DETERMINATION OF SUB-PHOTOSPHERE SOLAR ACTIVE REGION 3D MAGNETIC FIELD STRUCTURE FROM EMERGENCE OBSERVATIONS

by

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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF TABLES</td>
<td>iv</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>v</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>vii</td>
</tr>
<tr>
<td><strong>CHAPTER 1: INTRODUCTION</strong></td>
<td>1</td>
</tr>
<tr>
<td>OVERVIEW OF SOLAR ACTIVE REGIONS</td>
<td>1</td>
</tr>
<tr>
<td>OVERVIEW OF SOLAR ACTIVE REGIONS AND MAGNETIC FLUX EMERGENCE</td>
<td>4</td>
</tr>
<tr>
<td>STATEMENT OF WORK</td>
<td>7</td>
</tr>
<tr>
<td><strong>CHAPTER 2: DATA, OBSERVATIONS, AND PROCESSING</strong></td>
<td>9</td>
</tr>
<tr>
<td>SDO MISSION / HMI INSTRUMENT</td>
<td>9</td>
</tr>
<tr>
<td>ACTIVE REGION SELECTION</td>
<td>13</td>
</tr>
<tr>
<td>DATA SET SELECTION</td>
<td>19</td>
</tr>
<tr>
<td>DATA PROCESSING</td>
<td>21</td>
</tr>
<tr>
<td>3D VISUALIZATION</td>
<td>26</td>
</tr>
<tr>
<td><strong>CHAPTER 3: RESULTS</strong></td>
<td>32</td>
</tr>
<tr>
<td>OBSERVATIONAL DATA AND MODELING</td>
<td>32</td>
</tr>
<tr>
<td>3D VISUALIZATION OF MODELED ACTIVE REGION</td>
<td>43</td>
</tr>
<tr>
<td>EMERGENCE ANGLES AND HEIGHTS</td>
<td>45</td>
</tr>
<tr>
<td><strong>CHAPTER 4: INTERPRETATION OF RESULTS</strong></td>
<td>50</td>
</tr>
<tr>
<td>COMPARISON TO PREVIOUS OBSERVATION-BASED WORK</td>
<td>50</td>
</tr>
<tr>
<td>COMPARISON OF RESULTS TO EXISTING FLUX EMERGENCE MODELS</td>
<td>52</td>
</tr>
<tr>
<td><strong>CHAPTER 5: SUMMARY</strong></td>
<td>66</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>69</td>
</tr>
</tbody>
</table>
LIST OF TABLES

Table 1 – Polarity tilt angles vs. flux tube rise velocity ................................................................. 49
Table 2 – AR flux and geometric parameters summary ................................................................. 67
### LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1</td>
<td>Sunspot groups as viewed in the visible spectrum (image credit: NASA/Solar Dynamics Observatory)</td>
<td>1</td>
</tr>
<tr>
<td>Figure 2</td>
<td>Solar coronal loops (image credit: NASA/Solar Dynamics Observatory)</td>
<td>2</td>
</tr>
<tr>
<td>Figure 3</td>
<td>Flare and coronal mass ejection (image credit: NASA/Solar Dynamics Observatory)</td>
<td>3</td>
</tr>
<tr>
<td>Figure 4</td>
<td>Aurora viewed from the International Space Station (image credit: NASA/ISS Expedition 23)</td>
<td>4</td>
</tr>
<tr>
<td>Figure 5</td>
<td>Flux tube evolution (D’silva &amp; Choudhuri 1993)</td>
<td>5</td>
</tr>
<tr>
<td>Figure 6</td>
<td>Mean observed AR tilt angles (y-axis) vs. latitude (x-axis) (Hale, et al. 1919)</td>
<td>6</td>
</tr>
<tr>
<td>Figure 7</td>
<td>Emergence of flux into the corona (Zwaan 1985)</td>
<td>7</td>
</tr>
<tr>
<td>Figure 8</td>
<td>SDO spacecraft (Pesnell, et al. 2012)</td>
<td>10</td>
</tr>
<tr>
<td>Figure 9</td>
<td>AIA (171 Å) and HMI (magnetogram) imagery (image credit: NASA/Solar Dynamics Observatory)</td>
<td>11</td>
</tr>
<tr>
<td>Figure 10</td>
<td>HMI instrument (HMI Team 2012)</td>
<td>12</td>
</tr>
<tr>
<td>Figure 11</td>
<td>HMI Optical layout (Schou, et al. 2012)</td>
<td>13</td>
</tr>
<tr>
<td>Figure 12</td>
<td>Candidate AR 11149</td>
<td>14</td>
</tr>
<tr>
<td>Figure 13</td>
<td>Candidate AR (not numbered)</td>
<td>15</td>
</tr>
<tr>
<td>Figure 14</td>
<td>Candidate AR 11184</td>
<td>16</td>
</tr>
<tr>
<td>Figure 15</td>
<td>Candidate AR 11250</td>
<td>16</td>
</tr>
<tr>
<td>Figure 16</td>
<td>Candidate AR 11282</td>
<td>17</td>
</tr>
<tr>
<td>Figure 17</td>
<td>Candidate AR 11311</td>
<td>18</td>
</tr>
<tr>
<td>Figure 18</td>
<td>Candidate AR 11318</td>
<td>18</td>
</tr>
<tr>
<td>Figure 19</td>
<td>Candidate AR 11416</td>
<td>19</td>
</tr>
<tr>
<td>Figure 20</td>
<td>AR 11416 evolution with polarity peaks (x symbols) and centroids (+ symbols) overlaid</td>
<td>20</td>
</tr>
<tr>
<td>Figure 21</td>
<td>Magnetogram prior to (left) and after (right) derotation</td>
<td>21</td>
</tr>
<tr>
<td>Figure 22</td>
<td>Derotated and cropped reference image</td>
<td>22</td>
</tr>
<tr>
<td>Figure 23</td>
<td>AIA 4500 imagery of AR 11416</td>
<td>24</td>
</tr>
</tbody>
</table>
Figure 24 – Quiet sun histogram: (a) full scale, (b) zoomed to max=20 ....................................... 25
Figure 25 – 3D visualization of datacube ...................................................................................... 27
Figure 26 – Image slices as datacube inputs ................................................................................ 28
Figure 27 – Datacube viewed from +Y (left) and –Y (right) ......................................................... 29
Figure 28 – Datacube viewed from +Z .......................................................................................... 30
Figure 29 – Datacube with polarity peak values (left) and center/centroid (right) traces inlaid... 30
Figure 30 – Line-of-sight magnetic flux by polarity versus time .................................................. 32
Figure 31 – Polarity separation versus time................................................................................... 34
Figure 32 – Polarity separation speed versus time....................................................................... 36
Figure 33 – Tilt and radius of bipolar AR with flux peaks (x symbols) and flux centroids (+ symbols) overlaid........................................................................................................................... 37
Figure 34 – Tilt angle evolution ..................................................................................................... 38
Figure 35 – Center motion versus time .......................................................................................... 39
Figure 36 – Apparent motion of center relative to centroids ......................................................... 42
Figure 37 – Apparent motion of centroids (“_bar” entries) versus peak fluxes............................. 43
Figure 38 – Quality of fit for center and centroids (green lines) ................................................... 44
Figure 39 – Polarity radius vs. distance for v=0.1 km/s ................................................................ 46
Figure 40 – 3D contour plot and centroid trace of flux tube with displacement on z axis (0.1km/s rise rate assumed)......................................................................................................................................... 46
Figure 41 – 3D contour plot and centroid trace of flux tube with displacement on z axis (0.1km/s rise rate assumed) ......................................................................................................................................... 47
Figure 42 – Angle of flux emergence (centroids) .......................................................................... 48
Figure 43 – Flux rope shape and evolution of McMath 13043 (Tanaka 1991) ............................. 51
Figure 44 – Proper motion of polarity tracks and inferred 3D structure (Leka, et al. 2003) ......... 52
Figure 45 – Observed tilt vs. latitude vs. for thin flux tubes of different magnetic field strength (D'Silva & Choudhuri 1993) with overlays for AR 11416 tilt (×: centroid tilt, +: peak tilt) ..... 54
Figure 46 – Flux tube apex migration with time (left) and polar visualization (right) (Caligari, et al. 1995).............................................................................................................................................................. 55
Figure 47 – Results of thin flux tube simulation for origins in subadiabatic convection zone region (asterisks) and convective overshoot region (squares) (Caligari, et al. 1995) .................... 56
Figure 48 – Abbett, et al. (2000) simulated magnetograms vs. observed................................. 58
Figure 49 – Abbett, et al., (2000) simulated magnetogram for high twist flux tube compared with observation ................................................................. 59
Figure 50 – Resultant flux tube shape from Abbet, et al (2001) (rotation in +x direction) .......... 60
Figure 51 – Low Negative Twist case results (Fan 2008) .................................................. 62
Figure 52 – Shear velocity in an emergent flux tube (Manchester, et al. 2004) ............... 64
ABSTRACT

DETERMINATION OF SUB-PHOTOSPHERE SOLAR ACTIVE REGION 3D MAGNETIC FIELD STRUCTURE FROM EMERGENCE OBSERVATIONS

Brian Briggs, MS

George Mason University, 2012

Thesis Director: Dr. Jie Zhang

In this thesis, we study the emergence process of a bipolar solar active region NOAA 11416, a Hale class \( \beta \gamma \beta \) active region, and reconstruct its global 3D magnetic structure through a novel image-stacking technique. The emergence began on 8 February 2012, and data was taken through 11 February 2012. Magnetograms from the Solar Dynamics Observatory’s (SDO) Helioseismic and Magnetic Imager (HMI) are used in this study, and we take advantage of the unprecedented high imagery cadence offered by the observatory. We describe the selection process of observed candidate active regions covering the full data set since SDO first light. Using this magnetogram imagery, we visualize the detailed 3D magnetic structure by stacking the high-cadence SDO imagery and producing 3D isosurface plots, which yield extraordinary detail of the fine magnetic structure of the AR. The 3D structure shows a distinct “asymmetric \( \Lambda \)” shape to the emerging flux tube with tilt characteristic of Joy’s Law. This “asymmetric \( \Lambda \)” shape exhibited a differing slope of the leading and trailing legs, the leading leg being 61\(^\circ\) and the trailing leg 52\(^\circ\) for an assumed rise velocity of 100 m/s. Close examination of the 3D structure indicates a highly fragmented flux rope whose organization appears to increase as the eruption progresses. We also find that the leading polarity is more fragmented in magnetogram imagery, which appears to
support thin flux tube approximations and the results of analastic magneto-hydrodynamic simulations. Continuum imagery, however, shows the opposite situation, with the leading polarity being the more cohesive of the two. Additionally, the extracted AR parameters for center motion, tilt, polarity centroid separation, and flux per polarity are fit to functions, producing mathematical formulations for the 3D shape and magnetic flux of the flux tube. These functions not only accurately describe the shape and strength of the flux tube, but their exponential nature allows predictions of end points in terms of flux ($\sim 8 \times 10^{21}$ Mx), centroid separation (59.0 Mm), and centroid tilt angle ($18.5^\circ$). We find that both tilt angle evolution and movement of the center fit well to underdamped harmonic oscillator equations. The former indicates interplay between the Coriolis force and shear forces resulting from differing Lorentz forces beneath and above the photosphere, producing an initially anti-Joy’s-law tilt which rapidly changes to tilt following Joy’s law, overshoots, then settles back towards the final value. The center motion’s underdamped “overshoot, then settle” behavior, on the other hand, suggests interplay between magnetic tension of the rising flux tube and convective turbulence.
CHAPTER 1: INTRODUCTION

Overview of solar active regions

Solar active regions, or sunspots as they are more commonly known, are regions of intense magnetic field piercing the surface of the sun. The magnetic field is so strong (on the order of $10^3$ Gauss at the solar surface), that convection is inhibited, producing a significantly cooler section of the photosphere. This, in turn, produces a darker sunspot when viewed in the visible spectrum (shown in Figure 1).

Figure 1 - Sunspot groups as viewed in the visible spectrum (image credit: NASA/Solar Dynamics Observatory)
A closer look at the corona, particularly in the ultraviolet, shows that these regions of strong magnetic field expand into the solar atmosphere, producing pronounced looping structures where plasma tracks along the magnetic field lines (see Figure 2). This plasma is in turn heated by the magnetic energy in the magnetic structures, causing them to glow in the ultraviolet and soft X-rays.

Figure 2 – Solar coronal loops (image credit: NASA/Solar Dynamics Observatory)

Apart from their striking visible structure, solar active regions harbor a dark side. Unstable active regions are the source of some of the most energetic events in the solar system: solar flares and coronal mass ejections. These occur as a result of magnetic reconnection events in highly
convoluted active regions, which release stored magnetic energy within these loops in the form of hard X-ray radiation flares and/or large expulsions of solar plasma into deep space, called coronal mass ejections (CMEs).

![Figure 3 – Flare and coronal mass ejection (image credit: NASA/Solar Dynamics Observatory)](image)

These phenomena are of great scientific and engineering interest, as these events can result in geomagnetic and solar radiation storms which can adversely impact terrestrial communications, operation of spacecraft, and the health of astronauts beyond earth’s protective atmosphere and magnetosphere. Also, the charged particle clouds of earth-directed CMEs interact with the earth’s magnetic field and collide with the upper atmosphere at high latitudes, resulting in the remarkable glows of the Aurorae Borealis and Australis (see Figure 4).
The logical conclusion is that to understand the sun is to understand these phenomena, collectively known as space weather, and their impacts on the solar system and mankind at large. By extension, understanding the mechanics of our star serves to further the understanding of stars throughout the universe.

**Overview of solar active regions and magnetic flux emergence**

Solar active regions are highly concentrated regions of magnetic flux whose origins lie below the photosphere. The source of these intense fields is widely accepted to be the base of the solar convection zone, near the interface with the core (Choudhuri 1995, Zwaan 1987, Fan 2009b). Here, strong differential rotation between the radiative core and the convection zone, as well as stratified differential rotation within the convective zone itself, results in the production of toroidal magnetic flux tubes. These tubes are estimated to have initial magnetic field strengths on the order of $10^5$ Gauss (Fan 2009b).
It is generally believed an instability eventually develops at a particular point along the toroidal tube, as magnetic pressure begins to force plasma moving out the core of the flux tube, producing a region of low plasma density. This reduction in density results in the flux tube becoming buoyant which produces a radial propagation of a portion of the toroidal flux tube through the solar convection zone, a process illustrated in Figure 5.

![Figure 5 – Flux tube evolution (D’silva & Choudhuri 1993)](image_url)

Once in the convective zone, the Coriolis force causes the rising flux tube to deflect slightly poleward. Along with the poleward deflection, the Coriolis force is also the reason erupted active regions exhibit a tilt relative to the solar equator, with the leading polarity tilted towards the equator per Joy’s law (Hale, et al. 1919).
Figure 6 – Mean observed AR tilt angles (y-axis) vs. latitude (x-axis) (Hale, et al. 1919)

The reader should be cautioned, however against strict adherence to Figure 6, as this curve represents a statistical average and there is significant spread as a result of phenomena such as variation in active region magnetic field strength, vertical and helical convective turbulence, and even possible disconnection from the toroidal ring at the base of the convection zone (D'Silva & Choudhuri 1993, Longcope & Fisher 1996, Longcope, et al. 1998, Tóth & Gerlei 2003). Rather, we consider the “weak form” of Joy’s Law, in that this relationship is more qualitative than quantitative, to hold in nearly all cases.

Finally, we observe an eruption of flux into the solar corona, where the lower density plasma allows for dramatic expansion of the flux tube (illustrated schematically in Figure 7, shown in Figure 2).
Statement of work

While solar active regions have been observed and studied for about one century, our knowledge about its origin below the Sun and its emergence onto the surface remains rather limited. The aim of this thesis is to improve our understanding about solar active regions through a detailed study of the 3D magnetic topology and a sophisticated analysis of the emergence process using the latest unprecedented observations from the Solar Dynamic Observatory (SDO).

The methods in this thesis will be used to determine the overall shape of the structure (Ω-like or Λ-like), magnetic flux, motion of polarities, and the time evolution of the tilt of an active region. Additionally, we will employ a novel visualization technique to observe the fine magnetic structure of the flux tube based on its presentation in high-cadence magnetograms.

With these analyses and powerful 3D visualization techniques, we aim to confirm or refute numerous models of solar magnetic flux evolution and emergence processes within the convection zone, through the photosphere, and into the corona by comparing our findings with
output of thin flux tube, anelastic MHD, and ideal MHD numerical models. The tools developed and employed in this thesis can serve as the basis for detailed comparison of active regions to future numerical modeling efforts as well.
CHAPTER 2: DATA, OBSERVATIONS, AND PROCESSING

**SDO mission / HMI instrument**

**SDO Mission and Spacecraft**

The SDO spacecraft is a sun-watching spacecraft located in geosynchronous orbit at 102°W longitude. The mission and spacecraft was developed and managed by NASA/Goddard Space Flight Center as part of NASA’s Living With a Star program. The spacecraft was launched on 11 February 2010 and commissioned shortly thereafter, just before the onset of solar activity at the beginning of the current solar cycle. Its purpose is to study solar activity with the goals of achieving a greater understanding of the mechanisms that drive this activity. Its instruments observe the solar corona and visible magnetic structure in detail with the goal of uncovering clues to the underlying solar dynamo and mechanisms producing space weather (Pesnell, et al. 2012).
The SDO spacecraft has an instrument suite consisting of the Atmospheric Imaging Assembly (AIA), led by the Lockheed Martin Solar Astrophysics Laboratory, the Helioseismic and Magnetic Imager (HMI), led by the Stanford University Solar Physics Research Group, and the Extreme Ultraviolet Variability Experiment (EVE), led by the University of Colorado: Boulder Laboratory for Atmospheric and Space Physics. The AIA instrument observes the solar surface and corona in 10 different wavelengths. The HMI instrument observes the magnetic field on the photosphere. The EVE instrument images and monitors the UV output of the sun.

The placement of SDO in geosynchronous orbit allows for an unprecedented amount of data to be downlinked from a space-based solar observatory. Currently, SDO downlinks approximately 1.5
terabytes of data and high-resolution imagery at cadences 10s for AIA, 45s for HMI, and 10s spectrographic data for EVE (Pesnell, 2012).

Figure 9 – AIA (171 Å) and HMI (magnetogram) imagery (image credit: NASA/Solar Dynamics Observatory)

**HMI Instrument**

The Helioseismic and Magnetic Imager instrument is of prime interest for the current study. The HMI instrument focuses on wavelengths at and immediately around the Fe I absorption line (6173.3 Å). These filters allow for Doppler velocity measurement. Additionally, rotating waveplates allow measurement of the Stokes polarization parameters of the incoming light. These parameters allow for detailed measurement of line-of-sight and vector magnetic field components (see Zwaan 1987 for a detailed treatment of this). These magnetic field measurements can be used
to formulate a magnetogram, an image of the magnetic field strength at the photosphere (see Figure 9).

A cutaway of the HMI instrument and its optical components are shown in Figure 10 and Figure 11.

Figure 10 – HMI instrument (HMI Team 2012)
The HMI instrument is an evolution of the Michelson Doppler Imager (MDI) instrument that flew aboard the Solar and Heliospheric Observatory (SOHO) spacecraft. The HMI instrument affords improved resolution (0.5 arcsec/pixel versus 4) for a similar field of view (>2000 arcsec versus 2040), and imagery cadence (45s versus 90m) (Shou, et al. 2012).

**Active region selection**

This study initiated with a thorough search for active regions whose emergence was visible to the HMI instrument during the current solar cycle. More specifically, active regions with a distinct bipolar character were the goal of this search. As SDO is a relatively new observatory, having been commissioned in 2010, the data set was somewhat limited, although arguably of better
quality than previous space-based observatories. We used the ESA JHelioViewer tool to search through the HMI data set, looking for ARs that showed presentation on the magnetogram and continuum imagery, as often magnetic features masquerading as an AR on a magnetogram are merely surface effects. Continuum imagery verifies the presence of a visible sunspot which is more likely to represent a flux tube which initiated deeper within the sun.

The following is a listing of the ARs examined as identified by their National Oceanic and Atmospheric Administration (NOAA) designation. We present their dates of initial emergence observation and comment on their adequacy for the study. All images are magnetograms from the HMI instrument unless otherwise noted.

**11149** (1/20/2011) – While this AR exhibited a clear dominant negative and positive polarity, it was found to be a poor candidate on the grounds that there were numerous smaller polarities of opposite sign in the vicinity of the dominant polarities. Additionally, it emerged quite close to an existing active region 11147, which further would compromise the cleanliness of this study (see Figure 12).

![Figure 12 – Candidate AR 11149](image)

14
**Not Numbered** (2/15/2011) – This spot emerged just to the north west of AR 11161 but was not numbered. AR did begin to present as a bipole, however it exhibited a short lifetime (2 days). This suggests the bipole observed was more likely an ephemeral region (a result of surface effects) (Zwaan 1987) and not an AR produced by a toroidal flux tube. As it turns out, this short lifetime would have been problematic from the start, since it would have required a meridonal crossing to be useful to us. See *Error! Reference source not found.*.

![Figure 13 – Candidate AR (not numbered)](image)

**11184** (4/2/2011) – This AR initially appeared to be a good bipole, however its proximity to a strong intranetwork field could result in contamination of the study. Additionally, the leading polarity exhibited a mixed state towards the end of its emergence (see Figure 14).
**Figure 14 – Candidate AR 11184**

11250 (7/9/2011) – AR 11250 ended up being a decent quality AR. Unfortunately, its emergence began quite close to the edge of the solar limb. This will undoubtedly produce questionable magnetic field values, even after de-rotation, due to the line-of-sight nature of the magnetograms produced by the HMI instrument. As such, this AR was ultimately discarded.

**Figure 15 – Candidate AR 11250**

11282 (8/28/2011) – This AR produced a rather fascinating emergence, in that an initial bipole emerged, followed shortly thereafter by another bipole centered within the original. This bipole eventually expanded to merge with the first bipole. While this is potentially interesting in that it
may suggest a single, but fragmented flux tube emergence, use of this AR was decided against. Furthermore, its proximity to AR 11277 harbored further potential difficulties in segregating AR features during data processing.

![Figure 16 – Candidate AR 11282](image)

**11311** (10/3/2011) – AR 11311 was a good, quality AR and considered a valid candidate. The leading positive polarity was highly coherent, and although the trailing negative polarity was considerably more diffuse, it was still a good option (see Figure 17).
11318 (10/11/2011) – This AR was another high quality candidate for the study.

11416 (2/7/2012) – This AR was also found to be a high quality candidate bipolar AR, especially so since it proved to be considerably larger than 11311 and 11318. Ultimately, this AR was selected under the premise that a larger more coherent bipole would potentially yield finer details of the AR’s structure.
Our goal for selecting a data set was to obtain a series of magnetograms that encompass the quiet sun just prior to emergence, the initial emergence of the flux tube, a meridional crossing, and ending after the emergence appears to visibly approach a steady state condition. Our intent is to only capture enough of the emergence to process the data and model the AR emergence; a fully emerged flux tube in a steady state condition does not add utility.

With this in mind, we bounded by the dates and times for the data set of interest from 23:39 on 7 February 2012 (approximately 6 hours prior to visible cues of flux emergence) to 20:30 on 11 February 2012 (times in GMT). The active region crossed the meridian at 20:28 on 11 February, 2011. An image sequence of this emergence (after processing discussed later) is shown in Figure 20.
Next, we obtained the imagery from the Joint Science Operations Center (JSOC) at Stanford University. The JSOC has multiple levels of images available to the public. For this study, we have selected the Level 1.5 magnetogram images, which are the result of the Level 0 imagery (raw images) having been flat and bad pixel reduced to produce the Level 1 images, then further processed to produce the final magnetogram images. The images were obtained in Flexible Image Transport System (FITS) format, which is lossless and preserves precise pixel intensity information (in the case of the HMI magnetograms, the pixel intensity values represent magnetic field strength in Gauss). We took advantage of the high temporal resolution afforded by the observatory and obtained a data set with 45s cadence, extracting 7426 images.
**Data processing**

Data processing of imagery was performed using the Solar Software (SSW) suite of routines for IDL, developed collectively by the solar physics community and maintained by the Lockheed Martin Solar and Astrophysics laboratory.

**Derotation and Cropping**

Once the imagery was obtained, the first step was to de-rotate the images using the SSW drot_map routine. This process takes a full disk FITS image and applies appropriate geometric correction for solar curvature as well as the differential rotation of the photosphere. The output of this routine is a projection of the known curved surface onto a rectangular surface, similar to the process used to produce Mercator projections of spherical bodies, but corrected for photospheric differential rotation. This routine requires a reference image, for which we use the image taken at the time of crossing the meridian. The effects of derotation can be seen in Figure 21.

![Figure 21 – Magnetogram prior to (left) and after (right) derotation](image)

As HMI FITS data comes in 4096x4096 pixel arrays, operations on arrays this size are highly expensive computationally. Since we no longer require the full solar disk, and only the AR in
question and its immediate surroundings is of interest, the derotated arrays were cropped down to 596x398 before being written back to output FITS images. A view of a cropped and derotated image is shown in Figure 22; in this figure, the reference image is shown.

![Figure 22– Derotated and cropped reference image](image)

**Construction of 3D Datacube**

The next step in the process was to construct a 3D datacube. This involved the importing of the above processed FITS images as 2D arrays into IDL, then stacking them into a 3D array. This produces an array with dimensions of pixels (which can later be converted to distance) on the x and y axes and image number (which can later be converted to time) on the z axis. This datacube was used for all future image processing steps.
Ultimately, the computational expense of using all of the previous images, as well as the cropped 596x398 image dimensions, proved undesirably cumbersome to work with. Rather, the code was adjusted to further crop the pictures down to a final 409x255 pixel array, and only every 15th image was used. The final datacube size was 409x255x495, which corresponds to a 148Mm x 92Mm section of the photosphere with a 675s time step resolution.

This datacube was then exported to Visualization Toolkit (.VTK) format for 3D visualization (discussed in the next section) prior to executing the following processing steps.

**Finding of Polarity Maximum Field and Centroid Locations**

Each slice of the datacube (i.e. each newly-cropped image) was searched for maximum and minimum value locations, corresponding to the maximum positive and negative polarity value within the slice.

While it is tempting to use the maximum field value as the center point of a polarity, it becomes obvious when looking at the reference image (and later when visualized in 3D) that the polarities aren’t clean, symmetric tubes of flux. Rather, there is significant fragmentation and dispersion of the flux tube within the field of view, especially with regards to the leading polarity, which we will see in more detail in the 3D Visualization section. As an aside, this is in contrast to the continuum imagery, which shows a more cohesive leading polarity and more fragmented trailing polarity (see Figure 23).
Thus, there is strong desire to find a centroid of flux (or, the magnetic-flux-weighted centroid), which we expect to be more descriptive of the effective geometric center of the emerged flux, and which we will use in our analysis. This is given by:

\[
x = \frac{\sum_{n} x_n \cdot B_n}{\sum_{n} B_n}
\]

(1)

\[
y = \frac{\sum_{n} y_n \cdot B_n}{\sum_{n} B_n}
\]

(2)

Where:
- \( x \) is the x-coordinate of the centroid
- \( y \) is the y-coordinate of the centroid
- \( n \) is the pixel number
- \( x_n \) is the x-coordinate of the pixel under examination
- \( y_n \) is the y-coordinate of the pixel under examination
- \( B_n \) is the magnetic field value for the pixel under examination
Note that two such sets of computations are performed, one to locate the positive polarity centroid, and one to locate the negative polarity centroid.

This computation, however, was not completely straightforward. Because it effectively sums over the entire frame, quiet sun features can bias the positioning of the centroids towards the center of the frame. Thus, a cutoff value had to be specified, whereby pixels with a value below this cutoff were discarded.

To decide an effective cutoff value, we examined a histogram of the first frame in the sequence, which was well prior to the initial signatures of flux emergence. From here, a cutoff value of 200G was selected, which captures 99.9% of quiet sun features. While this value no doubt removes magnetic field measurements of the AR itself, it prevents quiet sun features from being counted towards the AR. Interestingly enough, while the peak in the distribution was at 0G (as we would expect), the histogram showed a distinct bias towards the positive polarity values, which was the driver for the cutoff value selected (see Figure 24). This bias is likely due to the relatively weak, but predominantly positive intra-network field in the vicinity of the AR.

Figure 24 – Quiet sun histogram: (a) full scale, (b) zoomed to max=20
Once the polarity maximum and polarity centroid locations were found, the locations were exported to a comma-separated file format (.CSV), and each slice of the datacube was appended with a visible marker to identify them throughout the time sequence. A selection of slices is shown in Figure 20.

**Magnetic Flux Computation**

The flux calculation is given by:

\[ \Phi = \sum_{n} B_n \cdot A_{\text{pix}} \]  

(3)

Where:
- \( \Phi \) is the magnetic flux (in Maxwells)
- \( B_n \) is the magnetic field value at a given pixel (in Gauss)
- \( A_{\text{pix}} \) is the area of the pixel (in square centimeters)
- \( n \) is the pixel number

Note that, once again, the above summation only includes pixels whose magnetic field values that are outside of the cutoff range, and a \( \Phi \) value is computed for both positive and negative polarities. \( A_{\text{pix}} \) is determined from the ancillary values RSUN_REF, RSUN_OBS, CDELT1, and CDELT2 stored in the FITS headers, corresponding to the reference radius of the sun (in meters), the observed radius of the sun (in arcseconds), and image scales in the x- and y-directions, respectively. The area size of each pixel is \( 1.30 \times 10^7 \text{cm/pixel} \).

**3D Visualization**

We now return to the output of the VTK export of the datacube discussed in the Construction of 3D Datacube section. This format allows us to import the data into Kitware’s Paraview software, which allows 3D visualization of data sets such as this. Once imported into Paraview, the
datacube can be visualized by producing a contour of an isosurface formed by pixel values of a certain amount. For this study, we formed this contour for isosurfaces of value +800G and -800G, although this selection was somewhat arbitrary; we found this value to be most illustrative of the intricacies of flux tube shape while eliminating unrelated quiet sun features. Furthermore, Paraview enabled a coloring of flux tube by value; in our case, red corresponds to the positive polarity and blue corresponds to the negative polarity. Figure 25 shows these isosurfaces in 3D, revealing much about the 3D structure of the AR as it emerges.
To help understand what we are seeing in Figure 25, the reader is directed to Figure 26. Recall that each z-coordinate value corresponds to a different time slice, and thus a different magnetogram image. Figure 26 shows how each slice can be used to generate the isosurface we see in Figure 25.

![Figure 26 – Image slices as datacube inputs](image)

The reader is again cautioned that the z-axis of the datacube is time, whereas the x and y axes are displacement. Also recall that the HMI instrument produces line-of-sight magnetograms, and so horizontal magnetic field components show up as zero. This is part of the reason why the polarity arches in Figure 25 (and beyond) appear to be thinning and disconnected at the top of the
emerging loop. Although Paraview enables us to lower the magnetic field value to produce a more complete loop, the 800G value chosen produces the clearest views of the structure.

On the following pages, we show Figure 27, which shows the contour plots from each direction along the y-axis (i.e. non-isometric views). Figure 28 shows the datacube from “above” the loop. Figure 29 shows the semi-transparent contour plot overlaid with the peak flux location traces, the centroid location traces, and the computed center of the centroids.
Figure 28 – Datacube viewed from +Z

Figure 29 – Datacube with polarity peak values (left) and center/centroid (right) traces inlaid
The contour views in 3D provide striking detail of the filamentary and fragmented nature of the AR.

The left panel in Figure 29 (polarity peak locations) shows an interesting feature of the AR: the polarity trace jumps approximately halfway into the emergence from one filament to another. When closely examining the contour, this is an obvious case of a fragmented flux tube where a weaker fragment initially emerges but is eventually taken over by and merges with a stronger, slower moving fragment behind it. This is also a good example of why use of centroids (right panel of Figure 29) are better media to describe the overall behavior of the AR, even if they do appear to be offset from the peaks at times.
CHAPTER 3: RESULTS

Observational Data and Modeling

Flux evolution of individual polarities

From the results of the section entitled Magnetic Flux Computation, the total magnetic flux can be seen in Figure 30, broken out by polarity, overlaid with a functional fit.

![Figure 30 – Line-of-sight magnetic flux by polarity versus time](image)
Initially we see what appears to be an asymptotic lift initially, followed by a steady climb in flux. This is believed to be due in part to the fact that only line of sight flux can be detected. (TBD – table with total flux injected, flux injection duration, injection rate)

Three different functional forms were explored to fit the magnetic flux evolution curves: $\Phi(t) = \arctan(t)$, $\Phi(t) = \tanh(t)$, and the logistic function $\Phi(t) = 1/[1+\exp(-t)]$. All three functions were able to fit the data with a determination coefficient ($R^2$) value of >0.99. Ultimately, the hyperbolic tangent form, with an $R^2$ value of 0.998 for the positive polarity and 0.997 for the negative polarity, was selected, shown in its exponential form and parameterized as follows:

$$
\Phi(t) = a \cdot \frac{e^{2[b(t+\delta)]} - 1}{e^{2[b(t+\delta)]} + 1} + c
$$

(4)

Where:

- $a = 4.02 \times 10^{21}$ Mx (positive polarity), $-3.86 \times 10^{21}$ Mx (negative polarity)
- $b = 0.0381$ hr$^{-1}$ (positive polarity), 0.0398 hr$^{-1}$ (negative polarity)
- $c = 3.87 \times 10^{21}$ Mx (positive polarity), $-3.64 \times 10^{21}$ Mx (negative polarity)
- $\delta = -50.4$ hr (positive polarity), -49.2 hr (positive polarity)

The exponential nature of the function allows us to determine the end point of photospheric magnetic flux for this bipole, which corresponds to the addition of the $a$ and $c$ parameters above. This value of $\sim 8 \times 10^{21}$ Mx falls under the umbrella of a large active region as defined by Zwaan (1987) and Cheung, et al (2010).

One can also view the $2b$ combination to be an inverse time constant of sorts, representing an e-folding time of 13.1 hrs and 12.6 hours for the positive and negative polarities respectively, however one should be cautioned that this represents an e-folding time starting at the function’s inflection point (located at $-d$, or $\sim 50$ hr for each polarity).
**Polarity separation**

When considering polarity separation, we define this as the distance from the simple geometric center between the two centroid locations, which is calculated for every frame and thus moves with time. Its definition here should be regarded as analogous to a radius (i.e. ½ of the total centroid-to-centroid distance). As such, the polarity separation with an overlaid curve fit shown in Figure 31 and is identical for both centroids. As data prior to 6hr was unreliable, it is truncated here. Specifically, the negative polarity centroid location could not be resolved prior to this time using the cutoff value from the Finding of Polarity Maximum Field and Centroid Locations section, and the positive polarity centroid location was erratic as well. As such, the resultant calculated separation value was meaningless.

---

**Figure 31 – Polarity separation versus time**
The polarity separation tends to follow an exponential function:

\[
\frac{r_f - r(t)}{r_f - r_0} = e^{-\frac{t(1+\delta)}{\tau}}
\]  

(5)

Where:

- \( r_0 = -28.3 \text{ Mm} \)
- \( r_f = 59.0 \text{ Mm} \)
- \( \delta = -2.35 \text{ hr} \)
- \( \tau = 76.9 \text{ hr} \)
- \( t: \text{ time (hours)} \)

This should not be surprising, since we are effectively dealing with a slowing emergence of a loop shaped structure. While this effectively produce a “kink” at the \( r=0 \) point (which would occur at \( t=-\delta \)), functions that are vertical at \( r=0 \) (e.g. \( r \sim t^{1/2} \) functions) have limits at \( \infty \). Like the damped oscillator fit for the x-coordinate of center motion, this function asymptotically decays and predicts an end point for the eventual maximum separation (the \( r_f \) parameter). For reasons identical to the center position motion, these fits only apply to \( t \geq 6 \text{ hr} \). Quality of fit is measured by the coefficient of determination, which has the value \( R^2=0.996 \).

Using the fit of Equation 5, we can obtain the speed of separation by performing a simple \( dr(t)/dt \) calculation, the results of which are shown in Figure 32. Keep in mind these resultant speeds are from the geometric center between the two polarity centroids; the speed of one centroid relative to the other is double the values shown.
We begin by first defining tilt with respect to bipolar active regions. Tilt is defined as the angle between the solar equator and the line drawn between the polarities. As before, we choose the polarity centroids of the AR as opposed to the location of maximum flux. This is illustrated in Figure 33.
With tilt defined, we now show the evolution of the tilt angle in Figure 34, once again truncating results prior to $t=6$ hr, and once again overlaying the fitted function discussed below.
An interesting feature in the tilt angle is that it initially starts out opposite to that of Joy’s law (high negative angle with respect to the equator), but rapidly corrects by nearly 100°.

The tilt angle follows the functional form of an underdamped oscillator:

$$\theta(t) = c \cdot e^{-\zeta \omega_0 (t + \delta)} \cdot \left[ a \cdot \cos \left( \omega_0 \cdot \sqrt{1 - \zeta^2} \cdot (t + \delta) \right) + b \cdot \sin \left( \omega_0 \cdot \sqrt{1 - \zeta^2} \cdot (t + \delta) \right) \right] + d$$

(6)

Where:

- $a = 0.55$
- $b = -1.1$
- $c = -150^\circ$
- $d = 18.5^\circ$
- $\zeta = 0.96$
- $\omega_0 = 0.099$ rad/hr
- $\delta = -7.31$ hr
- $t$: time (hours)
An interesting feature of this fit is that it predicts the eventual end point of the x-coordinate evolution, should it be allowed to progress further, which is the parameter $d$ above (18.5°). Quality of fit is given by $R^2=0.966$.

The computation shows an initial sharp climb in tilt angle, followed by a peaking and then reversal in direction. This tilt angle motion is visible with careful study of both the video and/or Figure 20, where the centroids (and peaks, for that matter) appear to visibly trace a logistic curve. While it is tempting to interpret this using a spring-mass-damper analogue, we find in the final subsection of Chapter 4 that the physics involved are fundamentally different than a simple underdamped oscillator.

**Motion of Center**

The motion of the geometric center (along with its fitted curves) of the AR is shown in Figure 35.
Like the tilt angle evolution, the motion of the x-coordinate of the center point (that is, the point at the geometric center between the flux centroids) best approximates the functional form of an underdamped harmonic oscillator. However, the parameters are different:

\[
\begin{align*}
a &= 0.7 \\
b &= -0.5 \\
c &= -25.3 \text{ Mm} \\
d &= 5.07 \text{ Mm} \\
\zeta &= 0.5 \\
\omega_0 &= 0.0977 \text{ rad/hr} \\
\delta &= 0 \text{ hr} \\
t: & \text{ time (hours)}
\end{align*}
\]

There is a most interesting contrast with the tilt angle evolution in that the damping coefficient is noticeably smaller, much less than the near-critical damping of the tilt angle evolution. As before, the function predicts an eventual end value (the \(d\) parameter).

The \(y\)-coordinate does not appear to move much as compared to the \(x\)-coordinate, and can reasonably be approximated with a linear relationship:

\[
y = 2.76 \text{ Mm} \tag{7}
\]

These fits are only valid for \(t \geq 6 \text{ hr}\). Prior to then, the center position could not be accurately resolved.

The quality of the \(x\)- and \(y\)-coordinate fits are given by \(R^2 = 0.885\) and \(R^2 = 0.009\) respectively. Although the \(y\)-coordinate has a poor quality of fit, we will see later this has very little impact on the overall shape model due to the small magnitude of the errors. We should, however, note that the general shape of the data does seem to suggest an underdamped harmonic oscillator as well.

This underdamped oscillator form of the center motion is interesting, as it suggests the flux tube rising through the convection zone behaves much like a mechanical spring-mass-damper system.
In this system, we would view the “spring” as the magnetic tension of the flux tube, the “mass” being the mass of plasma displaced by the flux tube, and the “damper” being friction forces within the convection zone plasma. The “forcing function” appears to be likely the result of turbulence within the convection zone. The natural frequency of the oscillator in a mechanical spring-mass-damper system, $\omega_0$, is given by $(k/m)^{1/2}$, with $k$ being the spring constant of the system and $m$ being the mass. The very small value of $\omega_0$ suggests a large displaced plasma mass relative to the magnetic tension force, which we would expect in high density regions between the base of the convection zone and the photosphere.

**Apparent surface motion versus vertical emergence of 3D structure**

If we were to examine the apparent center and centroids surface location versus time, we would see an initially distorted period between 6hr and 18hr where the negative polarity appears to remain rather constant in position whereas the positive polarity makes a significant lurch away. From 18hr to 60hr, this motion reverses course slightly, with the expansion of the bipole slightly favoring the negative polarity. Thereafter, another course reversal occurs, and motion in favor of the positive polarity occurs. This is illustrated in Figure 36.
If, on the other hand, we examine the centroid motion relative to the location of peak fluxes, we clearly see that simply observing the centroid motion doesn’t tell the full picture. As is evident from a cursory examination of Figure 37, we see a discontinuity at 48hr. This discontinuity appears to be the result of a stronger flux fragment that had followed the initial emergence. What is also interesting is the fact that in both flux fragments, the (leading) positive polarity exhibits a noticeably larger displacement over time than the (trailing) negative polarity, both for the initial and following flux tube. As is evident from the 3D visualization earlier (and viewing the associated video), the weaker fragment eventually merges with the stronger fragment.
When the above equations are combined, we return to Paraview to observe the quality of the fit in three dimensions. The curves of the center and centroids versus time are shown in Figure 38.
In general, we see a very good quality of fit ($R^2=0.978$ for the negative polarity, and $R^2=0.975$ for the positive polarity), except for the very start of emergence, where the coherence of the data itself noticeably breaks down. It is possible, if not likely, that the initial sweep of centroids and center at the top of the data cube is due to the imagery in question being closer to the western limb ($\sim 50^\circ$E apparent longitude at the start of emergence) than the meridian, considering we are dealing with a line-of-sight magnetic field detection.
**Emergence Angles and Heights**

We proceed to attempt to deduce the proper shape of the AR by converting the z axis from time to distance. A caveat of this process is that we have no exact way to measure the rise velocity of the flux tube, so we must therefore assume a velocity.

Reviewing literature of MHD simulations, we see a very large spread in computed rise velocities. Caligari, et al (1995) suggested a rise velocity of 0.5 to 1.0 km/s based on the results of his thin flux tube model. Fan (2008), on the other hand, suggested a slower rise rate of 0.1 km/s at the upper boundary of her simulation domain (16Mm below the photosphere) as part of the anelastic MHD simulations. Fan (2009a), when performing ideal MHD simulations at the photosphere, found a velocity of 2.6 km/s. Due to the fact that Fan (2009a) assumed a fairly weak flux tube and the anchoring of the tube shortly below the convection zone is unrealistic, a subsequent check of our using rise velocities at 1 km/s and above showing an extremely “sharp” AR, and the separation polarity separation speed was roughly 0.1 km/s, we opt to use a 0.1 km/s rise velocity.

We see in Figure 39 a total height of the section of the flux tube emerged prior to its emergence to be approximately 31Mm. The vertical scale was selected so as to provide the reader with a geometric perspective on the shape of the tube. Also, Figure 40 and Figure 41 show the apparent size of the contour from Figure 25 through Figure 29 when we consider a 0.1 km/s rise rate.
Figure 39 – Polarity radius vs. distance for \( v = 0.1 \) km/s

Figure 40 – 3D contour plot and centroid trace of flux tube with displacement on z axis (0.1km/s rise rate assumed)
Next we attempt to determine the angle of the flux tube polarities from the horizontal. Figure 42 shows the angle of the legs of the flux tube computed by taking the travel of the centroids between 5 data points, as the noise in every data point ends up producing a completely incoherent plot. Even with this smoothing, there exists quite a bit of “noise,” likely a product of convective turbulence.
Although the effect is difficult to see at first from the plots, it is possible to notice that the positive polarity remains at a higher angle, and thus produces more lateral travel relative to the starting point, than the negative polarity. This finding can be seen clearer in Figure 37 and Figure 39.

If we simply compare the initial and final locations of both polarities, we find that the leading polarity has an overall angle of $60.7^\circ$, and the trailing polarity has an overall angle of $30.3^\circ$ from the vertical. Since we have some uncertainty in the rise velocity, we present overall tilt angles for several rise velocities in Table 1.
### Table 1 – Polarity tilt angles vs. flux tube rise velocity

<table>
<thead>
<tr>
<th>Rise Velocity (km/s)</th>
<th>Pos. Polarity Leg Angle (degrees)</th>
<th>Neg. Polarity Leg Angle (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>86.8</td>
<td>-85.6</td>
</tr>
<tr>
<td>0.05</td>
<td>74.3</td>
<td>-69.0</td>
</tr>
<tr>
<td>0.10</td>
<td>60.7</td>
<td>-52.4</td>
</tr>
<tr>
<td>0.25</td>
<td>35.5</td>
<td>-27.5</td>
</tr>
<tr>
<td>0.50</td>
<td>19.6</td>
<td>-14.6</td>
</tr>
<tr>
<td>1.00</td>
<td>10.1</td>
<td>-7.4</td>
</tr>
</tbody>
</table>
CHAPTER 4: INTERPRETATION OF RESULTS

Comparison to previous observation-based work

Reconstruction of sub-photospheric flux rope shape

There has been little previous work with regards to attempts to reconstruct the 3D shape of emerging flux in active regions based upon emergence observations, however there are a few worth noting.

Tanaka (1991) closely studied a flare active $\delta$ group McMath 13043 in July of 1974, utilizing data from Big Bear, Okayama, Kitt Peak, and Mt. Wilson solar observatories. While Tanaka does not explicitly detail the cadence of observations, his presentation appears to suggest that of a one image per day set. Nevertheless, he quite intuitively deduced the subsurface structure of the highly twisted flux rope beneath the photosphere (reproduced in Figure 1).
Since the focus of this paper is on a stable, bipolar active region, it is difficult to draw a direct comparison between Tanaka’s work and that presented herein. However, it seems logical to conclude that the techniques presented for 3D visualization could have immensely benefited him, possibly confirming his inferences of the twisted flux rope’s shape.

Leka et al. (2003), on the other hand, used a technique largely analogous to the methods presented here. Using magnetograms from the Haleakala Stokes Polarimeter and the Imaging Vector Magnetograph at Mees Solar Observatory, the authors were able to reconstruct the shape of various sub-features of a magnetically complex active region NOAA 7260. The MSO instruments in question were able to produce spectroheliographs with a cadence of 20s, however the authors utilized imagery at 10 minute spacing. Although not the explicit goal of the paper, their analysis traced the paths of the peaks of individual polarities within the AR. One such trace is shown in reproduction in Figure 44.
Thus, the current techniques can be thought of as a furthering of the work of Leka, et al., i.e., not only tracing the path of the centers, but also fully reconstructing and visualizing the 3D structure.

**Comparison of results to existing flux emergence models**

**Thin flux tube models**

D’Silva and Choudhuri (1993) made use of the thin flux tube approximation to develop a model specifically for purposes of investigating the source of the tilt of active regions. In their work,
they simulated a flux tube whose rise begins at the base of the convection zone and rises under the influence of Coriolis force in a solid body rotation domain. In their study, they varied the magnetic field strength, drag (implying changes in diameter), and anchor footprint separation at the base of the tube.

Their primary finding was that in order to both buoyantly rise and produce tilt angles corresponding to those seen in observations, the flux tube needed a field strength of 60 to 160 kG. They also came to the conclusion that with decreases in footprint separation, magnetic tension of the loop plays an increasing role to slow the flux tube’s ascent, thus exposing it to more influence from the Coriolis force and producing a larger tilt.

Figure 45 shows the results of their computations versus observed values, with overlays for tilt angle of the AR of interest in this current study. We included the tilt angles for both centroid and peaks, since the tilt of the polarity peaks (darkest points) are more likely what others have considered as part of their observations.
Perhaps the most well-known thin flux tube analysis is that of Caligari, et al. (1995), whereby the authors performed numerical simulations using the thin flux tube approximation of the MHD equations. Their model was able to incorporate such effects as buoyancy, pressure, magnetic tension, drag, and rotational forces on a rising coherent tube of flux, although it was limited to ~0.95R\text{sun} due to the domain of the thin flux tube approximation.

Their model started with the initial assumption of a toroidal flux tube at either the convective overshoot layer just beneath the convection zone or the subadiabatic layer beneath the base of the convection zone. An artificially applied instability is generated at a point so as to become convectively unstable and begin to rise. The matter in the tube is found to drain downward as the tube rises, anchoring the flux tube at the base ahead of and behind the apex.
One feature of the result was an asymmetry that developed in tube, whereby the apex of the flux tube migrated in the direction opposite of rotation, developing an “asymmetric Λ” shape as shown in Figure 46. The authors attributed this due to conservation of angular momentum due to Coriolis force acting on a rising flux tube. Our observations see remarkable agreement with this prediction, especially with respect to the motions of the flux peaks (see the section entitled Apparent surface motion versus vertical emergence of 3D structure).

Another result produced by this model was the reproduction of the well known “Joy’s law” tilt of ARs with respect to the equator. While our observation matched the tilt in the qualitative sense (that is, leading polarity closer to the equator than the trailing), the tilt angle with respect to the centroids of 18.5º was very different for our emergence latitude of 17ºS than the results obtained from model runs originating in either the convective overshoot or the subadiabatic ranges. If, however, we consider the tilt angle of the peak fluxes (~8º as contrasted to the 18.5º obtained

---

**Figure 46 – Flux tube apex migration with time (left) and polar visualization (right)**

(Caligari, et al. 1995)
from use of centroids), which is probably more representative of historical observations, we see some agreement with the assumption of origins in the subadiabatic region of the convection zone (see Figure 47. Red X overlay denotes polarity tilt angle wrt peak fluxes; polarity tilt angle of centroids is off-scale at 18.5º). This, according to the authors, would suggest a somewhat weaker magnetic field than flux tubes anchored in the overshoot region.

![Figure 47](image)

**Figure 47 – Results of thin flux tube simulation for origins in subadiabatic convection zone region (asterisks) and convective overshoot region (squares) (Caligari, et al. 1995).**

**Anelastic 3D MHD models**

Although the thin flux tube approximation provides remarkable insight into the dynamics of the flux tube as a whole, it fails to answer questions regarding the fine structure of the flux tube. Specifically, it does not resolve the effects of plasma motions within the convection zone on the coherence of the tube or the presence or absence of twist (and effects thereof) in the tube. For
these, we turn to the anelastic approximation. Like the thin flux tube approximation, anelastic MHD breaks down just beneath the photosphere, where the speed of the rising flux tube ceases to be much lower than the local Mach number (Abbett, et al. 2000). While a notable limitation of the approximation, the computational expense of fully-compressible ideal MHD is cumbersome if not outright prohibitive. Much work on developing these models has occurred in the past decade or so, and we will examine a few of these relative to our observations.

Abbett, et al. (2000) employed this technique to examine the effects of twist on a tube of magnetic flux originating at the base of a rectangular, adiabatically stratified domain spanning 5.147 pressure scale heights. These simulations varied the lateral size of the computational domain, the initial length of the tube, the scale height, and the amount of twist initially present in the flux tube. The authors did not include convective turbulence or Coriolis force in their simulation.

Like previous studies, the authors came to the conclusion that a certain rate of twist is required to prevent fragmentation as the flux tube ascended. However, they also determined that the flux tubes shy of this critical twist value could remain coherent if the curvature of the rising loop was increased. Unfortunately for our observations, the contours of our flux tube do not outright suggest the presence of a twist in the field lines (nor reject it), at least not at the time of emergence. To answer this question, the usage of full vector data is needed.

The Abbett, et al. (2000) paper goes further on to show how their simulations at the upper boundary of their simulation would appear if viewed on a magnetogram. These results we can directly compare to our observations. Figure 48 shows the results of two of the twisted flux tube runs as compared to the observational results of the study.
The observations do not seem to clearly show the “tails” of the flux polarities, which was a feature of a rising, twisted flux tube as shown by the authors, although the “crescent” feature was certainly present. Also, in spite of the previous remark that our 3D contour showed no indication of twist, Abbett, et al., (2000) showed a final run that indicated a tilt on the magnetogram that was introduced as a result of a higher twist level ($q=0.25$), to which our observations bear some resemblance (see Figure 49).
The authors published another paper in the following year based on the same model, except this time with a modification of the anelastic momentum equation to include the noninertial \(-2\rho_0(\Omega \times \mathbf{v})\) term corresponding to effects of Coriolis force (Abbett, et al. 2001). Their simulation domain varied the latitude and peak flux of flux tubes rising from its base. Unlike their previous work, however, twist was not imparted onto the flux tube. The authors discovered that the Coriolis force alone was able to prevent the tube from developing transverse flows and fragmenting, alleviating the requirement for twist. This was a result of a strong axial flow along the flux tube, which prevented the flux tube from generating transverse (north-south) counter-rotating vortices resulting in a fragmentation of the flux tube seen in their 2000 work.

Another discovery from the newer model was that this axial flow was found to be counter the direction of rotation. This flow was also found to be stronger in the trailing polarity. With Coriolis force acting upon the asymmetry, the plasma moves preferentially towards the trailing
polarity, thus making it both more vertical and more coherent. This resulted in a shape not unlike our “asymmetric Λ” and that of the thin flux tube simulations performed by Caligari, et al (1995) (see Figure 50 for a reproduction of the “High Flux, Low Latitude” case results).

![Figure 50 – Resultant flux tube shape from Abbet, et al (2001) (rotation in +x direction)](image)

Their results also showed a migration of the leading polarity slightly more towards the equator than the trailing polarity due to Coriolis force, further confirming the physical basis of Joy’s Law and aligning with our model.

One of the authors of the above studies went on to publish another body of research in 2008 that took the next step by applying the model to a rotating, spherical shell domain (Fan 2008). This model simulated a solar convection zone in dimension, and the computational domain extended to 0.977R\text{sun}, where the anelastic approximation is predicted to break down.
This model produced a curious result, in that a flux tube would rise cohesively when the twist of the tube (right-handed in the northern hemisphere) was opposite the expected direction (left-handed in the northern hemisphere) relative to observations of active regions cited by the author, resulting in tilts opposite that of Joy’s law (Hale, et al. 1919). The Coriolis force, however, countered this force, pushing tilt back in the expected direction. Thus, the author concluded that a critical amount of twist opposite of that expected was required for both a cohesive rise while allowing Coriolis force to dominate and produce a proper tilt.

The simulations produced a significant amount of fragmentation of the flux tube during its rise during this “Low Negative Twist” case, losing significant amounts of flux during its ascent (reproduced in Figure 51), no doubt a result of the lower cohesion afforded by the lower twist.
This result has some interesting comparisons to our observations. First, we see a clear analogy in that we again see an “asymmetric Λ” shape encountered in the both the work of Caligari, et al. (1995), as well as in our observation. Also like Caligari, et al. (1995) and our AR, there is one more vertical leg of the flux tube, and one more inclined leg. We also see similarity in our AR in
that the more vertical leg is also the most cohesive, at least with respect to the magnetogram data (see contrasting white light image in Figure 23). Where we see a marked difference is that Fan’s (2008) model has this leg as the leading polarity, whereas our observations, the thin flux tube model of Caligari, et al. (1995), and the work of Abbet, et al (2001) both have this as the trailing polarity.

Fan (2008) also notes that there is an asymmetry in field strength between the leading and trailing polarities, with the leading polarity being 1.23x as strong as the trailing leg. We, too, see a similar disparity favoring the leading leg, although it is noticeably smaller (~1.14x). The author credits this to the asymmetric stretching of the legs producing a greater divergence of flow along the trailing leg (which is the less cohesive leg), driven by Coriolis force. A curious feature of our AR is that, if Fan’s logic were to hold, then it would be the trailing polarity (the more vertical and cohesive of the two) that should exhibit the greater peak field strength; clearly, this is not the case.

**Ideal MHD models at the photosphere**

As time has progressed, so too has the speed of computational resources, enabling computational modeling in the ideal MHD domain. This is of particular interest as modeling an emergence of flux through the photosphere into the corona is now possible.

We turn our attention to the work of Manchester, et al (2004), where a horizontal flux tube is simulated to rise from an initial location of 10 photospheric scale heights below the photosphere. Upon breaching the photosphere boundary, a significant shear flow develops which causes the polarities to initially emerge then separate along the latitude axis.
The authors attribute this shearing motion to the interaction of the Lorentz force with the gravitational stratification of the corona. That is, forces driving it to expand within the corona (whose density rapidly falls off) compete against those compressing it in a far denser solar envelope.

What is remarkable about this result is that it offers an explanation for the complex tilt angle evolution described in the AR Tilt Evolution section of this paper. Were the evolution due to Coriolis force alone, we would expect progression of tilt angle in a constant direction as the loop emerges. Instead, we clearly see the effects of this shearing force at work producing the reversal in tilt angle after the initial sharp climb.

Modeling efforts published by Fang, et al (2010) reveal similar results, even after adding detail. Here, the authors also used an ideal MHD model (the BATSRUS model developed by the
University of Michigan), although they included radiation terms, non-ideal equations of state, and an empirical coronal heating model.
CHAPTER 5: SUMMARY

In this study, we have presented a novel technique for visualization of active region emergence, whereby magnetograms acquired from the Solar Dynamics Observatory were stacked vertically to produce detailed contours of magnetic flux for active region NOAA 11416 as the region emerged. The same high time resolution magnetograms were further processed to yield an accurate figure for magnetic flux from the polarities. These data were further used to produce a detailed mathematical model of flux emergence for 3D shape, tilt, flux polarity positions, and motion of the AR for area-weighted flux centroid positions. We ultimately have determined the flux tube to exhibit an “asymmetric Λ” shape with a complex tilt angle evolution. A summary of relevant parameters can be found in Table 2. Comparisons to previous numerical simulations shows that this AR has much in common with thin flux tube approximations with respect to the overall shape and tilt of the AR, but comparisons with anelastic MHD simulations yield mixed results. Comparisons to ideal, compressible MHD numerical solutions show that shear force brought upon by differing Lorentz force contributions from sub-photospheric and coronal motions provide a good explanation for the complex tilt angle evolution of the AR.
Table 2 – AR flux and geometric parameters summary

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Flux</strong></td>
<td></td>
</tr>
<tr>
<td>Extrap. Peak Flux, + Polarity</td>
<td>$7.89 \times 10^{21}$ Mx</td>
</tr>
<tr>
<td>e-folding time¹, + Polarity</td>
<td>13.1 hr</td>
</tr>
<tr>
<td>Peak Flux Injection Rate, + Polarity</td>
<td>$1.53 \times 10^{20}$ Mx/hr</td>
</tr>
<tr>
<td>Extrap. Peak Flux, -Polarity</td>
<td>-$7.50 \times 10^{21}$ Mx</td>
</tr>
<tr>
<td>e-folding time¹, - Polarity</td>
<td>12.6 hr</td>
</tr>
<tr>
<td>Peak Flux Injection Rate, - Polarity</td>
<td>-$1.54 \times 10^{20}$ Mx/hr</td>
</tr>
<tr>
<td><strong>Polarity Separation</strong></td>
<td></td>
</tr>
<tr>
<td>Extrap. Max Polarity Separation</td>
<td>118 Mm</td>
</tr>
<tr>
<td>Max. Separation Velocity</td>
<td>0.408 km/s</td>
</tr>
<tr>
<td>e-folding time⁴</td>
<td>76.9 hr</td>
</tr>
<tr>
<td><strong>Tilt of Polarity Centroids</strong></td>
<td></td>
</tr>
<tr>
<td>Maximum</td>
<td>24.3°</td>
</tr>
<tr>
<td>Extrap. Final Angle</td>
<td>18.5°</td>
</tr>
<tr>
<td>Damping Constant of Oscillation</td>
<td>0.960</td>
</tr>
<tr>
<td>Natural Frequency of Oscillation</td>
<td>0.099 rad/hr</td>
</tr>
<tr>
<td><strong>Center Motion</strong></td>
<td></td>
</tr>
<tr>
<td>X-Coord. Max Travel</td>
<td>10.9 Mm</td>
</tr>
<tr>
<td>X-Coord. Extrap. Final Travel</td>
<td>5.07 Mm</td>
</tr>
<tr>
<td>X-Coord. Damping Constant of Oscillation²</td>
<td>0.500</td>
</tr>
<tr>
<td>X-Coord. Natural Frequency of Oscillation²</td>
<td>0.0977 rad/hr</td>
</tr>
<tr>
<td>Y-Coord. Max Travel</td>
<td>4.29 Mm</td>
</tr>
<tr>
<td>Y-Coord. Extrap. Final Travel</td>
<td>2.76 Mm</td>
</tr>
<tr>
<td><strong>Overall &quot;Λ&quot; Leg Angles</strong></td>
<td></td>
</tr>
<tr>
<td>+ Polarity, 0.1 km/s rise rate</td>
<td>60.7°</td>
</tr>
<tr>
<td>- Polarity, 0.1 km/s rise rate</td>
<td>-52.4°</td>
</tr>
</tbody>
</table>

¹ - e-folding time is for half of injection

² - Only X-coordinate motion determined $\zeta$ and $\omega_0$; Y was assumed linear
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REFERENCES

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Mr. Brian Briggs has been a student at the George Mason University since 2008. He began his education at the University of Central Florida, receiving a Bachelor of Science in Mechanical Engineering in 2003. Since receiving his BSME, Brian has worked for Swales Aerospace (presently ATK Space Systems) and Orbital Sciences Corporation as a thermal engineer, developing and testing thermal control subsystems for high-altitude research balloons as well as numerous scientific, technology demonstration and telecommunications spacecraft. Brian returned to school for graduate study in 2008, whereupon his studies have focused on astronomy and astrophysics, culminating in solar physics research with Dr. Jie Zhang in 2012.