Use of Strategy Maps and Virtual Coaching: A Case Study of a Teacher’s Development of Connections in Middle Grades Mathematics

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DEDICATION

To Evan and Josephine
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Common Core State Standards ................................................................. CCSSO
National Council of Teachers of Mathematics ........................................ NCTM
National Science Foundation ................................................................. NSF
Trends in International Mathematics and Science Study .......................... TIMSS
ABSTRACT

USE OF STRATEGY MAPS AND VIRTUAL COACHING: A CASE STUDY OF A TEACHER’S DEVELOPMENT OF CONNECTIONS IN MIDDLE GRADES MATHEMATICS

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George Mason University, 2015

Dissertation Director: Dr. Jennifer Suh

The implementation of math-talk appears simple and eloquent when facilitated by a skilled teacher, but daunting to those who try to implement it with little or no background on the behind the scenes practices. With the introduction of the Common Core State Standards, and the increased recommendation for mathematical discourse by the National Council of Teachers of Mathematics, teachers and administrators are feeling the need to teach mathematics differently. While previous research has identified specific practices that break down math-talk into manageable routines, it does not adequately address the practice of making connections.

This dissertation addresses the need for math-talk by providing ongoing math coaching that utilizes a visual tool for teachers called a strategy map, which targets the skill of making connections. Further, it will answer the following research questions:
1) In what ways do strategy maps support the teacher in making connections during the math-talk?

   (a) Given diverse student methods, how do strategy maps support teachers in making connections between students’ work?

   (b) What connections between students’ work emerge during the math-talk?

   (c) In what ways do the strategy maps and the math-talk develop together throughout a semester?

2) In what ways does virtual math coaching support the teacher in utilizing strategy maps as a tool to increase the level of math-talk?

This case study took place over one semester, and analyzed the way that one mathematics teacher used strategy maps while implementing math-talk in her classroom.
CHAPTER ONE

Importance

Many classrooms in the United States teach mathematics in traditional style, using mostly rote learning (Knapp, 1995). This style of learning forces students to memorize algorithms and procedures with minimal understanding or connections to the larger math concept. By the time students reach Algebra I, many do not have the ability to determine the correct algorithm to use in order to solve abstract problems. This leads to burnout, and a lack of interest in mathematics (Loveless, 2008; Spielhagen, 2006).

The National Council of Teachers of Mathematics (NCTM, 2000) recommends that teachers give students opportunities to reason about mathematics through discussion. Such discussions allow students the opportunity to share ideas and clarify understandings, develop convincing arguments regarding why and how things work, develop a language for expressing mathematical ideas, and learn to see things from other perspectives (NCTM, 2000). Further, the Common Core State Standards (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), which have been adopted by a majority of states, requires students to problem solve, and grapple with mathematical ideas, rather than only learn through rote teaching.

Mathematical discourse in the classroom is essential to student conceptual understanding (Hufford-Ackles, Fuson, & Sherin, 2004). Lewis (2002) and Ball (1991)
found that when students struggle with a rich task and have the opportunity to learn about conceptual ideas rather than rote learning, and there is more opportunity for mathematical discourse. Stein and Smith (2011) lay out a framework for educators, called the Five Practices, which breaks down the details of selecting this rich task, to achieving thoughtful, mathematical discourse. In their study, Stein, Engle, Smith, and Hughes (2008) show that when teachers use the Five Practices to anticipate student strategies, they are quicker to identify those strategies during the lesson, and thus better at selecting the student artifacts, and sequencing the order of presentation. It is only through this purposeful and thoughtful planning that teachers become more adaptive and successful at having mathematical discourse.

Stein and Smith (2011) further explain that even with the selection of a rich task, purposeful planning, and knowledge of the mechanics of math-talk, it is still sometimes difficult for teachers to facilitate rich mathematical discourse. This is because many teachers struggle in making connections between different strategies or algorithms. This skill requires teachers to identify the connections themselves, and then pose questions that probe students to identify those connections. While it requires less planning to allow students to present their strategies, it is significantly more challenging and requires more in depth planning to connect those responses to key mathematical ideas. Without making these connections, the mathematical discourse falls short and becomes a “show and tell” lecture directed by the students (Stein & Smith, 2011). West and Staub (2003) explain that if we want students to grasp the entire concept, they need to understand how these strategies relate to each other, and that they are not unique strategies, but instead similar
strategies. It is when students see the similarities in the different approaches, that they make connections with the larger mathematical concept. Therefore, there is a great need to further expand on students’ work. Students need to analyze the different strategies, models, and drawings that their peers derived to solve the rich mathematical task. These student-created strategies are the cornerstone of the discussion. Further, these strategies can be analyzed by the teacher along with the entire class to discover the similarities and differences, and the connections to each other and the mathematical objective.

I proposed an intervention to support teachers in exposing connections between different student strategies in order to bridge this gap. The intervention used ongoing professional development, through virtual math coaching, that utilized a tool called a strategy map to make a visual and explicit connection between students’ work. This tool supported teachers to anticipate several different strategies, and then sequenced these strategies in a meaningful way that was built upon connections. Further, this intervention supported the teacher in facilitating math-talk, to ensure that the connections were explicitly seen and heard by students.

Strategy maps (see figure 1) look similar to flow charts, with some unique elements that focus the teacher on identifying student connections. Strategy maps were chosen for this study for the ease of use, simplistic and visual layout, and focus on connections. They are similar, but more linear, than traditional concept maps. Novak and Cañas (2008) believe that concept maps support the designer to make connections to the larger concept because they allow the participant to play around with the concept. Teachers can implement strategy maps with drawings or post-it notes to rearrange ideas,
and constantly add and delete information. As the teachers examine student strategies, they can group them together, and use post-it notes to jot down similarities, differences, or other valuable information. Then, when they design their presentation of strategies, they will have more information at their fingertips. The strategy map can always be edited again, making this tool a living document. Strategy maps unique ability to visually see a plan and identify connections makes it the ideal tool to support teachers with this process.

Figure 1. Concept Map of a Concept Map

Hufford-Ackles (1999) studied math-talk in her dissertation. She researched how math-talk changed over one year in a school using a reformed math curriculum. She used
a math-talk framework to create a rubric to code teacher and student interactions and how they supported the math-talk classroom. I used her framework and coding techniques to determine if the use of strategy maps as a tool for identifying connections increased the level of mathematical discourse.

Currently, research has not addressed using a tool to specifically identify connections between student strategies or ensuring that the students’ work connects to the overall mathematical concept. The purpose of this investigation was to incorporate strategy maps in an ongoing professional development, as a tool to identify connections both between the students’ work and the overall mathematical concept, and to keep the math-talk focused on these connections. The significance of this research has the potential to affect all mathematics teachers who want additional support to manage mathematical discourse. If utilizing a strategy map is effective, the Five Practices (Stein & Smith, 2011) can be revised to incorporate this tool and support teachers in achieving those connections.

**Importance to the Researcher**

This research project has interested me for several years. My experience began as a middle school mathematics teacher. In the first year, I taught the way I was taught, through rote memorization and tricks. It was not long before I noticed that this was unsuccessful for the majority of my students. I began a Master’s Degree in Education at George Mason University that focused on problem solving, multiple representations, and mathematical discourse. My students and I made great progress, and found that after a
short adjustment period, my classes were more successful. Students were problem solving, and their confidence in mathematics was increasing.

Several years later, I became a school based mathematics coach. In this position, I supported teachers to incorporate problem solving, multiple representations, and math-talk into their classrooms. In some cases, the teacher and students adapted with ease and in other cases, this style of teaching felt very uncomfortable and forced. I was curious to understand why it was so easy for some teachers to change their pedagogical practices, and so difficult for others.

As I began a Ph.D. at George Mason University, I discovered a framework for math-talk (Hufford-Ackles et. al., 2004) that gave me clarity on the challenges teachers and students face when deciding to implement this in their learning. I explored this framework by observing and analyzing both teacher levels and student levels of math-talk in a variety of classrooms. I was beginning to notice that teachers and students needed to be at similar levels of math-talk in order for it to be successful. But this alone, was not the only determining factor. It seemed that there was an element within the teacher, and his or her ability to allow for uncertainty in the discussion.

I soon discovered the Five Practices (Stein, et al., 2008). The authors explicitly lay out five specific practices that adaptive teachers use to develop and facilitate a successful mathematical discussion. I began to implement these practices as I co-taught lessons side by side with other teachers. I noticed that many of these practices were manageable for all teachers, regardless of their teaching experience. As the teachers practiced these, a rich math-talk developed.
The Five Practices were a reliable stepping stone for beginning math-talk, but eventually the level of math-talk would plateau. Teachers became skeptical and were unsure if the students were gaining the necessary knowledge to be successful on their high stakes tests. It became apparent to me, that while the mathematical conversations were taking place, the level of deep understanding and connections varied widely. Even with various interventions, many lessons ended without a connection to different strategies or to the mathematical objective.

I grappled with this phenomenon for several years, and finally discovered a breakthrough in the use of concept maps. While I did not know they were concept maps at the time, I challenged teachers in a summer math methods course to use a ratio table in different ways to explore different strands of mathematics. As the week long professional development progressed, teachers had a chance to revisit their concept map several times. Each time, they added another element to the map. These elements could include different representations, pictorial and abstract connections to geometry, algebra, and other strands, story problems, drawings of manipulatives, etc. At the conclusion of the course, teachers were astounded by all the new learning they were able to transfer to this one simple ratio table.

My interest in concept maps peaked after this experience. I often wonder how this professional development could be improved. I thought about the power of connections and wondered if the teachers drew in cross-links and explicitly made connections amongst those elements if they would see how all of the mathematics is connected. This interest supported me in understanding strategy maps. This type of concept map was
more focused on the path a teacher took to make connections amongst students’ work and the overall math concept.

My work has focused so much on these three topics: math-talk, the Five Practices, and strategy maps. I believe that there is a link to be made between the three, and I am excited to explore this link.

**Statement of the Problem**

Math-talk is becoming an increasingly significant issue in education (Hufford-Ackles et al., 2004). Stein et al., (2008) has demonstrated that math-talk has become a more significant issue in recent years, and has created the Five Practices as a roadmap for educators to increase the level of math-talk in the classroom, but the practice of making connections amongst students’ work is still unresolved. Educators are still searching for strategies and tools that aid teachers in making these connections. In order to address this need, it is necessary to investigate tools that are designed to focus teachers on making connections.

**Purpose**

This study was intended to investigate the use of ongoing professional development through virtual math coaching that utilized a strategy map, as a tool that supports the teacher to make connections amongst students’ work. In this research, I collected and analyzed the strategy maps created by the teacher. I observed and described the way the strategy map supported the teacher in identifying and explicitly delivering connections amongst students’ work during the math-talk.
Research Questions

Research (which will be described in Chapter 2) has not addressed adequately this issue of connections, and the integral role it plays in math-talk. The purpose of this investigation, then, was to investigate the use of strategy maps, employing the following research questions:

1) In what ways do strategy maps support the teacher in making connections during the math-talk?

   (a) Given diverse student methods, how do strategy maps support teachers in making connections between students’ work?

   (b) What connections between students’ work emerge during the math-talk?

   (c) In what ways do the strategy maps and the math-talk develop together throughout a semester?

2) In what ways does virtual math coaching support the teacher in utilizing strategy maps as a tool to increase the level of math-talk?
Definition of Terms

For clarity purposes, the following terms used in this dissertation are defined here.

- A concept map is a graphical tool for organizing and representing knowledge.
- A node is an abbreviated graphical representation of a student strategy that is seen in a concept map.
- A cross-link is the line that connects nodes in a concept map. Cross-links include a word or phrase below the line that describes the connections between the nodes.
- A strategy map is a specific type of concept map that is created by the teacher or observer. It is a simplistic linear diagram that identifies the students’ work in nodes, and connections in the cross-links.
- A math-talk learning community is “a classroom community in which the teacher and students use discourse to support the mathematical learning of all participants and to understand and extend one’s own thinking as well as the thinking of others in the classroom” (Hufford-Ackles, et al., 2004, p. 2).
- A reform-based curriculum is a curriculum that focuses on rich tasks and investigations.
- Professional development is the term used for the individual and group teacher learning.
- A student strategy is the artifact of students’ work that shows their thinking, usually in multiple steps, as they solve the problem.
- An anticipated student strategy is an artifact created by a teacher or that shows predicted student thinking.
- A concrete strategy is a strategy that uses manipulatives to show a clear connection between pictorial and abstract strategies.
- A pictorial strategy is a strategy that uses diagrams, or pictures to show a clear connection between concrete and abstract strategies.
- An abstract strategy is a strategy that uses algorithms, numbers, and symbols to show a clear connection between concrete and pictorial strategies.
CHAPTER TWO

Concept Maps

Novak and Cañas (2008) describe concept maps as graphical tools for organizing and representing knowledge. They include concepts, usually enclosed in circles or boxes of some type, called nodes; and relationships between concepts indicated by a cross-link, a line connecting two nodes. Traditionally, concept maps use one of few words in the nodes and cross-links, however, many variations of concept maps allow for customization (see Figure 1) (Novak & Cañas, 2008, p. 4). Williams (1998) summarizes that concept maps unpack an individual’s knowledge within a particular domain, and can be used to assess the fluency and efficiency of their knowledge.

The origins of concept maps began in 1972 at Cornell University, where Novak sought to understand children’s’ knowledge of science (Novak & Musonda, 1991). Since the birth of these visual tools, many researchers have used them in both traditional and nontraditional ways to study the development of understanding a concept.

Types of concept maps. The creation of a traditional concept map is a complex task, which can be broken down into simple, meaningful segments. Novak and Cañas (2008) say that it is important to begin with a specific domain of knowledge. Since the concept map is dependent on the subject area, this context or question must first be identified. Next, a preliminary concept map is developed using tools that make editing
easy, as updating the concept map is an integral part of the creation of the map. Post-its, whiteboard, butcher paper, and even computer software support the creator with moving the concepts around easily. This is necessary to explore, edit, and revise the hierarchical organization as the creator begins to make meaning (Novak & Cañas, 2008). These concepts are represented by enclosed figures, called nodes, which are usually circles or squares. Next, the creator brainstorms the connections between the nodes. These connections can be similarities and differences, a chronological order, a subset, etc. Novak and Cañas (2008) believe that the use of these cross-links illustrates that the creator understands the relationship between the sub-domains of the concept map. Finally, the creator reviews the map, and goes through the process of learning and changing. That is, when the creator learns new information, they make the changes on the concept map to depict those changes. This editing cycle can always be revisited; therefore, concept maps are never finished (Novak & Cañas, 2008).

A concept map, with nodes, is a powerful tool in that the creator and observers can get a glimpse of what the learner is thinking. However, if the concept map contains only nodes, the map simply shows a brainstorming session. It is only through the more challenging task of introducing cross-links, that researchers can understand how the creator views these concepts and sub concepts in relation to each other. The cross-links represent the creativity and understanding of the connections between the nodes (Novak & Cañas, 2008). These two features: the hierarchical structure and connections made my cross-links are the catalyst for creative thinking and a meaningful concept map (Novak & Cañas, 2008). This is because in making cross-links, the creator is making connections
amongst the nodes. Simply put, the more connections that the creator can make, the better the understanding of the concept (Williams, 1998). Further, in two separate studies, Cardemone (1975) and Bogden (1977) found the importance of cross-links by analyzing the impact of eliminating them from the requirements of the concept maps. In both studies, only a minority of students were successful in creating concept maps that did not contain cross-links.

When creating a concept map, the developer must choose how they will represent the connection between the nodes. This connection will be either static or dynamic. In a static concept map, you will see descriptors that describe, define, categorize and organize the information in the nodes. A dynamic relationship is concerned with co-variation where two or more nodes will affect one another. This means, that the creator would show a change in quantity, quality, or state of the other concept (Derbentseva, Safayeni, & Cañas, 2004). When both static and dynamic are connections used, the creator can show an in depth understanding of the concept being studied (Novak & Cañas, 2008).

Currently, concept maps are used in a variety of ways. Novak (1990) emphasizes using hierarchical, but this was later questioned by Ruiz-Primo and Shavelson (1996) and Hibberd, Jones, and Morris (2002) who implemented diverse variations of concept maps. Derbentseva et al. (2004) prefer a cyclic concept map, which is where concepts feed into one another in a closed loop. They argued that a concept map with a cyclic structure encourages dynamic relationships that support thinking in systems (Derbentseva et al., 2004). While a hierarchical structure still represents the majority of concept maps, Harnisch, Sato, Zheng, Yamagi, and Connell (1994) explored spider maps, chain, linear,
and network variations of concept maps. Each variation of the concept maps may have a different structure, but all have been successful because of two common elements: nodes and cross-links. With the knowledge of different types of concept maps, creators can merge several within one concept map. For example, a hierarchical map could contain an embedded cyclical element. This allows the creator to fully extend their understanding in the most appropriate graphical representation (Harnish et al., 1994).

A final element in the variety of concept maps can be found in the purpose of creating the map. When the developer is creating a concept map for their own purpose to make sense of a concept, they will often use an open map. This means that there are no rules, no restrictions, and no master map. This allows the creator full freedom to summarize their understanding, and to question possible misconceptions. The creator can leave nodes blank or include notes on cross-links to continue further research. This style of an open map is necessary for any new knowledge or invention.

Often in educational settings, teachers will employ a closed concept map. This type of map uses strict rules, word banks, or even a template. The students are often trying to make sense of a concept using a very limited number of words and connections. Often this results in very similar maps amongst the students, with a master map as a guide for assessment. The use of a closed concept map has benefits in both time investment and focus. When there is a closed set of words to be used in the creation of the concept map, the developer only needs to focus on those terms and topics. While it may not be as in depth as an open concept map could be, for a beginner or young student, this focus can lead to more success in learning the structures of concept maps while
providing the teacher with insight to the students thinking. Hybrids of open and closed concept maps can be used to both expand a concept map based on prior knowledge, and still keep the focus on specific connections, such as similarities, or classifications (Liu, 2010).

**Concept maps and the use of prior knowledge.** Novak (1991) stresses the importance of using the tool of a concept map, to replace traditional rote teaching and learning. Concept maps do not allow the student to simply respond with a right or wrong answer, and instead require the student to grapple with a concept, and consider the way in which it connects to other prior knowledge. While this may be more time consuming, it is only through the grappling with these misconceptions that the students can refine their understanding. When the learner creates a concept map, they really learn the important things from the lesser important things and the student can explain and create new understanding.

Novak and Cañas (2008) state that when concept maps are used, they require a different type of learning than rote memorization, and meaningful learning takes place. This style of learning provides organization, structure, and has an emotional commitment to integrate new knowledge with existing knowledge. It includes multiple intelligences that challenge each developer to think differently (Gardner, 1983).

Coffey, Cañas, Reichherzer, Hill, Carff, and Suri (2003) summarize the use of concept maps as a way to facilitate meaningful learning. This is because they have been shown as effective ways to represent and communicate knowledge. When the designer carefully chooses the nodes and linking words, the maps can facilitate a meaningful way
to organize thinking and summarize a subject (Cañas, Hill, Carff, Suri, Lott, Eskridge, et al., 2004). Ford, Coffey, Cañas, Andrews, and Turner (1996), Coffey, et al. (2003), and Coffey, Hoffman, Cañas, and Ford (2002) found that concept maps can be a tool used during knowledge acquisition as a way to capture and share this new knowledge.

Novak and Cañas (2008) explain the importance of using concept maps to develop the working memory. Specifically, they explain the need to build on our existing knowledge. Ausubel, Novak, & Hanesian (1978) state “The most important single factor influencing learning is what the learner already knows. Ascertain this and teach him accordingly” (p. 2). If concept maps are used in conjunction with prior knowledge, the working memory is better able to work through new concepts. Thus, more information is able to be absorbed in a single learning session.

**Benefits for the creator.** The use of concept maps have shown such promise, that several researchers have experimented with them in the form of a master map. Cardemone (1975) created a master map in a mathematics topic to support his planning of the unit. He distributed copies of the map amongst his students, but the majority found the maps unhelpful for learning the topic. Similarly, Bogden (1977) found that when a professor created the master map for his genetics class, his students were confused by the concept map. In these studies, the master maps have been found most useful for the creator in both planning and assessment purposes.

Both the Cardemone (1975) and Bogden (1977) studies show the importance of the creator in constructing concept maps. While it may seem beneficial to give a student or teacher a concept map, it is only useful if they are the ones to create it. Later, Novak
(1990) found that once students had practice developing a concept map of the topic being studied, a master concept map was more helpful. However, in this study, the students still have an opportunity to practice this process, which brings us to the same conclusion that it is the process, not the product, which is useful. These studies show that the benefits of creating a concept maps lie in the creator and their new understanding. It is essential that teachers, curriculum developers, and researchers give learners the opportunity to struggle with the concept and to create their own concept maps.

**Concept maps and curriculum.** Concept maps have been shown to be an enormously useful tool in curriculum planning. Bascones and Novak (1985), Novak (1991), and Novak and Cañas (2008) found that when using a hierarchical map, curriculum developers can find the optimal sequencing of instructional material. The concept maps also transition curriculum developers through the more general topics to the specific. When writing curriculum, planners will first create macro maps to designate major ideas, then micro maps to show the knowledge structure of each of these major ideas (Novak & Cañas, 2008). Further, concept maps are beginning to emerge in classrooms through activities featured in textbooks and supplemental curriculum. Several science texts have included them in standard activities, Novak and Cañas (2008), and they are emerging in professional development courses for teachers.

**Concept maps in this research.** Williams (1998) used concept maps as a tool for students to explain their mathematical thinking. In his research, students used concept maps to successfully show relationships amongst functions, something that does not always lend itself to simple categorization. Similarly, in this study, when a teacher
develops the concept map, creates the nodes and cross-links, and predicts the outcome of the math-talk, he/she will have an in depth understanding the overall topic and the details and sequencing needed to present this topic.

Math-Talk Learning Communities

A math-talk learning community is defined as “a classroom community in which the teacher and students use discourse to support the mathematical learning of all participants and to understand and extend one's own thinking as well as the thinking of others in the classroom” (Hufford-Ackles, et al., 2004, p. 2). This discourse provides more than an explanation of facts or rules; it is an authentic exploration of ideas, and strategies, grounded by reasoning, rules and mathematical history. Through classroom discourse, students and teachers have the opportunity to break down and debate big mathematical ideas (Chapin, O’Connor, & Anderson, 2003). Classroom discourse in the mathematics class, or math-talk, is more than simply the teacher or students talking, it is rich in conversation, debate, and connections. This type of math-talk is rare in American classrooms, and, instead, is dominated by teacher lectures. Often, the only time students are talking in class is when they are answering a low level question in order to verify that they were paying attention (Chapin et al., 2003). The Trends in International Mathematics and Science Study (TIMSS) report showed the US lacking in mathematics and science knowledge, in part because US students could not think at higher levels or consistently solve problematic situations (Gonzales, Williams, Jocelyn, Roey, Kastberg, & Brenwald, 2008).
American students are not expected to engage in conceptual thinking or complex problem solving (Stigler & Hiebert, 1999). Instead, their school work focuses on repeating problems that their teacher has already instructed them on how to solve that day (Stein & Smith, 2011). “It is unrealistic to expect students to learn to grapple with the unstructured, messy challenges of today’s world if they are forced to sit silently in rows, complete basic skills worksheets, and engage in teacher-led ‘discussions’ that consist of literal, fact-based questions and answers” (Stein & Smith, 2011, p. 1). Social interactions and debate in the classroom are a necessary skill to be learned and practiced. This skill allows children the opportunity to rely on each other as resources, and to be held accountable, socially, for their mathematical knowledge. When high quality discussions dominate the mathematics classroom, students can communicate their ideas, debate them publicly, and evaluate their own unique mathematical thinking (Stein & Smith, 2011).

The teacher has an important responsibility to become the facilitator of these math-talk classrooms, and allow students the freedom and responsibility to discuss and develop the ideas that make up the mathematical ideas that they are exploring (Stein & Smith, 2011).

Vygotsky (1978) and Lave and Wenger (1991), tell us that complex knowledge and skills are learned through social interaction. This social interaction can be achieved while learning a mathematical concept through math-talk. Chapin et al., (2003) and the National Council of Teachers of Mathematics (2000) emphasize the need for math-talk; specifically it allows students to (1) listen to peers thinking, (2) clarify their thinking by putting it into words, and (3) to make meaning of misconceptions. The authors go further to explain that discussion teaches students logic at a very early age because it challenges
them to listen, think about the new information, and decide if their argument needs to be revised. Logic is a taught skill, not something we are born with, and it is through discussion that students improve their logical reasoning. Teachers can teach logic through discussions. When a student makes a claim, the teacher can guide the class by understanding the claim and soliciting examples or counterexamples, and teaching students how to debate arguments (Chapin et al., 2003). Students learn when they are the authors of the mathematical ideas taught in the classroom (Stein & Smith, 2011).

Even on a non-mathematical scale, math-talk increases the students’ confidence; which in turn makes them better at proving ideas and debating other representations. From a Vygotskian (1978) viewpoint, learning happens when support is provided for a student when they have had time to grapple with a problem. When the student struggles with a problem and discusses those challenges with other students in order to collaborate to find the solution, the student gains confidence and maturity, which lie in the zone of proximal development (Hufford-Ackles et al., 2004).

Hufford-Ackles et al. (2004) describe three themes of math-talk, each of which has four components and four levels of mastery which grow over time; (a) questioning, (b) explaining math thinking, (c) source of mathematical ideas, and (d) responsibility for learning (Hufford-Ackles et al., 2004).

In this model, the teacher, student, and community all play a part in the math-talk. While participants may be of varying levels, (for example, the teacher could be questioning at a high level, but many students may be answering on a low level, and simply trying to give the right answer) it is important to remember that this is a
continuum on which the math-talk community grows and communication increases. As a math-talk learning community grows, teachers and students progress through the four levels focusing on these four distinct components.

(a) Questioning – Shift from teacher as questioner to students and teacher as questioners (Hufford-Ackles et al., 2004, Appendix I).

(b) Explaining Mathematical Thinking – Students increasingly explain and articulate their math ideas (Hufford-Ackles et al., 2004, Appendix I).

(c) Source of Mathematical Ideas – Shift from teacher as the source of all math ideas to students’ ideas also influencing direction of lesson (Hufford-Ackles et al., 2004, Appendix I).

(d) Responsibility for Learning – Students increasingly takes responsibility for learning and evaluation of other sand self. Math sense becomes the criterion for evaluation (Hufford-Ackles et al., 2004, Appendix I).

Further, the math-talk can be analyzed using four distinct levels:

(0) Level 0: Traditional teacher-directed classroom with brief answer responses from students (Hufford-Ackles et al., 2004, Appendix I).

(1) Level 1: Teacher beginning to pursue student mathematical thinking. Teacher plays central role in the math-talk community (Hufford-Ackles et al., 2004, Appendix I).

(2) Level 2: Teacher modeling and helping students build new roles. Some co-teaching and co-learning begins as student-to-student talk increases.
Teacher physically begins to move to side or back of the room (Hufford-Ackles et al., 2004, Appendix I).

(3) Level 3: Teacher as co-teacher and co-learner. Teacher monitors all that occurs, still fully engaged. The teacher is ready to assist, but now in more peripheral and monitoring role (Hufford-Ackles et al., 2004, Appendix I).

Math-talk is a very thoughtful classroom behavior. When planning for math-talk, it is vital that the teacher plan their lesson carefully; paying special attention to the prompts they give students. Teachers must consider the questions they ask, and the level of answer they are eliciting. They need to consider the level of the students, and the experience that students have with math talk. It is easy to have a discussion without purpose, and the result is often difficult to manage. For talk to be productive, the teacher must plan carefully within the content of the math lesson (Chapin et al., 2003).

Selecting rich tasks. The challenges of setting up a math-talk community do not stop at thoughtful questioning; it also relies on problem or task selection. Ball (1991) found that selecting a rich task is an essential step towards allowing students and opportunity for true problem solving situation. Through her research on Lesson Study, Lewis (2002) found that when students struggle with a rich task and have opportunity to learn about conceptual ideas rather than rote learning, that there is more opportunity for mathematical discourse. However, this discourse must be nurtured with a teacher facilitator who supports a high level of thinking. “Opportunities for student learning are not created simply by putting students into groups, by placing manipulatives in front of them, or by handing them a calculator” (Stein, Smith, Henningsen, & Silver, 2000, p. 1).
Therefore, it is vital that teachers set up engaging tasks that include these items, but is rich enough to really pull the mathematics. The purpose of a task is to develop the ideas of the students (Stein et al., 2000). Pólya summarizes the role as the teacher as a balancing act. “The teacher should help, but not too much and not too little, so that the student shall have a reasonable share of the work” (Pólya, 1957, p.1). There are several mainstream curriculums published by leading publishers that contain investigations and tasks that are rich enough to engage in successful math-talk on a regular basis. Many of these curriculums sometimes referred to as reform based curriculums and funded through the National Science Foundation (NSF), have their roots based in research, the goals of NCTM, and focus on math-talk as an essential element to the learning. In order for one of these reform based curriculum to be considered “exemplary”, a unit or comprehensive curriculum must be consistent with the NCTM Standards, designed by groups of specialists in mathematical content and pedagogy, and revised based on field tests in various instructional settings (Borasi & Fonzi, 2002).

According to the NCTM (2000), rich task investigations should occur frequently in the mathematics classroom. Further, Pólya (1957) explains that the two most important roles of a teacher are to support the student in solving the problem and then develop his or her ability to solve future problem by him or herself. It is vital that educators offer many opportunities for students’ to problem solve and become engaged in multiple representations. The number of rich tasks offered in the classroom is directly related to the amount of logical reasoning displayed by a student. The type of task selected for discussion is the basis for the opportunity for learning during the discussion. It will
determine if the discussion will focus on complex thinking and reasoning, or
memorization and procedures (Stein et al., 2000). The authors break these tasks down
into four levels with increasing complexity: (a) memorization and (b) procedures without
connections are examples of low level skills. (c) Procedures with connections and (d)
doing the math are examples of high level skills (Stein et al., 2000).

While the number of problems completed in a class period could vary even within
one of these levels, the importance should be placed on the mathematics that the students
are discovering. Specifically, the new learning that is either told to them or explored on
their own, as mathematicians did years ago. Therefore, it is vital that the teacher plan
critically. Stein et al. (2000) explain that planning tasks evolve during a classroom lesson,
and before the lesson begins. During the beginning planning, the teacher may create a
table of misconceptions or off topic ideas that they expect students to discover. This
planning step can increase the success of clear delivery of the mathematics as the students
are summarizing.

It is vital that tasks are selected or created to match learner goals. If the goal is for
students to justify or explain the task should be rich enough to ensure that the student has
the opportunities to do so (Stein et al., 2000). Further, if the goal is for students to
increase speed and fluency, simpler tasks may be sufficient. There should be a well-
constructed balance of these different tasks. Focusing exclusively on any one type can
lead to a limited or disconnected understanding of mathematics (Stein et al., 2000). The
level of cognitive demand can be increased with discourse, multiple steps and use of
manipulatives. However, it requires the analyzer to go beyond superficial features such
as requiring the use of any of these supports, simply for the purpose of using those (Stein et al., 2000). The planning must ensure that the use of these devices is to increase the level of cognitive demand because it supports the learner in understanding. In short, any task can be examined by their cognitive demands, and these cognitive demands should be considered when choosing the task.

When analyzing mathematical tasks for the purpose of discussion, there are a few different lenses that must be observed. They can be examined in terms of variety of perspectives, representations, solutions, and ways of communication (Stein et al., 2000).

Given the need for rich tasks, many schools have adopted reform-based curriculums. Several reform-based curriculums have been granted through the NSF, and have been reviewed with scrutiny to ensure the opportunity for a higher level of thinking. These curriculums focus more on procedures with connections and doing mathematics than rote memorization skills. This provides teachers with ample tasks to choose from, in order to fit the needs of both the students and the trajectory of the curriculum. Even many traditional curriculums show value for rich tasks, and have these listed in additional materials, or in a unit project. While publishers and curriculum developers understand the need for rich tasks, it still requires teachers, and students to learn how to expand each other’s ideas to make the connections needed to see the larger mathematical concept. This is an area of professional development that is growing.

**Challenges of math-talk.** While the teacher or facilitator may agree that math-talk communities are an essential element in their classroom, there are some frequent frustrations and challenges. When students are the speakers, and their strategies are the
topic of discussion, teachers are introduced with an element of uncertainty (Chapin et al., 2003). This uncertainty should be addressed in the initial planning stages. The teacher, or group of teachers, should brainstorm examples of uncertainties or misconceptions they expect to observe. This allows the teacher to anticipate student reactions to the task, and to make sure the task is aligned to both the mathematical goal as well as the level of cognitive demand, allowing the students more opportunities for success. When math-talk is first explored, some students may disengage, and become discouraged with the introduction of more challenging tasks (Hufford-Ackles et al., 2004). In these cases, the planning session should pay special attention to the goals and tasks that the students are required to complete. The reason for the disengagement is either in the level of expectation of discourse and the classroom climate established, or the task given to the students. The teacher should empathetically understand the student, and ask questions that support the student in gaining confidence as well as explaining his or her mathematical thinking (Pólya, 1957).

Once a teacher becomes a facilitator, and relinquishes his or her classroom to the students so that they can discuss their ideas, he or she may find that the discussion is dominated with incorrect mathematics. The misconceptions are difficult to predict, and thus difficult to manage (Hufford-Ackles et al., 2004). Due to the nature of this anticipating planning stage, there is an opportunity to initiate group planning or coaching. In these planning sessions, more brainstorming should be applied to discover ahead of time the misconceptions that students will have, and the plan of action to route these students back to the correct mathematics.
When introducing math-talk to teachers, these challenges may seem daunting. It requires thoughtful planning, anticipating, and a level of adaptive teaching that may appear effortless when executed by an experienced teacher facilitator. To address these challenges, and to give teachers, new or veteran, support in learning to create a math-talk learning community, Stein and Smith (2011) created the Five Practices. The practices are manageable chunks that can be learned separately or together to develop the skill of facilitating a math-talk learning community.

**The Five Practices**

Stein and Smith (2011) created the Five Practices as a framework for planning that supports teachers’ in facilitating a math-talk learning community. These practices assist the teacher by providing some control within the uncertainty of student lead discussions. There are more requirements during the planning of the lesson, which anticipate instructional decisions made during the lesson, so that the teacher can predict certain outcomes and challenges ahead of time (Stein & Smith, 2011). The Five Practices are-

1. *Anticipating* likely student responses to challenging mathematical tasks;
2. *Monitoring* students’ actual responses to the tasks (while students work on the task in pairs or small groups);
3. *Selecting* particular students to present their mathematical work during the whole-class discussion;
4. *Sequencing* the student responses that will be displayed in a specific order; and
5. **Connecting** different students’ responses and connecting the responses to key mathematical ideas (Stein & Smith, 2011, p. 8).

Of the Five Practices, the most overlooked and important is connections. Traditionally, student presentations involve several student explaining their strategy and solution, without relating it to any other presentation. The goal in the connections practice is that students make connections amongst each other’s strategies, in order to develop powerful mathematical ideas (Stein & Smith, 2011). Not only is it the most important, it is also the most challenging. This is because teachers must craft questions that give the student the opportunity to explain their thinking. Then, those questions must go beyond clarifying, and must focus on making meaning of the different representations (Stein & Smith, 2011). Boaler and Humphreys (2005) say that the teacher’s questions need to challenge students and focus them to fully consider a specific mathematical concept. Questions must begin with what students know, and connect with their current way of thinking (Stein & Smith, 2011). “Without specific questions that make the connections, the lesson would become a show and tell and the key ideas would be lost” (Stein & Smith, 2011, p. 50).

There is a need for more professional development in this area of making connections and forming questions. One tool that can be used to address this is a concept map.

**Coaching to Support Teacher Math Knowledge**

In this research, I explored strategy maps to determine if they could be used as a tool to support teachers in making connections between students’ work, and therefore
increasing the level of math-talk in the classroom. To do this, I provided professional development to increase the teacher’s knowledge and coached the teacher as she was attempting to use concept maps. Teacher math knowledge played an integral part in this research, and there is not currently a widely used tool to assess the pedagogical practices of teacher math knowledge. While there are ample exams that assess content knowledge of the teacher, the participant in this research had various levels pedagogical content knowledge (Schulman, 1986), and various levels of adaptive teaching strategies (Corno, 2008). This knowledge encompassed “everything a teacher might know in teaching a particular topic, obscuring distinctions between teacher actions, reasoning, beliefs, and knowledge” (Ball, et al., 2008, p.394), and this knowledge is still being understood. In mathematics, pedagogical content knowledge and adaptive teaching strategies support the teacher with skills to understand different representations of a problem, different algorithms, and different ways of knowing (Corno, 2008; West & Staub, 2003). A teacher with these skills is more likely to remain flexible in the math-talk, while keeping focused on the overall goal of the lesson (Smith & Stein, 1998).

In order to increase a teacher’s knowledge, and to further his/her profession, teachers traditionally practiced annual professional development by attending a short workshop that focused on a specific objective, and would be complete once the workshop was over. Ball and Cohen (1999) found that such one-stop workshops were ineffective. Therefore, many school districts began supplementing or replacing these workshops with a coaching model (Campbell & Malkus, 2011). Coaching roles can be complex, and include a variety of roles (Campbell & Malkus, 2011). West and Staub (2003) use a
continuous model of professional development that is ongoing and collaborative, meaning that the coach and teacher work together during many instances throughout the school year to improve a teacher learning objective. Together, they plan, teach, and reflect on the lessons.

**Roles.** Before coaching begins, it is important to consider the many roles of coaches, and determine which roles optimize the goal of the coaching program. Too many coaching programs have been launched without looking at these roles and the balance between them (Knight, 2009). Knight (2009) further describes ten roles of coaches: data coach, resource provider, mentor, curriculum specialist, instructional specialist, classroom supporter, learning facilitator, school leader, catalyst for change, learner. While a coach may act in several of these roles, not all roles need to be balanced equally. That is why it is so important for the goal of the program to be clear. The coach will be more effective when they know how to balance their duties across these roles (Knight, 2009).

**Coaching skills.** The coach and teacher relationship is complex. Coaches must hone their skills in order to build a strong relationship based on trust and growth. Van Nieuwerburgh (2014) suggests three skills that the coach should practice and refine in order to be effective while building this relationship with the teacher. These skills are listening, questioning, and paraphrasing and summarizing. He recommends that coaches speak only 10-20% of the conversation, and spend the remainder of the time doing active listening. This involves listening curiously and asking open ended questions that invite the teacher to reflect in a thoughtful manner. By asking questions, the coach allows the
teacher the opportunity to reflect and advocate for his or her own personal growth. Further, by allowing the teacher to direct his or her growth, the coach does not fall into the common trap of giving advice too early. The final skill, paraphrasing and summarizing, require the coach to determine how to give feedback, and the timely manner in which it is given. This skill is related to the relationship between the teacher and coach. A mutually trusting relationship supports a safe space for feedback without the teacher becoming defensive. The feedback should be clear, objective, and specific. Positive feedback is also important as it can raise the teacher’s self-esteem and morale. However, in an effort to be positive, the feedback should never be diluted or vague (Van Nieuwerburgh, 2014). West and Cameron (2013) recommend rehearsing how the coach relays the feedback, especially when the feedback is a suggestion for improvement. Rehearsing these crucial conversations allows the coach practice word selection, tone, and body language (West & Cameron, 2013).

**Coaching heavy vs. coaching light.** There are a variety of ways in which coaches support teachers. Knight (2009) explored coaching heavy vs. coaching light in terms of effectiveness in improving teaching and learning. Coaching light refers to the style of coaching that values building relationships more than improving teaching and learning. In this style, the coach tries to integrate themselves with staff, and often do tasks for teachers that do not impact teaching and learning, but instead result in the coaches being accepted and liked by their peers. They avoid challenging conversations, and the feedback is usually vague and revolves around the behavior, rather than student learning.
Coaches who choose to coach heavy on the other hand, are committed to improving teaching and learning, even at the risk of not being liked. This type of coaching focuses on producing results rather than the teacher feeling supported (Killion, 2010). They coach all of the teachers in the building, and not simply volunteers. They do not support the teacher with trivial requests. They use data to determine how to coach planning, instruction, and assessment, and maintain high stakes interactions between the teachers. Coaching heavy has a “laser like focus” because the coach focuses all of his or her energy on a small space (Knight, 2009). Knight (2009) further explains that coaches will use a mixture of coaching heavy and coaching light. Early in the school year, the coach may implement coaching light as a way to build relationships. This is especially important if the coach is new, or there is a new teacher. However, once the relationship is established, which should only be a few months, coaching heavy needs to be implemented if the coach intends to improve teaching and learning.

**Content coaching.** When the goal of the coach is to improve teaching and learning by examining the content being taught, content coaching can be implemented. This style of coaching is similar to other coaching models because the coach focuses on instructional strategies and cultivates relationships among teachers. However, the conversations that take place revolve around conceptual content knowledge, and the way in which students access it (West & Cameron, 2013). When coaching is content focused, the coach is very knowledgeable in the content as well as knowledge of current learning theories and has a repertoire of instructional strategies aligned with those theories (Knight, 2009; West & Staub, 2003). Further, the content coach may support the teacher
with general teaching practices, but also has a focused objective to increase the level of content knowledge that students are obtaining (West & Staub, 2003). This means that the coach focuses on big ideas, structures, and/or essential questions relevant to their topic (West & Cameron, 2013). This is achieved by planning rich lessons that allow students opportunities to grapple with problems using reasoning and discourse. This coaching will support the teacher to cultivate those habits in their students (Knight, 2009).

Content coaching is specific to each teacher’s needs. West and Cameron (2013) suggests using the question, “What are the developmental needs of this teacher?” in order to determine the specific goals for that teacher. Once the goal is determined, the next step is to determine additional sub goals that can be achieved throughout the coaching process (West & Cameron, 2013). These sub goals provide a guide for the coach, so that he or she can remain focused on the specific goal, and not become distracted with improving other aspects of the teaching and learning.

**Virtual coaching.** Virtual coaching uses a variety of tools that can be used by the teacher and coach to collaborate together virtually, meaning that they do not meet face-to-face, and instead, interact using a computer or other digital device (Rock, Schoenfeld, Zigmond, Gable, Gredd, Ploessl, & Salter, 2013). The most widely used tool and method of virtual coaching is called “bug in ear” (Rock, Zigmond, Gregg, & Gable, 2011). This method of virtual coaching gives immediate feedback to the teacher while he or she is teaching the class. It is achieved by using real time recording tools. Typically, a teacher uses a camera to record her class, and an earpiece with a microphone to record what the teacher voice. This audio and visual information is sent to the coach, who can view it
remotely on a computer or other digital device. The coach responds back to the teacher, in real time, using a microphone. The coach’s comments are heard by the teacher through the earpiece (Rock, et al., 2011).

There are several benefits and drawbacks of using the bug in ear method. Teachers’ benefit from the immediate feedback, and enables them to make better decisions, revise a lesson, and learn about their teaching (Scheeler, McAfee, & Ruhl, 2004). Additionally, this model is less expensive financially because the coach does not need to travel, to and from the classrooms being observed. This travel time can be cumbersome in large divisions or urban settings with an abundance of traffic (Rock, et al., 2011). Further, the authors advise that this method should be handled with care to avoid the drawbacks related to building relationships. It is the coach’s role to support the teacher, rather than acting as Big Brother (Rock, et al., 2011). In my prior experience as a mathematics coach, I saw first-hand the impact that the bug in ear virtual coaching method had on four teachers. The virtual coach did not have a relationship with the teachers, and they expressed that they felt defensive with the coach’s feedback.

These benefits and drawbacks were considered when determining the coaching method for this research. I explored other tools that would allow the teacher to record herself and reflect back on her teaching as she watched her video at a different time. Charteris and Smardon (2013) found that teachers were more accepting of coaching suggestions when they were able to “see themselves” on video. Video recordings can be an objective way to provide this feedback because it is difficult for the teacher to reject what they are watching (Van Nieuwerburgh, 2014). The coaching became more about
the particular instance, and did not generalize the teacher’s overall teaching, but the way that she could improve the instance (Charteris & Smardon, 2013). This practice has been used for decades by expert athletes who are coached to improve a precise moment in their game (Finch, 2011). I explored two web based tools: Edthena and www.beasmartercookie.com. Both of these tools allowed the teacher to upload video, save it virtually to a cloud, and invite the researcher to view it. The site, www.beasmartercookie.com had the additional benefit of ease of use on an iPad device.

Summary

Research addressed the topics of concept maps, math-talk, the Five Practices, and coaching. Each of these topics has been effective in teacher professional development. In this project, I explored how each of these elements plays a role in a teacher’s development of connections in middle school mathematics.
CHAPTER THREE

Research Design

The purpose of this study was to explore how a graphic organizer, called a strategy map, could be used to support a teacher as she implemented the Five Practices in order to improve math-talk. This chapter describes the methodology used in this research. It provides detail about the participant, rationale for the case study approach, and procedures used to explore, collect and analyze the data. The following research questions guided this study.

1) In what ways do strategy maps support the teacher in making connections during the math-talk?

(a) Given diverse student methods, how do strategy maps support teachers in making connections between students’ work?

(b) What connections between students’ works emerge during the math-talk?

(c) In what ways do the strategy maps and the math-talk develop together throughout a semester?

2) In what ways does virtual math coaching support the teacher in utilizing strategy maps as a tool to increase the level of math-talk?
I used a phenomenological case study approach to focus on the learning trajectory of a teacher as she developed her math-talk goals. I explored the relationship between the teacher, the strategy maps that she created, and me, the participant researcher and my role in virtual coaching to determine how this relationship impacted the connections made during the math-talk. This project took place during a semester period during regular school hours in a private elementary/middle school. The semester long study examined the use of ongoing professional development through virtual math coaching that utilized a strategy map as a tool to support a teacher using the Five Practices to increase the level of math-talk in her class. The teacher who participated in the project taught her regularly scheduled classes and used planning time as she would without being involved in a study. For this study, the teacher increased her knowledge of current math-education trends in the following ways. She took a graduate level course that focused on the Five Practices, and how to incorporate these practices into teaching. Prior to the semester, she also had an introductory professional development session based on math-talk, the Five Practices, and how to use strategy maps. She engaged in a rich semester long planning, teaching, and reflecting process that focuses on honing the skill of math-talk. The planning and execution of math-talk also required the teacher to incorporate the Five Practices and develop a strategy map for each lesson. In addition, after each session, the teacher was virtually coached and supported in terms of how to better use the skill of developing strategy maps to be used for math-talk.
In the analysis, I show the journey of this teacher and the opportunities that arise to her because of the implementation of the virtual coaching on math-talk, the Five Practices, and strategy maps.

**Phenomenological Case Study**

A phenomenological case study approach was used in this research. Case studies explore the entire phenomenon taking place (Merriam, 2009). Yin (2009) further explains that case studies are “an empirical inquiry that investigates a contemporary phenomenon in depth and within its real-life context, especially when the boundaries between phenomenon and context are not clearly evident” (Yin, 2009, p. 18). Therefore, using this approach best illustrated the entire journey that this teacher used when exploring math-talk.

Many researchers have explored math-talk, and the factors that could influence math-talk are numerous. At the onset of the design phase, I determined to bound this case study by only including the role of the mathematics teacher, the strategy maps that she creates, and me, the participant researcher and my role in virtual coaching. This study did not explore other variables such as the students, other subject areas, achievement, or additional graphical tools similar to strategy maps. I gathered data through videos of the lesson, virtual coaching emails, her strategy maps, and researcher notes to determine the relationship between the teacher, strategy maps, and participant researcher and how this relationship influences connections during math-talk.

Prior to the study, I developed theoretical propositions of the teacher and implementation of math-talk to guide the data collection and analysis. There were
several important attributes of the teacher and implementation of math-talk that could be best explored using a case study. During the analysis, I determined that, indeed, there were unique teacher values that influenced her success. In order to fully explore these teacher values, and explain this teacher’s journey, I needed to analyze her teacher values in detail. For this reason, a case study was chosen to explain the phenomenon unfolding as she explored math-talk.

**Participants and Setting**

**Participant researcher.** I, the researcher in this study, am a participant and hold unique qualifications and bias. My previous employments include math teacher, math coach, and technology coach. In these roles, I have explored math-talk both in my own classroom and in the classes that I co-taught as a math coach. I struggled with the concept of math-talk for years. In my own classroom, the major dilemma was not providing enough summarization time at the end of the class to fully participate in a rich discussion. As a math-coach, it was easier to identify this area of improvement in other classes that I was observing, but still difficult to implement even with the revelation. I discovered the Five Practices while I was a math coach, and found it to be very useful. I learned that my frustrations were in the planning and implementation of math-talk. Some classes, I planned for up to twenty minutes of discussion, but there was nothing interesting to discuss. In other classes, I didn’t plan enough time, and we failed to summarize the objective of the lesson. The Five Practices gave a guide for planning that supported the teacher with anticipating the discussion. The co-teacher’s and I were more successful in implementing math-talk after I practiced the Five Practices. There was still
an area to be improved, and that was in making connections. The Five Practices were instrumental in giving us teachers the opportunity to predict the math-talk, but as students’ presented their work, the presentation was very similar to the show and tell that the authors of the Five Practices (Stein & Smith, 2011) reference. When the teacher and I would debrief the lesson, we frequently regretted that we didn’t make the connections more explicit. I wanted to find a tool that could focus us on making these connections during the discussion.

In addition to my full time employment, I also worked as an adjunct professor teaching mathematics methods courses for pre-service teachers and math specialists. Part of the curriculum for these courses was to create a concept map about rational numbers and proportional reasoning. The purpose of this was to show, mostly middle school teachers, that rational numbers and proportional reasoning exist in all strands of the curriculum, and not only the algebraic strand. In this early work on concept maps, I discovered the strengths of visual organizers. I explored with the idea of using strategy maps when I taught the graduate level course, which was a prerequisite for the participant of this research. Each class devoted an hour to solving a rich task, and presenting the findings using math-talk. I used the Five Practices, in a similar way as the participant in this study, to anticipate the math-talk with these adult math specialists. During the monitoring stage, I completed the strategy map, including the connections that I wanted to make explicit. After the math-talk, I disclosed this strategy map to the math specialists so that they could see the behind the scenes actions that took place. I did not know at the time that my participant would be in this course. I simply wanted to explore the
procedures that I would ask the participant to use, to anticipate any questions and misconceptions.

**Bias.** My history in mathematics education, as explained above, and strong desire to support teachers with a tangible way to make connections during math-talk, poses a bias in this research. To guard against this validity threat, I used several data sources to analyze and describe this case. In addition to these multiple data sources, I described the participant-researcher role in analyzing the data. This purpose is not to eliminate variance between me and the study, but rather to show how my values and beliefs influence the study (Maxwell, 2005).

**Recruitment.** The participant for this study was selected from students who completed a graduate level course that was based on incorporating the Five Practices in their math class. I was the instructor of this course; however, I did not intend to recruit from the students taking the course until the course was complete. At this time, I emailed the class expressing that I would be interested in participants for this study, and to contact me if they wanted to participate. Five of these teachers contacted me, and one teacher was available to complete all of the requirements needed during the selected time frame. This teacher was chosen because she was successful in the course, was eager to continue work in this area as a part of her annual teaching goal, and had the necessary time available to fulfill the requirements of the study. I contacted this teacher, via email, to ask her if she would like to participate in this study. Her eager response expedited the recruitment process.
**Teacher and school.** The participant in this study was a middle school mathematics teacher. She taught in a private religious school that is located less than an hour from the Washington DC Metro area. The student demographics include 20% African American, 6% Hispanic, 60% Caucasian, 12% Asian and 2% other, with 52% male and 48% female. The average class size was approximately 20 students. Students at this school were placed in their mathematics class through data obtained by standardized tests, and teacher recommendations. A vignette of the teacher is listed below. A pseudonym was created for this study to protect the privacy of the participant.

Cathleen chose teaching as a second career, after a successful career as an engineer. She had taught in private schools for two years before this study took place. She valued discipline, and an organized class, but believed that it was important for students to talk, develop ideas together, and derive mathematics in their own way, because that is how she saw successful engineers in her previous career. She didn’t think that students received enough time to talk about mathematics, and believed that they should have had these skills prior to middle school. She was ambitious to learn how to support her students in developing the skill of talking like an engineer, but was often frustrated with the chaos that seems to follow. She enjoyed learning the newest trends in math-education because she found them to be aligned with her goals as a math educator. When she took the graduate level course, she explained that her personal goal was to start thinking about math using pictures and diagrams, rather than with the “mathy” (abstract) algorithms. She thought that since engineers see problems in different ways, and they develop different solutions to those problems, that it was an important
skill to learn how to derive a solution without the teacher’s help. She also thought that students should learn to listen to each other, comment respectfully, and learn to let go of their strategy, if another presented strategy makes more sense. She wanted her students to think like engineers.

Materials

The teacher selected for this study was using a variety of curriculum materials. These include a traditional text, which she rarely used, a teacher’s copy of a reform-based curriculum called Connected Mathematics, and a variety of online resources. The purpose of using a reform-based curriculum was that it is necessary for the students to solve a rich task in order to create the opportunity for high levels of math-talk. In addition to the regular curriculum, a database of rich tasks was available for the teacher. This database of rich tasks provided real world problems that have been field tested to ensure that they had multiple points of accessibility. That is, it could be solved by a variety of grade levels, and could be solved using a variety of strategies and representations. “Rich tasks need to extend beyond computational problems and should enable students to formulate an understanding of mathematical concepts that integrates both conceptual and procedural knowledge” (Billings & Klanderman, 2010, p. 440). The daily curriculum and database of rich tasks were the majority of materials. In addition, post-it notes, graph paper, chart paper, markers, and other typical office supplies aided the teacher in planning and facilitating math-talk using strategy maps.
Procedures

The teacher engaged in multiple teacher education experiences that focused on strategy maps, math-talk, and the Five Practices. Through a graduate level course, professional development session, and semester long virtual coaching, she honed her skill of facilitating math-talk.

Graduate level course. The participant in this study had already completed a graduate level mathematics methods course that regularly presented the Five Practices of math-talk: anticipating, monitoring, selecting, sequencing, and connecting. I was the instructor of this course; however at the time I did not know that a student in this course would be the participant of this study. The course focused on non-traditional strategies of teaching and learning rational numbers, and each session began with a one hour problem solving experience using the Five Practices to present the solutions. A total of eight sessions included eight problems that were facilitated using the Five Practices. Further, I was transparent in terms of explaining the rationale for each part of the presentation. For example, I made a strategy map during the monitoring phase, selected and sequenced the presentation groups, and focused on the groups making connections during the presentation. I explained each of these steps to the students once the presentations were complete. Students participated in discussions after each of these problems to discuss the importance of the Five Practices. Further, students completed assignments and a Lesson Study that focused on them explaining and reflecting on their choice in each of the Five Practices.
Professional learning session: Strategy maps and math-talk. I administered this professional learning session prior to any data collection. In this session, the teacher learned the mechanics of strategy maps and the role they play in bridging the gap in the connections practice of the Five Practices. This consisted of a brief presentation of math-talk, the mechanics and uses of strategy maps, and how to use nodes and cross-links to develop questions that make connections between students’ work samples. An example of the strategy map can be seen in Figure 2.

![Strategy Map Example](image)

**Figure 2.** Example of a Strategy Map

The teacher engaged in a rich task and practiced creating a strategy map. Emphasis was placed on creating cross-links that focus on connections such as similarities and differences among students’ work, and big mathematical ideas (Charles, 2005). A short assessment of how the teacher used strategy maps was evidence of understanding (Saphier, Haley-Speca, & Gower, 2008). The lesson plan for this can be found in the appendix (See Appendix H).
Virtual coaching. Several options were considered when determining the method for virtual coaching. The most popular method, called “bug in ear”, uses a Bluetooth earpiece and microphone, along with a video camera to record the classroom and teacher voice. The coach watches the class remotely, and gives real time feedback to the teacher through her earpiece (Rock, et al., 2011). This option was not chosen because I wanted to be able to review the teacher’s video, code and analyze the session before giving feedback. Further, I wanted the teacher to be able to view herself at critical instances so that the feedback was relevant to the instance, and not her general teaching practices (Charteris & Smardon, 2013; Finch, 2011). The virtual coaching method done in this study used a cloud based online video uploading website where the teacher uploaded her video, and made it available for me, the virtual coach, to view. The coaching was implemented via email conversations. This allowed me to focus on word selection and eliminated the need to rehearse crucial conversations, and the way in which tone, and body language are used (West & Cameron, 2013).

As the primary researcher, I provided ongoing learning experiences, for the teacher, during virtual coaching held after each viewed session. After each class session, the teacher submitted her lesson to an online teaching website, www.beasmartercookie.com. I watched the videos, made notes, and gave feedback through email. We responded back and forth, through email, to discuss questions, strategies, and suggestions. This support addressed the mechanics of developing strategy maps, selection of students’ work and sequencing, connecting ideas to the big math concept, choice of phrases used in cross-links, questions developed based on the cross-
links within the strategy map, and specific ways to increase the level of math-talk. In order to stay focused with the support that I provided, I developed coaching topics. These topics were selected because of their accessible nature; meaning that they can be explored by educators who have various levels of experience and adaptive practices.

**Coaching topic 1: Strategy maps.** This topic addressed how the teacher used strategy maps. Specifically, how the teacher was able to create a strategy map that clearly indicated a node identifier and the chronological order of students’ work along with cross-links. Cross-links included a word, phrase, or statement that linked students’ work.

I supported the teacher with virtual coaching and one face-to-face learning session, by explicitly identifying when she included these elements of a strategy map or how to incorporate missing components of the strategy map, until they were observed regularly and mastered.

**Coaching topic 2: Similarities.** This topic addressed how the teacher focused on the use of similarities and differences in students’ work to make connections explicit during the math-talk.

I supported the teacher with virtual coaching by explicitly stating that the focus was on similarities and differences. After each session, I analyzed the video and strategy maps in terms of similarities and differences between students’ work. I also supplied phrases that she could use to probe student thinking towards making similarities and differences. I supported her with coaching topic 2 until it was regularly observed and mastered.
Coaching topic 3: Representations. This topic addressed how multiple representations (concrete, pictorial, and abstract) were used to make connections between students’ work. The teacher used the terms concrete, pictorial, and abstract in order to develop probing questions to make connections during the math-talk.

I supported the teacher with virtual coaching by explicitly stating that the focus is on identifying concrete, pictorial, and abstract examples. After each session, I analyzed the video and strategy maps in terms of concrete, pictorial, and abstract examples in students’ work. This coaching topic was a focus on for the entire length of the study.

Coaching topic 4: Examples. This topic addressed how the teacher identified examples and non-examples in students’ work and explicitly stating those in the math-talk.

I supported the teacher with virtual coaching for this time period by explicitly stating that the focus was on examples and non-examples. After each session, I analyzed the strategy maps and video in terms of examples and non-examples between the students’ work. I supplied the teacher with prompts that would probe students to discuss their misconceptions and think about examples and non-examples. This coaching topic was only introduced mid-semester, when other coaching topics had been mastered.

Coaching topic 5: Efficiency. This topic addressed how the teacher identified various levels of efficiency in students’ work. She used the term efficient in order to develop probing questions to make connections during math-talk.

I supported the teacher with virtual coaching by explicitly stating that the focus was on various levels of efficiency. After each session, I analyzed the strategy maps and
video in terms of various levels of efficiency between students’ work. I supplied the teacher with prompts that she could use to probe students into discussing their reason for deriving their strategy, in terms of efficiency.

Once all five coaching topics were presented and practiced, I continued to support the teacher in using each coaching topic regularly.

**Timeline.** A timeline of important dates can be seen in Table 1.

Table 1

<table>
<thead>
<tr>
<th>Date</th>
<th>Action Complete</th>
</tr>
</thead>
<tbody>
<tr>
<td>April/May/June</td>
<td>Candidates completed graduate level course</td>
</tr>
<tr>
<td>July</td>
<td>Identify participant and obtain informed consent from the teacher</td>
</tr>
<tr>
<td>September</td>
<td>HSRB Approval</td>
</tr>
<tr>
<td>September 23</td>
<td>Conduct Initial Professional Development</td>
</tr>
<tr>
<td>June 7</td>
<td>Observation 1 - Avocado Problem</td>
</tr>
<tr>
<td>October 19</td>
<td>Observation 2 - Bag of Marbles Problem</td>
</tr>
<tr>
<td>October 24</td>
<td>Observation 3 - Chicken and Sheep Problem - 6th Grade</td>
</tr>
<tr>
<td>October 29</td>
<td>Observation 4 - Chicken and Sheep Problem - 7th Grade</td>
</tr>
<tr>
<td>November 24</td>
<td>Observation 5 - Orange Juice Problem</td>
</tr>
<tr>
<td>December 4</td>
<td>Observation 6 - Grapple Juice Problem - 7th Grade</td>
</tr>
<tr>
<td>December 9</td>
<td>Observation 7 - Grapple Juice Problem - Algebra Class</td>
</tr>
</tbody>
</table>

**Teacher responsibilities.** The teacher continued her normal class and planning routine, with some modifications. This study required her to implement the Five Practices. This meant that during her planning period, she needed to verify that the problem in the curriculum is rich enough to provide an opportunity for problem solving and math-talk. Then she needed to anticipate students’ work by doing the problem in
different ways. During the lesson, she incorporated the other practices; monitoring, sequencing, and making connections.

The teacher also created a strategy map for each lesson. This was done during the lesson as part of the monitoring practice. That is, when students were solving the problem, her responsibilities included monitoring the students, identifying the mathematics behind each strategy that they were using, answering questions, supporting misconceptions, and additionally, completing a strategy map to be used for the presentation. The strategy map showed the student strategies (seen in the nodes), and a logical sequence with connections (see in the cross-links) (see Figure 2). This strategy map is authentic because it uses the strategies that are actually created by the students. After the lesson, I virtually coached the teacher to discuss the session, the coaching topics, math-talk, and the Five Practices.

**Data Sources**

Several sources of data were collected during this project including videos of the classes, virtual coaching emails, strategy maps, and researcher notes. The sources were used to triangulate the data collected for this project. The data sources were used to answer the research questions in the following ways:

**Videos.** All classes were recorded and the videos were posted to the online coaching website. The videos were analyzed to determine the level of math-talk, the number and complexity of connections made, and the coaching topics that were observed.
**Virtual coaching emails.** After each lesson was complete, I coached the teacher, over email, and prompted her to reflect on the lesson. We responded back and forth with questions, comments, and suggestions.

**Strategy maps.** The strategy map was created by the teacher. It mapped out the sequencing and rationale for the math-talk while identifying specific students, their strategies and connections.

**Researcher notes.** While reviewing the video, I made researcher notes about important instances that were not already aligned with coding. These could be body language, the level of respect students gave one another, and the willingness for the teacher to change her lesson. Additionally, included in the researcher notes for each session were the data analysis completed for each session. This data analysis, which is explained in this chapter, includes the math-talk rubric and analysis, coaching analysis, connections analysis, strategy map analysis, and the virtual coaching emails.

Table 2 outlines the data sources collected and the teacher and researcher responsibilities for each lesson cycle.
Table 2

*Data Sources Collected Per Lesson Cycle*

<table>
<thead>
<tr>
<th>Time</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before the lesson</td>
<td>The teacher planned the lesson using the <em>anticipating</em> practice. She identified that the task selected is rich enough for a successful math-talk by anticipating multiple student strategies.</td>
</tr>
<tr>
<td>As the lesson begins</td>
<td><strong>Video</strong> was turned on to record the lesson.</td>
</tr>
<tr>
<td>During the lesson -</td>
<td>The teacher used the <em>monitor</em> practice and circled the classroom to discover student strategies. These strategies were either anticipated during the teacher’s planning stage, or unexpected strategies. Then she used the <em>selecting</em> and <em>sequencing</em> practices to create a <em>strategy map</em> that identified the students’ work and connections she planned to address during the math-talk.</td>
</tr>
<tr>
<td>student work time</td>
<td>The teacher facilitated the math-talk as students presented their work. The teacher used the <em>connections</em> practice to focus on connections between the student strategies and the overall math concept.</td>
</tr>
<tr>
<td>During the lesson -</td>
<td><strong>Video</strong> was turned off, and uploaded to <a href="http://www.beasmartercookie.com">www.beasmartercookie.com</a>. The researcher coded the math-talk using a rubric created by Hufford-Ackles (1999) and made researcher <strong>notes</strong> to be discussed during the ongoing virtual learning sessions.</td>
</tr>
<tr>
<td>math-talk time</td>
<td>The researcher coded the math-talk for connections and coaching topic and made <strong>notes</strong> to be discussed during the coaching.</td>
</tr>
<tr>
<td>Post lesson</td>
<td>The researcher <strong>virtually coached</strong> the teacher. The researcher <strong>virtually coached</strong> the teacher in math-talk, the Five Practices, strategy maps, and coaching topics.</td>
</tr>
</tbody>
</table>
Data Analysis

This study included data from several sources; strategy maps, videos of classes, virtual coaching, and researcher notes. I began by coding the video using the math-talk rubric, created by Hufford-Ackles (1999) (see Appendix I) to determine the level of math-talk in each subcategory, and used the rubric matrix to display the results of each session. Then, I viewed the video again to identify instances when coaching topics were addressed, and used a matrix to display the results. Next, I viewed the video again and identified instances when connections were explicitly stated, either by the teacher or student, categorized them by sophistication, and used a matrix to display the results. Then, I analyzed the strategy map using the criteria for strategy maps which included format, identification of coaching topics, connections, and used a matrix to display the results. I used all of these results to create detailed anecdotal researcher notes with my findings for each session. I used the information in these researcher notes to focus the virtual coaching. Finally, the virtual coaching threads were added to the researcher notes to give a complete account of the class session, and follow-up coaching session. These completed researcher notes, which used all data sources, were analyzed using collapsed codes to understand themes that lead to change in Cathleen’s class.

Table 3 describes the use of each data set in relation to the research question. In this table, I have identified the data source, and analysis tool that I used to resolve each research question. These tools are explained in more detail in the section below.
Table 3

<table>
<thead>
<tr>
<th>Research Question</th>
<th>Data sources</th>
<th>Analysis tool</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1a) Given diverse student methods, how do strategy maps support teachers in making connections between students’ work?</td>
<td>Strategy Map</td>
<td>Strategy Map matrix</td>
</tr>
<tr>
<td>(1b) What connections between students’ work emerge during the math-talk?</td>
<td>Video Researcher Notes</td>
<td>Connections Matrix</td>
</tr>
<tr>
<td>(1c) In what ways do the strategy maps and the math-talk develop together throughout a semester?</td>
<td>Video Strategy Maps Researcher Notes</td>
<td>Hufford-Ackles math-talk rubric Strategy Maps Analysis</td>
</tr>
<tr>
<td>(2) In what ways does math coaching support the teacher in utilizing strategy maps as a tool to increase the level of math-talk?</td>
<td>Researcher Notes Virtual Coaching Strategy Maps</td>
<td>Coaching Analysis Strategy Map Matrix</td>
</tr>
</tbody>
</table>

**Coding math-talk.** This study replicates a similar study done by Hufford-Ackles (1999). I used the same framework that was presented in this earlier study, which includes rubrics to measure the level of math-talk.

A priori coding was based on several data sets: (a) teacher actions (examples and non-examples of reform-math practice), (b) evidence of a developing math community (examples of supportive, inclusive, and meaningful discourse-related interactions), (c) student actions (examples and non-examples of
doing reform math), and (d) evidence of increasing teacher knowledge (based on statements made in teacher interviews or distinct changes in practice).

Further coding used five components that capture the growth of the math-talk learning community over time: (a) questioning, (b) explanation of math thinking, (c) contributions to teaching and learning math content, (d) student responsibility of the learning of others as well as self, and (e) evaluation of students’ work. Within each attribute, development trajectories in teacher actions and student actions were identified. Each trajectory consists of four levels. This framework is titled: Levels of Math-Talk Learning Community: Teacher and Student Action Trajectories (Hufford-Ackles, 1999, p. 15).

The rubric that was created from this previous study was used to code the video for math-talk. After each of the seven sessions, I viewed the video and time stamped critical instances of observed math-talk. Using a video was especially helpful, because I was able to pause and replay critical instances until I had enough evidence to identify the time stamp with the criteria in a cell in the rubric. After I identified each critical instance by category and level, I identified the overall daily level of math-talk in each category as the greatest level of mastery. That is, in a single session, there may be evidence of level 0, level 1, and level 2 questioning, but if the teacher showed mastery of level 2, then level 2 was considered the daily level of math-talk for this category.

**Coding virtual coaching.** This research included virtual coaching after each of the seven sessions. Prior to the coaching, I asked the teacher to reflect on the lesson. The virtual interview protocol was semi-structured, and was guided by the following prompts:
Reflect on the lesson, and the connections made during the summary.

a) Did the summary go as planned? (If not, what was different? Refer to the strategy map. Why did this happen?)

b) Are you satisfied with the connections made during the lesson? (If yes, why? If no, why? Refer to the strategy map)

c) What aspects of preparing the strategy map supported you with math-talk?
Can you describe any aspects of preparing and using the strategy maps that did not help you with the math-talk?

Coding of the coaching sessions began by viewing the video after each session to identify and time stamp critical instances of changes in the lesson that was a product of coaching. Next, I coded these concepts into categories in order to unpack the virtual coaching and to determine what specifically could be seen in the class.

The virtual coaching also provided professional development addressing five coaching topics. It was predicted that as the teacher increased her knowledge of the five coaching topics, and practiced them in her class, the level of math-talk would increase. I coded the level of math-talk alongside the dates that each new coaching topic was introduced to determine if there were specific instances in this study that led to the increase of math-talk. This is discussed in terms of transferability, and specifically, how the introduction of new material influenced the level of math-talk.

**Analysis of coaching topics.** This study incorporated an element of virtual coaching. After each session, I coached Cathleen on her lesson, and introduced coaching topics to be used in her teaching. I originally chose a set of five coaching topics to be
used as a guide for the math coaching. This guide allowed for flexibility to pick and choose coaching topics based on the teacher’s need. This ensured that the coaching conversations were specific.

Coaching topic 1: Strategy Maps
Coaching topic 2: Similarities
Coaching topic 3: Representations
Coaching topic 4: Examples
Coaching topic 5: Efficiency

I analyzed the videos and strategy maps to determine the progress of each coaching topic: not observed, learning stage (it appeared in the coaching emails, and some evidence was viewed, but not mastered) and mastered stage. I then displayed this information on a matrix that corresponds to the day that each coaching topic was learned and mastered. This was used to determine if mastering a coaching topic correlated to increased math-talk.

**Analysis of strategy maps.** The strategy maps, which were created during the class by the teacher, were analyzed after each session. The teacher uploaded her strategy map, and I coded it with a priori codes and used a matrix to display the results. These a priori codes were: (1) strategy map format (does it include nodes and cross-links), (2) sequencing (does it include the order of presenters), (3) coaching topics (does it contain any of the coaching topics), and (4) connections (does it include connections to be stated). I wrote detailed notes, when applicable to explain how each of these criteria was mastered.
**Analysis of connections.** This study aimed at determining if creating strategy maps influenced the number of connections made during the math-talk. For this project, critical instances of connections were coded regardless of who made the connection; teacher or student. It is an aim of math-talk for students to state the connection, and therefore, identifying the student’s role in making connections was analyzed in the math-talk analysis.

After each session, I viewed the video and recorded time stamps of critical instances when connections were explicitly stated. Then, I identified the quantity and sophistication of the connections explicitly stated in the video. I coded the sophistication of the connections as low level or high level. A low level identifier is one that does not use the mathematical lesson to make the connection. A high level identifier is one that uses the mathematics lesson to form a connection to another student’s work. For example, low level identifiers are identifying the same operation, same chart, or same diagram. A high level identifier is identifying that different operations were used, but resulted in the same solution, or that a diagram is a visual representation of a symbolic algorithm.

Once the coding was complete, I displayed the results using a sample matrix. This was used to determine if there was a correlation between mastery of strategy maps, and increased quantity and sophistication of connections.

**Propositions**

The research questions in this study were accompanied with propositions based on research and prior experience. My relevant experience includes group professional
development, and individual one-on-one coaching. I have implemented several of the ideas and strategies listed in this research, including the Five Practices, the math-talk framework developed by Hufford-Ackles (1999), and general uses of concept and strategy maps. This however, was the first time I have implemented the use of a strategy map as a part of the Five Practices.

**Research question 1a.** Given diverse student methods, how do strategy maps support teachers in making connections between students’ work?

Early in the study, I expected the teachers to discover few connections; and instead, focus on having each student present their work. There would likely be more connections made in the strategy map than actually presented in class. From experience, this is because teachers often underestimate the amount of time it can take to unravel these connections, and are concerned with fairness in allowing all students the opportunity to present their work. As the study continued, I expected the teacher to identify connections based on the coaching topics introduced during our coaching sessions. When the teacher was focused on those coaching topics, I believed it would be observed during the math-talk.

**Research question 1b.** What connections between students’ work emerge during the math-talk?

Early in the semester, I expected very few connections to be explicitly stated. Instead, from experience, I’ve seen teachers focus on all the details of each student strategy, and run out of time, thus not effectively summarizing and making connections
between the students’ work. As the semester continued, and more coaching topics were introduced, I expected more connections to be addressed because the teacher would be focused on making connections, and the classroom climate for math-talk would be established.

**Research question 1c.** In what ways do the strategy maps and the math-talk develop together throughout a semester?

Early in the semester, I expected the teacher to review students’ work in a “show and tell” summary. I expected her to identify the connections and tell them to the students. These would be classified as lower levels of math-talk. As the semester continued and the teacher had more practice using strategy maps, I expected her to use more questioning to facilitate the students making the connections themselves. Further, I expected to see more connections as more coaching topics were introduced in the virtual coaching sessions.

**Research question 2.** In what ways does math coaching support the teacher in utilizing strategy maps as a tool to increase the level of math-talk?

It was through the virtual coaching sessions that the teacher gained one-on-one support. This coaching prompted the teacher to reflect on her teaching practices, and ways to increase the level of math-talk. I expected to begin with light coaching, where the discussions are polite and encouraging. As the semester continued, I expected to practice heavy coaching, where the teacher grapples with her teaching practice, and difficult conversations are brought to the surface.
**Transferability and Limitations**

The results from this case study focused on this teacher and her personal journey with her math-talk goal. While this study focused on a unique individual, in a unique situation, many of the results can be transferred within their limitations.

The analysis of the strategy maps was used to determine if the teacher (a) correctly implemented the use of strategy maps, (b) used cross-links to draw connections, and (c) used those connections in the math-talk. Furthermore, I analyzed the level of math-talk of the teacher using the framework provided by Hufford-Ackles (1999). My intention was to only coach the teacher on the use of the strategy maps, coaching topics, the Five Practices, and math-talk, but since I coached this teacher, one on one, there were more opportunities for support and attention to mastery than if this were to be replicated with a large group of teachers. Additional external validity limitations were found in the teacher as a unique individual. The teacher’s initial math-talk goal and motivations for her own professional growth influenced the level of effort and motivation she put towards this project. This teacher also had various levels of teacher training that may have given her an advantage when comparing to other teachers. Furthermore, the students brought with them their own advantages and disadvantages. Depending on their previous learning and expectations in math class, they may have come with a more or less willing attitude to share their thinking. Finally, I, the researcher, have had specific training in both mathematics and classroom discourse, and I may have unintentionally given additional support to the teachers through statements, body language, and gestures.
Validity

Multiple data sources were included to guard against validity threats (Maxwell, 2005). I will discuss these data sources, and their analysis to triangulate the data in order to explain how they are intertwined in this case study. In addition, I have determined the following validity threats, and included the ways that I intended to increase credibility and guard against these threats.

How can I determine what intervention (coaching, strategy maps, or other practice) resulted in a change in math-talk? To increase credibility, and to deal with this threat, I asked questions during the virtual coaching sessions to determine if there were external practices that she was doing in the class or planning that may have increased the level of math-talk. This was especially important after session 5 when I noticed an increase in math-talk over four categories. Since this was a significant jump, I probed her with additional questions to identify new practices that took place between sessions.

How can I remain transparent about my researcher bias? To increase credibility, I explained my bias in detail. I disclosed my previous work in coaching, the Five Practices, and math-talk, as well as my preference for visual tools such as graphic organizers.

How can I distinguish instances of math-talk and the trajectory of math-talk over the semester? To deal with this threat, I viewed the video to find examples of math-talk that are categorized in the rubric created by Hufford-Ackles (1999). By time stamping the video, and putting the timestamp in a cell of the rubric that corresponds to the specific
action, I verified that every critical instance belongs in that cell because of its category and level descriptors.

How can I distinguish instances of connections and the trajectory of connections over the semester? To deal with this threat, I viewed the video again only coding for instances when connections were explicitly stated. This allowed me to remain focused on one type of identification. After I identified them, I categorized them in terms of low level, high level, and big math concept. Criteria for each of these levels are described in the paper.
CHAPTER FOUR

In this study, a math teacher was introduced to a variety of teaching strategies, including strategy maps, which were aimed at increasing the connections presented during the math-talk. Over the course of the study, she practiced these theories and received virtual coaching on them, and refined them in order to change her role as a teacher, and the student’s role as learners. A portion of this study replicates a similar dissertation completed by Hufford-Ackles (1999). In her study, Hufford-Ackles transformed a classroom from traditional, teacher directed, to a classroom rich in student lead math-talk. She focused her research on developing a rubric for math-talk. I used strategy maps as an intervention to support the teacher in implementing math-talk, and used the rubric created in this previous study to determine the change in math-talk over the semester.

In this chapter I present the results from the data analysis to answer the following research questions:

1) In what ways do strategy maps support the teacher in making connections during the math-talk?

   (a) Given diverse student methods, how do strategy maps support teachers in making connections between students’ work?
(b) What connections between students’ work emerge during the math-talk?

(c) In what ways do the strategy maps and the math-talk develop together throughout a semester?

2) In what ways does virtual math coaching support the teacher in utilizing strategy maps as a tool to increase the level of math-talk?

Organization of Results

This study included data from several sources; strategy maps, videos of classes, virtual coaching, and researcher notes. I began by coding the video using the math-talk rubric, created by Hufford-Ackles (1999) (see Appendix I) to determine the level of math-talk in each subcategory, and used the rubric matrix to display the results of each session. Then, I viewed the video again to identify instances when coaching topics were addressed, and used a matrix to display the results. Next, I viewed the video again and identified instances when connections were explicitly stated, either by the teacher or student, categorized them by sophistication, and used a matrix to display the results. Then, I analyzed the strategy map using the criteria for strategy maps which included format, identification of coaching topics, connections, and used a matrix to display the results. I used all of these results to create detailed anecdotal researcher notes with my findings for each session. I used the information in these researcher notes to focus the virtual coaching. Finally, the virtual coaching threads were added to the researcher notes to give a complete account of the class session, and follow-up coaching session. These completed researcher notes, which used all data sources, were analyzed using collapsed
codes to understand themes that lead to change in Cathleen’s class. This study was complex, and used multiple theories, practices, and instruments. To determine why the math-talk in Cathleen’s class changed, I had to identify elements that lead to a change in her class. To determine this, I examined the completed researcher notes using the question, “What influenced change in the classroom?”

Analysis of Researcher Notes

Open coding began by identifying codes relevant to the change in the teacher’s classroom, that is, how the teacher used math-talk, the 5 practices, strategy maps, and the coaching topics to unpack the connections discussion during the math-talk. I used the completed daily researcher notes that included the daily analysis of math-talk, coaching topics, connections, strategy maps, and virtual coaching correspondence to identify codes. There were several concepts that can be identified in the following initial codes (a) strategy maps were created, (b) connections are explicitly stated, (c) the big math idea is the focus of the lesson, (d) students do the majority of the talking, (e) student presentations are the heart of the lesson, (f) student’s listen to one another, (g) the teacher asks probing questions, (h) misconceptions are discussed, (i) use of rich problems, (j) purposeful sequencing of student presentations, (k) students are expected to make connections between peers’ strategies, (l) teacher values student strategies, (m) teacher values multiple representations, (n) there is respect amongst students, (o) teacher willingly applies coaching advice.

Axial coding was then used to determine concepts that related to these codes. In this analysis, I classified each of the codes four concepts: strategy maps, math-talk, the
Five Practices, and teacher values. The final step in the analysis was to determine that when these concepts were used together, meaningful connections were observed and the level of math-talk increased.

There are strategies suggested in the research for the Five Practices that would support teachers making connections between students’ work, but even the authors write that this is the most difficult practice to master. There were more strategies suggested in the research for math-talk that supports teachers in facilitating a classroom with students discussing connections, but there were not any accountability for explicitly stating connections. This research considers the use of a strategy map to provide visual accountability for the teachers, but it depended on the teacher understanding the Five Practices and math-talk. Finally, it was discovered, that the teacher contained some important values that lead to her success. Each of these four themes was important and essential in making connections between students’ work. Therefore, it was the combination of multiple practices, theories, and tools that can account for this change.

This study was complex, and within each of the categories, there was additional data to be analyzed. In the next section, I present the analysis of strategy maps, math-talk, the Five Practices, and of coaching sessions that revealed teacher use of these practices and of her values, and explain how they are intertwined.

Vignette of Class Sessions

The seven sessions that were studied in this research are rich and complex. In each session, I analyzed the strategy maps, math-talk, the Five Practices, and virtual
coaching. To better understand the holistic observation, I have included a vignette of each class session, and how they developed over the course of the semester.

**Session 1.** In this session, Cathleen gives the students a rich task, and individual/pair time to develop strategies. She walks around the room and supports students who advocate for help. During the presentation, Cathleen appears to value students’ work, student thinking, and student explanation, and does not try to guide them to teacher thinking; rather she guides them to finishing their thoughts. She gives plenty of wait time to allow students to make sense of their work. She focuses on asking students questions about their work and asks them about their thinking. Cathleen prompts students to explain their strategies in detail, and gives them feedback in the form of corrections. There are no connections explicitly stated during the presentation.

**Session 2.** Cathleen continues to allow students ample time to grapple with the rich task. She monitors her students and answers questions. During the presentation, students’ work is center stage in the discussion. Four student posters hang in the front of the room, and she asks students, one at a time, to explain their work. Cathleen’s questioning extends above keeping students on task. Her questions ask students to explain their thinking. They are directed to one student, and the one student returns an answer. Each student explains their strategy, but do not reference any other student strategy to make connections. When Cathleen asks a group to make connections in terms of similarities, a student walks over to another student’s poster, pauses to read it over again, and makes a low level connection. Cathleen explicitly states another low level connection.
**Session 3.** Cathleen continues to allow students ample time to grapple with the rich task. She monitors her students and supports their questions, and attempts to use probing questions instead of simply answering. Students explain their work as the basis for the discussion. Cathleen frequently prompts the students to support them in unpacking their thinking, and rephrases the important math concept that the student just expressed (often, she says it with more accurate mathematical terms). The discussions are mostly between the presenter and the teacher, and the other students in the classroom are the audience. A student finds one low level connection. Cathleen asks students if they could see a connection between the different representations, but this is an unanswered question, and the connection is not explicitly stated.

**Session 4.** Cathleen continues to allow students ample time to grapple with the rich task. She monitors her students and supports their questions, and attempts to use probing questions instead of simply answering. During the presentation, students are becoming clearer in their math-talk. This is the first session when a student fully explained their strategy without the need for clarifying prompts from the teacher. One student explains reasoning using two different strategies. Another student presenter is also fluid in her explanation, and is excited to explain the similarities between her work and a previous presenter’s work. There is evidence that this student listened to the previous presentation because she was able to immediately identify and describe the connection, without pausing to review the poster. The discussion is still student to teacher though and does not include the entire class.
Session 5. Cathleen continues to allow students ample time to grapple with the rich task, and student norms are emerging. During this monitoring stage, students work independently for a few minutes, then they use phrases such as, “are you ready” to invite their partner to share as a pair. Cathleen monitors her students and supports their questions, and uses probing questions more regularly instead of simply answering.

During the presentation, there is evidence of student comfort and trust in her classroom. Students frequently interrupt each other (politely) to ask for clarification or to defend that they see a connection to their own strategy. Cathleen asked two pivotal questions several times: “Have you changed your mind? What would you think?” These questions result in students critiquing their work all while understanding the strategies made by the other students. Long lengths of time are spent with students conversing back and forth without Cathleen controlling the questioning. Cathleen focuses less on the solution to the problem, and more on what the numbers represented in each student strategy. For example, some students are exploring part: part, while others are using part: whole. Some students use decimals and percents while others use fractions. Cathleen asks a very simple follow up question: “So, what are the least orangey?” The student responses here lead the whole class into a long student-student discussion. Students are the facilitators of the lesson. They listen to one another and ask each other to explain their thinking. Further, presenters do not need to go to the front of the board. Students remain in their seats and refer to their work both orally and by pointing. This simple classroom arrangement allows for more discussion. Students use a variety of discussed strategies to defend and justify their solution to this follow up
question. There are four explicitly stated high level connections, and a big mathematical concept dominated the math-talk.

**Session 6.** Cathleen continues to allow students ample time to grapple with the rich task, student norms and monitoring stage are similar to session 5. The presentation begins with Cathleen asking students to explain their poster, and not simply read it. Student-student discourse is the focus of the presentation, and the flow of their explanations is clear. The order of student presenters begins with pictorial representations, and ends with abstract representations. Each student presenter identifies a similarity or difference between their work, and another student’s work before they sit down. These connections are high level, and focus on the mathematics and different representations.

**Session 7.** Cathleen continues to allow students ample time to grapple with the rich task, student norms and monitoring stage are similar to session 5 and 6. The presentations are very similar to session 6, and the flow of the presentations can be clearly seen. There is evidence that Cathleen thoughtfully sequenced students’ work, and the connections as seen as similarities are clear to students. Each presenter verbalizes these connections before returning to their seat. Cathleen comments on efficiency of an algorithm, but does not use a different tone or body language that would imply a valued preference to this efficiency. Again, the connections to the big mathematical concept dominate the discussion.

The following analysis will break apart each of these sessions to determine how each data source played a role in the increase of connections and level of math-talk.
Analysis of Strategy Maps

In this study, professional development on strategy maps was presented to Cathleen and it was to be used as a graphic organizer to support planning for the math-talk discussion. The strategy map originated from research in concept maps, which focuses on connections amongst concepts. The fundamental importance of the strategy map was to connect multiple concepts using cross-links and identifying the connection between them (see Figure 3). This strategy map includes three nodes (rectangles) that identify the math topic to be presented, and two cross-links (seen as lines between the rectangles) that identify the connections between these mathematical ideas.

Figure 3. Session 4 strategy map

The strategy maps were analyzed using a matrix analysis. The purpose for choosing this type of analysis is that I aimed to identify the level of mastery of each of the a priori codes: (1) strategy map format (does it include nodes and cross-links), (2) sequencing (does it include the order of presenters), (3) coaching topics (does it contain
any of the coaching topics), and (4) connections (does it include connections to be stated). I wrote detailed notes, when applicable to explain how each of these criteria was mastered.

For example, in Figure 3, a potential category for cross-link 2 was similarities. Within the category of similarities, the strategy map focused on the distributive property, a mathematical concept. The similarities in the strategy maps contained connections that were a higher level of understanding because they were mathematical concepts, rather than simply identifying identical numbers.

Once the coding was complete, I displayed the data using a matrix (see Table 4).
<table>
<thead>
<tr>
<th>Session</th>
<th>Strategy Map</th>
<th>Sequencing</th>
<th>Similarities &amp; Differences</th>
<th>Representations</th>
<th>Examples &amp; Non Examples</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Session 1</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Session 2</td>
<td>No</td>
<td>No</td>
<td>No, but Observed</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Session 3</td>
<td>Teacher Notes</td>
<td>Yes</td>
<td>Yes - math concepts</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Session 4</td>
<td>Yes - Created Together</td>
<td>Yes</td>
<td>Yes - math concepts</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Session 5</td>
<td>Yes</td>
<td>No</td>
<td>Yes - math concepts</td>
<td>Yes</td>
<td>Yes - notes misconceptions</td>
<td>No</td>
</tr>
<tr>
<td>Session 6</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes - math concepts</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Session 7</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes - math concepts</td>
<td>Yes</td>
<td>Yes - notes misconceptions</td>
<td>No</td>
</tr>
</tbody>
</table>

When I developed the study, I believed that teachers needed a tangible, simple, visual reminder to make connections in the math-talk. Stein et al., (2000) said that connections are difficult to make, and that there are few tools out there to support teachers with this. I showed and modeled the use of strategy maps to Cathleen and she agreed that it seemed useful and easy to do. After the first two observed lessons, I asked
her if she completed a strategy map. She said that she did not complete one. I reminded her that it was a requirement for being in the study. She then explained to me that she did not think it was as useful as she thought, so she was not using it. Instead, after the third session, she gave me a description of her facilitation notes.

“My strategy for presentations was to start with the admitted "guess and testers" that had a hard time explaining their starting point and adjustment reasons. I tried to present as a spectrum, increasing in richness of explanation/methods used. I had a reluctant team, so the simplest explanation went last, which actually worked pretty well as they were able to see what the other teams had to say. I left the one team with the chart for last, because they did such a good job of organizing what the others had done but could not articulate.”

I explained to her the importance of this graphic organizer to my research, and I believed that it was the most important element in the orchestrating of the discussion. She agreed to try to use it again. Before the fourth session, we developed a strategy map together based on the anticipated student responses that we had predicted together (see Figure 3). Cathleen seemed pleased with the map, and sounded excited for the next session. She thanked me for taking the time to do one together, and expressed her interest to continue doing them. The next observed session included a strategy map, except it did not use the style that I had modeled previously. Figure 4 was her first individual attempt at a strategy map.
I analyzed the organizer according to the strategy map codes, and discovered that it included many high level concepts and connections. It was not the diagram that I envisioned, but it had some incredible value to Cathleen. When I asked her about it further, she explained that it is really difficult to anticipate the connections ahead of time, and it was really easy to think of connections while the lesson unfolded. Her preference was to anticipate student solutions, and think about how they support the overall math concept, but she didn’t want to focus on smaller connections. She explained that she was more interested in the big math idea and focusing on this connection as opposed to connections between different methods. I encouraged her to use her own style of strategy maps, and to not request that she comply with the layout as originally intended. Her notes transitioned from a summary of each group, to looking similar to the originally intended strategy map. All the while, she focused on big math ideas, and connections to
the large math idea. She frequently mentioned that she wanted to focus on the big math idea, and not “small” connections between students’ work, but in the observations, the students explicitly made the “small” connections between the different strategies. Figure 5 shows her strategy map that was used on session seven.

![Strategy Map Image]

**Figure 5.** Session 7 strategy map

**Summary of themes from strategy maps.** Once the study was complete, and I coded the researcher notes, I discovered that the following codes were actions that the teacher did when creating a strategy map. I categorized these codes as Strategy Map. The following actions were observed under this category: a) strategy maps were created, b) connections are explicitly stated, and c) the big math idea is the focus of the lesson. The purpose of using a strategy map was to explicitly state connections (either by the teacher
or the student) between students’ work, so that the presentation is fluid, and not several
different disconnected strategies. Therefore, code a, strategy maps were created, and
code b, connections are explicitly stated, are codes related to this concept. Code c, the
big math idea is the focus of the lesson, refers to the way in which Cathleen used her own
style of strategy maps to identify the overall bigger mathematical concept.

**Code a: Strategy maps were created.** Early in the semester; Cathleen did not
create a strategy map for her lesson. As the coaching continued, Cathleen admitted that
she did not think they would be useful. On session 3, she sent me her notes on how she
selected and sequenced students’ work - an element of the strategy map. On session 4,
we created a strategy map together, which she admitted was useful. On session 5, she
created a visual that contained several elements of the strategy map. On session 6 and 7,
she created her own version of a strategy map that included the elements of identifying
the group, identifying the unique mathematics in the strategy, and the sequence for
student presenters.

**Code b: Connections are explicitly stated.** On session 1, Cathleen thought that
she had made connections, but those connections were not explicitly stated. The student
presentations were separated, with no connecting flow from one to another. On session
2, Cathleen implemented this portion of the coaching. She explicitly stated the
connections for the students. On session 3, one student identified their strategy of “guess
and check” but did not make mathematical connections, however Cathleen did explicitly
state them. On session 4, there were three connections, two of which focused on
similarities. On session 5, students identified four connections between strategies, and
Cathleen explicitly stated the overarching big mathematical idea, and how each strategy connects to this concept. On session 7, students are finding similarities and differences between each other’s posters, and using these similarities to explicitly state the connections. Beginning with session 5, Cathleen discussed with me that she thought it was more important to focus on the big mathematical idea, rather than “small” connections, such as similarities and differences, however, at this point the students were taking on the role of identifying those “small” connections.

**Code c: The big math idea is the focus of the lesson.** Starting on session 5; Cathleen focused her questions and statements on the overall big mathematical idea or objective. She explained to students that if they only understood how to solve the problem in one way, that they wouldn’t have learned all the material for this concept. She summarized the end of the presentation time with a brief synopsis of how each strategy is only a component of the overarching math concept.

**Analysis of Math-Talk**

A portion of this study replicated a similar dissertation completed by Hufford-Ackles (1999). I used the same framework that was presented in this earlier study - which includes a rubric to measure the level of math-talk in five components: (a) questioning, (b) explanation of math thinking, (c) contributions to teaching and learning math content, (d) student responsibility of the learning of others as well as self, and (e) evaluation of students’ work. The levels of each component were identified from level 0, traditional, teacher-directed, to level 3, reform, student-directed (Hufford-Ackles, 1999).
In Hufford-Ackles’ study, the teacher participant began the semester as a level 0 in all five components, questioning, explaining, math content, student responsibility, and evaluation. As the year progressed, she also progressed through the various levels until she reached the reformed teaching level 3 in all components. In this study, Cathleen did not begin at a level zero, and she had many advantages that could be the reason why she did not begin in a level zero. Cathleen began at a level 1. The following factors may have influenced the advanced baseline level.

1. Cathleen was pre-selected to participate in a graduate level course on teaching rational numbers using reform based teaching and learning.

2. During this course, Cathleen challenged herself to think in different ways. She focused her learning on modeling problems pictorially and abstractly.

3. She knew that the goals of this research were to observe teachers incorporating math-talk.

4. She knew that another goal was to uncover how teachers make connections amongst student strategies during the presentation time.

**Coding using the rubric.** In order to determine the trajectory of math-talk, I used the rubric designed by Hufford-Ackles (1999) (see Appendix I). After each of the seven sessions, I coded the video by identifying and time stamping critical instances when the class met the criteria listed in the rubric for the component and level. After coding a session, I used the time stamps to identify an overall level for each component. This level was the highest level mastered under each category. So, for example, if on a particular day, the session had evidence of mastering level 0 and level 2, the identified
level was 2. These double instances were rare, and were often in the questioning and student responsibility categories. It was observed in all classes, that Cathleen used teaching strategies, such as asking yes/no questions to select students to re-focus their attention, or for her to repeat and rephrase an important math concept that a student had mentioned. These actions are identified as level 0 in questioning and level 0 in student responsibility, but it was not the heart of the lesson.

When determining the trajectory of Cathleen’s class over the course of the semester, I used a table to visually represent how the sessions progressed through the four levels. Table 5 math-talk trajectory represents the overall level for each observed session.
Table 5

*Math talk trajectory*

<table>
<thead>
<tr>
<th>Level</th>
<th>Questioning</th>
<th>Explaining</th>
<th>Teaching Content</th>
<th>Student Responsibility</th>
<th>Evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Session 2</td>
</tr>
<tr>
<td>Level 1</td>
<td>Session 1</td>
<td></td>
<td></td>
<td></td>
<td>Session 1</td>
</tr>
<tr>
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<td>Session 2</td>
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Observing math-talk in a classroom and classifying instances as particular levels can be complex. The excerpts below are to support the reader with specific instances of questions and statements that were made by the teacher and students. Notes about the class, and body language of the speakers are also provided when applicable.

**Questioning.** In this component, the focus is on the questioner, and the way in which the class pursues mathematical information. Each level represents a shift that focuses from answers to thinking and shifts from the teacher as questioner to students as questioners.
**Level 0.** "Teacher asks questions of specific students, (mostly short-answer form). Questions often function to keep students listening. Students direct answers to the teacher only, no student-to-student math-talk" (Hufford-Ackles, 1999, Appendix A).

Teacher: “So, so that was a thirty three and a third percent decrease?”

Student: “yeah”

('Avocado' prob- presentation of last part. June 7, 2014. 0:20)

Teacher: “ok, so you did a subtraction problem?”

Student: “ya”

(Bag of marbles- difference or ratio? October 22, 2014. 0:12)

**Level 1.** “Teacher asks students questions about their thinking, focuses less on answers. Teacher rarely elicits an additional strategy. Students respond to probing by the teacher, some volunteering of thoughts. Other students listen passively or wait for their turn” (Hufford-Ackles, 1999, Appendix A).

Teacher: “And, did you assume that because it worked for one dollar, it would work for all prices?”

Student: “No, because we also tried it with two dollars. And, and also, you can think of it as percentage wise. Um, when you add twenty percent...”

('Avocado' prob- presented. Best discussion. June 7, 2014. 0:36)

Teacher: “It looks like there is a lot, I see something else, down at the bottom there Russ, and I know you were working very busily, what happened?”

(Bag of Marbles. October 19, 2014. 0:23)
Level 2. “Teacher continues to ask probing questions to prompt fuller explanations and elicit more strategies. She also facilitates student-student talk, e.g., “Everyone be prepared to ask a question about this student’s work.” Students ask questions of one another’s work on the board, often at the prompting of the teacher. Students listen to one another so they do not repeat questions” (Hufford-Ackles, 1999, Appendix A).

Teacher: “Do you have something to talk to Joey about?”

Student: “Yes, I do.”

Teacher: “What would you say to Joey?”

Student: “Well, the thing is, a lot of what he did was he kinda compared them but some people did it with sixteenths or one half, and one half is better, that’s all that I’m sayin.”

(Orangiest mixture. November 24, 2014. 0:24)

Teacher: “So, does everybody agree that there’s 2/3 of orange in A?

Student1: “yes”

Student 2: “yes”

Student 3: “noooooooo”

Student 4: “yes”

Student 5: “yeah”

Teacher: “I heard a no Charles.”

Student 3: “Well, what I think is happening is that you got the answer right, it is A, but um, the thing is that, 2 cups plus 3 cups equals 5 cups, wait, uh”
Student 2: “5 cups of the water and the orange juice.”
Student 3: “Oh, so it’s 2/5?”
Student 2: “yeah”
Student 3: “oh”

(Orangiest mixture. November 24, 2014. 2:20)

**Level 3.** “Teacher expects students to ask one another questions about their work. Her questions function to guide the discourse. Student-to-student talk is student-initiated, not dependent on the teacher. Students ask questions and listen to responses. (Note: many questions are “Why?” questions which require justification from the person answering). Students repeat their own or other’s questions until satisfied with answers” (Hufford-Ackles, 1999, Appendix A).

Teacher: “So, what are the least orangey?”

(Orangiest mixture. November 24, 2014. 7:20)

This is a level 3 question because of its follow-up nature. In the original problem, students were determining which orange juice concentrate to water ratio was the orangiest. By posing this problem, students relied on their strategy, or other student’s strategies to determine the answer in under a minute. Students are comparing each other’s work, and making body gestures that point to various posters. The session appeared disorganized as some students were thinking independently, others were working in pairs, and some students are merging their groups. Most importantly to this level, students are asking questions to each other to clarify and verify their strategies to ensure they work for this opposite problem.
**Examples of math thinking.** In this component, the focus is on the explainer, and the way the class moves from answers to mathematical thinking. Each level represents a shift that focuses on how students learn to explain and articulate their math thoughts and the classroom community grown to support students acting in central or leading roles.

*Level 0.* “No or minimal teacher elicitation of student thinking, strategies, or explanations; teacher expects answer-focused responses. Teacher may tell answers. No student thinking or strategy-focused explanation of work, just answers” (Hufford-Ackles, 1999, Appendix A).

No Example. Cathleen has always shown more than two student strategies. She has never submitted a lesson video that shows her at a level zero. When I explained the purpose of the study, and the responsibilities that she would do, she remarked about the probable reason that she was never a level 0. “If students are supposed to talk, then you need [students’ work] to talk about.” (Cathleen, initial professional development, September 23, 2014.)

*Level 1.* “Teacher probes student thinking somewhat. One or two strategies may be elicited. Teacher may fill in explanations herself. Students give information about their math thinking usually as it is probed by the teacher, (minimal volunteering of the thoughts). They provide brief descriptions of their thinking” (Hufford-Ackles, 1999, Appendix A).

No Example. Cathleen has always allowed students to voice their own strategies. It was observed, on every video, that students have control of explaining their
strategy. Cathleen does not try to explain any part, but simply supplies a probing question to support students with their explanations.

Level 2. “Teacher probes more deeply to learn about student thinking and supports detailed descriptions from students. Teacher open to and elicits multiple strategies. Students usually give information as it is probed by the teacher with some volunteering of thoughts. They begin to stake a position and articulate more information in response to probes. They explain steps in their thinking by providing fuller descriptions and begin to defend their answers and methods. Other students listen supportively” (Hufford-Ackles, 1999, Appendix A).

Teacher: “Talk to us about your problem solving.”

(Bag of marbles- difference or ratio? October 22, 2014, 0:00)

Teacher: “Explain your thinking.”

Student: “Um, every sheep, uh, there’s two chickens because, sheep have four legs and chickens have two legs. We went up to 72 legs, but there were too many animals, so we took away two chickens and added a sheep, which equaled 72 and animals equaled 23. Then, since we only had 72 legs, now, we had to um, take away another two chickens, and add a sheep and we got 74 legs and um, 23 animals. Um, our second way each one of these represents, um, two legs for chickens, and these are four legs for our sheep. And, it’s basically this, it’s this but in a table and we got nine chickens and fourteen sheep.

Teacher: “So, that is why you used tallies and circles?”
In this excerpt, the student explained two strategies that her small group used to solve the problem. The student completed the explanation of both strategies, including similarities between the two, before the teacher used any prompts. When the teacher, later prompts, it is for clarification purposes.

*Level 3.* “Teacher follows along closely to student descriptions of their thinking; may ask probing questions to make explanations more complete. Teacher asks students to think more deeply about strategies (e.g., by having them compare and contrast strategies). Students describe more complete strategies; they defend and justify their answers with little prompting from the teacher. Students realize that they will be asked questions from other students when they finish, so they are motivated and careful to be though. Other students support with active listening” (Hufford-Ackles, 1999, Appendix A).

Teacher: “But you did that differently than the other 3 groups.”

Student: “Well, um, this?”

Teacher: “Yeah, I don’t see that on any of the others.”

Student: “We multiplied by .4 because we realized that even when you divide, like when it was even when you divide, <the student points to a different group’s strategy of dividing> that the percentage stays the same. So, we multiplied it by .4 and got 6 oz. of grape.”

(Grapple-Algebra grp3. December 9, 2014, 1:52)
Teacher: “So, do you see that in any of the other?”

Student1: “They did a bunch of the basic things like we did.” <Student 2 & 3 agreeing>

Student 2: <Moving to group 3’s poster and referencing the top section> “All the first parts are basically the same.”

Teacher: “If you look down at the bottom left of the orange [poster]”

Student 2: “Oh yeah” <References a specific calculation> “This was kinda our thought process too. Like 6 had to be like 25 percent of the total, so divide by 4.”

Student 2: <References group 2’s poster> “And we basically did all the same as this one, except we multiplied it”

Teacher: “Instead of dividing by 4, you multiplied by the percent.”

Student 2: <References group 1’s poster> “And, we pretty much did this part of what they did, basically realizing that 6 had to be 25 percent.

Teacher: “Well, I thought it was interesting too, that you um, subtracted 15 from 24 to get the 9 oz. And, the others compared the 18 of the apple juice, and you compared the totals. You can still get the same answer.”

(grapple - alg grp4. December 9, 2014, 2:49)

**Contributions to teaching and learning math content.** In this component, the focus is on the teacher and students and the way each contributes to teaching and learning math content. Each level represents a shift from teacher as the source of all math content
to students contributing their ideas about content with confidence, and the way the teacher utilizes student ideas.

Level 0. “Teacher is physically at the board, usually chalk in hand, telling and showing students how to do math. Students in seats listen passively, then attempt to imitate the teacher” (Hufford-Ackles, 1999, Appendix A).

No Example. Cathleen’s classes were always student centered. She was physically in the back of the classroom. She was a level 2 when this research began.

Level 1. “Student thinking is valued by the teacher. She does some probing to assess where students are. Students’ thinking is included in their responses to probing by the teacher about math content” (Hufford-Ackles, 1999, Appendix A).

No Example. Cathleen’s classes were always student centered. She was physically in the back of the classroom. She was a level 2 when this research began.

Level 2. “Teacher physically begins to move to side or back of the room, asks more open questions, and allows discourse space for multiple explanations or strategies. Teacher is comfortable using student errors as opportunities for learning. Students exhibit confidence about their ideas and share their own thinking and strategies even if they are different from others. Student ideas sometimes guide the direction of the math lesson” (Hufford-Ackles, 1999, Appendix A).

Teacher: “Did you assume, that because it worked for one dollar, that it would work for all prices?”

Student 1: “No, because...”

Student 2: “Because we also tried it with two dollars.”
Students had the opportunity to explain their thinking, but Cathleen also held them responsible for fully explaining their strategy. After a student explained two different strategies, a symbolic strategy, and a pictorial strategy, she prompted the student to find similarities between the two.

Teacher: “Ok, and how, how did you, did you just draw the diagram that represented the numbers that you had already got or did you use your pictures toooo....”

(Chickens and sheep-7th (headband). October 29, 2014. 0:15)

**Level 3.** “Teacher allows for interruptions from students during her explanations; she lets students explain and “own” new strategies. (Teacher is still engaged and deciding what is important to continue exploring.) Teacher uses student ideas, methods as the basis for lessons or mini-extensions. Students interject their ideas as the teacher or other students are teaching. Students are confident that their contributions about math content are important to the lesson. Student ideas often guide the direction of the math lesson” (Hufford-Ackles, 1999, Appendix A).

Teacher: “So, do you see that in any of the other?”

Student1: “They did a bunch of the basic things like we did.” <Student 2 & 3 agreeing>

Student 2: <Moving to group 3’s poster and referencing the top section> “All the first parts are basically the same.”

Teacher: “If you look down at the bottom left of the orange [poster]”
Student 2: “Oh yeah” <References a specific calculation> “This was kinda our thought process too. Like 6 had to be like 25 percent of the total, so divide by 4.”

Student 2: <References group 2’s poster> “And we basically did all the same as this one, except we multiplied it”

Teacher: “Instead of dividing by 4, you multiplied by the percent.”

Student 2: <References group 1’s poster> “And, we pretty much did this part of what they did, basically realizing that 6 had to be 25 percent.

Teacher: “Well, I thought it was interesting too, that you um, subtracted 15 from 24 to get the 9 oz. And, the others compared the 18 of the apple juice, and you compared the totals. You can still get the same answer.”

(Student responsibility for the learning of others as well as self. In this component, the focus is on the student and how he/she takes responsibility for the learning of others as well as self. Each level represents a shift in how students increase their engagement, take ownership of learning for themselves, and grow to value the learning of others, and how it leads to active helping of others.

Level 0. “Teacher repeats student responses (originally directed to her) for the class. Students are passive listeners; they believe they will learn from the teacher and do not take responsibility for the learning of their peers” (Hufford-Ackles, 1999, Appendix A).

In this excerpt, the teacher rephrased a student. The student was quiet and shy.

(grapple - alg grp4. December 9, 2014, 2:49)
Teacher: “so that was the smallest difference right? That was an interesting approach.”

(Bag of marbles- difference or ratio? October 22, 2014, 1:04)

Level 1. “Teacher begins to set up structures to facilitate students listening to and helping other students. Students repeat what other students say or help another student with their work the teacher’s request. This helping mostly involves students showing how they solved” (Hufford-Ackles, 1999, Appendix A).

Student 1: “We went down two dollars.”

Student 2: “Yes, that is the same as ours because when you decreased it again, you get two dollars.”

('Avocado' prob- presentation of last part. June 7, 2014, 0:00)

Teacher: “What is that? Read that again?”

Student: “What is the total amount of the percent.”

('Avocado' prob- presented. Best discussion. June 7, 2014. 1:20)

Level 2. “Teacher encourages student responsibility for understanding the mathematical ideas of others. Students begin to listen to understand one another. When teacher requests, they explain other students’ board work and strategies in their own words. Helping involves clarifying other students’ ideas for themselves and others. Students listen actively so they do not repeat one another. Students also model teacher’s probing of their partners in pair work” (Hufford-Ackles, 1999, Appendix A).
The following excerpt shows a student finding similarities and differences between a diagram that she drew, and one from another group. When asked to find similarities, she immediately pointed to the other group’s diagram, and was ready and prepared to respond. This is evidence that she was listening and trying to understand another student’s work. When the teacher asks if the student can find any similarities, she says:

Student: “Well, in that one, it looks like they drew a diagram to show their work”. “And, Joseph’s group did the math the same.”

(Chickens and sheep-7th (headband). October 29, 2014. 0:33, 0:44)

In this excerpt, a student burst out of her seat and said:

Student: “I did the same strategy!”

(Chickens & Sheep - 6th grade. 1:44)

This is evidence that they are trying to understand each other’s work. Students are coming up on their own to show how their strategy is similar and making connections.

Level 3. “Teacher listens to students’ explanations and asks probing follow-up questions when necessary. Teacher supports students helping one another and helps and/or follows up when needed. Students listen to understand, then initiate clarifying other students’ work and ideas for themselves and for others during whole-class discussions as well as in small groups and pair work” (Hufford-Ackles, 1999, Appendix A).

By the end of the semester, Cathleen was emerging into level 3, but the level was not mastered. On session 5, there were several instances where students are eager to
begin to ask questions, but their questions turn into them explaining what they did. The teacher is the main questioner.

**Evaluation of students’ work.** In this component, the focus is on who does the evaluation, and how they evaluate. Each level represents a shift from teacher as the critic, helper, and supporter to students taking on this role as well and from answers to thinking.

*Level 0.* “Teacher responds to students’ answers by verifying the correct answer or showing the correct method. Individual students receive feedback on their work from teacher only” (Hufford-Ackles, 1999, Appendix A).

No Example. Cathleen began this study at a level 1.

*Level 1.* “Teacher evaluates work more deeply by asking follow-up questions about student methods and answers. The teacher alone gives feedback. Students describe to the teacher what each part of their work means in response to probing” (Hufford-Ackles, 1999, Appendix A).

Teacher: “Ok, ok. [Student finishes explaining his strategy] No, don’t just gloss over that. 20 is half of 40 so what?”

(Bag of marbles- fractions. October 22, 2014. 0:20)

Teacher: “Ok, so that would answer it, ok, good.”

('Avocado' prob- presented. Best discussion, June 7, 2014, 1:56)

*Level 2.* “Teacher asks other students questions about students’ work or whether they agree or disagree and why. Students evaluate their own work and that of
others. They make supportive comments about other students’ work as prompted by the teacher (or some volunteering) so they learn positive ways to evaluate and help others improve their work” (Hufford-Ackles, 1999, Appendix A).

Teacher: “Ok, so, you did a subtraction problem?”

Student 1: “yes.”

Teacher: “Ok, well, what else did you do there? Because I see the word ratio.”

Student 1: “Well, we put ratios becaussssssse, yeah”

[Both students at the board look at each other and give hand gestures that they are puzzled]

Student 2: “they did it the same”

Student 3: “no they didn’t”

Student 1: “yeah, so we subtracted the red and the blue each time.”

(Bag of marbles- difference or ratio? October 22, 2014, 0:28)

*Level 3.* “Teacher expects that students are responsible for co-evaluating of everyone’s work and thinking. Students autonomously and helpfully critique and evaluate each other’s and their own work and assist each other in understanding errors” (Hufford-Ackles, 1999, Appendix A).

By the end of the semester, Cathleen was emerging into level 3, but this level was not mastered. It was observed on session 5, 6, and 7, that students were proud of their work, and thus, their critique was that their way was better (for efficiency, simplicity, or representations) but it was never observed that the students fully critiqued all other student’s strategies.
Summary of themes focused on math-talk. Research on math-talk is an important element to this study. Since one of the aims of the study was to determine if the level of math-talk changed, I used a rubric developed in a similar dissertation about math-talk. Once the study was complete, and I coded the researcher notes I discovered that the following codes were actions that the teacher did that supported math-talk. I categorized these codes as Math-talk. The following actions were observed under this category: d) students do the majority of the talking at initial level of math talk, e) student presentations are the heart of the lesson, f) student’s listen to one another, g) the teacher asks probing questions, and h) misconceptions are discussed.

Code d: Students do the majority of the talking at initial level of math talk. This code was observed since session 1. Generally, the only time that the teacher spoke was to ask a question to clarify a student’s statement, ask for more information, or explicitly state a connection that may have not been verbalized or was vague. The teacher rarely explained students’ work.

Code e: Student presentations are the heart of the lesson. This code was observed since session 1, but evolved as the semester continued. In the early semester, the student presentations took at least 20 minutes (out of a one hour math class), and student presentations were the summary. As the semester unfolded, the teacher focused students to develop their own strategy to complete the problem, but held them responsible for understanding all of the presented strategies. Cathleen even discussed that the big math idea couldn’t be found in only one student’s work, but it was when you compared all the different strategies, that you could see how intertwined the topic had become. On
session 5, the students were discussing ratios. During the presentations, students discovered part: part ratios, part: whole ratios, fractions, decimals, and percents the connections between these representations, and proportions remained the same regardless of quantity. If a student only understood their own strategy, they would have discovered the solution, but they would not have seen the big idea about how ratios, decimals, fractions, percents, and proportions are connected.

**Code f: Student’s listen to one another.** This was observed starting on session 5, when students would respectfully interrupt one another to find similarities. The evidence was in the stopping of the conversation. On session 6 and session 7, additional evidence was seen when students were making connections to each other’s posters. They would explain their own strategy, and then physically move to another poster and point to identify the step that was similar to their thinking. This was done fluidly without pause, giving evidence that they had listened to the previous presenter.

**Code g: The teacher asks probing questions.** This was observed starting on session 1. The teacher would ask “how did you know”, “explain what you were thinking”. As the semester unfolded, Cathleen asked students to explain, validate, and find connections to other students. In all observed classes, Cathleen only clarified after she attempted a probing question.

**Code h: Misconceptions are discussed.** One of the coaching topics that was coached was examples and non-examples. Cathleen started to explore this coaching topic through misconceptions. Beginning on session 5, students presented their strategies, regardless of correctness. Misconceptions were explored, and the sequence of the
presentations, supported the incorrect student to identify similarities and discuss how to proceed.

**Analysis of the Five Practices**

This research used the Five Practices as a researched tool to support the teacher with the implementation of math-talk. The Five Practices include anticipating, monitoring, selecting, sequencing, and connections. Specifically, I analyzed the connections practice, and how it changed over the course of the study. The purpose of observing the connections in the classroom was to ensure that connections were explicitly stated, and not simply assumed that the students would make the connections. Therefore, all connections that were explicitly stated, either by the teacher or by the student, were analyzed.

**Connections.** Connections were analyzed by identifying how many specific connections were explicitly stated (either by the teacher or by the student) during the session, and by identifying the level of sophistication. A low level identifier is one that does not use the mathematical lesson to make the connection. A high level identifier is one that uses the mathematics lesson to form a connection to another student’s work. For example, low level identifiers are identifying the same operation, same chart, or same diagram. A high level identifier is identifying that different operations were used, but resulted in the same solution, or that a diagram is a visual representation of a symbolic algorithm. Further, later in the semester, the teacher focuses on the overall big mathematical concept, and connects students’ work to this concept. Since analysis was completed after each session, I was able to coach the teacher in some strategies to
increase the number of, and level of connections in the math discussion. Table 6 was created as a visual reference, to understand the trajectory of connections over the course of the semester.

Table 6

*Connections matrix*

<table>
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<tr>
<th>Session</th>
<th># of low level connections</th>
<th># of high level connections</th>
<th># of big mathematical concept connections</th>
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Since virtual coaching addressed the connections observed after each session, it is important to identify how the coaching may have influenced the teacher’s growth. The outline below shows the guidelines and specific instructions given to the teacher after each session.

**Session 1.** There were no observed connections. Students displayed their findings in a “show and tell” style of presentation. I coached her by explaining the
importance of making connections, and that she should say them out loud if students do not do so.

**Session 2.** There were two observed connections; both were low level identification, with no explanation. A student identified that they use a different operation, and the teacher identified that subtraction was in common. I coached her by saying that I would specifically be looking for connections. I gave her the following prompt to use with students: “What is similar about your poster and the other posters? What is different from your poster and the other posters?”

**Session 3.** A student made a low level connection, stating that his group also did a “guess and check.” I coached her by telling her that I would specifically be looking for connections and to continue to use the prompts, “What is similar about your poster and the other posters? What is different from your poster and the other posters?”

**Session 4.** There were three observed connections, one a low level identifier, and two high level identifiers. The low level connection used the same arithmetic, and the two high level connections identified connections amongst strategies. Both student presenters were able to find connections easily, and the second presenter showed body language of literally moving towards the other poster to identify the connections before the teacher even finished asking for connections.

1. “We both added, then subtracted.” (Sheep and Chickens-7th, October 28, 2014, 1:46)

2. “Well, it’s the same [as the previous group] because we both had 72 legs” (Sheep and Chickens-7th, October 28, 2014, 1:15)
3. “They also did a diagram, and these guys drew out the math.” The presenter in this video appears to be very confident and excited to identify and describe the similarities. (Chickens and sheep-7th, October 28, 2014, 0:52)

I coached her by commenting on how effortlessly it appeared that other students could find connections between their work and other strategies, and encouraged her to continue to probe for connections.

**Session 5.** There were many observed connections, and specifically four high level connections, but in this session, Cathleen does not focus on students making connections between each other’s work. Instead, Cathleen focused on how these connections build on the overall big mathematical idea: that you can compare ratios in different ways. Each student/small group of students solved the problem a slightly different way. Some students solved it using fractions, others using part: part ratios, other using decimals or percents, and there were different ways of comparing the fractions using student invented strategies and not necessarily the traditional (common denominator) algorithm. Cathleen stayed focused on this big idea and whenever she asked questions, it was to guide student to think about this idea of comparison. I coached her by encouraging her to continue to focus on making connections. Also, at the end of this video, Cathleen talks to the students about how she wished that she saw some different representations, and that nobody did a picture. She seems to have mastered the idea of the difference between concrete, abstract, and pictorial, as I saw evidence of this in the graduate level course. She would often say that she knew how to do it the “mathy” way, and that she wanted to do it using pictures. I suggested that when she plans a
lesson, that she anticipates a pictorial or concrete example so that she can support the students with these if they seem stuck. These different representations could lead to further connections.

**Session 6.** There were three high level connections and one overall big math idea connection. The focus of the lesson, again, is that the solution can be derived using multiple different strategies. The teacher focuses on the idea that the purpose of the math lesson is to understand and compare different strategies. Other mathematical objectives, such as arithmetic, formula setup, and math rules are embedded in each of the strategies. I coached her by encouraging her to probe the students to make connections before they end their presentation.

**Session 7.** There were four observed high level connections and one overall big math idea. This session was different, and had very different expectations for students. The focus of this lesson was for students to explain their work, and then find similarities and differences amongst their peers’ work. The purpose of this lesson though, was not necessarily in getting the right answer, but seeing that there were at least four different ways to arrive at that answer. Students were held accountable for identifying how their work was similar and different, and defending their strategy.

Cathleen teaches the students that there are many different ways of manipulating ratios, proportions, and percentages. The students derived a variety of strategies to solve the multi-step problem. Through her focus on similarities and differences, the following math concepts were explicitly stated.

- 4 divided by 10 is equivalent to multiplying by 0.4
• The ratio of a mixture remains constant regardless of the total quantity. If there is a smaller total of mixture, the ratio of each component is proportionately smaller.

In the final step of the problem, students needed to calculate how much more apple juice to add to a juice mixture in order for the percentage of apple to equal 75%. The following strategies were used, and explained in detail during the presentations. Cathleen even had students explain how their final step was similar or different from others.

\[
\begin{align*}
-18 - 9 &= 9 \\
-9 + 9 &= 18 \\
-24 - 15 &= 9 \\
-25/75 &= 6/(9+9) \\
\text{trial and error} &\ (6/9, 6/10, 6/15, 6/20, 6/19, 6/18) \quad 6/18 = 25/75, 9+9=18
\end{align*}
\]

Below are some instances that show how these connections unfolded.

Teacher: “How did you find how many of each type were in your glasses?”

Student: “We divided by 4”

Teacher: “So, you divided the ingredients? Ok, so you didn’t take the percent, that’s the other way [the previous group] had done it.”

(Grapple-Algebra grp2. December 9, 2014, 0:48)

Teacher: “What do you see on the girls’ chart that looks similar to the work that you guys did?”

Student: “This chart.”

Teacher: “Yes, a chart to organize your work. Look at their number 2, how is that different than what you guys did?”
Student: “Well, they did a direct percent”

Teacher: “ok”

Student: “We did the percent afterwards cause we just divided 6 um, out of 15 and then 9 out of 15 and that got our percents.”

(Grapple-Algebra grp2. December 9, 2014, 2:12)

Teacher: “But you did that differently than the other 3 groups.”

Student: “Well, um, this?”

Teacher: “Yeah, I don’t see that on any of the others.”

Student: “We multiplied by .4 because we realized that even when you divide, like when it was even when you divide, <the student points to a different group’s strategy of dividing> that the percentage stays the same. So, we multiplied it by .4 and got 6 oz. of grape.

(Grapple-Algebra grp3. December 9, 2014, 1:52)

Teacher: “So, do you see that in any of the other?”

Student1: “They did a bunch of the basic things like we did.” <Student 2 & 3 agreeing>

Student 2: <Moving to group 3’s poster and referencing the top section> “All the first parts are basically the same.”

Teacher: “If you look down at the bottom left of the orange [poster]”
Student 2: “Oh yeah” <References a specific calculation> “This was kinda our thought process too. Like 6 had to be like 25 percent of the total, so divide by 4.”

Student 2: <References group 2’s poster> “And we basically did all the same as this one, except we multiplied it”

Teacher: “Instead of dividing by 4, you multiplied by the percent.”

Student 2: <References group 1’s poster> “And, we pretty much did this part of what they did, basically realizing that 6 had to be 25 percent.

Teacher: “Well, I thought it was interesting too, that you um, subtracted 15 from 24 to get the 9 oz. And, the others compared the 18 of the apple juice, and you compared the totals. You can still get the same answer.”

(grapple - alg grp4. December 9, 2014, 2:49)

It was observed in session 5, 6, and especially in session 7, that making connections is the new focus of the session. Cathleen stated that she would rather make just one connection, than lots of connections between each student strategy. Cathleen identified big ideas as the importance of the lesson. On session 7, the big idea that was frequently stated, was that the solution could be found in a variety of ways, and that each of the strategies to find the solution were similar and different from each other. Those similarities and differences were explicitly described by the teacher and students in such a way that students made connections about the mathematical calculations and strategies, in order to make the connection to the big idea.
Summary of themes focused on the Five Practices. Research on the Five Practices is an important element to this study. Before the study began, Cathleen took a graduate level course in math education that focused on these Five Practices, anticipation, monitoring, selecting, sequencing, and connections. Before classroom observations took place, Cathleen participated in a professional development to explain the Five Practices, and the importance in using them for this study.

Once the study was complete, and I coded the researcher notes I found that the following codes were actions that the teacher did when relating to the Five Practices. I categorized these codes as the Five Practices. The following actions were observed under this category: i) use of rich problems, j) purposeful sequencing of student presentations, k) students are expected to make connections between peers’ strategies. Code i, the use of rich problems, is a prerequisite for using the Five Practices, and is stressed in its theory. Code j, purposeful sequencing of student presentations, and k, students are expected to make connections between peers’ strategies, are two of the Five Practices: sequencing, and connections.

Code i: Use of rich problems. This code was seen across all observed classes. This was a requirement for the research, and the researcher provided a database of real world problems that were field tested to ensure that they had multiple points of accessibility. That is, it could be solved by a variety of grade levels, and could be solved using a variety of strategies and representations. Each session used the entire math period of one hour to solve a problem. Because every lesson focused on one problem, there was ample time for students to derive their own strategy. Also, the problems were
relevant to the students, providing a real world, and age appropriate problematic situation.

**Code j: Purposeful sequencing of student presentations.** The teacher always had a plan for the order in which students presented their work. This code was observed in all classes, however the sequencing became more elegant as the teacher gained more experience, and the presentations became more fluid. Early in the semester, the teacher generally chose the “guess and check” strategies first, then chose the models and abstract representations, and finally students with low confidence, or only partially solved strategies. As the semester continued, and Cathleen implemented more coaching topics, her sequencing began to change to models and diagrams first, and abstract representations last. In the final session, there was a notable flow that took place during the presentation because of the order in which the students were selected.

**Code k: Students are expected to make connections between peers’ strategies.** Early in the semester, Cathleen asked students if they could find “any connections” (session 1) or “similarities and differences” (session 2), but it wasn’t until session 3 that she started holding students accountable for making these connections. On session 5, students began respectfully interrupting one another when they noticed a connection, and referenced their work while explaining it. The connection was identified, but often not explained; rather the student simply identified it as being similar, and then explained their work without referencing the connection again. On session 7, students identified and explained how their strategies are similar to other strategies. Since session 7 included a problem that used multiple steps, students were referencing portions of peers’ strategies
within their presentation. By the time a group had completed their presentation; they had referenced and explained the similarities of two other groups’ strategies.

Analysis of Coaching

**Analysis of coaching topics.** This study incorporated virtual coaching. After each session I coached Cathleen on her lesson, and introduced coaching topics to be used in her teaching. I originally chose a set of five coaching topics to be used as a guide to focus on connections for the math coaching. This guide allowed for flexibility to pick and choose coaching topics based on the teacher’s need. This ensured that the coaching conversations were specific.

- Coaching topic 1: Strategy Maps
- Coaching topic 2: Similarities
- Coaching topic 3: Representations
- Coaching topic 4: Examples
- Coaching topic 5: Efficiency

Prior to the initial professional development on strategy maps and the Five Practices, Cathleen had taken a graduate level course that focused on two of the coaching topics; coaching topic 2: similarities, and coaching topic 3: representations. At the beginning of each session, teacher participants were required to complete a problem using multiple representations, and then present their strategy to the class while identifying similarities and differences between other presented strategies. Cathleen excelled in both of these areas. She often held other teachers responsible for finding these similarities, or she would vocalize them herself. On several occasions, she would
tell me that she knew how to do it the “mathy way” (abstract representation), but that she wanted to learn how to do it with pictures (pictorial). She displayed motivation to develop her learning on these two coaching topics.

I observed seven sessions, and noted when Cathleen was learning and developing a coaching topic, and when that coaching topic was mastered. The following table can be used to understand when Cathleen mastered the coaching topics presented in this research. The following terms were used to describe three instances. Not observed is used to identify that this coaching topic was not observed during the lesson. Learning is used to identify that the coaching topic has been coached, but only partially observed. Finally, Mastered is used to identify the lessons where the coaching topic has been mastered according to the intended criteria. Table 7 was created to understand the trajectory of how Cathleen moved toward mastery of each coaching topic.
Table 7

Coaching topics matrix

<table>
<thead>
<tr>
<th>Session</th>
<th>Coaching topic 1 Strategy Map</th>
<th>Coaching topic 2 Similarities &amp; Differences</th>
<th>Coaching topic 3 Multiple Representations</th>
<th>Coaching topic 4 Examples &amp; Non-examples</th>
<th>Coaching topic 5 Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Session 1</td>
<td>Not observed</td>
<td>Not observed</td>
<td>Not observed</td>
<td>Not observed</td>
<td>Not observed</td>
</tr>
<tr>
<td>Session 2</td>
<td>Not observed</td>
<td>Learning</td>
<td>Not observed</td>
<td>Not observed</td>
<td>Not observed</td>
</tr>
<tr>
<td>Session 3</td>
<td>Learning</td>
<td>Learning</td>
<td>Learning</td>
<td>Not observed</td>
<td>Not observed</td>
</tr>
<tr>
<td>Session 4</td>
<td>Learning</td>
<td>Learning</td>
<td>Learning</td>
<td>Not observed</td>
<td>Not observed</td>
</tr>
<tr>
<td>Session 5</td>
<td>Mastered</td>
<td>Mastered</td>
<td>Learning</td>
<td>Learning</td>
<td>Not observed</td>
</tr>
<tr>
<td>Session 6</td>
<td>Mastered</td>
<td>Mastered</td>
<td>Learning</td>
<td>Not observed</td>
<td>Not observed</td>
</tr>
<tr>
<td>Session 7</td>
<td>Mastered</td>
<td>Mastered</td>
<td>Learning</td>
<td>Not observed</td>
<td>Learning</td>
</tr>
</tbody>
</table>

The outline below gives more detail about actual instances from the sessions, to support the reader to understand how she moved towards mastery of each coaching topic.

**Coaching topic 1: Strategy maps.** This topic addressed how the teacher used strategy maps. Specifically, how the teacher was able to create a strategy map that clearly
indicated a node identifier and the chronological order of students’ work along with cross-links. Cross-links included a word, phrase, or statement that linked students’ work.

_Not observed._ Cathleen did not submit a strategy map after sessions 1 and 2. We had a discussion about how the strategy map was the important and unique element to this research, and she committed to using them in the future.

_Learning._ Cathleen learned more about the strategy maps, and began to implement them on sessions 3 and 4. On session 3, she showed me her notes, but they did not contain any form of visual representation. The notes were in the form of sentences and contained a rationale for the mathematic to be presented, and a sequence for presentation. On session 4, we created a strategy map together, see Figure 3. The information in the strategy map was provided by the teacher, and I supported her in creating the visual. Once she saw this visual, she appeared satisfied and eager to create a strategy map for the upcoming lesson.

_Mastered._ After the fifth lesson, I observed Cathleen making her own form of a strategy map (see Figure 4). She had mastered the overall purpose of the strategy map - to make connections, but her strategy map is visually different. Her strategy map contained the students’ name for the title, a brief description of their strategy, and some notes about possible questions the students may have. On session 5, there was no indication that she has sequenced them in any order for presentation. By session 6 and 7 (see Figure 6 and Figure 5), she had included the sequencing. While the connections were not listed on her strategy map, she still explicitly announced them in each
session. Her notes supported her to understand the importance of each student’s contribution, which was seen in the session discussion as connections.

![Figure 6. Session 6 Strategy Map](image)

**Coaching topic 2: Similarities.** This topic addressed how the teacher focused on the use of similarities and differences in students’ work to make connections during the math-talk.

**Not observed.** On session 1, Cathleen did not have any instances during the presentation where students were responsible for identifying similarities or differences. Further, she did not explicitly identify any similarities or differences.
**Learning.** On session 2, I observed Cathleen explicitly identifying a similarity between two different group’s posters. She was comfortable with this coaching topic, so I coached her by giving her the following prompt, “What is similar about your poster and the other posters? What is different from your poster and the other posters?”, and encouraged her to use this prompt with her students. On session 3, there was ample evidence in the videos that she asks “what is similar” or “what is different”, but the students only responded with a basic similarity, or none at all. I advised her to continue to allow the students the opportunity to identify the similarities and differences, but if they do not identify them, she should say them explicitly. On session 4, I observed Cathleen focusing on this coaching topic. She did not allow the session to end without explicitly stating similarities. I coached her to ask students to not only identify the similarities, but also describe them. I also encouraged her to identify differences explicitly.

**Mastered.** There was a major shift on the purpose of math-talk observed on session 5. The focus of the lesson was not in simply finding an answer, but listening to one another’s strategies, figuring out what was similar and different, and why it was still mathematically sound. On session 6, discussing similarities and differences had become the heart of the lesson. Cathleen spent more time on the discussion, and less on the individual problem solving. During the discussion, students were held responsible for finding a similarity or difference in their work and another previous presenter before ending their presentation. Students were prepared and ready to tackle this part of the presentation. They appeared to have listened carefully to previous presenters, because
each group is quickly able to find similarities, and reference them on the other posters. Again, on session 7, this coaching topic was the heart of the lesson. Students were speaking about the similarities and differences with much more confidence, and speaking in detail about how their work is similar. They were referencing other posters, and specifically the step or number that is similar, and then showing what is different about their strategy.

**Coaching topic 3: Representations.** This topic addressed how multiple representations (concrete, pictorial, and abstract) were used to make connections between students’ work. The teacher used the terms concrete, pictorial, and abstract in order to develop probing questions to make connections during the math-talk.

**Not observed.** During sessions 1 and 2 students were engaged in finding multiple abstract representations. However, they did not engage in pictorial or concrete representations.

**Learning.** On session 3, students had both abstract and pictorial representations on their posters but they do not reference the pictorial representation during the presentation. On session 4, Cathleen’s students used the terms “picture” and “drawing” to refer to the pictorial representation, and the term “mathy” to refer to the abstract representation. I supported Cathleen by giving her the terms, concrete (manipulatives), pictorial (diagrams, drawings, etc.) and abstract (symbolic). I asked her to review these words with her students and see if they can be supportive during the math-talk. On session 5, Cathleen asked the students to think of multiple representations, but there were only various forms of abstract representations. She even used the term “representations”
with her students, but there were no pictures, diagrams, or concrete representations. On session 6 Students still preferred to show abstract representations, but they were also including a pictorial representation. The listening students saying comments such as “oh”, and “right there” when the presenting group referenced how their diagram connected to the symbolic mathematics. On session 7, the majority of the posters showed a pictorial representation as a model to show at least one step in their strategy. The students were also using the pictorial representations when identifying similarities and differences (a previous coaching topic). Students appeared to regard these diagrams as a useful way to explain their thinking. There were no negative comments, and instead supporting comments such as “you can see here what I mean”, or “If you look at this, the grape needs to stay the same.” Cathleen’s student did not use any concrete manipulatives in their problem solving; therefore, this coaching topic was never fully mastered.

**Coaching topic 4: Examples.** This topic addressed how the teacher identified examples and non-examples in students’ work and explicitly stating those in the math-talk.

*Not observed.* This coaching topic was not discussed with Cathleen until the completion of session 4.

**Learning.** Cathleen used misconceptions to explore non-examples beginning on session 5. In her strategy map, she noted when a student needed more support, and began a whole class conversation about a pair of boys who were confused about the next step in their strategy. I observed a very trusting environment, and the body language of the boys appeared to be eager, and accepting of the other students’ suggestions. The other
students used phrases such as “What we did was...” or “I think you could have...” and positive body language. This coaching topic was not observed in session 6 or 7, but there were also no misconceptions on those days.

**Coaching topic 5: Efficiency.** This topic addressed how the teacher identified various levels of efficiency in students’ work. She used the term efficient in order to develop probing questions to make connections during math-talk.

**Not observed.** During the math-talk on sessions 1-6, Cathleen did not appear to value one strategy over another. All strategies, whether long, short, detailed, or abbreviated, efficient, or not efficient, received the same amount of respect.

**Learning.** During session 7, Cathleen made a brief comment about how a poster showed an efficient method, but she doesn’t seem to value this efficiency. She did not praise the strategy or speak negatively about it, it was simply a comment.

**Emergence of teacher values as observed throughout the coaching sessions.**

Coaching is a very personal form of professional development. When coaching is successful, the coach first has a discussion with the teacher to identify areas that he/she would like to improve. Then, the coach takes those ideas, and compares them with areas that he/she also values. When the areas of growth are compatible with both the teacher and coach, the coaching is collaborative (Rawding & Wills, 2012). In this study, the participant was chosen because of our prior conversations. She showed an interest in improving math-talk, and several other qualities that I thought would be beneficial in the study. Once the study was complete and I coded the researcher notes, I discovered that the following codes were qualities that the teacher used to increase math-talk. I
categorized these codes as Teacher values. These were observed trends in Cathleen’s behavior that can be best identified as personal values. These values were already ingrained in her before she began the research, and continued to grow as the research progressed: l) teacher values student strategies, m) teacher values multiple representations, n) there is respect amongst students, o) teacher willingly applies coaching advice.

**Code l: Teacher values student strategies.** Beginning on session 1, the teacher used the student derived strategies for discussing how to find the solution. There was never an observed session where the teacher wrote a strategy or algorithm on the board, and required students to learn a traditional algorithm.

**Code m: Teacher values multiple representations.** Early in the semester, the teacher sequenced students with abstract, often traditional, strategies to begin the presentation. Then, the students with models, or less efficient strategies would follow. As the semester unfolded, on session 4, she changed the order, such that pictures, models, and tables were chosen first, and the abstract and traditional algorithms would present last. On session 6, Cathleen references the pictures and asks students to make connections. On session 7, most students have included a picture or diagram in their poster. More students included multiple of representations in their posters, which shows evidence that the teacher values these multiple representations.

**Code n: There is respect amongst students.** Beginning at session 1; students were respectful to one another. When comments were made about previous presenters, the
comments were positive and specific. On session 5, there are many instances of student-student discussion. This often led to interruptions. These were all done respectfully.

**Code o: Teacher willingly applies coaching advice.** This code was observed before the research began. I had taught the graduate level course prior to this research, and the teacher in this study was chosen from this course. Cathleen and I had several discussions during the course about how she was very interested in learning more about improving her teaching, and she was very willing to be coached. She would ask questions if she was unsure of the coaching suggestion, but would always attempt to implement the new suggested coaching topic. She was a very eager and willing learner.

**Analysis of teacher values.** Teacher values impacted this study. In this section, I describe how I coached Cathleen, while maintaining her vision of based on her values. After each lesson, Cathleen reflected on the lesson and gave me feedback using the following prompts. Note that the third and fourth prompt, about strategy maps, were only given after day four, when she implemented strategy maps.

Reflect on the lesson, and the connections made during the summary.

Did the summary go as planned? (If not, what was different? Refer to the strategy map. Why did this happen?)

Are you satisfied with the connections made during the lesson? (If yes, why? If no, why? Refer to the strategy map)

What aspects of preparing the strategy map supported you with math-talk?
Can you describe any aspects of preparing and using the strategy maps that did not help you with the math-talk?

Below is an outline of specific instances of Cathleen’s feedback about the lesson and strategy map. Also included is a brief overview of the coaching.

Early in the Semester (session 1 - session 3): Cathleen’s statements about the lesson were positive. She was pleased with the lesson and pleased with the mathematics that was discussed. When I asked her if she was pleased with the connections, she would refer back to how she was excited about the students showing their thinking about mathematics. She did not reference any actual connections that she intended to make, or that were actually made. She did not create a strategy map, and did not value it.

The lesson before session 4, we created the strategy map together. This was important to me, as it was the foundation of this research. I focused on the correct notation, and similarities and differences. After the lessons on session 4, Cathleen was pleased with the overall lesson. She could identify actual connections that were made. She mentioned that she did not think that small connections were necessary, and referred to them as the cross-links of the strategy map, and that she preferred to focus on larger connections to the big mathematical idea. Analysis of actual connections made in each session shows that she still made these smaller connections, as she stated were similarities, examples, etc., but that she didn’t think they were noteworthy, just something that teachers do.

On session 5, she was still satisfied with the lessons; in fact, she really seemed to enjoy them. She was asking for more problems, and said that they enjoyed listening to
“how the students really think about math instead of just doing calculations.” She was very satisfied with the overall big mathematical idea that students were discovering, and making connections to this big math idea. She thought that the strategy maps were not useful, but that the effort to create some sort of instructor’s notes for me was a good practice for her to get into. She said that it “made me think about what my students were doing, and how they related to one another.”

On session 6 and 7, she was very excited about the lesson, the student’s role, and the connections made. She enjoys her version of a strategy map and finds it to be a useful tool. I coached her by encouraging her to continue her work in these areas.

“Moment of convergence”: Analysis of session 5. After analyzing all the previous data, I noticed a dramatic shift the dynamics of Cathleen’s classroom on session 5. Session 5 is presented in terms of Cathleen’s growth in numerous categories. To do this, I described the observation, identified the components that had a positive shift, and then described instances that may have resulted in this growth.

Cathleen’s session began as usual, with one single math problem.

The problem. Jackson and Mariah are in charge of making orange juice for the school dance. They make the juice by using orange concentrate and mixing it with water. Unfortunately they forgot the recipe so they try a few options. Which mix will taste the most “orangey”? Explain your answer.

Mix A: 2 cups orange concentrate 3 cups cold water
Mix B: 1 cup orange concentrate 4 cups cold water
Mix C: 1 cup orange concentrate 2 cups cold water
Mix D: 3 cups orange concentrate 5 cups cold water

**Observation.** In this lesson, students worked individually for five minutes, and then they could work with a partner. The students used a norm created earlier in the semester, where they verified that their partner had enough individual time before they grouped together. This was done either through body language, or directly asking the partner. The students, in pairs, discussed their thinking. Most students only developed a partial solution individually, but in pairs, they unpacked one or both of the unique starting points, to find the solution to the problem. They used 10” x 12” whiteboards which allowed them to write large enough that both students could see the writing. Throughout the session, all pairs of students were engaged. If a partner was taking the lead, he or she used the norm of verifying that their partner was following their train of thought. Partners were discussing, asking questions, and explaining their thinking.

After ten minutes, Cathleen gathered her class together for a whole group discussion. The students were gathered around a large rectangular table. Cathleen was responsible for sequencing the group presentations, but allowed the students to do the talking. The first pair presented, but had difficulty defending their strategy. Just as they were finishing, another student respectfully interrupted, and rephrased their thinking more clearly. Then she asked the group why they did it their way, and stated that some groups did it differently. Then she explained her thinking. The girl was doing all the talking, except when Cathleen gave the occasional prompt for her to explain the rationale for the steps she chose to do. For example, “What are you doing when you are multiplying?” The student clearly explained her strategy, and even defended how she
knew that it was correct by checking it using reverse arithmetic. She was not even finished her explanation, when another student had his hand up, frantically waving. Cathleen asked if everyone agreed, and when there was a chorus of yeses, the boy with his hand up said no. Cathleen asked him to explain. He started by saying that he agrees with the previous students answer, but not the strategy used to get the answer. He explained his strategy, with reference to the steps that the previous girl used and compared and contrasted his strategy. Once he completed his explanation, Cathleen rephrased to focus on the mathematics.

Teacher: “So, he is doing a slightly different fraction than you are, you were doing orange to water, and he’s doing orange to...”

Student: “to all”

Teacher: “So, does that give us a different answer?”

Students chorus “No”.

As other groups presented their strategies, Cathleen asked the following question, “So, you were comparing what to what?” in an effort to focus students on the part: part comparison or part: whole comparison. The final group doubted the initial solution at the beginning of the presentation, and Cathleen sequenced this group to present last. This final group used reasoning to describe how two of the previous group’s strategies verified that the proposed solution was correct. Once Cathleen felt comfortable that all of the students understood at least one strategy to find the solution, she proposed a follow-up question. “So, which is the least orangey?” This required students to consider their previous strategies, and instead of figuring out which was the most orangey; they needed
to consider the least orangey. All of the students were talking, listening, or writing immediately. There were no students who displayed the “blank stare” body language. Some students joined a rather large and loud group to discuss a strategy on the board. There were a few students who worked in pairs, and one student who worked individually. The large group compared two strategies, and solved using both, to verify the solution. One pair of students in this large group decided that they liked a different strategy than the one they initially presented. Cathleen verified that every group found the solution, and gave the final summary. She explicitly stated the student strategies and the mathematical term associated with each strategy. She identified the following:

- part: part ratios
- part: whole fractions
- unit fractions
- decimals
- percents
- common denominator

She also discussed a big idea about proportions. Students were asked to think about if it made a difference that Mix D contained more liquid. The student who solved it using part: whole fractions defended that it didn’t make a difference about taste; only how many cups could be filled. Further, he said that once they determined the correct recipe, they would need to make a huge batch using the correct ratio.

*New teacher and student moves.* The following components were observed for the first time during session 5.
• Student to student initiated questions (Math Talk Rubric: Questioning - level 3)
• Students defend and justify multiple strategies (Math Talk Rubric: Math Thinking - level 3)
• Student strategies are the basis of the lesson (Math Talk Rubric: Math Content - level 3)
• Students agree and disagree with each other (Math Talk Rubric: Evaluation - level 2)
• Many high-level connections are explicitly stated (Connections Matrix: # of high-level connections)
• No low-level connections are explicitly stated (Connections Matrix: # of low-level connections)
• One big mathematical concept is explicitly stated and summarized (Connections Matrix: # of big mathematical concept connections)
• The coaching topic strategy map was mastered. (Coaching topics Matrix: Coaching topic 1 - Strategy Map)
• The coaching topic similarities and differences was mastered. (Coaching topics Matrix: Coaching topic 2 - Similarities and Differences)
• The coaching topic examples and non-examples were first learned. (Coaching topics Matrix: Coaching topic 4 - Examples and Non-Examples)
• The teacher created a strategy map. (Researcher Notes: Code a)
• The big math idea was the focus of the lesson. (Researcher Notes: Code c)
• Student presentations were the heart of the lesson. (Researcher Notes: Code e)
• Student’s listened to one another. (Researcher Notes: Code f)
• Misconceptions are discussed. (Researcher Notes: Code h)
• Students were expected to make connections between peers’ strategies. (Researcher Notes: Code k)
• There was respect amongst students. (Researcher Notes: Code n)

Session 5 was unique because all of these changes happened for the first time on this particular day, resulting in a large leap in the overall level of math-talk. I discuss this session further in chapter 5, and explain the elements that may have led to this leap.
CHAPTER FIVE

The purpose of this study was to better understand how strategy maps can support a teacher with the implementation of the Five Practices to increase the level of math-talk in the session. To accomplish this goal, a phenomenological case study was utilized to answer the following questions:

1) In what ways do strategy maps support the teacher in making connections during the math-talk?

   (a) Given diverse student methods, how do strategy maps support teachers in making connections between students’ work?

   (b) What connections between students’ work emerge during the math-talk?

   (c) In what ways do the strategy maps and the math-talk develop together throughout a semester?

2) In what ways does virtual math coaching support the teacher in utilizing strategy maps as a tool to increase the level of math-talk?

Research Question 1

In what ways do strategy maps support the teacher in making connections during the math-talk?
Novak and Cañas (2008) describe concept maps as graphical tools for organizing and representing knowledge. In this research, strategy maps were used, as a type of concept map, to support the teacher with making connections during the math-talk. This tool was a handwritten guide created to hold the teacher responsible for doing the Five Practices. That is, the teacher was asked to use the strategy map during the anticipating stage of planning. Once the lesson began, the teacher could have changed or recreated the strategy map using authentic students’ work which would be found during the monitoring and selecting stage. Then, she could sequence them, simply by drawing cross-links, or arrows, to designate order during the sequencing stage. Finally, below those cross-links, the teacher could write in the connections that she wanted to make between each set of presenters. An example of this can be seen in Figure 2. The teacher could edit the strategy maps at any point during the anticipating or monitoring stage. Since this editing cycle can always be revisited, concept maps are never finished (Novak & Cañas, 2008).

In this study, the teacher slowly integrated strategy maps into her lessons, creating her first map mid semester, and incorporating them regularly in subsequent lessons. She chose to use the strategy maps in a different way than originally intended. She used the components of the first four stages of the Five Practices: anticipating, monitoring, selecting, and sequencing, which created the ideal situation for students to identify connections between each other’s presentations. She did not complete the portion of the strategy map that includes connections on the cross-links, but this did not impact her ability to actually make connections. Strategy maps may have supported the teacher in
making connections during math-talk because they provided an authentic way for her to set up the ideal environment to find connections with ease.

**Research Question 1a**

Given diverse student methods, how do strategy maps support teachers in making connections between students’ work?

Strategy maps evolved during this study. In the beginning, Cathleen did not create strategy maps for her lesson, and did not find them useful for teaching and planning. On the third session, she submitted teacher notes that revealed that she was beginning to think about the sequencing of students’ work; however, the sequencing seemed to be based on student confidence, and not mathematics. We met face-to-face before the fourth session, and created a strategy map together. It was after this session, that Cathleen first shifted from low level connections, to explicitly stating two high level connections between student methods.

Session 5 is an important session in this study, because Cathleen’s class showed a dramatic shift in math-talk. This was the first day that Cathleen created her own strategy map. Her strategy map included some important elements, but missed others. During the math talk, a variety of diverse student methods were presented. During the presentation, there were four high-level connections, and no low-level connections. Session 5 is also the first time that there was one overall big math concept focusing on ratios.

Session 5 was the first time that Cathleen created her own strategy map, and she did so differently that I had taught her. Her strategy map was useful to her only when she was comfortable doing it her way, and not the prescribed way. Both the Cardemone
(1975) and Bogden (1977) studies show the importance of the creator in constructing concept maps. These studies show that the benefits of creating a concept maps lie in the creator and their new understanding.

Cathleen’s strategy map focused on student thinking. It included a student name, a brief description of the mathematics that the student completed, and additional teacher notes to know where a student may have struggled, or if they had a misconception, but it did not contain any sequencing or identification of connections to be stated during the math-talk. The practice of creating her own form of the strategy map focused her on the monitoring and selecting practice of the Five Practices. The strategy map was handwritten, and she knew that I would analyze it along with her growth in math-talk. During our coaching, I asked her if it helped that it was handwritten, and she explained that it held her accountable for really understanding what each student did, since she needed to identify the mathematics in the strategy. Previously, when she didn’t hand write a strategy map, she didn’t focus in as much detail about what each student was doing, how each student had a unique strategy, and the way that she hoped to expose this in the presentations. During this coaching session, I asked her if it would help to determine the order of student presentations, and she agreed that this would be useful.

Cathleen improved her strategy map on session 6 and 7 to include all the previous components and also arrows to represent the sequence of student presenters and notes about student misconceptions. Cathleen never fully implemented the idea of cross-links in the strategy map to identify the connection. Instead, the only evidence of connections was seen on the strategy maps as similarities and differences between students’ work, and
therefore it was essential to focus on similarities when sequencing students’ work. The focus of the student presentations became on finding similarities and differences between each other’s work. This focus even became a norm in her class. Students were expected to relate their work to another student’s work before they completed their presentation.

This new norm, and the thoughtful sequencing of students’ work, resulted in a change in flow during the math-talk. The following is an example of this flow observed in the math-talk on session 7: The first group used a table to display their data. The next group explained how their tables were similar, but that they chose a different strategy to arrive at their tables. The next group mentioned how their strategy was similar, but they drew a growing patterns picture instead of a table, but how the pictures and numbers represented the same thing. The final group connected their pictures with the previous groups growing patterns, but remarked on how the arithmetic was different, in fact, she discovered inverse operations. Cathleen summarized the big idea that the ratio of a mixture remains constant regardless of the total quantity. If there is a smaller total of mixture, the ratio of each component is proportionately smaller. This flow was completely built upon finding connections, in the form of similarities.

Strategy maps may have been useful in making connections during the math-talk when they were authentic, handwritten tools, that the teacher created herself, and include important characteristics such as identifying the mathematics used, areas where students needed teacher intervention, and sequencing of student presentations. Further, the teacher needed to focus on connections. Cathleen did this by thinking about how each student’s work was similar, and anticipating how these similarities would be
presented. Then she sequenced them so that students would have an easier opportunity to see their similarity with the previous student’s work.

In conclusion, strategy maps may have supported the teacher in making connections because they supported her with the flow of the students’ presentations. When she held herself accountable, by physically drawing out a strategy map, she was able to focus on selecting only specific students, and sequencing them in such a way that connections to similarities were more easily identified.

**Research Question 1b**

What connections between students’ work emerge during the math-talk?

The evolution of connections can be identified as three categories; low-level connections, high-level connections, and big mathematical concept connections. Low-level connections consisted of identifying the same operation, same chart, or same diagram. A high-level connection was identifying that different operations were used, but resulted in the same solution, or that a diagram is a visual representation of a symbolic algorithm. The big mathematical concept connection referred to when the class compared several high-level connections to develop a bigger understanding about a mathematical idea. The evolution of these connections, as observed in the classroom can be seen below.

In the early semester, sessions 1-3, Cathleen’s class did not focus on many connections, and the few times they did, they were low-level connections. On session 4 there were three observed connections, one a low-level, and two high-level. The low level connection used the same arithmetic, and the two high level connections identified
strategies. Both student presenters were able to find connections easily, and the second presenter showed body language of literally moving towards the other poster to identify the connections before the teacher even finished asking for connections.

Session 5 was an important session in this study, because Cathleen’s class showed a dramatic shift in math-talk. There were many observed connections, and specifically four high-level connections. In this session, Cathleen did not focus on students making connections between each other’s work, but instead, how these connections build on the overall big mathematical idea: that you can compare ratios in different ways. Each student/small group of students solved the problem a slightly different way. Some students solved it using fractions, others using part: part ratios, other using decimals or percents, and there were different ways of comparing the fractions using student invented strategies and not necessarily the traditional (common denominator) algorithm. Cathleen stayed focused on this big idea and whenever she asked questions, it was to guide student to think about this idea of comparison.

On session 6, there were three high-level connections and one overall big math idea connection. The focus of the lesson, again, is that you can find the solution using multiple different strategies. Cathleen focused on the idea that the purpose of the math lesson is to understand and compare different strategies. Other mathematical objectives, such as arithmetic, formula setup, and math rules are embedded in each of the strategies.

On session 7 there were four observed high-level connections and one overall big math idea. This session was different, and had very different expectations for students. The focus of this lesson was for students to explain their work, and then to find
similarities and differences amongst their peers’ work. The purpose of this lesson though, was not necessarily in getting the right answer, but seeing that there were at least four different ways to arrive at that answer. Students were held accountable for identifying how their work was similar and different, and defending their strategy.

In summary, there was a shift in connections beginning with session 4, and dramatically shifting in session 5. Cathleen focused on the big mathematical concept connection and more high-level connections were explicitly stated. Further, students were also expected to relate their work to another student’s work before they completed their presentation. This norm supported students with not only the opportunity, but the expectation to make connections.

**Research Question 1c**

In what ways do the strategy maps and the math-talk develop together throughout a semester?

To determine how strategy maps and math-talk develop together, I first analyzed the math-talk and how it developed over the semester. Cathleen grew in every category of math-talk, but she made her biggest growth on session 5. There could have been a variety of reasons to explain this growth. I examined the researcher notes to determine the role of strategy maps, virtual coaching, and other practices that could have supported the growth in math-talk. This section explains the role of strategy maps, while the other potential influencers are explained in research question 2.

This was the first session that Cathleen created her own style of a strategy map, independently. In the section below, I describe her growth in four of the categories of
math-talk and the ways in which strategy maps may have influenced this growth in math-talk.

**Questioning.** On session 5, the class shifted from level 1 questioning to level 3. Prior to session 5, the questioning was done by the teacher, and the purpose was to ask students about their thinking. On session 5, there was evidence of level 2 and level 3 questioning. Students were giving detailed answers about their thinking and they were questioning each other about instances within a strategy.

**Math thinking.** The class shifted from level 2 math thinking to level 3. Prior to session 5, the students explained their strategies after the teacher prompted them, but on session 5, their explanations became more elaborate, and used techniques such as comparing and contrasting other student’s strategies.

**Teaching and learning math content.** The class shifted from level 2 math content to level 3. Prior to session 5, the teacher stood at the back of the classroom, and allowed student strategies to be the focus of the lesson. On session 5, the teacher and students were sitting around a table, and the students used their strategies to debate, interrupt respectfully, and defend their strategies. Her role shifted from teacher to facilitator. Students were teaching each other how they developed their strategy, and how their strategy can be used to find the solution to the problem. The teacher only spoke to supports students with clarifying the mathematics. (i.e.: “is that part/part or part/whole?”)

**Evaluation.** The class shifted from level 1 evaluation to level 2. Prior to session 5, the teacher was responsible for letting students know when they have completed their
presentation. She would hold them responsible for answering her follow up questions. On session 5, there was a shift from the teacher doing the follow up questions, to students asking one another about their strategy. In this component of math-talk, the students were also responsible for agreeing and disagreeing with one another. The students asked the follow up questions, and a presentation was only complete when the students all agreed.

Strategy maps may have influenced the class’ growth in math-talk. Cathleen created her first original strategy map on session 5. This was also the session that had the greatest growth. There was a notable change in the presentation that can be contributed to the strategy map. On session 5, Cathleen did not allow every group to present their findings. Instead, she chose only 4 groups to present. She selected the presenters based on what she found during the monitoring stage, and selected only four students in order to prevent duplicate presentations. In terms of math-talk, this provided presenting groups more time to fully explain their strategy, and focused students to make connections about different ways to solve the problem, rather than simply identifying another group who did it the same way. Further, since they were four unique strategies, students used questioning skills and debate skills to understand one another’s thinking, and determine if the strategy was mathematically sound.

Cathleen began to develop a noticeable flow on sessions 6 and 7 by purposefully sequencing the student presenters. This was seen on the strategy map as arrows to connect the student groups. She was thoughtful of the presentation and the order in which students presented their strategies. Students continued to grow in math-talk by
discussing how their strategies were similar and different from others. They asked clarifying questions, agreed and disagreed, debated and defended their strategies, and made connections. By focusing on the order of student presentations, Cathleen set up the optimal opportunity for students to deeply explain their work, and make connections with previous student’s work.

**Conclusion**

Strategy maps focused the teacher in order to implement, thoughtfully, four of the Five Practices; anticipating, monitoring, selecting and sequencing. This implementation provided the ideal situation for students to identify connections between their works. The originally intended strategy map used cross-links to identify the connections between students’ work. Cathleen never used cross-links. Instead, she made connections the focus of her math lesson. She completely embraced the idea, and held students accountable for making connections. By the end of the semester, Cathleen facilitated a session where students were able to understand how all of the different strategies were connected to the overall big mathematical concept. The researcher notes were analyzed to find other reasons why connections were observed in the class, even though they were not viewed on the strategy maps. These are explained in research question 2.

Strategy maps may have supported the teacher in making connections during math-talk because they provided an authentic way for her to set up the ideal environment to find connections with ease. However, since connections were never a part of her authentic strategy maps, there are likely other explanations for why more connections were observed during math talk.
Research Question 2

In what ways does virtual math coaching support the teacher in utilizing strategy maps as a tool to increase the level of math-talk?

In this study, math coaching was used to support the teacher with the implementation of math-talk. I used a strategy map as a tool to hold the teacher responsible for thinking about each of the Five Practices, specifically making connections. In order to do this, I needed to focus the virtual coaching sessions in order to use heavy coaching (Killion, 2010) and content coaching (West & Staub, 2003). I developed five coaching topics, and used these topics to focus the virtual coaching sessions. The coaching topics were:

- Coaching Topic 1: Strategy Maps
- Coaching Topic 2: Similarities
- Coaching Topic 3: Representations
- Coaching Topic 4: Examples
- Coaching Topic 5: Efficiency

I introduced these topics slowly over time, so that the teacher would have one or two specific goals per session, to support her with identifying connections and increasing the overall math-talk.

In order to coach Cathleen on strategy maps, she first needed to create a strategy map. This was the first challenge of virtual coaching. Cathleen did not complete a strategy map until session 4. Throughout the coaching of sessions 1-3, I reminded Cathleen that I needed a graphic organizer and that the layout should include nodes and
cross-links. Within the cross-links, she should include the connections that she hoped to make during the lesson. On session 3, she seemed frustrated and provided her teacher notes. These notes showed me that she was thinking about the student presentations, but it was still a “show and tell” model, and did not fully implement the Five Practices. I asked her about the strategy maps, and she explained that her frustration was that she didn’t think they were useful. She felt that they were important to my research, and therefore was completing them as a favor to me. The only value that they held for her was that it helped her think about the sequencing of the students’ work. This told me that virtual coaching would only be effective for the strategy map if she fully understood the concept behind the tool. Therefore, before session 4, we met face-to-face.

During the face-to-face coaching for session 4 we created a strategy map together. This was done to make sure that Cathleen understood the strategy map, and the reason for each component. After working together, she expressed that she had a better understanding of strategy maps, and that she thought they were useful. When I observed session 4, I found that, for the first time, there were two high-level connections made between student strategies. One of these high-level connections was predicted ahead of time, when we were anticipating the student strategies during the face-to-face coaching.

Cathleen created a strategy map on session 5; however it was different from the strategy map directions that I supplied. Instead, it was an authentic strategy map that she created by herself to prepare for making connections during the math-talk.

Cathleen created this strategy map her own way, which showed a high level of understanding and thoughtful implementation of at least four of the Five Practices:
anticipating, monitoring, selecting, and sequencing. It was for this reason, that I coached her to continue making strategy maps using her style, and not request that she comply with the layout as originally intended.

Cathleen continued to create strategy maps for each additional session, using her own style. Her strategy maps even evolved to include sequencing and student misconceptions. These strategy maps included many of the elements that I had originally intended, but never included any sort of cross-link to explicitly identify connections to be stated during the math-talk. Her strategy maps did not explicitly identify connections, but Cathleen was able to maintain a focus on connections because of the virtual coaching component.

After each session, I collected data on math-talk, and specifically the quality and quantity of connections explicitly stated during the lesson. Then, I referred to the coaching topics to determine specific goals for Cathleen to focus on for the next session. Early in the semester, I focused on coaching Cathleen on coaching topic 1: strategy maps and coaching topic 2: similarities. I used direct phrases such as:

**Coaching topic 1: Strategy maps.** “Did you create a strategy map for this lesson? This is an important part of the study. Please let me know if you would like additional support with the strategy map.”

**Coaching topic 2: Similarities.** “I will specifically be looking for connections and to continue to use the prompts, ‘What is similar about your poster and the other posters? What is different from your poster and the other posters?’”
Once she mastered these coaching topics, I was able to coach her on the remaining topics. I continued to use direct, heavy coaching, while congratulating her on her success of previous coaching topics. Examples of those phrases are listed below.

**Coaching topic 3: Representations.** “I noticed that you were hoping to get a pictorial example when you asked students if anyone had a picture. If nobody in the class needs the pictorial example to fully understand the problem that is okay. If however, a student appears to be stuck, you can provide the following prompt during the small group time: ‘Can you show me using a picture?’ or even more specific, ‘Can you draw out the cups and the juice?’ Avoid drawing it for them, but instead, suggest that they draw, and listen to their ideas about how they could draw a model.”

**Coaching topic 4: Examples.** “I noticed that your class discussed a misconception at the end of the presentation time. The students who were confused seemed relaxed, and there seemed to be a sense of trust in your class. This is a good indication that you can discuss misconceptions and non-examples regularly during the presentations. Continue to foster this trusting environment.”

**Coaching topic 5: Efficiency.** “I would like for you to work on evaluation in terms of efficiency. Your students are ready to have a discussion about which strategy is the most efficient. This one is a challenging theme to manage as a teacher. The challenge will be in having the discussion about which one is the most efficient, while at the same time, valuing even the inefficient algorithms that may make the most sense. Do you remember the mango problem? The most efficient solution was actually the
The symbolic algorithm was very complex and inefficient. Remember to value both for different reasons.”

The virtual coaching done in this study may have supported Cathleen in making connections during the math-talk. Connections evolved, both in quantity and sophistication, even though those connections were not explicitly stated on the strategy map. This could be because the virtual coaching relied on coaching topics that were specifically chosen to focus on connections. After each session, Cathleen was provided feedback about the session, and connections were the focus of that feedback. We discussed ways to increase the quantity and quality of those connections, until Cathleen discovered that the most important purpose of the math session was that student connected all of the student strategies to the overall big mathematical concept. Cathleen reported this epiphany after session 5. This connection to the overall big mathematical concept was seen in session 5, and in each additional session.

Virtual coaching played an important role in supporting the teacher to use strategy maps, but I found another important role of virtual coaching. After each session, I analyzed the data to develop the coaching email. During the researcher notes coding, I discovered the theme; teacher values. These were trends that were observed in Cathleen’s behavior that can be best identified as personal values. These values were already ingrained in her before she began the research, and continued to grow as the research progressed: l) teacher values student strategies, m) teacher values multiple representations, n) there is respect amongst students, o) teacher willingly applies coaching advice.
Three of these teacher values show that Cathleen embraced the goals of this study. As she developed math-talk, and moved from level 0 to level 3, she allowed students to present their strategies using multiple representations in a respectful and trusting manner. Cathleen embraced these requirements from the beginning. I coached her in ways that supported these, but simply holding these values may have impacted her success in math-talk.

Cathleen also embraced the teacher/coach relationship. She was eager and willing to implement the suggestions made during the coaching sessions. If there were suggestions that needed further explanation, she asked questions. Cathleen’s eager attitude toward coaching may have also supported her success in math-talk.

In summary, virtual coaching is complex. In order to coach on strategy maps, Cathleen needed to fully understand the concept behind the tool, and to thoughtfully create one for each session. Once she was regularly creating the strategy maps, there was evidence of four of the Five Practices and the flow of her lessons greatly improved. In order to incorporate the fifth practice, connections, virtual coaching focused on specific topics to give Cathleen a targeted goal for making these connections in each following session. Cathleen’s teacher values played an important role during the coaching. Because she embraced the goals of the research, and was an eager participant, the coaching was collaborative and productive. As coaching topics were learned, the math-talk increased and connections were the focus of the presentations.
Implications for Further Research

**Strategy maps.** For this research, I designed a model of a strategy map that contained key elements from previously researched concept maps such as nodes and cross-links (Novak & Cañas, 2008). Cathleen did not find this tool useful when creating it as directed, but she was able to use the theory to create her own style of a strategy map. Cathleen did not ever incorporate the cross-links or any other form of written connection on her strategy map. It was solely the element of virtual coaching that supported Cathleen in making connections. Further research could determine if strategy maps are the appropriate location for teachers to explicitly write down the connections that they hope to make during the math-talk.

Through this research, I discovered that unless the participant understands, values, and embraces the theory behind strategy maps, it would not be a useful tool. Therefore, I believe that it is the theory of the strategy maps, and the elements of the tool that give it its importance. Further research could look at other models, and determine if teacher created models are more useful, or if there is one model fits all situations.

At the end of the semester, Cathleen used arrows on her strategy map to designate sequencing. This resulted in a logical flow as students transitioned their presentations. Research should also explore how sequencing is used in strategy maps, and how this element becomes important to the flow of the lesson.

**Connections.** When I established this research, I wanted to understand why connections were the most difficult practice (Stein & Smith, 2011) and how teachers could support students in making connections between each other’s math strategies. I
found that it was just as important to focus on the connection to the overall mathematical objective, because even if a teacher has a wonderful math-talk, if it is not specific about the objective, the students may not understand how these smaller connections relate to their math unit. Cathleen realized this mid semester and began implementing this in all her classes.

During the coaching sessions, I always reminded Cathleen that the focus of this research was on finding connections between students’ work. On session 1, she did not focus on this, and did not explicitly state connections. As the semester continued, she would make one or two connections, but they were low-level. On session 5, the class shifted, and there were not any low-level connections, and instead, four high-level connections. Further, on session 5, Cathleen focused on the big mathematical concept. I wrote her about these findings and asked why she thought the shift happened. Her response shows how much she values these connections to the big mathematical concept.

“I’m more interested in the kids knowing how everything is connected and I want them to see the big picture. They have been trained to think about what formula to use, but I want them to think of all the different ways. I want them to think like engineers, so they need to understand how their small part and the other small parts are related to the big picture, because the big picture is only going to get bigger and bigger.”

Further research should consider connections in two areas; connections between students’ work, and connections to the overall mathematical concept. Both of these
connections should be discussed during the math-talk, and it is possible that the math-talk rubric should be modified to extend to both types of connections.

**Other tools.** In this research, I implemented a tool called a strategy map to support the teacher with the implementation of the Five Practices in order to increase math-talk. The purpose of selecting this tool was to create accountability for implementing the Five Practices. Early in the semester, Cathleen did not use the strategy map, and there was little evidence that she had thought through the Five Practices. After she created the strategy map, the flow changed, and the Five Practices were observed in each session. Cathleen expressed that because the strategy map was handwritten and created during the monitoring phase, it held her accountable for the monitoring, selecting, and sequencing practices. Further research should investigate other tools to support teachers with the implementation of the Five Practices. These tools could be similar graphic organizers, or virtual web tools.

**Implications for Teacher Education**

The National Council of Teachers of Mathematics (NCTM, 2000) recommends that teachers give students opportunities to reason about mathematics through discussion. Such discussions allow students the opportunity to share ideas and clarify understandings, develop convincing arguments regarding why and how things work, develop a language for expressing mathematical ideas, and learn to see things from other perspectives (NCTM, 2000). Further, the Common Core State Standards (CCSS), which have been adopted by a majority of states, requires students to problem solve, and grapple with mathematical ideas, rather than only learn through rote teaching (National Governors
In order for students to become problem solvers, there needs to be more teacher education on how teachers can facilitate a session rich in math-talk. The following is a description of areas in teacher education that need to be addressed in order to support teachers in implementing a math-talk classroom.

**Math-talk paradigm shift.** Teacher education in math-talk begins with an important paradigm shift. Traditionally, the teacher physically stands at the front of the classroom and teaches a specific mathematical objective. The instruction is traditionally delivered by the teacher showing students how to use a procedure, followed by students practicing the procedure on several similar problems (Stein & Smith, 2011). Because this traditional style of teaching uses only one procedure, and that procedure was supplied by the teacher without student discovery, there is almost no opportunity for math-talk.

One of the reasons why this intervention was successful was that Cathleen already valued math-talk, and the coaching sessions were efficient because they did not need to focus on supporting the teacher to make this math-talk paradigm shift. This is especially important to teacher educators because it shows the how efficient and collaborate coaching can be when this challenge is eliminated. Had the teacher not valued math-talk, one of two outcomes would have likely taken place. (1) I would have spent valuable coaching time on supporting the teacher in exploring situations where discourse supported her own understanding of a topic in order to shift her perspective to value math-talk. (2) The teacher would have resisted the implementation of math-talk,
resulting in confusion in the classroom, and students not knowing how to show evidence of learning.

Cathleen showed evidence of understanding and implementing math-talk from the first observation. During session 1, she was physically seen in the back of the classroom. During session 3, she was making discussing similarities and differences. Because Cathleen began the study in this mindset, I did not need to convince her of the benefits of the goals of math-talk, instead, I would offer specific suggestions related to growth in a specific component of the math-talk rubric. She eagerly implemented those suggestions.

This is important to this research because an eager and willing participant would show more growth in a shorter amount of time. Cathleen wanted to learn how to teach using math-talk, and the Five Practices, and she even practiced this style of teaching when the session wasn’t observed. Because Cathleen already had the math-talk paradigm shift, we were able to immediately focus on other topics to improve the level of math-talk in her classroom.

Teacher education should focus on teaching teachers the rationale behind math-talk, and reform-based curriculum in order to provide an opportunity for the paradigm shift towards teaching using math-talk. Specifically, teacher education can focus on the following math-talk objectives:

- Questioning needs to shift to focus from answers to thinking and shifts from the teacher as questioner to students as questioners.
- Students need to learn to explain and articulate their math thoughts so that the classroom community will grow to support students acting in central or leading roles.
- There needs to be a shift from the teacher as the source of all math content to students contributing their ideas about content with confidence. The teacher should then utilize those student ideas.
- Students need to increase their engagement, take ownership of learning for them, and grow to value the learning of others.
- There needs to be a shift from teacher as the critic, helper, and supporter to students taking on this role as well and from answers to thinking (Hufford-Ackles, 1999).

**Problem selection.** Ball (1991) found that selecting a rich task is an essential step towards allowing students and opportunity for true problem solving situation. Teacher education could support teachers on problem selection. Specifically, this should include how to determine if a problem is rich enough for math-talk. For this research, I gave Cathleen a database of field tested questions. This database included questions that have appeared in various math professional development literatures and were chosen because they were rich problems that had multiple points of entry. That is, students with various mathematical abilities could solve the problem, and there was not an obvious single direction to solve the problem. This ensured that all students could begin working and that multiple strategies were likely to appear.
Even though Cathleen was given a database of rich problems, she still needed to choose the correct problem for her class. She needed to consider the mathematical objective, and the mathematical level of her students. After session five, I asked her how she chose a problem, she explained that, “choosing the problem can make or break math-talk”.

I've chosen problems pretty carefully-trying to select those that are not too easy, but where they have enough math background to approach a problem from different angles. This gives my different-learners the opportunity to shine, because they aren't all singing the same song! I've tried some that I thought would be a reach- the mango problem was a tough one- and was so pleased when they proudly unlocked the problem from two different directions!

Cathleen’s selection needed to consider student ability, current math topic, and language presented in the problem. Math education should focus on how to determine this selection, and how to scaffold a problem to meet the student needs of various mathematical or age levels.

**Classroom norms.** Math education research should consider the implementation of classroom norms and the way in which these norms enhance the math-talk environment. Yackel, Cobb, and, Wood (1993) found that small group discussions are more productive when norms are established. Cathleen used several norms in her class to support the implementation of math-talk. One of the overall aims of math-talk is for students to speak for themselves. This began before the actual presentation, during group
work. Students had their own voice, and knew when to ask questions. During the presentation, they explained their strategy, and thinking along the way, including misconceptions and ah-ha moments. They defended their strategy by answering other student’s questions. They compared their strategy with others, and talked about the mathematics with other students. Cathleen knew that there were several obstacles in the traditional classroom, so she developed norms that allowed the students to be the speakers.

**Pair group norms.** Cathleen established “pair norms” early in the semester. This was not anything that I coached her on, just something found during data analysis that was interesting, and I believe a benefit for math-talk. Whenever students worked in pairs, they verified that their partner had enough individual time before they grouped together. This was done either through body language, or directly asking the partner. Once they grouped together, both students had an opportunity to discuss their work before deciding on a path to continue working. If a partner was taking the lead, he or she used the norm of verifying that their partner was following their train of thought. Students often used the phrase “Does that make sense?” to verify that the other student was following their thinking. This was the first step in giving students a voice during the presentation. Students had the opportunity to practice speaking to one other person about their thinking. This practice was necessary in order to put their thinking into words and develop a flow.

**Student presenters.** Cathleen created a student centered norm from the very first session. Students understood that she would physically stand in the back of the
classroom, and they would present their posters by physically standing in the front of the classroom. The students were expected to explain their strategy and that the teacher would only ask clarifying questions. The students were the voice of their work.

**Interruptions.** Cathleen fostered student talk by limiting interruptions. She would only interrupt student presentations to focus on the mathematics presented. For example, she might ask the presenters to rephrase an important statement, or ask students how they arrived at a solution if their work was abbreviated. As the semester evolved, students were interrupting presentations to ask these same questions that were modeled earlier by Cathleen. The students used Cathleen’s prior modeling to learn how to interrupt respectfully, and the presenters did not appear to be frustrated with the interruption.

**Connections.** Discovering connections was the heart of the math class, especially near the end of the semester. On sessions 6 and 7, students were expected to make connections between each other’s work before they were permitted to sit in their chairs. Once this expectation was a norm, the transition between presentations was fluid and built upon previous student’s work.

**Trust.** Perhaps the most influential norm in Cathleen’s classroom was trust. Students appeared at ease in the classroom, and comments were almost entirely respectful. Cathleen modeled this early in the semester, but it was first tested during session 5 when student misconceptions were challenged and debated respectfully. When a student disagreed with the logic in a strategy, they asked questions. In this session, every student group contributed some new math concept to the whole class.
discussion. Cathleen continued to focus on the idea of quantity of strategies, and valued this over efficiency or the traditional algorithm.

**Implications for Math Coaches**

Math coaching is becoming common practice in school divisions, but the responsibilities of coaches and the tools that are available vary (Campbell & Malkus, 2011). Math coaches, who are looking for specific ways to increase math-talk, could use strategy maps as hands on, accountable way to implement the Five Practices, and increase the level of math-talk in the classroom. This study used virtual coaching and strategy maps to examine how the Five Practices could support the teacher in making connections and increase math-talk. Virtual coaching played an important role in this intervention. In this study, a mixture of coaching heavy and coaching light (Knight, 2009) was used to support Cathleen in creating the strategy maps. Cathleen did not send a strategy map for the first two sessions. When I analyzed the researcher notes, I discovered that I was coaching light in the early sessions. The coaching used phrases such as “Did you make a strategy map? If so, can you send it to me? This is the heart of my research, so I’m interested to find out if it supported you as a teacher.” Cathleen understood the theory of the Five Practices, but it wasn’t until we created a strategy map in session 4 that she was accountable for actually doing the Five Practices. On session 5, Cathleen created her own strategy map, and showed evidence of thoughtfully implementing the Five Practices, and writing them down on her version of a strategy map. This session showed a dramatic shift in the level of math-talk and the number of connections explicitly stated. This shift could have been due to Cathleen creating her
own style of a strategy map or in the way that I shifted to coaching heavy. The coaching that I used before session 5 was focused on the coaching topics. I explain evidence and give specific suggestions. The following is a sample of coaching heavy from session 4:

It is always a pleasure to watch your videos. I am already seeing a wonderful growth in your students as they explain their thinking. In both recent videos, the presenters were able to explain their strategy fluidly. They could walk through their procedure with little to no prompting. Both presenters were able to find similarities with other student strategies. The second video (girl with the headband) was so eager to find connections, that she was literally smiling and walking over to the next poster before you even finished your prompt. This shows evidence that your students understand that there is an expectation to identify similarities and to make connections. It is powerful to see this expectation already set so early in the school year.

Your students can identify similarities with ease. Your next step is to have them describe the similarities in detail. If a student says that another student use a diagram just like they did, you can ask them to describe where it was the same, and where it was different. Sometimes, it is as simple as one student used dots, and another use squares. Other times, it will be more difficult to describe the similarities and differences, but keep prompting them to try.

Another area to have students grow is in representations. Since your students are already using the terms “diagram” and “mathy”, it appears that they understand and value different representations. If you have manipulatives available, students
can work on concrete representations, and make connections to the diagrams (pictorial) and “mathy” (abstract/symbolic) representations.

On session 6 and 7, Cathleen’s strategy maps included all three of the previous Practices, but also included a fourth Practice, sequencing. On these sessions, Cathleen’s strategy maps included arrows to designate the order of presentations. Further, I maintained heavy coaching, and addressed specific instances when Cathleen focused on connections, or could have probed differently to allow connections to emerge. During the class session, there was a noticeable change in flow. This new flow was purposeful and transitions between student strategies were logical. This logical flow supported students to verbally state the similarities between each other’s strategies during their presentation. In this study, Cathleen did not ever include the fifth practice, connections, on her strategy map, however, her class made important connections on session 5, 6, and 7. This could have been because of the focus during the virtual coaching sessions. During these sessions, I provided Cathleen with specific quotes to ask students in order to reveal connections during the presentation. These quotes included prompts designed to allow students the opportunity to consider similarities, differences, multiple representations, misconceptions, and efficiency. Cathleen used these prompts to focus mostly on similarities and different representations.

The design and implication of this study was thoughtfully prepared. It is important for coaches who wish to utilize strategy maps to consider the deliberate setup
of this study, and how a different teacher and different coach may have different results. The following section details the rationale and outcome of the design.

**Focused coaching.** Math coaches are faced with a variety of challenges and roles (Knight, 2009). Depending on a school’s needs a math coach may be required to coach multiple content areas, new math content, new math pedagogy, or even data analysis and testing strategies. With so many potential responsibilities, it can be difficult for the coach to know how to balance his or her time across these various roles (Knight, 2009). In this study, I was focused on one goal: to increase math-talk by explicitly making connections between students’ work. Every coaching move was deliberately chosen to get her to focus on connections. There were certainly other areas that I could have chosen to coach. These include: math content that was missed during a problem, various manipulatives that could have been used to solve a problem, or classroom management strategies. The temptation to coach her on these, especially when she asked specifically for classroom management strategies, was difficult to overcome, but since I choose to implement content coaching (West & Cameron, 2013), and I remained focused on those goals, the implementation was successful. Cathleen understood that I would only comment on instances that supported our goal. By the third session, she only asked questions about this goal. This was an important step in making the coaching more efficient and direct, and an element that lead to an easy transition from coaching light to coaching heavy (Killion, 2010; Knight, 2009). After this point, all of our conversations revolved around our goal.
As a coach, it was important for me to understand how the theories of math-talk, the Five Practices, and strategy maps related to one another, and more importantly, how connections related to each of them. This understanding allowed me to create the coaching topics, a necessary component of content coaching (West & Staub, 2003) to be used to focus our coaching sessions.

**Coaching topics.** I chose the following coaching topics to focus the virtual coaching sessions on increasing math-talk through connections.

- Coaching Topic 1: strategy maps
- Coaching Topic 2: similarities
- Coaching Topic 3: representations
- Coaching Topic 4: examples
- Coaching Topic 5: efficiency

These carefully chosen topics ensured that my suggestions were specific and directly related to our goal. These topics were introduced slowly. A new topic was only introduced if Cathleen showed evidence of beginning that topic, or she had mastered the previous topic. Not all coaching topics were mastered. In fact, coaching topic 4: examples and non-examples was only observed at the end of the semester though student misconceptions, and coaching topic 5: Efficiency was only touched on during the last session. These were planned as final coaching topics, and were not expected to be mastered due to student needs. In the section below I describe the rationale for choosing coaching topics.
Coaching topic 1: Strategy maps. This topic was purposefully selected first because it was important that the teacher implement the strategy map since it was the focus of this study. It was an immediate focus, and quick mastery ensured that further coaching topics could be learned.

Coaching topic 2: Similarities. This topic was chosen because Cathleen had already experienced making connections though similarities during the graduate level course. Since I knew that she had prior knowledge in this area, and this topic directly supported our goal, it was an ideal beginning topic. This was coached simultaneously with coaching topic 1.

Coaching topic 3: Representations. This topic was chosen because Cathleen had already expressed her personal goal of trying to figure out traditional mathematical algorithms using diagrams and pictures, during the graduate level course. This topic was coached after she started showing evidence of implementing coaching topic 1.

Coaching topic 4: Examples. This topic was chosen as a way of looking at misconceptions through examples and non-examples. In my coaching experience, I have found that students first need to feel safe during their presentations. This meant that the teacher needed to show value to all student strategies, and not show negative judgment towards a group of students who solved the problem incorrectly. Instead, the teacher needed to show students that their mistakes were valued, and explored in order to further the understanding of all students. This coaching topic was introduced near the end of the semester. This allowed the teacher and students time to build this trust. Also, the previous coaching topics supported this safety because when the teacher found
connections through similarities and differences, or through multiple representations, the presentations were inherently different. Once the students learned that the teacher valued multiple strategies, they felt safe that their strategy would be valued.

*Coaching topic 5: Efficiency.* This topic was chosen because often in mathematics, it is important to understand the most efficient strategy. This was often the traditional algorithm. While the traditional algorithm was indeed efficient, it was very abstract, and students needed to understand how this abstract algorithm was derived.

This topic was only observed on session 7. During this session, Cathleen made a brief comment about how a poster shows an efficient method, but she didn’t seem to value this efficiency. She didn’t praise the strategy or speak negatively about it, it was simply a comment. Because this was the last coaching topic introduced, it was predictable that it would have been the last coaching topic observed. This was done for a purposeful reason - it was important to first set student norms. That is, students needed to feel comfortable in developing their own strategy and comfortable knowing that their strategy is just as valuable to the class as any other. In fact, Cathleen even expressed that she valued all students’ work, and she didn’t want them to try to do it “her” way. If this study were to continue for longer than a semester, I would expect that this coaching topic develops slowly. Students were still developing the norms that Cathleen fostered, and the next step in these norms would be to develop their own ways of thinking, but then to refine their way into an efficient and elegant algorithm.

The advantages of focusing on efficiency late in the semester were in the objectives of math-talk. Cathleen didn’t want her students to simply go straight to the
abstract solution. Similarly to her experiences in the graduate level course, she wanted her students to learn how to draw more diagrams and stop going right to the algorithm. She was skeptical about talking about efficiency and thought that this conversation would deter logical thinking. She decided that she would only discuss this when her students were ready.

If this study were replicated, I would recommend that the teacher and coach decide the coaching topics collaboratively. When the areas of growth are compatible with both the teacher and coach, the coaching is collaborative (Rawding & Wills, 2012). This guide allowed the coach to remain focused on the predetermined, specific, collaborative goal.

In summary, the coaching topics were chosen based on specific strategies that were implemented to support the overall goal. The topics were introduced slowly so that the teacher could maintain a focus on specific strategies. Also, there were coaching topics that were only introduced late in the process, such as efficiency.

**Reform based coaching.** Perhaps the biggest implication for coaching came from research about learning. Pólya (1957) describes the importance of students solving problems using their own strategies in order to make meaning. If the teacher simply showed students the traditional procedures, and expected them to create meaning through completing additional problems using this procedure, the student would never truly grasp the mathematics behind those procedures. Similarly, in this study, I found that when I tried to teach the procedure of creating a strategy map, Cathleen did not value it. It wasn’t until she tried to create her own strategy map, using her own thinking and style
that best fit her thinking, that she developed a tool that was authentically useful to
her. After she submitted her strategy map on session 5, I asked, “How do you decide to
do your organizer?” She explained that she knew that I wanted it to be done a certain
way, and she understood why it should be done that way, and was even specific about
having connections on the arrows. But, she didn’t find that to be useful to her. She said
that since she was teaching her students to solve problems in whatever way that they
found useful, that she would take that same advice. She told me that it was most useful
for her to write down each of the student’s names at the top, and a brief description
below. Some of these descriptions would be short, and some symbolic. Upon further
examination, I discovered that in each group, she had identified the strategy and any
similarities or differences that this group made. I reflected on this and discovered
something important about my coaching.

Students do not learn the big ideas of mathematics by practicing an algorithm
invented by someone else (Pólya, 1957). Instead, they learn the big ideas by developing
their own algorithm that is based off of meaning and context. It was a very humbling
experience for me to realize that I was trying to apply this “I show you how” method to a
teacher who is already using a constructivist approach with her students.

Once I had this epiphany, I decided to tell her to do it however she liked, and to
be able to explain this to me using whatever method she thought would be best. This
release of the prescribed strategy map resulted in creation of quality strategy maps and a
dramatic shift increasing the level of math-talk in her classroom.
The ideas behind reform based curriculum can be applied to coaching. This reform based coaching would require the coach to consider coaching the teacher in the same manner that they expect the teacher to teach her students. The coach would begin an equal partnership with the teacher, or even allowing the teacher to take the lead on the goals that are important to her. The coach would need to have a wealth of knowledge in mathematics pedagogy, so that when the teacher asks for support in an area, such as connections, that the coach has a plethora of strategies and suggestions to support the teacher in this goal.

Virtual coaching. Virtual coaching was the primary method of coaching the teacher in this study. This process required the teacher to record her lesson and upload the recording to a website, www.beasmartercookie.com. Then, I watched the video and coded it in terms of math-talk, number of connections, and development of coaching topics. I wrote an email to the teacher describing the findings. I also wrote suggestions for improving in each of these specific areas. My style of coaching began with coaching light, where I complemented her teaching style and built a relationship, but quickly moved to coaching heavy, where I was direct, with examples as evidence, and avoided judgment. The purpose of coaching heavy was not necessarily to be liked, but rather to improve teaching and learning (Knight, 2009). I included questions to consider about how she thought the lesson went in order to begin a dialogue about the lesson. Cathleen would respond to the email with brief follow up questions, usually about how to implement the suggestions if they were not clear. Virtual coaching had advantages and disadvantages that should be considered before choosing this style of coaching. The
The biggest advantages of virtual coaching were in terms of time and travel (Rock, et al., 2011). Other advantages that were found include time stamping specific instances and using video evidence, and the opportunity to reread the coaching email with an effort to coach specific without showing judgment. In each of these areas, virtual coaching was superior to traditional face-to-face coaching.

Coaching was done in a separate location from the teaching. There was no commuting time needed for the coach to arrive at the location of the classroom. Also, some classes did not always begin with the problem at the start of the class period. There were times when the teacher needed to discuss homework, field trip forms, or other housekeeping before the math problem began. Because this coaching was done virtually, the coach could simply skip these segments of the video and begin watching at the start of the lesson. After the lesson, the traditional coaching model requires the coach to meet face-to-face with the teacher to discuss the coaching. Again, since virtual coaching was done remotely, there was no need to consider commuting time. The face-to-face meetings are typically longer because of off topic casual conversation, while virtual coaching is shorter because it avoided this. The remote model of virtual coaching reduced the time commitment of coaching.

Virtual coaching implemented heavy coaching. This type of coaching focuses on producing results rather than the teacher feeling supported (Killion, 2010). Heavy coaching required the coach to be specific about instances where the teacher can improve her practice. One way that virtual coaching did this more effectively, was that the coach quoted and even time stamped the video for the teacher to use as a reference. The teacher
could then watch the video at that moment to understand the coaching comment with video context. This ability provided an environment for less judgment, and kept the comments and suggestions specific to the instance. At the end of the semester, Cathleen commented on how this was useful for her to understand how body language and how a student interruption could be respectful.

My general style for coaching was to comment on what she was doing (by providing examples) and how this action moved her class in a positive direction. I also identified the specific area of the math-talk rubric. Finally, I followed up with a strategy that supported further growth in these areas. The following is an excerpt from the coaching session after session 4.

Both presenters were able to find similarities with other student strategies. The [girl in the] second video was so eager to find connections, that she was literally smiling and walking over to the next poster before you even finished your prompt. This shows evidence that your students understand that there is an expectation to identify similarities and to make connections.

Your next step is to have them describe the similarities in detail. If a student says that another student used a diagram just like they did, you can ask them to describe where it was the same, and where it was different.

Virtual coaching has disadvantages to traditional face-to-face coaching. In session 4, I learned a very important lesson that was that virtual coaching was useful in areas that she had a good understanding. Cathleen did not have a good understanding of strategy maps, and my attempts at virtual coaching in this area were not successful. Before
session 4, we met face-to-face so that I could explain strategy maps differently, and she could practice making a strategy map in anticipation of session 4. After this face-to-face session, Cathleen understood the rationale for strategy maps, and created them for each following session.

Another disadvantage to virtual coaching is that it lacked the opportunity to build relationships. In this study, Cathleen and I had already established a relationship, and therefore, this relationship continued through the virtual coaching.

Virtual coaching was successful because both parties were vested in the overall goal. If the teacher was not interested in the virtual coaching, the teacher could have played a passive role in the feedback loop. In this study, Cathleen played an active role in the feedback. She responded to the questions, and showed evidence of reviewing the video and implementing suggestions. Virtual coaching has the potential to lack accountability if there is no follow-up. Therefore, a passive teacher may not have benefited from this style of learning.

**Teacher values.** Teacher values were the most important component of coaching in this study, and it is one that I didn’t have any control over. During the data analysis, I determined four teacher values that had an impact on this study; (1) teacher values student strategies, (2) teacher values multiple representations, (3) there is respect amongst students, (4) teacher willingly applies coaching advice. Cathleen was motivated and eager to learn more about math-talk. Her teacher values influenced the shift made in her class. Cathleen wanted to improve her math-talk, and she appeared to always report in a positive manner when I had questions about how she used the coaching moves.
**Independent practice.** Coaching was effective because there was a partnership between the teacher and the coach. This partnership required the coach to allow the teacher some independent time to implement the coaching suggestions.

In this study, Cathleen practiced math-talk in between observed sessions. This practice was obvious when I observed session 5. This session had a dramatic shift in the level of math-talk in almost all areas on the math-talk rubric.

The time period between session 4 and session 5 was almost a month, allowing for 16 school days. I investigated the events that happened during that non-observed time. After session 5, I asked Cathleen to think about her teaching. Specifically, I asked her “Do you teach this style regularly (i.e.: students solve it different ways, focus is on students’ work and math-talk)? If not, how often did you do it in the last month?”

“I have worked your problems and our math talk into class work 2-3 times each week- sometimes as a warm-up to another lesson. I am working hard to get the students to verbalized strategies and methods, to let go of memorization in favor of THINKING.”

I wondered if all teachers would practice as much as Cathleen. So, I decided to consider her teacher values and beliefs of teaching. Cathleen is a career switcher, and already had a successful career as an engineer. She believes that it is important for students to talk, develop ideas together, listen to each other, derive a solution their own way, and understand how their solution is part of the big picture. That is how she saw successful engineers in her previous career. She
also believes it is important for her to continue learning, and was pleased that her goals were aligned to the current trends in mathematics education. These teacher values are important to this research because the generalizations made about how Cathleen practiced during this one month period could be much different if a teacher did not share her values.

Between session 4 and session 5, Cathleen practiced 8-12 times. This practice could have been a problem that lasted the entire class period, or simply a warm up. When I coached her after session 5, she was very passionate about the big math idea. She thought that if students were to think like engineers, that they needed to see how their solution was just a small part of the big picture. She wanted her students to understand “the big picture, because the big picture is only going to get bigger and bigger.”

In summary, Cathleen’s class shifted on session 5 because she practiced math-talk. It was important for her to continue learning and challenging herself to teach math-talk. She valued all the components of this study; therefore, she was eager and willing to practice.

**Final Thoughts**

This complex study explored how a strategy map could be used to support a teacher with implementing the Five Practices, specifically connections, in order to increase the level of math-talk. The study was successful in that more connections were explicitly stated and the level of math-talk increased. Strategy
maps were used in partnership with virtual coaching. Additionally, the teacher was an eager participant who valued the goals of the study, and valued virtual coaching. The use of strategy maps and virtual coaching, along with an eager participant with compatible values, contributed to increased math-talk and overall success of this study.
APPENDIX A

IRB Approval

Office of Research Integrity and Assurance
Research Hall, 4400 University Drive, MS 6205, Fairfax, Virginia 22030
Phone: 703-993-5445, Fax: 703-993-9590

DATE: September 8, 2014
TO: Jennifer Sun
FROM: George Mason University IRB
Project Title: [638622-1] The Effects of Strategy Maps on Teacher's Ability to Make Connections During Math-Talk
SUBMISSION TYPE: New Project
ACTION: DETERMINATION OF EXEMPT STATUS
DECISION DATE: September 8, 2014
REVIEW CATEGORY: Exemption categories 1, 2

Thank you for your submission of New Project materials for this project. The Office of Research Integrity & Assurance (ORIA) has determined this project is EXEMPT FROM IRB REVIEW according to federal regulations.

Please remember that all research must be conducted as described in the submitted materials.

Please note that any revision to previously approved materials must be submitted to the ORIA prior to initiation. Please use the appropriate revision forms for this procedure.

If you have any questions, please contact Bess Dieljenbach at 703-993-4121 or edieffen@gmu.edu. Please include your project title and reference number in all correspondence with this committee.

This letter has been electronically signed in accordance with all applicable regulations, and a copy is retained within George Mason University IRB’s records.
APPENDIX B

IRB Approved Consent Forms

The Effects of Strategy Maps on Teacher’s Ability to Make Connections During Math Talk

INFORMED CONSENT FORM

RESEARCH PROCEDURES

This research is being conducted to study the ways in which professional development on strategy maps can support teachers with increased levels of math talk. In particular, the research will examine the development of the strategy maps and the ways they increase the number of connections made during a math lesson.

Prior to the research, you will participate in a graduate level course and a one hour professional development session to explain the research and procedures for planning your lessons. During the course of one semester, your strategy maps (part of your lesson plans) will be collected, and classes will be video-taped. Further, after each class, the researcher will interview you. The time commitment for each interview will vary between a few minutes to one hour. The total time commitment outside of your regularly scheduled class will not exceed five hours. The data will be analyzed throughout the course of the semester to give formative feedback and support.

If you agree to participate, you will be asked to allow the researchers to analyze your lesson plans, interviews, and video-recordings of your classes for researching purposes.

RISKS

There are no foreseeable risks for participating in this research.

BENEFITS

The benefits to you may include an increased understanding of math talk, and strategies to support implementation.

CONFIDENTIALITY

All data in this study will be confidential. Data collected in this study will be uploaded to a secure website: www.beasmartercookie.com. Each participant will manage their own account. This means that each participant can upload videos to their account and invite the researcher to view and comment on selected videos that they post for this study. Shared videos must remain available for up to five years. Once the research is complete, participants may choose to remove researcher access to their videos. The researcher account will be terminated within five years of the study.

Your name will not be included on any data, however, since video is used, you will be visible in the video.

IRB: For Official Use Only

Project Number: 638822-1
Additional data, such as interviews, comments, emails, transcripts, and strategy maps will remain confidential and stored on secured George Mason University property.

PARTICIPATION

Your participation is voluntary, and you may withdraw from the study at any time and for any reason. If you decide not to participate or if you withdraw from the study, there is no penalty or loss of benefits to which you are otherwise entitled. There are no costs to you or any other party. A reward of $75 will be distributed upon full completion of the study (approximately 20 weeks) to be spent on classroom supplies.

CONTACT

This research is being conducted by Theresa Wills as part of a dissertation. The principal investigator is Dr. Jennifer Suh, Associate Professor of Mathematics Education at George Mason University. You may contact them for questions or to report a research related problem at 703-774-7691 or 703-993-9119. You may contact the George Mason University Office of Research Integrity & Assurance at 703-993-4121 if you have questions or comments regarding your rights as a participant in the research.

This research has been reviewed according to George Mason University procedures governing your participation in this research.

CONSENT

I have read this form, all of my questions have been answered by the research staff, and I agree to participate in this study.

________________________________________
Name

________________________________________
Date of Signature

_______ I agree to video taping.

_______ I do not agree to video taping.
APPENDIX C

Session 1 Problem: The Avocado Problem

On Monday, Sammy the storekeeper decides to increase the price of avocados by 20%. On Tuesday, he increases this price by another 25%. a) What percent of the original avocado price is the price of avocados after both transactions? b) On Wednesday, Sammy decides to return the avocados to their original price. By what percent must he decrease the Tuesday price?
APPENDIX D

Session 2 Problem: Bag of Marbles Problem

Ms. Rhee’s math class was studying statistics. She brought in three bags containing red and blue marbles. Bag x contains 75 red and 25 blue marbles, bag y contains 40 red and 20 blue marbles, and bac z contains 100 red and 25 blue marbles. Ms. Rhee shook each bag. She asked the class, “If you close your eyes, reach into a bag, and remove 1 marble, which bag would give you the best chance of picking a blue marble?” Which bag would you choose? Explain why this bag gives you the best chance of picking a blue marble (Smith, Hughes, Engle, & Stein, 2009).
Session 3 and 4 Problem: Sheep and Chicken Problem

Farmer Cole has sheep and chickens. In all, she has 23 animals. One day, she counted the number of legs of her animals—and there were 74 legs. How many sheep and how many chickens are there?
Session 5 Problem: The Orange Juice Problem

Jackson and Mariah are in charge of making orange juice for the school dance. They make the juice by using orange concentrate and mixing it with water. Unfortunately they forgot the recipe so they try a few options. Which mix will taste the most “orangey”? Explain your answer.

Mix A: 2 cups orange concentrate 3 cups cold water
Mix B: 1 cup orange concentrate 4 cups cold water
Mix C: 1 cup orange concentrate 2 cups cold water
Mix D: 3 cups orange concentrate 5 cups cold water
APPENDIX G

Session 6 & 7 Problem: The Grapple Juice Problem

Ms. Johnson likes to mix grape juice with apple juice. She calls the mixture grapple juice. Ms. Johnson decides to mix a pitcher of grapple juice for Thanksgiving. She mixes 45 ounces of grapple juice that is 20 percent grape and 80 percent apple. After tasting the mixture, Ms. Johnson decides that it contains too much apple, so she adds 15 ounces of grape juice and mixes thoroughly. For Thanksgiving, she pours the juice from the contents of the whole pitcher into four large glasses, with the same portion in each glass. Elizabeth prefers grapple juice that is 25% grape and 75% apple. How many ounces of apple juice must she add to her glass to make her juice taste the way she likes it?
APPENDIX H

Professional Development Session: Professional Learning Session

The Five Practices

The following lesson has been adapted from: Smith, M., Hughes, E., Engle, R., & Stein, M. (2009). Orchestrating Discussions. Mathematics Teaching in the Middle School, 14(9), 549-556.

Goal: The teacher will be able to define, identify, and explore each of The Five Practices.

1) Welcome the teacher. Remind her of the study goals, outcomes, and teacher responsibilities.

2) Define the Five Practices

1. Anticipating likely student responses to challenging mathematical tasks;

2. Monitoring students’ actual responses to the tasks (while students work on the task in pairs or small groups);

3. Selecting particular students to present their mathematical work during the whole-class discussion;

4. Sequencing the student responses that will be displayed in a specific order; and
5. *Connecting* different students’ responses and connecting the responses to key mathematical ideas (Stein & Smith, 2011, p. 8).

3) Anticipating: Give the teacher the following task (Handout #1), and have her explore the anticipating practice by thinking of multiple ways of solving the problem. Remind her that the anticipating stage will include anything that she expects her students to do. This includes correct answers and misconceptions. Then, discuss this together.

4) Monitoring: Give the teacher the monitoring template tool (Handout #2) and sample students’ work (Handout #3). Have her analyze the students’ work and complete the strategy and who and what sections. Then, discuss this together.

5) Selecting: The teacher will decide which pieces of students’ work to discuss during the math-talk. Consider elements such as time, complexity, and similarity when choosing which items to discuss, and which to eliminate. Then, discuss this together.

6) Sequencing: The teachers will complete the order section of the monitoring template tool (Handout #2). Then, discuss this together.

7) Connection: The teachers will identify the connections they will make in the summary portion of their lesson. This will be discussed together.
8) Summary - Review the Five Practices with the teacher and remind her of her responsibility to the study in implementing The Five Practices on a regular basis.

9) Give the teacher the situation and students’ work (Handout #4).

10) Discuss the sequencing and rationale for the chosen sequencing.

11) Discuss and record major connections that need to be explicitly stated in the summary. Explain that these connections can be associated with the overall goal of the lesson, or simply connections that link one piece of student’s work to another.

12) Distribute Handout #5. Discuss how students’ work is represented as nodes, and how the connections are included as cross-links in the concept map.

14) Summary - A concept map is a tool used to support The Five Practices. It is a visual tool to aid the teacher in the sequencing and connections practices. It is important to explicitly verbalize the connections that are made on the concept map during the math-talk.
Ms. Rhee’s math class was studying statistics. She brought in three bags containing red and blue marbles. Bag x contains 75 red and 25 blue marbles, bag y contains 40 red and 20 blue marbles, and bag z contains 100 red and 25 blue marbles. Ms. Rhee shook each bag. She asked the class, “If you close your eyes, reach into a bag, and remove 1 marble, which bag would give you the best chance of picking a blue marble?” Which bag would you choose? Explain why this bag gives you the best chance of picking a blue marble (Smith, et al., 2009).
Handout #2

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<th>Who and What</th>
<th>Order</th>
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Situation: Students are working on a problem that requires them to represent $4(2 + x)$.

After monitoring, you have selected the following artifacts to present.

Student’s Work:
Handout #5
## APPENDIX I

Math-Talk Rubric

<table>
<thead>
<tr>
<th>Levels</th>
<th>Questioning: pursuing mathematical information (focus on questioner).</th>
<th>Explanation of math thinking: moving from answers to mathematical thinking (focus on explainer).</th>
<th>Contributions to teaching and learning math content.</th>
<th>Student responsibility for the learning of others as well as self.</th>
<th>Evaluation of student work: criteria and evaluator(s) (teacher, peers, self).</th>
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<td>Overview of shift over levels</td>
<td>Questioning shifts focus from answers to thinking and shifts from the teacher as questioner to students as questioners.</td>
<td>Students learn to explain and articulate their math thoughts and the classroom community grows to support students acting in central or leading roles.</td>
<td>Shift from teacher as the source of all math content to students contributing their ideas about content with confidence. Teacher utilizes student ideas.</td>
<td>Students increase their engagement, take ownership of learning for themselves, and grow to value the learning of others. This leads to active helping of others.</td>
<td>Shift from teacher as the critic, helper, and supporter to students taking on this role as well and from answers to thinking.</td>
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<td>Level 0</td>
<td>Teacher asks questions of specific students, (mostly short-answer form). Questions often function to keep students listening. Students direct answers to the teacher only, no student-to-student math talk.</td>
<td>No or minimal teacher elicitation of student thinking, strategies, or explanations; teacher expects answer focused responses, Teacher may tell answers. No student thinking or strategy-focused explanation of work, just answers.</td>
<td>Teacher is physically at the board, usually chalk in hand, telling and showing students how to do math. Students in seats listen passively, then attempt to imitate the teacher.</td>
<td>Teacher repeats student responses (originally directed to her) for the class. Students are passive listeners; they believe they will learn from the teacher and do not take responsibility for the learning of their peers.</td>
<td>Teacher responds to students' answers by verifying the correct answer or showing the correct method. Individual students receive feedback on their work from teacher only.</td>
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Note: Of course, there are other aspects of teaching math for understanding for which the teacher maintains authority. The teacher offers conceptual explanations, explains and/or models activities, helps children work together in pairs and groups, organizes peer helping, and generates enthusiasm for learning math in community. The teacher maintains authority over other traditional teacher functions such as managing materials (though children may help) and planning for content coverage as well.

<p>| Level 1 Teacher beginning to pursue student mathematical thinking | Teacher asks students questions about their thinking, focuses less on answers. Teacher rarely elicits an additional strategy. Students respond to probing by the teacher, some volunteering of thoughts. Other students listen passively or wait for their turn. | Teacher probes student thinking somewhat. One or two strategies may be elicited. Teacher may fill in explanations herself. Students give information about their math thinking usually as it is probed by the teacher, (minimal volunteering of thoughts). They provide brief descriptions of their thinking. | Student thinking is valued by the teacher. She does some probing to assess where students are. Students' thinking is included in their responses to probing by the teacher about math content. | Teacher begins to set up structures to facilitate students listening to and helping other students. Students repeat what other students say or help another student with their work at the teacher's request. This helping mostly involves students showing how they solved. | Teacher evaluates work more deeply by asking follow-up questions about student solution methods and answers. The teacher alone gives feedback. Students describe to the teacher what each part of their work means in response to probing. |</p>
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<th>Level 2 Teacher modeling and helping students build new roles. Some co-teaching and co-learning occurs. Teacher’s role becoming more peripheral in physical and discourse space.</th>
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<td>Teacher continues to ask probing questions to prompt fuller explanations and elicit more strategies. She also facilitates student-student talk, e.g., “Everyone be prepared to ask a question about this student’s work.” Students ask questions of one another’s work on the board, often at the prompting of the teacher. Students listen to one another so they do not repeat questions.</td>
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<td>Teacher probes more deeply to learn about student thinking and supports detailed descriptions from students. Teacher open to and elicits multiple strategies. Students usually give information as it is probed by the teacher with some volunteering of thoughts. They begin to stake a position and articulate more information in response to probes. They explain steps in their thinking by providing fuller descriptions and begin to defend their answers and methods. Other students listen supportively.</td>
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<td>Teacher physically begins to move to side or back of the room, asks more open questions, and allows discourse space for multiple explanations or strategies. Teacher is comfortable using student errors as opportunities for learning. Students exhibit confidence about their ideas and share their own thinking and strategies even if they are different from others. Student ideas sometimes guide the direction of the math lesson.</td>
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<td>Teacher encourages student responsibility for understanding the mathematical ideas of others. Students begin to listen to understand one another. When teacher requests, they explain other students’ board work and strategies in their own words. Helping involves clarifying other students’ ideas for themselves and others. Students listen actively so they do not repeat one another. Students also model teacher’s probing of their partners in pair work.</td>
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<td>Teacher asks other students questions about student work or whether they agree or disagree and why. Students evaluate their own work and that of others. They make supportive comments about other students’ work as prompted by the teacher (or some volunteering) so they learn positive ways to evaluate and help others improve their work.</td>
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<td>Level 3</td>
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REFERENCES


BIOGRAPHY

Theresa Wills graduated from Spotsylvania High School, Spotsylvania, Virginia, in 2000. She received her Bachelor of Science from Virginia Tech in 2004. She was employed as a mathematics teacher, mathematics instructional coach, and technology instructional coach in Alexandria City for eight years. She received her Master of Education in Mathematics Education Leadership from George Mason University in 2007.