Preparing Students with Disabilities for Algebra

Joseph Calvin Gagnon
Paula Maccini

The challenge for teachers to provide effective math instruction to students with disabilities is heightened by the most recent reform efforts by the National Council of Teachers of Mathematics Standards (NCTM, 2000). The NCTM standards emphasize the need to prepare all students for algebra beginning in kindergarten and progressing through each grade. As school districts and states increasingly adopt these standards (Parmar & Cawley, 1995), teachers must have the information necessary for successful implementation. Though knowledge of the standards is crucial, one study (Maccini & Gagnon, 2000) noted that almost half of special educators were unaware of the NCTM standards.

It is our goal, then, to provide specific instructional approaches and examples to assist teachers in developing the algebraic reasoning skills of students with disabilities, in light of the NCTM standards and empirically validated research.

This article discusses the NCTM standards and focuses on two key issues: (a) effective instructional strategies in algebra and (b) examples of effective instructional strategies for teaching algebraic reasoning at middle and high school levels that are consistent with the standards.

**NCTM Standards**

The new Principles and Standards for School Mathematics (NCTM, 2000) are based on five basic goals for students: (a) learning to value mathematics, (b) becoming confident in their ability to do mathematics, (c) becoming mathematical problem solvers, (d) learning to communicate mathematically, and (e) learning to reason mathematically.

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**Figure 1. Characteristics of Students with Learning Disabilities or Emotional Disturbance Related to Mathematics**

- Difficulty processing information which results in problems learning to read and problem-solve
- Difficulty with distinguishing the relevant information in story problems
- Low motivation, self-esteem, or self-efficacy to learn due to repeated academic failure
- Problems with higher-level mathematics that require reasoning and problem-solving skills
- Passive learners—reluctant to try new academic tasks or to sustain attention to task
- Difficulty with self-monitoring and self-regulation during problem-solving
- Difficulty with arithmetic, computational deficits

**Source:** From Best practices for teaching mathematics to secondary students with special needs: Implications from teacher perceptions and a review of the literature by Maccini, P., & Gagnon, J. C., 2000, Focus on Exceptional Children, 32(5), 1-22.
Learning to reason mathematically. The Principles and Standards involve a framework of six general principles, five content standards, and five process standards for achieving these goals.

**Six General Principles**

Consistent with the goals are six general principles of mathematics (NCTM, 2000).

1. The first, equity, is the assertion that “mathematics is for all students, regardless of personal characteristics, backgrounds, or physical challenges” (p. 12).

2. The second principle relates to curriculum and the belief that mathematics should be viewed as an integrated whole, as opposed to isolated facts to be learned or memorized.

3. The third principle, which relates to effective teaching, requires that teachers display three attributes: (a) a deep understanding of math, (b) an understanding of individual student development and how children learn math, and (c) the ability to select strategies and tasks that promote student learning.

4. The fourth principle is the view that students will gain an understanding of mathematics through classes that promote problem-solving, thinking, and reasoning.

5. The fifth principle provides the basis and support for continual assessment of student performance, growth, and understanding via varied techniques (e.g., portfolios, mathematical assessment of concepts embedded in real-world problems).

6. The final principle is written as a statement highlighting the importance of technology (e.g., computers, calculators) and the realization that use of these tools may enhance learning by providing opportunities for exploration and concept representation. Many educators, however, recommend that technology supplement teacher instruction, paper-and-pencil calculations, and mental calculations, rather than replace them.

**Five Content Standards**

Five content areas or “strands” are addressed in the NCTM standards: (a) number and operations, (b) algebra, (c) geometry, (d) measurement, and (e) data analysis and probability. These content strands extend across four grade bands (pre-kindergarten-2, 3-5, 6-8, 9-12) and have different value or weight within each band.

For example, the study of number and operations is highly emphasized in the first three grade bands, especially in pre-kindergarten-2 and 3-5. Interestingly, the study of algebra or “algebraic reasoning” is emphasized in all four grade bands. According to Van De Walle (2001), algebraic reasoning involves helping students to: (a) recognize, extend, or generalize patterns, and (b) communicate patterns or relationship generalizations via algebraic symbolisms (equations, variables, and functions).

**Five Process Standards**

The Principles and Standards also list five process standards or ways students should learn and apply mathematics across the curricular areas and grade bands. The first process standard, problem-solving, emphasizes the use of problem-solving contexts to help students build their mathematical knowledge (learning and “doing” math as students solve problems). This is the vehicle for new knowledge. The second standard, reasoning-and-proof process, involves logical thinking during problem-solving and considering if an answer makes sense. Communication, the third standard, refers to talking about, describing, explaining, and writ-

**Effective Teaching Strategies for Math Instruction**

- Teaching prerequisite skills, definitions, and strategies.
- Providing direct instruction in problem representation and problem solution.
- Providing direct instruction in self-monitoring procedures.
- Using organizers.
- Incorporating manipulatives.
- Teaching conceptual knowledge.
- Providing effective instruction.

**Effective Instruction**

To assist students with mathematical tasks and processes, as recommended by NCTM, the integration of these standards and documented “best practices” of how to teach math to students with disabilities is recommended. Specifically, researchers have determined that certain components of effective instruction positively influence the algebra performance of students with learning and behavioral disabilities (see Maccini et al., 1999). For example, Maccini et al. analyzed the literature focusing on teaching algebra to secondary students with learning disabilities and determined that successful interventions included variations of seven critical components: (a) teaching prerequisite skills, definitions, and strategies; (b) providing direct instruction in problem representation and problem solution; (c) providing direct instruction in self-monitoring procedures; (d) using organizers; (e) incorporating manipulatives; (f) teaching conceptual knowledge; and (g) providing effective instruction.

**Teach Prerequisite Skills, Definitions, and Strategies**

Before introducing a new concept, use quizzes or reviews to determine if students have the necessary prerequisite skills. Students with learning disabilities may lack knowledge of basic math terms and operations (Huntington, 2001).
In addition, many students with mild disabilities experience memory deficits and are considered passive learners. In such cases, provide direct instruction in foundational skills, definitions, and strategies.

Teaching students a first-letter mnemonic strategy enhances recall of general problem-solving steps with computational skills (Mercer & Miller, 1992). One such math strategy, DRAW (Mercer & Miller, 1992, p. 26), cues students to solve math problems involving computational tasks:

Discover the sign.
Read the problem.
Answer or DRAW a conceptual representation of the problem using lines and tallyis, and check.
Write the answer and check.

Another first-letter mnemonic strategy, STAR, is effective with older students with mild disabilities (Maccini & Hughes, 2000; Maccini & Ruhl, 2000). This strategy cues students to complete general problem-solving steps and related substeps and is based on the behaviors of expert problem-solvers (see Figure 2).

Although this article shows how this strategy is used with integer numbers, use of STAR may also be generalized to other tasks that require problem-solving skills. The four main steps of the strategy include:

Search the word problem (i.e., read the problem carefully, write down knowns/facts).
Translate the words into an equation in picture form (i.e., choose a variable, identify the operation, and represent the problem through manipulatives or picture form).
Answer the problem.
Review the solution (i.e., reread the problem, check the reasonableness of the answer).

Direct Instruction in Problem Representation and Problem Solution

Many students with learning disabilities experience difficulties with representing or visualizing a problem situation (Montague, Bos, & Doucette, 1991) and finding the solution (Algazzinie, O’Shea, Crews, & Stoddard, 1987). Thus, you should teach both problem representation (i.e., integrating the information from a word problem into a visual representation) and problem solution (i.e., applying appropriate procedures to derive the solution). For example, the first three steps of the DRAW strategy and the first two steps of the STAR strategy address problem representation.

To facilitate both problem representation and problem solution, provide students with questions or prompts on a card or structured worksheet. For example, the prompt "Draw a picture of the problem" cues students to identify and represent the problem. Similarly, the questions "Does the answer make sense? Why?" prompt students to check the answer.

Explicit Instruction in Self-Monitoring Procedures

Many students with learning disabilities experience difficulty with monitoring their problem-solving behavior (Montague et al., 1991). Teach students to ask themselves questions while problem-solving. First, model how to use prompts or questions from a structured worksheet by "thinking aloud" (i.e., reading and answering questions aloud), as students observe the self-questioning process (see box, p. 12, "Instructional Strategy Steps").
Teaching students a first-letter mnemonic strategy enhances recall of general problem-solving steps with computational skills.

Figure 3 shows an example of a structured worksheet, based on the first step of the STAR strategy. As shown, the worksheet lists the strategy steps and provides space for students to “check off” completed tasks.

Organizers

Many students with mild disabilities experience difficulty remembering or recalling information over time (Olson & Platti, 1996). In addition, these students may have difficulty identifying relevant information within a problem and organizing the information.

Using visual organizers, such as structured worksheets, prompt cards, or graphic organizers, helps all students analyze and solve problems. These organizers help students remember general problem-solving steps/substeps and the information within the problem (see Figures 4 and 5 for some examples of problems and the steps students use to solve them).

Manipulatives

Teachers may also incorporate the use of objects or other visuals to help students with problem representation. For example, students can use items from their environment (i.e., patterns within nature) when investigating patterns to build algebraic reasoning skills. Too, students can use algebra blocks, such as the Algebra Lab Gear (Picciotto, 1990), to help them visualize both numeric and variable amounts (see Figure 5).

Conceptual Knowledge

A concrete-semiconcrete-abstract (C-S-A) instructional sequence supports students’ understanding of underlying math concepts before learning “rules” (i.e., visual to abstract representations; see Figure 5). As Van De Walle (2001) stated, “If we emphasize only the procedural rules, there is little reason for students to attend to the conceptual justifications. Do not be content with right answers; always demand explanations” (p. 425).

During the initial phase of instruction (i.e., concrete), students represent the problem with objects. Students then advance to the semiconcrete phase of instruction and draw or use pictorial representations of the quantities. The abstract phase of instruction involves numeric representations, instead of pictorial displays. This instructional sequence can be successfully embedded within a problem-solving strategy. For example, the C-S-A sequence is integrated into the STAR strategy and the steps cue students to follow the graduated sequence.

Effective Teaching

Incorporating efficient and effective teaching components into the teaching routine promotes student learning and retention (Rosenshine & Stevens, 1986; see box, “Instructional Strategy Steps”). Researchers have found the following steps effective in math instruction of students with learning disabilities: modeling the task, providing guided and independent practice, frequent reviews, and corrective and positive feedback (Maccini et al., 1999; Maccini & Hughes, 2000; Maccini & Ruhl, 2000; Mercer & Miller, 1992; see Figure 4).

Teaching Algebraic Reasoning. The idea of teaching algebraic reasoning to students with mild disabilities as early as kindergarten, as recommended by NCTM, may seem like an unrealistic goal. But if we consider algebraic reasoning as the study (i.e., representing, generalizing, formalizing) of patterns within mathematics (Van De Walle, 2001), we can certainly approach this type of reasoning at many levels.

For example, simple repeating patterns are commonly taught to children as early as kindergarten. These patterns may include verbal patterns (i.e., the musical notes do, do, mi, mi, do, do), movements, and visual patterns using manipulatives. Students in the early elementary grades regularly engage in patterned gross motor movement activities. Such lessons can be linked to patterns as the basis for future exploration in algebraic reasoning.

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**Figure 3. Structured Worksheet**

**Strategy Questions**

Search the word problem

(a) Read the problem carefully

(b) Ask yourself questions: “What facts do I know?” “What do I need to find?”

(c) Write down facts

<table>
<thead>
<tr>
<th><strong>Write a check (√) after completing each task or question</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. I know there are 5 apples.</td>
</tr>
</tbody>
</table>

**Sources:** Maccini & Hughes, 2000; Maccini & Ruhl, 2000.

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**Representation, an NCTM process standard, refers to expressing math ideas/concepts through charts, graphs, symbols, diagrams, and manipulatives.**

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Instructional Strategy Steps

Step 1: Provide an Advance Organizer
Provide an advance organizer to (a) connect new information to previously learned skills, (b) state the new skill to-be-learned, and (c) provide the rationale introducing the new topic.

Step 2: Provide Teacher Modeling
Provide teacher "modeling" through two methods. First, "think aloud" to students while introducing a strategy. Then, "fade," or reduce, teacher prompts while involving students in application of the strategy. For example, following the teacher model, students answer questions and write down their responses using the graphic organizers or structured worksheets.

Step 3: Provide Guided Practice
Provide opportunities for students to practice the new strategy with teacher assistance. Fade teacher assistance until students can perform the task independently.

Step 4: Provide Student Independent Practice
Assess student mastery of the skills by providing problems without teacher prompts/assistance.

Step 5: Provide Feedback
Provide positive and corrective feedback throughout the lesson in five steps:
- Document student performance (e.g., calculate the percentage correct).
- Target error patterns/incorrect answers.
- Reteach, if necessary.
- Provide student practice with similar problems and monitor student performance.
- Close with positive feedback.

Step 6: Provide for Generalization
Provide prompts or questions to promote generalization to other
- Problem-solving situations.
- Content areas.
- Real-world situations.

Source: Adapted from Maccini & Hughes, 2000; Maccini & Ruhl, 2000; Mercer & Miller, 1992.

This process of applying a pattern to a variety of situations will reinforce the important understanding that a pattern can be maintained even when the materials change (Van De Walle, 2001).

Translate the words of a problem into an equation in picture form (i.e., choose a variable, identify the operation, and represent the problem through manipulatives or picture form).

Algebraic reasoning can be a focus of instruction for children through activities that focus on repeated patterns. Figures 4 and 5 provide examples of (a) repeated patterns (i.e., growing patterns; see Figure 4) and (b) integer operations that prepare students for more formal algebraic symbolisms and manipulations (i.e., solving word problems with integer numbers; see Figure 5). We provide these examples of teaching algebraic reasoning to illustrate the feasibility of effective instruction in algebraic reasoning. Though not exhaustive, these examples demonstrate the integration of NCTM Process Standards and the components of effective math instruction, (e.g., the components of effective teaching, general problem-solving strategies, and organizers).

Provide students with questions or prompts on a card or structured worksheet.

Building on Prior Knowledge. In presenting problems like these, your initial step is to assess the prerequisite skills needed by students to complete this problem. A brief examination of the problem reveals the need for students to have experience with multiplication and division, the concepts of growing and recursive patterns, and writing simple equations. In addition, students should have a working knowledge of the STAR strategy and some exposure to using a graphic organizer to solve word problems. These concepts can be informally assessed through discussion and review or more formally, through a short quiz.

Final Thoughts
The ideas presented here may prove helpful when designing and delivering algebra instruction to your students with mild disabilities. Students at all grade levels can learn to reason algebraically via engagement in problemsolving activities that include empirically-validated practices, such as the C-S-A continuum, graphic organizers, explicit instruction, manipulatives, and strategy instruction. Teachers, then, have the challenge to integrate these effective practices and the goals of the NCTM standards within their classrooms.

A concrete-semiconcrete-abstract (C-S-A) instructional sequence supports students' understanding of underlying math concepts before learning "rules."
Figure 4. Growing Pattern: Sample Problem

Sample Problem: Today, Walter borrowed $2.00 from his dad with the understanding that he would have to pay his dad “interest” each day. On the second day, Walter owed his dad a total of $4.00, on the third day, a total of $6.00, and on the fourth day, a total of $8.00. If Walter didn’t get his paycheck until the 25th day, how much would he owe his dad?

**Phase of Instruction**

1. Concrete Application:
   a. Students use paper money to represent the problem. Use the graphic organizer and count the money owed each day, up to the 25th day.
   b. Look for the relationships/patterns:
      - between each day and the number of dollars owed
      - across days (i.e., from one day to the next)

   
   ![Table](image)

   **Frame** | **Day 1** | **Day 2** | **Day 3** | **Day 4** | 25
   --- | --- | --- | --- | --- |
   **# of Dollars** | | | | |
   
2. Semiconcrete Application:
   a. Draw pictures of the money owed within each day of the graphic organizer.
   b. Count the dollars in each frame and write the total owed per frame (circled).
   c. Look for relationships/patterns:
      - between each day and the number of dollars owed (e.g., day 1: the number owed is $1 + $1 = $2, or D + D)
      - recursive patterns (i.e., from one day to the next, such as one day’s cost plus $2 or C + 2). Write the numbers under the organizer.

   ![Table](image)

   **Frame** | **Day 1** | **Day 2** | **Day 3** | **Day 4** | 25
   --- | --- | --- | --- | --- |
   **# of Dollars** | | | | |
   ![Chart](image)

**STAR Strategy**

Prompts students to:
- Search problem (read carefully, ask questions, write down facts); Translate the problem using paper money; Answer the problem using money; and Review the solution (reread the problem, check reasonableness, and calculations).

Prompts students to:
- Search problem (read carefully, ask questions, write down facts); Translate (represent) the problem via drawings, numbers, and an equation; Answer the problem using drawings and numbers; and Review the solution (reread the problem, check reasonableness, and calculations).
**Phase of Instruction**

3. Abstract Application:
   a. Write the total owed per frame in the graphic organizer.
   b. Look for relationships/patterns and write numerical representation:
      - between each day and the number of dollars owed (e.g., day 3: the number owed is $3 + $3 = $6, or $D + D$).
      - across days (i.e., from one day to the next, such as one day's cost plus $2 or $C + 2$). Write the numbers under the organizer.
   c. Apply the rule for the growing pattern to obtain the answer. Reread and check the answer for reasonableness.

**STAR Strategy**

Prompts students to:
- Search problem (read carefully, ask questions, write down facts); Translate the problem into numbers within the graphic organizer and an equation; Answer the problem; and Review the solution (reread the problem, check reasonableness of the answer, and calculations).

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**Figure 5. Division: Sample Problem with Integers**

Sample Problem: Suppose the temperature changed by an average of $-2^\circ F$ per hour. The total temperature change was $-16^\circ F$. How many hours did it take for the temperature to change?

**Phase of Instruction**

1. Concrete Application:
   Students use blocks to represent the problem.
   General guidelines: (inverse operation of multiplication)

   \[
   \begin{align*}
   \text{Algebra Tiles:} & \quad = 1 \text{ unit;} & \quad = 5 \text{ units;} & \quad = 25 \text{ units} \\
   \text{1) Students begin} & \quad \text{with no tiles on the workmat} & \quad \text{2) Students add 8 sets} & \quad \text{3) Students count the number of sets needed (8).}
   \end{align*}
   \]

   a. Count the number of sets of $-2$ needed to obtain $-16$.

2. Semiconcrete Application:
   Students draw pictures of the representations

3. Abstract Application:
   Students first write numerical representation:
   
   
   - $-16 \div -2 = x$, apply the rule for dividing integers to obtain $x = +8$, and reread and check the answer.

**STAR Strategy**

Prompts students to:
- Search problem (read carefully, ask questions, write down facts); Translate (represent) the problem via drawings and write down the equation; Answer the problem using drawings and write the answer; and Review the solution (reread the problem, check reasonableness, calculations).

Prompts students to:
- Search problem (read carefully, ask questions, write down facts); Translate the problem into an equation; Answer the problem (apply the rule for division integers); and Review the solution (reread the problem, check reasonableness of the answer and calculations).

*Source: Adapted from *The algebra lab* by Picciotto, H., 1990, Mountain View, CA: Creative Publications.*
“Do not be content with right answers; always demand explanations.”

References


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Joseph Calvin Gagnon, Doctoral Candidate; and Paula Maccini, Assistant Professor, Department of Special Education, University of Maryland, College Park.

Address correspondence to Joseph Calvin Gagnon, University of Maryland, Special Education, 1308 Benjamin Building, College Park, MD 20742 (e-mail: jcgagnon@bellatlantic.net).


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