THE EFFECTS OF EXPLICIT INSTRUCTION WITH MANIPULATIVES ON THE FRACTION SKILLS OF STUDENTS WITH AUTISM

by

Jugnu Agrawal
A Dissertation
Submitted to the
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of
Doctor of Philosophy
Education

Committee:

_____________________________________ Chair

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_____________________________________ Program Director

_____________________________________ Dean, College of Education and Human Development

Date: ________________________________ Spring Semester 2013
George Mason University
Fairfax, VA

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Jugnu Agrawal
Master of Education
George Mason University, 2007
Master of Science
Lady Irwin College, 1993

Director: Pamela Hudson Baker, Associate Professor
College of Education and Human Development

Spring Semester 2013
George Mason University
Fairfax, VA

DEDICATION

This is dedicated to my beautiful daughter Radhika and my loving husband Pradyumna who supported me in this rewarding journey.

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ABSTRACT

THE EFFECTS OF EXPLICIT INSTRUCTION WITH MANIPULATIVES ON THE FRACTION SKILLS OF STUDENTS WITH AUTISM

Jugnu Agrawal, Ph.D.

George Mason University, 2013

Dissertation Director: Dr. Pamela Hudson Baker

Students with autism can have discrepancy between their math performance and potential. A multiple-baseline across participants design was used to investigate the effects of explicit instruction with manipulatives on the conceptual and procedural knowledge of fractions of elementary school students with autism. This single-subject study included six 8- to 12-year-old students with mild to moderate autism who demonstrated math difficulties. There were five males and one female participant. Participants attended different schools within the same public school district in a Mid-Atlantic state.

This study investigated two different math concepts: addition and subtraction of like and unlike fractions. Two sets of data were analyzed to determine effectiveness of the independent variable (intervention). Both data sets consisted of baseline, intervention, generalization, and maintenance phases for the six participants. Before the intervention with explicit instruction with manipulatives, baseline data were collected for all

participants without manipulatives. Dependent measures included (a) conceptual knowledge of addition and subtraction of fractions (like and unlike), (b) procedural knowledge of addition and subtraction of fractions (like and unlike), (c) time taken to solve the problems (like and unlike), and (d) participant attitudes toward the intervention.

During intervention, the participants were taught addition and subtraction of fractions using explicit instruction with manipulatives. The average intervention time across participants was 278 minutes with each intervention session averaging 35 minutes. The number of sessions ranged from 5-11 due to staggered intervention. The conceptual and procedural knowledge of participants was measured during all phases of the study. Generalization and maintenance data were collected for all participants. Conceptual and procedural knowledge data of fractions was visually analyzed for level, trend, variability, overlap, immediacy, and consistency of data points. Overall findings of the study revealed that after the intervention: (a) five out of six participants improved in their conceptual knowledge of addition and subtraction of like fractions, (b) five out of six participants improved in their conceptual knowledge of addition and subtraction of unlike fractions, (c) five out of six participants could solve more addition and subtraction problems of like fractions accurately, (d) four out of six participants improved in their conceptual knowledge of addition and subtraction of unlike fractions, (e) all six participants explained their thinking for solving problems with like and unlike fractions during the generalization phase, and (f) all six participants accurately solved the problems with like fractions and unlike fractions during the generalization phase. Additionally, all participants could maintain their abilities to solve fraction problems 2 weeks (unlike

fractions) and 4 weeks (like fractions) after receiving instruction. There was no change in the time taken to solve problems with like fractions but the time taken for solving problems with unlike fractions increased. Participants reported positive attitudes toward the intervention and made real-life connections with fractions. Educational implications of this intervention and possibilities for future research are discussed.

1. INTRODUCTION

The provisions of the Education for All Handicapped Children Act in 1975 led to educational initiatives to ensure access to a free and appropriate public education (FAPE) for all students. The subsequent reauthorizations in 1990 and 1997 as the Individuals With Disabilities Education Act (IDEA), and Individuals With Disabilities Education Improvement Act (IDEIA) of 2004 increased federal funding (Yell, Drasgow, & Lowrey, 2005) to help states and local communities provide educational opportunities for approximately six million students with varying degrees of disability (National Dissemination Center for Children with Disabilities, 2010).

The requirements of No Child Left Behind (2001) demand that all schools show adequate yearly progress (AYP) and students with disabilities should be included in statewide assessments of reading and math. Students with disabilities can take alternate assessments with accommodations based on their individual needs and Individualized Education Plan (IEP) team decision. In addition to participating in statewide assessments, IDEIA (2004) requires that all students with disabilities should have access to and make progress in the general education curriculum (Rockwell, Griffin, & Jones, 2011). The National Research Council (2001) recommends that goals for educational services for students with Autism Spectrum Disorders (ASD) should be the same as those for typically developing children.

The recommendations of the National Council of Teachers of Mathematics (NCTM, 2000) specify that students should have an opportunity to develop understanding of mathematical concepts and procedures by engaging in meaningful math instruction. Still, the findings of a review done on math interventions for low-achieving students indicate that instruction for students with disabilities focuses on teaching computation skills and procedures rather than conceptual knowledge (Bottge, 2001). Additionally, the achievement gap for math between typically developing students and students with disabilities continues to increase because students with disabilities progress at a much slower rate as compared to their typically developing peers (Bottge, 2001; Cawley & Miller, 1989).

The National Assessment of Educational Progress (NAEP) report shows that math assessment scores of all students at the fourth and eighth grade level have improved when compared with previous years. However, students with disabilities continue to lag behind in their math performance. At fourth grade level, 45% of students with disabilities continue to perform below basic level as compared with 15% of students without disabilities; at the eighth grade level, 65% of the students with disabilities are performing at below basic level compared to 23% students without disabilities (U.S. Department of Education, National Center for Education Statistics [U.S.DOE-NCES], 2011).

A substantial need for research in investigating the effectiveness of academic interventions exists, especially math interventions for the students with autism. Among the studies done on interventions for students with autism, reading interventions are researched but math interventions have not received priority (Minshew, Sweeny, Bauman, & Webb, 2005).

Statement of the Problem

The Centers for Disease Control and Prevention (CDC, 2012) have estimated that an average of 1 in 88 children in United States has autism spectrum disorder (ASD). It affects all racial, ethnic, and socioeconomic groups but its incidence is 4 to 5 times more in boys than in girls. It is estimated that about 730,000 individuals between the ages of 0-21 have ASD (CDC, 2012). Autism is a neurobiological developmental disorder that is typically diagnosed by the age of 3. It affects three general areas: (a) social interactions, (b) communication skills, and (c) play skills and behavior (Prelock, 2006). Autism manifests itself differently in different individuals. No two individuals with autism are alike. Since autism involves a variety of characteristics and symptoms along a continuum, the term autism spectrum disorder (ASD) is more commonly used to refer to individuals with autism (Baker, Murray, Murray-Slutsky, & Paris, 2010).

Many students with autism have average mathematical skills (Chiang & Lin, 2007; Whitby & Mancil, 2009; Whitby, Travers, & Harnik, 2009) but approximately 23% of the students with high-functioning autism (HFA) have a learning disability in math (Mayes & Calhoun, 2006). In early years, the mathematical performance of students with autism is similar to that of typically developing children. In later years, they have good computational skills but difficulty with problem-solving skills that affects applied mathematical ability (Whitby & Mancil, 2009).

The researchers of the Special Education Elementary Longitudinal Study (U.S. Department of Education, Office of Special Education Programs [U.S.DOE-OSEP], 2008) report that students with disabilities receive language arts and mathematics instruction mainly in the general education settings; however, students with autism generally receive language arts and mathematics instruction in self-contained special education settings. Additionally, students with disabilities have goals related to academic performance, but students with autism have goals that focus primarily on social skills, communication, and behavior (U.S.DOE-OSEP, 2008).

Research done to study interventions for students with ASD has mainly been in the areas of language and communication (Buggey, 2005; Wert & Neisworth, 2003), social and adaptive behaviors (Buggey, 2007; Graetz, Mastropieri, & Scruggs, 2006; Nikopoulous & Keenan, 2004), and play skills (D'Ateno, Mangiapanello, & Taylor, 2003; Hine & Wolery, 2006). Academic interventions are rarely studied for students with ASD. Some studies have focused on reading interventions for students with ASD (Collins, Evans, Creech-Galloway, Karl, & Miller, 2007; Stringfield, Luscre, & Gast, 2011; Whalon & Hart, 2011; Whalon, Otaiba & Delano, 2009; Yaw et al., 2011). However, limited studies have been conducted on math interventions (Banda & Kubina, 2010; Banda, McAfee, Lee, & Kubina, 2007; Cihak & Foust, 2008; Eichel, 2007).

Two of the studies done with middle school students with autism spectrum disorders found no relationship between task-mastery and preference, although the use of high-preference (high-p) mathematics tasks increased task initiation for low-preference tasks (Banda & Kubina, 2010; Banda et al., 2007). The high-p strategy involved the presentation of two or three preferred academic tasks before the presentation of a nonpreferred academic task. Another study done with middle school students with ASD showed that students improved in their task completion skills when they were given reinforcement and choices through video recordings of preferred items and activities rather than tangible reinforcers (Mechling, Gast, & Cronin, 2006).

Two other studies found that the touch-point strategy is more effective than the number-line strategy to teach single-digit addition facts to three elementary school students with autism (Cihak & Foust, 2008; Eichel, 2007). In number-line instruction, the students used a number line to solve the problem, however, for the touch-point instruction the students were taught the dot position of the numbers one to nine. Eichel (2007) found that the performance of a 13-year-old student with autism improved with the use of the TouchMath strategy.

A paucity of research on evidence-based strategies to teach students with autism exists, especially in mathematics. Williams, Goldstein, Kojkowski, and Minshew (2008) report that 25% of the students with high-functioning autism exhibit math disability. The math problems of these students are similar to the ones demonstrated by students with Nonverbal Learning Disability (NLD). These problems include difficulties with abstract concepts (Donaldson & Zager, 2010), memory skills, organization, and language comprehension in word problems (Minshew, Goldstein, Taylor & Siegel, 1994). Based on these characteristics, Donaldson and Zager (2010) recommend direct instruction and concrete-representational-abstract (CRA) as effective strategies for teaching math skills to students with ASD.

A number of empirical studies have validated the use of an explicit instruction framework (Flores & Ganz, 2009; Fuchs et al., 2008; Mulcahy & Krezmien, 2009) and manipulatives to teach different math concepts to students with LD (Manches & O'Malley, 2012; McNeil & Uttal, 2009; Moch, 2001; Swan & Marshall, 2010). However, the use of these strategies has not been substantiated for students with ASD.

Significance of the Study

The outcomes of this study will be important in adding to the knowledge base critically needed by the districts that serve the teachers who teach students with autism. The alarming increase in the number of students with autism and the demand to educate these students in general education classes substantiates the need to investigate effective strategies to teach academic skills to students with autism.

In addition to being an evidence-based strategy, explicit instruction has several advantages that can be beneficial for students with autism. In explicit instruction framework, the new information is presented in sequential and conceptual format with constant progress monitoring, which aids with maintenance and generalization of the new concepts. The lesson format is predictable and familiar so it is easier for students with autism who have difficulty with transitions and changes (McCoy, 2011).

Concrete-representational-abstract (CRA) instructional approach is an evidence-based intervention that can enhance the math performance of students with learning disabilities. It involves three levels of learning: (a) concrete or hands-on instruction using manipulatives, (b) representation through pictures, and (c) abstract with numbers (Witzel, Riccomini, & Schneider, 2008). CRA has been used effectively to teach subtraction with regrouping (Flores, 2010; Mancl, Miller, & Kennedy, 2012), fractions (Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Jordan, Miller, & Mercer, 1999), geometry (Cass, Cates, Smith, & Jackson, 2003), algebra (Maccini & Ruhl, 2000; Witzel, 2005), and other math concepts to students with math difficulties (U.S. DOE-OSEP, 2000).

The first component of CRA instruction is the use of hands-on activities with manipulatives. The use of well-planned instruction using physical manipulatives allows students to become active participants in knowledge construction (Stein & Bovalino, 2001). When students use manipulatives to explore concepts, they are more engaged and motivated. They can participate in mathematical discourse, share their thinking and reflect on their learning. This leads to increased achievement and deeper understanding of the concepts studied (NCTM, 2008). Furthermore, manipulatives are effective across grade levels and skill levels (Gersten, Jordan, & Flojo, 2005). Although explicit instruction with manipulatives has been found to be effective for teaching math skills to students with disabilities, little is known about its effects for students with autism.

Students with autism have difficulty with language skills, especially with comprehension (American Psychiatric Association, 2000). They also exhibit abnormal development of the central executive function of the brain. They have problems with retrieval, working memory, controlling, planning, sequencing and switching activities (Firth, 2003; Hughes, Russell, & Robbins, 1994; McCoy, 2011). Language impairments and executive functioning issues interfere with the mathematical performance of students with autism because they have difficulty solving word problems (Donlan, 2007; Zentall, 2007).

Fractions are one of the most challenging mathematical concepts for students with and without disabilities in upper elementary and middle school (Bezuk & Cramer, 1989; McLeod & Armstrong, 1982). Students have difficulty with fractions because they fail to connect form and understanding. Form is the syntax (for e.g., symbols, numerals and algorithms) while understanding is the ability to relate mathematical ideas to real-world situations (Hiebert, 1985). The concept of fractions is especially hard for students with autism who have language, retrieval, and memory issues.

Despite the known mathematical difficulties of students with autism, little attention is given to developing and implementing effective mathematics interventions for elementary school students with autism. Pennington (2010) found no studies on math in a review on academic interventions for students with autism with computer-assisted instruction (CAI). In another review on video interventions in school-based setting for students with disabilities, researchers found only two studies on math out of 18 studies reported (Hitchcock, Dowrick, & Prater, 2003). Furthermore, previous research on math interventions with students with autism focused primarily on basic skills and computation (Cihak & Foust, 2008; Rockwell et al., 2011). These earlier investigations did not examine student performance on math skills and concepts required in upper elementary school settings, which are essential in meeting national and state standards as required by NCLB (2001) and IDEIA (2004). Moreover, there are currently no published studies on effective interventions for teaching fraction skills to students with mild to moderate autism.

Purpose of the Study

The overarching goal of this study was to document effective math instruction for learners with mild to moderate autism. The current study was specifically designed to address gaps in research literature by using explicit instruction with manipulatives to teach fraction skills to elementary school students with mild to moderate autism. Previous research on evidence-based interventions for students with autism is lacking, so the current study was planned based on research done with students with other disabilities (Butler et al., 2003; Gersten et al., 2009; Witzel, Mercer, & Miller, 2003). In the current study, the researcher investigated the effectiveness of using explicit instruction with manipulatives on the procedural and conceptual knowledge of addition and subtraction of like and unlike fractions of students with mild to moderate autism. The instructional components that have been found to be effective in previous research, were included in the current study: (a) the use of explicit instruction framework (Gersten et al., 2009; Witzel et al., 2003), and (b) the use of manipulative materials such as fraction circles and double-colored counting chips (Butler et al., 2003; Cramer, Behr, Post, & Lesh, 2009).

Research Questions

The purpose of this study is to examine the effects of an explicit instruction with manipulatives on the fraction skills of elementary school students with mild to moderate autism.

1. Is there a functional relation between the explicit instruction with manipulatives and an increase in level and trend of conceptual knowledge of

addition and subtraction of like fractions for elementary school students with mild to moderate autism?

2. Is there a functional relation between the explicit instruction with manipulatives and an increase in level and trend of procedural knowledge of addition and subtraction of like fractions for elementary school students with mild to moderate autism?
3. Is there a functional relation between the explicit instruction with manipulatives and an increase in level and trend of conceptual knowledge of addition and subtraction of unlike fractions for elementary school students with mild to moderate autism?
4. Is there a functional relation between the explicit instruction with manipulatives and an increase in level and trend of procedural knowledge of addition and subtraction of unlike fractions for elementary school students with mild to moderate autism?
5. Do elementary school students with mild to moderate autism show a change in the time taken for solving addition and subtraction problems with like and unlike fractions (procedural knowledge probes) after intervention with explicit instruction with manipulatives?
6. Do elementary school students with mild to moderate autism generalize the procedural and conceptual knowledge of addition and subtraction of like and unlike fractions to abstract formats?

7. Do elementary school students with mild to moderate autism maintain their procedural and conceptual knowledge of fraction skills (addition and subtraction of like and unlike fractions) overtime following the conclusion of the intervention phase?
8. What are the attitudes and perceptions of participants (students with mild to moderate autism) related to the explicit instruction with manipulatives for learning fraction skills?

Definition of Terms

The meanings of the terms can vary based on the different ways of measuring or manipulating the same variables (McMillan, 2004). In order to ensure consistency within the study and increase its internal validity, the terms in the study were operationalized. The following definitions are applicable for this study:

Autism: For this study, students were identified as having autism if they met federal definition criteria of the disability under IDEA (2004). According to IDEA,

> Autism means a developmental disability significantly affecting verbal and nonverbal communication and social interaction, generally evident before age three that adversely affects a child's educational performance. Other characteristics often associated with autism are engagement in repetitive activities and stereotyped movements, resistance to environmental change or change in daily routines, and unusual responses to sensory experiences. (§300.8(c) (1) (i))

Based on this definition, “autism does not apply if a child's educational performance is adversely affected primarily because the child has an emotional disturbance” (§300.8(c) (1) (ii)) and “a child who manifests the characteristics of autism after age three could be identified as having autism if the criteria in paragraph (c)(1)(i) of this section are satisfied”. (§300.8(c) (1) (iii))

Conceptual knowledge: the student’s ability to show or explain his or her reasoning or thinking to solve the fraction problems (addition and subtraction) with the use of manipulatives (e.g., fraction circles, colored chips, sticks), pictures, or words (Goldman, Hasselbring, & The Cognition and Technology Group, 1997). For example, while solving an addition or subtraction fraction problem, the participant can explain all the steps of the problem accurately.

Procedural knowledge: the student’s ability to solve a mathematical task or a problem (Bottge, 2001; Carnine, 1997; Goldman et al., 1997). For example, the participant can solve a given addition or subtraction fraction problem with the correct numerator and the denominator.

Explicit instruction: highly structured teacher-directed instruction in which new skills are introduced in small steps based on the student’s progress and understanding (Hudson & Miller, 2006).

Manipulatives: concrete three-dimensional objects (e.g., fraction circles, colored chips) used to help students understand abstract mathematical concepts (Hudson & Miller, 2006). “Manipulatives offer students the opportunity to explore concepts visually and tactilely, often through hands-on experiences" (McNeil & Jarvin, 2007, p. 310).

Advanced organizer: the first part of a lesson in which the teacher reviews the previously learned information, states the current lesson's objectives, connects them with the previous information, and explains the rationale for the lesson objective (Hudson & Miller, 2006).

Teacher demonstration: the part of the lesson in which the teacher shows or models the skill to the student (using manipulatives, pictures or numbers) and explains the steps verbally.

Guided practice: the part of the lesson where the student solves two or more problems related to the math skill taught earlier in the lesson with teacher assistance. The teacher gives prompts and cues.

Independent practice: the part of the lesson where the student independently solves a few problems related to the targeted skill (without teacher support).

Problem solving: the student's ability to solve a real life problem (word problem) by applying the skill or mathematical knowledge acquired in the lesson (Hudson & Miller, 2006).

Validation of materials: the process of review of like and unlike fraction probes by expert teachers for accuracy, content, and formatting on the probe sheets.

Delimitations

The delimitations of a study are characteristics that limit the scope of the study based on the parameters chosen by the researcher or are inherent to the design.

Delimitations could be considered as a “box” that the researcher has chosen for the study. In this study, the choice of participants and research questions limit the scope of investigation to upper elementary school students and fraction skills. This study does not examine lower elementary or secondary school students with mild to moderate autism. This study was limited to the students’ accessing the general education curriculum rather than the adapted curriculum. Further, this study examines conceptual and procedural knowledge of addition and subtraction of like and unlike fractions of students with mild to moderate autism; however, it does not examine the other fraction (multiplication and division) and math concepts.

This study is also delimited by the characteristics of single-subject research designs. The findings of single-subject research are limited to the participants of the study and cannot be generalized to a larger population (Gast, 2010; McMillan, 2004). Single-subject research designs are primarily used in research with exceptional children where the focus of change is an individual or a small group. Students with autism struggle with their academic performance due to differences in their learning needs and behaviors and receive services on a continuum (ranging from fully included to primarily self-contained).

2. LITERATURE REVIEW

Autism is like a puzzle and no two individuals with autism are alike so a spectrum is used to describe children with autism (Spencer & Simpson, 2009). The term *autism* is derived from the Greek word *autos* meaning self, which refers to limited ability to communicate and lack of response to people. Autism is a neurobiological developmental disorder that typically starts before the age of three. It affects a child's social interaction, communication skills, play skills and behavior (Prelock, 2006). Based on the age of onset, there are two different types of autism: congenital and regressive autism. Congenital autism is generally apparent from the beginning; however, children with regressive autism develop normally in the beginning and then begin to show regression in language and other skills (McCoy, 2011).

Foundational Perspectives

Several definitions related to autism exist. The medical model and the school-based model are the two most commonly used models to describe autism. This section will cover the medical definition, prevalence, causes, educational eligibility criteria, characteristics of students with autism and the school-based model in detail.

The Autism Society of America (ASA) defines autism as "a complex developmental disability that typically appears during the first three years of life and affects a person's ability to communicate and interact with others. Autism is a spectrum disorder that affects individuals differently and to varying degrees" (Autism Society of America [ASA], 2008, para. 1).

The Autism and Developmental Disabilities Monitoring (ADDM) network sponsored by the Centers for Disease Control and Prevention (CDC) collects data across the United States on children who are at risk for autism. Gender, race, and ethnicity seem to be important factors in determining the prevalence of autism. The most commonly cited prevalence rate for autism is 1 of every 88 individuals (1 in 54 boys), which is based on the data collected from 14 different states in 2008. The boy-to-girl ratio for autism prevalence was reported as 4.5:1 across all sites (CDC, 2012). The prevalence of ASD based on the results of a parent survey is 1 in 50 (U.S. Department of Health and Human Services, National Center for Health Statistics, 2013).

ADDM compared the 2008 data to 2006 data from 11 sites to determine the changes in the prevalence of autism and reported a 23% increase in the prevalence of ASD among 8-year-old children. Differences in the prevalence of autism based on race and ethnicity were also reported. The average prevalence for autism was reported to be much higher in non-Hispanic White children (12.0 per 1,000) as compared to non-Hispanic Black children (10.2 per 1,000) and Hispanic children (7.9 per 1,000) (CDC, 2012). No direct link or reason seems to account for the ethnic and racial disparities in diagnosis (Dyches, Wilder, Sudweeks, Obiakor, & Algozzine, 2004). According to U.S.DOE (2012), approximately 407,214 students between the ages of 6 and 21 receive special education services under autism classification across the United States.

CDC (2012) reported intellectual ability data for 70% of the students with ASD from 7 of the 14 sites. The data revealed that 38% of children with ASD were reported to be in the intellectual disability range (i.e., IQ < 70), 24% were in borderline range (IQ 71-85), and 38% had IQs scored in average or above average (>85) intellectual ability.

Powers defined autism as "a physical disorder of the brain causing a lifelong developmental disability" (as cited in Richard, 1997, p. 12). The brain of children with autism is wired differently (neurologically and biochemically) to attend to and respond to the incoming stimuli. Autism is usually diagnosed between 18 months and 36 months because the delay in developmental milestones is more pronounced at that time. Early intervention helps individuals with autism to develop coping skills to deal with this lifelong disability (Richard, 1997).

Leo Kanner, a psychiatrist at Johns Hopkins Medical Center, described detailed symptoms of autism based on the observations of his patients (Janzen, 1996). Kanner published a paper in 1943 with descriptions of the symptoms of autism based on his observations. He categorized these children as having early infantile autism (Kanner, 1943). A year after Kanner's description of autism, Asperger drew a parallel identifying children with similar symptoms related to social functioning but higher cognitive functioning (Prelock, 2006).

Autism and Related Disorders

The *Diagnostic and Statistical Manual of Mental Disorders* (*DSM-IV*) recommends a model of spectrum disorders under the category of Pervasive Developmental Disorders (PDD) (APA, 2000). The following section provides a brief description of each of the disorders included under PDD, including the five disorders included under this category.

Autistic disorder. According to *DSM-IV*, to be classified as an individual with autism, a total of six or more items from the diagnostic criteria should be present. These broad areas are:

- At least two of the conditions in the social interaction category are met;
- At least one of the conditions in the communication category is met;
- At least one of the conditions in the restricted, repetitive, or stereotyped patterns of behavior category is met;
- Delays or atypical functioning in social interaction, communication, or symbolic/imaginative play before the age of 3; and
- Disturbance is not due to Rett's or Childhood Disintegrative Disorder (APA, 2000, p. 75).

Asperger's disorder. German Psychiatrist Hans Asperger first identified this disorder in 1944 (Prelock, 2006). It is commonly referred to as Asperger syndrome or Asperger's. Students with Asperger's syndrome have strong and fascinating areas of interest and exceptional memory skills. They have stereotyped speech and have trouble with transitions (Kluth, 2008). At times, the terms Asperger's syndrome and high functioning autism are used interchangeably (Janzen, 1996). According to the *DSM-IV* (2000), the following criteria have to be met for a student to receive the diagnosis of Asperger's disorder:

- At least two of the conditions in social interaction category are met;

- At least one of the restricted, repetitive and stereotyped patterns of behavior are exhibited;
- Significant delays in social, occupational, or other areas of functioning;
- No significant delay in language;
- No significant delay in cognitive development before the age of 3; and
- Disturbance is ruled out due to other Pervasive Developmental Disorder or Schizophrenia (APA, 2000, p. 84).

Rett's disorder. This genetic disorder mainly affects females. It requires a very different set of criteria for diagnosis than the other spectrum disorders. Due to behavioral similarities between individuals with Rett's disorder and autism, it is included under the PDD criteria for the spectrum disorders as recommended by *DSM-IV*. Rett's disorder is characterized by decrease in head growth between 5 and 48 months, loss of hand skills between 5 and 30 months, intellectual disabilities, and communication deficits (APA, 2000, p. 89).

Childhood disintegrative disorder (CDD). This disorder was first described by Theodor Heller and was previously referred to as Heller's syndrome. Children with CDD like autism and Rett's, show impairments in social, motor and communication skill, however, it has an onset age (after 2) of much later than Rett's and autism. CDD differs from autism in four areas: (a) age of onset is much later than autism, (b) communication skills are impaired as compared to socialization skills in autism, (c) progressive regression in previously acquired skills, and (d) more severe impact on the individuals as compared with autism (APA, 2000, p. 92).

Pervasive developmental disorder not otherwise specified (PDD-NOS). Sometimes referred to as *atypical autism*, this category is used to classify individuals who meet some but not all criteria for autism. PDD-NOS is characterized by impairments in several areas: communication skills; socialization; or presence of stereotyped behavior, interests, and activities (APA, 2000, p. 86). Walker et al. (2004) compared the characteristics of children with PDD-NOS with those of autism and Asperger's syndrome. They found that on measures of level of functioning in communication, daily living and social skills, IQ, and age of acquisition of language, the scores of children with PDD-NOS were between those of children with autism and Asperger's syndrome.

Etiology

There are many speculations regarding the causes of autism. According to Janzen (1996), factors that result in abnormal development in central nervous system (CNS) can cause autism. Any disruptions in CNS can impair cognitive functioning, motor movements and learning. Developments in the brain and other related structures start in the early stages of fetus. Therefore, any interruptions to the growth of the fetus can influence later development and result in a disability.

It is hard to identify a single known cause of autism. Several conditions such as toxins or viruses, genetic factors, environmental factors, neurological, infectious, immunologic, and metabolic factors have been associated with autism (Gillberg & Coleman, 1992; Heflin & Alaimo, 2007; Janzen, 1996; McCoy, 2011; U.S. Department of Health and Human Services, National Institutes of Health [NIH], 2005). The findings of majority of the studies done to find the causes of autism indicate correlation but not causation to one single factor.

Genetic factors. Evidence from several twin and family studies supports the premise that genetic factors are one of the major underlying causes of autism (Abrahams & Geschwind, 2008; Muhle, Trentacoste, & Rapin, 2004). Muhle et al. found that prevalence of autism is 2% to 8% higher in siblings than in the general population. They also reported that in monozygotic twins (identical twins) with autism, both have autism 60% of the time; however, in dizygotic twins (fraternal twins) both have autism 0% of the time. These findings corroborated the findings of other studies (Chudley, Gutierrez, Jocelyn, & Chodirker, 1998; Trottier, Srivastava, & Walker, 1999). The National Institute of Child Health and Development report (U.S. DOE-NIH, 2005) suggests that more than 12 genes on different chromosomes could be responsible for causing varying degrees of autism.

Environmental factors. McCoy (2011) states that pollutants like industrial wastes have grave side effects. There are chemicals and other such products that have an adverse effect on the individuals who are genetically susceptible to autism. According to McCoy, pesticides are harmful for the central nervous system (CNS) of insects but whether theycan have the same effect on humans who are genetically predisposed to autism is still unknown. The Childhood Autism Risk From Genetics and the Environment (CHARGE) research studied the effect of environmental exposures and its impact on the genes as a cause of autism. The findings of this study provide evidence that there is a direct link between the immune system and central nervous system. Students with autism in the CHARGE study had lower levels of Cytokines as compared with their typically developing peers. Lower levels of Cytokines (proteins) seem to interfere with the immune system of students with autism and causes severe behavior issues (Hertz-Picciotto et al., 2006).

Stressors. Prenatal and perinatal stressors such as maternal smoking or drug use in early pregnancy, problems during labor and delivery, congenital malformations, maternal stress, and small for gestational age are some of the factors that are associated with occurrence of autism (Beversdorf et al., 2005; Claassen, Naude, Pretorius, & Bosman, 2008). Beversdorf et al. (2005) reported that the incidence of prenatal stressors was much higher in autism surveys than the control group surveys (32.4 per 100 for autism surveys and 18.9 for control surveys). Based on the findings of a dizygotic study, Claassen et al. (2008) reported that prenatal stress seems to contribute to the occurrence of autism.

Vaccines. Many people believe that vaccines cause autism. Many studies have been conducted to investigate this hypothesis; however, the findings of these studies are inconclusive. The Immunization Safety Review Committee published a report based on the research done to study the relationship between measles-mumps-rubella (MMR) vaccine and thimerosal-containing vaccine on autism. The report suggests that there is no causal relationship between vaccines and autism (Institute of Medicine, 2004). These findings are further substantiated by another study done recently to investigate the relationship between prenatal and infant ethylmercury exposure from thimerosal-containing vaccines and autism (Price et al., 2010).

Neuro-immune dysfunction syndrome (NIDS). NIDS has been recently identified as a possible cause of autism. “NIDS is a classification for disorders caused by a complex neuro-immune, complex viral, autoimmune-like illness affecting cognitive and body functions in children and adults” (Neuro-Immune Dysfunction Syndrome [NIDS] Research and Treatment Institute, 2011, para. 1). Some studies revealed higher level of autoantibodies in the central nervous system of children with early onset of autism as compared with the children with regressive autism (Ashwood, 2008; Wills et al., 2007). The NIDS institute suggests that disruption in the body’s immune system can cause a reduction of blood flow to certain parts of the brain. Reduction in blood flow can interfere with brain development. Breakdown in the immune functions hinders language development, auditory processing skills, and social skills (McCoy, 2011). NIDS could play a major role in the pathogenesis of autism. A history of familial autoimmunity poses a major risk factor in regression of children with autism (Molloy et al., 2006; Sweeten, Bowyer, Posey, Halberstadt, & McDougle, 2003).

Characteristics of Students with Autism

Although each child with autism is unique in his or her profile, some general characteristics are common among students with autism (Kluth, 2008; National Education Association [NEA], 2006). This section briefly delineates the most significant characteristics that students with autism share.

Movement differences. Students with autism could experience either atypical movement or a loss of typical movements. This sometimes leads to difficulties with motor planning, clumsiness, and excessive movements (e.g., rocking, hand flapping, pacing, hand wringing). Some sources refer to this as repetitive behaviors (Aspy & Grossman, 2007). They could also appear very stiff and robotic in their movements (Richard, 1997). The movement differences could interfere with running, stopping, and starting a movement (Kluth, 2008).

Sensory differences. Students with autism sometimes have unusual sensory experiences. They could have overly sensitive senses (hearing, smell, touch, sight, or taste). They notice fire alarms and other noises in their environment or are sensitive to food texture or seams and tags. On the other hand, in some cases a hearing loss might be suspected due to the inability of the students with autism to respond to sound (Kluth, 2008; Spencer & Simpson, 2009).

Communication differences. Language and communication skills are one of areas most significantly impacted in students with autism. They might have difficulties with expressive and receptive skills or comprehension. They also exhibit difficulties in understanding other person's perspective that is also referred to as "theory of mind" (Baron-Cohen, 2001). These deficits account for difficulties in sharing thoughts or ideas and understanding what is spoken to them or what they see (Prelock, 2006; Spencer & Simpson, 2009). Students with autism seem to benefit from the use of visuals and augmentative communication devices. Such tools facilitate their participation in-group activities and help them communicate with other adults and peers (Bondy & Frost, 2002).

Social interaction differences. Students with autism show varied social skills ranging from being very involved to very aloof (Murray, Baker, Murray-Slutsky, & Paris, 2009). They have difficulties deciphering the rules of social interactions, understanding social nuances, initiating and sustaining meaningful interactions and friendships, maintaining eye contact, and understanding others' nonverbal communication gestures. Therefore, they find interacting with others stressful and aversive. Sometimes the people around cannot understand how to interact with students with autism, so these students struggle socially (Aspy & Grossman, 2007; Kluth, 2008; Richard, 1997).

Special interests. Several individuals with autism have special interests in one or a variety of topics, sometimes to the point of obsession. Some of these interests are common across individuals with autism (e.g., *Thomas the Tank Engine*, fans, trains, and computers). The way educational teams deal with the special interests of students with autism varies. Special interests, if appropriate, should be encouraged and viewed positively. Activities utilizing their special interests allow students with autism to relax, refocus, and self-regulate their behaviors (Kluth, 2008).

Learning differences. Like the individuals with learning disabilities, individuals with autism have differences in the way they process information. Some individuals might have memory deficits while others may have visual-spatial problems. The problems of students with autism are similar to some of the difficulties faced by students with learning disabilities (Mooney & Cole, 2000). It is hard for teachers to understand the reason for students' nonresponsiveness in class. Sometimes it could be related to intellectual deficits but at times, it could be a lack of understanding or motivation. While working with students with autism, it is important for the teachers to be cognizant of these learning differences (Kluth, 2008).

Although there are several definitions of autism, school systems use the definition and criteria suggested by the Individuals With Disabilities Education Improvement Act (IDEA). The IDEA (2004) defines

> Autism as a developmental disability significantly affecting verbal and nonverbal communication, and social interaction, generally evident before age 3, that adversely affects a child's educational performance. Other characteristics related to autism are engagement in repetitive activities and stereotyped movements, resistance to environmental change or change in daily routines, and unusual responses to sensory experiences. (§300.8(c) (1) (i))

Additionally, "autism does not apply if a child's educational performance is adversely affected primarily because the child has an emotional disturbance" (§300.8(c) (1) (ii)) and "a child who manifests the characteristics of autism after age three could be identified as having autism if the criteria in paragraph (c)(1)(i) of this section are satisfied." (§300.8(c) (1) (iii)) The participants in the current study could have a medical diagnosis of ASD but they had to meet the school-based criteria to participate in this study.

The prevalence rates for autism were 5-15 per 10,000 in late '70s (Wing & Gould, 1979). Based on the low prevalence rate autism was considered a rare condition until recently and was referred to as a low incidence disability (Heflin & Alaimo, 2007). The number of cases of autism has increased at an alarming rate over the past few years. According to the CDC report, the average prevalence of autism increased 23% in 14 sites from 2006 to 2008 for the children aged 8 years (CDC, 2012). The reason for this increase in recent years is puzzling. There is great deal of controversy over the number of people diagnosed with autism. The increase in autism cases could be related to changes in the definition of autism to be more of a spectrum disorder, changes in diagnostic criteria, coexistence or comorbidity of other conditions with autism and difficulty in evaluation of very young children (Heflin & Alaimo, 2007; McCoy, 2011).

Since the prevalence of autism is increasing rapidly, there is a need to educate these students in general education classrooms. It is important to review the academic profiles of students with autism, math interventions for students with autism and math interventions for students with disabilities. The current study is planned using the effective instruction components drawn from the research done with students with other disabilities.

Academic Profiles of Students with Autism

Researchers have tried to assess the mathematical abilities of students with autism. It is hard to assess students with autism using standardized tests because behavioral issues might interfere with testing procedures. Whitby and Mancil (2009) reviewed the literature to study the academic achievement of students with HFA and Asperger Syndrome (AS) and to study their academic profiles. Five studies from 1994 to 2008 were included in the review on the academic profiles of students with HFA/AS. The findings of this review indicated that 80% of the students were males and had an IQ greater than 80. Further, students with HFA/AS have difficulties in comprehension, written expression, handwriting, higher order thinking, and problem solving. These deficits become more obvious as the academic tasks move from concrete or rote to more abstract level (Whitby & Mancil, 2009).

Whitby et al. (2009) indicated that in early years the mathematical performance of students with HFA/AS is similar to the performance of typically developing peers. Students with HFA/AS have good computation skills but they have difficulty with problem solving and reasoning, which affects their ability to apply the learned math skills to solve real-world problems. For example, difficulty solving word problems. These findings are consistent with the findings of other reviews done on the academic functioning of students with ASD (Chiang & Lin, 2007; Whitby & Mancil, 2009). An understanding of the academic profile and characteristics of students with ASD helps the teachers to plan instruction for these individuals. Since autism is a spectrum disorder even within this category, the students have a wide range of functioning (Whitby & Mancil, 2009). Students with autism exhibit great variability in their academic achievement ranging from significantly above average to far below grade level (Griswold, Barnhill, Myles, Hagiwara, & Simpson, 2002), which makes it extremely challenging for teachers to plan interventions for these students.

Chiang and Lin (2007) reviewed 18 articles to study the mathematical ability of individuals with Asperger syndrome (AS) and high-functioning autism (HFA). They found that only eight studies had used standardized tests to measure the mathematical ability of students with AS/HFA. The overall mean score of the participants from the eight studies included in the review was 92.5, indicating average mathematical ability of some students with AS/HFA. Mayes and Calhoun (2006) reported that approximately 23% of the students referred with HFA had a math disability.

Chiang and Lin (2007) recommend using an age-appropriate mathematical curriculum to teach students with HFA. Individual mathematical assessments should be conducted to collect individual data about each student's relative strengths and weaknesses. It is extremely important for students with autism to acquire functional skills. Functional skills require a combination of computation and problem-solving skills to solve real-life problems. Therefore, the mathematics instruction for students with autism should focus on basic skills, computation, and problem solving (Myles, Constant, Simpson, & Carlson, 1989).

Math Interventions for Students with Autism

Studies done on math interventions for students with autism are reviewed in detail in the following section. Two studies were done to investigate the effects of touch-point strategy on the math performance of students with autism (Cihak & Foust, 2008 Eichel, 2007). Eichel (2007) examined the effect of using touch-point strategy and real-life examples on one-to-one correspondence and coin identification of a 13-year-old boy with autism. Cihak and Foust (2008) compared the touch-point strategy and use of number line for teaching single-digit additions facts to three elementary school students with autism.

In the touch-point strategy, the numbers contain touch points or dots that correspond with the number. The numbers 1 through 5 have single dots but the numbers 6 to 9 use circles in addition to the dots to represent double touch points. Following the instruction with TouchMath Curriculum, the student showed improvements in his ability to independently do one-to-one correspondence and identify coins. The results of this study show that use of concrete material and manipulatives was a helpful strategy to teach one-to-one correspondence and coin identification to the student with autism (Eichel, 2007).

Cihak and Foust (2008) found that all the students completed 100% of the problems correctly when using the touch-point strategy. Students' skills to solve single-digit addition problems improved to an average of 72% with touch-point strategy and 17% with number line from the baseline score of 0.7%. Touch-point strategy was more effective than the number line strategy for teaching single-digit addition facts to students with autism (Cihak & Foust, 2008).

Two studies were conducted to examine the effect of presenting high-preference (high-p) tasks on task initiation and task completion of students with autism (Banda & Kubina, 2010; Mechling et al., 2006). According to Banda and Kubina (2010),

> The high-p strategy involves presentation of two or three academic tasks (i.e., tasks that are likely to be completed with high frequency) before the presentation of a nonpreferred academic task (i.e., tasks that a student can do but in which he or she does not frequently engage). (p. 81)

The first study investigated the effects of using high-p mathematics tasks for increasing initiation of low-preference (low-p) mathematics task with a middle student with autism. The student showed a preference for three-digit-by-three-digit addition problems. In the intervention phase, these addition problems were paired with corresponding missing addend problems. It was noted that the student's time to initiate low-p tasks improved from 7-12 seconds in the baseline to 2-3 seconds in the intervention phase. The findings of this study show that pairing low-p tasks with high-p tasks seems to reinforce the students and increases the likelihood of initiation of low-p tasks (Banda & Kubina, 2010).

The second study investigated the impact of presenting highly reinforcing items or activities through computer-based video technology, paired with choice on the duration of task performance of two middle school students with autism. In the first condition, the two students had to complete three tasks and a tangible reinforcer was provided after task completion. In the second condition following task completion, the students were given a choice of three highly preferred items that were delivered through computer-based video. In the second condition, the students were able to complete the three tasks in a shorter time. The first student reduced his task completion time from an average of 27.8 minutes in tangible reinforcement condition to an average of 11 minutes in the video reinforcement and choice condition. Similarly, the second student reduced his task completion time from an average of 53.8 minutes in the tangible reinforcement condition to an average of 28.8 minutes in the video reinforcement with choice condition. The findings of this study reveal that video presentation of reinforcer with choice leads to faster task completion. Motivation to complete the tasks seemed to increase when video reinforcement and choice were used (Mechling et al., 2006). This premise is supported by Temple Grandin, an individual with classical autism who attributes her success to motivation. At the age of 2, Temple was diagnosed as being brain damaged but with the help of her parents, school personal, and her own motivation, she became highly successful in her career (Cash, 1999).

Banda et al. (2007) conducted an investigation with five middle school students with autism to determine if these students showed preference for solving mathematical tasks that they had mastered when compared with nonmastered tasks. Mastery and preference assessments were done for each student with autism. Percentage of problems solved accurately was used as a measure of mastered tasks and percentage of problems chosen by the student was used to determine preferred tasks. First, the mastery assessment for addition problems (digit facts and word problems) was conducted followed by mastery assessment for subtraction. Similarly, preference assessment was first conducted for addition problems and then for subtraction problems. A task was classified as a "preferred task" when the student picked the task out of the two choices provided and solved it. The results of this study indicated that there was variability in the preference shown by the students. Two students showed preference for mastered tasks but one student showed preference for nonmastered tasks. Additionally, one student showed equal preference for both mastered and nonmastered tasks. It was also noted that some students preferred word problems while other students showed a preference for digit problems and some students showed an equal preference for both kind of problems (Banda et al., 2007).

Rockwell et al. (2011) used schema-based strategy instruction to teach addition and subtraction word problems to a fourth grade student with autism. Schema-based instruction used direct instruction with teacher modeling, guided instruction, independent practice, and continuous teacher feedback with visual strategies to teach students to solve word problems. A schema consists of a cognitive process in which the given quantities are compared and manipulated to obtain a new quantity (Carpenter & Moser, 1984; Gick & Holyoak, 1983). The student was taught group problems, change problems, and compare problems with addition and subtraction. Group problems consist of smaller parts, change problems consist of a beginning amount, change amount and an ending amount and compare problems consist of a larger amount, a smaller amount and a difference. The student's performance improved from 3.75 points at baseline to 5.75 points out of a possible 6 points for group problems, from 2 to 6 points for change problems and from 0 points to 6 points for compare problems. The student's performance on addition and subtraction problems improved drastically from baseline to after intervention for group problems, change problems, and compare problems. The student also used the appropriate strategy taught to solve each type of problem during maintenance and generalization phases. Additionally, the student's conceptual understanding of the problems seemed to have improved.

All the studies reviewed above show that some students with autism have average mathematical ability and they might benefit from using different math interventions. However, there seems to be a dearth of research in this area. All the studies used a single-subject design and included fewer than five participants. It is hard to make decisions about evidence-based interventions to teach math to students with autism based on the above studies. Since many students with autism are served in the general education setting, finding effective evidence-based practices to facilitate their math learning is essential. The few studies done in the area of math interventions for students with autism have either focused on lower level math skills like addition and subtraction or increasing task completion using high-preference math tasks. More research focusing on problem-solving skills, conceptual understanding and higher order mathematical concepts (e.g., fractions, word problems) is needed.

Math Interventions for Students With Disabilities

Mathematics is a crucial part of our lives that is used every day and at times even multiple times a day. Math learning and competency results in better job prospects for an individual and in turn positively affects the prosperity of the nation (U.S. Department of Education, National Mathematics Advisory Panel [U.S.DOE-NMAP], 2008). Mathematics is embedded in all aspects of our lives. According to *The Nation's Report Card: Mathematics 2009* (2009), students in grade 8 made gains in their math scores relative to their performance in 2007, however the average score of students in grade 4 remained unchanged from 2007 (National Center for Education Statistics [NCES], 2009).

Today, inclusion of students with disabilities in the general education curriculum is a prevalent choice, so finding effective interventions is a necessity. Cawley, Parmer, Yan, and Miller (1998) noted that math difficulties that become evident during elementary years continue through the middle school and students with math difficulties perform up to two grade levels behind their actual grade. Several reviews have been conducted to explore the effects of different instructional strategies on math performance of students with disabilities. Some reviews grouped the studies based on intervention used and others grouped them based on the mathematical concepts taught.

Maccini and Gagnon (2000) conducted a survey of secondary teachers to investigate their perceptions about effective mathematics instruction and accommodations for students with ED and LD. Additionally, researchers reviewed the literature on research-based mathematics interventions for secondary school students with mild disabilities (ED and LD). The results of the survey indicated that majority of the special education teachers emphasized effective instruction techniques to be helpful in teaching students with ED (19%) and LD (18%). However, majority of general education teachers reported the use of manipulatives (19%) and cooperative learning strategies (16%) to be more effective for this population. Based on the results of the survey, effective instruction practices, use of manipulatives to teach mathematics and using real-life problems to apply mathematical problems emerged as the three major recommendations for teaching math to students with ED and LD (Maccini & Gagnon, 2000).

Kroesbergen and Van Luit (2003) conducted a review of 61 studies on mathematics interventions for elementary students with special needs. This review is particularly helpful because all groups of students (mild disabilities, learning disabilities and mental retardation) with difficulties in learning math were included in this meta-analysis. The studies in this meta-analysis were grouped into following three categories of intervention: preparatory arithmetic (*n* = 13), basic facts (*n* =31), and problem solving (*n* =17). It was evident that basic facts is the most well researched area, however, no significant differences were found in the effect sizes of the three categories. The findings of this investigation revealed that interventions in basic skills had the highest effect size

(p = .001) and direct instruction seems to be most effective strategy for teaching these skills. This finding is consistent with those of the other meta-analyses (Maccini & Gagnon, 2000). The interventions with computer-assisted instruction had lower effect sizes than other interventions in the meta-analysis. These findings are in contrast to the findings of the meta-analysis conducted by Xin and Jitendra (1999).

Xin and Jitendra (1999) conducted a meta-analysis of word problem-solving interventions for students with learning problems. The effect sizes were calculated for the 14 group design studies and PNDs' were calculated for the 12 single-subject studies included in this review. The effect size for instruction in group-design studies was +.89 with strong effect sizes for maintenance (*ES* = +.78 and +.84) and generalization phases. PND for the single-subject studies was 100%. Studies were coded using the following four categories based on the intervention approach used: representation techniques, strategy training, computer-assisted instruction (CAI), and other. Representation techniques referred to using pictures, manipulatives, or verbal responses to solve word problems. Strategy training referred to explicit problem-solving strategies like direct instruction and self-regulation procedures and was the most frequently used intervention (*ES* = +0.77). CAI referred to using a computer or videodisc program for instruction and had the highest effect size (*ES* = +1.80). Additionally, the treatment duration and instructional arrangement seemed to affect the effect size of the intervention. Studies with long-term duration (greater than a month) and individually provided instruction yielded better effect sizes than studies with short-term intervention duration and small-group instruction.

Maccini, Mulcahy, and Wilson (2007) extended a previous review (Maccini & Hughes, 1997) on math interventions for secondary students with learning disabilities. The studies were divided into three different categories based on the instructional approach used: behavioral, cognitive, or alternate delivery systems. Behavioral interventions included more teacher-centered approaches (e.g., teacher-modeling, direct instructional approach) but cognitive approaches were more student-centered (self-regulation, mnemonics, or self-instruction). Alternate delivery systems included videodisc or computer-assisted instruction (CAI). The results of the review indicated that the most of the studies focused on problem solving, followed by algebra, basic skills, decimals, and geometry. The majority of the studies (11) focused on a combination of procedural and conceptual knowledge. This was a considerable increase from the previous review done by Maccini and Hughes (1997). One important finding of this meta-analysis was that the studies that resulted in significant gains in math skills for students with LD included parts of effective instruction framework including modeling, guided practice, independent practice, assessment and feedback (Maccini et al., 2007). The results of these meta-analyses indicate that CAI and effective instruction framework strategies hold promise for improving the math performance of students with disabilities; however, these two strategies have not been implemented together.

Explicit Instruction

Browder, Spooner, Ahlgrim-Delzell, Harris, and Wakeman (2008) conducted a meta-analysis on math instruction for students with significant cognitive disabilities. Of the 54 single-subject studies, 19 had all the quality indicators for research design recommended by Horner et al. (2005). The review included 493 students with disabilities across all studies out of which 24 were students with autism in 12 studies. Most of the studies in this review focused on number and operation skills (*n* = 6) and measurement (*n* = 13), but a few studies focused on other strands (algebra, geometry, data analysis, and probability) also as recommended by National Council of Teachers of Mathematics (NCTM).

Systematic instruction with explicit prompting and feedback to elicit a specific response or set of responses from the students was the most commonly used strategy (in 34 out of 54 single-subject studies) to teach math skills to the students. The findings of this review demonstrate that the research base to qualify systematic instruction as an evidence-based practice for students with significant cognitive disabilities is still emerging. Percentage of nonoverlapping data (PND) for the systematic instruction studies ranged from 59.0% to 100% with a median PND value of 92.15%. In vivo training had a PND of 100%, however, the median PND for reinforcing a student's correct response, stimulus prompting, and physical guidance was above 97%. Due to the limited number of group design studies, the results of those studies were not included in this review (Browder et al., 2008). The results of this meta-analysis indicate that more research is needed in the areas of algebra, geometry, data analysis, and probability.

Findings of the review done by Gersten et al. (2009) were consistent with those of Browder et al. (2008). Explicit instruction emerged as the most commonly used strategy in majority of the studies. Gersten et al. (2009) synthesized 42 studies with random control trials and quasi-experimental research on math instruction for students with LD.

They organized the literature using four main approaches: (a) explicit instruction, (b) use of heuristics, (c) student verbalizations of their mathematical reasoning (Think alouds), and (d) using visual representations while solving problems.

Explicit instruction was used to teach different strategies and mathematical concepts in 11 studies with an effect size of 1.22, which was significant. The four studies on heuristics had an average effect size of 1.56. The mean effect size for student verbalizations was 1.04 based on eight studies. The 12 studies for using visual representations had a smaller effect size (0.43) as compared to the other three instructional categories used in this review (Gersten et al., 2009). Gersten et al. noted that other instructional tools (e.g., visuals, student think alouds) were used in conjunction with explicit instruction in some of the studies.

The NMAP (2008) found:

> Explicit instruction with students who have mathematical difficulties has shown consistently positive effects on performance with word problems and computation. Results are consistent for students with learning disabilities, as well as other students who perform in the lowest third of a typical class. (p. xxiii)

Explicit instruction framework is a predictable format of the lesson plan based on the Strategic Math Series (Mercer & Miller, 1991). This framework has six sequential components, which are adapted and expanded in Table 1: (1) Advance organizer, (2) Teacher demonstration, (3) Guided instruction, (4) Independent practice, (5) Problem solving practice, and (6) Feedback (Butler et al., 2003; Maccini & Ruhl, 2000).

Table 1

Components of the Explicit Instruction Framework

Components	Description
Advance Organizer	The teacher will link the new lesson with the previously taught lesson by reviewing the objective of the previous day. The teacher will also identify the objective for that day's lesson and give a rationale for learning the skill.
Teacher Demonstration	The teacher will demonstrate the targeted skill while describing aloud the steps.
Guided Practice	The teacher will give prompts and cues to solve the few problems together using questions and answers.
Independent Practice	Student will independently solve a few problems related to the targeted skill.
Problem-solving practice	There will be two word problems. The student and the teacher will solve the first problem together and the student will solve the second one independently.
Feedback	The student's understanding will be checked by monitoring his work. At this stage the instructional decision will be made to either continue the lesson or go back to step one of the lesson. This will be contingent upon the student's performance on independent problems.

Concrete-Representational-Abstract (CRA)

The concrete-representational-abstract (CRA) sequence of instruction is an evidence-based practice that supports students' conceptual and procedural learning. Interventions that are found effective merge components from both explicit instruction framework and CRA (NMAP, 2008; Stein et al., 2006). This sequence begins with concrete-level lessons, during which the students use hands-on manipulatives to develop understanding of the math concepts. This is done through teacher modeling, guided practice, and independent practice of the concepts. Once the students gain mastery at solving problems using manipulatives, the instruction moves to the second phase of instruction with representations. During the representational phase, the student uses drawings, pictures, or representations of the manipulatives to solve the problems and demonstrate their conceptual understanding of the concepts. Once the student gains mastery at the pictorial level, the instruction progresses to the abstract level lessons, which include only numbers and symbols. Teacher modeling, guided practice, and independent practice for solving the problems are embedded in each phase of the CRA instructional sequence (Flores, 2010; Mancl, Miller & Kennedy, 2012). The growing body of empirical research supporting CRA validates its effectiveness as an evidence-based practice to teach numerous math concepts to students with math difficulties.

Subtraction with regrouping. Flores (2009, 2010) conducted two studies investigating the effects of CRA instruction on the computation performance of subtraction problems with regrouping of students with LD and students at-risk for math failures. Both studies were multiple-probe designs with six participants each and measured the fluency, math achievement, maintenance of subtraction with regrouping skills. After the CRA intervention, students in both studies showed gains in their performance and achieved the accuracy criterion on subtraction problems. In the first study, at 4-weeks maintenance, five out of six participants maintained their performance at or above criterion (Flores, 2009). In the second study, at 6-weeks maintenance, four out of six students were able to maintain their performance at or above the criterion level (Flores, 2010).

Mancl et al. (2012) used a multiple-probe across participants design to explore the effects of CRA procedure for teaching subtraction with regrouping to five fourth and fifth grade students with LD. They specifically determined the number of lessons needed in each phase to meet the mastery criterion for the subtraction problems with regrouping. The repeated measures consisted of the baseline and intervention probes. Each participant was instructed using 11 lessons which included five concrete, three representational, one strategy, and two abstract lessons. All five participants met the 80% accuracy criterion and showed gain percent scores ranging from 75.71 to 92.73 from baseline to intervention.

Multiplication. Morin and Miller (1998) used CRA and strategy instruction to teach multiplication facts and related word problems to three middle school students with mental retardation using a multiple-baseline design. The students were instructed using 21 scripted lessons. After the intervention, the percent gain scores were 40, 20, and 70 for the three participants respectively. There were only four occurrences of scores below 80% out of 63 lessons assessed. The findings of this investigation reveal that CRA holds promise for teaching multiplication facts and related word problems to students with mental retardation.

Fractions. Jordan et al. (1999) compared the effects of concrete to semiconcrete to abstract (CSA) instruction and instruction with textbook curriculum on the fraction concepts of fourth grade students with LD. Students were assessed on the following fraction concepts: (a) identification, (b) comparison, (c) equivalence, (d) subtraction, and (e) addition of fractions. A split plot analysis of variance (ANOVA) was used to measure differences between the two groups. The results of the posttest measures indicate that students in the CSA condition (F (1, 62) = 219.96, p = .0001) outperformed the students in the textbook curriculum instruction condition (F (1, 61) = 97.55, p = .0001) although both groups made significant gains.

Butler et al. (2003) conducted a study comparing the effects of CRA with representational-abstract (RA) on equivalent fraction concepts of middle school students with math disabilities. The CRA group used manipulatives for the first phase of instruction but the RA group used drawings during the first phase of instruction. Student achievement was measured using the subtests from the Brigance Comprehensive Inventory of Basic Skills-Revised (CIBS-R). Both groups received intervention with 10 scripted lessons on fractions using the explicit instruction format based on the Strategic Math Series (Mercer & Miller, 1991). The results of the study indicate that students in both groups improved on their fraction performance. Overall, mean scores of the CRA group were higher than the means of the RA group. Additionally, CRA group showed a better conceptual understanding of fraction equivalency than the RA group.

Algebra. Maccini and Ruhl (2000) used a multiple-probe across subjects design to investigate the effects of CRA on the algebraic subtraction of integers of three secondary students with LD. The percent accuracy on strategy use, problem representation, and problem solution were used as dependent measures. After the CRA intervention, all participants increased their percent of strategy use (from baseline to instructional phases), mean percent accuracy on problem representation (67.5, 66.25, and 46.25 percentage points) and problem solution (72.5, 56.25, and 46.25 percentage points).

Additionally, the participants generalized and maintained the skills at higher levels for the algebraic subtraction of integers. The percent accuracy on problem representation was 73% and 67% on problem solution on the near generalization and 29.3% and 28.7% for the far generalization respectively. These results show that the participants were able to generalize the skills at higher level immediately but had difficulty with generalization over extended time.

Witzel et al. (2003) compared the effects of explicit CRA instructional sequence with traditional instruction for teaching algebraic transformation equations to middle school students with LD or at-risk for difficulties in algebra in inclusive settings. The students in the two conditions were matched according to grade level, teacher, standardized math achievement scores and class performance. Data for this comparison study were analyzed using repeated measures analysis of variance on two levels of instruction (CRA vs. abstract) and across three testing conditions (pretest, posttest, and follow-up). Students in both conditions showed significant gains from pretest (CRA: M = 0.12, SD = 0.41; Abstract: M = 0.06, SD = 0.34) to the posttest (CRA: M = 7.32, SD = 5.48; Abstract: M = 3.06, SD = 4.37). However, students in the CRA instruction group outperformed their counterparts in traditional instruction on the posttest and 3-week follow-up measures (CRA: M = 6.68, SD = 6.32; Abstract: M = 3.71, SD = 5.21).

Witzel (2005) compared the efficacy of two procedural approaches: a multisensory algebra model with CRA instructional sequence and repeated abstract explicit instruction model in teaching linear algebraic functions to middle school students with math difficulties in inclusive settings. A pre/post follow-up design with random assignment of clusters by class (treatment and comparison) was used to compare the math achievement of the students. The results indicated that although students in the traditional instruction (abstract) outperformed the students in the CRA group on pretest scores (Abstract: $M = 0.57$, $SD = 1.12$; CRA: $M = 0.18$, $SD = 0.53$). However, on the posttests (Abstract: $M = 5.36$, $SD = 5.75$; CRA: $M = 8.26$, $SD = 7.65$) and follow-up tests (Abstract: $M = 5.51$, $SD = 5.97$; CRA: $M = 7.96$, $SD = 7.84$), the students in CRA group outperformed the students in the traditional instruction. The higher standard deviation scores for the treatment group indicate that the model has a gradual effect for the students with the linear algebraic functions.

Manipulatives

The use of manipulatives for teaching math emerged from the theories of Piaget (1970), Bruner (1986), and Skemp (1987). They all postulated that concept development follows a progression along a continuum from concrete manipulation to pictures and finally the abstract thought. Although several studies outlined above were done to explore the impact of CRA instructional sequence on learning of math concepts, fewer studies have investigated the impact of manipulatives on math learning in isolation. According to NCTM (2008), it is important for students to use concrete experiences and materials be able to understand mathematical skills and concepts. With the advancements in technology, a new set of web-based virtual manipulatives have emerged. For this section, only studies and papers addressing physical manipulatives as an intervention are included since these are most applicable to the current study.

Discrepant findings related to the use of manipulatives for teaching mathematics are present in the literature. Some researchers proposed that manipulatives aid learning by providing hands-on experiences, supporting memory and real-world application of math concepts, while others felt that it distracted the students and made learning difficult by placing more demands on the students (McNeil & Jarvin, 2007).

Clements (1999) questioned the meaning of the word *concrete* in using manipulatives. He suggested that manipulatives are the physical objects that can support learning. However, for that to happen, students have to reflect on their actions with manipulatives. In a way, they have to mentally, manipulate the ideas connected with the objects for meaningful learning to occur (Clements, 1999; Sarama & Clements, 2009). Kamii, Lewis, and Kirkland (2001) reiterated the idea that manipulatives by themselves do not reinforce learning but are beneficial when children use them as knowledge-creation tools.

Swan and Marshall (2010) replicated the study done by Perry and Howard (1997). They surveyed 820 teachers to gather information about the different types of manipulatives used and their usefulness. Similar to Perry and Howard’s findings, all elementary school teachers supported the use of manipulatives across grades and math concepts. However, they could not identify why the use of manipulatives supports math learning (Swan & Marshall, 2010).

Aburime (2007) explored the impact of using geometric manipulatives on the math achievement of high school students in Nigeria. A pretest and posttest design was used to study the impact of manipulatives in this group design study. The results of the pretest indicate that both groups started at similar levels of achievement, but the treatment group (*M* = 11.70) outperformed the control group (*M* = 9.89) on the posttest scores of math achievement.

Cass et al. (2003) explored the effect of manipulative instruction on the perimeter and area problem-solving skills of three secondary school students with LD. A multiple-baseline design across participants was employed in this study. The lessons followed the modeling, guided practice or prompting, and independent practice sequence and manipulatives were used in all stages of the lesson. In the baseline condition, all three students scored 0 on measures of perimeter and area problem solving. However, after the intervention with manipulatives, all students met the 80% criteria although the numbers of lessons to reach the mastery criteria varied. The three participants took 6, 7, and 5 days respectively to reach the 80% mastery criteria. Additionally, the participants maintained their problem-solving skills and generalized them to paper pencil tasks successfully.

Moch (2001) conducted a study with fifth grade students on using manipulatives during measurement lessons. As a follow-up activity, students wrote in their journal about their perceptions and experiences. Not only did the students make huge gains on their measurement unit posttests, they were enthusiastic and positive about using manipulatives for learning math. Manipulatives have been used to help students learn even complex concepts like three-dimensional coordinate systems (Koss, 2011).

The literature reviewed above indicates that manipulatives have to be presented within a framework of planned activities to enhance student learning. There is lack of rigorous empirical research on the use of manipulatives for teaching math to students with disabilities. Additional studies are needed to explore the effectiveness of manipulatives for teaching students across disabilities, math concepts, and grade levels.

Fractions

According to the National Assessment of Educational Progress (NAEP) report, 27% of the eighth grade students had difficulty shading 1/3 of a rectangle and 45% could not correctly solve a word problem with division of fractions (U.S. DOE, 2004). "Difficulty with fractions (including decimals and percent) is pervasive and is a major obstacle to further progress in mathematics, including algebra" (NMAP, 2008, p. xix). Proficiency with fractions is important for building mathematical concepts and for improving student achievement in later years (NMAP, 2008).

Learning fraction concepts is one of the most challenging tasks for students in middle and junior high schools (Bezuk & Cramer, 1989). Students have a basic understanding of fractional relationships in preschool and earlier grades but how this understanding translates to formal knowledge of fractions has not been well researched (NMAP, 2008). Sammons (2010) identified fractions as the "hot spot" for students in upper elementary grades. "Hot spots" are curricular concepts with which students struggle constantly (Sammons, 2010). The new rules related to fractions conflict with the ideas about whole numbers. Students have difficulty with comparing, ordering, adding and subtracting fractions. Students seem to lack procedural and conceptual understanding of fraction concepts. Since fraction concepts are challenging, more time should be spend developing a deeper understanding of fractions (Bezuk & Cramer, 1989).

The National Science Foundation sponsored the Rational Number Project (RNP) to conduct research on children's learning of fraction concepts (Post & Reys, 1979). As part of the RNP, students in fourth and fifth grade were taught fractions. The RNP curriculum provides an alternative approach to the one recommended in textbooks to teach fractions. RNP's belief is that students should learn fractions using manipulatives to develop concepts. They also support an emphasis on order and equivalence as the foundation for developing procedural and conceptual knowledge of fraction concepts.

RNP's lesson plans were based on the instructional model suggested by Lesh (1979). He suggested using an instructional model with five different components to plan the fraction lessons. The five components of the instructional model recommended by Lesh are: (a) manipulative models, (b) real-world situations, (c) pictures, (d) spoken symbols, and (e) written symbols (Cramer et al., 1997).

Misquitta (2011) synthesized 10 studies on instructional practices for teaching fractions to students at-risk for math difficulties. The effect sizes for the five group design studies included in the review ranged from -0.28 to 1.24. The percentages of nonoverlapping data for the two single-subject studies in the review were 96.6% and 100%. Based on the findings of the studies, the interventions for teaching fractions to struggling students were classified into four categories: (a) graduated sequence, (b) anchored instruction, (c) strategy instruction, and (d) direct instruction. Graduated sequence, direct instruction, and strategy instruction emerged as effective interventions for improving student achievement on fraction problems. Misquitta (2011) noted that based on this limited research, direct and explicit instructional strategies are effective for developing conceptual and procedural knowledge (related to fractions) of students who are at-risk for math difficulties (Misquitta, 2011). The findings of this meta-analysis are consistent with the findings and recommendations of NMAP (2008).

Conceptual and procedural knowledge of fractions. According to the recommendations of NMAP (2008), conceptual and procedural knowledge of fractions is directly linked with mastery of fractions. Instruction focused on conceptual knowledge of fractions is likely to improve the problem-solving skills of students. Conceptual knowledge is developing a deeper understanding of the mathematical concepts by linking new phenomenon to previously existing phenomenon and understanding the relationships and patterns among these different pieces of information (Miller & Hudson, 2007). For example, the student understands that multiplication and division have an inverse relationship. Therefore, they use this knowledge to check the answer to a multiplication problem by dividing the product with one of the multipliers. Conceptual knowledge also develops when students connect a newly learned math concept to a previously learned and stored concept. For example, the students understand place value of whole numbers but when they learn decimals, they connect the new math concept with the previously learned and stored math concept of place value (Hattikudur, 2011; Kridler, 2012; Miller & Hudson, 2007; Mulcahy & Krezmien, 2009).

The students understand the characteristics shared by the math concepts and can apply this knowledge to other situations and settings. For example, students understand the concept of elapsed time and they can manage time to complete their homework. Using manipulatives to teach math concepts enables students to develop the conceptual knowledge. Conceptual understanding of fractions is related to comparing magnitudes, understanding the concepts of parts and whole, equivalent fractions and representing different fractions on number lines (NMAP, 2008).

Miller and Hudson (2007) define “Procedural knowledge as the ability to solve a mathematical task” (p. 50). It is also defined, as the ability to follow step-by-step procedures to solve a math problem is the procedural knowledge (Bottge, 2001; Carnine, 1997; Goldman et al., 1997). Procedural knowledge can be used for solving problems ranging from simple addition and subtraction to complex word problems. The development of procedural knowledge has been researched extensively for students with LD (Brown & Frank, 1990; Case, Harris & Graham, 1992; Hattikudur, 2011; Kridler, 2012; Miller & Hudson, 2007; Montague, 1992; Mulcahy & Krezmien, 2009).

Procedural knowledge in fractions involves solving problems with addition, subtraction, multiplication and division of fractions (Misquitta, 2011). It also involves following the algorithms used to do computations for example, for addition of fractions with unlike denominators, finding the common denominator, changing the numerator based on the denominator, and then adding the numerators.

Conceptual and procedural knowledge of fractions supplements each other while solving problems of estimation, computation, and word problems. “One key mechanism linking conceptual and procedural knowledge is the ability to represent fractions on a number line” (NMAP, 2008, p. xix). Students should be instructed using a comprehensive curriculum that provides sufficient time for acquisition of conceptual and procedural knowledge, include multiple representations of fraction concepts (e.g., number lines) and include instruction in equivalence, magnitude, and other related tasks (NMAP, 2008).

Both conceptual and procedural knowledge are essential for improving math achievement of students with LD (NMAP, 2008). An ongoing debate persists regarding which of these two types of knowledge develops first and which one is more important. Researchers have come to realize that it is neither procedural nor conceptual knowledge alone; it is an integrated understanding of both conceptual and procedural knowledge that leads to math proficiency (Rittle-Johnson, Siegler, & Alibali, 2001).

Hallett, Nunes, and Bryant (2010) conducted a study to identify individual differences in combining procedural and conceptual knowledge to develop understanding of fractions. The researchers hypothesized that children can develop either conceptual or procedural knowledge first. A total of 318 fourth grade and fifth grade students from eight elementary schools participated in this study. Students completed an assessment of fractions knowledge on a computer. The items on the assessment were coded as either procedural or conceptual knowledge items. The conceptual items demonstrated understanding of fraction equivalence; however, procedural items demonstrated application of a procedure or a rule to solve the problem. The cluster analysis of the data resulted in five subgroups of children in the study: (a) lower procedural, (b) lower conceptual, (c) higher procedural-lower conceptual, (d) higher conceptual-lower procedural, and (e) higher related to understanding of fraction equivalence. The results of this study indicate that individual differences account for differences in the way the conceptual and procedural knowledge of the participants develops. The participants who rely more on conceptual knowledge might be at an advantage as compared with participants who rely on procedural knowledge.

Student Attitudes and Behaviors

Attitudes are defined as negative or positive feelings that a participant has toward a particular object or strategy (Goodykoontz, 2008). Behaviors account for the way students are going to act based on their attitudes. If a student has positive attitude, they will be engaged during instruction. Student attitudes toward math instruction seem to have an impact on whether or not students will learn and how much they will learn (Harackiewicz, Durik, Barron, Linnenbrink-Garcia, & Tauer, 2008). Nicolaidou and Philippou (2003) found a positive correlation between student attitudes and math achievement. These findings are supported by Lipnevich, MacCann, Krumm, Burrus, and Roberts (2011). They found that attitudes accounted for 25% to 32% variance in math achievement among middle school student. These results indicate that student attitudes could have an important role to play in math achievement.

Several methods have been used to gauge student attitudes and behaviors. Student surveys (Hoppe, 2010), questionnaires, or interviews (Butler et al., 2003; Nuangchalerm and Thammasena, 2009); parent surveys, questionnaires, or interviews (Rock & Thead, 2007); and student participation (Jones, 2009; Rock & Thead, 2007) in the classroom are recommended as some of the data indicators to measure student attitudes and satisfaction.

Hoppe (2010) used student and teacher surveys to collect data on student learning and attitudes to evaluate the effectiveness of an interdisciplinary math and science program. Butler et al. (2003) used attitude questionnaires to measure the attitude of middle school students toward mathematics instruction. Similarly, Nuangchalerm and Thammasena (2009) used learning satisfaction questionnaires to assess the satisfaction of second-grade students after they had learned science using the inquiry method. In the current study, parent and participant interviews and session recordings (student observations) were used to gauge participant attitudes, perceptions, and satisfaction related to the intervention.

The theory of planned behavior is used as a theoretical framework to explain participants' attitudes toward explicit instruction with manipulatives and the behaviors exhibited during instruction. This theory has been "designed to predict and explain human behavior in specific contexts" (Ajzen, 1991, p. 181). Based on this theory, performance or a behavior is influenced by intentions and the way individuals perceive behavioral control. Context has an important role to play in how the behavior occurs or what behaviors occur (Ajzen, 1991).

For the current study, this theory suggests an interaction among (a) attitudes the participant holds toward explicit instruction and manipulatives, (b) subjective norms regarding how the participants perceive the researcher and parent expectations that they can learn, and (c) perceived control given the predictable format or the routine of the explicit instruction framework. Subsequently, the participants' intention to participate may be influence such that the participant engages willingly in the behavior (see Figure 1). In the current study, this intention and related behavior could influence performance on the conceptual and procedural knowledge of addition and subtraction of fractions. For example, the participant may perceive control over the environment due to the explicit framework (routine) that could influence their attitudes and ultimately their behavior of math achievement.

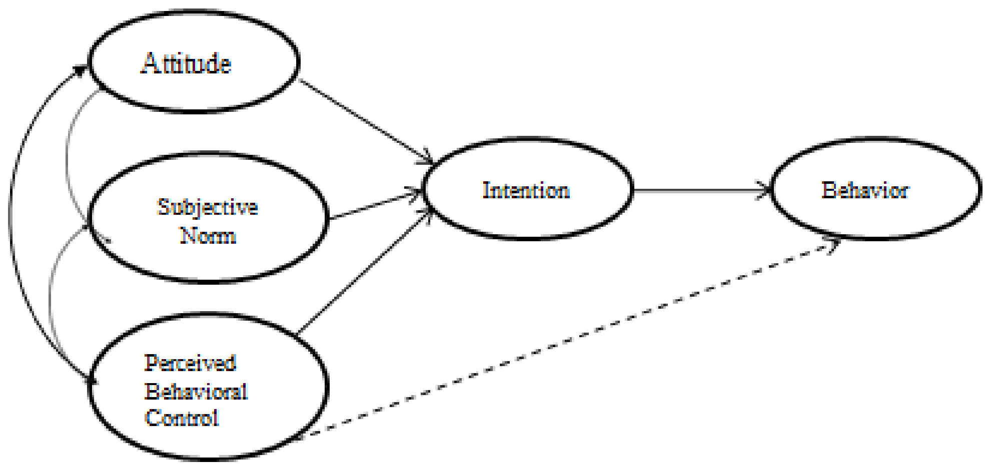

Figure 1 Theory of planned behavior- Attitudes: Participants' feelings towards the intervention (explicit instruction and manipulatives). Subjective norms: researcher's expectations that participants can learn. Perceived control: the predictable format of the explicit instruction provides a routine during the session. Intention: the participants' readiness to participate in the instruction and solve the fraction problems. Adapted from "The theory of planned behavior," by Ajzen, 1991, *Organizational Behavior and Human Decision Processes, 50*, p. 182.

Figure 1. Theory of planned behavior.

Conclusion

The literature reviewed in this chapter provides a foundation for understanding the current literature on autism, its characteristics, and math interventions for students with disabilities. Math interventions for students with autism should be planned based on the empirical evidence of effective interventions for students with LD and other disabilities as very few studies have been done on math interventions for students with autism, none of which investigated the effectiveness of explicit instruction with manipulatives for improving math achievement of students with autism. The current study investigated the effects of explicit instruction with manipulatives on the procedural and conceptual knowledge of like and unlike fractions of elementary school students with autism.

3. METHODOLOGY

This chapter includes the methods for the research study examining the effects of explicit instruction with manipulatives on the fraction skills (addition and subtraction of like and unlike fractions) of students with mild to moderate autism. Detailed information about participants, materials, independent and dependent variables, research design, treatment fidelity, interobserver agreement, social validity, and data analysis procedures is discussed in this section.

Research Design

Single-subject research design (SSRD) is well suited for the scientist practitioner as the individual is the focus of the study. SSRD allows researchers to study the effect of an intervention on an individual participant and address individual differences by making modifications to the existing intervention or using an alternative (Gast, 2010; Kratochwill et al., 2010). The recommendations of the National Research Council (2001) identified providing individualized intervention as one of the important components to plan an effective program for students with autism. It also specified that following three questions should be answered in educational research: (a) What is happening (i.e. descriptive research)? (b) Does it have an effect (i.e. causal relationship)? and (c) What led to the effect (i.e. processes that resulted in the effect)? SSRD has been widely used to evaluate the effectiveness of interventions especially for students with autism (Odom et al., 2003).

Multiple-Baseline Design

Single-subject research designs emerge from the literature on applied behavior analysis. There are specific characteristics of multiple-baseline design. In this design, there are usually two phases: baseline phase and intervention phase. *Baseline* refers to preintervention or existing conditions that are continuously measured prior to the introduction of the intervention. After a predictable pattern of performance is established, intervention is introduced to the first student. Once a stable performance is established after the intervention in the first student, the intervention is introduced to the second student. This pattern will continue until all the participants reach the intervention phase. In multiple-baseline studies, experimental control is established by replicating the intervention effect across tiers. Each tier could be an individual or group of individuals and the intervention should be replicated with at least three tiers to study the intervention effect (Gast, 2010).

SSRD is an experimental design in true sense due to its rigor and has several advantages. A multiple-baseline design across participants was selected for this study because:

- This design does not require return to baseline condition to demonstrate the effectiveness of intervention.
- The multiple-baseline will allow the researcher to investigate the effect of an intervention across participants in depth.

- This design will allow the researcher to measure the effect of the intervention on different participants (Gast, 2010).

Students with autism show variation in their performance depending on their behaviors, therefore, it is difficult to assess their true mathematical abilities (Whitby et al., 2009). Instructing students with autism is a challenging task because of their difficulties with social behaviors and communication issues (McCoy, 2011). Since very few studies have been done to investigate the academic interventions for students with autism particularly math, it was difficult to predict how students with mild to moderate autism would respond to a particular intervention or how long will they take to meet the mastery criteria for a particular phase. While a multiple probe design could also be used in this study, it would involve intermittent data collection rather than continuous measurement as in multiple-baseline design. It was the researcher's concern that a multiple probe design would not allow for continuously observing participants' performance and result in missing some important information in the unstudied context (Gast, 2010). Due to the above-mentioned limitations, the current study was planned using multiple-baseline design rather than a multiple probe design. A multiple-baseline design across six participants was chosen for this study. Six participants were assigned to four tiers, which allowed for multiple replications of the intervention and increased experimental control.

According to Gast (2010), single-subject research methodology allows researchers to respond to the individual differences and modify the intervention based on the participant needs. The initial purpose of this study was to investigate the impact of concrete-representational-abstract sequence of instruction with explicit instruction on the conceptual and procedural knowledge of like fraction of participants with mild to moderate autism. However, it was noted that after intervention with explicit instruction with manipulatives, the participants acquired the conceptual and procedural knowledge of like fractions and were able to generalize it to solve abstract problems of like fractions on a worksheet without manipulatives. Explicit instruction of the other two components (representational and abstract) was no longer necessary and was eliminated from the original study plan. The study was replicated with the same participants with unlike fractions to examine the functional relation between explicit instruction with manipulatives and improved performance (possibly generalized) to the solving of abstract problems of unlike fractions.

Design Standards for Single-Subject Research

According to the report developed by Kratochwill et al. (2010), the following requirements must be met for a study to be deemed as meeting design standards:

1. "The independent variable (i.e., the intervention) must be systematically manipulated with the researcher determining when and how the independent variable conditions change." (p. 14)
2. "Each outcome variable must be measured systematically over time by more than one assessor, and the study needs to collect inter-assessor agreement in each phase and on at least twenty percent of the data points in each condition (e.g., baseline, intervention) and the inter-assessor agreement must meet minimal thresholds." (p. 15)

3. “The study must include at least three attempts to demonstrate an intervention effect at three different points in time or with three different phase repetitions.” (p.15)
4. “For a phase to qualify as an attempt to demonstrate an effect, the phase must have a minimum of three data points.” (p.15)

In the current study, the intervention with explicit instruction with manipulatives was systematically manipulated by the researcher to study its impact on the conceptual and procedural knowledge of fractions of participants with mild to moderate autism. The baseline and intervention phases had five or more data points each and the study was replicated across six participants using two different conditions. Thus, there was an attempt to establish experimental control at three different points in time. Additionally, the interobserver agreement was assessed for 30% of data points in each phase resulting in 95.88% agreement. Therefore, this investigation meets the design standards set forth by Kratochwill et al. (2010).

Participants

This section provides details of inclusion criteria, human participants permission procedures, and the general characteristics of the individual participants. Students with mild to moderate autism who participated in this study directly are referred to as *participants* throughout the study. Other individuals involved in the study are addressed based on their profession or their engagement in the study. The other individuals in the study are parents of the participants, researcher, expert teachers (special education and general education) and independent observer(s).

Criteria for Participation

Students were included in the study based on the following criteria: (a) a school-based diagnosis of autism under the Individuals With Disabilities Education Improvement Act (IDEIA, 2004) resulting in eligibility for an Individualized Education Plan (IEP), (b) a documented deficit in math (e.g. IEP goal, IEP present level of performance, formal and/or informal assessment data), (c) between the ages of 8 and 12 years, (d) receiving math instruction in the general education curriculum with supplemental support from special education teacher in the school, (e) scored at least 80% on the screening test, and (f) agreed to participate by providing informed assent and consent granted by the parent or guardian.

Potential participants were excluded from the study if they (a) were not between the ages of 8 and 12 years, (b) did not have a documented math deficit, (c) received math instruction in special education setting using adapted curriculum, (d) earned a score of less than 80% on the screening test, or (e) did not give permission to the researcher to record the sessions.

Recruitment of Participants

The participants for this study were identified from parental contact initiated by the researcher. The researcher recruited participants by distributing the recruitment information through email, listservs, and personal contacts as well as snowballing. Some of the participants were recruited through snowballing, where the information about the study was shared by the individuals with other families who had children with autism (Atkinson & Flint, 2001). The researcher’s contact information was included on the recruitment letter (Appendix A), so interested parents contacted her directly. At this time, the researcher conducted a screening phone interview (Appendix B) with prospective participant's parent or guardian to check if the participant would qualify for the study based on the inclusion criteria. If the participant met the inclusion criteria, the researcher met individually with the parent or guardian of potential participant to get the consent and assent form signed prior to initiating the data collection.

Protection of Human Participants and Informed Consent

The Institutional Review Board (IRB) at George Mason University reviewed and approved all the methods and procedures for this study to protect the rights of the participants (Appendix C). If the participant met the inclusion criteria, the researcher met individually with the parents or guardians of participants at a mutually convenient location; the student also participated in this meeting. The researcher provided an overview of the project (including procedures and time commitment), completed the records review checklist, and answered questions. During this meeting, participants took the screening test, which was scored immediately. Detailed description of the screening test is provided in the materials section. If the participant met the 80% criteria on the screening test, signed consent was obtained from parent or guardian of the student. For the current study, all six participants scored 80% or more on the screening test. When the parent or guardian signed the consent letter, the researcher reviewed the letter of assent with the student. Once both consent and assent were given, the family was provided with copies of signed consent and assent forms for their records. The researcher made sure that the participants were aware that their participation in the study was voluntary, and they could withdraw from the study at any time and without specifying a reason. To maintain the confidentiality of the participants in this study, each participant was given a pseudonym. All identifiers were deleted so the participants and their families could not be identified.

Participants With Autism

Six elementary students with mild to moderate autism were selected for this study. The following section presents a detailed description of each participant's age, current level of functioning, current IEP goals, educational setting, strengths, weaknesses, and other demographic information. This information was collected from the parent interviews and the review of participants' educational records. Table 2 summarizes the demographic characteristics of the participants with autism, including hours of special education services received. It should be noted that hours of inclusion shown in the table represent a subset of the total special education hours since special education services follow the student into the general education setting.

Table 2

Participant Demographic Characteristics

Participant	Gender	Ethnicity	Age (in months)	VIQ	PIQ	Special ed. Hours/ Week*	Hours in Inclusion	Related Services
Jacob	Male	Caucasian	101	77	73	15	10	Speech/OT
Paulina	Female	Indian	112	63	99	26	20	Speech
Wilton	Male	Caucasian	103	73	69	29	20	APE/OT
Sam	Male	Indian	98	91	103	20	14	Speech
Kyle	Male	Caucasian	109	93	96	12	10	Speech
Brad	Male	Indian	121	58	69	19	12	Speech

Note. *Special education services were also provided in inclusive settings; VIQ = Verbal IQ, PIQ = Performance IQ, APE = Adapted Physical Education, OT = Occupational Therapy.

Jacob. Jacob is an 8-year and 5-month-old Caucasian boy. He is in third grade at a local public school. He is the oldest of the three siblings. Although Jacob started receiving special education services at the age of three through the Early Intervention Program, he received the formal diagnosis of autism only at the age of four.

His current IEP has goals related to communication, math, reading, writing and behavior skills. Jacob's attention skills, rigidity, and tendency to perseverate on different things interfere with his task completion. Jacob has difficulty with addition and subtraction with regrouping and with place value until thousands. His current math IEP goals are related to addition and subtraction with regrouping, identifying the place value of the given numbers in four-digit numbers and rounding numbers.

On the screening test, Jacob was able to solve all the addition and subtraction problems correctly. However, he could not solve the two fraction problems on the test.

These problems were included on the test to gauge the participant's familiarity with fractions.

Jacob receives 15 hours per week of special education support, out of which only five hours are in the special education setting. He also receives support from speech therapist, occupational therapist and adapted physical education itinerants. The analysis of the testing records indicated that for reading, Jacob has above average decoding skills but he is functioning two grade levels below for comprehension skills. He has difficulty retelling the events in a story, as he tends to forget specific details and names of the characters in the story. He has difficulty explaining his thinking. He participates in grade-level assessments with accommodations of flexible scheduling, small group size and visual aids for math.

He enjoys riding his bike and using the calculator to see patterns. He enjoys working with numbers and calendar math. He is fascinated with time and clocks from the different parts of the world and likes to see the international clocks on the iPad or iPod.

Paulina. Paulina is a 9-year and 4-month-old girl. She currently attends the neighborhood public school and is a fourth grade student. She is the older of the two siblings and her younger sibling has autism too. At the age of four years, she got a medical diagnosis of autism and started receiving special education services under this label. Paulina exhibits difficulty with attention skills although she tries extremely hard. She tends to perform better with visual supports in the instructional material.

She has goals related to reading, math, behavior, communication, writing, and social skills on her IEP. On the screening test, Paulina could solve the addition and subtraction problems with 80% accuracy but she could not solve the two fraction problems. She used TouchMath and made the dots on the numbers to solve the problems. Based on the present level of performance on her IEP and her testing reports, Paulina can add, subtract, and write numbers. However, she has difficulty with relationships such as greater than or less than and place value concepts. She has difficulty solving multiple-step problems. On her IEP, Paulina has math goals for reading and writing numerals through 10,000, identifying place value of each digit in a four-digit numeral, identifying relevant math vocabulary.

Paulina can decode well but has difficulty with inferential questions. She can answer simple questions directly stated in the text. She can sustain attention to a given task for up to 10 minutes without prompting. She maintains good eye contact and can engage in conversations about topics of interest. She has difficulty gaining attention appropriately from peers. In writing, Paulina can generate ideas but needs assistance to plan and edit her writing. Paulina receives 26 hours per week of special education support and 6 hours per month of speech therapy.

Wilton. Wilton is an 8-year and 7-month-old Caucasian boy. He is a third grade student with autism and ADHD. He attends the neighborhood public school. He is the older of the two siblings. He was formally diagnosed with autism at the age of 18 months by a developmental pediatrician and he started attending the Early Intervention Program at the age of three. He is currently on medication for attention skills. He responds to behavioral supports like token boards and positive reinforcement. Wilton receives in home Applied Behavior Analysis (ABA) services.

On the screening subtest, Wilton used TouchMath to solve the addition and subtraction problems. He scored 90% on the screening test; however, he could not solve the two fraction problems. Wilton's educational testing results revealed that he had scored in the below average range for math related subtests. He has difficulty identifying place value, comparing numbers, putting the greater than or less than symbol for the given numerals, and rounding numbers to the nearest hundred.

He has goals related to fine and gross motor skills, writing, math, communication, reading comprehension, behavior, and adapted physical education on his IEP. His math-related IEP goals include addition and subtraction of three-digit numerals without regrouping, rounding numbers to the nearest hundred, and comparing numbers and putting the correct symbol to show relationship between the given numerals.

Wilton has fleeting eye contact and has repetitive behaviors and restricted interests. He has a very strong interest in fans, lights, air conditioning units, and fire alarms. Wilton likes to watch videos of fans and fire alarms on YouTube on the computer. He exhibits considerable anxiety related to power cuts, fire drills and changes in routine. He has difficulty with social skills and maintaining reciprocal conversations. He tends to ask questions instead of making statements. He exhibits severe fine motor issues. He has difficulty writing numbers legibly.

Wilton has good memory skills and tends to remember events and dates. Wilton has a splinter talent for calculating the day of the week when provided with a date in history or in the future. He can decode well but has difficulty with comprehension skills and retelling the important events of a story or text. He tends to learn better with concrete representations and visual supports.

Sam. Sam is an 8-year and 2-month-old second grader with autism, who is currently being homeschooled. He attended public school until last year. At the age of 22 months, he got a medical diagnosis of PDD-NOS. He repeated preschool to help him bridge the gap with the age appropriate peers. At the age of 5 years and 4 months, he was found eligible for special education services under the autism and developmental delays labels due to pragmatic language and social skills deficits. Sam uses complete sentences to communicate his needs and feelings although his speech is sometimes unclear which sometimes makes it hard to understand him.

On the screening test, Sam could solve the addition and subtraction problems correctly but he did not attempt the fraction problems. He used his fingers to solve the addition and subtraction problems on the test. He has goals in the areas of math, reading comprehension, articulation, attention, conversation skills, and behavior on his IEP. The present level of performance on his IEP shows that Sam has difficulty with addition with regrouping and solving single-step word problems. He also has difficulty using doubling and other similar strategies for mental math. Sam can tell time to the nearest minute and can count and identify the value of a given set of coins. His current IEP goals include solving addition problems with regrouping and solving one-step and two-step word problems.

Similar to Jacob and Paulina, Sam is on grade level with decoding skills but has difficulty with reading comprehension. He participates in grade-level assessments with accommodations of flexible grouping and small group size. In math, Sam has difficulty solving word problems and explaining his thinking related to mathematical problem solving. He receives speech and second language services from the school.

When the task appears hard, Sam tends to give up easily without even trying. His usual response is “I don’t know.” He gets easily distracted and is very impulsive by nature. His mother reported that he is bullied easily and has difficulty advocating for himself. His mother also reported that he has low self-confidence that manifests in his ability to accept corrective feedback from adults.

He enjoys playing drums, juggling, and skateboarding. He also enjoys math and history and likes reading his history encyclopedia with important dates. He uses Legos to build different models of rockets and planes. He has a swing set and a drum set in his basement that he uses for relaxation. He has been taking drumming lessons since last two years with the same teacher.

Kyle. Kyle is a 9-year and 1-month-old Caucasian male student with autism and ADHD. He is third grade student at a public school. He has a twin brother and both repeated preschool. He receives support from a psychiatrist and takes medication for attention skills. He was diagnosed with autism by a psychiatrist at the age of 5 years.

On the screening test, Kyle was able to solve all the addition and subtraction problems with 100% accuracy. He used his fingers to solve the addition and subtraction problems. He attempted the two fraction problems but could not solve them correctly. Kyle has goals related to reading comprehension, writing, math, social skills, communication, and behavior skills on his IEP. His present level of performance on the

IEP shows that he can round numerals to the nearest hundred and identify place value up to hundred. He has difficulty with math vocabulary. He tends to get confused with operations in word problems. He also has difficulty recalling multiplication facts. The two math goals on his IEP are related to identifying the correct operation for the given word problems based on the vocabulary and recalling multiplication facts through the 12s table.

Kyle has difficulty with oral language skills and he does not like answering questions or explaining things. He tends to stammer when he is self-conscious and takes a long time to respond. He has difficulty with fine motor skills and organizing thoughts into writing. His mother reported that he is not a very social child and does not like playing outside with his friends. He takes time to process things but is accurate once he gets the concept. Kyle currently reads at grade level and can retell the important events of the story with details. Although he can tell the month, he has difficulty identifying the date. Kyle enjoys playing video games on the iPad and likes playing with superhero characters. He responds well to positive reinforcement.

Brad. Brad is a 10-year and 1-month-old Indian male. He is a fourth grade student at a public school. Brad is the older of the two siblings. He is a verbal and social child. He can engage in reciprocal conversations but sometimes tends to make off- topic comments. With reminders, he is able to attend to the task. He also engages in self-talk to calm himself. He is on gluten free and casein free diet that his mother reported has helped to improve his attention skills tremendously.

On the screening test, Brad scored 80% but was unable to solve the two fraction problems. He wrote his numbers very big on the screening test sheet. He used TouchMath to solve the problems on the screening test. For the two problems that he solved incorrectly, he put incorrect number of dots on the numbers, which resulted in an incorrect answer. He solved the fraction problems incorrectly. He has goals in the areas of reading comprehension, writing, math, fine motor skills, and communication skills on his IEP. The present level of math performance on the IEP indicates that Brad is able to count by 2, 5s, and 10s and tell time to the nearest half hour. He is able to identify coins and tell their value although he cannot count dollars. He continues to make errors in simple addition and subtraction. The math-related IEP goals include counting and telling the value of coins and dollars whose total value is less than five dollars, telling time to the nearest five minutes, and adding two-digit numerals with and without regrouping.

Brad has difficulty with comprehension skills. He is very prompt, dependent, and exhibits difficulty with his attention skills. He tends to make his letters and numbers very big. Although he can navigate independently through the school building for specials, lunch, and recess, he has difficulty understanding math concepts related to time, money, measurement, and fractions.

Brad is very fond of animals. He can navigate the Internet to find information about unique animals like Utakari, umbrella birds, etc. He enjoys art especially painting, paper folding and constructing things with Legos. He likes playing on the Wii. He attends tennis and swimming classes and receives private OT and speech services. He has recently started taking martial arts lessons, which his parents feel might help him with his attention skills.

Researcher. The researcher is a certified special education teacher for teaching K-12 students with multiple disabilities. She has more than 17 years' experience in providing educational services to students with autism and learning disabilities in classroom settings as well as in one-on-one private settings. She has worked extensively in providing remedial education in language arts, mathematics, and behavioral interventions. She has worked across settings, in team-taught and self-contained classes. The researcher provided instruction to all the participants and collected data.

Setting

This study was conducted in the Mid-Atlantic region of United States with a diverse population. According to the U.S. Census Bureau report (2010), this Mid-Atlantic region currently has 66% White, 11% Hispanic, 11% Black, 9% Asian, and 3% mixed race. The setting for participants varied as the researcher met with them in their home settings. The settings for each participant are described in detail at the end of this section. The study was conducted after school hours and over the weekends. Since students with autism may be sensitive to transitions and changes (Kluth, 2008), one-on-one instruction in a familiar setting (home) provided a positive learning environment for math instruction for the study.

Jacob

The setting for Jacob was the basement in his house. It was the same setting each time. Jacob sat with his back to the wall on a small circular table. The researcher sat to the left of Jacob. Basement was mainly used as a playroom for the children as it had toys and a trampoline. The setting was distraction free because neither his parents nor siblings came down while the sessions were in progress. It was cold in the basement so Jacob always wore his jacket during the sessions. The sessions with Jacob were primarily conducted after 4:00 p.m. on weekdays and around 2:00 p.m. on the weekends.

Paulina

The majority of the sessions were conducted in the dining room at Paulina's house. Paulina sat on the same chair next to the researcher each time. She sat on one side of the dining table facing the wall with her back to the kitchen. The activity in the kitchen did not seem to distract her as she sat with her back toward the kitchen. The dining room is attached to the living room where her brother sometimes watched TV while the sessions were in progress. Four sessions were conducted in Paulina's bedroom at her study table. The sessions with Paulina were conducted after 6:00 p.m. on weekdays and in the afternoons around 4:00 p.m. on the weekends.

Wilton

The intervention sessions were conducted in Wilton's bedroom in his house at a small rectangular table. He sat with his back to the wall on a small chair next to the researcher. Wilton's parents and brother did not come in the room while the lessons were in progress. Wilton's room has a bed, a bookshelf and the study table with a desk lamp and a floor lamp. The sessions with Wilton were conducted around 5:00 p.m. on weekdays and in the mornings on the weekends.

Sam

The intervention sessions were conducted in Sam's home in the dining room at the rectangular dining table. The dining room is separated from the kitchen with a wall. During the sessions, Sam's mom worked in the kitchen. Sam sat at the far end of the table facing the wall with his back toward the stairs. To the left of the kitchen is the living area with Sam's Legos and other toys. Sam's mom did not come to the dining area while the sessions were in progress. Since Sam was home schooled, the sessions with him were primarily conducted in the mornings around 10:00 a.m. on weekdays and weekends.

Kyle

All the intervention sessions were conducted in the dining room at Kyle's house. Kyle and the researcher sat in the same chair each time in the same position. Kyle sat facing the kitchen. For majority of the sessions, Kyle's parents were either working on the dining table on the other side or in the kitchen. The sessions with Kyle were conducted after 4:00 p.m. on weekdays and during late morning on the weekends.

Brad

All the intervention sessions were conducted in a room in the basement. Brad's parents and brother did not come in the room during the lessons. The room was furnished with a bed, study table with computer, and a small rectangular table with two chairs. The lessons were conducted on the small rectangular table. Every time Brad sat facing the bed with his back to the study table. The researcher sat on the right side of Brad each time. The sessions with Brad were conducted before school around 8:00 a.m. on weekdays and around 9:00 a.m. on the weekends.

Independent Variable

The intervention with explicit instruction with manipulatives was the independent variable for this study. Two different types of manipulatives (fraction circles and chips) were used since fractions can be a part of a whole (e.g., part of a circle) or part of a set (e.g., out of six). The intervention was used to teach addition and subtraction of like and unlike fractions to students with mild to moderate autism. Two scripted lesson plans were used for the intervention phase: (a) lesson plan for addition and subtraction of like fractions, and (b) lesson plan for addition and subtraction of unlike fractions (see Appendices D and E). The same lesson plans with different examples were used for each session during the intervention phase. The lessons for both conditions were based on the explicit instruction framework adapted from Strategic Math Series (Mercer & Miller, 1991) and the lessons suggested by Witzel and Riccomini (2008).

All the lessons were scripted to ensure consistency across participants. Each intervention lesson included the following components: (a) advanced organizer, (b) teacher demonstration or modeling, (c) guided practice, (d) problem-solving practice, and (e) independent practice.

During the advanced organizer component, the teacher script involved introducing the upcoming lesson by stating the lesson objective and sharing the instructional material, stating the rationale or the importance of the lesson and activating prior knowledge by reviewing what was learned in the previous lesson. The researcher used questioning strategy to illicit responses for the advanced organizer.

During the teacher demonstration or modeling component of each lesson, the teacher script involved think-alouds related to solving four fraction problems using fraction circles or chips depending on the lesson. The fraction problems included two addition and two subtraction problems. First the two addition problems were presented and then the subtraction problems. The researcher used the white dry-erase board and markers for the demonstrations. The researcher showed the participants how to read the fractions, set up the problem, represent the two fractions using fraction circles or chips, show the operation (addition or subtraction), using the fraction parts, and write the answer based on the manipulatives.

During the guided practice component of each lesson, the researcher provided verbal support and cues to help the participants as they solved the two guided practice problems presented to them. The participants used dry-erase board and markers to write the problems and used either fraction circles or chips to solve the problems. For the first guided practice problem, the researcher prompted the participants through each step of solving the problem. The researcher used questions to help guide the participants in solving the problems. For example, the researcher asked the participant, “What do we do first to solve this problem? That is right. Let us do that now. What do we do next to solve the problem? That is correct. Let us show that.” If the participant missed any steps or made mistakes, the researcher gave corrective feedback and helped the participant follow the correct procedure. The researcher provided fewer prompts or prompted as needed for the second problem.

During the problem-solving practice component of each lesson, the participant solved a real-life word problem using manipulatives. The researcher read the problem; the participant set up the problem, and identified the correct operation with the researcher's assistance. Then the participant solved the problem using the white board, markers, and manipulatives. The problems used for this component were real-life problems, usually using the name of the participant or siblings and friends. The same problem for the particular lesson was used across participants to keep the lesson consistent; only the names of the characters in the problem were individualized.

During the independent practice component of each lesson, the participant completed two probes (procedural knowledge and conceptual knowledge) using fraction circles or chips. Probes are described in detail in the materials section. No researcher feedback was provided as the participants solved these problems. When the participants completed the independent problems, the researcher scored them and recorded the scores. The researcher provided feedback related to any problems on either of the probes that were incorrect.

Dependent Variables

This study had three main dependent variables. The first dependent variable was the percentage of steps correctly stated or demonstrated based on the conceptual knowledge protocol for addition and subtraction of like and unlike fractions. The second dependent variable was the percent correct on the procedural knowledge probe. The third dependent variable was the time taken to solve the problems on the procedural knowledge probe. Test and Ellis (2005) have described three types of fractions. Type 1 fractions have like denominators, Type 2 fractions are unlike fractions but the smallest denominator can be divided into the largest denominator an even number of times, and Type 3 fractions are unlike fractions but the smallest denominator cannot be divided into the largest denominator an even number of times. For the current study, Type 1 fractions and Type 2 fractions were used. Responses for the conceptual knowledge probes were recorded on the conceptual scoring rubric for scoring. Responses for procedural knowledge probes were scored based on the participant responses on the worksheets. In addition, the time taken to solve the problems on the procedural knowledge probe was recorded. This process was repeated for conceptual and procedural knowledge and time taken for like as well as unlike fractions. Probes and scoring procedures are described in detail in the materials section. Additionally, data were collected to gauge participant attitudes toward the intervention.

Conceptual Knowledge

For the purpose of this study, the researcher used the following definition of conceptual knowledge. Conceptual knowledge is the participant's ability to show or explain their reasoning or thinking to solve the fraction problem (addition or subtraction) using manipulatives (fraction circles or colored chips), pictures, or words (Goldman et al., 1997, p. 200). For example, while solving an addition or subtraction fraction problem, the participant can explain all the steps of the problem accurately.

Conceptual knowledge of addition and subtraction of like and unlike fraction skills was chosen as a dependent variable in this study because many students with mild to moderate autism have difficulty understanding concepts beyond simple arithmetic.

Whitby et al. (2009) reiterated that many students with autism have good computation skills but they have difficulty with problem solving and reasoning. Some of them have difficulty applying previously learned math concepts to new situations. Their understanding is concrete and literal so they have difficulty processing abstract concepts (McCoy, 2011). The sessions were recorded using an iPad as the participant responses for conceptual knowledge probes were not evident on the worksheets. The researcher scored the conceptual knowledge probes during direct observations of participants completing the conceptual knowledge probes. Each participant's conceptual knowledge was measured by their ability to explain or show their thinking based on the conceptual scoring rubric. Researcher task-analyzed the steps for solving addition and subtraction problems with like and unlike fractions and created the rubrics. The two rubrics for like and unlike fractions were validated by the two expert teachers. Rubrics were changed based on the input from the experts. Detailed information for validation is included in the validation section. The steps on the scoring rubrics were different for like and unlike fractions. The following seven steps were included for like fractions: (1) participant represents/explains numerator of fraction 1 correctly, (2) participant represents/explains the denominator of fraction 1 correctly, (3) participant represents/explains numerator of fraction 2 correctly, (4) participant represents/explains the denominator of fraction 2 correctly, (5) participant represents the numerator for the answer correctly, (6) participant represents the denominator for the answer correctly, and (7) participant states that the denominator for the answer is the same as in the two fractions (Appendix F).

The following nine steps were included on the rubric for unlike fractions: (1) participant represents/explains the numerator of fraction 1 correctly, (2) participant represents/explains the denominator if fraction 1 correctly, (3) participant represents/explains the numerator of fraction 2 correctly, (4) participant represents/explains the denominator of fraction 2 correctly, (5) participant checks for the common denominator (by stating or looking), (6) participant multiplies the smaller denominator to get common denominators (if needed), (7) participant changes the numerator to match the denominator (by multiplying it with the same number as the denominator), (8) participant shows the denominator for the answer correctly, and (9) participant add/subtracts the numerators and represents/writes the numerator for the answer correctly (Appendix G).

Procedural Knowledge

Procedural knowledge of addition and subtraction of like and unlike fractions was chosen as the second dependent variable in this study. For the purpose of this study, the procedural knowledge was defined as the participant’s ability to solve a mathematical task or problem accurately (Bottge, 2001; Carnine, 1997; Goldman et al., 1997). For example, the participant can solve a given addition or subtraction fraction problem with the correct numerator, denominator, and the line in the middle to show the fraction. The participant scored one point each for the correct numerator, line in the middle, and correct denominator. The participants could score a maximum of three points for each problem correctly solved for like fractions. For the unlike fractions, the participants could score a maximum of five points for each problem correctly solved. They scored one point each for multiplying the smaller denominator to get a common denominator, correct numerator (by multiplying with the same number as denominator), correct numerator for the answer, line in the middle, and correct denominator for the answer. No points were given for a missing answer or unsolved problems.

According to Rittle-Johnson and Alibali (1999), procedural and conceptual knowledge are connected. Several studies have shown that gains in one type of knowledge might lead to gains in another. Little is known about the impact of explicit instruction with manipulatives on the procedural or conceptual knowledge of participants with mild to moderate autism. Therefore, the researcher chose to study the procedural knowledge separate from the conceptual knowledge.

Time Taken to Solve the Problems

The third dependent variable was the time taken to solve the problems (procedural knowledge probes) for like and unlike fractions. For the current study, the time taken was measured as the time to solve the 10 problems on the procedural knowledge probe. This was measured from the time the participant put the pencil to the paper to solve the 1st problem until he picked up the pencil after completing the 10th problem on the paper. The time included for writing the date was not counted toward the total time taken to solve the 10 problems. The researcher chose to measure the time to see if there were any changes in time taken to solve the problems during and after the intervention. The researcher also wanted to study if the time taken had any effect on the accuracy of the problems solved in each phase.

Participant Attitudes

In addition to the variables that could be directly measured, the researcher was interested in the attitudes and perceptions of participants toward instruction with manipulatives. This information was collected through interviews, field notes, and recordings of the sessions. All the baseline, intervention, generalization and maintenance sessions were recorded using an iPad to capture participant comments, behaviors, and performance. Qualitative data was used to validate the findings of the quantitative measures.

Materials

The materials for the study included recruitment flyers, letter of informed consent, and letter of participant assent. The content of the recruitment flyer served as the content for the recruitment email. Additional materials used were parent screening interview, participant-screening test, pre- and postintervention parent interview protocols, pre- and postintervention participant interview protocols, and field observation guide (Appendix H). Teaching materials included researcher-made fraction circles; store-bought colored chips; lesson plans with examples; and baseline (procedural and conceptual), intervention, generalization, as well as maintenance probes for like and unlike fractions. Detailed information about the probes is provided subsequently. Additionally, conceptual knowledge scoring rubric for like and unlike fractions, and treatment fidelity checklist (Appendix I) were also used. An iPad was used to record the sessions, a dry erase board with marker was used to present the problems, and a stopwatch was used to note the time taken to complete the procedural knowledge probes.

Preintervention and Postintervention Parent Interviews

Parent interviews were completed prior to and after the intervention. The preintervention parent interview included questions about demographic information, participant's current age, age at which the participant was diagnosed with autism, IQ, strengths, weaknesses of participants, kind of placement at school, academic performance level, grades, and questions about their child's math learning. Postintervention parent interviews were similar to preintervention interviews in a few items but included additional questions related to the effectiveness of the intervention, fractions, and items related to social validity. It also included about their child's feelings toward the intervention (if participant reported anything positive or negative about the intervention or their math learning). Both pre- and postintervention parent interviews were semistructured and included some open-ended as well as multiple-choice questions. The multiple-choice questions were constructed using a 5-point Likert scale (Appendices J and K).

Participant Screening Test

The participants took a screening test that included five single-digit addition problems, five single-digit subtraction problems, and two fraction problems (one addition and one subtraction of like fractions). According to NMAP (2008), the knowledge of basic math facts is a prerequisite skill to learn fractions. A score of 80% or more on the participant-screening test was used as one of the inclusion criteria to participate in the study. Only the addition and subtraction problems were scored. The two fraction problems were not counted toward the screening test score (Appendix L), but were used to document that potential participants were not already skilled in the target behavior. The potential participant took the screening test during the first meeting when the researcher completed the records review checklist.

Preintervention and Postintervention Participant Interviews

Participant interviews were completed pre- and postintervention. Preintervention participant interviews included questions about their interests, strengths, subjects they like most, etc. These interviews also served as a rapport builder. The postintervention interviews included specific questions about the participant experiences during intervention, perception of instruction with manipulatives, feelings toward fractions, math, and knowledge gained if any from the intervention. There were two types of items included on the participant interviews: open-ended questions and multiple-choice questions.

Some of the multiple-choice items used were adapted from Attitude Assessment Questionnaire used by Butler et al. (2003). Permission via email was obtained to use these items. These items were constructed using a 3-point Likert scale. Three pictures of faces that represented different feelings were provided as choices. The first picture was a smiley face, the second was a straight face (neither a smile nor a frown), and the third was a face with a frown. The three pictures were labeled as “Agree,” “Undecided,” and “Disagree.” Participants had to circle the picture of the face that best matched their feelings. All the questions were read to the participants (Appendices M and N).

Probes for Baseline, Intervention, and Generalization Phases

Since the current study had two different experiments embedded in it, two different types of probes were used. The probes for the first experiment had problems with addition and subtraction of like fractions and the probes for experiment two had problems with addition and subtraction of unlike fractions. The probes for baseline, intervention, generalization, and maintenance phases were similar in format and difficulty level. Two different probes were administered in each session to measure conceptual knowledge and procedural knowledge.

Like fractions. The researcher created a pool of questions with fraction items related to addition and subtraction of like fractions. The problems were modeled after the problems used in Rational Number Project (Cramer et al., Lesh, 2009). The problems from the question bank were randomly assigned to the different probes. Each of the probes were printed on an 8x11" piece of white paper with black printing on it. The probes were typed using Times New Roman font and 24-point font size. The problems were presented in two columns and all the problems were printed on one side of the paper. The appearance of the probes was suggested by the experts validating the materials as described in that section.

The procedural knowledge probe for like fractions included 10 problems. The problems for addition and subtraction of like fractions were presented in a mixed format. There were five addition and five subtraction problems on each procedural knowledge probe (Appendix O).

The conceptual knowledge probe had total four problems, two addition and two subtraction problems each. Fewer problems were presented on the conceptual probe because it required the participant to show and explain how he or she solved each problem. The conceptual knowledge probes were scored based on the scoring rubric for like fractions (Appendix P).

Unlike fractions. The fractions used for this study were Type 2 fractions (Test & Ellis, 2005). These are unlike fractions but the smallest denominator can be divided into the largest denominator an even number of times. Two different probes were administered in each session to measure conceptual knowledge and procedural knowledge. The procedures used for creating the probes for unlike fractions were similar to like fractions.

The procedural knowledge probe for unlike fractions also included 10 problems (Appendix Q). The problems for addition and subtraction of unlike fractions were presented in a mixed format. There were five addition and five subtraction problems on each procedural knowledge probe. All the probes were printed on 8x11" white paper in blank ink. The problems for both probes were presented in two columns using Times New Roman font and 20-point font size for the procedural knowledge probe. For procedural knowledge, six problems were presented on one side of the sheet and four problems were presented on the second side.

The conceptual knowledge probe had two problems each, an addition problem and a subtraction problem with a 24-point font size (Appendix R). Fewer problems were presented on the conceptual probe because it required the participants to show and explain how they solved each problem. The conceptual knowledge probes were scored based on the scoring rubric for unlike fractions.

Maintenance probes. In addition to the specific probes for like and unlike fractions, mixed probes for conceptual and procedural knowledge were also administered during the maintenance phase. The procedural knowledge probe had 10 problems. This probe had addition and subtraction problems with both like fractions and unlike fractions. The conceptual knowledge probe had four problems: two for like fractions and two for unlike fractions. A specific rubric was developed to score the mixed maintenance probes of conceptual knowledge of like and unlike fractions.

Validation of Probes and Rubrics

The problems for the conceptual and procedural knowledge probes of like and unlike fractions and the conceptual knowledge-scoring rubrics were checked by two expert teachers: a special education and a general education teacher. The general education teacher has 33 years' experience in teaching and has a master's degree in education. She holds an elementary school teaching license. The special education teacher has 15 years' experience working with students with disabilities in elementary school. She is highly qualified and holds a master's degree in special education.

The two teachers reviewed all the addition and subtraction problems of like and unlike fractions in the question bank. They checked these problems for content validity to ensure that all the problems were of similar difficulty level. They also reviewed the procedural and conceptual probes for both like and unlike fractions for face validity.

Face validity was the way the probes looked. The teachers recommended adding more space for the conceptual probes for unlike fraction because usually students needed more space to show their work for unlike fractions. Since participants in the study were students with mild to moderate autism and sometimes tend to write bigger, the teachers suggested adding extra space to account for these differences. Teachers recommended changing the denominators of some problems on unlike fractions. These problems seemed to be more difficult in level than the others because of bigger denominators. The denominators of those problems were changed.

The teachers reviewed the conceptual knowledge scoring rubrics also. No changes were recommended for the rubric for like fractions. An additional step of checking if the denominators are same or not by looking or stating was added to the unlike fractions rubric, based on the feedback from the expert teachers. Adding this step ensured that participants understood that the total number of parts had to be the same in both fractions.

Manipulatives

Two different manipulatives were used in this study to support student learning: (a) researcher-made fraction circles, and (b) colored chips. Two different types of manipulatives were used to teach fractions as part of a shape and part of a set. The researcher made the fraction circles using laminated construction paper of different colors. These fraction circles were modeled after the fraction circles used in the Rational Number Project (Cramer et al., 2009). Different colors represent different fraction pieces. Written permission via email was obtained to adapt the fraction circles. Additionally, colored chips with two different colors, red on one side and yellow on the other side, were also used to teach the concept. Although the fraction circles and colored chips were based on the manipulatives used in RNP, different colors were used for the fraction circles (see Figure 2.).

Procedures

The procedures delineated below include the activities completed before, during, and after the intervention. For each session, the researcher checked the iPad to ensure that it had enough memory and it was charged. For each session, the researcher set up the iPad to record the session.

Random Assignment of Participants to Tiers

Participants were randomly assigned to each tier of intervention. The random assignment of the participants to the four tiers of the multiple-baseline conditions was conducted using the following procedure. The researcher put the names of the six participants on index cards and placed each card in an envelope. Then the researcher wrote numbers one to four on separate cards. The researcher mixed up the envelopes and put one envelope each on the four numbers. The remaining two envelopes were placed on numbers three and four. So numbers one and two were matched with one envelop each; numbers three and four were matched with two envelopes each. The number determined the order in which the participants received the intervention in the staggered baseline design. One participant each was assigned to tiers one and two and two participants each were assigned to tiers three and four respectively.

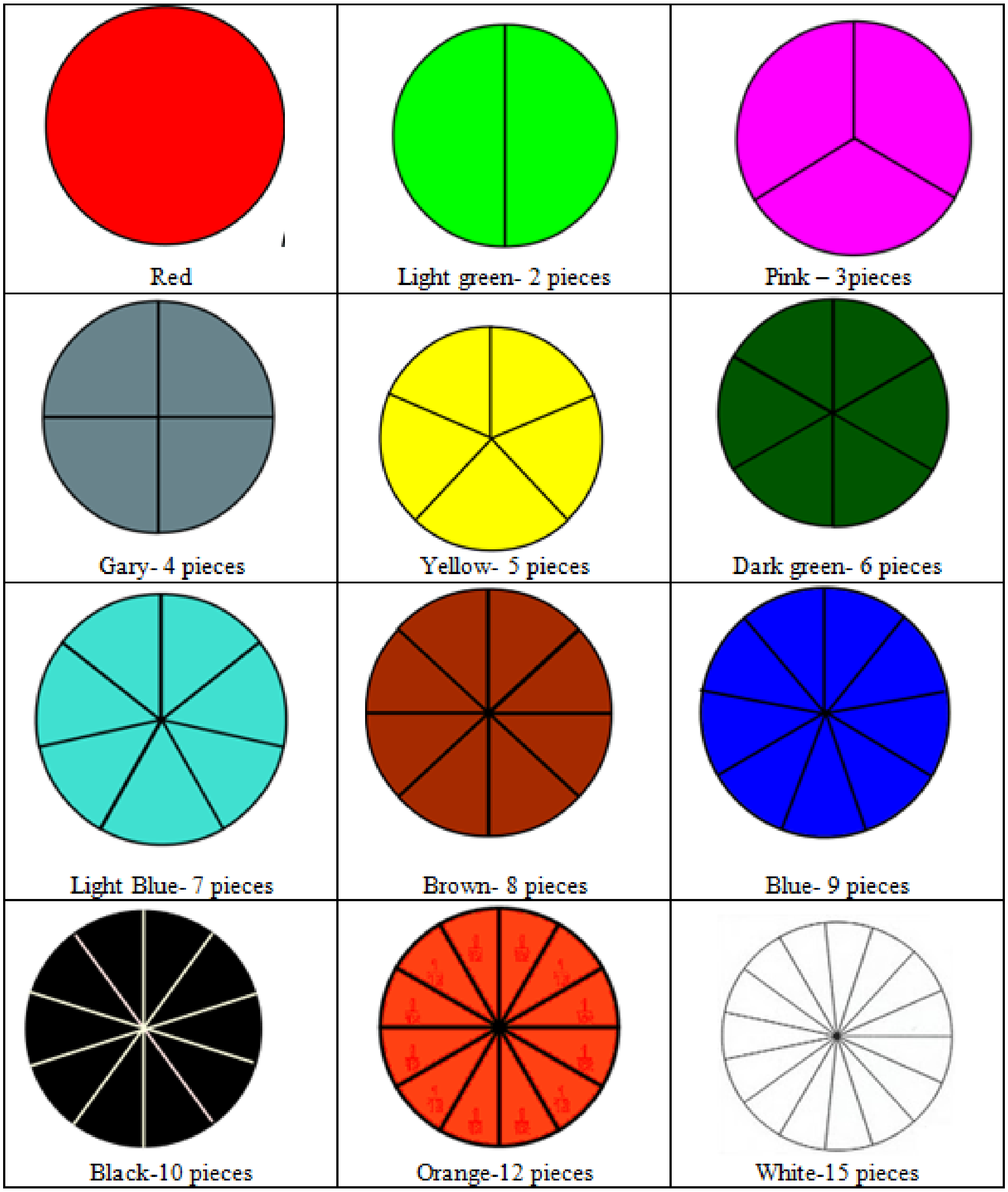

*Figure 2***.** Researcher-created fraction circles.

The intervention was introduced to each successive tier, two sessions after the introduction to the previous tier. The research base for math interventions is extremely limited and the few studies that have been done had only two to three participants each. Researchers have found that it is hard to assess the mathematical abilities of students with autism because behavioral issues might interfere with testing procedures (Whitby & Mancil, 2009). Based on the literature findings, in this multiple-baseline study, the intervention was introduced to the next tier, two sessions after the first tier rather than at the mastery criteria of the first participant. Intervention was withdrawn for all participants after the last tier had five data points in the intervention phase to meet the SCD design standards (Kratochwill et al., 2010). The current study was comprised of two experiments. The first experiment was on addition and subtraction of like fractions and the second experiment was on addition and subtraction of unlike fractions. Each experiment included baseline, intervention, generalization, and maintenance phases.

There were approximately 19 sessions in each experiment including the baseline, intervention, and generalization phase. Each session was a one-on-one session with the participant and scheduled for approximately 40-50 minutes but ended when the lesson plan was completed. The participant was reminded that the sessions would be recorded.

Experiment 1

Interviews. The researcher met with the individual families and completed the preintervention participant and parent interviews. Interviews served as rapport builders with the participants. During the interview session, the researcher interacted with the participants in their home setting. This session was used to make the participants comfortable with the researcher. After interviews were completed, baseline testing was started.

Baseline. In each baseline session, the researcher met with the participant in their home setting. During the baseline sessions, participants completed two probes without manipulatives. A procedural knowledge and a conceptual knowledge probe for addition and subtraction of like fractions were completed during each session. Baseline sessions were shorter than the intervention sessions and lasted 5 to 10 minutes each. The researcher told the participant, "I want you to complete these two worksheets and do the best you can." Participant selected the probe (conceptual vs. procedural) that he or she wanted to do first. After participant selected the probe, the researcher read the specific directions for the probe. The participant had to solve the problems on the procedural knowledge probe and they had to explain how they solved the problem for the conceptual knowledge probe. No assistance or feedback was provided to the participants as they completed the baseline probes. Session length varied based on the time taken by the participant to complete the probes.

Procedural knowledge probes were timed but the conceptual knowledge probes were untimed. The researcher recorded the time taken to solve the procedural probe using a stopwatch. The stopwatch was started when the participant touched the pencil to the paper to solve the first problem and stopped when the participant removed the pencil after completing the last problem on the baseline probe.

Intervention. After the first participant completed five baselines sessions, and given that the baseline data were stable for the first participant, the intervention was introduced to the first-tier participant while other participants continued in the baseline phase. The researcher was unsure of the participant's level of exposure to fractions, so the initial lesson was an introduction. During this lesson, the participant learned the concept of fraction, naming fractions and related vocabulary (numerator and denominator). For the independent phase of the initial lesson, the participant completed a worksheet on identifying numerator or denominator for the given fraction and reviewing the vocabulary terms.

After the initial lesson, the intervention was introduced to the first participant. Similar to the baseline sessions, the researcher met with the participants in their homes. The researcher sat next to the participant during instruction and introduced the lessons. The researcher followed the script and taught lessons as described above in the independent variable section (also available in Appendices D and E).

The intervention was introduced to each successive tier, two sessions after the introduction to the previous tier given the stability of baseline data. The research base for math interventions is extremely limited and the few studies that have been done had only two to three participants each. Researchers have found that it is hard to assess the mathematical abilities of students with autism because behavioral issues might interfere with testing procedures (Whitby & Mancil, 2009). Based on the literature findings, in this multiple-baseline study, the intervention was introduced to the next tier, two sessions after the first tier rather than the mastery criteria of the first participant. Intervention was withdrawn for all participants after the last tier had five data points in the intervention phase to meet the SCD design standards (Kratochwill et al., 2010). The total intervention time across participants ranged from 175 to 375 minutes across participants based on the number of sessions for the tier (range of sessions: 5-11). The average intervention time across participants was 278 minutes with each intervention session averaging 35 minutes.

Generalization. Immediately following withdrawal of intervention, three generalization probes were administered. Generalization procedures were identical to baseline procedures. The probes were similar in content and difficulty to the baseline probes and were completed without manipulatives and researcher assistance. For the procedural knowledge probe, the participant was instructed to solve the problems and for the conceptual knowledge probe, the participant was instructed to explain how they solved the problem. The reason that these probes are called generalization probes is because solving like fractions without the use of manipulatives showed that participants were able to generalize what they have learned in the intervention phase to working with the worksheet without the manipulatives.

Experiment 2

After the completion of the generalization phase for experiment one, experiment two was started. Procedures and phases (baseline, intervention, and generalization) for experiment two were identical to experiment one. Participants continued in the same tiers as experiment one. The only difference between experiment one and experiment two was the fraction content taught. Addition and subtraction of like fractions was taught during experiment one and addition and subtraction of unlike fractions was taught for experiment two. Immediately following the generalization phase of experiment two, maintenance data was collected.

Maintenance

The maintenance data were collected for both conceptual and procedural knowledge of addition and subtraction of like and unlike fractions. It was collected at 4 weeks for like fractions and 2 weeks for unlike fractions, after withdrawal of the intervention. Maintenance probes for both like and unlike fractions were similar to the baseline probes and followed the same format and procedures. In addition to the specific maintenance probes for experiment one and two (one probe with only like fractions and one probe with only unlike fractions), a mixed probe for procedural and conceptual knowledge was also administered. The mixed probe included addition and subtraction problems of like and unlike fractions. Participants completed the maintenance probes without manipulatives.

Interviews

After the completion of the maintenance phase, the researcher completed the postintervention participant and parent interviews. All the interviews were recorded. The researcher read the questions on the participant interviews to them to ensure consistency.

Validity and Reliability

In this section, information about fidelity of treatment and reliability of scoring of dependent measures is presented. Fidelity of treatment in this study refers to the procedures implemented during the intervention phase. Interobserver agreement for the baseline, intervention, generalization, and maintenance probes (procedural and conceptual) was also assessed.

Fidelity of Treatment

According to Horner et al. (2005), it is important to collect data demonstrating fidelity of implementation since independent variable is applied over time. This can be done directly by measuring the independent variable. Fidelity of implementation is also referred to as treatment reliability or treatment fidelity. By collecting data on fidelity of implementation, the researchers ensure that the results of a study can be attributed to the intervention.

Treatment fidelity was measured based on delivery of instructional components during the intervention session. A fidelity checklist listing the instructional components, use of an advance organizer (activating prior knowledge, stating the objective of the lesson, identifying the rationale of the lesson), implementing teacher modeling (demonstration), guided practice, problem solving, and independent practice was used.

Observer training. The observer was a retired elementary school teacher with 33 years of experience teaching all subjects to elementary school students. The researcher met with the observer in a quiet room at a public library. The researcher explained the five components of the lesson to the observer and shared the operational definitions of each component. Then, researcher and observer watched the training video created by the researcher while modeling a lesson. The observer completed the checklist based on the instructional components followed during the video. The researcher answered observer questions related to the rationale and objective in the advance organizer part.

Fidelity of treatment was calculated by dividing the number of completed steps by the total number of planned steps and multiplying it by 100. This was completed for 30%

of the sessions of like fractions and unlike fractions for each participant. Recordings of the intervention sessions were used to complete the fidelity checklists. The mean fidelity of treatment score was 97.22% (range = 85.71%-100%) for the all participants across like and unlike fractions: Jacob (*M* = 98.62%), Paulina (*M* = 97.62%), Wilton (*M* = 100%), Sam (*M* = 96.43%), Kyle (*M* = 100%), and Brad (*M* =96.43%). Fidelity of treatment was 85.71% for one of the sessions with Brad. During that session, it was noted that Brad made several off-topic comments about a new animal that he found on the Internet, which interfered with his attention skills. The researcher could not complete the initial steps of providing rationale for the lesson.

Interobserver Agreement

Scorer training. Two independent scorers were trained to score the probes across phases and participants for like and unlike fractions. The first observer was trained to score the conceptual probes and the second observer was trained to score the procedural probes for like and unlike fractions. The researcher met with the observers in a quiet room at a public library to conduct the training.

Conceptual. The observer was an instructional assistant in a special education classroom with 13 years' experience working with students with learning disabilities. The conceptual probes for each phase were scored using the videos because it required explanations of the steps that were not evident from the worksheets. The researcher first explained the scoring rubric for the like fractions to the observer. Then the researcher and the observer watched a researcher-created training video related to like fractions. Both researcher and observer scored the conceptual probe based on the recording and compared their scores. The agreement was 98.6% during the training session for like fractions, so no further training was conducted. The same process was repeated for conceptual knowledge of unlike fractions. The agreement for conceptual knowledge of unlike fractions was 92.3% during the training. The discrepancies were discussed to obtain 100% agreement.

The percentage of agreement for scoring was calculated using point-by-point formula, by number of agreements divided by the sum of agreements and disagreements and multiplying it by 100. This was completed for 30% of probes for each phase for all participants. The mean interobserver coefficient of agreement was calculated to be 95.14% (range = 92.85%- 97.45%) for conceptual knowledge of like fractions for all participants: Jacob (*M* = 93.87%), Paulina (*M* = 92.85%), Wilton (*M* = 95.4%), Sam (*M* = 95.4%), Kyle (*M* = 97.45%), and Brad (*M* = 95.92%). The mean interobserver coefficient of agreement was calculated to be 92.59% (range = 88.89%- 96.83%) for conceptual knowledge of unlike fractions for all participants: Jacob (*M* = 89.68%), Paulina (*M* = 88.89%), Wilton (*M* = 94.44%), Sam (*M* = 96.03%), Kyle (*M* = 96.83%), and Brad (*M* = 89.68%).

Procedural. The procedural knowledge probes for each phase were scored using the worksheets. The observer compared the probes against the answer key provided by the researcher for each probe. The observer was a general education teacher with nine years' experience working with students at an elementary school. The researcher explained and showed how the problems were scored to the observer. The scorer had to assign the points by comparing the participant's response on the probe with the answer key. The researcher and the observer scored a probe for like fractions independently and compared their results. There was 100% agreement for like fractions and 96% agreement for unlike fractions during the training session. The same process was repeated for the unlike fractions. There was 96% agreement for unlike fractions. The researcher and scorer discussed the differences in scoring. Once the training was completed, the observer scored 30% of the procedural knowledge probes for like and unlike fractions for each participant and phase.

The percentage of agreement for scoring was calculated using point-by-point formula, by number of agreements divided by the sum of agreements and disagreements and multiplying it by 100. For procedural knowledge of like fractions, the mean interobserver coefficient of agreement was calculated to be 99.3% (range = 98.8%-00%) for all participants: Jacob (*M* = 100%), Paulina (*M* = 100%), Wilton (*M* = 98.8%), Sam (*M* = 99.4%), Kyle (*M* = 98.8%), and Brad (*M* = 98.8%). The mean interobserver coefficient of agreement was calculated to be 96.48% (range = 94.29%-98.29%) for procedural knowledge of unlike fractions for all participants: Jacob (*M* = 94.57%), Paulina (*M* = 94.29%), Wilton (*M* = 97.14%), Sam (*M* = 96.86%), Kyle (*M* = 98.29%), and Brad (*M* = 97.71%). The observer had difficulty reading some of the numbers written on the probe, which made it harder for the observer to score those numbers. Matson, Matson, and Beighley (2011), found that students with autism have motor impairments. McCoy (2011) suggested that some students with autism have difficulty with reading and writing numbers. This explains some disagreements on scoring procedural probes.

Social Validity

Since the emergence of single-subject design, it has been important to address social validity especially when an intervention is used as an independent variable in the study (Horner et al., 2005). This data can be gathered by interviewing the participants and the parents or the guardians. Baer, Wolf, and Risley (1968) stated it is important that data be collected on “behaviors that are socially important rather than convenient for the study” (Gast, 2009, p.102). Social validity data can add to the intervention effectiveness data based on the responses of the participants. Social validity of a study can be improved by: (a) selecting dependent measures that are important, (b) showing that the intervention can be implemented by other personal across settings, (c) showing that the participants feel that the intervention is important, doable, and effective, and (d) that intervention produced the desired effect (Horner et al., 2005). In the current study, the social validity data was collected from the post intervention parent and participant interviews. Additional information on social validity emerged from field notes and session recordings.

Data Analysis

In order to investigate the functional relations and answer research questions addressed by the study, an analysis of the procedural, conceptual knowledge and the time taken to complete the probes was conducted. The effectiveness of the explicit instruction with manipulatives was determined through a visual analysis of data, percentage of nonoverlapping data (PND), and qualitative analysis of interviews, field notes and session recordings. Table 3 outlines the research questions, data collection procedures and corresponding analysis methods.

Visual Analysis

Visual analysis is the most commonly used method for analyzing data from a single-subject intervention study (Gast, 2010; Horner et al., 2005). It allows researchers to analyze data and find patterns or trends in an individual's data or small group's data. Data can be analyzed frequently using this strategy and results from data-analysis can be used to drive instruction or other related decisions. Since data is presented visually on a graph, it provides individuals with an opportunity to analyze the data independent of the actual analysis done for the study.

A visual analysis was conducted for the data collected for each dependent measure. The percentage of points earned for procedural and conceptual knowledge baseline, lesson, generalization, and maintenance probes for experiment one and two were charted using simple line graphs for each participant separately. These graphs served as a summary of the participant's performance. The data were analyzed across phases for each participant and across participants based on the following six features: (1) level, (2) trend, (3) variability, (4) immediacy of the effect, (5) overlap, and (6) consistency of data, for each participant and across participants (Kratochwill et al., 2010). Level is the average performance of a participant during a phase of the study. Trend refers to the slope of the best-fit line within each phase. Variability refers to the fluctuation in the performance of a participant relative to the mean or trend (Gast, 2010; Horner et al., 2005). Immediacy of the effect is the change in level between the last data points of the previous phase and the first data points in the subsequent phase for each participant. Overlap refers to the part of data in one phase that overlaps with the data in the next phase and can be analyzed using Percentages of Nonoverlapping Data (PNDs). Consistency refers to the similarity of data patterns within phases across participants.

Percentage of Nonoverlapping Data (PND)

PND is a nonparametric strategy used to analyze data from single-subject research to examine the change from once condition to the other by comparing the nonoverlapping data points from one phase to the other (Scruggs, Mastropieri, & Casto, 1987). It helps to compare the data of two adjacent conditions by calculating the percentage of nonoverlapping data points within phases of an intervention.

> If performance during an intervention phase does not overlap with performance during the baseline phase when these data points are plotted over time, the effects usually are regarded as reliable. The replication of nonoverlapping distributions during different treatment phases strongly argues for the effects of treatment. (Kazdin, 1978, p. 637)

PND’s were calculated using the following procedures. First, the range of data points of the baseline was determined. Second, the total number of data points for the intervention phase was counted. The third step involved determining the number of data points of intervention phase that fell outside the range of the baseline phase. Lastly, the number of data points of intervention phase that fell out of the range of baseline phase were divided by the total data points of intervention phase and multiplied by 100 to determine the percentage (Scruggs et al., 1987). The higher the PND, the more effective is the intervention. In this study, PNDs' were calculated individually for baseline to intervention, generalization, and maintenance phases for each participant and were used to describe the overlap of data in visual analysis.

Qualitative Analysis

The data from the interviews (parent and participant), field notes, and recordings of the sessions were analyzed to get information about social validity, participant attitudes, and perceptions related to the intervention. The interviews were semistructured and included multiple-choice and open-ended questions. Responses to open-ended questions for both parent and participant interviews were transcribed to obtain accurate responses. The qualitative analysis of interview transcripts and field notes was done using constant comparative analysis (CCA) technique (Merriam, 1998). The key words in the responses were highlighted and a matrix was created based on the categories and participant responses. Information related to setting, student behaviors and comments during sessions also emerged from the recordings and field notes. The repetitive themes across all interviews, field notes and recordings provided important information related to social acceptance and attitudes of the participants related to the intervention.

Summary

This chapter has provided a basis for understanding the study design, the participant characteristics, materials used, information about independent and dependent variables, procedures for all phases, data collection methods. Additionally, information related to the two attitudinal measures, parent and participant interviews were also included. Information from the data analysis for each participant and ancillary data is included in the next chapter.

Table 3

Methodology Summary

	Research Question	Type of Data Collection	Analysis Method
1.	Is there a functional relation between the explicit instruction with manipulatives and increase in level and trend of conceptual knowledge of addition and subtraction of like fractions for elementary school students with mild to moderate autism?	Baseline and intervention conceptual knowledge probes for like fractions	Visual analysis, PND
2.	Is there a functional relation between the explicit instruction with manipulatives and increase in level and trend of procedural knowledge of addition and subtraction of like fractions for elementary school students with mild to moderate autism?	Baseline and intervention procedural knowledge probes for unlike fractions	Visual analysis, PND
3.	Is there a functional relation between the explicit instruction with manipulatives and increase in level and trend of conceptual knowledge of addition and subtraction of unlike fractions for elementary school students with mild to moderate autism?	Baseline and intervention conceptual knowledge probes for unlike fractions	Visual analysis, PND
4.	Is there a functional relation between the explicit instruction with manipulatives and increase in level and trend of procedural knowledge of addition and subtraction of unlike fractions for elementary school students with mild to moderate autism?	Baseline and intervention procedural knowledge probes for unlike fractions	Visual analysis, PND
5.	Do elementary school students with mild to moderate autism show a change in the time taken for solving addition and subtraction problems with like and unlike fractions (procedural knowledge probes) after intervention with explicit instruction with manipulatives?	Time taken to complete procedural knowledge probes of like and unlike fractions	Visual analysis, PND

(continued)

Table 3. Methodology Summary (continued)

	Research Question	Type of Data Collection	Analysis Method
6.	Do elementary school students with mild to moderate autism generalize the procedural and conceptual knowledge of addition and subtraction of like and unlike fractions to abstract formats?	Procedural and conceptual knowledge generalization probes for like and unlike fractions	Visual analysis, PND
7.	Do elementary school students with mild to moderate autism maintain their procedural and conceptual knowledge of fraction skills (addition and subtraction of like and unlike fractions) overtime following the conclusion of the intervention phase?	Procedural knowledge and conceptual knowledge maintenance probes for like and unlike fractions	Visual analysis, PND
8.	What are the attitudes and perceptions of participants (students with mild to moderate autism) related to explicit instruction with manipulatives for learning fraction skills?	Pre- and postintervention parent and participant interviews, field notes and session recordings	Looked for patterns and triangulation of data sources

4. RESULTS

This chapter presents the results of the multiple-baseline single-subject research investigating the effectiveness of explicit instruction with manipulatives for increasing the conceptual and procedural knowledge of fractions of students with mild to moderate autism. The effects of explicit instruction with manipulatives are reported based on three dependent variables (procedural knowledge, conceptual knowledge of like and unlike fractions, and time taken) and additional measures for participant and parent perceptions and attitudes toward intervention.

Six participants with mild to moderate autism spectrum disorders participated in this study. Participants were randomly assigned to the four tiers of intervention and the intervention was introduced to each successive tier, two sessions after the introduction to the previous tier. One participant was assigned to tier one and one participant was assigned to tier two. Tier three included two participants and tier four had two participants. The current study is comprised of two experiments. The first experiment focused on addition and subtraction of like fractions and the second experiment focused on addition and subtraction of unlike fractions. Each experiment included baseline, intervention, generalization, and maintenance phases. Additional data were collected using participant and parent interviews, field notes, and video recordings of the sessions. In the baseline condition, data were collected through worksheets completed by participants. During the intervention phase, immediately following the teaching, the data were collected when participants completed worksheets using the manipulatives. The participants received an average of 278 minutes of instruction in fraction skills depending on the number of sessions (5-11) in the intervention tier. In the generalization phase, data were collected through completed worksheets without access to manipulatives and without explicit instruction of the abstract part of the C-R-A process. Maintenance data were collected for addition and subtraction of fractions with like and unlike fractions after 4 weeks and 2 weeks respectively, again without manipulatives and explicit instruction. Additional maintenance data were collected using worksheets with mixed problems of addition and subtraction of like and unlike fractions even though this blended skill had not been introduced at all. Data for the different sections are graphically represented and described across participants and for each participant using six different components: level, trend, variability, immediacy of change, overlap between phases, and consistency across phases (Kratochwill et al., 2010).

Experiment 1

For the first part of the study, the participants were taught addition and subtraction of like fractions using explicit instruction with manipulatives. Data were collected for conceptual knowledge, procedural knowledge, and time taken to solve the problems on the procedural knowledge probes for addition and subtraction of like fractions.

Conceptual Knowledge of Like Fractions

Conceptual knowledge was measured as the participant’s ability to explain or show their thinking based on the scoring rubric (described on page 80). The raw scores were then converted into percentages by dividing the points scored by the maximum points and multiplying it by 100. The percentages were calculated to obtain a consistent unit of measurement across items. Overall findings of conceptual knowledge indicate that after the intervention, five out of six participants could explain their reasoning or thinking for addition and subtraction of like fractions. During baseline, all participants had a mean of 0 for conceptual knowledge. On an average the participants increased in their level of conceptual knowledge to a mean of 54.7 (*SD* =13.57) during the intervention and generalized (*M* = 84.17, *SD* =14.53) the skills immediately following the withdrawal of the intervention (teaching and manipulatives). All participants maintained their performance after 4 weeks of withdrawal of intervention on the like fractions (*M* = 94.64, *SD* = 7.05) and mixed fractions probes (*M* = 88.25, *SD* = 17.05). The mean PND for all participants and across phases for conceptual knowledge of like fractions was 94.58%.

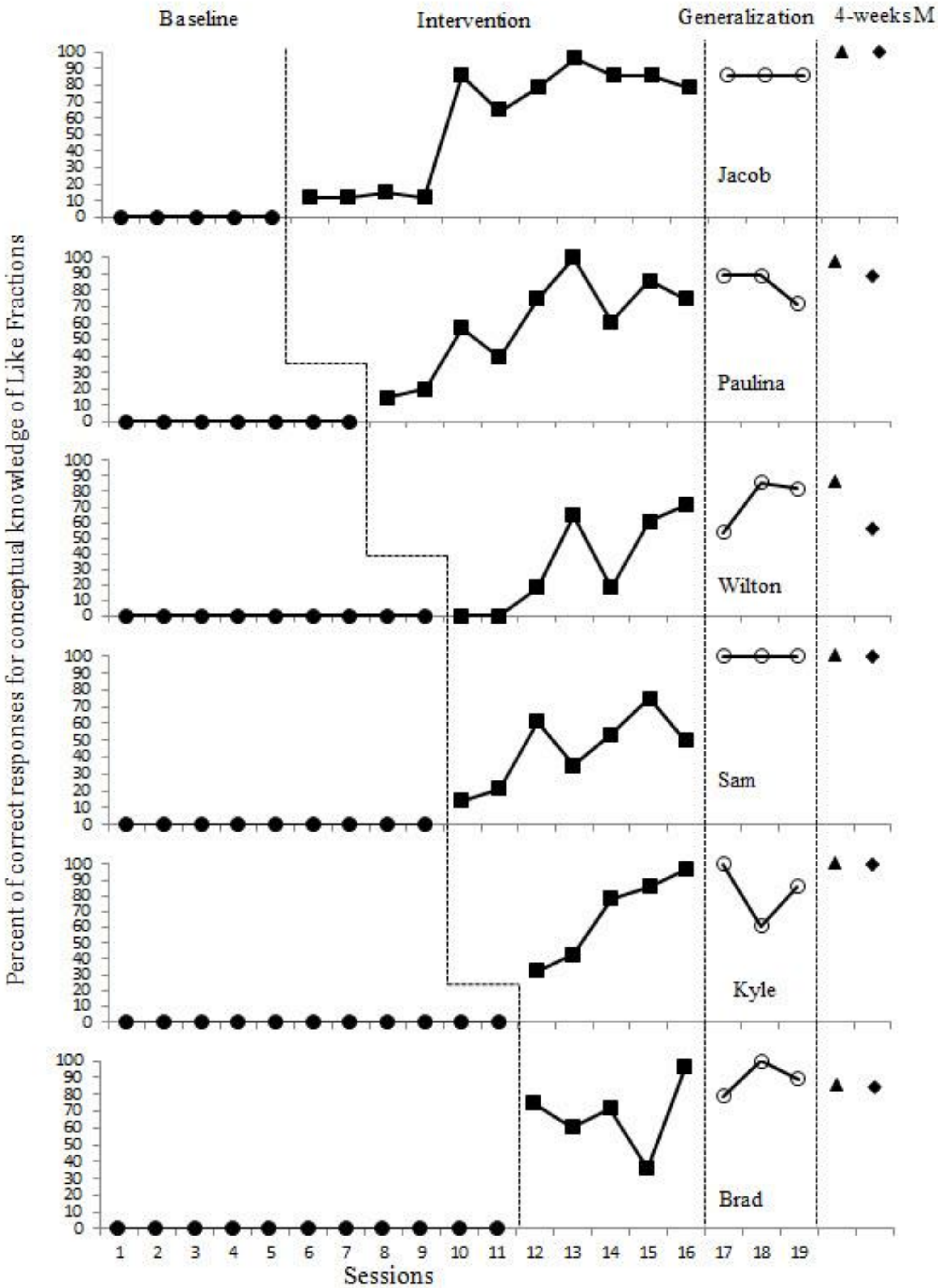

Figure 3. Conceptual knowledge of like fractions. The percentage of steps correctly explained for conceptual knowledge across baseline (●), intervention (■), generalization (○), four-week delayed maintenance (▲) and maintenance on mixed probe (♦) across six students with autism.

Figure 3. Conceptual knowledge of like fractions.

Jacob. As seen in Figure 3 (response to research question one), during all the sessions in the baseline condition, Jacob's performance on conceptual knowledge of addition and subtraction of like fractions was low (*M* = 0). There was no trend or variability in the baseline condition and his performance was consistently stable.

When the intervention was introduced, Jacob's performance demonstrated a change in level from baseline (*M* = 0.00, *SD* = 0.00) to intervention phase (*M* = 56.91, *SD* = 35.85). Immediately following the intervention, Jacob showed an upward trend in his conceptual knowledge. Compared to the trend line, the data had high variability. There was no immediacy of change from baseline to intervention. Jacob's performance increased rapidly after first four data points in the intervention phase. Overall, Jacob's performance was higher in the intervention phase as compared to the baseline phase. The PND was calculated to be 100% from baseline to intervention phase for Jacob.

Jacob generalized the conceptual knowledge of like fractions to abstract problems (*M* = 85.71, *SD* = 0.00) immediately following the withdrawal of intervention as seen in Figure 3 (response to research question six). The PND was calculated to be 100% from baseline to generalization and maintenance phases. Jacob maintained his performance at 100% for the conceptual knowledge after 4 weeks of withdrawal of intervention on the like fractions and mixed (like and unlike fractions) probes. His performance was consistently high across intervention, generalization, and maintenance phases.

Paulina. In response to research question one (Figure 3), Paulina demonstrated low performance across the seven sessions in the baseline condition (*M* = 0.00, *SD* = 0.00). Overall, Paulina's performance was flat and consistently stable in the baseline.

Upon introduction of the intervention, Paulina had a change in level between baseline (*M* = 0.00, *SD* = 0.00) and intervention (*M* = 58.65, *SD* = 29.06). Paulina showed small but immediate increase in her conceptual knowledge of like fractions that continued to increase gradually throughout the intervention phase (trend). Compared to the trend line, the data had somewhat high levels of variability. The PND was calculated to be 100% from baseline to the intervention phase for Paulina. Overall, the data in the intervention phase was much higher than the data in the baseline phase.

As seen in Figure 3 (research questions six and seven), Paulina generalized the conceptual knowledge of like fractions to abstract problems (*M* = 83.33, *SD* = 10.31), when the intervention was withdrawn. There was medium level of variability between the data points in the generalization phase with the last data point slightly decreasing compared to the other two data points. She maintained her conceptual knowledge at a high level even after 4 weeks of the withdrawal of the intervention on the like fractions (96.42%) and mixed fraction probes (88.89%). PND was calculated to be 100% from baseline to generalization and maintenance for Paulina. Overall, performance in the generalization and maintenance phases was much higher than in baseline and was consistent with the performance in the intervention phase.

Wilton. In response to research question one (Figure 3), Wilton's conceptual knowledge for like fractions was at a low level (*M* = 0.00, *SD* = 0.00) during the baseline condition. Across the nine sessions in baseline, Wilton's performance showed a flat trend. He had no variability and demonstrated low data in the baseline phase.

When the intervention was introduced, Wilton had a change in level between baseline (*M* = 0.00, *SD* = 0.00) and intervention (*M* = 33.16, *SD* = 31.25). Wilton had a gradual increase after the third session in the intervention phase showing an upward trend. Compared to the trend line, the data had somewhat high levels of variability. There was no immediacy of change from baseline to intervention but the performance started to increase after the second data point in intervention. The PND was calculated to be 71% from baseline to intervention for Wilton.

As seen in Figure 3 (research questions six and seven), Wilton generalized his conceptual knowledge to abstract problems (*M* = 73.81, *SD* = 17.62) at a level higher than baseline and intervention phases when the intervention was withdrawn. There was medium variability between the data points in the generalization phase with the last two data points increasing slightly compared to the first data point. At 4-week maintenance, he maintained his procedural knowledge at a medium level (85.71%) for the like fractions but his performance dropped on the mixed fraction probe (56.25%). PND was calculated to be 100% from baseline to generalization and maintenance phases for Wilton. Wilton's performance was consistently high across intervention, generalization, and maintenance phases although it decreased on the mixed maintenance probe.

Sam. In response to research question one (Figure 3), Sam's conceptual knowledge for like fractions was at a low level (*M* = 0.00, *SD* = 0.00) consistently during the baseline sessions. He had no variability across the baseline sessions because he scored 0 for all the baseline probes.

When the intervention was introduced, Sam had a change in level between baseline (*M* = 0.00, *SD* = 0.00) and intervention (*M* = 44.39, *SD* = 21.71). Sam had a small immediacy of change initially and then a continuously gradual increase in trend throughout the intervention phase. Compared to the trend line, the data had some variability across the intervention sessions. The PND was calculated to be 100% from baseline to intervention for Sam. His performance was consistently high across the intervention phase.

In response to research questions six and seven, Sam generalized his conceptual knowledge to abstract problems (*M* = 100, *SD* = 0) and attained the ceiling. There was no variability between the data points in the generalization phase. At 4-week maintenance, he maintained his conceptual knowledge at a high level (100%) for the like fractions and the mixed fractions probe. PND was calculated to be 100% from baseline to generalization and maintenance phases for Sam. His performance was consistently high across generalization and maintenance phases.

Kyle. In response to research question one (Figure 3), Kyle's conceptual knowledge for addition and subtraction of like fractions was at a low level (*M* = 0.00, *SD* = 0.00) consistently during the baseline sessions. He demonstrated no variability across baseline sessions. Overall, the data in the baseline sessions were consistently flat and stable.

When the intervention was introduced, Kyle had a change in level between baseline (*M* = 0.00, *SD* = 0.00) and intervention (*M* = 67.2, *SD* = 27.97). Kyle showed an immediacy of change in his conceptual knowledge which continued to increase gradually (upward trend) throughout the intervention session. Compared to the trend line, the data had very little variability and the PND was calculated to be 100% from baseline to intervention for Kyle. His performance was consistently high across intervention sessions.

In response to research questions six and seven, Kyle generalized his conceptual knowledge to abstract problems (*M* = 82.14, *SD* = 19.89) at a higher level than baseline and intervention phases. There was high variability in the generalization phase. The PND was calculated to be 100% from baseline to generalization and maintenance phases. Kyle maintained his performance at high level (100%) for conceptual knowledge even after 4 weeks of the withdrawal of intervention on the like fractions and mixed fraction probes. His performance was consistently high across generalization and maintenance phases.

Brad. In response to question one (Figure 3), Brad demonstrated low performance across the 11 baseline sessions (*M* = 0.00, *SD* = 0.00). He had no variability across the sessions in the baseline condition and the overall data were flat and stable.

When intervention was introduced, Brad had a change in level between baseline (*M* = 0.00, *SD* = 0.00) and intervention (*M* = 67.86, *SD* = 22.16). Brad's performance showed rapid immediate increase in his conceptual knowledge. Compared to the trend line, the data had some variability between the data points. He continued to score at a high level consistently in the intervention sessions. Brad's data demonstrated an upward trend in his performance on the conceptual knowledge of like fractions. The PND was calculated to be 100% from baseline to intervention for Brad.

As seen in Figure 3 (research questions six and seven), Brad generalized the conceptual knowledge of like fractions to abstract problems (*M* = 89.29, *SD* = 10.72), when the intervention was withdrawn. There was medium variability between the data points in the generalization phase. At 4-week maintenance, he maintained his conceptual knowledge at a high level on the like fractions and mixed fraction probes. PND was calculated to be 100% from baseline to generalization and maintenance for Brad. Overall, performance in the generalization and maintenance phases was higher than in baseline and was consistently high across maintenance and generalization phases.

Procedural Knowledge of Like Fractions

Procedural knowledge of the participants was measured as the points earned to solve the given math problems. The participants could earn up to 3 points for each problem solved correctly and a maximum of 30 points for the procedural knowledge probe. These points were converted into percentages for the ease of comparison of the scores. Overall findings of this study demonstrate that five out of six participants increased in their frequency to solve fraction problems correctly after the intervention. During baseline, participants had a mean of 19.06 (*SD* =14.15) for the procedural knowledge of addition and subtraction of like fractions that increased in level to a mean of 81.15 (*SD* = 21.1) after the intervention with explicit instruction with manipulatives. Overall, all six participants generalized the skills to abstract problems immediately after the intervention was withdrawn during generalization phase (*M* = 94.63, *SD* = 6.39). All participants maintained their performance even after 4 weeks of the withdrawal of intervention on the like fraction (*M* = 93.89, *SD* = 6.47) and mixed fraction (*M* = 80.95, *SD* = 24.19) probes. On the mixed fraction probes, the participants demonstrated extremely high variability with the scores ranging from 35.71% to 100%. The overall mean PND was calculated to be 95.48% across all participants and phases for procedural knowledge of like fractions.

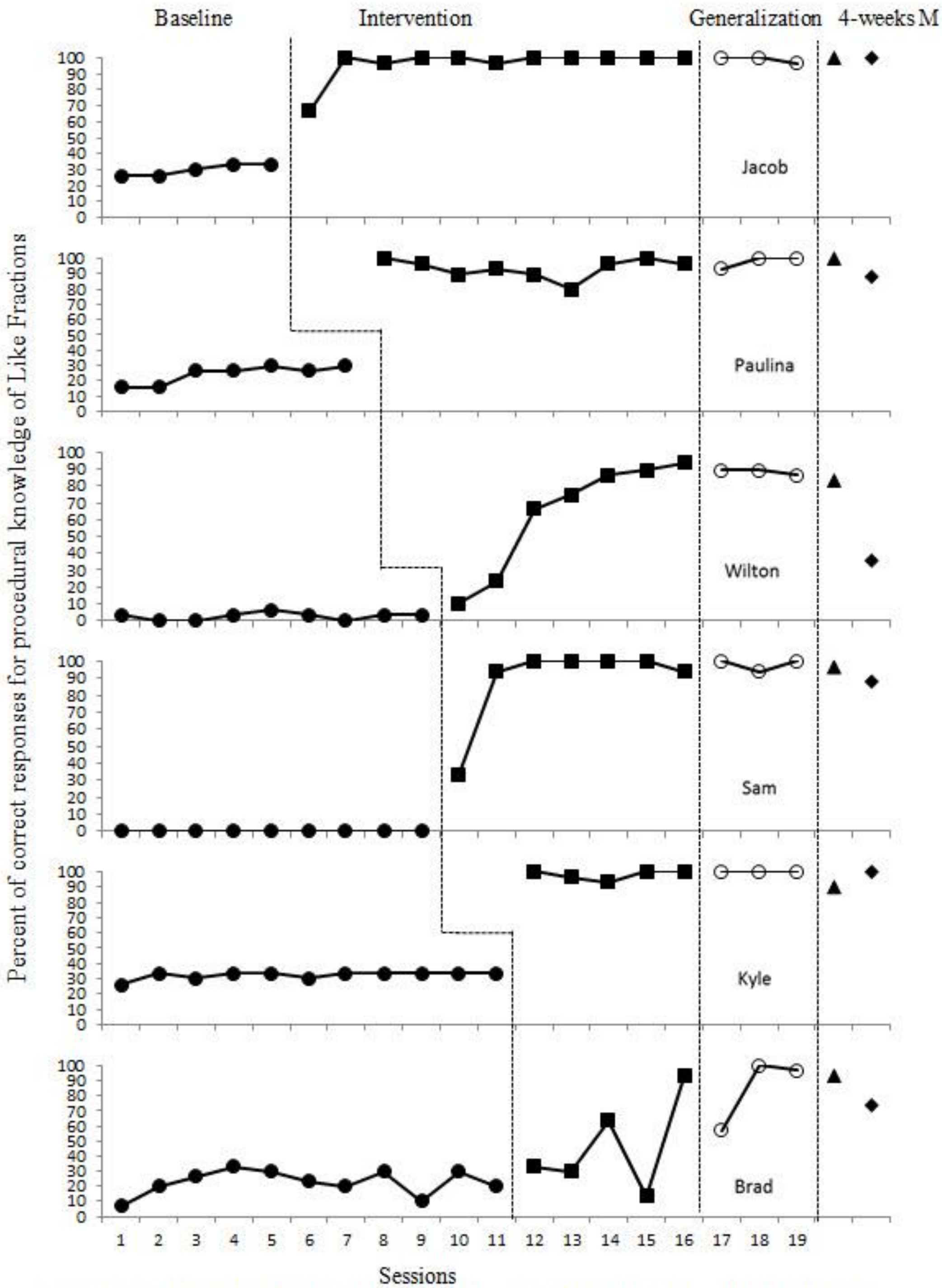

Figure 4. Procedural knowledge of like fractions. The percentage of steps correctly done for procedural knowledge across baseline (●), intervention (■), generalization (○), four-week delayed maintenance (▲) and maintenance on mixed probe (♦) across six students with autism.

Figure 4. Procedural knowledge of like fractions.

Jacob. As seen in Figure 4 (response to research question two), during all the sessions in the baseline condition, Jacob's performance on addition and subtraction of like fractions was low (*M* = 29.73, *SD*= 3.65). Across the five baseline sessions, Jacob demonstrated little variability and his performance was stable. Overall, the data in the baseline sessions were consistently flat and low.

When the intervention started, Jacob's performance depicted a change in level from baseline (*M* = 29.73, *SD* = 3.65) to intervention phase (*M* = 96.36, *SD* = 9.93). Jacob showed an immediate increase in his procedural knowledge from 33.33% to 66.67% that continued to increase showing an upward trend in the intervention phase. Compared to the trend line, the data had little variability. Jacob's performance showed an upward trend after the introduction of the intervention. Jacob's performance was consistent across the intervention phase. The PND from baseline to intervention phase was 100% for Jacob.

Jacob generalized the procedural knowledge of like fractions to abstract problems (*M* = 98.89, *SD* = 1.92) after the withdrawal of the intervention as seen in Figure 4 (response to research question six). There was minimal variability in the generalization phase. The PND was calculated to be 100% from baseline to generalization and maintenance phases. Jacob's performance was consistently high across phases (generalization, and maintenance). Jacob maintained his performance at 100% even after 4 weeks of the withdrawal of intervention on the like fraction and mixed (like and unlike fractions) probes.

Paulina. Paulina demonstrated low performance across the seven sessions in the baseline condition (*M* = 24.54, *SD* = 6.03). Paulina's performance showed slight increase in her procedural knowledge of addition and subtraction of like fractions starting with 16% and increasing to 30%. Although she had some variability across the sessions in the baseline condition, the baseline data were consistently low and stable.

Upon introduction of the intervention, Paulina had a change in level between baseline (*M* = 24.54, *SD* = 6.03) and intervention (*M* = 93.7, *SD* = 6.24). Paulina showed an immediate increase in her procedural knowledge of like fractions from 30% to 100% and maintained her performance at similar level throughout the intervention phase. Compared to the trend line, the data had low levels of variability and indicated a flat trend. The PND was calculated to be 100% from baseline to intervention phase for Paulina. Overall, the data in the intervention phase was consistently high.

As seen in Figure 4 (research questions six and seven), Paulina generalized the procedural knowledge of like fractions to abstract problems (*M* = 97.78, *SD* = 3.85), when the intervention was withdrawn. There was low variability between the data points in the generalization phase. She maintained her procedural knowledge at a high level (100%) even after 4 weeks of the withdrawal of the intervention although her performance dropped slightly for mixed fractions. PND was calculated to be 100% from baseline to generalization and maintenance for Paulina. Overall, performance in the generalization and maintenance phases was higher than in baseline and was consistently high across generalization and maintenance phases.

Wilton. In response to research question two (Figure 4), Wilton's procedural knowledge for like fractions was at a low level (M = 2.59, SD = 2.22) during the baseline condition. Across the nine sessions in baseline, Wilton's performance showed a flat trend. He had slight variability across the baseline sessions and consistently stable data in the baseline phase.

When the intervention was introduced, Wilton had a change in level between baseline (M = 2.59, SD = 2.22) and intervention (M = 63.57, SD = 33.53). Wilton showed a small immediate change in the performance. However, the data points after the second session showed a rapid increase in trend, which was consistent throughout the intervention phase. Compared to the trend line, the data had low level of variability. The PND was calculated to be 100% from baseline to intervention for Wilton.

As seen in Figure 4 (research questions six and seven), Wilton generalized his procedural knowledge to abstract problems (M = 88.89, SD = 1.92) at a level higher than baseline and intervention phases when the intervention was withdrawn. There was little variability between the data points in the generalization phase. At 4-week maintenance, he maintained his procedural knowledge at a medium level (83.33%) for the like fractions but his performance dropped on the mixed fraction probe (35.71%). PND was calculated to be 100% from baseline to generalization and maintenance phases for Wilton. Wilton's performance was consistently high across the generalization phase.

Sam. In response to research question two (Figure 4), Sam's procedural knowledge for like fractions was at a low level (M = 0.00, SD = 0.00) consistently during the baseline sessions. He had no variability across the baseline sessions because he scored 0 for all the baseline probes.

When the intervention was introduced, Sam had a change in level between baseline (*M* = 0.00, *SD* =0.00) and intervention (*M* = 88.57, *SD* = 24.56). Sam showed an immediacy of change and then a rapid increase in trend throughout the intervention phase. As compared to the trend line, the data had slight variability except the first data point. His performance was consistently high in the intervention phase. The PND was calculated to be 100% from baseline to intervention for Sam.

In response to research question six, Sam generalized his procedural knowledge to abstract problems (*M* = 97.78, *SD* = 3.85) at a level higher than baseline and intervention phases. There was little variability between the data points in the generalization phase. At 4-week maintenance, he maintained his procedural knowledge at a high level for the like fractions (96.67%) and the mixed fractions probe (88.10%). PND was calculated to be 100% from baseline to generalization and maintenance phases for Sam. He showed consistently high performance across generalization and maintenance phases.

Kyle. In response to research question two (Figure 4), Kyle's procedural knowledge for addition and subtraction of like fractions was at a medium level (*M* = 32.91, *SD* = 2.41) consistently during the baseline sessions. He demonstrated little variability across baseline sessions. Overall, the data in the baseline sessions were consistently flat and stable.

When the intervention was introduced, Kyle had a change in level between baseline (*M* = 32.91, *SD* = 2.41) and intervention (*M* = 98.00, *SD* = 2.99). Kyle showed an immediate increase in his procedural knowledge from 33.33% to 100%. Compared to the trend line, the data had little to no variability across the intervention sessions and consistently high data in the intervention phase. The PND was calculated to be 100% from baseline to intervention for Kyle.

In response to research questions six and seven, Kyle generalized his procedural knowledge to abstract problems (*M* = 100, *SD* = 0.00) at the ceiling level. There was no variability in the generalization phase. The PND was calculated to be 100% from baseline to generalization and maintenance phases. Kyle maintained his performance at 90% for procedural knowledge even after 4 weeks of the withdrawal of intervention on the like fractions and 100% for mixed fraction probes. Kyle's performance was consistently high across generalization and maintenance phases.

Brad. In response to question two (Figure 4), Brad demonstrated low performance across the 11 baseline sessions (*M* = 24.58, *SD* = 8.54). Although he had medium levels of variability across the sessions in the baseline condition, the baseline data were consistently low. Overall, the data in the baseline sessions were flat and stable.

When intervention was introduced, Brad had a change in level between baseline (*M* = 24.58, *SD* = 8.54) and intervention (*M* = 49.66, *SD* = 31.71). Brad's performance did not change immediately but showed a gradual upward trend for procedural knowledge. Compared to the trend line, the data had high levels of variability across the intervention sessions. The PND was calculated to be 60% from baseline to intervention for Brad. His performance was not consistently high across the intervention sessions.

As seen in Figure 4 (research questions six and seven), Brad generalized the procedural knowledge of like fractions to abstract problems (*M* = 84.44, *SD* = 24.11), when the intervention was withdrawn. There was high variability between the data points in the generalization phase with the first data point lower than the last two data points. He maintained his procedural knowledge at a high level (93.33%) even after 4 weeks of the withdrawal of the intervention on the like fractions and at medium level (73.81%) for mixed fraction probes. PND was calculated to be 100% from baseline to generalization and maintenance for Brad. Overall, performance in the generalization and maintenance phases was higher than in baseline.

Time Taken for Like Fractions

The time taken to solve the problems on the procedural knowledge probes for like fractions was noted using a stopwatch across all phases. The mean time taken to solve addition and subtraction problems with like fractions across participants was 2.79 min in the baseline phase. The participants scored 19.06% on an average in the baseline phase. There was medium variability in the time taken across data points in the baseline phase. Although, the average time taken to solve the problems on the procedural knowledge probes was similar during the intervention (*M* = 2.79, *SD* = 0.74) and generalization (*M* = 2.96, *SD* = 1.33) phases, the performance of all participants increased considerably as shown in Table 4. Overall, the participants scored 81.15% in the intervention and 94.63% in the generalization phase. Jacob, Paulina, and Kyle showed little variability, while Sam and Brad showed high variability, and Wilton showed medium variability in the time taken to solve the problems with like fractions from baseline to intervention phase.

Table 4

Time Taken vs. Procedural Knowledge Scores for Like Fractions

	Time Taken (in minutes)			Procedural Knowledge (%)		
	Baseline	Intervention	Generalization	Baseline	Intervention	Generalization
Participant	*M* (*SD*)	*M* (*SD*)	*M* (*SD*)	*M* (*SD*)	*M* (*SD*)	*M* (*SD*)
Jacob	1.60 (0.44)	3.44 (1.13)	3.00 (1.42)	29.73 (3.65)	96.37 (9.93)	98.89 (1.92)
Paulina	1.47 (0.29)	2.69 (0.85)	2.18 (0.12)	24.54 (6.03)	93.70 (6.24)	97.78 (3.85)
Wilton	2.63 (1.15)	2.36 (1.03)	2.18 (0.10)	2.59 (2.22)	63.57 (33.53)	88.89 (1.92)
Sam	5.26 (2.44)	2.34 (1.26)	4.37 (0.86)	0.00 (0.00)	88.57 (24.56)	97.78 (3.85)
Kyle	1.40 (0.54)	2.00 (0.46)	1.34 (0.20)	32.91 (2.41)	98.00 (2.99)	100.00 (0.00)
Brad	4.39 (1.71)	3.92 (1.52)	4.68 (0.50)	24.58 (8.54)	49.66 (31.71)	84.44 (24.11)
Total	2.79 (1.65)	2.79 (0.74)	2.96 (1.33)	19.06 (14.15)	81.15 (21.1)	94.63 (6.39)

Experiment 2

For the second part of the study, the participants were taught addition and subtraction of unlike fractions using explicit instruction with manipulatives. Data were collected for conceptual knowledge, procedural knowledge, and the time taken to solve the problems on the procedural knowledge probes for addition and subtraction of unlike fractions. The sessions for experiment two start at session 20 because this was a continuation of experiment one and prior to this; participants had already received 19 sessions of baseline, intervention, and generalization phases for like fractions.

Conceptual Knowledge of Unlike Fractions

The findings from the conceptual knowledge probes for addition and subtraction of unlike fractions show that five out of six participants improved in their ability to explain or show their thinking after the intervention with explicit instruction with manipulatives. During baseline, participants had a mean of 0.00 (*SD* = 0.00) for conceptual knowledge probes which increased in level to a mean of 67.25 (*SD* = 25.19) during the intervention. Overall, all six participants generalized (*M* = 91.05, *SD* = 12.8) the conceptual knowledge to generalization probes. All participants maintained the conceptual knowledge of unlike fractions (*M* = 90.74, *SD* = 14.77) and mixed fractions (*M* = 88.25, *SD* =17.05) 2 weeks after the withdrawal of intervention. The overall mean PND was 98.48% across all participants and phases for conceptual knowledge of unlike fractions.

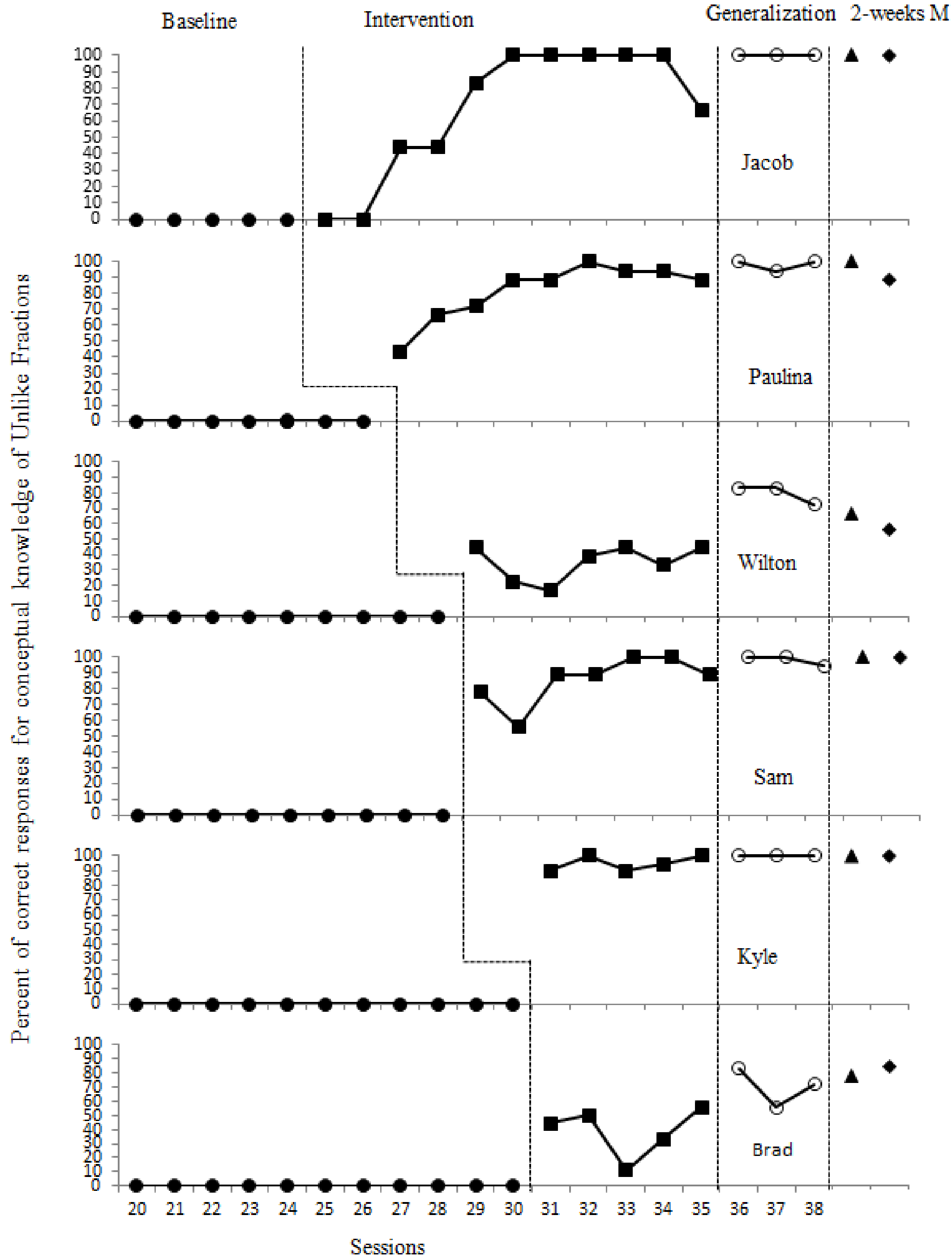

Figure 5. Conceptual knowledge of unlike fractions. The percentage of steps correctly done for conceptual knowledge across baseline (●), intervention (■), generalization (○), two-week delayed maintenance (▲) and maintenance on mixed probe (♦) across six students with autism.

Figure 5. Conceptual knowledge of unlike fractions.

Jacob. As seen in Figure 5 (response to research question three), during all sessions in the baseline condition, Jacob's performance on conceptual knowledge of addition and subtraction of unlike fractions was low (*M* = 0.00, *SD* = 0.00). Across the five baseline sessions, Jacob demonstrated no variability and his performance was stable. Overall, the data in the baseline sessions were consistently flat and low.

When the intervention started, Jacob's performance depicted a change in level from baseline (*M* = 0.00, *SD* = 0.00) to intervention phase (*M* = 67.06, *SD* = 39.67). Jacob showed a gradual upward trend in his conceptual knowledge during the intervention phase. There was no immediacy of change from baseline to intervention. Jacob's performance increased rapidly after first two data points in the intervention phase. As compared to the trend line, the data had some variability. The PND was calculated to be 89.81% from baseline to intervention phase for Jacob. Jacob's performance was consistent after an initial increase in the intervention phase.

Jacob generalized the conceptual knowledge of unlike fractions to abstract problems (*M* = 100, *SD* = 0.00) after the withdrawal of the intervention as seen in Figure 5 (response to research question six). There was no variability in the generalization phase. The PND was calculated to be 100% from baseline to generalization and maintenance phases. Jacob's performance was consistently high during the intervention, generalization, and maintenance phases. At 2-week maintenance, Jacob maintained his performance at high level (100%) for the unlike fractions and mixed fraction probes.

Paulina. As seen in Figure 5 (response to research question three), Paulina demonstrated low performance across the seven sessions in the baseline condition (*M* =

0.00, *SD* = 0.00). She showed no variability and her performance was stable. Overall, the data in the baseline sessions were flat and low.

Upon introduction of the intervention, Paulina showed a change in level between baseline (*M* = 0.00, *SD* = 0.00) and intervention (*M* = 82.04, *SD* = 17.86). Paulina showed an immediate change in her conceptual knowledge of unlike fractions. As compared to the trend line, the data had low variability. Paulina demonstrated a gradual upward trend in her performance. The PND was calculated to be 100% from baseline to intervention phase. Overall, the data in the intervention phase was consistent and much higher than the baseline phase.

As seen in Figure 5 (research question six and seven), Paulina generalized the conceptual knowledge of unlike fractions to abstract problems (*M* = 98.15, *SD* = 3.21), when the intervention was withdrawn. There was minimal variability between the data points in the generalization phase. She maintained her conceptual knowledge at a high level on unlike fractions (100%) and at 88.89% for the mixed fraction probes after 2 weeks of withdrawal of intervention. The PND was calculated to be 100% from baseline to generalization and maintenance phases. Paulina had consistently high performance across phases (generalization and maintenance).

Wilton. In response to research question three (Figure 5), Wilton's conceptual knowledge for unlike fractions was at a low level (*M* = 0.00, *SD* = 0.00) during the baseline condition. Across the nine sessions in baseline, Wilton's performance showed a flat trend. He had no variability across the baseline sessions and consistently low data in the baseline phase.

When the intervention was introduced, Wilton had a change in level between baseline (*M* = 0.00, *SD* = 0.00) and intervention (*M* = 34.92, *SD* =11.43). Wilton had an immediate increase in his performance but his performance followed an inconsistent and variable pattern in the intervention phase. His performance showed an upward trend after the introduction of intervention. As compared to the trend line, the data had medium level of variability. The PND was calculated to be 100% from baseline to intervention for Wilton. Wilton's performance was consistent across intervention sessions except two data points.

As seen in Figure 5 (research questions six and seven), Wilton generalized his conceptual knowledge to abstract problems (*M* = 79.63, *SD* = 6.41) at a level higher than baseline and intervention phases. There was some variability between the data points in the generalization phase with the last data point showing a slight decrease in performance. At 2-week maintenance, he maintained his conceptual knowledge at a medium level (66.67%) for the unlike fractions and low level for the mixed fraction probe (56.25%). PND was calculated to be 100% from baseline to generalization and maintenance for Wilton. His performance was consistently high during the generalization phase.

Sam. In response to research question three (Figure 5), Sam's conceptual knowledge for unlike fractions was at a low level (*M* = 0.00, *SD* = 0.00) consistently during the baseline sessions. Across the nine baseline sessions, Sam demonstrated no variability and his performance was stable and flat.

When the intervention was introduced, Sam had a change in level between baseline (*M* = 0.00, *SD* = 0.00) and intervention (*M* = 85.71, *SD* = 15.33). Sam had an immediate increase in the first two data points and then a rapid increase in trend throughout the intervention phase. As compared to the trend line, the data had some variability. His performance was consistently high in the intervention phase. The PND was calculated to be 100% from baseline to intervention phase for Sam.

In response to research questions six and seven, Sam generalized his conceptual knowledge to abstract problems (*M* = 98.15, *SD* = 3.21) at a level higher than baseline and intervention phases. There was low variability between the data points in the generalization phase. At 2-week maintenance, he maintained his conceptual knowledge at a high level (100%) for the unlike fractions and the mixed fraction probes. PND was calculated to be 100% from baseline to generalization and maintenance for Sam. His performance was consistently high in the generalization and maintenance phases.

Kyle. In response to research question three (Figure 5), Kyle's conceptual knowledge for addition and subtraction of unlike fractions was at an extremely low level (*M* = 0.00, *SD* = 0.00) during the baseline sessions. He demonstrated no variability across baseline sessions. Overall, the data in the baseline sessions were consistently flat and stable.

When the intervention was introduced, Kyle had a change in level between baseline (*M* = 18, SD = 2.82) and intervention (*M* = 94.84, *SD* = 5.06). Kyle showed an immediate increase and upward trend in his performance on conceptual knowledge of unlike fractions. Compared to the trend line, the data had low variability. His performance was consistently high in the intervention phase. The PND was calculated to be 100% from baseline to intervention for Kyle.

In response to research questions six and seven, Kyle generalized his conceptual knowledge to abstract problems (*M* = 100, *SD* = 0.00) at the ceiling level. There was no variability in the generalization phase. The PND was calculated to be 100% from baseline to generalization and maintenance phases. At 2-week maintenance, Kyle maintained his performance at high levels for conceptual knowledge on the unlike fractions and mixed fraction probes. His performance was consistently high across the generalization and maintenance phases.

Brad. In response to question three (Figure 5), Brad demonstrated extremely low performance across the 11 baseline sessions (*M* = 0.00, *SD* = 0.00). He had no variability across the sessions in the baseline condition because he earned a score of 0.00% across all baseline sessions. Overall, the data in the baseline sessions were flat, low, and stable.

When intervention was introduced, Brad had a change in level between baseline (*M* = 0.00, *SD* = 0.00) and intervention (*M* = 38.89, *SD* = 17.57) phases. Brad’s performance showed an immediate increase a flat trend in his conceptual knowledge after the introduction of the intervention. Compared to the trend line, the data had high variability across the intervention sessions. The PND was calculated to be 100% from baseline to intervention for Brad. His performance was consistently high in the intervention phase except one session.

As seen in Figure 5 (research question six and seven), Brad generalized the conceptual knowledge of unlike fractions to abstract problems (*M* = 70.37, *SD* = 13.98), when the intervention was withdrawn. There was medium level of variability between the data points in the generalization phase. He maintained his conceptual knowledge at a medium level on the unlike fractions (77.78%) and high level on the mixed fraction probes (84.38%). PND was calculated to be 100% from baseline to generalization and maintenance for Brad. His performance was consistent across phases (intervention, generalization, and maintenance). Overall, performance in the generalization and maintenance phases was considerably higher than in baseline.

Procedural Knowledge of Unlike Fractions

Overall findings related to the procedural knowledge of addition and subtraction of unlike fractions reveal that all participants could solve more addition and subtraction problems with unlike fraction problems correctly after the intervention. The participants could earn up to 5 points for each problem solved correctly and a maximum of 50 points for the procedural knowledge probe. Similar to like fractions, these points were converted into percentages for the ease of comparison of the scores. During baseline, participants had a mean of 12.8 (SD = 9.94) for procedural knowledge of unlike fractions which increased to 62.19 (SD = 28.99) after the intervention and was at 84 (SD = 18.06) in the generalization phase. All participants maintained the procedural knowledge of unlike fractions (M = 83.67, SD = 25.6) and mixed fractions (M = 80.95, SD = 24.19) 2 weeks after the withdrawal of intervention. The overall mean PND was calculated to be 89.79% across all participants and phases for procedural knowledge.

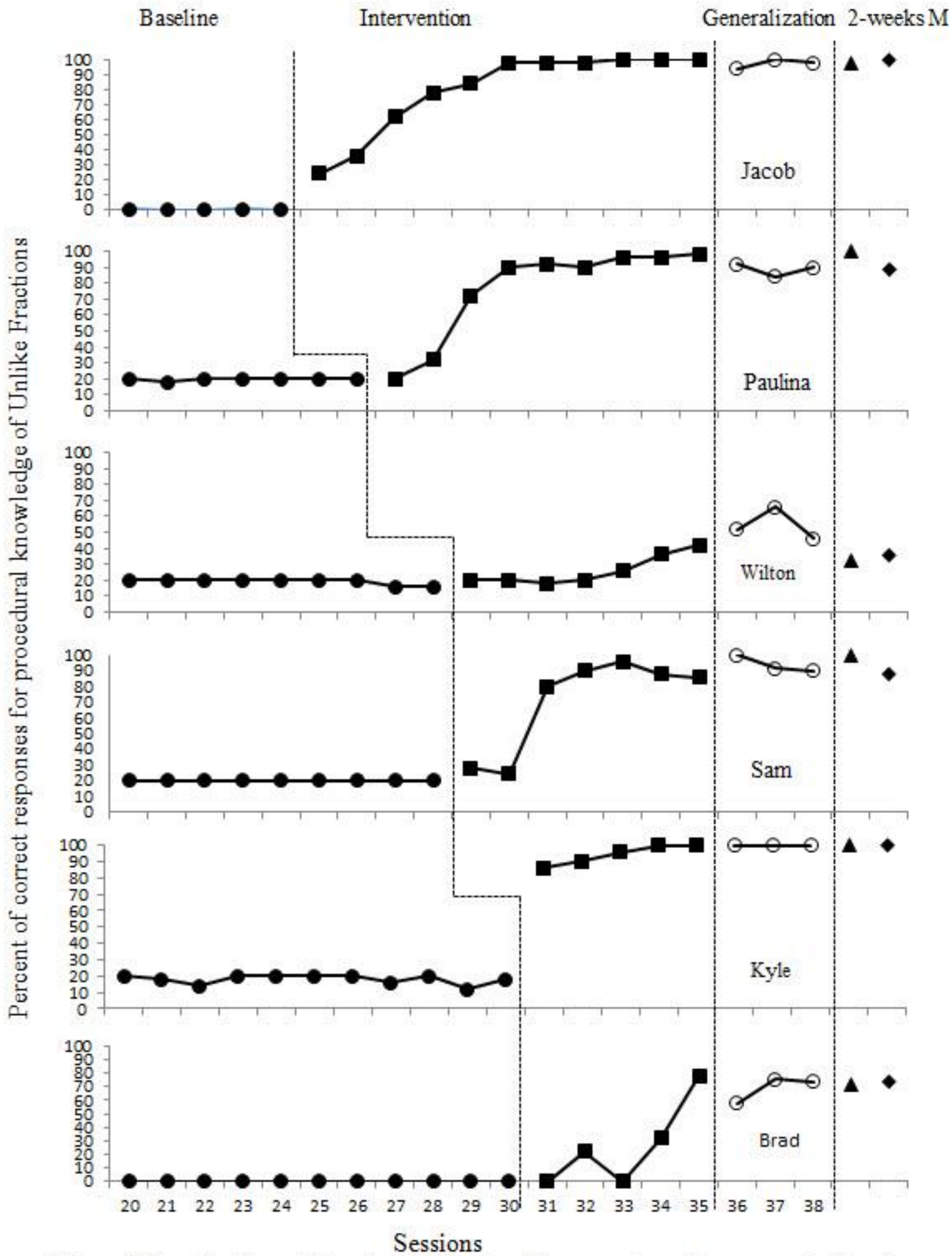

Figure 6. Procedural knowledge of unlike fractions. The percentage of steps correctly done for procedural knowledge across baseline (●), intervention (■), generalization (○), two-week delayed maintenance (▲) and maintenance on mixed probe (♦) across six students with autism.

Figure 6. Procedural knowledge of unlike fractions.

Jacob. As seen in Figure 6 (response to research question four), during all sessions in the baseline condition, Jacob's performance on addition and subtraction of unlike fractions was low (M = 0.00, SD = 0.00). Across the five baseline sessions, Jacob demonstrated no variability and his performance was stable. Overall, the data in the baseline sessions were consistently flat and low.

When the intervention started, Jacob's performance depicted a change in level from baseline (M = 0.00, SD = 0.00) to intervention phase (M = 79.82, SD = 27.55). Jacob showed a small but immediate of change in his procedural knowledge that continued to increase in the intervention phase. Compared to the trend line, the data showed some variability. His performance showed a gradual upward trend after the introduction of the intervention. The PND was calculated to be 100% from baseline to intervention phase for Jacob and his performance was consistent across the intervention phases.

Jacob generalized the procedural knowledge of unlike fractions to abstract problems (M = 97.33, SD = 3.05) after the withdrawal of the intervention as seen in Figure 6 (response to research question six). There was low variability in the generalization phase. The PND was calculated to be 100% from baseline to generalization and maintenance phases. He showed consistently high performance across phases (generalization and maintenance). Jacob maintained his performance at 98% even after 2 weeks of the withdrawal of intervention on the unlike fractions and 100% for the mixed (like and unlike fractions) probes.

Paulina. As seen in Figure 6 (response to research question four), Paulina demonstrated low performance across the seven sessions in the baseline condition (*M* = 19.71, *SD* = 0.76). She showed minimal variability and her performance was stable. Overall, the data in the baseline sessions were flat and low.

Upon introduction of the intervention, Paulina showed a change in level between baseline (*M* = 19.71, *SD* = 0.76) and intervention (*M* = 76.22, *SD* = 29.62). Paulina showed a gradual increase in her procedural knowledge of unlike fractions although there was no an immediate change in her performance after the intervention was introduced. Compared to the trend line, the data had some level of variability. Paulina demonstrated an upward trend in her performance. The PND was calculated to be 88.89% from baseline to intervention phase. Overall, Paulina's performance in the intervention phase was consistent and higher than the baseline phase.

As seen in Figure 6 (research questions six and seven), Paulina generalized the procedural knowledge of unlike fractions to abstract problems (*M* = 88.67, *SD* = 4.16), when the intervention was withdrawn. There was little variability between the data points in the generalization phase. She maintained her procedural knowledge at a high level (100%) on unlike fractions and at 88.09% for the mixed (like and unlike fractions) probes after 2 weeks of withdrawal of intervention. She demonstrated consistently high performance during phases. The PND was calculated to be 100% from baseline to generalization and maintenance phases.

Wilton. In response to research question four (Figure 6), Wilton's procedural knowledge for unlike fractions was at a low level (*M* = 19.11, *SD* = 1.76) during the baseline condition. Across the nine sessions in baseline, Wilton's performance showed a flat trend. He had low levels of variability across the baseline sessions and consistently low data in the baseline phase.

When the intervention was introduced, Wilton had a change in level between baseline (*M* = 19.11, *SD* = 1.76) and intervention (*M* =26.00, *SD* =9.38). Wilton did not show an immediate increase in his performance. Wilton's performance data showed a gradual increase in trend on the data points at the end of intervention phase. Compared to the trend line, the data had low levels of variability. The PND was calculated to be 42.86% from baseline to intervention for Wilton. His performance was consistent in the intervention phase.

As seen in Figure 6 (research questions six and seven), Wilton generalized his procedural knowledge to abstract problems (*M* = 54.67, *SD* = 10.26) at a level higher than baseline and intervention phases. There was some variability between the data points in the generalization phase. At 2-week maintenance, he maintained his procedural knowledge at a low level (32%) for the unlike fractions and mixed fraction probe (35.71%). PND was calculated to be 100% from baseline to generalization and maintenance for Wilton. Wilton's performance was not consistent during the generalization and the maintenance phases.

Sam. In response to research question four (Figure 6), Sam's procedural knowledge for unlike fractions was at a low level (*M* = 20, *SD* = 0.00) consistently during the baseline sessions. Across the nine baseline sessions, Sam demonstrated no variability and his performance was stable. Overall, the baseline data were low and flat.

When the intervention was introduced, Sam had a change in level between baseline (*M* = 20.00, *SD* = 0.00) and intervention (*M* = 70.28, *SD* = 30.65). Sam had an immediate slight increase in the first two data points and then a rapid increase in trend throughout the intervention phase. As compared to the trend line, the data had medium variability. Sam's performance was consistently high in the intervention phase. The PND was calculated to be 85.71% from baseline to intervention for Sam.

In response to research questions six and seven, Sam generalized his procedural knowledge to abstract problems (*M* = 94.00, *SD* = 5.29) at a level higher than baseline and intervention phases. There was some variability between the data points in the generalization phase. At 2-week maintenance, he maintained his procedural knowledge at a high level (100%) for the unlike fractions and the mixed fractions probe (88.1%). PND was calculated to be 100% from baseline to generalization and maintenance for Sam. His performance was consistently high during the generalization and maintenance phases.

Kyle. In response to research question four (Figure 6), Kyle's procedural knowledge for addition and subtraction of unlike fractions was at a low level (*M* = 18, *SD* = 2.82) consistently during the baseline sessions. He demonstrated little variability across baseline sessions. Overall, the data in the baseline sessions were consistently flat and stable.

When the intervention was introduced, Kyle had a change in level between baseline (*M* = 18, *SD* = 2.82) and intervention (*M* = 94.4, *SD* = 6.22). Kyle showed an immediate change in his procedural knowledge from 18% to 86%. He showed an upward trend in his performance. As compared to the trend line, the data had little variability across the intervention sessions. Kyle's performance was consistently high in the intervention phase. The PND was calculated to be 100% from baseline to intervention for Kyle.

In response to research questions six and seven, Kyle generalized his procedural knowledge to abstract problems (*M* = 100, *SD* = 0.00) at the ceiling level. There was no variability in the generalization phase. The PND was calculated to be 100% from baseline to generalization and maintenance phases. Kyle maintained his performance at 100% for procedural knowledge even after 2 weeks of the withdrawal of intervention on the unlike fractions and 100% for mixed fraction probes. His performance was consistently high during the generalization and maintenance phases.

Brad. In response to question four (Figure 6), Brad demonstrated low performance across the 11 baseline sessions (*M* = 0.00, *SD* = 0.00). He had no variability across the sessions in the baseline condition because he earned a score of 0.00% across all baseline sessions. Overall, the data in the baseline sessions were flat, low, and stable.

When intervention was introduced, Brad had a change in level between baseline (*M* = 0.00, *SD* = 0.00) and intervention (*M* = 26.4, *SD* = 32.04). Brad did not show an immediate change in his performance after the introduction of the intervention. His performance showed an upward trend in his procedural knowledge on the last two data points. Compared to the trend line, the data had high variability across the intervention sessions. The PND was calculated to be 60% from baseline to intervention for Brad. His performance was not consistent across the intervention sessions.

As seen in Figure 6 (research questions six and seven), Brad generalized the procedural knowledge of unlike fractions to abstract problems (*M* = 69.33, *SD* = 9.87). There was some variability between the data points in the generalization phase. He maintained his procedural knowledge at a medium level on the unlike fractions (72%) and mixed fraction probes (73.81%). PND was calculated to be 100% from baseline to generalization and maintenance for Brad. Overall, performance in the generalization and maintenance phases was higher than in baseline. His performance was consistent in the generalization and maintenance phases.

Time Taken for Unlike Fractions

The time taken to solve the problems on the procedural knowledge probes for unlike fractions was noted using a stopwatch across all phases. The mean time taken to solve addition and subtraction problems with unlike fractions across participants was 2.45 min in the baseline phase. The participants scored 12.80% on an average in this phase. The time taken by Jacob, Paulina, and Sam had medium variability from baseline to the intervention phase. Wilton, Kyle, and Brad had low variability in the time taken to solve the problems for unlike fractions. Overall, the average time taken to solve the problems as well as the performance of all the participants increased considerably during the intervention (*M* = 12.69, *SD* = 3.51) and generalization (*M* = 10.05, *SD* = 5.67) phases just like the procedural knowledge scores as shown in Table 5.

Table 5

Time Taken vs. Procedural Knowledge Scores for Unlike Fractions

	Time Taken (in minutes)			Procedural Knowledge (%)		
	Baseline	Intervention	Generalization	Baseline	Intervention	Generalization
Participant	*M* (*SD*)	*M* (*SD*)	*M* (*SD*)	*M* (*SD*)	*M* (*SD*)	*M* (*SD*)
Jacob	2.85 (2.00)	16.75 (8.34)	9.48 (5.71)	0.00 (0.00)	79.82 (27.55)	97.33 (3.05)
Paulina	3.55 (1.13)	16.63 (5.20)	8.30 (0.15)	19.71 (0.76)	76.22 (29.62)	88.67 (4.16)
Wilton	2.46 (0.63)	11.81 (6.74)	12.15 (0.57)	19.11 (1.76)	26.00 (9.38)	54.67 (10.26)
Sam	2.01 (1.30)	10.35 (2.35)	6.63 (1.66)	20.00 (0.00)	70.29 (30.65)	94.00 (5.29)
Kyle	2.05 (0.81)	7.85 (3.16)	3.65 (1.47)	18.00 (2.82)	94.40 (6.22)	100.00 (0.00)
Brad	1.75 (0.50)	12.79 (8.28)	20.07 (1.22)	0.00 (0.00)	26.40 (32.04)	69.33 (9.87)
Total	2.45 (0.67)	12.69 (3.51)	10.05 (5.67)	12.80 (9.94)	62.19 (28.99)	84.00 (18.06)

Participant Attitudes

Data were collected on participants' attitudes toward instruction with explicit instruction with manipulatives using pre- and postintervention participant interviews, anecdotal notes, and recordings of the instructional sessions. All the pre- and postintervention interviews were recorded and transcribed. The field observations revealed that five out of six participants knew the steps of explicit instruction and verbalized the steps unprompted during the instructional lessons. Four out of six participants found it difficult to explain their thinking. Six of the participants could use the words, numerator, denominator, and equivalent fractions while explaining their thinking on the conceptual knowledge probes for unlike fractions. Participants named several places where they could use fractions outside the school context. These responses helped to establish the social validity of the intervention.

Jacob. In response to the interview questions, Jacob responded that he is good at doing math and fractions and he found it easy to understand fractions. He felt that it was time consuming to use manipulatives to solve fraction problems. He reported that he had gotten better at doing fractions after the lessons because of practice. Additionally, he reported that he liked doing fraction problems because he could do multiplication and he enjoys doing that. He commented that it is easier to do fraction problems without explaining.

During one instructional lesson, he wrote his observations on the dry erase board (Figure 7). On another occasion, he created a word problem with fractions using the names of his friends. He was undecided about his feelings toward the use of manipulatives. During the observations, it was noted that Jacob knew the multiplication facts but he had difficulty showing the multiplication facts using manipulatives. When he had to double the denominator, he added two chips rather than doubling the chips during guided instruction phase of unlike fractions.

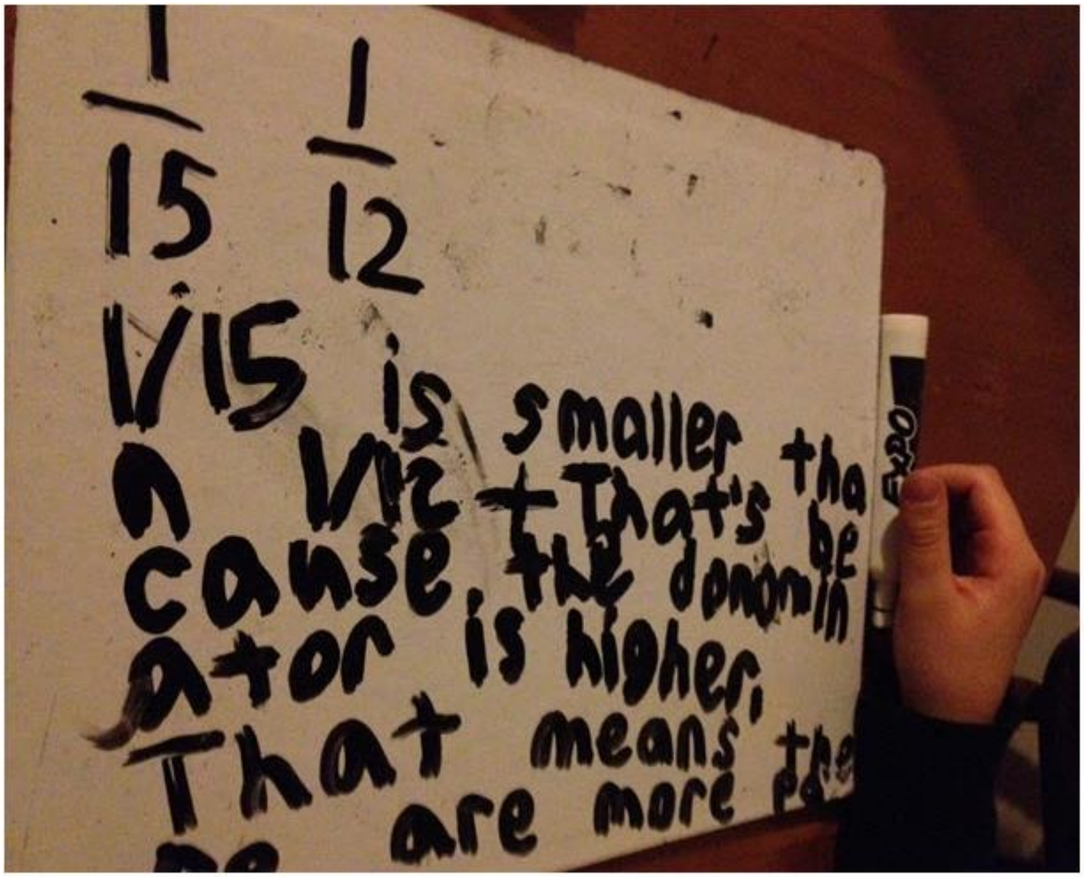

Figure 7. Jacob’s observation.

Jacob’s dad shared that he talked to him about the fraction chips and circles. He is also attempting to make his own fraction problems when they visit a restaurant over the weekend and with time. Jacob’s dad felt that he had become more confident about solving problems with unlike fractions and liked the instruction with manipulatives. Jacob had told his dad, “I am doing fourth grade math now.” Jacob’s dad felt that it is very important to learn about fractions to apply them to other situations.

Paulina. In response to the interview questions, Paulina reported that she enjoyed working with fraction circles. She could show addition of fractions using fraction circles.

She felt that she had gotten better at solving fraction problems after the lessons and found fraction problems easier to do.

Paulina knew all the steps of the explicit instruction and verbalized them during several instructional sessions. She commented that she preferred working with fraction circles to the chips. On several occasions, she requested to play with the fraction circles after the completion of the lesson. During one session, she wrote about using fraction circles and a pie (Figure 8). She used a lot of self-talking to help her work out the fraction problems.

Paulina's mother shared that she had positive feelings related to the use of manipulatives. One day while cutting cake at the birthday party, Paulina related fractions to the cake. Paulina's mother said, "She is very visual, so objects make it easier for her to understand." Paulina's mother felt that it is important to learn about fractions because it will help her with sharing things with her friends. It is also important for day-to-day activities.

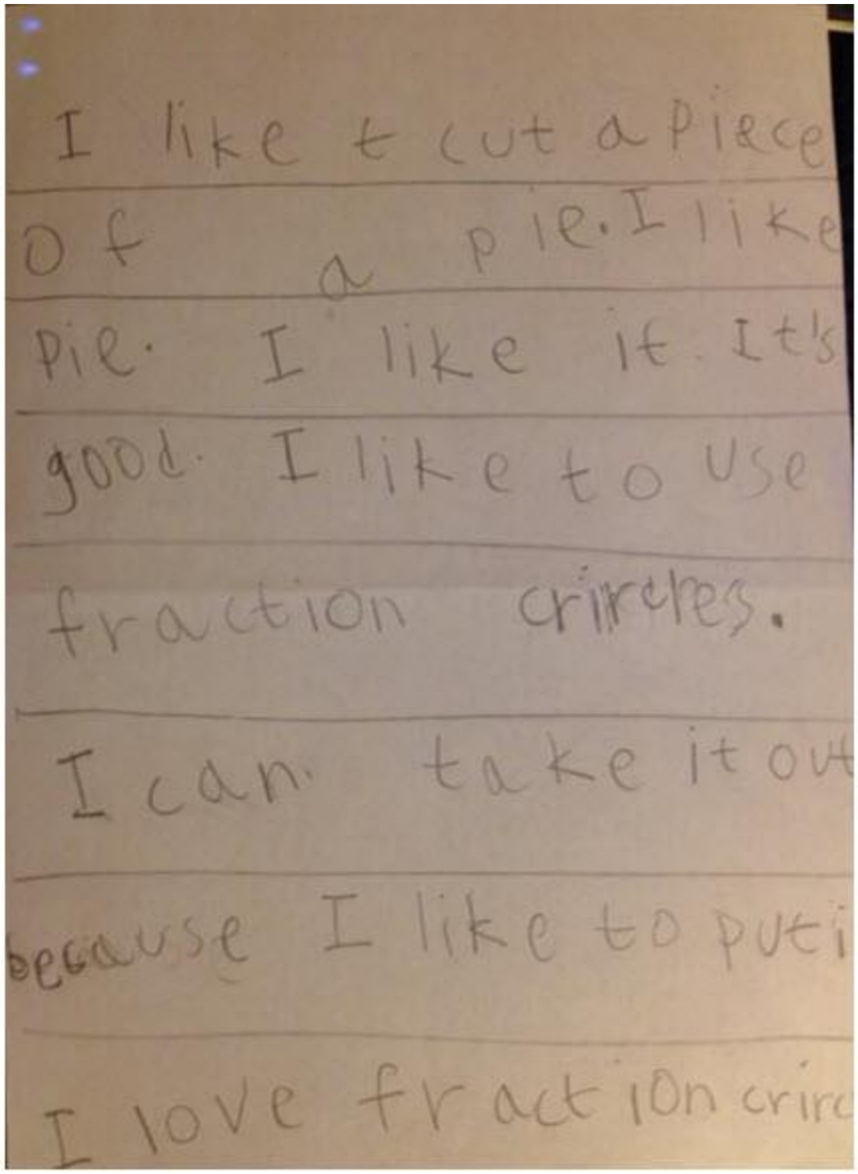

Figure 8. Paulina's writing about fractions.

Wilton. In response to the interview questions, Wilton responded that he is good at doing math specifically fractions. He felt that they are easy and it was fun working on fraction problems. He reported that he liked working with manipulatives and that he has gotten better at solving fraction problems after the lessons. He felt that it is important to learn about fractions because we use them at school, home and at the restaurant to share things.

In many sessions, Wilton verbalized the steps of explicit instruction even before the lesson was started. On several occasions, he requested to watch the recordings where he was working with fraction chips. He reported that he could use fractions to share the pizza with his family at Bertucci's restaurant. Initially, Wilton used a multiplication chart to solve the problems with unlike fractions but during the generalization phase, he could solve the problems without using the multiplication chart. During three sessions, Wilton needed prompting to continue with the work during the guided instruction phase.

Wilton's mother shared that he has gotten better at doing fraction problems and multiplication after the lessons. On the preintervention interview, she had reported his performance on fractions as being fair but she reported it as excellent on the postintervention interview. She shared that when they ordered pizza for dinner, Wilton looked at the pizza and said, "I am going to eat one-sixth of that pizza." He enjoys doing math specially fraction problems. She felt that the instruction with manipulatives was beneficial for Wilton and he enjoyed it. She reported that he talked about the lessons even after the intervention was over. Wilton's mother felt that it was important for him to learn about fractions so he could share things with his brother and help with measuring stuff for the recipes.

Sam. In response to the interview questions, Sam responded that he is good at doing fractions and it is fun to work on fraction problems. He felt that he had gotten better at solving fraction problems after the lessons because he could change the denominators of unlike fractions and make them same. He said that he enjoyed changing fractions to equivalent fractions. He said that he preferred to solve 6 problems instead of 10 problems on the test because 10 were too many. Sam responded that learning about fractions helps him solve difficult math problems. He can use it in cooking and for solving time and money problems.

During the instructional sessions, Sam used multiplication chart to solve the problems with unlike fractions but he was able to complete the generalization probes without the chart. He reported that he could use fractions to solve time and measuring problems. He said, “I can make a cake now because I can use fractions.” During one of the sessions, he wanted to share a donut with his mom and the researcher so he said, “I am going to divide the donut into three parts and each one will get one-third.” During one of the instructional sessions, he said, “Sometimes, it is hard for me to explain my thinking.” He knew the steps of explicit instruction and verbalized them even prior to the start of the session.

Sam’s mom shared that his performance on solving fraction problems has improved after the intervention and he feels comfortable doing fraction problems. She felt that instruction with objects was effective because Sam could relate the fraction circles with pizza. She shared that during their visit to a bookstore; Sam picked a book on fractions and started working on it when they got home. She felt that because of the intervention, Sam had gotten better at explaining his thinking for other kind of math problems too.

Kyle. In response to the interview questions, Kyle reported that solving fraction problems has become easier for him. He enjoys doing math and does not think that it is a waste of time. He responded that he liked using fraction circles and chips to solve fraction problems although some of the steps were hard to explain. He felt that learning about fractions would help him do better in school. He shared that he liked that fractions can be reduced and denominators can be changed to make them same. He reported that fractions were above his grade level. He had learned to solve fraction problems where the denominators were not the same. Kyle shared that it is important to learn about fractions because then he could solve difficult problems at school.

It was observed that during the initial lessons, Kyle did not even attempt to explain his mathematical thinking. At times, he would stammer while trying to explain his thinking. On the later sessions, Kyle explained all the steps of explaining fraction problems.

Kyle's mom shared that his ability to solve fraction problems has improved after the instruction with objects. She rated his feelings toward using objects as excellent. She said, "I think it helped him visualize the abstract concepts and apply them." She felt that the intervention helped to improve Kyle's self-confidence.

Brad. In response to the interview questions, Brad shared that fractions are hard to understand. He did not like using chips to solve the fraction problems. He felt that he had gotten better at solving fraction problems after the lessons and he can use fractions in math at school. He felt that it was important to learn fractions but it was very hard for him.

During the lessons, Brad made several off-topic comments. He needed reminders to focus on the lesson and continue working. He did not attempt the problems when he was unsure of the method. This was especially seen during the baseline phase of the unlike fractions. He said, “I don’t like this. This is too hard.” He used a multiplication chart to solve problems with unlike fractions. He shared that it was easier for him to do like fractions than unlike fractions. Brad’s mom shared that it is important to learn about fractions as it helps with day-to-day things but Brad did not like learning fractions with objects. She shared that Brad does not like math.

Summary

This chapter included information from the data analysis related to the overall performance of all the participants in graph form as well as individually for the three dependent measures for like and unlike fractions. Additionally, the findings from the qualitative data analysis of the parent and participant interviews were also included. The following chapter includes a discussion of the findings, limitations of the current study, and suggestions for practitioners and researchers.

5. CONCLUSIONS AND RECOMMENDATIONS

The primary purpose of this study was to investigate the effectiveness of explicit instruction with manipulatives for improving the conceptual and procedural knowledge of addition and subtraction of like and unlike fractions of participants with mild to moderate autism. This study also examined the attitudes and perceptions of the participants related to the intervention.

The results of this study reveal that elementary school students with mild to moderate autism benefit from explicit instruction with manipulatives to learn how to solve addition and subtraction problems of like and unlike fractions. Four design criteria for evaluating the single case research designs were applied to this study (Kratochwill et al., 2010). The first criterion for single case design is that the intervention must be systematically manipulated. In the current study, multiple-baseline design was used to stagger implementation of explicit instruction with manipulatives across the four tiers of intervention for the six elementary school participants with mild to moderate autism.

The second criterion for single-case designs is that outcome variables or dependent variables (baseline, intervention, and other probes) must be measured systematically over time by more than one assessor and that interassessor agreement data should be collected in each phase for at least 20% of the data points (Kratochwill et al., 2010). Within each phase of the study, an interassessor agreement score between 80 and 90% is considered as an acceptable score (Kratochwill et al., 2010). In the current study, interassessor agreement exceeded the 20% recommendation for both interobserver agreement and fidelity of treatment (i.e., 30%). The percentage of agreement in this study exceeded the acceptable range of 80 to 90% for fidelity of treatment (i.e., 97.22%), for scoring of conceptual knowledge of like (i.e., 95.14%) and unlike (i.e., 92.59%) fractions, and procedural knowledge of like (i.e., 99.3%) and unlike (i.e., 96.48%) fractions.

The third criterion for single-case multiple-baseline designs is that there should have been at least three attempts to demonstrate an intervention effect at three different points in time or with three different phase repetitions (e.g., at least three baseline conditions for multiple-baseline design) (Kratochwill et al., 2010). The current study had three tiers with one participant each in the first two tiers and two participants each in tier three and tier four. Each participant had a baseline phase.

The fourth design criterion for single-subject designs is that each phase must have at least three data points to qualify as an attempt to demonstrate an effect. A multiple-baseline design must have a minimum of six phases with at least five data points in each phase (Kratochwill et al., 2010). In the current study, there were 12 phases with at least five data points for each dependent measure. Therefore, the current study met evidence standards related to the design used based on the criteria recommended by Kratochwill et al. (2010) for single-subject research design.

Summary of Findings

Since the current study meets the design standards, the findings are evaluated relative to Kratochwill et al.'s (2010) evidence of effectiveness criteria:

1. Five out of six participants could explain or show their thinking (conceptual knowledge) correctly related to solving addition and subtraction problems with like fractions after participating in the intervention, indicating moderate evidence of effectiveness;
2. Five out of six participants could explain or show their thinking (conceptual knowledge) related to solving problems with unlike fractions after participating in the intervention, indicating moderate evidence of effectiveness;
3. Five out of six participants could solve more problems with addition and subtraction of like fractions correctly after participating in the intervention, indicating moderate evidence of effectiveness; and
4. Four out of six participants could solve more problems with addition and subtraction of unlike fractions correctly after participating in the intervention, indicating moderate evidence of effectiveness. The other two participants also showed promising results after participating in the intervention.

The findings related to the generalization phase, based on the Kratochwill et al.'s (2010) evidence of effectiveness criteria, indicate:

1. All six participants explained their thinking for solving problems with like fractions on the generalization probes correctly, indicating moderate evidence of effectiveness;

2. All six participants explained their thinking for solving problems with unlike fractions on the generalization probes correctly, indicating moderate evidence of effectiveness;
3. All six participants accurately solved the problems with like fractions on the generalization probes, indicating moderate evidence of effectiveness; and
4. All six participants accurately solved the problems with unlike fractions on the generalization probes, indicating moderate evidence of effectiveness.

In reference to the maintenance of the fraction skills: (a) all participants maintained the conceptual knowledge of addition and subtraction of like fractions over a 4-week period, (b) all participants maintained the conceptual knowledge of addition and subtraction of unlike fractions over a 2-week period, (c) all participants maintained their skills for solving problems with like fractions over a 4-week period, and (d) all participants maintained their skills for solving problems with unlike fractions over a 2-week period. Additionally, all participants maintained their conceptual and procedural knowledge of addition and subtraction of like and unlike fractions on mixed probes even though this skill was not explicitly taught during intervention.

Based on the comments made by the participants during sessions and their responses to the interview questions, it can be concluded that they liked the intervention with explicit instruction with manipulatives. They were able to make real-life connections with fractions and were able to state the places where they could apply their fraction skills. Participants participated in the instruction without the use of tangible reinforcers.

Conclusions and Implications

The findings of the study are discussed based on the dependent variables of conceptual knowledge (research questions one and three), procedural knowledge (questions two and four), and time taken (research question five) to solve the problems. The generalization (research question six) and maintenance (research question seven) results are discussed along with the dependent measures. Additionally, the attitudes and perceptions of participants (research question eight) toward the individual components of the intervention are also discussed.

Conceptual Knowledge

Conceptual knowledge was measured as the participant's ability to explain or show the steps of solving addition and subtraction problems (like and unlike fractions). Low performance on the baseline probes indicated the participants' lack of conceptual knowledge of fraction skills. Language skills are one of the most significantly impacted areas in students with autism (Heflin & Alaimo, 2007; Spencer & Simpson, 2009). Language difficulties could have accounted for the participants' inability to explain their thinking related to fractions in the baseline condition.

Like fractions. Based on the research done with students with learning disabilities (Maccini & Gagnon, 2000; Maccini et al., 2007) and students with autism (Rockwell et al., 2011), it is evident that explicit instruction with teacher modeling, guided instruction, problem solving, independent practice, continuous teacher feedback, and the use of manipulatives helps to improve some students' conceptual understanding of math concepts taught directly. Increasing math achievement of participants with mild to moderate autism after explicit instruction with manipulatives is worthwhile because this indicates that students with mild to moderate autism can learn in inclusive classrooms if explicit instruction with manipulatives is used. Some students with autism have average mathematical abilities and they benefit from using evidence-based practices that have been validated with students with learning disabilities.

One possible explanation for the gains in the conceptual knowledge of the participants was that explicit instruction lesson format and the scripted lessons made the instruction *routine* for the participants with autism. Researchers recommend that repeated patterns, routines, and scripts help students with autism organize their thoughts and provide them with a routine that they can follow (Janzen, 1996). The theory of planned behavior also shows that when participants have perceived behavioral control (due to the routine), their readiness to participate in the instruction and solve math problems increases (Ajzen, 1991). Five participants showed an ascending trend line in the intervention phase for conceptual knowledge of like fractions, but Brad's performance was not consistent with the others although he showed improvements. Individual differences in learning styles can sometime account for differences in performance (McCoy, 2011). These findings are validated by Brad's responses on the interview. Brad shared that he did not like learning math with objects. All participants were able to generalize and maintain their conceptual knowledge skills for like fractions after 4 weeks. This shows that explicit instruction with manipulatives can help to develop a deeper, long-lasting understanding of the math concepts taught.

Unlike fractions. Based on the results of the conceptual knowledge of like fractions, it was anticipated that participants would perform higher on the baseline probes of unlike fractions, but five of the six participants obtained 0 on the conceptual knowledge baselines of unlike fractions. These findings can be explained based on the literature because students with autism who demonstrate conceptual or procedural knowledge deficits have difficulty with application of previously learned or mastered concepts (Heflin & Alaimo, 2007). Their ability to generalize information from one situation to another is impaired and instruction has to be started from step one, every time a new problem or concept is introduced (McCoy, 2011).

The findings and performance of participants on conceptual knowledge of unlike fractions were consistent with that of like fractions. All participants showed gains in their conceptual knowledge after the intervention, but Jacob's performance was extremely low for the first two points. One plausible explanation for Jacob's delayed gains is his sensory sensitivities. These findings are validated by the observational notes. Jacob complained of being tired and cold during those sessions, which could have hindered his ability to attend to instruction (Kluth, 2008; Spencer & Simpson, 2009).

Conceptual knowledge of fractions is crucial for participants to solve problems with like and unlike fractions. Without it, the use of algorithms for solving fraction problems (especially addition and subtraction of unlike fractions) might be more difficult because the participants with autism would not understand the "why" behind the procedures (Flores, 2009; Rittle-Johnson & Alibali, 1999). With manipulatives, the participants could visually see that total number of pieces (denominator) in either shapes or sets needed to be the same before manipulating the selected parts (numerators). Research indicates that use of visual aids assists with attention to task and retention of the content for students with autism (Janzen, 1996; McCoy, 2011). The participants physically showed their knowledge and understanding of conversion of denominators to get common denominators rather than doing it by rote memory. The use of manipulatives provided the participants with a scaffold through which they could easily transfer their knowledge to generalization probes (Sarama & Clements, 2009).

The explicit instruction activities enabled them to interact with the manipulatives in a meaningful way (Sarama & Clements, 2009). Based on the results obtained from the visual analysis related to level, trend, variability, overlap, immediacy, and consistency for conceptual knowledge (like and unlike fractions), there appears to be a causal relation between increased student math achievement and intervention (Kratochwill et al., 2010). All participants were able to generalize and maintain their conceptual knowledge of unlike fractions after a 2-week delay, indicating deeper understanding of concepts learned which could be attributed to the interactive nature of explicit instruction. The participants were engaged in hands-on activities in all parts of the lesson plan during the intervention sessions.

Procedural Knowledge

The overall performance of the participants was better on the procedural knowledge probes than the conceptual knowledge probes. One possible explanation for this difference is that for the procedural knowledge, the participants had to solve the problems correctly, but conceptual knowledge problems required participants to explain their math thinking. Students with autism show significant delays in their language skills, which could have affected their performance on the tasks that placed language demands on the students (American Psychiatric Association, 2000). In addition, the highly structured rule-based aspects of mathematics often appeal to individuals with autism who are highly structured themselves (McCoy, 2011; Prelock, 2006). Comments made by the participants during the intervention sessions demonstrated preference for procedural tasks. Sam said, “I don’t like to explain what I am thinking.” Brad shared, “It is hard to talk.”

Like fractions. All participants showed immediate and consistent gains but Brad showed variability in his performance on procedural knowledge probes during intervention and generalization phases. This pattern was similar to his performance on the conceptual knowledge probes. One plausible explanation for the variability in Brad’s performance could be individual differences. It was observed that in the sessions when his performance was not consistent with other days, he perseverated on the new animal that he had discovered on the Internet that seemed to interfere with his attention skills (Kluth, 2008). All participants were able to generalize their procedural knowledge and solve more problems correctly than baseline without manipulatives. All participants maintained high levels of math achievement even after 4 weeks.

Unlike fractions. Procedural knowledge baseline scores were higher than conceptual knowledge baseline scores for unlike fractions for four participants. Visual analysis of the data indicated that all participants showed immediate increase in math achievement (solved more problems correctly) for unlike fractions after the intervention.

It seemed harder for the participants to add and subtract unlike fractions because they had little or no knowledge of multiplication facts. First building fluency with math facts and then using this knowledge to teach complex skills is better than trying to teach all parts together (Kubina, Young, & Kilwein, 2004). In the case of Brad, it was noted that although his performance improved, the fine motor and motor planning issues hindered his ability to demonstrate his knowledge on unlike fraction probes (APA, 2000; Prelock, 2006). Wilton could explain the steps but made calculation errors, which affected his performance on the unlike fractions. It would have been beneficial for Wilton to practice multiplication facts along with the intervention to build fluency with math facts before performing multistep tasks (addition and subtraction of unlike denominators).

Based on the results obtained from the visual analysis related to level, trend, variability, overlap, immediacy, and consistency for procedural knowledge (like and unlike fractions), there appears to be a causal relation between participant math achievement and intervention (Kratochwill et al., 2010). All participants were able to generalize their fraction skills and solve more problems correctly, although Wilton showed a downward trend in his performance on generalization probes. This downward trend could be attributed to his difficulty with multiplication facts.

At 2-week maintenance, all participants were able to maintain their math achievement at high levels. Sam, Brad, and Wilton used a multiplication chart to solve the procedural knowledge probes during sessions but they did not use a multiplication chart during generalization and maintenance phases. This shows that there were additional gains for participants with mild to moderate autism. Not only were they able to solve more problems with unlike fractions correctly, they also learned the multiplication facts. These findings are consistent with the literature on teaching multiplication facts using explicit instruction (Morin & Miller, 1998).

Time Taken to Solve the Problems

No major changes were noted in the time taken to solve the problems for like fractions across phases and participants, although all participants improved in the frequency of problems solved correctly. These findings are consistent with the findings of other studies done on subtraction with regrouping (Flores, 2009, 2010) and multiplication facts (Morin & Miller, 1998). The time taken to solve addition and subtraction problems of unlike fractions increased dramatically from the baseline phase for all participants and so did the accuracy of the problems solved. This indicates that in the baseline phase, participants were solving the problems without understanding them. After instruction as their conceptual knowledge improved, they plausibly made connections and took longer to solve the problems but solved them accurately.

Student Attitudes and Perceptions

The data from the qualitative sources and interviews indicated that all participants responded well to explicit instruction and instruction with manipulatives without external or tangible reinforcers. These findings are consistent with other research that students with autism can learn the same curriculum as their general education peers without external reinforcement or special adaptations (Colasent & Griffith, 1998; Kurth & Mastergeorge, 2012). Relative to the theory of planned behavior, the participants had positive attitudes toward the intervention in general. During most of the sessions, the participants were aware of the lesson format which was evident from their comments "First you will do some problems, then we will do some problems and in the end, I will do some problems all by myself." The positive attitudes of the participants toward the explicit instruction seem to emerge from the perceived behavioral control that they had due to lesson structure and format. The structure of the lesson was consistent across all sessions for like fractions and unlike fractions which made it easier for the participants to understand the routine. This also gave them more control over their environment that led to increased math achievement. Majority of the participants reported positive feelings toward the use of manipulatives in instruction; however, they seem to like explicit instruction more.

The data from the anecdotal notes of observations and the session recordings showed that participants used longer sentences for explanations during the intervention, maintenance, and generalization probes for conceptual knowledge of like and unlike fractions. The participants used vocabulary terms (numerator, denominator, and equivalent fractions) related to fractions consistently to explain their mathematical reasoning. These findings were validated by the parent responses to the interviews. Parents of four participants shared that they had seen an increase in the language use of their children after the intervention. Two parents reported that their children showed increased self-confidence after the intervention. Additional studies should be conducted to explore the effect of explicit instruction with manipulatives on language use of students with mild to moderate autism.

Limitations and Future Research

Although the results of this study are encouraging, the findings should be considered in the light of several limitations. First, the instruction was delivered to the participants one-on-one in their home setting without other students and typical classroom distractions. Autism is a spectrum disorder and with the increase in the prevalence of autism, more students with mild to moderate autism are included in the general education classrooms (Chiang & Lin, 2007). Many of these are likely to receive math instruction in inclusive settings with increased distractions. Additional research is needed to determine if the same results can be achieved with the explicit instruction with manipulatives in general education settings.

Second, additional sessions of intervention should have been conducted for conceptual knowledge of like fractions for Brad to establish a trend line, because he showed variability in his performance. The results of the generalization phase across participants are promising but additional sessions should have been conducted to include at least five data points during this phase.

Third, the current study had only six participants and only one participant was female, which limits the generalizability of the results of this study to a larger population. Additional research is needed to determine if the same results can be achieved across participants and female participants.

A fourth limitation of this study was that manipulatives were embedded within the explicit instruction framework; therefore, it was hard to judge the effectiveness of either of these components. Researchers should replicate this study with each component individually isolated to determine the effectiveness of each for students with mild to moderate autism.

A fifth limitation of this study relates to the type of fractions used in the two experiments. Test and Ellis (2005) have described three types of fractions. Type 1 fractions have like denominators, Type 2 fractions are unlike fractions but the smallest denominator will divide into the largest denominator an even number of times, and Type 3 fractions are unlike fractions but the smallest denominator will not divide into the largest denominator an even number of times. For the current study, Type 1 and Type 2 fractions were used. Additional research is needed to determine if similar results can be achieved for Type 3 fractions also with the explicit instruction with manipulatives.

A sixth limitation of this study was that no other studies have been conducted using explicit instruction with manipulatives to teach fractions or other math concepts to students with mild to moderate autism. Although the results of this study are promising, they warrant further research. Additional research is needed to determine if this strategy will yield similar results for teaching other math concepts to students with mild to moderate autism (place value, subtraction with regrouping, algebra, etc.).

A seventh limitation of this study was that word problems were not included on the probes. Further research is needed to investigate the effects of explicit instruction with manipulatives on word problems with fractions. An eighth limitation of this study is that the current study only explored the concrete part of the CRA instructional sequence because the participants mastered the concepts with the use of manipulatives alone. Further research is needed to investigate if the other two components (representational and abstract) individually will achieve similar results. Additionally, future studies should be conducted to examine the total intervention time relative to previous CRA research and fluency of solving the fraction problems by examining the change in time throughout the intervention.

Educational Implications for Future Practice

The current study has implications for teachers of elementary school students with mild to moderate autism. First, results of this study indicate that students with mild to moderate autism can master grade-appropriate objectives when instructed with explicit instruction with manipulatives and without special behavioral accommodations. The participants in the current study improved in their math performance with instruction only and without tangible reinforcers. The participants in the last tier of both experiments acquired the fraction skills in five lessons, which is reflective of the length of a typical elementary instructional math unit. While more research is needed, moderate indicators of effectiveness indicate that explicit instruction with manipulatives holds promise for teaching students with mild to moderate autism. The explicit instruction with manipulatives may be effective in inclusive classrooms, special education resource rooms, and self-contained classrooms, where the teacher can provide explicit instruction to individual students or small groups of students.

Because basic fact errors were evidenced in both like (addition and subtraction facts) and unlike (multiplication facts) fractions, it may be helpful for teachers to provide ongoing practice related to these facts even though a higher-level skill (e.g., finding the least common multiple i.e., L.C.M.) is being taught to students with mild to moderate autism.

In the current intervention, one participant had fine motor issues so he had difficulty writing his numbers legibly. Therefore, he obtained a lower score when the independent observer scored his probes. In the classroom situation, students with mild to moderate autism might benefit from instructional accommodations of using a multiplication chart or a scribe to dictate their answer or thinking, or using a keyboard to show the steps of math problems in the independent phase of explicit instruction to account for fine motor or motor planning issues.

Summary and Conclusions

Access to general education curriculum and evidence-based instructional practices in fractions is necessary for improved mathematics performance among students with autism. Federal mandates support the need to improve mathematics performance of students with autism (IDEIA, 2004; NCLB, 2002; NCTM, 2000; NMAP, 2008), but limited research has been done on the math interventions of students with mild to moderate autism. Prior to this study, there was no research on interventions targeting the conceptual and procedural knowledge of like and unlike fractions with elementary school students with mild to moderate autism. This study provides evidence that these students can improve performance on grade-appropriate fraction skills objectives when taught using explicit instruction with manipulatives, regardless of the behavior and academic difficulties. The participants were able to maintain and generalize the learned skills.

The present study adds to the limited literature on math interventions for students with mild to moderate autism, but continued research is needed to identify effective approaches to teach math concepts to students with mild to moderate autism. Explicit instruction with manipulatives provides educators with a framework to plan effective math lessons for students with mild to moderate autism and instruct them in inclusive settings using the general education curriculum. A set of empirically supported evidence-based math practices will likely improve the math achievement of students with mild to moderate autism.

APPENDICES

APPENDIX A. RECRUITMENT LETTER

Recruitment Flyer/ Email

This project is being conducted for a doctoral dissertation at George Mason University. The purpose of this project is to examine the effects of using concrete-representational-abstract instruction, which is the use of objects, pictures, and numbers, for teaching fraction concepts to students with Autism. This project will help researchers and educators to understand the effect of using this approach for teaching fractions to students with Autism and hopefully help your child improve his/her fraction skills.

Who is Eligible: A child who . . .

- Has a school-based label of Autism that results in eligibility for an Individualized Education Plan (IEP)
- Has a documented deficit in math
- Is between the ages of 8 and 12 years
- Is receiving math instruction in the general education curriculum with support from a special education teacher
- A score of 80% or more on the screening test

What is Involved:

- A screening phone interview
- A screening test
- A follow-up meeting to go over documents and procedures
- If your child is eligible, he/she will receive a minimum of 20 sessions of 30-40 minutes each of individualized instruction in different fraction concepts using concrete-representational-abstract instruction and related progress monitoring activities.

To participate or for more information, please contact **Jugnu Agrawal at 571-277-8085 or jagrawal@gmu.edu**

Approval for the use of this document EXPIRES

OCT 0 2 2013

10/1/2012

Protocol # 7772
George Mason University

APPENDIX B. SCREENING PHONE INTERVIEW

Just as a reminder, since you have already seen the flyer about this project, I am conducting a study for a doctoral dissertation at George Mason University. The purpose of my project is to examine the effects of using concrete-representational-abstract instructional sequence, which is the use of objects, pictures and numbers for teaching fractions to students with Autism. This project will help us to understand the effect of using this approach for teaching fractions and, hopefully, help your child improve his/her fraction skills. If eligible, your child will receive a minimum of 20 sessions of about 30-40 minutes each over approximately 4 to 8 week period of individualized instruction in different fractions concepts using concrete-representational-abstract instruction and related progress monitoring activities. Is this something that you might be interested in working with me to offer your child?

If so, then I have a few questions for you . . .

If not, thank you for your time.

1) How should I address you?

2) How old is your child?

Target age 8-12 years at the time of screening interview

3) What is your child's date of birth?

Verify age

4) Does your child have an IEP to receive special education services at school?

Target is YES

5) What is the disability category under which your child receives special education services?

Target is AUTISM

6) On the IEP, is your child exempt from any state testing?

Target is NO

7) What are the academic areas in which your child has a difficulty?

Target is MATH

8) How do you know?

PLOP, assessment data, IEP goal, classroom reports

If child appears to be eligible:

9) Let me confirm, you are still interested in your child participating in this project?

10) What time/location would be convenient for you? Please plan to provide documentation- eligibility packet, IEP, recent testing report etc.

If child does not appear to be eligible:

Because of __________, your child does meet the criteria for this project. Thank you so much for your time.

APPENDIX C. IRB INFORMED CONSENT FORMS

Letter of Informed Consent-Parent or Guardian

RESEARCH PROCEDURES

This study is being conducted to study the effects of using concrete-representational-abstract instruction, which is the use of objects, pictures, and numbers, for teaching fraction concepts to students with Autism. This research will add to the literature and findings on how concrete-representational-abstract instruction can be effectively used to build the procedural and conceptual knowledge of fractions of students with Autism.

The purpose of this letter is to ask for your permission to work with your child on a one-on-one basis for 30-40 minutes for each session for a minimum of 20 sessions. Your child will participate in these sessions over a period of approximately four to eight weeks to learn more about fractions. The number of sessions each week will be determined based on your availability and preferences. The study involves instruction in fractions and completion of mathematical tasks related to fractions. Additionally, interviews will be conducted with you and your child before and after the concrete-representational-abstract instruction. Each interview will last approximately 10-30 minutes. The pre-intervention interview will include general questions about your child, his disability, strengths, and interests and about their math learning. The post-intervention interview will include questions related to the intervention and your child's math learning.

RISKS

There are no foreseeable risks for participating in this study.

BENEFITS

There are no direct material benefits to you or your child as a participant. The intent is for your child to benefit indirectly from participation by learning new skills to solve fraction problems that can generalize for him/her as he/she works on fraction problems in school. It is predicted that concrete-representational-abstract instruction with explicit instruction framework will enhance the mathematical performance of participants with Autism, which will contribute to the general knowledge of the special education field as further evidence that this practice is effective.

CONFIDENTIALITY

Approval for the use of this document EXPIRES

OCT 0 2 2013

10/1/2012

Protocol # 7772

George Mason University

All data collected for this study will be confidential. You and your child will be assigned a code so that only the researcher knows your and you child's identity; your name and your child's name will not be noted in any reports of the results of the study. While all sessions and interviews will be video recorded, the CDs, related notes, and interview protocols will be kept in a locked cabinet until they can be destroyed.

PARTICIPATION

Your participation in this study is voluntary. You and your child may withdraw from this study at any time and for any reason. If you decide not to participate or if you withdraw from the study, there is no penalty or loss of benefits to which you are otherwise entitled.

CONTACT

This research is being conducted by Jugnu Agrawal, Doctoral Candidate at George Mason University under the direction of Dr. Pamela Baker, Assistant Professor at George Mason University. Dr. Baker may be contacted at 703- 993-1787. You may contact George Mason University Office of Research Integrity & Assurance at 703-993-4121 if you have any questions or comments regarding your rights as a participant in the research.

This research has been reviewed according to George Mason University procedures governing your participation in this research.

CONSENT

I have read this form and agree to participate in this study.

______________________________ Date______________________

(Parent/Guardian Signature)

Please check one

_________ I agree to videotaping _________I do not agree to videotaping

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of this document
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OCT 0 2 2013

Protocol #7772
George Mason University

10/1/2012

Letter of Participant Assent

Hi. I am Mrs. Agrawal from George Mason University. I am doing a project to find better ways to help children learn fractions using objects (such as fraction circles, chips, etc.), pictures and numbers.

We will work together on math activities so I can learn more about the way kids learn math. I will also ask you some questions. I will collect every activity sheet that you work on and video tape our sessions. Your name will not show up in anywhere. The videos, your answers to the questions and your work will be kept in a locked cabinet until we are finished, then they will be destroyed.

If you do not want to be in this project at any time for any reason, you can let me know. You don't have to talk to me if you don't want to. If you change your mind after we start talking and want to stop that is OK. I will not get mad and nothing will happen to you.

I will be working with my teacher, Dr. Baker, from George Mason University. I look forward to working with you on this project and thank you for your help.

ASSENT

Please check one

________ I agree to video taping

_______ I do not agree to video taping

Approval for the use of this document EXPIRES

OCT 0 2 2013

Protocol #7772

George Mason University

GEORGE MASON UNIVERSITY

Office of Research Integrity and Assurance
Research Hall
4400 University Drive, MS 6D5, Fairfax, Virginia 22030
Phone: 703-993-4121; Fax: 703-993-9590

TO: Pamela Baker, College of Education and Human Development

FROM: Aurali Dade
Assistant Vice President, Research Compliance

PROTOCOL: 7772 Research Level: Doctoral Dissertation

PROPOSAL NO.: N/A

TITLE: The Effects of Computer-Assisted Video Modeling on Fraction Skills of Students on the Autism Spectrum

DATE: October 3, 2012

Cc: Jugnu Agrawal

On October 3, 2012, the George Mason University Institutional Review Board (GMU IRB) reviewed and approved the continuation of the above-cited protocol as submitted following expedited review procedures.

Please note the following:

1. Copies of the final approved consent documents are attached. You must use these copies with the IRB stamp of approval for your research. Please keep copies of the signed consent forms used for this research for 3 years after the completion of the research.
2. **Any modification to your research (including the protocol, consent, advertisements, instruments, funding, etc.) must be submitted to the Office of Research Integrity & Assurance (ORIA) for review and approval prior to implementation.**
3. Any adverse events or unanticipated problems involving risks to subjects including problems involving confidentiality of the data identifying the participants must be reported to the ORIA and reviewed by the IRB.

The anniversary date of this study is 10/2/2013. **You may not collect data beyond that date without GMU IRB approval.** A continuing review form must be completed and submitted to the ORIA 30 days prior to the anniversary date or upon completion of the project. In addition, prior to that date, the ORIA will send you a reminder regarding continuing review procedures.

If you have any questions, please do not hesitate to contact me at 703-993-5381.

APPENDIX D. SCRIPTED LESSON PLAN FOR LIKE FRACTIONS

Lesson Objective: Student will learn to add and subtract fractions with like denominators using manipulatives.

Materials: Manipulatives, pencil and paper, activity sheet with problems for teacher modeling and guided practice and independent activity sheet.

Advance Organizer:

Objective and activate prior knowledge: In this lesson, we will learn to add and subtract fractions with like denominators using objects. All the problems that we solve today will have like denominators.

For example, <u>4/5 and 1/5.</u> Like denominators means the same denominator or the same number at the bottom in the fractions. In this fraction, we have 5 as the denominator in both fractions, so the fractions have a like denominator. I will solve some problems for you first and then we will solve some together. At the end of the lesson, you will add and subtract some fractions using fraction circles on your own.

Rationale: It is important for you to learn to add and subtract fractions because fractions are used in everyday life outside of school. For example, cooking involves addition of fractions when the recipe needs to be doubled or tripled. Fractions are also used in construction of models, etc.

Teacher Demonstration:

Problem 1: ($\frac{1}{6} + \frac{2}{6}$ Addition problem)

- When we add fractions, we add the numerators but the denominators stay the same. This is because we are adding parts of an equally cut circle. Therefore, before we begin adding, we have to be sure that we have common (or like) denominators. Like denominators means same denominators. Let us read our first problem. One sixth plus two sixths (Teacher reads the problem out loud).
- Show the problem using fraction circles. On one side of the plus sign, we have one-sixth (teacher reads fraction 1) and on the other side, we have two-sixths (teacher reads fraction 2). Now we will put an equal sign at the end of the problem.
- Do we have a common denominator (same denominator) in both fractions? Yes (pointing to 6 {circle with lines to show the total parts} in both fractions).
- So we put the common denominator as the denominator in the new fraction (Teacher places the circle with six parts to show the denominator for the answer).

- Now we add the numerators in the two fractions. (Teacher demonstrates the addition of 1 and 2 and writes 3 in the answer). Remember, we discussed that when we are adding fractions, the common denominator stays the same.
- So our new fraction after addition is $\frac{3}{6}$ (shows the new fraction using fraction circles). $\frac{1}{6} + \frac{2}{6} = \frac{3}{6}$

Problem 2: ($\frac{1}{4} + \frac{2}{4}$ Addition problem)

- Remember, when we add fractions we add the numerators and the denominators stay the same. Let us read our problem. One fourth plus two fourths (Teacher reads the problem out loud).
- Let us set up the problem using fraction circles. On one side of the plus sign we have one-fourth (teacher reads fraction 1) and on the other side, we have two-fourths (teacher reads fraction 2). Now we will put an equal sign at the end of the problem.
- Do we have common denominators in the both fractions? Yes (pointing to the denominator 4 in both fractions).
- So we put the common denominator as the denominator in the new fraction (Teacher places the circle with four parts to show the denominator for the answer).
- Now we can add the numerators in the two fractions i.e. 1 and 2. Remember, we discussed that when we are adding fractions, the common denominator stays the same. We do not add the denominators.
- What is our new fraction? $\frac{3}{4}$ (shows using fraction circles).

Good. $\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$

Problem 3: ($\frac{3}{4} - \frac{1}{4}$ Subtraction problem)

- Now we are going to subtract these fractions. In subtraction, also the common denominator stays the same just as in adding. Let us read our problem. Three fourths minus one fourth (teacher reads the problem out loud).
- Let us set up this problem using fraction circles. On one side of the minus sign, we have three fourths and on the other side, we have one fourth. Now we will put an equal sign at the end of the problem.
- Do we have common denominators in both fractions? Yes (teacher points to

4, the common denominator in both fractions).

- So we put the common denominator 4 as the denominator in the new fraction (Teacher places the circle with 4 parts to show the denominator for the answer).
- Now we can subtract the numerators of the two fractions. i.e. 3-1 Remember similar to addition, the common denominator stays the same.
- What is our new fraction? Great work. So $\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$ (shows the answer using fraction circles).

Problem 4: ($\frac{4}{5} - \frac{4}{5}$ Subtraction problem).

- Let us read out the problem. Fourth fifths minus four fifths (teacher reads the problem out loud).
- Let us set up this problem using fraction circles. On one side of the minus, we have four fifths and on the other side, we have four fifths. We will put an equal sign at the end of the problem.
- Do we have common denominators? Yes (pointing to the 5 in both fractions).
- So we put the common denominator, 5 as the denominator in the new fraction (Teacher places the circle with 5 parts to show the denominator for the answer).
- Now we can subtract the 2nd numerator, which is 4 from the first numerator which is also 4. Remember similar to the previous problem, the common denominator stays the same. We do not subtract the denominators.
- What is our answer? $\frac{0}{4}$ (the teacher says the answer).

So $\frac{4}{5} - \frac{4}{5} = \frac{0}{5}$ (shows the answer using fraction circles).

Guided Practice: Let's try a few problems together:

Problem 5: ($\frac{4}{7} + \frac{2}{7}$ Addition problem)

Let us read our problem. Four sevenths plus two sevenths (teacher and student read the problem together)

Are we adding or subtracting? Adding.

What does that tell us about the denominator? The common denominator, 7 stays the same in the answer or in the new fraction.

Which numbers do we add? The numerators.

Let us show this using fraction circles. (The teacher shows the problem with assistance from the student). On one side of the plus sign, we have four sevenths and on the other side is two sevenths. We will put an equal sign at the end of the

problem.
Do we have common denominators? Yes (student points to 7, the denominator).
What should we do before we start adding? (Puts the denominator for the answer)
Put the circle with 7 parts in the denominator of the answer.
Good. Now we can add the numerators of the fractions (4+2).
When adding fractions, what do we do with our parts? (Move all the parts in the numerators of our original fractions to the numerator in the answer).
What is our new fraction? (Teacher and student name the fraction together)
Therefore, $\frac{4}{7} + \frac{2}{7}$ is $\frac{6}{7}$.

Problem 6: ($\frac{6}{8} - \frac{4}{8}$ Subtraction problem).

Let us read our problem. Six eighths minus four eighths (Teacher and student read the problem together).
Are we adding or subtracting? Subtracting
What does that tell us about the denominator? The common denominator remains the same in the new fraction i.e. 8
Which numbers do we subtract? The second numerator from the first numerator. 6-4.
Let us set up the problem using our fraction circles. (The teacher shows the problem with assistance from the student). On one side of the minus, we have six eighths and on the other side, we have four eighths. We will put an equal sign at the end of the problem.
Do we have common denominators? Yes, (student points to the denominators and reads 8).
What should we do before we start subtracting? (The teacher puts the denominator for the answer with the help of the student i.e. circle with 8 parts to show the denominator for the answer). Good. Now we can subtract.
When subtracting fractions, what do we do with our parts? Put the same number of parts as in the numerator of our first fraction (6).
Good. Now how many parts do we need to take away? (Student verbalizes the numerator of the second fraction, 4).
What is our new fraction? (The teacher with the help of the student says the answer).
So, $\frac{6}{8} - \frac{4}{8}$ is $\frac{2}{8}$

Problem-solving practice:
Word Problem: Let us look at this problem. Brenda ate $\frac{3}{4}$ of a candy bar during lunch. She ate 1/4 of the same candy for snack in the evening. How much

candy did Brenda eat? We will show our answer using fraction circles.
Let us start by setting this up as an addition problem.
How much candy bar did Brenda eat during lunch? $\frac{3.}{4}$
How much did she eat for snack? $\frac{1.}{4}$
Is it addition or subtraction? Addition
Let us start by setting up this addition problem. $\frac{3}{4} + \frac{1}{4} =$
Let's read our problem. Three fourths plus one fourth.
What does that tell us about our denominator? The common denominator remains the same in the new fraction.
Which numbers do we add? The numerators in both the fractions.
Let's show this using fraction circles.
Three fourths on one side of the plus and one fourths on the other side of the plus.
Place an equal sign at the end of the problem.
Do we have a common denominator? (Participant points to the common denominator). Yes, so we should move the common denominator to our answer (move the circle with four parts to the denominator of the answer). Good, now we can add.
When adding numerators, what do we do with our fraction parts? (move the fraction parts from the numerator of the two fractions to the answer)
Good. What is our new fraction? $\frac{4}{4}$
How much candy did Brenda eat? $\frac{4}{4}$

Independent Practice:
Now you are going to add and subtract some fractions on your own. Remember, you will be showing your answer using the manipulatives. Make sure that you write your answers on the paper.

Feedback: The teacher will score the independent practice activity sheet for conceptual and procedural knowledge and then, review the problems that the participant missed on the independent activity sheet.

APPENDIX E. SCRIPTED LESSON PLAN FOR UNLIKE FRACTIONS

Lesson Objective: Student will learn to add and subtract fractions with unlike denominators using manipulatives.
Materials: Manipulatives (fraction circles), pencil and paper, activity sheet with problems for teacher modeling and guided practice and independent activity sheet.

Advance Organizer:

Objective and activate prior knowledge: In this lesson, we will learn to add and subtract fractions with unlike denominators using fraction circles. All the problems that we solve today will have unlike denominators. For example, 1/3 and 2/9. Unlike denominators means the fractions with different denominators or different number at the bottom in the fractions. In this fraction, we have 3 as the denominator in the first fraction and 9 as the denominator in the second fraction, so the fractions have unlike denominators. Remember, before we add or subtract fractions the denominators have to be _______? I will solve some problems for you first and then we will solve some together. At the end of the lesson, you will add and subtract some fractions using fraction circles on your own.

Rationale: It is important for you to learn to add and subtract fractions because fractions are used in everyday life outside of school. For example, cooking involves addition of fractions when the recipe needs to be doubled or tripled. Fractions are also used in construction of models, etc.

Teacher Demonstration:
Problem 1: 1/9 + 2/3 (Addition problem)

- Let us read this problem together. One ninth plus two thirds. Show the problem using fraction circles. On the left side of the plus sign, we will show one-ninth (using fraction circle with 9 parts). On the right side of the plus sign, we will show two-thirds (using fraction circle with 3 parts).
- Do we have common denominators? No. Therefore, before adding we must find a common denominator. To do this we need to divide the second circle into 9 parts instead of 3. Is there a way we can use multiplication to make 9 from 3. Yes, we can multiply it by 3 or make it 3 times. OR, We can find multiples of 3 and see if we have one that is same as 9.
- So, we can divide the fraction into 9 equal parts instead of 3. It is very important to remember that whatever we do to the denominator, we have to do the same to the numerator. So what should we do here in the numerator?
- Multiply by 3. Good. Therefore, 2 times 3 is 6 for our second numerator. What is our new addition problem? 1/9 + 6/9.

- Do we have common denominators? Yes. Now we can add.
- Remember, we first write/show our common denominator for the answer (using circle with 9 parts) and add the numerators. We move the numerators of the two fractions to the right side of the equal sign to show the numerator for the answer.
- What is our new fraction? 7/9

Problem 2: ½ + ¼ (Addition problem)

- Let's read this problem together. One-half plus one-fourth. Show the problem using fraction circles. On the left side of the plus sign, we will show one-half (using fraction circle with 2 parts). On the right side of the plus sign, we will show one-fourth (using fraction circle with 4 parts).
- Do we have common denominators? No. Therefore, before adding we must find a common denominator. To do this we need to find the multiples of the denominators to get a common number. Is there a way we can use multiplication to get 4 from 2? Yes, we can multiply it by 2 or make it 2 times. OR, We can find multiples of 2 and see if we have one that is same as 4.
- So, we can divide the fraction into 4 equal parts instead of 2. It is very important to remember that whatever we do to the denominator, we have to do the same to the numerator. So what should we do here in the numerator?
- Multiply by 2. Good. Therefore, 1 times 2 is 2 for our first numerator. What is our new addition problem? 2/4 + 1/4.
- Do we have common denominators? Yes. Now we can add. Remember, we first write/show our common denominator for the answer (using circle with 4 parts) and add the numerators. We move the numerators of the two fractions to the right side of the equal sign to show the numerator for the answer.
- What is our new fraction? 3/4 (shows using fraction circles with 4 parts)

Problem 3: 2/3-1/6 (Subtraction problem)

- Let us read our problem together. Two-thirds minus one-sixth (teacher reads the problem aloud).
- Let us set up this problem using fraction circles. On one side of the minus sign, we have two-thirds (shows using fraction circles) and on the other side, we have one-sixth (shows using fraction circle). Now we will put an equal sign at the end of the problem.
- Do we have common denominators in both fractions? No (teacher points to the denominators in both fractions). Therefore, before subtracting we must find a common denominator. To do this, we have to find common multiples in the two denominators 3 and 6. Let us look at the smallest denominator,

which is 3. Is there a way we can change it to 6 using multiplication? Yes, we can do 3 times 2. Therefore, we can divide the circle into 6 equal parts instead of 3.

- It is very important to remember that whatever we do to the denominator, we have to do the same to the numerator. So what should we do here in the numerator? Multiply by 2. Good. Therefore, 2 times 2 is 4 for our first numerator. What is our new subtraction problem? 4/6 – 1/6
- Do we have common denominators now? Yes. So we can subtract. So we put the common denominator as the denominator in the new fraction (Teacher places the circle with total parts to show the denominator for the answer).
- Now we can subtract the numerators of the two fractions. 4-1=3
- What is our new fraction? Great work. So4/6-1/6 = 3/6. (shows the answer using fraction circles).

Problem 4: ¾ -1/8 (Subtraction problem).

- Let us read our problem together. Three-fourth minus one-eighths. (teacher reads the problem aloud).
- Let us set up this problem using fraction circles. On one side of the minus sign, we have three-fourths (shows using fraction circles) and on the other side, we have one-eighth (shows using fraction circle). Now we will put an equal sign at the end of the problem.
- Do we have common denominators in both fractions? No (teacher points to the denominators in both fractions). Therefore, before subtracting we must find a common denominator. To do this, we have to find common multiples in the two denominators 4 and 8. Let us look at the smallest denominator, which is 4. Is there a way we can change it to 8 using multiplication? Yes, we can do 4 times 2. Therefore, we can divide the circle into 8 equal parts instead of 4.
- It is very important to remember that whatever we do to the denominator, we have to do the same to the numerator. So what should we do here in the numerator? Multiply by 2. Good. Therefore, 3 times 2 is 6 for our first numerator. What is our new subtraction problem? 6/8 – 1/8
- Do we have common denominators now? Yes. So we can subtract. So we put the common denominator as the denominator in the new fraction (Teacher places the circle with total parts to show the denominator for the answer).
- Now we can subtract the numerators of the two fractions. 6--1=5
- What is our new fraction? Great work. So 6/8 --1/8 = 5/8. (shows the answer using fraction circles).

Guided Practice: Let's try a few problems together:
Problem 5: (2/6 + 7/12 Addition problem)
Let us read our problem. (teacher and student read the problem together)
Let us show this using fraction circles. (The teacher shows the problem with assistance from the student).
Do we have common denominators? No
Before we add, we must find a common denominator.
How do we find the common denominator? Use basic multiplication to find common multiples of the original denominators (6 and 12).
Let's look at the denominators we have now. Let's look at the smallest denominator (6).
Is there a way we can use multiplication to get 6 to 12, our other denominator? Yes, we can multiply by two. Very good.
How many parts will we divide the first circle into? 12
Can we add yet? No
What else do we need to do? Multiply the numerator of the first fraction by 2. How many parts will that give us for the numerator of the first fraction? 4.
What is our new addition problem? 4/12 + 7/12.
Do we have common denominators? Yes (student points to the denominators and says the number in the denominator).
What should we do before we start adding? (Puts the denominator for the answer) Good. Now we can add.
When adding fractions, what do we do with our parts? (Move all the parts in the numerators of our original fractions to the numerator in the answer).
What is our new fraction? 11/12. (Teacher and student name the fraction together)
Problem 6: 2/4 –1/4 (Subtraction problem)
Let us read our problem. (teacher and student read the problem together)
Are we adding or subtracting? Subtracting
What do we do first? Look at the denominators of the two fractions.
Do we have a common denominator? Yes.
What does that tell us about the denominator? The common denominator remains the same in the new fraction.
Which numbers do we subtract? The second numerator from the first numerator.
Let us set up the problem using our fraction circles. (The teacher shows the problem with assistance from the student).
Do we have common denominators? Yes, (student points to the denominators and reads the number for the denominator).
What should we do before we start subtracting? (The teacher puts the denominator for the answer with the help of the student). Good. Now we can subtract.
When subtracting fractions, what do we do with our parts? Put the same number of

parts as in the numerator of our first fraction.
Good. Now how many parts do we need to take away? (Student verbalizes the numerator of the second fraction).
What is our new fraction? (The teacher with the help of the student says the answer).
So 2/4 – 1/4 is? 1/4.

Problem-solving practice:
Word Problem: Let us look this situation. You borrowed ½ cup of milk you're your friend to add to the 2/8 cup you already have. If you combine the milk, how much will you have? We will use fraction circles to show our answer. (Word problem will be written here and teacher will read it aloud to the participant).

- To set up this problem, we will have ½+2/8 because we are combining the milk.
- Let's read this problem together. One-half plus two-eighths.
- Show the problem using fraction circles.
- Do we have common denominators? No. Therefore, before adding we must find a common denominator.
- How do we find a common denominator? To do this we need to find the multiples of the denominators to get a common denominator. Let's look at the smallest denominator which is 2. Is there a way we can use multiplication to get from 2 to 8? Yes, we can multiply it by 4 to make it 8. OR, We can find multiples of 2 and see if we have one that is same as 8.
- So, we can divide the fraction into 8 equal parts instead of 2. It is very important to remember that whatever we do to the denominator, we have to do the same to the numerator. So what should we do here in the numerator?
- Multiply by 4. Good. Therefore, 1 times 4 is 4 for our first numerator. What is our new addition problem? 4/8 + 2/8.
- Do we have common denominators? Yes. Now we can add. Remember, we first write/show our common denominator for the answer (using circle with 8 parts) and add the numerators. We move the numerators of the two fractions to the right side of the equal sign to show the numerator for the answer.
- What is our new fraction? 6/8 (shows using fraction circles with 8 parts)
- You will have 6/8 cup of milk.

Independent Practice:
Now you are going to add and subtract some fractions on your own. Make sure that you write your answers on the paper.

APPENDIX F. SCORING RUBRIC FOR CONCEPTUAL KNOWLEDGE OF LIKE FRACTIONS

Put + on the blank space if the student gives a correct response and put – if the student gives an incorrect response or no response

Student ID________ **Score**_______________

1) The student represents the numerator of fraction 1 correctly	_________
The student represents the denominator of fraction 1 correctly	_________
The student represents the numerator of fraction 2 correctly	_________
The student represents the denominator of fraction 2 correctly	_________
The student represents the numerator for the answer correctly	_________
The student represents the denominator for the answer correctly	_________
The student states that the denominator for the answer is the same as in the two fractions.	_________
2) The student represents the numerator of fraction 1 correctly	_________
The student represents the denominator of fraction 1 correctly	_________
The student represents the numerator of fraction 2 correctly	_________
The student represents the denominator of fraction 2 correctly	_________
The student represents the numerator for the answer correctly	_________
The student represents the denominator for the answer correctly	_________
The student states that the denominator for the answer is the same as in the two fractions.	_________
3) The student represents the numerator of fraction 1 correctly	_________
The student represents the denominator of fraction 1 correctly	_________
The student represents the numerator of fraction 2 correctly	_________
The student represents the denominator of fraction 2 correctly	_________

	The student represents the numerator for the answer correctly	_________
	The student represents the denominator for the answer correctly	_________
	The student states that the denominator for the answer is the same as in the two given fractions.	_________
4)	The student represents the numerator of fraction 1 correctly	_________
	The student represents the denominator of fraction 1 correctly	_________
	The student represents the numerator of fraction 2 correctly	_________
	The student represents the denominator of fraction 2 correctly	_________
	The student represents the numerator for the answer correctly	_________
	The student represents the denominator for the answer correctly	_________
	The student states that the denominator for the answer is the same as in the two given fractions.	_________

APPENDIX G. SCORING RUBRIC FOR CONCEPTUAL KNOWLEDGE OF UNLIKE FRACTIONS

Put + on the blank space if the response is correct and put – if the response is incorrect or if there is no response.

Student ID________ **Score**_______________

Scorer ____________ **Lesson**_______________

1) Participant represents/explains the numerator of fraction 1 correctly	_________
Participant represents/explains the denominator of fraction 1 correctly	_________
Participant represents/explains the numerator of fraction 2 correctly	_________
Participant represents/explains the denominator of fraction 2 correctly	_________
Participant checks for the common denominator (by stating or looking)	_________
Participant multiplies the smaller denominator to get common denominators (if needed)	_________
Participant changes the numerator to match the denominator (by multiplying it with the same number as the denominator)	_________
Participant shows the denominator for the answer	_________
Participant add/subtracts the numerators and represents/writes the numerator for the answer	_________
2) Participant represents/explains the numerator of fraction 1 correctly	_________
Participant represents/explains the denominator of fraction 1 correctly	_________
Participant represents/explains the numerator of fraction 2 correctly	_________
Participant represents/explains the denominator of fraction 2 correctly	_________
Participant checks for the common denominator (by stating or looking)	_________
Participant multiplies the smaller denominator to get common denominators (if needed)	_________
Participant changes the numerator to match the denominator (by multiplying it with the same number as the denominator)	_________
Participant shows the denominator for the answer	_________
Participant add/subtracts the numerators and represents/writes the numerator for the answer	_________

APPENDIX H. FIELD OBSERVATION GUIDE

Student ID ______________ Lesson/Session: ________ Time: _________

Area	Notes
Setting (details about the room, furniture, people, size of the room, placement of the room, free of distractions, etc.)	
Participant behaviors during instruction (Participant engagement, comments, reactions, verbalizations related to intervention, feelings expressed, body language, receptiveness, etc.)	
Participant behaviors while taking the assessment (way to approach the problem- drew picture or used manipulatives etc.)	

APPENDIX I. TREATMENT FIDELITY CHECKLIST

Put a '+' if you observe the step in the instruction process and '--' for a step that was not followed during the instruction.
Participant ID____________
Lesson Number _________
Score___________

1) Advance Organizer a) Activate prior knowledge	
b) State Objective of the lesson	
c) Identify Rationale of the lesson	
2) Teacher demonstration	
3) Guided Practice of Fraction concepts	
4) Problem Solving	
5) Independent Practice	

APPENDIX J. PREINTERVENTION PARENT INTERVIEW

Name____________________ Date_______________________

1) What is your child's name?

---

2) What grade is your child in?

---

3) Has your child repeated a grade?

---

4) Which school does your child attend?

---

5) How did you first know that your child had Autism? (Medical vs. Educational)

---

6) Who diagnosed your child with Autism Spectrum Disorder (medical)? Who referred your child for evaluation in the school (education)?

---

7) At what age was your child diagnosed with autism spectrum disorder (medical)? When was your child labeled at school (education)?

---

8) How many hours of special education support does your child receive from the school?

---

9) In what setting does your child receive special education support at school for the majority of the time at school?

10) Does your child receive any related services? If yes, which ones?

11) Which subjects does your child like best?

12) Which subjects does he/she not like?

13) What does your child like to do in his/her free time?

14) What content is being covered in class in math currently? Are you aware? If not, can you please ask the teacher and let me know next time?

15) Is there anything else that you would like to tell me about your child's learning especially related to math?

Please rate each of the following items by circling your answer to the questions below.

16) How would you rate your child's performance in math?

Poor	Fair	Good	Very Good	Excellent
1	2	3	4	5

17) How would you rate your child's performance on fractions?

Poor	Fair	Good	Very Good	Excellent
1	2	3	4	5

18) How would you rate your child's feelings towards math?

Poor	Fair	Good	Very Good	Excellent
1	2	3	4	5

Thank you for your time!

APPENDIX K. POSTINTERVENTION PARENT INTERVIEW

Name: __________________ Date: ___________________

Please rate each of the following items by circling your answer to the questions below.

1) How would you rate your child's performance in math?

Poor	Fair	Good	Very Good	Excellent
1	2	3	4	5

2) How would you rate your child's performance on fractions?

Poor	Fair	Good	Very Good	Excellent
1	2	3	4	5

3) How would you rate your child's feelings towards math?

Poor	Fair	Good	Very Good	Excellent
1	2	3	4	5

4) How would you rate the effectiveness of the instruction with manipulatives for teaching fraction skills to your child?

Poor	Fair	Good	Very Good	Excellent
1	2	3	4	5

5) How would you rate your child's feelings towards the instruction with manipulatives for fractions?

Poor	Fair	Good	Very Good	Excellent
1	2	3	4	5

6) Overall, how would you rate your experience participating in the project?

Poor	Fair	Good	Very Good	Excellent
1	2	3	4	5

7) Has ________ (insert child's name) talked to you about the instruction?

____

8) If yes, what did __________ (insert child's name) say?

____

9) Do you think your child liked learning fractions using objects (for e.g. fraction circles, chips, etc.)? Why or why not?

____

____

10) Is it important to learn about fractions? If yes, why?

____

____

11) Is there anything else you want to share (about instruction, lessons etc.)?

____

Thank you for your time!

APPENDIX L. PARTICIPANT SCREENING TEST

Student ID: ________________ Date: ________________________

Solve the following problems:

$4 + 3 =$	$7 + 6 =$
$2 + 8 =$	$5 + 7 =$
$8 + 4 =$	$7 - 3 =$
$10 - 4 =$	$9 - 6 =$
$13 - 8 =$	$14 - 9 =$
$\frac{5}{6} + \frac{1}{6} =$	$\frac{4}{7} - \frac{2}{7} =$

APPENDIX M. PREINTERVENTION PARTICIPANT INTERVIEW

1) What is your name?

___

2) How old are you?

___

3) When is your birthday?

___

4) What do you like to do in your free time?

___

5) Can you tell me more? (Depending on the student's answer to the previous question, this question will be used to get more information about the student's interest).

___

6) What are your strengths or tell me the things that you think you are good at?

___

7) Which subjects do you like best? Why do you like _____?

___

8) Which subjects do you like the least? Why?

___

9) What are you learning in math at school?

___

10) How do you do math at school?

___

11) Do you do math on paper or use other methods?

___

12) If the answer to the previous question is other methods. Can you tell me more? (Depending on the student's answer to the previous question, this question will be used to get more information about the methods used at school for learning math).

___

For the next, few items read each question carefully. Circle the answer that you think best matches your own feelings.

13) I am good at doing math.

Agree	Undecided	Disagree
☺	😐	☹

14) I am good at doing fractions.

Agree	Undecided	Disagree
☺	😐	☹

15) Learning fractions is a waste of time.

Agree	Undecided	Disagree
☺	😐	☹

16) Fractions are easy.

Agree	Undecided	Disagree
☺	😐	☹

17) It is hard to understand fractions.

Agree	Undecided	Disagree
☺	😐	☹

18) It is fun working on fraction problems.

Agree	Undecided	Disagree
☺	😐	☹

Thank you for your time!

APPENDIX N. POSTINTERVENTION PARTICIPANT INTERVIEW

Student ID: ______________________ Date: ______________________

Read each question carefully. Circle the answer that you think best matches your own feelings.

1) I am good at doing math.

Agree	Undecided	Disagree
☺	😐	☹

2) I am good at doing fractions.

Agree	Undecided	Disagree
☺	😐	☹

3) Learning fractions is a waste of time.

Agree	Undecided	Disagree
☺	😐	☹

4) Fractions are easy.

Agree	Undecided	Disagree
☺	😐	☹

5) It is hard to understand fractions.

Agree	Undecided	Disagree
☺	😐	☹

6) It is fun working on fraction problems.

Agree	Undecided	Disagree
☺	😐	☹

7) I liked learning about fractions with objects (for e.g. fraction circles, chips, etc.).

Agree	Undecided	Disagree
☺	😐	☹

8) If yes to the previous question then the student will answer this question. What did you like most about learning fractions with objects and why?

____

____

9) If undecided or disagree to question 7 then the student will answer this question. What did you like least about learning fractions with objects and why?

____

____

10) What did you like the best about learning fractions?

____

____

11) What did you like the least about learning fractions?

____

____

12) Do you think you have gotten better at solving fraction problems after these lessons? If yes, how?

____

____

13) You can tell me any other things that you want to share with me about your learning or about the lessons and learning fractions.

____

____

Thank you!

APPENDIX O. PROCEDURAL KNOWLEDGE PROBE FOR LIKE FRACTIONS

Solve the following problems and write your answers on the paper.
Student ID: ____________________ Date: ________________

$\frac{1}{4} + \frac{1}{4} =$	$\frac{3}{6} - \frac{2}{6} =$
$\frac{9}{12} - \frac{8}{12} =$	$\frac{3}{8} + \frac{2}{8} =$
$\frac{2}{5} + \frac{1}{5} =$	$\frac{8}{11} - \frac{5}{11} =$
$\frac{9}{12} - \frac{5}{12} =$	$\frac{2}{6} + \frac{5}{6} =$
$\frac{7}{13} - \frac{2}{13} =$	$\frac{1}{3} + \frac{2}{3} =$

APPENDIX P. CONCEPTUAL KNOWLEDGE PROBE FOR LIKE FRACTIONS

Student ID: _______________ Date_________________

Show me how to solve the following problems. As you solve the problems, tell me what you are doing or did to solve the problem.

$\frac{7}{9} - \frac{3}{9} =$	$\frac{3}{4} - \frac{1}{4} =$
$\frac{3}{5} - \frac{2}{5} =$	$\frac{6}{8} + \frac{1}{8} =$

APPENDIX Q. PROCEDURAL KNOWLEDGE PROBE FOR UNLIKE FRACTIONS

Solve the following problems and write your answers on the paper.

Student ID: ______________ Date: ______________

$\frac{3}{5} + \frac{3}{10} =$	$\frac{2}{5} - \frac{1}{10} =$
$\frac{1}{2} - \frac{1}{6} =$	$\frac{1}{6} + \frac{10}{12} =$
$\frac{5}{6} + \frac{2}{12} =$	$\frac{2}{4} - \frac{3}{8} =$

$\frac{3}{10} - \frac{1}{5} =$	$\frac{3}{5} + \frac{4}{10} =$
$\frac{3}{4} + \frac{3}{12} =$	$\frac{3}{12} - \frac{1}{6} =$

APPENDIX R. CONCEPTUAL KNOWLEDGE PROBE FOR UNLIKE FRACTIONS

Show me how to solve the following problems. As you solve the problems, tell me what you are doing or did to solve the problem.

Student ID: ___________________ Date: ________________

$\frac{6}{14} - \frac{2}{7} =$	$\frac{1}{8} + \frac{1}{2} =$

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CURRICULUM VITAE

Jugnu Agrawal was born and raised in New Delhi, India. She earned her B.S. in Child Development from Lady Irwin College in 1991 and her M.S. in Child Development in 1993 from the same college. She also earned a M.Ed. from George Mason University in 2007. She began her work in India as a Special Education Teacher at Educare, a private school for students with learning disabilities in New Delhi, India. She currently works with Fairfax County Public Schools in Virginia, where she has several roles including Special Education Teacher, Technology Outreach Program Support, and Mentor for new Special Education Teachers.