BEYOND MATHEMATICS INTERVENTIONS: A LOOK AT THE PERCEPTIONS AND THOUGHT PROCESSES OF SECONDARY STUDENTS WITH LEARNING DISABILITIES OR AT RISK FOR MATHEMATICS DIFFICULTIES

by

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A Dissertation Submitted to the Graduate Faculty of George Mason University in Partial Fulfillment of The Requirements for the Degree of Doctor of Philosophy in Education

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Date: April 21, 2015

Spring Semester 2015 George Mason University Fairfax, VA
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DEDICATION

This is dedicated to my loving husband Cornell Dayne, who was supportive, encouraging, and remained positive throughout this journey. He was willing to help me and stand by my side no matter how hard the experience seemed. He was willing to stay up during my sleepless nights so I would not have to stay up alone. I would also like to thank my friends and family who understood my absences and encouraged me to achieve my goals.
ACKNOWLEDGEMENTS

The desire to delve deeper into an interest that drives my very existence has led me to pursue a Ph.D. in Education. For this, I am very grateful to have been blessed with the opportunity to complete this dissertation. As a result, I acknowledge and recognize that this journey would not have been possible without the guidance of the creator and heavenly father. Most fittingly, many thanks also go to the individuals who have had faith in the contribution and meaning of the current study to the literature and research community, particularly my committee chair, Dr. Kelley Regan, and my committee members, Dr. Peggy Weiss and Dr. Anna Evmenova. These individuals have made this journey possible in various ways and through their unique contributions.

First, I would like to express my deepest appreciation to my advisor and chairperson, Dr. Kelley Regan, who continually conveyed an aura that was always welcoming, helpful, understanding, optimistic, and thoughtful. Dr. Regan has been a part of my journey through the doctoral program for the past four years and has witnessed my growth as a professional student, researcher, and advocate for students. Dr. Regan has been a key instrument of guidance through the milestones demonstrated in portfolios I, II, and III. Therefore, I am very appreciative of her hard work in encouraging and expecting persistance, research quality, and nothing but stellar work. Without her guidance and encouragement, this dissertation would not have been possible.

Second, I would like to thank Dr. Peggy Weiss for her faith in ensuring my progress through this program. Dr. Weiss was gracious enough to step in during the last year of this program with her wealth of knowledge to help me reach my goal. Her expert knowledge and support throughout this process has been instrumental in helping not only with this dissertation but also helped to increase and widen my experience as a teacher. Dr. Weiss served as my supervisor as I completed my student teacher internship. Under her guidance, I was able to obtain confidence and experience in teaching preservice teachers and teachers. In addition, Dr. Weiss’s knowledge of the literature on students with learning disabilities has helped to guide me through the dissertation process.

Third, I would like to thank Dr. Anna Evmenova for her contributions to the completion of this dissertation. Dr. Evmenova was able to step into a key role as a committee member during the last year of this program, and I would like to thank her for her continued support of the current study. I am extremely thankful for her expert knowledge and contributions to my research. Her kind words, supportive attitude, and encouraging persona have truly been a blessing. Dr. Evmenova’s expertise on the literature, research...
methods, and experience in her work with doctoral students has been a blessing over the last few months of the dissertation process.

Fourth, I would like to thank Dr. Earle Reybold, Dr. Margo Mastropieri, and Dr. Tom Scruggs for their guidance and for serving as references for me throughout this program. I thank Dr. Reybold for being my mentor as I worked to increase my knowledge in qualitative research, and Drs. Mastropieri and Scruggs for their early contributions to my overall research knowledge and procedural methods.

Fifth, I would like to give a special thanks to my family members who understood and supported my decision to pursue a doctoral degree. The Hays, the Bents, and the Daynes have been an understanding and supportive family throughout these past years as I served my time as a doctoral student, and especially during these past months of completing this dissertation.

Finally, I would like to thank my friends, colleagues, and all past professors who have contributed in more ways than one can imagine.
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ABSTRACT

BEYOND MATHEMATICS INTERVENTIONS: A LOOK AT THE PERCEPTIONS AND THOUGHT PROCESSES OF SECONDARY STUDENTS WITH LEARNING DISABILITIES OR AT RISK FOR MATHEMATICS DIFFICULTIES

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George Mason University, 2015

Dissertation Director: Dr. Kelley Regan

The purpose of this mixed-methods research study was to understand the perceptions and thought processes of a group of secondary students with learning disabilities (LD) or at risk for mathematics difficulties. Using quantitative and qualitative methods, the researcher explored the phenomenon of how a group of students perceived mathematics and thought mathematically, which included measuring students’ attitude toward mathematics, knowledge of math, and students’ thought process when performing mathematical tasks. Data were collected through a demographic questionnaire, math attitude inventory, a math diagnostic test, a clinical interview, and a semistructured interview. A convenience sampling followed by a purposeful selection was used in selecting students from secondary schools in a northeastern part of the United States. This study consisted of six students diagnosed with a specific learning disability (LD) and two students at risk for mathematics difficulties. The quantitative data were analyzed using
descriptive statistics, while the qualitative data were analyzed through an inductive analysis consisting of a constant comparative analysis method. A collective set of common procedural and mathematical errors were identified across participants and addressed. In addition, three themes that influenced the perceptions of students with LD and students at risk for mathematics difficulties about math were identified: (a) motivational factors, (b) teacher characteristics, and (c) instructional approaches. Implications for future research, policy makers, schools, and teachers are discussed. Limitations and suggestions for further research are also presented.
CHAPTER ONE

Mathematics is a subject that all students will encounter throughout their academic experience as a fundamental aspect to all areas of daily life (Calhoon & Fuchs, 2003; Xin, Jitendra, & Deatline-Buchman, 2005). One’s understanding of mathematical awareness affects successful functioning at home, school, on the job, and in the community (Xin et al., 2005). Mathematics education guarantees access to an extensive spectrum of careers, educational options, and participation in various—if not all—areas of society (National Council of Teachers of Mathematics [NCTM], 2009a, 2009b; Xin et al., 2005). Currently, statistical data suggests that a higher level of mathematical and technical skills is necessary for most jobs and unquestionably in the future (Witzel, Mercer, & Miller, 2003; Xin et al., 2005). Furthermore, a shared understanding or a common knowledge of mathematics is as important for students who will enter the workplace as it is for those who will pursue further study in mathematics (Arslan, Canli, & Sabo, 2012; NCTM, 2009b). Consequently, proficiency in mathematics is an essential prerequisite for all students entering secondary school with the intention of graduating with a diploma.

Despite the increasing importance of proficiency in mathematics, internal assessments continue to indicate that eighth-grade students and secondary students in the United States experience difficulty in mathematics and score below many of their peers
internationally (Bottge et al., 2004). More specifically, students with learning disabilities (LD) and those at risk for mathematics difficulties are faced with even greater challenges when it comes to learning, recalling, and demonstrating an in-depth understanding of various mathematical concepts (Cortiella, 2011; Cortiella, & Horowitz, 2014).

**Background and History**

In 2003, 2007, and 2011, data from the Trends in International Mathematics and Science Study (TIMSS) and the Program for International Student Assessment (PISA) indicate that students in the U.S. perform below the level of many industrialized countries in science and mathematics. In addition, recent TIMSS reports in 2003, 2007, and 2011 (Gonzales et al., 2008; Martin, Mullis, Gonzales, & Chrostowski, 2004; Provasnik et al., 2012) show that Grade 8 students in the U.S. performed above the international average in both mathematics and science. However, at age 15, U.S. students performed below the international average in mathematics literacy, and scored similar to the international average in reading and science literacy. Although some progress has been noted in reading, students in the U.S still lag behind their international counterparts in mathematics despite an unremitting national effort to improve performance.

The National Center for Educational Statistics (NCES) revealed that between 2000 and 2013, the exclusion rate of secondary students’ mathematics assessments dropped significantly nationally (2013). This means that more students with learning disabilities (LD), despite the nature of their disability, were required to participate in state assessments with appropriate accommodations. The National Assessment of Educational Progress (NAEP) indicated that 67% of Grade 8 students with disabilities scored below
the basic level in mathematics compared to 26% of students without disabilities (NCES, 2009). According to NAEP, in both 2011 and 2013, these students’ average math scores increased by less than 1%. Specifically, 12th-grade students who reported only completing Algebra I since high school typically scored lower on the national assessment than students who reported completing higher level courses such as Geometry, Algebra II, and Calculus or Trigonometry (NCES, 2009, 2013). It is evident that algebraic concepts, which are mostly taught between 7th grade and 9th grade, are still not fully understood by some 12th-grade students. For example, 12th-grade students are expected to (a) identify formulas to solve problems, (b) evaluate functions at a given point, and (c) evaluate expressions with fractional exponents. According to the NAEP (NCES, 2009, 2013) Algebra continues to remain the highest level math course taken by a number of 12th-grade students and many of these students continue to demonstrate difficulty in achieving proficiency in this content area. As a result, it is not surprising that national experts are concerned that the U.S. will lose its standing as an international leader in mathematics (Butler, Miller, Crehan, Babbitt, & Pierce, 2003). The low math performance (compared to international peers) and the overall math performance of U.S. students, including students with disabilities, continue to generate national concern.

Consequently, a desperate need and startling trend for all students to produce higher test scores—despite their disabilities—has swept the nation. Teachers and students are under tremendous pressure to demonstrate students’ competency on the general math and reading standardized assessments. School administrators, classroom teachers, and students have scrambled to meet the goals of one of the most influential educational
reforms over the past three decades: the No Child Left Behind Act (NCLB) of 2001 (Mabry & Margolis, 2006). The NCLB stated that all students, regardless of race or disability, should demonstrate 100% proficiency in math and reading by 2014. The law increased the focus on accountability for student progress by establishing performance standards designed to hold schools, teachers, and school districts responsible and accountable for all student achievement, including students with LD (Porter, Linn, & Trimble, 2005).

Currently, in the 2014-2015 school year, over 90% of state schools have not met NCLB tenets (U.S. Department of Education [U.S. DOE], 2013). As a result of the trends prior to 2014, President Obama realized the repercussions of the NCLB Act, and provided an alternative to all states in 2011 by awarding them waivers under the NCLB law known as the Elementary and Secondary Education Act (ESEA) flexibility (U.S. DOE, 2013), granting states flexibility in meeting these essential requirements. According to the 2013 U.S. DOE report, states may qualify for the waivers if they agree to develop state plans that will better prepare students for college. Certain states have chosen to adopt a system that ties teacher evaluations to end-of-year student assessments; others have implemented statewide programs to increase graduation rates. In return, upon agreeing to such terms, states have been permitted some flexibility for the tenets of NCLB (i.e., 100% proficiency for all students in math and reading by 2014). According to the U.S. DOE (2013), over 95% of states applied and approximately 90% were granted an NCLB waiver. Despite this shift of expectations, school systems, teachers, and
students with and without disabilities remain under pressure to demonstrate proficiency in math and reading on a yearly basis.

Consequently, there is no denying that as students continue to score low on national assessments, fall short in closing the achievement gap, and fail to achieve 100% proficiency in math and reading, educational leaders are working overtime toward improving the educational system. As a result of painstaking educational policies, an increase in student and teacher expectations and rigorous standards have been introduced to the educational system.

In response to the concerns regarding students not achieving or demonstrating proficiency in mathematics, the NCTM (n.d., 2009b) developed standards that address both the content standards and process standards in mathematics. These standards were developed to better prepare students for the challenges of high stakes assessments. According to the NCTM (2009b), content standards focus on providing concepts that all students in K-12 should learn prior to advancing in mathematics. For example, seventh-grade students should be well adapted to preestablished benchmarks in concepts such as number properties and operations, measurement, geometry, data analysis, statistics, probability, and algebra prior to advancing to eighth grade. Process standards reflect conceptual understanding, mathematics reasoning, and problem-solving concepts.

The implementation of these new standards represented an attempt to improve the overall level of students’ mathematical knowledge through the enforcement of higher and more rigorous standards and mathematics expectations. The TIMSS data provided results supporting the need to improve the mathematics curriculum. The data shows that there is
a significant difference between the performances of students at the fourth-grade level versus students at the eighth-grade level. In fourth and eighth grades, the TIMSS test covers various areas such as number properties and operations, measurement, geometry, data analysis, statistics, probability, and algebra. The scores of students who participate in these assessments are compared to those of their international peers and as previously stated, as students in the United States approach the eighth grade, the difference in their scores compared to their international peers’ increases. Likewise, data from the NCTM report (Grønmo, Lindquist, Arora, & Mullis, 2015) revealed that the largest portion of problems for the fourth-grade test was number properties and operations (40%), and the largest portion of problems for the eighth-grade assessment focused on algebra (30%).

Students are expected to demonstrate a greater understanding of numbers and operations at an earlier age. For example, in fourth grade, students focus mostly on number properties and operations. However, as students approach middle school and high school, they are expected to transition their mathematical way of thinking to a more abstract understanding. Therefore, in order to prepare students to better perform on standardized assessments and fare better on international assessments, NCTM has increased the level of knowledge and reasoning required in each grade level. Given the improvements made to both the content and process standards, which were designed to foster a more in-depth understanding of math content, students will be expected to approach a higher level of mathematical reasoning and understanding.

Consequently, as students move up in grade level, they are expected to develop advanced skills and work with abstract information. Thus, algebraic concepts are
considered to be an entrance into developing the ability to think beyond what is visible, tangible, or easily represented at a concrete level (Witzel et al., 2003). These concepts are an important part of the overall math curriculum and therefore, the development of these concepts does not occur in one specific course, as stated by NCTM (2008). Rather, algebraic concepts are introduced and developed over the course of a student’s elementary, middle, and high school years. For example, students in Grade 9 through 12 should be able to use algebra expressions, also known as symbolic representation, to represent situations and to solve problems instead of using numeric expressions, involving only numbers, to represent and solve mathematical problems (NCTM, 2014). However, prior to Grade 9 and as early as kindergarten, teachers are expected to cultivate an understanding of number fluency for students and to demonstrate an understanding of mathematical relationships. Students are also expected to use their knowledge of visual representation to describe problems, generalize patterns, and solve equations. Consequently, NCTM has described algebraic skills as powerful, useful, and one that “open doors and are evident in many professions and careers” (NCTM, 201, p. 1). Thus, there is a strong emphasis placed on conceptual understanding developing over time, rather than a procedural step-by-step understanding. As stated earlier, the focus of each NCTM standard is an emphasis on conceptual understanding (Maccini & Gagnon, 2002). Students are expected to know, understand, relate, and apply these standards. For many students, especially students with LD and students at risk for mathematics difficulties, understanding mathematical content on a conceptual level is more difficult than simply memorizing steps and procedures.
In fact, a major provision of the reauthorization of the Elementary and Secondary Education Act (ESEA) under the 2002 law, now known as NCLB, and the reauthorization of the Individuals with Disabilities Education Act (IDEA) in 2004, is that students with disabilities have equal access to the general education curriculum as their peers. The current ESEA flexibility of 2011 continues to uphold the original tenets that no child should be left behind and the ESEA enforces the principle that students, despite their disabilities, will be assessed annually and “counted” as a part of the schools’ overall performance score. This means that students with learning disabilities (LD) and students at risk for mathematics difficulties, despite their noted areas of difficulties, will continue to be held to the same academic standards for all students with or without disabilities (Cortiella & Horowitz, 2014; NCLB, 2001).

**Statement of the Problem**

Students with LD or at risk for mathematics difficulties (NCES, 2004, 2009, 2013) have one of the highest secondary dropout rates among their peers (Bottge, Rueda, Grant, Stephens, & LaRoque, 2010; Cortiella & Horowitz, 2014; National Council on Disabilities [NCD], 2008, 2014) and one of the lowest passing rates on standardized math tests (Bottge, 1999; Bottge, Rueda, et al., 2010). Bottge, Heinrichs, Chan, and Serlin (2001) noted that many students with disabilities perform below their average peers in elementary schools, fall further behind as they move up in grade level, fail to catch up in high school, and eventually drop out of school. Consequently, school systems are constantly seeking solutions such as best practices, interventions, and additional accommodations to help students with LD to complete high school successfully.
Although this is not an alarming or shocking new reality, this issue has remained an ongoing problem for the past few decades despite state, local, and district incentives.

According to the U.S. DOE (2013), students with LD continue to top the list as being one of the most difficult subgroups to close the achievement gap in the nation on math assessments. Additionally, the data supporting the adversities that students with disabilities experience continues to show that most students with LD are living in poverty when compared to students in the general population (Cortiella & Horowitz, 2014). These students also have a greater chance of exhibiting poor performance in school and poor outcomes in their adult life (Cortiella & Horowitz, 2014). This knowledge has led to various actions by lawmakers, one of which places pressure on the educational system. School systems are required to provide appropriate and rigorous instruction to better prepare students for the future and to be competitive counterparts among their peers in the international community. Therefore, school systems are required to provide appropriate and challenging education for all students, including students with LD and those at risk for mathematics difficulties. Nevertheless, the task of better preparing students to pass required assessments so that students move closer toward graduation, continues to be a difficult and daunting task.

Consequently, the U.S. DOE (2010) has increased graduation requirements for all students and at the same time, NCTM has changed state benchmarks and added more mathematics requirements (2009b). Because mathematics requirements are now higher for all students at the secondary level, these students are expected to demonstrate an extensive level of understanding mathematics before meeting graduation requirements.
For instance, students with and without LD are required to take and pass Algebra I, Geometry, and Algebra II courses in order to meet graduation requirements (NCTM, 2014), but in the past were only required to pass Prealgebra, Algebra I and Geometry classes. Existing data show that students with LD and students at risk for mathematics difficulties are at a higher risk of failing to meet these mathematics requirements compared to students without disabilities (Cortiella, 2011; Cortiella & Horowitz, 2014).

**Student Perceptions**

Intervention research in the area of mathematics has identified promising practices to support students with LD and/or those at risk for mathematics difficulties (Axtell, McCallum, Bell, & Poncy, 2009; Bottge, Heinrichs, Mehta, & Rueda, 2004; Bottge, Rueda, LaRoque, Serlin & Kwon, 2007; Mayfield & Glenn, 2008). Whereas qualitative studies have involved observations, analysis, and a focus on what students think about math and how students perform mathematics tasks (e.g., Howard & Whitaker, 2011; Jenkins, 2010; Lewis, 2010), there are far fewer investigations. Researchers such as Ginsburg (1981) and Even and Tirosh (2008) have delved deeper into students’ perceptions of mathematics and have argued that all students are not created equally, meaning students learn and interpret information differently. Researchers state that while quantitative research in the area of mathematics may treat all students as equals, qualitative research in this area has focused more on students’ individual and unique characteristics and perceptions.

Although research-based mathematical interventions are practiced, they might not be enough to support secondary students through the new sequence of mathematics
needed for graduation. Further investigations are needed to better understand how students with LD and students at risk for mathematics difficulties perceive mathematics. As researchers such as Ginsburg (1997b) stated, understanding how students think about, interpret, and perform mathematic problems is important to educators. Higher mathematics expectations for all secondary students have enhanced the need for an even greater understanding as to why students with LD and students at risk for mathematics difficulties continue to fall behind their state, national, and international peers. Understanding student perceptions is needed to better understand and aid students in their overall mathematical performance, improve mathematical achievement, and foster mathematical growth and development. This is important in the movement forward for students with disabilities and students at risk for mathematics. It is vital to note that what appears to be challenging for students to understand mathematically is not always obvious to even the most skilled teachers of math. Thus, students’ feedback regarding their understanding, interpretations, and how they go about mathematical tasks can impact teachers’ instruction and students’ mathematical growth and development. Further research is necessary to better assess the perceptions and mathematical thought process of students with LD and those at risk for mathematics difficulties.

**Significance of the Problem and Characteristics**

The expectations discussed above may be difficult for many secondary school students, but for students with LD or at risk for mathematics difficulties, they present an even greater challenge. Researchers have indicated that students with LD and students at risk for mathematics difficulties often develop serious deficits in mathematics as they
move toward higher level mathematics courses (Witzel et al., 2003). The National Center for Learning Disabilities reported that approximately 2.4 million students are diagnosed with LD and receive special education services in our schools, representing 41% of all students receiving special education (2010). Also, nearly 50% of secondary students with LD perform three or more grade levels below their enrolled grade in essential academic skills, while approximately 44% of students with LD perform below grade level in math (NCLD, 2010, 2011, 2014). These students are more likely to experience problems in mathematics because of the uniquely specific challenges they encounter when solving mathematical problems (Jitendra, Griffin, McGoey, & Riley, 1998; NCLD, 2014).

Similarly, many students at risk for mathematics difficulties demonstrate similar characteristics to students with LD and perform below average according to their classroom teacher (Witzel et al., 2003). Furthermore, as these two groups of students move closer toward the secondary grades, the achievement gap between them and their general education peers widens (Cortiella, 2011; Jitendra et al., 1998).

More specifically, most students diagnosed with a LD in mathematics exhibit difficulties with conceptual procedures (i.e., mathematical reasoning and problem solving) (Bottge, Rueda, et al., 2010; Naglieri & Johnson, 2000; Xin et al., 2005), computational skills (i.e. addition, subtracting, multiplication, and division skills) (Bottge, Rueda, et al., 2010; Calhoon, Fuchs, & Hamlett, 2000), and abstract thinking (i.e., understanding and completing algebraic equations) (Witzel et al., 2003). In addition to those difficulties, students with disabilities experience many mathematical challenges including the identification of relevant information (Bottge, Grant, Stephens, & Rueda,
recognizing different problem types, organizing information, following procedural steps, applying self-monitoring skills, checking answers after completion (Naglieri & Johnson, 2000), and retaining short-term and long-term memory (Bottge, Grant, et al., 2010). Furthermore, students with LD are more susceptible to experiencing poor motivation, poor self-esteem, poor work habits, and withdrawal from classroom activities (Howard & Whitaker, 2011; Naglieri & Johnson, 2000).

Unfortunately, these characteristics of students with LD and students at risk for mathematics difficulties not only impact students’ ability to use and understand numbers and symbols, but they also impact how students feel about mathematics. Howard and Whitaker’s (2011) study of first year college students enrolled in Prealgebra and Algebra courses, and Fennema, Franke, Carpenter, and Carey’s (1993) study consisting of elementary school students, argued that students with LD tend to develop negative feelings toward math, which may also lead to a lack of motivation and desire to learn. Although the two studies involved individuals at two very different academic levels, their research findings identified motivation as a strong factor in student learning of mathematics. Howard and Whitaker (2011) also state that students who lack motivation are less likely to pay attention in class, participate, or volunteer answers, and thus do not perform well on standardized tests.

On the other hand, limited qualitative research exits in the area of mathematics with secondary students with disabilities. It also appears that few studies are interested in students’ perceptions of mathematics. More specifically, it is difficult to locate qualitative
research involving (a) students with learning disabilities, (b) students at risk for mathematics difficulties, (c) students at the secondary level, and (d) mathematics. As a result, further research is needed to better assess and understand the perceptions and mathematical thought processes of secondary students with LD or at risk for mathematics difficulties.

The Present Study

Researchers need to explore the reasons why students with LD and students at risk for mathematics difficulties continue to struggle with prealgebraic concepts, the transition from arithmetic to abstract thinking, and/or algebraic concepts with prerequisite skills including an understanding of numbers and number operations. This is important because as pressure is placed on school districts and teachers to better prepare students to pass standardized tests and graduate high school, all students—including students with disabilities—are expected to demonstrate proficiency on the high stakes state tests in mathematics. In this situation, student needs may not be adequately met if vital information regarding student perceptions about mathematics is not available. Every day, students with disabilities and students at risk for mathematics difficulties are unfortunately placed at a disadvantage in learning situations that are unconducive to meeting their learning needs. More specifically, many of these students are currently in learning environments in which their voices, their thoughts, or their feelings about mathematical tasks are not heard or used as the primary source in data collection. For example, in typical math intervention studies many researchers (Jitendra, Star, Rodriguez, Lindel, & Someki, 2011; Maccini & Ruhl, 2000; Naglieri & Johnson, 2000; Witzel et al.,
have involved students at risk for mathematics difficulties and/or those students with LD. In these studies, procedures have included administering a pretest, introducing the intervention, administering one or more posttests, and then finally evaluating and reporting findings. In these studies, students are tested, taught, and retested. However, students are not asked about their understanding or perceptions about math that may possibly affect their academic performance. Although various studies of mathematics have been conducted to determine interventions for low achieving students, this absence continues to be a trend in the research studies. More research should specifically target students’ perceptions of mathematics. As Ginsburg (1997a, 1997b) stated, understanding a student’s way of thinking about math can effectively shape how mathematic educators approach instruction.

Thus, the present study investigated the perceptions and thought processes of a group of students with LD and students at risk for mathematics difficulties. The researcher’s primary goal was to gain a better understanding of the attitudes or challenges that one group of secondary students perceive when completing problems. That is, the researcher’s goal was to understand how a specific group of students see, interpret, approach, and complete a given math problem. The idea of mathematical thinking refers not to an ability to demonstrate mathematical reasoning skills but rather to understand the students’ mathematical thought processes as they solve a problem. A student’s mathematical thinking, which may or may not be correct, is based on the student’s interpretation.
Consequently, this study satisfies Maxwell’s (2005) definition of an intellectual goal. Through a mixed-methods research design, the researcher sought to understand the perceptions and thought processes of students with learning disabilities and students at risk for mathematics difficulties. The quantitative data was used to determine students’ cognitive abilities regarding mathematical calculations and problems solving ability. The qualitative data was also merged to better understand student meaning of the math problem, the unique experiences of math reasoning for each participant, and the circumstances leading to the outcome (Maxwell, 2005). The goal of this study was to understand and gain further insight as to what was happening when a student with a LD or a student at risk for mathematics difficulties attempts to solve a math problem and why a student makes certain decisions when presented with a mathematical problem.

**Research Questions**

The following research questions guided this study:

RQ1. What are the perceptions and attitudes of a group of secondary students with learning disabilities and students at risk for mathematics difficulties about mathematics?

RQ2. How well does a group of secondary students with learning disabilities and students at risk for mathematics difficulties understand important concepts and symbols of algebra, and how they are used?

RQ3. What does a group of secondary students with learning disabilities and students at risk for mathematics difficulties find to be the most challenging about mathematics?
Definition of Terms

For the purpose of this study, the following definitions were used.

*Algebra:* The National Council of Teachers of Mathematics (NCTM) defines algebra as “a way of thinking and a set of concepts and skills that enable students to generalize, model, and analyze mathematical situations” (2008). Algebra provides a logical approach to investigating connections, as well as describing, organizing, and understanding mathematical concepts as they relate to the world (NCTM, 2008).

*Attitude Measure:* An attitude measure is a type of assessment used to measure students’ attitudes toward mathematics, and to also determine the underlying extent of the students’ beliefs by closely examining their responses (Tapia & Marsh, 2004).

*Cognitive Perspective:* Cognitive perspective is a concept derived from cognitive psychology, which is the study of how individuals understand, learn, recall, and think about information (Sternberg, 2003). An individual interested in students’ cognitive perspectives about mathematics would seek to study how students perceive numbers, shapes, operations, and symbols. Additionally, that person would seek to understand factors such as why some students remember certain facts but cannot recall others, or how students learn and process meaning from learned information.

*Clinical Interview:* This type of interview involves dialogue and retrieval of perceptions. Clinical interviews seek to discover “what kind of logical structure or information-processing routine or strategy underlies [a] child’s performance” (Ginsburg, 1977, p. 62). A clinical interview tends to be flexible, individualized, and conversational,
and can potentially provide an accurate perspective of how a child constructs, retrieves, and interprets knowledge (Arias, Schorr, & Warner, 2010).

**Content Standards:** According to the NCTM (n.d.), content standards describe the five strands of math content that all students in Grades K-12 should learn: (a) number and number operations, (b) algebra, (c) geometry, (d) measurement, and (e) data analysis and probability. For each content standard, the benchmarks are written and described with examples that demonstrate what the standard should involve for Grades K-12 (NCTM, n.d.).

**Conceptual Knowledge:** Conceptual knowledge refers to an understanding of the relationship between mathematical concepts and abstract symbols (Doabler et al., 2012). Students can demonstrate an understanding based on conceptual knowledge by exhibiting an ability to use previously taught mathematical concepts to relate, build, and create new connections and conclusions (Lobato, Hohensee, Rhodehamel, & Diamond, 2012).

**Diagnostic Math Test:** A math diagnostic test refers to a type of assessment generated to gather and establish a detailed understanding of what is known and unknown, as it relates to a particular concept in math, by each child (Glencoe McGraw-Hill, 2000).

**Mathematical Thinking:** Mathematical thinking can be defined in various ways; however, in this study the idea of mathematical thinking refers to a student’s approach to mathematical ideas and mathematical tasks that may or may not involve mathematical symbols. It is not necessarily based on accuracy of thinking but on how students use prior knowledge and experiences of mathematics to create mathematical perceptions and
formulate mathematical conclusions. This can also be defined as what students think about before, during, and after the process of solving a math problem (Empson & Jacobs, 2008; Steinberg, 1996; Steinberg, Empson, & Carpenter, 2004). Specifically, mathematical thinking will refer to how students make sense of mathematics (Jenkins, 2010).

**Mathematical Reasoning:** The term mathematical reasoning is defined differently across the literature. However, in this study, mathematical reasoning, unlike mathematical thinking, refers to an accurate way in which students use prior knowledge, logic, and reasoning skills to obtain mathematical answers. Sumpter (2013) presented a definition of mathematical reasoning which described this concept as the use of mathematical terms (e.g., numbers, operations, and variables), and concepts (e.g., mathematical ideas, practices, and norms) to perform a sequence which consists of starting a problem and ending with an answer. In other words, for the purposes of this study, as Sumpter argued, mathematical reasoning “comprises a set of practices and norms that are collective not merely individual or idiosyncratic, and rooted in discipline” (2013, p. 1119).

**Prealgebra:** In this study, this term refers to the transition from arithmetic (numbers and operations) to abstract thinking involving the use of variables and symbols. This is generally the math course that is taken as an introduction to abstract algebraic concepts and an algebra course.

**Procedural Knowledge:** Procedural knowledge refers to the ability to complete math procedures by following various steps with fluency and ease (Doabler et al., 2012).
Students with procedural knowledge can usually produce answers to mathematical facts mentally. In contrast, students without procedural knowledge usually depend on various strategies to solve simple mathematical problems.

*Procedural Fluency:* Procedural fluency is defined as the ability to solve math procedures with accuracy and ease (Maccini, Strickland, Gagnon, & Malmgren, 2008).

*Process Standards:* Process standards reflect a conceptual understanding rather than rote memorization. They involve an ability to use prior knowledge to perform mathematics reasoning and apply problem-solving concepts. Process standards involve approaches that: (a) focus on mathematical instructions that enable students to build knowledge and apply problem-solving strategies to solve mathematical problems; (b) recognize when to develop mathematical arguments, and apply research techniques to proof and evaluate mathematics; (c) communicate mathematical thinking articulately to adults and peers; (d) connect and relate mathematical concepts to real-world situations; and (e) develop representations to organize and communicate mathematical ideas (NCTM, n.d.).

*Psychological Perspective:* Psychological perspective refers to the perspective of students based on their feelings toward mathematics as a result of an accumulation of their individual and unique experiences.

*Secondary School Students:* Secondary school students are a range of students in Grades 6-12. In this study, secondary school refers to students in Grades 6, 7, 9, 11, and 12 – the grade levels of this study’s participants.
Specific Learning Disability: According to the Individuals with Disabilities Education Act (IDEA) of 2004, a specific learning disability is a disorder in one or more of the basic psychological processes involved in understanding or in using language, spoken or written, that may manifest itself in an imperfect ability to listen, think, speak, read, write, spell, or do mathematical calculations. The term includes such conditions as perceptual disabilities, brain injury, minimal brain dysfunction, dyslexia, and developmental aphasia (p. 11).

Likewise, the National Center for Learning Disabilities (NCLD) describes a specific learning disability as more than a problem with learning, and states that it is a neurological disorder that prevent the brain from adequately or easily obtaining, processing, storing, and responding to information (NCLD, n.d.).

Students at risk for mathematics difficulties: This term generally refers to students who fail to achieve basic levels of proficiency in core subjects in mathematics (NCES, 1992). However, in this study, the participants who are referred to as “at risk” have a documented history of performing poorly in math classes (e.g., documentation evidenced by K-12 mathematics report card, standardized test scores, and/or student, parent, or teacher concerns). Each participant was identified by his or her parent as a student at risk for mathematics difficulties because he or she (a) had a documented history of receiving failing grades on mathematics assessments, as evident by progress reports and report cards; (b) received concerns from classroom math teachers; and (c) had an understanding that his or her child has agreed and expressed difficulty understanding various concepts
and procedures in mathematics. Overall, the term “at risk” describes students who are in danger of not achieving academic success in mathematics.
CHAPTER TWO

In an effort to help educators improve the mathematical performance of students with learning disabilities and students at risk for mathematics difficulties, the purpose of this study was to explore factors that potentially influence the performance of a group of students beyond the implementation of interventions. Specifically, the current study focused on better understanding the perceptions and mathematical thought processes of a group of students with learning disabilities or at risk for mathematics difficulties who are not successful in their current mathematics courses. Because this group of students continues to demonstrate difficulties in mathematics, even after intervention, it is becoming increasingly important to better understand the phenomenon from the perspective of the students (Ginsburg, 1997b; Howard & Whitaker, 2011; Sumpter, 2013).

Both mathematics interventions and student perceptions are important concepts to be recognized and explored. Exploring the available literature, including both quantitative and qualitative literature, serves an important role in the development of educators’ understanding of students’ perceptions of mathematics. As a result, this chapter summarizes the literature related to mathematics interventions, student perceptions, motivation, and mathematical thought processes. First, a brief overview of the intervention research in mathematics is presented. Second, the findings of previous
research relevant to the study of students’ perceptions about mathematics and mathematical thinking are addressed extensively. Third, this chapter reviews the potential effects of motivation on student academic progress according to the existing literature. A summary concludes this chapter.

**Intervention Research in Mathematics**

Research-based mathematics instruction is progressively becoming a key component of mathematics education reform policies (e.g., NCLB) in classrooms around the United States (Superfine, Kelso, & Beal, 2010). These educational reform policies are based on published research knowledge about how students currently learn various math concepts, how students should learn mathematical content, and what students should know in order to demonstrate proficiency in a specific content area.

From 2000–2012, approximately 15 group design studies and 7 single-subject design studies were published in major peer-reviewed journals such as *Exceptional Children, Council for Exceptional Children, Learning Disabilities Research and Practice, Learning Disability Quarterly, The Journal of Special Education, Teaching Exceptional Children*, and *Journal of Learning Disabilities* describing interventions in secondary mathematics for students with LD. Each study targeted specific mathematical skills. Student participants consisted of students receiving math remediation, students considered at risk for mathematics difficulties, or students without disabilities. See Appendix A for a detailed summary of each study.
Effective Instructional Approaches for Teaching Math

A total of eight specific instructional strategies were noted in these studies. Among the list of instructional strategies used across the studies were the schema-based instructions (SBI), concrete-representational-abstract instruction (CRA), enhanced anchored instruction (EAI), peer-assisted learning strategies (PALS), explicit instruction, video modeling, the planning, attention simultaneous, and successive (PASS) strategies, and graphic organizers. The SBI approach uses explicit instructions to help individuals understand the structure of a particular problem in math (Jitendra et al., 2009). CRA instructions are based on gradually advancing students through a series of three stages (concrete, semiconcrete, and abstract) (Butler et al., 2003). EAI uses technology to help students develop their computation and problem-solving skills (Bottge et al., 2001; Bottge, Rueda et al., 2010). PALS involve using students to teach or reinforce particular concepts to their peers (Calhoon & Fuchs, 2003). These instructional approaches used strategies that targeted students’ procedural skills, conceptual understanding, and problem-solving abilities.

First, studies included instructional approaches that used primarily explicit instructional techniques designed to enhance students’ procedural skills. These instructional techniques focused on introducing the skill, modeling the procedures, providing corrective and immediate feedback, providing opportunities for independent practice, and reinforcement (Ives, 2007; Maccini, Mulcahy, & Wilson, 2007; Mayfield & Glenn, 2008). Specifically, approaches involved instructional demonstration of a step-by-step strategy for solving a problem, introduction of steps that were related to a
specific type or category of problems, and instruction which involved the need for students to perform the same steps exactly as they were introduced in order to solve specific or similar types of problems with guidance and without. Students model these steps as needed to enhance procedural accuracy and math computational skills. Additionally, other intervention studies used specific and direct instruction to help students better understand the concept. For instance, interventions may include a type of explicit instructional method with lessons consisting of a verbatim scripted intervention known as Direct Instruction (DI) (Ives, 2007; Woodward & Brown, 2006). These studies included the use of (a) explicit instruction (Mayfield & Glenn, 2008); (b) visual modeling (Bottge, Rueda, et al., 2007); and (c) graphic organizers (Ives, 2007).

Mayfield and Glenn (2008) used an explicit instructional strategy to teach students to attain answers to problems involving linear equations and exponents. The authors used a single-subject design to determine the effects of five instructional interventions: cumulative practice, tiered feedback, feedback plus solution sequence, review practice, and transfer training. The instructional phase consisted of providing students with worksheets containing problems for each target skill. These worksheets consisted of explanation and examples of step-by-step procedures of how to attain the targeted skills. The authors reported that practice consisted of cumulative review, along with constant feedback, which led to a small increase in problem-solving skills. On the other hand, they noted that teaching students how to transfer certain skills to math situations was more significant in its ability to promote an increase in problem-solving skills.
In another example, Ives (2007) used graphic organizers to improve students’ understanding of how to approach and solve math problems involving linear equations. Students in both the experimental and the control group were taught through direct instruction and strategy instructions to help students produce significant results after the experiment. In this quasi-experimental study, students were randomly separated into two groups. In the experimental group, students were taught how to use the graphic organizer for more than one situation and then asked to demonstrate how the graphic organizer would work for a similar problem. The graphic organizers were used to enhance the students’ ability to organize his or her thoughts then accurately apply a correct method to solve specific types of problems consistently.

Second, these studies also included interventions designed to primarily enhance memory, promote understanding of concepts, develop problem-solving skills, and increase self-monitoring skills and self-instruction strategies (Maccini et al., 2007). These studies included: (a) mnemonic strategy instruction (Test & Ellis, 2005), (b) graduated instructional sequence (i.e., CRA, CSA, and RA) (Butler et al., 2003; Maccini & Hughes, 2000; Maccini & Ruhl, 2000; Scheuermann, Deshler, & Schumaker, 2009; Witzel, 2005; Witzel et al., 2003), (c) cognitive strategy instruction (Naglieri & Johnson, 2000), (d) SBI (Jitendra, Dipipi, & Perron-Jones, 2002; Jitendra et al., 2009; Xin et al., 2005), and (e) problem-solving strategies (e.g., Planning, Attention Simultaneous, and Successive (PASS) strategies). For example, mnemonic strategies involved students using symbols, pictures, phrases, or words to remember a concept or routine for finding a solution to a problem (Test & Ellis, 2005). Studies involving the use of these strategies in mathematics have
shown that mnemonic strategies are an effective type of intervention strategy for students with and without LD (Gersten et al., 2009; Maccini et al., 2007; Test & Ellis, 2005). These types of devices have been noted for their extensive use across the general curriculum, their flexibility, and their adaptive nature to various subjects (Mastropieri & Scruggs, 1991, 1998). Additionally, the use of a graduated instructional sequence approach involves instructions based on gradually advancing students through a series of three stages (concrete, semiconcrete, and abstract) (Butler et al., 2003). This eventually provides students with an opportunity to gain a better understanding of abstract problems, gradually moving from concrete to abstract.

Test and Ellis (2005) implemented a mnemonics instructional approach to improving students’ learning, specifically to teach students fractions. The authors used a single-subject design to determine the effects a mnemonic strategy called LAP to improve students’ ability to remember how to solve fraction problems with different denominators. The instruction phase consisted of providing students with instructions to remember the letters of the LAP strategies (L-Look at denominator and sign, A-Ask yourself the questions, “Will the denominator divide into the largest denominator an even number of times?, and P-Pick your fraction type)” (p. 15). These steps were modeled by the teachers, and students were given an opportunity to practice with guidance from the teachers. Test and Ellis reported that all but one student achieved mastery on the mnemonic steps and maintained above 80% during the maintenance phase when later evaluated. The student was better able recall procedural steps and better monitor accuracy of problems (Test & Ellis).
In another instance, Witzel (2005) examined the effectiveness of CRA instructional sequence for middle school students. Witzel implemented a graduated instructional sequence to teach students how to solve algebraic equations. A graduated instructional sequence consists of instructional procedures that generally have three phases: (a) concrete items, such as a manipulative, to first introduce the overarching concept (e.g., colored chips, unifix cubes, candy); (b) semiconcrete items that may consist of pictures that can be used to represent the same concept (e.g., drawings such as dots, tally marks, circles); and (c) abstract items that consist of using symbols to represent numbers. In Witzel’s study, 34 students with LD were matched with 34 other students with LD. The students in the experimental group were taught using the CRA sequence, and students in the control group were taught in a traditional manner (explicit instruction without the use of manipulatives). The findings revealed that students who were taught using the CRA sequence demonstrated a better understanding of how to solve algebraic equations. However, the findings also indicated that both groups increased significantly, with slightly more improvement noted in the CRA group. Other studies incorporated the use of a graduated instructional strategy and used mathematical concepts such as ratios and proportions (Jitendra et al., 2009), solving algebraic equations (Scheuermann et al., 2009; Witzel, 2005; Witzel et al., 2003), and word problems (Maccini & Hughes, 2000).

Third, studies included interventions involving the use of a specific type of procedure that involved primarily student-centered activities. The instructional approaches used in these studies were used to maintain students’ attention and foster practice of math problems through the use of interactive activities. These studies involved instruction other
than the explicit instructional techniques used by the classroom teacher to teach, model, guide, and support student learning. In this learning environment, the teacher is not the primary source of instructional delivery. Instead, the teacher’s purpose is to facilitate student activities and keep students on task. These studies included interventions such as peer-assisted learning strategies (PALS) (Calhoon & Fuchs, 2003) and enhanced anchored instruction (EAI) (Bottge et al., 2001; Bottge et al., 2004; Bottge, Rueda, LaRoque et al., 2007; Bottge Rueda, Serlin et al., 2007). Thus, the types of intervention used involved peer tutoring and video modeling to provide classroom instruction. For example, Calhoon and Fuchs (2003) examined the effects of PALS and curriculum-based measures (CBM) on the mathematics performances of students at the secondary level with LD. Ninety-two students were randomly assigned to a treatment or a control group. The students were taught to participate in PALS by the classroom teacher, given the opportunity to practice, and provided with guidelines to follow when working with peers. Students worked on problems involving math computational skills and concept/application math skills. The results indicated that students in the experimental group significantly outperformed students in the control group for math computation skills. However, there was no difference in performance for concept/application skills between the experimental and the control groups.

In another instance, Bottge, Heinrichs, Mehta, and Hung (2002) examined the effect of EAI on problem solving and computation instruction. Students in the treatment groups received instruction using videodisc problems. During these sessions, students worked together to solve mathematical problems and then were introduced to characters on
a video who instructed them further on how to solve each problem. During this process, the teacher provided guidance on how to use the technology and kept students on task. However, the instructions provided to the students were provided via interaction with peers and through the video. Students were asked to apply their knowledge by performing tasks such as constructing and racing cars. In the control group, students planned a trip and used different math strategies to complete their task. The results demonstrated that students who received EAI instructions improved in their problem-solving skills.

**Targeted Skills Supported by Math Interventions**

In general, experimental studies demonstrate that the students’ ability to understand mathematical functions and basic algebra concepts depends heavily on their ability to do math computations fluently and accurately, their understanding of concepts, and their ability to problem solve (Gersten et al., 2009; Maccini et al., 2007). Therefore, these specific mathematical skills are targeted to help create effective interventions. The literature shows that many experimental studies for secondary students with LD are designed to focus on: (a) math computation and fluency (b) conceptual skills, and (c) problem-solving skills (Bottge, Rueda et al., 2010; Calhoon & Fuchs, 2003; Fuchs, Bahr, & Reith, 2001; Gersten et al., 2009; Ives, 2007; Jitendra et al., 2009; Maccini et al., 2007; Witzel, 2005; Witzel et al., 2003; Xin et al., 2005).

First, students’ ability to master math skills and concepts is highly dependent on their ability to solve and respond accurately to math problems (Poncy, Skinner, & Jaspers, 2007). Math computation and fluency involve an ability to calculate correct answers consistently. Interventions usually use procedures designed to enhance skills
through the practice of math facts, completing the same problem multiple times, using strategies to reduce errors, and corrective feedback and prompts designed to teach students how to attain, check, and consistently produce correct answers. Therefore, interventions are straightforward and representative of a specific standard or benchmark (Axtell et al., 2009; Poncy et al., 2007; Test & Ellis, 2005). For instance, Axtell et al. (2009) used an instructional strategy referred to as Copy, Cover, and Compare (CCC) to increase the procedural fluency of a group of 36 middle school students (13 in the control group and 23 in the experimental group). Eleven students had been identified with LD and all other participants were enrolled in the study due to mathematics weaknesses. Each participant was randomly assigned to either a control or experimental group. The students from each group were first administered a curriculum-based measurements (CBM) probe. Students in the experimental group were given a folder with sets of division problems of up to 148 problems. Each student was given 1 minute and 20 seconds to complete the first page of math problems. After the students finished the first page, the teacher showed the students the answers and asked each student to circle the first five problems that were either answered incorrectly or not answered. Using a new page, the students were then asked to copy the correct answers to those problems using a CCC matrix. Then, the students were asked to cover the answers with their hands, copy the problem again, and complete the problem on a new sheet of paper. Students were then asked to look at the questions and then the answer five times each. Students completed the same problems multiple times and each time compared their answers to the correct answers. In total, students were provided with 30 opportunities to give a written response and 125
opportunities to respond vocally. The students in the control group were given basic math problems to practice until their CBM probes. The results of this study showed that CCC strategy was effective in increasing student procedural fluency and accuracy of division math fact when compared to their peers in the control group.

Second, studies targeting conceptual understanding involved the use of a flexible, integrated (Maccini et al., 2007), and useful understanding of mathematical concepts. Interventions that focus on developing and measuring conceptual understanding aim to teach students how to develop an understanding of abstract mathematics (Scheuerman et al., 2009; Witzel, 2005; Witzel et al., 2003). Conceptual measures in this type of intervention involve solving word problems, interpreting inequality symbols and meaning, and the ability to explain the underlying concepts rather than merely following steps to produce accuracy. Researchers interested in conceptual understanding are focused on how students think about mathematics, the thought process involved in this procedure, and an emphasis on methods used during problem solving tasks (Jenkins, 2010). In order to demonstrate conceptual understanding, students should be able to understand meaning rather than perform rote memorization (Maccini et al., 2007), understand the makeup of the problem, and understand the math ideas as opposed to facts (Maccini et al., 2008). Overall, to demonstrate conceptual understanding, students need to articulate why as well as the how. For instance, in Montague, Enders, and Dietz (2011), middle school students with LD were taught a cognitive strategy designed to teach cognitive processing skills through the use of word problems. Students were taught to “read, paraphrase, visualize, hypothesize, estimate, compute, and check” (p. 263)
mathematical problems. The students in the experimental group outperformed the control group.

Similarly, Naglieri and Johnson (2002) used a cognitive strategy to teach 19 middle school students with LD how to accurately understand and compute basic operational problems. A single-subject design was used to teach a specific strategy: Planning, Attention Simultaneous, Successive (PASS). The instruction phase consisted of providing students with an introduction to the meaning and activities to be performed during each phase of using the strategy. Students were expected to verbalize strategies and various probes were used to generate active thinking throughout the problem. The steps began as soon as the students received the worksheet and continued through to the end of completing each problem. The researchers reported that from baseline to intervention, students obtained a 95% improvement on the final assessment.

Third, the studies targeting problem-solving skills focused on the students’ ability to demonstrate problem-solving abilities using math computation skills, procedural fluency, and conceptual knowledge. Problem solving refers to the entire process of integrating prior knowledge in tactful ways to maneuver word problems. Therefore, student participants in these studies demonstrated an understanding of problem solving by showing how they would evaluate the situation, interpret meaning, plan a strategy, and solve the problem (Jitendra et al., 2002; Jitendra et al., 2009). For example, Xin et al. (2005) examined the effects of two problem-solving instructional strategies—SBI and general strategy (GBI)—on mathematical word problem solving for 22 middle school students. The study consisted of students with LD and students at risk for failing math. In
both conditions, students received explicit instruction from the teacher and were guided through problems before being presented with independent work. Overall, in the two conditions, the students were taught how to solve the problem using a general four-step problem-solving procedure of reading and understanding the problem, planning, solving, and checking the completed work. However, the SBI group was taught how to identify the problem type to choose the correct schema diagram to represent the problem, and the GBI group was taught to identify the type of problem then draw semiconcrete pictures to present the information before solving the problem. The results indicated that students who used the SBI strategy outperformed the students in the GBI condition. In general, these types of intervention studies typically focused on providing students with strategies to solve different types of problems with the intention of examining their ability to apply concepts to new situations.

Overall, these instructional approaches have been proven to structure, guide, and improve mathematics instructions for students with and without LD. These studies have improved outcomes by helping to guide instructional procedures. Subsequently, interventions were implemented to evaluate whether or not students had mastered a particular type of skill (i.e., mathematics computational skills, mathematics problem-solving skills, mathematics fluency, and math conceptual skills), retrieved the skill when needed, and completed the mathematical problem accurately (i.e., basic computations involving integers, equations in one variable, functions involving two variables, linear equations) (Scheuermann et al., 2009; Witzel, 2005; Witzel et al., 2003). However, addressing only targeted skills and specific mathematical concepts can be
limiting in different ways. For instance, because mathematical understanding depends on various skills that can vary from one concept to another, an understanding of one concept or skill may not necessarily lead to an understanding of multiple mathematical concepts or concepts beyond those taught during the intervention.

Despite the interventions available, some students with LD and mathematics difficulties continue to show limited success in mathematics. Ginsburg (1997b) has shown that students’ mathematical performances may be influenced by their experiences and perceptions about math. Therefore, an understanding of students’ way of thinking about math is an important component in students’ academic progress and may help to increase educators understanding of how they might plan and execute mathematical tasks.

What Do We Not Know?

As Ginsburg (1997a, 1997b) and Howard and Whitaker (2011) have stated, the absence of students’ voices in research has left a gap in the literature. Although the number of quantitative intervention studies that exist on LD in the area of mathematics at the secondary level has steadily increased over the years, the topic of learning disabilities in mathematics also requires serious attention from qualitative researchers (Ginsburg, 1997b). An understanding of students’ perceptions about math and mathematical thinking can help guide teachers’ choice of interventions more effectively so that they plan lessons accordingly and better incorporate a variety of strategies to effectively teach math to students with LD (Even & Tirosh, 2008; Fennema et al., 1993; Fennema et al., 1996; Ginsburg, 1997b; Ginsburg & Seo, 1999; Jenkins, 2010). This may impact students’ level of motivation, which can be related to academic performance (Howard & Whitaker,
2011; Sumpter, 2013) and students’ desire to progress academically (Fennema et al., 1993; Howard & Whitaker, 2011; Lucas & Fugitt, 2009).

**Literature Search Procedures**

A comprehensive literature search was executed to identify studies related to the mathematics perception and mathematical thinking of students with LD and mathematics difficulties. This search consisted of a systematic electronic search conducted of the following six databases: Academic Search Complete, Education Resources Information Center (ERIC), InfoTrac, EBSCO Host, Psychology and Behavior Science, and PsycInfo. The databases were searched from 1993 to 2013 using the following keywords: mathematics, learning disability, at risk for math difficulty, math disability, mathematics learning problems, mathematics computation, high incidences disabilities, effective intervention, qualitative math, interview with students with learning disabilities, perceptions of student with disabilities about math, feelings about math, secondary math qualitative research, and high school students with math difficulty. This resulted in seven articles that met the criteria.

Then, an ancestral search was conducted from the reference sections of each article that was obtained which resulted in three articles. Third, in order to optimize search results, the second and the third authors were used as keyword descriptors in all previously mentioned databases resulting in one additional study. Finally, a hand search was conducted of education journals, including Behavioral Disorders, Exceptionality, Exceptional Children, Learning Disability Quarterly, Learning Disabilities Research and Practice, Remedial and Special Education, Journal for Research in Mathematics.
A final total of nine studies were identified for inclusion.

**Criteria for Inclusion**

This literature review examined and analyzed studies that had been specifically designed to describe and evaluate the mathematical thinking of secondary students with LD or at risk for mathematics difficulties. In order to be considered for the current review, studies needed to meet the following criteria: (a) be published in a peer-reviewed journal between January 1993 and November 2013, (b) include secondary students (i.e., Grades 6-12), with LD and/or students at risk for mathematics difficulties as defined by the school system, local, state, and federal government where the study was conducted; and (c) involve the perspectives of students with LD or at risk for mathematics difficulties. Both qualitative and mixed-methods studies were included.

Since the number of qualitative research articles that focused on the perceptions of secondary students with LD about mathematics was very limited, the qualitative literature review was broadened to include mathematical studies involving elementary through high school students (Clarke, 2001; Fennema et al., 1996; Lewis, 2010; Sumpter, 2013), precollege students (Howard & Whitaker, 2011), teachers, and preservice teachers (Jenkins, 2010).

**Studies Included**

The search procedures resulted in nine qualitative and mixed-methods studies. Each study was included because of its contribution to the literature in the researchers’ exploration of students’ thought process and mathematical thinking, and in the
methodological approach to their research findings. All nine studies were identified as meeting the previously stated criteria and included seminal researchers commonly cited for conducting qualitative studies in mathematics with secondary students (Fennema et al., 1993; Fennema et al., 1996; Ginsburg, 1981; Ginsburg & Seo, 1999; Howard & Whitaker, 2011; Jenkins, 2010; Lewis, 2010; Lobato et al., 2012; Sumpter, 2013).

Characteristics of Studies

The included studies were published between 1993 and 2013 with fewer than 25% of these studies conducted in the 20th century (Fennema et al., 1993; Fennema et al., 1996). Of the 103 respective numbers of students in these nine studies, three studies included the number of students with LD (Fennema et al., 1993; Fennema et al., 1996; Lewis, 2010). A total of three students identified with LD were reported. One study was conducted in a self-contained classroom (Fennema et al., 1993). However, Fennema et al. (1993) did not report the total number of students enrolled in each self-contained classroom. The remaining studies included general education students and students at risk of mathematics difficulties. Additionally, the research was conducted with elementary, middle, and secondary students’ school students; one study was conducted at the precollege level using recently graduated high school students. Overall, limited research was conducted at the secondary level, especially with students with LD. Appendix B contains a summary of these studies.

Of the nine studies, six were considered only qualitative, and three were mixed methods. The studies consisted of both students and teachers (Fennema et al., 1993). The interviews conducted with teachers were incorporated because they (a) added an
understanding of students’ problem solving and conceptual understanding, (b) involved
teachers’ participation in dialogues with students as they worked through problems and
reflected on past math experiences, and (c) incorporated or evaluated the effectiveness of
developing teachers’ understanding of students’ mathematical perspectives on
instructional practice (Clarke, 2001; Fennema et al., 1993). In general, the studies’
research questions and purposes ranged from an exploration of students’ perceptions of
mathematics to teachers’ perceptions of the impact of understanding students’
perceptions about mathematics on student achievement (Clarke, 2001; Howard &
Whitaker, 2011; Fennema et al., 1996; Lewis, 2010; Lucas & Fugitt, 2009). The
academic content consisted of prealgebra and algebra skills (i.e., adding and subtracting
integers, solving algebraic equations, understanding mathematical symbols, solving
proportions, and combining like terms).

The review of these studies is organized in two primary sections followed by a
summary. The first section includes a description of the studies. The second section
describes the components of each study by describing its purpose, methodological
approach, and results. Finally, the third section concludes with a summary of how
students’ perspectives, student motivation, and teachers’ perspectives about math and
problem solving can inform the type and the quality of math instruction needed for
students with LD and those at risk for mathematics difficulties.

Mathematical Thought Processes of Students

Overall, the research on students with mathematics difficulties focused on matters
related to cognitive processing, specifically, mathematical thinking (Fennema et al.,
1993, Fennema et al., 1996; Jenkins, 2010; Lewis, 2010) and issues related to psychology (students’ feelings about and toward mathematics) (Clarke, 2001; Howard & Whitaker, 2011; Lobato et al., 2012; Lucas & Fugitt, 2009; Sumpter, 2013). The components of these studies are presented in terms of their primary approach to better understanding students’ mathematical thinking.

More specifically, the psychology of thought processes and the cognitive approach to understanding students’ perception of mathematics is best described by Ginsburg (1981), who argued that understanding students’ knowledge of mathematics should not only involve the pursuit to test and interpret questions correctly answered by students. Ginsburg argued that understanding students’ knowledge of mathematics should also involve dialogue with students regarding their perceptions about math and math problem solving. Ginsburg contended that teachers’ understanding of how the students they teach solve a variety of problems is correlated to the students’ achievement. He further stated that when students are taught math in classrooms, they are usually taught using symbols and numbers—a process that can lead to a lack of meaning for some of these students (Ginsburg, 1981). He stated that these methods are usually taught quantitatively and follow step-by-step procedures designed to establish procedural knowledge in the students’ approach to solving a problem. The demonstration of students’ abilities to complete mathematical problems can be used to help determine how students process mathematics.

Ginsburg (1981) added that a gap exists between the presented materials or the instruction students receive and what students actually learn. In his work, he indicated
that a majority of teachers’ present instruction is based on what these teachers think they know about students and their perceived understanding of students’ prior knowledge. As Ginsburg stated, symbols and numbers can have an abstract meaning for students and sometimes prevent students from problem solving or being taught about critical concepts. Ginsburg believed that in order to promote meaningful learning in students, teachers should strive to design lessons or instructions that complement students’ actual way thinking. He contended that this particular drive to gain such knowledge can lead to a better future for math educational practices.

**Students’ Mathematical Thinking**

One way to analyze students’ mathematical thinking is to focus primarily on a cognitive or systematic approach to analyzing and observing student mathematical work samples. For example, Lewis (2010) explored the qualitative differences in the work process of students with LD in mathematics when solving math problems. The researcher conducted a longitudinal, qualitative case study to understand the LD through an exploration of common errors and student explanations of mathematics task completion procedures. The longitudinal study was conducted with data from over a course of four years, collected on a weekly basis. Data collection consisted of student achievement test scores, classroom observations, videotaped tutoring sessions, and math scores collected. This study consisted of one participant diagnosed with a specific LD in mathematics. The student participated in the study from age 13 to 18 years, 8th through 12th grade.

The participant was videotaped for a total of 30 tutoring sessions: 11 sessions using the computer-based program Math Master, and 19 sessions on fraction reduction.
using a researcher-created intervention designed to use the student’s knowledge of factors (e.g., 3 and 5 are factors of 15). An error analysis was conducted based on how the student responded to questions on Math Master, a computerized program. This test was generated to evaluate multiplication facts.

As a result, during the analytic portion after the intervention was taught, the researcher conducted an error analysis of multiplication facts and fraction reduction. The error analysis identified errors involving basic operations (e.g., $7 \times 4 = 21$ instead of $7 \times 4 = 28$) and in the process of reducing fractions through knowledge of listing multiples of each numbers (e.g., four: 4, 8, 12…). In addition, videotaped data from tutoring sessions and evaluated effects of the intervention approach were analyzed qualitatively. As a result, the student’s sessions were assessed, work was analyzed for common errors, and problem-solving strategies were assessed on a regular basis. Findings revealed the student with LD made errors similar to errors made by students without LD in mathematics. The student with LD had difficulty recognizing signs and understanding operations and thus continued to rely on strategy. Lewis (2010) also stated that once the student’s problems had been identified and understood, it was possible to effectively help the student.

Similarly, Jenkins (2010) conducted a qualitative exploratory research study using a structured interview process of students. The researcher trained preservice teachers to conduct semistructured interviews using a sample of middle school students, Grade 6 through 8, who were enrolled in a math course. The interviews adhered to existing procedures of clinical interview processes for the cognitive tasks (Jenkins, 2010). Each interview was conducted in three rounds, all sessions were videotaped, and anecdotal
notes were recorded. A preservice teacher was paired with another teacher and interviewed three students. One teacher assumed the role of the interviewer and the other the role of the recorder. Students were interviewed using age-appropriate math problems, mathematical thinking was observed, and an understanding of mathematical concept was analyzed. In order to observe students’ mathematical thinking, students were asked to demonstrate an understanding of problems involving finding the area and perimeter. Findings revealed that students lacked a sufficient knowledge of the mathematic procedures required to accurately explain their thinking articulately. Students also had difficulty in connecting mathematical facts to real-life situations and, among the middle school students interviewed, there was a significant difference in the way students made sense of each problem. As result, Jenkins (2010) contended that structured interviews can lead to further awareness of how students make sense, process, and understand math. Additional results also revealed that understanding the knowledge of students is an effective tool that teachers can use to implement classroom instructions (Jenkins, 2010). Teachers reported that developing an understanding of the different ways in which students interpret the same problems can lead to better understanding students’ mathematical thinking.

In another study, Fennema et al. (1993) described how an awareness of children’s mathematical thinking can be generated by using a cognitive science research paradigm. Fennema et al. explored how a teacher used research on student thinking in basic math (addition, subtraction, division, and subtraction) in a classroom using a mixed-methods study. The first and the second years of the study consisted of an experimental focus and
for year three and four, a case study approach was implemented in self-contained classes. Although this was a mixed-methods study, a large component of the data collection procedures consisted of gathering qualitative data. This study incorporated the use of a Cognitively Guided Instructional (CGI) method that helped the teacher in practice gain better understandings of how students learn. Data were collected over four years by one particular teacher, starting with a first-grade class in a self-contained classroom. Fennema et al. (1993) focused on the teacher’s process, including classroom teaching and what the children appeared to learn. A student interview component was also used to collect data. Nine students (including one student with LD) were randomly selected from the class and interviewed to assess their mathematical knowledge and skills. These interviews included problem sets consisting of number facts and problem-solving questions, mostly subtraction and addition. The findings showed that the teacher observed and listened to the students’ strategies, and used the knowledge gained from understanding how the students thought as they solved different types of problems to inform their next instruction. Students were able to use different strategies such as algorithms, and some used a regular counting strategy to solve word problems. As reported by the classroom teacher, an understanding of how students think was vital in teaching mathematics instruction. In the results, the teacher reported that the student information gathered was helpful and improved the delivery of the curriculum to the first graders, and it helped structure her teaching to better help students learn more. Fennema et al. (1993) interpreted the teacher’s use of students’ knowledge to aid in constructing the curriculum as having a positive impact on students’ ability, and motivation, to solve problems.
In a later study, Fennema et al. (1996) did not work directly with students, but instead focused on how general and special education teachers’ use of children’s knowledge could affect student mathematical thinking. This mixed-methods study, containing a large qualitative component, involved the use of a teacher developmental program. The study followed a group of 21 teachers from Grades 1-3 over a period of four years with the use of tape recorders to document students’ problem-solving strategies, and focused on basic addition, subtraction, multiplication, and division problems. The teacher used a CGI model that involved observing and recording the perspective of students’ basic number concepts and how they usually think through number and number operations mathematical tasks. The teacher development program consisted of helping teachers discover how to gain an understanding of the development of students’ mathematical thought processes. This was done not only to identify the differences between the problems and the strategies students used to solve them, but also to effectively teach them. Children’s problem-solving, conceptual understanding, and computational skills were assessed. Overall, after the first year, the teachers reported that adjustments made to instructional lessons, as a result of student interviews, were observed in the improvement made by her students. The findings revealed that toward the end of the study, teachers were able to witness how understanding students’ mathematical thinking helped them to positively affect the mathematical performance of students. They also reported that they have started to believe that their role was not just to tell students what to do, but to give the students an opportunity to develop knowledge through their perceived experiences (Fennema et al., 1996).
Students’ Attitudes Toward Math

In a qualitative phenomenological study, Howard and Whitaker (2011) examined the perceptions of what they referred to as newly successful mathematics students: students whose academic history suggested academically low grades in mathematics. The purpose of their research was to interview students about their perceptions regarding what they thought hindered their ability to perform well in math and what enabled them to succeed. Students were asked to assist researchers in identifying qualities in mathematics teaching experiences that could affect students’ attitudes toward math. This study consisted of 14 college-level students enrolled in a developmental Prealgebra or Algebra course. Data were collected through student interviews, classroom observations, reflective journals, and student assessment scores for exams, quizzes, and homework assignments (Howard & Whitaker, 2011).

The findings revealed that all 14 students were able to identify a point in their interactions with math that had a negative impact on their perception of mathematics. They were also able to identify positive situations or experiences they believed had led to a change for the better in terms of understanding various math concepts. More specifically, Howard and Whitaker (2011) stated that students with negative attitudes toward math are usually prone to be unsuccessful in math classes. For example, students in the study were able to talk about a time when they did poorly in math and stated that they were usually unmotivated to try during this period in time. This study suggests sometimes students’ mathematic performances can be based on nonacademic influences such as students’ feelings or concerns about their ability. Therefore, their research
demonstrated that motivation (high or low) can affect students’ performance, strategies taught to students, and teachers’ actions. For example, if students are highly motivated they are more likely to be motivated to complete a more difficult task (Howard & Whitaker, 2011). The research findings revealed that knowledge of how students’ experiences impact their effort and motivation in the classroom can help teachers be better able to teach students effectively.

In another study, Clarke (2001) used one-on-one student interviews to help guide preservice teachers. In this qualitative descriptive study, a random sample of 40 students without LD (grade 3 through 4) participated in Early Numeracy Research Project (ENRP). The student participants were enrolled in Grade 3-4. The interviews were conducted in three 30- to 40-minute sessions. The purpose of this project was to better understand the mathematical thinking of students while assessing their knowledge of measurement, number, and number sense on various mathematical tasks. Therefore, the interviews provided a tool for assessing the student participants’ way of thinking about these various tasks. Students were interviewed prior to introduction of new materials, during the instruction of some, and after the instruction of most new lessons for the year. As a result, all interviews took place in a three-week time span over two months, and again in eight months. Interviews were completed at the beginning of the year and at the end of the school year. Data were collected primarily through semistructured interviews. During this study, preservice teachers were involved in the process of interviewing each student. From the first set of interviews, teachers were provided with a sense of what typical students knew and could complete. The focus of the researcher was primarily to
understand student perceptions about math, but also to identify and analyze students’ mathematical growth over time. Clarke contended that common themes emerged from the data collected. Over time, students were able to better explain their reasoning for solving certain problems in a particular way. The teachers who reviewed the original interview data reported that they were better able to teach students the content in a more individualized way, which was based on the respective students’ first interview responses.

In a similar study, Sumpter (2013) conducted a case study to understand and highlight the types of beliefs that influence students’ decisions in reasoning while solving mathematics problems. In this study, the mathematics thought process of four students (ages 17-18) was studied. Data included a questionnaire and work samples that were collected during a think-out-loud session with each student. Each 30-minute “talk solving” session was video recorded to show the students’ written work as they were asked to think aloud. In order to further explain the students’ way of solving and thinking, an interview was conducted immediately after each session. Each student also answered a 23-item Likert scale questionnaire. Two days after the first set of information was collected, the researcher conducted another 20-minute semistructured interview to clarify students’ behavior during the think-out-loud sessions. The data were transcribed, problem-solving sessions were interpreted, and a thematic analysis was used to represent patterns and meaning from the collected data. As a result of the researcher’s analytic process, three themes emerged: safety, expectations, and motivations. Students felt safer completing the task with familiar strategies such as rote memorization and completed
problems better when they expected to succeed and were motivated by familiarity and beliefs. The researcher noted that participants’ belief that mathematics tasks should be completed in a specific manner and that familiar strategies are easier to use, resulted in math reasoning that is not always accurate. Overall, through a case study method, this study revealed that students’ own beliefs tend to dominate students’ mathematical reasoning, thus making it even more difficult for students to perform tasks when problems arise (Sumpter, 2013).

Similarly, Lucas and Fugitt (2009) conducted qualitative research to better understand the perceptions of a group of individuals about mathematics. The study consisted of understanding the perceptions of math and math education in a Midwest region of the United States. Surveys were used to collect data and community members were divided and interviewed based on participant availability. Secondary school students, college-bound students, and adults participated in the study (Lucas & Fugitt, 2009). The survey results indicated that although many students feared the concepts of mathematics, eventually these students gradually understood and recognized the need and importance of math. Both students and adults in this study reported that teachers with bad tempers or attitudes had a negative impact on their attitude toward math. Participants stated they believed these types of teachers can also have a negative impact on students’ mathematical achievement. Participants also discussed a need to make sense of the idea that some students with LD remain on track to take Algebra when the knowledge of computational facts are unknown (e.g., \(2 \times 12 = 24\)). Likewise, these participants shared a common belief that all students may not benefit from taking the same courses. Overall,
this study revealed that many participants believed that the quality of a school and the characteristics of teachers are an important part of encouraging and keeping students engaged in difficult mathematical tasks (Lucas & Fugitt, 2009).

Lobato et al. (2012) focused on the role of psychological examinations in understanding students’ conceptual educational goals related to quadratic functions and rate of change. This mixed-methods study included 24 eighth-grade students in 15 hours of instruction on quadratic functions using the pivotal intermediate concepts (PIC) to shape conceptual learning goals (CLGs). This concept involves the use of imagery or mental images to encourage meaning of primary interpretations of situations and symbols for students. Each student participated in a 75-minute clinical interview, was videotaped, their work collected, and samples were analyzed. The results of this study demonstrated that psychological analysis can be used to help identify mathematical learning goals (Lobato et al., 2012). Teachers were better able to understand how students approach more complex problems involving quadratic equations based on their perceptions of the mathematical task. The research demonstrated that students interpret and communicate understanding of various concepts differently. Students were taught the idea of distance, time, and rate, but most students were not able to demonstrate an understanding of terms such as acceleration in regard to speed. As a result, students were unable to perform the necessary tasks required to determine the speed of an object based on their understanding of these terms. However, through dialogue and interpretation knowledge, a deeper understanding of student perceptions was reached. The findings also indicate that what appears to be challenging for students to understand, and what should be an explicit
target or goal for students, may not be apparent or obvious to knowledgeable adults. Thus, their findings demonstrate the importance of systematic psychological analysis to identify students’ perceived understanding, interpretation, and mental operations of mathematical tasks.

Conclusions

Overall, the studies described in this section consisted of a mixture of students with LD, students in general education, teachers of students with mathematics difficulties, and adults who shared their understanding or perceptions of math based on their experiences or through the eyes of their child. The nine qualitative and mixed-methods studies are summarized in Appendix B. Their results suggest that an understanding of psychological and cognitive perceptions of students in mathematics can increase the ability of teachers to better prepare, understand, and incorporate the needs of students into instructional practices (Clarke, 2001; Lewis, 2010; Sumpter, 2013). Teacher and student interviews also suggest that the performance of students with LD at the middle and secondary levels can improve given an understanding of the unique student perspectives about math. As a result, these studies revealed how the perceptions of students can inform the type of instruction provided by classroom teachers and the quality of math instruction provided in the classroom.

For example, Clarke (2001) reported that teachers interviewed after the study reported that they were better able to teach and meet the unique needs of each student. Study findings show that students were better able to explain their reasoning for solving specific problems after participating in dialogues with their teachers. These teachers felt
empowered with useful knowledge for how to proceed with instruction. Likewise, Fennema et al.’s (1993) research findings indicated that knowledge of students’ thinking helps to make effective instructional classroom decisions. Teachers in this study reported that students were able use various strategies to conduct problem-solving tasks. Fennema et al. (1996) also reported similar findings but added that cultivating an understanding of children’s mathematical thinking can also help teachers to make fundamental changes to instruction. Students’ perceptions about mathematics can aid in the identification of problems that may prohibit growth. However, with the identification of such problems, researchers can help students effectively with more fitting intervention practices (Lewis, 2010; Lobato et al., 2012).

The perceptions of students about mathematics can inform instruction, and thus may improve academic outcomes and provide a better understanding of how students with LD and those at risk interpret mathematics. Lewis (2010) sought to understand students’ thinking as it related to how they use problem-solving techniques. Through interviews and observations, Lewis noted that common mathematical errors or hang-ups can impact students’ ability to focus on problem solving in mathematics. However, with knowledge of such factors’ existence, it is possible to help students improve. Similarly, Lucas and Fugitt (2009) reported that individual perceptions about mathematics can have a great deal of impact on effort and student motivation. In their research, participants shared that the attitude of teachers, the quality of one’s learning environment, and the value placed on mathematics can greatly impact students’ motivation and ability to succeed academically (Lucas & Fugitt, 2009; see also Sumpter, 2013).
This knowledge of qualitative research in mathematics can help provide teachers with a deeper understanding of how students perceive math and how they think through the process of problem solving. The perceptions of students about math and math thinking can impact what teachers target in the classroom and their approach to instruction (Ginsburg & Seo, 1999). Ultimately, this type of information can allow teachers to be more sensitive as to how students process math computations and mathematic problem solving.

Although the current literature consists of qualitative studies about the mathematical perspectives of students and teachers about mathematics, it lacks the examination of a substantial amount of students with LD at the secondary level. It is important for educators to seek knowledge on how to create, modify, and adapt instruction so that it interconnects with students’ mathematical thinking (Ginsburg, 1997a, 1997b).

**The Impact of Motivation**

Additionally, across the literature on student perceptions, the concept of motivation and its potential influence on student performance was commonly cited. Researchers found that teachers’ understanding about students’ motivation toward math and perception about mathematics have been shown to influence classroom instruction, activities, and lesson plans (Fennema et al., 1993; Fennema et al., 1996; Howard & Whitaker, 2011; Middleton & Spanias, 1999; Sumpter, 2013). Therefore, student motivation toward math and perceptions about math instruction/math teachers are key data sources for the current study.
Student motivation continues to be a concern of mathematics educators which is supported by the idea that researchers have found those students’ experiences and attitudes influence student ability to achieve success (i.e., Howard & Whitaker, 2011; Middleton & Spanias, 1999). Middleton and Spanias (1999) imply that students who believe that there are limits to what they can learn will be apprehensive and reluctant to learn seemingly difficult material. Motivation, attitude, and perceptions are closely related (Howard & Whitaker, 2011). Thus, the importance of understanding more clearly how motivation can potentially influence perceptions about mathematics can provide useful information for educators committed to student success. Consequently, researchers also agree that how the feeling or attitude students developed toward learning can significantly impact how students learn and what they learn. For students with LD or math difficulties to be successful, a change in attitude, belief, or perception is often needed (Howard & Whitaker, 2011).

As it relates to attitude and perceptions, motivation is defined by Middleton and Spanias (1999) as the reason behind an individual’s motive for acting in a particular way in a given situation. Motivation stems from an individual’s idea of what is important and whether or not to participate in the movement toward pursuing a specific task or goal. Middleton and Spanias refer to two types of educational motivation: academic intrinsic motivation and extrinsic motivation. Academic intrinsic motivation refers to a student’s need to participate in learning for personal development. Students who are intrinsically motivated participate in academic activities and are engaged because they want to learn and because they find these activities enjoyable. On the other hand, academic extrinsic
motivation refers to students who participate in the same academic activities for what they can potentially gain. Students who are extrinsically motivated are interested in obtaining rewards (i.e., good grades, praise, etc.) or wish to stay away from discomforting situations (i.e., bad grades, belittled, etc.). As a result, the research in the area of academic motivation for students focuses on either extrinsic motivation, intrinsic motivation, or both types motivation. The influence of motivation, whether from an intrinsic or extrinsic mindset, has proven to affect how and what students can potentially learn (Middleton & Spanias, 1999).

Along with the realization that students can be motivated intrinsically or extrinsically, researchers have also found that when students see themselves as understanding math, they are more engaged and value mathematics more than students who do not feel confident about their mathematics capabilities (Howard & Whitaker, 2011; Middleton & Spanias, 1999). In addition, in a meta-analysis conducted by Middleton and Spanias (1999) about motivation in mathematics education for student achievement, the researchers identified major findings that suggest a reduction of a positive attitude toward math can be better understood through a look at a classroom teacher’s demeanor and the structure of the classroom environment. Howard and Whitaker (2011) and Lucas and Fugitt (2009) both found that students are more motivated to work through challenges in math classes when they feel a sense of support from the classroom teachers and when the classroom environment is positive.

According to Sumpter (2013), students are more likely to attempt and use problem-solving methods when they are motivated intrinsically and prefer to solve
problems using the teacher’s method or directions when motivation is based on more extrinsic factors. The literature demonstrates that motivation has always been a concern of educators and implies that attitude and perceptions are closely related to motivation. For instance, the attitude a student develops toward learning a specific problem can impact how well a student learns the material and is willing to work through difficult problems. The more information educators are able to use about students’ ability, motivation, and perception, the better teachers are able to implement effective lesson planning, lesson presentation, and instructional approaches (Ginsburg, 1997b; Howard & Whitaker, 2011; Lewis, 2010; Middleton & Spanias, 1999; Sumpter, 2013).

**Research Limitations in the Current Literature**

Extant studies in the literature on the mathematical thinking and perceptions of students about mathematics demonstrate and display sound methodology. The studies’ designs are clearly stated, the participants’ background information and the studies’ interventions are described, and data collection and data analysis procedures are explained. However, several limitations exist in the current literature in this area, which impose questionable restrictions on the generalizability of these studies. In a brief review of the effectiveness of various mathematical interventions, it is evident that mathematical intervention strategies can improve student understanding of math concepts. However, researchers argue that intervention research is merely numbers and still requires a voice (Ginsburg, 1997b; Lewis, 2010; Whitaker & Howard, 2011). Ginsburg and Seo (1999) stated that voices of the participants can provide an even greater and more in-depth understanding of the phenomenon. Although research demonstrates the positive impact of
better understanding students’ mathematical perceptions, limitations in the literature still exist.

First, very few studies exist to better understand the mathematical thinking and perceptions of secondary students. Instead, most studies have focused on students at the elementary and middle school levels. Specifically, there seems to be a need to design research studies that evaluate how secondary students with LD or at risk for mathematics difficulties process, interpret, and perform mathematical tasks. The representation of a more diverse population of students can lend to an even wider source of knowledge and also add to the existing literature.

Second, mathematical concepts progressively become more difficult as students complete each level of math requirements. Thus, the clinical interview process of assessing the mathematical thinking of secondary students with LD and students at risk for mathematics difficulties may be limited. If students do not understand a problem or a component of the problem, they will be unable to talk out loud during a clinical interview. For example, students learn in kindergarten how to add, subtract, multiply, and divide single-digit numbers; as they progress, they learn how to work with double- and triple-digit numbers. However, concepts build on previously taught information, requiring that students develop a good understanding of the fundamental knowledge prior to advancing. Many students lack this foundation, especially those with LD. While researchers such as Scheuermann et al. (2009) believe and recommend that math instruction should be focused on procedural understanding of fundamentals, the NCTM standards require a need to focus on conceptual understanding. Therefore, researchers are
limited to the types of questions that can be used to assess students’ mathematical thought processes.

With that said, many difficulties exist at the secondary level, when students are expected to have a basis for higher level subjects such as Algebra, Geometry, and Algebra II. Currently, students are required to pass at least Algebra to graduate high school with a standard diploma (NCTM, 2014). Therefore, a limitation exists on the difficulty level of content examined in the previous studies. Lewis (2010) stated that students are more reluctant to perform or talk about mathematical tasks that require a higher level of reasoning skills or simply deemed difficult by the participant. Therefore, there is a limit to how much researchers can deduce about the process of how students reason through mathematical tasks. Consequently, very few studies assessed the mathematical thinking of students using higher level concepts such as solving systems of equations and factoring; instead, they used lower level concepts such as solving equations, proportions, and fractions.

Third, the majority of studies were conducted for students enrolled in lower level math classes. Current studies in this area lack the shared experiences of students enrolled in higher level math classes such as Geometry and Algebra II. As more and more students with LD enroll in higher level math courses, researchers should also seek to better understand these students’ thought processes at these levels. As geometry presents more visual and deductive challenges and Algebra II also presents even more abstract and complex sets of problems for many students with LD, further investigation in higher levels mathematics course are needed for this population of students.
Fourth, there is a need for greater specificity in providing relevant information about student participants. Future studies should include more specific information such as socioeconomic status and educational background (i.e., current classroom size, previous math course completed, and course grade, etc.) to enhance the generalizability and/or transferability of the study. Adding such information about a study’s participants adds to the study’s ability to be replicated, and increases the transferability or applicability of the study across a broader population of students. As Merriam (1998) states, providing sufficient information can help readers decipher the degree to which a given situation matches that of the research, and thus whether or not the finding(s) can be applied. There is a need to include more students with learning disabilities in secondary studies, and an even greater need to provide more descriptive information about students with learning disabilities such as age, previous test results, setting, and disability categories, in addition to socioeconomic background.

A final limitation is the participant’s ability to recall and draw from previous experiences and respond to interview protocols. As stated by Howard and Whitaker (2011), participants’ experiences can be affected by the time of event, their perceptions and feelings about what happened, and the consequences of their actions. Thus, the researchers’ ability to better understanding the participants’ perceptions are limited to what students are able to remember and share during the interview process, making it critical for researchers to carefully consider what students were taught, and the time of the school year the interview is conducted.
Summative Educational Implications

Based on the results of this literature review, many mathematics instructional approaches and strategies are available for students who have been identified as having LD, including instructional strategies that were found to be very effective for students at the secondary level. However, it is evident that more research is needed to identify the psychological and cognitive perspectives (mathematical thinking) (Ginsburg, 1997a, 1997b) of students with LD. As stated by Ginsburg and Seo (1999), it is not only important to evaluate what is known about students with a math disability, it is also important to understand students’ dispositions on learning math, their perceived ability to understand algebraic concepts and symbols, and their cognitive processes in solving algebraic concepts and problems.

The literature also demonstrated that, at the secondary level, the demand for schools to produce higher and more mathematically competent students is rapidly increasing. For students with LD and students at risk, it has become more and more difficult to live up to the demands of the school system. Students with LD and at risk often lack mathematics problems-solving, computational, and problem application skills to demonstrate competence in various core subjects (Axtell et al., 2009; Bottge, Grant et al., 2010; Bottge, Rueda et al., 2010; Calhoon, Fuchs, & Hamlet, 2000; Cortiella, 2011; Cortiella & Horowitz, 2014). As of 2009, the NCTM standards have changed and are currently more focused on students achieving mathematics proficiency through an ability to demonstrate conceptual skills rather than computational skills. This literature analysis revealed that students continue to demonstrate mathematics difficulty in being able to
correctly think and explain computational problems, even at the middle and upper secondary levels (Howard & Whitaker, 2011; Lewis, 2010; Sumpter, 2013). In addition, the current studies examine the ability of students to think about computational problems and meaning of symbols but lack an examination of how students think and communicate about higher level math problems. Consequently, further knowledge of how students with LD and at risk for mathematics difficulties think and reason about higher level math problems is needed. It is therefore important for educators and lawmakers to seek a better understanding of how secondary students may or may not understand higher level mathematical concepts needed to complete courses such as Algebra and Geometry.

Likewise, further research is needed to better understand student perspectives of mathematics to guide interventions, procedures, and classroom instruction. This literature review implied that studies would be more insightful if they incorporated student interviews and classroom observations. As Ginsburg and Seo (1999) stated, the goal of research should be to understand how students, especially students with LD, feel about math and approach learning and problem solving. Lobato et al. (2012) argued that students’ views of mathematics can greatly affect their overall performance and understanding. Likewise, knowing how students understand math can be one of the greatest tools used to reach those students. While literature in the field of teaching mathematics exists in the area of validating evidence-based practice, there is a need for more literature that helps the field to better understand student perspectives in order to provide effective classroom instruction. Similarly, Jenkins (2010) stated that knowledge of how students think and work through mathematics is a vital component of the
development of teacher’s pedagogical knowledge. It can be used to guide and drive instructional practices. The knowledge of how students think about math can also steer interventions that are based on students’ needs and can result in students’ improved overall mathematical performance. As a result, this knowledge can also serve as an important tool for college and university teacher preparation programs.

In particular, qualitative studies maintain that students are more motivated to learn when teachers appear to understand their individual situations. Furthermore, while researchers claim to understand what students with LD need, it is important for them to also consider how students feel about mathematics, and how they capture and make sense of taught materials (Howard & Whitaker, 2011; Sumpter, 2013). As Ginsburg and Seo (1999) stated, knowledge about specific students can help educators compete with the effects of external influences, such as environment and students’ constantly increasing experiences, both academic and nonacademic, that can negatively impact students’ understanding.

In short, there is a gap in combining research on intervention and student mathematical thinking or perceptions about math. Therefore, this study explored the following research questions:

RQ1. What are the perceptions and attitudes of a group of secondary students with learning disabilities and students at risk for mathematics difficulties about mathematics?
RQ2. How well does a group of secondary students with learning disabilities and students at risk for mathematics difficulties understand important concepts and symbols of algebra, and how they are used?

RQ3. What does a group of secondary students with learning disabilities and students at risk for mathematics difficulties find to be the most challenging about mathematics?

**Theoretical Framework**

According to Maxwell (2005) and Creswell (2008), the researcher’s paradigmatic views are important in mixed-methods research inquiry; otherwise, it can become unclear why two different methods are being used and the benefits of using them. Therefore, the quantitative portion of this study focused on the traditions of positivism, which seek to provide facts through testable measures (Greene, 2007). This paradigmatic view seeks to answer questions and find a universal truth from an objective stance. However, this type of inquiry does not provide a means of understanding various factors, such as individual’s feelings and perceptions (Creswell, 2008; Greene, 2007; Patton, 2002). Thus, the positivist’s views of quantitative research were combined with the phenomenological views of qualitative research.

For mixed-methods researchers, it is also important to understand the argument presented by Fisher and Stenner (2011), who argued that a phenomenological method “has the potential to integrate qualitative and quantitative concerns” (p. 1). They also stated that this method works toward a “uniform criteria of substantive meaningfulness and mathematical rigor” (p. 1). This perspective would allow a researcher to seek an
understanding of a group of students’ perceptions about mathematics, and thus aided in developing the theoretical framework for this study.

Despite Fisher and Stenner’s (2011) view regarding the benefits of using the phenomenological method, this mixed-methods research involved views from both a quantitative angle and a qualitative perspective. Thus, both different and more traditional approaches of quantitative and qualitative methods of data collection and analysis were used to understand, explain, and answer the research questions, and helped to guide the researcher’s development of this study. The methods used in this study were chosen as tools deemed most useful in better understanding the presented phenomenon.

**Phenomenology as a Philosophy and Methodology**

Creswell (2008) stated that a qualitative study stems from the philosophy of phenomenology and places an emphasis on the experiences and the interpretations of the participants involved in the study. As a result, researchers conduct phenomenological studies that consist of particular procedures specific to identifying the essence of the phenomenon being explored through the eyes of the participants. Merriam (1998) recognized that an investigation which is based on a phenomenological perspective—one that focuses “on the essence or structure of an experience (phenomenon)” (p. 15)—lends itself to the use of qualitative research seeking to gain a better understanding of the phenomenon.

The theoretical perspectives of phenomenology look at the smallest unit of the phenomenon and what makes an experience or the essences of the experiences (Patton, 2002). This philosophical lens looks at how individuals conceptualize actions,
interactions, and objects to make meaning of an experience. The idea of making meaning of an experience can also be looked at as what goes into the shared meaning of individuals.

Within the theoretical framework of phenomenology, this research sought to use methods and techniques that focused on describing how individuals experience certain events, such as math. Therefore, this study sought to have the participants describe in detail their learning environments, circumstances, understandings, and feelings about a particular personal or shared math experience. As Patton (2002) stated, anything that contributes to an individual’s consciousness or awareness of a phenomenon is of interest to a phenomenologist. For instance, if a researcher designed a phenomenological study that focused on the essences of the participants’ specific experience, the researcher-driven design would include in-depth interviews, observations, and possible data collections such as photos, documents, or information received from a direct experience. As a result, this study focused on in-depth interviews, observation of student work, student assessments, and student background information as primary modes for understanding students’ experiences.

The phenomenologist as researcher approaches the research as a tool that provides information purely on the participants’ perceptions and not what it may appear to mean, resemble, or reflect. The researcher is interested in using approaches that generate how experiences or meaning become reality for the individual. The phenomenologist asserts that, through carefully chosen research methods and close attention to detail, the goal of phenomenological study is not to compare perceptions to the external knowledge of the
participant but to focus on consciousness and perception (Willis, 2007). Consequently, a phenomenological mental model helped to guide this study primarily because it provided insight into the philosophy of understanding the experiences of students’ perceptions of mathematics through in-depth and accurate descriptions of their past and current experiences (Patton, 2002; Willis, 2007).

**Positivism as a Philosophy and Methodology**

Patton (2002) respected and recognized that an investigation that is based on a positivist point of view allows for the use of quantitative research. A minimal explanation of this particular lens is simply that it seeks a universal truth. A positivist seeks to generalize ideas, create general theories, and categorize phenomenon. Positivists also believe that the social world is independent of an individual’s knowledge of its existence. This paradigm seeks to answer questions with facts and seeks a universal truth. Generated from this idea was the idea of quantitative research; that is, research designed to capture the facts through testable measures (Greene, 2007).

Therefore, from a positivist perspective, this research used quantitative data sources (i.e., clinical interview items, math diagnostic test) to better understand the mathematical thought processes of a group of secondary students with or without LD. The goal was to find out exactly what the student participants understood or perceived when given a specific math task. These data sources sought to capture the facts in terms of students’ current knowledge of subject matter. Table 1 provides a summary of the purposes of the mixed-methods study by method.
Table 1

Summary of the Purposes of the Mixed-Methods Study by Method

<table>
<thead>
<tr>
<th>Purpose</th>
<th>Quantitative</th>
<th>Qualitative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Goal</td>
<td>Student’s current math abilities</td>
<td>Student perspective</td>
</tr>
<tr>
<td></td>
<td>Student’s current attitude toward math</td>
<td></td>
</tr>
<tr>
<td>Philosophical Roots</td>
<td>Positivism</td>
<td>Phenomenology</td>
</tr>
<tr>
<td>(Theoretical)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Design</td>
<td>Structured</td>
<td>Emergent</td>
</tr>
<tr>
<td>Sample</td>
<td>Nonrandom</td>
<td>Convenience sampling</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Purposeful selection</td>
</tr>
<tr>
<td>Data Collection</td>
<td>Math attitude inventory</td>
<td>Researcher primary instrument</td>
</tr>
<tr>
<td></td>
<td>Math diagnostic test</td>
<td>Interviews</td>
</tr>
<tr>
<td></td>
<td>Demographic questionnaire</td>
<td></td>
</tr>
<tr>
<td>Data Analysis</td>
<td>Numerical</td>
<td>Emic, Inductive</td>
</tr>
<tr>
<td>Potential Validity Threats</td>
<td>External and internal</td>
<td>External (internal validity)</td>
</tr>
<tr>
<td></td>
<td>Generalizability (transferability)</td>
<td>Generalizability (transferability)</td>
</tr>
<tr>
<td></td>
<td>Researcher bias, subjectivity, sole researcher</td>
<td>Researcher bias, subjectivity, sole</td>
</tr>
<tr>
<td>Results</td>
<td>Nonparametric data, descriptive statistics</td>
<td>In-depth quotations, themes, categories</td>
</tr>
</tbody>
</table>

Summary

This chapter examined the literature relating to the knowledge of mathematics interventions and the perceptions and attitudes of students with and without LD, and students at risk for mathematics difficulties. Characteristics and mathematical tendencies of students with LD and students at risk for mathematics difficulties were also discussed.
and examined. Overall, the literature served as a source of knowledge for the availability and effectiveness of mathematics interventions, and as a source for how an understanding of students’ thought processes and perceptions about mathematics can positively influence instructional practices. The literature on mathematics intervention serves as a guide to better understanding how methods such as the use of explicit instructional practices, mnemonics, and a graduated instructional sequence improve the understanding of various mathematical concepts for students who experience mathematics difficulties. However, more importantly the researchers found that, in addition to evidence-based practices, factors such as student experience and motivation, and student perception and attitude, can be directly tied to the overall achievement or success of students with LD and students at risk for mathematics difficulties. It is important to note that while there are a number of research investigations in the area of secondary mathematics interventions for students with LD, there is a lack of research investigations on students’ perceptions and thought processes of mathematics. Furthermore, understanding students’ perceptions regarding their mathematical thinking and attitude about mathematics may better inform mathematical interventions and instructional strategies for teaching students with LD and students at risk for mathematics difficulties.
CHAPTER THREE

This chapter discusses the methods for the research study. The organization of this chapter is as follows: (a) research design, (b) researcher subjectivity, (c) participants, (d) participation selection procedures, (e) data sources, (f) data collection and procedures, (g) data management, and (h) data analysis. Each component of the methodology is described individually.

Research Design

A mixed-methods approach was used to better understand the thought processes and perceptions of a group of secondary students with learning disabilities and those at risk for mathematics difficulties. The intent of this study was to help educational leaders, researchers, and teachers to understand students’ mathematical thinking more clearly (Jenkins, 2010). This study explored the following questions:

RQ1. What are the perceptions and attitudes of a group of secondary students with learning disabilities and students at risk for mathematics difficulties about mathematics?

RQ2. How well does a group of secondary students with learning disabilities and students at risk for mathematics difficulties understand important concepts and symbols of algebra, and how they are used?
RQ3. What does a group of secondary students with learning disabilities and students at risk for mathematics difficulties find to be the most challenging about mathematics?

**Rationale for Research Design**

The data sources for the current mixed methods study included a demographic questionnaire, a student math attitude inventory, a researcher-constructed math diagnostic test, a clinical interview, and semistructured student interviews. As stated by Maxwell (2005), a mixed-methods approach consists of a combination of research paradigms used in an effort to examine the differences or similarities in the findings in order to understand the problem more comprehensively. Furthermore, Mundia (2012), Maxwell (2005), and Creswell (2008) argue that researchers usually conduct mixed methods research because of four main advantages: (a) it incorporates both qualitative and quantitative methods, (b) it provides a comprehensive review of the phenomenon, (c) the data collection is not limited to one method, and (d) it is a method of data source triangulation.

Consequently, the researcher chose to use a triangulation mixed methods design in which both quantitative and qualitative data are collected simultaneously (Creswell, 2008; Greene, 2007; Mundia, 2012) to provide an opportunity for a more comprehensive analysis of collected data. This triangulated research design helped to provide the researcher with an in-depth description and explanation of the phenomenon being studied. The rationale that underlies using this method is that each form of data collection
will be used to support the weakness that may exist in the other forms of data collection (Creswell, 2008).

As a result, each component of this research created a clear working relationship, which helped to attain the research goals (Maxwell, 2005) and helped to unite the techniques and strategies used for interviewing, selecting the location and participants, collecting the data, and analyzing the data. As stated by Greene (2007), mixed methods research can provide an opportunity for triangulation of data sources and methods in various ways, as triangulation “seeks to enhance validity and credibility through convergence and corroboration” (p. 43). Thus, this study used a triangulation mixed methods design that sought to explore, describe, and explain how a group of secondary students with learning disabilities and those at risk for mathematics difficulties think through math problems and perceive mathematics.

**Participants**

This study was conducted with individuals from similar school systems or regions in the United States who followed the same mathematics curriculum schedule and sequence. Students in this region followed a sequence that included courses Math 1-7 (mostly arithmetic), Math 8 (mostly prealgebraic concepts), Algebra, Geometry, Algebra II, and an option to enroll in Precalculus, Calculus, or a Statistics course. As a result, the participants chosen for this study included students who: (a) had an individualized educational plan (IEP), or (b) were identified by the parent as at risk for mathematics difficulties, (c) attended a middle or high school, and (d) were currently enrolled in or had completed one of two of the following courses: Prealgebra (i.e., including Math 7,
Math 8), Algebra, or Geometry course, or all three courses. The data were collected during the middle of a 10-month school year for each student when the student was enrolled in one of the above-mentioned courses. In addition to having a documented IEP, the participants needed to have a mathematics-related IEP goal, and/or demonstrate difficulty in mathematics based on parental reports and/or math teachers’ reports as noted through previous report cards. Students at risk for mathematics difficulties were identified by the parent because (a) the student had a documented history of receiving failing grades on mathematics assessments (e.g. progress reports, report cards), (b) the student had received concerns from classroom math teachers, and (c) he or she had an understanding that his or her child has agreed with and expressed difficulty understanding various concepts and procedures in mathematics. The term “at risk” described students who were in danger of not achieving academic success in mathematics. Each participant chose a pseudonym or fictitious name to conceal his or her identity. The researcher wanted to assure the participants understood that a conscious effort was taken to protect their rights to privacy (Glesne, 2006).

**Participant Selection Procedures**

After obtaining George Mason University’s Institutional Review Board’s (IRB) approval, participants were recruited for this study. Based on the goal and the purpose of this research, a convenience sampling strategy, followed by the use of a criterion-based purposeful sampling strategy, was used to carefully select each participant for this study. Potential participants were initially recruited electronically. The researcher e-mailed a letter that summarized the study to individuals who were responsible for disseminating
information through listservs of various organizations. The organizations included the George Mason Ph.D. in Education listserv, the Special Education listserv, and the Parent Education Advocacy Training Center listserv (PEATC) (see Appendix C). Each contact person of these organizations agreed to send the recruitment letter through his or her respective listserv. The recruitment letter identified the purpose of the study, the criteria for the study, and a briefing of what the data collection process would entail.

Following listserv recruitment, the parents who responded to the recruitment e-mails were contacted via e-mail or phone by the researcher. Nine parents responded to the recruitment e-mail and five participants were chosen as potential participants. Four participants did not qualify due to age and/or math ability. The first round of participant selection was based on convenience sampling. In order to increase the sample size, the researcher implemented snowball recruitment procedures. That is, the parent of each interviewed participant was asked if he or she knew anyone else who might be a potential fit for the study (Creswell, 2008). If so, the parents were asked to pass the information on to another parent. As a result, three parents were obtained from this method. Each of the three parents contacted the researcher, which then allowed the researcher to follow-up via phone and e-mail to further assess for interest in this study. These three additional participants were identified. After identifying potential participants, a script describing the research and its objective, explaining the meaning of participation in the research, and the length of time required for the study was read to all of the interested parents during a phone conversation. Eight student participants and their parents then met the researcher at a
preferred setting (e.g., at their home, a coffee shop, or at the library). Parents provided informed consent and students provided assent.

**Description of Participants**

Participant information was collected through the use of a demographic questionnaire designed to obtain data about the research participants. The demographic questionnaire was the first document presented to each participant. Each participant was then given the choice of having the questionnaire read out loud or completing it independently, without any reading assistance. The demographic questionnaire contained questions regarding age, gender, grade level, and classroom size. Four of the eight participants were at the middle school level and four at the high school level. Among them, four students were female and four students were male. Six out of eight students were diagnosed with a learning disability and two were students identified as at risk for mathematics difficulties. The average mean age for all the participants was 14.78 (SD = 1.98). There was an even distribution of students across grade levels: one participant at the 7th-grade level, two participants at the 8th-grade level, one participant at the 9th-grade level, one participant at the 10th-grade level, and two participants at the 12th-grade level. Four students were enrolled in a Prealgebra course, one in Algebra I, one in Geometry, and two were enrolled in or completed Algebra II. Out of the eight participants, no participant reported receiving tutoring during the time of the study. Table 2 contains a summary of each participant’s demographic information, and a description of each participant is presented after the table.
### Table 2

**Student Demographic Information and Math Diagnostic Test Findings**

<table>
<thead>
<tr>
<th>Name (pseudonym)</th>
<th>Age</th>
<th>Gender</th>
<th>Ethnicity</th>
<th>Grade</th>
<th>Math class/latest math course</th>
<th>Current class grade</th>
<th>Service delivery</th>
<th>Class size</th>
<th>Disability</th>
<th>Area(s) of difficulty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hive</td>
<td>14</td>
<td>M</td>
<td>Caucasian</td>
<td>9</td>
<td>Algebra</td>
<td>D</td>
<td>Team-taught</td>
<td>&gt;15</td>
<td>At risk</td>
<td>*Mathematics</td>
</tr>
<tr>
<td>Stacy</td>
<td>17</td>
<td>F</td>
<td>Hispanic</td>
<td>12</td>
<td>Algebra II</td>
<td>C</td>
<td>General Education</td>
<td>&gt;15</td>
<td>LD</td>
<td>Mathematics</td>
</tr>
<tr>
<td>David</td>
<td>14</td>
<td>M</td>
<td>Hispanic</td>
<td>8</td>
<td>Math 8 (Prealgebra)</td>
<td>D</td>
<td>Team-Taught</td>
<td>&gt;15</td>
<td>LD</td>
<td>Mathematics</td>
</tr>
<tr>
<td>Rosie</td>
<td>13</td>
<td>F</td>
<td>Hispanic</td>
<td>7</td>
<td>Math 8 (Prealgebra)</td>
<td>B</td>
<td>General Education</td>
<td>&gt;15</td>
<td>LD</td>
<td>Mathematics</td>
</tr>
<tr>
<td>Hunter</td>
<td>14</td>
<td>M</td>
<td>Caucasian</td>
<td>8</td>
<td>Math 8 (Prealgebra)</td>
<td>D</td>
<td>Team-Taught</td>
<td>&gt;15</td>
<td>LD</td>
<td>Mathematics</td>
</tr>
<tr>
<td>Carl</td>
<td>15</td>
<td>M</td>
<td>African American</td>
<td>10</td>
<td>Geometry</td>
<td>C</td>
<td>Self-contained</td>
<td>8 or less</td>
<td>LD</td>
<td>Mathematics</td>
</tr>
<tr>
<td>Brook</td>
<td>18</td>
<td>F</td>
<td>Caucasian</td>
<td>12</td>
<td>Algebra II</td>
<td>B</td>
<td>General Education</td>
<td>&gt;15</td>
<td>At risk</td>
<td>(504 plan)</td>
</tr>
<tr>
<td>Dylan</td>
<td>13</td>
<td>M</td>
<td>Caucasian</td>
<td>8</td>
<td>Prealgebra</td>
<td>F</td>
<td>Self-Contained</td>
<td>8 or less</td>
<td>LD</td>
<td>Mathematics</td>
</tr>
</tbody>
</table>

*Note.* * Student at risk for mathematics difficulties.
**Brook.** Brook was an 18-year old Caucasian female student in the 12th grade. At the time of the study, Brook was enrolled in a Statistics course and recently completed Algebra II. Brook was considered by her parents as at risk for mathematics difficulties. According to her parents, Brook sometimes struggled to understand various algebraic concepts and frequently ignored signs and symbols. As reported by the parents, Brook tended to benefit more from one-on-one attention and repetition. At the time of the study, her parents started the process of obtaining a 504 plan due to chronic health reasons that were negatively impacting her ability to perform well academically, especially in mathematics.

**Carl.** Carl was a 15-year-old African American male student in the 10th grade. At the time of the study, Carl was enrolled in a team-taught Prealgebra course. According to Carl’s individualized educational plan (IEP), Carl was diagnosed with a learning disability in middle school. Additionally, he demonstrated a severe discrepancy between achievement and intellectual ability in the areas of mathematics, reading, and writing. In addition, one of his parents reported that Carl struggled to retrieve mathematical facts and complete procedures on mathematics assessments. His parents also reported that Carl’s math grade showed some improvement but on an inconsistent basis.

**David.** David was a 14-year-old Hispanic male student in the eighth grade. At the time of the study, David was enrolled in a team-taught Prealgebra course. According to David’s IEP, he was diagnosed with a learning disability in elementary school. He demonstrated a severe discrepancy between achievement and intellectual ability in the areas of mathematics and reading. In addition, his parents reported that David struggled
to follow oral directions and tended to have difficulty verbally recalling mathematical definitions but had the ability to demonstrate understanding of mathematical facts and procedures on paper. David’s parents also reported that they were concerned that over the years his progress in math has been inconsistent.

**Dylan.** Dylan was a 13-year-old Caucasian male student in ninth grade. At the time of the study, Dylan was enrolled in a self-contained Prealgebra math course. According to his IEP, Dylan was diagnosed with LD while attending elementary school. He demonstrated a severe discrepancy between achievement and intellectual ability in the area of mathematics calculation and reasoning. Likewise, Dylan’s parents expressed that Dylan had always needed extra help in mathematics. They reported that he demonstrated progress in small settings and had managed to remain very positive about his mathematics abilities despite performing consistently with low grades on tests and quizzes.

**Hive.** Hive was a 14-year-old Caucasian male ninth-grade student. At the time of the study, Hive was enrolled in a general education Algebra course. Hive was identified by his parents as being at risk for mathematics difficulties and thus had no documented disability at the time of the study. According to a parent, Hive has had noticeable problems with mathematics in middle school and needed assistance from the teacher and tutors on a regular basis. His parent also expressed that Hive’s math teachers had consistently reported that he had a tendency to forget information easily, needed to retake assessments on a regular basis, and depended heavily on after-school review sessions in order to retain information.
Hunter. Hunter was a 14-year-old Caucasian male eighth-grade student. At the time of the research, Hunter was enrolled in a team-taught Prealgebra course. According to Hunter’s IEP, he was diagnosed with a learning disability in mathematics and reading comprehension. He demonstrated a severe discrepancy between achievement and intellectual ability in the area of mathematics reasoning. According to a parent, Hunter was recommended for this research because of his inconsistent success in math over the past few years. His parent also stated that for years, Hunter’s math teachers had consistently stated that he had the tendency to forget procedural steps and needed additional time assessments on a regular basis.

Rosa. Rosa was a 12-year old Hispanic female student in the seventh grade. At the time of the study, she was enrolled in a team-taught Prealgebra math course. According to her IEP, Rosa was diagnosed with a learning disability in mathematics while enrolled in middle school after demonstrating difficulty in her subject-area classes. In addition, she demonstrated a severe discrepancy between achievement and intellectual ability in the areas of mathematics reasoning. Her parents reported that Rosa was very verbal but tended to have difficulty demonstrating mathematical facts and procedures on paper.

Stacy. Stacy was a 17-year-old Hispanic female student in the 12th-grade student. At the time of the study, Stacy recently completed Algebra II in self-contained settings and was not enrolled in math class. According to her IEP, Stacy was diagnosed with LD in mathematics reasoning while in middle school after her teachers and parents noted that she had experienced difficulty retaining new vocabulary terms used in her classes. Her
mother explained that she had continued to worry about Stacy’s performance in math classes because of the unenthusiastic way in which Stacy described her classes and assignments on a regular basis. Stacy’s mom hoped that the study would help Stacy share some of her experiences with someone who could possibly help other students struggling with math like her daughter.

**Participant Relationship with the Researcher**

Glesne stated that “no matter how qualitative researchers view their role, they develop relationships with research participants” (2006, p. 138). Therefore, before beginning the interviews, a consideration of the relationships was addressed. Although the researcher was concerned about authentic disclosure from participants, there was an attempt to avoid what Maxwell (2005) categorized as a participant who was intellectually involved but not willing to reveal any personal details. The researcher sought to create a “relationship that allowed access to ethically gain information that can answer the research questions” (Maxwell, 2005, p. 83). This was attempted by dividing the data collection process into two sessions. The first session involved the students completing the less interactive data—the demographic questionnaire, the math attitude inventory, and the researcher-constructed math diagnostic test—on their own. Minimal dialogue occurred between the researcher and the participant. During this time, the researcher was able to begin establishing a rapport with the students in preparation for the second session, which consisted of the clinical and semistructured interviews; the latter required more interaction between the participant and the researcher.
Research Setting

This study was conducted in a northeastern region of the United States. The research setting was based on the individual preference of participating students. Therefore, the setting varied across participants. One student chose to meet at a public library, which provided a more professional feel during the interview and may have affected how the students answered each question. Five participants choose to meet at home and the other two in a coffee shop. Choice of location was left up to the participant with the intent of using the participant’s chosen atmosphere as a tool to help obtain more in-depth information pertaining to the purpose of the study.

Data Sources

The data sources for the current mixed methods study included: (a) a demographic questionnaire, (b) a math attitude inventory, (c) a researcher-constructed math diagnostic test, (d) semistructured student interviews, and (e) clinical student interviews. According to Creswell (2008), it is important to use different data sources to enhance the fidelity and quality of a study by corroborating findings across data sources.

Demographic Questionnaire

The demographic questionnaire was used to gather specific information about the student participants. The questionnaire consisted of 18 demographic items/questions presented in a variety of formats. It had 16 multiple-choice questions (e.g., I am currently enrolled in: (1) a middle school, (2) junior high school, or (3) high school), 1 close-ended (I have a math tutor (1) yes or (2) no), and 1 fill-in-the-blank question (I am ___ years old). Some fill-in-the-blank were questions were included to provide information that
contributed to understanding students’ attitude toward mathematics (e.g., I am expecting the following grade in my current math class of an A, B, C, D, F, or I do not know; I work on assignments during math class always, most of the time, about half the time, once in a while, almost never, or never). Overall, responses provided the student’s age, grade, sex, previous math grades, expected grade, and a general overview of each student’s attitude toward math class and math teachers, perceptions of their own understanding of math concepts, and attitude toward grades received in mathematic courses. The questionnaire was read and completed independently by each student. This data source was designed to last for a total of 15-20 minutes. The questionnaire responses enhanced the opportunity for the researcher to observe similarities and themes that surfaced in each interview during session two (see Appendix D).

**Math Attitude Inventory**

The math attitude inventory was modified from the Attitudes Toward Mathematics Inventory (ATMI) designed by Tapia and Marsh (2004) to investigate the students’ attitudes toward mathematics. The original ATMI inventory consisted of 40 items on a 5-point Likert-scale (1 = strongly disagree, 2 = disagree, 3 = neutral, 4 = agree, and 5 = strongly agree).

The original ATMI scale created by Tapia and Marsh (2004) has a reliability coefficient alpha of .97 for the entire inventory and consisted of items representative of four factors inclusive of attitudes toward mathematics: anxiety, confidence, enjoyment, and motivation. The calculated Cronbach’s alpha showed an internal value for each of the four factors with a range from .89 -.95 (Tapia & Marsh, 2004) (see Appendix E). The
modified ATMI was reduced by the researcher from 40 items to 20 items in order to reduce the amount of time students would take to complete the inventory. An expert reviewed the modified version of the ATMI. Although the original factors were not maintained after the reduction process, the process of reducing the number of questions involved choosing five items representative of each factor. These categories of questions were used to better understand whether students were motivated about mathematics, valued mathematics, enjoyed mathematics, and were confident in their mathematics abilities. The modified version of the inventory was designed to take each student 15-20 minutes to complete. Sample items for each factor are shown in Figure 1.
<table>
<thead>
<tr>
<th><strong>Value</strong></th>
<th><strong>Self-Confidence</strong></th>
<th><strong>Enjoyment</strong></th>
<th><strong>Motivation</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Question 1: Math is a very worthwhile and necessary subject.</td>
<td>Question 6: My mind goes blank and I am unable to think clearly when working with mathematics</td>
<td>Question 12: Math is dull and boring.</td>
<td>Question 11: I have usually enjoyed studying math in school.</td>
</tr>
<tr>
<td>Question 3: Math is important in everyday life.</td>
<td>Question 8: Math makes me feel uncomfortable.</td>
<td>Question 15: I am happier in a math class than in any other class.</td>
<td>Question 18: I plan to take as much math as I can during my education.</td>
</tr>
<tr>
<td>Question 4: Math is one of the most important subjects for people to study.</td>
<td>Question 9: When I hear the word math, I have a feeling of dislike.</td>
<td>Question 16: Math is a very interesting subject.</td>
<td>Question 19: The challenge of math appeals to me.</td>
</tr>
<tr>
<td>Question 5: Middle or high school math courses would be very helpful no matter what I decide to study.</td>
<td>Question 10: Math does not scare me at all.</td>
<td>Question 17: I am willing to take more than the required amount of math in school.</td>
<td>Question 20: I think studying advanced math is useful.</td>
</tr>
</tbody>
</table>

*Figure 1. Modified Attitudes Toward Mathematics Inventory (ATMI) questions by category based on original ATMI survey by Tapia and Marsh (1996).*
Math Diagnostic Test

The diagnostic test was used to determine each student’s current level of understanding of various math concepts leading up to taking an Algebra course. The diagnostic test was created by the researcher who used a modified version of the Glencoe McGraw-Hill (2000) diagnostic test to assess students’ mathematical skills. The original Glencoe McGraw-Hill diagnostic test was designed to accurately place students in middle school Algebra courses, and to provide diagnostic information about students’ mathematical knowledge or mastery of skills. Additionally, the original diagnostic assessment was also used by specific school systems, individual teachers, and parents to assess the mathematical skills of students currently taking or who will take a Prealgebra course and Algebra course. The original Glencoe McGraw-Hill diagnostic test consists of 30 multiple-choice questions with four content strands: number and number operation, data analysis, geometry, and algebra. These content strands reflect the targeted skills identified in the literature as a focus of intervention studies for Grades 6 through 12 in mathematics. Modifications made by the researcher included a reduction in the number of strands included and the number of questions from each of the four content strands to reduce the amount of time needed for each participant to complete the diagnostic test. Particular items were selected from each strand of the diagnostic test so that the final copy of the modified math diagnostic test included 15 items: six items representing the number and number operations strand, two items representing the geometry strand, one item from the data and analysis strand, and six items from the algebra strand. The modified diagnostic test was designed to take each student a total of 30-45 minutes to
complete. This diagnostic test provided the researcher with a better understanding as to how students performed on specific math skills. The test also allowed the researcher to subsequently conduct a clinical interview with each student after a diagnostic error analysis was completed (Ginsburg, 1997a) (see Appendix F).

**Diagnostic error analysis.** The diagnostic error analysis rubric consisted of 12 items generated from the 12 common mathematical errors frequently made by students with learning disabilities and students at risks for mathematics difficulties (Butler et al., 2003; Ginsburg, 1997b; Naglieri & Johnson, 2000). The items included on the rubric were taken from previous literature on the noted characteristics of students with mathematics difficulties: using mathematical symbols, making calculation errors, recalling mathematical terms, and completing problems after starting correctly. Using the diagnostic error analysis, the researcher recorded the frequency with which each student made the error on the rubric by checking the corresponding box next to the description of the error. See Appendix G for further detail.

**Clinical Interview**

The clinical interview is a type of interview method used to assess the students’ cognitive abilities when computing a math problem. The clinical interview session was audio recorded by the researcher and students were also provided with additional paper to show the steps completed for each problem. Therefore, as students approached each task, they were asked to “think-out-loud” (Ginsburg, 1997a); that is, students were prompted to talk through their steps as they completed the mathematical problems. Both Ginsburg (1997a) and Lobato et al. (2012) referred to this as a clinical interview. In this study the
clinical interview was conducted 1:1 to gather additional information regarding student understanding of each of the previously mentioned strands (i.e. number and number operations, geometry, algebra, and data analysis). As a result, different types and quantities of questions where chosen for each student based on math diagnostic test scores per strand and question type. For instance, if a student received one or more problems incorrect from a specific strand, the student was asked questions from that strand during his or her clinical interview. If a diagnostic test question was \(2x + 3 = 9\), a corresponding or similar clinical interview question would be \(3x + 4 = 10\). This process was designed to take each student a total of 20-30 minutes and each student completed approximately 10 math problems. After the clinical interview, the clinical interview error analysis rubric (Appendix I) and the process standard analysis rubric (Appendix J) were used to analyze students’ responses to the questions that were asked. These rubrics were designed to aid in evaluating students’ ability to read, interpret, reason, and communicate mathematical concepts.

**Clinical interview error analysis rubric.** The clinical interview error analysis rubric consisted of five areas identified by researchers (Bottge, Grant et al., 2010; Bottge, Rueda et al., 2010; Maccini et al., 2007; Test & Ellis, 2005) that help students with mathematics difficulties to improve their mathematical skills. This rubric differed from the math diagnostic error analysis rubric in two ways: (a) the clinical errors rubric was designed to analyze verbal and observed student response, and (b) verbal errors and explanations during the think-out-loud sessions differed from student to student and also from the written errors made on paper (e.g., students were allowed to try again with
prompts, students were able to explain a problem verbally). Thus, the clinical interview analysis rubric was also designed to capture a possible difference between what students demonstrated on paper and what they were verbally able to share about a specific math problem. More specifically, each item on the rubric contained specific levels or ratings, which were marked as: 1 - Far Below Standard, 2 - Below Standard, 3 - Standard, 4 - Proficient, and 5 - Advanced (see Appendix I). For instance, students who scored a 1 (far below standard) (a) did not read or restate the problem, (b) were not able to identify the problem, and/or (c) did not recognize the problem. On the other hand, students who scored a 5 (advanced) were able to (a) adequately, clearly, and confidently identify and restate problems; (b) appropriately use advanced math language; and (c) provide additional details needed to demonstrate a good understanding of the presented problem. See Appendix I for the clinical interview error analysis rubric.

**Process standards analysis rubric.** The process standards analysis rubric consisted of five items generated from the standards set forth by the NCTM (n.d.) for problem solving, reasoning and proof, communication, connection, and representation. Similar to the clinical interview error analysis rubric, each item on the rubric contained specific levels or ratings per strand. These ratings across strands were marked as: 1 - Far Below Standard, 2 - Below Standard, 3 - Standard, 4 - Proficient, and 5 - Advanced (see Appendix J). For instance, students who scored a 1 (far below standard) on the problem-solving strand (a) showed no understanding of the problem, nor (b) did they attempt to complete the problem. On the other hand, students who scored a 5 (advanced) were able to (a) show an excellent understanding of the problem, (b) used more than one
strategy that produced a correct answer, and (c) solved problems without errors. Therefore, the highest possible score a student could receive was a total of 25 points and the lowest score a student could receive was a total of 5 points.

**Semistructured Interview**

An in-depth, semistructured, 1:1 interview was selected because it fit the nature of the research questions and helped to reveal how individuals interpreted their actions (Johnson, 2001). The interview protocol consisted of 17 open-ended questions that allowed each participant to voice his or her own opinions. For example, items included questions requiring each student to share his or her feelings about math (e.g., how do you feel about math?) and to express strategies he or she felt has been helpful over the years (e.g., what are some of the things that you like about a teacher that you find to be most helpful?). Based on the type of questions asked, participants were asked to elaborate, provide more details, or give examples to elicit more information as needed (see Appendix K). This session was audio recorded by the researcher and field notes were taken at appropriate times. The semistructured interview session lasted an average of 15-20 minutes.

Table 3 summarizes the research questions and the various data sources used to inform each research question.
### Table 3

*Research Questions Addressed by Various Data Sources*

<table>
<thead>
<tr>
<th>Research questions</th>
<th>Attitude tests</th>
<th>Diagnostic test</th>
<th>Demographic questionnaire</th>
<th>Semistructured (A) and clinical interview (B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>What are the perceptions and attitudes of a group of secondary students with learning disabilities and students at risk for mathematics difficulties about mathematics?</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X(A)</td>
</tr>
<tr>
<td>How well does a group of secondary students with learning disabilities and students at risk for mathematics difficulties understand important concepts and symbols of algebra, and how they are used?</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>What does a group of secondary students with learning disabilities and students at risk for mathematics difficulties find to be the most challenging about mathematics?</td>
<td>X</td>
<td></td>
<td></td>
<td>X(B)</td>
</tr>
</tbody>
</table>

**Data Collection Procedures**

Prior to the start of the data collection process, the first step was to obtain approval from the George Mason University Institutional Review Board (IRB).
Permission was granted for each of the eight participants to respond to the demographic questionnaire, complete a math attitude inventory, a math diagnostic test, and to participate in a clinical and semistructured interview with the researcher. These measures were administered in two separate sessions with each participant individually.

**Session One**

The first session collected data with limited interactions between the participant and the researcher. The demographic questionnaire, math attitude inventory, and the researcher-constructed diagnostic test were completed over the course of 60-90 minutes. During this time, the researcher was able to build a rapport with the students in preparation for the second session.

**Demographic questionnaire.** The questionnaire was provided in person after the consent and assent forms were signed. The researcher described the purpose of the demographic questionnaire and asked each student if he or she would like the questions read out loud. Two students preferred the questions read out loud and the other six students read and completed the questionnaire independently. The questionnaires took students an average of 5-10 minutes and provided the researcher with the opportunity to target and ask specific questions during part two of the interview process.

**Math attitude inventory.** After students completed the demographic questionnaire, they were given an attitude inventory to determine how they felt about math. In a similar manner to the procedures for the demographic questionnaires, the researcher described the purpose of the math attitude inventory and asked each student if he or she would like the questions read out loud. The same two students requested that
the questions be read out loud while the other students completed the inventory independently. On the math attitude inventory, student participants responded to each question on a scale of 1-5 (1 = strongly dislike and 5 = strongly agree). The inventory took students an average of 5-10 minutes to complete. This measure was used to corroborate information captured from the students’ interviews.

**Math diagnostic test.** The researcher-constructed diagnostic test was given to each participant after the demographic questionnaire and math attitude inventory were completed and collected. This diagnostic test took students an average of 30-45 minutes. The students were provided with a four-function calculator, two sheets of scrap paper, and two pencils. Each student was asked to inform the researcher if he or she needed a problem read out loud. However, no prompts were given during this assessment.

**Session Two**

The second session consisted of the clinical and semistructured interviews. For each student, session two only occurred after the researcher graded and/or summarized the demographic questionnaires, the math attitude inventory, and the math diagnostic test completed in session one. As a result, the math diagnostic test was graded and an error analysis was completed for each student’s work. The researcher evaluated the diagnostic test prior to starting the clinical interview because each student’s noted error was used to determine the types of problems used during the clinical interview. Thus, the clinical interview questions were individualized based on the student participant’s identified errors and demonstrated standard area difficulty. This math diagnostic test analysis rubric consisted of the (a) type of procedure applied, (b) appropriateness of procedures used, and
(c) whether or not the procedural steps were correctly executed. The second session interviews were audio recorded by the researcher and lasted an average of 30-45 minutes per participant. Students were presented with preselected problems for the clinical interview.

**Clinical interview.** After the first session and the math diagnostic tests were graded and analyzed for errors, each student participated in a 20- to 30-minute clinical interview. The participants were reminded that the interviews would be audiotaped and that they could have sheets of paper that were divided into different sections for space to show work. The participants were also provided with a pencil and a four-function calculator.

During the clinical interviews, participants were asked to solve algebraic problems aloud, identify symbols and operations, and discuss their strengths and weaknesses. The clinical interview questions were similar to questions used during the math diagnostic tests and were taken directly from the original diagnostic test. The researcher carefully compiled a list of up to two similar problems that matched the math diagnostic test. These questions were presented on individual notecards to each participant. Similar to the diagnostic test, it consisted of the four standard strands (i.e., number and number operations, data analysis, geometry, and algebra). The questions presented were also based on the error analysis based on the students’ math diagnostic tests and consisted of approximately 10 problems. Each student was asked the same interview questions for each problem (see Appendix H). However, the probes, follow-up questions, and conversations were personalized for each individual student, based on
individual student response (Ginsburg, 1997a). During this time, the researcher also observed and recorded the students’ gestures during the clinical interview.

**Semistructured interview.** The last part of session 2 consisted of a semistructured interview that lasted for an average of 10-15 minutes. During the semistructured interview process, the researcher collected field notes, which consisted of using a notepad to write notes of questions that required follow-up and the researcher’s thoughts during the interview. The researcher also used probing as an attempt to encourage each participant to provide more in-depth information (i.e. “Why?”, “Can you explain further?” “Can you share another example?”). Subtle probes such as pausing also encouraged each participant to continue his or her response. The researcher also invited the participant to provide more examples in order to encourage in-depth information.

After the interviews were transcribed, participants were given the option to verify their semistructured interview transcriptions. Five out of the eight participants verified the overall accuracy of the transcriptions, but three participants did not take the option because they were satisfied with their overall contribution.

**Data Analysis**

Data analysis occurred both during and after the data collection process. Questionnaire data were analyzed quantitatively and math diagnostic tests and clinical interviews were analyzed both quantitatively and qualitatively. Interviews were analyzed qualitatively. The analysis of each of the data sources is described in this section.
Data Management

Stake (1995) and Morrow (2005) agreed that every researcher should have a data storage system. Throughout the research and at the completion of this study, all data sources including math diagnostic tests, math attitude inventory, field notes, recorded interviews, transcriptions, and consent forms were kept in a secure file cabinet located at George Mason University and/or at the home of the researcher. These data sources did not include the names of the participants, only a pseudonym. The audiotapes and transcripts will be kept for a minimum of five years and will then be shredded. In order to uphold research quality, constant awareness of participants’ personal information and research data was and will always be a top priority.

Demographic Questionnaire

The demographic questionnaire was analyzed using the Statistical Package for Social Sciences (SPSS) Version 22. The mean and standard deviation for student participants were calculated separately in SPSS for each individual item in the questionnaire. The use of total numeric average responses accompanied the questionnaire findings in the written report. The mean provided the average responses across the participants and the standard deviation helped analyze how each individual response spread about the mean. Additionally, the researcher analyzed items answered from the questionnaire to describe how the various individual characteristics (e.g., age, gender, race, and demographics) related to student attitude and/or feelings.
Math Attitude Inventory

The math attitude inventory was modified from the Attitudes Toward Mathematics Inventory (ATMI) and was scored based on the ATMI’s standardized scoring criterion and set procedures for grading, analysis, and interpretation. The math attitude inventory results were entered into SPSS to calculate the frequency and average responses across participants.

Math Diagnostic Test

Each question of the math diagnostic test evaluated the participants’ ability to perform a given mathematical task based on each question’s assigned strand. Therefore, this assessment was graded according to the answers provided by the publisher of the original test. In addition, the answer key to assess the student’s answers and identify the types of questions students are able to complete and/or the problems they have difficulty completing was based on the aforementioned preestablished types of questions used to evaluate student abilities. Refer to the last page of Appendix F for the math diagnostic test answer key. A percentage score of accuracy was determined for each individual student. Therefore, an overall score was determined for each participant which was also accompanied by a score for each of the four strands. For example, a percentage of accuracy score was calculated for number correct (NC) divided by the total number (TN) of items, and then results were multiplied by 100 \( \left( \frac{NC}{TN} \times 100 \right) \). Similarly, a percentage score was calculated for the number of questions correct per strand, that is the NC per strand divided by the TN items per given strand, multiplied by 100 \( \left( \frac{NC}{TN} \times 100 \right) \). The strand with only one question (the data analysis strand) was recorded as either 0% or
100% correct. In addition, all student work was collected and analyzed qualitatively when possible for common errors, techniques, and steps used to solve each problem. This process was conducted through manual observation based on the researcher’s prior knowledge and the use of a diagnostic error analysis of student work (see Appendix G).

Clinical Interviews

The clinical interviews were conducted during the second session of the study and were analyzed both quantitatively and qualitatively. Similar to the observation of student work that took place after the math diagnostic test, the researcher also analyzed the students’ work after the clinical interviews. Student observations also took place during the clinical interviews and notes were recorded. As a result, the researcher listened to and observed students as they completed each problem. Specifically, students were observed when they first received a problem, as they explained their steps, and as they attempted to or completed a problem. Observations such as the participants’ gestures were recorded next to each problem. In addition to the observational notes taken during the early stages of the interview, the researcher listened to each audiotape twice, followed along with the original observations, and made additional notes or edits as needed. The researcher used two rubrics to analyze student errors: an error analysis rubric and an analysis by the five process standards. The clinical interview error analysis rubric (Appendix I) was used to identify the common errors made by each participant.

Semistructured Interviews

The analysis portion of the qualitative data sources (semistructured interviews) followed a constant comparative analysis approach and thus contained components of
grounded theory (Corbin & Strauss, 1990). This involves a process of “gathering data, sorting it into categories, collecting additional information and comparing new information with emerging categories” (Creswell, 2008, p. 443). As a result, many steps were taken to carefully analyze the data, which involved constantly reading, comparing, connecting, and relating fractures of student interview data to make meaning of the phenomena. In this study, the data were analyzed simultaneously during the data collection process. Therefore, as suggested by Maxwell (2005), the researcher listened to, transcribed, read, and took notes of what was heard or read. During and after the transcription of the interviews the researcher listened to each interview several times which provided an additional opportunity to analyze to the data. Throughout this process, the researcher searched for and identified patterns, and attempted to identify similarities across each participant’s interviews.

Since the goal of the research was to reach a deeper understanding of a group of secondary students’ perceptions about mathematics, themes were developed and analyzed using a constant comparative analysis. The themes were informed by the demographic questionnaire, semistructured interviews, and math attitude inventory. Therefore, this approach consisted of using a specific set of procedures for data collection and analysis to inductively formulate conclusions and/or themes.

The first few steps involved transcribing each interview, categorizing the data by factorizing, and analyzing the key words and phrases of participants using open and axial coding (Maxwell & Miller, 2008). Then, during thematic categorizing, components of different connecting strategies were used to form conclusions. This final step used
components of different connecting strategies to form a concise conclusion about the data. This allowed the researcher to “develop a well-integrated set of concepts that [may] provide a thorough theoretical explanation” (Corbin & Strauss, 1990, p. 5) with an emic approach.

Transcription. The researcher began the transcription process by formatting a Microsoft Word document in single-space format, with .5-inch left margins and a 3-inch right margin, which made the format convenient for the coding processes. The right margin was spacious enough to take notes and categorize, analyze, or summarize the data in the researcher’s handwriting. Numbers and bold writing represented questions and regular-sized font represented participants’ responses.

The researcher transcribed each participant’s entire interview to ensure that the participants’ voices would be heard. In this in-depth qualitative research, it was important not to select parts of the interview to transcribe and omit others, as choosing to do this could “lead to premature judgment about what is important and what is not” (Seidman, 2006, p. 115). Seidman also stated that choosing not to transcribe parts of the interview may lead to misinterpretation of data because of lost perspectives and ideas from the participant. Given the importance of including all of the participants’ perspectives, each interview was transcribed verbatim.

Open coding. After the transcription process, the first procedure consisted of open coding analysis: breaking down the data into smaller fractures, searching for similarities and differences, and grouping concepts together into different categories and subcategories (Corbin & Strauss, 1990). Although a specific coding method was not used
to perform open coding, the method in which the researcher felt most comfortable was applied. As a result, each line of the interview transcriptions was read carefully, with the purpose of the study’s research questions in mind. Phrases and words stated by the participant that were perceived to hold significant value were highlighted. Multicolored highlights were used by the researcher to identify different keywords or phrases and the right margin was used to write down similarities and differences. This helped to prevent the urge to move on to any thematic analysis or axial coding. The goal in this stage was to group together concepts directly within the participants’ interviews and/or across participants’ interview transcriptions.

In addition to categorizing, ideas or concepts stated by the participants were constantly compared and contrasted. This consisted of asking comparative questions as the data were read and highlighted. Corbin and Strauss (1990) stated that this type of internal dialogue serves as an excellent way to guide researchers through the categorizing process of open coding. As a result, this process helped to bring awareness to various components of each category, and helped to identify different categories as they arose.

**Axial coding.** After open coding, the next step was to perform axial coding. This involved the use of prior knowledge, meaning that the researcher used an understanding of words and experiences in education to categorize and analyze the conditions that led each participant to use certain words or phrases to express their thoughts (Corbin & Strauss, 1990). With that purpose in mind, the researcher examined the previously highlighted information from each participant’s transcript and attempted to perform what Corbin and Strauss called a “coding paradigm” (1990, p. 13). This entailed separating all
phrases and words into similar categories, and then relating each category to a bigger picture or conclusion (Corbin & Strauss, 1990). This was accomplished by extracting and writing the identified highlighted categories on a separate sheet of paper, which reduced the data to be analyzed. The researcher looked for guidance from the perceived meaning of the words or phrases used and grouped them according to likeness of meaning. For instance, if each word or phrase in a given category suggested the same meaning as in another category, then the two categories were grouped together. The researcher related and compressed each category into emerging themes, thus merging each category into a smaller category or theme.

The process of axial coding was influenced by the researcher’s perception and interpretation of the data. However, throughout this process, the researcher used a memo pad to take notes and to help keep external thoughts outside of the analytic process. An important distinction was placed in the researcher’s subjectivity during analysis in order to focus only on interpreting meaning from the point of view of the participant. Thus, the relationship that was determined in this process was verified repeatedly against additional data collected and analyzed (Corbin & Strauss, 1990). Constant comparative analysis depends on the researcher’s commitment to constantly verify conclusions with the data collected so that the conclusions or relationships that could not be verified were revised as new data were collected (Corbin & Strauss, 1990).

**Thematic categorizing.** Similar to the selective coding process used in Corbin and Strauss (1990), a final connecting strategy was used to unify all categories and subcategories, including categories with descriptive information. However, this presented
a few problems of its own. Like Corbin and Strauss (1990) suggested, some researchers have difficulty reaching a central theme or category. Following Corbin and Strauss, a systematic approach leads to further confirmation of theory or provides instances in which the theory does not apply. This was the most difficult question to answer during this process: How does one generate “one core category” (Corbin & Strauss, 1990, p. 14) from the data. However, in the current study, adequate coding meant leading to more than one theme or category that represented the data collected during the semistructured interviews. Table 4 presents a summary of the data analysis type and how it corresponded to the research questions.
Table 4

Data Analysis Type Corresponding to Each Research Question

<table>
<thead>
<tr>
<th>Research questions</th>
<th>Primary data source</th>
<th>Sample</th>
<th>Method of analysis</th>
<th>Specific tool</th>
</tr>
</thead>
<tbody>
<tr>
<td>RQ 3</td>
<td>Demographic Questionnaire</td>
<td>Grade level, sex, expected math grade, math class, setting, work completion, class attendance</td>
<td>Descriptive statistics (SPSS frequency)</td>
<td>SPSS</td>
</tr>
</tbody>
</table>
| RQ 1               | Math Attitude Inventory | • I think math is important in everyday life.  
• Math does not scare me.  
• I learn mathematics easily.  
• Math is dull and boring. | Test-based findings Diagnostics | Researcher   |
| RQ 2               | Clinical Interview   | • Solve the problem 4x - 6 = 14  
• What does the \( X \) mean in the problem above?  
• What do the values in the coordinate pair (6, 5) represent?  
• Do you recognize this symbol (e.g., \( >, <, = \)) | Error analysis  
Number of questions solved correctly | Researcher  
Diagnostic Test Guide |
| RQ 3               | Semistructured Interview | • Can you share a situation in which you liked math?  
• Can you share a situation in which you disliked math?  
• Who was or is your favorite math teacher and why? | Comparative analysis/Thematic analysis | Researcher   |
**Trustworthiness and Validity**

As Morrow (2005) and Patton (2002) stated, quantitative and qualitative research studies exist in different forms and are approached from various paradigmatic and epistemological points of views. Therefore, researchers and the research audience may examine, interpret, and make sense of qualitative research differently. Thus, assessing qualitative research through a uniform perspective or criteria can potentially lead to unfair conclusions or raise ethical concerns about the quality of the research. Consequently, this study fully describes all details and procedures conducted during the study. Likewise, the quantitative portion of this study required the use of numerical interpretation with quantitative responses. As a result, the researcher conducted procedures and followed specific research protocols to enhance the trustworthiness and validity of the current mixed-methods study.

**Qualitative Trustworthiness**

The trustworthiness of the current study was established with the use of multiple data sources: demographic questionnaire, math attitude inventory, and semistructured interviews. As recommended by Brantlinger, Jimenez, Klingner, Pugach, and Richardson (2005), a number of data sources were used for triangulation. In addition, verification methods such as an audit trail, member checks, and thick, detailed descriptions were also employed. These methods helped to reduce the issues related to trustworthiness.

Triangulation of data sources and methods used by the researcher can provide an opportunity to use more than one method or source to study the problem and corroborate findings from multiple sources (Creswell, 2008; Merriam, 1998). In this study, themes
found during the coding process were analyzed and compared across all data sources. This use of triangulation helped to verify the data findings, which in turn strengthened the trustworthiness of the research findings and reduced typical qualitative research validity threats (Merriam, 1998). Verifying or confirming findings using various methods and data sources can strengthen the research by providing various means to study a phenomenon (Brantlinger et al., 2005; Greene, 2007; Maxwell, 2005).

Consequently, the researcher kept an audit trail that consisted of the order of research procedures. This trail consisted of detailed recording of how the data were collected, how the data were analyzed, and how various conclusions were derived during the study (Merriam, 1998). This process also provided an additional method to include detailed descriptions throughout the study.

To further ensure the credibility and trustworthiness (Creswell, 1998), a member check was completed to ensure that the participants’ views were expressed from their perspectives. This involved going back to five out of the eight participants after the semistructured interviews to verify the accuracy of what they expressed (as noted above, the other three participants were offered the opportunity but declined). Throughout the study, the researcher attempted to restate or summarize participants’ comments. The researcher presented and read the interview transcription or an interpretation of interview transcription to the five participants prior to the final analysis and explanation of findings. Participants were provided with the opportunity to verify the correctness or errors in the researchers’ interpretation of their semistructured interview, which aided in demonstrating research credibility.
Quantitative Validity

Validity in quantitative research refers to the soundness of the research conducted (Creswell, 2008). In general, quantitative research is usually based on numeric facts that are accurate, measurable, and precise. In this mixed methods research the researcher reported detailed descriptions of measures with corresponding grading criterion and employed the use reliability scoring.

Reliability Scoring

The researcher scored all measures using either an answer sheet or a rubric design to equally evaluate each student’s performance. In addition, a math teacher was trained by the researcher to identify common errors using the same rubrics as the researcher. The math teacher was trained using examples of errors identified on the math diagnostic test and clinical interview rubrics. During the training session, the math teacher verified the findings of two randomly selected students by scoring the students’ work collected after the diagnostic test, then comparing it to the researcher’s findings.

After training was completed for the diagnostic test, the math teacher assessed the entire set of student diagnostic assessment work samples. The scorer used an error analysis rubric specifically made to evaluate students’ work collected from the math diagnostic tests. After all samples were scored, a reliability score was calculated by taking the number of corresponding agreements divided by the quantity of agreements and disagreements. This score was multiplied by 100 to obtain a percentage score. An average score of 90% was calculated by subtracting the 9 disagreements from 51 total
responses. The result, 46 agreements, was then was divided by 51 and then multiplied by 100 to obtain the reliability score.

The scorer also used two clinical interview rubrics to (a) analyze the common errors on each problem completed by students after the oral interviews, using both the students’ tape-recorded answers and work shown on a separate sheet of paper, and (b) evaluate students’ ability to meet the process standards outlined by NCTM. The two clinical interview rubrics were developed to score the clinical interview’s audiotaped “think-out-loud” sessions and the corresponding work completed by each participant. Following the same calculations for percentage reliability of the diagnostic test error analysis, the scorer completed scoring for two out of eight random students using the clinical interview error analysis rubric and then the process standard rubric. An average of 75% interrater reliability score was obtained with a 1:4 for disagreements to agreements for the two randomly chosen students. As a result of the interrater reliability score of 75%, a special education math teacher was also trained and asked to perform the same task as the first scorer. After scoring of the same two student samples, a score of 95% was achieved, resulting in an average interrater reliability score of 85%.

**Researcher Subjectivity: Contributions of Prior Experiences**

As recommended by Maxwell (2005), before starting a research study (especially a mixed-methods study with a qualitative component), it is important for the researcher to consider his or her role as a sole constructor and interpreter of possible subjective data. Researchers should recognize and identify their connections to any previous understandings and knowledge of the phenomenon being explored. Therefore, it is
important for researchers to provide their audience with information that may give deeper insight about the research approach, researcher results, and conclusions, which can perhaps lead to even further interpretation. This is a critical component of understanding and eliminating misapprehensions among the research audience (Stake, 1995).

**Personal Experiences**

The development of my researcher perspective/conceptual framework started with my childhood experiences. As I looked back at my past, I realized that the majority of my professional discussions were centered on a particular awareness. This awareness developed in the late 1980s, as I became aware of my existence in an elementary classroom. It was then that I can first recall my struggle to understand the differences among the rates at which my peers achieved academic success. At an early age, I realized that while some students were able to demonstrate that they understood previously taught content, others were not able to adequately show academic progress. I grew up in a poor region of a small island in the Caribbean. At times many of my peers and I found it difficult to attend school because of various circumstances at home. Nevertheless, I grew to understand the importance of attending school and cherished the thought of one day changing my life through education. I now understand that each individual child can be forced to make choices based on their given circumstances and unique experiences.

**Teaching Experiences**

As a special education teacher, I am now currently in a position to help students with LD progress academically. Therefore, I entered this research with extensive background knowledge of issues pertaining to special education students and teachers.
that contributed to my perspective on the characteristics of students with LD. However, as the years passed and various laws evolved, so did the math requirements for all students per NCTM. I have observed teachers scramble to help students pass the tests, and I have seen students fail to achieve passing scores. As a classroom teacher at the time of the study, I was concerned about how students with LD or those who are at risk for mathematics difficulties were affected by these higher standards imposed on all students including those with LD, and what can be done to provide more support for them, in addition to using best practices and interventions in the classroom.

**Research Experiences**

As a researcher, I seek to discover the truth. I seek to find sound explanations. I seek to find explanations and hope to find academic “fix-alls” that can possibly be validated across different settings. However, I also realize that context and human interactions matter. I realize that the truth can be based on the observer, the interpreter, and the participants. I believe in multiple truths and I also believe that the social context of any research should not be separated from the outcome. I believe that past experiences and interactions matter in the development of how individuals become aware of the social realm in which they exist.

As I reflected on these experiences, I realized that all of my experiences related to the context of my research would affect how I approached it. I understood that based on my ontological beliefs about my reality and experiences, I needed to understand the background or context of individual and group perspectives, and possible connections, not merely the general interpretation and assumptions of results or findings. I was
interested in the background, context, and perceptions of those being researched. I also understood that individuals’ interpretations of an experience can be based on their perceived reality, how they learn, and the values that they place on various elements in reality. These notions stem from my epistemological belief that I acquire knowledge through my experience, interaction, and observation. All of these assumptions reflect my views of how I have come to know and understand the world. Consequently, from a phenomenological stance, my own views and understanding of how students learn and feel about mathematics are shaped by my experiences. My perceptions and assumptions are, in turn, based on my interpretation of reality and had to be recognized in order to perform this study.

As I entered this study, I realized that these experiences may also affect how I approached my research methodology. Based on my ontological interpretation of my experiences, I was aware of a need for the voices of students to be heard so as to be able to be used by teachers when constructing lessons. I have my own perspectives on what contributes to a student’s ability to understand and conceptualize math concepts. I interpret my experiences based on my perceived reality, how I learn, and values that I place on actions and things. As a result, during this study, it was important for me to monitor and be aware of subjectivity during the data analysis process (Glesne, 2006). I not only had to work toward understanding my participants’ perspectives, but also increase my own awareness of how I could impact my research outcomes (Glesne, 2006). Therefore, data were collected and analyzed systematically with an awareness of my subjectivity.
Summary

This chapter provided a detailed description of the mixed methods research used in the current study to determine the perceptions and thought processes of a group of secondary students with learning disabilities or at risk for mathematics difficulties. The methods used were guided by the purpose of the study and a set of procedures that were carefully organized to investigate the phenomenon. Five data sources were used during the study: demographic questionnaire, math attitude inventory, researcher-constructed math diagnostic test, clinical interviews, and semistructured interviews. A detailed description of the instrumentation, methods for data collection, trustworthiness and credibility, and method of analysis were provided. Chapter 4 provides a detailed report and description of the findings of this study in relation to its guiding research questions.
CHAPTER FOUR

The purpose of this mixed methods research study was to understand the mathematical thinking of a group of secondary students with learning disabilities and those at risk for mathematics difficulties. In other words, what do students think about mathematics? In addition, it was the hope of the researcher to shed light on some of the issues surrounding the documented low achievement of this student population in the area of mathematics. Although educators may be aware of math interventions that are effective for students, many researchers such as Ginsburg (1997a, 1997b) agree that the voices of these students should be heard, recognized, and taken seriously. Therefore, one of the primary purposes of this study was to provide educational leaders, teachers, and aspiring teachers with insight into the current thoughts and mathematical thinking of students with learning disabilities and students at risk for mathematics difficulties.

Overall, this chapter presents the findings obtained from the descriptive statistics and qualitative analyses. As a result, this chapter separately addresses findings by research question and by the data source(s) that inform each research question. First, findings from the student demographic questionnaire, the math attitude inventory, and the semistructured interviews are used to inform research question number one. Second, participant performance on the math diagnostic test is used to inform the second research question. Finally, descriptive statistics from the math diagnostic error analysis rubric are
used to inform the third research question. The clinical interview findings are also
presented to inform research question three.

The following research questions guide the presentation of each finding:

RQ1. What are the perceptions and attitudes of a group of secondary students
with learning disabilities and students at risk for mathematics difficulties
about mathematics?

RQ2. How well does a group of secondary students with learning disabilities
and students at risk for mathematics difficulties understand important
concepts and symbols of algebra, and how they are used?

RQ3. What does a group of secondary students with learning disabilities and
students at risk for mathematics difficulties find to be the most challenging
about mathematics?

Research Question One Findings

The first research question was, “What are the perceptions and attitudes of a
group of secondary students with learning disabilities and students at risk for
mathematics difficulties about mathematics?” The following section provides an analysis
of the major findings that were discovered across all participants regarding research
question one. The data sources that were essential in understanding the student
perceptions and attitudes include the demographic questionnaire (questions 11-22), the
math attitude inventory, and the semistructured interviews. Participants’ attitudes about
math are shared first. Next, an analysis of the interview data revealed student perceptions
of math, math teachers, and teachers’ teaching styles.
**Participants’ Attitudes and Perceptions About Mathematics**

The demographic questionnaire was used to collect participant information (i.e., age, grade level, and gender) and to gain insight about what students might prefer or not prefer in regard to learning mathematics. Additionally, the math attitude inventory was used to further inform students’ attitudes and perceptions of mathematics. Item 9 of the demographic questionnaire identified that students within the sample reportedly expected a mean grade point average (GPA) for their current math class of 2.63 ($SD = 1.06$). Questions 10-18 and a summary of the answers are found in Table 5.
No student participants had a math tutor. Also, the majority (six out of eight) of participants reported that they worked harder in math class than other students. It was also evident that half of the participants believed it was very important for them to do well in math class while the remaining thought it was somewhat important. However, only one student reported that he or she stayed after school for help on a regular basis. In addition, students reported completing classwork and homework assignments.
inconsistently and five reported that they never or almost never stayed after school for assistance with math.

**Math attitude inventory.** The math attitude inventory also informed each student’s attitude toward mathematics. SPSS was used to determine reported frequencies of items from the inventory. Responses were made on a 5-point scale. Table 6 includes all 20 items from the inventory and the number of participants who selected each response.
Table 6

*Frequency for Math Attitude Inventory by Item*

<table>
<thead>
<tr>
<th>Item</th>
<th>Strongly agree</th>
<th>Agree</th>
<th>Uncertain</th>
<th>Disagree</th>
<th>Strongly disagree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Math is a very worthwhile and necessary subject.</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2. I get a great deal of satisfaction out of solving a math problem.</td>
<td>0</td>
<td>6</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3. Math is important in everyday life.</td>
<td>5</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4. Math is one of the most important subjects for people to study.</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5. Middle or high school math courses would be very helpful no matter what I decide to study.</td>
<td>1</td>
<td>6</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6. My mind goes blank and I am unable to think clearly when working with mathematics.</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>7. Studying math makes me feel nervous.</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>8. Math makes me feel uncomfortable.</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>9. When I hear the word math, I have a feeling of dislike.</td>
<td>0</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>10. Math does not scare me at all.</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>11. I have usually enjoyed studying math in school.</td>
<td>0</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>12. Math is dull and boring.</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>13. I would prefer to do an assignment in math than to write an essay.</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>14. I really like math.</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>15. I am happier in a math class than in any other class.</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>16. Math is a very interesting subject.</td>
<td>0</td>
<td>3</td>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>17. I am willing to take more than the required amount of math in school.</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>18. I plan to take as much math as I can during my education.</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>19. The challenge of math appeals to me.</td>
<td>0</td>
<td>3</td>
<td>1</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>20. I think studying advanced math is useful.</td>
<td>0</td>
<td>5</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

*Note. N = 8.*
The math attitude inventory revealed that overall students agreed or strongly agreed that mathematics was important in everyday life, while two students were uncertain. Two students disagreed with the statement that math “does not scare me” while two students were uncertain and two others agreed that math does not scare me. Five students strongly agreed that they would prefer to do an assignment in math over a writing assignment. Four students agreed that they usually enjoyed studying math in school while four students were uncertain with this statement. Only one student agreed with the statement that, “I am willing to take more than the required amount of math in school” while four students disagreed with the statement. Overall, students showed that they had a lot of uncertainty regarding math. The majority was not certain as to whether or not math was an interesting subject and half were uncertain if math was dull or boring. Nevertheless, six participants agreed or strongly agreed that math is a worthwhile and necessary subject and also that mathematics is an important subject for people to study. Finally, four participants agreed that they really liked math while four disagreed with the statement.

**Summary of math attitude inventory.** The modified math attitude inventory provided an overview of students’ attitudes toward math. Many responses fell into the uncertain category. The math attitude inventory revealed that the majority of participants agreed that math was worthwhile and four students reported that they liked math. Four participants did not like the challenge of math and would not choose to take more than the required amount of math in school.
**Semistructured interviews.** The analyses of the semistructured interviews are reported in this section. The process of open coding resulted in 10 major categories that informed research question one: (a) initial student feelings, (b) types of challenges, (c) reasons for liking math, (d) reasons for disliking math, (e) effective classroom activities, (f) positive things about math teachers, (g) negative things about math teachers, (h) description of helpful math teachers, (i) specific help teachers provide, and (j) studying and practicing math.

The first category, “initial student feelings,” referred to the present consciousness of how students felt about math if they were able to use two to three words to describe their initial thoughts on the subject of math. The words students used to describe their initial thoughts resulted in a wide range of responses that were clearly either negative or positive. For instance, students used some contradictory words such as “challenging” but “interested,” “boring” but “necessary,” “frustrating” but “fun,” and “important” yet “difficult.” For example, David stated that math is important “because if you have to pay a bill you have to add and multiply…but one time in math class I was taking a preassessment and I didn’t know…I got upset.” Words that presented a view of a negative or positive attitude toward math were grouped into this category. Table 7 contains additional examples of phrases contributing to this category.

The second category, “challenges,” referred to what participants identified as challenging about math. This category involved the types of math problems students found difficult, situations in which problems got more difficult, reasons why math was difficult, and what teachers “do” to make math difficult. Students reported that math was
challenging because it had too many steps. For instance, Stacy stated, “sometimes if you miss one step, you miss the whole question.” Dylan stated that math can be “a little stressful” and “when you keep trying and getting it wrong and you can’t understand…it gets frustrating.” As a result, all phrases coded in this category consisted of reasons that students gave as they talked about the everyday challenges they faced in math class (see Table 7).

The third category, “reasons for liking math,” reflected the data that describe what students liked and appreciated about math. This category involved statements that began with “I liked math when….” These statements told the interviewer about the specific times when students were able to say to themselves that math class was “fun,” “enjoyable,” and pleasing to attend. Often, the experiences shared by the participants were situations that reassured their understanding of the classroom material. For instance, Rosa stated, “I like [math] because it is fun to do and I remember it.” Both David and Rosa referred to strategies they like to use. For example, David stated that “strategies help us so I can remember…” and Rosa stated that problems “that have steps are easier to do and remember.” She also expressed that “practice on the board and when the teacher tells us what we do wrong” were times when she found math enjoyable. Students reported they prefer having specific steps to follow and opportunities to check their answers and get feedback (see Table 7).

The fourth category, “reasons for disliking math,” encompassed phrases that students used to describe the reasons for not enjoying math and/or specific situations to support their dislike. The phrases that made up this category contained many references
to negative teacher attitudes, bad grades, and specific types of math problems. Stacy spoke about a teacher, stating that “last year, I had the worst teacher. She was really just…and every time someone did badly on a test she would be like, ‘you want to take this class again?’” Rosa stated, “I don’t get it when she [teacher] explains and gets so angry.” David also stated, that “I don’t like math when I got a bad grade on a test.” This category was generated to represent the specific reasons students provided to describe situations when they did not like math (see Table 7).

The fifth category, “presentation of material,” consisted of comments that described a teacher’s presentation of math content lessons from the perspective of the participants. Students used phrases such as “when we do the warm-up,” “explaining,” and “board work.” The researcher, after identifying a pattern of these types of words, recognized that these comments were collectively representing the experiences that may take place during teacher’s presentation of a math lesson or new math concept. For example, Carl reported enjoying when the teacher “explain[s] everything to us before she gives a real test and when she teaches us other steps.” Hunter and Dylan both talked about enjoying when teachers came over to their desk. Dylan stated, my teacher “was good because she just came over to my desk and if I had a question she would just like kind of help me…each time I had a question she would and just like explain it until I understood.”

The sixth category, “positive things about math teachers,” consisted of words that each participant used to describe their math teachers such as “patient,” “no attitude,” and “reliable.” For instance, Stacy referred to a teacher she had for Algebra who “would offer
to study with [her] and make sure that [she is] studying the right way.” Dylan mentioned that his favorite math teacher “was funny, he always cheer[s] you up when you have a trouble with math.” Autumn mentioned that she enjoyed her math teacher in eighth grade because “she was patient and she was always willing to help…she was also encouraging and knows how to treat us kids.”

Similar to the 6th category (positive things about math teachers), the 7th (negative things about math teachers), 8th (description of helpful math teachers), 9th (specific help teachers provide), and 10th (effective study and practice math) categories were also arranged together by their meaning, action, feeling, or event. Table 7 contains a description of the words and phrases that made up each category during open coding.
Table 7

Open Coding of Perceptions of Students with Learning Disabilities and Students At Risk for Mathematics Difficulties

<table>
<thead>
<tr>
<th>Open code</th>
<th>Properties: Examples of participants’ emic responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Feelings</td>
<td>Positive: Challenging, Fun, Important, Good, Helpful, Interesting</td>
</tr>
<tr>
<td></td>
<td>Negative: Acceptable, Frustrating, Difficult, Lots of thinking, Boring</td>
</tr>
<tr>
<td>2. Challenges</td>
<td>Content: Simplify, Fractions, Too many steps, Signs and equations, After basic math steps, Memorizing steps</td>
</tr>
<tr>
<td></td>
<td>Teacher: Rushing teachers, Yelling teachers, Angry teachers</td>
</tr>
<tr>
<td>3. Reasons for liking math</td>
<td>When I know it, When I get it, I am good at it, When it’s fun</td>
</tr>
<tr>
<td>4. Reasons for disliking math</td>
<td>Content: Fractions, Word problems</td>
</tr>
<tr>
<td></td>
<td>Results: Bad grades, Bad teachers, When it is hard, Pushy teachers</td>
</tr>
</tbody>
</table>

(continued)
<table>
<thead>
<tr>
<th>Open code</th>
<th>Properties: Examples of participants’ emic responses</th>
</tr>
</thead>
</table>
| 5. Effective classroom activities | Warm-ups  
Helping others  
Explaining  
Going over things  
Smart board  
Partners  
Board work  
Step-by-step work  
Practice |
| 6. Positive things about math teachers | Characteristics: Fun  
Patient  
No attitude  
Helpful  
Reliability  
Actions: Explains things  
Stays after  
Goes over it  
No attitude  
Repeats things  
Answers questions |
| 7. Negative things about math teachers | Characteristics: Mumbling  
He won’t explain  
Discouraging teacher  
Negative  
Discouraging  
Not explaining enough  
Actions: Grades  
Rushing  
Fast  
Yelling  
Too much HW [homework] |
| 8. Description of helpful math teachers | Patient  
Nice  
Encouraging  
Helps individuals  
Makes you feel good  
Goes over things multiple times  
Helps  
Funny  
Fun |

(continued)
<table>
<thead>
<tr>
<th>Open code</th>
<th>Properties: Examples of participants’ emic responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>9. Specific help teachers provide</td>
<td>Comes over to help</td>
</tr>
<tr>
<td></td>
<td>Goes over again</td>
</tr>
<tr>
<td></td>
<td>Shares</td>
</tr>
<tr>
<td></td>
<td>Shows notes</td>
</tr>
<tr>
<td></td>
<td>Helps</td>
</tr>
<tr>
<td></td>
<td>Stays after with me</td>
</tr>
<tr>
<td></td>
<td>Comes to the desk to explain</td>
</tr>
<tr>
<td></td>
<td>Helps with study skills</td>
</tr>
<tr>
<td>10. Effective study and practice math</td>
<td>Short Problems</td>
</tr>
<tr>
<td></td>
<td>Multiple choice</td>
</tr>
<tr>
<td></td>
<td>Board work</td>
</tr>
<tr>
<td></td>
<td>Worksheets</td>
</tr>
<tr>
<td></td>
<td>Warm-ups</td>
</tr>
<tr>
<td></td>
<td>Partner work</td>
</tr>
</tbody>
</table>

The open codes were then condensed and categorized into the following six categories: positive things about math, negative things about math, positive things about math teachers, negative things about math teachers, positive impact of teachers’ attitude, and negative impact of students’ attitude (see Table 8).
### Table 8

**Axial Coding of Perceptions of Students with Learning Disabilities and Students At Risk for Mathematics Difficulties**

<table>
<thead>
<tr>
<th>Axial codes</th>
<th>Properties: Examples of participants’ emic responses</th>
</tr>
</thead>
</table>
| 1. Positive things about math | Attitude: Challenging  
Fun  
Important  
Good  
Helpful  
Interesting  
Activities: Short Problems  
Multiple choice  
Board work  
Worksheets  
Warm-ups  
Partner work  
Smart board work  
Partners  
Step-by-step work  
Teacher: Helps others  
Explains  
Goes over things  
Is Nice |
| 2. Negative things about math | Content: Simplify  
Fractions  
Too many steps  
Signs and equations  
After basic math steps  
Memorizing steps  
Word problems  
Attitude: Acceptable  
Frustrating  
Difficult  
Lots of thinking  
Boring | (continued)
<table>
<thead>
<tr>
<th>Axial codes</th>
<th>Properties: Examples of participants’ emic responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>3. Positive things about math teachers</td>
<td>Actions: Explains things</td>
</tr>
<tr>
<td></td>
<td>Stays after</td>
</tr>
<tr>
<td></td>
<td>Goes over it</td>
</tr>
<tr>
<td></td>
<td>No attitude</td>
</tr>
<tr>
<td></td>
<td>Repeats things</td>
</tr>
<tr>
<td></td>
<td>Answer questions</td>
</tr>
<tr>
<td></td>
<td>Goes over again</td>
</tr>
<tr>
<td></td>
<td>Shares</td>
</tr>
<tr>
<td></td>
<td>Shows notes</td>
</tr>
<tr>
<td></td>
<td>Stays after with me</td>
</tr>
<tr>
<td></td>
<td>Explains at my desk</td>
</tr>
<tr>
<td></td>
<td>Helps with study skills</td>
</tr>
<tr>
<td></td>
<td>Goes over things multiple times</td>
</tr>
<tr>
<td></td>
<td>Attitude: Makes me feel good</td>
</tr>
<tr>
<td></td>
<td>Helps</td>
</tr>
<tr>
<td></td>
<td>Funny</td>
</tr>
<tr>
<td></td>
<td>Fun</td>
</tr>
<tr>
<td>4. Negative things about math teachers</td>
<td>Characteristics: Mumbling</td>
</tr>
<tr>
<td></td>
<td>He won’t explain</td>
</tr>
<tr>
<td></td>
<td>Discouraging teacher</td>
</tr>
<tr>
<td></td>
<td>Negative</td>
</tr>
<tr>
<td></td>
<td>Discouraging</td>
</tr>
<tr>
<td></td>
<td>Yelling</td>
</tr>
<tr>
<td></td>
<td>Rushing</td>
</tr>
<tr>
<td></td>
<td>Fast</td>
</tr>
<tr>
<td></td>
<td>Actions: Rushing teachers</td>
</tr>
<tr>
<td></td>
<td>Yelling teachers</td>
</tr>
<tr>
<td></td>
<td>Angry teachers</td>
</tr>
<tr>
<td>5. Positive impact of student attitude</td>
<td>Characteristics: Fun</td>
</tr>
<tr>
<td></td>
<td>Patient</td>
</tr>
<tr>
<td></td>
<td>No attitude</td>
</tr>
<tr>
<td></td>
<td>Helpful</td>
</tr>
<tr>
<td></td>
<td>Reliable</td>
</tr>
<tr>
<td></td>
<td>Patient</td>
</tr>
<tr>
<td></td>
<td>Nice</td>
</tr>
<tr>
<td></td>
<td>Encouraging</td>
</tr>
<tr>
<td></td>
<td>Knowledge: When I know it</td>
</tr>
<tr>
<td></td>
<td>When I get it</td>
</tr>
<tr>
<td></td>
<td>I am good at it</td>
</tr>
<tr>
<td></td>
<td>When it’s fun</td>
</tr>
<tr>
<td>6. Negative impact of student attitude</td>
<td>Results: Bad grades</td>
</tr>
<tr>
<td></td>
<td>Bad teachers</td>
</tr>
<tr>
<td></td>
<td>When it is hard</td>
</tr>
<tr>
<td></td>
<td>Pushy teachers</td>
</tr>
</tbody>
</table>
The six new categories formed during axial coding were further condensed to identify more generalizable themes with the goal of finding a central theme. This process consisted of unifying categories previously identified through thematic categorizing. After the process of inductive analysis was applied and constant analysis of the existing categories was exhausted, three themes that could not be unified without further losing the meaning were identified. This process if represented in Figure 2. Each theme is explained with excerpts from participants.

Figure 2. Semistructured interview data analysis process.
**Theme 1: Exposure to motivational factors.** The theme “exposure to motivational factors” was informed by the language used by all eight students in describing how they felt about mathematics. That is, the researcher found that students’ exposure or lack of exposure to motivational factors influenced their attitude toward math. For example, feedback was a motivational factor. Elements of feedback included grades they received, the checkmarks received for obtaining the correct answers, positive feedback, and/or immediate gestures of satisfaction from the teacher. Students expressed greater motivation about math when they were successful and less motivation during times of confusion or frustration. Participants also reported that they felt good about mathematics when they had proof and reassurance that they understood the lesson. Experiences included getting problems correct, being able to explain certain problems to their peers, and participating in class activities by volunteering correct answers. Students described these situations using very descriptive phrases such as “I like math when I get it,” “I always felt like I enjoyed math when I knew what I was doing,” and “at one point [when] I was really good at it…that was the class that I actually liked to go to.” All of the students during the interview were able to identify a time when they liked math, and each situation was based on how well they understood the mathematical concept. In addition, students were also able to recall the experience vividly, giving specific detail about the grade level, the situation and at times the topic that was presented. Here are typical student responses that represent the development of this theme (names used are pseudonyms):

**Stacy.** Last year, I really enjoyed going to math class because we weren’t just sitting down and doing math—we had like stations and every station we had like a
different problem and we would talk about different problems. So it wouldn’t feel like I was forced to just sit down and look at the paper. For example, our teacher would have problems on the board and we would go around and pick which problem you wanted to do. And for example, number 10 was something that I was really good at and I was really good at explaining it in others ways that my classmates would understand as well…I never felt like I was going to class. I always felt like I enjoyed it because I knew what I was doing.

David. I like math when I get the answer right. Then I get excited about math because I am good at it. I liked it better and I enjoyed it when I knew how to do the problems.

Dylan. Yes…I was learning percent and discounts and all that and I was enjoying this. I was excited about going to class.

Brook. When I was in elementary school…I thought I liked it because I was always good at it and it was simple and more like basic. I was more excited about it and even in Algebra…I thought it was fun. I enjoyed math when I did like Algebra…I really enjoyed it because at first I was like struggling a lot and in the beginning of the year I was struggling a lot and my teachers was working a lot. But she worked so hard with me to get me so much better at Algebra by the end of that year I knew everything so well. She had worked with me all year and like I ended up passing the SOL and it felt so good.

Carl. At one point, when I was really good at it so whenever I was in school that was a class that I actually like to go to. Also, I enjoyed it when I get everything on
a test or quiz. I wanted to try because like it’s better for me and the people around me.

Conversely, students also expressed that they were not motivated to do math at times nor did they like to attend class when they did not understand the material being presented. They explained that they “just didn’t get it.” One student mentioned that it becomes frustrating and discouraging “when you keep on trying and you keep on getting it wrong and you can’t understand.” Students expressed that math was not enjoyable when they did not get problems correct or when they did not get immediate positive feedback. Students also expressed that during this time, they would enjoy math more if teachers did not respond using a high-pitched voice or respond in an aggressive manner, and if they were not able to see the frustration or look of disappointment on the teacher’s face. Participants expressed their lack of enjoyment for math in very specific ways.

**David.** I got a C- on my test because they were some questions that I didn’t know so I just guessed. I then had to stay after school.

**Brook.** This is after I had been sick and I came back and was trying to learn what I needed to learn to like pass the test or something. It was factoring and I am horrible at factoring so it was like Algebra II so I stayed after to work with the teacher on the lesson involving factoring and once it got to the point that we needed to factor, the teacher says now factor this and the whole time she was like, “You don’t know how to factor this? You don’t know how to factor this?” And I don’t do anything in my head because when I was younger I just didn’t learn my
multiplication table so I just didn’t learn it so I rely on like a calculator a ton…I felt like so stupid.

**Stacy.** I didn’t like math my freshman year. I hated it so much. I think I took Prealgebra or something and I think it was just because it was introducing me to algebra and it was just very different than middle school and looking at shapes and angles. Since it was new that’s why it was very hard I think it was because it was new…and even though I was looking I just didn’t get it the first time.

**Theme 2: Characteristics of teachers.** The characteristics of students’ math teachers influenced students’ attitudes toward mathematics. Participants expressed that negative teachers created anxiety and that poor student attitudes resulted because of the stressful environment, criticism, and negative behaviors that math teachers displayed in the classroom. Findings revealed that the characteristics of teachers can influence a student’s willingness to make an effort in class. In addition, a strong influential factor is whether a student liked math class. When describing times they did not like math, many students included in their comments whether or not they thought the teacher was a good teacher. One student said that “I had a bad teacher.” Another said, “My teacher yelled too much.” Students even said “I think I hated the teacher, not the class.” For the students in this study, how they characterized their math teachers impacted their comfort level with math. Students explained these sentiments like so:

**Hunter.** Last year, I had a math teacher that I didn’t like. He just didn’t care. He would just like explain it really badly then give us a worksheet then walk away. He didn’t walk around and help individual students.
**Rosa.** Sometimes, like other teachers they go, “How do you not remember this” and they do that making the students mad, scared. I think they should actually like a little calm down.

Evidently, students are aware when teachers are negative and, for many students, this affects how well they learn the material or even enjoy math class. Stacy expressed what she does in such a situation.

> When I feel a teacher is being annoyed when I have a question, I back out and I don’t ask questions. It really depends on the teacher…my freshman year, I didn’t like the teacher. She was kind of like strict to the point where I felt like I had a question and I didn’t want to tell her because I knew what she would say, “why weren’t you listening?” I prefer a teacher that actually cares and is not just what you call it, numbers and grades, for the students and actually knows the students’ weaknesses and strengths and what they are good at and always compliments them and lets them know what they are good at and what they need to work on.

Other students expressed similar concerns. For instance, Carl reflected that teachers “have to have patience,” and that they have to be a “reliable person to talk to.” In addition, when Carl was asked, “Is there anything you would change about a math class you did not like?” he replied, “I would probably change the teacher.” He also explained that “I really don’t ask her much questions because she gets an attitude. She gets impatient.” Brook described an experience with a teacher after being absent from school for a few days. Brook explained:
It was probably more of the teacher in the math class, more than a specific math thing. I think I have had so many negative teachers that have been so mean to me that…like one my teachers that I had last year. I felt like so stupid and on top of being sick for so long and on top of all my stress and just getting out being in out of it…and just being really mean about the fact that I couldn’t factor and all I needed her to do was say like, “here this is how you do it. Let’s do a review together” to have a whole review about factoring so I would feel more confident about it but instead she was just like being so rude about it and she never reviewed it with me and she never helped me and she just said, “why don’t you know it” and I just sit here struggling and in tears by the time I walk out I was crying because I was like…I just felt she made me feel so stupid and horrible.

Brook was very passionate about how teachers affected her attitude toward math and went on to express more about her feelings. She also recounted that:

Every year except for one year in Algebra I did not like [math] because last year I had the worst teacher, she was really just pushing through everything and every time someone did bad on a test she would be like “you want to take this class again.” Her catch phrase was like “do you want to be in this class again?” And for every bad grade a student got or if a student was not doing good she would tell them over and over again “you are going to be in this class next year” and I don’t know it just wasn’t a good environment to learn it was just rushed and if you did bad you did bad and it’s like she wasn’t trying to help. I think throughout the year my bad experiences with math is because it because of the bad teachers and bad teaching.
Similarly, Stacy shared an experience from her freshman year:

My freshman year teacher was very stressful...if only she was less stressful. She was always like, “you have a test coming up; you have a quiz coming up” and since that was my weakness that worried me even more even though the test wasn’t until next week. So while we are learning the lesson I would think about the test coming up instead of focusing on what we are learning. I would be unable to focus on what’s on the board and then she tells me, “oh you have a test.”

When students expressed their feelings about math and their experiences with math, they also included their experiences with teachers. Although the majority of the conversations involved students’ negative experiences in math, there were also a few positive stories about teachers who were helpful and served to support students’ liking for math, more so. Stacy shared a positive experience with her Algebra teacher:

When I was taking Algebra, my teacher was always willing to stay after. I was going through a really tough week and emotional week, at home and school. I was doing really badly and it was at the end of the quarter and so the end the quarter is always crucial for your grades. So I was really nervous not really about what my grade was going to end up to be but I went to my teacher and she was so patient and she was like...it was like a Friday and usually teachers hate staying after on a Friday and she let me in her office and study with her on the quiz I didn’t do well on, and she let me take the quiz and right after I studied I did an A and she didn’t even average out the quiz grades, she just gave me the grade. And the fact that she was so helpful whenever I needed her help she would help me and I ended up doing
well and I ended up passing advance on the SOL and everything just because she was so patient to always help and I think the reason why was because she saw that I was really trying and so she was so willing to go too far extremes to help me do well.

Overall, the characteristics of classroom teachers are important factors that should be considered in educational planning and practices. These students prefer teachers who understand their strengths and weaknesses. Throughout the interview, students shared experiences regarding a time they did not enjoy math. Those experiences were centered on a negative anecdotal situation leading back to an experience with the classroom teacher. However, other factors also influenced students’ perceptions about mathematics.

**Theme 3: Instructional approach.** Student participants expressed an appreciation for the instructional approach classroom teachers embraced when teaching math. For example, participants’ teachers were complimented for “explaining more.” Students noted that “I like it when she explains it” in various ways and when “she gives us multiple practice” of the same thing and “goes over it.” Hunter reported that he was always happy with math “if they explain it, if they give enough details and [give] steps.” Other student participants described their preferences for specific instructional practices used during math class:

**Carl.** Like up front, when she is up front. I would sit up front and she would be on the smart board. Like, I like to visually see what she is doing. [With us,] she tries to go back over and make sure that we understand it.
**Stacy.** I like to see the problems in front of me. I am also a visual learner…so just a lot of repetition and explaining. That’s the only way I will understand math. I can’t see it just one time. For example, when you are doing a long problem, we do it multiple times and then you do it again and then add the second steps.

As Stacy explained, many of these students prefer to see math problems multiple times before trying a problem independently. In Stacy’s explanation, she alluded to a preference for explicit instruction or direct instruction, a type of teaching style that involves the use of guided practice and immediate feedback. David referred to a warm-up exercise. He mentioned that he liked when “she gives us warm-ups because it helps you get started for the day.” This type of teaching style, which is typically an introductory part of the lesson, prepares the learner for guided practice.

Further, Brook and Rosa both described a situation in which the teacher provided example problems and then gave students an opportunity to practice and check their work for accuracy:

**Brook.** Honestly for math, what works best for is when a teacher is…the lesson is on the smart board and you get the notes in front of you and you just do the problems with the class, with the teacher, and just going along step by step just doing it…like learning it just like that throughout class with each problem progressively getting harder and the teacher is doing it with you on the board step by step and you can write down the problem so like when you do homework you can go back and look at each step. I think using the smart board and going through like problems together throughout the lesson is the best way for me.
**Rosa.** I like it when [my teacher] explains it…if I don’t get a problem she explains it, the whole thing and gives me like hits on it and how to do it. And when she teaches it she teaches us hints and ways that really help. We write them down and sometimes she gives us worksheets and just like goes over them and that helps me a lot. We get morning work…everyone has to do it and she goes over it with us and if we get it wrong she go over it again and she explains it and she does on it on the smart board or the projector.

Math problems were understood by the participants when teachers took the time to explain each problem, and when the opportunity was given to correct each problem. Similarly, the style in which teachers review materials, or present a new concept or lesson, was recognized by participants. Furthermore, throughout the interviews, students made reference to instructional approaches which included explicit instructional techniques. The student participants were able to talk about how when teachers used these instructional strategies, they understood the presented materials better. Likewise, when an unfamiliar math problem was taught, students appreciated the time that teachers spent completing problems that were visually displayed, accessible, and clear.

**Supplemental Analyses**

In addition to the three themes, some students also expressed an awareness that math is important in life. One student stated, “Math is important to life because it will help me pay my bills in the future.” Another stated “I know math is important because I want to be like my dad…he is a financial guy.” Stacy, a student who worked at retail store expressed that:
Math is important because for example, what if I took the job at Edible Arrangements and I didn’t have the basic math? It would have been horrible. I would have been confused. I would have been embarrassed especially in the workforce, if [I didn’t] know the basic math at least.

The students made clear connections between math and career choices.

**Summary of Research Question Number One Findings**

The researcher was able to identify students’ attitudes about mathematics and perceptions. In so doing, student participants also shared information regarding their classroom teachers and the instructional approaches used. The math attitude inventory revealed that students were usually uncertain about their feelings about math but generally thought that math was a worthwhile subject. Similarly, questions 10-18 of the demographic test also revealed that students thought math was important but they inconsistently completed assignments and rarely stayed after school or asked for assistance with math. Throughout the semistructured interviews, students articulated situations in which they were more likely to enjoy math class. The participants expressed how excited they were to attend class when they understood the material, experienced getting problems correct, and felt confident about new material. In addition, they expressed displeasure for teachers who were negative and appreciated teachers who were patient, positive, and approachable. Students referenced instructional approaches that they preferred in math classrooms such as those involving guided practice and constructive feedback from the teacher.
Research Question Two Findings

The second research question for this study was, “How well does a group of secondary students with learning disabilities and students at risk for mathematics difficulties understand important mathematical concepts and symbols of algebra, and how they are used?” To inform this question, participants were asked to complete diagnostic test items which assessed students’ ability to demonstrate an understanding of important math concepts, the meaning of algebraic symbols, and an ability to interpret and perform mathematical tasks. The data source used to obtain a basic understanding of what students did or did not know was the math diagnostic test (see Appendix F for the instrument). An error analysis was then completed for each problem on the test.

Math Diagnostic Test Findings

Overall, the math diagnostic test revealed that participants demonstrated a greater understanding of the geometry and algebra strands. Students answered problems related to the number and number operations and data analysis strand with less accuracy. Specifically, on the number and number operation strand, students had a mean percent score of 57% correct; on the data analysis strand, a mean percent score of 50% correct; on the geometry strand, a mean percent score of 81%; and on the algebra strand, a mean percent score of 67% correct. Across students, the overall mean test score was a raw score of 8.87 ($SD = 3.04$) out of a possible 15 and a percent score of 55.87% accuracy. Table 9 contains a summary of the overall descriptive statistics of the math diagnostic test scores by raw score and percent score.
Table 9

*Overall Math Diagnostic Test Score Across Participants*

<table>
<thead>
<tr>
<th>Diagnostic raw score</th>
<th>Diagnostic percent score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>8.62</td>
</tr>
<tr>
<td>Median</td>
<td>9.50</td>
</tr>
<tr>
<td>Mode</td>
<td>10.00</td>
</tr>
<tr>
<td>Std. Deviation</td>
<td>3.02</td>
</tr>
<tr>
<td>Range</td>
<td>8.00</td>
</tr>
<tr>
<td>Minimum</td>
<td>4.00</td>
</tr>
<tr>
<td>Maximum</td>
<td>12.00</td>
</tr>
</tbody>
</table>

The possibility of numeric scores on this math diagnostic test were 0-15 but the actual scores obtained ranged from 4-13. Each problem was worth one point. The math diagnostic test scores indicated a large range of scores. The lowest score was 4 problems correctly solved (Dylan, 26% accuracy) and the highest score was 12 problems correctly solved (Carl, 80% accuracy). This is shown in Table 9. The mean and standard deviation was also compared across age, gender, and grade level, presented in Table 10.
Table 10

Means and Standard Deviation for Math Diagnostic Test

<table>
<thead>
<tr>
<th>Variable</th>
<th>M (%)</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gender</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Male</td>
<td>7.80 (53%)</td>
<td>3.19</td>
</tr>
<tr>
<td>Female</td>
<td>9.67 (64.4%)</td>
<td>.58</td>
</tr>
<tr>
<td>Age</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age 14 and below</td>
<td>8.00 (53.3%)</td>
<td>3.36</td>
</tr>
<tr>
<td>Age 15 and above</td>
<td>9.00 (60%)</td>
<td>2.00</td>
</tr>
<tr>
<td>Grade Level</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grade 9 and below</td>
<td>8.00 (53.3%)</td>
<td>3.36</td>
</tr>
<tr>
<td>Grade 10 and above</td>
<td>9.00 (60%)</td>
<td>2.00</td>
</tr>
<tr>
<td>Math Class</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Algebra and Prealgebra</td>
<td>7.60 (50.6%)</td>
<td>3.05</td>
</tr>
<tr>
<td>Geometry and above</td>
<td>10.00 (66%)</td>
<td>0.00</td>
</tr>
<tr>
<td>Overall</td>
<td>8.87 (59%)</td>
<td>3.04</td>
</tr>
</tbody>
</table>

The descriptive breakdown of each student’s percent score by strand is displayed in Table 11. Further description by students is provided below.

**Dylan.** Dylan scored a 4 (26.66%), and was more than four points below the overall average of 8.87 (59%). His scores by content standard were: number and number operations strand 2 out of 6 problems correct, geometry strand 1 out of 2 problems correct, algebra strand 3 out of 6 problems correct, and data and analysis strand 0 out of 1 problem correct.

**Carl.** Carl scored a 13 (86%), and was more than four points above the overall average of 8.87 (59%). His scores by content standard were: number and number operation 4 out of 6 problems correct, geometry strand 1 out of 2 problems correct,
algebra strand 6 out of 6 problems correct, and data analysis strand 1 out of 1 problem correct.

**David.** David scored a 12 (80%), and was more than three points above the overall average of 8.87 (59%). David’s scores revealed distinct differences between his lowest and highest strand result. His lowest score was in the number and number operations category and his highest scores were in the two content strands of geometry and data analysis. David’s scores by content standard were: number and number operation 4 out of 6 problems correct, geometry 2 out of 2 problems correct, algebra 5 out of 6 problems correct, and data analysis 1 out of 1 problem correct.

**Brook.** Brook scored a 10 (66%), and was almost two points above the overall average of 8.87 (59%). Brook scored highest in the algebra and the geometry strand. Her scores by content standard were: number and number operation 4 out of 6 problems correct, geometry 2 out of 2 problems correct, algebra 6 out of 6 problems correct, and data analysis 0 out of 1 problem correct.

**Hunter.** Hunter scored a 7 (46%), and was more than one point below the overall average of 8.87(59%). Hunter’s highest scores were in the number and number operations and algebra strands. His scores by content standard were: number and number operation 4 out of 6 problems correct, geometry 0 out of 2 problems correct, algebra 4 out of 6 problems correct, and data analysis 0 out of 1 problem correct.

**Stacy.** Stacy scored a 9 (60%) and was almost one point above the overall average of 8.87(59%). Stacy’s highest scores were in geometry and data analysis. Her scores by content standard were: number and number operation 3 out of 6 problems correct, geometry 2 out of 2 problems correct, algebra 4 out of 6 problems correct, and data analysis 1 out of 1 problem correct.
correct, geometry 2 out of 2 problems correct, algebra 4 out of 6 problems correct, and data analysis 1 out of 1 problem correct.

**Rosa.** Similarly, Rosa scored a 9 (60%) and was almost one point above the overall average of 8.87(59%). Rosa’s highest scores were in geometry and algebra. Her scores by content standard were: number and number operation 2 out of 6 problems correct, geometry 2 out of 2 problems correct, algebra 4 out of 6 problems correct, and data analysis 0 out of 1 problem correct.

**Hive.** Finally, Hive scored a 5 (33%) on the math diagnostic test and was almost four points below the overall average of 8.87 (59%). Hive’s scores by content standard were: number and number operation 0 out of 6 problems correct, geometry 2 out of 2 problems correct, algebra 3 out of 6 problems correct, and data and analysis 1 out of 1 problem correct.

Table 11

*Percentage of Accuracy by Strand Correct for Each Student*

<table>
<thead>
<tr>
<th></th>
<th>Number and number operations (%)</th>
<th>Algebra (%)</th>
<th>Geometry (%)</th>
<th>Data analysis (%)</th>
<th>Overall (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brook*</td>
<td>66.66</td>
<td>100.00</td>
<td>100.00</td>
<td>00.00</td>
<td>66.66</td>
</tr>
<tr>
<td>Carl</td>
<td>66.66</td>
<td>100.00</td>
<td>100.00</td>
<td>00.00</td>
<td>80.00</td>
</tr>
<tr>
<td>David</td>
<td>66.66</td>
<td>100.00</td>
<td>50.00</td>
<td>100.00</td>
<td>80.00</td>
</tr>
<tr>
<td>Dylan</td>
<td>30.00</td>
<td>50.00</td>
<td>50.00</td>
<td>100.00</td>
<td>26.66</td>
</tr>
<tr>
<td>Hive*</td>
<td>00.00</td>
<td>50.00</td>
<td>100.00</td>
<td>100.00</td>
<td>33.33</td>
</tr>
<tr>
<td>Hunter</td>
<td>66.66</td>
<td>66.66</td>
<td>100.00</td>
<td>00.00</td>
<td>46.66</td>
</tr>
<tr>
<td>Rosa</td>
<td>33.33</td>
<td>50.00</td>
<td>100.00</td>
<td>00.00</td>
<td>66.66</td>
</tr>
<tr>
<td>Stacy</td>
<td>33.33</td>
<td>66.66</td>
<td>100.00</td>
<td>100.00</td>
<td>60.00</td>
</tr>
</tbody>
</table>

*Note.* * indicates students at risk for mathematics difficulties.
Overall, three out of eight participants were able to solve all six algebra problems correctly while six of the participants were able to solve one or both of the geometry problems correctly. Of these, the geometry problem that involved applying a given formula (e.g., in Item 5, students needed to apply the formula \( A = \frac{1}{2} h(a + b) \)) was the one completed accurately. On the other hand, most students demonstrated difficulty solving problems in the number and number operations strand. This strand involved a students’ ability to add, subtract, divide, and multiply in order to solve rational numbers (i.e., integers and fractions). For example, in Item 7, students were asked to solve \((-4)(-2)(-3) = ?\). In Item 2, students were asked to subtract fractions \( \left( \frac{5}{4} - \frac{5}{6} = ? \right) \).

**Math Diagnostic Test Error Analysis**

Using the students’ diagnostic test results, the types of errors made by more than one student on each problem were closely examined. The researcher developed and applied an error analysis rubric containing 12 common errors that students make when solving math problems. Students’ errors on the math diagnostic test (Questions 1 through 15) were exhibited across math problems. Some students may have gotten the same problems incorrect, but they did not necessarily make the same errors. An error analysis conducted for each question on the math diagnostic test is explained below. Each error is identified and a figure included with students’ representative examples of errors is provided. The presentation below includes the diagnostic error and provides the correct answer.
**Question 1 and question 2.** Questions 1 and Question 2 (Number and Number Operation strand) required students to complete a multiplication or a subtraction problem using two fractions. These fractions consisted of a different numerator and/or a different denominator. Some students experienced difficulties with these problems because they (a) did not recognize what they were being asked to do in the problem, (b) completed the problem using the correct strategy but did not reduce the final answer, or (c) applied another strategy to solve the problem by attempting to cross multiply the denominator with the numerator. Any of these three errors may have been exhibited. The following three errors were noted on the rubric for Question 1 and 2. Figure 3 displays an example of similar errors made by two students.

- Difficulty recognizing different problem types.
- Difficulty completing last step to problem.
- Applying incorrect strategy.
Figure 3. Questions 1 and 2: Student sample and brief error description.

**Question 3.** Question 3 (Number and Number Operation strand) required students to apply the distributive property with both a number and a variable. This required
students to demonstrate an understanding of the order of operations as well as to recall the rule of combining like terms. Some students experienced difficulty with (a) completing the problem due to not performing the last step and (b) using the incorrect strategy needed to solve the problem. Therefore, students applied a strategy but it was not problem-specific. From the diagnostic test error analysis, the errors checked for students who did not solve this problem are listed below:

- Difficulty completing last steps to a problem.
- Applying incorrect strategy.

Figure 4 displays examples of each type of error.
Error: David started the problem correctly by multiplying 3 and 5 to get 15. However, failed to complete the problem by multiplying the 3 and X. Thus, he exhibited difficulty completing the problem.

Correct Answer: 15 + 3x

Error: Rosa completed this problem incorrectly by applying an incorrect strategy. The strategy used by Rosa is used when the problem requires a solution for a variable. Rosa is currently using what she referred to as a T-strategy to solve for the variable (x). She drew an upside down T to separate the two sides of the equation.

Correct Answer: 15 + 3x

Figure 4. Question 3: Student sample and brief error description.

Question 4. Question 4 (Number and Number Operation strand) asked students to identify a given percent from a representation. Two students had difficulty using the model to represent the percent of the shaded region. More specifically, the students did not make a connection between how to interpret a shaded region of the representation to identify a percent. These students were not able to completely identify the problem and therefore did not remember how to solve the problem correctly. Figure 5 displays an example of each type of error made by students. The common error checked on the rubric for this question was:

- Difficulty recognizing different problem type.
Ex 1. Question 4 (Dylan)

What percent of the model is shaded?

- a. 3%
- b. 15%
- c. 25%
- d. 30%

**Error:** Dylan appeared to have counted the number of shaded rectangles however, failed to choose the correct answer. Thus, he exhibited difficulty completing the problem by using the correct strategy.

**Correct Answer:** 15%

*Figure 5. Question 4: Student sample and brief error description.*

Ex 2. Question 4 (Stacy)

What percent of the model is shaded?

- a. 3%
- b. 15%
- c. 25%
- d. 30%

**Error:** Hive completed this problem incorrectly by applying an incorrect strategy. Hive attempted to count all the rectangles, indicated by the 20% written. However, after not being able to find the correct answer, Hive counted the number of shaded rectangles (3).

**Correct Answer:** 15%

Question 5. Question 5 (Geometry strand) asked students to find the area of a trapezoid by using a formula and evaluating the expression. Some students had difficulty using the formula to evaluate the problem. More specifically, students did not make a connection between the numbers included on the trapezoid (i.e., h = height, b = base) and the formula presented in the problem. Figure 6 displays an example of an error made by a student. The error checked on the rubric for this question was:

- Difficulty completing last step to problem.
Ex1. Question 5 (Carl)

What is the area of the trapezoid below with height 6 centimeters and bases 3.2 centimeters and 7.4 centimeters? (The formula for the area of a trapezoid is $A = \frac{1}{2}h(a + b)$.)

- a. 21.4 cm$^2$
- b. 31.8 cm$^2$
- c. 34.0 cm$^2$
- d. 63.4 cm$^2$

**Error:** In this example, Carl used the formula correctly. However, he had difficulty applying the formula in the final calculation. He replaced the number values correctly but failed to apply the formula by failing to complete the final steps of formula. After replacing the numbers, Carl did not perform the operations necessary to obtain an accurate answer.

**Correct Answer:** 34.8 cm$^2$

*Figure 6. Question 5: Student sample and brief error description.*

**Question 6.** Question 6 (algebra strand) asked student to solve a multistep linear and quadratic equation in two variables, including justifying steps used to simplify expression and solve the equation. Specifically, students were asked to add polynomials of similar and different terms. Students had difficulty adding integers that were of different signs. *Figure 7* displays examples of each type of error made by students.

Common errors were:

- Difficulty understanding abstract thinking (i.e., understanding algebraic expressions and equations).
- Not applying rules of negative and positive signs and variables.
Questions 7 and 8. Question 7 and Question 8 (Number and Number Operations strand) asked the students to add and subtract integers. Students had difficulty recalling the rules of using negative and positive integers. Figure 8 displays examples of this error made by students. The error made was:

- Not applying rules of negative and positive signs and variables
Question 9. Question 9 (algebra strand) asked students to demonstrate their understanding of exponents and powers by simplifying a numerical expression involving positive exponents. Students experienced difficulty applying the correct technique needed to solve problems involving exponents. One student added the base or number below the exponent to the same number of times stated by the exponent. For example, to solve the problem $5^3$, he added five, three times ($5+5+5$), and obtained the answer 15 instead of the correct answer, $5\times5\times5$, which equals 125. Figure 9 displays an example of this type of error. The following error was noted:

- Applying incorrect strategy.
Ex1. Question 9 (Rosa)

Error: Rosa applied the an incorrect strategy to solve this problem. She added 3 four times instead of multiplying $3 \times 3 \times 3 \times 3$.

Correct Answer: 81

Figure 9. Question 9: Student sample and brief error description.

**Question 10.** Question 10 (algebra strand) asked students to demonstrate an understanding of solving two-step equations. It was apparent that the students who made these errors had difficulty completing the correct procedural steps or did not recognize the problem, thus being unable to recall the procedure needed to solve the problem. Figure 10 displays examples of these error made by students. The following errors were noted:

- Difficulty recognizing different problem types.
- Difficulty following procedural steps.
Ex 1. Question 10 (David)

Solve the equation $4x - 5 = 7$.

Ex 2. Question 10 (Rosa)

Solve the equation $4x - 5 = 7$.

**Error:** David started the problem correctly and applied an appropriate strategy. However, he failed to complete the final step of the problem, which was to divide by 4 on both sides. He had difficulty following the procedural steps.

**Correct Answer:** $X=3$

**Error:** Rosa also applied the correct strategy. However, by ignoring the coefficient (4), she failed to complete the procedural step of dividing by 4 on both sides of the equation to get the final answer.

**Correct Answer:** $X=3$

*Figure 10. Question 10: Student sample and brief error description.*

**Question 11.** Question 11 (algebra strand) asked students to identify and extend geometric and arithmetic sequences, interpret relationships within a table, and solve an arithmetic problem. As seen in Figure 11, Hive had difficulty interpreting the arithmetic pattern. He chose to use the opposite of the last number given in the box before the empty space. This error was made by only one of the participants. Figure 11 displays Hive’s attempt to complete the problem and his error. The following error was noted:

- Difficulty completing the problem.
Ex1. Question 11 (Hive)

This is a function table for \( f(n) = 2n - 1 \). What is the missing value?

<table>
<thead>
<tr>
<th>( n )</th>
<th>( 2n - 1 )</th>
<th>( f(n) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2(0) - 1</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>2(1) - 1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2(2) - 1</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>2(3) - 1</td>
<td></td>
</tr>
</tbody>
</table>

\( \boxed{-3} \)

\( \boxed{b. 4} \)

\( \boxed{c. 5} \)

\( \boxed{d. 6} \)

\( \boxed{2 (3)} \)

**Error:** Hive was the only student to complete this question incorrectly. It appeared that he was not able to think of the correct strategy needed to complete the problem. It also appeared that he identified an incorrect pattern. Therefore, the conclusion was that Hive had difficulty completing the problem. He replaced \( n \) with 3 but forgot to subtract 1.

**Correct Answer:** 5

*Figure 11.* Question 11: Student sample and brief error description.

**Question 12.** Question 12 (algebra strand) asked students expected to represent quantitative descriptions algebraically. Figure 12 displays an example of an error made by a student for this question. The following errors were noted:

- Difficulty applying self-monitoring skills and checking answers after completion.
- Difficulty understanding abstract thinking (i.e., understanding algebraic expressions and equations).
Error: Stacy, like David, made the same error and chose the same answer. They both were unable to demonstrate and understand abstract thinking needed to fully express the algebraic expression correctly. In addition, they did not re-read the problem to verify answer.

Correct Answer: 12a + d

Question 13. Question 13 (Data Analysis strand) asked students to demonstrate an understanding of how to make comparisons and inferences using information displayed in a graph. All students had difficulty interpreting the meaning of the term “greater than,” which hindered their ability to make the correct inferences needed to interpret the problem. Therefore, most students selected the answer that represented the opposite of “greater than” (the least). This error was made by 50% of the participants. Figure 13 displays an example of the type of error made by students. The following error was noted:

- Difficulty recalling terms
Ex 1. Question 13 (Hive and Hunter)

On the graph below, the solid line shows Company A’s profits. The dashed line shows Company B’s profits. In what year are Company A’s profits greater than Company B’s?

Error: Both Hive and Hunter made the same error and chose the same answer. It appeared that they did the opposite of finding the year Company A was “greater than” Company B. Instead, both students chose the opposite. Therefore, the conclusion was that the students had difficulty recalling the term used.

Correct Answer: The year 2002

Figure 13. Question 13: Student sample and brief error description.

Question 14. Question 14 (Geometry strand) asked students to demonstrate an understanding of how to graph ordered pairs in a coordinate plane. All students were able to complete this problem correctly by choosing the correct set of points that represented the given graph. However, only one participant was able to show how he arrived at the correct answer: Hive completed the problem by first creating a chart. Figure 14 displays Hive’s method of completing the problem.
Error: Hive was the only student to complete this question by showing his work correctly. It appeared that he was able to think of a correct strategy needed to complete the problem. It also appeared, from the table he created, that he identified a correct pattern.

Correct Answer: B

Figure 14. Question 14: Student sample and brief error description.

Question 15. Question 15 (algebra strand) asked students to find the solutions to linear equations, using the variables X and Y. Students demonstrated an understanding of what was required to solve the problem and chose a correct strategy but could not execute the chosen strategy correctly. Figure 15 displays examples of each type of error made by students. As a result, the errors identified were:

- Difficulty completing last step to problem.
- Applying the incorrect strategy.
Summary of Research Question Two Results

In summary, students were able to solve basic algebra problems (e.g., one-step equations, two-step equations involving substituting a number for a missing variable), and geometry problems (e.g., remembering how to apply formulas to find area and perimeter). On the other hand, students had an overall difficulty solving problems within the number and number operations strand which involved calculation of fractions, decimals, and percents. In addition, students also displayed difficulty in the data analysis strand. Some students struggled to interpret charts and graphs.

The results showed that the most frequently found error was that students appeared to have a difficult time applying the correct strategy to solve a problem. This
involved applying cross-multiplication to a fraction that required the students to multiply fractions. Another example of this common error appeared in the exponent problem, where students had difficulty understanding the meaning of the exponent and what it meant in an equation. The second most frequent error was that although students demonstrated that they were able to start a problem and use a correct strategy, they had difficulty completing the problem. The third involved a difficulty with understanding abstract thinking (e.g., failing to understand how to represent algebraic expressions and to solve equations involving the use of a variable). Overall, students demonstrated some recognition of the problem type but could not execute the correct strategy needed to obtain the correct answer. Therefore, in order to further understand this phenomenon, the researcher conducted a clinical interview, referred to as a think-out-loud session.

**Research Question Three Findings**

The third research question was, “What does a group of secondary students with learning disabilities and students at risk for mathematics difficulties find to be the most challenging about mathematics?” The purpose of this research question was to better understand the challenges students face as they attempt to solve mathematical tasks. Challenges were initially identified from the graded and analyzed math diagnostic test. To further understand the results of the diagnostic test that revealed problem types that students found to be challenging, the clinical interviews were the primary data source.

The clinical interviews were conducted in session two, after the math diagnostic test was completed and scored. The clinical interview was the first task conducted during the second session with the participants in the study (see Appendix H). Each interview
lasted an average of 35 minutes. As previously stated, data from the math diagnostic test was used to guide questions asked during the clinical interviews. Each question was chosen specifically based on the students’ previously documented strengths and weaknesses from the math diagnostic test. For example, one of the participants, Carl, incorrectly solved a type problem on the math diagnostic test. As a result, both an easier and a more difficult version of the problem type were selected for Carl to complete during the clinical interview. Students were able to work through approximately 10 problems. The primary difference between this exercise and the completion of the math diagnostic test was that the students were probed to “think out loud” and to answer specific questions about their thoughts while completing the clinical interview math problems. Consequently, the clinical interview provided an opportunity for the researcher to listen to each participant’s mathematical thinking.

**Clinical Interview Rubrics**

Two different rubrics were used by the researcher to analyze the clinical interviews. The clinical interview error analysis rubric was used to analyze the verbal portion of the interview. This involved the analysis of the various types of errors that students made as they spoke about and solved each math problem. The clinical interview error analysis rubric identified verbal errors in students’ procedural explanations. The second rubric, the process standards rubric, was used to evaluate the students’ ability to read, interpret, reason, and communicate mathematical concepts while thinking out loud. The process standards rubric is also based on the NCTM process standards (problem solving, reasoning and proof, communication, connection, representation). The clinical
interview rubrics were used to analyze student work samples collected during the clinical interview, while simultaneously listening to the verbal explanations of students. Students were recorded during the clinical interviews.

**Clinical Interview Error Analysis Rubric**

Findings from the clinical interview error analysis rubric highlighted two key areas of math difficulty among the participants: number and number operations standards, and algebra standards. Although six common errors were shared among the group and appeared most frequently from the math diagnostic test, this section explains the results in terms of the commonly found errors displayed during the clinical interviews.

When the clinical interview error analysis rubric was applied, four common errors were identified: (a) difficulty recognizing problem types and given information, (b) difficulty applying the correct strategy, (c) difficulty obtaining accuracy of mathematical work (i.e., computational errors, including difficulty applying negative and positive signs), and (d) difficulty with an explanation of the solution, mostly involving abstract thinking. Noticeably, all four errors were also commonly found errors in the math diagnostic test.

Overall, the clinical interview error analysis rubric indicated that students consistently made errors ranging from (1) far below average to (4) proficient (see Appendix I for the standard point scale).

**Clinical Interview Process Standard Rubric**

Most participants’ scores fell between a standard (3) and below a standard (2) across the standards. More specifically, the average students’ process standard was a
standard (3) and students either made an attempt or made very little attempt to solve math problems. Scores ranged from a 1 - Far below (made no effort) to a 3 - Standard (made no effort) across each participant. Students were unable to demonstrate an advanced understanding (exceptional demonstration) for each problem, scoring below a 5 on the process standard rubric. First, the researcher will present a detailed definition of a commonly identified error made by students. Next, a representative example of that type of error will be presented and reflect the students’ mathematical thinking. Finally, the dialogue that ensued between the researcher and the student during this problem-solving interview will be presented.

**Challenges Identified**

During the clinical interview process, the primary challenges included recognizing problem type, difficulty recalling terms, difficulty applying a correct strategy, computational errors, and difficulty understanding abstract thinking. Each of the challenge areas are discussed below.

**Recognizing problem type and difficulty recalling terms.** All eight students with learning disabilities and students at risk for mathematics difficulties experienced some difficulty recognizing the type of one or more problems they were asked to solve. Thus, students were unable to start the problem or demonstrate their ability to apply previously learned strategies. Although some were able to recognize the prompts, more specifically, the language used in the prompts, students were unable to recognize the different tasks that were present within the problem. These required the students to be able to recognize a type of task required and apply a specific strategy in order to solve the
question correctly. Students also demonstrated difficulty recalling the terms of the components of the each problem, which also contributed to their inability to complete the problem accurately. To represent this, while solving the problem, the following responses were transcribed during the clinical interview between the researcher (R) and the students’ chosen pseudonym in response to the clinical interview items. Figure 16 displays Item 2 from the clinical interview questions.

<table>
<thead>
<tr>
<th>Clinical Interview Item 2 (Compare fractions, decimals, percent, and scientific notation):</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Item 2: Arrange the four numbers shown in order from least to greatest.</td>
<td></td>
</tr>
</tbody>
</table>

![Figure 16. Clinical interview Item 2.](image)

In Item 2, all participants were asked to order the numbers presented in a different form. On the math diagnostic test, 100% of students incorrectly solved Questions 1 and 2 that involved fractions. Overall, the clinical interview error analysis rubric showed that students demonstrated difficulty with identifying this type of problem because of the many different types of numbers the problem involved. More specifically, examples demonstrating the indicated type challenge are presented below based on participants’ analyses error(s).

**David.** For instance, David was unable to recognize different forms of numeric representation (i.e., percent, decimals, fractions, and scientific notation) within Item 2
during the clinical interview on Item 2 (see Figure 16). At first, David had difficulty starting the problem and appeared to lack the prerequisite skills needed to start and complete the problem. David needed to understand the difference in the values of numbers written in percent, decimal, fraction, or scientific notation form. As previously mentioned, David was a 12-year-old male student currently enrolled in a middle school Prealgebra course. David was diagnosed with a LD in math reasoning and reading comprehension while in elementary school. During the first session, David was very excited to meet and talk to me about mathematics. At the start of the second session, David also expressed excitement about completing the study and had many questions about the type of questions I would ask during the interview. The following dialogue with David is in response to the Clinical Interview Item 2:

R: If you recognize this problem, what type of problem is this?

David: It looks familiar. I am not sure.

R: Read the problem. What do you think?

David: We should put the problem from least to greatest.

R: What should you do first?

David: I should solve this first. (pointing to the fraction)

R: Then? As you solve it, can you say your steps out loud?

David: (working on the problem silently)

David: (pointing to 6.7%) Is this a decimal?

R: Yes.

David: Okay…perfect...
David: Okay, so you move the decimal two spaces.

R: Where?

David: To the right. Oh, wait! Is it to the right or to the left? To the left! No, to the right.

R: Correct. OK.

R: (pointing to $6.7 \times 10^{-3}$) What is this called?

David: I am not sure. I don’t know the names but I know how to do them. So, you put the number in order. This goes first (pointing to the 0.067 which came from 6.7%). This goes first.

In the end, David was unable to provide the name of all the different types of numbers presented in the problem. However, after reading the problem, he was able to figure out that he had to put the numbers in order from least to greatest. During the interview, he was silent for most of the time as he worked on changing the numbers from their current state to a decimal. This was a good strategy but he had difficulty changing the scientific notation, $6.7 \times 10^{-3}$, into a decimal. David’s answer was incorrect but he was able to use a strategy that, when accurately used, would produce the correct answer. However, because David was not able to recognize the differences between scientific notations and a decimal, he was unable to complete the task.

Process standard analysis and score. On a number point scale, David’s process standard score and analysis was primarily standard, with a total score of 17 out of 30 and an average score of 3.4 out of an average score of 5. David’s process standard score is in parentheses after the math standard and analysis is below.
Problem Solving (3)  David demonstrated some understanding of the problem by independently using a strategy. He made less than two errors.

Reasoning and proof (3)  David was able to come up with a plan to solve the problem. He changed the fraction and the percent into a decimal and arranged them in order using the decimals. However, he was unable to obtain the correct answer. As a result, David demonstrated a possible start to some form of reasoning required to solving the problem.

Communications (2)  David was able to use some important information from the problem to verbally explain his answer. With assistance, he performed and explained the beginning of the problem but had difficulty completing the problem correctly.

Connections (3)  David made the connection to compare the numbers need to all be in decimal form. For example, David’s first step was to convert the percentages to decimals.

Representation (3)  Independently, David attempted to use mathematical representations to record and communicate, with less than two errors. David demonstrated how to change the percent to a decimal accurately by placing an arrow from the decimal as he moved it two spaces left. David was able to clearly show a portion of his work as he verbally explained how to solve the problem.
**Stacy.** Similarly, when Stacy was presented with the same problem, she also was unable to identify the problem, thus was not immediately able to draw from prior knowledge to solve the problem. Stacy was a 17-year-old senior currently enrolled in a public high school. She was diagnosed with a learning disability in mathematics in middle school when her teachers and parents noted that she was experiencing difficulty demonstrating an ability to retain new vocabulary terms used in her classes. Stacy made five errors on the math diagnostic test from the algebra and number and number operations content standards. She made errors such as failing to complete procedural steps, failure to recognize problem types, and difficulty with abstract thinking. The dialogue below occurred regarding Item 2:

R: If you recognize this problem, what type of problem is this?

Stacy: Umm. It’s kind of like you have to list them in order

R: *(pointing to each part of the problem)* What does this mean?

Stacy: A percent.

R: Then this one…

Stacy: A decimal.

R: This one…

Stacy: A fraction.

R: And this one?

Stacy: *(Silence)* Hmmm, I don’t know what it’s called.
At this point, Stacy had difficulty remembering what the scientific notation was called and what should be done in order to complete the problem. The researcher continued with asking Stacy:

R: What would be the first thing you would do when you see this problem?
Stacy: I would multiply $10^3$ and multiply it by 6.7.

R: How would you go about completing this problem?
Stacy: I would then figure out what the decimal is for this (pointing to the fraction $\frac{6}{9}$), and this (pointing to the decimal).

Stacy did not, however, point to 6.7%. To Stacy, this problem was already in decimal form. The researcher continued:

R: Can you show and tell me how you complete this problem?
Stacy: I would put the 0.0067, then 0.67. Then, 6.7%, then $\frac{6}{9}$.

The correct answer for this problem was to list the numbers like this: $6.7 \times 10^{-3}$, 6.7, 0.67, then $\frac{6}{9}$.

*Process standard analysis and score.* On a number point scale, Stacy’s process standard score and analysis was primarily standard, with a total of 14 out 25 and an average score of 2.8 out of an average score of five. Stacy’s process standard score and analysis is below.

Problem Solving (3) Stacy demonstrated some level of reasoning. However, her work consisted of two or less errors.
<table>
<thead>
<tr>
<th>Reasoning and proof (3)</th>
<th>Stacy was able to reason through solving the problem by using prior knowledge to evaluate what was known about the different parts of the problem.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Communications (3)</td>
<td>Stacy was able to communicate how she solved the problem using some mathematical language such as “power,” “decimal,” and “fraction.”</td>
</tr>
<tr>
<td>Connections (3)</td>
<td>Stacy was also able to talk about changing the numbers into decimal form and she was able to accomplish changing the fraction into a decimal.</td>
</tr>
<tr>
<td>Representation (2)</td>
<td>Stacy was unable to rewrite the problem in order, using the original format of the numbers.</td>
</tr>
</tbody>
</table>

In brief, Stacy was not able to identify all of the problem types. She was able to identify the fraction, the decimals, and the percent. She was not able to recognize the scientific notation or convert the percent to a decimal format. She attempted to talk through solving the problem by rearranging the number but was unsuccessful because she was unable to recognize the different parts or components of the problem.

**Dylan.** In the same way, Dylan was unable to recognize the problem type until he attempted to work on each individual part of the problem. The researcher prompted him to read the question. As previously stated, Dylan was a 13-year-old freshman student enrolled in a public high school. He was diagnosed with a learning disability in mathematics and received services in a self-contained classroom. He made 11 errors on the math diagnostic test. He experienced difficulty recalling terms, difficulty recognizing
different problem types, difficulty applying correct strategies, difficulty understanding abstract thinking, applying a formula to a specific problem, and he made computational errors involving negative and positive signs. The dialogue below regarding Item 2, went as follows:

R: If you recognize this problem, what type of problem is this?
Dylan: I am not sure.

R: Read the problem then tell me what type of problem it is?
Dylan: Oh, I remember this – it says arrange the problem from least to greatest. I remember you would change them to decimals and then to percent and then see what decimal is bigger then what percent is bigger. Like 6 divided by 9 is a decimal and then you get a percent. See, I can do this.

R: Okay. Can you tell me how you go about completing the problem?
Dylan: Ahhh. (Silence)

R: You can use your calculator to help you complete this problem.

Dylan: (Silence as student work on problem and then approached the scientific notation)

R: (pointing the scientific notation) Do you know what this is?
Dylan: No, I don’t remember ever doing that in school.

R: This is called scientific notation.

Dylan: Oh…actually, I remember that. I actually did that like once.

Dylan was able to read Item 2 and was able to use the exact same words in telling the researcher what he would do or how to proceed with the problem. He recognized the
percent and the fraction in the problem. Therefore, Dylan used prior knowledge to help remember that he had to change the fraction to a decimal. However, not stopping there, after changing the fraction to a decimal, he immediately changed it to percent. Although Dylan was aware that he needed to change the fraction, in doing this he forgot what to do about answering the original question. In addition, he also had trouble recalling how to use the scientific notation in the problem.

*Process standard analysis and score.* On a number point scale, Dylan’s process standard score and analysis was primarily standard, with a total of 14 out 25 and an average score of 2.8 out of an average score of five. Dylan’s process standard score and analysis is below.

<table>
<thead>
<tr>
<th></th>
<th>Dylan understood what was needed to better represent the numbers in order. However, two or less errors were noted.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem Solving (3)</td>
<td>Dylan demonstrated the ability to reason through this problem but was unable to make a decision about two numbers in the problem.</td>
</tr>
<tr>
<td>Reasoning and proof (3)</td>
<td>Dylan was able to communicate what he knew (e.g., decimals, percent, and fractions) with assistance. However, he was not able to communicate how to successfully solve the problem using the 6.7% versus the scientific notation. He had less familiarity with the scientific notation.</td>
</tr>
<tr>
<td>Communications (2)</td>
<td></td>
</tr>
</tbody>
</table>
Connections (3) Dylan made a good connection with the conversion procedures for the fraction so that he could compare the value to the other numbers he knew.

Representation (1) Dylan was not able to write down his steps during this process. He completed most of his work using the calculator. He was able to verbally talk about which ones were smaller and which were bigger but did not represent this information in the correct order.

**Hunter.** In contrast, Hunter stated that he did not remember what type of problem Item 2 was but he was able to talk through and demonstrate some of the steps needed to obtain the correct answer. Hunter was a 14-year-old male eighth-grade student currently attending a public high school. He was currently enrolled in a team-taught Prealgebra course at the time of the study. Hunter was diagnosed with a learning disability in math. He made eight errors on the math diagnostic test. He experienced difficulty recalling terms, difficulty completing last steps to the problem, difficulty applying the correct strategy, difficulty applying self-monitoring skills and checking answers after completion, difficulty understanding abstract thinking, applying a formula to a specific problem, and he made computational errors involving negative and positive signs. The dialogue below occurred regarding Item 2 went as follows:

R: If you recognize this problem, what type of problem is it?

Hunter: I have no idea

R: Okay, what would be the first thing that you would do?
Hunter: I would change the fractions into decimals.

R: Then?

Hunter: I don’t know.

R: Read the problem again.

Hunter: Oh. Hmm…

R: *(pointing to the scientific notation)* Do you recognize this?

Hunter: Yes, but I don’t know the math name for it. I know what you would do.

R: Okay.

Hunter: You would count 1…2…3 *(moving the decimal)*

R: Why would you put the decimal there?

Hunter: Because you count 1…2…3. Oh, I would go that way.

R: Why would you put it there now?

Hunter: Because it has a negative.

R: Can you tell me what else you can do to finish the problem?

Hunter: So, we have a tie, this one would be first then….

As illustrated by the example, Hunter had difficulty problem solving because he did not read the problem carefully. He read the problem again but still had some difficulty. He recognized that he should change the fractions to decimals. However, he did not acknowledge or make a move to change the scientific notation to a decimal until it was pointed out by the researcher. With probing questions, he was able to change the scientific notation to a decimal and eventually corrected his mistake. Eventually, Hunter completed the problem with less than two mistakes. He had difficulty figuring out why
two answers were the same (6.7% and .67) In addition, an error existed in his conversion of the percent 6.7% to a decimal.

*Process standard analysis and score.* On a number point scale, Hunter’s process standard score and analysis was primarily standard, with a total of 13 out 25 and an average score of 2.6 out of an average score of 5. Hunter’s process standard score and analysis is below.

**Problem Solving (3)** Hunter demonstrated an understanding of the problem by independently selecting a strategy.

**Reasoning and proof (3)** Hunter was independently able to start some form of reasoning required to solve the problem.

**Communications (2)** With assistance, Hunter was able to select and use some important information from the problem to attempt explain the problem. He was prompted to refer to the numbers as percent and scientific notation.

**Connections (3)** Hunter was able to use at least one aspect of the problem to connect it to another mathematical topic. He made the connection that the different numbers would be easier to compare if they were all in decimal form.

**Representation (2)** Hunter was unable to fully demonstrate mathematical representation of the solution. He attempted to write out his first step, however after converting the scientific notation to a
decimal, he was unable to correctly show his work process using the other numbers, neither verbally nor on paper.

**Difficulty applying a correct strategy.** Seven out of eight students with learning disabilities and students at risk for mathematics difficulties in the study experienced difficulties applying a correct strategy to complete problems with one or more steps. Errors with applying a strategy included errors such as mixing up the steps in solving one-step equations or two-step equations, applying the distributive property correctly, or correctly completing the order of operations. For example, one student recalled an order of operations mnemonic device called PEMDAS (P-arenthesis, E-xponent, M-ultiplication, D-ivide, A-dding, and S-ubtract). However, students may at times apply the correct strategy to only a portion of the problem, and then switch to an incorrect strategy later in the problem. Figure 17 represents a problem from the clinical interview that requires application of the order of operations.

<table>
<thead>
<tr>
<th>Clinical Interview Item 3 (Number and Number Operations):</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item 3: What is the value of this expression?</td>
</tr>
<tr>
<td>$(−17) − (−26) ÷ 2$</td>
</tr>
</tbody>
</table>

*Figure 17. Clinical interview Item 3.*

All participants were asked to evaluate Item 3, presented in Figure 17. On the math diagnostic test, students showed a difficulty with computational problems involving the use of negative integers, subtraction, and other mathematical operations. Overall, the clinical interview error analysis rubric showed that students were able to identify clinical
interview Item 3 as an order of operations problem or as a problem containing “parentheses” but at the same time demonstrated difficulty in using the correct strategy to solve this problem.

**Hive.** Hive completed this problem by using a tool that was most familiar and easily accessible. He decided to use the calculator and appeared to be very confident and comfortable using this tool to solve the entire problem. As previously stated, Hive was a 14-year-old male ninth-grade student, enrolled in a public high school at the time of the study. He was enrolled in a general education Algebra course. Hive was a student at risk for mathematics difficulties who had noticeable problems with mathematics during middle school. He made nine errors on the math diagnostic test. He experienced difficulty recalling terms, recognizing different problem types, difficulty completing last steps to the problem, difficulty understanding abstract thinking, difficulty applying a formula to a specific problem, and made computational errors involving negative and positive signs. The dialogue below was in response to the problem in Figure 17:

R: How would you go about completing this problem?

Hive: *(Taking the calculator up)* I would do -17 minus 26 divided 2.

R: And then?

R: I get -21.5.

Hive demonstrated that he did not know or remember the strategy (i.e., PEMDAS) to use while completing a problem with multiple operations (i.e., addition, subtraction, division, or multiplication).
**Process standard analysis and score.** On a number point scale, Hive’s process standard score and analysis was primarily below standard, with a total of 7 out 25 and an average score of 1.4 out of an average score of five. Hive’s process standard score and analysis is below.

**Problem Solving (2)** Showed little understanding of the problem. No use of problem-solving strategy existed beyond the use of a four-function calculator.

**Reasoning and proof (2)** Demonstrated a possible start to some form of reasoning. He read the problem and rephrased what the problem was asking and explained by reading the order in which he would enter the number directly from the prompt. This process was incorrect so no example of accurate mathematical reasoning existed.

**Communications (1)** No evidence of communicating the problem exists. He verbally read the steps from the prompt as he stated how he would enter the problem on the calculator. He also ignored a negative sign during this process along with the parentheses.

**Connections (1)** No connection was made to any other subject or topic.

**Representation (1)** No representation besides the use of a calculator was used to solve this problem.

**Carl.** Similarly, Carl understood that it was a calculation problem. However, he applied the wrong strategy in the beginning of the problem. Recall, Carl was a 15-year-old male 10th-grade student at a public high school at the time of the study. He
was currently enrolled in a team-taught Prealgebra course. Carl was diagnosed with a learning disability in math. He made five errors on the math diagnostic test. He experienced difficulty recognizing different problem, difficulty completing the last steps of the problem, difficulty applying the correct strategy, difficulty understanding abstract thinking, and difficulty applying a formula to a specific problem. The dialogue below was in response to clinical interview Item 3 (see Figure 17):

R: If you recognize this problem, what type of problem is this?

Carl: Distribution problem.

R: Why?

Carl: Because you have to distribute the negative outside the parentheses.

Carl noticed that he would have to distribute the subtraction sign (−) outside of the parentheses to the −26. The interviewer continued with the next question.

R: What is the first thing you would do when you see this problem?

Carl: I would get my calculator.

R: Okay, then?

Carl: I would write the problem down.

R: Is there a part of the problem you would do first?

Carl: The parentheses, the negatives.

R: So, tell me how you would solve this problem.

Carl: I am subtracting 17 and 26.

At that moment, Carl began to complete the problem incorrectly. He did not apply the correct strategy. In this problem, Carl should have first divided 26 by 2. As evident in
Carl’s verbal steps, although he is aware that in the order of operations parentheses are addressed foremost, he was not able to apply the strategy that would aid in solving this problem correctly. After Carl completed the problem, the researcher asked:

R: What part of the problem did you find easiest?
Carl: The division part.
R: Was there a part of the problem you found most difficult?
Carl: The parentheses.
R: Why?
Carl: It would have been easier if I had a chance of using the [graphing] calculator.
R: What does your answer mean? (Carl’s answer was 4.5.)
Carl: It is a decimal.
R: What does that tell you?
Carl: I don’t know.

In brief, Carl may have been confused about the parentheses surrounding the (-17) and the (-26), so he was unable to apply a strategy correctly. Although Carl was able to obtain the correct answer, he completed the problem by ignoring the second negative sign. Therefore, his communication and representation of the problem was incorrect but the sequence of numbers he entered into the basic calculator yielded a correct answer. He also mentioned that the problem would be easier with the use of his calculator, referring to his graphing calculator.
Process standard analysis and score. On a number point scale, Carl’s process standard score and analysis was primarily standard, with a total of 15 out 25 and an average score of 3. Carl’s process standard score and analysis is below.

Problem Solving (3) Carl showed an understanding of the problem and independently chose an effective strategy. He pointed out the parentheses as the part of the problem he would complete first.

Reasoning and proof (3) Carl demonstrated a possible start to some form of reasoning required to solve the problem. He looked at the parentheses and also the location of the negative signs.

Communications (3) Carl selected and used some important information from the problem to explain his answers. He referred to terms such as parentheses, negative signs, and to the symbol that meant division.

Connections (3) Carl attempted but was not fully able to accurately connect at least one aspect of the problem, or to relate or connect the problem to similar types problems based on prior experience.

Representation (3) Carl attempted to create mathematical representation(s) to display the problem. He correctly wrote the problem down and attempted to demonstrate the distribution of the negative sign outside the parentheses. However, his representation was not completed in the correct order.
**Brook.** In contrast, Brook recognized and started the problem with confidence but focused on the wrong strategy while completing the problem. Brook was a 12th-grade student enrolled in a public high school at the time of the study. She was currently enrolled in a team-taught math course beyond Algebra II. Brook was a student at risk for mathematics difficulties. She made five errors on the math diagnostic test. She incorrectly applied a specific strategy, had difficulty applying formulas, and had computational errors involving negative and positive signs. The dialogue below was in response to the clinical interview Item 3:

Brook: This is like negative problems, just adding and subtracting…

R: What would be the first thing that you would do with this problem?

Brook: I notice the two negative signs next to each problem so that means -17 + 26 and that 9 and then divided by 2 and that is 4.5.

R: Why did you add first?

Brook: That’s just how I learned how to do it.

At that moment, Brook used an incorrect strategy to solve this problem. Similar to Carl, she decided to complete the calculation of the problem with the 17 and the 26 first.

*Process standard analysis and score.* On a number point scale, Brook’s process standard score and analysis was primarily standard, with a total of 14 out 25 and an average score of 2.8. Brook’s process standard score and analysis is below.

Problem Solving (3) Brook showed an understanding of the problem and independently chose an effective strategy.
Reasoning and proof (2) Brook demonstrated a possible start to some form of reasoning with some assistance.

Communications (3) Brook selected and used some important information from the problem to explain her answers. She referred to the location of the two negative signs and was able to explain using mathematical term what the two signs meant.

Connections (3) Brook attempted but was not fully able to accurately connect at least one aspect of the problem, relate or connect the problem to other problems based on prior experience in math. Brook connected the two negative signs in the middle of the problem to the multiplication of two negative numbers (a negative times a negative will equal a positive).

Representation (3) Brook attempted to create mathematical representation(s) to display. She wrote the problem down and she also changed the two negative signs to a positive sign but was not able to represent the remaining problems in paper sequentially or without errors.

**Rosa.** Rosa was able to state the correct procedural steps accurately until she got to the portion of the interview where she had to demonstrate her understanding by completing the work step by step. Rosa identified a correct strategy to use (i.e., PEMDAS) but applied the wrong strategy at the beginning of the problem. As a result, she was unable to obtain the correct answer. Rosa was a 12-year-old seventh-grade
student enrolled in a public middle school at the time of the study. She was currently in a
team-taught Prealgebra math course. Rosa was diagnosed with a learning disability in
mathematics reasoning. She made six errors on the math diagnostic test. She experienced
difficulty recalling terms and applying the correct strategy needed to solve certain
problems. The dialogue below was in response to the problem above:

R: What would be the first thing that you would do in this problem?

Rosa: I would just do the problem by using PDAS.

R: Can you write it down?

Rosa: Yes. (*She writes PEMDAS*)

R: So, what would you do first?

Rosa: 17 minus 26.

R: Okay.

Rosa: Then 9 divided by 2 and then we get a decimal.

Rosa definitely planned on using the correct strategy. She then wrote down the
word PEMDAS next to the problem, however, completed the wrong part of the problem
first by subtracting 17−26 (ignoring the negative signs). This is representative of a
computational error, involving students forgetting or ignoring negative and positive signs.
This showed that although she remembered a correct strategy and identified it by name
(PEMDAS), she applied the strategy incorrectly.

*Process standard analysis and score.* On a number point scale, Rosa’s process
standard score and analysis was primarily standard, with a total of 14 out 25 and an
average score of 2.8. Rosa’s process standard score and analysis is below.
Problem Solving (3)  Rosa showed an understanding of the problem and independently chose an effective strategy.

Reasoning and proof (2)  Rosa demonstrated a possible start to some form of reasoning with some assistance.

Communications (3)  Rosa selected and used some important information from the problem to explain her answers. She verbalized a strategy (PEMDAS).

Connections (3)  Rosa attempted but was not fully able to accurately connect at least one aspect of the problem, or to relate or connect the problem to other math problems based on prior experience with PEMDAS.

Representation (3)  Rosa attempted to create mathematical representation(s) to display and communicate the problem. Rosa wrote down here chosen strategy and attempted to show her work in sequential order with two or less errors.

Similarly, in Item 14 (Figure 18) of the clinical interview, two out of four students applied an incorrect strategy to solve the problem.

Clinical Interview Item 14 (Algebra Strand):

Item 14: Solve and Graph?

-4x > 4

*Figure 18. Clinical interview Item 14.*
In this example, Stacy and David both applied the wrong strategy. They added 4 to both sides of the greater than sign. Rather, the students needed to divide by -4 on both sides of the inequality.

**Computational errors.** Six students who are at risk for mathematics difficulties and those with learning disabilities in this study made computational errors. These students required the use of a calculator but demonstrated difficulty using the calculator to compute some problems accurately. Students were given the use of a four-function calculator but not a graphing calculator. The researcher wanted to focus the clinical interviews more on the students’ process as they thought through a problem, rather than calculation skills (i.e., addition, subtraction, division, and multiplication). Although students were given the use of a four-function calculator and were capable of using signs, most errors occurred while working with negative and positive numbers. Students ignored what the negative signs represented.

In clinical interview Item 14 (Figure 18), some students applied the correct strategy but, due to a computational error, were not successful in solving the problem accurately. The most frequently made computational error was ignoring the negative signs. Students ignored the negative sign in front of the four. As Hunter solved this problem, he explained:

Hunter: You have to divide by -4 on both sides.

R: Then?

Hunter: You get... *(He wrote down X>1)*
Ignoring the negative sign was not the only computational mistake Hunter made. This mistake may have caused Hunter to also forget to change the sign from (> to (<), a procedure that takes place after dividing or multiply an inequality by a negative number. In addition, Hunter was not the only participant to make this mistake, as shown by Brook’s approach to the clinical interview Item 9 (Figure 19).

**Clinical Interview Item 9 (Algebra Strand):**

Item 9: Solve each linear equation

\[ \frac{K}{-5} + 7 = 22 \]

*Figure 19. Clinical interview Item 9.*

**Brook.** Brook correctly applied a strategy by first subtracting 7 on both sides of the equation. However, she then immediately multiplied both sides by 5, ignoring the negative value. Students were asked to think out loud while simultaneously solving the problem. However, in all cases, the students who made computational errors were not able to talk through the problem with an understanding that would allow students to apply a self-correcting strategy. For example, Brook solved the above problem, she explained:

Brook: First, you subtract 7 on both sides.

R: OK...

Brook: And then you multiply by 5 on both sides.
Brook was able to apply a correct strategy but ignored the negative sign during her computation of the problem and again during her step-by-step explanation of how to solve the problem.

**Difficulty using symbolism and understanding abstract thinking.** Six students in the study had difficulty using symbolism and understanding abstract thinking, which includes effectively interpreting algebraic expression and inequalities that may involve variables, numbers, symbols, and signs. Most of these student participants exhibited difficulty with interpreting the meaning of a variable and or symbols (i.e., <, >, =). Clinical interview Item 8 involved abstract thinking of inequality and representing this knowledge graphically. Figure 20 displays Item 8 from the clinical interview questions.

Clinical Interview Item 8 (Algebra):

Item 8: Solve the given problem and graph.

\[ X + 5 > 8 \]

*Figure 20. Clinical interview Item 8.*

**Rosa.** Rosa explained and worked through this problem effectively until she had to represent her answer using a number line. Rosa explained:

Rosa: You draw your upside down T.

R: OK…

Rosa: You minus 5 here and here.
Rosa: Then, you get… (writes done $X>3$)

R: Ok, can you graph this?

Rosa: Hmmm, no.

Evidently, Rosa used an applicable strategy to solve the problem correctly. However, she was unable to graph the answer. In the same manner, Stacy was able to solve the problem but was unable to interpret the meaning of her final answer. As a result, she was unable to graph the problem correctly on the number line. In order to graph this problem correctly, Stacy should have started at the 3 and then drawn the line to the right. Stacy’s response to the problem was as follows:

R: How would you go about solving this problem?

Stacy: You would minus 5 on both sides.

R: OK…

Stacy: Then, you graph. Oh, wait. You have to do 8-5.

R: OK.

Stacy: On the line, you start on “0” and go this way. (drawing an arrow to the right)

In another problem, students demonstrated difficulty representing word expressions algebraically. This was a skill that again required students to consider the use of variables, numbers, and symbols. For example, clinical interview Item 14 represented students challenge with using symbolism and understanding abstract thinking. Figure 21 displays Item 15 from the clinical interview questions.
Clinical Interview Item 14 (Algebra Strand):

Item 15: Write the following algebraically:

a. Three times a number increase by 10.

b. Nine less than a number.

Figure 21. Clinical interview Item 15.

**Hunter.** Hunter was able to use his understanding of terms in the clinical interview Item 15 such as “algebraically.” He also interpreted “a number” to mean that a variable must be used to answer the question. Although Hunter was able to correctly think out loud and respond with “three times a number increased by 10”, for Item 15 a., he was unable to accurately represent the algebraic expression “Nine less than a number.” All participants, including Hunter, were unable to correctly represent Item 15b. As Hunter explained:

Hunter: Nine less than a number.

R: Yes.

Hunter: Would be this, 9-X.

Stacy, Rosa, and David all responded with 9-X as their final answers. Brook responded using with 9 > X and Hive only wrote 9 minus. However, the correct answer was X-9.

In another instance, students were asked to solve an algebraic equation and explain their answers. While half of the students were able to provide an acceptable
answer, the others did not understand the meaning of the variable found except for “that’s the answer.” For instance:

R: What does this mean? *(pointing to the X)*

Hive: The answer, a letter. *(smiles)*

The students had difficulty understanding abstract thinking and interpreting representations of an algebraic equation.

**Summary of Research Question Three Findings**

Research question number three was designed to further determine what was most challenging about math for students with learning disabilities and students at risk from mathematics difficulties. Findings demonstrate that students in this sample had difficulty with the following: (a) identifying problem type, (b) recalling terms, (c) applying a correct strategy, (d) computing accurately (e.g., negative signs), and (e) using symbolism and understanding abstract concepts. Results demonstrate that students continued to have difficulty in both the content and in process standard areas. For instance, from the “think-out-loud” clinical interview sessions, students consistently demonstrated an inability to identify problem type or a specific portion of a problem. Also, many often failed to use a correct strategy. Even though correctly identifying a problem is often tied to a student’s ability to choose a correct strategy, this is not necessarily a determinant of whether students will apply the strategy accurately.

More so, the clinical interview also brought about the awareness that many of these students were unaware of how to use mathematical language when communicating about math. Students were at times very confused about the name or type of problem with which
they were presented. As a result, those students needed to be asked probing questions in order to get started on choosing a specific strategy to use. Most students had difficulty recalling the terms, “order of operations,” “exponents,” “scientific notation,” and “inequalities.” They also had difficulty explaining their answers and reasoning as to why they chose a specific approach to solve the problem.

Another common observation was that many of these students were not able to apply a self-correcting strategy as they may have talked through or explained their work during the interview. Some students were able to self-correct when they were confident about their chosen strategy; however, they applied an incorrect strategy while solving the problem. However, those who ignored the negative signs appeared to ignore the negative signs during their explanation of the problem they completed as well. Overall, a constant challenge that the students demonstrated during the interview process was the inability to clearly and accurately identify, demonstrate, explain, and interpret mathematical tasks using correct mathematical language.
CHAPTER FIVE

The purpose of this study was to investigate the perceptions and thought processes of a group of secondary students with learning disabilities in mathematics and students at risk for mathematics difficulties. The first research question listed below was examined in order to gain a better understanding of the perceptions and thought processes behind students’ attitude toward mathematics. The second was included to gain an understanding of how well the students in this study understood symbols of algebra, and the use of concepts in mathematics. The third question was included to delve deeper into the results identified by research question two by asking students to think out loud while completing math problems. The research questions were:

RQ1. What are the perceptions and attitudes of a group of secondary students with learning disabilities and students at risk for mathematics difficulties about mathematics?

RQ2. How well does a group of secondary students with learning disabilities and students at risk for mathematics difficulties understand important concepts and symbols of algebra, and how they are used?

RQ3. What does a group of secondary students with learning disabilities and students at risk for mathematics difficulties find to be the most challenging about mathematics?
In the first section of this chapter, the findings of the study are summarized and discussed in terms of their relative relationship to the current literature. Then, a discussion of findings by research questions is presented. Finally, the limitations of the study and implications for future research are addressed.

**Discussion of Findings Relative to Current Research Literature**

Since the implementation of the National Council of Teachers of Mathematics’ (NCTM) new standards of learning (2009a, 2009b), six studies with mostly qualitative components were found to be published for secondary students with learning disabilities and students at risk for mathematics difficulties. At the time of this study, intervention studies dominated the literature. These studies involve mathematics and students with learning disabilities, while the qualitative studies continue to remain scarce in the research community (Sumpter, 2013). Furthermore, in the last decade, limited qualitatively studies have been conducted to better understand the perceptions of students who experience difficulties in the area of mathematics (Howard & Whitaker, 2011; Sumpter, 2013).

Consequently, the current study was designed to address the gap in the research literature of mathematics by giving students with LD and students at risk for mathematics difficulties a voice and an opportunity to be further understood. Students should be able to communicate about math in different ways (Wadlington & Wadlington, 2008) and, as stated by researchers such as Baroody & Ginsburg (1990), Fennema et al. (1993), and Howard & Whitaker (2011), researchers should also work toward understanding students’ perceptions of math. This study, along with others (Baroody & Ginsburg, 1990;
Ginsburg, 1997b; Jenkins, 2010; Whitaker, 2011), demonstrate that students’ beliefs and misconceptions or understandings about mathematics cannot be identified solely by simply evaluating students using a math assessment. These researchers emphasized the need to further understand what students think about math in order to provide quality instruction and to direct pedagogical methods. The current study incorporated an explicit research method and utilized different data sources to address the perceptions of a group of secondary students with learning disabilities and students at risk for mathematics difficulties.

Important findings emerged from this study. Collectively, findings revealed that (a) motivation plays a major role in students’ effort toward math, (b) teachers’ characteristics greatly affect students’ understanding of mathematical concepts, (c) instructional approaches used by teachers influence students’ perception of mathematics, and (d) think-out-loud sessions while students complete math problems can shed light on the challenges that contribute to the poor mathematical achievements of students with learning disabilities and students at risk for mathematics difficulties. Each of these findings will be discussed below.

**Motivation**

Findings from the current study suggest that motivation plays a role in student effort and work efficacy. As supported by research, many students’ attitudes toward math are directly connected to their level of motivation (Howard & Whitaker, 2011; Lipnevich, MacCann, Krumm, Burrus, & Roberts, 2011; Middleton & Spanias, 1999) and willingness to perform mathematical tasks (Howard & Whitaker, 2011). In this study,
most participants shared their positive and negative experiences related to their feelings about math. Previous research has demonstrated that as students experience success with math, their level of motivation increases (Mundia, 2012) and motivation positively influences their attitude toward math (Howard & Whitaker, 2011). Throughout the study, students referred to situations in which they liked math and when they were also successful in math class. During semistructured interviews, students appeared genuinely enthusiastic about math as they shared these successful experiences with the researcher. This level of enthusiasm was also evident as students talked through problems that they were successful with during the clinical interviews.

Perceptions and attitudes of mathematics for students are closely related to how well students perform in math (Howard & Whitaker, 2011; Lewis, 2010; Lucas & Fugitt, 2009). Students in the current study appeared proud and excited when recalling their success in math. Students expressed that they were more willing to attend math class and that they were motivated to learn new concepts, participated in classroom discussions, and enjoyed learning when they were successful. In contrast, during unsuccessful situations in math class, students, including those in this study, have expressed that they were more likely to “shut down” during classroom lectures and independent work (Howard & Whitaker, 2011). This aligns with Ginsburg’s (1997b) observation that students appear to appreciate learning and are more engaged with interest and enthusiasm when they feel motivated by their accomplishments. Experts in the field (Ginsburg, 1997b; Howard & Whitaker, 2011; Lipnevich et al., 2011; Mundia, 2012; Sumpter, 2013) also conclude that students who are motivated toward an academic subject or task are
also most likely to have a positive attitude toward that task or subject area. Increasing ways for students with LD or students at risk to experience success with mathematics is critical.

Fortunately, the literature of mathematical interventions has sought to enhance students’ confidence in mathematics by introducing interventions and strategies that students can use to be successful when solving mathematical problems. During the semistructured interviews, students in this study often referred to motivational factors such as immediate feedback, the ability to check answers, and the use of positive or corrective feedback. These motivational factors are consistent with the research-based interventions identified in the literature to ensure accurate computation and fluency (Poncy et al., 2007; Test & Ellis, 2005; Woodward & Brown, 2006). Using an instructional approach designed to teach students how to use mnemonics to help remember facts and follow basic procedural steps, self-monitor, and check answers is another research-based practice that supports students’ success with math (Maccini et al., 2007). In the current study, participants were favorable toward positive corrective feedback and receiving instructions that clearly demonstrated the mathematical process and problem-solving steps. Explicit instruction can serve as a motivational factor during mathematics. The current study’s findings align with Lewis (2010), Howard and Whitaker (2011), and Sumpter’s (2013) work that students with LD obtain satisfaction when they are able to solve a math problem. Further, Howard and Whitaker found that students tend to experience a feeling of self-confidence when they successfully complete mathematical tasks.
Teachers’ Characteristics

As stated by Lucas and Fugitt (2009), the personality, attitude, and characteristics of math teachers impact student learning. Lucas and Fugitt revealed that math teachers’ negative attitude and unpleasant classroom behaviors impact students. Similar to Lucas and Fugitt’s study, participants in this study expressed a feeling of dislike for math if they had a teacher who appeared to be unapproachable, negative, or simply displayed a bad attitude. For instance, many students reported that it was difficult to ask for help from a teacher who frequently raised his or her voice or made them feel poorly about themselves. Alternatively, teachers characterized as patient, helpful, and humorous were received positively by students. The characteristics of math teachers can have an impact on students’ understanding of concepts or their willingness to make an effort to better understand a math concept. Research suggests that teachers should be mindful of their personal interaction with students and their role as mediums through which students gain pertinent information (Howard & Whitaker, 2011; Lucas & Fugitt, 2009; Sumpter, 2013).

Instructional Approaches

Current research has identified effective evidence-based practices for teaching math to students with learning disabilities. For students with learning disabilities and students at risk for mathematics difficulties, research suggests that direct instruction, when performed correctly, is one of the most effective ways of instructing students in mathematics (Gagnon & Maccini, 2001; Maccini et al., 2008; Mayfield & Glenn, 2008; Strickland & Maccini, 2010). Remarkably, when most students in this study were asked about the type of presentation style they preferred most in their math classes, they were
able to describe a type of explicit instruction clearly without second-guessing themselves. Students expressed that they were more likely to understand math when the teacher provided them with multiple examples, explained well, gave them opportunities to try problems, and then assisted them as needed. This represents an instructional approach in which students are taught techniques for problem solving via modeling, guided practice, and supporting student learning with multiple opportunities to practice. More specifically, students described how it helps to first watch the teacher perform problems on the board. They also explained that it helps them if teachers provide problems for them to do on their own and then help them work through each problem (e.g., guided practice).

As they continued to describe the type of instructional approaches that were most effective for them, they also included presentation styles that were not helpful, such as when the teacher only showed them one problem, and/or expected them to work independently. Students also explained classroom situations in which they were presented with a worksheet and expected to work independently without assistance for an entire class period, on a daily basis. According to various researchers (Gagnon & Maccini, 2001; Ives, 2007), providing students with multiple examples and providing corrective feedback is most effective for learners with a disability or for those at risk for mathematics difficulties. Likewise, research states that mathematical structure can provide an opportunity for students to encourage self-regulation and possibly solve more problems correctly (Butler, Beckingham, & Lauscher, 2005).
Mathematical Challenges

The mathematical challenges demonstrated by the current study were evident by participants’ dialog during the think-out-loud sessions. These sessions involved a verbal assessment in which students completed various math problems. Students were provided with the use of a calculator in this study because without the use of the calculator, students’ opportunity to respond may greatly decrease and their level of frustration may in turn increase (Wildmon, Skinner, Watson, & Garrett, 2004). In addition, Axtell et al. (2009) noted that students who master mathematics computational skills are better able to progress in mathematics. Further, students who are provided with this support will have a better chance of understanding more abstract problems that require mathematical reasoning. So, given a calculator, the clinical interviews served as a way to provide great insight into each participant’s mathematical thought processes and abilities as they completed mathematical tasks. Researchers in mathematics have used this method in order to better understand students’ cognitive processing approaches to mathematical tasks (Arias et al., 2010; Ginsburg & Seo, 1999; Lobato et al., 2012; Mundia, 2012; Steinberg et al., 2004).

Based on NCTM process standards and the new criteria evaluating the standard of learning, the researcher was interested in discovering how students reasoned through tasks. The clinical interview intended to determine what strategies students employed. How well did they mathematically communicate their thoughts? What connections did they make to mathematical concepts across content? How well did they use mathematical representation to demonstrate (or arrange) and solve the problems? The current study
confirmed previous findings (Baroody & Ginsburg, 1990; Sumpter, 2013) that students may be exposed to more procedural teaching methods rather than instructional methods that are geared toward reasoning and conceptual understanding. The researcher found that most participants had difficulty identifying a problem type but they were able to recognize a component of the problem that triggered their use of a specific strategy or procedure. As a result, student participants often explained strategies that were inappropriate and/or did not fit a particular task, attempted to solve problems using the correct strategy, or at times, unsuccessfully verified answers to solutions that were one small error away from being accurate. Consequently, it was apparent that many skills were not clearly understood by students.

**Discussion of Findings by Research Question**

The demographic questionnaire and the math attitude inventory provided data to inform the first research question. The data analysis revealed information about the perceptions of the study’s participants. It also provided an opportunity for the researcher to corroborate findings across multiple data sources.

**Research Question One: Perceptions and Attitudes**

Research Question One was: What are the perceptions and attitudes of a group of secondary students with learning disabilities and students at risk for mathematics difficulties about mathematics? Overall, findings are consistent with past research that suggests that students are generally in agreement that math is very important in today’s society (Lucas & Fugitt, 2009). However, despite this acknowledgement, students’ personal attitudes toward math are based on various classroom experiences that develop
over time. As Lipnevich et al. (2011) state, students’ attitudes toward academia tend to have the ability to positively influence their grades. Evidence from previous studies has demonstrated that attitudes toward math have been known to impact individual performance in mathematics (Howard & Whitaker, 2011; Lobato et al., 2012). Similarly, findings from this study as evidenced by the math attitude inventory revealed that participants’ attitudes toward math paralleled their motivation toward math. This finding was further supported by the semistructured interviews, which demonstrated that students’ attitudes toward math were influenced by the characteristics of the classroom teacher and by the instructional approach used in the classroom to teach math. Student participants were motivated more so by success with problem solving or task completion; positive interactions with teachers; and the ability to hear, see, and participate in the teacher’s presentation of new concepts. The current study found that classroom teachers who were patient, understanding, mild-mannered, approachable, entertaining, and willing to help were preferred. Students were able to easily share their experiences about teachers who were most effective in helping them to achieve some academic success in math. In contrast and consistent with previous research, students expressed a strong dislike for teachers who had a negative attitude during class, or were unapproachable, ill-mannered, and impatient with students (Lucas & Fugitt, 2009; Howard & Whitaker, 2011). Furthermore, students shared that teachers who were not willing to help them were a deterrent for personal improvement in math class. To further illustrate, students shared that at times when they would like to ask a question for clarification, they were usually deterred by math teachers who appeared bothered or annoyed.
Another finding suggests that participants may not prefer to do math because they demonstrate a difficulty with reading the instructions or interpreting the language of math that explains how to complete problems. When there was a need to translate specific math terms in problems, students failed to demonstrate an understanding of the mathematical task. This disconnect between the written language and mathematics was noted for the participants in this study as well as for many students with LD and students at risk for mathematics difficulties (Bottge, Rueda, LaRoque et al., 2007; Lobato et al., 2012). Similarly, in Lobato et al.’s study (2012), students were asked to demonstrate their understanding of the word “acceleration” in term of its relationship to speed. Although students knew the meaning of the terms “acceleration” and “speed,” they were unable or uncomfortable with the transfer of their working language to a mathematical exercise. Likewise, students in the current study demonstrated difficulty understanding the meaning of mathematical terms embedded in certain problems (e.g., greater than, less than, and scientific notation).

Along these same lines, the type of instructional approaches may influence students’ attitudes toward mathematics. The current study’s findings showed that student perceptions and attitudes are shaped by the type of instruction they perceive to be most effective or ineffective to use during math. As previously mentioned, all students reported that they preferred instruction that involved the teacher at the smart board explaining the problem, followed by the teacher providing students with independent work, and then the teacher reviewing the instructions or helping students individually. Research supports what was evident in this study: Students enjoy math when the teacher clearly explains
and reviews problems until students can work independently (Jitendra et al., 2009; Witzel et al., 2003). Students in this study also explained that they enjoyed the visual representation on the smart board and completing the same task at the same time as the teacher. Findings of this study indicate that the participants were being exposed to one type of teaching style on a regular basis. The instructional approach students described and preferred teachers to use was explicit instruction. Explicit instructional methods such as direct instruction are deemed effective practices for teaching math (Maccini et al., 2007; Woodward & Brown, 2006).

Although students preferred a structured format, participants in this study appeared to lack exposure to other types of instructional practice. This may have prevented them from accessing the curriculum more effectively. For example, student participants did not exhibit or verbally describe strategies or instructional approaches that have a research base, including concrete-representational-abstract (CRA), peer-assisted learning strategies (PALS), or the enhanced anchored instruction (EAI) strategies. As Ginsburg (1990) states, not all students are alike, and despite the fact that standardized assessments test all students the same, students are presented with different experiences and may interpret or make meaning of the same information differently, based on their interpretation of shared experiences.

**Research Question Two: Concepts and Symbols**

Research Question Two was: How well does a group of secondary students with learning disabilities and students at risk for mathematics difficulties understand important concepts and symbols of algebra, and how they are used? Overall, student participants
revealed a tendency to be confused or unaware of the symbols used in math, particularly, for algebraic concepts. This was evident in the math diagnostic test and also during the clinical interview. Students were mostly confused by the meaning of variables and symbols such as the “greater than (>)” and the “less than (<)” signs. Likewise, the concepts that students had the most difficulty with were from the number and number operations and algebra strands. At times, students were able to solve and obtain a correct answer for an algebraic problem but unable to explain the solution. Additionally, students expressed discomfort with abstract problems involving algebraic concepts, referring to them as being challenging, hard, and frustrating. This relates to Ginsburg’s (1990) and Sumpter’s (2013) findings that sometimes providing students with multiple practices may lead to pointless endeavors. The math diagnostic test revealed that participants experienced difficulties with variables, variables with exponents, scientific notation, and problems involving algebraic expressions. As Witzel (2005) stated, students with learning disabilities and students at risk for mathematics difficulties often demonstrate difficulty understanding abstract concepts. In a similar manner, during the clinical interviews, students verbally expressed confusion for all problems that consisted of combining like terms, especially those with exponents (e.g., $XY^2$). Additionally, if students were able to solve these types of problems, they experienced a difficult time explaining the meaning or reason for their answers.

**Research Question Three: Challenges**

Research Question Three was: What does a group of secondary students with learning disabilities and students at risk for mathematics difficulties find to be most
challenging about mathematics? In previous research, Witzel (2005) reported that students similar to this study’s target population found abstract problems more challenging to make sense of or to conceptualize. Ginsburg (1990) stated that when students fail to make the connection between written symbols and manipulatives or concrete examples, math has very little meaning to children or adolescents. Therefore, students will have difficulty making a connection with math to their outside experiences, especially if they are not taught the connection. During the think-out-loud sessions in this study, not once were students able to explain the meaning of the variable in terms of connecting a concept to their personal life experience.

Another concept that appeared to be challenging to students was their understanding of fractions. In a sense, fractions appeared to be of abstract meaning to students. Therefore, students had difficulty understanding, using, and interpreting fractions. This was also a noted concern of students in previous research (Jitendra et al., 2009; Naglieri & Johnson, 2000; Test & Ellis, 2005). In addition, the math diagnostic test also revealed that students were missing the underlying concept of what a fraction was and how it functioned. For instance, students were presented with two types of fraction problems: adding fractions with different denominators and multiplying fractions with different denominators. The math diagnostic results showed that none of the students were able to obtain the correct answers to these questions and, even more noticeable, none of the students were able to choose a correct strategy to begin the problem. These observations reinforce past research that argues for students to gradually move from
concrete thinking in order to develop a good understanding of abstract thinking (Butler et al., 2003; Jitendra et al., 2009).

In addition to mathematical challenges faced by the students in this study, the researcher did note student recollection of strategies. For instance, students recalled the mnemonic (Test & Ellis, 2005) strategy taught to solve problems consisting of multiple operations (e.g. PEMDAS). This instructional approach has been noted to aid memory of multiple steps to problem solve (Maccini et al., 2007). However, the current study illustrates that although students recollected the correct strategy (Ginsburg, 1981), they were unable to complete the problem and/or apply the strategy effectively.

Alternatively, in addition to the instructional approaches that students in the current study demonstrated being exposed to, it was evident that students were not exposed to a variety of instructional approaches highlighted as effective or that were evidence-based practices. For example, students referenced a type of instructional approach that included the use of a mnemonic strategy (e.g., PEMDAS) but did not make pictorial representations or schematic representations of numbers and/or operational procedures. In addition, students made reference to teachers using explicit instruction in their descriptions, which involves teaching math using procedural steps for students to work through in order to facilitate a desired outcome. Although an abundance of math interventions are identified as effective for students with mathematics difficulties, participants in this study did not reference any interventions involving instructional approaches that were not teacher directed (i.e., peer assisted learning and enhanced anchored instruction).
Implications for Practice

The changes to the NCTM mathematics requirements for students with learning disabilities and those at risk for mathematics difficulties requires that teacher planning and delivery of instruction for students with learning disabilities be designed so that the students are better able to meet these new requirements and progress academically through the general education curriculum. While research continues to overlook the significance of the perceptions and mathematical thinking of students with learning disabilities (Ginsburg, 1997a, 1997b; Howard & Whitaker, 2011; Sumpter, 2013), students with learning disabilities will continue to fall behind their peers on mathematics assessments. Further research inclusive of more students with LD and those at risk for mathematics difficulties is needed to help understand their mathematical cognitive processes. This study demonstrated that an understanding of a student’s cognitive processing can effectively inform teacher planning and instruction and even teacher preparation programs.

First, in practice, educators should take an opportunity to understand that their actions influence how students perceive math. Teacher characteristics play an important role in how students with mathematics difficulties work through and seek to further understand mathematical tasks. In this study, participants were able to point out events that negatively or positively affected their performance in math. Students also revealed that their desire to learn is influenced by a teacher’s attitude. Students can be impacted either negatively, which can involve students shutting down in class, or positively, which
can encourage students to persist through frustrations and move toward a better understanding of the math problems.

Similarly, educators should also seek ways to build a rapport with students. Many studies (Howard & Whitaker, 2011; Jenkins, 2010; Lucas & Fugitt, 2009; Sumpter, 2013) revealed that students are more likely to work through their frustrations in a classroom that is built on support and trust. Educators should seek opportunities to get to know students’ likes and dislikes, better understand their weaknesses and strengths, and also work to establish an effective method of communicating with students at all times. This also implies providing more than one opportunity to talk to students about their feelings toward success, their math performance, and their progress throughout the school year.

Knowing how students think about and interpret math information is an important component of learning and is a common trait among effective classroom teachers. Knowledge of students’ perceptions about math can steer teacher planning, instructional practices, and the selection of classroom accommodations for students with LD. This knowledge can not only drive the practices (Jenkins, 2010) of classroom teachers but it can also aid policy makers’ decisions about how to better structure the evaluation of students with LD. The NCTM standards require these students to be skillful with abstract concepts when they have yet to demonstrate a clear understanding of concrete mathematical concepts. Student feelings and attitudes toward math can also be used to better design the curriculum to help meet the needs of a diverse student population.

Furthermore, knowing students’ preferences for the various interventions that are commonly used can inform teachers’ planning of instruction (i.e., mnemonics, explicit
This study’s findings revealed that many students were being exposed to the same type of intervention, but despite their effectiveness (Maccini et al., 2007), other interventions or instructional approaches should also be introduced and practiced to engage students on a regular basis.

Second, findings of this study have strong implications for teacher education and preparation programs. Teacher preparation programs need to prepare educators to teach and employ techniques (e.g., verbal praise, immediate feedback) deemed beneficial for students with math difficulties. In addition, teacher disposition matters. Patience, helpfulness, and encouragement support students who struggle to grasp and fully understand mathematical concepts. Researchers (Fennema et al., 1996; Howard & Whitaker, 2011; Lucas & Fugitt, 2009; Sumpter, 2013) have been able to demonstrate that a better understanding of student perceptions of math can influence teachers’ instructional practices which may in turn improve student performance. Teacher preparation programs train preservice teachers to become better educators and as a result, these teachers are exposed to a tremendous amount of information regarding best classroom practices. However, institutions responsible for educating educators should also work toward reinforcing that not all students are alike (Ginsburg, 1997a). Students may perceive their math experiences and instructions differently, and therefore interpret and process information differently. Thus, understanding this concept can help preservice teachers to consider the varied needs of students and in so doing, better guide instruction and individualize their instruction. Furthermore, practical application of this study may also be useful for parents of students who continue to demonstrate low achievements in
mathematics. This study can provide parents with insight into some of the factors that might influence their child’s performance in mathematics. As a result, parents may be able to better advocate for their children in schools.

**Limitations**

The current study did have limitations. First, this study’s delimitations impose questionable restrictions on its generalizability or transferability. This study was limited to a particular number of participants from a specific school district. Therefore, the number of participants and area of representation was partial to only eight participants located in a single state and school district. Although this commonality aided in defining the specific unit of analysis of the case to be studied (Merriam, 1998), it also limited the range of participants to those who share a common curriculum, demographics, and instructional situations. Although this study provided a thick description (Cho & Trent, 2006) of the participants and context in which the study was conducted, this study was locally bound. Therefore, similar to Howard and Whitaker’s (2011) study, the results are limited to the particular population of students who had difficulty in mathematics and collectively had similar feelings and similar experiences. Consequently, as stated by Cho and Trent (2006), qualitative studies do not assert transferability of findings but provide an opportunity for research findings to be understood more generally rather than being textually bounded.

Second, participants’ current account of experiences and attitudes toward math might have been tainted by the brief length of this study. Researchers state that prolonged involvement (Merriam, 1998) provides the researcher with an opportunity to report an
in-depth description of the study and its participants. Furthermore, this prolonged engagement allows the researcher to better account for the various qualitative research assumptions that state that reality consists of different dimensions which are emergent in nature (Merriam, 1998). This particular study was conducted in two sessions lasting an average of two hours per participant. Notwithstanding prolonged engagement, participants’ perceptions of mathematics have been proven to change over time based on their level of successful and unsuccessful experiences. As a result, a prolonged study that assessed students’ mathematical knowledge over a period of a year or more, and/or evaluated students’ perception of math through semistructured interviews or through the use of journaling, could provide tremendous knowledge as to how students’ specific experiences tend to affect perceptions or attitudes toward mathematics over time.

Third, the data sources in this study were limited to a demographic questionnaire, a diagnostic test, a clinical interview, and a semistructured interview. The clinical interviews, semistructured interviews, and the semistructured interviews were recorded using a tape recorder and field notes were written by the researcher. In the existing qualitative research literature related to this line of study, most researchers not only conducted more than one interview, recorded classroom observations (Howard & Whitaker, 2011; Lewis, 2010), and repeated clinical interview sessions (Fennema et al., 1993; Fennema et al., 1996), but in some cases the clinical interviews were videotaped for each student (Lewis, 2010). Therefore, because the current study lacked multiple interviews, videotaped recordings, and observations of participants, students may not have had opportunity to fully expose their tone, feelings, behaviors, or mathematical
performance. By including multiple interviews, students would have had another opportunity corroborate or clarify previously started perceptions. Also, by recording each clinical interview or including classroom observations and descriptions, the analysis may have led to new findings or served to verify the existing research findings.

Fourth, students in this study were not able to begin solving some problems and, as a result, did not provide written work for a selected answer. As a result, the researcher was not able to analyze student work samples for every problem. As previously stated, although NCTM (2009b) standards require focusing on students’ conceptual understanding, this study demonstrated that many secondary-level students with LD continue to demonstrate limited skills beyond basic-level math, including skills of Prealgebra, Algebra, Algebra II, Statistics, or Trigonometry courses. Students also continue to demonstrate an inability to explain, connect, reason, and represent higher level mathematics problems.

Fifth, this study was limited to a number of participants who were enrolled in a secondary school and had completed Prealgebra and Algebra courses. In order to obtain a better understanding of what students know algebraically and beyond, future research should focus on recruiting only participants who are enrolled in classes beyond Algebra I such at Geometry, Algebra II, Statistics, and possibly Trigonometry courses.

Sixth, as revealed in Jenkins’ (2010) study, the reluctance or inability of participants in this study to think out loud while working on mathematical tasks hindered what could be deduced about the cognitive processing of secondary students with learning disabilities and students at risk for mathematics difficulties. Throughout the
clinical interviews, it appeared that participants were challenged to think out loud about their problem solving skills for a few reasons. Students had difficulty (a) identifying problem type, (b) recalling terms, (c) applying a correct strategy, (d) computing accurately (e.g., negative signs), and (e) using symbolism and understanding abstract concepts. Since difficulty recognizing problems by name and using mathematical language might have prevented students from fully expressing how they solved a problem, future qualitative studies should explore how teachers can further develop student knowledge of mathematical language through classroom understanding.

**Implications for Future Research**

Future research should incorporate a wider range of opportunities for students to be better able to demonstrate an understanding of higher level math. Future studies may include an increasing number of participants and should be replicated in different school districts and/or different states. As stated by Stake (1995), a single case study can add to the body of literature, thus increasing the generalizability of research findings. However, as previously mentioned, not many studies consider the perceptions of students with LD and/or students at risk for mathematics difficulties; therefore, the use of a multicase study approach (Stake, 1995) or a multisite design (Merriam, 1998) could be employed by future researchers to better understand the phenomenon across different cases (e.g., different age groups, gender, and geographic locations). This would allow individuals to better apply the situation to a greater range of other circumstances (Merriam, 1998). Specifically, future studies may include the following:
1. Examine the effects of using student perceptions and attitudes to guide teachers’ instructional practices, individualize instruction, and implement interventions.

2. Expand the current body of qualitative research regarding students’ perceptions and mathematical thinking toward math across various locations and populations of students.

3. Evaluate the effect of teachers’ characteristics on students’ performance and attitude toward math.

4. Examine the influence of student perceptions about mathematics on student math performance.

5. Understand how students’ ability to learn new topics is influenced by their prior knowledge, preconceptions, and attitudes toward math.

6. Seek to discover and identify the factors that effectively motivate students who are struggling in math.

7. Explore how understanding students’ perceptions about mathematics and mathematical thinking can foster mathematical growth.

8. Evaluate how think-out-loud sessions can lead to a greater understanding of how students with LD and students at risk for mathematics difficulties approach and solve math problems.

9. Seek to better understand the thought processes and mathematical thinking of a large group of students with LD in mathematics.
10. Examine how students’ perceptions about math and mathematical thinking differ by varying demographic characteristics (e.g., age, gender, and ability level).

**Conclusion**

Students with LD and students at risk for mathematics difficulties are required to demonstrate proficiency with various mathematics concepts throughout their schooling. Secondary students with LD and students at risk for mathematics difficulties can experience difficulty in understanding basic-level abstract math concepts with even further difficulty in understanding higher level abstract math concepts. However, despite this awareness, these students are required to meet rigorous standards by passing Algebra I, Geometry, and Algebra II as they progress through high school. While effective math interventions are available to assist students’ understanding of math concepts, many teachers choose to not introduce these interventions to their students. Additionally, the use of interventions is simply not enough. The current study reveals that asking students to share their perceptions and thought process regarding math can be very informative.

The findings of this study provide an in-depth description of a group of students’ perceptions of mathematics and provide insight into better understanding students’ mathematical thinking (Clarke, 2001). The study revealed that participants associated their feelings and attitude toward mathematics with their performance in math. Attitudes and perceptions were also influenced by their classroom teachers and the instructional approach employed by the classroom teacher. The think-out-loud sessions revealed that students appeared to have more success when there was a specific procedure involved—a
step-by-step way to solve a problem (Maccini et al., 2007). However, as expected, students demonstrated difficulty explaining their thinking (Ginsburg, 1997b). These findings revealed that communication may be one of the greatest challenges that students with learning disabilities and students at risk for mathematics difficulty are faced with when presented with mathematical tasks.

Further research is crucial to improving the mathematical performances of low-achieving students. Consequently, this research noted that future studies should focus on how to improve the cognitive processing of secondary students and how to develop the mathematical language of secondary students. This study not only adds to the current literature on students with learning disabilities and students at risk for mathematics difficulties, it also highlights the challenging areas of math upon which students need an opportunity to improve. The study also highlights areas of improvement for teachers such as the need to develop a connection with students, understand their individual strengths and weakness, and provide multiple opportunities for students to talk about math. For educators, knowing students’ perceptions of math is relevant in order to guide instruction, plan effective instruction, and create a more conducive environment for students to be more successful. Teacher awareness of students’ attitude and perceptions of math can also aid teachers to better individualize instruction for students who are struggling with math. Findings of this study encourage teachers to understand that when students feel comfortable about mathematics, students are then more motivated to learn.

This mixed methods approach demonstrated the importance of reaching beyond the use of mathematical interventions. Findings showed that it is important for educators
to look at how students take in and interpret mathematical information and to consider how various factors can affect or alter students’ willingness to learn or care about mathematical concepts. More specifically, with the challenges that students with LD and teachers of special education students face, it is crucial that special educators better understand their students to better inform their instruction.
## APPENDIX A

### SUMMARY OF MATH INTERVENTION FOR SECONDARY STUDENTS WITH LD AND AT RISK FOR MATHEMATICS DIFFICULTY

Table A1

<table>
<thead>
<tr>
<th>Author (year)</th>
<th>Sample and Setting</th>
<th>Instructional Content/ Instructional Focus</th>
<th>Research Design/ Intervention</th>
<th>Dependent Variable</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axtell, McCallum, Bell, &amp; Poncy, 2009</td>
<td>$N = 36$</td>
<td>Computation and operations</td>
<td>Pretest/Posttest</td>
<td>Criterion Reference</td>
<td>(a) Cognitive strategies planning vs. Control (d = 1.11)</td>
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<tr>
<td></td>
<td>LD = 11</td>
<td>Mathematics Fluency and Computation</td>
<td>Random assignment</td>
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<tr>
<td></td>
<td>Grade: 8</td>
<td></td>
<td>Explicit Instruction</td>
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<td></td>
<td>Ages: 13-15</td>
<td></td>
<td>Traditional/Direct Instruction</td>
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<tr>
<td></td>
<td>Mean age: 13.8</td>
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<td></td>
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<tr>
<td>Bottge, Heinrichs, Chan, &amp; Serlin, 2001</td>
<td>$N = 75$</td>
<td>Linear function, line of best fit, variables, rate of change (slope) Conceptual Focus</td>
<td>Pretest/posttest group design (a) video-based instruction (b) text-based instruction</td>
<td>Researcher developed problem solving test; WRAT-III arithmetic subtest (a) = (b) on problem solving measure, greater gains for students in remedial class; (a) = (b) on computation measure for students in prealgebra classes; Decline in computation scores for students in remedial class receiving (b).</td>
<td></td>
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<tr>
<td>Author (year)</td>
<td>Sample and Setting</td>
<td>Instructional Content/ Instructional Focus</td>
<td>Research Design/ Intervention</td>
<td>Dependent Variable</td>
<td>Results</td>
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<tr>
<td>Bottge, Heinrichs, Mehta, &amp; Rueda, 2004</td>
<td>N = 93, LD = 13, Other disabilities: 30, Grade Mean: 6, Mean Age: 11.5</td>
<td>Fractions Conceptual Knowledge Enhanced Anchored Instruction (EAI) Math problem solving</td>
<td>Quasi-experimental Random assignment</td>
<td>Criterion Reference</td>
<td>Math computation EAI vs. Typical (d = .05), Math Problem Solving EAI vs. Typical (d = .10), Mathematics Comprehension (d = .21)</td>
</tr>
<tr>
<td>Bottge, Rueda, Grant, Stephens, LaRoque, 2010</td>
<td>N = 54, LD = 12, Other disability: 13, Mean Grade: 6.78, Mean Age: 12</td>
<td>Linear Function Computation and Operations</td>
<td>Quasi-experiment, random assignment</td>
<td>Norm and Criterion</td>
<td>(a) Math computation (d = 1.46), (b) Mathematics Problem Solving (d = .02)</td>
</tr>
<tr>
<td>Bottge, Rueda, LaRoque, Serlin, &amp; Kwon, 2007</td>
<td>N = 100, LD = 100, Grades: 6 - 12, Ages: NS, Special ed. Classroom</td>
<td>Linear function, line of best fit, variables, rate of change (slope) Conceptual Focus</td>
<td>Pretest/posttest group design</td>
<td>Researcher developed problem solving test; ITBS computation solving measure; effect size was large</td>
<td>(a) &gt; (b) on problem solving test; ITBS computation solving measure; effect size was large (d = 1.08)</td>
</tr>
<tr>
<td>Author (year)</td>
<td>Sample and Setting</td>
<td>Instructional Content/ Instructional Focus</td>
<td>Research Design/ Intervention</td>
<td>Dependent Variable</td>
<td>Results</td>
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</tbody>
</table>
| Bottge, Rueda, Serlin, Hung, & Kwon, 2007 | $N = 128$  
LD = 12  
Grade: 7  
Ages: NS  
1 inclusive classroom, 1 prealgebra classroom, 4 typical classrooms | Linear function, line of best fit, variables, rate of change (slope)  
Conceptual Focus | Quasi-experimental group design  
(a) video-based instruction | Researcher developed problem-solving test | All students demonstrated improvements; students with LD demonstrated larger improvements on algebraic tasks. No difference between students with LD and without LD on maintenance. |
| Butler, Miller, Crehan, Babbitt, Pierce, 2003 | $N = 42$  
LD = 42  
Mean Grades: 6-8  
Classroom Mean Age: 12.7 | Fractions  
Prealgebra | Quasi-experimental design  
Nonrandomized control-group design  
Pre-/Posttest | Scores on subtests of BCIBS-R | Overall, higher mean scores for CRA group compared to the RA |
| Calhoon & Fuchs, 2003 | $N = 92$  
LD = 68  
Mean Grade: 10.30  
(9-12)  
Mean Age: 13  
Resource Room | Math Computation  
Math Application  
Math Comprehension | Pre-/Posttest Random assignment  
Peer Assisted Learning (PAL) + Curriculum –based measures (CBM) | Math Operation Test  
Math Concepts and Application Test (Revised)  
Tennessee Comprehensive Test (statewide assessment) | (a) = Math computation PALS + CBM vs. Control  
(d = .40)  
(b) Math Application (d = -.01)  
(c) Mathematics Comprehension (d = -.29) |
| Fuchs, Bahr, & Reith, 2001 | $N = 20$  
LD = 20  
Mean Grade: 8  
Mean Age: 16.20 | Computation and ratios  
Cognitive strategy | Quasi-experimental.  
Random assignment | Criterion Reference | Assigned Goals/ Noncontingent vs. Self Goals (d = -.88)  
Assigned Goal vs. Self Goals (d = .46)  
Assigned Goals/contingent vs. Self Goal/contingent (d = -.62) |
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<tr>
<th>Author (year)</th>
<th>Sample and Setting</th>
<th>Instructional Content/ Instructional Focus</th>
<th>Research Design/ Intervention</th>
<th>Dependent Variable</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ives, 2007</td>
<td>N = 40</td>
<td>Linear systems of equations in two variables (Study 1) and three variables (Study 2)</td>
<td>Two group comparison</td>
<td>Researcher developed test, teacher-generated test</td>
<td>(a) &gt; (b) in Study 1 and Study 2 posttests (a) &gt; (b) in Study 1 maintenance test; large effect sizes for both studies</td>
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<tr>
<td></td>
<td>LD and/or ADHD = 40</td>
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<td>Grades: 7-12</td>
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<td>Ages: 13-19</td>
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<td></td>
<td>Special ed. Classroom</td>
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<tr>
<td>Jitendra, Dipipi, &amp; Perron-Jones, 2002</td>
<td>N = 4</td>
<td>Ratio and Proportions, Problem solving</td>
<td>Single subject</td>
<td>Criterion Reference</td>
<td>Schema-Based Instruction (SBI)</td>
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<tr>
<td></td>
<td>LD = 4</td>
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<td>Grade: 8</td>
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<td>Age: 13.75</td>
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<tr>
<td>Jitendra et al., 2009</td>
<td>N = 148</td>
<td>Ratio and Proportions Problem Solving</td>
<td>Quasi-experimental Schema-based Instruction (SBI)</td>
<td>Norm Reference Test</td>
<td>SBI vs. Control (d = .55)</td>
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<tr>
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<td>LD = 14</td>
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<td>Grade: 7</td>
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<td></td>
<td>Age: 12.76</td>
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<tr>
<td>Maccini &amp; Hughes, 2000</td>
<td>N = 6</td>
<td>Word problems involving addition, subtraction, multiplication, and division of integers Conceptual + Procedural Focus</td>
<td>Multiple-probe single subject design across participants instructional package: strategy instruction + CSA instructional sequence</td>
<td>Researcher develop problem representation and problem solution measures</td>
<td>Significant improvements in problem across participants and problem types; Large effect sizes (PND = 90% for representation; PND = 72% for solution)</td>
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<tr>
<td></td>
<td>LD = 6</td>
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<td>Grades: 9-12</td>
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<td></td>
<td>Age: 14-18</td>
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<td>Individualized instruction</td>
<td></td>
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<tr>
<td>Maccini &amp; Ruhl, 2000</td>
<td>N = 3</td>
<td>Word problems involving subtraction of integers Conceptual + Procedural Focus</td>
<td>Multiple-probe single subject design across participants instructional package: strategy instruction + CSA instructional sequence</td>
<td>Researcher developed problem representation and problem solution measures</td>
<td>Significant improvements in problem representation and problem solution across participants; Large effect size (PND = 67% for representation; PND = 94% for solution)</td>
</tr>
<tr>
<td></td>
<td>LD = 3</td>
<td></td>
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<tr>
<td></td>
<td>Grade: 8</td>
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<tr>
<td></td>
<td>Ages: 14-15</td>
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<tr>
<td></td>
<td>Individualized instruction</td>
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<tr>
<td>Author (year)</td>
<td>Sample and Setting</td>
<td>Instructional Content/ Instructional Focus</td>
<td>Research Design/ Intervention</td>
<td>Dependent Variable</td>
<td>Results</td>
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<tr>
<td>Mayfield &amp; Glenn, 2008</td>
<td>N = 3</td>
<td>Problem solving involving target skills of variables, exponents, and linear equations</td>
<td>Single subject design across skills replicated across three participants (a) cumulative practice (b) tiered feedback (c) feedback + solution sequence (d) review (e) transfer training</td>
<td>Researcher developed target skills tests, problem solving test</td>
<td>Limited increases in accuracy after (a) and (c). Consistent improvements in all participants after (e) with large effect size (PND = 96%)</td>
</tr>
<tr>
<td>Naglieri &amp; Johnson, 2000</td>
<td>N = 19</td>
<td>Fractions</td>
<td>Experimental Cognitive Strategy</td>
<td>Cognitive Assessment System (CAS) 10-minute phases Math worksheets</td>
<td>Students with low planning score on CAS demonstrated a greater change from baseline to intervention</td>
</tr>
<tr>
<td>Scheuermann, Deshler, &amp; Schumaker, 2009</td>
<td>N = 14</td>
<td>One-variable equations</td>
<td>Single subject, Multiple probe across students Explicit Inquiry Routine = content sequencing, scaffold inquiry, modes of representation</td>
<td>Researcher-developed word problem test and concrete manipulation test; Key Math revised</td>
<td>Mean of 95% accuracy on final instructional word problem probe; large effect size (PND = 93%); Mean of 88.6% accuracy on manipulation Significant improvement on KeyMath (effect size .54)</td>
</tr>
<tr>
<td>Test &amp; Ellis, 2005</td>
<td>N = 6</td>
<td>Fraction</td>
<td>Single subject</td>
<td>Norm Reference Test</td>
<td>All participants maintained gain over 6 weeks. 100% of the students also agreed that the strategy used was easy to learn.</td>
</tr>
<tr>
<td>Author (year)</td>
<td>Sample and Setting</td>
<td>Instructional Content/ Instructional Focus</td>
<td>Research Design/ Intervention</td>
<td>Dependent Variable</td>
<td>Results</td>
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</tr>
</tbody>
</table>
| Witzel, Mercer, & Miller, 2003 | $N = 358$  
LD = 34  
Grades: 6-7  
Ages: 12.3  
Inclusive classroom | Algebra  
Linear equations  
Conceptual + Procedural Focus | Pretest/posttest/ follow-up design with random assignment of clusters  
(a) CRA instruction  
(b) abstract instruction | Researcher-developed assessment | (a) > (b); Effect size was large (0.97) |
| Witzel, 2005 | $N = 231$  
LD = 49  
Grade: 6,7  
Age: 12.16 | Algebra, solving one and two equation  
Mathematics fluency/ computation | Pretest/posttest design  
(follow-up test)  
Cognitive strategy  
(Behavioral Instructional Approach/Goal structured)  
(a) CRA | Norm References | Treatment group > control on post test |
| Woodward & Brown, 2006 | $N = 53$  
LD = 35  
Total Disability = 39  
Grade: 6  
Age: 11 | Computations and Operations | Quasi-experimental, random assignment, pretest-posttest design  
Cognitive Strategy planning vs. Control | Norm Reference Test  
CRA vs. RA  
(d = 1.14) | Cognitive Strategy planning vs. Control |
APPENDIX B

SUMMARY OF PERCEPTIONS AND THOUGHT PROCESSES OF STUDENTS ABOUT MATHEMATICS

Table B1

Review of Perceptions and Thought Processes of Students About Mathematics

<table>
<thead>
<tr>
<th>Author (Date)</th>
<th>Purpose/Research Questions</th>
<th>Research Design</th>
<th>Participants</th>
<th>Data Sources</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clarke, 2001</td>
<td>Seek to identify processes for supporting and enhancing mathematics learning in the early years of school</td>
<td>Qualitative descriptive study</td>
<td>40 students Elementary School Students</td>
<td>Interviews</td>
<td>Students were able to better explain their reasoning for solving certain problems in a particular way. Teachers reported being better able to teach students.</td>
</tr>
<tr>
<td>Fennema, Carpenter, Franke, Levi, Jacobs, &amp; Empson, 1996</td>
<td>To better understand children’s mathematical thinking and possible benefits to teachers</td>
<td>Mixed Methods Longitudinal study (4 years) (a) Year 0 baseline data about teachers Years 1-3 (CGI) (b) Teacher observation, instruction, interviews, and paper pencil instruments</td>
<td>21 teachers</td>
<td>(a) Cognitive guided instructional levels of CGI (b) Interviews</td>
<td>(a) + (b) Findings suggests that the developing an understanding of children’s mathematical thinking can be a productive basis for helping teachers to make the fundamental changes (a) + (b) Change in a teacher’s level of CGI instruction resulted in the achievement of students</td>
</tr>
<tr>
<td>Author (Date)</td>
<td>Purpose/Research Questions</td>
<td>Research Design</td>
<td>Participants</td>
<td>Data Sources</td>
<td>Results</td>
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<tr>
<td>Fennema, Franke, Carpenter, &amp; Carey, 1993</td>
<td>How knowledge of children’s thinking in mathematics derived by using a cognitive science research paradigm</td>
<td>Mixed Methods</td>
<td>1 classroom teacher with classroom over 4 years</td>
<td>(a) Experimental Cognitively Guided Instruction (CGI)</td>
<td>(a) + (b) Knowledge of student thinking helps to make effective instructional decisions</td>
</tr>
<tr>
<td></td>
<td>To understand teachers’ use of children’s thinking to make instructional decisions.</td>
<td>(a) Experimental</td>
<td>(b) Case study Years 3 &amp; 4</td>
<td>(b) Semistructured interview</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>Years 1 &amp; 2</td>
<td>(c) 9 students (1 LD)</td>
<td>Likert-type instruction</td>
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<tr>
<td></td>
<td></td>
<td>(b)</td>
<td>(c) Standard interview (word problem involving adding and subtracting)</td>
<td>(c) + (b)</td>
<td></td>
</tr>
<tr>
<td>Howard &amp; Whitaker, 2011</td>
<td>What common phenomena accompany students’ shift from unsuccessful to successful math experiences?</td>
<td>Qualitative Phenomenological study</td>
<td>14 students College remedial Prealgebra and Algebra courses</td>
<td>(a) Interviews of the student participants</td>
<td>Turning point—students experience a negative turning point based on difficulty and bad grades.</td>
</tr>
<tr>
<td></td>
<td>Examines the perspectives and experiences of newly successful developmental mathematics students.</td>
<td>(b)</td>
<td>(b) Classroom observations of students participants</td>
<td>(b) Classroom observations of students participants</td>
<td>Motivation exists when students did well in math and had successful experiences and vice versa. Ex. Understanding improves motivation.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(c)</td>
<td>(c) Reflective journal kept by the researcher</td>
<td>(d) Students’ assessment scores for exams, quizzes, and homework.</td>
<td>The development and use of strategies improved students’ success.</td>
</tr>
<tr>
<td>Author (Date)</td>
<td>Purpose/Research Questions</td>
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<tr>
<td>Jenkins, 2010</td>
<td>To advance teachers understanding of students as learners of mathematics.</td>
<td>Qualitative case study design</td>
<td>2 prospective teachers + 3 middle school students Grades 6-8</td>
<td>(a) Structured Interviews (b) Clinical Interviews</td>
<td>(a) Evidence of evaluative listening Evidence of interpretive listening (b) Superficial understanding of perimeter and area that, for the most part, is limited to direct application of the corresponding computational algorithms Insufficient communication skills for explaining their thinking clearly Difficulty applying perimeter and area of attempting to connect the concepts to real life situations</td>
</tr>
<tr>
<td>Author (Date)</td>
<td>Purpose/Research Questions</td>
<td>Research Design</td>
<td>Participants</td>
<td>Data Sources</td>
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<tr>
<td>Lewis, 2010</td>
<td>(a) Does the student error patterns match that documented for students with MLD in Mazzocco et al. (2008)?</td>
<td>Longitudinal qualitative case study</td>
<td>1 Student age: 13-18 years 8th grade through 12th grade</td>
<td>Achievement test scores, classroom observation, interviews, weekly videotaped tutoring</td>
<td>(a) Demonstrated the same type of errors (b) Relied on a strategy (c) Once the student’s problems have been identified and understood, it is possible to help student more effectively with a typical strategy</td>
</tr>
<tr>
<td></td>
<td>(b) How does the student solve multiplication problems, and what does this reveal about her patterns of errors?</td>
<td></td>
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<tr>
<td></td>
<td>(c) In what ways can remediation address these kinds of math fact errors?</td>
<td></td>
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<tr>
<td>Lobato, Hohensee, Rhodehamel, &amp; Diamond, 2012</td>
<td>(a) To improve and understand students conceptual learning of quadratic functions</td>
<td>Mixed methods Algebra (experimental + qualitative interviews)</td>
<td>24 eighth-grade students</td>
<td>(a) Instruction on Quadratic functions (15 hours) (b) Clinical interviews (75 minutes)</td>
<td>(a) Student demonstrated an understanding of 5 concepts involving quadratic functions. Based on identified perceptions, conceptual learning goal was achieved for most students. (b) Psychological analysis Identified mathematical ideas</td>
</tr>
<tr>
<td>Author (Date)</td>
<td>Purpose/Research Questions</td>
<td>Research Design</td>
<td>Participants</td>
<td>Data Sources</td>
<td>Results</td>
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<tr>
<td>Lucas &amp; Fugitt, 2009</td>
<td>(a) To understand the perceptions of people concerning math and math education</td>
<td>Folknography</td>
<td>148 total students (youth), teachers, and parents</td>
<td>Interviews</td>
<td>(a) Math helps students develop good logical thinking</td>
</tr>
<tr>
<td></td>
<td>(b) To understand the perceived quality of math and math education in a specific school district</td>
<td>Qualitative study design</td>
<td></td>
<td>Questionnaires</td>
<td>(b) Youths prefers math teachers who are more positive, pleasant, and kind to teachers who appear to be ill-tempered with a bad attitude in the classroom</td>
</tr>
<tr>
<td></td>
<td>(c) To understand the impact of math and math education on the future success of students after graduation</td>
<td></td>
<td></td>
<td></td>
<td>(c) Math prepares youth for success in college and a better chance of obtaining a good career</td>
</tr>
<tr>
<td>Author (Date)</td>
<td>Purpose/Research Questions</td>
<td>Research Design</td>
<td>Participants</td>
<td>Data Sources</td>
<td>Results</td>
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<tr>
<td>Sumpter, 2013</td>
<td>Purpose: to analyze secondary students mathematical task solving reasoning with a focus on student arguments for strategy choice and answers</td>
<td>Case study design</td>
<td>40 students</td>
<td>Questionnaires</td>
<td>(a) Expectations of mathematics goals influence students achievement</td>
</tr>
<tr>
<td></td>
<td>Research Question: What types of belief are indicated in students’ central decision in their reasoning while solving problematic situations?</td>
<td></td>
<td>Age 17-18</td>
<td>Semistructured interviews</td>
<td>(b) Motivation-intrinsic such as problems solving methods students like best.</td>
</tr>
<tr>
<td></td>
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<td></td>
<td>11th- and 12th-grade students</td>
<td>Think-out-loud Interviews</td>
<td>(c) Motivation- extrinsic such as proceeding as teacher prefers and obtaining the answer.</td>
</tr>
<tr>
<td></td>
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<td></td>
<td>(d) Safety or security involves the idea that students feel safer with a calculator or using a specific method.</td>
</tr>
</tbody>
</table>
APPENDIX C

INSTITUTIONAL REVIEW BOARD SUBMISSIONS

Institutional Review Board
Application Form

Instructions:
1. CITI certification (www.citiprogram.org) must be completed for all team members at the time of application submission.
2. Complete all sections and required addenda. Submit one complete package with all via IRBNet.
3. Projects with funding/proposed funding must include a copy of the grant application or proposal.
4. Research may not begin until you have received notification of IRB approval.
5. Handwritten and incomplete forms cannot be accepted.

1. Study Title: What do kids think about math problems and problem solving?

2. Study Investigators
   A. Principal Investigator (must be faculty/staff and meet PI Eligibility, University Policy 4012)
      Name: Dr. Margo Mastroperoi
      Department: Graduate School of Education, Special Education
      Mail Stop: 1F2 Phone: 703.993.4136 E-mail: mmastrop@gmu.edu
   B. Co-Investigator/Student Researcher
      Name: Damali Hay
      Department: EDSE, Doctoral Program
      Mail Stop/Address: 6318 Draco Street, Burke, VA 22015 Phone: 917-374-3815
      E-mail: dhay@masonlive.gmu.edu

   C. Are there additional team members? No ☒ Yes ☐ If yes, attach Addendum I to list additional team members

   D. Do any investigators or team members have conflicts of interest related to the research? No ☐ Yes ☒ If yes, explain

3. Study Type: ☐ Faculty/Staff Research ☐ Doctoral Dissertation ☐ Masters Thesis
   ☐ Student Project (Specify ☐ Grad or ☐ Undergrad) ☐ Other (Specify)

4. Complete Description of the Study Procedures
   A. Describe the aims and specific purpose of the study:
      According to the United States Department of Education, students with learning disabilities continues to top the list as being one of the most difficult sub-group to close the achievement gap in the nation. At the same time, the number of students with learning disabilities who are found eligible for services appears to be increasing. Since school systems are required to provide appropriate education for students with LD, the task of better preparing students to pass a state required assessment which moves students closer towards graduation, continues to be a difficult and daunting task.

      The Department of Education (DOE) has increased graduation requirements for all students. At the same time, NCTM changed and increased mathematics requirements. According to the literature, students without LD are behind their international peers, and students with LD continue to fall even further behind. As a result, investigations are needed to better understand the perceptions of students with a learning disability regarding math and how they perceive completing math problems. If there will be a change, an even greater understanding of the issue is needed. Even and Tirosh (2003) state, “all students are not created equally”, meaning students will learn and interpret information differently. Research needs to focus more on students’ current way of thinking. New information of students’ perception of math and how they think through math tasks is needed in order to address
the issue. Thus, there is very little doubt that further research is needed to assess students’ psychological and cognitive perception of mathematics.

The purpose of this study is to understand why students with learning disabilities continue to demonstrate a high difficulty in mathematic reasoning from a qualitative perspective where the voice of the students can be heard and recognized. Information will be acquired through a survey questionnaire, diagnostic test, attitude measure inventory, interviews, and student work samples.

The research questions are:

- What are the feelings and attitudes of students with learning difficulties about mathematics?
- How well do students understand important concepts, symbols of algebra, and how they are used?
- What do students find to be most challenging about mathematics?
- What are students’ perceptions about the differences between effective or ineffective classroom teachers?
- What types of instructions are most effective or ineffective for students with learning difficulties?

B. Provide a COMPLETE description of the study procedures in the sequence they will occur including the amount of time each procedure will take (attach all surveys, questionnaires, standardized assessment tools, interview questions, focus group questions/prompts or other instruments of data collection):

This is a mixed method research design (quantitative and qualitative) research design in which the responses from the participants will be analyzed to describe, understand and determine themes about the phenomenon. Therefore, the study procedures will be chosen to gather appropriate data for the study. The study procedures will following the sequence described below. The data for the case study will be collected through a variety of sources. Data sources included: (a) multiple choice survey questions, (b) student math attitude assessment, (c) a researcher constructed diagnostic test covering pre-algebra and algebra I contents, and (d) semi-structure student interviews (with a component of clinical interview).

This measure will be administered in two sessions.

Session 1:

1. Survey questionnaire (5-10mins). The purpose of the survey questionnaire will be to primarily gather demographic information before the assessments and interviews are conducted. The survey will consist of both closed and open-ended questions directed to students with learning disabilities. This survey will address questions related to student’s age, grade, sex, pervious math grades, expected grade, and a general overview of student’s attitude towards math class, attitude towards math teachers, understanding of math concepts, and attitude towards grades received in mathematic courses. From these questions, I hope to get a feel for possible similarities and themes that may surface during each interview.

2. Student math attitude inventory (15-20mins). After parents and students sign the consent and
assent form to participate in the study, each student will be given a math attitude test to
determine how students feel about math as a subject. This test will be administered orally or on
paper to each student by the researcher. The math attitude measurement will be modified
from the Attitudes towards Mathematics Inventory (ATMI) designed by Tapia, M. and Marsh
(2004). This inventory will be designed to investigate the essential dimensions of student’s
attitudes towards mathematics. The inventory consists of 40-item scale. The ATMI consists of 5-
point Likert-scale questions with 1 (strongly disagree), 2 (disagree), 3 (neutral), 4 (agree), and 5
(strongly agree). This inventory was designed to assess four factors (i.e., anxiety, confidence,
enjoyment, and motivation).

The original scale created by Tapia, M. and Marsh (2004) has a reliability coefficient alpha of .97
in which factor analysis using a Varimax rotation produce a four factors (i.e., self-confidence,
value of mathematics, enjoyment of mathematics, and motivation). The Cronbach alpha which
was calculated showed an internal for each factor ranged from .89 -.95 (Tapia, M. & Marsh,
2004). Tapia and Marsh (2004) factor analysis revealed that factor one was designed to measure
self-confidence with a Cronbach alpha of .95. This factor consisted of 15 items (e.g., “studying
mathematics makes me feel nervous”, “I am able to solve mathematics problems without too
much difficulty”). The second factor was designed to measure the value of math with a Cronbach
alpha of .89. This factor consisted of 10 items (e.g., “Mathematics is one the most important
subject for people to study”, “high school math courses would be very helpful no matter what I
decide to study”). The third item measured enjoyment with a Cronbach alpha of .89. This factor
consisted of 10 items (e.g., “mathematics is dull and boring,” “I am happier in a math class than in
any other class”). The fourth factor was designed to measure motivation with a Cronbach alpha
of .88. This factor consisted of 5 items (e.g., “I am willing to take more than the required amount
of mathematics”, I plan to take as much mathematics as I can during my education”).

3. Student math diagnostic test (20-30 minutes). The diagnostic test will be used to determine
student current level of understanding of various math concepts leading up to taking an algebra
course. Students will also be assessed to determine their current level of math concepts. The
test will be a modified version of the McGraw-Hill/ Glencoe diagnostic test will be used to assess
students’ mathematical skills. Therefore, only a selective number of questions from selected
strands will be used. This test was to designed to assist academic settings accurately place
students in middle school and algebra courses and also to provide diagnostic information about
students mathematical knowledge or mastery of skills. This test can be used to place students
and tests mathematic skills for student taking currently taking or will take a pre-algebra courses
and algebra course.

Session 2:

4. The clinical interview (20-30 minutes) will be used to assess the students’ cognitive abilities
because as students approach each task they will be asked to “talk-out-loud” (Ginsburg), talk
through their steps as they complete mathematical problems.

5. Interviews (5-10 minutes). In-depth, semi structure interview will be selected because it fits the
nature of the research questions and will help reveal how others interpret their actions (Johnson,
2001). In-depth interviews will be use primarily when the researcher “seeks to achieve the same
deep knowledge and understanding” (Johnson, 2001, p.1006) as the participants. As a result, an
Interview protocol will be developed for the interview process. All interviews will be video-taped and recorded for further review. Furthermore, each participant’s time will be taken into consideration and will be a priority. The interviews will consist of open-questions that allow each participant to voice his or her own opinions. Based on the type of questions asked and received, participants will be asked to elaborate, provide more details, or give examples. The semi-structure interviews will also include what Ginsburg (1997) and Lobato et al. (2012) refers to as clinical interview.

C. Describe the target population (age, sex, ethnic background, health status, etc.):

1. Summarize the inclusion/exclusion criteria for participation in the study:

Each participant will be chosen either because he or she is a student in a special education program or student in general education program. In addition, each participant matches at least all of the following criteria: (a) has an individual education plan (IEP), (b) currently a student in a Virginia public schools, (c) has completed one of the following courses—pre-algebra, algebra, geometry, or all three course, and (d) a middle or high school student.

More specifically, Inclusion criteria are students in all of the following categories:

- Each participant will be chosen either because he or she is a student with special educational needs.
- Students will be used for this study because the goal of the study is to understand the psychological and cognitive perspectives of students with learning disabilities about mathematics.
- Each participant will meet all of the following criteria, therefore the student must:
  - Have an Individual education plan (IEP)
  - Currently a student in a Virginia public school
  - Have completed a pre-algebra, algebra, geometry, or all three course
  - Currently a middle or secondary high school student.

2. Are there any enrollment restrictions based on gender, pregnancy, race or ethnic origins?
   - [ ] Yes  [ ] No  If yes, please describe the process and reasons for restriction(s):

3. Do you have a relationship to any participants? [ ] Yes [ ] No  If yes, please describe the relationship and how you will manage any possibility of undue influence:

4. Estimated number of subjects (may use a range):

   Approximately 6-20 individuals will participate in this study. Participant characteristics include students under the 21-years-old who are in sufficiently good health to attend school.

5. Estimated amount of total participation time per subject:

   The total amount of time per participant will be 55 - 90mins. This will take place in two sessions. The first session will be between 30 - 50 minutes and the second session will be between 25-40 minutes.

D. Where will the study occur (list all study sites and collaborators)? The study will take place in the students’ home or in a nearby library.

E. Describe other approvals that have been/will be sought prior to study initiation (facility authorizations, biosafety review, IRB approval from collaborating institutions, etc.). There will be other approvals that will be sought prior to the study initiation. E-mails consent to use the previously listed list serves (e.g., T/TAC, PEATC, GMU Phd, Sped Edu) will be included in this
5. Recruitment and Consent

A. Describe the processes used for selecting subjects and the methods of recruitment, including use of advertising. Include when, how, and by whom the subjects will be recruited (attach all recruitment materials including flyers, emails, SONA posting, scripts, etc.)?

A contact person who is responsible for what goes through the list serves of various organization will be contacted by e-mail with a letter (entitled Dear PEATC..., etc) summarizing the study. The organizations include, George Mason PhD in education and Special education list serves, and the Parent Education Advocacy Training enter (PEATC).

After this contact person agrees to send an e-mail through it's listserve, participants will then be identified by first asking the contact person to send the recruitment letter via e-mail to its listserver. By sending an e-mail (the recruitment letter) through the George Mason PhD in education and Special education list serves and the Parent Education Advocacy Training enter (PEATC) this will help to recruit parents of with students with a specific learning disability in mathematics.

This e-mail will consist of a list of the purpose of the study, the criteria for the study, and a briefing of what the data collection process will entail. This will result in a pool of applicants who will be contacted for permission to explain the study and request permission. Parents will then be contacted via phone, email, or in-person formats may be used for initial identification and further recruitment. After identifying potential participants, the script describing who I am, what I am seeking to do, what their participation in the research would be, and how long their participation may last will be provided (either in writing or verbally). Participants and parents of participants who indicate they are interested in contributing to this study will then be provided an Informed Consent and absent form, which describes further the confidentiality and nature of how this research will be conducted and data analyzed.

The following mode of contact may be used in the recruitment process and providing information to participants.

- **Snowball sample:** Once I conduct initial interviews with participants using one of the two methods above, I will attempt to "snowball" from these initial contacts. This means I will ask the parent of each interviewed participant if they know anyone else who might be important to interview as part of this study. If so, I would then ask these parents to pass on my information to another parent. If this individual responds with interest, I will then contact him/her via phone or e-mail to see if they were interested in participating in this study. In our initial conversations with these "snowballed" contacts, we will follow this same procedure stated in point number 1.

B. Describe the consent process including how and where the consent will take place, who will conduct the consent process, and information that will be discussed with and distributed to subjects (attach all consent documents):

Following listserve recruitment, the parents who respond to the recruitment e-mails will be contacted via email or phone. After identifying potential participants, I will call the person. I will introduce myself and my study, what I am seeking to do, what their participation in the
research would be, and how long their participation may last. Participants and parents of participants who indicate they are interested in contributing to this study will be met in person at their home, coffee shop, or library. Each participant and parent will then be provided Informed Consent and Assent forms, which describes further the confidentiality and nature of how this research will be conducted and data analyzed. The consent and assent form will also be discussed with the participant and parent.

This process will be conducted prior to the start of the study and will be conducted a location convenient for the participant, which may be at the parent’s house or chosen location such as a coffee shop or library. If potential participants agree to be apart of the study, then the parent will sign and date two copies of the Informed Consent, one of which he/she will keep. After the parent signs the consent form then the child will be asked to sign the Assent form.

C. Is a waiver of signature of Informed Consent being requested? ☐ Yes ☑ No

If yes, complete the following:
1. This waiver is being sought because (check one):
   ☐ The only record linking the subject and the research would be the consent document AND the principal risk would be potential harm resulting from a breach of confidentiality.
   ☐ The research presents no more than minimal risk of harm to subjects AND involves no procedure for which written consent is normally required outside of the research context.
2. Explain why the waiver of signature is being requested:

6. Privacy & Confidentiality
A. How will you protect the privacy of the participants and the confidentiality of the data obtained?
Any study records that identify participants will be kept confidential by the researchers. Also participants name are required for participant contact, once the names are not longer required, participants will be replaced by pseudonym, letters or number.
B. What individually identifiable information will be collected?
   Participants name are required for participant collected for contacting the the participants.
However, once the names are no longer required, participants names will be replaced by a pseudonym. No identifiable information will be collected.

C. Where will the data be stored (Copies of records should be stored on Mason property)?
C. Once the interviews are complete, the data from the audio tape will be transcribed. These transcriptions will not include the names of the participants, only a pseudonym. If the participants do not want to be audio tape, the responses from the interview questions will be written during the interview. After the transcripts are generated, the tapes will be kept in a secure file cabinet located in a file cabinet on George Mason and/or at the home of the researcher (Damali Hay) property. At the completion of the write-up of the study, the tapes will kept safe secure in a secure location, locked in a file cabinet on George Mason campus or in a safe and secure file cabinet, locked in the home of the researcher (Damali Hay).
D. How long will the data be stored?
The audio tapes and transcript will be kept for a minimum of 5 years and will then be shredded.
E. What, if any, are the final plans for disposition/destruction of the data (data must be retained for at least 3 years after the study ends)?
The audio tapes and transcript will be kept for a minimum of 5 years and will then be shredded.

F. Will results of the research be shared with the participants? ☐ Yes ☐ No If yes, describe how this will be accomplished: Parents will be provided with a summary of the report via e-mail or snail mail.

G. Will individually identifiable information be shared with anyone outside of the research team (if yes, please explain and be sure to include this information in the consent form)? ☐ Yes ☐ No If yes, please explain:

7. Risks
   A. Summarize the nature & amount of risk if any (include side effects, stress, discomfort, physical risks, psychological and social risks):

   No potential physical, psychological, social, or legal risks will be taken by participants.

   B. Estimate the probability if any (e.g. not likely, likely, etc.) that a given harm may/will occur and its severity:

   There is a 0% probability that potential physical, psychological, social, or legal risks will occur.

   C. What procedure(s) will be utilized to prevent/minimize any potential risks?

   Participant will be given the correct information before the study and will not be misinformed or uninformed about the true nature of the project. No deception will be used.

8. Benefits
   A. Describe any probable benefits (if any) of the research for the subject(s) (Do not address compensation in this section):

   There are no direct benefits from participating in this study. However, this study may add to the body of existing knowledge by exploring how student perceive and approach mathematics task.

   B. Describe the benefits to society and general knowledge the study is likely to yield:

   This study may provide the following benefits to society:

   Provide information that can enhance the use of evidence-based practices with students with learning disability in math.

   This study will help to provide school officials with the importance of students’ perceptions to lesson planning.

   Provide a better understanding of how students think and perceive mathematics and math problem solving.

9. Financial Information
   A. Is there any external funding or proposed funding for this project? ☐ Yes ☐ No If yes, funding agency and OSP # (attach grant application)

   B. Are there financial costs to the subjects? ☐ Yes ☐ No If yes, please explain:

   C. Will subjects be paid or otherwise compensated for research participation? ☐ Yes ☐ No If yes, please respond to the following questions:

   1. Describe the nature of any compensation to subjects (cash, gifts, research credits, etc.):

   The subjects of my research will be compensated for their participation. However, this will be in the
form of a gift card. Students will receive a $15 iTunes/Dollar Tree gift card as a token of gratitude for participating in the study.

In addition, based on the times that the participant may be available to participate in the study snacks (i.e., chips, crackers, candy, water, etc.) will be provided to the student during a break or after the study.

2. Provide a dollar amount/research credit amount, if applicable:
Each gift card will be work $15 and will be presented as an iTunes/Dollar Tree gift card.

The number of gift card needed will be for approximately 20 students, therefor the total dollar amount will be approximately $300.

3. When and how is the compensation provided to the subject?
The participants will be provided the gift card after the student has completed their participation in the study, whether before or at the end their time needed.

As the gift card is distributed, a log with the recipient information (pseudonym) will be kept.

However, the snacks will be made available to the students during the the data collection. Receipts will be kept of snacks purchased and distributed to students.

4. Describe partial compensation if the subject does not complete the study:
Each student will be provided with the giftcard whether or not they complete the study.

5. If research credit, what is the non-research alternative to research participation?

<table>
<thead>
<tr>
<th>10. Special Topics</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Will the study involve minors? ☑Yes ☐No</td>
</tr>
<tr>
<td>If yes, complete addendum A</td>
</tr>
<tr>
<td>B. Will the study involve prisoners? ☐Yes ☑No</td>
</tr>
<tr>
<td>If yes, complete addendum B</td>
</tr>
<tr>
<td>C. Will the study specifically target pregnant women, fetuses, or neonates? ☑Yes ☑No</td>
</tr>
<tr>
<td>If yes, complete addendum C</td>
</tr>
<tr>
<td>D. Will the study involve FDA regulated drugs (other than the use of approved drugs in the course of medical practice)? ☑Yes ☑No</td>
</tr>
<tr>
<td>If yes, complete addendum D</td>
</tr>
<tr>
<td>E. Will the study involve evaluation of the safety or effectiveness of FDA regulated devices? ☑Yes ☐No</td>
</tr>
<tr>
<td>If yes, complete addendum E</td>
</tr>
<tr>
<td>F. Will false or misleading information be presented to subjects (deception)? ☑Yes ☑No</td>
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<tr>
<td>If yes, complete addendum F</td>
</tr>
<tr>
<td>G. Will participants be audio or videotaped? ☑Yes ☐No</td>
</tr>
<tr>
<td>If yes, complete addendum G</td>
</tr>
<tr>
<td>H. Will the research involve other potentially vulnerable participants (e.g. disabled or addicted individuals, populations engaging in illegal behavior)? ☑Yes ☑No</td>
</tr>
</tbody>
</table>
If yes, complete addendum H

I. Will the research be conducted outside of the United States? ☐ Yes ☒ No
   If yes, complete addendum I

11. Investigator Certification

I certify that the information provided in this project is correct and that no other procedures will be used in this protocol. I agree to conduct this research as described in the attached supporting documents. I will request and receive approval from the IRB for changes prior to implementing these changes. I will comply with all IRB policies and procedures in the conduct of this research. I will be responsible for ensuring that the work of my co-investigator(s)/student researcher(s) complies with this protocol. I understand that I am ultimately responsible for the entire conduct of this research.
DATE: February 4, 2014

TO: Margo Mastropero

FROM: George Mason University IRB

Project Title: [509366-2] What do kids think about math problems and problem solving?

SUBMISSION TYPE: Amendment/Modification

ACTION: APPROVED

APPROVAL DATE: February 4, 2014

EXPIRATION DATE: October 20, 2014

REVIEW TYPE: Expedited Review

Thank you for your submission of Amendment/Modification materials for this project. The George Mason University IRB has APPROVED your submission. This submission has received Expedited Review based on applicable federal regulations.

Please remember that all research must be conducted as described in the submitted materials.

Please remember that informed consent is a process beginning with a description of the project and insurance of participant understanding followed by a signed consent form. Informed consent must continue throughout the project via a dialogue between the researcher and research participant. Federal regulations require that each participant receives a copy of the consent document.

Please note that any revision to previously approved materials must be approved by the IRB prior to initiation. Please use the appropriate revision forms for this procedure.

All UNANTICIPATED PROBLEMS involving risks to subjects or others and SERIOUS and UNEXPECTED adverse events must be reported promptly to the Office of Research Integrity & Assurance (ORIA). Please use the appropriate reporting forms for this procedure. All FDA and sponsor reporting requirements should also be followed (if applicable).

All NON-COMPLIANCE issues or COMPLAINTS regarding this project must be reported promptly to the ORIA.

The anniversary date of this study is October 20, 2014. This project requires continuing review by this committee on an annual basis. You may not collect date beyond this date without prior IRB approval. A continuing review form must be completed and submitted to the ORIA at least 30 days prior to the anniversary date or upon completion of this project. Prior to the anniversary date, the ORIA will send you a reminder regarding continuing review procedures.
Please note that all research records must be retained for a minimum of three years, or as described in your submission, after the completion of the project.

If you have any questions, please contact Bess Dieffenbach at 703-993-4121 or edieffen@gmu.edu. Please include your project title and reference number in all correspondence with this committee.

Addendum A – Minors

1. Describe the order of the recruitment process (parental and child) and how the study will be presented to minors:
The recruitment process will start by using the internet as way of reaching out to potential participants. Therefore, participants will initially be identified by first sending an e-mail through the George Mason PhD in education and Special education list serves, and the Parent Education Advocacy Training enter (PEATC) to recruit parents of students with a specific learning disability in mathematics.

This e-mail will consist of the purpose of the study, the criteria for the study, and a briefing of what the data collection process will entail. This will result in a pool of applicants (parents) who will be contacted, if interested, as a potential parent of a student participant. Parents will then be contacted via phone, email, or in-person formats may be used for initial identification and further recruitment. After identifying potential participants, I will talk about who I am, what I am seeking to do, what their participation in the research would be, and how long their participation may last will also be provided (either in writing or verbally). If parents indicate that they would like their child to participate in the study, both the participants and parents of participants, who indicate that they are interested in contributing to this study will then be provided an Informed Consent form (for parental permission) and an assent form (for the child’s permission). The parental consent will be obtained prior to obtaining the child’s consent. Each form will be describe the confidentiality and nature of how the research will be conducted.

More specifically, each consent form will be explained to the parent and the child as it reads on the form and then I will answer any questions asked by either the parent or the child. This process will be conducted prior to the start of the study and will be conducted a location convenient for the participant, which may be at the parents house or chosen location such as a coffee shop or library. If potential participants agree to be apart of the study, then the parent will sign and date two copies of the Informed Consent, one of which he/she will keep. After the parent signs the consent form then the child will be asked to sign the Assent form.

2. Describe the assent process including how you will obtain active assent from minors (this process should be appropriate to the age/maturity of the child) and permission from parents. If requesting a waiver of active assent or permission, describe the specific reasons for this request:

The assent process will include speaking with the child about who I am, what I am seeking to do, what their participation in the research would involve, and how long their participation may last will be provided (both in writing or verbally). The assent form which includes a detailed description of the purpose, procedures, and rights during the study. Each will be explained further and students will be encouraged to ask questions for clarification if needed. After the assent form is read by the participant and explained by me, the child will be asked to sign the document with an understanding that he or she can quit at any time and nothing bad will happen to him/her.

3. Does this study pose greater than minimal risks to the minors? □ Yes □ No
   If yes, ensure that section 7D and 7E on the application form completely describe potential benefits to individuals and/or society.

4. Does the research involve children who are wards? □ Yes □ No
   If yes, describe the appointment of an advocate where applicable. Additionally, fully describe guardianship and anticipated interactions with the guardian:
Subject: THE VOICE OF YOUR CHILD NEEDS TO HEARD!!

Seeking the perspective of students “at-risk” for achieving academic success in mathematics and students with a learning disability in mathematics for a research entitled “What do kids think about math problems and problem solving?”

Dear Parent/Student:

As the parent of a student “at-risk” or a student with learning disability in mathematics, the perspective of your child is very important to teachers of special education students and also to our current educational system.

I am seeking your help in finding students to participate in a research study designed to acquire information from a students who tend to do poorly in mathematics. Participation consists of responding to a survey, an interview with open-ended questions, and completing a diagnostic test. This can be completed in two sessions. One session will last for approximately 30 to 50 minutes and the second for 25 to 40 minutes. Students will be asked to share their feelings, attitude towards math, and mathematical thinking.

This study will not only help me to better understand perspectives of students with learning disability but it could also assist educators, administrators, and other researchers plan more individual lessons to better and more effectively teach the curriculum.

I’m seeking students who are:

- Currently in middle or high school
- “at-risk” for achieving academic success in mathematics
- Diagnosed with a learning disability in mathematics
- Currently living in Virginia

If you would be kind enough to recommend a student to participate in this study, please respond to this e-mail within one week. The participation of your child is very important and it will mean a great deal to the existing research in the field of special education. The voice of your child needs to be heard. I will follow up immediately to set up a date, time, and location for the interview.

***Also, please note that the information provided during the interview will be reported as anonymous and at no time will any name be associated with the information provided.

Your help is greatly appreciated.

Sincerely,

Damali Hay
Doctoral Candidate, Special Education Teacher
George Mason University
917-374-3815
dhay@masonlive.gmu.edu

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IRB: For Official Use Only

Project Number: 509366-2
Date Approved: 2/4/14
Approval Expiration Date: 10/20/14
RESEARCH TITLE:
What do kids think about math problems and problem solving?

RESEARCH TOPIC:
Beyond mathematics interventions: A look at the psychological and cognitive perspectives (mathematical thinking) of middle and secondary students with learning disabilities about pre-algebra and algebra

RESEARCH PROCEDURES
This research is being conducted to understand the psychological and cognitive mathematical perspective of middle and secondary students with a specific learning disability in mathematics. If you agree to allow your child to be a participant, he or she will be asked to participate in a variety of activities such as a multiple choice survey questionnaire, a math attitude assessment, modified diagnostic test covering pre-algebra and algebra I contents, and a semi-structured student interviews (with a component of clinical interview). The study will take place in 2 sessions. Session 1 will last for approximately 30-50 minutes. Session 2 will last for approximately for 25-45 minutes. The total time needed by all participants will range from 55 – 90 minutes. I would also like to collect some of your child’s achievement test score and math grades from you.

RISKS
There are no foreseeable risks for participating in this research.

BENEFITS
There are no direct benefits from participating in this study. However, this study may add to the body of existing knowledge by exploring how student perceive and approach mathematics task.

CONFIDENTIALITY
The data in this study will be confidential. Your child’s identity will never be revealed in the data or report. Numbers will be substituted for names and original links to names will only be available to project staff during the study. Links to names will be destroyed on study completion. As result, the following steps will be taken during the study: (1) family name will not be included on the collected data nor will it be used in the study; (2) a pseudonym, letter, or number will be placed on all personal documents collected data; (3) through the use of an pseudonym, the researcher will be able to link your interview and documents to your identity; and (4) only the researcher and the principal designee will have access to the identification key. Data will be maintained in secure location, locked in cabinets in office at GMU.

In addition, once the interviews are complete, the data from the audio tape will be transcribed. If the participants chose to be audio taped, the transcripts will be kept confidential. After the transcripts are generated, the tapes will be kept in a secure file cabinet. At the completion of the write-up of the study, the tapes will kept safe secure in a locked file cabinet. The audio tapes and transcript will be kept for a minimum of 3 years and will then be destroyed.

PARTICIPATION
Participation is voluntary and at any time participants may withdraw from the study at any time and for any reason. If your child decides not to participate or decides to withdraw from the study, there is no penalty or loss of benefits to which he/she are otherwise entitled. There are no costs to you or any other party.

CONTACT
This research is being conducted by Damali Hay, a PhD candidate at George Mason University. I can be reached at 917-374-3815 for questions or to report a research-related problem. Dr. Margo Mastropieri will be supervising the research. Therefore, you may also contact Dr. Margo Mastropieri (Dissertation Chair) at 703-587-6583 or via e-mail at nmastrop@gmu.edu. You may also contact the George Mason University Office of Research Integrity & Assurance at 703-993-4121.

This research has been reviewed according to George Mason University procedures governing your participation in this research.

Please note that each interview will be recorded using a recording device.

☐ I understand that this interview will be audio taped.
☐ I do not want to be audio taped.

Print Name Here: ___________________________ Signature: ___________________________
TITLE: What do kids think about math problems and problem solving?

Assent Form for Student

I, ____________________________, understand that my parents (mom and dad) / guardian have/have given permission (said it is okay) for me to take part in this researcher project. This research will be done by Damali Hay, a student at George Mason University (GMU) who wants to understand how students feel, think, and complete math problems.

RESEARCH PROCEDURES: WHAT WE ARE DOING
This research being conducted is to understand what students think about math problems and math problem solving. If you participate in this study, you will be asked to participate in a variety of activities such as a multiple choice survey questionnaire, a math attitude assessment, modified diagnostic test covering pre-algebra and algebra I contents, and a semi-structure student interviews (with a component of clinical interview). I would also like to collect some of your achievement test score and math grades from you.

The study will take place in 2 sessions. Session 1 will last for approximately 30-50 minutes. Session 2 will last for approximately for 25-45 minutes. The total time needed by you will be about 55 – 90 minutes.

RISKS: WHAT COULD HAPPEN TO YOU
There are no risks for taking part in this study. Nothing will happen to you.

BENEFITS: WHAT'S IN IT FOR YOU
You will be given a choice of an iTunes gift OR a dollar store gift card of $15 to participate in the project. Snacks will also be provided during the time you participate in the study.

RESEARCH PROCEDURES: WHAT WE ARE DOING
The researcher (Damali Hay) will not tell anyone about your participation in this study. You do not have to (talk to me, fill out the survey, etc.) if you don’t want to. If you change your mind after you start and want to stop that is OK. I will not get mad and nothing will happen to you.

CONFIDENTIALITY
The data in this study will be confidential. Your identity will never be revealed in the data or report. Numbers will be substituted for names and original links to names will only be available to project staff during the study. Links to names will be destroyed on study completion. As result, the following steps will be taken during the study: (1) family name will not be included on the collected data nor will it be used in the study; (2) a pseudonym, letter, or number will be placed on all personal documents collected data; (3) through the use of an pseudonym, the researcher will be able to link your interview and documents to your identity; and (4) only the researcher and the principal designee will have access to the identification key. Data will be maintained in secure location, locked in cabinets in office at GMU.

CONTACT
This research is being conducted by Damali Hay, a PhD candidate at George Mason University. I can be reached at 917-374-3815 for questions or to report a research-related problem. Dr. Margo Mastropieri will be supervising the research. Therefore, you may also contact Dr. Margo Mastropieri (Dissertation Chair) at 703-587-6583 or via e-mail at mmastrop@gmu.edu. You may also contact the George Mason University Office of Research Integrity & Assurance at 703-993-4121.

This research has been reviewed according to George Mason University procedures governing your participation in this research.

Please note that each interview will be recorded using a recording device.

☐ I understand that this interview will be audio taped.

☐ I do not want to be audio taped.
RE: Request for IRB submission
Gary R Galluzzo [ggalluzz@gmu.edu]

You replied on 9/11/2013 11:39 AM.

Sent: Monday, September 09, 2013 3:13 PM
To: dhay; Margo A Mastropero [mmastrop@gmu.edu]

Damali,

I would be happy to send your letter to the Ph.D. listserv. Once you receive IRB approval, please send me an updated letter (if required by IRB, etc.), and I will send it out.

Gary Galluzzo

-----Original Message-----
From: dhay [mailto:dhay@masonlive.gmu.edu]
Sent: Monday, September 09, 2013 7:56 AM
To: Gary R Galluzzo; Margo A Mastropero
Subject: Request for IRB submission

Dear Dr. Galluzzo,

I hope all is well. I am days away from submitting my IRB dissertation application. However, I am contacting you prior to submitting the application to request your permission to send a participant recruitment letter through the GMU PhD listserv. My goal is to find a few participants for this study through the university. Attached are the letters that I would ask you to send on my behalf.

As a part of my IRB application, I would like to include your permission letter. If permission is granted, please let me know through a letter (email) that can be submitted to IRB.

Thank You,

Damali Hay
Re: Hello- following-up
Michael M. Behrmann [mbehrman@gmu.edu]

You replied on 9/12/2013 1:47 PM.
Sent: Thursday, September 12, 2013 12:49 PM
To: dhay

Damali, I can give you permission to use the Spec Ed list serve but I am not able to provide access to the TTAC email list.

On Sep 11, 2013, at 7:39 AM, dhay wrote:

> Hi Dr. Behrmann,
>
> I hope all is well. I hope my last e-mail was clear. I would like to have your permission to recruit participants using both the T/TAC and Sped Ed (for the current and past students) list serves.
>
> However, my sample letter which was attached in the first e-mail was addressed to T/TAC. This letter will be addressed to the appropriate organization in the e-mail that will be sent to you after IRB's approval.
>
> Please let me know if permission will be granted to use the list serves.
>
> Thank you so much.
>
> Damali Hay
RE: please reconsider
Suzanne Bowers [bowers@peatc.org]

Sent: Tuesday, September 10, 2013 4:39 PM
To: dhay

I spoke with Nichole, who is the person you spoke with. She was waiting for you to send her the request.

The earliest we can send this out on our listserv is November. Hope that helps!

Suzanne Bowers, Executive Director
Parent Educational Advocacy Training Center (PEATC)
(800) 869-6782
www.PEATC.org
"Building Positive Futures for Virginia's Children"

CONFIDENTIAL
This PEATC message is meant for the exclusive use of the intended recipient and may contain information that is privileged, confidential or legally exempt from disclosure. If you have received this message in error, please notify the sender immediately by e-mail or by telephone at (703) 923-0010 and delete message.
Survey- Questionnaire
Instructions: Circle or write in your answers

1. I am a:
   1. Girl
   2. Boy

2. I am ________________ years old.

3. I am currently attending:
   1. Middle school
   2. Junior high school
   3. High school

4. I am currently in grade:
   1. 6
   2. 7
   3. 8
   4. 9
   5. 10
   6. 11
   7. 12

5. I am currently enrolled in:
   1. Math 7
   2. Math 8
   3. Pre-Algebra
   4. Algebra
   5. Other, please specify ____________

6. I am currently in a:
   1. Small classroom (with 2 teachers)
   2. Small classroom (with 1 teacher)
   3. Large classroom (with 2 teachers)
   4. Large classroom (with 1 teacher)

7. I have:
   1. 8 or less students in my class
   2. About 12 -15 students in my class
   3. More than 15 students in my class

8. My overall GPA is about:
   1. (2.0 to 2.9)
   2. (3.00 to 3.24)
   3. (3.25 to 3.49)
   4. (3.50 to 3.74)
   5. (3.75 to 4.0)
9. I am expecting the following grade in my current math class:
   1. A
   2. B
   3. C
   4. D
   5. F
   6. I do not know

10. Compared to other students in my math class, I am:
    1. In the top 10%
    2. Above average
    3. About average
    4. Below average
    5. In the bottom 10%

11. Compared to how hard other students work at mathematics, I work
    1. Very hard
    2. Hard
    3. Not so hard
    4. Not hard
    5. I do not try

12. I work on math assignments during class:
    1. Always
    2. Most of the time
    3. About half the time
    4. Once in a while
    5. Almost never
    6. Never

13. I work on math homework assigned at home:
    1. Always
    2. Most of the time
    3. About half the time
    4. Once in a while
    5. Almost never
    6. Never

14. I stay after school for help from my math teacher:
    1. Always
    2. Most of the time
    3. About half the time
4. Once in a while
5. Almost never
6. Never

15. My parents, neighbor, brother, or sister help me with my math assignments:
   1. Always
   2. Most of the time
   3. About half the time
   4. Once in a while
   5. Almost never
   6. Never

16. I have a math tutor:
   1. Yes
   2. No

17. I attend math class every day:
   1. Always
   2. Most of the time
   3. About half the time
   4. Once in a while
   5. Almost never
   6. Never

18. I think it is ___________ for me to do well in math class?
   1. Very important
   2. Somewhat of important
   3. Not really important
   4. Not important
APPENDIX E

MATH ATTITUDE INTERVENTION MEASURE (MODIFIED VERSION)

<table>
<thead>
<tr>
<th>NAME:</th>
<th>SCHOOL:</th>
</tr>
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</table>

**ATTITUDES TOWARDS MATHEMATICS INVENTORY**

**Directions:** Please complete the following questionnaire by placing a CROSS \( \times \) in the appropriate box.

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<thead>
<tr>
<th></th>
<th>strongly agree</th>
<th>agree</th>
<th>neutral/not applicable</th>
<th>disagree</th>
<th>strongly disagree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Math is a very worthwhile and necessary subject.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. I get a great deal of satisfaction out of solving a math problem.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Math is important in everyday life.</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>4. Math is one of the most important subjects for people to study.</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>5. Middle or high school math courses would be very helpful no matter what I decide to study.</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>6. My mind goes blank and I am unable to think clearly when working with mathematics.</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>7. Studying math makes me feel nervous.</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>8. Math makes me feel uncomfortable.</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>9. When I hear the word math, I have a feeling of dislike.</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>10. Math does not scare me at all.</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>11. I have usually enjoyed studying math in school</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>strongly agree</td>
<td>agree</td>
<td>uncertain/not applicable</td>
<td>disagree</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>----------------</td>
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<td>--------------------------</td>
<td>----------</td>
</tr>
<tr>
<td>12</td>
<td>Math is dull and boring.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>I would prefer to do an assignment in math than to write an essay.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>I really like math.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>I am happier in a math class than in any other class.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>Math is a very interesting subject.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>I am willing to take more than the required amount of math in school.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>I plan to take as much math as I can during my education.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>The challenge of math appeals to me.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>I think studying advanced math is useful.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX F

MATH DIAGNOSTIC TEST (MODIFIED)

Diagnostic Test

Directions:

- Circle OR write the correct letter to the answer on the line.
- Please show all work on the blank paper provided.

1. \[ \frac{3}{8} \cdot \frac{4}{9} = \ ? \]
   a. \( \frac{1}{6} \)  
   b. \( \frac{1}{5} \)  
   c. \( \frac{1}{3} \)  
   d. \( \frac{7}{17} \)

2. \[ \frac{5}{4} - \frac{5}{6} = \ ? \]
   a. 0  
   b. \( \frac{5}{24} \)  
   c. \( \frac{5}{12} \)  
   d. \( \frac{25}{12} \)

3. \[ 3(5 + x) = \ ? \]
   a. \( 8 + x \)  
   b. \( 15 + x \)  
   c. \( 15x \)  
   d. \( 15 + 3x \)
4. What percent of the model is shaded?
   a. 3%
   b. 15%
   c. 25%
   d. 30%

5. What is the area of the trapezoid below with height 6 centimeters and bases 3.2 centimeters and 7.4 centimeters? (The formula for the area of a trapezoid is $A = \frac{1}{2}h(a + b)$.)
   a. 21.44 cm$^2$
   b. 31.8 cm$^2$
   c. 34.04 cm$^2$
   d. 63.44 cm$^2$

6. Use the algebra tiles below to simplify the polynomial expression $5x - 2 - 3x + 5$.
   a. $2x - 3$
   b. $2x + 3$
   c. $8x + 3$
   d. $8x + 7$
7. \((-4)(-2)(-3) = \_\_\_\_\_\_
\ a. 24 \\
\ b. 11 \\
\ c. 5 \\
\ d. 24

8. \(-18 - (-6) = \_\_\_\_\_\_
\ a. -24 \\
\ b. -12 \\
\ c. 12 \\
\ d. 24

9. \(3^4 = \_\_\_\_\_\_
\ a. 7 \\
\ b. 12 \\
\ c. 64 \\
\ d. 81
10. Solve the equation $4x - 5 = 7$.
   a. $\frac{1}{2}$
   b. 3
   c. 8
   d. 12

11. This is a function table for $f(n) = 2n - 1$. What is the missing value?

   \[
   \begin{array}{|c|c|c|}
   \hline
   n & 2n - 1 & f(n) \\
   \hline
   0 & 2(0) - 1 & -1 \\
   1 & 2(1) - 1 & 1 \\
   2 & 2(2) - 1 & 3 \\
   3 & 2(3) - 1 & \\
   \hline
   \end{array}
   \]

   a. $-3$
   b. 4
   c. 5
   d. 6
12. Which algebraic expression matches the verbal expression, "the amount of money in Tad's account if he starts with $s$ dollars and adds $d$ dollars each week for 12 weeks"?
   a. $12s + d$
   b. $s + 12d$
   c. $12(s + d)$
   d. $12ds$

13. On the graph below, the solid line shows Company A's profits. The dashed line shows Company B's profits. In what year are Company A's profits greater than Company B's?

   a. 1999
   b. 2000
   c. 2001
   d. 2002
14. Which set of ordered pairs represent points on the line that is graphed below?

![Graph of a line with points marked]

a. (0, -6), (0, 2), (6, 4)
b. (0, -6), (2, 0), (4, 6)
c. (-6, 0), (0, 2), (4, 6)
d. (0, 6), (2, 0), (6, 4)

15. Which two ordered pairs are both solutions to the equation $y = -2x - 3$?

a. (0, -3), (1, 5)
b. (2, -1), (-1, -1)
c. (2, -7), (1, 5)
d. (0, -3), (1, -5)
<table>
<thead>
<tr>
<th>Problem</th>
<th>Answers (Glencoe connection)</th>
<th>Content Strand</th>
<th>Focus and Objective</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.A</td>
<td>1.A</td>
<td>Number and Operation</td>
<td>Multiply, divide fractions</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>The student will:</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6.6 a) multiply and divide fractions and mixed numbers:</td>
</tr>
<tr>
<td>2.C</td>
<td>2.C</td>
<td>Number and Operation</td>
<td>Add and subtract fractions</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>The student will:</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6.6 a) multiply and divide fractions and mixed numbers:</td>
</tr>
<tr>
<td>3. D</td>
<td>5. D</td>
<td>Number and Operation</td>
<td>Distributive property</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>The student will apply the following properties of operations with real numbers:</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>7.16 b) the distributive property.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>The student will:</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6.2 b) identify a given fraction, decimal, or percent from a representation</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>The student will:</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>A.1 The student will represent verbal quantitative situations algebraically and</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>evaluate theses expressions for given replacement values of the variables.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>7.7 The student will compare and contrast the following quadrilaterals on properties;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>parallelogram, rectangle, square, rhombus, and trapezoid.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>7.13 The student will evaluate algebraic expressions for given replacement values of the variables</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>
| 6. C | 11.C | Algebra | Simplify polynomials with models, combine like terms  
A.4 b) The student will solve multistep linear and quadratic equations in two variables, including justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets.  
A.1 b) adding, subtracting, multiplying, and dividing polynomials. |
| 7.A | 14.A | Number and Operation | Multiply and divide integers  
7.3 b) The student will add, subtract, multiply, and divide integers. |
| 8. B | 15. B | Number and Operation | Add and subtract integers  
7.3 b) The student will add, subtract, multiply, and divide integers. |
| 9. D | 17. D | Algebra | Exponents and power  
8.1 a) The student will simplify numerical expressions involving positive exponents, using rational numbers, orders of operation, and properties of operations with real numbers. |
Solve simple equations  
7.14 a) The student will solve one-step and two-step linear equations in one variable.  
8.14 a) The student will solve |
<p>| 11. C | 22. C | Algebra | Function table |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th>6.17 The student will identify and extend geometric and arithmetic sequences.</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.12 a) The student will represent relationships with tables, graphs, rules, and words.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.14 a) The student will solve</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12. B</td>
<td>27. B</td>
<td>Algebra</td>
<td>Write expressions and equations</td>
</tr>
<tr>
<td>A.1 The student will represent verbal quantitative situations algebraically and evaluate theses expressions for given replacement values of the variables.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13.D</td>
<td>30. D</td>
<td>Data Analysis</td>
<td>Scatter plots, positive, negative, or no correlation</td>
</tr>
<tr>
<td>8.13 a) The student will make comparisons, predictions, and inferences, using information displayed in graphs;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.11 The student will graph ordered pairs in a coordinate plane.</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>8.16 The student will graph a linear equation in two variables.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15. D</td>
<td>32. D</td>
<td>Algebra</td>
<td>Represent functions and relations, solve equations with two variables</td>
</tr>
<tr>
<td>8.14 The student will make connections between any two representations (tables, graphs, words, and rules) of a given relationship.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A.7 The student will investigate a linear function and their characteristics both algebraically and graphically, including</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>A.7 e) finding the values of a function for elements in its domain.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## APPENDIX G

### MATH DIAGNOSTIC ERROR ANALYSIS RUBRIC

**Error Analysis of Student Work (pencil and paper)**

<table>
<thead>
<tr>
<th>Diagnostic Test</th>
<th>Characteristic of Students with &quot;at-risk&quot; and students W/LD</th>
<th>Check and Indicate number of times (if applicable)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Difficulty using symbolism such as (identifying =, &lt;, &gt;, parenthesis) (Ginsburg, 1997; Ginsburg &amp; Seo, 1999). Ex. Symbols cause confusion or misunderstanding of problem.</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Calculation errors (computational error) (Bottge et al., 2010) not involving frequently forgetting negative sign. Ex. Makes an error due to basic calculation errors.</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Difficulty recalling terms (Bottge et al., 2010) Ex. Instructions hinders progress</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Difficulty recognizing different problem types (Bottge et al., 2010). Ex. Failed to remember problems.</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Difficulty completing problem (upon starting correctly). Ex. Started problem but failed to complete problem.</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Applied incorrect strategy. Ex. Use a strategy that is most likely for another problem.</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Difficulty following procedural steps (Naglieri &amp; Johnson, 2000). Ex. Missing steps hinders progress and steps are out of order.</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Difficulty applying self-monitoring skills and checking answers after completion (Naglieri &amp; Johnson, 2000). Ex. Problem appears correct but 1 to 2 minor errors exists due to poor self-checking skills.</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Difficulty understanding abstract thinking (i.e. understand algebraic expressions and equations) (Witzel, Mercer, &amp; Miller, 2003). Ex. Does not understand the placement of variables, how to combine like terms, variable with exponents, and solving for the a missing variable, etc.</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Write numbers illegible (Miller &amp; Mercer, 1999). Ex. Handing writing is hard to follow and may have led to an error.</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Difficulty applying formula. Ex. Formula exists but student has difficulty substitution each number for the correct variable and or correctly completing the operations.</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Computational errors, involving student not applying the rules of negative and positive signs, and variables. Ex. The student ignores negative signs and forgets about variables, when working with numeric and algebraic expressions and equations.</td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX H

CLINICAL INTERVIEW PROTOCOL

Clinical Interview Coverage (Based on student’s level on performance on diagnostic test)

Clinical Interview

Potential questions that may follow given mathematical tasks:

1. Do you recognize this problem? If so, what type of problem is this?
2. What does this component of the problem mean (e.g., <, >, ≤, ≥, =, variables, ( ), exponents, bases)?
3. What is the first thing you would do when you see this type of problem? Next?
4. How would you go about completing this problem?
5. Can you complete this problem another way? If so, please explain?
6. Please explain the meaning of the answer? What does your answer mean?
7. Was there one part (s) of this problem that you found easier to do? Why?
8. Was there one part (s) of this problem that you found more difficult to do? Why?
9. Did you use any strategies or tricks to remember this type of problem?

Number and Number Sense

Question 1 (Compare fractions, decimals, etc.)

6.2 The student will
d) will compare and order fractions, decimals, and percents

7.1 The student will
a) compare and order fractions, decimals, percents, and numbers written in scientific notation

8.1 The student will
b) compare and order decimals, fractions, percents, and numbers written in scientific notations

1. Which number could represent point $K$?

   $\begin{array}{c}
   0 \quad K \quad 1 \\
   \end{array}$

   a) 0.08   b) 8/10   c) 0.8%   d) 3/8

2. Arrange the four numbers shown in order from least to
Questions 2 (Operations with integers)

7.3 The student will
   a) model addition, subtraction, multiplication, and division of integers; and
   b) add, subtract, multiply, and divide integers

1. **What is the value of this expression?**
   
   \((-17) - (-26) ÷ 2\)

2. **What is the value of this expression?**
   \((-2)(6)\)

**Question 3 (Exponents and Square roots)**

6.5 The student will

   The student will investigate and describe concepts of positive exponents and perfect squares.

8.1 The student will
   a) Simplify expressions involving positive exponents, using rational numbers, order of operations, and properties of operations with real numbers; and

1. \(2^2 + 7 - 1\)

2. \((5 + 3) \times 6 ÷ 2\)
Question 4 (Operations with Polynomial expressions)

A.2. The student will perform operations on polynomials including
   a) applying the laws of exponents to perform operations on expressions
   b) adding, subtracting, multiplying, and dividing polynomials

1. \((8X^2 + 6X - 5) + (X^2 - X + 9)\)

2. \((2x^{-3})(4x^{-4})\)

3. \(\frac{15x^{-5}y^7}{3x^2y^{-8}}\)

6.10 The student will
   d) describe and determine the volume and surface area of a rectangular prism

7.5 The student will
   b) solve practical problems involving the volume and surface area of rectangular prism and cylinders

8.7 The student will
   a) investigate and solve practical problems involving volume and surface area of prism cylinders, cones, and pyramids;

1. What is the volume of the shape below?

Height = 5 cm
Length = 4 cm
Width = 3 cm
6.11 The student will:
   a) Identify the coordinate of a point in a coordinate plane; and
   b) graph ordered pairs in coordinate plane

8.11 The student will solve practical area and perimeter problems involving composite plane figures

1. Identify the location of the point \((10,0)\).
   a. In Quadrant I
   b. In Quadrant III
   c. On the x-axis
   d. On the y-axis

2. Select the two points that are located on the y-axis.
   \((0,0)\) \((0,1)\) \((1,1)\) \((-1,-1)\) \((1,0)\)

3. Find the area and the perimeter of the following:

   \[
   \begin{array}{|c|}
   \hline
   24 \text{cm} \\
   \hline
   2 \text{cm} \\
   \hline
   \end{array}
   \]
Probability and Statistics (Data Analysis)
Question 6:
8.13 The student will
a) make comparison, predictions, and inferences, using information displayed in graphs; and
b) construct and analyze scatter plots

Geometry

Question 6:

1. When were the most number of Tulips sold?

Tulip Sales

<table>
<thead>
<tr>
<th>Months</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dozens of Tulips Sold</td>
<td>165</td>
<td>185</td>
<td>195</td>
<td>215</td>
<td>195</td>
<td>185</td>
<td>175</td>
</tr>
</tbody>
</table>

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Algebra (One-step and two-step equations and inequalities)

Questions 7:
6.18 The student will solve one-step linear equations in one variable involving whole numbers. Coefficients and positive rational solutions

6.20 The student will graph inequalities on a number line.

7.14 The student will
a) solve one-step inequalities in one variable
b) solve practical problems requiring the solution of one and two step linear equations

7.15 The student will
a) solve one-step inequalities in one variable and:
   b) graph solutions to inequalities on a number line

1. Solve the equation:
   a. \( X + 5 = 14 \)
   b. \( 3M = 9 \)

2. Solve and graph on a number line:
   a. \( X + 5 > 8 \)

   \[
   \begin{array}{c}
   \quad < \\
   -5 -4 -3 -2 -1 0 1 2 3 4 5 \\
   \end{array}
   \]
   b. \( -4X > 4 \)

   \[
   \begin{array}{c}
   \quad < \\
   -5 -4 -3 -2 -1 0 1 2 3 4 5 \\
   \end{array}
   \]
Question 8: (Two-step and multi-step equations and inequalities)

8.15 The student will
   a) solve multistep linear equations in one variable on one and two sides of the equation;

   b) solve two-step linear inequalities and graph the results on a number line; and
   c) identify properties of operations used to solve an equation

A.5. The student will solve multistep linear inequalities in two variables including
   a) solving multistep linear inequalities algebraically and graphically

1. Solve each linear equation.
   a. \(3M + 27 = 18\)
   b. \(2 + 13n = -8 + 8n\)
   c. \(5(2.9 + x) = 8.3\)
   d. \(\frac{k}{5} + 7 = 22\)

2. Graph the solution to \(-5a - 7 \leq 12\) on a number line.

Questions 9: (Expressions and Operations)

7.13 The student will
   a) Write verbal expressions as algebraic expressions and sentences as equations and vice versa; and
   b) Evaluate algebraic expression for given replacement values of the variables
7.16 The student will apply the following properties of operations with real numbers
   a) the distributive property

1. Write the following algebraically:
   a. Three times a number increased by 10
   b. Nine less than a number

2. Evaluate the algebraic expression if \( X = 2 \) and \( Y = 4 \)
   a. \( XY \)
   b. \( X + 2y \)

3. Simplify the expression: \( 6(X + 5) \)

Questions 10 (Pattern and Functions)

7.12 The student will represent relationships with table, graphs, rules, and words

8.14 The student will make connections between any two representations (tables, graphs, words, and rules) of a given relationship.

8.16 The student will graph a linear equation in two variables

8.17 The student will identify the domain, range, independent variable, dependent variable in a given situations
1. Complete the following
   
   a. What is the slope of the line?
   b. What is the y-intercept of the line
   c. Write the equation of the line in \(Y=MX+b\) form

2. List the DOMAIN and the RANGE:
   
   a) \{ (7, 3), (-2, -3), (5, 8), (7, 2) \}  Domain: _______ Range: _______

   b) \{(8, 11), (3, 14), (0, 9), (10, 14) \} Domain: _______ Range: _______
<table>
<thead>
<tr>
<th>Error Analysis Techniques (Criteria)</th>
<th>1 Far Below Standard</th>
<th>2 Below Standard</th>
<th>3 Standard</th>
<th>4 Proficient</th>
<th>5 Advanced</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clinical Interview Questions (#1, 2)</td>
<td>Cannot identify the problem</td>
<td>(e.g., student is limited use of correct math language, and may need 3 to 4 probing questions)</td>
<td>With limited use of math language.</td>
<td>Appropriate use of basic math language.</td>
<td>Appropriate use of advance math language.</td>
</tr>
<tr>
<td>Use of Strategies</td>
<td>Unable to identify any strategy (i.e.).</td>
<td>Unable to approach problems without assistance.</td>
<td>Independently: Uses somewhat appropriate strategies to solve problem, with 1-2 probing questions.</td>
<td>Independently: Identifies a strategy that was used to complete the problem.</td>
<td>Independently: Identify more than one strategy that was used to complete the problem accurately.</td>
</tr>
<tr>
<td>Accuracy of Mathematical Work</td>
<td>No work/No solution</td>
<td>Many errors (3-4). Needing plenty of assistance to show steps.</td>
<td>Minimal errors (Less than 2), with assistance. Missing details. Expansion of steps and work needed.</td>
<td>Solution is accurate. Some steps and work are needed but end results are accurate.</td>
<td>Solution is accurate. All steps and additional details are shown and accurate.</td>
</tr>
<tr>
<td>Clinical Interview Questions (#5, 6)</td>
<td></td>
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</tr>
<tr>
<td>Organization</td>
<td>No steps are present for each solution.</td>
<td>Steps were present in an illegible form. Organization was present with some assistance. The work present was difficult to follow. Consists of more than 3-4 errors.</td>
<td>Steps were presented. Organization was attempted with numerous errors. The work present is difficult to follow. Consists of less than 2 errors.</td>
<td>Steps were presented. Organized and consists of easy to follow steps, with some missing steps. The work was present with no errors.</td>
<td>Steps were clearly and accurately presented. Very organized in a straight-forward, detailed, clear, and easy to follow manner. The work was present with no errors.</td>
</tr>
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</tr>
<tr>
<td>Clinical Interview Questions (#3,4,5)</td>
<td>No explanation provided.</td>
<td>Unable to explain without a lot of assistance. Explanation demonstrates very little understanding of the underlying concept.</td>
<td>Moderately explains process and reason for solution with little than 2 probing questions. Explanation demonstrates some understanding of underlying concept.</td>
<td>Independently, explains solution with some missing details Explanation demonstrates an understanding of the underlying concept.</td>
<td>Clearly and accurately explains their own process, reason, and meaning to solution. Explanation demonstrates complete understanding of the underlying concept.</td>
</tr>
<tr>
<td>Reflection and Explanation of solution</td>
<td>No explanation provided.</td>
<td>Unable to explain without a lot of assistance. Explanation demonstrates very little understanding of the underlying concept.</td>
<td>Moderately explains process and reason for solution with little than 2 probing questions. Explanation demonstrates some understanding of underlying concept.</td>
<td>Independently, explains solution with some missing details Explanation demonstrates an understanding of the underlying concept.</td>
<td>Clearly and accurately explains their own process, reason, and meaning to solution. Explanation demonstrates complete understanding of the underlying concept.</td>
</tr>
<tr>
<td>Clinical Interview Questions (#6,7,8)</td>
<td>No explanation provided.</td>
<td>Unable to explain without a lot of assistance. Explanation demonstrates very little understanding of the underlying concept.</td>
<td>Moderately explains process and reason for solution with little than 2 probing questions. Explanation demonstrates some understanding of underlying concept.</td>
<td>Independently, explains solution with some missing details Explanation demonstrates an understanding of the underlying concept.</td>
<td>Clearly and accurately explains their own process, reason, and meaning to solution. Explanation demonstrates complete understanding of the underlying concept.</td>
</tr>
</tbody>
</table>

Highest Possible point 25 points
**Clinical Interviews Rubric by Process Standards (Based on NCTM standards)**

(Verbal Responses)

<table>
<thead>
<tr>
<th>Process Standards</th>
<th>1 Far Below Standard</th>
<th>2 Below Standard</th>
<th>3 Standard</th>
<th>4 Proficient</th>
<th>5 Advance</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Criteria)</td>
<td>(Does not make an effort)</td>
<td>(makes very little attempt)</td>
<td>(Makes an attempt)</td>
<td>(Ability to demonstrate)</td>
<td>(Exceptional demonstration)</td>
</tr>
<tr>
<td>Problem Solving</td>
<td>Shows no understanding of the problem.</td>
<td>Shows little understanding of the problem.</td>
<td>Shows an understanding of the problem.</td>
<td>Shows a good understanding of the problem.</td>
<td>Shows an excellent understanding of the problem.</td>
</tr>
<tr>
<td>Did not try a strategy</td>
<td>With assistance and prompts, problem was attempted.</td>
<td>Independently, used a strategy.</td>
<td>Independently, used a strategy that works.</td>
<td>Independently, used more than one strategy that worked.</td>
<td>Independently, used more than one strategy that worked.</td>
</tr>
<tr>
<td>(I did not understand the problem)</td>
<td>Multiple errors (3-4) exist</td>
<td>(I only understand a part of the problem)</td>
<td>No errors exist.</td>
<td>No errors exist.</td>
<td>No errors exist.</td>
</tr>
<tr>
<td>Reasoning and Proof</td>
<td>Does not demonstrate an understanding or recognition of problem.</td>
<td>With assistance: Demonstrates a possible start to some form of reasoning required to solve the problem.</td>
<td>Independently: Demonstrates a possible start to some form of reasoning required to solve the problem.</td>
<td>Independently: Can provide reasoning that is correct, with little errors.</td>
<td>Independently: Can use reasoning and proof to accurately solve a problem in one or more ways using details.</td>
</tr>
<tr>
<td>Reasoning neither strategies nor effort does not exist.</td>
<td>Demonstrates some reasoning strategy exists, with 3-4 errors.</td>
<td>Demonstrates some reasoning strategy exists with less than 2 errors.</td>
<td>Applied reasoning strategy exists with no errors.</td>
<td>Use multiple reasoning strategy(ies) exist with no errors.</td>
<td></td>
</tr>
<tr>
<td>Connections</td>
<td>(My mathematical thinking is not correct)</td>
<td>(Some of my mathematical thinking is correct with help)</td>
<td>(Some of my mathematical thinking is correct)</td>
<td>(All of my mathematical thinking is correct)</td>
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<td>--------------------------------------------</td>
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</tr>
<tr>
<td></td>
<td>With assistance:</td>
<td>Independent:</td>
<td>Independent:</td>
<td>Independent: Café and all important information from problem.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Can select and use some important information from the problem to explain answers.</td>
<td>Can select and use some important information from the problem to explain answers.</td>
<td>Can communicate and perform all problems. Show all steps clearly and accurately.</td>
<td>Can communicate and perform all problems. Show all steps clearly and accurately.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Can communicate and perform the beginning of the problem.</td>
<td>Can communicate and perform some of the problem.</td>
<td>Uses the correct vocabulary and symbol with less than 2 errors.</td>
<td>Uses the correct vocabulary and symbol.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Uses some correct vocabulary and symbol with lot assistance. 3-4 errors exit.</td>
<td>Uses the correct vocabulary and symbol.</td>
<td>Explanation shows confusion or lack clarity.</td>
<td>Uses the correct vocabulary and symbol.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Can explain some thinking with prompts.</td>
<td>Independent:</td>
<td>Independent:</td>
<td>Independent: Café and all important information from problem.</td>
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<td>Independent: Café and all important information from problem.</td>
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<tr>
<td></td>
<td>(I cannot talk about this math problem)</td>
<td>(Independently, I used math language and/or notation to communicate/think-aloud.)</td>
<td>(Independently, I used math language and/or notation to communicate/think-aloud.)</td>
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<tr>
<td></td>
<td>(I do not know)</td>
<td>(Independently, I used math language and/or notation to communicate/think-aloud.)</td>
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<td></td>
<td>No connections are made.</td>
<td>With assistance:</td>
<td>Independent:</td>
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<td></td>
</tr>
<tr>
<td>Representation</td>
<td>Attempts, at least one aspect, to relate or connect the problem to other subjects/topic based on prior experience.</td>
<td>Attempts, but was not fully able to accurately connect, at least one aspect of the problem, to relate or connect the problem to other subjects/topic based on prior experience.</td>
<td>Makes all accurate connections and solved the problem correctly.</td>
<td>Makes all accurate connections and observations needed to accurately solve and expand solution.</td>
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<td>--------------------------------------------------------------------------------------------------</td>
<td>--------------------------------------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>(I do not recall any aspect of this problem)</td>
<td>(I did not notice anything about the problem or the numbers in my work)</td>
<td>(Independently, I noticed something about my math work)</td>
<td>(I noticed something in my work, and used that to extend my answers and/or I showed how this problem is like another problem.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No attempt is made to create mathematical representation(s).</td>
<td>With assistance: Some attempt is made to create mathematical representation(s) to display and communicate problem solving technique.</td>
<td>Independently: Some attempt is made to create mathematical representation(s) to display and communicate problem solving technique.</td>
<td>Independently: Create mathematical representation(s) to record and communicate problem solving technique with no errors or minor errors.</td>
<td>Independently: Create abstract or symbolic representation(s) to record, communicate, analyze relationships, expand thinking, and clearly interpret problem.</td>
<td></td>
</tr>
<tr>
<td>(I did not use math representation to help solve the problem and explain my work)</td>
<td>(I did not use math representation to help solve the problem and explain my work)</td>
<td>(I tried to use math representation to help solve the problem and explain my work, with assistance)</td>
<td>(I tried to use math representation to help solve the problem and explain my work, but made mistakes)</td>
<td>(I used math representation to help solve the problem and explain my work in more than one way)</td>
<td></td>
</tr>
</tbody>
</table>

Highest Possible point 25 points
APPENDIX K

SEMISTRUCTURED INTERVIEW PROTOCOL

Semi- Structure Interview Questions

1. How do you feel about math or algebra class?
2. What words would you use to describe math?
3. What do you find challenging about math?
4. Can you share a situation in which you thought you liked math?
5. Can you describe a time that you did not like math, and why?
6. Can you describe a type of teaching or presentation of the materials in class that you like best?
7. What types of math problems do you tend to remember the most?
8. What are some of the things that you like about a teacher you find most helpful?
9. What are some of the things you dislike about a teacher you do not find helpful?
10. So, who was your favorite math teacher and why?
11. In your current math class, what kind of thing(s) is your math teacher doing to help you in class?
12. What type of practice, worksheets, or homework assignments do you find more helpful?
13. Is there anything you would change about your math classes?
   a. How would you change it?
   b. Why?
14. Do you think math is important? If so, why do you think math is important? If not, why don’t you think math is important?
15. What type of things do you think math will be good for in the future?
   a. What do you want to do in the future, as a job, or a career?
16. Think of a time you did not enjoy math class. Tell me what happened and why.
17. Now, think of a time you did enjoy math class. Tell me what happened and why.
REFERENCES


280


288


BIography

Damali Hay Dayne received her Bachelor of Science in Economics from Virginia Polytechnic Institute in 2003. She went on to receive her Master of Arts in Curriculum and Instruction at Virginia Polytechnic Institute in 2004, concentrating in Special Education. She currently teaches at a high school and manages her own private tutoring business.