with one another.

ACPs), professional development, and thorough reflection on teaching practices must all be considered when trying to bring about change in a group's teaching practices.

### **Conclusion 2: Students Can Do ACPs as They Are Intended**

The participants in this study learned or became more aware that their students are capable of doing challenging problems with little to no Algebra. They also learned or became more aware of their own ability to promote the Process Standards with students, review necessary pre-requisite knowledge, and begin developing conceptual understanding of a new topic among their students by doing a single activity. The experience of doing the ACPs with their students and seeing the results in their lessons and students' work was paramount in helping them realize the effectiveness of ACPs.

As Grant, Hiebert, and Wearne (1998) determined with elementary school teachers, simply telling teachers about the positive effects of Standards-based approaches to learning is not sufficient. Teachers need to experience the impact such methods of teaching have on their students, and they need to see the benefit of such approaches to their own teaching. The high school teachers in my study were not only told of the benefit of Standards-based practices prior to implementing ACPs, but they were also trained on how to use them. However, based on their opinions given after the first professional development that the ACPs should be administered in the middle or at the end of a unit of study, the participants still did not realize the benefit administering them at the beginning of the unit. It was through the experiences of having their own students do the ACPs prior to units of study, seeing how the students were required to review and implement prior skills while doing the ACPs, and witnessing how the ACPs benefited

their instruction that helped the participants realize (or become more aware of) what their students were able to do without prior instruction.

The necessary pre-requisite skills to do the ACPs (i.e., graphing points, scaling axes, finding the area of a rectangular pig pen, etc.) significantly concerned many of the participants. They felt the need to refresh the students on these skills already taught in previous mathematics classes prior to administering the ACPs, and this was one of the primary reasons why many of the participants thought they should be administered later in units of study. What many of the participants discovered was that the process of doing the ACPs also acted as a review of previous material. They were able to “kill many birds with one stone” as stated by one participant by having their students explore future topics of Algebra, review past material, and promote the Process Standards. The participants also learned that it was possible to do all of this in a time-efficient manner. More specifically, they realized that they could cover the Algebra II material while also promoting the Process Standards.

What sets this study apart from others is that the participants not only learned or become more aware of how to teach from a Standards-based approach, but they also realized that they could do so while reviewing past topics and prepping for the required topics of Algebra II dictated by state requirements. The participants learned that review of past material could be done in cognitively demanding tasks that promote the ways of thinking and reasoning they all fundamentally wished to develop with their students. Finally, they discovered that their students could do these problems with little to no

preparation, and the benefits of doing so were worthwhile for staging future learning in a meaningful context.

This study further validated the adage of learning by doing. By administering the ACP, the participants recognized their students' abilities to do cognitively demanding tasks. Further, they learned they could cover a lot of material as well as promote the Process Standards without sacrificing too much time. In fact, all participants stated that they intend to use the ACPs in the future. The findings of this study suggest that the participants would not have come to these conclusions without taking the initiative to take part in the professional development and administer the ACPs to their classes. Glasersfeld's (1993) statement about teachers becoming enthralled in such methodologies after making a serious effort to implement them seems to be true, but what must one do in order to get teachers to make that initial, serious effort?

### **Conclusion 3: Factors Influencing Adoption of an Innovation**

Three key factors played a role in the participants' adoption of ACPs. First, they learned they could cover past material and prepare for future material with little loss of time. Second, the participants saw significant value in being able to reference the ACPs when introducing new topics. Third, they trusted and respected me, the creator of the ACPs and the facilitator of the professional development. In the following sections, I discuss these factors and make conclusions about how my findings can be used to encourage teachers to implement Standards-based approaches in their classrooms.

#### **Personal Meaningfulness**



In the second group reflection, Shareese stated that ACPs would “be dead in the water” if they did not have any value to her or other mathematics teaching. Her frank response led me to think about overall teacher interest in professional developments given by school administration. If professional development is not sufficiently meaningful to teachers, is it “dead in the water?” As stated in conclusion two, the participants’ discovered that the ACPs encompassed many of their goals in teaching mathematics. They were able to review past material, prepare their students for future material, and foster the Process Standards. As Ken stated, the ACPs “spoke directly” to the participants as Algebra II teachers – The professional development series about ACPs was meaningful to the participants.

As referenced earlier, ACPs are reflective of Lesh’s MEAs, and an important requirement of MEAs is that they foster personal meaningfulness in those working with them (Lesh & Doerr, 2003). Specifically, “Is the question worth solving? Is it truly a real-life situation? Do students find it interesting? Will they be able to give their own ideas on the problem or will they be expected to follow the teacher’s line of reasoning (p. 43-44)?” Similar to these meaningfulness requirements for students, the ACPs met the meaningfulness requirements for the participants in this study. They covered many of their needs as Algebra II teachers while also fostering the Process Standards. The participants’ desire to foster meaningful skills such as reasoning and communication with their students and still meet state requirements was a major factor in their adoption of ACPs.

## **Referencing**

In Clark and Lesh's study on middle school teachers who worked through several iterations of concept maps on MEAs (2003), the researchers found that the teachers in the study that demonstrated the greatest growth were those that identified the interrelationships between skills and concepts covered in a given MEA. Specifically, the teachers who were able to identify the interrelationships made the greatest move towards a Standards-based approach to teaching mathematics.

For the participants in my study, the identification of the skills covered in the ACPs and their ability to reference those skills when introducing a new topic in their daily lessons played the largest role in them adopting the ACPs. In both sets of interviews and group reflections, the most common theme among the participants was the value they placed on the referencing quality of the ACPs. Their ability to introduce a topic in the units of study by referring back to a specific aspect of the ACPs seemed to be the most appealing attribute. This reflects the personal meaningfulness that participants in the study placed on the ACPs after they discovered the ACPs could enhance their ability to cover necessary material. However, how much the participants shifted towards a Standards-based approach due to the professional development on, and administration of ACPs was not conclusive.

The difference between my study and Clark and Lesh's (beyond my use of high school teachers) was they sought to determine factors that caused shifts in their participants' beliefs about Standards-based approaches to teaching while I sought to determine factors that contributed to participants' adoption of a Standards-based practice. I wanted to know how to get a Standards-based approach's foot in the classroom door. In

addition to exploring how such approaches impact teachers' beliefs and perspectives, I wanted to know what motivated the participants to try the new practices in the first place. My findings suggest that creating problems such as ACPs that foster the Process Standards and that also meet the specific subject matter (Algebra I, Geometry, Algebra II, etc.) needs of teachers is a worthwhile method for encouraging teachers to shift more towards a Standards-based approach to teaching.

### **Trust and Respect**

Innovations are new ideas or practices introduced into a community by an individual (Rogers, 2003). In my study, ACPs are an innovation I introduced to a small community of high school, Algebra II teachers. The participants (teachers) found the ACPs meaningful and useful for meeting both personal goals and state standards while also fostering the Process Standards. As discussed above, the value they placed on the ACPs was the primary factor in their adoption; however, there were two other factors that guided the process. First, the participants' willingness to initially try out the innovation was mainly due to the fact that I, a person they knew and trusted, introduced the innovation. Secondly, the professional development I created to train teachers on ACP was conducive to the participants' needs and desires as teachers. In the remainder of this section, I will relate the process I went through in my study to Roger's principles of diffusion to emphasize the importance of the person diffusing an innovation and lay the foundation for my proposal for conducting professional development in an educational setting.

**The innovator diffusing the innovation.** As discussed above, innovations are new ideas or practices introduced into a community by an innovator (Rogers, 2003). According to Rogers, in order for an innovation to be diffused, early adopters must bring it to the early majority, after using it themselves and proving that it is worthwhile to adopt. I clearly did not pioneer the idea of introducing cognitively demanding tasks at the beginning of a unit of study, so I am not considered what Rogers calls an innovator. Instead, I am an early adopter who learned of the practice from the research of more “cosmopolitan” educators. The participants in the study are considered the early majority who could serve as the critical mass of teachers necessary to complete the diffusion of ACPs among all of the Algebra II teachers.

In general, Rogers describes the early adopter as a leader within a community that innovators seek to diffuse their ideas or products (*loc 5717*). The early majority respects the innovators opinions, as they view them as people who carefully evaluate an innovation before adopting it as their own. The participants in this study mentioned that I was a person they knew and trusted. They knew of my studies, my practices, and my past leadership position in their school district. I was a district leader for the study’s school site for a year. Specifically, I was the curriculum specialist for 6 – 12 mathematics, but decided to return to the classroom after determining I did not prefer administration work and missed the classroom. Their willingness to take the time to implement ACPs in their classrooms was not due to their initial knowledge or understanding of them. In fact, the participants did not fully understand the value of ACPs until well after the study began.

Their willingness to use ACPs was based on their opinion that I would not ask them to do something that was “a waste of time.”

The value of Standards-based approaches has been extensively researched, albeit mostly on K – 8 students and teachers, and determined to be beneficial for learning mathematics and problem solving (Elstein, Lesh & Doerr, 2003; Greeno, 1998; Lesh et al., 2000; Lesh & Doerr, 2000; Lesh, Hoover, & Kelly, 1993; Lesh & Yoon, 2004; Lester, Garafalo, & Kroll, 1989; Lester & Kehle, 2003; Schoenfeld, 1992) but many of these approaches were not implemented in U.S. mathematics classes (Hiebert, 2005). What I concluded from my findings was that research offering evidence of valuable improvement in teaching practices and student mathematical achievement is not sufficient for convincing teachers to change their practices. Teachers must either personally see the research (innovations) in practice or know a person they trust who has implemented the practices successfully. A person who acts as a bridge between research and new practices and teachers is an essential component for fostering change in teaching practices.

**Professional development.** Another key feature that affected the adoption of ACPs was how they were presented to the participants. In addition to being a meaningful and useful tool, the participants were also amenable to the small-group environment of my professional development. Their complaints of school-wide professional developments being too broad, not specific to their subject matter, or not relevant to them as teachers were not applicable for my professional development. My sessions worked around the participants’ schedules (the participants had a common planning period that

made this easy to do) was very specific to their courses, and was not mandated. Instead of feeling an administrator who did not understand their subject matter was wasting their time, the participants felt like they were doing something of their own accord – something they wanted to do. Though very obvious, I concluded that teachers taking part in a professional development they were interested in were more likely to implement the innovation presented in the professional development.

### **Summary**

All of the participants adopted ACPs for use in their Algebra II classes. Their stated reasons for adopting ACPs were that the ACPs blended well with their subject matter, had useful referencing qualities, and met their desires for fostering the Process Standards while also meeting their needs to cover the state standards. The fact that I was the one who brought the innovation to the participants made it easier for them to initiate a trial of the ACPs because I was a colleague they knew and trusted.

My findings indicate that the diffusion of innovations such as ACPs requires a bottom-up approach to professional development where teachers are given choices about their professional development by people (early adopters) they know and trust. Administrators wishing to implement change in their schools must identify these early adopters and determine if the desired changes align with the early adopter's personal and professional goals. If facilitators can find a sufficient number of these early adopters, then professional development can be more meaningful and effective to teachers.

## **Open Question**

As noted above, a key factor to the participants' adoption of ACPs was the referencing qualities for introducing new topics. The participants all commented on this factor repeatedly. I wonder, however, whether the participants appreciated this factor more than the value of the ACPs to foster the Process Standards. The participants all mentioned their beliefs that the ACPs fostered all or most of the Process Standards, but this study did not seek to determine the degree they valued this over the referencing qualities.

It is clear that ACPs would have been "dead in the water" had they not met the participants' needs to meet state standards; however, the primary goal of ACPs is to foster the Process Standards and aid students in becoming better problem-solvers. The degree to which the participants valued the ACPs as a means of developing the Process Standards is pertinent, as use of ACPs can become "traditionalized" and scaffolded if the participants did not place a sufficiently high value on the ACPs use to foster the Process Standards. The value of ACPs as a means to promote the Process Standards could diminish over time if the participants viewed the ACPs solely as a means of introducing a new topic; scaffolding them too much in order to save time and ultimately removing the students' need to gather, organize, and interpret data from the process of solving the ACPs.

Determining the value the participants placed on the ACPs' capacity to foster the Process Standards would have been a valuable means of predicting their correct, future administration. Since this was not a component of the study, it remains to be seen

whether the participants will continue administering the ACPs as they were intended. Finding a way to measure the value participants place on the ACPs' capacity to promote the Process Standards would be a worthwhile addition to any future studies similar to mine.

### **Implications for Future Research**

Chapter II describes a significant number of studies, also referenced in this chapter, that provide evidence of the value Standards-based practices have on students' ability to problem-solve. The studies also demonstrate that teachers can be persuaded to adopt these practices as their own. However, most of these studies focus on k – 8 grades and did not address the concerns and perspectives of high school mathematics teachers. In my study, I extended the research on Standards-based approaches to teaching mathematics to high school teachers and found they were quite amenable to such approaches. Using a single Standards-based practice (ACP), the participants were able to address the Process Standards in their classrooms by using a cognitively demanding task while still covering the topics and skills mandated by the state. Their ability to address both sets of standards in a timely manner made it easier for them to adopt the ACPs as their own.

Ultimately, this study provided evidence that Standards-based practice are effective means of developing problem solving with high-school students, and that teachers concerns and perspective can be shifted to a favorable view of using such practices. This evidence suggests that more comprehensive studies can be conducted to determine the effects such practices have on student achievement and students' problem-



solving capabilities. In chapter I, I discussed how the PISA exam rates 15-year-olds from all over the world on their problem-solving abilities. I suggest a study that administers a past version of this test to a large sample of 15-year-old students in a middle-sized school district on an annual basis. At the same time, the school district should begin introducing ACPs to its sixth grade classes using the same approach I did with the Algebra II teachers in this study. As the years progress, ACPs and related professional developments should be added to subsequent grade levels while the grade levels that already have ACPs in place continue refining these tools in professional learning communities.

By the time the initial group of sixth grade students turn 15, they would have had three or four years of experience with mathematics that incorporate such practices. Comparing the PISA scores of the initial group with the preceding groups could give insight to the effectiveness this specific Standards-based approach has on students' problem-solving abilities. Further, correlations between the years that students worked with ACPs and their state standardized test scores can also offer valuable information to researchers, policy makers, and educators.

### **Implication for Future Practice**

In my study, the participants valued the way I conducted the professional development in comparison to how it is conducted at the study site. This led me to believe that school divisions using the study site's professional development model should consider a different approach to professional development in mathematics. Based

on the participants' negative view of the top-down approach<sup>21</sup> the study site's administration uses for conducting professional development and the positive view the participants had for the small groups focused on mathematics, a bottom-up approach to professional development should be considered. To be clear, a top-down approach is not necessarily a poor model, especially since school districts that wish to focus on improving in a specific area can benefit from focusing their faculty in a particular direction. However, such an approach can risk alienating teachers by not fully taking their opinions and needs into consideration. Further, it has the ability to become too generalized in the process of trying to deliver it to an entire faculty.

The participants spoke of such concerns in the group reflections and emphasized their appreciation for the small-group nature of my professional development and its emphasis on areas in mathematics particularly important to them. This was a significant factor leading to the participants' adoption of ACPs. Further, the ACPs came from someone they knew and trusted as opposed to a district-level administrator they had never met. I suggest that school districts take these two factors into serious consideration when designing district-wide models for diffusing a practice they wish their teachers to use – whether implementation of Standards-based approaches to teaching, or any other innovation.

I propose that when district leaders select an innovation they wish their teachers to use that subject-matter specialists adjust the innovation to fit the needs of a particular subject. Further, I recommend that the subject-matter specialists identify the early

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<sup>21</sup> A top-down approach to PD is where administrators at a school or district level determine what their teachers will pursue in regards to their professional growth.

adopters (teacher leaders) in specific departments (i.e., mathematics, English, history, etc.) at each school site and use them as diffusers of the innovations. The early adopter does not have to be a department chairperson, but he or she must be someone who believes in the innovation and is willing to use it in his or her classroom. It is not enough to simply dictate that a department chair require the use of an innovation by the teachers within his or her department. Further, the early adopter must be a veteran member of the faculty that is well known and respected among the other teachers. After the early adopters were trained on the innovation, they would implement the innovation in their classes, give testimony to its value, and seek out other members of their department interested in implementing it themselves. Training the early adopters on how to work with their peers as well as conduct professional development would help the early adopters optimize the number of colleagues they recruit. Using a similar approach to my professional development, if the early adopter could diffuse the innovation in a manner amenable to teachers, there would be a strong chance that the department would adopt the innovation and teachers would fully use it in their classrooms.

This would be a slow process, but it would allow district leaders to focus their school districts in the direction they wish by using a bottom-up approach to professional development. Based on what I discovered from the participants in my study, teachers' adoption of an innovation must be their choice and not the choice of unknown district leaders. By taking the time to tailor a general innovation to the needs of individual departments and having respected members of those departments diffuse the innovation

to their colleagues, district leaders can likely enhance the effectiveness and adoption of an innovation they feel is pertinent to the future direction of their school districts.

### **A Microscopic Approach to a Macroscopic Issue**

The findings of my study are not limited to diffusing Standards-based practices such as ACPs to mathematics teachers. Any practice a school division, state, or even national organization wishes to implement must consider Rogers (2003) factors for adoptions. For example, if the Department of Education wishes to successfully diffuse the Common Core Standards to teachers across the US, then they must help teachers see the following: (a) that there is an advantage in using them over what they are presently using; (b) that they are compatible with teachers' existing practices; (c) they are simple and not overwhelming to use; and (d) there is little risk in trying them out. Further, teachers must be able to observe the Common Core Standards successfully implemented in other teachers' classrooms.

There is no "silver bullet" that can accomplish the task of diffusing something as significant as national standards to all teachers in the US. It must be a process that takes into account the slow nature of social change. If kindergarten took more than 50-years to diffuse throughout the US (Rogers, 2003, *loc* 1815) and Tetracycline took 15 (Rogers, 2003, *loc* 1860), then advocates of large-scale educational changes must be prepared for a lengthy implementation. The findings of my very, small-scale study seem to suggest that humans are amenable to change if given the time consider the innovation being diffused and to see it successfully put into practice. Organizations such as the Department of Education may wish to consider a bottom-up approach to diffusing the Common Core

Standards as opposed to a top-down approach, so individual teachers have the opportunity and time to digest all that is being diffused. Not giving teachers this opportunity may result in the failure of the Common Core or any other innovation like it.

### **Limitations**

A major component of this study was whether the participants would adopt the ACPs presented. All participants stated in their interviews and/or group discussion that they would use the ACPs during the subsequent school year; however, this dissertation was completed prior to the beginning of the next school year, so it was not possible to determine if the participants actually used the ACPs again. To truly measure the degree to which the participants adopted ACPs as their own pedagogical tool for fostering the Process Standards, the length of my study would need to be extended to at least a second year, if not a third.

Asking the participants after both units of study in subsequent years how they administered the ACPs would give a true indication of overall adoption of the innovation. By waiting until after participants teach their units on quadratics and polynomials in following years to ask if they used the ACPs in the beginning of the units, I could determine if they were still enthusiastic about them. However, asking about the ACPs prior to those units could possibly encourage them to use the ACPs when they would not have done so on their own. Due to the timeframe for the data collection portion of this study, this data could not be gathered in subsequent years, and more thorough evidence of participants' adoption of ACPs could not be determined.

## **Final Thoughts**

This study arose from the need to disseminate well-researched practices on fostering problem solving in classrooms across the US. Based on the evidence provided by NCTM (2000) and Hiebert (2005), valuable practices for developing a Standards-based approach to teaching mathematics such as MEAs (Lesh & Doerr, 2003) have not been widely implemented in U.S. schools. We as a nation are falling behind on assessments of problem-solving abilities and these abilities have a relationship to the economic stability of a nation (Hanushek & Woessmann, 2010). These facts emphasize the imperative need to improve the problem-solving abilities of students in the US.

The purpose of this study was not to identify new practices, but to find a way to effectively disseminate an already established and effective practice to teachers in a manner that would increase the probability they would adopt it and use it with their students. I found that adapting an MEA to specific topics in Algebra II such that state-mandated topics were addressed in conjunction with NCTM's Process Standards was an effective means of reaching my goal. By taking the needs of teachers into account and addressing their immediate concerns of state-mandated standards being taught in a timely manner, I was able to help them incorporate meaningful, cognitively-demanding problems into their teaching. The fact that I, the facilitator of the professional development on ACPs, was a colleague the participants knew and trusted aided in their initial willingness to try ACPs in their classrooms. The full adoption of the ACPs by all the participants was due to the ACPs referencing qualities, how the professional

development was conducted, and characteristics of the tasks that fostered the Process Standards.

If other teachers in the US are like those in this study, they already have a desire to incorporate NCTM's Process Standards in their classrooms whether they know them explicitly, implicitly, or haphazardly. Determining methods and practices for fostering problem solving in U.S. classrooms that mesh with our culture of incessant, standardized testing is an important step in developing students who are mathematically literate, critical thinkers willing to engage in constructive and reflective activities beneficial to the advancement of our nation and the world at large.

## APPENDIX 1

### Survey of Beliefs and Concerns Regarding Anchored Conceptualizing Problems

*For the following statements, please choose a number from one to five, one being you strongly disagree and five you strongly agree. If you believe the questions are not applicable to you, please choose NA.*

- |    |  |   |   |   |   |   |    |
|----|--|---|---|---|---|---|----|
| 1. | I am concerned about implementing these ACPs properly. (1)   | 1 | 2 | 3 | 4 | 5 | NA |
| 2. | I am concerned that I do not know how to solve these ACPs myself. (1)  | 1 | 2 | 3 | 4 | 5 | NA |
| 3. | I am concerned that I do not know how to solve these ACPs without using Algebra. (1)                         | 1 | 2 | 3 | 4 | 5 | NA |
| 4. | I am concerned that I will feel inclined to help my students with these ACPs more than I am supposed to. (1) | 1 | 2 | 3 | 4 | 5 | NA |
| 5. | I believe my students do not have enough prior knowledge necessary to solve these ACPs. (2)                  | 1 | 2 | 3 | 4 | 5 | NA |
| 6. | I believe the students will not be able to do this ACP. (2)  | 1 | 2 | 3 | 4 | 5 | NA |
| 7. | I believe these ACPs are not suitable for this grade level. (3)  | 1 | 2 | 3 | 4 | 5 | NA |



8. I believe these ACPs are better suited for a lower-level course. (3)  
1                    2                    3                    4                    5                    *NA*
9. I believe these ACPs are better suited for a higher-level course. (3)  
1                    2                    3                    4                    5                    *NA*
10. I believe these ACPs will positively impact how my students learn the content of the related unit of study. (3)  
1                    2                    3                    4                    5                    *NA*
11. I believe these ACPs will have additional benefit to my students beyond teaching content specific to the curriculum. (3)  
1                    2                    3                    4                    5                    *NA*
12. I believe this ACP will take too much time for my students to complete in a 90-minute block. (4)  
1                    2                    3                    4                    5                    *NA*
13. I believe these ACPs will hinder my ability to cover all of the content I must teach for the given school year. (4)  
1                    2                    3                    4                    5                    *NA*
14. I do not believe ACPs should be given to students at the beginning of the unit of study. (4)  
1                    2                    3                    4                    5                    *NA*
15. I am excited about using these ACPs. (5)  
1                    2                    3                    4                    5                    *NA*
16. I have used teaching strategies/approaches to learning similar to ACPs before. (5)  
1                    2                    3                    4                    5                    *NA*
17. I believe that such strategies/approaches to learning should be taught as frequently as possible. (5)  
1                    2                    3                    4                    5                    *NA*

18. I will incorporate other strategies for learning mathematics that will aid the students in doing these problems. (5)
- |   |   |   |   |   |    |
|---|---|---|---|---|----|
| 1 | 2 | 3 | 4 | 5 | NA |
|---|---|---|---|---|----|
19. I know what the National Council of Teachers of Mathematics (NCTM) Standards-based approaches to learning are. (6)
- |   |   |   |   |   |    |
|---|---|---|---|---|----|
| 1 | 2 | 3 | 4 | 5 | NA |
|---|---|---|---|---|----|
20. I am comfortable working with NCTM's Standards-based approaches to learning. (6)
- |   |   |   |   |   |    |
|---|---|---|---|---|----|
| 1 | 2 | 3 | 4 | 5 | NA |
|---|---|---|---|---|----|
21. I believe that ACPs are closely aligned with the standards and focal points of NCTM. (6)
- |   |   |   |   |   |    |
|---|---|---|---|---|----|
| 1 | 2 | 3 | 4 | 5 | NA |
|---|---|---|---|---|----|

What do you think is an appropriate time to administer this ACP (i.e., before, during, or after the unit it encompasses)?

Please list any other concerns, interests, and/or opinions you have about using ACPs as a pedagogical tool in your classrooms.

*1 – Teacher Self-Efficacy*

*2 – Teachers' Beliefs about Students' Ability to Work with ACPs*

*3 – Teachers' Beliefs about ACPs*

*4 – Organizational Concerns*

*5 – Adopter Category for Roger's Diffusion Theory*

*6 – Understanding of Standards-based approaches to learning as determined by NCTM*

## APPENDIX 2

### Teacher Questionnaire About Administration of ACPs

*In order to better tailor the next professional development on Anchoring Contextualizing Problems to your needs and those of the other participants, please briefly respond to each of the question below.*

1. In general, how do you feel your administration of the ACP went? What went well? What did not go well?
2. Now that you have administered the ACP, what would you change about it?
3. What would you have done differently if given an opportunity to administer the ACP again?
4. Is there any preparation that you would have added to lessons prior to the ACP (i.e., skills for problem solving, practice with word problems, more on graphing, etc.)?
5. Do you feel that the dynamics of the specific class to which you administered the ACP had any impact on the way you taught it?

## APPENDIX 3

### Letter to Prospective Participants

Dear Teacher,

I am in the final stages of my doctoral work and am preparing to implement the study for my dissertation. The study will begin in September and end in later January or early February. It is a qualitative research project, which requires the participation of Algebra II teachers. I would very much appreciate it if you would consider taking part. Below is a brief outline of the schedule of events and the data I will need to collect from you.

#### **September 2013**

1. I will send a mathematical task to you that I call an Anchored Contextualizing Problem (ACP)
2. I will send a survey to you asking about your opinions, concerns, and perspectives regarding the ACP
3. I will observe your teaching style within the first two weeks of the 2013-2014 school year
4. A professional development on the administration of the ACP will be given
5. You will take the survey again

#### **October/November 2013**

1. You will be interviewed individually about your concerns and perspectives about using the ACP
2. You will administer the ACP to your Algebra II class and complete a questionnaire about your administration of the ACP
3. You will collect a cross-section (varying qualities of work) of student work
4. You will take part in a second professional development where all participants will reflect on the implementation of the ACP in their classes, discuss student work, and be trained on another ACP
5. You will take the above-mentioned survey again

#### **December/January 2013-2014**

1. You will administer the second ACP to your Algebra II class and complete a questionnaire about your administration of the ACP
2. You will collect a cross-section (varying qualities of work) of student work
3. You will take part in a final group reflection where we will discuss student work and general thoughts on the use of ACPs
4. You will complete the survey one last time
5. You will take part in an individual interview

In all, you will give six to 10 hours of your time (administration of ACP in your classroom included) to this study. I understand that this is significant, but if you can take the time to do so, I would greatly appreciate it.

I will be approaching you in person soon about the study to determine if you will take part. If you would like to reply via email you are welcome to do so.

Thank you,

Brad Rankin  
Math Teacher  
T.C. Williams High School

## APPENDIX 4

### Consent Form

Teachers,

The following is a consent form outlining the study in which you will be taking part. Your participation is completely voluntary, and you are allowed to drop out of the study for any reason at any time. The data, including recordings of interviews and group reflections, collected during this process will remain secure at all times. A pseudonym will be given in lieu of your actual name during the study and in my dissertation. All indicators of your identity will be kept out of my dissertation, and information that allows me to code you to your pseudonym will remain encrypted and secure for three years, after which I will destroy the information.

The following is a review of the time commitments for this study:

#### **September 2013**

1. I will send a mathematical task to you that I call an Anchored Contextualizing Problem (ACP) (*approximately 20- to 30-minutes to go over*)
2. I will send a survey to you asking about your opinions, concerns, and perspectives regarding the ACP (*approximately 30- to 40-minutes to complete*)
3. I will observe your teaching style within the first two weeks of the 2013-2014 school year (*No time lost on your part*)
4. A professional development on the administration of the ACP will be given (*1.5 hours*)
5. You will take the survey again (*approximately 30- to 40-minutes to complete*)

#### **October/November 2013**

1. You will be interviewed individually about your concerns and perspectives about using the ACP (*30- to 60-minutes*)
2. You will administer the ACP to your Algebra II class and complete a questionnaire about your administration of the ACP (*One full block*)
3. You will collect a cross-section (varying qualities of work) of student work (*five-minutes*)
4. You will take part in a second professional development where all participants will reflect on the implementation of the ACP in their classes, discuss student work, and be trained on another ACP (*1.5 hours*)

5. You will take the above-mentioned survey again (30- to 40-minutes)

**December/January 2013-2014**

1. You will administer the second ACP to your Algebra II class and complete a questionnaire about your administration of the ACP (*One full block*)
2. You will collect a cross-section (varying qualities of work) of student work (*five-minutes*)
3. You will take part in a final group reflection where we will discuss student work and general thoughts on the use of ACPs (*1 to 1.5 hours*)
4. You will complete the survey one last time (30- to 40-minutes)
5. You will take part in an individual interview (30- to 60-minutes)

Please acknowledge the following information by signing your name below.

The time commitments for this study will be eight to 10 hours over the course of five months. Participation is not mandatory and you may drop out for any reason at any time. Your actual name will not be used in any publication of the study such that any views and/or opinions you make will not be connected to you. The interviews and group reflections will be recorded for data collection and they will only be accessible by Brad Rankin, the independent company transcribing the recordings, and Dr. Margaret Hjalmarson (Brad Rankin's dissertation committee chair). There are no risks or direct benefits for participating in this study.

If you have any questions regarding the research, you may contact Brad Rankin by phone at (703) 489.7725 or by email at [Bradley.Rankin@acps.k12.va.us](mailto:Bradley.Rankin@acps.k12.va.us). Dr. Margaret Hjalmarson can be contacted via email at [MHjalmar@gmu.edu](mailto:MHjalmar@gmu.edu).

X \_\_\_\_\_

## APPENDIX 5

### ACP for Quadratic Functions Unit

Old McDonald wants to build a *rectangular* fence for his pigs, and he is asking you to help him determine the dimensions he should choose in order to give his pigs the **most** space to roam and graze. He wants to build the fence alongside a river, so he only needs three sides of fencing to enclose the pigs. He has 300 feet of fencing with which to build his fence. In order to do the most thorough job for Old McDonald, you need to do the following:

1. Draw a diagram of the fence
2. Create a list with at least eight different dimensions and their corresponding areas and show all of the work for each calculation in your list
3. Graph your data comparing the widths you chose for each possible fence to their corresponding areas
4. Identify the largest area you were able to find
5. Write a detailed description explaining how you calculated the area of the fence given 300 feet of fencing
6. Write a letter to Old McDonald telling him how you think he should arrange his fence in order to get the greatest area for his pigs to roam and graze



## APPENDIX 6

### Interview Questions

1. What do you know about Standards-based approaches to teaching mathematics?
2. Did you have any misconceptions of what Standards-based approaches to learning were prior to the professional development?
3. What are your opinions about Standards-based approaches to teaching mathematics?
4. How often do you include Standards-based approaches in your lessons?
  - a. If you do not use them often, then what are some reasons why?
5. What do you think of the ACP as a Standards-based approach?
6. Do you think it can be a useful tool in helping your students learn mathematics?
  - a. What about the specific use of this ACP to help students learn about quadratics?
7. What do you think about administering the ACP prior to starting the unit on quadratics?
8. Will you do anything to prepare your students for interacting with this form of learning prior to administering the ACP?
9. How often do you think you will reference the ACP when teaching the skills and concepts of quadratics?

10. If you had been presented with this method of teaching earlier, do you think you would have tried it out on your own without the impetus of a colleague, me, asking you to do so?

11. What are some problems you see arising when administering the ACP?

What about possible problems later in the year due to the ACP?

## APPENDIX 7

### ACP for Polynomial Functions Unit

You are about to take a short vacation to Miami and wish to save some money by only packing a carry-on piece of luggage. The airline with which you are traveling has a simplistic way of determining whether you are allowed to take your carry-on with you or pay to have it placed in the baggage compartment of the plane. They measure the length, width, and height of each carry-on to see if the sum of these measurements is less than 40-inches.

All of the carry-ons you have looked at have lengths that are 10 inches greater than their widths. In order to get the carry-on with the greatest volume, you will need to find a bag whose dimensions (length, width, and height) follow these guidelines. In order to do so, you will need to do the following:

1. Draw a diagram of a piece of luggage and label the length, width, and height
2. Generate a list of *at least* eight different sets of dimensions for carry-ons that fit the guidelines in paragraph two and calculate the volume of each
3. Graph your data using the widths of your carry-ons as your x-axis and the volumes as your y-axis
4. Identify the largest volume you were able to find
5. Write a detailed description explaining how you calculated the volume of each carry-on given only 40 inches of combined length, width, and height

## APPENDIX 8

### *Teacher Self-Efficacy*

	S1	S2	S3	D(S2-S1)	D(S3-S2)	D(S3-S1)
<b>Hartley</b>						
Q1	4	4	4	0	0	0
Q2	4	1	3	-3	2	-1
Q3	4	1	3	-3	2	-1
Q4	2	3	3	1	0	1
<b>Mike</b>						
Q1	3	1	1	-2	0	-2
Q2	3	1	1	-2	0	-2
Q3	3	1	1	-2	0	-2
Q4	4	5	1	1	-4	-3
<b>Ken</b>						
Q1	1	1	1	0	0	0
Q2	1	1	1	0	0	0
Q3	1	1	1	0	0	0
Q4	0	1	3	1	2	3
<b>Nicole</b>						
Q1	4	2	NR	-2	NR	NR <sup>22</sup>
Q2	2	1	NR	-1	NR	NR
Q3	2	1	NR	-1	NR	NR
Q4	4	2	NR	-2	NR	NR
<b>Shareese</b>						
Q1	3	2	1	-1	-1	-2
Q2	3	1	1	-2	0	-2
Q3	3	1	1	-2	0	-2
Q4	5	3	3	-2	0	-2
				-1.1	0.0625	-0.9375

<sup>22</sup> Nicole did not submit the third survey, so data requiring this survey was not available.

*Teachers' Beliefs about Students' Ability to Work with ACPs*

	S1	S2	S3	D(S2-S1)	D(S3-S2)	D(S3-S1)
<b>Hartley</b>						
Q5	4	2	3	-2	1	-1
Q6	2	2	2	0	0	0
<b>Mike</b>						
Q5	4	2	2	-2	0	-2
Q6	2	3	1	1	-2	-1
<b>Ken</b>						
Q5	3	2	3	-1	1	0
Q6	2	2	1	0	-1	-1
<b>Nicole</b>						
Q5	2	1	NR	-1	NR	NR
Q6	1	2	NR	1	NR	NR
<b>Shareese</b>						
Q5	3	3	2	0	-1	-1
Q6	3	3	2	0	-1	-1
				0.2222222222	-0.375	-0.857142

*Teachers' Beliefs about ACPs*

	S1	S2	S3	D(S2-S1)	D(S3-S2)	D(S3-S1)
<b>Hartley</b>						
Q7	1	1	1	0	0	0
Q8	1	1	1	0	0	0
Q9	1	1	1	0	0	0
Q10	4	5	5	1	0	1
Q11	5	5	4	0	-1	-1
<b>Mike</b>						
Q7	3	1	1	-2	0	-2
Q8	3	3	3	0	0	0
Q9	4	3	3	-1	0	-1
Q10	3	4	2	1	-2	-1
Q11	4	4	5	0	1	1
<b>Ken</b>						
Q7	1	1	1	0	0	0
Q8	1	1	1	0	0	0
Q9	1	1	1	0	0	0
Q10	5	4	4	-1	0	-1
Q11	5	4	5	-1	1	0
<b>Nicole</b>						
Q7	1	1	NR	0	NR	NR
Q8	1	1	NR	0	NR	NR
Q9	1	1	NR	0	NR	NR
Q10	5	5	NR	0	NR	NR
Q11	5	5	NR	0	NR	NR
<b>Shareese</b>						
Q7	2	2	1	0	-1	-1
Q8	2	2	1	0	-1	-1
Q9	2	2	1	0	-1	-1
Q10	4	4	4	0	0	0
Q11	4	4	4	0	0	0
				-0.12	-0.2	-0.35

*Organizational Concerns*

	S1	S2	S3	D(S2-S1)	D(S3-S2)	D(S3-S1)
<b>Hartley</b>						
Q12	5	4	3	-1	-1	-2
Q13	4	4	1	0	-3	-3
Q14	2	1	2	-1	1	1
<b>Mike</b>						
Q12	3	2	3	-1	1	0
Q13	4	3	3	-1	0	-1
Q14	3	3	3	0	0	0
<b>Ken</b>						
Q12	2	2	1	0	-1	-1
Q13	2	2	3	0	1	1
Q14	3	2	1	-1	-1	-2
<b>Nicole</b>						
Q12	2	1	NR	-1	NR	NR
Q13	1	1	NR	0	NR	NR
Q14	1	1	NR	0	NR	NR
<b>Shareese</b>						
Q12	3	4	2	1	-2	-1
Q13	3	3	2	0	-1	-1
Q14	2	2	1	0	-1	-1
				-0.33333	-0.58333	-0.83333

*Adopter Category for Roger's Diffusion Theory*

	S1	S2	S3	D(S2-S1)	D(S3-S2)	D(S3-S1)
<b>Hartley</b>						
Q15	4	4	5	0	1	1
Q16	2	3	3	1	0	1
Q17	4	4	3	0	-1	-1
Q18	5	5	5	0	0	0
<b>Mike</b>						
Q15	4	5	5	1	0	1
Q16	3	4	5	1	1	2
Q17	3	3	3	0	0	0
Q18	3	3	4	0	1	1
<b>Ken</b>						
Q15	4	4	4	0	0	0
Q16	5	4	5	-1	1	0
Q17	5	4	4	-1	0	-1
Q18	5	5	5	0	0	0
<b>Nicole</b>						
Q15	5	5	NR	0	NR	NR
Q16	3	3	NR	0	NR	NR
Q17	4	4	NR	0	NR	NR
Q18	4	4	NR	0	NR	NR
<b>Shareese</b>						
Q15	4	4	4	0	0	0
Q16	3	3	3	0	0	0
Q17	4	4	4	0	0	0
Q18	4	3	3	-1	0	-1
				0	0.1875	0.1875



*Understanding of Standards-based approaches to learning as determined by NCTM*

	S1	S2	S3	D(S2-S1)	D(S3-S2)	D(S3-S1)
<b>Hartley</b>						
Q19	1	1		0		
Q20	3					
Q21						
<b>Mike</b>						
Q19	3	1	2	-2	1	-1
Q20	2	2	2	0	0	0
Q21	3		3			0
<b>Ken</b>						
Q19	3	3	4	0	1	1
Q20	4	4	5	0	1	1
Q21	4	4	4	0	0	0
<b>Nicole</b>						
Q19	NA	NA	NA	NA	NA	NA
Q20	NA	NA	NA	NA	NA	NA
Q21	NA	NA	NA	NA	NA	NA
<b>Shareese</b>						
Q19	4	3	4	-1	1	0
Q20	3	3	4	0	1	1
Q21	4	4	4	0	0	0
				-0.3333	0.625	0.2222

## APPENDIX 9

### Exit Interview Questions

1. When watching/discussing your lesson on absolute values, I noticed that you approached it in a traditional/more standards-based approach. Would you agree with my assessment?
2. When I asked you about Standards-based approaches to learning in our earlier interview, you originally saw them as \_\_\_\_\_ . What comes to mind now?

**I would like to read the NCTM's Process Standards to you. They are five, overarching standards that the NCTM views as integral to k-12 mathematics education.**

- **Problem Solving** – Instructional programs from prekindergarten through grade 12 should enable all students to: a.) Build new mathematical knowledge through problem solving; b.) Solve problems that arise in mathematics and in other contexts; c.) Apply and adapt a variety of appropriate strategies to solve problems; d.) Monitor and reflect on the process of mathematical problem solving.
- **Reasoning and Proof** – Instructional programs from prekindergarten through grade 12 should enable all students to: a.) Recognize reasoning and proof as fundamental aspects of mathematics; b.) Make and investigate mathematical conjectures; c.) Develop and evaluate mathematical arguments and proofs; d.) Select and use various types of reasoning and methods of proof.
- **Communication** – Instructional programs from prekindergarten through grade 12 should enable all students to: a.) Organize and consolidate their mathematical thinking through communication; b.) Communicate their mathematical thinking coherently and clearly to peers, teachers, and others; c.) Analyze and evaluate the mathematical thinking and strategies of others; and d.) Use the language of mathematics to express mathematical ideas precisely.
- **Connections** – Instructional programs from prekindergarten through grade 12 should enable all students to: a.) Recognize and use connections among

mathematical ideas; b.) Understand how mathematical ideas interconnect and build on one another to produce a coherent whole; c.) Recognize and apply mathematics in contexts outside of mathematics.

- **Representation** – Instructional programs from prekindergarten through grade 12 should enable all students to: a.) Create and use representations to organize, record, and communicate mathematical ideas; b.) Select, apply, and translate among mathematical representations to solve problems; c.) Use representations to model and interpret physical, social, and mathematical phenomena.

1. **[General]** To what degree did you adopt these standards as your own prior to taking part in this study?
2. **[General]** How often do you feel you were able to promote these standards prior to the study; to encourage/require students demonstrate them?
3. **[General]** How did you do so?
4. **[General]** [If any suggestion of time or state standards (preconceived notions of what a Standards-based approach to learning mathematics is) getting in the way then...] Do you feel that these are standards with which you would have preferred dealing?
5. **[Research Question1]** In all honesty, how many students, in your opinion, develop these skills in an average mathematics classroom?
6. **[Research Question1]** Do you think you have developed them sufficiently with your own students over the course of your career?
7. **[Research Question1]** What are your concerns and/or perspectives on working towards these process standards as a personal/professional goal?
8. **[Research Question1]** Would you say that the ACPs encompass some or many of these process standards?
9. **[Research Question1]** If so, to what degree?
10. **[Research Question1]** Have the use of ACPs allayed any of your concerns about teaching towards these process standards? If so, in what way?
11. **[Research Question1]** Has your use of the ACPs changed your perspective on teaching towards these process standards?

**At this point, let us refer to a Standards- Based Approach to learning mathematics as means of teaching all students to be problems solvers through**

**rich, cognitively demanding tasks. From these tasks, students can learn the skills and procedures traditionally taught in mathematics within the context of something meaningful rather than learn the skills first in the hope they will culminate in the ability to solve rich, cognitively demanding tasks.**

12. **[Research Question2]** Prior to implementing ACPs in your classroom, what would your opinion on this statement have been?
13. **[Research Question2]** How do ACPs relate to this statement?
14. **[Research Question2]** Have the use of ACPs taught you anything about what it means to pursue a Standards-Based Approach to learning mathematics as it pertains to NCTM's process standards?
15. **[Substantive]** Do you feel that these Process Standards have been on your mind, in some form, but have not culminated in a method for addressing them in your classroom?
16. **[Substantive]** How has the process of learning about ACPs affected your knowledge of standards-based approaches to learning as they pertain to the Process Standards?
17. How have the ACPs affected your views on teaching mathematics?
18. How have the ACPs affected your teaching?
19. **[Theoretical]** As noted in your previous interview, the ACPs are not instruments you would have used on your own or at least not at the beginning of a unit of study. Are they something you would use now? At the beginning of a unit of study?
20. **[Theoretical]** What made you come to this conclusion?
21. **[Theoretical]** When we identified as a group the skills that the ACPs encompassed, did this have any impact on your views of ACPs? If so, how?
22. **[Diffusion]** How have the ACPs affected your future plans for teaching Algebra II?
23. **[Diffusion]** Will you use these ACPs, in the chronological order suggested, when you teach these topics (Quadratics and Polynomials) next year?
24. **[Diffusion]** If given another one for this year, would you use it? [If yes, offer them the video game ACP].

**[Diffusion]** With complete honesty, would you say you have adopted this approach to teaching as your own? To what degree?

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## **CURRICULUM VITAE**

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