DEVELOPING RICH TASKS: INFLUENCE ON PLANNING AND IMPLEMENTING MATHEMATICS INSTRUCTION

by

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Dedication

This is dedicated to my loving husband Tom, our two sons, J.D. and Evan, their wives, Felicia and SaraAnn, and my parents, Doyle and Jane Hudson.

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I would like to thank the many friends, relatives, and supporters who have made this happen. My husband, Tom, who was there for me every step of the way, from driving me to George Mason for classes to reading my many papers. My daughter-in-laws, SaraAnn and Felicia, for helping me by proofreading my papers and their wonderful husbands, J.D. and Evan, for being tolerate as they did so. Brad Rankin, Trina Campbell, and Dr. Jackie Getgood for assessing activities using the unit rubric to insure validity and consistency. Drs. Margret Hjalmarson, Jennifer Suh, and Erin Peters Burton for their invaluable help and support through the many revisions.

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List of Abbreviations

Buck Institute of Education ..... BIE
Cognizant and Experienced Team ..... CET
College Preparatory Mathematics ..... CMP
Interactive Mathematics Program ..... IMP
National Council of Teachers of Mathematics ..... NCTM
Partnership for 21st Century Skills P21
Passionate and Energetic Team ..... PET
Project-Based Learning ..... PBL
Rich Task Prep ..... RTP
Sample Algebraic Modules ..... SAM
University of Virginia - School of Continuing and Professional Studies ..... UVA-SCPS
Virginia Standards of Learning ..... SOL

Abstract

Developing Rich Tasks: Influence on Planning and Implementing Mathematics Instruction

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George Mason University, 2013

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Teachers are continually confronted with meeting the needs of all students and preparing them to be successful on state end-of-course assessments while changing or evolving to be more of a facilitator of student-centered. Lessons are to be relevant for the students, engaging them so they are active participants in their learning. However, many teachers have never experienced rich tasks or observed a classroom where this approach is customary. The purpose of this study is to discover if activities during a professional development on rich task creation will influence the Algebra I teacher team's planning of instruction and facilitating lessons.

A descriptive case study approach is used to focus on the teachers and their growth during the Summer Institute and the school year lesson study. The question that drives the research is: How does the Summer Institute on rich tasks influence the teacher's implementation of the lesson study process? Secondary research questions

delve into the participants' growth and actions during the professional development and the lesson study:

1. What did teachers perceive as characteristics of "high cognitive demand" tasks prior to and after professional development?
2. What were the goal(s) of the teacher and teacher team prior to, during, and after the professional development?
3. How did the group-focused experiences of rich tasks influence teacher practices and beliefs during lesson study?
4. What challenges and/or constraints did teachers encounter when implementing rich tasks in the lesson study?

The focus is on the teacher's willingness to use a lesson study approach that combines quantitative data (content assessment and planning quality) with qualitative data (interviews, field notes, journaling, lesson study briefing and debriefing, and observations). There are two phases to the study. The first phase is a Summer Institute in which teachers will learn about and develop rich tasks; the second phase is the implementation of the task through a lesson study approach in which teachers will self-evaluate.

The results of the study showed that teacher change is more complex than just having teachers experience and develop rich tasks. Developing the course flow that correlates with the big ideas needs to be understood by the teachers and have teacher buy-in, support and time need to be a top priority with all leaders in agreement, and teachers

need to plan the units and lessons in writing so thoughts may be thoroughly explored and explained.

Chapter One

Passing required courses in mathematics is expected of all students graduating from high school. Student A is a sophomore taking Algebra I during the first block of the day. Student B is an eighth grade student taking Algebra I and on track to take higher and more rigorous math classes in high school. Teachers in both of these classrooms know about the National Council of Teachers of Mathematics (NCTM) Process Standards (2000) and are encouraged by central office leadership to provide instruction that is reform based and student-centered. Despite the teachers' awareness of constructivist pedagogy, let us assume that they still approach their classes differently. Teacher A wants to break down the concepts so that students can follow steps easily outlined for a procedure. Teacher B wants students to think, problem solve, own the mathematics, and to question their actions and responses. Students in both classes are still taking the same course, Algebra I, so they should have the opportunity to learn the material at the same level of rigor with the same content. However, how a teacher views the curriculum, textbooks, and other resources with respect to their beliefs play a role in how the students receive and participate in their own learning. The role of the teachers of students A and B are typical situations in secondary schools.

Teachers Creating Rich Tasks

Educating teachers about alternative approaches to teaching mathematics is needed to insure that more students will have the opportunity to learn the concepts and apply them to their daily lives. Teaching mathematics using rich tasks allows entrance into the activity at multiple levels, with multiple paths possible to obtain the resulting solution (Grootenboer, 2009; NRICH, 2007). The purpose of this study is to discover if activities during a professional development on rich task creation will influence the Algebra I teacher team's planning of instruction and facilitating lessons. Question that drive the research is: How did the Summer Institute on rich tasks influence the teacher's implementation of the lesson study process? Secondary research questions delve into the participants' growth and actions during the professional development and the lesson study:

1. What did teachers perceive as characteristics of "high cognitive demand" tasks prior to and after professional development?
2. What were the goal(s) of the teacher and teacher team prior to, during, and after the professional development?
3. How did the group-focused experiences of rich tasks influence teacher practices and beliefs during lesson study?
4. What challenges and/or constraints did teachers encounter when implementing rich tasks in the lesson study?

Rich task development is not a step-by-step procedure. The creation of the tasks involves creativity and a very thorough understanding of the content along with connections.

Creation of rich tasks for this study will initially be on slope, how it is taught, categories of slope (Stump, 1996; 1999), and connections to other content. Slope was selected due to teacher lack of understanding the concept as revealed by Stump (1996, 1999). Various categories of slope will be addressed by looking at the "bigger picture" and determining a relevant and interesting approach for students through involvement in a rich task (Grootenboer, 2009; NRICH, 2007). A lesson study approach will be taken in order for teachers to self-assess the rich task they create while investigating instructional implementation and the involvement and acceptance by the students (Lewis, 2002; 2010; Lewis, Perry & Hurd, 2004; Perry & Lewis, 2008; REL Northwest, 2012; Stigler & Hiebert, 1999).

Buck Institute for Education (BIE, 2009) gives five principles for designing project-based lessons (PBL). These are still not steps, but rather a questioning sequence. The first principle is to begin with the end in mind. This would be similar to McTighe and Wiggins (2011) work discussed in *The Understanding by Design Guide to Creating High-Quality Units* where developing the big idea and knowing what the end result should be drives the development of instruction. Knowing what the students are to have gained in content knowledge will give the "big picture." Developing the driving question is the second principle. The question should be intriguing to students, problematic for them on their level, and not easily answered. Planning the assessment comes next by deciding on the final project that will elicit answers, involve problem solving skills, and methods that involve the chosen mathematics. The fourth principle is creating a timeline

for the project that organizes the tasks in a pacing that will lead to a result in the timely manner. The final principle is being a guide on the side.

This project will begin by assessing teacher knowledge of the content, slope, and then determining individual and team goals. The teacher team will then have the foundation needed to determine the big ideas they will be teaching throughout the course and the driving question for a unit of study, ideas that correlate with BIE principles (2009). Also aligning with the BIE principles will be the opportunity for the teacher team to experience a rich task, acknowledging the time line given to complete the task, and the role of the facilitator. The first phase, the summer institute lasting for four days, will focus on the participants actions and interactions in all events. In addition to experiencing rich tasks, participants will also be exposed to the lesson study process. A second phase looks at the implementation and evaluation of a rich task through the lens of the teacher during the lesson study approach.

The Context of the Study

Warde (1960) posited that when people are challenged with real world, relevant problems that need to be solved, it is a time when the best possible learning and growth occur. Rich tasks will be the conduit used in this research to bring real world, relevant problems or those that are unique to the students to provide all with the opportunity to learn mathematics.

Going along the continuum of methodologies from traditional to constructivist-based has at the far left direct instruction then proceeds to rich tasks, project-based, then at the opposite end of the spectrum, problem-based learning. Direct instruction is

teacher-centered with a focus that is typically on skill proficiency whereas problem-based lessons are student-centered with a focus on application and understanding. As one proceeds along the continuum so does a teacher's increase in concern of time, the concern about student ability to complete lengthy projects, and a concern that content knowledge will be gained by the student (Wu, 1997). It becomes increasingly more difficult for teachers to accept, practice, and put into action the vision and expectations of methodologies as they become more constructivist (Wu). Table 1 compares rich tasks to project-based learning. Information for the table was gathered from University of Virginia - School of Continuing and Professional Studies (UVA-SCPS, 2011) summer institute for the new capstone course in the state of Virginia.

Table 1

Comparison of Rich Tasks to Project-Based Learning

	Rich Tasks	Project-Based Learning
Class Time:	3-8 hours	More than 8 hours to several weeks
Cognitive Demand:	Medium level Procedures with connections	Medium to high level Problems that are relevant, practical or have more real-world connections
Final Product:	Teacher is specific about the final product expectations (ex: presentation to the class that addresses all of the expectations of the task). Multiple methods may be used to solve.	Final Product: A menu of final product choices is provided by the teacher (ex: presentation to the class, poster presentation, written paper, etc.) Students need to determine the best approach to share and illustrate the problem and solution.
Data:	Provided by the teacher OR students may be directed where to find the information.	Problem leads to students knowing what data to collect. Students are responsible for gathering or finding data.
Explore:	Teacher directed with some autonomy given to students.	Teacher lead but students work in groups using the information provided by the teacher. Teacher taking on more of a consulting position.

As can be seen in Table 1, project-based teaching and learning puts a great deal more responsibility on the student to look at the whole problem before breaking the problem down into parts, to study the content on their own with minimal assistance from the teacher, and to apply their past and newly founded knowledge to real-world situations

(Southern Regional Education Board (SREB), 2000). On the continuum of learning (UVA-SCPS, 2011) beginning with direct instruction there exists modeling of the mathematics which may be seen in all levels and applications of the mathematics at all levels. Tasks become more student directed as one progresses toward reform-based instruction, moving forward to include, support and correlate with the NCTM Process Standards (2000) and 21st Century skills (Partnership for 21st Century Skills (P21), 2012).

A comparison between working characteristics of project-based learning and rich tasks is provided in Table 1. Project-based units begin with the end in mind, acknowledging the objectives and goals for the unit first, followed by determining the driving question for the unit, leads to determine the resulting task to meet all the items (BIE, 2009). The same approach will be true for rich tasks (UVA-SCPS, 2011). However, with rich tasks one is only planning backward and developing a driving question for an activity that will last from three to eight hours. The final project is presented in a more specific manner for rich tasks whereas project-based learning activities are more open ended and may need a timeline in order for students, and facilitators, to stay on task. An example of a rich task is seen in figure 1.

The Hexagonal Train

Some Town Mono-Rail (STMR) has been working to develop a unique train that is unlike any train that we see today. Each of the cars in the train is the shape of a regular hexagon. When the cars are linked together there is no gap between them. Each of the cars is connected on one side thereby creating a train. After the cars have been linked together to form a train workers will place a connection belt around the perimeter of the entire train. The belts are made to specific lengths which match the number of cars in each train. Your task is to determine how the company can determine the length of the belt for any train of *n* length.

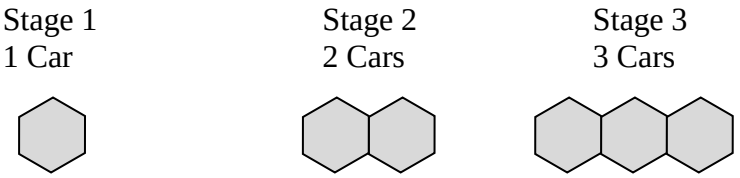
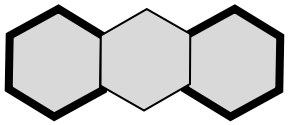


Figure 1. *Stages of Growth for the Hexagonal Train.*

Students are given hexagonal pattern blocks to form the first three stages of growth of the cars in the train. Multiple entry points and methods are recognized when students verbalize the growth in various ways such as keeping the perimeter of the two end cars as constants and relating the exterior edges of the middle cars to the stage, recognizing that the two ends remain constant (2 sets of 5 sides) and relating all of the exterior edges of the cars to the stage, see figure 2.

Stage 3
3 Cars



Ends = 10

3 cars – 2 end cars = 1 car left in the middle

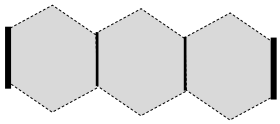
Middle cars each have 2 sides with 2 edges or a total of 4 edges

Result is Perimeter = 10 + 4(Stage – 2)

Figure 2. *Method 1 for Representing Stages of Growth for the Hexagonal Train.*

A second method may just hold the front and back segments constant and relate the remaining edges to the stage, see figure 3.

Stage 3
3 Cars



Ends = 2

3 cars: 2 sides with 2 edges for a total of 4 edges per car.

Result is Perimeter = 2 + Stage(4)

Figure 3. *Method 2 for Representing Stages of Growth for the Hexagonal Train.*

There are many ways to visualize the growth, including the two revealed above, with all leading to the same end result, perimeter = 4(stage) + 2. Students create a table of values, a graph representing the data, analyze the function and its graph for properties, and evaluate and predict for various number of cars. Each group of students will create a poster illustrating and explaining how they viewed the growth along with the various representations. The task may take two blocks or about 3 hours to complete and may be used at the beginning of a unit on linear relationships. Cognitive demand for the task is of medium level, there are connections but there are also guidelines to help students determine the equation even though they approach it differently. The data are developed with the guidance of the teacher with an explanation of how to build the train. For guidance the teacher encourages pattern recognition and verbalizing the pattern.

In contrast, Campaign for Governor: Commonwealth of Virginia, a unit developed for the Mathematics Capstone Course in the state of Virginia (https://sites.google.com/site/mathematicscapstonecourseunits/home/) is an example of a project-based unit:

Campaign for Governor: Commonwealth of Virginia

> In today's economy voters are constantly concerned with how government funds will be budgeted. Your task, as a team of the Candidate and staff members, will be to take a $35 trillion budget and determine how it should be spent based off of your state's concerns and needs.
> Your team will be running against other candidate teams in our class. On Election Day, voters (your classmates) will be afforded the opportunity to vote for the candidate team they believe made the best decisions with budget spending based off of the mathematics researched and presented.

> To guide you with the project expectations a rubric is attached.
> You will have 10 days to prepare your campaign presentation for
> Election Day on _____.

Students are given the rubric of expectations which is discussed and explained thoroughly. In their teams, students must make several decisions based on their research on what is needed in their state which includes thirty cities with regard to items such as transportation, water and sewer, education, or recycling. Working in collaborative teams, the teacher is the consultant, providing guidance and being the timekeeper so that the project does not stall. The cognitive demand of the project is high as the students must not only decide what to research but also interpret the data, make decisions on how to illustrate their stance, and be able to justify their position to their classmates during the election process.

Rich tasks, which are shorter in implementation time span, expect students to apply current knowledge, and at the same time encourages students to construct new content knowledge, will be less complicated than project-based tasks to create and implement due to the overall characteristics of the tasks (Grootenboer, 2009; NRICH, 2007; UVA-SCPS, 2011). Using rich tasks is one approach that is aimed at engaging students in relevant and interesting mathematical problem solving activities (Grootenboer, NRICH). The characteristics of a rich task lead to students being successful by having multiple entry points into the situation and multiple pathways to obtain a solution.

All students in public schools take required mathematics courses so we then assume that they have or will have an opportunity to learn the material. The phrase

"opportunity to learn" is also applied to situations of equity such as race, gender, or age. Byrnes (2003) discusses the idea and states that there needs to be a willingness to learn the material and to be engaged in the classroom instruction as well as being able to take advantage of all learning opportunities. Stipek (2002) claims that instruction effects student engagement and their enjoyment of mathematics. Rich tasks, being based on relevant, interesting topics for students; topics that are engaging while allowing students to enter into the problem at different levels, provides all students with the opportunity to learn (New Basics Branch, 2001; NRICH, 2007; Piggott, 2004). The connections made by focusing on the "big idea" instead of individual and independently taught concepts will aid in involving students in owning their learning (Bruner, 1977; Stipek).

Teacher Knowledge Concept Map

Guskey's (1986, 2002) concept map for changing teacher beliefs through professional development is based on applying what is learned in the classroom and watching how the students react and interact with the lesson. Observing student interactions with the lesson is also the basis of lesson study **(**Lewis, 2002) where teachers are assigned a student or group of students to acknowledge responses and comments. The concept map, Figure 4, shows where this study falls, the reversed print portion, within Guskey's research.

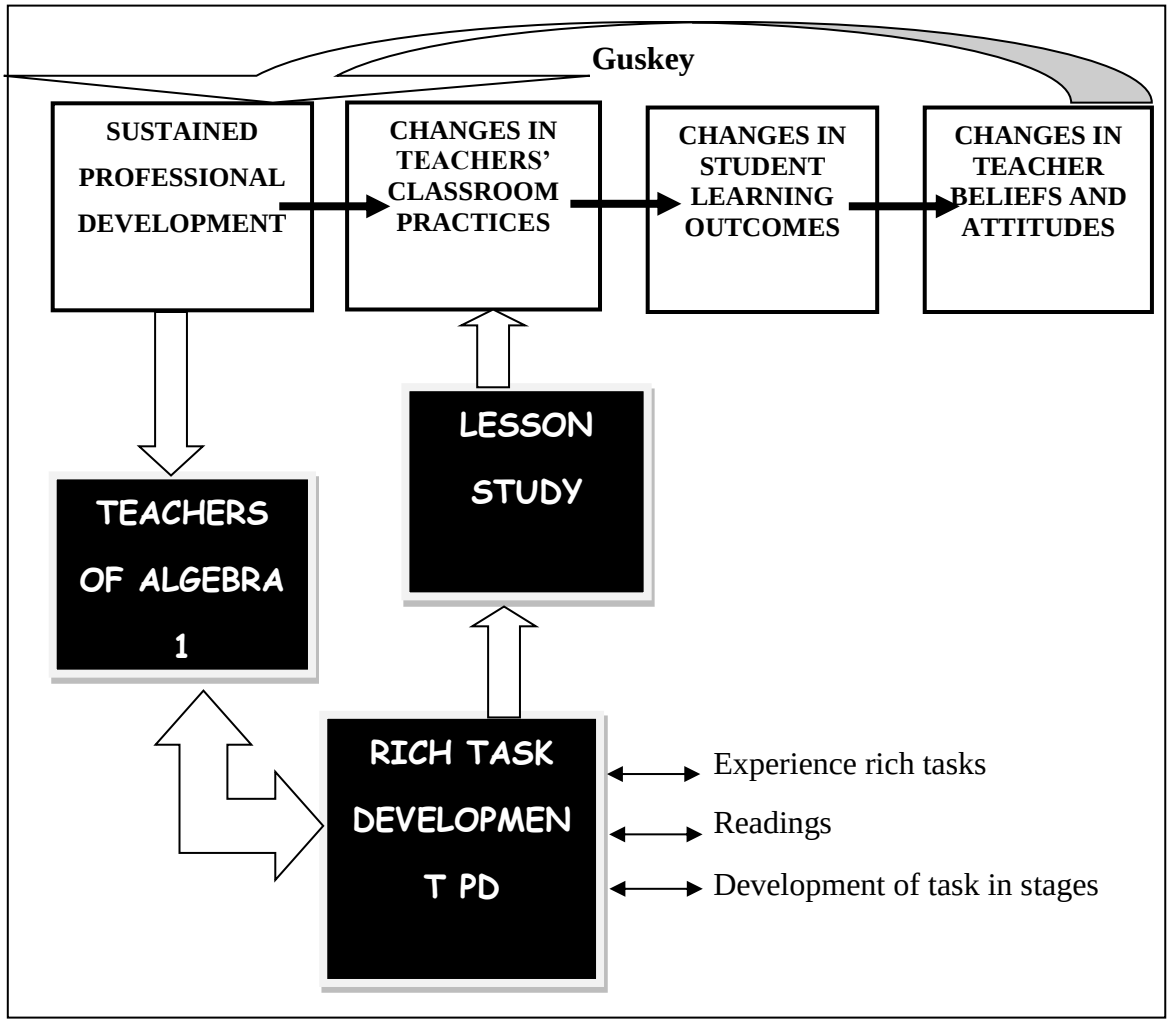


Figure 4. *Concept map of teacher knowledge.*

Teachers need to complete rich tasks so that they will be able to experience the cognitive dissonance like their students (Loucks-Horsley, Stiles, Mundry, Love, & Hewson, 2010). There is more to a rich task than giving students an activity to complete. The learning environment, the questions students and teachers ask, the interactions between students and between the teacher and student all play a role (NRICH, 2007). The summer institute follows Derry, Wilsman, and Hackbarth's (2007) usage of SAM's

(sample algebraic modules), along with readings and discussions, to increase algebraic thinking. The product, a rich task involving slope and later in the institute a unit of their choice, will provide teachers with the opportunity to determine all concepts that are to be built into the task, what the goal will be for students upon completion of the task, and acknowledge all of the multiple paths to determine a solution and the importance of each.

Lesson study will help teachers focus on what the students are doing during the lesson along with the facilitator actions and reactions to what is going on in the classroom (BIE, 2009; REL Northwest, 2012; Lesson Study Research Group, 2012). Fernandez speaks about the focus of lesson study being from the inside of the school, about what is going on in the classroom from the students actions (2000). The teacher needs to have at the forefront of their planning what needs to be going on in the classroom but also the goals for the lesson that you want your students to have gained by the end of the unit need to correlate with those of the school. Lesson study is there to support teachers in their endeavors to teach the state standards and to increase student achievement but with a focus on students learning the content and lesson quality.

What is Missing in the Research

Several research articles address project-based learning, the teacher role and the student role during implementation (Moylan 2008; Piggott, 2004), achievement results for student of various academic abilities (McCaffrey, Hamilton, Stecher, Klein, Bugliari, & Robyn, 2001; NRICH, 2007; Ridlon, 2009), elements needed for implementation (Bays, Reys, & Reys, 1999; Kolmos, 2010, Piggott, 2004), and beliefs of those involved (Clarke, Breed, & Fraser, 2004; Rickard, 1995, Rogers, Cross, Gresalfi, Trauth-Nare, &

Buck, 2011; Wu, 1997). BIE (2009) even gives guidance on how to proceed through the development of a project-based unit of instruction by asking questions based on the five principles. Changing teacher beliefs and actions through collaborating with their peers on project-based units were questionable (Gill & Hoffman, 2009). Manouchehri and Goodman (2000) discuss how a teacher must enter a "zone of practice that is ambiguous and uncertain" (p. 30) when working with project-based lessons which tends to hold them back from accepting the reform vision. A lack of confidence, content knowledge, and pedagogical content knowledge was stated by Taplin and Chan (2001) as holding teachers back from accepting reform. Wilson and Lloyd (1995) revealed that the teachers in their study feared that students would not make the needed connections between concepts.

Behind all of the reasons that hold teachers back is time and students' ability to be successful on mandated tests. So looking at the comparison between rich tasks and project-based learning, it may be assumed that rich tasks are somewhat closer to traditional teaching methods with less time involved in the task than project-based units of study. If teachers become adept at creating and implementing rich tasks, then the next step would be to progress toward project-based teaching and learning. Change is gradual but also we need to consider the "jump" that is expected of those involved. Schoenfeld (2009) stated "What we teach not only affects how students see themselves, but it also affects their trajectories through life" (p 28). Project-based teaching and learning has shown in research to be beneficial to students but not if teachers are not accepting and capable of implementing the units. Teachers may need to take smaller steps with smaller

units (rich tasks) and sets of concepts. This includes knowing the cognitive demand of a question or problem as revealed by Stein, Smith, Henningsen, and Silver (2000) and looking at the curriculum through the lens of "big ideas" (SREB, 2000)

Definitions

BELIEFS: When provided with evidence beliefs can change (Cooney, Shealy, & Arvold, 1998). Philipp (2007) states that beliefs are lenses through which individuals interpret their surroundings. Trujillo (2010) states that beliefs can be influenced by what a person experiences, they can be flexible.

PEDAGOGICAL CONTENT KNOWLEDGE: "The blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction" (Shulman, 1987, p. 8).

PROJECT-BASED LEARNING (PBL): Lessons include the four C's: critical thinking and problem solving, communication, collaboration, and creativity and innovation according to P21 (2012) as well as NCTM Process Standards (2000) and the Six A's (SREB, 2000) of authenticity, academic rigor, applied learning, active exploration, adult relationships, and assessment.

PROBLEM SOLVING APPROACH: Reasoning, gathering and organizing data needed to answer the question, communicating ideas and results, reflecting on the approach and the result, and cycling through the procedure as many times as needed (Thompson, 1992).

REFORM MATHEMATICS: Pedagogy that begins and focuses on the learner's mathematical conceptions and misconceptions (Ball & Cohen, 1999) as given in Trujillo

(2010). Based on the constructivist theory where the learner is engaged in "doing" mathematics and meeting the NCTM Process Standards (NCTM, 2000).

RICH TASKS: Overall characteristics come from Grootenboer (2009) and NRICH (2007). It is an activity where there are multiple entry points, multiple approaches and representations, a driving question that connects the mathematics to the scenario, application of prior knowledge to the exploration, the scenario is relevant or interesting to those involved, the cognitive demand is at the minimum level of procedures with applications (Stein, Smith, Henningsen, & Silver, 2000), problem solving skills applied, NCTM Process Standards (2000) are addressed, data to be used are either given or directions to where it may be found, and 21st Century Skills (P21, 2012) applied.

SOCIAL CONSTRUCTIVISM: Every individual constructs their own knowledge according to their environment (Trujillo, 2010).

The following literature review dives into rich tasks, professional development, slope, and lesson study to gain a background and foundation for the study.

Chapter Two

This chapter describes the research related to rich tasks and professional development, including lesson study. The first section will describe the characteristics of a rich task, why it is applicable for all students in today's global society, and what goes in to developing a rich task in order to answer the research question: How did the Summer Institute on rich tasks influence the teacher's implementation of the lesson study process? Secondary research questions delve into the participants' growth and actions during the professional development and the lesson study:

1. What did teachers perceive as characteristics of "high cognitive demand" tasks prior to and after professional development?
2. What were the goal(s) of the teacher and teacher team prior to, during, and after the professional development?
3. How did the group-focused experiences of rich tasks influence teacher practices and beliefs during lesson study?
4. What challenges and/or constraints did teachers encounter when implementing rich tasks in the lesson study?

The second major section is on professional development that is viewed as productive, taking into consideration teacher beliefs, support needed from administrators, and promoting student-centered instruction. Transforming teachers from traditional views

and actions to reform-based will also be addressed. Secondary issues (slope and lesson study) will be discussed briefly as both play a role in the research. Slope is the foundation of the lessons during the Summer Institute and lesson study is the avenue taken to discuss what the teachers have learned and are implementing in their classrooms. All of the topics discussed are to inform myself, the researcher and facilitator of the Summer Institute, so that a better understanding of the participants' actions and interactions with the team will be gained for an end result, an increase in student knowledge, understanding, and appreciation of mathematics.

Rich Task

Characteristics. A rich task is an activity where learning is being done actively versus an activity that applies what has been taught using direct instruction. Characteristics include an activity that has multiple entry points and pathways to the solution (Grootenboer, 2009; NRICH, 2007). The pathways allow for students to represent the situation in multiple ways. Group work is encouraged so that the students can make connections between the views and work of their peers as well with the mathematics they are working within the problem. Resources are shared by the students and may even be given to them versus the students searching for the data. The cognitive demand of the task encourages students to perform and produce quality results as they are engaged and motivated to complete the task as it is seen as relevant and real to their daily lives. NRICH adds additional characteristics that include that the task should be accessible for all students, encourage students to ask their own questions, require students to provide reasons and justifications for their thinking and procedures, and increase

content knowledge instead of repeating what they have learned. The task should build a student's confidence and increase or build critical thinking skills. Rich task characteristics are the foundation of project-based learning.

Upholding the NCTM Process Standards (2000) and 21st Century Skills (P21, 2012) may be seen in standards-based instruction, reform-based instruction, project-based learning, or problem-based learning (PBL). P21 skills include the four C's: critical thinking and problem solving, communication, collaboration, and creativity and innovation. These skills, along with the process standards of connections, communication, problem solving, reasoning, and representations, show a great deal of overlap between them and the rich task characteristics. More research has been conducted on project-based learning than on rich tasks. Project-based learning characteristics include all of the above and the Six A's (SREB, 2000) of authenticity, academic rigor, applied learning, active exploration, adult relationships, and assessment. SREB stresses that one needs to consider the big picture before the parts, to observe what the students are doing, to try the rich task themselves to experience what is expected, and then reflect. The challenge when creating a rich task is to connect the mathematics to real world situations. Even though there is more research on project-based learning I am concentrating on rich tasks since it is the foundation of the UVA professional development project (UVA-SCPS, 2011). Rich tasks are one step toward changing teacher beliefs about teaching and learning along the continuum from teacher-centered instruction to student-centered, toward meeting all the needs of today's students.

Another definition or characteristics of rich tasks may be found in the New Basic Project (New Basics Branch, 2001; Queensland State Education, 2004; 2006) in Queensland, Australia. They include those mentioned above as well as being transdisciplinary (draws on another subject while maintaining the value of the connections and content), should span a considerable amount of time in the curriculum, and the workload of the teacher is reasonable. A task is not considered rich if the transdisciplinary aspect is not present according to the New Basic Project.

The operating definition for rich tasks used for this study is taken from Grootenboer (2009) and NRICH (2007). The key points are multiple entry points, multiple approaches and representations, a driving question that connects the mathematics to the scenario, application of prior knowledge to the exploration, and a scenario relevant to those involved. Cognitive demand and the final project criteria are also part of the definition. A student's mathematical knowledge should increase through their involvement in the rich task. The task should be classified as "procedure with connections" or "doing math" as per the level of cognitive demand (Stein, Smith, Henningsen, & Silver, 2000). Throughout the tasks, students engage in applying problem solving skills. The final product includes the application of the NCTM Process Standards (2000) and 21st Century Skills (P21, 2012) with students working in teams. Data for the task may be given to students or they may be told where to find the information. Students may also receive guidance from the facilitator through the exploration.

Rich tasks' characteristics, as seen by the participants, will be assessed prior to the intervention and during the lesson study process. What about the teacher's

viewpoints on the creation process? Their thoughts on the strategies, their individual efficacy, and the collective efficacy of the team of teachers will be addressed during the intervention through observations and journaling (participants and researcher) but also followed up and triangulated during the lesson study process. As stated previously, there is not a set of procedures used to create a project-based unit and, likewise, the rich tasks. Content knowledge, creativity, and the teachers' experiences will be their "handbook" along with guiding questions. Guiding questions will assist teachers, leading to an opportunity for all students to learn mathematics.

Opportunity to learn for all students using rich tasks. Rich task development is not just for the advanced student (NRICH, 2007; Piggott, 2004; Ridlon, 2009). The learning environment is what makes a rich task rich (NRICH, 2007). How the task is presented to the students, what type and how much support does the teacher provide students, what types and levels of questioning does the teacher employ, and the role of the student in the classroom all contribute to making the task rich. Piggott (2004) states that teachers need to offer students an opportunity to experience mathematics, seeing themselves as mathematicians, problem solvers, and capable of owning their learning. Teachers, in their planning, need to think how each student may reach their own zone of proximal development (Vygotsky, 1978). Characteristics such as multiple entry points and paths allow students to enter into the problem and at the same time follow a route which they may be somewhat comfortable and at the same time stretch their knowledge. Moylan (2008) states that PBL units of study are important to develop a student's sense of "self", a student's "skills", and prepare students to be productive citizens in "society",

the three S's. This will be true for rich tasks as well, encouraging students to recognize mathematics in their surroundings, increasing their confidence, and improving their skills.

Just like the learning environment needs to be conducive to rich tasks and providing the opportunity for students to learn, so does the teacher's choice of the right problem given at the right time and allowing or permitting the usage of the right tools (Piggott, 2004). The best made plans can become worthless if the implementation and facilitation are not successful. For example, the groups or teams of students need to be put together with some thought (Blumenfeld, Marx, Soloway, & Krajcik, 1996). Norms (Lewis, Perry, & Hurd, 2009), what support will be given to the teams, and the makeup of the membership of the teams are some of these considerations. The academic ability of the students and their strengths and weaknesses play a role in the success of the team. This is where the teacher needs to know his or her students in order to establish productive teams. Knowing the students goes further than their grades and should include personality and academic strengths and weaknesses. Piggott (2004) stresses that a rich task is enrichment for everyone and should saturate all instructional aspects. Knowing the students will help the teacher guide the students to be successful. Blumenfeld, Marx, Soloway, and Krajcik (1996) also state that in order for the task to be successful and engaging for students, there needs to be connections made between prior knowledge and the lesson content, have multiple pathways and solutions, and contain a problem that will interest students.

Rich tasks workable for teachers and students. Queensland State Education (2004) research, The New Basic Project, sought answers to whether the rich task trial was viable and would lead to the changes they wanted with instruction and student achievement? The researchers also asked if the project would be accepted by the stakeholders along with being feasible on an extended basis. The longitudinal study involving 59 schools wanted to improve the outcome of student achievement, encourage improvement of teacher practices and approaches to teaching and learning. The overall important outcome was an increase in student achievement but also teachers worked together and had meaningful discourse about instruction, teaching, and learning. Teachers realized that collaboration and communication are needed in order for the desired outcome to be realized. Ridlon's (2009) research looks at low and high achieving students that have experienced project-based learning units.

Problem-based units were given to sixth-grade students over a two-year period during the same nine weeks of instruction (Ridlon, 2009). The experimental design study looked at lower achieving students during the first year and a mixed group of students the following year. Their results revealed that the low achievers showed greater levels of increased achievement. Problem based learning was viewed by Ridlon (2009) as a success if the approach was understood and implemented properly. The focus of their study was on student interactions with the content. Teaching students that there are different ways to think and that the students needed to explain their thinking was the approach the teacher took when implementing the problems. Parent and student attitudes

were positive during their engagement in the activity as students stated that they felt that their opinions were heard; rich tasks were workable in the eyes of the students.

Beliefs, teachers and students, play a role in problem-based instruction with regard to achievement and acceptance of practices according to Clarke, Breed, and Fraser (2004). The study was based on using Interactive Mathematics Program (IMP) with 182 high school students in California. Goals for the PBL approach included affective outcomes as well as cognitive. Comparisons were made between the students who were divided into the experimental group, using IMP, and the control group, who were taught using a traditional approach. IMP students rated themselves higher in mathematical ability and had more positive attitudes toward the course. There were higher SAT scores for females. The experimental group was shown to value their learning experiences over sitting through lectures, and valued collaborating with their peers. In order to have the experimental group deem the experience as "workable", teacher beliefs and goals need to be conducive to the approach.

McCaffrey, Hamilton, Stecher, Klein, Bugliari, and Robin (2001) also based their research on IMP with tenth grade students, comparing the instructional approach with College Preparatory Mathematics (CMP) and traditional approaches. The research focused on the effects of what teachers learned during a professional development on student achievement. Results showed that frequent use of reform practices increased test scores for students taking IMP or CMP courses, not true for those taking the traditional course. McCaffrey, et al. (2001) concluded that professional development needs to

include curriculum and instructional concepts and practices in order to make the approach workable.

These studies were all based on problem-based practices or reform-based mathematics (Clark, Breed, & Fraser, 2004; McCaffrey, et al, 2001; Ridlon, 2009). Overall, these studies have shown that better test scores were seen of students involved in PBL. Better attitudes toward mathematics and the course are also acknowledged. Positive results were shown for all students but greater growth was revealed for the lower achieving students, providing all students with the opportunity to learn meaningful mathematics (Ridlon, 2009). The practice has still not become common within the mathematics classroom even with the positive outcomes. So how does one create a task that meets the above characteristics and provides the opportunity for all students to learn mathematics?

Rich task development. Information on rich tasks is available through NRICH (2007) and the New Basic Project (New Basics Branch, 2001; Queensland State Education, 2004; 2006). These documents list characteristics of a rich task but focus on implementation and teacher/student results. The book that is closest to providing procedures to develop a rich task are from BIE (2009) which gives steps, principles, and forms to create a PBL. BIE (2009) state that the essential elements of a PBL include content that is substantial, 21st century skills (P21, 2012) that are similar to the NCTM Process Standards (2000), a driving question that is open-ended, students acknowledge a need to learn the material, choices are given for the product and the path, feedback is given and may lead to adjustments to the plan or result, and the final product is presented

to audience. Forms are provided for items such as the project overview, a teaching and learning guide, and a project work report for group accountability. PBL's are not just fun activities or an activity where learning is applied (BIE, 2009). Students are "pulled through the curriculum by a meaningful question to explore, an engaging real-world problem to solve, or a design challenge to meet (p. 4)." This happens when students explore and investigate, leading to a desire to learn the material.

As shown above there are a lot of similarities between rich tasks and PBL so the BIE forms (2009) are helpful but still lacking. For instance, PBL is a much larger "rich task" where the entire unit or topic of study is taught through the project. A rich task, in contrast, is a part of the larger unit; the task(s) is embedded in the unit to provide relevance and encourage students in their learning endeavors. It is a part of the bigger picture, the unit. NRICH (2007) and the New Basics Project (New Basics Branch, 2001; Queensland State Education, 2004; 2006) listing of characteristics is not the same as steps to create a task.

Rich tasks is not just the task itself but consideration needs to be given to the affective development as well as the cognitive development of students' experiences (Clark, Breed, & Fraser, 2004). In addition, teacher and student beliefs play a role in the development and implementation of rich tasks (Clark, Breed, & Fraser, 2004; Edwards, 2011; Rickard, 1995; Rogers, Cross, Gresalfi, Trauth-Nare, & Buck, 2011). Beliefs play a role in how teachers interpret the implementation, what they believe is evidence of success, and what materials and approaches they encourage students to use when working on units. Wu (1997) shared some concerns that students will not experience the beauty of

mathematics that is seen in its structure and cohesiveness unless it is explicitly given. Concern for problem solving over computation and rules was expressed by Rickard (1995). Edwards (2011) also reveals a concern for classroom management issues that might be addressed in the development of the units. Teachers viewed their method of instruction through the lens of classroom management and the concerns it may or may not create. Again, the teacher's beliefs and goals influence what they see as important in a student's mathematical growth, including management of the class.

Steps to developing rich tasks were not explicitly given, only characteristics. Besides the development of the tasks, one must consider the learning environment and teacher and student beliefs on teaching and learning mathematics. With the desire to create rich tasks that meet the above characteristics, provide students with an opportunity to learn mathematics, and address concerns about development of the units comes a need for professional development during creation and implementation that supports the teachers in their journey, encourages collaboration, promotes changing teacher beliefs and practices, and increases teacher knowledge.

Professional Development

If the beliefs of the teachers are similar, then collaborative sessions could be more productive and focused on the same goals. But a lot comes into play during collaboration. Did the teachers have professional development sessions embedded in their planning to educate them on the expectations of reform-based lessons? Is there an expectation that teachers will plan together and create common assessments so that all teachers of the same course will have the same objectives and level of rigor? Do teachers

come together to discuss assessment results and how to help those students who are not as successful as they should be? The big question is if the teachers believe that they are supported in their efforts of implementing reform-based lessons.

Productive professional development characteristics. Guskey (1986; 2002) states that there are two factors that are needed for successful professional development: determining what will motivate the teachers to become involved in the professional development and the process a teacher goes through as they change. Teachers want sessions that are usable, wanting to attend and take home ready to use activities, documents or lessons. This will help to motivate teachers along with the teachers being a part of the planning of the professional development, helping to determine what they need to learn (Orrill, 2006). Professional development sessions need to provide teachers with practical ideas but the goals of the professional development need to match those of the participants and need to be centered on promoting student achievement (Guskey, 1986, 2002; Orrill, 2006; Stigler & Hiebert, 1999). Guskey (1986; 2002) considers change in teachers as an orderly progression from staff development to a change in teachers' classroom practices, this leads to a change in student learning outcomes and finally to a change in the teachers' beliefs and attitudes. The change in student learning is seen as a precursor to changing how teachers believe students learn and about teaching and learning methodologies. Stigler and Hiebert (1999) state that change in instruction, transforming from teacher-centered to student-centered teaching and learning needs to take place in the classroom which confirms Guskey's concept of change. Student learning includes increased achievement but also affective results such as an increase in

attendance, becoming more involved in the class, and attitudes that are more positive toward the class and the subject.

When planning professional development sessions the facilitator needs to recognize that change takes time (Guskey, 1986; 2002). It is a gradual process with all concepts presented and/or facilitated in a clear concise manner (Guskey, 1986, 2002; Stigler & Hiebert, 1999). Personal concerns that the teachers have must also be addressed. The facilitator of the professional development must be credible in the teachers' eyes in order for them to take it seriously. Teachers need to have feedback on their progress that is regular and constructive (Guskey, 1986) while also having time to reflect and share with others their concerns and experiences (Stigler & Hiebert, 1999), especially problem solving approaches (Orrill, 2006).

Cwikla's (2002) study addressed characteristics that influence teachers' reactions and interactions with students and what activities affect teacher's classroom practices. Four hypotheses were considered: 1) when teachers' thinking and learning is centered around how students' think and learn then instructional practices improve; 2) environments that promote collaboration lead to improved instructional practices; 3) instructional improvement changes occur in small increments; and 4) experimentation and investigation lead to instructional improvement. Overall, the goal is for teachers to increase their understanding of student thinking so that there will be a change in instructional practices that are based on the National Council of Teachers of Mathematics (NCTM) Process Standards (2000).

Analyzing what affects teachers' instruction in professional development was studied by Desimone, Porter, Garet, Yoon, and Birman (2002). The study involved 207 teachers in thirty schools and five different states. Characteristics of high-quality professional development was seen as common in past research but none on the effect a specific characteristic might have on improving teaching and student achievement. The study looked at six key features that were split between structural and core features. Structural features include the reform type, the duration of the activity, and the collective participation of teams of teachers. Core features include active learning, promoting coherence with goals, standards, and assessments, and, lastly, a focus of content. Stretched over a three year period, the participants in the longitudinal study had taught the same course every year. The survey in year three was an attempt to explain the practices of the teachers, with respect to the key features, and their professional development experiences during year two while controlling the practices of the teachers with the survey results from year one. Results of this study by Desimone et al. (2002) revealed the importance of teacher teams working together cohesively to change instructional practices as well as the teachers being actively involved in the learning opportunities.

Boaler and Staples (2008) conducted a case study of reform movement vs. traditional and problem solving approaches on three schools that they called Greendale, Hilltop, and Railside. The focus was on Railside as they implemented the reform movement approach. The other two schools were split between traditional and the problem solving approaches. Railside teacher's focused on beginning algebra courses

with a curriculum oriented to mathematical reform involving student's conceptual understanding, collaborative work, multiple representations, math talk, and making connections between algebra and geometry. The teachers acknowledged different approaches and solution routes as they employed open task problems that permitted entry levels for different student abilities. All the students at Railside took Algebra I in their first year of high school and teachers held high expectations for all.

Students at Railside came from diverse backgrounds, had the largest English Language Learners of all three high schools, more students qualified for free/reduced lunch, lowest percentage of parents with college degrees, and the lowest student achievement. At the beginning of the year Railside staff tested the incoming students on middle school skills and then a post test at the end of the year. Results showed that Railside students were approaching the levels of achievement of the other two high schools. Improvement continued through year two but not as notable in year 3 as that course was not developed with the same vision and the teachers had less experience. Differences between Black, White, and Latino students at Railside began to vanish while the achievement gaps between ethnicities remained the same at Greendale and Hilltop High School's. Railside teachers believed that their students could achieve and had the support from their administration. They were given the opportunity to step out and try new methods, saw the positive effects on the students' attitudes and growth, which in turn helped to change their beliefs.

Video-based professional development. Videos were used in several research studies to aid in teachers acknowledging various types of instruction as well as their own

instruction (Breyfogle, 2005; Derry, Wilsman, & Hackbarth, 2007; Stigler & Hiebert, 1999). Breyfogle (2005) videotaped a teacher as he interacted in the classroom with his students. The teacher started out being defensive and explaining his actions, progressed to questioning his actions, and being frustrated with himself. The final state was exploring the concept and how he was interacting with students. Breyfogle (2005) believed that this final state indicated that the teacher was becoming more thoughtful about his actions, like an outsider looking in.

Derry, Wilsman, and Hackbarth (2007) used videos as case studies to allow teachers to examine various situations of interactions between students and teachers algebraic understanding. Teachers compared and interpreted tasks that demonstrated fluency with representations and algebraic thinking based on the NCTM *Principles and Standards for School Mathematics* (2000). Twenty-two teachers during a four-day span generalized strategies by watching the videos, reading articles about algebraic thinking, and participating in sample algebraic modules (SAM). Participating in group activities led the teachers to a deeper understanding, comparing and contrasting strategies used with the various representations and solutions. Teachers were also encouraged to reflect on the mathematics and pedagogy. The results of the study showed that teacher pedagogical content knowledge increased but content knowledge did not. Reflections revealed a change in beliefs but teachers would only reflect when requested to do so. Videos may be used to encourage change and reflection but also to document reform and why or why not students are successful (Stigler & Hiebert, 1999).

Teacher and student beliefs. The videos above aided in changing teacher beliefs regarding instruction and algebraic thinking (Breyfogle, 2005; Derry, Wilsman, & Hackbarth, 2007; Stigler & Hiebert, 1999). But not all beliefs are subject to change. Cooney, Shealy, and Arvold (1998) discuss beliefs that are backed by evidence and those that are not in their study of fifteen preservice teachers. A survey, along with interviews and journaling, led to the researchers understanding that if a belief is backed by evidence, then by changing and/or challenging the evidence beliefs can and will change. Cross' (2009) qualitative research involving five high school algebra teachers led to a categorization of beliefs in contrasting viewpoints such as computational versus a way of thinking, demonstrations versus guidance, and practice versus understanding. How a teacher responds and elicits discourse, selects activities, and assesses students is based on his or her beliefs. Does the teacher believe that correct computations and procedures are more important than students understanding the concept, that he or she is the giver of information or is the guide on the side, and whether rote drill is the way a student learns mathematics or if being able to explain and apply the concept is more important. These questions lead a teacher in their planning for student success. Cross (2009) states that a teacher's decisions regarding instruction are formed by the beliefs they hold. He suggested ongoing, sustained professional development for support.

Cwikla's (2002) research, professional development based on how teachers implemented new curriculum materials, spanned a three year period with 110 middle school teachers. Half way through the professional development, participants completed a survey about how students learn and on their constructivist beliefs, their thoughts on

activities that encouraged mathematical thinking, as well demographic information. Her findings showed that years of teaching experience, advanced degrees, and content knowledge affected their insight into student thinking and their beliefs. Harel and Lim's (2004) two year study involved middle school and high school teachers whose schools had large population of low income students. They were looking at teachers' knowledge, content and pedagogical, and how their knowledge level influenced their teaching. During a unit on functions, classrooms of students were observed along with teacher interviews, taping of lessons, and acknowledging goals and objectives formed the data gathered. The researchers determined that a teachers' way of understanding mathematics versus their way of thinking were equally important and both influenced by teacher content and pedagogical content knowledge.

Dweck (2010) discuss two types of beliefs held by teachers, fixed mind-set and growth mind-set. Her findings were based on a prior study by Blackwell, Trzesniewski, and Dweck (2007) that spanned five years and involved 373 seventh grade students. The students were given a questionnaire about their theory of intelligence, goals, responses to failure, and beliefs about their efforts. They found that students with a growth mind-set were more successful despite negative situations or failings of any sort. Dweck (2010) states that teachers can teach students, based on their own beliefs, that their intelligence is static, either they are able to do the mathematics or they are not, a fixed mind-set. The alternative, a growth mind-set, is a belief that intelligence can be developed through various modes of learning. Students that are taught or have the growth mind-set improved their grades, focused more on learning the material, showed more effort, and

bounced back more readily after setbacks. Teachers having the growth mind-set helped to close achievement gaps. Educating and changing beliefs of teachers to have the growth mind-set has not been studied at this time.

Support by principal and peers. After initial professional development sessions, teachers need continued support as they engage in new practices (Becker, Pence, & Pors, 1995; Guskey, 1986; 2002; Orrill, 2006). Becker, Pence, and Pors (1995) observed a school system that was transforming their sequence of mathematics courses so that all students took Algebra I in the ninth grade. Teachers attended a program that was sustained throughout the school year to support their efforts of differentiating instruction for all students in the Algebra I courses. The researchers wanted to know which aspects influenced the teachers the most and why some chose to continue with the program. Five teachers were interviewed with all agreeing with the concept of Algebra I for all ninth graders. Becker, Pence, and Pors (1995) concluded that the longer teachers were in the program resulted in changes in their beliefs and actions regarding student-centered teaching and learning. The teachers that stayed with the program for the full year did so for the support.

It is when teachers put new ideas in to practice that they need guidance (Guskey, 1986; 2002). Guskey's model of teacher change reinforces the idea that change takes time and therefore support. He states that change increases a teachers' anxiety level and they might even feel threatened; teachers fear students might not learn the necessary concepts and they would in turn be embarrassed. Feedback that is positive, encouragement to continue efforts, and support that is united with pressure to continue

the path of change is needed for the transformation. Time to discuss the successes and concerns that are encountered as well as new ideas is essential, it is what the teachers want and appreciate.

Support is also acknowledged when principals preserve instruction time and planning time. The purpose of Gill and Hoffman's (2009) study was to investigate planning time discourse to gain insight into teacher decisions and how they might be related to the teacher's beliefs about the subject, teaching, and learning with four eighth grade mathematics teachers. Their findings raised more questions about whether team planning helped to correct misunderstandings about student learning and teaching and if collaborative session outcomes aid teachers in adopting new practices and changing beliefs that they currently do not hold leading to a more student-centered classroom.

Promoting changes in beliefs, teaching, and learning. Reform does not occur just because a new curriculum or textbook is introduced (Cohen & Ball, 2001; Cwikla, 2002; Manouchehri & Goodman, 2000). Cohen and Ball (2001) go further to state that a curriculum does not equate to changes with instruction. Elements of instruction include perceptions held by the teachers and students, their interactions, the tasks they engage in, and the learning environments in their study of two teachers and their third grade classrooms. Change is a more complex issue that is multifaceted; a new textbook or curriculum does not equate to transformations with teachers and student learning.

Manouchehri and Goodman (2000) add to the complexity by stating a teacher's content knowledge and beliefs influence how he or she uses the textbook. Their qualitative study, investigating the implementation and evaluation of a standards-based

textbook, involved two seventh grade classrooms over a two year period. Teachers follow the text but the researchers concluded that they need to understand and share the vision of the text, know the central idea of the unit or chapter, be able to reason mathematically, make connections between the concepts of various sections and chapters, and have the confidence to learn new approaches and concepts. With a textbook, teachers have the choice of rote drill or deeper thinking questions to promote conceptual understanding. Their choice of problems and/or examples is based on their beliefs about how students learn as the teacher will put their own spin on resources (Cross, 2009).

Transforming teachers. Most teachers do not willingly accept reform according to Taplin and Chan (2001) in their study with fourteen preservice teachers. Teachers were presented with content and pedagogical content problems, responded in journals, reflecting on their ability to problem solve and make changes to instruction. The researchers state that teachers shy away from accepting reform initiatives because of a lack of pedagogical content knowledge, lack of confidence, lack of creativity, accountability, and a lack of knowledge of new expectations.

Project-based, reform, learning takes time and there is a concern that students will not have the knowledge to achieve all that is expected (Wilson & Lloyd, 1995). Wilson and Lloyd (1995) studied three mathematics teachers, all considered traditional instructors, and ten ninth grade students over a period of six weeks. They looked at pedagogy and the teacher and student beliefs on pedagogical authority. The study revealed that teachers will struggle more with transforming their beliefs and actions than

their students; teachers perceived that student resistance was stronger than it actually was. Actions of students that did resist lessened with time and involvement in the task.

So how do we transform teachers? Guskey (1986; 2002) states that professional development needs to be embedded in the teachers day-to-day activities so that they can try new methodologies with the support of knowledgeable facilitators. Teachers engaged in sustained professional development and collaborative planning will have a better understanding of the content and pedagogy while acknowledging their personal understanding and those of their peers (Zambo & Zambo, 2008).

Individual and collective efficacy was investigated by Zambo and Zambo (2008) with fourth through tenth grade teachers from two different school systems where one school had low student performance and the other one had a high student performance. Individual efficacy improved during professional development; collective efficacy did not increase. The professional development influenced teacher skills and knowledge which changed their perceptions of their own effectiveness and how they relate and instruct students. Professional development did nothing for how teachers looked at their peers, their content knowledge, pedagogical knowledge, and what they brought to the planning of instruction.

The change process is cyclic. Guskey (2002) discusses teachers attending professional development and trying what they have learned in their classrooms. Sustained embedded professional development provides teachers with time to acknowledge student's actions, supplying the evidence for the teachers to want to try to do more. The more teachers attempt or practice reform actions, the more they will have

the evidence that it works. Professional development needs to be sustained so teachers will continue to be supported.

Along with understanding that change takes time is the need for teachers to have a vision or goal(s) for mathematics in their schools, providing an opportunity for all students to learn mathematics (Loucks-Horsley, Stiles, Mundry, Love, & Hewson, 2010). All stakeholders will go through times of disequilibrium and cognitive dissonance. Gaining knowledge (content and pedagogical content) needs to be part of professional learning in order to support the work of teachers in their classrooms (Cohen & Ball, 2001; Cross, 2009; Cwikla, 2002; Derry, Wilsman, & Hackbarth, 2007; Guskey, 1986, 2002; Orrill, 2006; Taplin & Chan, 2001). A teacher's confidence will increase as content knowledge and pedagogical content knowledge increases which will have a cyclic effect on students; students will gain confidence in their efforts and actions which will in turn flow back to the teachers (Guskey, 2002; Zawojewski, Chamberlin, Hjalmarson, & Lewis, 2008)

Opportunity to learn. Dweck's (2010) fixed and growth mind-set is affected by teacher beliefs. Teachers having a fixed mind-set hinder student learning as they have already determined if a child will or will not be successful in mathematics. The growth mind-set is needed when developing rich tasks that will have multiple entry points and paths to solve, thereby reaching more students and giving them the opportunity to learn and be successful. Teachers with the growth mind-set, open to learning new approaches, will see rich task as an avenue to reach all students so that they will have the opportunity to learn the mathematics. Schoenfeld (2009) states, "When teachers are well supported in

teaching for understanding and have good curricular materials to use, children really do learn (p.19)." Knowing oneself as a teacher, having a fixed or growth mind-set, and getting to know our students have life-long affects. Why do we want all students to learn mathematics? Is it due to our beliefs that all should learn and are capable of learning, that it is a social justice, or is it self-interest (Secada, 1989)? We need to answer these questions for ourselves in order to plan for instruction as our beliefs affect student learning and their futures.

Teachers' Understanding of Slope

What methods, examples, or analogies do teachers use when presenting the concept of slope to students? Stump (1996; 1999) found that there was a difference between pre-service and in-service teachers in how they approached teaching slope. Participants took a Mathematical Survey (see Appendix A) which led to Stump identifying seven categories of slope: functional property, physical property, geometric property, algebraic ratio, parametric concept, trigonometric concept, and the calculus concept. Of the first four main categories listed, the functional property was least mentioned by in-service teachers and the physical property was least mentioned by pre-service teachers. The Mathematical Survey, an assessment recognizing various types of slope and correct computations, given by Stump (1996; 1999) revealed weaknesses in the trigonometric conceptions with in-service teachers doing twice as well as pre-service teachers. All participants did well with the parametric and functional concepts. The physical, geometric, and algebraic ratios were close between the two groups, however, determining the initial condition and slope in a given situation was 57% and 62%

respectively for in-service teachers. If a teacher is not fluent in all categories then their instruction will be lacking and students will not be encouraged to pursue all avenues.

Stump (1996; 1999) and Cavey, Whitenack, and Lovin (2006) both wrote about the content knowledge of a teacher affecting their instruction. Cavey, Whitenack, and Lovin (2006) studied an Algebra I teachers' understanding of linear functions and the content knowledge that was called up when teaching the lesson. Their research question investigated the mathematical knowledge that teachers draw from to support students' developing mathematical understanding. Findings revealed that the teacher needed to move fluidly between various representations of slope and to make connections to mathematical concepts. What they were missing in their analysis is the teacher's voice to gain a perspective of why she chose to respond a certain way or apply specific representations at that time.

Walter and Gerson (2007) also studied the concept of slope and teaching through collaboration and reflection with twenty-five elementary teachers over a three year period. The researchers looked at teacher performance to gain understanding of their knowledge and how they communicate their knowledge through the interrelations between activities. Personal agency, the obligation, accountability, and option to select teaching and learning activities based on prior knowledge, is inseparable with social dynamics in the classroom and within collaborative settings. The professional development nurtured the personal agency of the teachers which resulted in the teacher's experiencing the sense of making choices, communicating mathematics, and understanding mathematics with minimal facilitator interactions.

Altogether a teacher‘s lack of content knowledge will not only effect instruction but also their ability to develop a rich task that makes connections to other content and has multiple entry and pathways to solve. Teachers will need to research content in an effort to insure their understanding of the material but also to search out methods to present the material. Slope was chosen for this study due to the concerns in research of teacher understanding of this content but also to model what needs to be done to develop a rich task involving this content.

Lesson Study

Lesson study was brought to teachers’ and researchers’ attention in 1999 when Stigler and Hiebert (1999) published *The Teaching Gap* (Perry & Lewis, 2008). The researchers relayed information from the Third International Math and Science Study video study of teachers in the United States, Germany, and Japan’s eighth grade classrooms to our awareness. Lesson study was one chapter in the book. The culture within the United States is quite different from Japan and therefore lesson study has been adopted with caution and at a slower pace. When Japan adds a new topic to a curriculum, it is only the basic objectives that are given (Lewis, 2010). Teachers and researchers join forces to research and develop lessons. After a year or so, the lessons that have been developed are openly presented to the public for comment. Teachers are not caught up in the specifics of what is added but develop the concepts through the lesson study process.

Lesson study is a method to improve instruction by observing students (Lewis, 2002). Using a team approach, teachers join together to determine goals for the whole child. It is a time to dig deep into the content, determine questions to probe students to

higher order thinking, and to develop strategies and to anticipate students' actions and the questions they might ask.

The process begins with teachers setting goals (Lewis, 2002; Lewis, Perry, & Hurd, 2004). The goals include those for the lesson, the unit, the bigger picture with respect to the content, and the long term development of the student. Teachers will then research the content and map out the entire unit. Selecting one lesson to be the research lesson will lead to determining the product of the lesson, writing questions that will probe and promote thinking skills, and acknowledging the "look fors" in the lesson. A briefing session prior to the lesson is conducted with all team members and the "knowledgeable other" who is knowledgeable about the content. One member of the team will be the presenting teacher for the lesson while the other members become observers. Lewis (2002) suggests that each observer should be assigned a student or groups of students. Groups may consist of struggling students, students that get the answer quickly and become bored, and students who might have language difficulties. After the lesson is presented, teachers will come together once more to debrief. The presenter of the lesson begins by sharing what he or she feels was good, bad, or unexpected. Each member will take a turn sharing their observations as well. Discussions will guide making changes to the lesson and planning next steps.

Summary

Rich task is not a new idea in education but very under used. All of the characteristics of rich tasks are what experts say that students need in order to become productive citizens (Grootenboer, 2009; NRICH, 2007; Piggott, 2004). Teachers are

pressed by administrators to obtain passing scores on state mandated tests and therefore are hesitant to try new ideas. One would think that the pressure for students to perform successfully would enhance and raise the level of need by teachers for methods to help them in their endeavors.

Characteristics of rich tasks may be found in the literature (Grootenboer, 2009; NRICH, 2007) but not explicit steps as to how to create one or suggestions as how to approach the creation. Rich tasks, as well as PBL, were shown to be successful and to increase student achievement but have not become commonplace in the mathematics classroom. Is the creation of the task so enormous or overwhelming that teachers do not want to learn or continue implementing them? The New Basic Project (Queensland State Education, 2006) revealed that after two years of un-mandated participation, more students were completing rich tasks than when it was an expectation however the researchers did not elaborate on why. May it be assumed that teachers had experienced success, "evidence" as seen by Cooney, Shealy, and Arvold (1998), leading them to continuing with the changes in instruction?

The research on professional development and slope reveals that content knowledge and pedagogical content knowledge that is sustained and supported lead to teacher change. Professional development is multi-faceted with teacher beliefs playing a role in their willingness and acceptance to participate. Teachers, administrators, and parents want students to be successful in mathematics so they will be productive citizens, able to communicate and apply problem solving skills in their futures. Rich tasks are the first step, the foundation of project-based lessons, which will enable students to learn and

practice the skills needed and for teachers to gain confidence and support with their endeavors.

These concerns and desires lead to the current study by presenting, developing, and supporting teachers as they learn, practice, and reflect on rich task creation using the lesson study approach in their classrooms. Teachers will acknowledge rich task characteristics while attending a summer institute to learn how to create a rich task. They will also learn what is lesson study and how might using the lesson study process help them on their journey to providing all students with the opportunity to learn mathematics. Interactions between the teacher team will hopefully aid in gaining insight into their transformation. Teachers, as lifelong learners, will see the need to continue learning, as with the concept of slope in this study.

Chapter Three

Research Questions

Questions for this study revolved around teachers' acceptance of and willingness to change actions on planning and implementing instruction. Our world has changed drastically due to technology so there is a need to take mathematics education and research into the twenty-first century. Students need to apply their knowledge and be problem solvers, using technology readily available to them. This led to a concern about how to engage and motivate students to want to learn mathematics. Rich tasks have been defined previously as being relevant to student lives and challenging students to think (NRICH, 2007). Lesson study focuses on instruction and how the students perceive and participate in their learning (Lewis, 2002; Lewis, 2010; Lewis, Perry, & Hurd, 2004; Perry & Lewis, 2008). Putting rich tasks and lesson study, as the mode of professional development, together was the foundation for this study.

Both lesson study and rich tasks have content knowledge as an important element in the process of teachers learning to implement creative and challenging lessons (Lewis, 2002; Lewis, 2010; Lewis, Perry, & Hurd, 2004; Perry & Lewis, 2008). Increasing teacher content knowledge and pedagogical content knowledge has shown to affect instruction and ultimately student achievement (Cwikla, 2002; Derry, Wilsman, & Hackbarth, 2007; Manouchehri & Goodman, 2000). Rich task development, the process

of lesson study, the increase in content knowledge, and acknowledging their own beliefs and goals need to be addressed during professional development. Therefore the purpose of this study was to discover if strategies during professional development on rich task creation affected Algebra I teachers and teacher team actions when planning instruction and facilitating lessons using the lesson study approach. The content that was focused on during the professional development was slope which is traditionally taught using formulas and a geometric approach. Teachers need to have a deep conceptual understanding of the content to in turn affect student learning and expectations of success. Slope was the conduit through which rich task creation was approached and lesson study the conduit by which rich tasks development was learned. As the researcher and facilitator of the professional development, I acknowledge that one summer institute will not alter the way a teacher approaches instruction but it is one small step toward changing to be more student-centered. It is hoped that teachers will recognize and begin to apply big ideas and rich tasks into their lessons. The question that drove the research was: How did the Summer Institute on rich tasks influence the teacher's implementation of the lesson study process? Secondary research questions are:

1. What did teachers perceive as characteristics of "high cognitive demand" tasks prior to and after professional development?
2. What were the goal(s) of the teacher and teacher team prior to, during, and after the professional development?
3. How did the group-focused experiences of rich tasks influence teacher practices and beliefs during lesson study?

4. What challenges and/or constraints did teachers encounter when implementing rich tasks in the lesson study?

Participants and Setting

The research was conducted in a Virginia school district which has one high school. Participants, Algebra 1 teachers, consisted of five general educators and two special educators from the high school and two teachers from the alternative education setting. The number of Algebra 1 teachers changes yearly according to student needs. For the upcoming school year, 2012-2013, the school division is planning on all Algebra 1 students taking the course daily for the entire school year while at the same time keeping the class enrollment numbers low which in turn has raised the number of teachers needed for the course. The information in Table 2 is about Algebra 1 teachers:

Table 2

Algebra I Teacher Characteristics for 2012-2013

		Degree			Number of Years in Education				
	Count	Bachelor's	Master's	Degree in Math Education	0 – 5 Years	5 – 10 Years	10 – 15 Years	15 – 20 Years	>20 Years
General Education	5			2	1	2		1	1
Special Education	2	1	1	0	1	1			
Alternative Education	2	2		0	1	1			

There are about one hundred teachers in the high school of approximately one thousand students. This rural school district's high school was fully accredited but did not make AYP, adequate yearly progress, for the past two years. The results of the state standards of learning assessment in Algebra I may be seen in Table 3 (data retrieved online from VDOE at https://p1pe.doe.virginia.gov/reportcard/).

Table 3

Algebra I Results 2011-2012

	2010-2011			2011-2012		
	State	All Students	Students with Disabilities	State	All Students	Students with Disabilities
Passed advanced	28%	4%	2%	7%	0%	0%
Passed proficient	66%	83%	65%	68%	46%	14%
Passed	94%	86%	67%	75%	46%	14%
Failed	6%	14%	33%	25%	54%	86%

Algebra I was the lowest for the 2011-2012 school year of the three end of course assessments which included Algebra I, Geometry, and Algebra II as shown in Table 4 (data retrieved online from VDOE at https://p1pe.doe.virginia.gov/reportcard/). The assessment for the 2011-2012 school year was based for the first time on the new standards of learning. Algebra I was also below the state average pass rates for the last three years.

Table 4

End of Course Results for All Students

	2009-2010		2010-2011		2011 – 2012	
	School District	State	School District	State	School District	State
Algebra I	82%	94%	86%	94%	46%	75%
Geometry	76%	88%	76%	87%	65%	74%
Algebra II	92%	91%	96%	91%	88%	69%

My Role in the Study

My role as leader was that of researcher and facilitator throughout this study of participants in a nearby school district. In order to perform in both roles, the leader must have a firm grasp of content knowledge, pedagogical content knowledge, state and local standards that embrace standards-based instruction, and knowledge of teacher change (Alger, 2005). My role, then, was to guide the participants throughout the professional development by providing them with knowledge, experiences needed for growth, and support. Blumenfeld, Marx, Soloway, and Krajcik (1996) discussed the importance of creating successful groups that are encouraged to discuss concepts and actions. Having common goals and considering academic and affective outcomes was also in the hands of the leader (Clarke, Breed, & Fraser, 2004). Content knowledge and pedagogical content knowledge was gained by the leader through participation in coursework when fulfilling requirements for a degree of Doctor of Philosophy in Mathematics Education Leadership.

Additionally the researcher has taught most high school courses up to and including A.P. Calculus BC, has sought additional course work and professional development in teaching calculus through a reform-based approach, and has been a practicing Texas Instruments Regional Trainer for eight years. The leader/researcher also served as the secondary mathematics coordinator for another public school district in the same state which provided the opportunity to continue researching methods of reaching students through reform based practices and working with teachers on the methodology to do so. Observing teachers during planning sessions and during instruction have helped to gain insight into the promotion of reform-based lessons. All of these observations and experiences have assisted in growth as a leader but also with questioning why teachers minimally embrace student-centered instruction.

As the leader, I provided the agenda and resources for each of the Summer Institute days and the guidance for the lesson study during the school year. Promoting journaling on the part of the participants occurred as free writes and with requested topics so that they might put their feelings down on paper. I, as the facilitator of the sessions, did the same along with field notes recording my thoughts about participant interactions. Some of my biases included having already introduced reform-based instruction to another local county with varied levels of growth shown. I needed to remain cognizant that every group of teachers is different, with different needs, desires, and expectations, keeping an open mind on the participant's actions and reactions to the activities and to each other. In this study I was not familiar with the school district's curriculum guides, administration, or student body. I needed to listen carefully to what was being said in

order to respond accordingly. Reflecting on the work was important for the participants and for the researcher.

Theoretical Framework

The overall approach to this research was a descriptive case study (Yin, 2003) of teacher's actions to activities during a Summer Institute on developing rich tasks and during the lesson study where a rich task lesson was presented to students and modified. Using Baxter and Jack's (2008) article as a guide to determining the type of case study, the unit of analysis, the boundaries, and the propositions helped to narrow and focus the study. The unit of analysis was the participant's actions and activities they engaged in during the Summer Institute and with each other as they worked together as a team on rich tasks and planned a rich task and a unit of instruction. Boundaries assisted in keeping the focus of the study manageable. Baxter and Jack (2008) discussed setting boundaries where the time and the activity is the boundary. The boundary for this study was that the participants were Algebra I teachers in the same school system participating in the same professional development, the Summer Institute and Lesson Study during the school year. Propositions, according to Yin (2003), help to focus the collection of data by providing a direction and depth of the study. For this study the propositions were the teacher's beliefs about teaching and student learning, the teacher team goal(s) that was influenced by their personal goals, their knowledge of the content of the unit, rich tasks, and cognitive demand, and the teacher's instructional practices.

Learning occurred within the environment of the participants and with interaction between the participants. Cooper (2009) shared four principles that would be part of a

Vygotskian classroom. The first one is that learning is a social activity where those involved collaborate. Social activity is followed by lesson planning and curriculum decisions that are based on the zone of proximal development where students maximize their learning. The third principle is based on the activity being real and relevant to the students. Extending learning to the community beyond the classroom is the last principle given by Cooper (2009). Vygotskian principles reinforced the selection of a descriptive case study as the participants in the study were in their own environment and the study focused on the interactions between the participants.

The teacher team was expected to research concepts, learning from each other, as they put the elements together to create a rich task that was applicable to their students. As they developed the rich task, they were to take into consideration multiple entry points, multiple pathways to solve the problem, and the discourse they wanted to promote between the students that will elevate their learning further than the "sit and get" classroom. At the same time, the teacher team experienced the task as their students will in the classroom thereby enabling the teachers to acknowledge student actions and anticipating their response.

This study followed Derry, Wilsman, and Hackbarth's (2007) design research on algebraic thinking, Stump's (1996) survey research that analyzed content knowledge of pre-service and in-service teachers, and Lee's (2010) study on project-based learning with a geometric foundation. Derry, Wilsman, and Hackbarth's (2007) research involved video case studies that teachers viewed examining various situations of teacher and student algebraic thinking. Their research began during a summer workshop and

followed through the next school year. Goals of the summer workshop were to compare and interpret tasks that demonstrate fluency with representations and to generalize the strategies employed to encourage algebraic thinking. The foundation of their actions was based on the NCTM *Principles and Standards for School Mathematics* (2000, p. 37-40). Teachers involved in Derry, Wilsman, and Hackbarth's (2007) study solved problems illustrating an aspect of algebraic thinking in tasks called Sample Algebraic Modules (SAM) along with reading articles that encouraged reflection on the topic and watching videos of classes engaged in algebraic thinking. Altogether, teachers completed five SAM's throughout the school year. During the same time, researchers collected artifacts that illustrated the algebraic thinking, multiple representations employed, and the discourse between teachers. SAM's were traded in for rich tasks in this study so that teachers might experience what their students will experience, the cognitive dissonance of deciding how to approach the problem and then following through with the plan.

Stump's (1996) research surveyed pre-service and in-service teacher's knowledge of slope that included content knowledge and pedagogical content knowledge. Her survey revealed seven different categories of slope along with the type most frequently referred to and the type that was lacking as revealed in the artifacts. Comparison of the usage of the slope categories by the two groups was assessed in the interview and in the survey on the mathematical concept of slope. What teachers' believed was the most important for students to be successful was determined through the survey and interview conducted by Stump (1996). Content knowledge is such an important part of rich task development which is why research on teachers' knowledge of slope was conducted.

Teachers need to investigate topics even though they are confident in their knowledge, content, and pedagogy. Recognizing the knowledge one's peers have and are willing to share will allow the collective efficacy of the team to grow.

Lee's (2010) project-based research was a cross-disciplinary unit involving mathematics, history, and English courses. Project-based units are the macro-instructional approach whereas a rich task is a micro-instructional approach (UVA-SCPS, 2011). Project-based units are typically interdisciplinary while a rich task may remain within one subject area. One of Lee's (2010) research questions centered on how two teachers designed a project-based unit that is relevant to the students as well as rigorous. The case study included observations, field notes, and interviews. Informal interviews were conducted after daily lessons to discuss and reflect on the student and teacher actions and interactions. Data analysis was approached using grounded theory techniques due to the cyclic nature of the data collection.

Due to the unavailability of not being able to "shadow" the participants during this study and conduct informal, frequent, and probing interviews, a lesson study approach was taken with the professional development. The product, a unit of study that was tangible and useful for teachers beyond this research was the desired result of the planning, rich tasks experiences, and lesson study approach that was introduced during the Summer Institute.

The Summer Institute and Lesson Study

The research had two phases: the Summer Institute professional development and the school year Lesson Study (see Appendix B for agenda). Due to scheduling, the

Summer Institute was split with three days in June, the end of the school year, and one day in August during the work week prior to school starting in the fall.

Processes/strategies that were used to introduce rich tasks and lesson study made up the Summer Institute. Professional development was planned for the teachers to learn about cognitive demand **(**Stein, Smith, Henningsen, & Silver, 2000), NCTM Process Standards (2000), and the Rule of 5. In addition, the participants revisited the Algebra I curriculum to develop goals for the course and to determine big ideas for units of study. Learning about and creating driving questions pulled the unit of study together to meet objectives and goals. Finney, Demana, Waits, and Kennedy (2003) brought the idea of Rule of 5 into the classroom with their Calculus and Pre-calculus textbooks. Their books focus on looking at the subject using algebra, graphs, numbers (tables), verbal, and concrete representations, the Rule of 5. Given one of these representations, students are able to make connections to the others, justifying their relationships through verbal and written communication. The five representations correlate with the NCTM Process Standards (2000) of representations, reasoning and justification, problem solving, communications, and connections.

The foundation for units of study that included one or more rich tasks was created by the participants using a modified Frayer Model format and was completed during the professional development as a result of their involvement. Sections of the Frayer Model included pre-requisites, objectives and standards, rich tasks, and assessments. The center of the model contained the title for the unit of study and the driving question. It was hoped that the process that the teachers, in collaborative groups, used during the

professional development on rich tasks will be repeated with different topics throughout the school year thereby meeting the criteria of generalizing what they have learned through other content. Participants were expected to apply the process from the Summer Institute as they planned a future unit of study for a topic that was taught at the beginning of the school year. Ultimately, the "process" needed to be viable and not overwhelming for the teachers so they would see its usefulness which is why the unit plan template (see Appendix C) was chosen instead of the lesson plan template for the units of study. Learning more about the topic, the connections to other mathematical concepts and to real-world situations, assists teachers in developing rich tasks that are engaging and relevant for students.

Procedures

Introduction and foundation setting for the study. I met with the Algebra I teachers to explain the research and requested their participation prior to the Summer Institute. Prior to the meeting the prospective participants were requested to bring with them a lesson or unit that each teacher had implemented in their classroom instruction on slope however only three were collected. The informational meeting was followed by scheduling interviews with seven of the nine participants using the Initial Interview Guiding Questions (see Appendix H) so that a discussion of beliefs about teaching slope, planning lessons or units, and engaging students may occur. All interviews were completed prior to the Summer Institute beginning. Two teachers that could not schedule a time were from the alternative education program. The Mathematical Survey (see

Appendix A) (Stump, 1996), was also distributed with the request that it be completed and turned in on the first day of the Summer Institute.

Professional Development Agenda

Table 5 below outlines the events for the high school Algebra I teachers for the Summer Institute in June, and one day in August during the school year work week, as well as the lesson study presentations during the first semester of the school year.

Table 5

Professional Development Agenda

Day	Event
1	Summer Institute – June
	Journal prompt example followed by journaling about personal goals
	Course goal determined
	Journal relating personal goals with course goal
	Cognitive demand activity
	Norms for the professional development
	Flow of the course into big ideas
	Rich Task #1 – Hexagonal Train
	Flow of the course into big ideas revisited
	Journal - rich task experiences influence planning, practices and beliefs
	Reading assigned: Why Is Teaching With Problem Solving Important to

	Student Learning? (NCTM, 2010)
2	Discuss readings and compare to rich tasks
	Rich Task #2 – Enlarging Forts
	Journal – Peer and self-evaluation and challenges and constraints
	Topic research discussed
	Unit plan on Linear Functions
	Determine units of study for the first semester of the school year
	Frayer Model – brainstorm pre-requisites, objectives, goals, assessments, and rich tasks
	Lesson Study process introduced
	Journal – How do participants perceive using rich tasks in their classrooms
	Reading assigned: Helping Students Connect Functions and Their Representations (Moore-Russo & Golzy, 2005); Connecting Procedural and Conceptual Knowledge of Functions (Davis, 2005).
3	Discussion of readings
	Journal – goals and role anticipated by teacher in the classroom.
	Rich Task #3 – The Towers
	Journal – Compare rich tasks characteristics and Peer and self-evaluation
	Frayer Model – continue brainstorming with a focus on the rich tasks.
	Flow of the unit. Research that is needed.
	Discuss school year work day schedule

	Exit survey (electronic – sent by email)
4 SY	School Year Work Week – August
	Review ideas, Frayer Model and brainstorming events from Summer Institute in June
	Work on unit flow and share with entire group
	Continue working on unit flow and rich tasks.
	Exit Survey
SY	School Year Lesson Study
	Passionate and Energetic team presentation
	Debriefing, lesson modification – October
	Exit survey
	Repeat Lesson Study, Cognizant and Experienced Team – November

Data Collection

Quantitative data. The Mathematical Survey (see Appendix A) developed by Stump (1996) on slope was modified and given at the beginning of the summer institute in order to evaluate content knowledge and pedagogical content knowledge of the teachers. The survey, as used by Stump (1996), contained problems from other sources. The first question in the survey explores the teacher's definition of slope, what characterizes slope, and pre-requisite knowledge needed in order to be able to learn about slope. The following question, where several situations and problems are given, requires the participant to determine whether each example represents a linear function. The last

group of questions necessitates the participant working on problems focusing on the many categories of slope. Questions 3-5 are modified items from the text, *UCSMP: Algebra* by McConnell, Brown, Eddins, Hackworth, and Usiskin (1990), question 6 is modified from an article by McDermott, Rosenquist, and van Zee (1987), and question 7 is from the text, *Calculus*, by Hughes-Hallet et al. (1992). Remaining questions in the survey were created by Stump. Additional modifications to the survey have been made to also delve into rich task characteristics by adding a table of ten items to rank using a Likert scale from 0 to 4 with 0 indicating a no response and 4 a response of yes. The rich task questions were based on research listing characteristics by Grootenboer (2009) and NRICH (2007) and correlate with the planning rubric. Moylan's (2008) three "S's", along with Southern Regional Education Board's (SREB, 2000) Six A's, were also embedded in this set of questions. Survey results provided information on teacher knowledge of slope, content, and pedagogy, which in turn influenced activities for the Summer Institute sessions.

Lesson/units that the teacher used when teaching slope were collected prior to the Summer Institute. These artifacts were analyzed using the planning rubric (see Appendix F) created to assess the quality of the document. The rubric is a synthesis of characteristics given by Grootenboer (2009) and NRICH (2007), cognitive demand levels by Stein, Smith, Henningsen, and Silver (2000), Moylan's (2008) sense of self, student skills, and society relevance, and SREB's (2000) Six A's. Inter-rater reliability for the planning document was obtained in a pilot study which assessed lessons on slope and rate of change. Three Math education experts, and myself, rated several lessons and activities

that were obtained from teachers using the rubric that evaluated the documents using characteristics of rich tasks. Characteristics were consolidated into fourteen items for a possible total of sixty points and assessed on a Likert scale from 0-4**.** Each level in the rubric contained detailed remarks as to what the scale value indicated for the given criteria. The results of the pilot study are shown in Table 6. The higher the score, the better quality of the printed lesson, according to the characteristics assessed. Evaluator's scores were similar with an average deviation on each of the items shown below in the table.

Table 6

Pilot Trial Results for Planning Rubric

	Activity Average (60) – Researcher		Activity Average (60) – Experts		Average Deviation per Item
	Points	Percentage	Points	Percentage	
Lesson 1	23	38.33%	21.0	35.00%	1.067
Lesson 2	23	38.33%	25.5	42.50%	0.933
Lesson 3	41	68.33%	42.0	70.00%	0.200
Lesson 4	41	68.33%	40.3	67.22%	1.733
Lesson 5	44	73.33%	46.5	77.50%	1.067

The planning rubric (see Appendix F) was also used to assess the quality of the unit plan (see Appendix C) that the team was to create for the lesson study during the first semester of the school year. Comparisons were to be made between the initial plans that were turned in and the final unit created by the team.

Qualitative data. Interviews, participant's journals, and audio and video recording of the Summer Institute along with the debriefing of the school year lesson presentation made up the qualitative data collection. The focus of all data was on the propositions: participant's beliefs about teaching and student learning, the teacher team goal that was influenced by their personal goals, their knowledge of the content of the unit, rich tasks, and cognitive demand, and the teacher's instructional practices.

Prior to the Summer Institute initial interviews (see Appendix H for interview protocol) were conducted, taking what is learned about the participants through their lesson plan on slope and delving further into their beliefs, their practices in the classroom, their personal goals and goals for teaching and learning, and their objectives for the turned in lesson on teaching slope. Initially, interview prompts centered on how various representations of slope are presented and/or explored by students. In addition to the content prompts, questions explored the development of the lesson. For example, did the teacher research the topic? If so, how and what did they find out? Did the task have multiple entry points, multiple strategies to solve, and use twenty first century skills? Additional questions were asked of each participant to further explain their viewpoints. Guiding questions, as defined by Creswell (2008), contain instructions or notes for the interviewer as well as specific questions to be asked from each interviewee. These questions insured that all interviews attempted to gather information in the same way and on the same topics. Interviews dove into creation and facilitation of the lesson or unit through the eyes of the teacher. Patton (2002) states that an interview is a form of intervention where the questions elicit comments about the interviewee's thoughts,

content knowledge, and pedagogical content knowledge that leads to their reflecting on their comments and actions. The initial interview laid the foundation for the activities during the Summer Institute as well as the final interview questions.

On the first day of the Summer Institute, participants were given a red, green, or blue notebook that contained journal prompts, rich tasks, and a linear and quadratic function unit plan. The color of the notebook was to aid in data collection as the red notebooks were for the Algebra I general education teachers, green was given to the special educators, and blue to the teachers from the alternative education center. Later the notebooks were collected by the team so differences in data between the two groups could be acknowledged. Going over what was a good journal response was done on the first day so the depth of their thoughts could be captured. This was followed by the participants completing a journal about their goals for teaching and learning mathematics as well as for the professional development. Following the journaling, participants determined whether items on a list of characteristics were indicative of traditional instruction or of reform-based instruction. A debate on some of the items ensued and included calculator usage and classroom management. Going over and determining the rigor in problems for the cognitive demand activity followed. These last two activities are where the team's characteristics began to be noticed.

Comments made in the journal, as well as the Mathematical Survey results, aided in promoting and extending the participants content knowledge and making connections to assist in the rich task creation. They were also helpful when triangulating what has been observed during the creation process of learning about rich tasks and the lesson

study process. The written data revealed content and pedagogical content knowledge of the participants while the interviews attempted to decipher and extract teacher thoughts on content knowledge, rich tasks development, and facilitation of the tasks for a better understanding of the written remarks.

Additional data were collected during the lesson study as a team member presented the lesson for their peers and me. The entire team was tasked with developing the rich task but only one teacher presented the lesson. Remaining team members observed groups of students that were selected by the presenting teacher. After the lesson, the team members met to discuss their observations. The presenting teacher led the discussion with his or her beliefs about how the lesson went. In turn, each member shared their observations about how the students engaged in the lesson, if the cognitive demand of the task was maintained or destroyed, and if the goals and objectives were reached. All of this discussion was audio taped and transcribed. The team was to make changes to the task according to the discussion.

The second team repeated the process of briefing, presenting the lesson and observing the students, and debriefing. Changes to the lesson were determined by the observations made by those participating. The lesson study session closed with some questions that were asked of the group to encourage them to reflect on the experience, as well as how and if the lesson study approach to creating rich tasks is viable for future planning sessions. Some of the questions and observations to be made include:

- What are the actions and reactions of the teacher during the implementation of the task from the lens of the team?

- Did the teacher fill the role of facilitator or giver of knowledge and procedures during the task?
- Did the implemented task include the intended characteristics of a rich task according to the team?
- How did the written lesson compare to the implemented lesson according to the team? If it was different, why was it so?

Table 7 shows the intersections between the research questions and the various instruments that will be used to answer the questions. A table organizing the intersections between the events during the Summer Institute and the school year lesson study, the research question, and the artifact or data gathered is found in Appendix B.

Table 7

Correlation Between Research Questions and Instruments

Research Questions	Journaling	Unit Planning Rubric	Observ-ations	Lesson Study	
				Briefing	Debrief–Final Interview
1. What do teachers perceive as characteristics of "high cognitive demand" tasks prior to and after professional development?	Participant	X	X	X	X
2. What are the goal(s) of the teacher and teacher team prior to, during, and after the professional development?	Participant	X	X	X	X
3. How do the group-focused experiences of rich tasks influence teacher practices and beliefs during lesson study?	Participant Researcher		X	X	X
4. What challenges and/or constraints do teachers encounter when implementing rich tasks in the lesson study?	Participant Researcher		X	X	X

Methods of Analysis

Planning artifacts were assessed using the planning rubric (see Appendix F). The rubric assesses the documents according to: cognitive demand, multiple entry points, multiple paths and solutions, authenticity, connectedness, multiple representations, academic rigor, active exploration, applied learning, group work, and assessment. Total scores ranged from 0 to 60 points with the larger the result the higher quality of the document as per rich task characteristics. Comparisons were made between the scores on the initial artifact created by one individual and the final documents created by the team. Final document quality score was triangulated with the presentation of the lesson, journal prompt responses, comments made during the debriefings, and the focus interview. The results are applicable to answering the first research question on whether the participants know and recognize rich tasks characteristics.

The journal prompts, researcher journaling from the observations of the summer institute, lesson study presentations, and the interviews were coded according to techniques given by Saldana (2009) and Creswell (2008). All of the above was transcribed and coded by reading and re-reading the documents. A matrix was created for each of the research questions and the corresponding statements from the participants for the initial interviews. The matrix was repeated with the transcripts from the Summer Institute and again with the lesson study presentation. These statements were then coded for patterns or themes in the participant's remarks. Each participant was coded separately using descriptive and structural coding (Saldana, 2009). Additional rounds of coding grouped the codes into categories using pattern coding. Pattern coding assisted in

developing major themes, patterns of relationships and expectations. Changes in themes determined through coding of the initial interviews, journal prompts, researcher journaling, and observation of the planning session were compared to the lesson study briefing and debriefing, focus interview, researcher journaling, and observations made during the events of the school year.

Research question #2 regarding goals was answered using participant journal prompt responses, lesson study briefings and debriefings, and the final group interview. Participant remarks were taken from the transcribed items above and triangulated between the items to determine consistency. Rich task effects on future planning practices and beliefs, research question #3, and challenges and/or constraints, research question #4, are answered by the journal prompt responses, observations, and lesson study briefings and debriefings. Consistency was acknowledged between the spoken or written responses and those that are observed.

The primary research question: How did the Summer Institute on rich tasks influence the teacher's implementation of the lesson study process was addressed using the responses of the four secondary questions. Key aspects were to determine if the written or spoken words were consistent with the observed, if the themes garnered from the transcribed events lend themselves to a desire to facilitate instruction that meets state and local standards, and if the debriefings provide evidence from student observations that coincided with the teachers' viewpoints.

Chapter Four

This chapter contains the data collected during a Summer Institute and lesson study during the school year regarding rich task creation and implementation. The purpose of this study was to discover if strategies during the professional development on rich task creation will affect Algebra I teachers and teacher teams when planning instruction and facilitating the lesson. All data were collected to answer the question "how did the Summer Institute on rich tasks influence the teachers' implementation of the lesson study process?' Pseudonyms have been used for the teacher's names throughout this study. The following four secondary questions will assist in answering the research:

1. What did teachers perceive as characteristics of "high cognitive demand" tasks prior to and after professional development?
2. What were the goal(s) of the teacher and teacher team prior to, during, and after the professional development?
3. How did the group-focused experiences of rich tasks influence teacher practices and beliefs during lesson study?
4. What challenges and/or constraints did teachers encounter when implementing rich tasks in the lesson study?

The unit of analysis was the participants' actions to the activities they engaged in during the Summer Institute and with each other as they developed a unit and lesson, leading to the rich task. Boundaries set for the study were those teachers who will be in the study, Algebra I teachers, and the time when the data was collected. The study focused on the propositions of teacher beliefs about teaching and learning mathematics, teacher and teacher team goals, participant content knowledge, the levels of cognitive demand, and the participant experiences, creation and implementation of rich tasks.

Prior to Summer Institute

Data gathered. Prior to the Summer Institute, data gathered included an initial interview Mathematical Survey (see Appendix A) about slope from seven of the participants, and a unit or lesson involving slope from only three of the participants. During the Summer Institute additional data collected included journal entries, transcribed audio and visual tapes, and artifacts from the rich task and lesson/unit creation. School year data included audio tapes of the debriefing session of the lesson study presentations.

Lesson or unit analysis. Instead of turning in the requested lesson or unit plan, three of the participants turned in an activity without the corresponding lesson/unit that they used when teaching slope. Each was asked if they had a lesson plan that went with the activity and all responses were negative. The activities were scored by me and another individual using the Unit Plan Rubric (Appendix F). Out of sixty possible points the average for these activities as determined by myself and the other individual was 23 and 24 respectively with a correlation of assigning the same score per item at 87%.

This was my first encounter that the Algebra I teachers did not write lesson plans or unit plans. It was also not an expectation of the administration. The activities that were turned in were created to be completed after the material was taught with students regurgitating what they knew with new values. All activities were to be approached using the same method with step by step guidelines to obtain the one answer. Their view on activities correlated with a statement made during the Summer Institute by Denise that she wanted tasks that was approachable by the students and where students could be led into the task with small successes. Time, a concern expressed by many of the participants, was their justification for why the activities were not higher in cognitive demand.

Initial interviews. For the first secondary research question, the initial interviews set the foundation of teacher beliefs and understanding of the characteristics of "high cognitive demand" tasks prior to the Summer Institute beginning. Three themes stood out from the participants' remarks about high cognitive demand tasks: student engagement, ability to communicate, and varied approaches to instruction.

Student engagement was mentioned by six of the seven interviewed. Delving further into what each teacher meant when they stated that they wanted students engaged in their class I found that everyone was not using the same definition. To Ellen, engagement meant that the students needed to be doing something and listed things like homework, working on a handout, or completing a guided note taking worksheet. Jane stated that engagement was an activity with explicit instructions to guide the students. It was even mentioned that students were engaged if they did not put their heads down or

sleep. Lessons that encouraged engagement were said to be interesting and relevant to the students with Brendan stating that the lesson could be told in the form of a story with "…a beginning, a main act, and an ending." Underlying all definitions of student engagement is that the students are "doing something".

Communication was more universally discussed as a 21st century skill that contributes to high cognitive demanding tasks. Being clear and explicit was a communication expectation of the teachers for their role in student learning. Garth stated that "If they will give the effort, then to me it is my fault if they do not succeed," referring to the ability of the teacher to communicate their knowledge to the student. However Carl did mention that "...when the students are asking [mathematical] questions and they want to know what is happening" is the ultimate goal for communicating mathematically as well as having students engaged. Using technology was also viewed as a mode of communication and as raising the level of cognitive demand of the task by the PET teachers. To increase communication many of the teachers asked students "why" to encourage explanations.

Even though varied approaches were mentioned, five of the seven participants stated that a non-negotiable characteristic was the need for students to know and memorize formulas. The slope formula was referred to as the "safety net." If students could "plug and chug" using the "safety net" then they could determine slope given any information. Only Amber stated that memorizing the slope formula would not be high on a list of characteristics and that "…if they are not getting it, graph it and move on."

Connecting to the real world, using technology, and relating the content to the student's lives to increase student learning and engagement was not high on their priority list.

Question 2, what were the goal(s) of the teacher and teacher team prior to the professional development, revealed themes centered on procedural skills versus conceptual understanding, and the Virginia Standards of Learning (2009). Brendan, Carl, Denise, and Jane's goals evolved around students understanding the procedures, with all of the teachers concerned that students have the "skill set" so that they could solve the problems. These teachers wanted the students to have the skills but to also be able to explain what they did. The explanation was in reference to what step needs to be taken. Conceptual understanding was mentioned by only Amber. Probing to determine what she meant by conceptual understanding led to an explanation that it was deeper than performing the steps of a computation, it was understanding what they did and being able to relate the concept to other situations. The SOL assessment was foremost on the minds of Brendan, Denise, and Garth as they expressed a goal of the students being successful on the end of course test. This was illustrated by Brendan's comments that the goal was to have students "…know the basic stuff they would need for the SOL" and after that was accomplished then Brendan would "…want to move toward them having a deeper understanding of the material." Skills are first in priority for Brendan, understanding coming in second.

The third question, "How did the group-focused experiences of rich tasks influence teacher practices and beliefs during lesson study," was approached during the initial interview to understand teachers' practices and beliefs going into the study.

Practices revealed themes of connections and "what works;" beliefs revealed themes of relationships and the differences between the "young and old" teachers. Amber, Brendan, Carl, Denise, and Ellen wanted to connect their lessons to the real world and make connections between procedures so students will be better prepared for the SOL tests. "…The basic stuff…" mentioned by Brendan was the primary concern for instruction and student learning. Basics as the focus may be a result of teacher content knowledge and pedagogical content knowledge. Many of the teachers stated that connections to the real world were important but Brendan revealed that it was "…hard to do with equations, inequalities, and very difficult with quadratics." Brendan even referred to the slope formula as $y = mx + b$ instead of the difference in the dependent values divided by the difference in the independent values. Several lacked knowledge of connections between multiple representations of a situation. This became evident with a discussion about factoring quadratics. Carl stated, "What do you do [besides] use a routine way of teaching it". Amber stated, "…show them the pattern and they just follow the pattern." Neither teacher made the connection between the factors, solutions, table of values, graph of the function, and the link to real world situations. Several comments were made about how to present the pattern for factoring. Finally Jane brought up algebra tiles. Questions ensued about how to use the tiles, what if the function had a greatest common factor, and how do students transfer their use of manipulatives to paper and pencil.

The interviews revealed that research by the teachers to assist with connections, content knowledge, and pedagogical content knowledge was minimal, and if it was done,

teacher actions concentrated on finding activities and worksheets to promote skill proficiency. Brendan, a PET teacher, stated that research was done "…more often than I like to admit," as the feeling of inadequacy with the content was acknowledged. To further the feeling of inadequacy Brendan stated that "…there are some things that we did this year that I haven't seen since I was in high school." Coming from another discipline, he was concerned about remembering everything he learned in high school and all the steps that the teacher had used when presenting the topic. Carl recognized the importance of doing more research since being involved in a higher education program and spoke about "math hopelessness" versus "math helplessness." Four of the teachers, Amber, Denise, Garth, and Jane, stated that they relied on what they had done in the past. What worked and what they already had available was usually what they used in the classroom. They fell back to what was comfortable and easier for them as they prepared for a lesson.

Beliefs seemed to cluster around relationships with all making some type of remark that relationship building was important. This became more obvious when asked about their beliefs about all students learning algebra, remarks centered on a positive response but who was responsible for their learning was different. Some statements included: "if a student tries then they will be successful," it is "…up to the students to learn," and the "focus on those who want to learn." Three of the teachers, Brendan, Ellen, and Garth, stressed that they believed the relationships they built with the students led to the students wanting to learn the material to please them but also because the students began to have faith in themselves and their ability to be successful in the class.

Ellen summed up many of the comments when she said "Relationships is the key, getting more one on one and establishing that relationship is so important. If we can get the students to open up and ask questions then we have come a long way,"

All of the teachers thought they had a "growth mind set" but many of their remarks fell back to the "fix mind set" belief. Stating that they believed all students could learn the mathematics was followed by statements that put limitations on their comments. These include comments by Garth, Jane, and Amber: "If they will give the effort then…", "I still try to engage all of them but I do a lot of focusing on those that want to learn and I have been successful", to referring to the "dumb students".

Comments were also made about the young (Amber, Brendan, Carl, and Jane) and older (Denise, Garth, and Ellen) groups of teachers. Carl stated that "they" differentiated more with technology and that their counterparts did not. Amber stated that "They [older, seasoned teachers] tend to do what they have been doing for years and I am trying to do new stuff and branch out in other ways." This was not the only difference in teaching and learning practices. Classroom management between the groups was brought up several times during the interviews. There was a concern about losing control, acknowledgement that they did not have good control, or lack of understanding about how to maintain control.

The last research question dealt with challenges and constraints when implementing rich tasks. For the initial interview I attempted to delve into the teachers thoughts and concerns about the new SOL, professional development, and support. Time was mentioned overwhelming by everyone. Not enough time to plan, to collaborate, or

to learn about the new material. The Algebra I team did meet at lunch once each week but most of the teachers stated that it was not productive. Responses varied when questioned about support with time being brought into the discussion again. Most stated that if support involved money, going to a conference or some other type of professional development, then there was no support. Carl went as far as to say "I believe the people and administration are against professional growth." The consensus among the Algebra I team was that the administration was supportive in that they did not try to tell them how to teach, had faith in them and their abilities, and did not interfere with instruction. Time to assist with instructional endeavors was to be done on teacher time without the support of built in time by the administration.

Teachers overwhelmingly agreed that their biggest form of assistance was from Doyle, a mathematics specialist, who was acknowledged for knowing the material and one with whom they felt comfortable coming into their classroom. It was said that Doyle was concerned with more than the SOL scores; Amber expressed that he "…is our biggest support". Doyle did know the content but was not comfortable with putting "it all together," in reference to group activities in the secondary classroom and with how to create and implement rich tasks. When discussing the new standards, Carl, Ellen, and Garth expressed a concern about the retention of the material by the students and the time needed to teach all that was expected. This led to discussions about making connections, rigor, and if all the students in Algebra I received the same level of rigor and opportunity to learn the concepts. Only two of the teachers, Amber and Brendan, felt that rigor and opportunity to learn for all was a challenge for the team. Raising rigor was viewed as

happening when using technology and when making connections to real world. Comments were made by Ellen that "we're all going on the same curriculum guide and plan" so it was assumed that the students received the same level of rigor in the content. I gave the scenario that if a set of twins were taking Algebra I and were placed into two different teachers classrooms, would they experience the same level of rigor? It was acknowledged Jane that some teachers may go into more depth but the depth was in explaining the procedures in varying ways. Amber stated that "They [older teachers] tend to do what they have been doing for years and I am trying to do new stuff and branch out in new ways. Some of us are trying to do a little bit of both." Even with different approaches, participants, for the most part, believed that students were receiving the same level of rigor. The biggest constraint was knowing how to raise the overall rigor of their curriculum which lead to comments about time and opportunity to have professional development that was meaningful.

Mathematical Survey results. The Mathematical Survey was assessed for correctness of computations for each of the problems and how each participant viewed the richness of the problems according to rich task characteristics (Grootenboer, 2009; NRICH, 2007**),** cognitive demand (Stein, Smith, Henningsen, & Silver, 2000), NCTM Process Standards (2000) and 21st Century Skills (P21, 2012). Stump's (1996; 1999) work provided the answer key to assess the computational aspect of the six problems. Table 8 reveals the results of the survey turned in by seven of the participants with the "answer" column indicating whether the participant got the answer correct and the "show

Table 8

Mathematical Survey Results – Computations for Tasks

	Question 3		Question 4		Question 5		Question 6		Question 7		Question 8		Average Number Correct / Complete	
	Answer	Show Work	Answer	Show Work	Answer	Show Work	Answer	Show Work	Answer	Show Work	Answer	Show Work	Answer	Show Work
GE 1	Y	N	Y	Y	Y	N	N	N	N	N	N	N	0.50	0.17
GE 2	Y	Y	N	N	Y	N	N	N	N	Y	N	N	0.33	0.33
GE 3	N	N	Y	Y	N	Y	Y	Y	Y	Y	Y	Y	0.67	0.83
GE 4	N	N	Y	Y	Y	Y	Y	Y	N	N	N	N	0.50	0.50
GE 5	Y	Y	Y	Y	Y	Y	Y	Y	N	N	N	N	0.67	0.67
AVG	0.60	0.40	0.80	0.80	0.80	0.60	0.60	0.60	0.20	0.40	0.20	0.20	0.53	0.50
SE 1	N	N	N	N	N	N	N	N	N	N	N	N	0.00	0.00
SE 2	Y	N	N	N	Y	Y	Y	Y	N	N	N	N	0.50	0.33
AVG	0.50	0.00	0.00	0.00	0.50	0.50	0.50	0.50	0.00	0.00	0.00	0.00	0.25	0.17
AVG ALL	0.57	0.29	0.57	0.57	0.71	0.57	0.57	0.57	0.14	0.29	0.14	0.14	0.43	0.40

work" column specifying whether all work for the problem was shown as requested. Averages showed how many teachers got a problem correct or showed their work for all six problems. General education (GE) teachers were separated from the special education (SE) teachers to determine if there were any distinctions between the two. The problems focused on different aspects of slope as discussed by Stump. Question 3 and 5 concentrated on slope as a function and an algebraic ratio. Parametric, the meaning of the parameter of an equation, was the emphasis for question 4 and 7. Question 6 also focused on function as well as slope from geometric and physical aspects. The last question, question 8, was based on trigonometry and whether participants recognized and could apply the tangent function.

Question seven, along with question number eight, revealed results that were the lowest in computational correctness as well as completing all that was requested, including explanations. This question required manipulation of a complex equation into the slope intercept format. Participants showed the most success on the fifth question, determining the rate of change given the coordinate points. This is a very straightforward question and typical of a traditional classroom. Out of six free response questions Ken, one of the special education teachers, did not get any of the slope problems correct; Garth, a general education teacher, got two correct, and Amber and Ellen got three correct.

A knowledgeable teacher of content and rich tasks, and myself, formed a researcher team correlating our responses to the rich tasks matrix and then comparing our agreed upon results with those of the participants. Table 9 and 10 renders the results of

how teachers rated the six problems with respect to rich task characteristics. The reported values for Table 9 relate the percentage of participants that scored a specific problem at the same level as the answer key. The average number of participants that

Table 9

Mathematical Survey Results – Percentage of Teachers in Agreement with Each Task Characteristics

	Question 3	Question 4	Question 5	Question 6	Question 7	Question 8	Average
Procedures with connections or doing math?	0.00	0.29	0.29	0.14	0.14	0.14	0.17
Apply and/or develop higher order thinking skills	0.57	0.43	0.43	0.43	0.14	0.29	0.38
Apply problem solving skills and ask questions	0.29	0.29	0.29	0.43	0.29	0.29	0.31
Multiple entry points and pathways	0.29	0.29	0.29	0.29	0.00	0.29	0.24
Apply prior knowledge	0.14	0.00	0.14	0.29	0.29	0.29	0.19
Present solutions and justify responses	0.14	0.00	0.14	0.29	0.14	0.29	0.17
Relevant, interesting, and/or applicable	0.29	0.43	0.14	0.00	0.00	0.14	0.17
Group work and responsibility	0.14	0.14	0.14	0.00	0.00	0.14	0.10
Reflections	0.29	0.29	0.14	0.14	0.00	0.57	0.24
Average	0.24	0.24	0.22	0.22	0.11	0.27	0.22

Table 10

Mathematical Survey Results – Percentage of Teachers Range Agreement with Task Characteristics

	Question 3	Question 4	Question 5	Question 6	Question 7	Question 8	Average
Procedures with connections or doing math?	0.43	0.71	0.43	0.57	0.29	0.29	0.45
Apply and/or develop higher order thinking skills	1.00	0.86	0.71	0.71	0.57	0.57	0.74
Apply problem solving skills and ask questions	0.86	0.86	0.71	0.71	0.57	0.57	0.71
Multiple entry points and pathways	0.43	0.86	0.43	0.71	0.3	0.29	0.52
Apply prior knowledge	0.86	0.57	0.86	0.57	0.57	0.43	0.64
Present solutions and justify responses	0.43	0.29	0.14	0.71	0.14	0.43	0.36
Relevant, interesting, and/or applicable	0.57	0.57	0.57	0.43	0.29	0.29	0.45
Group work and responsibility	0.14	0.14	0.14	0.43	0.00	0.43	0.21
Reflections	0.71	0.43	0.29	0.43	0.14	0.43	0.41
Average	0.60	0.59	0.48	0.59	0.33	0.41	0.50

chose the same Likert scale value for each of the rich task characteristics ranged from 10% to 38%. Group work and being responsible for themselves and the team was the lowest in agreement. This is followed by three categories with an average of 17%: procedures with connections or doing mathematics, present solutions and justify responses, and the question is relevant, interesting, and/or applicable to the students. Applying and/or developing higher order thinking skills revealed the highest correlation, 38%. Table 10 took into account the range of values that are one below to one above the answer key selection thereby taking into account the variability of how individuals perceived the characteristic. The characteristics with the lowest average value, 21%, and the highest average value, 74%, did not change from those which were exactly as the answer key. The characteristics with the second and third lowest values included present solutions and justify response (36%) and being reflective (41%).

Table 11, rating the rich task characteristics, expressed the average Likert scaled score in relationship to the score determined by the researcher team. All participants' scores for a characteristic on a specific question was totaled and averaged then compared to the score determined by the researcher team. Of the 54 items being scaled, 24 of the items showed a discrepancy between researcher team score and average score of participants of about 1.5. The questions that showed the largest number of discrepancies was number six, slope approached as a function including geometric and physical properties, and number eight, the trigonometric approach. This table also revealed the characteristics that had low and high correlations. For five of the problems participants

Table 11

Mathematical Survey Results – Rating Task Characteristics (Average Participant Score Rating/Researcher Score Rating)

	Question 3	Question 4	Question 5	Question 6	Question 7	Question 8
Procedures with connections or doing math?	2.429/2	1.857/2	2.143/2	1.857/3	0.714/2	1.143/3
Apply and/or develop higher order thinking skills	2.429/2	2.143/2	2.286/2	2.571/4	1.571/2	2.000/4
Apply problem solving skills and ask questions	2.857/2	2.286/2	2.429/2	2.571/4	1.429/2	2.000/4
Multiple entry points and pathways	2.857/3	2.286/3	2.429/4	2.429/4	1.000/1	1.571/4
Apply prior knowledge	2.857/3	2.571/3	2.429/3	2.143/4	1.714/3	1.857/4
Present solutions and justify responses	2.000/0	2.143/0	2.714/1	2.429/4	1.286/0	1.857/4
Relevant, interesting, and/or applicable	1.714/2	1.286/2	1.714/2	1.286/3	1.000/0	1.000/4
Group work and responsibility	2.429/0	2.143/0	2.000/0	2.429/3	1.429/0	1.714/1
Reflections	1.143/0	1.000/0	1.571/2	1.714/4	0.857/0	1.571/4

disagreed with the researcher team on presenting solutions and justifying their responses. Procedures with connections and doing mathematics along with applying prior knowledge showed discrepancies with the same three problems, numbers 6, 7, and 8. Inconsistencies with the characteristic, reflecting on the problem, were seen between numbers 3, 6, and 8. Participant understanding of the characteristic, present solutions and justify responses, did not correlate with the researcher team. Questions 6 and 8, overall revealed the greatest disparity between participant and researcher team ratings.

During the Summer Institute

First day activities. The three day Summer Institute in June began the day after the study was introduced to the teachers and the initial interviews were conducted. The agenda (see Appendix B) reveals the flow of activities for the first three days of activities and the one day in August during the school year work week. On the first day of the three-day Summer Institute, participants positioned themselves into two groups by their choice. One group was to be video-taped and the other audio-taped. No single group was chosen specifically for a mode of recording but instead was chosen out of convenience for the camera setup.

One team was made up of four younger teachers, Amber, Brendan, Carl, and Jane with less than five years of experience who considered themselves more technologically savvy. Jane was the special education teacher for that group. Two of the four on the other team were experienced general education teachers, Denise and Garth, with more than fifteen years in the field. Ken was the special education teacher on the experienced team. Both teams also included an alternative education teacher, Francis on the younger

team and Ken on the experienced team. Three of the four in the experienced team were also coaches for various athletic teams in the school. They named themselves but for the sake of anonymity they will be called the "Passionate and Energetic Team" (PET) for the younger team and the more experienced team will be referred to as the "Cognizant and Experienced Team" (CET).

During the first day the two teams began showing distinct personalities. There was a great deal of teasing between the two teams during the entire Summer Institute which centered around age/experience, technology usage, and modes of teaching. Viewpoints in teaching became apparent as well. When working on the characteristics of traditional and reform based classrooms discussions arose around calculator usage, classroom management, and even notes and lectures. Garth, a CET teacher, expressed concerns about classroom management when he said "…once I advocate my control then I am in trouble…I am concerned I can't get it back." PET teachers mentioned a concern of classroom management but not about losing it, admitting that it was a weakness but more concerned about how to facilitate group work. Amber, a PET teacher, was against calculator usage but was for technology usage whereas all the CET teachers were uncomfortable with any form of technology. The teams filled in a chart with the characteristics of traditional and reform practices (see Table 12) with each team placing the characteristics under the column they believed best fit.

Table 12

Traditional and Reform-Based Characteristics as Viewed by Participants Beginning of Summer Institute

Traditional Characteristics	Both	Reform-Based Characteristics
More problems to practice and for homework	Student engaged in thought provoking questions	Critical thinking skills
Notes and lectures	More practice	Peer teaching
Teacher-centered		Engaging
Rote drill		Group work/Collaboration
Direct instruction		Student-centered
Well organized/run classroom		Activities
Lower level questioning		Mnemonics
Worksheets		Make connections
Teacher directed instruction		Inquiry based
Graphing calculators		Problem solving
		Real world applications

As a whole group each characteristic was discussed and a decision was made for the characteristic to remain in the column where it was placed, move to the opposite column, or placed in a new column titled "both". For a move to be made, participants provided a

justification as to why. After discussions the following changes were made: a well-organized classroom and graphing calculators were changed to be under both, mnemonics was moved to traditional instruction.

Goals, the focus of question two, was approached from an individual viewpoint and then through collaboration, from a course viewpoint. The participants completed a journal prompt requesting them to expound on their goals for the professional development and their goals for teaching mathematics. Individually, participant's goals ranged from a desire to find new approaches to teach algebra and increase student understanding to ideas to help with the state assessments and an opportunity to collaborate with peers. This was noted by Denise on day one journaling that the goal was to "steal ideas to enhance my presentation" to day three where the goal comment was "to include activities to represent the 5 Es and employ multiple methods."

As the researcher, I needed to know what the teachers expected to get out of the institute and their vision for working with mathematics instruction. Manouchehri and Goodman (2000) discussed that teachers need to share their vision, just giving them a lesson, a curriculum guide, or textbook will not change teacher practices. Orrill's (2006) study revealed that the goals of the workshop need to match or compliment the facilitator's goals in order for the professional development to be effective.

Goals for the teacher teams and mathematics instruction needed to come before determining the flow of the Algebra I curriculum as participants needed to plan instruction based on big ideas and goals. Rich tasks may be used with any curriculum but teachers need to think through the year so that all concepts will be addressed in a

meaningful way. Restructuring the curriculum allowed the teachers to make connections to something that was relevant for students in a unit of instruction and to select a rich task that addressed all goals and objectives (BIE, 2009; Fernandez, 2000). Each participant had already written in their journals about their personal goals. Working in teams, the participants began discussing a goal statement for the Algebra I course. PET wanted to just list the rich task characteristics and process standards while CET wanted to make it short and simple, "solid goals for success". Both teams discussed how the state assessment could be placed in the department goal but then decided to not look at the product but the journey to get to the product. The final draft of their goal statement was "As teachers of Blank High School we will successfully increase student's conceptual and real-world understanding of mathematics." Along with the goal statement, the group wanted to bullet some items so they could maintain a focus on what was meant by their goal. These included spiraling, teaching to the big ideas, NCTM Process Standards, connections, collaboration, student engagement, and the Rule of 5. The course goal was written on chart paper and placed in the room.

A journal written on day one by the participants gave them the opportunity to reflect on how their personal goals related to the course goal. One of the PET members and two of the CET members believed that their individual goal correlated with the course goal. A change in goals to better correlate with the course goal as the professional development progressed was noted in two of the journal entries. After experiencing rich tasks all of the participants saw a need to plan so that the goal and characteristics of rich tasks may be implemented throughout a unit of study.

Cognitive demand knowledge was also information needed for discussions on rigor and rich tasks. An activity on cognitive demand levels (Stein, Smith, Henningsen, & Silver, 2000) allowed teachers to determine levels of demand given a set of exercises, discuss how to raise the level of questions so they become higher in rigor, and how the cognitive demand of a lesson or question may be destroyed.

For the cognitive demand activity the participants were split into three teams so more responses and interactions could take place. The participants determined the level of cognitive demand using tasks, A through P, created by Stein, Smith, Henningsen, and Silver (2000). Recording their responses on chart paper resulted in the participants interpreting the chart as a scale and placed their responses accordingly (see Table 13). Each of the problems, A through P, was discussed according to the placement given by Stein, Smith, Henningsen, and Silver. The level of cognitive demand according to the researchers is indicated by an A on Table 13. Each task was discussed as to the level determined by the researchers in comparison to the level determined by the participants and why or why not they correlated. Both of these activities, reform vs. traditional classroom characteristics and the cognitive demand activity, were an effort to help the participants define reform-based mathematics and levels of rigor, all leading to rich task characteristics. By the end of the Summer Institute all participants were aware of and recognized characteristics of a rich task as noted on their journal entries. Following the participants determining the cognitive demand of the various exercises another activity was conducted to determine norms for the group since the sessions were going to be group based.

Table 13

Cognitive Demand of Problems

	High Level					Low Level				
A	X	A			X		X			
B							XXX	A		
C	X									XXA
D								A		XXX
E	XXX				A					
F								A		XXX
G					X	X		A		X
H			X					A		X	X
I			XX		A					X
J		A				XXX				
K									X	X	XA
L				X		AX		X		
M	X			X		A		X		
N							X	A		XX	
O	X				A		X	X		
P		A					X	X	X		

The norms were decided by having all the participants brainstorm actions that were acceptable and desired and then narrowing them down and grouping them into a usable list. The final list of norms included respect, quality work, and responsibility. After developing the norms, the participants began looking at the flow of the standards for the course, placing them into units of study. This was accomplished with a card game where all the concepts were written on index cards. With the course goal previously determined in mind, the participants applied that goal to the flow of concepts. Each group of concepts was to build on previous knowledge while maintaining the overall goal while thinking about units of study versus chapters in their textbooks. Questions they needed to ask themselves as they went through this activity related to determining if it was a logical flow and did it make sense to them? What connections could they make between the ideas in the unit? The participants struggled with developing the big ideas so a break was taken and the first rich task was presented so they might experience a task that includes multiple concepts and multiple representations.

As the facilitator, I set the stage and introduced the rich task. The task, The Hexagonal Train, began with introducing the concepts by exploring them in small investigative activities. Participants investigated combining like terms, graphing and plotting points, and translating verbal to symbolic expressions and equations and vice versa. This was followed with pattern blocks being given to the participants that allowed them to recreate the rich task scenario and as a team encouraged them to brainstorm verbally the pattern of growth. "Students" at this time would have prior knowledge on how to create a table of values, plot points, and the idea of stages of growth, all concepts

investigated previously in other units or during the exploration. All of these pre-requisite concepts would be "available" to the participants as well as the pattern blocks, so that they could have the same experiences as their students.

The first rich task, Hexagonal Trains, was difficult for the participants. As a group we went over the scenario and had time for the participants to ask questions for clarifications, none were asked. Teams were requested to build the scenario concretely first using the pattern blocks. Using the pattern blocks had to be strongly encouraged as the participants wanted to jump to the abstract. This was followed by having the participants transfer the concrete to pictorial on their papers. The guiding advice for the participants was to encourage them to look at the third stage to determine the pattern and then apply their pattern of growth to all of the stages. When left to rely on their groups to determine a verbalization of the growth many questions arose such as "will you have to connect the trains together? and "does the belt go down between here [in between the hexagons] or does it go just around the exterior sides?" The teachers wanted to place a link between the cars. Other remarks made included asking for more specific instructions and requests to tell them what to do. Upon reflecting back on the task Brendan stated "we did not know what YOU wanted."

Most of the teachers decided on the rule for the growth of the train by using two data points instead of determining the growth and understanding what and how the equation related to the concrete. When Amber spoke up to share her verbalization of the growth, the rule representing the situation was given. Asked to relate it to the concrete, the response was based on basic skills, the y-intercept and the slope. In addition to

verbalizing the growth of the train, the task encouraged the teachers to use representations in multiple ways and making connections between them. Once they determined a pattern of growth, discussion began. If the pattern was recursive, participants were asked to determine the value at the fiftieth stage and if their thinking would help them determine the answer.

Finally Carl spoke up to share a pattern of growth. To illustrate what he meant he went to the white board and used colors to help everyone visualize his statements. Carl stated that every train had a front and back that never changed and illustrated this by drawing two red vertical lines on the board. Then for each car he illustrated that there were V's, made up of two lines for a total of four lines per car (see Figure 5 below).

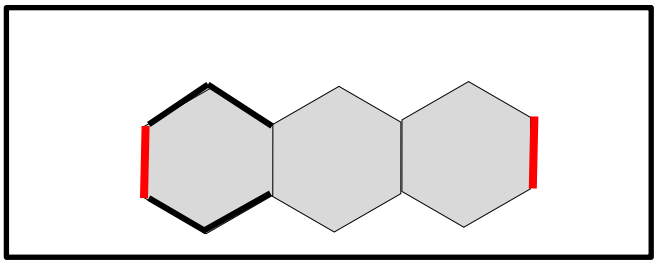


Figure 5. *Stage 3 Pattern of Growth for the Hexagonal Train.*

Carl verbalized the growth by saying "front plus the back plus four times the car". Translating the verbalized growth into symbols he wrote, $4c + 2 = p$. After illustrating the growth with stage 3 he went on to verify his conjecture with stage 2 and 4. This gave the remaining teachers the idea of how to express the growth using constants and patterns. A table of values was created along with the graph so connections could be

made between all representations, the characteristics of the function, and of the scenario were discussed. A difference was made between the function representing the situation as being continuous but that the situation was not. This led to discussions about the abscissa and ordinates as well as discrete and continuous functions. The teachers had a blank look on their faces. Denise asked "What is an abscissa?" and Amber asked "What do you mean by discrete?"

After the task was completed the whole group compared the experience with the characteristics of a rich task. Participants acknowledged multiple paths and entry points, a higher level of cognitive demand, multiple objectives, multiple representations, and the relevancy of the scenario. The teams also listed the number of objectives covered and determined the driving question that was answered. Discussions on how the task related to the NCTM Process Standards (2000), 21st Century Skills (P21, 2012), and cognitive demand (Stein, Smith, Henningsen, & Silver, 2000) encouraged the teachers to make connections to rigor. Now that the participants had experienced a rich task and determined a goal for the Algebra I teams, discussions ensued regarding where in the curriculum the rich task could be inserted. This became the lead in to looking at the Algebra 1 curriculum and restructuring the flow of concepts. The flow of concepts that the teams determined that would help them make connections and use multiple representations are listed in Appendix I.

The rich tasks during the Summer Institute sessions were similar to Derry, Wilsman, and Hackbarth's (2007) idea of Sample Algebraic Modules (SAM). Teachers experienced a rich task on one or more types of slope; read articles on rich task

implementation; and reflected on the readings as they related to the task. Whereas Derry, Wilsman, and Hackbarth (2007) used video case studies for the participants to experience and/or analyze, this study concentrated on having the participants experience the rich task through involvement in the activity, reflecting on the concepts being addressed, and the cognitive demand of the task. The first day ended with a request for the participants to read an article about rich tasks and problem solving and complete a journal on their experiences during the rich task. A CET member journaled that "Just as a student, [there is a] fear of failure or not meeting expectations." A second CET member recorded in a journal that "I found it was very interesting how you could do one task and gain many different uses for the data." "I felt frustrated by putting the pattern into words. I'm really good at expressions and equations but tend to struggle with pictorial and abstract representations" was recorded in a journal by a PET member.

Activities for day two. On the second day the participants discussed the readings and reviewed what they had discovered about rich tasks, their characteristics, the NCTM Process Standards, and the learning environment. This was followed by an opportunity for each participant to journal about "How has the rich task experience influenced you in the following ways: planning, practices in the classroom, and beliefs on teaching and student learning?" A second question asked the participants to anticipate challenges and/or constraints they might experience or expect when implementing rich tasks. Responses ranged from personal concerns to more global concerns. For example, a concern was expressed about "working outside of my comfort zone" to how to deal with

the "flexibility of the students/parents with a new change in dynamics of learning and the methods that learning is expressed."

As in the first rich task, I, as the facilitator, set the stage and introduced the second rich task, Enlarging Forts. This time the participants were asked to put one of their methods on chart paper and present their approach to the rest of the group. This second rich task came much easier to the participants on day two. Both teams dove into the task using the manipulatives to help them look for patterns of growth. Excitement could be felt in the room as a team discovered one approach and began to look for a second method. They appeared to be more comfortable trying to use the manipulatives, to try and to fail, as illustrated in a comment by PET member: "This is interesting, the previous stage fits inside the next stage… No it's every other one that fits." After presenting the one approach, each team asked if they could present another method as they wanted to share all they had found.

A comparison between the two rich tasks, The Hexagonal Train and Enlarging Forts, led to participants acknowledging the different approaches, the conducive environment that was needed for implementation, what was meant by linear, and how it can be viewed using multiple representations. With more experience the participants were able to verbalize and determine the function that represented the growth without using formulas with less stress. Some content concerns arose about the connections between and about the representations and the driving question. One of the discrepancies in thought centered around whether the scenario represented graphically should have discrete data points or a line. This led to team discussions about what is meant by

discrete data. Both tasks involved perimeter of a linear model, however, when comparing the rich tasks it was mentioned by two participants that the Hexagonal Train task involved area.

After the second rich task was completed the participants free wrote to record their thoughts and feelings about their experience as a "student" and as a member of the team when completing the rich task by responding in their journals. Comments expressed the changes with teacher practices that will need to come about, "Rich tasks will force us to expand planning….sharing, group planning, and classroom feedback (experiences) will need to be reviewed and revisited." Changes will need to be addressed with students as well was expressed as "Students are capable of learning any concept that they put their energy into." Students need to take ownership of the changes and the expectations. The teams also completed, individually, a peer and self-evaluation form (see Appendix L). Participants had an opportunity to expound on their team efforts and what did or did not work. This was their first opportunity to peer and self-assess. The mass majority of responses were very favorable about their peers, sharing equal work loads, helped to keep the team focused on the task, and contributed to the discussions with scores on the rubric at strongly agree and agree. A score of disagree was mentioned by one person that a peer was quiet and did not contribute much.

They then came out of student mode and into teacher mode to discuss the characteristics (Grootenboer, 2009; NRICH, 2007), NCTM Process Standards (2000), and the learning environment regarding the task and their experiences, individually and as a team. As the "teacher", participants discussed assessment of the task and the usage

of a rubric. All of the characteristics were recorded on chart paper and displayed in the room. After charting the characteristics and comparing the tasks, participants discussed the categories of slope that could be brought to a students' attention through the task along with listing all concepts addressed. The importance of researching the topic was stressed by relating the content of the rich task to the results revealed on the Mathematics Survey (Stump, 1996).

Linear functions, a unit plan that included both rich tasks, was shared and discussed with the participants. This unit on linear functions was based on the Five E approach: engage, explore, explain, elaborate, and evaluate. Each section was reviewed so that all the participants could see how the big idea was developed. To begin the unit the teacher would need to engage the students, explorations followed that set the stage for the rich task and provided students with the needed content, next were notes for the teacher explaining the task to the students which lead to the final project elaborated on in detail, and lastly, how the task was to be evaluated. Teacher notes for the unit were given in the document as a guide to what should be expected from students. The unit that the two rich tasks had been embedded in was the format for the plan that was to be used in the lesson study. This format would assist the participants in pulling their thoughts together and in maintaining their course goal.

The teams of teachers began thinking about their own rich task and unit of study for the first semester of the next school year using the big ideas they determined from the first day activities, the course goal, and their experiences from the rich task. The passionate and energetic team (PET) chose solving equations and inequalities for their

unit; the cognizant and experienced team (CET) chose solving systems of equations and inequalities for their unit. Their ideas were organized and recorded on chart paper using the modified Frayer Model template as shown in Figure 6.

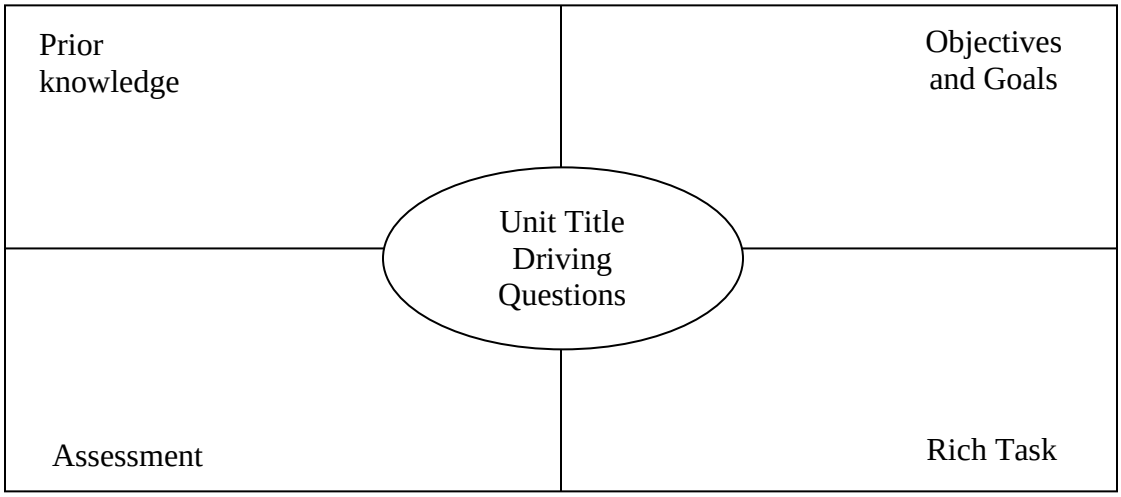


Figure 6. *Rich task organization template.*

Each team approached filling in the Frayer Model differently. PET had some difficulty staying on task with Amber stating, "Let's just get this done." This could be seen in the product as objectives were Virginia Standards (2009) numbers such as A1.2 for Algebra 1, second bullet, versus the standard written out in the teacher's own words. The same happened with the NCTM Process Standards (2000). PET just stated "NCTM Standards" while CET gave a listing of them. There was also an incomplete listing of pre-requisites. The rich tasks were a listing of those we had done in the last two days with no additional suggestions. In comparison, CET took time to delineate the objectives in short statements, brainstormed pre-requisites, and stated how they would apply the NCTM

Process Standards. This team also brainstormed ideas for rich tasks such as movie attendance and cost, concession stand cost and profit, and car cost and gas mileage. The two teams shared what they had recorded on their chart paper and it was noted by PET how in-depth the CET went in their response. A PET member commended the team with their ability to come up with ideas for future tasks and how they worked together.

Pulling the participants' ideas together led to a discussion about lesson study, the process and rationale behind engaging in the collaborative, structured planning sessions. Participants were requested to read two articles on connections and the second day ended with everyone journaling about how they envisioned working with rich tasks in their classrooms and how would they incorporate the thinking process used during the rich task in the development of their units of study? Denise wrote about beginning with lower level thinking and building to the higher levels as "these methods will also be new and different for our students." Amber wrote favorably about the "brain dump and sharing" but then added "our department is full of stubborn people who get into arguments where both people argue the same point using different words and think they are disagreeing so I tend to avoid collaborating with them." She also expounded on using the tasks in the classroom: "I love the idea!" However this was also followed up with a concern about classroom management skills needing to improve while keeping the students "focused and working." Garth recorded in his journal that the tasks need to be at a level of understanding for the students and that time needs to be built into planning to do the tasks.

Last day of the Summer Institute. The third day began with a discussion of the readings and how they related to rich tasks characteristics and the expectations of teachers and students. One article they read related connections between procedural fluency and conceptual understanding; the second article related connections to functions and their representations. Journaling for the first time of the day was about their goals for the unit of study as well as the role that they anticipated they would play. Everyone remarked in their journals in some manner that rich tasks correlated with their personal goals of wanting students to learn and be engaged.

Another task, The Towers, was begun during the morning session of the third day. Task one and two had been based on linear functions while task three was about a quadratic function. Participants were given snap blocks to model the scenario, then were requested to transfer their concrete model to a pictorial representation. Stage two consisted of a tower of two blocks with a base of one block on each of the four sides of the tower. With each successive stage a block was added to the tower as well as to each leg on the base. Three stages of the tower may be seen in Figure 7.

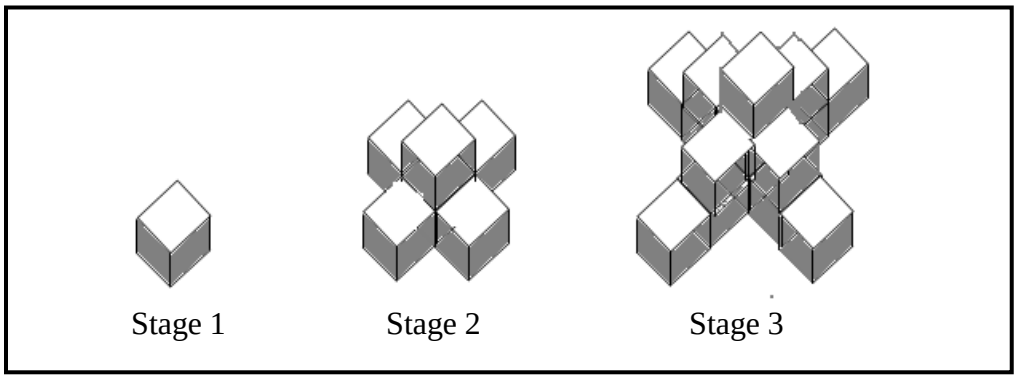


Figure 7. *Growth of The Towers*

The Towers was challenging to both teams but their enthusiasm carried through to find several approaches which they put on chart paper and presented to the whole group. CET discussed that the model was now three dimensional which was very different from the first two tasks. Recursively both teams expressed the growth of adding four to each stage and followed it up with a table of values to realize it was not linear. A PET member got the idea of pulling the tower apart and putting it into another form, a rectangle, to see if that could be replicated. Other members of the team copied her actions with other stages and determined it a success. "Here's what we got. We took our stuff and manipulated them into rectangles starting with stage three. And this is stage squared and this one is stage squared but you're missing one stage so that gives you stage squared and another stage squared minus a stage (see Figure 8).

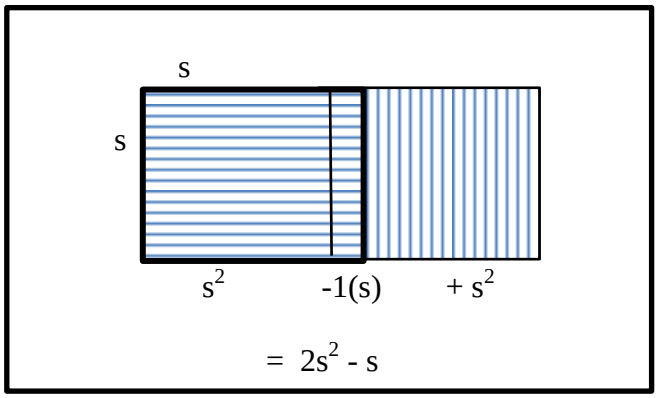


Figure 8. *Visual Interpretation of the Growth of The Towers*

The importance of research was again stressed as a way to learn about the other approaches and applications of quadratic functions. Quadratics being related to the area

model was key to helping discover the approaches and at the same time was recognized as a weakness of the participants. This was noted when I mentioned the area model in relationship to multiplication in elementary school. Reference was made to how elementary students learned multiplication using area models and how we, as high school teachers, could build on that knowledge. Participants compared the linear rich tasks to the quadratic task of the day. Differences were noted between the linear and quadratic functions, how to determine if the function was linear or quadratic using a table of values and determining the constant second difference, the graph in the shape of a parabola, and the standard form and vertex form of the function. The characteristics of the quadratic function was compared to the linear function when graphed, how the table of values related to the model, how to create a chart that reveals the growth in words and numbers, and the rich task characteristics that were apparent in each.

Discussions followed the presentations about using multiple approaches and manipulatives to learn about quadratics. It was asked "do we have any of these blocks in the department?" The patterning for this task was different and there was a need to pull from content knowledge and connections that were unlike the linear scenarios. Introducing the task was the same, participants completed the task as "students" then came out of student mode and into teacher mode to discuss the characteristics of the rich task (Grootenboer, 2009; NRICH, 2007), the application of the NCTM Process Standards (2000), and the learning environment. Participants were given the quadratic functions unit document to analyze the components and determine all of the concepts students would be engaged with during the unit.

A mid-day journal prompt was given for the participants to complete which gave them an opportunity to compare and contrast all three rich tasks and discuss the last task, relating it to the flow in a unit of study. There were comments about the last task being area or if it was still about perimeter. Brendan expressed his thoughts again about the importance of planning, "In order to meet goals, objectives, and rich task creation, we need a solid plan which lays out an organized framework and pacing guide for the units." Participants also completed a second peer and self-evaluation (see Appendix L) in an effort to see how the team was evolving. Comments on this evaluation were more brutally honest with additional remarks about working, or not working, together as a team.

During the afternoon of the third day, participants had time to brainstorm ideas about their chosen unit of study that included what they already recorded on the Frayer model such as the driving question, prior knowledge, and the objectives and goals for the topic. Their discussions within their teams centered on the flow for their unit of study which they will complete for the first semester of the school year. Using the unit template to guide their thoughts on how to develop the unit of study, the teams talked about how to engage the students in the unit, what explorations or lessons need to be determined, and what is the rich task(s) that they would like to develop for the unit and the lesson study presentation. Researching their topic was stressed so that the unit would not be based on using the same approaches as was the norm in the past. The school year work day agenda was discussed and an exit survey was explained and given to the participants electronically.

Day 4, school year work week. In August, as the participants entered the room, excitement about the new year was apparent as the teachers caught up on summer activities, class schedules, and plans for the year. Administration had built in a common planning time for everyone but Carl for this school year. The focus for this session, day 4, was on collaborative team work on their unit of study. The day began with a review of the Summer Institute topics, cognitive demand, course goal, rich task characteristics, and the norms for the group. Teams reviewed their progress to date using the Frayer Model they created in June and their notes on the unit development. A goal was established that the teams would work on and present the flow of their units prior to lunch. Participants began filling in the unit plan with the topic, content to be covered, prior knowledge, objectives, goals, driving question, modes of assessment, and rich task possibilities, all of which had been determined using the Frayer Model.

The flow and development of the units was difficult as CET wanted to use a rich task throughout and PET was resorting back to traditional instruction. The CET got busy discussing an overall plan of using a rich task to engage and present/investigate the concepts for solving a system of equations that would be used throughout the unit. They called this rich task the "rich task prep" or RTP. The plan was to present the task and have their students use what they already knew to determine a solution and the meaning behind the answer. Participants agreed that the students would probably use substitution or graphing to find the solution which is where they planned to begin. Their plan of action may be seen in Appendix J. One of their members wanted to begin the unit with a review but this was vetoed by the remaining team members as being redundant since they

had already graphed and used substitution in several previous units of study. This member also wanted to break down the concepts and teach the method prior to letting the students try. Again, the approach was vetoed but, as with the first suggestion, it was discussed why they did not agree and how the students would have more of a buy-in if they "discovered" their learning.

PET approached the development of the unit very differently. A day by day approach was attempted but after two hours of encouraging them to look at the big idea and working with what they had already developed during the summer, the team was still debating what to teach on day one, day two, day three….of the unit. This team began planning to present or provide an activity on concepts that they had originally listed as pre-requisites on the Frayer Model. When their attention was called to the repetition of concepts, Jane and Brendan stated that "My kids are not going to remember it if I do a little bit every day." The discussion was that they needed to do one concept so their students would understand the one aspect, then they could go on to another. A review of the course goal and the objectives previously stated did not influence the PET team in their approach to planning as they continued their day by day flow without acknowledging what the big idea was or how they could implement the ideas discussed during the Summer Institute. While listening to this team you would hear "I" numerous times, "I can get them to…., I can slowly transition them…, I can go back…"

By the time the PET team was to present to the whole group the unit flow was sketchy but some thoughts were down on paper (see Appendix K). The two teams proceeded to present how they expected to flow through their units. A CET member got

up and went to the board to illustrate the flow. The enthusiasm in the presentation was apparent as the RTP was introduced, sharing how the team expected to execute the task, how it would engage the students, and how they would use it to lead the students to learn new material. PET members, on the other hand, stayed seated during their presentation and gave a general flow for their unit. Each team had the opportunity to ask questions and make suggestions. A suggestion was given to PET members to hold off on using data that would create a scatter plot until the students had worked with data that had a perfect correlation to the function. Other obligations by the team members resulted in the afternoon being cut short but some time was spent on solidifying their units of study. Day four concluded with a discussion of what is workable for them as a team as they did not want to develop the unit plan or lesson plan. Doyle, the Math Specialist, said that they would work it out. Also discussed how or what to do in the future to sustain developing rigorous tasks, the time factor, knowledge of how to create rich tasks, and what to expect during the lesson study presentation. Expectations for the preparation for the presentation as well as the day of the presentation were discussed. A lesson plan and how and where it fits into the unit of study was requested from each team. For the day of the presentation participants were told of the procedure: brief participants as a reminder of the lesson, giving me a copy of the unit/lesson plan and supporting materials, and assigning team members to observe students or groups of students. It was stressed that as an observer they were not to interact with the students. Sharing observations during the debriefing was discussed and how it will lead to modifications in the lesson. Participants

were also requested to complete a second exit survey (see Appendix E) which they completed prior to leaving.

Throughout the Summer Institute, the participants were video-taped or audio taped to record their remarks and/or interactions as a team. Stigler and Hiebert (1999) discussed the usage of videotapes as a mode for reflection in order to document student success. In the case of this study, videotapes were used to document teacher interactions as a team to include their words and actions. Videotapes were transcribed as if they were audio tapes. Teacher actions were noted to clarify the verbal responses and usage of pronouns, when being transcribed.

Participant beliefs. Question 3 had two aspects, participants' beliefs and practices with rich tasks and the lesson study approach helped with lesson implementation. Beliefs by PET members included comments such as graphing calculator usage lowers the cognitive demand of the problem, reform-based ideas were not implemented in the classroom due to the expectations of the state assessment, and any type of group work raised the level of rigor. Brendan and Carl stated that they liked experiencing the rich task so they would know "…where the kids are coming from and where they might get stuck." At the same time Carl stated that "it can be hard to motivate kids to keep trying when the answer is not immediately apparent, " and that "...we really do need to be more patient…"

Participant practices. Regarding practices within the classroom, Brendan, a PET member, believed that concepts needed to be covered prior to introducing a rich task so that students will be able to experience success. The statement was made that we

"…want them to regurgitate what we have done." Team members did acknowledge that the rich tasks became more enjoyable and accessible the more they experienced them. Discussions within PET on day three flowed around planning. They believed that rich task implementation was possible without having to plan an entire unit. Brendan commented that "Planning would be more burdensome by needing to think through all of the objectives and approaches." Members spoke about using their own plans as they have typically done and inserting rich tasks where applicable. Day four brought thoughts of students being unable to complete the rich tasks with multiple representations with Jane stating, "I'm not sure if my students would be able to take it from the equation, graph, to one in only one variable." However Carl took the approach that he was "…trying not to think of the lower level kids…I'm trying to think coming into my class that they're all higher level kids." Amber even referred to students who might have difficulty with the rich tasks as the "dumb kids."

The peer evaluation from Amber at the beginning of the Summer Institute was "My team is awesome." By day three the peer evaluation from the same teacher contained a statement that the department was "full of stubborn people…I tend to avoid collaborating with them," and there was a "Need to work on communicating and keeping egos in check." This sentiment was echoed by Brendan, another team member, with the statement "…as a whole we were a dysfunctional group." Additional comments included a need to work together and not be lone rangers. Francis was the only member of PET that did not journal about communication, staying on task, and listening to each other. Her remark was that there were "some great ideas" and that their discussions were "not

competitions." Teams of teachers within a school do not have the option of picking their team members. However, if an administrator is aware of ineffective groups then maybe there could be guidance on what is expected during collaboration as well as administrative participation.

On the last day of the professional development, the PET team resorted back to discussing specific concepts and how they were going to sequence them in a unit. Brendan was concerned about how he was going to tell the students what they were learning, the objectives for the day. The team even discussed how many objectives they were going to have to recognize for each days lesson when they wrote on the board, ASWBAT, a student will be able to… The beliefs of CET members took on a different spin.

CET member conversations centered around the instruction, the students, and their work with the students as rich tasks were implemented. Statements were made about urgency to change to meet assessment needs but also a concern for special needs students and their abilities to complete a task. Classroom management was also addressed, a fear of failure and of losing control of the students. Denise revealed her beliefs when she stated "…that there is a fear factor of changing and doing these tasks for us that are traditional…the fear is if we can come up with these great ideas that are embedded and have the connections and what if it does not go so well." The team believed that together they could create and carry out a rich task. Denise and Ellen spoke of the experience of working on a rich task as necessary so they could be prepared to address the issues prior to implementation. It was stated that by doing so it "lets us know

the problems before the students found them so we could prepare better to address those issues." Garth spoke about the difficulty of approaching concepts using multiple methods, the Rule of 5, and manipulatives where possible. "We have done it a certain way and it has produced results…we feel comfortable doing it in a way that we think we will get the results out of it." Together the team realized that being uncomfortable was part of growing and learning. As Denise, the unofficial team leader, summed it up when she said she was "…trying to get it in my mind how I will present this, how I will teach this, how I'm going to build all this up."

When thinking about practices, the CET members noted a concern about finding appropriate tasks to fit the needs of the unit and that would contained multiple representations. Some ideas that the team discussed when working with rich tasks include a conscientious effort to plan on including them in the units of study. "The concepts in the map are helping us to go deeper," was stated by Denise as it provided connections, encouraged multiple representations, and problem solving. A comment made by Denise was that they [teachers] would need to change their thoughts on how to approach a topic and the questions they would need to ask when she journaled the following statement:

> I think the fact that the questioning techniques have changed on the recent SOL's to expand thinking and reasoning, suggests that teaching and student learning need to change and grow. I think the rich task lesson will lead to this expansive learning and allow

students to make the necessary connections in problem solving and learning.

Challenges and constraints. The last research question addresses the challenges and constraints experienced by the team members as they journey through learning about, creating, and implementing rich tasks. Both teams expressed challenges and concerns in their journals and exit slips that were similar. The major challenges are listed in Table 14 along with a count from each team that expressed the concern. Many of the items have been previously addressed however further notations will follow for some of the issues.

Difficulty staying on task was mentioned more by CET members as they were concerned about interruptions and other tasks taking priority in the classroom and with their planning time. Team dynamics came into the picture as a challenge for the CET members as they listed planning that is productive due to a need to have collaborative times established in the daily schedule and a request for support of the mathematics specialist being available during these times. PET listed planning that is not productive due to the interactions and communication concerns between the members. The difficulty level being too high for students was reported equally by both teams with the difference being who listed the challenge. With the CET members it was the special and alternative educators; with the PET members it was mainly the general education teachers.

The difficulty level leads into the challenge of keeping students engaged throughout the task. Brendan, PET member, summed up the team's beliefs and concerns

by stating that the "Students can find them frustrating…give up easily when frustrated." CET members were concerned about selecting the task that was appropriate for the lesson

Table 14

Challenges and Constraints per Team

	PET	CET
Fear of failure	1	2
Difficult staying on task	1	4
Finding appropriate tasks to meet needs	1	2
Maintaining a growth mind set	2	1
Time is a concern but the experience is worth it	3	4
Time is a concern and question that it is worth it	2	
Time to implement rich tasks in the classroom	2	2
Time when state assessments weigh heavy on teacher evaluations	1	2
Content knowledge is needed by students prior to working on rich tasks	2	1
Teacher content knowledge		1
Rich tasks are a burden to include when planning	2	1
Planning that is productive	1	3
Planning that is not productive	4	
Student ability – difficulty level too high	3	3
Keeping students engaged throughout the task	3	2
Student and parent resistance	1	1

and student needs. Denise stated she wanted to "make sure students can experience simple success early, then move to more in-depth work on activities."

Lesson Study

School year lesson study. Prior to the school year I stayed in contact with the teachers via email to determine when I could observe them facilitating instruction of the rich tasks on the lesson study topic chosen during the Summer Institute. The lesson presentation was to be taught during the first semester of the school year. Planning with the school administration for this event was imperative as all participants needed to attend the briefing, lesson presentation, and debriefing as part of the lesson study process. Even though communication existed, all members of each team were not able to attend the lesson study presentation and debriefing. Doyle, the mathematics specialist for the high school, was my liaison to planning the participation and availability of the teachers for the lesson presentation. There was to be a briefing of the lesson prior to it being taught so that all were reminded of the lesson and given directions of the students they were to observe. Due to time limitations the briefing was not possible. Observers were assigned a group of students as they entered the room and given all handouts at that time. After the presentation of the lesson there was a debriefing which was held during their common planning time. The presenting teachers were requested during the Summer Institute and reminded in emails to bring to the debriefing three samples of student work that portrayed the various approaches to solving the rich task that were taken by students. None were brought to either debriefing as neither presenting teacher collected the work.

Participant observer comments and guiding questions that were answered during the debriefing assisted in the decision as to what modifications were to be made to the existing lesson. This includes the teacher team discussing whether the course goals were met, if the characteristics (Grootenboer, 2009; NRICH, 2007) of the rich task were implemented with the students, did the level of cognitive demand (Stein, Smith, Henningsen, & Silver, 2000) remain at the intended level, and was the NCTM Process Standards (2000) present. All needed to be taken into consideration when determining what changes needed to be made to the lesson in order to meet all of the above.

Other questions asked during the debriefings and exit survey (see Appendix E) was to see if and how the teacher's views had changed, what research was done on the topic, were there any other units being put together using the same format, the team's thinking about the development of rich tasks, the effects it had on instruction, and future thoughts on planning with as many questions posed with regard to the presented lesson. The exit survey (see Appendix E) was to be completed individually by the participants.

Presentation by PET. The first lesson study presentation was by PET on their unit of solving equations and inequalities. A team of five had been reduced to a team of three. The alternative educator and one general educator were excluded due to situations within the school district. This team of three, Amber the presenter, Brendan another mathematics teacher, and the special educator, Jane, decided to use Enlarging Forts as their rich task. With this choice they were assured that it met the characteristics of a rich task. This decision was made by the presenter of the lesson study, Amber, and relayed to the other members who agreed with her decision. This was the team that struggled to

pull their unit together on the fourth day of the Summer Institute. For the lesson study there was no additional research done, no unit plan to pull the unit together, nor any lesson plan written. The chosen task was never modified to fit the needs of the unit on solving equations and inequalities. The only adjustment was to spread out the document so students would have more room to write.

When the team was questioned about their experiences and those of their students prior to the lesson study with regard to rich tasks their responses were disappointing. The presenter's class of students had experienced one other lesson that would have been considered a rich task and it was done the day before the lesson study. No one else revealed any experiences with rich tasks. The biggest concern from the team about the lesson presentation was that it was too much for the students even though there were some students who discovered patterns and got excited about their success. Brendan stated that he believed that the task was "much too high a level for the students." Amber stated that "it was nice to hear other ways to approach the lesson and what they would have done differently so I can draw some inspiration from others." She also stated "that it is hard to see and understand all of the beauty if you do not know the basics first." This was in defense to not doing more with rich tasks with her students. Amber and Brendan did state in their exit surveys that they were trying to keep the Rule of 5 in their lessons. Brendan shared his beliefs thus far by stating "Unfortunately, it has not gone well so far. The students generally are unable to make connections between what they are doing and what they are supposed to be learning."

Most of the questions on the exit slip for the team were not relevant since they chose a task from the Summer Institute and did not research, create, or modify a task. When asked about the characteristics that they wanted to get across to the students and that they felt was very important resulted in two of the members stating its "open-endedness." Facilitating the lesson, Amber stated that her expectation of the students was to "stare at the wall, to whine, and comment that it didn't make sense." The expectation of the students was to be self-fulfilling when Amber made the remark that "this is the hard part." This was stated to the students at the portion of the lesson where students were to describe the growth. Amber also stated that she "anticipated getting frustrated."

Students on the other hand did get involved and did not whine. Ellen did step in, prompting and giving the group she was watching hints. She stated during the debriefing that she "hated to see them struggle." Students were hesitant to work together but as the class proceeded on more began talking with each other about the task. A recursive approach was the first model of growth determined by all the groups. Amber prompted the groups separately to encourage them to find an explicit rule for the growth of the fort. The lesson ended with Amber telling the students to create the table and graph, no discussion about the rules representing the growth.

At the debriefing everyone shared the responses from students. It was apparent to all participants that the approach used during the lesson study was not familiar to the students. With that in mind, the students still tried to find the rule. It took a while but students did begin discussing the task and the growth of the fort. In the presented lesson students drew pictures of each stage which led to a suggested modification to give them

blocks so they could model the stages. Doyle, the math specialist, commented about the involvement of the students and the success of the lesson. She asked the group to take into consideration that this was the second time students have been given a rich task and look what the students did today and what would they be able to do if this was a common practice.

Presentation by CET. CET had been reduced to a team of one so Denise, the leader of the team, was now making all decisions. Denise found and modified a lesson to meet rich task characteristics and be a mode of presentation that correlates with the requests for the lesson study. The lesson was about equations and inequalities involving Thanksgiving Dinner. Amber and Brendan were able to attend the presentation along with Ellen, a special educator. All attendees acted in the role of observers during the facilitation of the task without stepping in to prompt. Students came into class and sat in groups. This was a new type of seating arrangement that Denise had implemented for the school year to encourage group discussions and one that she still struggles with daily. She stated that "I am trying to find the correct grouping and improve my teaching and working with the groups." In preparation for the unit, Denise stated that she had not looked at the course goal since our August meeting nor the documents produced at that time. When I asked if she used the RTP and the unit flow that their team developed during the August session as the mode of presentation for the unit she replied that she had resorted to what was comfortable but was trying to keep the multiple representations in lessons when possible.

During the facilitation of the lesson you could tell that the students were still working on collaborating. One group of four boys worked in teams of two for a while and then began discussing their approaches as a group of four. They were explaining their individual approaches to each other and provided clarification and proof when one of the members did not understand. All but one of another team of five students worked together sharing approaches and thinking aloud. Later the observers found out that the one that was not participating did not "like" her team and had requested during an earlier class to be moved. A group in the back of the room made up of two boys and two girls had pulled their desks around so they were not forming a group. Two in this group, a boy and a girl, did share their thoughts on the task while the other two worked individually. One of the boys who worked alone became the major distraction of the class. We later found out that the girl that was working alone was quiet and did not usually need assistance in the class. In general all students were involved with the lesson, discussing approaches and sharing their methods when requested with Denise and their peers.

During the debriefing the principal and all the observers attended but one, Ellen. Observer comments began with Brendan who focused his views on the one student that was a distraction. He stated that Denise's attention was on this student often and to the detriment of the rest of the students. I shared my observations of the four boys and how their collaboration started out slowly but in the end included all in discussions. Amber shared about the group of five and even though the one girl did not participate she did benefit from the remaining team's comments. The debriefing ended with a discussion

about how to modify the lesson and the only comment was to do more of the same so students would be comfortable with sharing.

As we got ready to leave the debriefing, Denise shared with the principal how she was concerned about changing her instruction to fit the characteristics of rich tasks and the possible effects on the state assessment. The principal remarked that we needed to concentrate on good instruction and the scores would come. He also acknowledged that change takes time and practice. I encouraged everyone to collaborate on lessons that did increase rigor, include all levels of cognitive demand, and was student-centered.

Growth of Four Participants

Mathematics instruction is changing but knowledge about how it changes with respect to pedagogy and pedagogical content knowledge is lacking specificity. An overview of four of the participants and the two groups, PET and CET, will be discussed prior to a discussion about the answer to the research questions and future implications. Lesson study presentations, the culminating activity, is the portion of this research where all aspects of the study correlate for a stronger result and firm foundation for the future or as a source of what was missing and/or needed. To start off discussing the findings I chose four of the participants to pull together their views on the research questions from the initial interview through to the lesson study presentation.

Foundation setting. Amber and Brendan, members of PET, had worked together on curriculum in the past but did not view instruction the same way. Amber taking the lead was mentioned by many of the participants during the initial interviews. This "position" as lead Algebra 1 teacher seemed to be by default, as she made curriculum

decisions even though she was one of the youngest in teaching experience in mathematics. She made the decisions on pacing and the flow of the curriculum which was reviewed by her peers who accepted them without question. Amber was also the creator of the benchmarks by default as well. Brendan would meet with Amber when developing the curriculum in the past but also defaulted to her as his background is in a different subject. This resulted in uneasiness when both teams were expected to have input into the flow of big ideas and in their struggles with writing the unit and lesson plans. As referenced previously, a member of CET revealed that independence over the years led to teaching using a peer's pacing, plans, with any additional directions. Amber was the peer that was referenced. Prior to Amber coming to the high school, the participant stated that the textbook was their source for pacing and suggestions for pedagogy.

Amber, a member of PET. Amber's goal, as given during the initial interview, was to have the students "…conceptually understand what is happening." By the end of the summer, Amber was struggling with the development of the unit plan based on the big idea of solving equations and inequalities as well as the time involved to think through their plan of action. Amber was overheard during the August session saying "Let's just get this done." Her actions and beliefs came to fruition when she panicked and had the students complete a rich task the day before the lesson study presentation so they would have some idea of the expectations. The students, up to that time, had not experienced a rich task nor an activity of any type that entailed some of the characteristics. When asked about the goal of the lesson study rich task she replied, "I

wanted to let the students guide the discovery and completion with as little help from me as possible." This statement reinforced the personal goal made during the interview however her actions did not correlate with her words.

Learning mathematics via an integrated approach was a major influence on Amber's beliefs and the backing of her goals and desires regarding instructional practices. She was taught using big ideas but is lacking the pedagogical knowledge and pedagogical content knowledge to make it happen in her own classroom. Because of her role in the past as the lead for Algebra 1, many still expected her to continue in that role with the new approach of rich tasks and teaching to the big ideas however Amber was not wanting nor willing to take on this new role. Instead she resorted to activities and instructional practices of the past for her unit and lesson study presentation, "fly[ing] by the seat of my pants". Her references to the "dumb kids" also revealed that even though she stated that all students could learn mathematics that some students were at such a low level that it would take a long time to do so. Amber also struggled with some content knowledge such as what was meant by discrete or how to use the rule of five within the unit on solving equations and inequalities.

Challenges about the inner workings of the department and of collaborative planning by Amber revealed that the Algebra 1 team went in "circles" with "a lot of miscommunication." Classroom management skills was a challenge for Amber when providing instruction traditionally so she was also concerned about how to manage a group of students that were interacting with each other and had more freedom in their approaches to discovery of concepts. During the initial interview, Amber voiced a desire

to facilitate instruction and use rich tasks. Her words and actions during the sessions did not concur with her original statements. She used many excuses such as not enough time or support/encouragement to do so. These struggles affected her willingness to develop the units and lessons as she would need to research topics and instructional practices as well as develop the tasks, lessons, and units. I was unable to note changes in the tasks she had done in the past in comparison to what she was doing during the lesson study as Amber did not turn in a lesson plan or activity prior to the Summer Institute beginning.

Brendan, a member of PET. Brendan's goals at the beginning of the Summer Institute divulged that there was the necessity for his students to "…know the basic stuff they would need for the SOL" but evolved to a desire of "keeping things interactive and fun …want to get lessons to a point that they are engaging for the students," all due to the new expectations of the standards of learning for Virginia. Throughout the Summer Institute in June and August, Brendan continued to state his desired goals but at the same time verbalize his discontent over planning a unit and lessons in writing. In the past, Brendan consulted with Amber "during the development of the lessons and offered occasional small suggestions." He continued to play this role as a member of PET and during the creation of the unit/lesson for the lesson study presentation. Brendan is aware of the varying needs of his students but stated that he found it difficult to meet their needs. Adding rich tasks and teaching with the big idea in mind only increased his concerns about instruction. He spoke of the rich tasks as activities that "force" students to use their prior knowledge and critical thinking skills. In order for these practices to come to realization, Brendan believed that there needed to be a solid plan which laid out

an organized framework and pacing guide for the units of study. He wants everything planned and ready to be used. Brendan also lacked content knowledge and admitted that he searched the web to learn and/or renew his understanding as he had not worked with many of the concepts since high school. This lack of understanding led to his inability to view instruction that allowed students to think critically and approach big ideas in multiple ways along with struggles to develop rich tasks.

A concern by Brendan was also expressed about the Algebra 1 team planning collaboratively. He stressed that many of the teachers wanted "…to do their own thing" and that everyone would need to willingly participate. Brendan also voiced a lack of support. As a teacher coming from another discipline, he believed it would be beneficial to have someone to guide and provide constructive criticism of instruction and student learning.

Brendan did turn in an activity using roller coasters without a lesson plan prior to the Summer Institute beginning. This initial task could have been student centered however Brendan explained that he led them through each step and expected them to do the tasks as shown in class. The task gave pictures of numerous roller coasters with height and horizontal length given for each of them. Students were expected to determine the rate of change, repeatedly, using the slope formula. Brendan believed this to be an interactive lesson which in essence it was just a handout to practice recognizing and writing values in a ratio of rise over run. His belief in what is considered to be an interactive lesson is shown in remarks made when planning the unit of study for solving equations and inequalities. In this unit he planned for very distinct concepts, such as

gathering like terms, and then wanted an activity that had the students practice the skill using the same approach as practiced in class.

Denise, a member of CET. Having a goal that the students would have the skill set and could actually accomplish getting a problem correct was Denise's initial view and also remained with her throughout the study. Denise also saw that with the new standards and expectations that she needed to change her approach to teaching and learning, stating that she wanted "…to see things done differently." She also had a concern about the state scores on the end of course assessments. Even though Denise stated that she believed that rich tasks would help improve the scores, she was concerned that if they did not improve or went flat that the administration would be upset. This concern remained in the forefront of her thinking throughout the Summer Institute and school year activities. Her buy-in to including rich tasks into instruction that was based on big ideas was evident when their team began planning using the Frayer Model and when developing the unit plan. Denise led the members into filling in the Frayer Model very explicitly with objectives, ideas for rich tasks, and ideas for assessments. When developing the unit, Denise was the lead in initiating the concept of the rich task prep, RTP.

Although Denise believed that if the teacher was committed to teaching to the big ideas and in rich tasks she acknowledged that her practices had minimally changed. She had a fear of failure that led to a desire for activities that would help her change her practices to align more with her beliefs. There was also a desire for support as she mentioned that their math specialist was the most helpful and one that was the least

intimidating. Due to her belief and desire for change, Denise did rearrange her classroom at the beginning of the school year to be in groups so that the students could communicate with one another. This arrangement kept the notion of students communicating mathematics at the forefront of her thoughts and led to many more activities in the classroom. Even though the seating arrangement was a reminder, Denise had not consciously focused on the course goal or on the idea of rich tasks. The lesson study presentation brought her thoughts on the topic to the forefront while also becoming aware that the activities she had done with her students did correlate with many of the rich task characteristics. She shared the example given previously regarding plotting points and writing equations of lines.

Challenges that Denise expounded on ranged from those about the Algebra 1 team but also about herself. She did not want to prejudge a student's ability to complete a task but also wanted to build student confidence in their problem solving abilities. At the same time these concerns also applied to herself. She did not want to prejudge her own ability to facilitate a task with her students or in her competence to encourage and provide opportunities for students to apply problem solving skills. This is where Denise wanted to rely on the math specialist to help provide that support and encouragement.

Denise did not turn in a task at the beginning of the Summer Institute so I am unable to relate her past activities to those she is doing this year with her Algebra 1 students. Verbally she spoke about the lesson on coordinate points and placing students in groups, a big step on the road to change.

Ellen, a member of CET. Ellen is a special educator completing her first year on the job working with Algebra 1 teachers and students. Her goal was to have the student "…comprehend what we are doing…" She spoke about breaking the material down into smaller, workable concepts so that those with special needs could be successful. Ellen acknowledged the idea that rich tasks could allow this group of students an opportunity to approach a problem on their own level but at the same time she believed that they needed guidance doing so. Lacking an understanding of the standards of learning was brought forth when Ellen stated that "if the teacher understood the standards, the concepts, then the students will be successful."

Ellen's priorities during the Summer Institute, as well as the school year lesson study, were on her responsibilities as a special educator. She had to leave the summer sessions numerous times to deal with various situations of closing files for the school year and then opening the school year in August. Ellen attended the presentation of the lesson study for both teams but not the debriefing. During the presentations, Ellen did not remain "an observer". She pulled her chair up to the group of students she was to observe and began prompting them.

Ellen did turn in an activity prior to the Summer Institute. This was a step by step worksheet on determining the rate of change, slope, given certain data. She stated that the worksheet was interactive with students working on it alone thereby raising the level of rigor for the student. Ellen did not assist in developing the task for the lesson study but did support Denise in her efforts to use the RTP.

Doyle, the Mathematics Specialist

A brief discussion with Doyle revealed his willingness but also his anxiety of how to help the Algebra 1 teachers in their endeavors. He stated that he lacked the knowledge of how to make all the connections as well as the experience of working with rich tasks and forms of problem based learning. The specialist expressed a concern that the state provided institutes for one person from each school system to attend a one day session on rich tasks in 2011 and a one day session on performance based assessments during the fall of 2012 was not enough. A one day institute, or even four days, was not enough to build confidence of teachers to go forward in their growth. He believed that this study provided an opportunity to create an awareness of expectations of the state standards but more is needed for the teachers to be comfortable developing, implementing, and reflecting on their actions. Doyle was also placed briefly back in the classroom to cover for an unexpected long term absence prior to the lesson study presentations. Between the unexpected teaching duties, and his own lack of confidence and knowledge, the Algebra 1 teachers did not receive much, if any, support and encouragement to change their practices.

Emerging Themes

Three themes emerged during the study and centered around correlation of beliefs and practices, understanding the foundations of big ideas, and being able to thoroughly process what is to be taught in a unit or lesson but writing it out. Prior to implementing a new approach to teaching and learning, teachers needed to understand and buy into the big ideas, thus the theme of traditional concept flow versus big ideas. Members of both

groups were challenging as they proposed to be innovative and creative but at the same time exhibited traditional traits to planning and instruction, skeptical in the beginning but rose to the challenge only to fall back to past modes of planning and instruction, all leading to the theme of the ideal versus the implemented. Teachers customary planning habits versus lesson study planning practices is an emerging fourth theme. This last theme ties the ideas of the previous themes together.

Traditional concept flow versus big ideas. Before teachers can be expected to teach using the Rule of Five and the NCTM Process Standards (2000) they need to understand and take the time to construct a flow of concepts built around major ideas. This was not easy as came to light during the study. The participants did not know nor understand what is meant by teaching and learning via big ideas. Their idea of developing a scope and sequence of topics meant following the textbook or doing what they have always done. Before proceeding or expecting teachers to implement rich tasks they must have a foundation and understanding of why and how the lessons are put together. Cognitive demand, discussions on traditional versus reform teaching and learning characteristics, and experiencing rich tasks was helpful. In this study the teachers experienced three rich tasks with two that would be part of the same unit. They then chose two different units and were expected to develop the big idea using the characteristics of rich tasks, Rule of Five, and the NCTM Process Standards. While attempting to develop the units the majority of teachers were still struggling with teaching isolated concepts to some degree versus teaching using the big ideas. The teachers were not comfortable with the change in the traditional flow of concepts even

though they were the creators of the written curriculum. They may have benefited from experiencing more rich tasks with continued discussions on what concepts are included, representations and their connections, and more time on how units of study using multiple concepts are put together. The teachers viewed mathematical instruction as linear, one concept at a time, versus pulling concepts together as a big idea and approaching teaching and learning as an iterative process. Going from teaching isolated concepts to the big ideas is a giant step.

Ideal versus the implemented. Prior to the study Denise, Garth, and Ellen considered themselves to be traditional. Lecture, rote practice and drill, and skill proficiency was the focus in their classrooms. The remaining participants considered themselves innovative and technologically savvy but at the same time conducted their classrooms just like the traditionalists. Participants were led through activities that restructured the curriculum and then experienced three tasks that were characteristic of the rich tasks. The tasks supported what some were stating verbally, allowing them to experience what they were expressing; at the same time opening others to the views that are expected as given in the Virginia Standards of Learning (2009).

Through the rich tasks experienced during the Summer Institute, the participants were able to become familiar with the critical thinking skills needed as well as learning via big ideas. Participants were also learning content at this time. Discrete data, continuous data, abscissas, ordinates, and the relationships between the various representations are some that were most prevalent. They realized that the ideal instruction included implementing the Rule of Five and the NCTM Process Standards

(2000). After the Summer Institute the same approach to teaching and learning occurred with all acknowledging the ideal classroom situation. Teachers acknowledged the quality of the rich tasks but without the support and encouragement of change, without the time and structured collaboration, they were unable to follow through implementation. Their beliefs had not totally changed as they did not have the evidence that the approach worked nor the time or support to obtain the evidence.

Customary planning habits versus lesson study planning practices. As noted previously teachers in Blank High School did not have the expectation that lesson plans or unit plans be written. They were only expected to write on the board ASWBAT (all students will be able to…) which is a listing of the standard of the lesson. Ken questioned how he was to write all of the standards on the board that are now involved in a lesson. Teachers were requested to begin writing a unit plan on the third day of the Summer Institute in an attempt to bring their ideas together from research and best practices. This was halted when Doyle suggested that they discuss writing out plans, unit and lessons, when they met at the beginning of the school year. Teachers did complete the Frayer Model during the summer and during the work week session they began working on laying out a flow for the unit.

On day four, notes were jotted down on paper so if the group decided on writing plans then they would have the foundation but otherwise they still had their thoughts recorded. CET's notes exuded what was the hoped for launch of writing the unit plan; PET's notes were lacking. Prior to returning for the lesson study presentation emails were sent to the participants to encourage them to research their topic, continue building

on their work on the unit plan, and to write the unit plan and lesson plan. Encouragement was also given for the participants work on the rich task; reminders of the characteristics, the Rule of Five, and the NCTM Process Standards were included in the emails. Doyle was also included in the emails as he was their support during collaborative planning sessions and their encouragement when attempting new approaches in the classroom. Upon return for the lesson study presentations, it was quickly recognized that the group decision was not on writing out unit or lesson plans. I was only handed the rich task for both presentations. No unit or lesson plans.

Without having the opportunity to think through the entire unit the teachers did not anticipate some of the actions, teachers and students, nor were able to plan the implementation of the tasks as part of the whole. During PET's presentation Amber knew she had to present the task, have the students work on developing a rule, and have them create a table and graph. But that was all she had thought about. Amber did not plan on how to present the task, was she going to read the scenario and discuss it or have the students read it first. She ended up passing out the task and giving them an overview which lead to telling them to develop a rule for the growth. This did not encourage the students to draw what they knew nor was their previous planning as to using manipulatives. Amber ended up going to each group and suggesting that they draw each stage of growth. Amber even had her students complete a similar rich task the day before the lesson study presentation as she was skeptical of their ability to perform on the day of the lesson study presentation.

CET's ideas of using RTP's never came to fruition. Instead they turned to what they had done in the past and just included the task as an additional item to be completed. They did not discuss the specifics of how to use the rich task and the role it was to play in their units so it just became another "thing" to do, taking up valuable time. Therefore the task we observed was a repeat of the previous day with just a change in values. Unlike the PET presentation, the CET presentation was new to the students but Denise did say that she was trying to include multiple representations in all lessons. Denise's students were also used to sitting and discussing mathematics, getting more proficient with their conversations as the year proceeded.

Conclusion

Behind the three themes: traditional concept flow versus big ideas, ideal versus the implemented, and customary planning habits versus lesson study planning practices, are the underlying threads of support, time, and effective collaboration. Teachers need support and encouragement as they develop the big ideas, change their habits in the classroom, and develop new units of study. Time, uninhibited by other duties, is needed. Along with time comes the support so that the teachers make the best use of the collaborative sessions. This leads to collaboration that is guided and has a purpose that is supported by administration. Chapter 5 will sum up the study, discuss the relationship to the research, and propose future implications.

Chapter Five

Summation of the Study

Change is not easy, however this group of teachers volunteered to participate in the study as they saw a need for assistance with their instruction, and subsequently their state end-of-course test scores. The need was apparent but the mode to provide the evidence for changing habits as discussed by Cooney, Shealy, and Arvold (1998) was not in alignment to what the teachers possibly had in mind. Support, as discussed by several researchers (Becker, Pence, & Pors, 1995; Guskey, 1986; 2002; Orrill, 2006), along with evidence, was the missing link in the growth of the teachers. Teachers needed to have the support of the administration and someone who could guide and provide a critique of their actions; someone who understood "the ideal" and the "lesson study planning practices." Those in support position needed to be aware of rich tasks, their characteristics, and the expectations of teachers and students in a student-centered mathematics classroom.

The culminating activity, the lesson study presentations, revealed that the "implemented" did not match the "ideal". Lesson study was attempted through the simplest and most accessible method possible thereby not fully engaging in the approach; using rich tasks they had already experienced without any modification to meet the current unit of study. During the professional development, the teachers in attendance

were immersed in the big ideas but the real world took over during the school year. Old habits continued. It was then difficult for them to follow a written curriculum where big ideas were the focus when they were attempting to teach using isolated concepts. It seemed that the lesson presentations using the rich tasks were considered a one-time deal for many of the teachers; a formality to complete the professional development versus a step in changing to be a more student centered teacher.

Professional Development Implications

In order for a professional development to be a success many aspects need to come together like the "perfect storm." Meeting the needs of all involved, knowing the group dynamics, timing of the sessions, expectations of the events of the sessions as well as the product, and being able for participants to have the opportunity to gather evidence or artifacts of their efforts are just a few of the elements. The Summer Institute and all subsequent sessions were arranged with the assistant superintendent of the school system and were believed to be what the teachers needed. Administration also made the teachers aware of the Algebra I scores and how they needed to be improved. It was therefore deemed by central office that the professional development was what the teachers needed and that the professional development I had to offer would meet their needs. In the future, prior to developing a professional learning experience, it would be beneficial for the teachers to complete a needs assessment versus central office deciding what was needed. If possible, it would also be beneficial to visit classrooms, attend department meetings, and collaborative sessions. Time to get to know where the participants were

coming from and what they would be bringing to the sessions needs to be built into the overall plan.

Knowing the participants would also be helpful when dealing with group dynamics. The PET participants had difficulty getting along, everyone wanted to be the "chief". Even though the two groups in this study were self-selected, in the future separating the groups would have given them the opportunity to work with teachers that they do not normally. Ultimately, all of the teachers need to work together as a collaborative team to plan Algebra I lessons and units. This leads into expectations of events and the timing of the sessions.

It would be valuable for all involved, assistant superintendent, mathematics specialist, department or course leader, principal, and professional development facilitator to meet together to discuss the sessions and to obtain their buy-in on expectations so the opportunities for change from traditional concept flow to big ideas will be the same for all. Deciding what is expected during collaborative sessions, lesson planning, the end product, and when and where all events will be held would set the stage that all know what is expected. Support personnel would know and be able to encourage teachers, having the same end result in mind and all placing the sessions as a priority for all.

Lastly, the issue of gathering evidence or artifacts to discuss need to be included in the initial planning and expectations of the events. Evidence, or expectations of the sessions, is part of the planning process but important enough to be listed separately. Cooney, Shealy, and Arvold (1998) place a lot of importance on teachers being able to

gather evidence, and having discourse regarding the evidence, during collaborative sessions. The evidence or artifacts will encourage discussions, promoting discourse about student engagement and meeting student needs. At the same time, evidence or artifacts will provide the means of enabling change in a teacher's beliefs and practices.

In the end, we are dealing with many individuals with different beliefs and practices when teaching mathematics. The goals of the participants in this study and those of the professional development, on the surface, did complement each other. However, the lack of a cohesive plan of action where all support individuals were working toward the same result did not permit the depth of the goals to come to fruition.

Relationship to Research

NRICH (2007) wrote about the implementation of rich tasks being more than just the activity, it included the learning environment. This was a concern for participants as they plan during the school year without the support of knowledgeable individuals. This study revealed that teachers need support with more than content. How to handle the unknown approaches students might come up with, how to plan for higher cognitive demand tasks, and how to assess students using performance based tasks are just a few concerns of teachers in the throes of change. Group work (Blumenfeld, Marx, Soloway, & Krajcik, 1996) and norms (Lewis, Perry, & Hurd, 2009) were also a concern that teachers need to address. Teachers with traditional classrooms with students seated in straight rows will need assistance with how to handle classroom management surrounding these issues. Acknowledging that developing norms would encourage students to have some ownership in the "rules of the game" for group work and aid in

classroom management need to be addressed and supported with embedded professional development. Concerns about classroom management (Edwards, 2011) and how to pull together productive groups could have been approached using their buy-in to the rules/norms. Teachers needed to view a classroom as productive where students would be talking about mathematics, go in different directions with their approaches, and communicate mathematically in contrast to a more "orderly" classroom where students followed the same procedures and obtained the same answers. The ideal versus the implemented were at odds; knowing what should be done but having a fear of doing so. At the same time customary planning habits were easy to fall back into and, without support, teachers did not pursue lesson study planning practices.

Applicable rich tasks were mentioned by several of the participants. Those involved in creation of rich tasks for the participants in The New Basic Project (Queensland State Education, 2004) sought for the tasks to be readily usable. This leads back to comments referred to previously regarding a desire by the participants for a framework to be developed by the state using the big ideas with all the details for units of study that are ready to use. Like those involved in The New Basic Project (Queensland State Education, 2004), both teams involved in this study ended up using tasks that they had already experienced or were given, however the PET and CET participants did not have the awareness or support of how to fit the task into their units with specific details. Even given the tasks, planning practices and expectations need to be more in line with lesson study planning practices.

Beliefs and goals of the participants needed to be complimentary in order for the professional development experience to be "workable" as stated by Clarke, Breed, and Fraser (2004). As noted previously, the participants wrote and reflected about how and if their personal goals, the course goals, and their beliefs did or did not work together. In writing all seemed to be workable however the participant's dispositions, their beliefs toward a student-centered classroom, did not always fit into their view of quality instruction. Many remained concerned about computations, that skill proficiency would be lost as students' attention would be more on problem solving as was Wu (1997) who spoke about the beauty of mathematics being missed when the focus is not on computation. These concerns could be addressed through the lesson study process with teachers collaborating and discussing details of instruction and in the end assessing whether their actions were conducive to students learning to the level of rigor and application as desired.

The cyclic effect of Guskey's (1986; 2002) theory was not possible with the limitations of four days in June and August and the school year lesson study presentations. Guskey wrote of the importance of teachers obtaining feedback on the progress while Stigler and Hiebert (1999) shared the importance of having time to reflect and share concerns and experiences. Guskey and Stigler and Hiebert's ideas could all be addressed with the teachers using the lesson study approach to planning. The ideal could be a focus as teachers reflect, obtain feedback, and share experiences. All of which could have been obtainable if the mathematics specialist was available to give the support.

Doyle was not available nor did he feel competent to provide teachers with feedback or lead discussions reflecting and sharing their experiences.

Railside High School teachers in Boaler and Staples (2008) study did collaborate and had a curriculum set up that was oriented to mathematical reform that included conceptual understanding, multiple representations, making connections, and communicating mathematically. Again, this group had a curriculum already created and they just needed to understand the expectations and implementation. The differences between Railside and Blank High School teachers were their acceptance and desire to implement the big ideas along with having the support needed with their planning practices when implementing new ideas. The teachers in the Boaler and Staples (2008) study believed that all of their students could be successful in mathematics, a growth mind-set (Dweck, 2010), and they also had support from those in administration. Blank High School teachers in this study stated that they also held the growth mind-set for their students but their words and actions did not uphold their spoken beliefs. The ideal did not match the implemented.

Derry, Wilsman, and Hackbarth's (2007) video case study was similar to this study with the switch of videos with rich tasks. Their study revealed that the awareness created through watching the videos with respect to pedagogical content knowledge increased but content knowledge did not. In this study, the participants became aware of the big ideas in mathematics, multiple representations, and the NCTM Process Standards (2000) for a unit on linear functions. Content knowledge also increased. Concepts missed in the Mathematical Survey such as continuity, the relationships between

representations, and the multiple meaning of slope were included in the rich tasks experienced by the participants during the Summer Institute. The survey was not given as a post assessment as the questions were incorporated into the professional development and the linear function unit. While the content and pedagogical content knowledge was addressed for the linear functions unit, time was not built into the professional development for the participants to have the opportunity to go to the same depth of investigation and cognizance with their chosen units of study nor did they have the desire or support to do so afterward. Teachers need some guidance as they investigate content and pedagogy. Manouchehri and Goodman (2000) discussed the depth of teacher content knowledge, pedagogical content knowledge, and how their beliefs influence how they implement instruction. In this study the participants did not research the concepts with respect to content and pedagogy. Participants also did not express the unit of study in writing using the lesson study planning practices thereby missing opportunities to discuss and delve further and deeper into the content and pedagogy which would have influenced their instructional practices.

Participants individually stated they could develop rich tasks provided the time and given guidance and support. Collective efficacy was another issue as PET struggled working together. This follows Zambo and Zambo's (2008) discussion on individual and collective efficacy. As the Summer Institute progressed it became more evident by the remarks made in the participants' journals that the strong personalities and beliefs did not come together to form a productive team. This raises concerns about how to get teams of teachers to productively work together to produce a product.

The lesson study approach (Lewis, 2002; Lewis, Perry & Hurd, 2004) was followed for this study with participants beginning by creating a goal statement for their course and developing a set of norms for group interactions. Research of their topics was missing for both teams as they lacked support and encouragement to do so. Their choice was to take the easier route, to use what they were given and not investigate additional approaches to providing/facilitating instruction. In the case of this study the participants also needed to align topics into big ideas during the professional development and it was more difficult for them to do than anticipated so it took more time. They struggled with the traditional flow of concepts versus understanding how to put together the big ideas. Expressing their thoughts in writing continued to be a challenge. Attending a professional development for four days with no embedded, follow-up sessions or support proved to be unproductive. The sessions were not a "quick fix" but did open their eyes to rich tasks and their characteristics. Trying to push ideas on teachers did not work, there needed to be time to assimilate the concepts and provide support during implementation.

The mathematics specialist was another concern as he sympathized with the teams about having to write up the unit and lesson plan, stating that "we will come up with a compromise". The compromise was that neither was written therefore logically thinking through the units was not further developed. Teachers resorted back to what was comfortable and what they already had with the exception of inserting the one rich task into their plans. Rich tasks became one more step: warm ups, the lecture, practice, and rich tasks. The actions of the mathematics specialist show that all need to be in agreement on the outcome and the product resulting from the sessions.

Taplin and Chan's (2001) study with pre-service teachers led to the recognition of five areas that were behind teachers not accepting reform. A lack of pedagogical content knowledge was the first area. Teachers need someone to lead/guide them through experiences, making connections, discussing approaches to instruction, and making relationships between the representations. All of which a mathematics specialist could lead or facilitate to support those in the change process. The second area, a lack of confidence, comes from never experiencing reform approaches. Teachers look to the mathematics specialist for guidance which did not work out in the case of Blank High School. A lack of creativity was the third area. Many mathematics teachers view their subject as linear and one that revolves around computations. Providing or facilitating instruction differently than was tradition is hard for them to accept. This is where evidence of success as discussed by Cooney, Shealy, and Arvold (1998) is important to change in instruction. Not deeply understanding rich tasks and big ideas effects planning and ultimately implementation.

The fourth area, accountability, played a huge part in this study. Teachers in the midst of change are concerned about students' scores on school/district benchmarks and state assessments becoming flat or decreasing and then in return what will the administration say in response to the scores. Support is needed from administrators that they recognize change takes time, that they respect the time needed for planning, and that they provide the support of learning content and pedagogical content knowledge. The last area, lack of knowledge of the new expectations, links back to accountability and lack of confidence. Brendan stated that during the first rich task he didn't know what I

wanted. He asked for explicit directions during the rich task and later questioned how one could encourage and support students so that the work, and the product, would be acceptable and what was desired. Lacking experience with students led to the fear of the unknown. Brendan struggled because he did not get step-by-step directions and feared how he would handle the same fear with students.

Future Implications

At the beginning of the study it was acknowledged that four days of professional development would not bring about substantial change, however if it was combined with continued embedded professional development by those supporting mathematics then change could happen. Long-term goals are needed for change to take place. Goals may match on the surface but teachers need individuals to support the change and to guide them in their endeavors. Teachers need to acknowledge that the professional development on rich tasks and standards-based reform is not just a fad, one that is here for a while and then back to the norm. Administrators are obligated to changing their instructional roles and those of the teachers and thereby supporting their needs.

Professional development sessions should be held when there are no other conflicts and all intended participants are able to be present for all events. It would be beneficial if the principal could be included in the professional development sessions or at least be available for a debriefing of the events and expectations. It is imperative that the support personnel, the mathematics specialist, also attend all sessions. If all had met prior to the sessions beginning and Doyle had known that he was supported by and in

agreement with all facilitators, then it would have helped him to stand firm and to be an encourager in the expectations of the teachers.

This leads to the role of the mathematics specialist and the educational training they receive. Certified mathematics specialists, such as the one employed at Blank High School, receive training in kindergarten through eighth grade. One would think that the conceptual understanding of concepts received during this training would aid in the upper level courses in high school. However, one needs to consider content knowledge and pedagogical content knowledge for high school courses. In the future, training for high school specialists should be considered so that teachers at that level might receive appropriate assistance in meeting student needs.

Schools might also consider the expectations of teachers in their buildings. Teachers at Blank High School are not accustomed to planning. They did not have lesson plans or activities to share nor did they want to write out anything during the professional development. Teachers in transition may verbally state that they value the tasks, but actions do speak louder than words. There needs to be an expectation of a product-driven process that will assist the teachers in thinking through the process and expectations of themselves and their students. The product in this case would be the unit and lesson plans. Taylor (2000) stated in his work with pre-service teachers that writing out the unit and lesson plans and perhaps developing and using a checklist will help to "ensure that you have thought it through well enough to do a competent and complete job (p. 253)." He went on to say that many teachers had a general idea of what they were teaching on a specific day and did not believe it needed to be written out. Documenting and thinking

through how they were going to promote discourse and the task needed to include the expectations or focus of the task, how it was to be implemented and how to flow through the task, and the expectations of the teacher and the student during the lesson. Writing out plans will enable teachers to insure that they address all state and national standards (Taylor, 2000). Professional development sessions then need to have a product driven expectation.

Taylor (2000) stated that the "most important lesson, however, is that teacher educators, like everyone else, need to practice what they preach (p. 255)". So that a teacher can teach using methods that they have not experienced, it is necessary that they have the opportunity to model it for themselves. This modeling begins with thorough planning. With rich tasks and performance-based assessments, it is essential that teachers need to plan, and to plan collaboratively as outlined in the lesson study planning process (Lewis, 2002). What priorities need to take precedence when establishing a culture of change? For Blank High School, priorities such as writing out unit or lesson plans and of collaborating with peers would have been helpful and provided an expectation for results.

The mathematics specialist mentioned that the state of Virginia provided a one day institute on rich tasks in 2011 and a one day institute on performance assessments in 2012. With these institutes, all were aware of the state's expectation with regard to instruction however there is still a need to get teachers to be comfortable and competent with rich tasks and teaching to the big ideas. The lesson study approach aided the session by establishing goals and norms, and researching the topic. Promoting writing the unit plan and lesson plans (Lewis, 2002) would have provided the teachers with more support

and confidence. Mathematics teachers traditionally plan independently and reflect, if required, independently on their lesson presentations. Applying the lesson study approach would encourage the teachers to work as a collective unit to plan as well as share their thoughts on the lesson presentation during the debriefing. Administrators need to be educated on the lesson study approach and its benefits to the teachers and students.

Developing units of study and individual lessons take time, content knowledge, pedagogical content knowledge, and the ability to make connections within the subject, across disciplines, and within a real world context. At this time teachers do not have many opportunities to experience student-centered teaching and learning using rich tasks. There is a need for the state to come together to develop a scope and sequence of big ideas and then develop units and lessons that meet the standards, state and national. Training for the support personnel could then be conducted using a train the trainer model so that they could better support the mathematics teachers in all schools. Providing a curriculum flow of concepts using big ideas and rich tasks with all supporting documents would in turn help to provide the same level of expectations and rigor to all students in the commonwealth.

Limitations

This study was limited in the time spent involved in the professional development by participants. In a small school district, many individuals take on numerous roles. Participants were pulled out of the sessions to deal with special education issues, to attend another meeting, or to close out the school year. The teachers participating in the

study taught many different courses, one of which was Algebra I, and performed many extra duties. Professional development was something that the teachers normally had to seek out on their own as it was not provided by the district. Professional development that is embedded within instruction was not practiced. Teachers were used to going to, and paying for, professional development at conferences of their choosing therefore they attended and listened to topics of their choice. Another limitation is that the mathematics specialist was the only one for the middle and high school. His main area had been the middle school but the algebra results on assessments were forcing him to assist in the high school. Even though teachers in smaller school systems perform multiple roles, one must determine what is in the best interest of the students and the priorities of the school system.

Possible Professional Development Changes

In addition to conducting a needs assessment and providing time for teachers to collaborate, the professional development conducted for future sessions could be altered to be more of an embedded professional development. There could still be a session prior to school starting where the teachers would go through the cognitive demand activity and rich tasks could be introduced. Analyzing the concepts built into the task along with the characteristics of the task could be recorded for future reference. Tasks would need to be approached as an exploration where the teachers are recognizing the multiple concepts being included in the tasks as well as how to approach presenting the tasks. Discussions and journaling of goals and the vision for the course could still be done prior to the school year.

During the school year rich tasks would be presented but at a slower pace. Planning weekly meetings at first and then later on switching to monthly meetings where the rich task that the teachers would experience during the session would contain some of the objectives being taught in their classrooms at that time. Follow up support for the teachers to facilitate the task with their students would help to build their confidence and provide evidence for continued growth or to help answer concerns and questions. This would include lesson study presentations that could occur with the teachers selecting one of the rich tasks they experienced and working to make it the best task. Teachers would begin discussions on modifying the curriculum to better enable the implementation of rich tasks. Where the curriculum flow of big ideas was completed in an afternoon during this study, future sessions would allow the participants to think about the changes and alter them over several meetings.

Second semester meetings could continue the rich task experiences but also begin creating their own rich tasks and presenting them in a lesson study format. During the last quarter, there could be an expectation that teachers would begin writing out unit plans with a goal of having the basics in writing for all units for the first quarter of the school year. Over the summer the teachers could meet to continue their work on second quarter topics. Embedded professional development needs to continue during the following school year with teachers receiving assistance and encouragement in their classrooms and during collaborative times using the lesson study planning process. Teachers need to continue experiencing standards-based learning and rich tasks as it is important to change but there still needs to be support and encouragement for those

learning to teach using this approach. Therefore there needs to be an emphasis on collaboration with a possible structure to the expectations.

Conclusion

Additional research may be conducted for those in the throes of change regarding the benefits of writing collaborative unit and lesson plans for rich task creation. Research would be about the importance of writing out units of study and the specific lesson plans to meet state and national standards, and in this case rich task development, creation, and implementation versus having an overview of what the unit is about and shooting from the hip to deliver instruction. Combining written plans with lesson study could also be researched. What we teach and how we teach mathematics will make a difference to those students sitting in the desks. As previously referred to, Schoenfeld stated that "What we teach not only affects how students see themselves, but it also affects their trajectories through life" (2002, p 28). We, as teachers, hold many lives in our hands so we have an obligation to provide them with the best education in mathematics that we can.

APPENDICES

APPENDIX A: MATHEMATICAL SURVEY

I.

1. Answer the following using "slope" as the content.

A) Define slope? Give one or more examples to illustrate your definition.

B) In the real-world what does slope represent? Give one or more examples to illustrate your definition.

C) Pre-requisites needed by students?

D) How would you present slope in the classroom?

2. Which of the following are examples of linear functions? Circle YES or NO for each.

A.

x	y
1	2
2	4
3	8
4	16
5	32

YES / NO

B.

x	y
-5	12
-10	17
-15	22
-20	27
-25	32

YES / NO

C. $y = -|2x|$ YES / NO

D. $y = -2/x$ YES / NO

E. The ratio of the number of ice cream drumsticks bought to the number of éclairs purchased at the local grocery store is 5:2.

YES / NO

F. $y = 10$ YES / NO

G. Mechanical pencils cost $1.25 at the school store while plain wooden pencils cost $0.25 cents each. I buy a combination of both types of pencils and spend $6.00. YES / NO

H. Tom has been asked to keep track of the number of boys and the number of girls that purchase chocolate milk at lunch. He notices that they are about the same. YES / NO

I. A virus is spreading throughout your school. The school nurse stated that students with the virus is increasing by about 10% every day.

YES / NO

J. If p is the probability that an event will occur then $q = 1 - p$ is the probability that an event will not occur. YES / NO

3. At the age of 2 month old Jingles, my pet shih tsu, weighed 5 pounds and by the time he was 1 year old he weighed 12 pounds

A) How fast did Jingles weight increase from the age of 2 months to age of 1 year?

B) On a scale from 0 – 4, with 4 representing "Yes" to the question and 0 representing "No".

	0	**1**	**2**	**3**	**4**
Would you consider the question to encourage procedures with connections or doing mathematics?					
Does the question encourage students to apply and/or develop higher order thinking skills and knowledge?					
Does the question involve students applying problem solving skills and ask questions?					
Are there multiple entry points and multiple pathways to obtain the solution?					
Is there a driving question?					
Are students expected to apply prior knowledge?					
Are students expected to present their solutions and justifying their responses?					
Is the question relevant, interesting, or applicable to students?					
If used in the classroom would students be expected to work in groups, being responsible for self and team?					
Do students reflect on their work?					

4. Evan takes an entrance test and earns a score of 125. Everyone can take the test twice so he studies. He figures that for every hour he studies that his score will increase by 2 points.

A) If h is the number of hours Evan studies for the second test and s is his final score then determine an equation to represent the situation.

B) On a scale from 0 – 4, with 4 representing "Yes" to the question and 0 representing "No".

	0	**1**	**2**	**3**	**4**
Would you consider the question to encourage procedures with connections or doing mathematics?					
Does the question encourage students to apply and/or develop higher order thinking skills and knowledge?					
Does the question involve students applying problem solving skills and ask questions?					
Are there multiple entry points and multiple pathways to obtain the solution?					
Is there a driving question?					
Are students expected to apply prior knowledge?					
Are students expected to present their solutions and justifying their responses?					
Is the question relevant, interesting, or applicable to students?					
If used in the classroom would students be expected to work in groups, being responsible for self and team?					
Do students reflect on their work?					

5. The following two points give information about the cost to send a package: (2 pounds, $3.50) and (5 pounds, $9.80).

A) Calculate the slope.

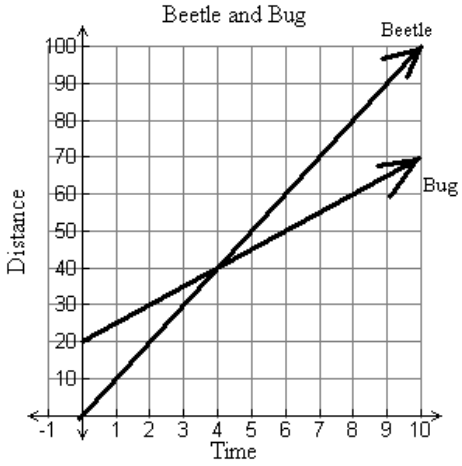
B) Describe what the slope stands for in the context of the situation.

C) On a scale from 0 – 4, with 4 representing "Yes" to the question and 0 representing "No".

	0	**1**	**2**	**3**	**4**
Would you consider the question to encourage procedures with connections or doing mathematics?					
Does the question encourage students to apply and/or develop higher order thinking skills and knowledge?					
Does the question involve students applying problem solving skills and ask questions?					
Are there multiple entry points and multiple pathways to obtain the solution?					
Is there a driving question?					
Are students expected to apply prior knowledge?					
Are students expected to present their solutions and justifying their responses?					
Is the question relevant, interesting, or applicable to students?					
If used in the classroom would students be expected to work in groups, being responsible for self and team?					
Do students reflect on their work?					

6. The figure shown below shows a position versus time graph for the motions of two toy race cars labeled Beetle and Bug that are traveling along the same track.

A) At the instant when $t = 2$ sec, is the speed of Beetle greater than, less than, or equal to the speed of Bug? Explain your reasoning?



B) Does Beetle ever have the same speed as Bug? If so, at what times? Explain your reasoning.

C) On a scale from 0 – 4, with 4 representing "Yes" to the question and 0 representing "No".

	0	**1**	**2**	**3**	**4**
Would you consider the question to encourage procedures with connections or doing mathematics?					
Does the question encourage students to apply and/or develop higher order thinking skills and knowledge?					
Does the question involve students applying problem solving skills and ask questions?					
Are there multiple entry points and multiple pathways to obtain the solution?					
Is there a driving question?					
Are students expected to apply prior knowledge?					
Are students expected to present their solutions and justifying their responses?					
Is the question relevant, interesting, or applicable to students?					
If used in the classroom would students be expected to work in groups, being responsible for self and team?					
Do students reflect on their work?					

7. Jay made soup that he brought to a boil and it now needs to be placed in a refrigerator. However the refrigerator is unable to handle soup that hot. From Jay's classes in high school he knows that Newton's Law of Cooling might help him. He places the hot soup in a sink of cold running water. Given the formula: $l - l_o = al_o(t - t_o)$, where l_o is the initial temperature of the soup, l is the standing temperature of the water in the sink, a is a constant value, t is time, and t_o is the initial time.

A) Express Newton's Law of Cooling, l as a linear function of t.

B) Find the slope and the y-intercept.

C) On a scale from 0 – 4, with 4 representing "Yes" to the question and 0 representing "No".

	0	**1**	**2**	**3**	**4**
Would you consider the question to encourage procedures with connections or doing mathematics?					
Does the question encourage students to apply and/or develop higher order thinking skills and knowledge?					
Does the question involve students applying problem solving skills and ask questions?					
Are there multiple entry points and multiple pathways to obtain the solution?					
Is there a driving question?					
Are students expected to apply prior knowledge?					
Are students expected to present their solutions and justifying their responses?					
Is the question relevant, interesting, or applicable to students?					
If used in the classroom would students be expected to work in groups, being responsible for self and team?					
Do students reflect on their work?					

8. A graph on the coordinate plane shows a straight line that makes an angle of 60^{o} with the x-axis and passes through the point (3,1).

A) Is it possible to find the slope of this line? YES OR NO
If yes, what is the slope? Show work and explain your reasoning.

If no, why is it not possible to find the slope?

B) On a scale from 0 – 4, with 4 representing "Yes" to the question and 0 representing "No".

	0	**1**	**2**	**3**	**4**
Would you consider the question to encourage procedures with connections or doing mathematics?					
Does the question encourage students to apply and/or develop higher order thinking skills and knowledge?					
Does the question involve students applying problem solving skills and ask questions?					
Are there multiple entry points and multiple pathways to obtain the solution?					
Is there a driving question?					
Are students expected to apply prior knowledge?					
Are students expected to present their solutions and justifying their responses?					
Is the question relevant, interesting, or applicable to students?					
If used in the classroom would students be expected to work in groups, being responsible for self and team?					
Do students reflect on their work?					

NAME _____

II. Participant Background Information

1. Number of years teaching mathematics _____

2. Number of years teaching _____

DEGREES HELD	MAJOR	MINOR
Bachelor's		
Master's		
Higher than Master's level		

3. Check the courses you have taught and complete the table for those that are applicable.

COURSES	NUMBER OF YRS TAUGHT COURSE	LAST YEAR TAUGHT
Algebra I		
Algebra II		
Geometry		
Pre-Calculus / Math Analysis		
Calculus		
Statistics		

4. What additional mathematics content courses have you taken beside those for your degree?

5. Additional Comments:

APPENDIX B: PROFESSIONAL DEVELOPMENT AGENDA, RESULTING ARTIFACTS, AND RESEARCH QUESTION

ACTIVITY	ARTIFACT	RQ#
INITIAL MEETING		
Introduce the study	-----	
Obtain lesson/unit involving slope	Lesson/unit on slope	1, 3
Interview	Interview (initial planning, high cognitive demand characteristics)	1, 2, 3
Mathematical Survey	Mathematical Survey- Assess content knowledge and rich task characteristics	1, 3
PHASE I – SUMMER INSTITUTE	Each day's activities video and audio taped to record expressions and discussions	
DAY 1		
Example of Journaling	-----	
Journal Prompt – task high in cognitive demand, characteristics defined, goal for the task	Journal - Individuals recognize rich task characteristics	1, 2
Journal Prompt – goal for professional development, goal for mathematics teaching and student learning	Journal – individual goals	2
Cognitive Demand Activity – Understand and implement levels of cognitive demand)	Individual scoring of a set of items (high or low). Revise problem as a group to be high in cognitive demand. Begin recognizing	1, 3

	group interactions to revise the given question.	
Norms	List of Norms – Acknowledge what is important to the team for successful interactions.	3
Department Goal	Group goal(s) for teaching and student learning	2
Curriculum Big Ideas Activity	Chart paper and cards revealing units of big ideas determined by the group	1, 2, 3
Rich Task – The Hexagonal Train	Interactions between members working on task. Chart paper/presentation of task response	1, 3
Reading : Why is teaching with problem solving important to student learning? (NCTM, 2010)		
Day 2		
Reading Discussion	-----	
Discussion about the task day 1: Characteristics, NCTM Process Standards, Learning Environment	Verbal discussion – recognize characteristics	1, 2, 3, 4
Mid-day Journal – What were your experiences with the rich task (emotions, challenges, and successes)? How will these experiences influence your planning and instruction?	Journal response	2, 3, 4
Research of topic (slope) – Relate to Mathematical Survey results	Verbal discussion	2, 3
Rich Task – Enlarging Forts	Interactions between members working on task. Chart paper/presentation of task response	1, 2, 3
Discussion about the task: Characteristics, NCTM Process Standards, Learning Environment	Verbal Discussions – recognize characteristics in the rich task	1, 2, 3, 4
Unit Planning	Lesson on slope and	1, 2, 3, 4

	lesson for presentation	
Lesson Study – what is it? Introduction of Unit Plan	Verbal discussions	2, 3
Frayer Model	Chart Paper	1, 2, 3, 4
Determine topic for lesson during the first month of the next school year (SY)	Group interaction about how the group determined the topic.	2, 3
Readings: Connecting procedural and conceptual knowledge of functions (Davis, 2005); Helping students connect functions and their representations (Moore-Russo & Golzy, 2005).		
Journal Prompt – Do your individual goals correlate with the group goals? Why or why not? Do your individual goals correlate with rich tasks (activity and implementation)?	Summary in writing individually. Group response to same question on chart paper	1, 2
Day 3		
Reading Discussion	-----	
Rich Task – The Towers	Interactions between members working on task. Chart paper/presentation of task response	1, 2, 3
Discussion about the task: Characteristics, NCTM Process Standards, Learning Environment	Verbal Discussions – recognize characteristics in the rich task	1, 2, 3, 4
Compare/contrast all three rich tasks	Discussions, Chart paper of comparisons	1, 2, 3
Mid-day Journal – What were your experiences with the rich task (emotions, challenges, and successes)? How will these experiences influence your planning and instruction?	Journal response	3, 4
Journal Prompt – How do you envision using rich tasks in your classroom?	Individually record in journals. Discussion as a team and record on chart paper.	2, 3, 4
Unit Planning - Work on SY – Driving question, prior knowledge, objectives, and goals	Group interactions and discussions. Chart paper with items listed.	2, 3, 4

	Resources used or need requested by team.	
Exit Survey	Survey turned in per participant.	1, 2, 3, 4
Day 4 (SY Work Week)		
Review Summer Institute ideas		
Continue working on unit and rich task Discuss and share units with each other.	Group interactions and rich task document	1, 2, 3, 4
Exit Questions	Exit questions – individual to determine personal challenges and successes	2, 3, 4
PHASE II – SCHOOL YEAR LESSON STUDY		
First Presentation		
Collect lesson plan and supporting materials	Lesson plans	1, 2, 3, 4
Briefing and Debriefing of the Lesson Study	Video tape to record discussions and actions, student artifacts	1, 2, 3, 4
Observation of the rich task lesson	Field notes, unit/lesson plans	2, 3
Modify lesson	discussion, video taped	1, 2, 3, 4
Second Presentation		
Collect lesson plan and supporting materials	Lesson plans	1, 2, 3, 4
Briefing and Debriefing of the Lesson Study	Video tape to record discussions and actions student artifacts	1, 2, 3, 4
Observation of the rich task lesson	Field notes, modified unit/lesson plans	2, 3
Modify lesson	discussion, video taped	1, 2, 3, 4
Exit Questions	Exit questions – individual to determine personal challenges and successes	1, 2, 3, 4

APPENDIX C: UNIT PLAN TEMPLATE COURSE

Unit Overview		
Title of Unit:		**Number of Class Hours:**
Context: Summary of the issue, challenge, investigation, or problem.		
Other Subject Areas/Disciplines Addressed:		
Driving Question:		
Mathematics Content Addressed:		
Unit Goals:		
Unit Objectives:		
Prior Knowledge:		

College and Career Readiness/21st Century Skills											
T: Taught **E: Expectation** **A: Assessed**	Collaboration	T	E	A	Research	T	E	A	Critical Thinking/Decision Making	T	E	A
	Communication (Oral and/or Written)	T	E	A	Technology	T	E	A	Other: (Describe)	T	E	A
Launch/Engagement:											
Evaluation:	**Formative Assessments**	Interview							Practice Presentations		
		Mathematicians Journal							Notes		
		Preliminary Plans/Outlines/Prototypes							Checklists		
		Rough Drafts							Concept maps		
		Field Tests							Other:		
		Quizzes									
	Summative Assessment	Written Products, with a rubric without a rubric							Peer Evaluation, with a rubric		
		Oral Presentation with a rubric							Self Evaluation, with a rubric		
		Written Test							Other		
Resources Needed:	**Materials:**										
	Equipment/Technology:										
	On-site people, facilities:										
References:											

CALENDAR

Day 1	Day 2	Day 3	Day 4	Day 5
Day 6	**Day 7**	**Day 8**	**Day 9**	**Day 10**
Day 11	**Day 12**	**Day 13**	**Day 14**	**Day 15**

Algebra I Instructional Plan		
Unit Title: **Driving Question:**		
ENGAGE (LAUNCH) **Time:**		Mathematician Journal Prompts: Key Questions:
EXPLORE Teacher provides guidance through direct instruction or explorations to prepare students with the knowledge and skills to engage in the task. **Time:**		Mathematician Journal Prompts: Key Questions:

EXPLAIN Teacher introduces the rich task(s) of the unit. **Time:**	Rich Task #1:	Mathematician Journal Prompts: Key Questions:
ELABORATE Expectations of the Explore and Explain sections - Lesson or Rich task(s). **Time:**		Mathematician Journal Prompts: Key Questions:
EVALUATE Formative/ Summative Assessments for the unit, lessons, and rich task(s). **4 hours**		Mathematician Journal Prompts: Key Questions:

APPENDIX D: SUMMER INSTITUTE EXIT SURVEY

CREATION PROCESS	RESEARCH QUESTION
1. Has being a part of the professional development changed your views on	
a. teacher practices? Explain	3
b. lesson development? Explain	3
2. When developing the rich tasks, what was helpful? (Process, Activity, Discussion?)	3
3. Do you believe the time involved will be worth the results? For teachers? For students?	2, 3
4. What would you change about the professional development upon reflection?	
5. Do you believe you can develop additional rich tasks and/or units of study that include rich tasks after the summer institute? Elaborate.	2, 3
6. Team Members: Discuss their helpfulness and support.	3
7. Have your goals for mathematics teaching and learning changed during the summer institute? Elaborate on what has or has not changed.	2
8. How have your experiences evolved as you were exposed to the professional development activities (emotions, challenges, and successes)?	3, 4
9. How will the experiences during the summer institute influence your planning and instruction?	2, 3, 4

10. How have your beliefs about teaching and student learning evolved during the summer institute?	3
11. Name characteristics of rich tasks.	1
12. Do you believe some of the characteristics of rich task play a more important role in teaching and student learning?	1, 2, 3
13. Do you believe some of the characteristics of rich task play a less important role in teaching and student learning?	1, 2, 3

APPENDIX E: SCHOOL YEAR EXIT SURVEY

LESSON STUDY DEBRIEFING QUESTIONNAIRE

CREATION PROCESS AND IMPLEMENTATION	RESEARCH QUESTION
MEMBERS OF THE TEAM – TO BE COMPLETED AS A TEAM CED / PET	
1. Describe the unit – what lessons/activities have already been done prior to the lesson presentation? (objectives, goals, expectations)	1, 2, 3
2. Describe the lessons/activities that will come after the lesson presentation.	1, 2, 3
3. Were there certain rich tasks characteristics that you made a conscious effort to include?	1, 3
4. What are the expectations of the students during the lesson study presentation? (Actions)	1, 2, 3
5. What research was conducted on your topic?	3
6. What are the expectations of the teachers during the lesson study presentation? (Actions)	1, 2, 3
7. Have you and your team referred to the department goal this year? Could you describe when/where?	2, 3
8. What were some challenges or constraints you encountered?	4

Name _____

CREATION PROCESS AND IMPLEMENTATION	RESEARCH QUESTION
To be completed by the team member individually.	
9. Describe your role in developing the unit of study	2, 3
10. Describe your role in developing the rich task lesson/presentation.	2, 3
11. What are your expectations of yourself in planning for the lesson study?	3, 4
12. What are your expectations of the presenter of the lesson? (Actions)	3, 4
13. Have you collaborated on other lessons to the depth that was needed for the lesson study presentation? Describe the collaborative session (topic, what you discussed, expectations)	3
14. Describe how being involved in the professional development effected your practices in the classroom?	3, 4
15. Describe your beliefs about teaching mathematics.	
16. Describe your beliefs about students and their ability to learn mathematics.	
17. Describe a lesson that you have facilitated this school year that is high in cognitive demand. Were any rich task characteristics included? If so, which ones?	1, 2, 3
18. Will you continue collaborating with a team of algebra teachers and implementing rich tasks? Expound on why or why not?	3, 4
19. What would help you on your journey of developing lessons that are high in cognitive demand, meets NCTM Standards, state standards, and characteristics of rich tasks?	3, 4
20. Do you come together as a team to discuss how to better meet student needs according to their actions and reactions to a lesson? Please expound on your answer.	3, 4

21. Do you believe you are capable of developing a lesson involving a rich task? Why or why not?	1, 3, 4
22. Do you believe that by working in a team that together you are capable of developing a lesson involving rich tasks? Why or why not?	3, 4
23. Do rich tasks and student centered lessons enable students to become more engaged in the lessons and the learning of the concepts? Why or why not?	1, 2, 3
24. What are some of the challenges and/or constraints you might have experienced when facilitating rich lessons?	4

During Debriefing:

1. Discuss the actions of the students – those you observed.
2. What portion of the lesson did you think went as planned (according to the lesson plan or discussions you had as a team and during the briefing)?
3. What portion of the lesson did not go as planned (according to the lesson plan or discussions you had as a team and during the briefing)?
4. What are the actions and reactions of the teacher during the implementation of the task from the lens of the team?

APPENDIX F: UNIT PLAN RUBRIC

	4	3	2	1	0
COGNITIVE DEMAND:					
1. Does the task assist a student to **increase** content **knowledge**?	Students will apply knowledge and attain additional knowledge from other areas in the content areas or other disciplines.	Students will apply knowledge and attain additional knowledge related to the content being studied.	Students will apply knowledge already attained to the content being studied.	Students will apply knowledge already attained, mocking previous procedures.	Does Not Exist
2a. Are students able to make **connections** within the course, between content areas, and/or with the real world?	Students make connections within the course, between content areas, and/or with the real world?	Students make connections between content within the course and are told what the connections are to the real-world.	Connections are made with assistance from the teacher that is between content only.	No connections are made with content and/or the real-world.	Does Not Exist/ No Evidence
2b. Are the students able to make **connections** between content, school, and community expressed using multiple mathematical	Multiple mathematical representations are used to make connections between content,	Connections between content, school, and community are expressed using mathematical	Multiple mathematical representations are used to make connections between content	Multiple mathematical representations are not used to make any type of connections.	Does Not Exist/No Evidence

representations?	school, and the community.	representations.	only.		
3. Does the task encourage students to apply **problem solving skills** and/or develop higher order thinking skills?	The task encourages students to apply problem solving skills and develop higher order thinking skills.	The task requires students to apply problem solving skills and higher order thinking skills already practiced.	The task requires students to apply lower level thinking skills already learned	The task requires students to use skills already learned without applying any additional skills.	Does Not Exist/No Evidence
4. Does the task encourage students to **pose questions** as well as solve problems?	The task encourages students to pose questions as well as solve problems.	The task encourages students to solve problems by making decisions.	The task encourages student to solve the problem but not to explore deeper.	The task is procedural, step-by-step directions given.	Does Not Exist/No Evidence
EXPLORATION – THE TASK					
5. Does the task enable a student to enter into solving at various levels (**multiple entry points**)?	The task enables a student to enter into solving the problem at many different levels.	The task is based on students apply only one method.	The task gives students a choice of methods but also provides specific directions.	The task is procedural with step by step directions.	Does Not Exist/No Evidence
6. Are there **multiple pathways** to obtain a solution as well as multiple solutions to the task due to decisions students make?	There are multiple pathways to obtain the solution. Student's decision making leads to multiple solutions to the task.	There are multiple pathways to obtain the solution that leads to multiple solutions.	There are multiple pathways to the solution but only one solution is possible.	There is only one pathway to the solution and only one correct response.	Does Not Exist/No Evidence
7. Does the **driving question** given in the plan **connect** the mathematics to the	The driving question connects the mathematics to		There does not exist a driving question but there		Does Not Exist/No Evidence

situation in the problem?	the situation in the problem.		is a connection between the mathematics so that one can determine the objective.		
8. Does the task require students to explore using **prior knowledge applied** to new content?	The task requires students to explore using prior knowledge that is applied to new content.		The task requires students to explore using only the information given and the mathematics expressed.		Does Not Exist/No Evidence
9. Is the task based on a problem or question that is meaningful to students (**relevant**)?	The task is an applied problem that is applicable and meaningful to students.	The task is a word problem that requires students to make decisions but is not meaningful to the student.	The task is a word problem requiring known procedures and is not meaningful to the student.	The task is rote drill and procedural with no meaning.	Does Not Exist/No Evidence
FINAL PRODUCT					
10. Are students expected to **communicate** their knowledge using reasoning skills, **justifying** their responses with representations and written or verbal	The task product expects students to communicate their knowledge using reasoning skills, justifying their responses with	The task product expects students to communicate their knowledge using reasoning skills using a specific representation to	The task product expects students to communicate their knowledge using reasoning skills, with guided steps to aid in their	The task product expects students to communicate their knowledge with an answer that is not justified.	Does Not Exist/No Evidence

statements.	representations and written or verbal statements.	justify the statement.	justification.		
11. Are students expected to **present** their solutions, justifying their responses?	Students are expected to present their solutions and justify their responses.	Students share their solutions informally with or without justifications.	Students provide their solution with justifications.	Students provide their solution without justification.	Does Not Exist/No Evidence
12. Are students expected to work in **groups**, being responsible for self and team?	Students work in groups with self and peer evaluations completed at the conclusion of the task.	Students work in groups but are assigned responsibilities while still completing self and peer evaluations at the conclusion of the task.	Students work in groups but are assigned responsibilities.	Students may work individually to complete the task.	Does Not Exist/No Evidence
13. Is the **data** for the exploration **given** or students told where to find the data?	Data is given to the student or they are told where they will find the data.				Does Not Exist/No Evidence
14. Does the task provide a **rubric** of expectations to **guide** student learning?	Students receive a rubric of expectations that they may or may not have helped create to guide their learning. Explained thoroughly.	Students receive a rubric as part of the task handout. No explanation given or input obtained from students.	Students will be told expectations as they progress through the task.	Students are to produce the answer without expectations or rubric given for guidance.	Does Not Exist/No Evidence

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APPENDIX G: JOURNAL PROMPTS

DAY 1:

A.

1. What are your goals for this professional development?
2. What are your goals for teaching and learning mathematics?
3. Name some characteristics of a lesson that is academically challenging to students.

B.

1. How do your individual goals correlate with the "team" goal?

C.

1. What are your experiences with the rich task:
   <u>As a participant--</u>
   - Yourself: Emotions, challenges, successes,;
   - Future: students in your classroom: challenges, successes, how to implement, student role

   <u>As a member of the Team</u>
   - when completing the task as a "student"
   - as a teacher working as part of the "team" (your practices and beliefs)
   - thoughts on implementing task with students

2. Do your individual goals correlate with the rich tasks (the activity and the implementation)? Why or Why not?

Day 2:

A.

1. Has the rich task experience influenced you in the following ways:
   - Planning
   - Practices in the classroom
   - Beliefs on teaching and student learning?
2. Challenges or constraints you have experienced or anticipate when using rich tasks?

B.

1. What are your experiences with the rich task:
   <u>As a participant--</u>
   - Yourself: Emotions, challenges, successes,
   - Future: students in your classroom: challenges, successes, how to implement, student role

   <u>As a member of the Team</u>
   - when completing the task as a "student"
   - as a teacher working as part of the "team" (your practices and beliefs)
   - thoughts on implementing task with students

   Peer and Self Evaluation – based on Hexagonal Team Task

C.
1. How do you envision using rich tasks in your classroom?
2. How do you perceive using the "thinking process" for developing units of study?

Day 3:

A.
1. What would be your goal for students when planning any unit of study?
2. What are your thoughts about your role when planning for a unit of study? (Individual and team)
3. Describe your thoughts when implementing a rich task.

B.
1. How do the characteristics compare between Hexagonal Trains, Enlarging Forts, and The Towers?
2. Discuss your thoughts about our approach to the unit on Quadratics?
   - Determining if it was quadratic,
   - Deciding on the objectives and goals of the unit,
   - What rich tasks can we create from what we already have, and
   - What do we need in order to meet goals, objectives, and rich task creation?

C.

Peer and Self Evaluation – based on The Towers task.

APPENDIX H: INITIAL INTERVIEWS GUIDING QUESTIONS

Beliefs about student learning:

1. Growth mind-set or fixed-mind set?
2. Can all students learn algebra?
3. What would be helpful to enable all students to have an opportunity to learn algebra?
4. Define student engagement?

Slope lesson:

1. When introducing or presenting the lesson, what representations do you use?
2. What examples (activities, investigations, analogies) do you use?
3. What are your goals for the unit/lesson?
4. What are your objectives for the unit/lesson?
5. What role does the slope formula play in algebra?
6. How does your unit involving slope develop?
7. Did you do any research on the topic? If so, how and what did you find out?
8. What difficulties have you experienced with students learning about slope?

Tasks that are engaging academically, use prior knowledge, and help students to understand future concepts:

1. What are the characteristics of a lesson or activity where the students are engaged academically?
2. Did the task have multiple entry points?
3. Did the task have a possibility for students to apply multiple strategies to solve?
4. Did the students apply twenty first century skills?
5. Did the teachers have professional development sessions embedded in their instruction to educate them on the expectations of standards-based lessons?

Teacher collaborative practices:

1. Do the teachers have common planning?
2. What is your normal procedure for planning?
    a. What do you use as a guide?
3. Do teacher teams create common assessments?
4. Is there a concern that instruction between classrooms is equivalent?
5. Are lessons and expectations at the same level of rigor?
6. Are similar connections made between concepts regardless of the classroom a student is in?
7. Do teachers come together to discuss assessment results?
8. Do teacher confer with each other about students who are not as successful as they should be?
9. Do teachers believe that they are supported in their efforts of implementing standards-based lessons?

APPENDIX I: BIG IDEAS FOR ALGEBRA I

PARENT FUNCTIONS

- Evaluate
- Graphs and Tables
- Function Characteristics

LINEAR FUNCTIONS

- Repeat and apply Parent Functions Unit: Evaluate, Graphs &Tables, & Function Characteristics
- Modeling
- Simplify
- Slope
- Write Equations and Curve of Best Fit

SOLVE LINEAR EQUATIONS & INEQUALITIES

- Modeling
- Solve Linear Equations
- Solve Linear Inequalities

SOLVE SYSTEMS OF EQUATIONS & INEQUALITIES

- To Be Applied to Systems of Equations and Inequalities:
  Equations, Inequalities, Slope Concepts, Writing equation of a line
- System of Equations
- Systems of Inequalities

VARIATIONS

- Modeling
- Simplify, Solve, Evaluate
- Recognize, Graph, and Write Variations

POLYNOMIALS (Exponents, Radicals, Polynomial operations)

- Evaluate and Simplify
- Polynomial Operations
- Exponents & Radicals

QUADRATICS – FACTORING & SOLVING

- Apply polynomial operations
- Factoring
- Solve Quadratics

QUADRATIC FUNCTIONS

- Modeling
- Evaluate and Simplify
- Function Characteristics
- Curve of Best Fit

STATISTICS

- Descriptive Statistics
- Mean Absolute Deviation/Standard Deviation

APPENDIX J: CET UNIT FLOW

Cognizant and Experienced Team (CET) Systems of Equations and Inequalities		
	Warm Up (Do Now)	Lesson Ideas
Day 1	DN: Plotting points, Graph, Table, Write Linear Equations	Guided Direct Instruction (30): In four groups, A, B, C, and D. Do the same as the DN with group A and B working with the same scenario or scenarios that can become a system. Equations written in slope-intercept form and then manipulate to standard form. Repeat for groups C and D. Each set of students will draw a graph and overlay the second graph to discuss what the point of intersection means. Direct Instruction (30): Use graph, table, points, and equation to do the following: ■ Given slope intercept form, use substitution to solve. ■ Given standard form, use elimination to solve. If time permits give them a system of equations graphically and have students determine the equations and the system, acknowledging the system of equations. Possible practice – abstract exercises.
Day 2	DN: Solve by graphing 3-5 problems. Last problem higher in cognitive demand to lead into lesson.	RTP #1 (30): Group activity that is similar to the lesson study rich task. Table of values, graph, write equations – for two situations within the one scenario. Come together with product to determine the solution for a "system". May need to stop and do a lot more checks and/or guidance. Direct Instruction: Revisit substitution with practice.
Day 3	DN: Solve by substitution 3-5 problems.	RTP #2: Very similar to #1. This time more on their own and could possibly be a performance assessment. Come together with product to determine the solution for a "system". Direct Instruction: Revisit elimination with practice.
Day 4	DN: Solve by elimination 3-5	RTP #3: Similar to the first two RTP's but this time each team will receive both scenarios to complete using

	problems	all methods. No Direct Instruction. Practice solving with all three methods. Possibly a quiz.
Day 5	DN: Rich Task Lesson Study Day. Group activity. Thinking of starting off with a map of some sort and use for the engagement.	RT: Continue with the task to follow through to answer questions using various solving methods. Will have questions to answer and some evaluation type questions as well. Closure of some sort.
Day 6	DN: Go back to elementary school concepts with basic inequalities involving numbers.	Guided Direct Instruction: Phrases that illustrate inequalities and possibly from a scenario, RTP #1. Symbolization for graphing solutions on number line and the coordinate plane – using scenario will mean more to students. Practice solving inequalities and the meaning of the solution.
Day 7	DN: Inequalities continued	RTP #2 Inequalities: Possibly use a rich task already completed but changed to an inequality. Students illustrate solutions graphically and algebraically. Practice solving inequalities and the meaning of the solution. Determine which method to use given a situation or abstract exercise. Possible assessment.

APPENDIX K: PET UNIT FLOW

Passionate and Energetic Team (PET) Solving Equations and Inequalities	
Day 1	Start out doing a mixture of some word problems and algebraic expressions using algebra tiles, hands on equations, or a balance. Something that is tactile and visual for the students. Verbal → pictures. Evaluate and solve using the pictorial representations.
Day 2	Revisit Day 1 and get into the algebraic representation of it. Translate from the algebra tiles to an expression or equation. Working with the pictures from the algebra tiles and solving using the tiles while at the same time transforming their actions with the tiles to algebraic manipulations. Evaluate using the pictorial representations and algebra.
Day 3	Rich task previously completed. Students have already determined the pattern using the Rule of 5 and wrote the rule --use for evaluating and solving, --review of characteristics, --add meaning to the result of the evaluation and solving.
Day 4	Rich task for lesson study. Gave example of the rope tying activity.
Day 5	Rich Task for lesson study. Can you get two pieces of rope to be the same length – not taking as far as a system but discussing the meaning of equality and solving.

APPENDIX L: PEER AND SELF-EVALUATION

Peer Evaluation

The following is a list of statements to be answered by you about your partners. Think carefully about assigning values for each of the following statements. Put an "x" in the box that applies.

My partner _____	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
Contributed positively to discussions					
Did an equal portion of the workload					
Helped to keep me focused on the task					
Was respectful of my ideas and opinions					
Is someone I would work with again					

My partner _____	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
Contributed positively to discussions					
Did an equal portion of the workload					
Helped to keep me focused on the task					
Was respectful of my ideas and opinions					
Is someone I would work with again					

Elaborate on your selections above.

Self Evaluation

The following is a list of statements to be answered by you about yourself. Think carefully about assigning values for each of the following statements. Put an "x" in the box that applies.

I, _____ (insert name here)	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
Contributed positively to discussions					
Did an equal portion of the workload					
Helped my partners focus on the task					
Was respectful of my partner's ideas and opinions					

Elaborate on your selections above.

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Curriculum Vitae

Pamela R. Hudson Bailey graduated from George Washington High School, Charleston, West Virginia, in 1976 and received her Bachelor of Science in Mathematics Education from West Virginia State College in 1994. She was employed as a teacher in Westmoreland County for nine years and in Stafford County for three years prior to becoming the Secondary Mathematics Coordinator for Spotsylvania County Schools in 2006. Pamela is currently an instructor at George Mason University. She received her Master of Arts in Diverse Learner from the University of Phoenix in 2000.