

NUMICON INSTRUCTION AS A SUPPLEMENTAL MATHEMATICS
INTERVENTION FOR KINDERGARTEN STUDENTS

by

Melissa C. Jenkins
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Committee:

_____ Chair

_____ Program Director

_____ Dean, College of Education
and Human Development

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Students

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by

Melissa C. Jenkins
Master of Science
Old Dominion University, 2004
Bachelor of Arts
George Mason University, 1995

Director: Frederick J. Brigham, Associate Professor
College of Education and Human Development

Summer Semester 2013
George Mason University
Fairfax, VA



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DEDICATION

This work is dedicated to my incredibly supportive husband Chuck, and my wonderful parents Bill and Margaret.

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As I look back on the events that led to the completion of this project, I realize how fortunate I am to be surrounded by people who believe in me. This whole Ph.D. program has been quite a journey. I've experienced challenges and achievements, but I have never been alone. I am so thankful to all of the people who offered support along the way.

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TABLE OF CONTENTS

	Page
List of Tables	ix
List of Figures	x
Abstract	xi
1. Introduction.....	1
Sustained Reform	2
Conceptual Knowledge for All Students.....	3
Number Sense as a Foundation for Mathematics Learning	5
Purpose of the Present Study.....	6
Mathematics Instruction with Numicon.....	7
Definition of terms.....	11
2. Review of the Literature	15
Literature Search Procedures	15
Number Sense Theory.....	16
Number sense development.....	18
Teaching number sense.	24
Summary of Number Sense Theory	30
Number Sense Intervention Research	31
Search procedures.....	31
Overview of the research.	32
Core instruction.	33
Supplemental instruction.	40
Summary of Findings from Number Sense Intervention Research.....	52
Numicon as an Intervention for Number Sense	55
Numicon research.	55
Overall findings related to Numicon.	58
3. Methods.....	60

Setting.....	61
Participants	62
Students.	62
Researcher/Intervention provider.	69
Materials.....	70
Number sense intervention.	70
Phonics instruction.	73
Behavior management.	73
Dependent Measures	74
The Number Sense Brief.	74
Quantity discrimination.	75
Quantity	77
Procedures	77
Screening.	78
Group formation.	78
Baseline data collection.	79
Baseline phonics intervention.....	79
Number sense intervention	80
Generalization and maintenance.....	81
Reliability	82
Procedural Fidelity	82
Social Validity.....	83
Data Analysis	83
Summary of Methods	85
4. Results.....	87
Procedural Fidelity	88
Quantity Discrimination.....	88
Group analysis.	91
Individual analysis.....	100
Quantity–Numeral Correspondence.....	119
Group analysis.	122
Individual analysis.....	129

The Number Sense Brief	148
Group analysis.	150
Individual analysis.	152
Social Validity	161
Student interviews.	161
Teacher questionnaire.	167
Summary of social validity findings.	170
Summary of Results	171
5. Discussion and Recommendations	173
Summary of Findings	174
Quantity discrimination.	175
Quantity	180
General number sense skills.	185
Social validity.	190
Educational Implications	191
Limitations	194
Recommendations for Future Research	196
Conclusions	197
Appendices	200
Appendix A. Numicon Lesson Plans	200
Appendix B. Numicon Fidelity Checklists	219
Appendix C. Phonics Lesson Plans	229
Appendix D. The Number Sense Brief	245
Appendix E. Quantity Discrimination Measures	247
Appendix F. Quantity–Numeral Correspondence Measures	253
Appendix G. Intervention Schedule	262
Appendix H. Student Interview	265
Appendix I. Teacher Questionnaire	266
References	267

LIST OF TABLES

Table	Page
1 <i>Student Characteristics</i>	64
2 <i>Data Sources and Analyses by Research Question</i>	86
3 <i>Quantity Discrimination Means and Trendline Slopes by Group and Phase</i>	91
4 <i>Quantity Discrimination Means and Trendline Slopes by Student and Phase</i>	100
5 <i>Quantity–Numeral Correspondence Means and Trendline Slopes by Group and Phase</i>	122
6 <i>Quantity–Numeral Correspondence Means and Trendline Slopes by Student and Phase</i>	130
7 <i>Group Means on the Number Sense Brief Before and After Intervention</i>	150
8 <i>Student Performance on the Number Sense Brief Before and After Intervention</i>	153

LIST OF FIGURES

Figure	Page
1. Numicon Shapes representing the numbers 1 through 10.	10
2. Group means on the quantity discrimination measure.	90
3. Dylan's performance on the quantity discrimination measure.	103
4. Devin's performance on the quantity discrimination measure.	104
5. Aurora's performance on the quantity discrimination measure.	106
6. Seth's performance on the quantity discrimination measure.	108
7. Steven's performance on the quantity discrimination measure.	110
8. Lacey's performance on the quantity discrimination measure.	112
9. Adam's performance on the quantity discrimination measure.	114
10. Charlie's performance on the quantity discrimination measure.	116
11. Daniella's performance on the quantity discrimination measure.	117
12. Group means on the quantity–numeral correspondence measure.	121
13. Dylan's performance on the quantity–numeral correspondence measure.	132
14. Devin's performance on the quantity–numeral correspondence measure.	133
15. Aurora's performance on the quantity–numeral correspondence measure.	135
16. Seth's performance on the quantity–discrimination measure.	137
17. Steven's performance on the quantity–numeral correspondence measure.	139
18. Lacey's performance on the quantity–numeral correspondence measure.	141
19. Adam's performance on the quantity–numeral correspondence measure.	143
20. Charlie's performance on the quantity–numeral correspondence measure.	145
21. Daniella's performance on the quantity–numeral correspondence measure.	146
22. Pre- and postintervention performance on the Number Sense Brief.	160

ABSTRACT

NUMICON INSTRUCTION AS A SUPPLEMENTAL MATHEMATICS INTERVENTION FOR KINDERGARTEN STUDENTS

Melissa C. Jenkins, Ph.D.

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Dissertation Director: Dr. Frederick J. Brigham

A single-case multiple baseline design was used to evaluate the effects of supplemental mathematics instruction with Numicon on the number sense skills of kindergarteners with math difficulties. Nine students (6 males) with a mean age of 5 years 9 months participated in the study. All of the students had classroom math difficulties as evidenced by performance on classroom assessments. Three students were also classified as being at risk for math failure based on their performance on a validated number sense screening. One additional student was classified as being in the borderline at-risk range of performance based on the same screening.

Baseline data were collected using fluency measures of quantity discrimination and quantity–numeral correspondence. Students received phonics instruction during the baseline phase to control for the effects of small group instruction. Groups entered the intervention phase of the study once stable baseline data were established on the quantity

discrimination measure. Intervention consisted of 20 minutes of supplemental mathematics instruction over 10 consecutive school days. The mathematics intervention lessons were derived from the *Numicon Intervention Programme* (Atkinson, Tacon, & Haseler, 2011). Daily probes using the quantity discrimination and quantity–numeral correspondence measures were conducted during the intervention phase of the study. The same measures were administered to evaluate maintenance of those skills one week after the intervention ended. Additionally, the screening measure used at the onset of the study was administered again at the conclusion of the study as a measure of generalization.

Visual analysis of the quantity discrimination and quantity–numeral correspondence measures was conducted to evaluate level, trend, stability, consistency across similar phases, and immediacy of effect. The percentage of nonoverlapping data was also calculated as a measure of effect. Student scores from the pre- and postintervention administrations of the number sense screening were also compared.

This study found evidence of gains in quantity discrimination skills for kindergarten students with mathematics difficulties and moderate evidence that the gains were associated with the Numicon intervention. The small gains also evident in the quantity–numeral correspondence data could not be clearly attributed to the Numicon intervention. Large gains were evident in general number sense skills for students in each of the three categories of math difficulty. Furthermore, there was strong evidence of social validity indicating that both teachers and students felt that the intervention was effective and enjoyable.

This study adds to the emerging body of number sense intervention literature. Educational implications are described emphasizing the role of screening and supplemental intervention in Response to Intervention frameworks. Recommendations for future research are also discussed.

1. INTRODUCTION

Mathematics education in the United States has been marked by shifting pedagogic perspectives and practices since the 1930s (Cole & Wasburn-Moses, 2010; Moreno-Armella & Waldegg, 1993; Woodward & Montague, 2002). The timeline of math education reveals pendulum-like shifts between traditional and reform-oriented mathematics curricula with additional variation in practice among general and special educators (Woodward, 2004). The curricular shifts often reflected political and social changes, as well as emerging learning theories (Walmsley, 2007), while variations in classroom practices reflected differences in teacher education, as well as delays in the adoption of new learning theories by classroom teachers (van Garderen, Scheuermann, Jackson, & Hampton, 2009; Walmsley, 2007).

As a result of these varied perspectives and practices, mathematics education in the United States was simultaneously dichotomous and eclectic. The dichotomy was apparent in the distinct foci of traditional mathematics education, which emphasized teacher-directed instruction focused on computation, as compared to reform instruction which was typically student-directed with a problem-solving focus (Walmsley, 2007). Eclecticism was evident in what was characterized as an excessively broad curriculum in which too many topics were taught at a superficial level, while no unified pedagogy for

math instruction was developed to support that curriculum (Little, 2009; Woodward & Montague, 2002).

Sustained Reform

Although there were multiple reform movements between 1930 and 1980 (Walmsley, 2007), very few resulted in the same degree of sustained impact as the reform movement initiated in the early 1980s following the publication of *A Nation at Risk* (United States Department of Education, 1983). In that report, the National Commission on Excellence in Education expressed that our democratic society and ability to compete in a global market were jeopardized by low academic standards and low achievement relative to other industrialized nations. Specifically, in terms of mathematics, the National commission on Excellence in Education found that only one third of 17-year-olds could “solve a mathematics problem requiring several steps” and that remedial mathematics courses “constitute[d] one-quarter of all mathematics courses taught” in U.S. public colleges (United States Department of Education, 1983, p. 11).

The National Council of Teachers of Mathematics (NCTM) became a leading force in the reform movement that followed. The NCTM advocated a shift away from memorizing facts and algorithms to a problem-solving approach using multiple representations and real-life examples to improve student performance in mathematics (Woodward & Montague, 2002). In 1989, the NCTM released the first *Principles and Standards in Mathematics Education* which, as the title suggests, included several principles intended to guide math instruction (NCTM, 1989, 2000; Woodward, & Montague, 2002). Most notable among the principles were the equity principle, which

established the assumption that all students can succeed in complex mathematics; the call for a well-articulated and coordinated curriculum; and emphasis on active construction of math knowledge (NCTM, 2000; Woodward & Brown, 2006).

The NCTM standards and the associated constructivist teaching practices were the subject of intense debate throughout the 1990s and into the 2000s (Cole & Wasburn-Moses, 2010; Davison & Mitchell, 2008; Klein, 2007). At the same time additional studies, such as the Third International Mathematics and Science Study, continued to show evidence that students from the US were not achieving at levels commensurate with their peers from other developed countries (United States Department of Education, 2000). A consensus began to emerge recognizing the value of conceptual knowledge to facilitate mathematical applications and support long-term retention of math skills for most students (Fuchs & Fuchs, 2001). That value was reflected in the proposed Common Core State Standards (National Governors Association Center for Best Practices, 2010), and the individual mathematics curricula proposed by some states that did not adopt the Common Core, including Virginia (Virginia Department of Education, 2009). While the focus of mathematics curricula became increasingly clear, instructional methods, especially for students with disabilities and other math difficulties, became the next topic of scrutiny and debate.

Conceptual Knowledge for All Students

Montague (2003), Steffe (2011), and Woodward and Montague, (2002), among others have advocated for constructivist teaching methods over procedural methods for the development of deep conceptual knowledge of mathematical concepts. However,

there is evidence that many students with math difficulties struggle when purely constructivist methods are used. In a study of constructivist-oriented classrooms, Baxter, Woodward, and Olson (2001) found that students with low math achievement demonstrated minimal academic engagement during whole-class discussions and took on nonmathematical tasks during small group activities unless skilled teachers employed differentiated instructional strategies. Additional research found that students with math-related learning disabilities exhibit specific characteristics, not present in their typically performing peers, which make accessing constructivist instruction challenging (Bryant, Hartman, & Kim, 2003). These characteristics include frequent difficulty with basic procedures, such as retrieval of facts from memory, response automaticity, and understanding vocabulary (Bryant, Bryant, & Hammill, 2000).

The National Mathematics Advisory Panel (USDOE, 2008) concluded that neither purely constructivist, nor purely procedural instruction will meet the needs of students with math difficulties. Researchers have been pressed to identify instructional practices that can support students with math difficulties in problem-solving, conceptually oriented curricula. The USDOE report advocated instruction that “simultaneously develop[s] conceptual understanding, computational fluency, and problem-solving skills” (2008, p. xix). That report provided a clear picture of what needs to be accomplished, but not how to accomplish it and concluded that more research is needed “that identifies effective instructional practices and materials” and “mechanisms of learning” (USDOE, 2008, p. 63).

Number Sense as a Foundation for Mathematics Learning

Many experts in the field of mathematics education view number sense as a foundational skill that is required for conceptual understanding, computational fluency, and problem-solving skills (e.g., Bobis, 2008; Gersten & Chard, 1999; Jordan, Kaplan, Olah & Locuniak, 2006). In fact, Gersten and Chard suggested that number sense is as critical to the development of math skills as phonemic awareness is to reading.

Additionally, the National Mathematics Advisory Panel (USDOE, 2008) highlighted number sense of whole numbers and fractions as two of the three foundational skills that all students require to ultimately achieve proficiency in algebra.

Number sense is particularly critical for students who have risk factors for math difficulties or math disabilities. There is evidence that early math problems are associated with number sense difficulties in the areas of number knowledge, number relationships, efficient counting strategies, and number combinations (Bryant, Bryant, Gersten, Scammacca, Funk, et al., 2008). Furthermore, number sense difficulties in the primary grades are associated with poor math achievement at both the elementary and secondary levels (Locuniak & Jordan, 2008; USDOE, 2008). Number sense measures in kindergarten are particularly well correlated with calculation fluency, math applications, and overall math performance through third grade (Jordan, Glutting, & Ramineni, 2010; Jordan, Kaplan, Locuniak, & Ramineni, 2007; Locuniak & Jordan, 2008). In fact, these studies found that number sense measures administered in kindergarten were more predictive of later mathematics achievement than age, gender, socioeconomic status, reading ability, oral vocabulary, memory, and spatial reasoning.

There is evidence that some fundamental number sense skills such as subitizing, basic counting, and magnitude comparisons emerge before formal education begins for many students (Dowker, 2008a; USDOE, 2008). However, there is also evidence that students enter kindergarten with varying degrees of number sense (Jordan, Glutting, Dyson, Hassinger-Das, & Irwin, 2012). Differences are associated with a variety of factors including age, gender, and socioeconomic status (Jordan et al., 2007).

While number sense is valued as a construct and there is strong evidence that it is aligned with later mathematics achievement, there has not been sufficient research to identify evidence-based practices for number sense instruction or intervention in the early elementary grades (Bryant, Bryant, Gersten, Scammacca, & Chavez, 2008). The research has, however, provided insight into some potentially promising practices for young students with math difficulties. These include student verbalization of math strategies, concrete and pictorial representations of concepts, and explicit instruction for both procedural and conceptual development (Bryant, Bryant, Gersten, Scammacca, Funk, et al., 2008).

Purpose of the Present Study

Many in the field of mathematics education recognize the value of a conceptual understanding of mathematical concepts for all students. Yet, there is also acknowledgement that traditional instructional practices from the fields of general and special education have limited effectiveness in developing conceptual knowledge for students with math difficulties. The existing literature reveals that students with math difficulties engage in math learning in ways that are different from their typically

performing peers. These differences may make accessing conceptually oriented math activities more difficult.

Number sense has been identified as a foundational skill that is required for the development of conceptual knowledge and more complex mathematical activities. Both general and special educators recognize the need for early intervention in this area (Chard et al., 2008). However, the body of intervention research is rather limited and evidence-based practices have not been delineated. There is some evidence that interventions incorporating verbalization within a structured instructional sequence that progresses from concrete to abstract can benefit students with mathematics difficulties. Additionally, multiple researchers have described the value of providing explicit and systematic instruction to help students build and elaborate mental number lines (e.g., Chard et al., 2008; Gersten & Chard, 1999; Griffin, 2004). This study proposes to evaluate an instructional program that incorporates each of those elements.

Mathematics Instruction with Numicon

Numicon is a commercially available mathematics program published by Oxford University Press. It features multiple leveled kits addressing skills ranging from early number awareness (e.g., counting and numeral recognition) to multiplication and division. Numicon was designed for implementation in coordination with other mathematics activities/curricula (Atkinson, Tacon, & Wing, 2008). It can be used for whole class or small group instruction with students at all levels of mathematics achievement.

The hallmark of Numicon instruction is the use of special manipulatives called Numicon Shapes. Numicon Shapes are colorful plastic plates that represent the numbers 1 through 10 (Figure 1). These critical elements of Numicon instruction demonstrate patterns and relationships in the number system. However, the authors emphasize that the value of the program lies in the variety of instructional activities that are intended to broaden children's understanding of number (Atkinson et al., 2008). Specifically, these activities include structured presentation of number concepts using a sequence of concrete to abstract instruction, opportunities for verbalization and problem solving using real world scenarios, and counting activities paired with number lines.

While the framework of Numicon instruction is well-aligned with theories related to effective number sense intervention, there is no peer-reviewed published research documenting the effectiveness of the program. This study seeks to evaluate the effectiveness of supplemental instruction using Numicon for improving the number sense skills of kindergarten students with varying degrees of math difficulty. Six research questions will be addressed:

- 1) Is there a functional relation between supplemental mathematics instruction with Numicon and quantity discrimination skill for kindergarten students with mathematics difficulties?
- 2) Do kindergarten students with mathematics difficulties maintain quantity discrimination skills one week after the conclusion of supplemental instruction with Numicon?

- 3) Is there a functional relation between supplemental mathematics instruction with Numicon and performance on counting and numeral recognition tasks for kindergarten students with mathematics difficulties?
- 4) Do kindergarten students with mathematics difficulties maintain counting and numeral recognition skills one week after the conclusion of supplemental instruction with Numicon?
- 5) Do kindergarten students with mathematics difficulties demonstrate gains in their general number sense skills following supplemental instruction with Numicon?
- 6) Do kindergarten students with different degrees of mathematics difficulty exhibit different trends in their responsiveness to supplemental mathematics instruction with Numicon?

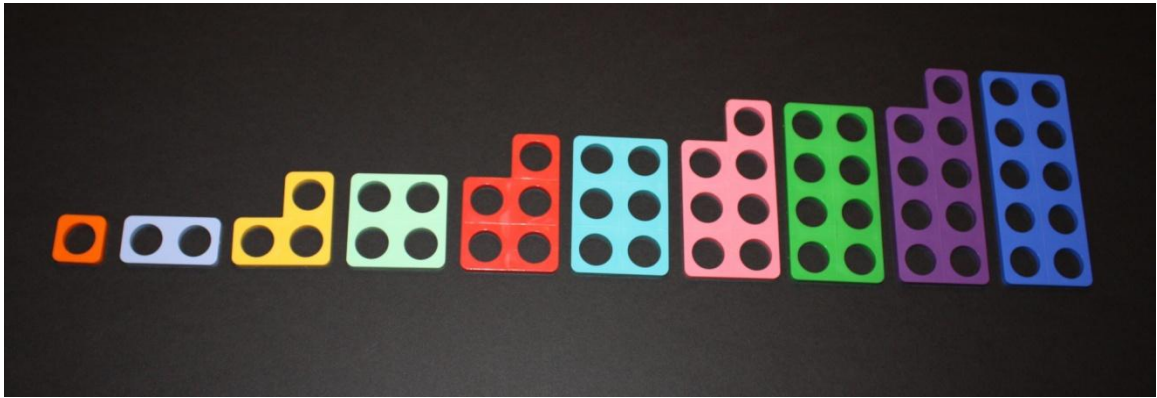


Figure 1. Numicon Shapes representing the numbers 1 through 10.

DEFINITION OF TERMS

at risk: The student participants in this study were classified as being at risk for mathematics failure based on their performance on the Number Sense Brief (NSB) (Jordan, Glutting, & Ramineni, 2008). The NSB is a validated screening measure that was demonstrated to be predictive of mathematics proficiency on a validated high-stakes state math assessment. Based on cut scores established by Jordan, Glutting, Ramineni, and Watkins (2010), a score of 15 or lower on the preintervention administration of the NSB resulted in a student being described as at risk for mathematics failure. The term *at risk* was used to describe the students with a high degree of mathematics difficulty in this study.

borderline at risk: Students were described as being in the borderline at risk category if they earned between 16 and 18 points on the preintervention administration of the NSB. This score range was determined to be an indicator of potential risk because it was within one standard deviation of the cut score that resulted in the at-risk classification established by Jordan, Glutting, Ramineni, and Watkins (2010). *Borderline at risk* is used in this study to describe the students with a moderate degree of mathematics difficulty.

classroom math difficulties: This term was used to describe students who were nominated for participation in this study based on teacher observations that they

consistently performed at or below the 25th percentile on classroom mathematics assessments. All of the student participants in this study exhibited classroom math difficulties. Some were also found to have a more substantial degree of difficulty as determined by their performance on the NSB. The term *classroom math difficulties* was used to describe students with the lowest degree of mathematics difficulty in this study.

concrete to abstract instruction: This phrase describes an instructional sequence in which students are taught mathematical concepts through a series of representations. The sequence begins with tangible objects and ends with purely symbolic representation (i.e., written numerals and operations signs). A series of intermediary representations (e.g., photographs, drawings, tally marks) is often used, as well. The most commonly described concrete to abstract instructional sequences are concrete-semiconcrete-abstract (CSA) and concrete-representational-abstract (CRA).

concreteness: This term is used to describe concrete or semiconcrete representations. Representations are considered to have a greater degree of *concreteness* if they are perceptually rich and have details that are similar to the objects they represent (Heddens, 1986). A representation that lacks these details is described as *generic*. Toy objects, photographs, or detailed drawings of objects have a high degree of concreteness.

generic: This term refers to concrete or semiconcrete representations that lack the perceptual details of the tangible object that they stand for. Pegs, cubes, line drawings, and tally marks are examples of generic representations.

number combinations: *Number combinations* is a term used in recent math literature to describe simple mathematical calculations using addition or subtraction (Fuchs et al.,

2010). The term is synonymous with *math facts*; but is intended to reflect the idea that sums and minuends within 20 can be efficiently solved using a variety of strategies.

number sense: For the purpose of this study, the term *number sense* refers to “a child’s fluidly and flexibility with numbers, the sense of what numbers mean, and an ability to look at the world and make comparisons” (Gersten & Chard, 1999, p. 19). The term includes a broad array of early math skills. These include, but are not limited to, verbal counting, counting objects, recognizing written numerals, associating numerals with quantities, making judgments about the relationships between numbers or quantities, composing and decomposing numbers, solving number combinations, and determining the reasonableness of results (e.g., Berch, 2005; NCTM, n.d.).

quantity discrimination: Quantity discrimination (also called magnitude comparison) is a number sense skill that refers to an “ability to discern quickly the greatest number in a set, and to be able to weigh relative differences in magnitude efficiently—e.g., to know that 11 is a bit bigger than 9, but 18 is a lot bigger than 9” (Gersten, Clarke, Haymond, & Jordan, 2011, p. 5).

quantity–numeral correspondence: Quantity–numeral correspondence is a number sense skill that refers to a student’s ability to determine the number of objects in a set and identify the written numeral associated with that quantity. The skill is also used when students use concrete or semiconcrete representations to solve number combinations or word problems.

representation: The depiction of an object, event, or scene (Richardson, 1999).

structured representation: Objects or images that visually present numeric information using “instantly recognizable patterns” that allow students to see the relationships between numbers (Haseler, 2008, p. 232). Examples of structured representations include five- and ten-frames, Cuisenaire Rods, rekenreks, and Numicon Shapes.

subitize: This term describes a number sense skill that allows individuals to make “rapid and accurate perceptions of small quantities” (Berch, 2005, p. 334).

2. REVIEW OF THE LITERATURE

This chapter provides a synthesis of the current understandings of number sense, first by providing an overview of the theory behind number sense and then by identifying and evaluating research of number sense intervention. The section on theory is intended to (a) define number sense and describe the factors associated with early number sense development; (b) describe the characteristics of students who have math difficulties associated with number sense; and (c) investigate the features of effective number sense instruction. The review of the intervention literature evaluates the existing number sense research with special attention to the proposed features of effective instruction identified in the theoretical literature. Finally, Numicon, a number sense intervention program, is described and placed in the context of number sense theory and research.

Literature Search Procedures

A literature search of EBSCO, APA PsycNet, ProQuest, and ERIC was conducted using combinations of the terms *number sense*, *numeracy*, *intervention*, and *instruction*. The search was limited to peer-reviewed articles published between January 1985 and December 2012. Titles and abstracts of the identified publications were examined to determine if the articles addressed mathematics learning at the elementary school level (i.e., kindergarten through fifth grade, and sixth grade if that grade was included in an elementary school). Intervention articles were only included in this review if a

mathematics intervention was the primary independent variable. Articles that addressed behavioral interventions, such as self-regulation, were excluded from this review to maintain focus on cognitive and academic causes of and interventions for mathematics difficulties.

The selected articles were read and categorized as being primarily focused on number sense development, classroom implementation of number sense strategies, number sense assessment, or intervention research. The initial database search yielded 24 articles with a developmental focus, four that were practitioner oriented, nine that focused on assessment, and 13 that described intervention research. The reference list from each of those articles was used to conduct an ancestry search. Frequently occurring topics or key words (e.g., representation, CRA) were also used to search for additional related articles.

The articles discussing number sense development, assessment, and classroom implementation were grouped together and synthesized to provide a broad overview of number sense theory. A total of 41 articles contributed to that synthesis. Thirteen number sense intervention studies were analyzed were evaluated separately.

Number Sense Theory

The concept of number sense was first outlined by Dantzig in 1954 (Berch, 2005). The term has come to be widely used in mathematics education, yet there is no single consistently applied definition (Gersten, Jordan, & Flojo, 2005; Lago & DiPerna, 2010). In fact, Berch identified 30 different components of number sense derived from definitions in 21 reports published between 1954 and 2004.

The difficulty in arriving at a consistently applied definition for number sense may stem from the fact that math educators and cognitive scientists have traditionally examined the construct from different perspectives. Many cognitive scientists have described number sense as an innate cognitive ability present at a basic level in infants (Berch, 2005; Dehaene, 2001). Conversely, math educators have described number sense as a skill that develops through informal and formal learning experiences (e.g., Fuchs, Fuchs, & Karns, 2001; Jordan et al., 2012). Recently, experts from both fields have concluded that biological and experiential factors contribute to distinct elements of number sense, which can be categorized as nonverbal or symbolic (Jordan et al., 2012; Dehaene, 2001). The nonverbal element includes the ability to instantly recognize quantities less than four (subitize) and make general magnitude comparisons for visually presented quantities. This foundational element of number sense is thought to be biologically influenced (Berch, 2005; Dehaene, 2001). The symbolic element of number sense includes the ability to associate quantities with abstract symbols such as words or written numerals. Symbolic number sense is described as a higher order skill that is heavily influenced by experience and instruction (Berch, 2005; Dehaene, 2001; Jordan et al., 2012).

In examining the vast array of published number sense definitions, two stand out for allowing the inclusion of both biological and experiential influences. Dehaene, a neuroscientist, wrote that the term *number sense* describes “our ability to quickly understand, approximate, and manipulate numerical quantities” (2001, p. 16). Gersten and Chard, educational researchers, described number sense as a term that reflects “a

child's fluidity and flexibility with numbers, the sense of what numbers mean, and an ability to look at the world and make comparisons" (1999, p. 19). These definitions account for the broad range of skills often associated with number sense. The NCTM description of those skills adds clarity to the definitions:

Students with number sense naturally decompose numbers, use particular numbers as referents, solve problems using the relationships among operations and knowledge about the base-ten system, estimate a reasonable result for a problem, and have a disposition to make sense of numbers, problems, and results. (NCTM, n.d., p. 3)

Gersten and Chard's (1999) definition of number sense will be used in this study since it most closely relates to number sense intervention for kindergarten-aged children. NCTM's description of number sense skills will be used as additional guidance in operationalizing behaviors associated with number sense development. Further operationalization of age-appropriate number sense skills for kindergarten students is dependent upon an understanding of typical development and factors that may impede that development.

Number sense development. There is evidence that typically developing children follow a common progression in developing number sense between the ages of three and nine (Griffin, 2004). The NCTM described the progression as "moving from the initial development of basic counting techniques to more sophisticated understandings of the size of numbers, number relationships, patterns, operations and place value" (2000, p. 79). The progression typically begins before age four, when many children develop

quantity awareness and counting knowledge as two unrelated concepts (Siegler & Booth, 2004). These concepts emerge as the result of contextual learning experiences (Gersten et al., 2005). Emerging quantity awareness is evident when young children make broad comparisons of the quantities of concrete items using terms such as “more” or “less” (Jordan, Kaplan, Olah, & Locuniak, 2006). The comparison is typically made using a visual reference (e.g., which group of items is taller or bigger).

As quantity awareness emerges, many children also develop counting knowledge. They learn the verbal counting sequence and then begin to apply the concepts of one-to-one correspondence and cardinality when counting objects (McGuire, Kinzie, & Berch, 2012). They may begin to respond to the question “How many?” by using the last number stated in the counting sequence. However, in the early stages of counting knowledge the response does not always reflect an understanding that the spoken number word is consistently associated with the same quantity or that the number word is unrelated to the nature of the objects that were counted (McGuire et al., 2012).

At some point during kindergarten most children will begin to integrate quantity awareness and counting knowledge to form a mental number line (Jordan et al., 2006). This development reflects an emerging sense of the relationships between numbers and quantities. At this stage subtle, yet important, differences are evident even in typically developing students’ number sense skills. Gersten and Chard (1999) highlighted these differences by describing how kindergarten-aged children differ in their responses when asked to describe the relationship between 5 and 8. Some children are able to explain that 8 is 3 more than 5. Others know only that 8 is more than 5. Another group of

children cannot automatically identify the magnitude of the difference between the two numbers, but can figure it out using concrete objects. Finally, there is a group of children who are unable to describe any relationship between 8 and 5. Each of these responses indicates a different degree of integration of quantity awareness and counting knowledge.

As students continue with formal schooling, most add to their number sense skills by associating symbols (e.g., Arabic numerals) with spoken number words and quantities. By the age of 8 or 9, they further develop their mental number lines to include two-digit numbers and begin to have a sense of place value (Griffin, 2004). Each of these milestones is critical to the future development of more complex mathematical ideas including calculations and problem-solving (USDOE, 2008).

Factors influencing number sense development. As with all developmental tasks, not all children experience a steady progression in developing number sense skills. Researchers have identified a variety of factors associated with variance in number sense performance among young children. These factors include age, gender, socioeconomic status, reading ability, cognitive functioning, and personality (Chard et al., 2008; Gersten et al., 2005; Jordan et al., 2007, Van Luit & Schopman, 2000). In some cases early math difficulties may be related to delays in the development and application of procedures rather than to more persistent characteristics (Bryant, 2005). Of the more static factors, age, gender and socioeconomic status have been subject to the most in-depth research with young children, while reading ability has been one of the most widely researched fluid factors.

In a longitudinal study of number sense growth in kindergarten, Jordan et al. (2006) found that age was a significant predictor of number sense skill. Specifically, the researchers found that older children began kindergarten with slightly stronger number sense skills than their younger peers and maintained that advantage over the course of the school year. They concluded, however, that the effect was small in terms of practical impact.

In the same study, Jordan et al. (2006) found gender effects showing a small, but reliable advantage for boys over girls on overall number sense, nonverbal calculations, and estimation. The finding held constant even when controlling for socioeconomic status, age, and reading ability. The researchers reported that the differences were small in practical terms, but aligned with findings from math research in later elementary grades. They hypothesized that the gender effect could be associated with differing achievement rates, differences in spatial cognition, socialization, or motivational factors.

Socioeconomic status was the most widely reported factor associated with number sense development. Multiple researchers found that students from low socioeconomic backgrounds have lower number sense performance and are at greater risk for long term math difficulties than their peers from midrange or high socioeconomic backgrounds (e.g., Bryant, 2005; Chard et al., 2008; Jordan et al., 2006, 2007). These differences are evident in kindergarten. Specifically, students from low socioeconomic backgrounds were found to be “overrepresented in an empirically defined group of children with low performance and flat growth in number sense over four time points during kindergarten” (Locuniak & Jordan, 2008, p. 452). These deficits are thought to be associated with

fewer at-home experiences with numbers and counting relative to those experienced by children from midrange socioeconomic backgrounds (Gersten & Chard, 1999). Gersten et al. (2005) concluded that effective number sense instruction in preschool, kindergarten, or first grade could potentially eliminate the achievement gap associated with socioeconomic status.

Reading difficulties were also identified as a factor associated with early and persistent math difficulties (Gersten & Chard, 1999; Jordan et al., 2006). Specifically, students with combined math and reading difficulties were shown to have significantly slower development in mathematics skills than their peers with math difficulties only (Jordan, Kaplan, & Hanich, 2002). The greatest differences were found in counting procedures, number combinations, and story problems and were shown to persist even when controlling for IQ and socioeconomic status. Gersten and Chard reported that the simultaneous occurrence of math and reading difficulties was already well established by the mid-1990s. However, the relationship between the two variables has been the subject of discussion. Berch (2005) suggested that problems with reading and math might be linked by difficulties in recognizing the associations between words, written numbers, and other symbolic representations. Alternatively, Jordan et al. (2002) suggested that the increased weaknesses in overall math performance for students with combined math and reading difficulties relative to students with only math difficulties may be associated with weaknesses manipulating visual representations of quantity. The authors also concluded that the frequent co-occurrence of the two difficulties may reflect general school-readiness skills or home learning experiences. Regardless of the association, reading

difficulties were described as a risk factor for mathematics difficulties (Jordan et al., 2006).

Characteristics of students with number sense related math difficulties. Students with math difficulties are clearly a heterogeneous group, not only in the factors associated with their performance difficulties, but also in the manifestations of those difficulties. In fact, Bryant et al. (2000) identified 33 learning behaviors described as being characteristic of students between the ages of 8 and 18 who had mathematics difficulties. Although the data described students in first through twelfth grade, six of the behaviors of concern were directly related to early number sense (e.g., reading and writing numbers, counting on fingers, remembering number words). Further examination of the literature on mathematics difficulties reveals the close connection between number sense difficulties and general mathematics achievement difficulties. These can generally be described as difficulties with counting knowledge, number knowledge, quantity representation, and number combinations. In the area of counting knowledge, researchers have documented that some students with math difficulties do not understand counting principles (e.g., the order irrelevance principle), make verbal counting errors, and use inefficient counting strategies (Bryant et al., 2000; Bryant, Bryant, Gersten, Scammacca, Funk, et al., 2008; Gersten et al., 2005). In the area of number knowledge, researchers have described difficulties with naming, reading, and writing numbers and recognizing the reasonableness of answers (Bryant et al., 2000; Gersten et al., 2005). Students have also demonstrated difficulties representing numbers with objects or pictures to solve number combinations or word problems (Bryant et al., 2000; Gersten & Chard, 1999).

Finally, automatic retrieval of number combinations has been identified as a frequent area of concern (e.g., Bryant et al., 2000; Locuniak & Jordan, 2008).

Given that long-term mathematics difficulties are closely associated with deficits in early number sense skills, many math educators and researchers have advocated for early number sense screening and intervention (Berch, 2005; Gersten & Chard, 1999; Jordan et al., 2007, Van Luit & Schopman, 2000). These processes may reduce the risk of long-term math difficulties for many and facilitate meaningful participation in conceptually oriented curricula for students who do develop math difficulties (Chard et al., 2008; USDOE, 2008).

Teaching number sense. There is little research evidence documenting instructional programs that teach number sense skills in the early grades using strategies that can transition to more formal elementary math (Chard et al., 2008). Classroom practice and emerging research in this area have been guided by principles of effective instruction derived from learning theory and research in other areas of mathematics (Griffin, 2004b). Four themes emerged from a review of practitioner-oriented and theoretical publications regarding number sense instruction. These include the need for (a) systematic and explicit instruction; (b) concrete to abstract representation; (c) number line experiences; and (d) verbalization of concepts and processes (e.g., Bryant & Bryant, 2012; Bryant, Bryant, Gersten, Scammacca, Funk, et al., 2008; Chard et al., 2008; Gersten et al., 2005; Griffin, 2004b).

Systematic and explicit instruction. Extensive evidence supports the use of systematic and explicit instruction for students with learning difficulties in a variety of

subject areas (Doabler & Fien, 2013). The process has documented effectiveness for teaching mathematical concepts including computation and problem solving (Bryant & Bryant, 2012). The term *systematic* refers to instructional planning designed to build on students' prior knowledge. Learning activities are structured to allow students to use their existing knowledge to make connections to new material (Griffin, 2004). The term *explicit* refers to a teaching sequence in which teachers model new content, provide guided practice opportunities, and then transition students to independent practice (Bryant & Bryant, 2012; Doabler & Fien, 2013). Systematic and explicit instruction also includes frequent academic feedback (Bryant & Bryant, 2012; Doabler & Fien, 2013).

Some have perceived that systematic and explicit instruction prevents students from constructing their own knowledge. However, Gersten and Chard described the sequence as a balanced approach to mathematical learning in which “students (a) learn the conventions, language, and logic of a discipline such as mathematics from adults with expertise; and (b) actively construct meaning out of mathematical problems (i.e., try a variety of strategies to solve a problem” (1999, p. 18). This balance provides students with learning disabilities the structure and information needed to help them learn, while also allowing them to actively construct meaning from learning activities.

Concrete to abstract representation. There is widespread support for the use of representations (models) when teaching number sense concepts (Griffin, 2004b; Gurganus, 2004). Multiple representations have been encouraged. However, there is evidence that a sequential progression from concrete to abstract representation is particularly effective for students who struggle with math concepts (Bryant & Bryant,

2012; Flores, 2010; Locuniak & Jordan, 2008). Early studies refer to the concrete-semiconcrete-abstract (CSA) sequence (Peterson, Mercer, & O'Shea, 1988). In recent years, most of the research on this sequence has focused on the Concrete-Representational-Abstract (CRA) sequence (e.g., Mancl, Miller, & Kennedy, 2012; Witzel, Riccomini, & Schneider, 2008). In the CSA/CRA sequence three sequential levels of instruction are used to present new material. At the concrete level, students are introduced to new math concepts by manipulating concrete objects, such as straws, buttons, coins, or place value blocks. At the representational level students use drawings to replace the concrete objects. Finally, students are taught to use numbers and symbols to represent mathematical concepts at the abstract level (Witzel et al., 2008). Although this is the most frequently described sequence in recent research, there has long been recognition that the concrete and representational stages actually include a continuum of representations (Brown, McNeil, & Glenberg, 2009; Heddens, 1986), which can vary in degrees of concreteness and structure.

Concreteness. Although we typically associate the term *concrete* with solid objects that can be manipulated, Heddens (1986) suggested that one-dimensional representations (i.e., pictures or drawings) can have varying degrees of concreteness, as well. Representations are considered to be more concrete (or pictorial) when they are perceptually rich and have details that are similar to those in the objects they represent. Representations are viewed as less concrete (or pictorial) when they are generic (Heddens, 1986; Kaminski, Sloutsky, & Heckler, 2009). For example, consider how a student could represent cookies when solving a word problem. Real or toy cookies are

very concrete in the sense that they can be physically manipulated and contain a lot of perceptual information (e.g., size, shape, color, texture). Plastic chips, which are concrete in the sense that they can be manipulated, but lack most of the perceptual detail associated with real cookies, could also be used. These are concrete in the sense that they can be manipulated, but lack most of the perceptual detail associated with real cookies. Conversely, a photograph of a cookie cannot be manipulated, but may contain more perceptual detail than the plastic chips. Finally, circles or tally marks, which cannot be manipulated and have virtually none of the perceptual details associated with real cookies, could be drawn to represent cookies. Plastic chips, circles, and tally marks would be the most generic representations of cookies, while real cookies, toy cookies, or pictures of real cookies are concrete/pictorial representations.

A discussion of concreteness within the concrete and representational phases of the CRA sequence is important because there is evidence that use of highly concrete representations for mathematics instruction may inhibit learning and reduce generalization (Brown et al., 2009). Specifically, students using the more concrete/pictorial types of representation often focus on idiosyncratic and sometimes irrelevant details that distract them from the mathematical concepts (Mulligan, 2011). This distraction from what is important is thought to occur because “superficial features . . . may compete with relational structure for attention...making the detection of relations more difficult than it would be in an abstract, generic format” (Kaminski et al., 2009, p. 152). Therefore, some have proposed that generic representations in the concrete and

representational phases are more useful than concrete/pictorial representations in helping students make the transition to abstract mathematics.

Structure. In addition to having varying degrees of concreteness, representations can simultaneously vary along a continuum from unitary to structured. Unitary representations include single objects (or pictures of objects) that are counted individually (e.g., coins, cubes, pegs, buttons). Conversely, structured representations present numeric information visually, using “instantly recognizable patterns” that allow students to see the relationships among numbers (Haseler, 2008, p. 232). Five- and ten-frames, Cuisenaire Rods, rekenreks, and Numicon Shapes are examples of structured representation (Haseler, 2008; McGuire et al., 2012). In each case, the representations can be instantly associated with quantities without counting and the relationships between numbers are visibly evident. For example when using ten-frames, the number 7 is clearly 2 more than 5 and 3 fewer than 10.

The arrangement of dots on dice or dominoes is also sometimes considered to be a structured representation because the images are instantly recognizable. However, the images do not consistently provide information about the relationships among numbers. For example, the typical dot presentations for 5 and 6 do not convey the idea that 6 is 1 more than 5. Similarly, base-10 blocks can become structured representations for quantities beyond 10. However, unitary counting is still required for all quantities below 10 and when trading tens/rods for ones/units (Haseler, 2008). While these representations are instantly recognizable as quantities for many adults and high-

achieving students, they may not be meaningful to children with math difficulties (Tournaki, Bae, & Kerekes, 2008).

A discussion of the differences between unitary and structured representations is important because of the difficulties associated with counting for many students with math difficulties (Bryant et al., 2000; Bryant, Bryant, Gersten, Scammacca, Funk, et al., 2008). Instruction with structured representations allows students to reduce their reliance on counting and instantly recognize quantities (Haseler, 2008). Van Luit and Schopman found that the visual patterns in structured manipulatives “may shorten elaborate counting strategies” and “facilitate the transition from concrete to abstract” (2000, p. 28). Additionally, there is evidence that use of structured representations is associated with higher levels of mathematics proficiency (Hegarty & Kozhevnikov, 1999; Kozhevnikov, Hegarty, & Meyer, 2002; van Garderen, 2006). The patterns evident in structured representations support generalization and an awareness of the part–whole relationships of numbers (Bobis, 2008), which can in turn facilitate computation and problem solving. These two skills are critical for students with mathematics difficulties (Fuchs et al., 2010).

Number line experiences. Number line experiences can be, but are not always, included in the concrete to abstract instructional sequence. Given that the creation of a mental number line is so closely aligned with number sense performance (e.g., Griffin, 2004a), this element of number sense warrants separate discussion. According to Chard et al. (2008) number lines allow students to see visual patterns in written numbers, associate written numerals with spoken number words, develop quantity discrimination

skills, and complete number before/number after activities. Griffin (2004a) described various formations of number lines commonly used in Western culture (vertical, horizontal, and dial) and indicated that by working with these number lines children develop understanding of numbers in real world contexts (e.g., tape measures, thermometers, clocks). These number line experiences are relevant to long-term mathematics performance because the development of a child's mental number line has been described as "the critical big idea necessary for solving addition and subtraction problems in later mathematics learning" (Chard et al., 2008, p. 13).

Verbalization. Some have described the language of math as one of the most critical areas of instruction for students with mathematical difficulties (Bryant, 2005; Witzel, Ferguson, & Mink, 2012). For students to become comfortable with the language of math teachers must introduce vocabulary using mathematically precise, but age-appropriate, definitions (Griffin, 2004; Witzel et al., 2012). Additionally, students should have opportunities to engage in mathematical discussions with peers in the context of games and problem-solving scenarios (Griffin, 2004a; 2004b). Teachers can encourage students to discuss their own mathematical reasoning through the use of effective questioning and prompts. Students may clarify their own thinking when given the opportunity to verbalize their understanding of mathematical concepts and procedures (Heddens, 1986).

Summary of Number Sense Theory

Number sense is viewed as foundational to more complex mathematical understanding and is well correlated with mathematics performance through high school.

There is evidence that certain risk factors are associated with delayed number sense development and that performance differences are evident in kindergarten. There is also a strong belief among mathematics educators that number sense skills can be remediated through explicit and systematic instruction. Instructional variables associated with effective number sense instruction include concrete to abstract representation, number line experiences, and verbalization of mathematical concepts and processes.

Number Sense Intervention Research

Number sense theory provides a framework for evaluating the number sense intervention research. The goal of this study is to gain an understanding of the effectiveness of an intervention that emphasizes the use of structured representation for developing number sense skills for kindergarten students with math difficulties. A literature review was conducted to identify number sense interventions that have been used in the elementary grades. The intervention research was analyzed to identify the presence of instructional factors that have been associated with effective number sense instruction with specific attention given to the use of varied types of representations.

Search procedures. As previously mentioned, intervention articles were identified through a database search of EBSCO, APA PsycNet, ProQuest, and ERIC. Search terms included combinations of number sense, numeracy, intervention, and instruction. The search included peer-reviewed articles published between January 1985 and December 2012. Ancestry searches of all identified articles were used to identify additional publications. The reviewed articles were limited to those in which a

mathematics intervention was used as the independent variable and a measure of number sense was used as the dependent variable. The articles were further delimited to include only studies in which elementary school students were the participants.

Overview of the research. Thirteen number sense intervention articles were identified. All of the studies were completed after 2000 and 77% of the research was published in 2008 or later. A total of 1,648 student participants were involved in the collective research, with 771 receiving some type of number sense intervention. A variety of elementary grade levels were represented. Interventions for students in kindergarten made up the largest portion of the research (46%). First graders were included in 23% of the research and third graders in 15% of the research. Second and sixth graders were each included in 8% of the number sense intervention research.

Many of the interventions were designed to be implemented within a multitiered instructional framework such as Response to Intervention (e.g., Chard et al., 2008; Bryant, Bryant, Gersten, Scammacca & Chavez, 2008). Tiered frameworks are intended to match the instructional needs of individual students to resources available within a school (Gadke, Cates, & Swerdlik, 2012). These frameworks facilitate the early identification of students with learning differences, provide structured systems of intervention, and can reduce the risk of later learning disabilities (Bryant et al., 2011). Response to Intervention (RtI) is widely used in reading, but is just emerging as a practice for mathematics. Bryant et al. (2011) emphasized the importance of identifying and validating practices to support RtI in mathematics. With that in mind, the studies reviewed here are classified as either core instruction or supplemental instruction.

Core instruction. Core instruction is also called Tier 1 instruction in RtI frameworks. Core instruction is provided to all students within a grade level and should use evidence-based practices (Gadke et al., 2012). Five of the thirteen number sense studies identified for this review were designed to be implemented as core instruction. The studies evaluated the use of peer-assisted learning, contextual applications of number sense, specialized representations, and multielement instructional packages.

Peer-assisted learning. Fuchs et al., (2001) investigated the effects of peer-assisted learning strategies (PALS) on the mathematics development of kindergarten students with a variety of academic achievement levels. Twenty teachers from five schools were randomly assigned to either the PALS condition or a business-as-usual control condition. Students within the classrooms were described as high achieving, medium achieving, or low achieving based on pretest performance on the Stanford Early School Achievement Test (SESAT). Additionally a group of students was classified as having disabilities if they had been found eligible or referred for special education services. A total of 248 students participated in the study. Students from each of these achievement groups had nearly equal representation in the treatment and control conditions.

The PALS intervention was implemented twice per week for 20 minutes over a 15 week period. The intervention replaced a component of the core instruction in the intervention classrooms, thereby ensuring that the treatment and control groups had equal amounts of mathematics instruction. Teachers were trained to implement PALS with their students. The student participants engaged in mutual tutoring activities following

the PALS activity sheets. Activities emphasized number concepts, number comparisons, adding and subtracting concepts, and written addition and subtraction. The activities progressed from work with numbers 0–5, then 0–10, and finally 10–20. The students used number lines with number arrays below each numeral. Base-10 materials (beans and popsicle sticks) were also incorporated into the activities. The beans were likely used as a generic representation of quantity. Although the specific base-10 activities are not well described, it can be assumed that the materials were used for unitary counting the sets 0–5 and 0–10 and for structured representation for the numbers 10–20.

The SESAT was used as the primary dependent variable. The SESAT was administered as both a pre- and posttest to all of the students. The data were evaluated using an ANOVA. The authors concluded that the growth of students who received the PALS intervention significantly exceeded the growth of the student in the business-as-usual condition with an effect size of .24. Additionally, although the growth differences were not statistically significant among students from different achievement groups, the authors concluded that PALS seemed to have greater effect for students from the medium achievement group (effect size = .53), low achievement group (effect size = .46), and students with disabilities (effect size = .41). The effect size for high achieving students was -.20. The authors indicated that many of the high achieving students were approaching the ceiling of the SESAT and the number sense instruction may not have been appropriate for them. In general, the findings of this study supported the use of peer-assisted learning to develop number sense and early math calculations for kindergarten students with low or average achievement.

Conceptual applications of number sense. Yang and Wu (2010) used an experimental design to evaluate the effect of embedding number sense instruction through contextual problem solving into the general curriculum to improve number sense performance for third-grade students in Taiwan. Sixty students, from two third-grade classrooms participated in the study. Classes were randomly assigned either the intervention condition or a business-as-usual condition. The mean age of the students was 8-5 years. They attended schools representing a range of socioeconomic backgrounds.

Mathematics instruction was provided by the regular classroom teacher for 20 sessions lasting 40 minutes each in both the intervention and comparison conditions. The teacher in the comparison condition only used textbook activities. In the intervention condition, the teacher supplemented the regular textbook with contextualized number sense activities. The number sense activities required the students to apply existing knowledge of common measures (e.g., the height or weight of a third grader) to select reasonable answers for mathematics story problems.

Data were collected using a researcher-developed number sense test that was administered before intervention, after the first instructional unit, and after the second instructional unit. The results were analyzed using ANCOVA with the pretest as the covariate. The researchers found a significant difference between the intervention and comparison groups after the first unit, $F(1,57) = 16.03, p < .05$, and the second unit, $F(1,57) = 21.387, p < .05$. The effect size for the intervention group was .82 after the first unit and .99 after the second unit. It is important to note the researcher-developed

measure was closely aligned with the intervention. Qualitative data were also collected by interviewing each student after the first and second instructional units. The researchers found that the students from the intervention group applied number-sense based problem solving strategies with greater frequency than their peers from the comparison group. However, both groups were equally effective in arriving at correct answers to the problems. This research suggests that students can be taught to apply number sense strategies to efficiently solve applied mathematics problems. A limitation is the requirement that students be familiar with common measures that can be used as referents for these types of problems.

Specialized representations. Yang and Tsai (2010) investigated the effect of embedding technology (virtual manipulatives) into core instructions on the number sense skills of sixth-grade students from Taiwan. Two classes from an elementary school with eight sixth-grade classes were randomly selected for participation in the study. Both classes had 32 students. One class was assigned to the intervention condition; the other utilized the standard curriculum. Both classes were taught by the same teacher.

Students in the intervention condition received fraction instruction that incorporated virtual manipulatives. Math instruction was provided for 40 minutes four times per week over a four-week period. The teacher followed identical lesson plans for both classes. However, the intervention group used technology and the comparison group did not. It is not clear if nontechnology based representations were used in the comparison condition.

Pre- and posttest data were collected using a number sense measure developed by the first author. The measure evaluated five elements of number sense for fractions including quantity comparison, use of benchmark numbers as referents, use of varied representations, operations with fractions, and the ability to determine the reasonableness of an answer. Minimal information regarding the nature of the questions was provided. One point was awarded for a correct answer and use of number sense strategy. Rules-based strategies earned .5 point. The data were analyzed using ANCOVA with the pretest scores as the covariate. A significant difference was found between groups, favoring the intervention group, $F(1,62) = 7.713, p = .007$. Additionally, there was no significant difference between the pre- and posttests for the comparison group, while the intervention group did perform significantly better on the posttest. A survey measuring student attitudes toward mathematics learning revealed that the two groups were not statistically different before intervention. However, there were statistically significant differences between the two groups after intervention, with the intervention group reporting more positive attitudes toward mathematics than the comparison group. The findings from this study suggested that the use of technology may benefit students in acquiring and applying number sense skills for problem solving, while also improving student attitudes towards mathematics instruction.

Multielement instructional packages. Van Luit and Schopman (2000) investigated the effects of a structured early numeracy program on the number sense skills of kindergarten children with special education needs in the Netherlands. Participants included 124 Dutch children who had been found eligible for special

education services, but did not have significant sensory problems, motor problems, or “clear signs of severe mental retardation” (Van Luit & Schopman, (p. 2000), p. 29). All student participants had both language and behavior problems and performed below the 25th percentile on a standardized early numeracy assessment. The students were divided into two equal sized groups. Data analysis revealed no significant differences between the groups on key demographic characteristics.

Trained research assistants instructed the participants in the intervention group in small groups using hands-on activities, instructional coaching, and realistic problem-solving activities. The instruction was for 30 minutes per session two times per week over a six-month period and replaced regular classroom instruction, which ranged from one to two hours per week. The concrete-semiconcrete-abstract instructional sequence was used to build number sense with quantities up to 15 and to solve addition problems. In the concrete phase real objects (i.e., bananas) were used. In the semiconcrete phase perceptually detailed pictures of the real objects were used. Students were also taught to use tally marks as a structured representation of quantity in the semiconcrete phase of instruction. Students in the comparison condition received one to two hours of mathematics instruction per week in their special education classroom using designated special education curriculum materials.

A standardized early numeracy test was used as a pre- and posttest measure to compare the performance of the students in the two groups. A significant difference was found on the overall numeracy measure with students in the experimental condition outperforming students in the comparison condition, $t(124) = 3.29, p = .001$. The effect

size was calculated to be 0.66 for that measure. There was no significant difference between the two groups for performance on a transfer measure. This study indicates that the concrete to abstract sequence was effective in improving general number sense skills for students with special education needs in the Netherlands. However, the students were not able to apply the new skills to related tasks.

Chard et al., 2008 conducted a study to evaluate the feasibility of implementing a core kindergarten mathematics curriculum focused on developing early number sense, geometry, measurement, and vocabulary. The effect of the program on student learning was evaluated in combination with teachers' satisfaction with the program. The student participants included 254 students from 14 schools. Fifty-two percent of the participants received free or reduced-cost lunch, 14% were classified as minorities, and 6% received ELL services.

Regular classroom teachers were trained to implement the *Early Learning in Mathematics Program* (ELM) as a replacement for their regular core mathematics instruction. ELM consisted of 100 lessons, at 30 minutes each, focused on the use of mathematical models, math vocabulary, and procedural fluency. Modeling followed the CRA sequence. The authors described use of unitary generic items for the concrete phase, transitioning to number lines or dots in the representational phase. Ten-frames and tally marks were also used as structured representations of numbers for counting tasks. The comparison group was a business-as-usual condition where teachers used a variety of approaches to address the state mathematics standards.

Data were collected using the SESAT and analyzed using ANCOVA with pretest scores as the covariate. The authors described a significantly different performance between the ELM classrooms and the business-as-usual classrooms ($t = 2.18$, $df = 9$, $p = .0571$) and an effect size of .26. There was no significant difference in effect for low achieving (i.e., below the 25th percentile) or average achieving students. As a feasibility study, the researchers sought the teachers' perceptions of the program. Teacher interviews indicated that the teachers were very satisfied with the program.

Supplemental instruction. Supplemental instruction can be provided at Tier 2 or Tier 3 in RtI frameworks. Tier 2 instruction includes targeted group intervention designed to meet the specific learning needs of children who are not successfully progressing in the curriculum with the supports provided through Tier 1 (Gadke et al., 2012). Tier 3 instruction is individualized intervention designed to meet the unique learning needs of students who have not been successful with Tier 2 intervention. Intervention time and frequency-of-progress measurement typically increase with progression through the tiers. Evidence-based practices are expected at each level. Eight of the studies that were identified for this literature review evaluated supplemental interventions that could be used at the Tier 2 level. These studies evaluated the use of specialized representations, computer-assisted instruction, and multielement interventions.

Specialized representations. Tournaki et al. (2008) examined the effect of teaching number sense with a rekenrek on addition and subtraction fact accuracy for students with learning disabilities. The rekenrek is a structured concrete manipulative

with two rows of 10 beads. Half of the beads in each row are red and half are white. The structure of the rekenrek is intended to help students see numeric relationships using 5 and 10 as referents.

The participants in this study included 45 first-grade students with learning disabilities who received instruction in self-contained classes. All of the students were found eligible for special education services using the discrepancy model. All also demonstrated combined mathematics and reading difficulties. The mean age of the students was 7.6.

Three conditions were employed in the study. The first and second groups followed identical instructional sequences. Supplemental instruction was provided for 30 minutes five times per week over a three-week period. Both groups used unitary counters paired with five-frames, ten-frames, and double ten-frames. Group one also used the rekenrek as an additional representation. The third group received no supplemental instruction.

Pre- and posttest data were collected using a researcher-developed assessment of first-grade addition and subtraction concepts. The assessment evaluated addition facts with two or three addends, subtraction with minuends up to 20, missing number equations, and word problems. The data were analyzed using an ANOVA. Significant differences were found among the three methods of instruction, $F(2,44) = 16.93$, $p < .001$. A Scheffé post-hoc test revealed that the group using the rekenrek performed significantly higher than the students in either of the other groups and that there was no significant difference between groups two and three. The effect size for the group using

the rekenrek was calculated to be 2.27. The findings of this study support the idea that structured concrete manipulatives can improve number sense related calculation skills for students with disabilities, and that the manipulatives may have greater impact than unitary counters with semiconcrete structure (i.e., five- and ten-frames).

Computer-assisted instruction. Baroody, Eiland, Purpura, and Reid (2012) evaluated computer-assisted discovery learning of two different addition strategies on the addition performance of kindergarten students who were identified as being at risk. Twenty-eight students participated in the study. They were classified as being at risk based on performance at or below the 25th percentile on the *Test of Early Mathematics Ability, 3rd Edition* (TEMA-3) or a district identified risk factor (e.g., low socioeconomic status, minority status, ELL, low birth weight, ADHD). The participants ranged in age from 5 years 1 month to 6 years.

The participants were assigned to one of two treatment conditions. One group was designated to receive strategy instruction for adding 0 or 1 (add 0/1 group), the other to receive strategy instruction for doubles addition facts. Both groups participated in prerequisite training activities that involved subitizing with dot patterns, adding dot patterns, counting, and identifying one more than a given number. Group specific training followed the prerequisite training and lasted for nine weeks. The intervention for the add 0/1 group involved computer-assisted discovery learning of plus 0 or plus 1 patterns through sequenced trials. The intervention for the doubles group used computer-assisted discovery learning through skip counting by two and doubles addition fact trials.

An addition fact fluency measure was used as one dependent variable. The doubles facts on that measure served as a control for the add 0/1 group, while the plus 0 and plus 1 facts served as a control for the doubles group. ANCOVA were used to analyze the fluency measures using the pretest results as a covariate. The results showed that the add 0/1 group mastered significantly more plus 0 and plus 1 facts (practiced and unpracticed) than participants from the doubles group. However, there was no significant difference between groups on fluency with small doubles facts (e.g., $1 + 1$, $2 + 2$). The researchers did report greater gains in large doubles facts (e.g., $6 + 6$) for the doubles group. The TEMA-3 was also analyzed as a posttest using pretest results as the covariate. Students from the add 0/1 group significantly outperformed the doubles group on the TEMA-3, $F(1,25) = 5.246$, $p = 0.031$. The effect size was calculated to be .92. The gains for the add 0/1 group appeared to be aligned with questions asking students to identify the number after a given number. The authors concluded that computer-assisted discovery learning can be an effective tool for improving number combination fluency for kindergarten students with risk factors, and that the add 0/1 strategy, seemed to support students in learning rules rather than memorizing facts. It is important to note that although all students had risk factors, no data were provided indicating how many of the students actually performed in the at-risk range on the TEMA-3.

Wilson, Revkin, Cohen, Cohen, & Dehaene (2006) also evaluated the impact of computer-assisted instruction on number sense skills including counting, subitizing, quantity discrimination, and number combinations. The researchers used a single group pretest/posttest design with no control group as an open trial for the program. The

participants included nine students from France with a mean age of 8.1 years. All of the students performed below the 37th percentile on the arithmetic subtest of the WISC ($M = 12^{\text{th}}$ percentile) and were classified as students with math difficulties.

The students received a maximum of 10 hours of intervention over a three-week period using “The Number Race” software developed by the authors. The computer-assisted learning program provided training in quantity comparison and emphasized the relationship between quantities, written numbers, and number words. The program also provided instruction in addition and subtraction facts using concrete representations.

The researchers found significant improvement in the participants’ fluency with subitization of quantities up to 3 ($F(1,7) = 19.1, p = .003$), but no significant gains for quantities between 4 and 8. The authors reported a marginal improvement for counting skills, statistically significant increases in fluency for magnitude comparisons ($F(1,8) = 19.7, p = .002$), and no improvement for base-10 concepts. Accuracy with subtraction facts increased, but addition fact accuracy did not. In short, the study indicated some potential for growth in early subitization skills, magnitude comparisons, and subtraction fact accuracy.

Wilson, Dehaene, Dubois, and Fayol conducted additional research on the effectiveness of “The Number Race” (2009). In this second study the researchers evaluated the impact of the program on the number sense skills of kindergarten students from two French schools in low socioeconomic status communities. Fifty-three students participated in the study. The mean age was 5.6 years. The first language of 43 percent of the students was not French.

A crossover design was used to evaluate the program. The students were divided into two groups. For the first half of the study one group received instruction using the mathematics software and the other received instruction using reading software. Instructional conditions reversed for the second half of the study. In the mathematics condition students completed six sessions at 20 minutes each. In the reading condition students completed four sessions at 20 minutes each.

Student performance was measured before intervention, at the midpoint of the study, and after intervention. Measures of symbolic quantity discrimination (i.e., verbal and written number comparisons), nonsymbolic quantity discrimination (i.e., identify the larger quantity of dots), counting, and addition were evaluated. The quadratic interaction was significant for the verbal and written quantity discrimination tasks, $F(1,51) = 5.28$, $p = .03$, reflecting a significant decrease in quantity discrimination errors for both groups following the mathematics intervention. There was no significant difference in results associated with the intervention for quantity discrimination tasks using images, suggesting that the program improved symbol-quantity associations rather than quantity awareness. The quadratic interaction was also significant for counting tasks, $F(1,51) = 4.88$, $p = .03$. However, this result reflected a decrease in counting errors during the reading intervention, not the mathematics intervention. In terms of addition performance, the students showed consistent decreases in addition fact errors across time regardless of intervention condition. The findings of these two studies indicate that this software program may be useful for improving student performance on quantity–numeral correspondence measures and symbolic quantity discrimination measures.

Multielement interventions. Bryant, Bryant, Gersten, Scammacca, and Chavez (2008) evaluated the impact of small group supplemental instruction on the development of number sense, operations, and quantitative reasoning of first and second graders with math difficulties. Participants included 126 first graders (26 receiving supplemental instruction) and 140 second graders (25 receiving supplemental instruction). A regression discontinuity design was used to determine if the intervention resulted in closing the gap between the performance of the students with math difficulties who received the tutoring and their typically performing peers who received no supplemental instruction. Students were identified as being in need of the supplemental intervention if they scored at or below the 25th percentile on the *Texas Early Mathematics Inventories – Progress Measure* (TEMI-PM) at the start of the school year. Demographic data for the student participants revealed a diverse representation of racial/ethnic backgrounds in both the at-risk and not at-risk groups. Additionally, students from low socioeconomic backgrounds made up 37% of the at-risk group and 33% of the not at-risk group in first grade. Students from low socioeconomic backgrounds made up 37% of the at-risk group and 42% of the not at-risk group in second grade.

The supplemental instruction was delivered in the form of small group tutoring sessions to same-ability students within each grade level. The first graders completed an average of 64 sessions and the second graders completed an average of 62 sessions across 18 weeks. All sessions were 15 minutes long and occurred three to four times per week. Researcher-trained tutors using scripted lessons provided the explicit and systematic instruction. The content was aligned with state standards for each grade and emphasized

number concepts, place value, and addition and subtraction combinations. Activities included strategy instruction for addition and subtraction and use of the concrete-semiconcrete-abstract instructional sequence. Unitary counters and base-10 models were used in the concrete phase. Number lines and hundreds charts were used in the semiconcrete phase. Instruction emphasized numbers 0–99 for first-grade students and 0–999 for second-grade students.

Pre- and posttest scores for the total TEMI-PM were analyzed using regression discontinuity. There was a significant main effect for the second-grade students ($b = .19$, $p = .018$), with specific gains in addition and subtraction skills and no significant effect on the other three subtests. There was no significant effect of the intervention on any portion of the TEMI-PM for the first graders. The researchers theorized that first graders needed additional instructional time and a follow-up study was conducted to evaluate this hypothesis.

Bryant, Bryant, Gersten, Scamacca, Funk, et al. (2008) extended the study by using the intervention with first graders with additional instructional time. Participants included 161 students, 42 of whom received intervention. In this extension study the first graders received 20 minutes of instruction four times per week over 23 weeks. The researchers demonstrated a significant main effect for total math performance ($\beta = .21$, $p = .014$) with specific gains on the Number Sequences, Addition/Subtraction, and Magnitude Comparison portions of the Texas Early Mathematics Inventory: Progress Monitoring measure. The combined studies indicated that well-structured supplemental

instruction in 15 to 20 minute sessions four times per week can narrow the performance gap for students with math difficulties in first and second grades.

Bryant et al. (2011) conducted another evaluation of the effects of supplemental intervention on the mathematics performance of first-grade students with math difficulties. In this study the researchers administered multiple probes over a three-week period to identify students who had true math difficulties and to reduce the inclusion of students who were falsely identified as being at risk. The yearlong study started with 224 students and ended with 204 primarily due to students moving out of the school system. Of the 204 participants, 139 received intervention and 65 were assigned to a comparison group. The participants represented a broad range of demographic variables (e.g., ethnicity, ELL status, socioeconomic status).

The students in the intervention group received approximately 30 minutes of supplemental instruction focused on numbers and operations for 88 lessons. All instruction was provided to small groups of students. Activities were designed to promote conceptual and procedural understanding of counting, number knowledge, number relationships, and base-10 concepts. Additionally, addition and subtraction were taught using part-part-whole relationships, related facts, and strategy instruction (e.g., counting on, doubles + 1). Explicit and systematic instruction was guided by daily progress monitoring. Vocabulary was emphasized and taught through generic unitary representations (cubes) and base-10 blocks in the concrete phase. Number lines and ten-frames were used in the representational phase. A behavior management system was also in place.

Pre- and posttest data were collected using the TEMI-PM as a proximal measure and *Texas Early Mathematics Inventories – Outcome* (TEMI-O) as a distal measure. The TEMI-PM places greater emphasis on number sense related skills, while the TEMI-O emphasizes computation and procedures and is designed to align with the state mathematics standards. The data were analyzed using ANCOVA with the pretest data from the TEMI-PM as the covariate. Subtest and composite scores were analyzed. Statistically significant differences were found on the TEMI-PM with an effect size of .50. There were also significant differences on all of the TEMI-PM subtests with the exception of magnitude comparisons (i.e., quantity comparisons). There was no significant difference between groups on the TEMI-O after controlling for Type 1 error. The findings of this study confirm that multielement supplemental instruction can improve overall number sense related mathematics skills for young students with mathematics disabilities.

Jordan et al.(2012) evaluated the impact of a multielement supplemental intervention on the overall number sense skills of kindergarten students from low-income schools. A total of 132 students participated in the experimental design. Student participants were randomly assigned to a number sense group, a language group, or a business-as-usual control group. Males and females were represented fairly evenly, making up 51% and 49% of the total sample, respectively. The sample was also diverse in terms of race/ethnicity. Forty-eight percent of the students were identified as African American, 41% Hispanic, 10% Caucasian, and one student was biracial. Nearly 25% of

the participants received ELL services. The mean age of the students was 5 years 5 months.

Intervention took place in small groups over an eight-week period, with 30 minute sessions provided three times per week. The mathematics intervention focused on whole number competencies and was aligned with the Common Core State Standards (CCSS). The activities focused on place value, identification of numerals and quantities greater than 20, determining one more or one fewer than a given number, and written calculations. Scripted lessons were used to provide systematic and explicit instruction. Key vocabulary was selected and defined for each lesson. The activities included modeling of number concepts by pairing concrete representations with symbolic representations. Unitary generic representations were most commonly used (i.e., Unifix cubes). The students were also taught to use their fingers as a structured representation relating to the quantity five. Students were taught to “make the numbers 1–5 with their fingers automatically, without counting” (Jordan et al., 2012, p. 653) and to use the counting on strategy for quantities up to 10. They were also taught to use their fingers for addition and subtraction. Number lines were also used to support the development of counting principles and magnitude comparisons.

Students in the language group received instruction at the same frequency as the mathematics intervention group. The language intervention used literature to expose students to mathematics vocabulary (e.g., before, after, big, small, part, whole) along with general vocabulary. This group was intended to serve as a control for small group instruction. A third group of children received no specialized instruction. The business-

as-usual group participated in regular mathematics instruction in the general education classroom.

Data were collected before intervention, immediately after completion of the intervention, and eight weeks after the intervention. The NSB (Jordan, Glutting, & Ramineni, 2010) was used as a broad measure of number sense achievement. The Woodcock Johnson-Brief Math (WJ-B) was used as a standardized measure of overall mathematics achievement. A vocabulary measure of 15 words (12 intervention related and three generic) was also administered. The data were evaluated using 22 multilevel models. The NSB data were analyzed as total scores and domains. The WJ-B data were also analyzed as total scores and domains. The results showed that the number sense intervention group outperformed the control group on all of the math measures. The differences were statistically significant in 14 of the 20 mathematics comparisons. There were no significant differences on any measure between the language group and the business-as-usual control group. Effect sizes were calculated comparing the number sense intervention group to the business-as-usual group. The mean effect size for the number sense group on the mathematics measures was 1.12. The effect size for total NSB score was 1.80 immediately after intervention and 0.63 on the delayed posttest.

These results indicate that the number sense intervention was more effective than either the language intervention or regular classroom instruction in improving the general number sense skills of kindergarten students from low socioeconomic backgrounds. It is important to note that these students were considered to be at risk for mathematics difficulties by virtue of attending a school that served primarily low-income students.

However, no data were reported regarding how many of the student participants performed in the at-risk classification on the NSB. Therefore, the impact of this intervention for students who have or are at risk for math difficulties cannot be determined.

Summary of Findings from Number Sense Intervention Research

The earliest identified number sense intervention for elementary school children was conducted in 2000. An increased focus on number sense intervention became apparent in 2008 and subsequent years. A majority of the studies (69%) evaluated the impact of a number sense intervention for students in kindergarten or first grade, reflecting the emphasis on early identification and intervention. The intervention research was well-aligned with the documented need for practices to support RtI in mathematics. Five of the intervention studies identified for this review were implemented as core instruction. An effect size could be calculated for four of the core interventions. The mean effect size for core interventions was 0.54 ($SD = 0.33$, range 0.24 to 0.99). Eight of the studies evaluated practices that were implemented at the level of Tier 2 interventions. Effect sizes could be calculated for four of those studies, as well. The mean effect size for supplemental interventions was 1.37 ($SD = 0.81$, range 0.5 to 2.27).

Multielement interventions made up the greatest portion (46%) of the number sense intervention research. These interventions included multiple instructional variables (e.g., varied representations, verbalization, number line activities, behavioral interventions) with no clear emphasis on any single factor. They also often addressed a

broad range of number sense skills. The mean amount of intervention time was 2220 minutes for the core instructional programs and 1588 minutes for the supplemental interventions. Effect sizes could be calculated for four of the studies. The mean effect size for multielement interventions was 0.81 (range 0.26 to 1.80), indicating a strong positive effect of intervention. Positive outcomes were most frequently identified for number combinations. There were mixed effects for quantity discrimination (i.e., magnitude comparisons).

Interventions that incorporated computer-assisted-instruction made up 23% of the number sense intervention research. All of these studies were implemented as supplemental instruction. Intervention time was reported for two of the studies. The mean intervention time was 360 minutes (range 120 to 600). An effect size, 0.92, could only be calculated for one study (Baroody et al., 2012). The computer-assisted interventions were effective in improving number combinations, subitizing skills, and quantity discrimination.

Specialized representation of number concepts was used in two studies (15%). One study used virtual manipulatives as an element of core instruction. The other used structured concrete representation (a rekenrek) as an element of supplemental instruction. The average intervention time for interventions incorporating specialized representation was 545 minutes (range 450 to 640). An effect size could only be calculated for the supplemental intervention. It was 2.27, indicating that the concrete structured representation had a very strong effect on addition and subtraction performance. The

core instruction using virtual manipulatives resulted in improved number sense applications over a comparison group.

One study utilized peer-assisted learning as an element of core instruction. The total intervention time was 600 minutes. The intervention resulted in an effect size of 0.24, reflecting small gains on a measure of general mathematics readiness skills. Greater gains were evident for children with low and average mathematics achievement when the data were disaggregated. The final study paired number sense instruction with conceptual applications as an element of core instruction. That intervention was implemented for 800 minutes and had an effect size of 0.82.

Collectively the interventions were effective in addressing a broad range of number sense skills (e.g., counting, quantity discrimination, number combinations). The grand mean effect size for all of the number sense interventions was 0.94 ($SD = 0.68$, range 0.24 to 2.27). There was strong support for multielement interventions, which included several of the elements of effective instruction that were identified in the theoretical literature. The mean effect size for these interventions was 0.81. Numicon intervention includes many of these instructional elements, as well. However Numicon instruction emphasizes the use of structure manipulatives to represent mathematical concepts in a manner similar to what was done in a study conducted by Tournaki et al. (2008). The intervention in that study resulted in the greatest gains of all those identified in this review, as determined by an effect size of 2.27. The supplemental instruction emphasized the use of a structured generic manipulative (i.e., a rekenrek) to develop

number combination skills for students with learning disabilities. These findings suggest that further exploration of number sense intervention with Numicon is warranted.

Numicon as an Intervention for Number Sense

Numicon is a mathematics program designed for the primary elementary grades. The program has leveled instructional kits that can be used in the general education classroom; there are two programs specifically designed for Tier 2 and Tier 3 intervention, as well as publications describing how the materials can be used to teach mathematics to students with Down syndrome. Numicon instruction features many of the characteristics of effective number sense instruction identified in theory and research. These include explicit and systematic instruction employing counting and number line activities, opportunities for children to verbalize mathematical concepts while focusing on applied problem solving, and a concrete to abstract instructional sequence. The use of Numicon Shapes and other structured manipulatives distinguish Numicon instruction from many other multielement instructional packages.

Numicon research. Numicon instruction is well-aligned with the number sense intervention theory. The database search for number sense interventions did not identify any studies in which “Numicon” was used as the independent variable. Therefore a second search was conducted using “Numicon” as the search term with no limitations. The database search yielded one dissertation (Rouse, 2009) and one article, which had elements of a case study for an individual student, as well as a group of schools (Saunders, 2005). Both articles were primarily descriptive. They are summarized below.

The dissertation study evaluated the impact of lessons from the Numicon Foundation Kit on the development of early mathematics skills for three students with Down syndrome (Rouse, 2009). The student participants included one female, aged 8-11 years, who was in the third grade at a public school. She attended both general and special education classrooms. Most of her academic instruction was provided in the special education classroom. Before the intervention, this student was described as being able to count from 1 to 10, recognize the numerals from 1 to 10, and match quantities of objects to numerals between 1 and 10. A second female student, aged 11-5 years, was homeschooled. Anecdotal reports indicated that she could count from 1 to 100, count money, tell time on a digital clock, and complete multiplication with single digits. This student was reported to struggle with addition and subtraction with two-digit numbers. The third student was an 11-6 year old male who was also homeschooled. Before the intervention, anecdotal reports indicated that he could count to 100, tell time on a digital clock, and complete some addition and subtraction.

The researcher provided one-on-one intervention for 20 sessions lasting 30 minutes each. The sessions were conducted in a library or in the students' homes during the summer break. The lessons were drawn directly from the Numicon Foundations Kit, and the Numicon Assessment Signposts were used to determine when each student was ready to progress to a new lesson. A variety of hands-on activities were employed including a task in which the students were required to create a mental image of a Numicon Shape that was hidden in a bag or box by touch and match the imaged shape to a model. Students were also asked to cover a 10 by 10 square pegboard using mixed

Numicon Shapes. This task required them to determine which shapes would fit together to cover a given amount of space. The students also manipulated the Numicon Shapes to solve addition and subtraction facts.

No quantitative data were collected in this study. All three students were reported to be able to associate the Numicon Shapes with the corresponding name and numeral. The younger student was able to add single digits using the shapes. The older two students were able to solve single digit addition and subtraction facts with the shapes and occasionally demonstrated the ability to solve the facts without the shapes. These two students were also reported to recognize the pattern in which adding one to any given shape made the next shape and the inverse activity in which subtracting one from any shape made the previous shape. While the author of this study felt that using the Numicon materials improved the performance of the three students, the study was severely limited by the absence of baseline data.

In a second article regarding Numicon, Saunders (2005) reported on the implementation of Numicon intervention in five schools in the United Kingdom that had not made the gains on numeracy goals required by the national standards. The program was implemented with Year 3 students who received instruction using Numicon materials for 60 minutes a week over a ten-week period. Trained teaching assistants provided the instruction with small groups of up to four students. No additional details regarding the implementation were provided.

The implementation results were reported as a percentage of improvement made by the whole group of participating students on a variety of different math skills. It is not

clear what measure was used to evaluate progress. Gains ranged from 10% for counting out groups of objects to 39% for recognizing halves and doubles. Additionally, data were provided for a single student who was reported to have earned a score of 36 out of 53 possible points on a pretest and 51 out of 53 on the posttest. Qualitative data for this student indicated that he felt happier about math instruction following the instruction with the Numicon materials. The study was clearly limited by the lack of a control condition and limited descriptions of both the intervention and dependent measures.

While the perception of Numicon instruction reported in both of these articles appeared to be positive, no quantifiable evidence was provided concerning the program's effectiveness for students with mathematics difficulties.

Overall findings related to Numicon. Numicon instruction is well-aligned with number sense intervention theory. However, there is very little research on the use of Numicon as a program to support learners with math difficulties. The existing publications indicate that Numicon instruction has the potential to improve number sense skills for students with math difficulties or disabilities in the primary grades. However, systematic evaluation is needed. Numicon includes key elements of effective instruction identified and documented in theoretical and research publications. This method emphasizes the use of generic structured representation of numbers in a manner that is closely aligned with uses discussed in a study by Tournaki et al., (2008). That study found significant and large performance gains in number combination skills for first graders with learning disabilities. Additional research is clearly needed to determine if

Numicon instruction can improve the number sense skills of young students with math difficulties.

3. METHODS

A single-case multiple baseline design was used to investigate the effects of the supplemental instruction with Numicon on the development of number sense skills for kindergarten students with varying degrees of math difficulties. Numicon instruction emphasizes multiple representations of number concepts and follows a concrete to abstract instructional sequence. Number sense instruction with Numicon begins with unitary concrete manipulatives, then transitions to structured concrete manipulatives before imagery is taught as a form of representation. The single-case design was selected as a systematic method for documenting the presence of a functional relation while conducting the intervention under typical educational conditions (Horner et al., 2005; Scruggs, Mastropieri, & Regan, 2006). Additionally, single-case multiple baseline designs have been used to describe the effectiveness of mathematics interventions for students with disabilities and those classified as being at risk for math failure (e.g., Cassel & Reid, 1996; Flores, 2010; Maccini & Ruhl, 2000).

Participants in this mathematics intervention study received 20 minutes of supplemental mathematics instruction for 10 sessions over a six-week period. All mathematics intervention was based on the *Numicon Intervention Programme* (Atkinson et al., 2011). The students received phonics instruction when they were not receiving mathematics intervention to control for variances in supplemental mathematics activities

that may have occurred in the students' classrooms and to maximize the benefits of participating in the study.

Setting

This study was conducted at an elementary school located in a rural school district in a mid-Atlantic state. The school served approximately 470 students in prekindergarten through fifth grade. Two center-based programs for students with disabilities were housed within the school building. These included a program for students with moderate intellectual disabilities and an early childhood special education program. The school also had two resource programs for students with high-incidence disabilities and one for English Language Learners (ELL).

A review of the school report card from the state department of education website indicated that the school had met Adequate Yearly Progress (AYP) standards for the three years before the study and was fully accredited. Disaggregated proficiency data from the 2011–2012 school year revealed that Students with Disabilities, English Language Learners, and students from economically disadvantaged backgrounds (Gap Group 1) did not meet the Annual Measureable Objective (AMO) set forth by the state in mathematics. Specifically, 33% of the students from this group had passed the state mathematics assessments. The AMO was set at 47%. The same pass rate was documented for African-American students (Gap Group 2), meaning that they also had performed below the AMO. These performance deficiencies did not count against the school's accreditation status because the number of students in these groups was classified as being too small to evaluate. However, the school administrators targeted

mathematics instruction and intervention, with particular emphasis on number sense, as areas for school-wide improvement on their annual school improvement plan.

Participants

This section provides information on the student participants and the intervention provider/researcher. The participant selection process is described along with detailed information on the participants' demographic and learning characteristics. Pseudonyms are used to protect student confidentiality.

Students. The number sense intervention was implemented with students from the three kindergarten classes at the school. Kindergarteners were selected for intervention based on evidence that kindergarteners enter school with varying degrees of number sense (Jordan et al., 2007) and that performance on number sense measures in kindergarten is highly predictive of later mathematics achievement (Jordan, Glutting, & Ramineni, 2010; Locuniak & Jordan, 2008). Additionally, in keeping with the tenets of Response to Intervention, it is important to conduct mathematics screening in the early elementary years to identify students in need of support before they experience failure (Brigham & Jenkins, 2012).

The kindergarten classrooms had between 18 and 22 students. All three classes were taught by certified teachers who had been teaching kindergarten at the school for at least seven years. An instructional assistant was also present in each kindergarten classroom for core academic instruction.

The teachers selected potential student participants who consistently performed at or below the 25 percentile on classroom assessments of counting and numeral

recognition. Informed consent documents were sent home to the parents of those students through the standard home-school communication methods. All of the students who returned the consent form were given the NSB (Jordan et al.,2008) as a more thorough screening and descriptive measure. Students were allowed to participate in the intervention regardless of their performance on the NSB.

Descriptive data for each student were drawn from the educational files. The parent or guardian provided demographic information on school enrollment forms. Information about special services (ELL, Special Education, supplemental intervention) had generally been documented in the file by the child's teacher or an administrator. Data regarding socioeconomic status were not typically available, as that information was kept confidential by the school system. The number of hours that each student attended preschool in the year before kindergarten was also reported. Table 1 summarizes the descriptive data for each student.

Table 1
Student Characteristics

Student	Gender	Ethnicity	Age	Services	Pre-K Hours
Adam	Male	White	6-2	None	30+
Aurora	Female	White	5-5	RtI Reading	0
Charlie	Male	Black	5-4	RtI Reading	30+
Dylan	Male	White	5-11	None	30+
Daniella	Female	Hispanic	5-7	RtI Reading	15–30
Devin	Male	White	5-10	None	30+
Lacey	Female	White	5-8	None	30+
Seth	Male	White	6-1	RtI Attention	30+
Steven	Male	White	5-10	None	30+

Adam. Adam was a white male, aged 6 years 2 months. Adam had no history of special education services or referrals for intervention. He attended preschool for more than 30 hours per week in the year before beginning kindergarten. Adam did not have any documented attendance issues.

Adam completed the NSB at the onset of the study. He earned 15 points, indicating that he was in the at-risk range of performance on that measure. Adam

demonstrated a relative strength in the area of number knowledge, earning 6 of the 7 possible points for that domain. This result suggests that he had a relatively strong sense of the relationships between numbers. Counting knowledge was also an area of relative strength. Adam earned 5 of the 7 possible points for that domain. He was able to verbally count past 50. Adam had difficulty with number recognition and tasks that required calculations (verbal or nonverbal).

Aurora. Aurora was a white female, aged 5 years 5 months at the time of the study. She did not attend preschool before kindergarten. Aurora began receiving supplemental instruction in phonics and prereading skills in November of her kindergarten year. She did not receive any other educational services. Attendance records indicated that Aurora had been absent eight times and tardy eight times over a three-month period in the first semester of the school year.

Aurora earned 19 out of 33 possible points on the NSB indicating that she was not at risk for math failure. She was classified as a student with classroom math difficulties for the purpose of this study. Aurora demonstrated strong counting skills, earning all 7 points for that domain. She also demonstrated a strength in solving addition word problems. She was able to count to 34. Aurora was not able to identify any two-digit written numerals. She also had difficulties with nonverbal calculations and number combinations.

Charlie. Charlie was a black male, aged 5 years 4 months at the onset of the study. He had attended Head Start for more than 30 hours per week before his kindergarten year. Charlie had been referred to the school's Response to Intervention

team to address broad social and academic delays. He was receiving intervention in early reading and phonics skills. School records indicated that Charlie had attendance issues as evidenced by 10 tardies and three absences recorded over a three-month period in the first semester of school.

Charlie's score of 12 points on the NSB placed him in the at-risk range. Charlie demonstrated a relative strength in counting knowledge, earning 5 of the 7 possible points for that domain. He was able to verbally count to 12, but did not demonstrate an understanding of stable counting order when counting objects. Charlie demonstrated a relative strength in nonverbal calculations, earning 3 out of 4 possible points for that domain. Charlie did not recognize any written numerals or demonstrate knowledge of the relationships between numbers. He did make a concerted effort during the screening and would typically repeat the last number that was stated in the questions on the number knowledge portion of the assessment.

Dylan. Dylan was a white male, aged 5 years 11 months at the beginning of the study. He had no history of special education services or referrals for intervention. Dylan had attended Head Start for more than 30 hours per week in the year before he started kindergarten. Attendance records indicated that Dylan had been tardy 10 times over a six-week period in the first semester of school

Dylan completed the NSB before the intervention. He earned 20 points, indicating that he was not at risk for math failure. Dylan was classified as a student with classroom math difficulties for the purpose of this study. He demonstrated strong number knowledge skills, successfully answering all seven questions in that domain. He also

correctly answered all of the questions in the nonverbal calculations domain. Dylan was able to count to 39. He did not recognize any two-digit numerals. He also had difficulty with story problems and number combinations.

Daniella. Daniella was a Hispanic female, aged 5 years 7 months. She had no history of special education or ELL services. Daniella was referred to the Response to Intervention team for remediation and monitoring of her early reading skills. Daniella had attended preschool between 15 and 30 hours per week before starting kindergarten. Attendance records indicated that there were no attendance concerns.

Daniella earned a score of 14 on the NSB, indicating that she was in the at-risk range of performance on that assessment. She demonstrated a strength in counting knowledge by correctly answering all seven of the questions from that domain. She was able to verbally count up to 39. Daniella had difficulty naming written numbers, and was not able to solve any story problems or number combinations.

Devin. Devin was a white male who was 5 years 10 months old at the onset of the study. Devin had attended preschool for more than 30 hours per week before he began kindergarten. He did not receive any special or supplemental educational services. Attendance records showed that Devin did not have any ongoing attendance concerns.

Devin performed well on the NSB, earning 24 out of 33 points. He was not in the at-risk range of performance on that measure and was classified as a student with classroom math difficulties for this study. Devin was able to count to 29. He demonstrated strong counting and number knowledge skills. He was able to compare the relative value of two single-digit numbers, solve nonverbal addition and subtraction

calculations, and solve some addition word problems involving common objects. Devin had difficulty identifying two-digit numbers.

Lacey. Lacey was a 5 year 8 month old white female at the onset of the study. She had attended preschool for more than 30 hours per week before beginning kindergarten. She did not receive any special or supplemental educational services. Her attendance records indicated that Lacey had been tardy 10 times over a four-month period in the first semester of school.

Lacey successfully completed 17 out of 33 tasks on the NSB. This score was in the borderline at-risk classification used for this study. She was able to count to 47 and was able to solve nonverbal calculations. Lacey had difficulty identifying two-digit numbers, solving story problems, and completing number combinations.

Seth. Seth was a 6 year 1 month old white male at the start of the study. He had no history of special education services, but was referred to the Response to Intervention team to address behavioral and attention needs. Seth had a medical diagnosis of ADHD. He had attended preschool for more than 30 hours per week before kindergarten. His attendance records did not indicate any concerns.

Seth earned 19 points on the NSB and was classified as a student with classroom math difficulties for the purpose of this study. Seth demonstrated a strength in solving story problems by successfully answering all five of the questions in that domain. He also had a relative strength in counting knowledge. He was able to count up to 19. Seth had difficulty identifying two-digit numbers, completing nonverbal calculations, and solving number combinations.

Steven. Steven was a white male, aged 5 years 10 months. Steven was raised in a home in which Arabic was spoken, although English was his first language and he did not require ELL services. Steven had no history of special education services or any referrals for intervention. He had attended preschool for more than 30 hours before beginning kindergarten. There was no indication of attendance concerns in his file.

Steven was given the NSB at the beginning of the study. He earned 20 out of 33 possible points. He was classified as a student with classroom math difficulties for the purpose of this study. He exhibited strengths in areas such as performing nonverbal calculations and solving addition word problems involving common objects. He had difficulty naming two-digit numbers, often reversing the digits (e.g., identifying 13 as 31). He also had difficulty with knowledge of number relationships. Steven was able to verbally count to 12.

Researcher/Intervention provider. I both provided the intervention and carried out the research. I am a certified special education teacher with endorsements in Learning Disabilities, Emotional Disabilities, and Intellectual Disabilities. Before beginning the intervention, I taught at the elementary school level for nine years, providing both special education services and prereferral intervention. Additionally, I was trained in the intervention by the authors of the *Numicon Intervention Programme* and am a certified Numicon trainer.

Materials

A variety of materials were used for both the number sense intervention and the phonics intervention. All of the Numicon materials were purchased through the publishers of the Numicon program. In addition, I made many of the materials used for the phonics intervention. The materials, described below, were stored in the classroom where the supplemental instruction was provided.

Number sense intervention. All number sense intervention was provided through small group instruction for 20 minutes each day over 10 intervention sessions. One third of the instructional time was dedicated to counting and number line activities, while two thirds of the instructional time incorporated the use of structured representation to teach number patterns and relationships. The number sense intervention used materials and activities from the *Numicon Intervention Programme* (Atkinson et al. 2011). The *Numicon Intervention Programme* was designed for the equivalent of Tier 2 and Tier 3 intervention for students with math difficulties in the United Kingdom. The program was developed for implementation by trained teachers or teaching assistants who are monitored by a supporting teacher.

Numicon instruction is grounded in the idea of providing multiple representations for mathematical concepts to help children develop a strong concept image of numbers and operations. Concept image is described as an individual's "totality of impressions" of an abstract idea (Elliott, 2010, p. 4). For example, our concept image of "four" may be composed of our experiences rote counting to four, a quantity of four objects, the number 4 on a school bus, the age of a child, sums of four, a mental picture of four dots on a die,

and so on. A concept image is determined by the combination of all of an individual's experiences.

Numicon instruction uses special manipulatives, referred to as Numicon Shapes, to provide concrete and semiconcrete representations of mathematical concepts during instruction to support students in developing rich concept images. The Numicon Shapes are structured (pattern-based) manipulatives that visually portray patterns such as one more and alternating even and odd numbers. The shapes are used as a visual scaffold to develop student awareness of pattern within the number system (Atkinson et al., 2011). Other pattern-based manipulatives, such as Cuisenaire Rods, can also be used in the *Numicon Intervention Programme* to provide a range of experiences that facilitate a broader concept image. (This study only used the Numicon Shapes due to time constraints and the young age of the participants.) As students progress in the program they are encouraged to develop mental images of the Numicon Shapes and other manipulatives and to mentally manipulate those images to support conceptual understanding and solve problems (Atkinson et al., 2011).

Teacher materials. A variety of Numicon materials were used in the intervention lessons. These included two sets of Numicon Shapes, 1 through 10, with magnets affixed to the back and a large magnetic white board. The magnetic sets of Numicon Shapes and the magnetic white board allowed the teacher to maintain correct orientation of the shapes while modeling their use for the whole group. The teacher also had a large preprinted Numicon number line displaying the Numicon representations of the numbers 1–21. The number line was part of the Numicon kit and was used for modeling and group

work. Numicon numeral cards, 1–50, were required for some of the intervention lessons. The Numicon Number Track, also part of the kit, was used in the lessons. The Track is made of sturdy cardboard and displays the numbers 1 to 100, divided into groups of 10. The tracks can be used to make number lines of various lengths or to build a hundreds chart.

I used researcher-developed lesson plans (Appendix A) derived from the *Numicon Intervention Programme* using the implementation manuals from *Numicon Level One* and *Closing the Gap with Numicon*. Each lesson plan included instruction in counting/number line use and number patterns and relationships. The lesson plans identified and defined key vocabulary, indicated required materials, directed the intervention provider to a description of the activity in the Numicon Teaching Guide, and provided approximate timing for each portion of the lesson. In addition to the lesson plan, a fidelity checklist (Appendix B) was used for each lesson. The fidelity checklists provided an outline of the lesson sequence and were intended to serve as a prompt for the intervention provider, as well as a measure of treatment fidelity.

Student materials. Each student received a small canvas bag containing the Numicon Shapes 1 through 10. Additional bins of mixed Numicon Shapes were available for sharing. Approximately 80 Numicon Pegs were stored in small plastic baskets and made available for group use during specified activities. Each student had two sharpened pencils which, I kept until the students needed them in order to minimize distraction.

Phonics instruction. I provided phonics instruction, using direct instruction, on the days when students did not receive instruction in the number sense intervention (e.g., during baseline and maintenance phases). Phonics instruction was selected for the baseline and maintenance conditions to control for the effects of small group instruction, maximize the benefits of participating in the study for all of the students, and reduce the likelihood of additional overlapping mathematics instruction in the classroom.

Teacher materials. I used researcher-developed lesson plans (Appendix C) that focused on kindergarten level words. Each lesson plan described activities in letter-sound awareness, sight word recognition, and phonics/phonemic awareness. A script, timelines, and materials were all provided for each activity. I made flash cards from index cards with the kindergarten sight words and letters.

Student materials. The students had individual dry erase boards, markers, and erasers, which I kept stored away until needed. The students did not require any other materials.

Behavior management. A behavior management system was used during the baseline assessments, number sense intervention, and phonics instruction. “Ready to Learn” behaviors were briefly described at the beginning of each lesson or activity. Differential reinforcement using a point system was used to encourage student engagement and minimize off-task behavior. The students were familiar with verbal praise as the classroom teachers consistently used this type of praise during whole group instruction. The point system was incorporated to help all of the students stay on task during the intensive and fast-paced instruction. I recorded points on the dry erase board

with markers and calculated each student's points at the end of the instructional session. I also used praise and encouraged students to achieve at the same level or better during the next instructional session.

Dependent Measures

Three primary types of dependent measures were used in this study, all of which evaluated different elements of number sense. Additionally, student interviews and teacher questionnaires were used to evaluate social validity. Each measure is described in detail below.

The Number Sense Brief. All student participants were given the NSB (Jordan et al., 2008) before the intervention to measure their overall number sense skills. The NSB (Appendix D) is an untimed measure consisting of 33 items that evaluate different aspects of number sense including one-to-one correspondence, cardinality, number recognition, quantity association, and nonverbal addition/subtraction calculations, number combinations, and story problem solving (Jordan, Glutting, Ramineni, & Watkins, 2010). It was demonstrated to be a highly effective screening measure for identifying kindergarten and first-grade students who are likely to develop math difficulties (Jordan, Glutting, & Ramineni, 2010). Diagnostic validity analyses for the NSB using repeated measures analyses of variance and receiver operator curve (ROC) analyses demonstrated that kindergarten and first-grade scores on the NSB were predictive of mathematics proficiency on a validated high-stakes state mathematics test taken in third grade (Jordan, Glutting, Ramineni, & Watkins, 2010). Specifically, a score at or below 15 on the NSB in November or 16 in February of the kindergarten year was

found to distinguish students who passed the state-administered mathematics test from those who failed that test. Additionally, the NSB was found to have strong test–retest reliability with coefficients ranging from .81 to .86 with test administrations two months apart.

The preintervention administration of the NSB was used to identify the participants who were at risk for developing math difficulties and to provide information about the specific number sense skills that were challenging to each of the students. A postintervention administration of the NSB was used as a generalization measure by evaluating near-transfer of the number sense skills that were emphasized in the Numicon instruction, as well as far-transfer skills such as nonverbal calculations, story problem solving, and number combinations that were not specifically targeted by the Numicon intervention.

Quantity discrimination. Curriculum-based measures (CBM) have been demonstrated to be effective methods for evaluating mathematical knowledge and progress of students in kindergarten (Clarke & Shinn, 2004; Seethaler & Fuchs, 2010). CBM can be used to evaluate a variety of number sense skills. Some CBM are particularly strong predictors of mathematics achievement (Clarke & Shinn, (2004); Lembke & Foegen, 2009; Methe, Hintze, & Floyd, 2008). Quantity discrimination (also referred to as magnitude comparison) has been identified as a key construct for assessments of number sense (Gersten et al., 2011). Brief screenings using quantity discrimination have shown strong correlations with mathematics achievement on a variety of standardized measures including the Stanford Early Achievement Test ($r =$

.62), Terra Nova ($r = .49$), Early Math Diagnostic Assessment: Math Reasoning ($r = .53$), and Early Math Diagnostic Assessment: Numeral Operations ($r = .75$) (Clarke & Shinn, 2004; Gersten et al., 2011; Jordan, Glutting, Ramineni, & Watkins, 2010; Seethaler & Fuchs, 2010). Researchers have suggested that timed measures of quantity discrimination are more effective than untimed measures in detecting students who are at risk for math difficulties. Berch identified fluency as a “crucial” variable that “can reveal subtle yet important differences in numerical information processing that may not be tapped by assessing accuracy alone” (2005, p. 335).

Based on the findings from the CBM research, a one-minute fluency probe of quantity discrimination (Appendix E) was used to monitor student progress and make decisions about transitions between phases of the study. The quantity discrimination probes consisted of 40 pairs of numbers (0–20) placed side by side in a box. The two numbers in each pair were separated by a vertical line. The students were asked to look at the two numbers and circle the number that was greater or bigger. Fluent completion of this type of task involves numeral recognition, association of quantity with a given numeral, and understanding of the concepts “greater/bigger” and “less than/smaller” (Bryant & Bryant, 2011).

Three forms of the quantity discrimination probe were used. Each form included the same number pairs in a different order. The number pairs were derived by randomly selecting numbers from 0–20 (Clarke & Shinn, 2004). The quantity discrimination probes were presented with a blank cover sheet and 20 number pairs on each of the following two pages. Scores were recorded as answers correct per minute.

Quantity–numeral correspondence. Dowker (2008) and Gersten et al. (2011) have also described counting principles and efficient counting strategies as number sense skills that are critical for the development of later arithmetic skills. Therefore, a fluency task evaluating quantity–numeral correspondence was also administered during the baseline and intervention sessions. The quantity–numeral correspondence probes consisted of 18 sets of dot configurations that represented quantities between 1 and 20. The students determined the number of dots in each image and circled the corresponding number from a field of four choices. Fluent completion of this type of task involves skills such as seeing and recognizing numerals, counting with one-to-one correspondence, subitizing, and counting on (Bryant & Bryant, 2011). The students had one minute to respond to as many images as they could. Scores were recorded as answers correct per minute.

Three forms of the quantity–numeral correspondence probe were used. Each form had four pages. The first page was a blank cover page. The remaining pages each had six sets of dot configurations. The second page included configurations showing quantities 1 through 5. The third page included quantities 5 through 10. The fourth page included quantities 10 through 20. The quantity–numeral correspondence probes are located in Appendix F.

Procedures

Consent was obtained from the university and school district review boards, the school principal, teachers, and parents before the study began. Student assent was also

obtained. The study was conducted on consecutive school days over a six-week period using a multiple baseline across participants design. All of the instructional sessions lasted 20 minutes. The students received either 15 or 20 minutes of instruction in each session. The remaining five minutes of the shorter instructional sessions were used for number sense assessment. The intervention schedule is provided in Appendix G.

Screening. The NSB (Jordan et al., 2008) was individually administered to each student on the first day of the study. The screening was intended to provide additional documentation of the student's need for mathematics intervention in the area of number sense. The NSB provided descriptive information about each student's number sense skills and identified students who were at risk. Students with a score below 16 on the NSB were categorized as being at risk for the purpose of this study following the guidelines established by Jordan, Glutting, Ramineni, and Watkins (2010). Students with scores between 16 and 18 were described as being in the borderline at-risk classification. This score range was determined to be an indicator of potential risk because it was within one standard deviation of the November cut score for the NSB as established by Jordan, Glutting, Ramineni, & Watkins, 2010

Group formation. Students who returned parent consent forms were placed into a group based on their availability during one of the three established intervention periods. Students were scheduled for intervention at a time that did not conflict with their core instruction in reading or math. The scheduling also accommodated students who received supplemental instruction in reading. (The school did offer supplemental mathematics instruction at the kindergarten level.)

Baseline data collection. Baseline data for all of the students were collected using the quantity discrimination and quantity–numeral correspondence probes for three consecutive school days. No supplemental instruction (either phonics or number sense) was provided on the first three baseline days. The groups did receive instruction in the behavioral expectations and routines for the group activities. The first group to have stable baseline data on the quantity discrimination measure began the intervention on the fourth day of the study. Stable baseline was defined as the absence of an upward slope and no more than 2 points of variation over at least three data points. Although there is no established definition for stable baseline, this definition is aligned with guidance set forth by experts in the field (e.g., Kratochwill, et al., 2010; Kazdin, 1978). The second group entered the intervention phase on the ninth day of the study and the third group entered the intervention phase on the 14th day of the study. Both groups had established stable baselines on the quantity discrimination probe before entering intervention.

Baseline phonics intervention. As groups entered the intervention phase of the study, the remaining groups received instruction in phonics skills, which was provided for 15 to 20 minutes each day. The shorter instructional sessions occurred on days where baseline data were collected. Baseline probes were conducted no more than twice per week after the first group began intervention.

The phonics lessons included approximately five minutes of flashcard work addressing letter-sound correspondence and sight word reading. During the letter-sound correspondence portion of the lesson, individual letters were shown on flashcards. The teacher provided an auditory prompt cueing the students to chorally respond with the

sound made by the letter. If students responded correctly, the teacher repeated the correct response with a praise statement. If any of the students made an error, the teacher stated the correct sound and cued the students to repeat the sound. The same process was used during the sight word portion of the lesson.

Phonics-phonemic awareness activities were conducted for 10 to 15 minutes of the lesson. The teacher guided the students in segmenting the individual sounds in verbally presented consonant-vowel (c-v) and consonant-vowel-consonant (c-v-c) words. The group then identified the letters that made each sound. Initially, the teacher wrote the letters on a dry erase board. The students then had the opportunity to write the word on their own dry erase boards.

Number sense intervention. Each Numicon intervention lesson included approximately 15 minutes of instruction and five minutes of assessment. The instruction followed my lesson plans and used activities drawn from the Numicon Teaching Guide. Each lesson included counting and number sense activities.

The counting portion of the lessons involved various levels of circle counting, in which each student took turns saying the next number in the counting sequence. In early circle counting activities students counted from 1 to 10. They later progressed to circle counting from 1 to 30, and finally 1 to 50. Students also counted backward from 10 and were asked to start the counting sequence from a number other than 1. The Numicon Number Tracks were laid out in number line or hundreds chart formation for all of the circle counting activities. The Number Tracks provided a visual cue and were intended to help students associate the number names with written numerals.

The number sense portion of the lessons included a variety of activities in which the Numicon Pegs and Numicon Shapes were used as concrete representations of numbers. The activities were sequenced to guide the students from unitary counting (i.e., counting the holes or placing pegs in the holes of Numicon Shapes) to automatic association of the Numicon Shapes with numbers. On the seventh mathematics intervention lesson the students were taught to develop mental images of the shapes and patterns. The final three Numicon lessons emphasized association of the mental images with quantities, spoken numbers, and written numerals. Students were taught to use the “one more” pattern to build numbers greater than 10 on the final intervention lesson.

Fidelity checklists were used during the intervention instruction and provided the researcher with a prompt and brief description of each activity during the lesson. The researcher used the checklists to ensure that key vocabulary was defined, all planned lesson activities occur, and the specified materials were used. The quantity discrimination and quantity–numeral correspondence probes were administered at the end of each Numicon intervention session.

Generalization and maintenance. The NSB was individually administered to each student as a generalization measure between five and seven days after each group completed the mathematics intervention. The NSB included both near-transfer and far-transfer number sense tasks. The Numicon Shapes were in storage baskets in the classroom where the students completed the NSB, but they were not used during the generalization assessment.

The students also completed the quantity discrimination and quantity–numeral correspondence probes between six and seven calendar days after the intervention was completed as a measure of maintenance. During the maintenance period, groups one and two completed the remaining phonics lessons. Group three completed the generalization assessment and social validity interviews.

Reliability

All of the quantity discrimination and quantity–numeral correspondence probes were scored by the researcher and another trained evaluator using a key. Any discrepancies in the scoring were resolved to arrive at 100% inter-rater reliability before the data were entered into Microsoft Excel™ for analysis.

Procedural Fidelity

The intervention instruction used detailed lesson plans highlighting key vocabulary and directed the researcher to activity descriptions provided by the program publishers. The researcher used a treatment checklist during each lesson to ensure that all components were completed. The checklist was used to monitor fidelity in following the prescribed lesson sequence, using key math vocabulary, using the specified manipulatives, and completing instruction within the prescribed time parameters. One third of the Numicon intervention lessons for each group were videotaped and reviewed using the treatment fidelity checklist. Inter-rater reliability was calculated using the checklists completed during the instruction and those completed using the videotapes.

Social Validity

Social validity was evaluated through student interviews and teacher questionnaires completed at the end of the intervention. The student interviews used a structured interview format. The questions are provided in Appendix H. The students were asked how they liked using the Numicon materials, what they learned, and how the math intervention sessions were different from math instruction in their classrooms. Seven of the nine interviews were videotaped with parent consent. Two interviews were audiotaped due to parent requests that their children not be videotaped.

The participating teachers were asked to complete one questionnaire for each of their students who participated in the intervention. The teacher questionnaires included statements about perceived changes in the students' performance on number sense tasks in the classroom and whether the student requested Numicon materials for classroom activities. (See Appendix I.)

Data Analysis

Data from each of the probes were entered into Microsoft Excel™. Group means for each probe were calculated and graphed to allow visual analysis using the group as the primary unit of analysis. The data were evaluated for level, stability, variability, and trend (Kennedy, 2005). The percentage of nonoverlapping data (PND) between the baseline and intervention phases was also calculated (Scruggs, Mastropieri, & Casto, 1987). The guidelines set forth by Kratochwill et al. (2010) were used to determine if there was sufficient evidence of a functional relation between the supplemental instruction with Numicon and student performance on the fluency probes. This analysis

included evaluating the data for stable baseline, consistency across similar phases, immediate changes in level at the onset of intervention, and demonstration of effect at three different points in time (Kratochwill et al., 2010). Additionally, trendline slopes were calculated for the entire baseline phase, final three data points of the baseline phase, and the intervention phase for each group. Comparisons of the trendline slopes across phases provided additional information on changes in student performance.

The data were also evaluated at the individual level to describe each student's response to the intervention and to determine if different response patterns existed among students with different degrees of math difficulty. Visual analyses were used with the individual data with emphasis on level, stability, variability, and trend. PND was also calculated between the baseline and intervention data for individual students.

Finally, the NSB data were analyzed at both the group and individual level as a measure of generalization. Pre- and postintervention means were compared for each domain, near- and far-transfer tasks, and total score. These data were analyzed at the group and individual level. Effect sizes were calculated for the intervention groups and risk category groups using Cohen's d (Cohen, 1992). The individual NSB data were also analyzed to determine if any students experienced changes in their risk category following the intervention. Pre- and posttest NSB scores were also analyzed using the Wilcoxon signed-rank test for dependent samples. The Wilcoxon signed-rank test is a nonparametric test that calculates differences in paired scores on a dependent variable and is appropriate for small samples (Dimitrov, 2009).

Summary of Methods

A single-case multiple baseline design was used to evaluate the effect of supplemental intervention with Numicon on the number sense skills of kindergarten students with varying degrees of math difficulty. The study was conducted over a six-week period and included 10 mathematics intervention lessons. Nine students participated in the study. Five were classified as having classroom math difficulties. One student was described as being in the borderline at-risk category and three students were classified as being at risk for math failure. Fluency measures of quantity discrimination and quantity–numeral correspondence served as the primary dependent variables. The NSB (Jordan, et al., 2008) served as a measure of generalization. Analyses were conducted at the group and individual level. Table 2 provides a summary of the research questions, data sources, and analyses. Finally, social validity was evaluated using student interviews and teacher questionnaires.

Table 2
Data Sources and Analyses by Research Question

Research Question	Data Source	Data Analyses
1. Is there a functional relation between supplemental mathematics instruction with Numicon and quantity discrimination skill for kindergarten students with mathematics difficulties?	Quantity discrimination fluency measure	Visual analysis of group data Phase means Trendline slopes Percentage of nonoverlapping data
2. Do kindergarten students with mathematics difficulties maintain quantity discrimination skills one week after the conclusion of supplemental instruction with Numicon?	Quantity discrimination fluency measure	Visual analysis of group data Phase means
3. Is there a functional relation between supplemental mathematics instruction with Numicon and performance on counting and numeral recognition tasks for kindergarten students with mathematics difficulties?	Quantity–numeral correspondence fluency measure	Visual analysis of group data Phase means Trendline slopes Percentage of nonoverlapping data
4. Do kindergarten students with mathematics difficulties maintain counting and numeral recognition skills one week after the conclusion of supplemental instruction with Numicon?	Quantity–numeral correspondence fluency measure	Visual analysis of group data Phase means
5. Do kindergarten students with mathematics difficulties demonstrate gains in their general number sense skills following supplemental instruction with Numicon?	The Number Sense Brief	Gain scores Effect size calculations using Cohen's <i>d</i> Significance testing using Related-samples Wilcoxon Signed Rank test
6. Do kindergarten students with different degrees of mathematics difficulty exhibit different trends in their responsiveness to supplemental instruction with Numicon?	Individual student data from the quantity discrimination measure, quantity–numeral correspondence measure, and the Number Sense Brief	Visual analysis of fluency data Comparison of phase means by degree of math difficulty Effect sizes on the Number Sense Brief by degree of math difficulty

4. RESULTS

This chapter presents the results of the single-case multiple baseline research study evaluating the impact of supplemental mathematics instruction with Numicon on the number sense skills of kindergarten students with mathematics difficulties. The supplemental mathematics instruction with Numicon was provided in a small-group format over 10 consecutive school days. The primary dependent variables were fluency on quantity discrimination and quantity–numeral correspondence measures. The quantity discrimination and quantity–numeral correspondence probes were administered between three and seven times during the baseline phase, between nine and 10 times during the intervention phase, and once during the maintenance phase. Maintenance data were collected between six and seven calendar days after the intervention phase was completed. Additionally, a general number sense assessment (The Number Sense Brief) (Jordan et al., 2008) was administered before and after the intervention to evaluate general improvement in number sense skills. Finally, student interviews and teacher surveys were conducted to evaluate social validity.

Intervention decisions were made based on group performance. Therefore, the analyses will be presented using the group as the primary unit of analysis. Analyses of the individual student data will also be presented to address research questions related to response patterns among students with different learning characteristics.

Procedural Fidelity

I used a treatment checklist during each lesson as a visual prompt of the lesson activities, critical math vocabulary, and required manipulatives. The completed checklists also provided a measure of procedural fidelity. A fidelity score was established for each supplemental mathematics lesson by dividing the number of completed tasks by the total number of tasks associated with the lesson. Ten of the 30 lessons were also videotaped and reviewed using the treatment checklists to enable verification of the fidelity scores. The review indicated a high rate of agreement between the checklists completed during instruction and those completed using the videotapes (IRR = 97%).

The overall degree of treatment fidelity for the supplemental mathematics instruction was high ($M = 95.6\%$). Comparisons of the fidelity scores for each group across lessons revealed that similar degrees of fidelity were present. Treatment for group one was administered with an average of 95.1% fidelity, group two was administered with 98.3% fidelity, and group three was 93.3% fidelity.

Quantity Discrimination

A quantity discrimination probe was used as one measure of number sense skill across all phases of the intervention. Fluent completion of quantity discrimination tasks indicates that a student recognizes written numerals, associates quantity with numerals, and understands the concepts “bigger” and “smaller” (Bryant & Bryant, 2011). Three forms of the quantity discrimination probe were used, as described in chapter 3. Each form consisted of 40 pairs of numbers (0–20) placed side by side in a box. Students were

given one minute to circle the bigger number from as many pairs as possible. The quantity discrimination probes were administered to each group of students before the baseline or intervention instruction that was scheduled in all cases except the first intervention session for each group. The quantity discrimination probe was administered after instruction on the first intervention session.

I scored the quantity discrimination probes each day using a key. Correct responses per minute and accuracy were recorded. An observer confirmed the scoring using the same key. Any scoring discrepancies were resolved to achieve 100% agreement before the data were graphed using Microsoft Excel™. Visual analysis was used to evaluate the data for level, stability, variability, and trend (Kennedy, 2005). The PND between the baseline phase and the intervention phase was also calculated as a measure of effect (Scruggs et al, 1987).

The results are first reported using the group as the primary unit of analysis because intervention decisions were made based on group outcomes rather than individual student performance. Figure 2 is a graphic display of the quantity discrimination group means for all three groups. Table 3 provides means and slopes for each group during the baseline and intervention phases of the study. Additionally, the slopes of the trendlines for the final three baseline points of each group are presented in keeping with the premise that “the last few data points contain the most essential information regarding the level of behavior before a phase change” (Kennedy, 2005, p. 197).

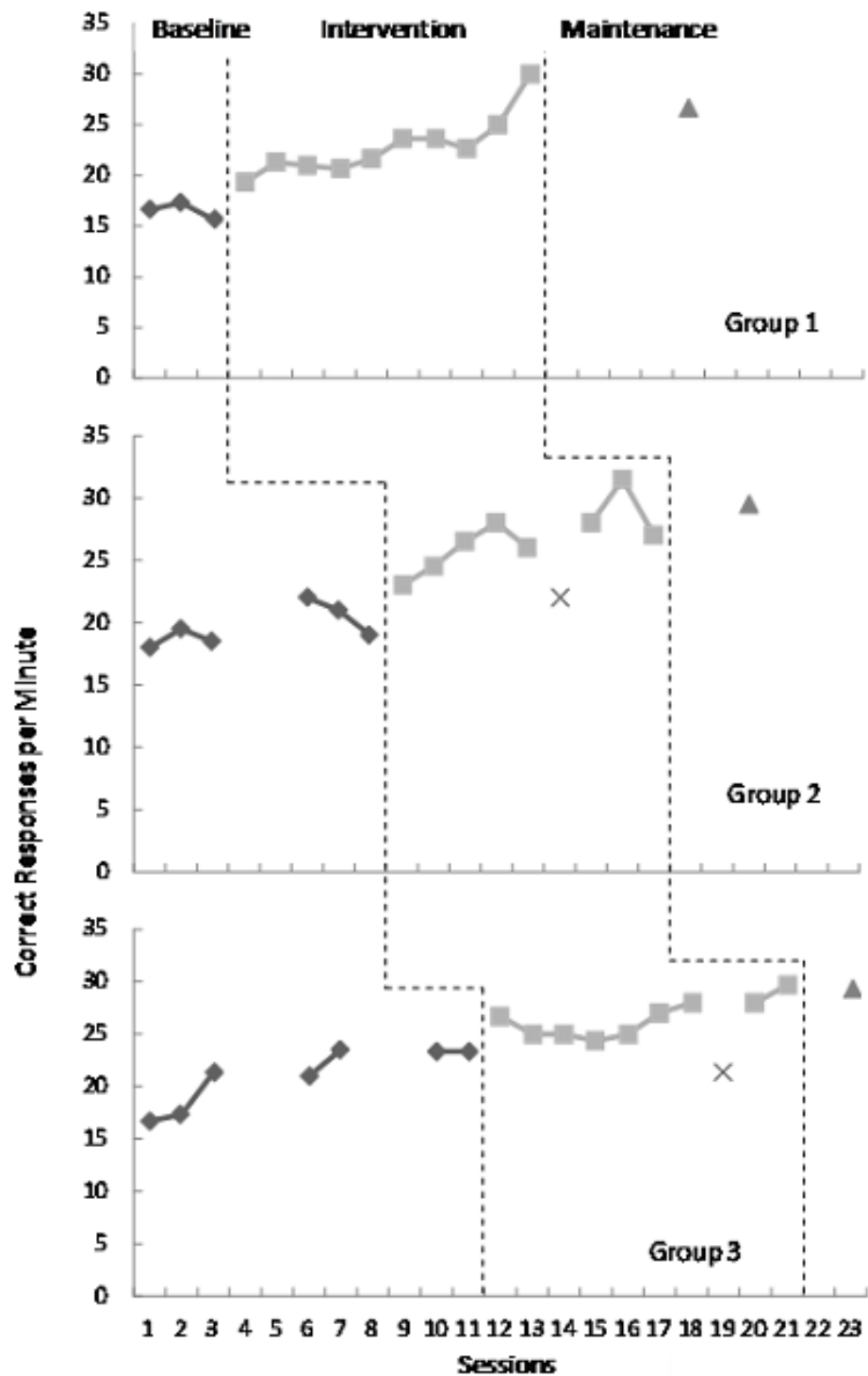


Figure 2. Group means on the quantity discrimination measure.

Table 3

Quantity Discrimination Means and Trendline Slopes by Group and Phase

Group	Baseline Mean	Baseline Slope (Total)	Baseline Slope (Final 3 Points)	Intervention Mean	Intervention Slope (Total)
Group 1	16.56	-0.55	-0.55	22.9	0.86
Group 2	19.67	0.30	-1.50	26.81	0.66
Group 3	20.93	0.64	-0.05	26.52	0.45

Group analysis. Nine students were divided into three groups of three students each. Students were assigned to a group that received intervention at a time that did not conflict with their core classroom instruction in reading or math. The group assignments were also designed to accommodate students who received supplemental instruction in reading. Adam, Dylan, and Daniella were in group one; Charlie, Devin, and Lacey were in group two; and Aurora, Seth, and Steven were in group three.

Group means were calculated using data from all of the students in a group, except in circumstances where a student had frequent absences that affected the variability of the scores. This scenario occurred with Charlie in group two, who was absent for one of the six baseline sessions and four of the 10 intervention sessions. As the student with the lowest fluency scores, the presence or absence of his data created variability around the group means that did not reflect the performance of the other two students in his group.

Group one. The group means on the quantity discrimination probe were calculated using the data from all three students in group one (Adam, Dylan, and Daniella). The students completed three baseline probes. The baseline data were stable with a mean of 16.56 correct responses per minute ($SD = 0.85$, range 15.66 to 17.34). The final baseline data point was slightly lower than the previous two points, creating a downward trend for the phase (slope = -0.55).

An immediate level change was evident at the onset of the intervention. The mean of the first three intervention data points (20.55) indicated a gain of nearly four correct responses per minute over the baseline mean. The overall phase was marked by an upward trend (slope = 0.86). The upward trend continued with an accelerated rate of growth in intervention sessions eight through 10 (slope = 3.66). The mean for correct responses per minute during the intervention phase was 22.9 ($SD = 3.01$, range 19.33 to 30). PND was calculated to be 100% from baseline to intervention for group one.

The maintenance data for group one students were collected seven calendar days after the last intervention session. It is notable that Adam and Dylan had both been sick and absent for three school days before administration of the maintenance probe. The group mean of 26.67 correct responses per minute on the maintenance assessment was higher than 90% of the intervention data points.

Group two. The means for group two were calculated using two of the three participating students (Devin and Lacey). As noted above, Charlie's data were not included in the calculation of group means for data analysis due to his frequent absences. It is important to note, however, that since he was only absent once during the baseline

phase of the study, I used his data during the study to determine if group two had a stable baseline. Charlie was absent for four of the ten intervention sessions and it became evident that inconsistent inclusion of his data inflated the variance around the group means, rendering them unstable. Therefore his data were only analyzed individually. Additionally, a special classroom activity conducted on the day session 14 took place contributed to the students being more off-task than usual. As a result, one student (Lacey) was less focused and completed markedly fewer assessment items than usual. Her score on that day was more than 2.5 standard deviations below her average performance. Therefore, the data from session 14 were excluded from the calculation of group means. That data point is represented on Figure 2 by the symbol x.

Six baseline assessments of quantity discrimination were administered to group two. When the study was in progress I determined that the baseline was stable by calculating the mean for group performance on the five sessions when all three students were present. At that point the overall baseline mean was 14.33 correct responses per minute ($SD = 1.00$, range 13 to 15.33). When Charlie's data were removed from the final analysis one baseline data point was greater than the group mean by 2.33 points. (Stable baseline was defined in chapter 3 as no more than 2 points of variation from the mean over at least three data points.) Aside from that one data point, the remaining five baseline data points were stable even when Charlie's data were excluded. A gradual upward trendline represents the overall phase (slope = 0.30). However, a downward trend was evident across the final three data points (slope = -1.50). Mean performance

on the quantity discrimination measure in baseline, excluding Charlie's data, was 19.67 correct responses per minute ($SD = 1.54$, range 18 to 22).

An immediate level change was evident at the onset of intervention for group two. The mean of the first three intervention data points (24.67) marked an average improvement of four correct responses per minute over the mean of the last three baseline data points (20.67). The phase was characterized by an upward trend (slope = 0.66) with some variability. The mean for correct responses during the intervention phase was 26.81 ($SD = 2.84$, range 23 to 31.5). PND between the baseline and intervention phases was 100%.

Maintenance data for group two were collected six calendar days after the last intervention session. The mean of 29.5 correct responses per minute was higher than 89% of the intervention points. The maintenance data point reflected a continuation of the trend established during the intervention phase.

Group three. Data from all three participants (Aurora, Seth, and Steven) in group three were used for the overall group analysis. However, data from session 19 were excluded from the calculation of the intervention mean due to one student's behavior. This student purposely worked slowly on the quantity discrimination measure and achieved a score that was three standard deviations below her average performance. Her performance falsely deflated the group mean for that session. That data point is represented on Figure 2 by the symbol x.

Seven baseline probes of quantity discrimination were administered to group three. The overall trend had an upward slope (0.64). The trend was largely influenced by

gains between sessions one and three. The data became stable from point three forward. The trendline for the final three baseline points was nearly horizontal with a very small downward slope (-0.05). The overall mean for the baseline phase was 20.93 correct responses per minute ($SD = 2.87$, range 16.67 to 23.5).

A slight, but immediate level change occurred at the onset of intervention. The mean of the first three intervention data points (25.56) was just over two correct responses per minute higher than the mean of the last three baseline data points (23.39). A mild upward trend was evident in the overall intervention phase (slope = 0.45). The overall intervention mean for group three was 26.52 correct responses per minute ($SD = 1.81$, range 24 .33 to 29.67). PND between baseline and intervention was 100%.

Maintenance data were collected six calendar days after the last intervention session. The mean for maintenance was 29.33 correct responses per minute. This result was nearly level with the last intervention data point and higher than all previous intervention data points.

Results across groups. Analysis of the group data from the quantity discrimination measure provides insight into whether a functional relation exists between supplemental mathematics instruction with Numicon and quantity discrimination skills for kindergarten students with mathematics difficulties. Kratochwill et al., (2010) identified four factors that are critical to the determination of a functional relation: stability of the baseline data, consistency in the data across similar phases, immediate level changes at the onset of intervention, and a demonstration of effect at three different points in time. In addition to these factors, percentage of nonoverlapping data has also

been described as a measure of effect (Scruggs et al., 1987). Each factor is analyzed below.

Stable baseline data. For the purposes of this study, the quantity discrimination data were characterized as stable when there was no upward slope and no more than two points of variation from the phase mean over three consecutive data points. All three groups demonstrated a stable baseline on the quantity discrimination measure before beginning the intervention phase of the study.

Consistency across similar phases. The baseline data for groups one and two followed a similar pattern. In both cases the data were stable throughout the phase and a downward trend was evident in the last three baseline assessments. Group three presented a different baseline pattern. This group demonstrated large gains between baseline points one and three, but stabilized from point three forward. The final three baseline data points had minimal variability and a very slight downward trend. Overall the mean for the quantity discrimination baseline phases across groups was 19.64 correct responses per minute ($SD = 2.62$).

The three groups exhibited similar data patterns in the intervention phase. All of the groups demonstrated a change in level and an upward trend was present, though the rate of gain differed for each group. The intervention data were relatively stable with the exception of occasional outlying data points that could be explained by classroom events or student behavioral needs. These points are denoted by the symbol x on Figure 2 . The intervention mean across groups, was 25.27 correct responses per minute ($SD = 3.05$).

The maintenance data indicated that all three groups maintained quantity discrimination skills that were higher than baseline performance even after instruction was withdrawn for one week. Additionally, the mean maintenance score for each group was higher than a majority of the intervention scores. The mean for the maintenance phase across groups was 28.5 correct responses per minute ($SD = 1.59$).

Immediacy of effect. Immediacy of effect was determined by comparing the mean of the last three baseline data points with the mean of the first three intervention data points for each group (Kratochwill et al., 2010). Immediate level changes were present in the intervention phases of all three groups. The gain from the mean of the last three baseline data points to the mean of the first three intervention data points across groups was 3.39 correct responses per minute.

Demonstrations of effect. An effect is demonstrated “when the data pattern in one phase (e.g., an intervention phase) differs more than would be expected from the data pattern observed or extrapolated from the previous phase (e.g., a baseline phase)” (Kratochwill et al., 2010, p. 17). The data for groups one and two provide a clear demonstration of effect in the transitions from baseline to intervention. In both cases there is an immediate and marked level change at the onset of intervention followed by an upward trend. The data patterns observed in the intervention phases for those groups would not have been predicted from the baseline data patterns.

Group three also provides evidence of an effect, though the influence of that effect in determining the presence of a functional relation is lessened by the gains made early in the baseline phase. The gains between points one and three created an upward

slope for the overall phase, though the data stabilized and a very slight downward trend was evident across the final three baseline data points. In spite of those circumstances, a small but immediate change in level was present at the onset of intervention and the phase was characterized by a gradual upward trend providing a third demonstration of effect.

Overall percentage of nonoverlapping data. Measures evaluating the degree of overlap between phases in a single-case design contribute to the analysis of visually presented data (Kennedy, 2005). One such measure is percentage of nonoverlapping data (PND). The PND was calculated across the three groups by counting the number of intervention data points that exceeded the highest baseline data point from the preceding baseline session and dividing that number by the total number of intervention data points (Scruggs et al., 1987). The PND across intervention groups for the quantity discrimination measure was 100%.

Summary of group data analysis for research question one. This study sought to determine if there was a functional relation between supplemental mathematics instruction with Numicon and quantity discrimination skills for kindergarten students with mathematics difficulties. The analysis of the group data for the quantity discrimination measure supports the conclusion that there was moderate evidence of a functional relation between the intervention and quantity discrimination skills for kindergarten students with mathematics difficulties. The support for the functional relation was mitigated by the early baseline data for group three which created an upward trend in the phase before the data stabilized.

The evidence supporting a functional relation includes demonstrations of immediate effect upon transition from the baseline to intervention phases for all three groups. These effects occurred at three different points in time. Additionally level changes from baseline to intervention were consistently present across the groups. All three groups also demonstrated an upward trend in the number of correct responses per minute on the quantity discrimination measure during the intervention phases. Finally PND between the baseline and intervention phases was calculated to be 100%.

Summary of the group data for research question two. This study also sought to determine if kindergarten students with mathematics difficulties would maintain their quantity discrimination skills one week after the conclusion of the supplemental instruction with Numicon. Each of the three groups of students made gains in their mean quantity discrimination skills over the course of the research study. There was moderate evidence of a functional relation between those gains and the supplemental instruction with Numicon. Additionally, the groups continued to demonstrate performance at or near the high scores achieved during the intervention phase of the study one week after the intervention ceased. Specifically, the maintenance score for group one was higher than 90% of the intervention data and 100% of the baseline data. The maintenance score for group two was higher than 89% of the intervention data and 100% of the baseline data. The maintenance score for group three was higher than 90% of the intervention data and 100% of the baseline data. This result suggests that the students maintained the gains in quantity-discrimination skill achieved during the supplemental instruction with Numicon.

Individual analysis. Data for each student were graphed separately to enable analysis of the individual student's response to the intervention and to determine if students with differing levels of math difficulty presented different response patterns.. Graphic representations of the data are presented in Figures 3 to 11. Additionally, a comparison of the trendline slope for each student by phase of the study is presented in Table 4. Scores from the NSB (Jordan et al., 2008) were used to classify students as being at risk, borderline at risk, or having classroom math difficulties. The performance of individual students within each of those categories is discussed below.

Table 4
Quantity Discrimination Means and Trendline Slopes by Student and Phase

Student	Baseline Mean	Baseline Slope (Total)	Baseline Slope (Final 3 Points)	Intervention Mean	Intervention Slope (Total)
Adam	22.00	2.50	2.50	24.56	0.49
Dylan	13.33	-2.50	-2.50	21.40	1.24
Daniella	14.33	-1.50	-1.50	23.10	0.78
Charlie	5.60	0.47	-0.36	6.67	-0.17
Devin	21.80	1.11	0.46	32.11	0.93
Lacey	18.00	0.12	-0.5	21.75	0.40
Aurora	22.57	0.83	-0.04	22.38	-0.39
Seth	21.14	0.62	0.81	31.90	0.94
Steven	17.83	0.23	-0.64	24.40	0.50

Students identified as having classroom math difficulties. All of the students who participated in the study were classified as having classroom math difficulties. Teachers referred students for the supplemental instruction with Numicon based on low achievement on classroom mathematics measures. Some of the students were found to also have more substantial mathematics difficulties as demonstrated by their performance on the NSB (Jordan et al., 2008). Those students were classified as being at risk or borderline at risk and their outcomes are described later.

Five students were classified as only having classroom math difficulties. They were Dylan, Devin, Aurora, Seth, and Steven. Collectively, the students in this classification had a mean score of 19.36 correct responses per minute during baseline, 26.44 correct responses per minute during intervention, and 29.8 correct responses during the maintenance phase. Each student's individual performance is described below.

Dylan. Dylan earned 20 points on the NSB (Jordan et al., 2008) before the intervention. This score indicated that he was well outside the range of being at risk for math failure. Dylan was placed in group one for the study. He was present for all of the baseline, intervention, and maintenance sessions. It is notable that he was absent for three days before the maintenance assessment.

Dylan's data on the quantity discrimination measure are graphically presented in Figure 3. Dylan exhibited notable variability in correct responses per minute during the baseline phase of the study ($M = 13.33$, $SD = 4.04$, range 9 to 17). The phase was marked by a steep downward turn on the third baseline probe, which resulted in a downward trend for the phase (slope = -2.50).

An immediate level change was present once the supplemental instruction with Numicon began. Dylan's mean performance for the first three intervention data points was 17.67 correct responses per minute, marking an average gain of over 4 correct responses per minute from the baseline phase. An upward trend was present in the intervention phase (slope = 1.24). The overall mean for the phase was 21.4 correct responses per minute ($SD = 4.58$, range 14 to 32). PND between the baseline and intervention phases was 90%.

Dylan completed the maintenance assessment seven calendar days after the last intervention session. He had been sick for three days before the assessment and it was not clear if his illness impacted his performance on the maintenance measure. Dylan had 25 correct responses per minute on the maintenance assessment, which was greater than 90% of the intervention data points. Dylan's quantity discrimination data suggest that he benefited from supplemental instruction with Numicon.

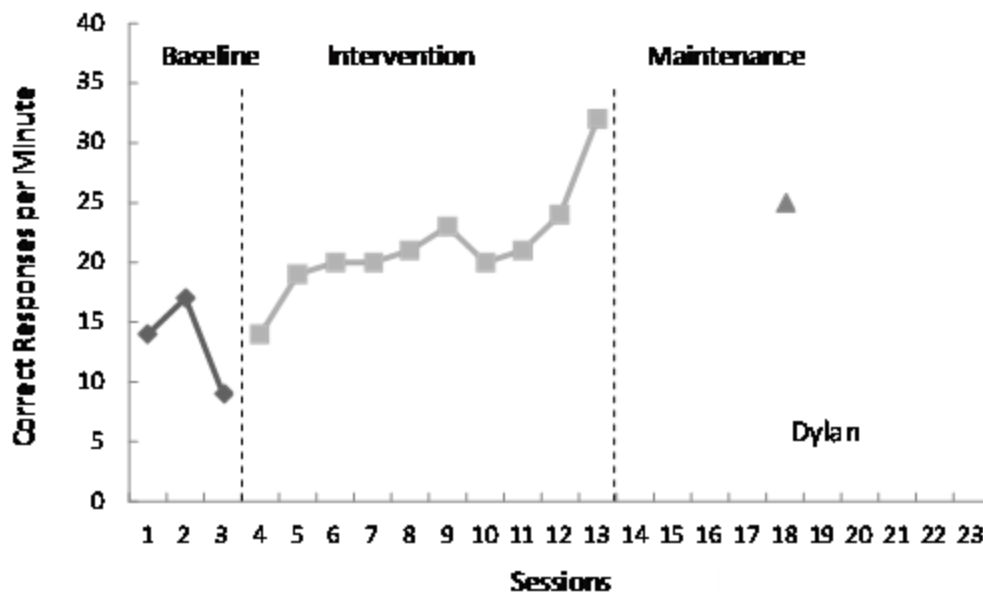


Figure 3. Dylan's performance on the quantity discrimination measure.

Devin. Devin, who was in group two, had the strongest performance of all of the participants on the NSB (Jordan et al., 2008) with a score of 24. Devin was absent for one of the baseline assessments, but was present for all of the intervention and maintenance sessions.

Devin's performance on the quantity discrimination measure is graphically presented in Figure 4. Devin established an upward trend in the baseline phase with minimal variability outside of that trend. The greatest gains in his baseline performance occurred between the first and third sessions, with more moderate gains across the final three data points. The trendline slope across the entire baseline phase was 1.11. The trendline for the final three baseline data points was 0.46. His mean performance for the phase was 21.8 correct responses per minute ($SD = 3.11$, range 18 to 25).

Devin continued to make gains during the intervention phase (slope = 0.93). His mean performance during intervention was 32.11 correct responses per minute (SD = 3.02, range 27 to 37). There was 100% PND from baseline to intervention. However, this result was attributable to the growth pattern established in baseline.

Devin completed the maintenance assessment six days after his last intervention session. He completed 36 correct responses per minute, demonstrating maintenance of the previously established growth. Devin's data pattern suggests that his gains were not attributable to the Numicon intervention.

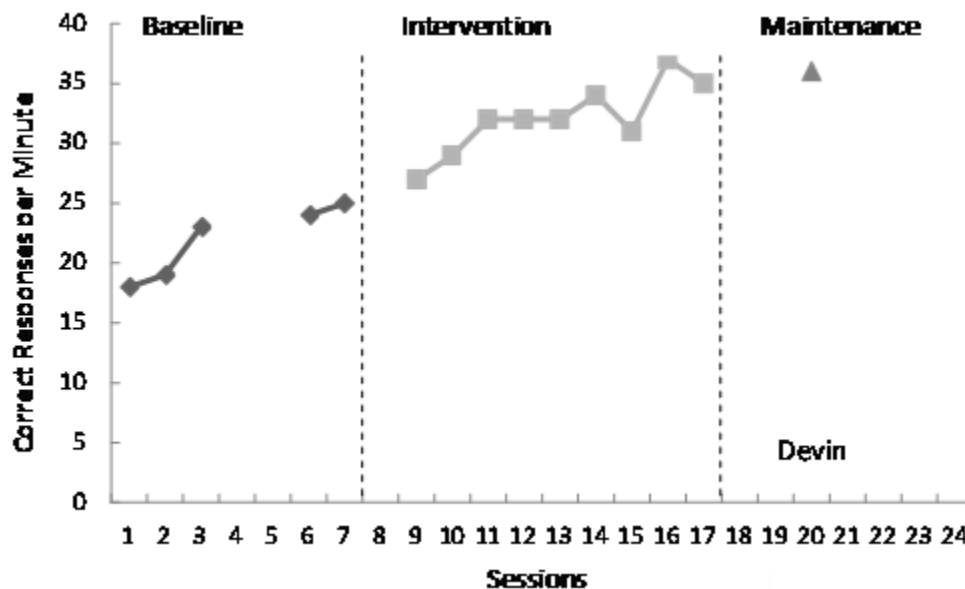


Figure 4. Devin's performance on the quantity discrimination measure.

Aurora. Aurora accurately answered 19 questions on the NSB (Jordan et al., 2008) and was classified as having classroom math difficulties for the purpose of this

study. She was assigned to group three for the intervention. Aurora was absent for one baseline assessment and one intervention assessment. Aurora's data from session 19 were excluded from the analysis. She was upset when the fluency assessments were administered on that day. She purposely worked slowly on the quantity discrimination measure and then refused to complete the numeral-quantity correspondence task. Her performance on the quantity discrimination measure for session 19 was nearly three standard deviations lower than her average for the phase. The data point is represented on Figure 5 by the symbol x.

Aurora experienced growth during the baseline portion of the intervention, with the greatest gains being evident in the early portion of the phase. The overall phase was characterized by an upward trendline (slope = 0.83). Her rate of growth stabilized and the trendline for the final three baseline points had a slope of -0.04. Aurora had a mean of 22.57 correct responses for the baseline phase ($SD = 3.87$, rate 17 to 28).

Aurora exhibited an immediate level change and downward trend at the onset of intervention. The trendline for the intervention phase had a downward slope (-0.39). Her participation and performance were highly variable and were impacted by her mood. Anecdotal notes taken during the intervention sessions indicate that she was cooperative on some days and resistant on others. Aurora completed a mean of 22.38 correct responses per minute ($SD = 3.62$, range 17 to 30) on the quantity discrimination probe during the intervention phase. PND between the baseline and intervention phases was 13%.

Aurora was given the maintenance assessment six calendar days after the last intervention session. Her maintenance score of 25 correct responses per minute was higher than all but her first intervention score, but overlapped with scores achieved during the baseline phase. Aurora's data pattern suggests a negative impact of intervention. She made several comments during the follow-up interviews that provide insight into this occurrence. They are discussed later.

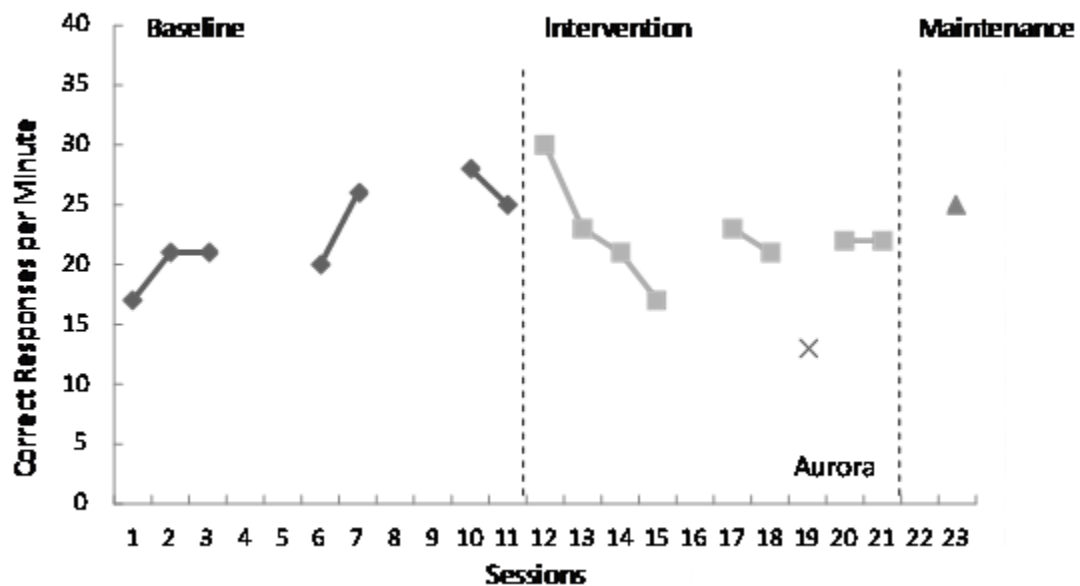


Figure 5. Aurora's performance on the quantity discrimination measure.

Seth. Seth earned 19 points on the NSB (Jordan et al., 2008) before the intervention and was classified as a student with classroom math difficulties. Seth was a member of group three. He was present for all baseline, intervention, and maintenance sessions.

Figure 6 is a graphic presentation of Seth's performance on the quantity discrimination measure across phases of the study. Seth made strong gains between the first and third administrations of the baseline probe. His performance was stable across the remaining four baseline probes. An upward trend was present for the overall baseline phase (slope = 0.62), and was also present across the final three baseline data points (slope = 0.81). Seth's mean performance for the phase was 21.14 correct responses per minute ($SD = 3.08$, range 16 to 24).

An immediate level change was present when Seth began intervention. The mean of the first three intervention data points reflected a gain of 6.33 correct responses per minute over the last three baseline data points. The intervention phase was characterized by an upward trendline with some variability (slope = 0.94), which reflected acceleration in the rate of gain over baseline once intervention began. Seth's mean performance for the intervention phase was 31.9 correct responses per minute ($SD = 3.45$, range 27 to 37). PND between the baseline and intervention phases was 100%.

Seth was given the maintenance assessment six calendar days after the last intervention session. He completed 38 correct responses per minute. His performance on that probe was higher than any previously achieved score. Seth's data pattern suggests that he derived benefit from the supplemental intervention with Numicon. His previous rate of growth increased with the intervention.

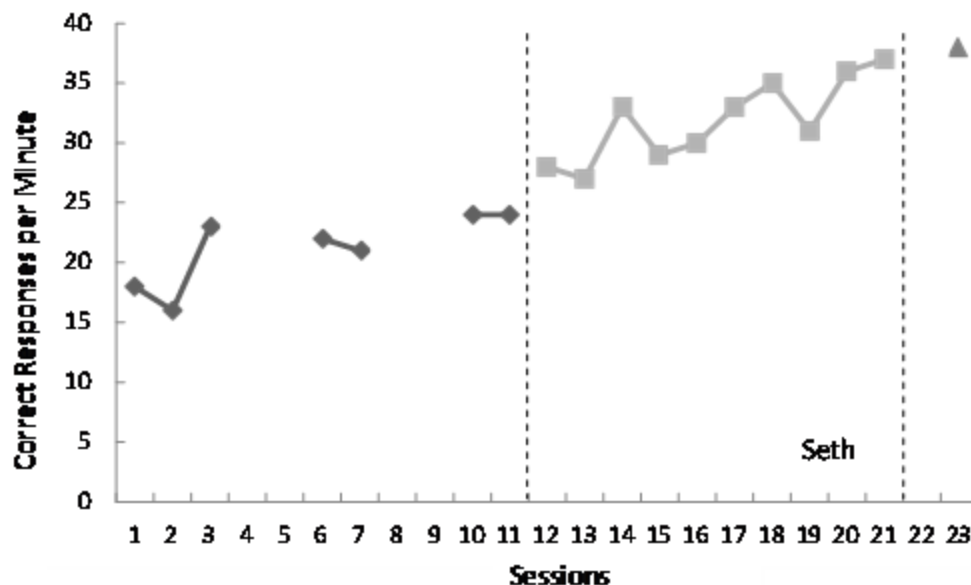


Figure 6. Seth's performance on the quantity discrimination measure.

Steven. Steven earned 20 points on the NSB (Jordan et al., 2008) before the intervention. He was identified as a student with classroom math difficulties for the purposes of this study and was assigned to group three. He was absent for one baseline probe, but was present for all of the intervention and maintenance sessions.

Steven's performance on the quantity discrimination measure is graphically portrayed in Figure 7. Steven demonstrated gains between the first and third baseline data points establishing a very slight upward trend for the phase. The trendline for the entire phase had a slope of 0.23. However, his rate of performance decreased over the final three baseline data points and the trendline slope for that portion of the phase was -0.64. Steven's mean performance for the baseline phase was 17.83 correct responses per minute ($SD = 2.48$, range 15 to 21).

Steven demonstrated an immediate level change at the onset of intervention with a mean 22.67 correct responses per minute for the first three interventions sessions. This marked a gain of 3.67 points over his average performance on the last three baseline data points. Steven continued to make gains through the intervention phase, though there was notable variability in his performance. The trendline documenting his growth over the course of the entire intervention phase had a steeper slope than his baseline trend (slope = 0.50). His mean performance on the quantity discrimination measure during the intervention phase was 24.4 correct responses per minute ($SD = 3.5$, range 20 to 30). PND between the baseline and intervention phases for Steven was 70%.

Steven completed the maintenance assessment six calendar days after the last intervention session. He completed 25 correct responses per minute. His performance on the maintenance assessment was slightly higher than the mean of his performance during intervention and was higher than any data point during the baseline phase. The overall pattern of Steven's data from the quantity discrimination measure suggests that changes in his performance were associated with the intervention.

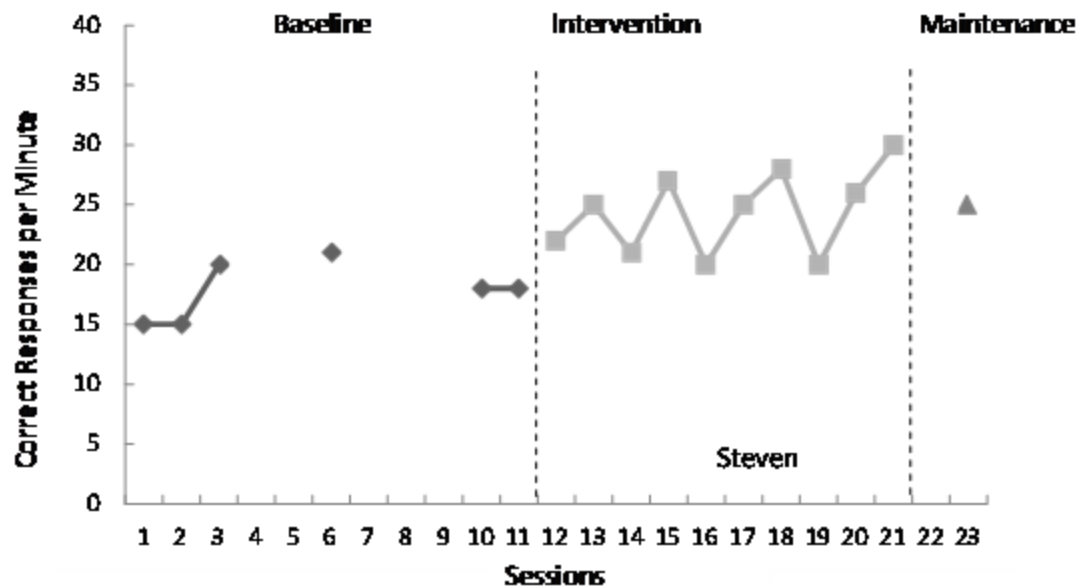


Figure 7. Steven's performance on the quantity discrimination measure.

Students identified as borderline at risk. The NSB (Jordan, Glutting, Ramineni, & Watkins, 2010) does not provide a cut score for students in the borderline at-risk range. For the purposes of this study students were described as being borderline at risk if they earned between 16 and 18 points on the NSB. This score range was determined to be an indicator of potential risk because it was within one standard deviation of the November cut score for the NSB as established by Jordan et al. (2010). Lacey's performance placed her in this range.

Lacey scored 17 on the NSB (Jordan et al., 2008) and she was assigned to group two. She was present for all the baseline, intervention, and maintenance sessions. Lacey's data from session 14 were excluded from the analysis as a special classroom activity had caused her to be unusually distracted. Her performance on the quantity

discrimination measure administered on that day was more than 2.5 standard deviations below her average performance for the phase. That data point is represented on Figure 8 by the symbol x.

Lacey showed some variability in her performance on the baseline quantity discrimination measures, though the overall trend for the phase was fairly flat (slope = 0.12). A downward trend was evident over the final three baseline data points (slope = -0.50). Lacey completed a mean of 18 correct responses per minute during the baseline phase ($SD = 2.28$, range 14 to 20).

Lacey continued to demonstrate variability in her performance during the intervention phase. A change in level was evident and the overall phase was characterized by an upward trend (slope = 0.40). Her mean score for the phase was 21.75 correct responses per minute ($SD = 2.82$, range 19 to 26). PND between the baseline and intervention phases was 50%.

Maintenance data were collected six calendar days after intervention had ceased. Lacey completed 23 correct responses per minute. This score was higher than anything she had achieved in baseline and higher than 66% of her scores during intervention. Visual analysis of Lacey's data provided no clear evidence of a functional relation.

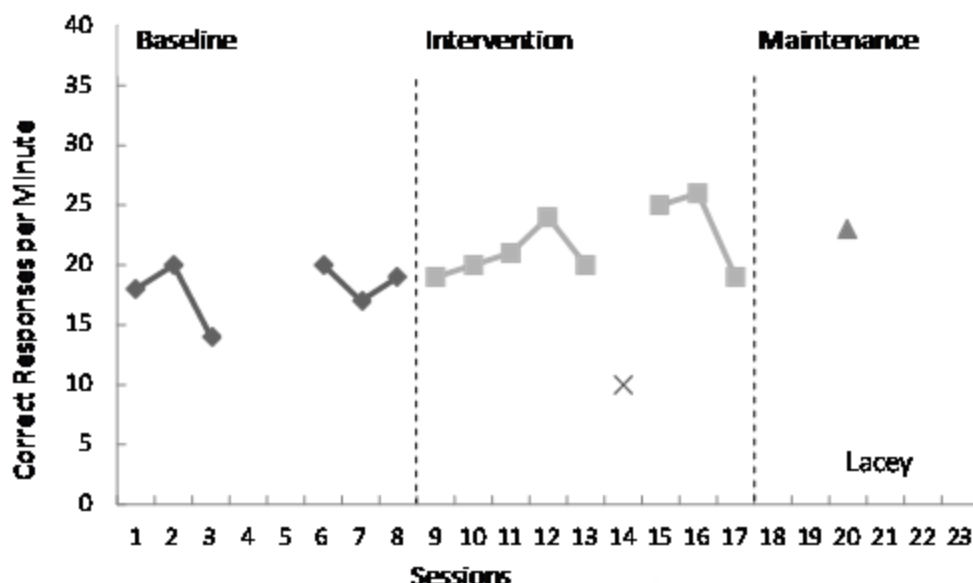


Figure 8. Lacey's performance on the quantity discrimination measure.

Students identified as being at risk. Students were identified as being at risk for math failure if their performance on the NSB (administered in January) fell below the cut score identified for November of the kindergarten year. (January cut scores were not established.) Jordan, Glutting, Ramineni, and Watkins (2010) established the cut score at 15. Adam, Charlie, and Daniella met this criterion.

Collectively, the students in this classification had a mean score of 13.98 correct responses per minute on the quantity discrimination measure during baseline, 17.87 correct responses per minute during intervention, and 23.33 correct responses during the maintenance phase. Each student's individual performance is described below.

Adam. Adam accurately completed 15 out of the 33 questions on the NSB (Jordan et al., 2008), and was therefore classified as a student in the at-risk range of performance. He was assigned to the first group. Adam was present for all three of the

baseline measures administered to group one. He was present for nine of the 10 intervention sessions.

Figure 9 provides a graphic representation of Adam's performance on the quantity discrimination measure. Adam demonstrated an upward trend in baseline that did not stabilize before his group entered the intervention phase (slope = 2.50). His mean performance for the baseline phase was 22 correct responses per minute (SD = 2.65, range 19 to 24).

Adam experienced a decrease in level at the onset of intervention, but then continued with a gradual upward trend. The overall slope of the intervention phase was 0.49. Adam averaged 24.56 correct responses per minute during the intervention phase (SD = 2.30, range 20 to 27). PND between the baseline and intervention phases was 78%.

Adam completed 26 correct responses per minute on the quantity discrimination probe administered seven calendar days after the last intervention session. Although the performance was well within the range of what would have been predicted based on his performance in the intervention phase, it was notable that he had been sick and was visibly lethargic when the maintenance probe was administered. Visual analysis of Adam's data does not clearly indicate that he benefited from the supplemental instruction.

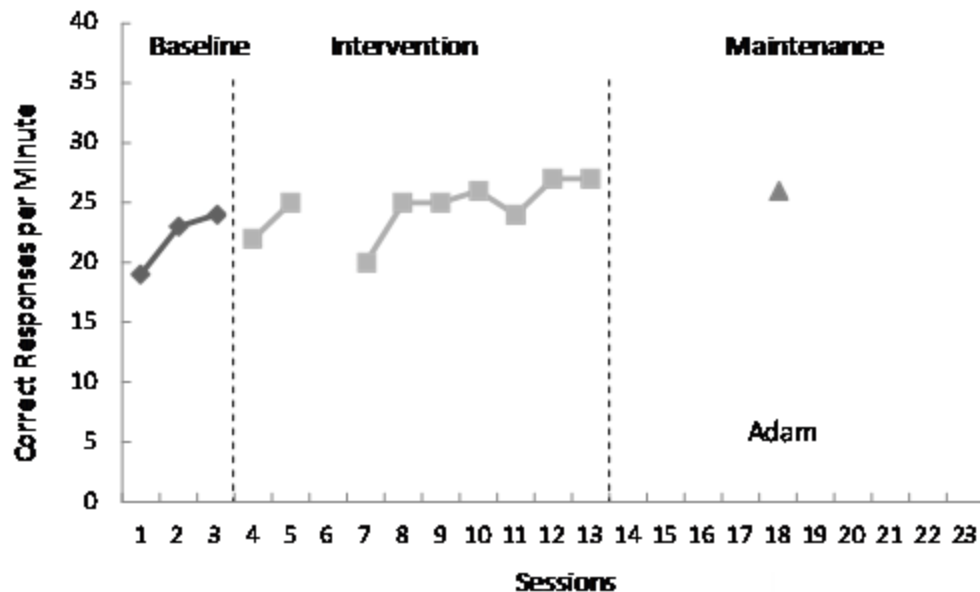


Figure 9. Adam's performance on the quantity discrimination measure.

Charlie. Charlie earned 12 of the 33 possible points on the NSB (Jordan et al., 2008), placing him solidly in the at-risk category. Charlie was assigned to group two for this study. He was absent for one of the six baseline sessions and four of the 10 intervention sessions. Three of these absences were due to illness. On two occasions he was present at school, but had fallen sound asleep just before the intervention.

Figure 10 provides a graphic representation of Charlie's performance on the quantity discrimination measure. His performance across phases was highly variable and there was anecdotal evidence that he was guessing (e.g., saying numbers that were unrelated to those he was circling). The percentage of accurate attempts was calculated for each phase in addition to correct responses per minute to address that concern.

Accuracy scores below 50% on the probes were considered indicative of guessing since there were only two response options per item.

Charlie's baseline data were highly variable. The overall trend for the phase had an upward slope (0.47). However, there was a downward trend across the final three data points (slope = -0.36). Charlie had a mean of 5.6 correct responses per minute ($SD = 3.13$, range 3 to 9) during the baseline phase. His responses were correct an average of 46% of the time in that phase ($SD = 21.69\%$).

Variability continued to be evident in the baseline phase. The overall trendline for the intervention phase had a downward trend (slope = -0.17). Charlie had a mean of 6.67 correct responses per minute ($SD = 4.46$, range 3 to 15) and averaged correct selections 51.83% of the time ($SD = 14.85\%$) during the intervention phase.

Charlie took the maintenance assessment six days after intervention was completed. He completed 15 correct responses per minute on that assessment, with 65% accuracy. PND was calculated at 17% between the baseline and intervention phases. In summary, there was no evidence of effect on the quantity discrimination measure for Charlie.

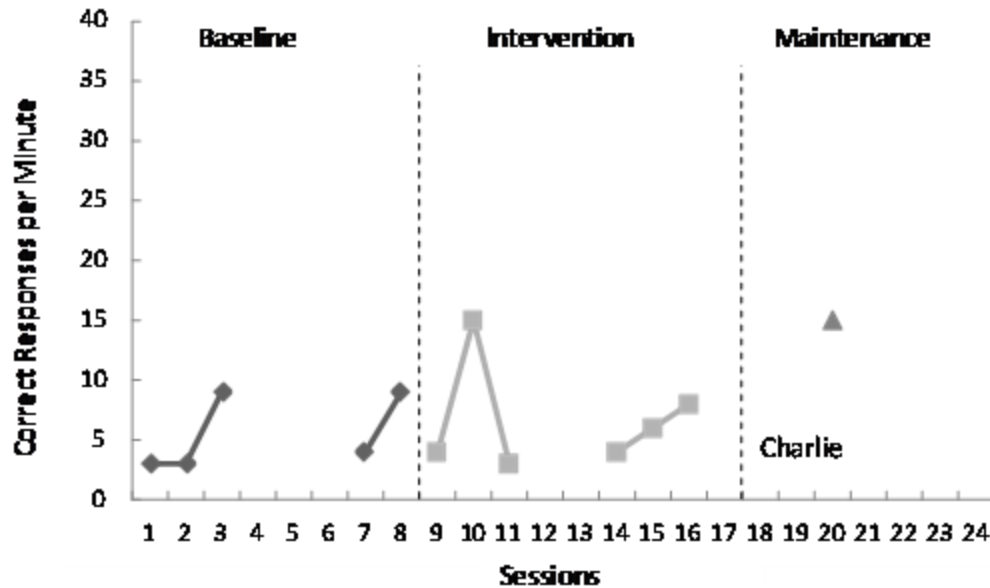


Figure 10. Charlie's performance on the quantity discrimination measure.

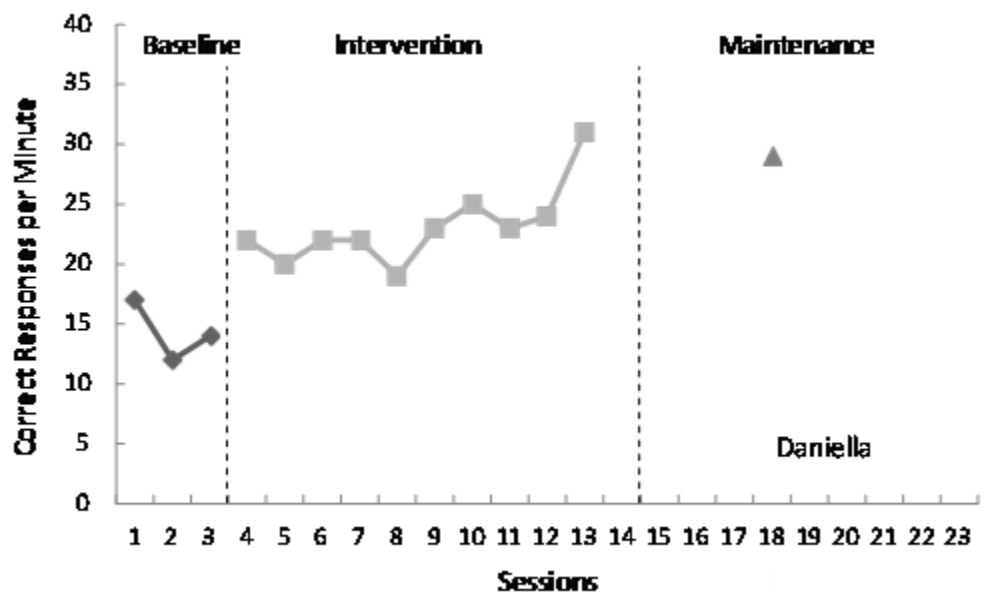
Daniella. Daniella scored 14 on the NSB (Jordan et al., 2008) indicating that she was at risk for math failure. She was assigned to group one. She was present for all of the baseline, intervention, and maintenance sessions.

Figure 11 provides a graphic representation of Daniella's performance on the quantity discrimination measure. Daniella exhibited some variability in the baseline phase with an overall downward trend (slope = -1.50). Her mean performance during baseline was 14.33 correct responses per minute ($SD = 2.53$, range 12 to 17).

Daniella experienced an immediate level change at the onset of intervention. Her first three intervention data points ($M = 21.33$) were 7 correct responses per minute higher than baseline, on average. The phase was characterized by an upward trend with some variability (slope = 0.78). Daniella had a mean of 23.1 correct responses per

minute ($SD = 3.28$, range 19 to 31) during intervention. PND between the baseline and intervention phases was 100%.

Daniella completed the maintenance assessment seven calendar days after her last intervention session. She demonstrated maintenance of the quantity discrimination skill with a score of 29 correct responses per minute on the assessment. Daniella's quantity discrimination data suggest that she derived benefit from the supplemental instruction with Numicon.



Summary of individual analysis for research question six. This study sought to determine if kindergarten students with different degrees of mathematics difficulty would exhibit different trends in their responsiveness to supplemental mathematics instruction

with Numicon. The findings from the individual analysis of student performance on the quantity discrimination measure provided one data source for addressing that question.

The individual analysis revealed a variety of response patterns among the students. Four of the nine students made quantity discrimination gains that were aligned with the start of the Numicon intervention. Two students made gains that were not clearly associated with the intervention. Two students never established a clear pattern of responding and one student showed a decrease in performance that was aligned with the start of intervention.

The various response patterns continued to be evident when students were grouped according to their degree of math difficulty. Five students were classified as having classroom math difficulties. Of those students, three (Dylan, Seth, and Steven) made quantity discrimination gains associated with the onset of intervention. One of the students (Devin) made steady gains across the phases of the study with no noticeable change in the pattern of performance between baseline and intervention. Aurora's demonstrated a decrease in performance associated with the start of the intervention.

Only one student (Lacey) was classified as being borderline at risk. Interpretation of her data was negatively impacted by variability in both the baseline and intervention phases. Given the variability and the fact that she was the sole student in the borderline at-risk category, no clear conclusion regarding the impact of supplemental instruction with Numicon for students in this category could be drawn.

Evaluation of the data for the students in the at-risk category provided further support for the premise that response patterns on the quantity discrimination measure

were not associated with degree of math difficulty. Three students were identified as being at risk (Adam, Daniella, and Charlie). Adam made mild gains across the phases of the study that could not be associated with the onset of intervention. Daniella made gains that were well-aligned with the start of the intervention. Charlie's data included such variability that no clear pattern of responding could be established. These findings indicate that degree of math difficulty, as determined by performance on the NSB (Jordan et al., 2008), was not associated with a specific response pattern on the quantity discrimination measure.

Quantity–Numeral Correspondence

A quantity–numeral correspondence fluency task was used to evaluate the students' use of accurate and efficient counting strategies. The quantity–numeral correspondence measure consisted of 18 sets of dot configurations representing quantities from 1 to 20. The students were given one minute to determine the number of dots in as many configurations as possible. Fluent completion of this task indicates that a student can recognize numerals, count with one-to-one correspondence, and apply counting strategies such as subitizing and counting on (Bryant & Bryant, 2011). Students may also apply place value knowledge to identify configurations with more than 10 dots. Three forms of the quantity–numeral correspondence probe were used, as described in chapter 3. The probes were administered before the baseline or intervention instruction for all sessions except the first intervention session. Students completed the quantity–numeral correspondence probe after instruction for intervention session one.

I scored the quantity–numeral correspondence probes daily, using a key. Scores were recorded as the number of correct responses per minute. All of the scores on the quantity–numeral correspondence measures were verified by a second observer, using the same key. All scoring discrepancies were resolved to reach 100% agreement before the data were graphed. As with the quantity discrimination measure, visual analysis was used to evaluate the data for level, stability, variability, and trend (Kennedy, 2005). The PND between the baseline and intervention phases was also calculated (Scruggs et al.,1987).

The visual analysis of the group data for the quantity–numeral correspondence measure is presented first. Figure 12 provides a graphic representation of the group performance. Table 5 provides means and slopes for each group during the baseline and intervention phases of the study. The slopes of the trendlines for the final three baseline points of each group are also presented.

It is important to note that intervention decisions were not made using the quantity–numeral correspondence data. The decision to move a group into the intervention phase of the study was made using the quantity discrimination data because that skill is more closely correlated with mathematics achievement than quantity–numeral correspondence (Gersten et al.2011). As a result, some groups entered intervention before a stable baseline trend was established on the quantity–numeral correspondence measure.

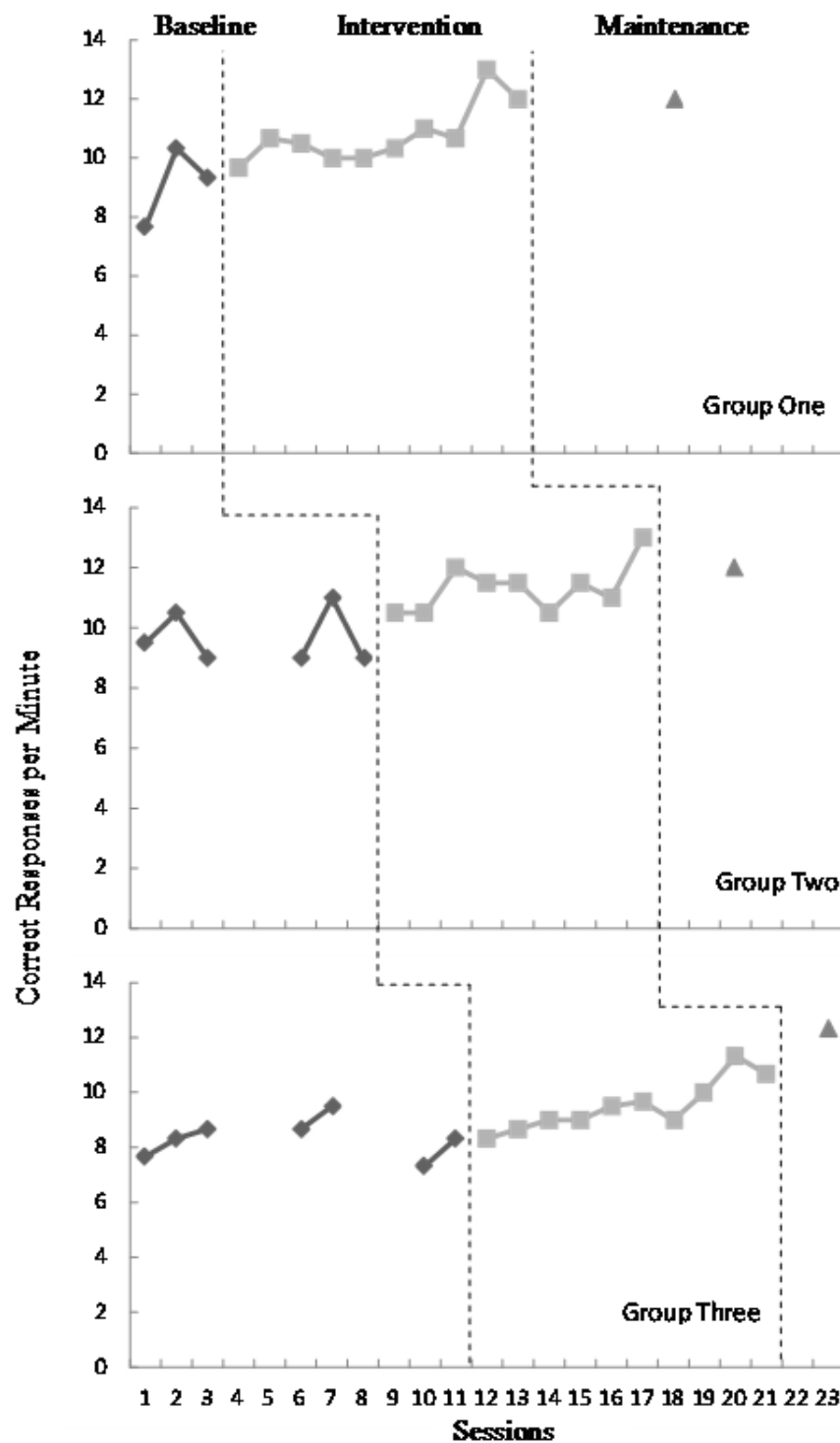


Figure 12. Group means on the quantity-numeral correspondence measure.

Table 5

Quantity–Numeral Correspondence Means and Trendline Slopes by Group and Phase

Group	Baseline Mean	Baseline Slope (Total)	Baseline Slope (Final 3 Points)	Intervention Mean	Intervention Slope (Total)
Group 1	9.11	0.83	0.83	10.78	0.25
Group 2	9.67	-0.01	0.00	11.33	0.16
Group 3	8.36	-0.01	-0.39	9.52	0.27

Group analysis. As discussed previously, students were placed into one of three groups based on their availability during the specified intervention periods. Each group had three students. Adam, Dylan, and Daniella were in group one. Charlie, Devin, and Lacey were in group two (although Charlie’s data are not included in the group analysis, as discussed previously). Aurora, Seth, and Steven were in group three.

Group one. Data from all three participating students (Adam, Dylan, and Daniella) were used to calculate the group one means for the quantity–numeral correspondence measure. The students completed three baseline probes. An immediate gain was evident from the first to the second probe, establishing an upward trend for the phase. However, the third baseline point was lower than the previous point. The trendline for the phase had a slope of 0.83. The baseline mean was 9.11 correct responses per minute ($SD = 1.34$, range 7.67 to 10.33).

The overall intervention phase was characterized by a gradual upward trend (slope = 0.25). There was minimal variability in the early part of the phase; however, the rate of gain increased over the final three intervention data points (slope = 0.67). This

change in performance aligned with the seventh intervention lesson, which introduced the idea of using knowledge of number patterns and the Numicon Shapes to quantify sets without using unitary counting methods. The change in performance becomes apparent when the mean from the first six intervention points ($M = 10.20$) is compared to the final four intervention data points ($M = 11.67$). The mean for the entire phase was 10.78 correct responses per minute ($SD = 1.01$, range 9.67 to 12). PND between the baseline and intervention phases was 70%.

The students in group one completed the maintenance assessment seven calendar days after the last intervention. As previously noted, two of the students had been sick for three school days before the maintenance probe. It is unclear if their performances were impacted by illness. The maintenance data point was equal to the last intervention data point and higher than 90% of the intervention data points, indicating that the students maintained their quantity–numeral correspondence skills in the absence of the supplemental instruction.

Group two. Group two means on the quantity–numeral correspondence measure were calculated using data from Devin and Lacey. As noted previously, Charlie’s data were omitted from the group analysis because they were highly variable. Additionally missing data points from his absences created variance around the group means that did not reflect the other students’ performances. Charlie’s performance on the quantity–numeral correspondence measure is discussed in the individual analysis.

Six baseline assessments were given to the students in group two. The overall pattern of the data was stable: no single data point varied from the mean by more than 1.5

correct responses per minute. The line of best fit for the baseline phase was nearly horizontal (slope = -0.01). The trendline for the final three intervention points was perfectly horizontal (slope = 0.00). The baseline mean was 9.67 correct responses per minute ($SD = 0.88$, range 9 to 11).

An immediate level change occurred upon entry into the intervention phase, as evidenced by a comparison of the final three baseline data points ($M = 9.67$) and the first three intervention data points ($M = 11$). An upward trend characterized the overall phase (slope = 0.16) with an increase in the rate of gain across the final three intervention data points (slope = 0.75). Mean performance for the intervention phase was 11.33 correct responses per minute ($SD = 0.83$, range 10.5 to 13). PND between the baseline and intervention phases was 56%.

The maintenance assessment was administered six calendar days after the last intervention session. The mean performance of 12 correct responses per minute was higher than all but two of the intervention data points, suggesting that the quantity–numeral correspondence skills had been maintained in the absence of intervention. The maintenance data point was also higher than any of the baseline data points.

Group three. Data from all three participants (Aurora, Seth, and Steven) were used to derive the means for group three. The group completed seven baseline probes. Initial gains were followed by a decrease in performance which resulted in a nearly horizontal trendline for the phase overall (slope = -0.01). A downward trend was evident in the final three baseline data points (slope = -0.39). The mean performance on the

quantity–numeral correspondence measure during baseline was 8.36 correct responses per minute ($SD = 0.71$, range 7.33 to 9.67).

An upward trend began at the onset of intervention and was evident throughout the phase (slope = 0.27). The data were stable within that trend. The mean performance of 10.25 correct responses per minute ($SD = 0.93$, range 8.33 to 11.33) reflected a change in level relative to the group's performance during baseline. This change was largely influenced by the data from the final four intervention probes. A change in the level and slope of the data were evident within the phase when the first six intervention probes ($M = 9.03$) were compared to the final four intervention probes ($M = 10.25$). This change occurred after students completed the seventh intervention lesson. Additionally, there was a slight increase in the rate of growth across the final three intervention data points (slope = 0.34). PND between the baseline and intervention phases was 40%.

Results across groups. The group data for the quantity–numeral correspondence measure were analyzed to determine if there was evidence of a functional relation between supplemental mathematics instruction with Numicon and quantity–numeral correspondence skills for kindergarten students with mathematics difficulties. Visual analysis was used to identify stable baseline data, consistency in the data across similar phases, immediate changes in level at the onset of intervention, and demonstration of effect at three different points in time (Kratochwill et al., 2010). These factors are associated with the determination of a functional relation. The PND was also evaluated as a measure of effect (Scruggs et al., 1987).

Stable baseline data. For the purposes of this study, the quantity–numeral correspondence data were characterized as stable when there was no upward slope and no more than two points of variation from the phase mean over three consecutive data points. A stable baseline was not established for group one. The baseline phase was characterized by an upward trend. The baseline phases for groups two and three did meet the definition of stability.

Consistency across similar phases. All three groups exhibited periods of improved performance followed by periods of decreased performance in the baseline phase of the quantity–numeral correspondence measure. The first group completed only three baseline data points and had an upward trend for the phase, although the final baseline point was lower than the second baseline data point. Groups two and three completed six and seven baseline data points, respectively. Both had a slight downward slope during the baseline phase. The overall mean for the baseline phases across the three groups was 8.99 correct responses per minute ($SD = 1.03$).

The data patterns for the three groups were similar across intervention phases. All three groups experienced a level change between baseline and intervention. The groups also displayed gradual upward trends at the onset of the intervention and increased the rate of growth following the seventh intervention lesson. The overall mean for intervention across the groups was 10.52 correct responses per minute ($SD = 1.18$).

The groups were very similar in their performance on the maintenance assessment. The maintenance scores of all three groups were higher than any baseline data point. The maintenance data points were also higher than a majority of the

intervention data points for all of the groups. The overall mean for maintenance across the groups was 12.11 correct responses per minute ($SD = 0.19$).

Immediacy of effect. An immediate change in level from baseline to intervention was only evident in group two. The mean of first three intervention data points reflected a gain of 1.33 correct responses per minute over the mean of the final three baseline data points. Groups one and three demonstrated a pattern of gradual gains at the onset of intervention. These gains were not sufficient to constitute an immediate level change. The data revealed that the overall change in level for groups one and three was heavily influenced by the data collected after the seventh intervention session.

Demonstrations of effect. The data from groups two and three provided demonstrations of effect between the baseline and intervention phases. Both groups evidenced downward trends in performance during the baseline phases, followed by upward trends in performance during the intervention phases. Group one did not provide a clear demonstration of effect due to the upward trend established during baseline. The third baseline data point reflected a downward turn in the group's performance; however, additional data points would have been required to establish a predictable pattern of responding. Absent this predictable pattern in baseline data, a clear association cannot be established between the gains achieved during intervention and the supplemental instruction with Numicon.

Overall percentage of nonoverlapping data. The PND was calculated across the three groups for the quantity-numeral correspondence measure. The PND for the

individual groups ranged from 40% to 70%. The overall mean PND between the baseline and intervention phases across the three groups was 55%.

Summary of group analysis for research question three. This study sought to determine if a functional relation was evident between supplemental mathematics instruction with Numicon and performance on counting and numeral recognition tasks for kindergarten students with mathematics difficulties. The quantity–numeral correspondence measure was used to evaluate counting and numeral recognition. The group data do not provide sufficient evidence to establish a functional relation between the supplemental instruction with Numicon and the quantity–numeral correspondence skills of kindergarten students with mathematics difficulties. The identification of a functional relation was precluded by the upward trend established during baseline for group one, the absence of immediate changes in level for groups one and three, and the low PND across the groups.

Summary of group analysis for research question four. This study also sought to determine if kindergarten students with mathematics difficulties would maintain their counting and numeral recognition skills one week after the conclusion of the supplemental instruction with Numicon. Each of the three groups made small gains in their mean quantity–numeral correspondence skills over the course of the research study. There was not sufficient evidence to attribute those gains to the supplemental instruction with Numicon. The groups did continue to demonstrate performance at or near the high scores achieved during the intervention phase of the study one week after the intervention ceased. Specifically, the maintenance score for group one was higher than 80% of the

intervention data and 100% of the baseline data. The maintenance score for group two was higher than 78% of the intervention data and 100% of the baseline data. The maintenance score for group three was higher than any data point in either the baseline or intervention phases of the study. Although it cannot be determined if the gains on the quantity–numeral correspondence measure were associated with the intervention, the small group instruction, or some other factor, the groups of students generally maintained the gains in the absence of the additional support.

Individual analysis. The quantity–numeral correspondence data for each student were analyzed individually to draw conclusions about each student’s response to the intervention. The individual data were also analyzed to determine if different response patterns were evident among students who were identified as being at risk, borderline at risk, or having classroom math difficulties. Figures 14 to 22 provide a graphic display of individual student data on the quantity–numeral correspondence measure. Additionally, Table 6 provides a comparison of the means and trendline slopes by student across the phases of the study.

Table 6
Quantity–Numeral Correspondence Means and Trendline Slopes by Student and Phase

Student	Baseline Mean	Baseline Slope (Total)	Baseline Slope (Final 3 Points)	Intervention Mean	Intervention Slope (Total)
Adam	10.33	0.50	0.50	11.33	0.21
Dylan	8.67	2.00	2.00	11.00	-0.03
Daniella	8.33	0.00	0.00	9.80	0.45
Charlie	5.00	0.39	0.07	4.83	-0.25
Devin	9.60	0.25	0.12	11.56	0.25
Lacey	9.83	-0.13	0.00	11.11	0.07
Aurora	8.86	0.28	-0.16	8.13	0.34
Seth	8.43	-0.01	-0.12	10.89	0.14
Steven	7.5	-0.32	-0.21	9.11	0.30

Students identified as having classroom math difficulties. Five students—Dylan, Devin, Aurora, Seth, and Steven—were described as only having classroom math difficulties. For the purpose of this study, this classification indicated that the students were performing at or below the 25th percentile on classroom math assessments, but not in the at risk or borderline at risk range on the NSB (Jordan et al, 2008).

Collectively, the students in this classification had a mean score of 8.61 correct responses per minute during baseline ($SD = 0.76$), 10.14 correct responses per minute

during intervention ($SD = 1.45$), and 12.2 correct responses during the maintenance phase ($SD = 1.10$). Each student's individual performance is described below.

Dylan. Dylan was in group one for the study. His quantity–numeral correspondence data are graphically presented in Figure 13. Dylan completed three baseline assessments and made strong gains between the first and second baseline probes. These gains created a steep upward trend in his baseline phase ($slope = 2.0$). His baseline mean was 8.67 correct responses per minute ($SD = 2.31$, range 6 to 10).

Dylan continued to make upward gains in the first three intervention data points. His performance on the remaining intervention probes was highly variable. In contrast to the baseline phase, a very slight downward trend was visible for the intervention phase ($slope = -0.03$), though the overall level of the intervention data remained higher than the baseline. Dylan performed at a mean rate of 11 correct responses per minute ($SD = 1.05$, range 9 to 12). PND between the baseline and intervention phases was 70%.

Dylan completed the maintenance probe seven calendar days after the last intervention session. He completed 11 correct responses per minute on the maintenance probe. This result was within the range of his performance during the intervention phase of the study, and one response per minute higher than any of his baseline scores. Dylan's data, primarily due to their instability, do not support a clear conclusion regarding the impact of supplemental instruction with Numicon on his quantity–numeral correspondence skills.

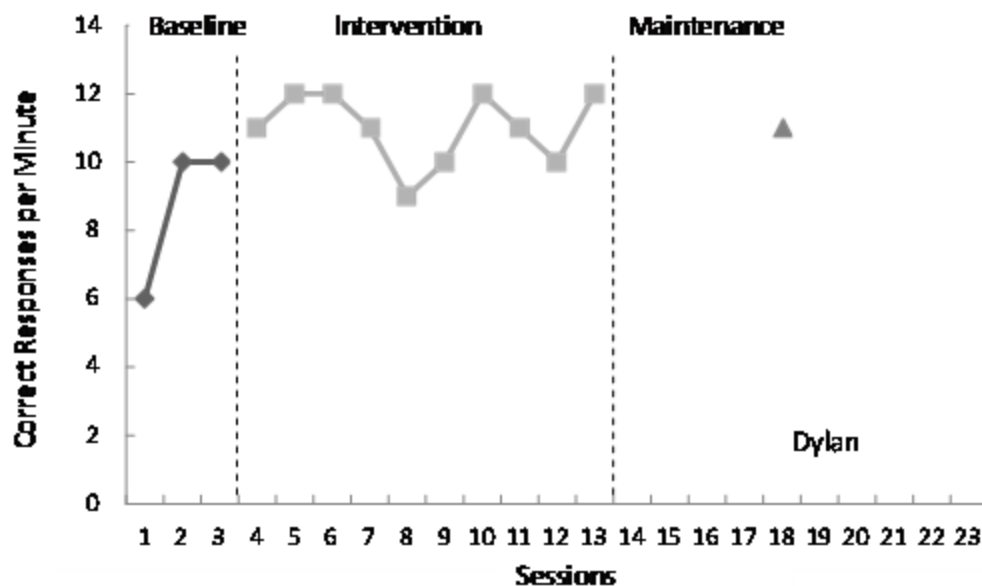


Figure 13. Dylan's performance on the quantity-numeral correspondence measure.

Devin. Devin was a member of group two. His data for the quantity-numeral correspondence measure are presented in Figure 14. Devin was present for five of the six baseline quantity-numeral correspondence probes administered to his group. Devin established an upward trend early in the phase. The overall trendline for the phase had a slope of 0.25. The trendline across the final three baseline data points was 0.12. His mean performance was 9.6 correct responses per minute ($SD = 1.14$, range 8 to 11).

Devin maintained the upward trend established in baseline throughout the intervention phase (slope = 0.25). His mean for the intervention phase was 11.56 correct responses per minute ($SD = 1.13$, range 10 to 14). PND between the baseline and intervention phases was 44%

Devin completed the quantity–numeral correspondence maintenance assessment six days after his last intervention session. He completed 13 correct responses per minute. His performance on the maintenance assessment was aligned with the trendline established in the baseline phase. Devin’s data pattern indicates that his improved performance on the quantity–numeral correspondence measure was not attributable to the intervention.

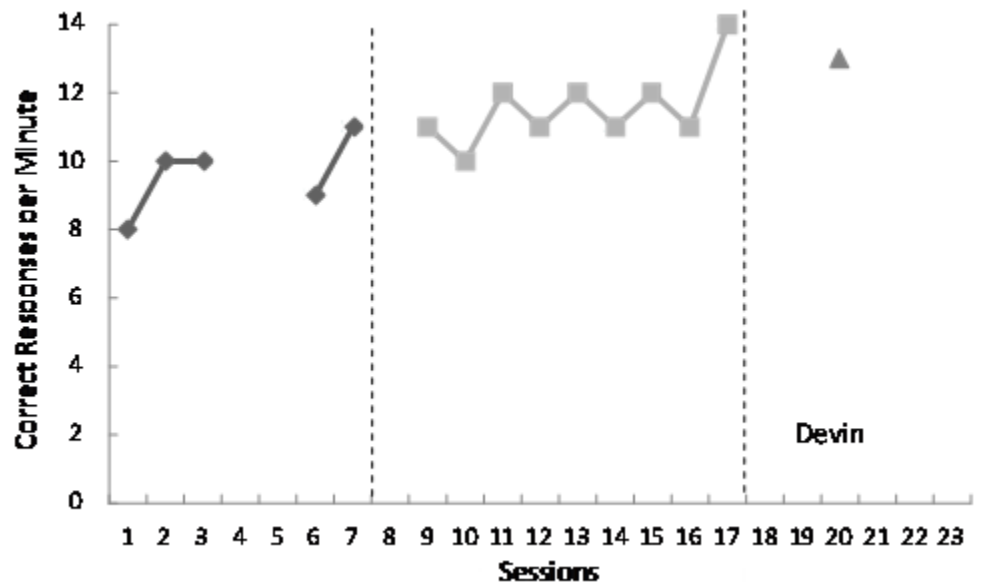


Figure 14. Devin's performance on the quantity–numeral correspondence measure.

Aurora. Aurora received intervention with group three. She completed all seven of the baseline probes given to this group. Her quantity–numeral correspondence data are presented in Figure 15. Aurora established and maintained an upward trend in the baseline phase, with the exception of her final baseline data point, which was lower than

the previous three points. The trendline for the entire phase had a slope of 0.28, while the trendline across the final three baseline data points had a slope of -0.16. Aurora had a mean of 8.86 correct responses per minute ($SD = 1.35$, range 7 to 11).

Aurora completed 8 of the 10 intervention probes administered to her group. She was absent for one and refused to complete the quantity–numeral correspondence probe for session 19. Aurora experienced an immediate level change at the onset of intervention with an average decrease of three correct responses per minute from the final three baseline data points to the first three intervention data points. Her performance improved over subsequent administrations of the probe and an upward trendline characterized the phase (slope = 0.34). However, her overall performance in the intervention phase was lower than her performance in the baseline phase. Aurora’s mean performance in the intervention phase was 8.13 correct responses per minute ($SD = 1.25$, range 7 to 10). PND between the baseline and intervention phases was 0%.

Aurora completed the maintenance assessment six calendar days after the last intervention session. She completed 11 correct responses per minute. Her maintenance performance was aligned with the trend that she established during the intervention phase and was equal to her highest baseline data point. Aurora’s data pattern suggests that her performance on the quantity–numeral correspondence measure was negatively impacted by the supplemental instruction with Numicon.

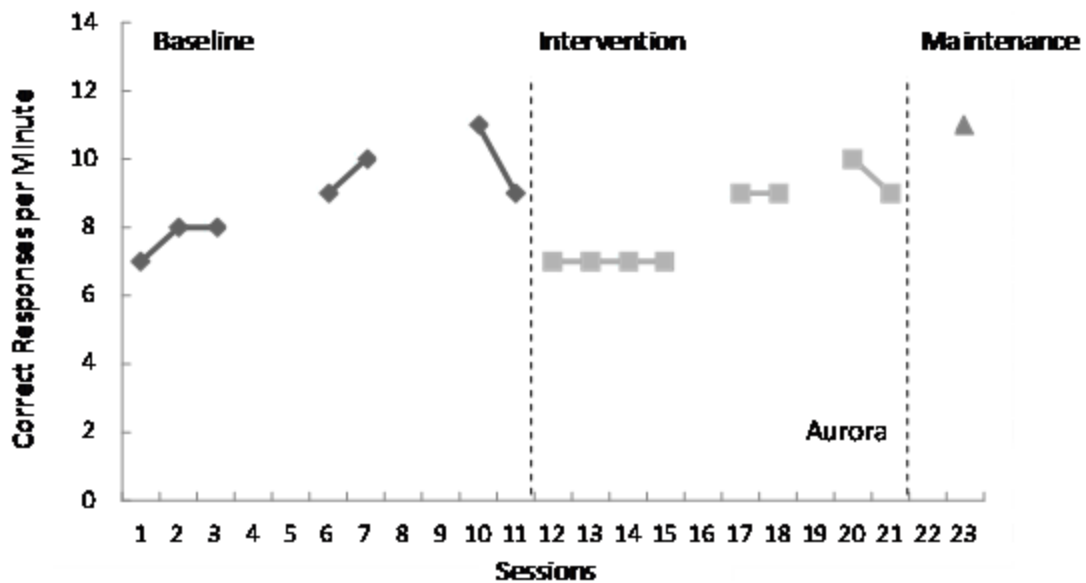


Figure 15. Aurora's performance on the quantity-numeral correspondence measure.

Seth. Seth was a member of group three. His data on the quantity-numeral correspondence measure are presented in Figure 16. Seth completed all seven of the baseline probes administered to his group. His performance was highly variable. The overall trendline for Seth's baseline data was nearly horizontal (slope = -0.01), with an increase in the downward slope over the final three baseline data points (-0.12). His mean for the baseline phase was 8.43 ($SD = 1.90$, range 5 to 10).

Seth completed nine of the 10 intervention probes. He was absent for one intervention session. Seth experienced an immediate change in level at the start of intervention. The mean of his first three intervention points was 2.67 correct responses per minute higher than the mean of the last three baseline points. An upward trend was evidenced in Seth's intervention data with much less variability than in the baseline phase

(slope = 0.14). Additionally, Seth's rate of gain increased after he had completed the seventh intervention lesson, as seen in the final three intervention data points. The mean for Seth's performance during the intervention phase was 10.89 correct responses per minute ($SD = 0.78$, range 10 to 12). PND between the baseline and intervention phases was 67%.

Seth completed the maintenance assessment six calendar days after his last intervention session. He completed 13 correct responses per minute. This result was better than any previous score in either the baseline or intervention phases. Anecdotal notes recorded on the day that Seth took the maintenance assessment indicate that he described, for the first time, recognizing that the pattern of the dot configurations on the quantity–numeral correspondence measure aligned with the patterns in the Numicon Shapes. Seth's data pattern suggests that his improved performance on the quantity–numeral correspondence measure was associated with the intervention.

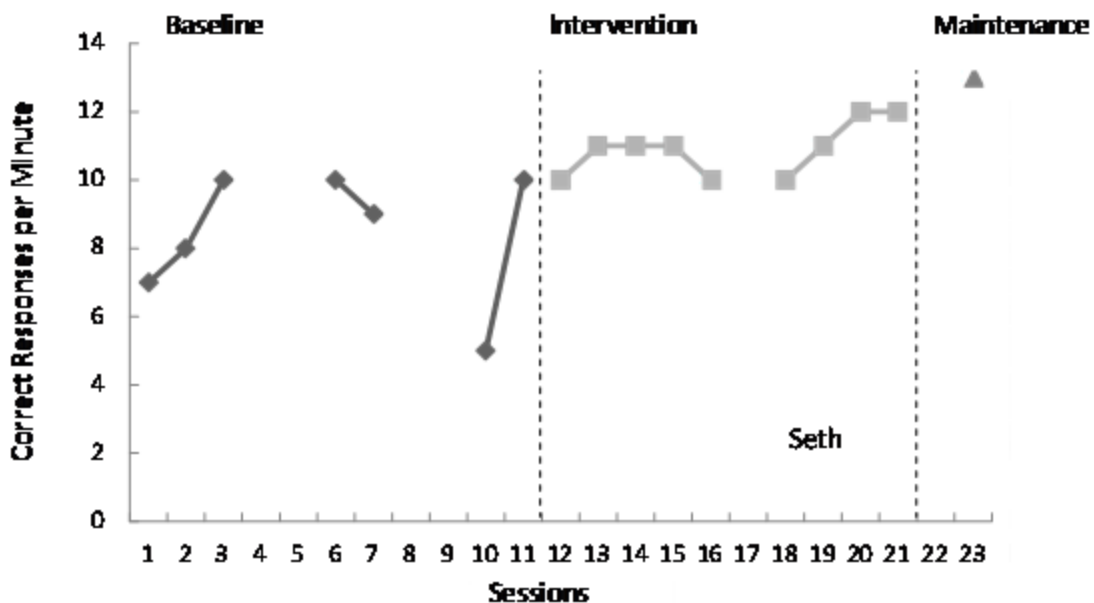


Figure 16. Seth's performance on the quantity-discrimination measure.

Steven. Steven was a member of group three. His data for the quantity-numerical correspondence measure are presented in Figure 17. He completed six of the seven baseline probes administered to his group. Steven demonstrated a downward trend in the baseline phase. The overall trendline slope was -0.32. The slope of the final three baseline data points was -0.21. The data were stable within that trend. His mean for the baseline phase was 7.5 correct responses per minute ($SD = 1.38$, range 6 to 9).

Steven was absent for one of the intervention probes. His level improved immediately upon entry into intervention. The mean of his first three intervention data points was 3 correct responses per minute higher than the mean of his last three baseline data points. The intervention phase was marked by an overall upward trend (slope = 0.30), though he did experience a decrease in performance on the first probe after his absence. Additionally, Steven experienced a positive change in level and rate of

progress within the intervention phase across the final three intervention probes. This change aligned with the intervention session in which the concept of quantifying without counting was introduced. Steven's mean performance for the intervention phase was 9.11 correct responses per minute ($SD = 1.54$, range 7 to 12). PND between baseline and intervention was 33% due to the high scores that Steven achieved on the first two baseline probes.

Steven completed the maintenance assessment six calendar days after the last intervention session. He completed 13 correct responses per minute. His maintenance performance was higher than any of the scores that he achieved in either the baseline or intervention phases, which suggested continued growth in his quantity–numeral correspondence skills. Steven's data pattern generally suggests that his performance gains on the quantity–numeral correspondence measure were associated with the intervention. However, his strong performance in the first two baseline measures raises questions about other factors that may have influenced his performance.

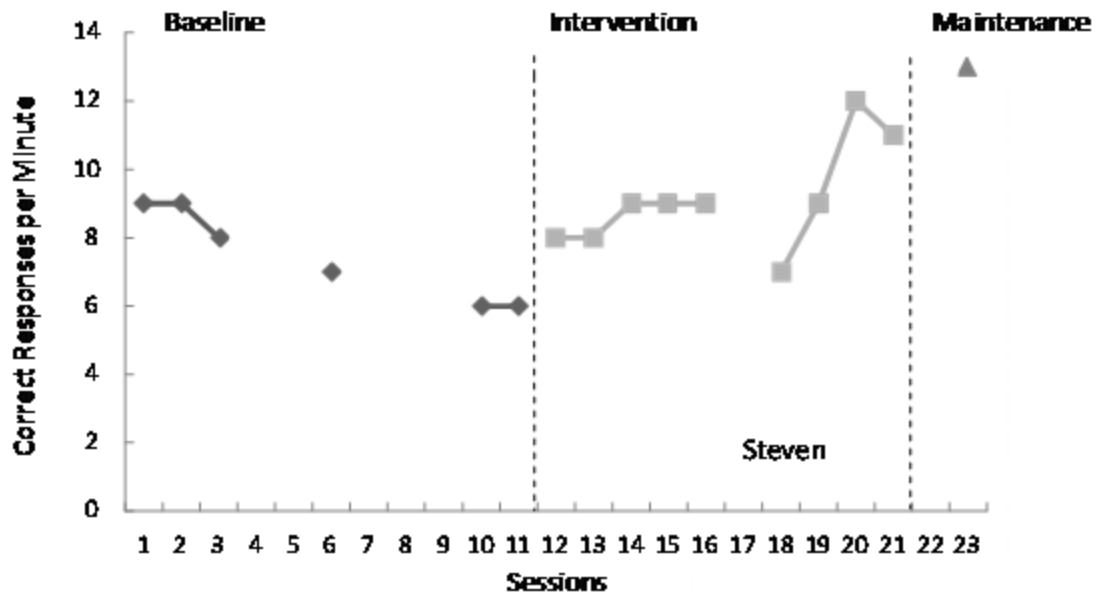


Figure 17. Steven's performance on the quantity-numeral correspondence measure.

Students identified as being borderline at risk. Students were classified as being in the borderline at-risk range if they scored between 16 and 18 points on the NSB (Jordan et al., 2008). A score in this range was within one standard deviation of the cut score that identified students who were at risk (Jordan, Glutting, Ramineni, & Watkins, 2010). One student in this study was classified as being borderline at risk.

Lacey was assigned to group two. Her performance on the quantity-numeral correspondence measure is presented in Figure 18. Lacey was present for all of the baseline, intervention, and maintenance sessions. She completed six baseline probes. Her performance resulted in a downward trend for the overall phase (slope = -0.13). The trendline representing the final three data points was flat (slope = 0.00). She completed a mean of 9.83 correct responses per minute ($SD = 1.33$, range 8 to 11).

Lacey began an immediate upward trend in performance at the start of the intervention phase. Her mean performance on the first three intervention probes was 1.33 correct responses per minute higher than her performance on the last three baseline probes. Although this result did not constitute an immediate change in level, because two of the three probes were within the range of the baseline data, it did reflect an immediate change in the pattern of responding. Lacey had not previously demonstrated an upward trend across three consecutive probes. The intervention phase was characterized by a mild upward trend (slope = 0.07), though there were two consecutive days in which Lacey's performance decreased. These lower data points (sessions 13 and 14) corresponded to classroom activities that distracted Lacey, with the result that she completed fewer items on all of the probes administered on those dates. (Data from session 14 were removed from the quantity discrimination analysis because they were well outside the range of Lacey's average performance. In this case, there was a decrease relative to the previous data points, but the scores were in line with her overall performance in the phase.) Lacey returned to her previous rate of performance in the subsequent sessions. Her mean performance for the intervention phase was 11.11 correct responses per minute ($SD = 0.78$, range 10 to 12). This result constituted a change in level relative to her performance in baseline and reflected greater stability of the intervention data. PND between the baseline and intervention phases was 33%.

Lacey completed the maintenance assessment six calendar days after intervention. She completed 11 correct responses per minute. Her performance aligned with the pattern established during the intervention phase. Lacey's data did not support a clear

conclusion regarding the impact of supplemental instruction with Numicon on her quantity–numeral correspondence skills.

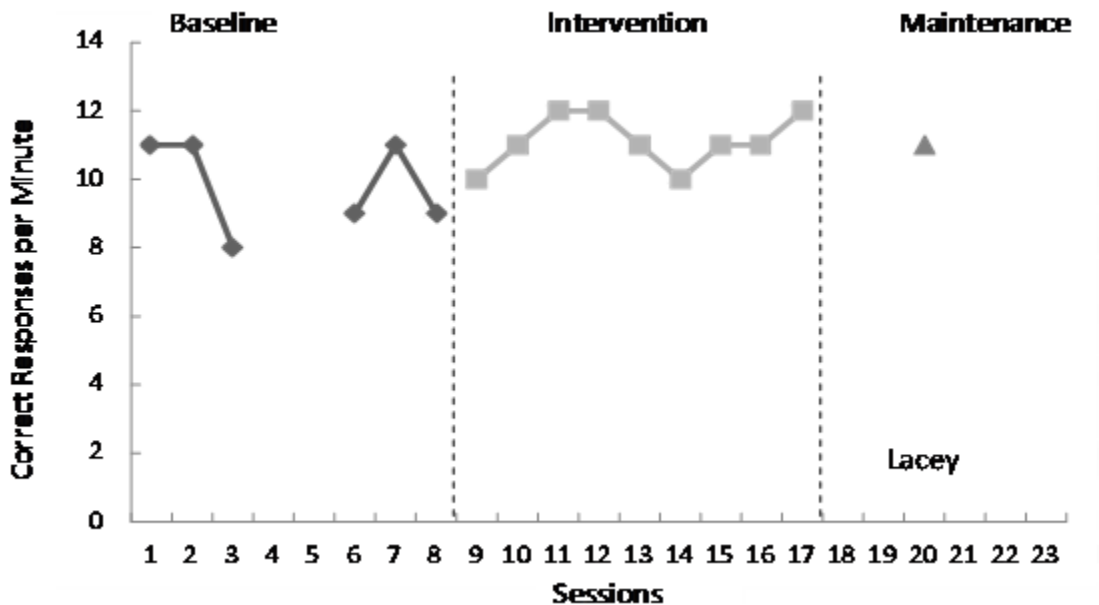


Figure 18. Lacey's performance on the quantity–numeral correspondence measure.

Students identified as being at risk. Students were classified as being at risk for mathematics failure if they earned a score of 15 or less on the NSB (Jordan et al., 2008). Adam, Charlie, and Daniella scored in this range.

Collectively, the students in this classification had a mean score of 7.89 correct responses per minute during baseline ($SD = 0.2.69$), 8.82 correct responses per minute during intervention ($SD = 3.12$), and 9.67 correct responses during the maintenance phase ($SD = 4.93$). Each student's individual performance is described below.

Adam. Adam was a member of group one. His quantity–numeral correspondence data are presented in Figure 20. Adam was present for all of the baseline assessments and nine of the 10 intervention assessments administered to his group. He completed three baseline probes and the last data point was 1 point higher than the previous two data points, creating an upward trend for the phase (slope = 0.50). Additional data points were needed to determine if the gain reflected a change in the pattern of performance or variation associated with the administration of the measure. The mean of Adam’s baseline performance was 10.33 correct responses per minute ($SD = 0.56$, range 10 to 11).

Adam’s performance varied more during the intervention phase than in the baseline phase. He exhibited a slight, but immediate change in level at the onset of intervention. His mean performance on the first three intervention probes was .67 correct responses per minute higher than the mean of his baseline probes. The intervention phase was characterized by an overall upward trend interrupted by two data points (sessions 9 and 11). The trendline slope for the intervention phase was 0.21. His mean performance on the quantity–numeral correspondence measure was 11.33 correct responses per minute ($SD = 1.41$, range 9 to 13). PND between the baseline and intervention phases was 50%.

Adam completed the maintenance assessment seven calendar days after the final intervention session. He had been sick and was visibly lethargic when the maintenance assessment was given. Adam completed 13 correct responses per minute, matching his highest scores during the intervention phase. Although Adam made gains in his

performance on the quantity–numeral correspondence measure over the course of the study, there was not sufficient evidence to link those gains to the Numicon intervention.

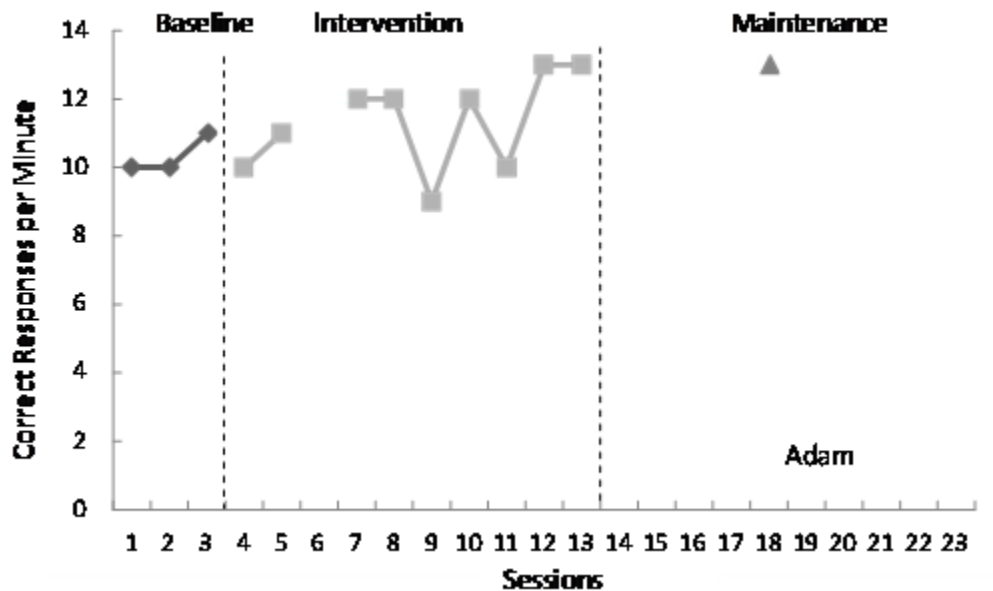


Figure 19. Adam's performance on the quantity–numeral correspondence measure.

Charlie. Charlie was assigned to group two. Figure 20 provides a graphic representation of his performance on the quantity–numeral correspondence measure. As noted previously, Charlie was frequently absent from the supplemental sessions. He was present for five of the six baseline probes that were administered to group two. He made gains in each consecutive administration of the baseline probe, creating an upward trend for the total phase (slope = 0.39). A very slight upward slope was also present for the trendline representing his final three baseline data points (0.07). His mean performance

on the quantity–numeral correspondence measure in baseline was 5 correct responses per minute ($SD = 1.58$, range 3 to 7).

Charlie’s scores on the quantity–numeral correspondence measure during the intervention phase were within the range of performance established during baseline. There was, however, a downward trend in the data (slope = -0.25). Charlie’s mean performance during the intervention phase was 4.83 correct responses per minute ($SD = 0.98$, range 3 to 6). There were no nonoverlapping data points between the baseline and intervention phases.

Charlie completed the maintenance assessment six calendar days after the last intervention session. He completed four correct responses per minute. The pattern of Charlie’s data suggest that there was a negative relation between his performance on the quantity–numeral correspondence measure and his participation in supplemental instruction with Numicon.

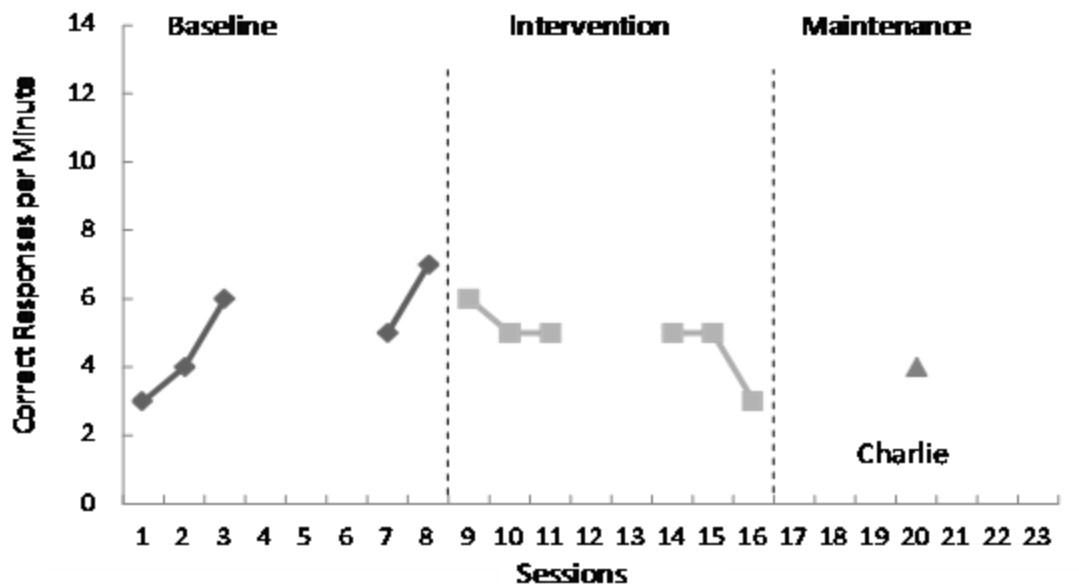


Figure 20. Charlie's performance on the quantity-numeral correspondence measure.

Daniella. Daniella was assigned to group one. Her data are presented in Figure 21. She was present for all of the sessions and completed three baseline probes. Variability was evident in the phases. She completed 7 correct responses per minute on the first and third probes and 11 correct responses per minute during the second probe. Overall, the trendline for the baseline phase was flat (slope = 0.00). Her mean score was 8.33 correct responses per minute ($SD = 2.31$, range 7 to 11).

Daniella's intervention phase data showed an upward trend (slope = 0.45). There was some variability in her performance. Her mean score for the intervention phase was 9.8 correct responses per minute ($SD = 1.87$, range 7 to 13). The high point in her baseline data resulted in a low PND between the baseline and intervention phases. PND was calculated to be 20%.

Daniella was given the maintenance assessment seven calendar days after the last intervention session. She completed 12 correct responses per minute. This score was higher than 80% of her intervention scores and all of her baseline scores, indicating that Daniella maintained her quantity–numeral correspondence skills in the absence of the intervention. The lack of stability in baseline prevented a solid conclusion regarding the role of intervention in her quantity–numeral correspondence gains.

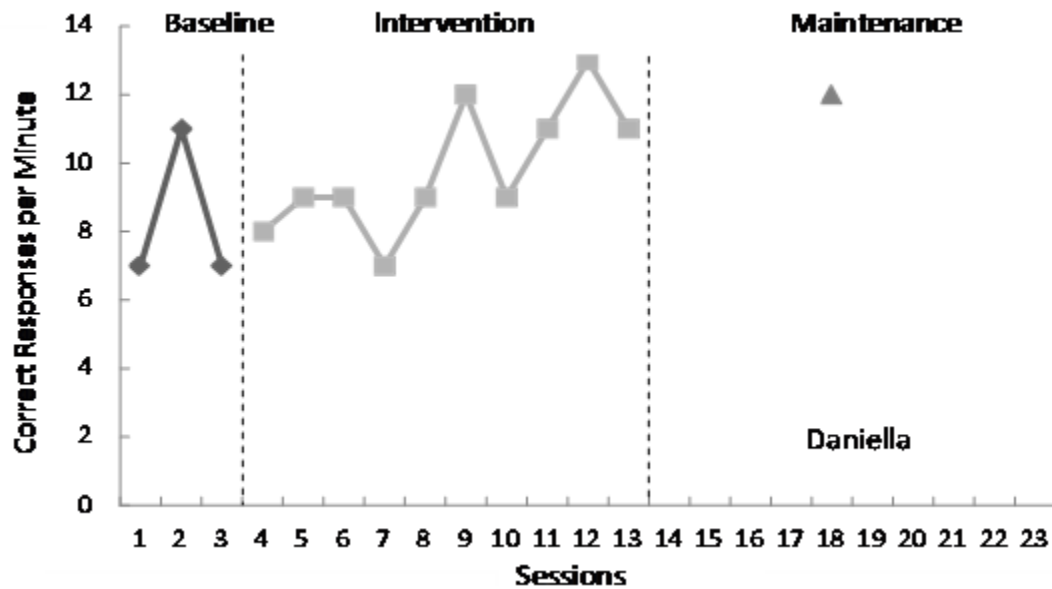


Figure 21. Daniella's performance on the quantity–numeral correspondence measure.

Summary of individual analysis for research question six. This study sought to determine if kindergarten students with different degrees of mathematics difficulty would exhibit different trends in their responsiveness to supplemental mathematics instruction with Numicon. The findings from the individual analyses of student performance on the

quantity–numeral correspondence measure address that question. Although, the group analysis of the quantity–numeral correspondence data did not provide sufficient evidence to establish a functional relation between supplemental instruction with Numicon and quantity–numeral correspondence skills, individual analysis indicated that some individual students did benefit from the instruction.

As with the quantity discrimination measure, a variety of response patterns were evident across individual students. Two of the nine students showed gains in performance aligned with the start of intervention. One student made gains across the phases of the study, suggesting that the supplemental instruction was not the source of improvement. The results were inconclusive for four students, primarily due to variability in the baseline and/or intervention phases. Finally, the performances of two students showed decreases associated with the start of the intervention.

Examination of these response patterns revealed that gains on the quantity–numeral correspondence measure only occurred for students who were described as having classroom math difficulties. Two students in that classification (Seth and Steven) made gains associated with the intervention. One student (Devin) established a pattern of gains in baseline and continued that pattern throughout the study. Other response patterns were also evident in the group of students with classroom math difficulties. One student (Aurora) experienced a decrease in performance and one (Dylan) had no clear pattern of responding.

Most of the students in the borderline at-risk and at-risk classifications experienced variability that prevented a clear conclusion regarding the effect of

supplemental instruction with Numicon on their quantity–numeral correspondence skills. This was true for Lacey, in the borderline at-risk category, as well as Adam and Daniella in the at-risk category. Charlie, described as being at risk for math failure, experienced a decrease in his performance associated with the start of the intervention.

The results of the individual student analysis reveal that multiple response patterns were present across students. They further show that degree of math difficulty may play some role in a student’s responsiveness to the intervention in terms of improving quantity–numeral correspondence skills. Specifically, only students in the classroom math difficulties classification made clear gains on the measure.

The Number Sense Brief

The NSB is an individually administered screening tool for early identification of math difficulties (Jordan et al., 2008). It is composed of 33 questions across six domains of number sense. The questions evaluate counting knowledge and principles, number recognition, number knowledge, nonverbal addition and subtraction, addition and subtraction story problems, and addition and subtraction number combinations (Jordan, Glutting, & Ramineni, 2010). The NSB has been demonstrated to have a strong correlation with general mathematics achievement in first and third grades (Jordan, Glutting, & Ramineni, 2010) and to be predictive of student success on a third-grade state-mandated mathematics test (Jordan, Glutting, Ramineni, & Watkins, 2010).

The NSB was first administered to all of the participating students at the beginning of January, just before the baseline phase of the study. The results were used to describe each student’s degree of math difficulty. The NSB was administered to each

of the students again in late February as a measure of generalization. Each student was given the assessment individually between four and seven days after they had completed the intervention. Three domains (counting knowledge and principles, number recognition, and number knowledge) were aligned with the content of the Numicon instruction but required a different application of the skills used on the quantity discrimination and quantity–numeral correspondence probes. These domains were classified as near-transfer tasks. The remaining three domains (nonverbal addition and subtraction, addition and subtraction story problems, and addition and subtraction number combinations) were not closely aligned with the Numicon instruction. Successful completion of those questions required students to apply number sense knowledge to basic calculations. These domains were classified as far-transfer tasks.

Although pre- and postintervention data were available from the NSB for each student, it was not appropriate to attribute the changes in performance to the supplemental instruction with Numicon. The changes may have reflected the intervention, classroom instruction, maturation, or a combination of those variables. That being said, Jordan, Glutting, Ramineni, and Watkins (2010) established test–retest reliability for the NSB across four time periods ranging from two to eight months apart within the kindergarten year. Reliability coefficients ranged from .78 to .86, with higher stability for shorter intervals. This finding indicates that under typical instructional conditions (i.e., no supplemental instruction or intervention) student performance on the NSB was fairly stable across time. Therefore, the changes in student performance on the NSB that were observed over the six-week duration of this study merit discussion.

Both group and individual analyses were conducted, in keeping with the other measures used in this study. The group analysis is presented first. Table 7 provides pre- and postintervention group means for the near-transfer and far-transfer domains, as well as total NSB performance.

Table 7
Group Means on the Number Sense Brief Before and After Intervention

Group	Near-Transfer Tasks (18)		Far-Transfer Tasks (15)		Total Score (33)	
	Pre	Post	Pre	Post	Pre	Post
Group 1	12.33	15.33	4.00	6.33	16.33	21.00
Group 2	10.67	13.00	6.67	8.33	17.67	21.33
Group 3	11.67	14.67	8.00	9.67	19.33	24.33

Group analysis. Gains were evident from the pre- to postintervention administrations of the NSB for all three groups. Effect sizes were calculated for each group using the standard deviation for students who were not at risk in the longitudinal study by Jordan, Glutting, Ramineni, and Watkins (2010). The standard deviation for students who were not at risk was selected over the standard deviation for students who were at risk because all of the intervention groups in this study contained at least one student who did not fall into the at-risk category. Group one (Adam, Dylan, and Daniella) had the lowest overall mean of the three groups before the intervention with a mean score of 16.33 points ($SD = 3.21$). Following intervention, the mean for group one was 21.00 points ($SD = 1.00$). The effect size for group one was 1.07. The students in

group two (Charlie, Devin, and Lacey) had a preintervention mean score of 17.67 points ($SD = 6.02$). They improved to a mean of 21.33 points ($SD = 7.21$) following the intervention, resulting in an effect size of .84. Group three (Aurora, Seth, and Steven) had the strongest overall performance with the least variation among group members on the NSB before the intervention. They averaged 19.33 points ($SD = 0.58$). The postintervention mean for group three was 24.33 points ($SD = 2.52$). The effect size for group three was 1.15.

The group results from the NSB were further analyzed by comparing the mean gains for each group on the near- and far-transfer tasks. Group one gained 3.00 points on the near-transfer tasks of the NSB between the first and second administrations of the assessment, and 2.33 points on the far-transfer tasks. Group two gained 2.33 points on the near-transfer tasks and 1.66 points on the far-transfer tasks. Finally, group three gained 3.00 points on the near-transfer tasks and 1.67 points on the far-transfer tasks.

Finally, the NSB pre- and posttest data were compared using the Wilcoxon signed-rank test for dependent samples. The posttest NSB scores ($M = 22.11$) were significantly higher than the pretest NSB scores ($M = 17.78$), $T = 45$, $p = .008$.

Summary of group analysis of the Number Sense Brief for research question five. This study sought to determine if kindergarten students with mathematics difficulties would demonstrate gains in their general number sense skills following supplemental instruction with Numicon. All three groups demonstrated improved performance of general number sense skills as measured by performance on the NSB. The mean effect size for the three groups was 1.02. Additionally, all three groups made

greater gains in the domains classified as near-transfer skills than in those classified as far-transfer skills. Although the gains in general number sense skills cannot be fully attributed to the supplemental instruction with Numicon, previously established test–retest reliability suggests that the intervention may have contributed to the improved performance.

Individual analysis. The NSB data were analyzed individually to evaluate growth for each student in terms of generalized number sense. The individual data were also analyzed to determine if any students with different degrees of math difficulty had similar response patterns. Table 8 provides pre- and postintervention scores for each student across the six number sense domains, as well as the total score on the NSB.

Table 8
Student Performance on the Number Sense Brief Before and After Intervention

Name	Counting Knowledge (7)		Number Recognition (4)		Number Knowledge (7)		Nonverbal Calculations (4)		Story Problems (5)		Number Combinations (6)		Total (33)	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Adam	5	6	2	3	6	7	2	4	0	1	0	1	15	22
Dylan	6	5	0	2	7	7	4	3	2	3	1	1	20	21
Daniella	7	7	1	2	3	5	3	4	0	1	0	1	14	20
Charlie	5	6	0	0	3	3	3	3	0	0	1	1	12	13
Devin	7	6	0	3	6	7	4	4	2	3	5	4	24	27
Lacey	6	7	1	3	5	4	4	4	0	3	1	3	17	24
Aurora	7	7	0	2	4	5	2	4	4	5	2	4	19	27
Seth	6	6	1	2	5	6	2	3	5	5	0	2	19	24
Steven	6	7	1	2	4	7	3	3	2	2	4	1	20	22

Students identified as having classroom math difficulties. Five students were classified as having classroom difficulties at the onset of the study. These students had a mean score of 20.4 points ($SD = 2.07$, range 19 to 24) on the NSB administered before the intervention. The mean score on the postintervention administration was 24.2 points ($SD = 2.77$, range 21 to 27). The mean effect size for students with classroom math difficulties was 0.87. Individual student performance is described below.

Dylan. Dylan had an overall score of 20 on the NSB before the intervention. He demonstrated particular strengths in the areas of counting knowledge, number knowledge, and nonverbal calculations. His weakest performance was in the area of number recognition. He was not able to identify any two- or three-digit numerals and did not earn any points for that domain.

Dylan earned 21 points on the NSB after intervention. The change in score reflected losses in some domains and gains in others. He demonstrated 1 point losses in counting knowledge and nonverbal calculations. His performance improved from zero to two correct responses in the number recognition domain. He also successfully answered one additional story problem. Dylan's overall performance did not change substantially between the preintervention and postintervention administrations of the NSB. In both cases, he surpassed the cut scores that indicate risk or borderline risk.

Devin. Devin had the strongest preintervention NSB score, 24 points, of all the students in this study. He correctly answered all of the questions in the counting knowledge and nonverbal calculations domains. He also had strengths in number knowledge and number combinations, earning 3 out of 4 and 6 out of 7 points,

respectively. Devin's weakest preintervention performance was in the number recognition domain. He was not able to identify any two- or three-digit numbers and therefore earned no points for that domain.

Devin gained 3 points on his overall NSB score between the first and second administrations. The change was associated with 1 point losses in the areas of counting knowledge and number combinations. He gained 3 points in number recognition and 1 point each in number knowledge and story problems. Devin's performance on both administrations of the NSB surpassed the cut scores indicating risk or borderline risk for math failure.

Aurora. Aurora correctly answered 19 questions on the first administration of the NSB. She demonstrated a strength in counting knowledge by answering all of the questions in that domain correctly. She correctly answered four of five questions each in the number knowledge and story problems domains. Aurora's weakest performance was in the number recognition domain, where she did not answer any questions correctly.

Aurora correctly answered 27 questions on the NSB administered after intervention. She gained 2 points each in the number recognition, nonverbal calculations, and number combinations domains. She gained 1 point each in the number knowledge and story problem domains. Neither of Aurora's scores on the NSB indicated risk or potential risk for math failure.

Seth. Seth earned 19 points on the initial administration of the NSB. He demonstrated a strength in solving story problems, correctly answering all of the questions in that domain. He also correctly answered six of the seven questions in the

counting knowledge domain. Seth was not able to solve any number combinations and was only able to identify one of the numerals in the number recognition domain.

Seth correctly answered 24 questions on the second administration of the NSB. His pre- and postintervention scores were equal in the counting knowledge and story problems domains. He made a 2 point gain in number combinations, and 1 point gains in number recognition, number knowledge, and nonverbal calculations. Seth's performance on both administrations of the NSB indicated that he is not at risk for math failure.

Steven. Steven earned 20 points on the preintervention administration of the NSB. He demonstrated relative strengths in counting knowledge where he earned 6 out of 7 possible points, and nonverbal calculations where he earned 3 out of 4 possible points. Steven correctly answered one question in the number recognition domain, indicating that this was his weakest skill.

Steven correctly answered 22 questions on the postintervention administration of the NSB. His score decreased by 3 points in the number combinations domain. He gained three points in the number knowledge domain, and 1 point each in counting knowledge and number recognition. Steven's performance on both administrations of the NSB indicated that he was not at risk or bordering risk.

Students identified as being borderline at risk. Lacey was the only student classified in the borderline at-risk range of performance for this study. She improved her performance by 6 points on the pre- and postintervention administrations of the NSB. This change resulted in an effect size of 1.61. Her performance is described below.

Lacey earned 17 points on the initial administration of the NSB. She demonstrated a strength in nonverbal calculations by correctly answering all four of the questions in that domain. Lacey also had a relative strength in counting knowledge, where she correctly answered six out of seven questions. Lacey's weakest performance on this assessment was in the story problems domain. She was not able to answer any of those questions correctly. Lacey also had some difficulty with number recognition and number combinations.

Lacey made a 7 point gain between the first and second administrations of the NSB, with a total of 24 points. She decreased by 1 point in the number knowledge domain. She gained 3 points in the story problems domain, 2 points each in the number recognition and number combinations domain, and 1 point in the counting knowledge domain. These gains moved Lacey out of the borderline at-risk category. Her performance was more than two standard deviations higher than the at-risk cut score for February.

Students identified as being at risk. Three students were described as being at risk based on their preintervention performance on the NSB. The preintervention mean for all three of those students was 13.67 ($SD = 1.53$, range 12 to 15). The three students had a mean gain of 4.67 points between the first and second administrations of the assessment, bringing the postintervention mean to 18.33 points ($SD = 4.73$, range 13 to 22). When Charlie's scores were excluded, due to his frequent absences from the intervention sessions, the mean gain score for the group of students who were classified as at risk increased to 6.5 points, resulting in a mean effect size of 2.0. When Charlie's

scores were included, the mean effect size for the group was 1.44. The individual performance of each student from this group is described below.

Adam. Adam earned 15 points on the initial administration of the NSB. He demonstrated a relative strength in the number knowledge domain by correctly answering six of the seven questions. Adam was not able to solve any story problems or number combinations. He correctly identified two of the four numerals presented in the number recognition domain.

Adam earned 22 points on the second administration of the NSB, making gains in each of the six domains. He correctly answered all questions in the number knowledge and nonverbal calculations domains. He improved his score in counting knowledge to six out of seven correct responses. He also improved his number recognition skills, identifying three out of the four numerals that were presented. Adam's performance on the second administration of the NSB indicated that he was no longer in the at-risk or borderline at-risk categories.

Charlie. Charlie earned 12 points on the initial administration of the NSB. He had a relative strength in nonverbal calculations where he earned 3 out of 4 points. Charlie also earned 5 out of 7 points in the counting knowledge domain. He was not able to correctly answer any of the questions in the number recognition or story problem domains.

Charlie achieved a 1 point gain between the first and second administrations of the NSB. The additional point was earned in counting knowledge. Charlie maintained

his level of performance in all of the other domains. His score on the second administration of the NSB indicated that he continued to be at risk for math failure.

Daniella. Daniella correctly answered 14 questions on the initial NSB. She correctly answered all seven questions in the counting knowledge domain. Daniella also had a relative strength in nonverbal calculations. She correctly answered three out of four questions in that domain. Daniella was not able to solve any story problems or number combinations. She was able to identify one of the numerals presented in the number recognition portion of the assessment.

Daniella earned 20 points on the second administration of the NSB. She maintained a perfect score in the counting knowledge domain. She improved her performance by 2 points in the number knowledge domain and 1 point each in number recognition, nonverbal calculations, story problems, and number combinations. Daniella's performance on the second administration of the NSB indicated that she was no longer in the at-risk or borderline at-risk categories.

Summary of individual analyses of the Number Sense Brief for research question six. This study sought to determine if kindergarten students with different degrees of mathematics difficulty would exhibit different trends in their responsiveness to supplemental mathematics instruction with Numicon. The NSB data provided a third source of information to address that question. The individual analysis of the data from the NSB provided insight into each student's ability to apply number sense skills to an array of classroom tasks. As the data in Table 8 show, all of the students showed some improvement in their total NSB scores regardless of their degree of math difficulty.

Analysis of the gain scores and effect sizes by degree of math difficulty revealed that students in the at-risk and borderline at-risk classifications made greater overall gains than the students who exhibited classroom math difficulties. Additionally, as shown in Figure 22, three of the four students who were in the at-risk or borderline at-risk classifications no longer scored in these ranges on the second administration of the NSB. The one student who remained in the at-risk range was absent for four of the 10 intervention sessions. The analysis of the individual data provides further evidence of improvement in general number sense skills for most of the students who participated in the study, with greater gains apparent for the students who had the weakest preintervention performance.

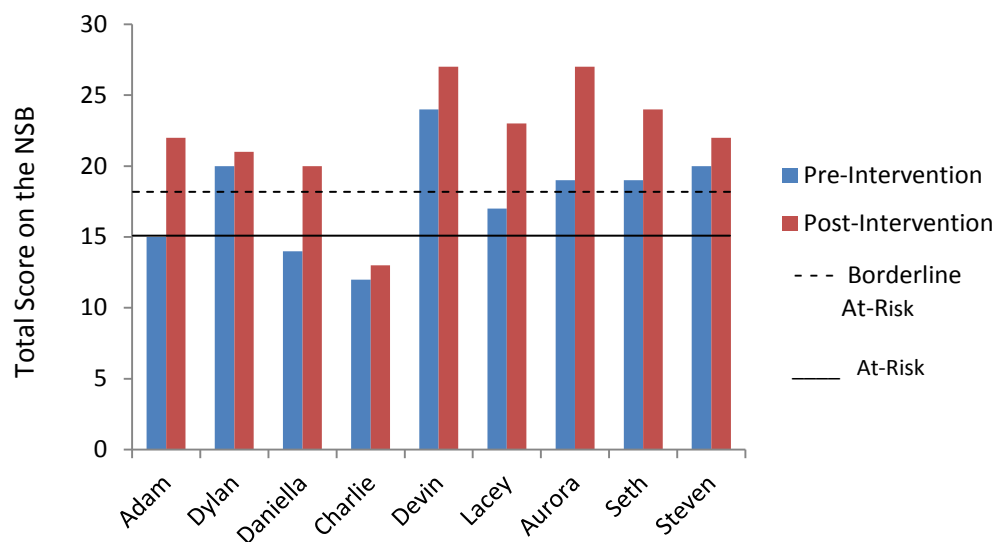


Figure 22. Pre- and postintervention performance on the Number Sense Brief.

Social Validity

“Social validity is the estimation of the importance, effectiveness, appropriateness, and/or satisfaction various people experience in relation to a particular intervention” (Kennedy, 2005, p. 213). In this study, social validity was evaluated by examining student and teacher perceptions of how much the students learned and how much they enjoyed the intervention. These questions were addressed through student interviews and a teacher questionnaire.

Student interviews. The student interviews were conducted with each student individually between one and seven days after the completion of the intervention. A structured interview format was used and incorporated six guiding questions. The students were interviewed in the classroom used for intervention. Numicon materials were visible in the room, but were not specifically offered to most of the students during the interview. Charlie was the exception. The Numicon Shapes were directly presented to him when it became evident that he was having difficulty verbalizing responses to the interview questions. Many other students independently reached for the materials or asked to use them during the interviews.

Seven of the nine interviews were video recorded. Two interviews were audio recorded to respect parents' wishes that their children not be video recorded. I transcribed all of the interviews and then read the transcriptions to identify each student's response to the six interview questions and determine if themes were present across students. Descriptions of the findings for each question are presented below.

Question one. The students were asked how the intervention classes differed from the way they usually learned math in their regular classroom. This question was very difficult for many of the students and often required getting the students to describe the activities for math in their regular classes and then asking them if those activities were the same or different from what was done in the intervention math classes.

The students consistently described “counting” as an activity they did in their regular math class. The term “counting” most frequently referred to counting objects, but sometimes referred to written numerals. In describing their regular math instruction five students mentioned using a variety of unitary counters, including bugs, spiders, blocks, erasers, and pictures of objects such as butterflies, buses, and cars. Two students (Aurora and Steven) specifically described using “toys” for math activities in their regular classes. Aurora also described using her fingers to count.

A majority of the students identified the Numicon Shapes as something that was used in the intervention classes but not in their regular classes. Five students specifically mentioned the shapes, though many had difficulty remembering the word “Numicon.” The students would point to the shapes or describe them with phrases like “the hole things.” Four students also commented that the fluency measures (“brain races”) were different from activities that they did in their regular math classes.

Question two. The students were asked to identify their favorite part of the intervention math classes. They offered a broad array of responses to this question. Five students mentioned that they liked using the Numicon Shapes. They enjoyed counting with the shapes and ordering them from 1 to 10. Daniella stated, “I like to get them and

put them out really quickly.” When questioned how she put them in order really quickly, Daniella described stacking one shape over the other to find the shape with one more hole than the previous shape. Seth explained that he liked to put them in order by making “stair steps.” Three of the students also mentioned that they enjoyed playing “Swaps” with the Numicon Shapes.

In addition to work with the Numicon Shapes, some students also mentioned that they liked working with the Numicon Number Tracks. Adam stated that he liked using the “sticks” to “count to 10 and then 40 and then 50.” His comment referenced the intervention activities in which the Number Tracks were used to identify patterns in the written numerals. Lacey stated that she enjoyed using the Number Tracks to count “jewels.” Steven summarized why he liked working with the Numicon Shapes and the Number Tracks when asked what he liked about them. He replied, “Cuz’ they are fun.”

Several students also commented that they enjoyed the assessment activities associated with the intervention. Three students (Adam, Devin, and Aurora) stated that they enjoyed the “brain races.” (I used “brain race” to describe the fluency probes in child-friendly language.) Aurora explained that she liked trying to go all the way to the end of the paper. Three other students stated that they enjoyed activities that were part of the NSB (Jordan et al., 2008): Dylan enjoyed the counting tasks with the dinosaur puppet and Seth and Steven both enjoyed the nonverbal calculations tasks.

Question three. The students were asked if there was anything they did not like about the intervention math classes. The students were generally positive about the classes, mentioning only a limited number of things that they did not like. Three students

specifically said that there was nothing they did not like. Seth summarized, “Actually, I liked everything.”

Two students (Dylan and Devin) identified minor behavioral issues that arose during the course of the intervention. Daniella and Lacey reported that they did not like a task that was difficult for them. In Daniella’s case it was the nonverbal calculations task on the NSB (Jordan et al., 2008). She stated that she did not like the task the first time they did it, but liked it more on the second administration because “I got 100!” Lacey did not like the Feely Bag activities with the Numicon Shapes because “sometimes I would make mistakes.” Steven stated that he did not like counting stars on the NSB, but did not explain why.

Aurora stated that she did not like working with the Numicon Shapes at all. When asked why she stated, “Because I want to have fun playing. I just like to play and not do numbers.” I asked her if that was why she was grumpy on some days when she came to the intervention classes. She agreed and added that on some of the days she wanted to see her mom.

Question four. When asked if the Numicon Shapes helped them learn, all nine students indicated that the Numicon Shapes did help them learn something. When asked to explain what they learned the students often independently took the Numicon Shapes from the storage baskets and modeled their learning.

The students most frequently demonstrated that they could order the Numicon Shapes from 1 to 10 using a variety of strategies. Four students identified different visually represented patterns as being helpful in ordering the shapes. Lacey described

using the alternating “smooth shape, bump shape” pattern. Daniella modeled identifying the next shape in the sequence by locating the shape that had one more hole than the previous shape. She said, “I found out when you put one on top of the other it can go in the right place, and it looks just like the other, but it needs one more.” Seth and Aurora both described using the “stair step” pattern to determine the order of the shapes. Devin explained that he could order the shapes more quickly when he did not count the holes. Charlie modeled putting the shapes in order, but worked backward from the 10 shape. He was not able to verbalize what he was doing or why.

Four students indicated that they learned to use the Numicon Shapes to represent numbers. Three of those students made statements indicating that they had developed an association of the various shapes with quantities. When asked to elaborate on what he learned about numbers with the shapes Adam stated, “I could make a picture in my head.” The researcher asked him to describe the picture that he had in his head for the number five. Adam stated that it was a “bump shape” and “It has five holes.” Seth made a similar comment when ordering his shapes. He held up the blue shape and stated, “This is a 10....because it’s the shape of 10. It’s the biggest shape.” When asked what the “shape of 10” meant, he put the shape down and modeled using two fingers in the air to fill five rows of imagined holes. Devin explained that the shapes helped him work faster, “but I don’t count. I know.”

Some students also indicated that the Numicon Shapes could be used to represent numbers greater than 10. Lacey demonstrated building a representation of 11 by putting the blue shape and orange shape together. Devin stated that the shapes could be used to

“count all the way to 10... or all the way to 100.” He further explained, “If you want to make 100 you have to take more shapes.” Aurora said that the shapes “helps us go all the way to 100...and 30 hundred.”

Two students used the Numicon Shapes to compare the relative size and or quantity of numbers. When asked what he had learned with Numicon Shapes, Steven pulled out three random shapes from the bag and explained, “This one is bigger and this one is smaller...and this one is the most smaller.” Seth did something similar. He initially held up the blue shape, identifying it as 10 and the biggest shape. When asked to clarify, he pulled out two other shapes and said “It’s bigger than this one and this one.” The researcher prompted him to use number words by saying, “So we could say 10 is bigger than...” to which Seth immediately replied “6” and raised the teal shape. He then stated that “8 is bigger than 7” while modeling with the shapes.

Question five. When asked if they would like to use Numicon Shapes in their regular classrooms, seven of the nine students immediately responded that they would. They described wanting to play games and build numbers. Devin said he wanted them in his classroom because “It’s super fun to play swaps and Numicon games.” Steven explained that “It’s real fun” and he could show other kids “about size.”

Adam initially said that he did not want to use Numicon Shapes in his classroom. When asked why he stated, “Because we don’t have them.” When asked if he thought Numicon Shapes should be purchased for his regular class, he said, “yes” and added that he would want to use them “for counting.”

Aurora also stated that she did not want to use Numicon Shapes in her regular class. When asked why she said, “because it’s boring” and “we don’t get to play.” She added that the students get to play more in her regular class than in the intervention class.

Question six. The students were asked if there was anything else they wanted to share about the intervention math classes or the Numicon Shapes. Six of the students had nothing else to add. Lacey reiterated that she enjoyed playing the swaps game and would like to play it in her regular classroom. Two students expanded on things that they had learned. Daniella said, “I think it gets me a little bit more remembering.” She described how the intervention classes helped her with work in her regular classroom. Steven explained how he could see the circles created by the Numicon Shapes on the surface of the table below the shape. He indicated that he could sometimes determine how many circles were on the table without counting saying, “If there’s easy ones, that means you know.”

Teacher questionnaire. The three kindergarten teachers who participated in the study completed questionnaires at the conclusion of the study. The questionnaires consisted of five statements referencing the student participants’ application of number sense skills in the classroom and one statement referencing the students’ desire to use Numicon Shapes during regular class activities. Teachers were asked to identify their level of agreement with each statement using a five-item Likert scale. The teachers were also asked an open-ended question inviting them to share any comments or observations about their students’ participation in the math intervention.

Each teacher completed one questionnaire for every participating student in their class, for a total of three questionnaires per teacher. The questionnaires were analyzed by assigning a score to each of the response options. A response of Strongly Agree was assigned 5 points, Agree was 4 points, Not Sure was 3 points, Disagree was 2 points, and Strongly Disagree was 1 point. The mean and modal responses for each question were calculated. A description of the results for each closed-response question is provided below.

Statement one. Teachers were asked to express their level of agreement with this statement: “The student demonstrated improvement in rote counting skills.” The mean response for this question was 4.33 ($SD = 0.87$, $Mode = 5$, range 3 to 5) indicating general agreement that students improved rote counting skills after participating in the intervention. The response Strongly Agree or Agree was selected for seven of the nine students. The teachers marked Not Sure for two students.

Statement two. The teachers expressed their level of agreement with this statement, “The student demonstrated an improvement in his/her ability to count sets of objects.” The mean response for this question was 4.22 ($SD = 0.97$, $Mode = 5$, range 2 to 5). The teachers generally agreed that their students showed increased skill in counting sets of objects after participating in the intervention. Specifically, the teachers responded Strongly Agree or Agree for eight of the nine participating students. One teacher chose Disagree for one of her students.

Statement three. Teachers were asked to express their agreement with the statement: “The student demonstrated improvement in his/her ability to name written

numerals.” The mean response for this question was 3.78 ($SD = 1.09$, $Mode = 4$, range 2 to 5). The teachers expressed some uncertainty as to whether the students collectively made gains in numeral recognition. Further analysis of these data revealed that the teachers responded using Strongly Agree or Agree for seven of the nine participants. The teachers disagreed with the statement for two students.

Statement four. Teachers expressed their level of agreement with the statement: “The student demonstrated an improved ability to count backward from 10.” The mean response for this question was 3.78 ($SD = 0.97$, $Mode = 4$, range 2 to 5). The teachers expressed some uncertainty as to whether the students collectively improved their ability to count backward. They responded Strongly Agree or Agree for six students, Not Sure for two students, and Disagree for one student.

Statement five. Teachers expressed their degree of agreement with the statement: “The student demonstrated a better understanding of the relationship between numerals and quantities.” The mean response for this question was 3.78 ($SD = 0.83$, $Mode = 4$, range 2 to 5). The teachers expressed some uncertainty as to whether the students collectively made gains in understanding the relationship between numerals and quantities. The teachers selected Strongly Agree or Agree in response to this statement for seven of the nine students. One teacher indicated she was unsure for one student and another indicated that she disagreed with this statement for one student.

Statement six. The teachers were asked to express their level of agreement with the statement: “The student expressed a desire to use Numicon Shapes during regular

class activities.” The teachers disagreed with this statement for all of the students. None of the students had asked to use Numicon Shapes in their regular classrooms.

Open-ended question. The final question invited teachers to share their observations regarding each student’s participation in the math intervention classes. Only one of the three teachers chose to respond to this question. For each student she wrote a two- or three-word phrase indicating that her students enjoyed the supplemental instruction with Numicon.

Summary of social validity findings. The student interviews and teacher questionnaires were used to evaluate student and teacher perceptions of what the students learned and whether they liked the intervention. The student interviews revealed that all of the students could identify or model something that they had learned by using the Numicon Shapes. They modeled a variety of skills including ordering the shapes using various patterns, making size or magnitude comparisons, and representing numbers with the shapes. The teacher questionnaire revealed that the teachers agreed or strongly agreed that most students had improved their counting skills, their ability to name written numerals, and their understanding of the relationships between quantities and numerals. In some cases the teachers were unsure if some students had improved their abilities to count backward or understand the relationships between quantities and written numerals.

The student interviews also revealed that the majority of the students enjoyed the supplemental instruction with Numicon. Many students identified specific Numicon activities as their favorite part of the intervention classes. Only two of the students referred to an instructional task that they did not like in the supplemental math classes.

Additionally, all of the students except Aurora expressed a desire to use Numicon in their regular classroom. The limited number of written comments on the teacher questionnaires also indicated that the students enjoyed the intervention math classes.

Summary of Results

The analyses presented in this chapter evaluated the effect of supplemental instruction with Numicon on the quantity discrimination skills, quantity-numeral correspondence skills, and generalized number sense skills of kindergarteners with different degrees of math difficulty. The group results provided moderate evidence of a functional relation between the Numicon instruction and student performance on quantity discrimination measures. Additionally, the data suggested that students maintained their quantity discrimination skills one week after the intervention ended.

The group results did not provide evidence of a functional relation between the Numicon instruction and student performance on quantity–numeral correspondence measures. The groups of students did make small gains on the measure over the course of the study that could not be specifically associated with the intervention. The gains persisted one week after the completion of the intervention.

Analyses of the group and individual results from the NSB (Jordan et al., 2008) indicated that the students improved their general number sense skills following the supplemental instruction with Numicon. Gains were slightly greater for the number sense skills that were closely aligned with the Numicon intervention as compared to the skills that were not specifically addressed by the intervention. Additionally, three of the four students who were classified as at-risk or borderline at-risk based on the results from

the initial administration of the NSB no longer scored in either risk category after the Numicon intervention.

Finally, individual analyses were used to determine if students with different degrees of math difficulty exhibited different trends in their responsiveness to the supplemental instruction with Numicon. There was no evidence of a response pattern associated with degree of math difficulty on the quantity discrimination measure. There was evidence of a pattern on the quantity–numeral correspondence measure.

Specifically, only students with the lowest degree of risk (classroom math difficulties) experienced gains on the quantity–numeral correspondence measure. A response pattern was also evident, to some degree, on the NSB. Although individual students from each category of mathematics difficulty made large gains in performance between the pre- and postintervention assessments, students from the at-risk and borderline at-risk categories made the largest gains.

5. DISCUSSION AND RECOMMENDATIONS

Mathematics educators and researchers have come to recognize the value of conceptual understanding, computational fluency, and problem-solving skills for all students (Fuchs & Fuchs, 2001). However, neither purely procedural, nor purely constructivist instructional methods have been successful in developing those skills for students with math difficulties (Baxter et al., 2001; USDOE, 2008). Educators and researchers have been pressed to identify interventions that can facilitate mathematical competency for all students. Number sense has been identified as a foundational skill associated with long-term mathematics achievement in each of those important areas and could therefore be critical to early identification and intervention (Bobis, 2008, Gersten & Chard, 1999).

There is evidence that a wide range of variables contribute to number sense difficulties and that these difficulties can be identified as early as kindergarten (Chard et al., 2008; Gersten et al., 2005; Jordan et al., 2007). In the last decade significant progress has been made in the development and validation of number sense screenings that provide efficient means for identifying students who require intervention (Gersten et al., 2011). Yet, number sense intervention research is still quite limited and no evidence-based practices have been identified (Chard et al., 2008). Learning theory and research in other areas of mathematics have contributed to the identification of instructional factors that are likely

to be associated with effective number sense intervention. These include (a) systematic and explicit instruction, (b) concrete to abstract representational sequences, (c) number line experiences, and (d) verbalization of mathematical concepts and processes (e.g., Bryant & Bryant, 2012; Bryant, Bryant, Gersten, Scammacca, Funk, et al., 2008; Chard et al. 2008). These factors are currently being explored in the emerging body of number sense intervention research.

The purpose of this study was to evaluate the effectiveness of supplemental instruction with Numicon on the development of number sense skills for kindergarten students with varying degrees of math difficulty. Numicon intervention employs many of the instructional factors associated with number sense development, but it is unique in emphasizing structured representation of numbers and quantity.

Nine students participated in a single-case multiple baseline design to evaluate the effect of Numicon intervention on quantity discrimination skills, quantity–numeral correspondence skills, and general number sense skills. This chapter summarizes the findings from that study, discusses the instructional implications of those findings, and makes recommendations for future intervention research.

Summary of Findings

Six research questions were addressed by this study. Overall, the data revealed that

- 1) There is moderate evidence of a functional relation between supplemental instruction with Numicon and gains in quantity discrimination skills;

- 2) Students maintained quantity discrimination skills that were at or near the levels achieved during intervention after the intervention had been discontinued for one week;
- 3) There is no evidence of a functional relation between supplemental instruction with Numicon and small gains that were achieved in quantity–numeral correspondence skills over the course of the study;
- 4) The small gains achieved in quantity–numeral correspondence skills over the course of the study were maintained after the intervention had been discontinued for one week;
- 5) The students collectively made large gains on a measure of general number sense skills following the supplemental intervention with Numicon; and
- 6) Students with different degrees of mathematics difficulty exhibited different trends in their responsiveness to intervention on the quantity–numeral correspondence measure and NSB, but not the quantity discrimination measure.

Further discussion of these findings is presented below, arranged by dependent variable. Group findings are discussed first, as those results were used for the determination of a functional relation. The discussion of individual student results for each dependent variable follows, taking advantage of the unique opportunity that single-case research provides to evaluate the characteristics of students who responded to the intervention and those who did not (Kratochwill et al., 2010).

Quantity discrimination. Quantity discrimination was one of the primary dependent variables for this study. It was selected due to its identification as a key

construct in number sense development and its strong correlation with mathematics achievement on a variety of standardized measures (e.g., Clarke & Shinn, 2004; Gersten et al., 2011). Data from fluency probes of quantity discrimination were evaluated using visual analysis. Guidelines established by Kratochwill et al. (2010) were used to evaluate the group data for the presence of a functional relationship.

Overall, the group data provided moderate evidence that the supplemental intervention with Numicon was associated with improvement in quantity discrimination skills for the participating kindergarten students and that students maintained their performance gains for one week in the absence of intervention. The strength of the conclusion was moderated by early baseline gains for group three which created an upward trend in the phase before stabilizing across the final 4 points. This presence of a functional relation was supported by demonstrations of immediate effect upon transition from baseline to intervention, level changes between the baseline and intervention phases, and upward trends in the intervention data for all three groups. PND between the baseline and intervention phases was calculated to be 100%. According to guidelines set forth by Scruggs and Mastropieri (1998), a PND greater than 90% suggests that an intervention is very effective.

The practical effects of the intervention on quantity discrimination performance are supported by findings from a study of early numeracy indicators. Lembke and Foegen (2009) evaluated the quantity discrimination performance of 126 kindergarten students from six classrooms in two states in the fall, winter, and spring. The sample included students with diverse learning characteristics (e.g., high, average, and low

academic achievement; special education; ELL). The mean kindergarten performance on the winter assessment in that study was 18.83 correct responses per minute ($SD = 9.51$). Group means from the intervention phase for all of the student groups in the present study exceeded the mean performance of the students in the Lembke and Foegen study, suggesting that the students were collectively performing in the average range or slightly above when compared to the students in the larger study.

Patterns of student responsiveness on the quantity discrimination measure.

Individual student data from the quantity discrimination measure were evaluated to determine if patterns of performance were evident among students representing different degrees of mathematics difficulty. The data did not reveal any pattern that was associated with student achievement level as determined by the NSB. This may reflect Van Luit and Schopman's (2000) finding that students with mathematics difficulties are a heterogeneous group, even when performing similarly on mathematics measures. Further evaluation of the data for responders and nonresponders provides examples of this phenomenon.

Responders. Dylan and Daniella are strong examples of responders representing students from the classroom math difficulties and at-risk groups, respectively. Dylan had a baseline mean of 13.33 correct responses per minute with a downward trend (slope = -2.50) on the quantity discrimination measure. He improved to a mean of 21.4 correct responses per minute with an upward trend (slope = 1.24) during the intervention phase. Although Daniella was in the achievement category indicating greater mathematics difficulty, she had a higher baseline mean than Devin with 14.33 correct responses per

minute and a downward trend (slope = -1.50). She improved to 23.1 correct responses per minute and demonstrated an upward trend (slope = 0.78) during the intervention phase.

Pretest scores on the NSB show that although Dylan and Daniella performed at different levels on the total NSB, they both had areas of weakness in the skills that are most closely aligned with the Numicon intervention. Dylan had very weak performance in number recognition and was unable to identify any of the two-digit numbers presented in the NSB assessment. Daniella had weaknesses in number knowledge (i.e., number relationships) and number recognition. These three skills were targeted by the Numicon intervention and were necessary for successful completion of the quantity discrimination task. Therefore, these students' positive responses may have been the result of the intervention that targeted specific areas of weakness within their differing overall number sense achievement levels.

Nonresponders. Three categories of nonresponders were evident in the quantity discrimination data: students who had stable performance across phases of the study, students who displayed wide variability in performance across the phases of the study, and students who experienced decreases in performance during the intervention phase. Adam, who was in the at-risk achievement category, provides an example of the first nonresponsive pattern. Charlie, also in the at-risk achievement category, provides an example of the second nonresponsive pattern. Aurora, a student in the classroom math difficulties achievement category, provides an example of the third nonresponsive pattern.

Although Adam's total NSB score before the intervention indicated that he was at risk for math failure, he demonstrated strengths in the three NSB domains most closely associated with the Numicon intervention and the quantity discrimination measure. In fact, no student from any achievement category had a higher score in those domains. Adam's lack of response to the intervention as measured by the quantity discrimination measure likely indicates that his number sense difficulties were not associated with the skills most closely aligned with completion of that task. His difficulties were more likely associated with applications of those number sense skills to calculations-related tasks. Therefore, large gains on the quantity discrimination measure would not be expected.

Charlie's data present a stark contrast to Adam's, although both students were in the same achievement category. Charlie had the lowest total score on the NSB before the intervention with weaknesses across all of the domains. Charlie also had language and reading difficulties and was absent for four of the 10 intervention sessions. The variability in Charlie's data likely indicates that the quantity discrimination assessment was not in his zone of proximal development. Charlie was not able to consistently recognize written numerals less than 10 and did not associate written numerals with quantities. Some of his difficulties seemed to be aligned with Berch's (2005) suggestion that some students, especially those with combined reading and math difficulties, struggle with the associations between number words, written numbers, and quantities. Charlie required instruction focused on unitary counting and number recognition skills beyond the level of intensity provided in this intervention.

Although Aurora's pretest performance on the NSB placed her in the achievement category representing the lowest level of mathematics difficulty, she had weaknesses in number recognition and number knowledge skills targeted by the Numicon intervention. Aurora made initial gains in the baseline phase with some variability. However she experienced a steep decrease in performance over the intervention phase with additional variability related to her mood. Aurora's lack of responsiveness on the quantity discrimination measure seems to be associated with personality, another factor identified by Van Luit and Schopman (2000). Anecdotal records show that Aurora was initially excited about participating in the intervention group and activities. However, she became disenchanted. In her interview, Aurora described the intervention activities as "boring" and said she wanted to play, not work. It is possible that Aurora's lack of responsiveness as measured by the quantity discrimination measures reflected her disinterest in the activities.

Quantity–numeral correspondence. Quantity–numeral correspondence skills were also evaluated as a primary dependent variable for this study. This measure was selected based on evidence that counting principles and efficient counting strategies are critical to the development of later arithmetic skills (Dowker, 2008b; Gersten et al., 2011). Data from fluency probes of quantity–numeral were evaluated using visual analysis with guidelines established by Kratochwill et al. (2010).

The group data from the quantity–numeral correspondence measure did not provide sufficient evidence to conclude that there was an association between the Numicon intervention and student performance on the quantity–numeral correspondence

measures. This finding was primarily due to an upward trend established during the baseline phase by group one and the absence of immediate level changes for groups one and three. Additionally, PND for the group data was 55%, indicating questionable effectiveness according to the guidelines established by Scruggs and Mastropieri (1998).

The lack of baseline stability for group one was a critical factor that reduced the chance of identifying a functional relation between quantity–numeral correspondence and supplemental instruction with Numicon. The lack of stability occurred because the decision to move groups into the intervention phase of the study was based on the quantity discrimination data, which were more closely correlated to long-term mathematics achievement. Small gains were evident in the group means over the course of the study, but these could not be attributed to the intervention. The students maintained performance on the quantity-discrimination measure after the intervention ceased.

Although there was not sufficient evidence to document a functional relation between the Numicon intervention and student performance on the quantity–numeral correspondence measure, the group data revealed a pattern of improved performance after the seventh intervention lesson. That lesson introduced the concept of “knowing how many without counting.” Students were taught to use knowledge of number patterns and relationships visible in the Numicon shapes to quickly evaluate quantities. An increase in the slope of the data is evident for all three groups after this lesson, suggesting that greater gains might have been evident if data collection had continued for a longer period of time.

Patterns of student responsiveness on the quantity–numeral correspondence

measure. Individual student data from the quantity–numeral measure were evaluated to determine if patterns of performance were evident among students representing different degrees of mathematics difficulty. The data revealed that nonresponsiveness was evident in students representing all of the levels of math difficulty. However, only students who were described as having classroom difficulties made gains on the quantity–numeral correspondence measure.

Responders. Seth and Steven both demonstrated responsiveness to intervention on the quantity–numeral correspondence measure. They were both classified as students with classroom math difficulties based on NSB performance. Their baseline patterns differed, but they responded to the intervention in a similar fashion.

Seth exhibited high variability in his baseline performance, though the trend was marked by a nearly horizontal slope (-0.01). His mean performance for the baseline phase was 8.43 correct responses per minute. Seth experienced a small level change at the onset of intervention and demonstrated increased stability in his intervention data. An increasing trend in his performance became evident after the seventh intervention lesson, which introduced the idea of using pattern and imagery to know how many without counting. His intervention mean was 10.98 correct responses per minute. Notably, Seth increased his performance to 13 correct responses per minute on the maintenance assessment. Anecdotal records indicate that he recognized that the dot configurations on the quantity–numeral correspondence measure were similar to the Numicon Shapes.

Steven's baseline data differed from Seth's in that he experienced a consistent downward trend in the baseline phase (slope = -0.32). His mean performance on the quantity-discrimination measure was 7.5 correct responses per minute during baseline. Steven experienced an immediate change in level upon entry into intervention. He made small gains over the first five intervention sessions, and then increased his rate of growth after the seventh intervention lesson. His intervention mean was 9.11 correct responses per minute. The phase was marked by an upward trend (slope = 0.30). Like Seth, Steven's performance on the maintenance assessment was higher than any score achieved during the intervention phase.

Both Seth's and Steven's achievements seem to be associated with the development and application of efficient counting strategies, adding support to the existing evidence that these strategies are critical for students with math difficulties (Bryant et al., 2000; Bryant, Bryant, Gersten, Scammacca, Funk, et al., 2008; Gersten et al., 2005). In the early phases of the intervention anecdotal notes indicate that both students applied a consistent unitary counting method, resulting in improvement over baseline performance. In the probes given after the seventh intervention lesson, both students began to use the pattern-based quantity recognition skills (i.e., subitizing). This evolution resulted in fluency gains on the task. Both students also made comments indicating that they were using this strategy during the follow-up interviews.

Nonresponders. Two types of nonresponders were evident among students from all three achievement groups. These included students who did not establish a consistent pattern of responding and students whose performances decreased after intervention.

The data for Adam, Dylan, and Daniella provide examples of inconsistent patterns of responding in both the baseline and intervention phases. The variability in performance may be related to the well-documented counting difficulties that students with math difficulties experience (Bryant et al., 2000; Bryant, Bryant, Gersten, Scammacca, Funk, et al., 2008; Gersten et al., 2005). The students were familiar with and employed unitary counting techniques at the onset of the study. Unitary counting had been emphasized in their classroom instruction since the start of school. It is likely that the students would have continued to use that strategy regardless of its accuracy or efficiency until they became comfortable with a new strategy. As previously mentioned, the use of pattern-based quantity recognition techniques were not introduced until the seventh intervention lesson. Adam and Daniella had some of their strongest scores after that lesson. This suggests that additional time and/or instruction with the strategy may have resulted in gains for these students.

Lacey's data also demonstrated general instability; however, her behavioral pattern differed from those discussed above. Lacey was particularly susceptible to distraction and her performance on the fluency measures may have been impacted by environmental factors. This was clearly evident during the quantity discrimination probe administered on session 14, which was "Pizza Chef Day" for the kindergarten students. Lacey was visibly distracted and performed more than two standard deviations below her average performance for that phase. Her performance on the quantity-numeral correspondence measure on session 14 also reflected a decrease in performance, though

not as large. Lacey's variability suggests that some students may have difficulty with fluency measures due to attention issues.

Finally, Aurora and Charlie both experienced decreases in performance associated with the intervention phase. As previously discussed, Aurora's performance seemed to be associated with her mood. She expressed that the intervention activities and assessments were "work" when she wanted to play. Aurora began to evidence an increasing trend in performance on the quantity-discrimination intervention probes when she began comparing her performance to that of her peers. In her interview, she described being motivated to be the first student to complete the entire quantity-discrimination fluency probe.

As noted in the discussion on quantity discrimination performance, a variety of factors may have contributed to Charlie's lack of responsiveness to the intervention. The quantity-numeral correspondence measure was a more developmentally appropriate task in that it focused on counting and number recognition, the skills Charlie most needed to develop. However, Charlie continued to struggle with counting quantities up to three and matching those quantities to written numerals. Clearly, he needed additional intensive instruction counting unitary objects before making the transition to the use of structured representations as described by Tournaki et al. (2008).

General number sense skills. The NSB (Jordan et al., 2008) was used as a measure of general number sense skill. Data were collected just before the baseline phase of the study and after intervention was completed. Although a causal relationship could not be fully established, the results warranted discussion because the measure was

previously demonstrated to have strong test–retest reliability over periods ranging from two to eight months apart, with higher stability for shorter intervals (Jordan, Glutting, Ramineni, & Watkins, 2010). The test-retest reliability findings indicated that student performance on the NSB was fairly stable across time under typical instructional conditions (i.e., no supplemental instruction or intervention). The current study involved two administrations of the NSB that were approximately six weeks apart. This time period minimizes the likelihood of either maturation or a practice effect contributing to changes in performance. Therefore, the identified changes can most reasonably be attributed to some combination of classroom instruction and the supplemental intervention, which is perfectly in line with the goals of Tier 2 supplemental interventions.

Comparisons of the pre- and postintervention scores revealed large positive effects (Cohen, 1992). The mean effect size across groups was 1.02. The strength of this effect is particularly interesting when the finding is compared to findings in other studies with much longer intervention times. Students in this study received 200 minutes of supplemental mathematics instruction. The resulting effect size was much larger than that found by Bryant et al. (2011) when they evaluated the effect of a multielement tutoring program that involved 2,640 minutes of intervention time. First graders in that study gained .5 standard deviations on a broad measure of number sense skill. Conversely, the findings of the current study were smaller than those found by Jordan et al. (2012) in their evaluation of a multielement supplemental intervention for kindergarteners. However, the students in that study received 720 minutes of

supplemental instruction between administrations of the NSB. The mean effect size for that intervention was 1.8. These data suggest that supplemental instruction with Numicon resulted in large gains with a relatively small amount of intervention time.

In addition to improvement on total NSB scores, group data were analyzed to identify gains in the domains that represented near- and far-transfer domains for the Numicon intervention. Counting knowledge, number recognition, and number knowledge were classified as near-transfer domains, as they were closely aligned with the Numicon instruction. Nonverbal calculations, story problems, and number combinations were classified as far-transfer domains. All three groups made greater gains on the near-transfer domains than on the far-transfer domains. The greatest mean gains were evident in the number recognition domain. This finding was in contrast to the results described by Fuchs et al. (2007) in a study that evaluated growth rates of low-performing first graders on a variety of curriculum-based measures of number sense. Fuchs et al. found minimal growth on measures of number identification and counting. They attributed the finding to a ceiling effect. The findings from the current study provide evidence that kindergarteners still have considerable room for growth in basic counting and number identification skills. The findings also demonstrate that supplemental intervention can be useful in building those skills to prepare kindergarten students who have math difficulties for first grade.

Patterns of student responding on the measure of general number sense skills.

Effect sizes were calculated for individual students using the standard deviation established for students in either the at-risk or not at-risk categories on the NSB in the

longitudinal study conducted by Jordan, Glutting, Ramineni, and Watkins (2010).

According to Cohen (1992), an effect size of .20 to .49 represents a small effect, .50 to .79 represents a medium effect, and a finding greater than .80 represents a large effect.

Five students, representing each of the three achievement categories, demonstrated large effects ranging from 1.15 to 2.15. One student, from the classroom math difficulties category, had a medium effect of .69. Three students, representing the at-risk and classroom math difficulties categories, had small effects ranging from .23 to .46.

Responders. Large effect sizes were evident in students from each of the achievement levels. However, Adam and Daniella, who were in the at-risk classification, made the largest gains in performance. Adam's pre- to postintervention performance gains reflected an effect size of 2.15. He was relatively strong in the near-transfer domains on the first administration of the NSB, but he also made gains in each of those domains following intervention. Additionally, Adam made gains in each of the far-transfer domains. Daniella demonstrated weaknesses in two of the near-transfer domains (number recognition and number knowledge) at pretest. She made gains in both of those areas, as well as all of the far-transfer domains. The total gains in her pre- to posttest performance reflected an effect size of 1.85. The postintervention NSB scores indicated that neither of these students performed in the at-risk or borderline at-risk classifications after intervention. The results for these students provide evidence that supplemental intervention can reduce student risk for mathematics difficulties.

Aurora had the third highest individual effect size on the NSB. This finding was surprising given that her fluency results suggested a negative effect of intervention.

Aurora's gains reflected an effect size of 1.84, suggesting that she did derive benefit from the supplemental instruction. The contrast between the fluency results and the NSB results may be explained by the nature of the measures. As mentioned previously, Aurora did not seem to like anything that she perceived as work. The fluency measures, paper and pencil tasks administered daily during the intervention, likely felt like work. The NSB, however, was administered in an individualized testing session. Standard administration called for the use of a puppet and presentation of the tasks in a game-like format. It is possible that Aurora enjoyed the NSB more and therefore put more effort into completing the tasks to the best of her ability.

Nonresponders. Charlie's NSB data provided additional evidence that the supplemental instruction with Numicon was not sufficient to address the factors that placed him at risk for math failure. His difficulties with the concepts were compounded by frequent absences from the intervention sessions. He made minimal improvement between the first and second administrations of the NSB. The effect size was calculated to be 0.31. Charlie provides evidence of the need for more intensive and individualized instruction, as is often provided in Tier 3 of RtI models.

The NSB data did not provide evidence of a positive effect of intervention for Dylan, though clear gains were evident in his quantity discrimination performance. Dylan had strong performance on two of the three near-transfer domains of the NSB before the intervention. He had little room for gain on the counting knowledge and number knowledge domains, which were emphasized in the Numicon intervention. He did experience gains in the number recognition domain, which was a notable weakness

for him at pretest and a skill that was emphasized in the Numicon intervention. The supplemental intervention may have improved Dylan's ability to recognize written numerals, which in turn resulted in a positive effect on his performance on the quantity discrimination measure. If this is the case, the number line activities that were incorporated into the supplemental instruction may have been more beneficial to Dylan than the structured representations.

Social validity. Social validity was formally evaluated through student interviews and teacher questionnaires completed at the end of the study. Both measures were intended to examine student and teacher perceptions of how much the students learned and how they enjoyed the intervention. The two qualitative data sources added to the fluency and NSB data indicating that most of the participants derived some benefit from the supplemental instruction. All of the students were able to describe and model something that they had learned as a result of the intervention. Many students demonstrated newly acquired knowledge of the relationships between numbers by using the Numicon Shapes. The teachers agreed that most of the students had improved their counting skills, number recognition skills, and understanding of quantity-numeral discrimination skills. Charlie and Lacey were the exceptions to this finding. Charlie's teacher was not sure if he had made gains in counting skills and disagreed with statements indicating that he had made improvements in his recognition of written numerals or understanding of the relationships between quantities and numerals. Lacey's teacher was not sure if she had made gains in counting skills and disagreed with the statement that she had made improvements in her ability to name written numerals.

Both sources of qualitative data also indicated that most of the students enjoyed the activities incorporated into the supplemental instruction. Most students reported that they enjoyed working with the Numicon Shapes and would like to have them in their regular classrooms. Aurora was the exception to this finding, though there were some aspects of the supplemental instruction that she did report enjoying.

All of the students provided evidence that the Numicon shapes helped them engage in conversations about number concepts. During the student interviews the Numicon Shapes were stored in baskets near the interview table, and were not presented to the students. A majority of the students asked to use the Numicon Shapes or independently retrieved the shapes from their baskets when responding to the questions in the interview. The students used the shapes to model and describe a variety of number patterns and relationships between numbers.

Social validity of the supplemental instruction with Numicon was further supported by events that occurred after the research study was completed. The kindergarten teachers ordered Numicon materials to be incorporated into classroom activities. The school principal also began developing plans to use Numicon as one element of Tier 2 intervention for students across grade levels who experience difficulties with number sense.

Educational Implications

The findings of this study have implications for assessment and intervention practices related to young children with mathematics difficulties. This study, in combination with previous intervention studies, provides support for the use of

multielement supplemental instruction to improve number sense skills for kindergarten students with early signs of math difficulties. This particular supplemental intervention paired explicit and systematic instruction, counting and number line activities, and opportunities for verbalization with structured representations of number concepts. The intervention was effective in eliciting large and significant gains on a general number sense measure in a relatively short amount of time, as evidenced by a gain of more than one standard deviation in just 200 minutes of instruction. This result is particularly meaningful to school-based applications where teachers and administrators are challenged to identify intervention time that does not take away from core instruction.

The results of this study also provided evidence of the need for multitiered systems of intervention. The students in this study had varying degrees of difficulty with mathematics. Many made number sense gains associated with the intervention. In fact, three of the four students who were described as being at risk or borderline at risk based on their preintervention NSB performance, were no longer in any risk category after intervention. However, one student showed little to no evidence of improvement on any of the number sense measures. That student had a variety of interacting risk factors (e.g., age, reading/language difficulties, frequent absences) that inhibited progress. He needed more intensive and individualized instruction than his peers.

This study also provided evidence of the need for efficient number sense screenings to facilitate the early identification of students with math difficulties and identify the specific number sense skills that require remediation. The need for teacher-friendly screening tools was made particularly evident by a teacher who commented that

she did not have a reliable way of measuring student progress that would allow her to knowledgeably respond to all of the questions on the social validity questionnaire. The teachers in this study clearly had a sense that some students were struggling more than their peers, but did not have a reliable way of identifying those students or evaluating their needs. Curriculum-based measures such as the quantity discrimination probe used in this study may provide an efficient tool for class-wide screening, while the NSB can provide more detailed information about the specific skills deficits and provide guidance for instruction. Use of these measures can be implemented with brief teacher training.

Furthermore, effective use of number sense screenings can improve the efficiency and effectiveness of supplemental intervention. The quantity discrimination and quantity–numeral correspondence measures can be administered to an entire class in less than five minutes. Results from the measures can be used to identify students who are not performing at levels commensurate with their peers. Follow-up assessment using the NSB can provide more detailed information regarding the specific number sense skills that are challenging to those individual students. That information can be used to develop skill-specific intervention groups with targeted Numicon activities.

As demonstrated by this study, Numicon intervention can be implemented successfully under typical school conditions. The program is appropriate for core and supplemental instruction in the primary grades. It can also be used to provide intensive mathematics intervention to students with disabilities. Teachers can gain acquire a general understanding of how to use the program and materials with one or two days of

training. Detailed knowledge of the program can be acquired with three to five days of training.

Overall, this study adds to the emerging body of intervention research for young students with math difficulties. It provides evidence that number sense difficulties can be identified through easy-to-use screening measures. It also demonstrates the effectiveness of supplemental instruction for remediating those difficulties.

Limitations

This study had several limitations. External validity is a primary limitation associated with the use of single-case designs (Kennedy, 2005). The nine kindergarteners who participated in this study had a variety of demographic traits. However, they by no means represent the broad array of variables that contribute to academic success or difficulty. Additional replication is needed to more accurately identify the characteristics of students who will respond to this intervention.

This study was also limited by the absence of socioeconomic status (SES) data. SES is one of the most widely recognized factors contributing to early mathematics difficulties. Anecdotally, some students in this study were known to be from low socioeconomic backgrounds. However, the absence of SES data for all students prevented an examination of the effectiveness of this intervention for those students who are known to be at increased risk of math failure.

Another limitation of this study was inherent in the design, which emphasized the value of quantity discrimination data over quantity–numeral correspondence data. This design element resulted in unstable baseline data for one group, thereby reducing the

chances of identifying a functional relation between the intervention and quantity–numeral correspondence. The decision to prioritize the quantity discrimination data over the quantity–numeral correspondence data was based on evidence that quantity discrimination measures are more closely correlated to long-term outcomes. Therefore, quantity discrimination was identified as the more critical variable, overall. However, the quantity–numeral correspondence measure was more suited to students with the lowest levels of number sense skill (e.g., Charlie) and should be considered a primary variable for students who are just learning to count objects and identify written numerals.

A fourth limitation involved the number of probes in the baseline and maintenance phases. Group one entered the intervention phase of the study after three baseline probes of quantity discrimination were administered and found to be stable. Kratochwill et al. (2010) suggest that 5 baseline points provide greater evidence of baseline stability. However, Horner et al. (2005) suggested that 3 were satisfactory. Five baseline data points would have provided additional evidence of stability in the quantity discrimination data and may have resulted in stabilization of the quantity–numeral correspondence data for group one. However, given the time limitations of this study, 3 stable baseline points were determined to be sufficient to move the first group into the intervention phase. The remaining groups had at least five baseline probes.

This study also included only one maintenance probe, administered one week after intervention ceased. Additional maintenance probes over a longer period of time would have provided greater evidence that the acquired skills had been maintained. However, time constraints also limited the feasibility of multiple maintenance probes.

When considering this study as an evaluation of Numicon, it is important to note that the students received a very small sampling of the activities and strategies available within the Numicon program. In fact, the full *Numicon Intervention Programme* (Atkinson et al., 2011) is conducted over a 12 week period and addresses calculation skills in addition to building number sense. Therefore, the results of this study must be considered as evidence of the effectiveness of the specific Numicon activities used and not as a measure of the effectiveness of Numicon instruction overall.

Finally, the Numicon intervention used in this study included multiple instructional elements associated with effective instruction and structured representation of number concepts. It is not clear which of these variables or combinations of variables contributed to the gains that were achieved. The study only allows conclusions to be drawn regarding the intervention as a package.

Recommendations for Future Research

Future research can be used to address the limitations of the present study. Additional replication as a single-case design with kindergarten participants from varied backgrounds would extend the generalizability of this study. When replicating the study, it would also be beneficial to extend the duration to allow an increased number of baseline and maintenance probes.

Alternatively, a group experimental or quasiexperimental study would contribute to the external validity of the intervention program and allow more intervention or maintenance sessions over a shorter period of time. The use of a group design would also allow the introduction of a third condition to evaluate the specific role of the structured

manipulatives in improving number sense skill. The study conducted by Tournaki et al. (2008) provides an example of an effective design that delineates these instructional variables.

The data from the quantity–numeral correspondence measures indicated that further exploration of the role of Numicon instruction in improving this skill would be warranted. The measure is more appropriate than the quantity discrimination task for students who are just learning to count and recognize written numerals. Therefore, it would be beneficial to conduct research using the quantity–numeral correspondence measure as the primary dependent variable when working with very young students or those with the most significant number sense deficits.

Additionally, the group data from the quantity–numeral correspondence measure suggested that students who were already familiar with unitary counting began to make fluency gains after they were taught to use knowledge of number patterns and relationships to quickly evaluate quantities. It would be useful to extend the current study to determine if additional Numicon instruction results in the development of more accurate and efficient counting strategies for students with math difficulties.

Conclusions

The present study is the first known evaluation of Numicon instruction using a systematic research design. The study found evidence of gains in quantity discrimination skills for kindergarten students with mathematics difficulties. There was moderate evidence that the gains were associated with the supplemental instruction using Numicon. Additionally, the data revealed that the students maintained those gains for one week in

the absence of intervention. Small gains were also evident in student performance on quantity–numeral correspondence measures. However, there was no evidence that these gains were associated with the Numicon intervention.

The study found large and gains in general number sense skills as measured by pre- and postintervention performance on the NSB. The gains were important in that most of the students classified as being at risk or borderline at risk for math failure on the first administration of the NSB were no longer in either risk category after intervention. Additionally, the students collectively improved their performance on the NSB by just over one standard deviation. The presence of this degree of gain with only 200 minutes of intervention is very promising.

In addition to the academic gains that were evident, most of the students enjoyed the intervention and were able to describe or model specific skills that they learned as a result of their participation in the program. Teachers also observed that most students made gains in number sense skills that were evident in the regular classroom. In fact, the teachers had such a positive perception of the intervention that they purchased Numicon materials for classroom use after the study ended.

The findings from this study contribute to the emerging knowledge of effective practices for number sense intervention with kindergarten students. The results add to the existing research in supporting the use of supplemental instruction to remediate mathematics difficulties. The results also provide additional evidence that multielement instructional packages including explicit and systematic instruction, concrete to abstract

instructional sequences, number line experiences, and verbalization of mathematical concepts do benefit students with mathematics difficulties.

Appendix A. Numicon Lesson Plans

Numicon Lesson Plan—Intervention Session 1

Timing	Objectives and Activities	Resources and Materials
5 min.	Quantity Discrimination —1 min. fluency Quantity Numeral Correspondence —1 min. fluency	Form A
1 min.	Ready to Learn Behaviors —“Quiet mouth, Still body, Listening ears, Eyes on the teacher”	- Picture cues of behaviors - White board & marker
4 min.	Counting & Number Line Activity: Kit 1 NNS 1A—Circle counting activities (Bullet Points 1,2,4, & 5) Objective: The student will count forward 1–10, backward from 10, and identify numerals 1–10 on a number line. Key Vocabulary: forward, backward What am I looking and listening for? Students are able to count 1–10. Students can identify the next number in the sequence when starting from a number other than 1. Students can count backward from 10 using a number line as a reference. Students can identify any numeral between 1 and 10 on a number line.	Class Touch Math Number Line or 1 Number Track for each student
8 min.	Number Patterns & Relationships Activity: CtG 2B, 3A (Activity 1 from each) Objective: The student will identify the smaller or larger Numicon Shape from a field of two to four using touch. Key Vocabulary: Numicon Shape, big, bigger, biggest, large, larger, largest, small, smaller, smallest	- Numicon Shapes 1–10 - Feely bag

2 min.	What am I looking and listening for? Students are able to compare and describe two Numicon Shapes using terms of relative size. For example, “The orange shape is smaller than the blue shape.” Students can use their sense of touch to compare shapes.	
	Closure/Wrap Up: Ask each student to hold up two Numicon Shapes, tell how many holes are in each shape, and state which shape has more holes.	

Lesson plan format adapted from Atkinson et al., 2011

Numicon Lesson Plan—Intervention Session 2

Timing	Objectives and Activities	Resources and Materials
5 min.	Quantity Discrimination —1 min. fluency Quantity Numeral Correspondence —1 min. fluency	Form B
1 min.	Ready to Learn Behaviors —“Quiet mouth, Still body, Listening ears, Eyes on the teacher”	- Picture cues of behaviors - White board & marker
4 min.	Counting & Number Line Activity: Kit 1 NNS 1A—Circle counting activities (Bullet Points 1,2,4, & 5) Objective: The student will count forward 1–20, backward from 10, and identify numerals 1–20 on a number line. Key Vocabulary: forward, backward What am I looking and listening for? Students are able to count 1–20. Students can identify the next number in the sequence when starting from a number other than 1. Students can count backward from 10 using a number line as a reference. Students can identify any numeral between 1 and 20 on a number line.	Class Touch Math Number Line or 1 Number Track for each student
8 min.	Number Patterns & Relationships Activity: CtG 2B, 3A, 3B (Activity 1 from each) Objective: The student will identify the smaller or larger Numicon Shape from a field of two to four using touch. Key Vocabulary: Numicon Shape, big, bigger, biggest, large, larger, largest, small, smaller, smallest What am I looking and listening for? Students are able to compare and describe two Numicon Shapes using terms of relative size. For example, “The orange shape is smaller than the blue shape.” Students can use their sense of touch to compare shapes.	- Numicon Shapes 1–10 - Feely bag
2 min.	Closure/Wrap Up: Ask each student to hold up two Numicon Shapes, tell how many holes are in each shape, and state which shape has more holes.	

Lesson plan format adapted from Atkinson et al., 2011

Numicon Lesson Plan—Intervention Session 3

Timing	Objectives and Activities	Resources and Materials
5 min.	Quantity Discrimination —1 min. fluency Quantity Numeral Correspondence —1 min. fluency	Form C
1 min.	Ready to Learn Behaviors —“Quiet mouth, Still body, Listening ears, Eyes on the teacher”	- Picture cues of behaviors - White board & marker
4 min.	Counting & Number Line Activity: Kit 1 NNS 1A—Circle counting activities (Bullet Points 1,2,4, & 5) Objective: The student will count forward 1–20, backward 1–10, and identify numerals 1–20 on a number line. Key Vocabulary: forward, backward What am I looking and listening for? Students are able to count 1–20. Students can identify the next number in the sequence when starting from a number other than 1. Students can count backward from 10 using a number line as a reference. Students can identify any numeral between 1 and 20 on a number line.	Class Touch Math Number Line or a Number Track for each student
8 min.	Number Patterns & Relationships Activity: CtG 4A—Activity 1 and 4B—Activity 1 (Swaps) Objective: The student will put Numicon Shapes (1–10) in order according to size and identify incorrectly ordered shapes based on pattern. Key Vocabulary: first, next, before, after, last, in between, pattern What am I looking and listening for? Students are able to order the shapes by comparing relative size. Students recognize that there is a <i>growing pattern</i> as shapes progress from left to right; each shape is <i>one more</i> or one bigger than the previous shape. (Students often describe this as a stair step pattern.)	Numicon Shapes 1–10 for each student

2 min.	Students can use relative size or recognition of the growing pattern to identify shapes that are in the wrong location.	
	Closure/Wrap Up: Ask students to explain how they can tell if the Numicon Shapes are in the wrong place in the number line.	

Lesson plan format adapted from Atkinson et al., 2011

Numicon Lesson Plan—Intervention Session 4

Timing	Objectives and Activities	Resources and Materials
5 min.	Quantity Discrimination —1 min. fluency Quantity Numeral Correspondence —1 min. fluency	Form A
1 min.	Ready to Learn Behaviors —“Quiet mouth, Still body, Listening ears, Eyes on the teacher”	- Picture cues of behaviors - White board & marker
5 min.	Counting & Number Line Activity: Kit 1 NNS 1A—Circle Counting Activity 1 Objective: The student will rote count forward to 30, backward from 10 using a number line, and count up to 30 objects. Key Vocabulary: forward, backward What am I looking and listening for? Students are able to count 1–30 and backward from 10 using a number line as a reference. Students can identify the next number in the sequence when starting from a number other than 1. Students understand that the last number they say when counting objects tells how many objects are in the group (cardinality). Students can locate numbers 1–30 on a number line.	Number line
7 min.	Number Patterns & Relationships Activity: CtG 6A, Activities 1 and 2 Objective: The student will assign number names to the Numicon Shapes. Key Vocabulary: how many What am I looking and listening for? Students are able to order the shapes by comparing relative size. Students recognize that there is a <i>growing pattern</i> as shapes progress from left to right; each shape is <i>one more</i> or one bigger than the previous shape. (Students often describe this as a stair step pattern.) Students recognize that each shape is assigned a number name based on the number of holes it has &	- Numicon Shapes 1–10 for each student 10 Numicon Pegs for each student - Numicon Number Line

2 min.	objects it can hold.	
	Closure/Wrap Up: Ask each student to select their favorite shape, put pegs into the holes, count the number of pegs, and identify the shape by its number name.	

Lesson plan format adapted from Atkinson et al., 2011

Numicon Lesson Plan—Intervention Session 5

Timing	Objectives and Activities	Resources and Materials
5 min.	Quantity Discrimination —1 min. fluency Quantity Numeral Correspondence —1 min. fluency	Form B
1 min.	Ready to Learn Behaviors —“Quiet mouth, Still body, Listening ears, Eyes on the teacher”	- Picture cues of behaviors - White board & marker
5 min.	Counting & Number Line Activity: Kit 1 NNS 1A—Circle Counting Activity Objective: The student will rote count forward to 30 and backward from 10 using a number line. Key Vocabulary: forward, backward What am I looking and listening for? Students are able to count 1–30 and backward from 10 using a number line as a reference. Students can identify the next number in the sequence when starting from a number other than 1. Students understand that the last number they say when counting objects tells how many objects are in the group (cardinality). Students can locate numbers 1–30 on a number line.	Number line
7 min.	Number Patterns & Relationships Activity: CtG 6A, Activities 1 and 2 Objective: The student will assign number names to the Numicon Shapes. Key Vocabulary: how many What am I looking and listening for? Students are able to order the shapes by comparing relative size. Students recognize that there is a <i>growing pattern</i> as shapes progress from left to right; each shape is <i>one more</i> or one bigger than the previous shape. (Students often describe this as a stair step pattern.) Students recognize that each shape is assigned a number name based on the number of holes it has &	- Numicon Shapes 1–10 for each student 10 Numicon Pegs for each student - Numicon Number Line

2 min.	objects it can hold.	
	Closure/Wrap Up: Ask each student to select their favorite shape, put pegs into the holes, count the number of pegs, and identify the shape by its number name.	

Lesson plan format adapted from Atkinson et al., 2011

Numicon Lesson Plan—Intervention Session 6

Timing	Objectives and Activities	Resources and Materials
5 min.	Quantity Discrimination —1 min. fluency Quantity Numeral Correspondence —1 min. fluency	Form C
1 min.	Ready to Learn Behaviors —“Quiet mouth, Still body, Listening ears, Eyes on the teacher”	- Picture cues of behaviors - White board & marker
5 min.	Counting & Number Line Activity: Kit 1 NNS 1A—Circle Counting Activity Objective: The student will rote count forward to 30 and backward from 10 using a number line. Key Vocabulary: forward, backward What am I looking and listening for? Students are able to count 1—30 and backward from 10 using a number line as a reference. Students are pronouncing the teen numbers correctly (-teen, not -ty). Students can identify the next number in the sequence when starting from a number other than 1. Students understand that the last number they say when counting objects tells how many objects are in the group (cardinality). Students can locate numbers 1–30 on a number line.	Number line
7 min.	Number Patterns & Relationships Activity: CtG 7A—Activity 1 Objective: The student will match numerals to the Numicon Shapes and order the shapes using size or pattern. Key Vocabulary: order, match, smaller, bigger, more, less, next What am I looking and listening for? Students are able to order the shapes by comparing relative size. Students recognize that there is a <i>growing pattern</i> as shapes progress from left to right; each shape is <i>one more</i> or one bigger than the previous shape.	- Numicon Shapes 1–10 for each student - Numerical Cards 0–10 for each student (Photocopy Master 11)

	(Students often describe this as a stair step pattern.) Students are able to identify Numicon Shapes that are not in the correct location using pattern. Students recognize that each shape is assigned a number name based on the number of holes it has and objects it can hold. Students can match the numerals to the Numicon Shapes.	
2 min.	Closure/Wrap Up: “Speed Round”—Draw two numeral cards from the pile, ask students to quickly point to the Numicon Shape that matches the numeral and state the name. Ask students which shape is bigger/more.	

Lesson plan format adapted from Atkinson et al., 2011

Numicon Lesson Plan—Intervention Session 7

Timing	Objectives and Activities	Resources and Materials
5 min.	Quantity Discrimination —1 min. fluency Quantity Numeral Correspondence —1 min. fluency	Form A
1 min.	Ready to Learn Behaviors —“Quiet mouth, Still body, Listening ears, Eyes on the teacher”	- Picture cues of behaviors - White board & marker
5 min.	Counting & Number Line Activity: Kit 1 NNS 1A—Circle Counting Activity Objective: The student will rote count forward to 30 and backward from 10 using a number line. Key Vocabulary: forward, backward What am I looking and listening for? Students are able to count 1–30 and backward from 10 using a number line as a reference. Students are pronouncing the teen numbers correctly (-teen, not -ty). Students can identify the next number in the sequence when starting from a number other than 1. Students understand that the last number they say when counting objects tells how many objects are in the group (cardinality). Students can locate numbers 1–30 on a number line.	Number line
7 min.	Number Patterns & Relationships Activity: CtG 7A—Activity 2 Swaps Objective: The student will match numerals to the Numicon Shapes and order the shapes using size or pattern. Key Vocabulary: order, match, smaller, bigger, more, less, next What am I looking and listening for? Students are able to order the shapes by comparing relative size. Students recognize that there is a <i>growing pattern</i> as shapes progress from left to right; each shape is <i>one more</i> or one bigger than the previous shape.	- Numicon Shapes 1–10 for each student - Numerical Cards 0–10 for each student (Photocopy Master 11)

	(Students often describe this as a stair step pattern.) Students are able to identify Numicon Shapes that are not in the correct location using pattern. Students recognize that each shape is assigned a number name based on the number of holes it has and objects it can hold. Students can match the numerals to the Numicon Shapes.	
2 min.	Closure/Wrap Up: “Speed Round”—Draw two numeral cards from the pile, ask students to quickly point to the Numicon Shape that matches the numeral and state the name. Ask students which shape is bigger/more.	

Lesson plan format adapted from Atkinson et al., 2011

Numicon Lesson Plan—Intervention Session 8

Timing	Objectives and Activities	Resources and Materials
5 min.	Quantity Discrimination —1 min. fluency Quantity Numeral Correspondence —1 min. fluency	Form B
1 min.	Ready to Learn Behaviors —“Quiet mouth, Still body, Listening ears, Eyes on the teacher”	- Picture cues of behaviors - White board & marker
4 min.	Counting & Number Line Activity: Kit 1 NNS 1A—Whole Class Counting; Activities for Counting and Numeral Recognition Objective: The student will count forward to 50, backward from 10 using a number line, and count up to 50 objects. Key Vocabulary: forward, backward What am I looking and listening for? Students are able to count 1–50 and backward from 10 using a number line as a reference. Students are pronouncing the teen numbers correctly (-teen, not -ty). Students can identify the next number in the sequence when starting from a number other than 1. Students are transitioning between the decades correctly. Students can locate numbers 1–50 on a number line.	- Number line - Numeral Cards 0–30
8 min.	Number Patterns & Relationships Activity: CtG 7B—Activity 3 Feely Bag Objective: The student will match Numicon Shapes to number names and written numerals without counting. Key Vocabulary: match, bigger, smaller, flat shape (even number), bump shape (odd number), mental picture. What am I looking and listening for? Students are able to describe the relative size of a shape using touch. Students can determine if a shape is a bump shape	- Numicon Shapes 1–10 for each student - Additional set of Numicon Shapes 1–10 to be placed in the Feely Bag (Start with 3 to 5 shapes in the bag at a time) - Numeral Cards 0–10 for each student (Photocopy Master 11)

2 min.	(odd number) or flat shape (even number) by touch. Students can create a mental picture of the shape that they are feeling in the Feely Bag and match that shape to a model and numeral. Students refer to the Numicon Shapes by their number names.	
	Closure/Wrap Up: Ask students to select two “bump shapes,” say the number name, and identify which number is bigger/more. Repeat with two “flat shapes.”	

Lesson plan format adapted from Atkinson et al., 2011

Numicon Lesson Plan—Intervention Session 9

Timing	Objectives and Activities	Resources and Materials
5 min.	Quantity Discrimination —1 min. fluency Quantity Numeral Correspondence —1 min. fluency	Form C
1 min.	Ready to Learn Behaviors —“Quiet mouth, Still body, Listening ears, Eyes on the teacher”	- Picture cues of behaviors - White board & marker
5 min.	Counting & Number Line Activity: Kit 1 NNS 1A—Whole Class Counting; Activities for Counting and Numeral Recognition Objective: The student will count forward to 50, backward from 10 using a number line, and count up to 50 objects. Key Vocabulary: forward, backward What am I looking and listening for? Students are able to count 1–50 and backward from 10 using a number line as a reference. Students are pronouncing the teen numbers correctly (-teen, not -ty). Students can identify the next number in the sequence when starting from a number other than 1. Students are transitioning between the decades correctly. Students can locate numbers 1–50 on a number line.	- Number line - Numeral Cards 0–50
7 min.	Number Patterns & Relationships Activity: CtG 7B—Activity 3 Feely Bag Objective: The student will match Numicon Shapes to number names and written numerals without counting. Key Vocabulary: match, bigger, smaller, flat shape (even number), bump shape (odd number), mental picture. What am I looking and listening for? Students are able to describe the relative size of a shape using touch.	- Numicon Shapes 1–10 for each student - Additional set of Numicon Shapes 1–10 to be placed in the Feely Bag (Start with 3 to 5 shapes in the bag at a time) - Numeral Cards 0–10 for each student (Photocopy Master 11)

2 min.	Students can determine if a shape is a bump shape (odd number) or flat shape (even number) by touch. Students can create a mental picture of the shape that they are feeling in the Feely Bag and match that shape to a model and numeral. Students refer to the Numicon Shapes by their number names.	
	Closure/Wrap Up: Ask students to select two “bump shapes,” say the number name, and identify which number is bigger/more. Repeat with two “flat shapes.”	

Lesson plan format adapted from Atkinson et al., 2011

Numicon Lesson Plan—Intervention Session 10

Timing	Objectives and Activities	Resources and Materials
5 min.	Quantity Discrimination —1 min. fluency Quantity Numeral Correspondence —1 min. fluency	Form A
1 min.	Ready to Learn Behaviors —“Quiet mouth, Still body, Listening ears, Eyes on the teacher”	- Picture cues of behaviors - White board & marker
5 min.	Counting & Number Line Activity: Kit 1 NNS 1A—Whole Class Counting; Activities for Counting and Numeral Recognition Objective: The student will count forward to 50 and backward from 10 using a number line. Key Vocabulary: forward, backward What am I looking and listening for? Students are able to count 1–50 and backward from 10 using a number line as a reference. Students are pronouncing the teen numbers correctly (-teen, not -ty). Students can identify the next number in the sequence when starting from a number other than 1. Students are transitioning between the decades correctly. Students can locate numbers 1–50 on a number line.	- Number line - Numeral Cards 0–50
7 min.	Number Patterns & Relationships Activity: Kit 1 NNS 2A—Activity 1 Objective: The student will build, order, and identify Numicon Shapes representing numbers up to 19. Key Vocabulary: teen numbers, ten/10-shape What am I looking and listening for? Students refer to the Numicon Shapes and combinations by their number names. Students recognize that the pattern of one more continues beyond 10. Students see that each teen number is made of a 10	- Numicon Shapes 1–10 for each student - Extra 10-shapes

2 min.	and some more (a unit)	
	Closure/Wrap Up: Teacher shows the Numicon Shapes for 9, 10, and 11. Ask each student to build and name the next number in the sequence. (One student builds 12, the next 13, etc.)	

Lesson plan format adapted from Atkinson et al., 2011

Appendix B. Numicon Fidelity Checklists

Appendix B

Numicon Fidelity Checklist—Intervention Session 1

Directions: Put a check mark in the box to indicate that the target behavior was completed.

Underlined vocabulary must be used to count as completed behavior.

Ready to Learn

- ☐ Teacher reviews ready to learn behaviors before beginning lesson.

Number Sense Activities

- ☐ Teacher explains the **Counting objective** in student-friendly language.
Objective: The student will count forward 1–10, backward from 10, and identify numerals 1–10 on a number line.
- ☐ Teacher **leads group in counting** 1 to 10 while pointing to the numerals on a number line.
- ☐ Teacher engages the class in **circle counting to 10**, with each child saying the next number in the sequence.
- ☐ Teacher **leads group in counting** backward from 10 while pointing to the numerals on a number line.
- ☐ Teacher engages the class in **circle counting backward from 10** using a number line, with each child saying the next number in the sequence.
- ☐ Teacher explains the **Patterns & Relationships objective** in student-friendly language.
Objective: The student will identify the smaller or larger (bigger) Numicon Shape from a field of two to four using touch.
- ☐ Teacher **displays the shapes 1 and 10** and asks students which shape is bigger.
- ☐ Teacher **models** finding the bigger or smaller Numicon Shape in the Feely Bag using 1 and 10.
- ☐ Teacher provides **guided practice** finding the bigger or smaller Numicon Shape in the Feely Bag. 4 and 6; 4 and 3; 4, 5, and 6
- ☐ Counting and Patterns & Relationships Activities were completed in approximately 15 minutes.

Closure/Wrap Up

- ☐ Teacher engages students in a **closure activity** by asking each student to hold up two Numicon Shapes, tell how many holes are in each shape, and state which shape has more holes.
- ☐ Teacher ensures that the following key ideas are mentioned in closure.
 - ☐ Each shape has a different number of holes.
 - ☐ The bigger shapes have more holes.
 - ☐ The smaller shapes have fewer holes.

Teacher: _____

Date: _____

Observer: _____

Time: _____

Number of Boxes Checked: ____ / ____

Fidelity Score: _____

Observer Comments:

Numicon Fidelity Checklist—Intervention Session 2

Directions: Put a check mark in the box to indicate that the target behavior was completed.

Underlined vocabulary must be used to count as completed behavior.

Ready to Learn

- ☐ Teacher reviews ready to learn behaviors before beginning lesson.

Number Sense Activities

- ☐ Teacher explains the **Counting objective** in student-friendly language.
Objective: The student will count forward 1–20, backward from 10, and identify numerals 1–20 on a number line.
- ☐ Teacher **leads group in counting** 1 to 20 while pointing to the numerals on a number line.
- ☐ Teacher engages the class in **circle counting to 20**, with each child saying the next number in the sequence.
- ☐ Teacher **leads group in counting** backward from 10 while pointing to the numerals on a number line.
- ☐ Teacher engages the class in **circle counting backward from 10** using a number line, with each child saying the next number in the sequence.
- ☐ Teacher explains the **Patterns & Relationships objective** in student-friendly language.
Objective: The student will identify the smaller or larger (bigger) Numicon Shape from a field of two to four using touch.
- ☐ Teacher **displays the shapes 9 and 3** and asks students which shape is bigger.
- ☐ Teacher **models** finding the bigger or smaller Numicon Shape in the Feely Bag using 9 and 3.
- ☐ Teacher provides **guided practice** finding the biggest or smallest Numicon Shape in the Feely Bag with three or four shapes in the bag.
4, 5, and 6; 3, 5, 7, and 9
- ☐ Counting and Patterns & Relationships Activities were completed in approximately 15 minutes.

Closure/Wrap Up

- ☐ Teacher engages students in a **closure activity** by asking each student to hold up two Numicon Shapes, tell how many holes are in each shape, and state which shape has more holes.
- ☐ Teacher ensures that the following key ideas are mentioned in closure.
 - ☐ Each shape has a different number of holes.
 - ☐ The bigger shapes have more holes.
 - ☐ The smaller shapes have fewer holes.

Teacher: _____

Date: _____

Observer: _____

Time: _____

Number of Boxes Checked: ____ / ____

Fidelity Score: _____

Observer Comments:

Numicon Fidelity Checklist—Session 3

Directions: Put a check mark in the box to indicate that the target behavior was completed.

Underlined vocabulary must be used to count as completed behavior.

Ready to Learn

- ☐ Teacher reviews ready to learn behaviors before beginning lesson.

Number Sense Activities

- ☐ Teacher explains the **Counting objective** in student-friendly language.
Objective: The student will count forward 1–20, backward from 10, and identify numerals 1–20 on a number line.
- ☐ Teacher **leads group in counting** 1 to 20 while pointing to the numerals on a number line.
- ☐ Teacher engages the class in **circle counting to 20**, with each child saying the next number in the sequence.
- ☐ Teacher **leads group in counting** backward from 10 while pointing to the numerals on a number line.
- ☐ Teacher engages the class in **circle counting backward from 10** using a number line, with each child saying the next number in the sequence.
- ☐ Teacher explains the **Patterns & Relationships objective** in student-friendly language.
Objective: The student will put Numicon Shapes (1–10) in order according to size and identify incorrectly ordered shapes based on pattern.
- ☐ Teacher **models putting the Numicon Shapes in order** starting with the 1-shape and using the one more pattern to identify which shape comes next.
- ☐ Teacher provides opportunity for **guided practice ordering the shapes from 1–10**.
- ☐ Teacher **models** playing the Swaps game including a description of taking a mental picture of the ordered Numicon Shapes and using the one more/stair step pattern to determine which shapes have been swapped.
- ☐ Teacher provides **guided practice** playing the Swaps game.
- ☐ Counting and Patterns & Relationships Activities were completed in approximately 15 minutes.

Closure/Wrap Up

- ☐ Teacher asks students to explain how they can tell if the Numicon Shapes are in the wrong place in the number line.
- ☐ Teacher ensures that the following key ideas are mentioned in closure.
 - ☐ The shapes get larger as you move from left to right.
 - ☐ You can see the stair step pattern.
 - ☐ Every shape has one more hole than the shape before it.

Teacher: _____

Date: _____

Observer: _____

Time: _____

Number of Boxes Checked: ____ / ____

Fidelity Score: _____

Observer Comments:

Numicon Fidelity Checklist—Session 4

Directions: Put a check mark in the box to indicate that the target behavior was completed.
Underlined vocabulary must be used to count as completed behavior.

Number Sense Activities

- ☐ Teacher explains the **Counting objective** in student-friendly language.
Objective: The student will count forward 1–30, backward from 10, and identify numerals 1–30 on a number line.
- ☐ Teacher **models** counting groups of objects by placing them along a number line.
- ☐ Teacher provides **guided practice** counting groups of up to 30 objects using a number line.
- ☐ Teacher **removes all but 10 items** from the number line and leads the group in **counting backward from 10**.
- ☐ Teacher explains the **Patterns & Relationships objective** in student-friendly language.
Objective: The student will assign number names to the Numicon Shapes.
- ☐ Teacher provides each student with a set of Numicon Shapes 1–10 and assists them in ordering the shapes then identifying them using number names.
- ☐ Teacher **models** placing pegs in the holes of one of the Numicon Shapes, counts the pegs, and explains that the shape represents the number because it can hold that many pegs.
- ☐ Teacher has each **student select a Numicon shape, fill the holes with pegs and determine the number that the shape represents**.
- ☐ Teacher asks students to **find the number on the number line**.
- ☐ Counting and Patterns & Relationships Activities were completed in approximately 15 minutes.

Closure/Wrap Up

- ☐ Teacher asks each student to select their favorite shape, put pegs in the holes, and identify the shape by its number name.
- ☐ Teacher ensures that the following key ideas are mentioned in closure.
 - ☐ Each Numicon Shape stands for (represents) a number.
 - ☐ The number of holes in the shape tells us the number that the shape represents.

Teacher: _____

Date: _____

Observer: _____

Time: _____

Number of Boxes Checked: ____ / ____

Fidelity Score: _____

Observer Comments: _____

Numicon Fidelity Checklist—Session 5

Directions: Put a check mark in the box to indicate that the target behavior was completed.

Underlined vocabulary must be used to count as completed behavior.

Ready to Learn

- ☐ Teacher reviews ready to learn behaviors before beginning lesson.

Number Sense Activities

- ☐ Teacher explains the **Counting objective** in student-friendly language.
Objective: The student will count forward 1–30, backward from 10, and identify numerals 1–30 on a number line.
- ☐ Teacher **leads group in counting** 1 to 30 while pointing to the numerals on a number line.
- ☐ Teacher engages the class in **circle counting to 30**, with each child saying the next number in the sequence.
- ☐ Teacher **leads group in counting backward from 10** while pointing to the numerals on a number line.
- ☐ Teacher engages the class in **circle counting backward from 10** using a number line, with each child saying the next number in the sequence.
- ☐ Teacher explains the **Patterns & Relationships objective** in student-friendly language.
Objective: The student will match numerals to the Numicon Shapes and order the shapes using size or pattern.
- ☐ Teacher provides **guided practice** ordering the shapes from 1–10.
- ☐ Teacher points out the one more and bump shape/flat shape (even and odd) patterns and encourages students to make a mental picture of what the Numicon Shapes look like when they are ordered correctly.
- ☐ Teacher **reminds students that they can determine the number** represented by a shape by placing pegs in the holes and counting the pegs.
- ☐ Teacher **models using pattern** to determine the number represented by a Numicon shape.
- ☐ Teacher has each **student select a Numicon shape** and determine its number by **filling the holes with pegs or using pattern**.
- ☐ Teacher asks students to **find the number on the number line**.
- ☐ Counting and Patterns & Relationships Activities were completed in approximately 15 minutes.
- ☐ Counting and Patterns & Relationships Activities were completed in approximately 15 minutes.

Closure/Wrap Up

- ☐ Teacher engages students in a **closure activity** by quickly naming a number and asking students to touch the Numicon shape that represents that number.
- ☐ Teacher ensures that the following key ideas are mentioned in closure.
 - ☐ Each Numicon Shape stands for (represents) a number.
 - ☐ We can determine the number by counting the holes or using the number patterns.

Teacher: _____

Date: _____

Observer: _____

Time: _____

Number of Boxes Checked: ____ / ____

Fidelity Score: _____

Observer Comments:

Numicon Fidelity Checklist—Session 6

Directions: Put a check mark in the box to indicate that the target behavior was completed.

Underlined vocabulary must be used to count as completed behavior.

Ready to Learn

- ☐ Teacher reviews ready to learn behaviors before beginning lesson.

Number Sense Activities

- ☐ Teacher explains the **Counting objective** in student-friendly language.
Objective: The student will count forward 1–30, backward from 10, and identify numerals 1–30 on a number line.
- ☐ Teacher **leads group in counting** 1 to 30 while pointing to the numerals on a number line.
- ☐ Teacher engages the class in **circle counting to 30**, with each child saying the next number in the sequence.
- ☐ Teacher **leads group in counting backward from 10** while pointing to the numerals on a number line.
- ☐ Teacher engages the class in **circle counting backward from 10** using a number line, with each child saying the next number in the sequence.
- ☐ Teacher explains the **Patterns & Relationships objective** in student-friendly language.
Objective: The student will match numerals to the Numicon Shapes and order the shapes using size or pattern.
- ☐ Teacher provides **guided practice** ordering the shapes from 1–10.
- ☐ Teacher points out the one more and bump shape/flat shape (even and odd) patterns and encourages students to make a mental picture of what the Numicon Shapes look like when they are ordered correctly.
- ☐ Teacher **guides students in naming written numerals 1–10** and reminds students that the Numicon shapes represent numbers.
- ☐ Teacher **models identifying a written numeral**, then using pattern or the number of holes in a Numicon shape to identify the shape that represents the numeral.
- ☐ Teacher **provides guided practice matching numerals to Numicon shapes**.
- ☐ Counting and Patterns & Relationships Activities were completed in approximately 15 minutes.

Closure/Wrap Up

- ☐ Teacher engages students in a **closure activity** by asking each student to select two numeral cards from the pile, select the Numicon shape that matches the numeral, and determine which shape is bigger/more.
- ☐ Teacher ensures that the following key ideas are mentioned in closure.
 - ☐ The Numicon shapes represent written numbers.
 - ☐ We can match written numbers to Numicon shapes.
 - ☐ We can find the bigger number and the bigger shape.

Teacher: _____

Date: _____

Observer: _____

Time: _____

Number of Boxes Checked: ____ / ____

Fidelity Score: _____

Observer Comments:

Numicon Fidelity Checklist—Session 7

Directions: Put a check mark in the box to indicate that the target behavior was completed.

Underlined vocabulary must be used to count as completed behavior.

Ready to Learn

- ☐ Teacher reviews ready to learn behaviors before beginning lesson.

Number Sense Activities

- ☐ Teacher explains the **Counting objective** in student-friendly language.
Objective: The student will count forward 1–30, backward from 10, and identify numerals 1–30 on a number line.
- ☐ Teacher engages the class in **circle counting to 30**, with each child saying the next number in the sequence.
- ☐ Teacher **leads group in counting backward from 10** while pointing to the numerals on a number line.
- ☐ Teacher engages the class in **circle counting backward from 10** using a number line, with each child saying the next number in the sequence.
- ☐ Teacher explains the **Patterns & Relationships objective** in student-friendly language.
Objective: The student will match numerals to the Numicon Shapes and order the shapes using size or pattern.
- ☐ Teacher provides **guidance as students** order the shapes from 1–10.
- ☐ Teacher points out the one more and bump shape/flat shape (even and odd) patterns and encourages students to make a mental picture of what the Numicon Shapes look like when they are ordered correctly.
- ☐ Teacher **guides students in naming written numerals 1–10** and reminds students that the Numicon shapes represent numbers.
- ☐ Teacher **provides guided practice matching numerals to Numicon shapes**.
- ☐ Teacher shows students how to play **SWAPS game with Numicon shapes and numerals**.

- ☐ Counting and Patterns & Relationships Activities were completed in approximately 15 minutes.

Closure/Wrap Up

- ☐ Teacher engages students in a **closure activity** by asking drawing two numeral cards from the pile, then asking the students to select the Numicon shapes that match the numerals, and determine which shape is bigger/more.
- ☐ Teacher ensures that the following key ideas are mentioned in closure.
 - ☐ The Numicon shapes represent written numbers.
 - ☐ We can match written numbers to Numicon shapes.
 - ☐ We can find the bigger number and the bigger shape.

Teacher: _____

Date: _____

Observer: _____

Time: _____

Number of Boxes Checked: ____ / ____

Fidelity Score: _____

Observer Comments:

Numicon Fidelity Checklist—Session 8

Directions: Put a check mark in the box to indicate that the target behavior was completed.

Underlined vocabulary must be used to count as completed behavior.

Ready to Learn

- ☐ Teacher reviews ready to learn behaviors before beginning lesson.

Number Sense Activities

- ☐ Teacher explains the **Counting objective** in student-friendly language.
Objective: The student will count forward 1–50, backward from 10, and identify numerals 1–50 on a number line.
- ☐ Teacher engages the class in **choral counting to 50**, with each child saying the next number in the sequence.
- ☐ Teacher engages the class in **circle counting to 50**, with each child saying the next number in the sequence.
- ☐ Teacher **leads group in counting backward from 10** while pointing to the numerals on a number line.
- ☐ Teacher explains the **Patterns & Relationships objective** in student-friendly language.
Objective: The student will match Numicon Shapes to number names and written numerals without counting.
- ☐ Teacher has all students **order their Numicon Shapes from 1–10**.
- ☐ Teacher **defines a mental picture** as something you can see in your mind even when it's not there and provides examples (i.e., the location of your bed in your bedroom, the color of your dog or cat).
- ☐ Teacher **explains** that **mental pictures** can be used to match Numicon Shapes to number names without counting.
- ☐ Teacher **models** placing shapes 1–4 in the Feely Bag, selecting a numeral to locate, and using touch to locate the shape that matches the numeral.
- ☐ Teacher **models drawing a number card from the pile describing a mental picture** of the Numicon Shape and finding it in the Feely Bag using touch. (Ex. “The 3-shape is going to be a bump shape it’s pretty small.”)
- ☐ Teacher provides **guided practice** as **students play the Feely Bag** game.
- ☐ Counting and Patterns & Relationships Activities were completed in approximately 15 minutes.

Closure/Wrap Up

- ☐ Teacher engages students in a **closure activity** by asking students to select two “bump shapes,” tell the number name, identify which is bigger/more, and identify a feature of the shape that can help make a mental picture.
- ☐ Teacher ensures that the following key ideas are mentioned in closure.
 - ☐ Numicon Shapes stand for numbers.
 - ☐ We can use patterns and mental pictures of the shapes to know how many without counting.

Teacher: _____

Date: _____

Observer: _____

Time: _____

Number of Boxes Checked: ____ / ____

Fidelity Score: _____

Observer Comments:

Numicon Fidelity Checklist—Session 9

Directions: Put a check mark in the box to indicate that the target behavior was completed.

Underlined vocabulary must be used to count as completed behavior.

Ready to Learn

- ☐ Teacher reviews ready to learn behaviors before beginning lesson.

Number Sense Activities

- ☐ Teacher explains the **Counting objective** in student-friendly language.
Objective: The student will count forward 1–50, backward from 10, and identify numerals 1–50 on a number line.
- ☐ Teacher engages the class in **choral counting to 50**, with each child saying the next number in the sequence.
- ☐ Teacher engages the class in **circle counting to 50**, with each child saying the next number in the sequence.
- ☐ Teacher **leads group in counting backward from 10** while pointing to the numerals on a number line.
- ☐ Teacher explains the **Patterns & Relationships objective** in student-friendly language.
Objective: The student will match Numicon Shapes to number names and written numerals without counting.
- ☐ Teacher has all students **order their Numicon Shapes from 1–10**.
- ☐ Teacher **defines a mental picture** as something you can see in your mind even when its' not there and provides examples (i.e., the location of your bed in your bedroom, the color of your dog or cat).
- ☐ Teacher **explains** that **mental pictures** can be used to match Numicon Shapes to number names without counting.
- ☐ Teacher **models** placing shapes 5–10 in the Feely Bag, selecting a numeral to locate, and using touch to locate the shape that matches the numeral.
- ☐ Teacher **models drawing a number card from the pile describing a mental picture** of the Numicon Shape and finding it in the Feely Bag using touch. (Ex. “The 3-shape is going to be a bump shape. It’s pretty small.)
- ☐ Teacher provides **guided practice** as **students play the Feely Bag** game.
- ☐ Counting and Patterns & Relationships Activities were completed in approximately 15 minutes.

Closure/Wrap Up

- ☐ Teacher engages students in a **closure activity** by asking students to select two “flat shapes,” tell the number name, identify which is bigger/more, and identify a feature of the shape that can help make a mental picture.
- ☐ Teacher ensures that the following key ideas are mentioned in closure.
 - ☐ Numicon Shapes stand for numbers.
 - ☐ We can use patterns and mental pictures of the shapes to know how many without counting.

Teacher: _____

Date: _____

Observer: _____

Time: _____

Number of Boxes Checked: ____ / ____

Fidelity Score: _____

Observer Comments:

Numicon Fidelity Checklist—Session 10

Directions: Put a check mark in the box to indicate that the target behavior was completed.
Underlined vocabulary must be used to count as completed behavior.

Ready to Learn

- ☐ Teacher reviews ready to learn behaviors before beginning lesson.

Number Sense Activities

- ☐ Teacher explains the **Counting objective** in student-friendly language.
Objective: The student will count forward 1–50, backward from 10, and identify numerals 1–50 on a number line.
- ☐ Teacher engages the class in **choral counting to 50**, with each child saying the next number in the sequence.
- ☐ Teacher engages the class in **circle counting to 50**, with each child saying the next number in the sequence.
- ☐ Teacher **leads group in counting backward from 10** while pointing to the numerals on a number line.
- ☐ Teacher explains the **Patterns & Relationships objective** in student-friendly language.
Objective: The student will build, order, and identify Numicon Shapes representing numbers up to 19.
- ☐ Teacher orders the Numicon Shapes from 1–10 commenting about the pattern in which each shape has one more hole (or is worth one more) than the shape before it.
- ☐ Teacher **asks students what number comes next** and how it might be made. Teacher emphasizes the one more pattern and makes eleven showing using a 10-shape and 1-shape. Teacher continues to ask how to build the remaining numbers up to 19.
- ☐ Teacher guides students in counting up to 19 while pointing at the Numicon Shapes.
- ☐ Counting and Patterns & Relationships Activities were completed in approximately 15 minutes.

Closure/Wrap Up

- ☐ Teacher engages students in a **closure activity** by showing the shapes for 9, 10, and 11 and asking each student to build the next numbers in the sequence. (One student builds 12, one builds 13, etc.)
- ☐ Teacher ensures that the following key ideas are mentioned in closure.
 - ☐ We can build numbers that are bigger than 10 by combining two Numicon shapes.
 - ☐ Each shape is one more than the shape before it.

Teacher: _____

Date: _____

Observer: _____

Time: _____

Number of Boxes Checked: ____ / ____

Fidelity Score: _____

Observer Comments:

Appendix C. Phonics Lesson Plans

Phonics Lesson Plan—Session 1

Timing	Objectives and Activities	Resources and Materials
1 min.	Ready to Learn Behaviors —“Quiet mouth, Still body, Listening ears, Eyes on the teacher”	• Picture cues of behaviors • White board & marker
5 min.	Flashcards Letter-Sound Identification: The student will verbally identify the sounds made by individual letters shown on flashcards using choral response. Error Correction: “This sound is _____. What sound?” Sight Word Recognition: The student will verbally identify kindergarten sight words presented on flashcards using choral response. Error Correction: “This word is _____. What word?”	• Alphabet flashcards • Sight Word Cards (a, and, like, me, the, see, my, with, is, to, go, you, she, he)
10–12 min.	Phonics/Phonemic Awareness Objective: The student will identify individual sounds within consonant-vowel (c-v) and consonant-vowel-consent (c-v-c) words to spell and read. Student Objective: Listen for the sounds /a/ and /e/ to write and read words. Complete the following activities for each of these words: bag, bed, bad, red, mad, am 1) Segmenting—“The first word is _____. What word?” Students respond chorally. “Yes, _____. Let’s count the sounds in the word _____.” Teacher models saying the words slowly while counting/tapping the sounds with fingers. “<i>How many sounds were in the word _____?</i>” 2) Spelling—“There were _____ sounds in the word _____, so I know it has at least _____ letters. I’m going to make a line on my dry erase board for each letter. Let’s figure out what letters are needed to spell this word. The first sound in _____ is _____. What letter makes that sound?”	• Word List • Dry erase boards, markers, & erasers • Alphabet desk-top or alphabet linking chart • Dice with words—big, small, corners, capitals

	Students respond chorally. <i>“Yes, the letter ____ says _____. That is the first letter in the word ____.”</i> Teacher writes the letter on the first line. <i>“What’s the next sound in the word _____.”</i> Teacher guides students through identifying all of the sounds and letters in the word and writes them on the dry erase board. <i>“What word did we write?”</i> Students respond chorally. <i>“Yes, _____. Let’s spell _____ together.”</i> Students chorally spell the word while looking at the model on the dry erase board. 3) Writing—<i>“Now it’s your turn to write the word on your dry erase board. Let’s roll the die to see how we should write this word.”</i> Roll the die. <i>“The die says to write the word (<u>big, small, in capitals, on the corners</u>). Write the word. Make sure you listen for the sounds and put down the letters you hear. When you are done writing your word, check my dry erase board to see if you spelled it just right.”</i> Monitor student spelling and provide prompts as needed. Repeat steps 1–3 with different words until there are about 2 minutes remaining in the session. Ask students to erase the words on their boards before introducing a new word. Teacher will keep all of the words on her dry erase board. Closure/Wrap Up: <i>“Before we leave, I’d like each one of you to tell me one word that you learned how to read and spell today. You can erase the word from my dry erase board before lining up.”</i>	
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Phonics Lesson Plan—Session 2

Timing	Objectives and Activities	Resources and Materials
1 min.	Ready to Learn Behaviors —“Quiet mouth, Still body, Listening ears, Eyes on the teacher”	• Picture cues of behaviors • White board & marker
5 min.	Flashcards Letter-Sound Identification: The student will verbally identify the sounds made by individual letters shown on flashcards using choral response. Error Correction: “This sound is _____. What sound?”	• Alphabet flashcards • Sight Word Cards (a, and, like, me, the, see, my, with, is, to, go, you, she, he)

	Sight Word Recognition: The student will verbally identify kindergarten sight words presented on flashcards using choral response. Error Correction: “This word is _____. What word?”	
10–12 min.	Phonics/Phonemic Awareness Objective: The student will identify individual sounds within consonant-vowel (c-v) and consonant-vowel-consent (c-v-c) words to spell and read. Student Objective: Listen for the sounds /a/ and /e/ to write and read words. Complete the following activities for each of these words: bag, bed, bad, red, mad, am 1) Segmenting—“The first word is _____. What word?” Students respond chorally. “Yes, _____. Let’s count the sounds in the word _____.” Teacher models saying the words slowly while counting/tapping the sounds with fingers. “<i>How many sounds were in the word _____?</i>” 2) Spelling—“There were ____ sounds in the word _____, so I know it has at least ____ letters. I’m going to make a line on my dry erase board for each letter. Let’s figure out what letters are needed to spell this word. The first sound in _____ is _____. What letter makes that sound?” Students respond chorally. “Yes, the letter _____ says _____. That is the first letter in the word _____.” Teacher writes the letter on the first line. “<i>What’s the next sound in the word _____.</i>” Teacher guides students through identifying all of the sounds and letters in the word and writes them on the dry erase board. “<i>What word did we write?</i>” Students respond chorally. “Yes, _____. Let’s spell _____ together.” Students chorally spell the word while looking at the model on the dry erase board. 3) Writing—“Now it’s your turn to write the word on your dry erase board. Let’s roll the die to see	• Word List • Dry erase boards, markers, & erasers • Alphabet desk-top or alphabet linking chart • Dice with words—big, small, corners, capitals

	<i>how we should write this word.”</i> Roll the die. <i>“The die says to write the word (<u>big, small, in capitals, on the corners</u>). Write the word. Make sure you listen for the sounds and put down the letters you hear. When you are done writing your word, check my dry erase board to see if you spelled it just right.”</i> Monitor student spelling and provide prompts as needed. Repeat steps 1–3 with different words until there are about 2 minutes remaining in the session. Ask students to erase the words on their boards before introducing a new word. Teacher will keep all of the words on her dry erase board. Closure/Wrap Up: <i>“Before we leave, I’d like each one of you to tell me one word that you learned how to read and spell today. You can erase the word from my dry erase board before lining up.”</i>	
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Phonics Lesson Plan—Session 3

Timing	Objectives and Activities	Resources and Materials
1 min.	Ready to Learn Behaviors —“Quiet mouth, Still body, Listening ears, Eyes on the teacher”	• Picture cues of behaviors • White board & marker
5 min.	Flashcards Letter-Sound Identification: The student will verbally identify the sounds made by individual letters shown on flashcards using choral response. Error Correction: “This sound is _____. What sound?” Sight Word Recognition: The student will verbally identify kindergarten sight words presented on flashcards using choral response. Error Correction: “This word is _____. What word?”	• Alphabet flashcards • Sight Word Cards (a, and, like, me, the, see, my, with, is, to, go, you, she, he)
10–12 min.	Phonics/Phonemic Awareness Objective: The student will identify individual sounds within consonant-vowel (c-v) and consonant-vowel-consent (c-v-c) words to spell and read. Student Objective: Listen for the sounds /i/ and /u/ to write and read words. Complete the following activities for each of these words: in, up, big, bug, did, dug 1) Segmenting—“The first word is _____. What word?” Students respond chorally. “Yes, _____. Let’s count the sounds in the word _____.” Teacher models saying the words slowly while counting/tapping the sounds with fingers. “<i>How many sounds were in the word _____?</i>” 2) Spelling—“There were ____ sounds in the word _____, so I know it has at least ____ letters. I’m going to make a line on my dry erase board for each letter. Let’s figure out what letters are needed to spell this word. The first sound in _____ is _____. What letter makes that sound?” Students respond chorally. “Yes, the letter _____ says _____. That is the first letter in the word _____.”	• Word List • Dry erase boards, markers, & erasers • Alphabet desk-top or alphabet linking chart • Dice with words—big, small, corners, capitals

	Teacher writes the letter on the first line. <i>“What’s the next sound in the word ____.”</i> Teacher guides students through identifying all of the sounds and letters in the word and writes them on the dry erase board. <i>“What word did we write?”</i> Students respond chorally. <i>“Yes, _____. Let’s spell _____ together.”</i> Students chorally spell the word while looking at the model on the dry erase board. 3) Writing—<i>“Now it’s your turn to write the word on your dry erase board. Let’s roll the die to see how we should write this word.”</i> Roll the die. <i>“The die says to write the word (<u>big, small, in capitals, on the corners</u>). Write the word. Make sure you listen for the sounds and put down the letters you hear. When you are done writing your word, check my dry erase board to see if you spelled it just right.”</i> Monitor student spelling and provide prompts as needed. Repeat steps 1–3 with different words until there are about 2 minutes remaining in the session. Ask students to erase the words on their boards before introducing a new word. Teacher will keep all of the words on her dry erase board. Closure/Wrap Up: <i>“Before we leave, I’d like each one of you to tell me one word that you learned how to read and spell today. You can erase the word from my dry erase board before lining up.”</i>	
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Phonics Lesson Plan—Session 4

Timing	Objectives and Activities	Resources and Materials
1 min.	Ready to Learn Behaviors —“Quiet mouth, Still body, Listening ears, Eyes on the teacher”	• Picture cues of behaviors • White board & marker
5 min.	Flashcards Letter-Sound Identification: The student will verbally identify the sounds made by individual letters shown on flashcards using choral response. Error Correction: “This sound is _____. What sound?” Sight Word Recognition: The student will verbally identify kindergarten sight words presented on flashcards using choral response. Error Correction: “This word is _____. What word?”	• Alphabet flashcards • Sight Word Cards (a, and, like, me, the, see, my, with, is, to, go, you, she, he)
10–12 min.	Phonics/Phonemic Awareness Objective: The student will identify individual sounds within consonant-vowel (c-v) and consonant-vowel-consent (c-v-c) words to spell and read. Student Objective: Listen for the sounds /i/ and /u/ to write and read words. Complete the following activities for each of these words: in, up, big, bug, did, dug 1) Segmenting—“The first word is _____. What word?” Students respond chorally. “Yes, _____. Let’s count the sounds in the word _____.” Teacher models saying the words slowly while counting/tapping the sounds with fingers. “<i>How many sounds were in the word _____?</i>” 2) Spelling—“There were ____ sounds in the word _____, so I know it has at least ____ letters. I’m going to make a line on my dry erase board for each letter. Let’s figure out what letters are needed to spell this word. The first sound in _____ is _____. What letter makes that sound?” Students respond chorally. “Yes, the letter _____ says _____. That is the first letter in the word _____.”	• Word List • Dry erase boards, markers, & erasers • Alphabet desk-top or alphabet linking chart • Dice with words—big, small, corners, capitals

	Teacher writes the letter on the first line. <i>“What’s the next sound in the word ____.”</i> Teacher guides students through identifying all of the sounds and letters in the word and writes them on the dry erase board. <i>“What word did we write?”</i> Students respond chorally. <i>“Yes, _____. Let’s spell _____ together.”</i> Students chorally spell the word while looking at the model on the dry erase board. 3) Writing—<i>“Now it’s your turn to write the word on your dry erase board. Let’s roll the die to see how we should write this word.”</i> Roll the die. <i>“The die says to write the word (<u>big, small, in capitals, on the corners</u>). Write the word. Make sure you listen for the sounds and put down the letters you hear. When you are done writing your word, check my dry erase board to see if you spelled it just right.”</i> Monitor student spelling and provide prompts as needed. Repeat steps 1–3 with different words until there are about 2 minutes remaining in the session. Ask students to erase the words on their boards before introducing a new word. Teacher will keep all of the words on her dry erase board. Closure/Wrap Up: <i>“Before we leave, I’d like each one of you to tell me one word that you learned how to read and spell today. You can erase the word from my dry erase board before lining up.”</i>	
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Phonics Lesson Plan—Session 5

Timing	Objectives and Activities	Resources and Materials
1 min.	Ready to Learn Behaviors —“Quiet mouth, Still body, Listening ears, Eyes on the teacher”	• Picture cues of behaviors • White board & marker
5 min.	Flashcards Letter-Sound Identification: The student will verbally identify the sounds made by individual letters shown on flashcards using choral response. Error Correction: “This sound is _____. What sound?” Sight Word Recognition: The student will verbally identify kindergarten sight words presented on flashcards using choral response. Error Correction: “This word is _____. What word?”	• Alphabet flashcards • Sight Word Cards (a, and, like, me, the, see, my, with, is, to, go, you, she, he)
10–12 min.	Phonics/Phonemic Awareness Objective: The student will identify individual sounds within consonant-vowel (c-v) and consonant-vowel-consent (c-v-c) words to spell and read. Student Objective: Listen for the sounds /o/ and /i/ to write and read words. Complete the following activities for each of these words: on, it, pop, sit, stop, wig 1) Segmenting—“The first word is _____. What word?” Students respond chorally. “Yes, _____. Let’s count the sounds in the word _____.” Teacher models saying the words slowly while counting/tapping the sounds with fingers. “How many sounds were in the word _____?” 2) Spelling—“There were ____ sounds in the word _____, so I know it has at least ____ letters. I’m going to make a line on my dry erase board for each letter. Let’s figure out what letters are needed to spell this word. The first sound in _____ is _____. What letter makes that sound?” Students respond chorally. “Yes, the letter _____ says _____. That is the first letter in the word _____.”	• Word List • Dry erase boards, markers, & erasers • Alphabet desk-top or alphabet linking chart • Dice with words—big, small, corners, capitals

	Teacher writes the letter on the first line. <i>“What’s the next sound in the word ____.”</i> Teacher guides students through identifying all of the sounds and letters in the word and writes them on the dry erase board. <i>“What word did we write?”</i> Students respond chorally. <i>“Yes, _____. Let’s spell _____ together.”</i> Students chorally spell the word while looking at the model on the dry erase board. 3) Writing—<i>“Now it’s your turn to write the word on your dry erase board. Let’s roll the die to see how we should write this word.”</i> Roll the die. <i>“The die says to write the word (<u>big, small, in capitals, on the corners</u>). Write the word. Make sure you listen for the sounds and put down the letters you hear. When you are done writing your word, check my dry erase board to see if you spelled it just right.”</i> Monitor student spelling and provide prompts as needed. Repeat steps 1–3 with different words until there are about 2 minutes remaining in the session. Ask students to erase the words on their boards before introducing a new word. Teacher will keep all of the words on her dry erase board. Closure/Wrap Up: <i>“Before we leave, I’d like each one of you to tell me one word that you learned how to read and spell today. You can erase the word from my dry erase board before lining up.”</i>	
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Phonics Lesson Plan—Session 6

Timing	Objectives and Activities	Resources and Materials
1 min.	Ready to Learn Behaviors —“Quiet mouth, Still body, Listening ears, Eyes on the teacher”	• Picture cues of behaviors • White board & marker
5 min.	Flashcards Letter-Sound Identification: The student will verbally identify the sounds made by individual letters shown on flashcards using choral response. Error Correction: “This sound is _____. What sound?” Sight Word Recognition: The student will verbally identify kindergarten sight words presented on flashcards using choral response. Error Correction: “This word is _____. What word?”	• Alphabet flashcards • Sight Word Cards (a, and, like, me, the, see, my, with, is, to, go, you, she, he)
10–12 min.	Phonics/Phonemic Awareness Objective: The student will identify individual sounds within consonant-vowel (c-v) and consonant-vowel-consent (c-v-c) words to spell and read. Student Objective: Listen for the sounds /o/ and /i/ to write and read words. Complete the following activities for each of these words: on, it, pop, sit, stop, wig 1) Segmenting—“The first word is _____. What word?” Students respond chorally. “Yes, _____. Let’s count the sounds in the word _____.” Teacher models saying the words slowly while counting/tapping the sounds with fingers. “How many sounds were in the word _____?” 2) Spelling—“There were ____ sounds in the word _____, so I know it has at least ____ letters. I’m going to make a line on my dry erase board for each letter. Let’s figure out what letters are needed to spell this word. The first sound in _____ is _____. What letter makes that sound?” Students respond chorally. “Yes, the letter _____ says _____. That is the first letter in the word _____.”	• Word List • Dry erase boards, markers, & erasers • Alphabet desk-top or alphabet linking chart • Dice with words—big, small, corners, capitals

	Teacher writes the letter on the first line. <i>“What’s the next sound in the word ____.”</i> Teacher guides students through identifying all of the sounds and letters in the word and writes them on the dry erase board. <i>“What word did we write?”</i> Students respond chorally. <i>“Yes, _____. Let’s spell _____ together.”</i> Students chorally spell the word while looking at the model on the dry erase board. 3) Writing—<i>“Now it’s your turn to write the word on your dry erase board. Let’s roll the die to see how we should write this word.”</i> Roll the die. <i>“The die says to write the word (<u>big, small, in capitals, on the corners</u>). Write the word. Make sure you listen for the sounds and put down the letters you hear. When you are done writing your word, check my dry erase board to see if you spelled it just right.”</i> Monitor student spelling and provide prompts as needed. Repeat steps 1–3 with different words until there are about 2 minutes remaining in the session. Ask students to erase the words on their boards before introducing a new word. Teacher will keep all of the words on her dry erase board. Closure/Wrap Up: <i>“Before we leave, I’d like each one of you to tell me one word that you learned how to read and spell today. You can erase the word from my dry erase board before lining up.”</i>	
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Phonics Lesson Plan—Session 7

Timing	Objectives and Activities	Resources and Materials
1 min.	Ready to Learn Behaviors —“Quiet mouth, Still body, Listening ears, Eyes on the teacher”	• Picture cues of behaviors • White board & marker
5 min.	Flashcards Letter-Sound Identification: The student will verbally identify the sounds made by individual letters shown on flashcards using choral response. Error Correction: “This sound is _____. What sound?” Sight Word Recognition: The student will verbally identify kindergarten sight words presented on flashcards using choral response. Error Correction: “This word is _____. What word?”	• Alphabet flashcards • Sight Word Cards (a, and, like, me, the, see, my, with, is, to, go, you, she, he)
10–12 min.	Phonics/Phonemic Awareness Objective: The student will identify individual sounds within consonant-vowel (c-v) and consonant-vowel-consent (c-v-c) words to spell and read. Student Objective: Listen for the sounds /a/ and /o/ to write and read words. Complete the following activities for each of these words: and, on, can, not, tap, top 1) Segmenting—“The first word is _____. What word?” Students respond chorally. “Yes, _____. Let’s count the sounds in the word _____.” Teacher models saying the words slowly while counting/tapping the sounds with fingers. “<i>How many sounds were in the word _____?</i>” 2) Spelling—“There were ____ sounds in the word _____, so I know it has at least ____ letters. I’m going to make a line on my dry erase board for each letter. Let’s figure out what letters are needed to spell this word. The first sound in _____ is _____. What letter makes that sound?” Students respond chorally. “Yes, the letter _____ says _____. That is the first letter in the word _____.”	• Word List • Dry erase boards, markers, & erasers • Alphabet desk-top or alphabet linking chart • Dice with words—big, small, corners, capitals

	Teacher writes the letter on the first line. <i>“What’s the next sound in the word ____.”</i> Teacher guides students through identifying all of the sounds and letters in the word and writes them on the dry erase board. <i>“What word did we write?”</i> Students respond chorally. <i>“Yes, _____. Let’s spell _____ together.”</i> Students chorally spell the word while looking at the model on the dry erase board. 3) Writing—<i>“Now it’s your turn to write the word on your dry erase board. Let’s roll the die to see how we should write this word.”</i> Roll the die. <i>“The die says to write the word (<u>big, small, in capitals, on the corners</u>). Write the word. Make sure you listen for the sounds and put down the letters you hear. When you are done writing your word, check my dry erase board to see if you spelled it just right.”</i> Monitor student spelling and provide prompts as needed. Repeat steps 1 – 3 with different words until there are about 2 minutes remaining in the session. Ask students to erase the words on their boards before introducing a new word. Teacher will keep all of the words on her dry erase board. Closure/Wrap Up: <i>“Before we leave, I’d like each one of you to tell me one word that you learned how to read and spell today. You can erase the word from my dry erase board before lining up.”</i>	
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Phonics Lesson Plan—Session 8

Timing	Objectives and Activities	Resources and Materials
1 min.	Ready to Learn Behaviors - “Quiet mouth, Still body, Listening ears, Eyes on the teacher”	• Picture cues of behaviors • White board & marker
5 min.	Flashcards Letter-Sound Identification: The student will verbally identify the sounds made by individual letters shown on flashcards using choral response. Error Correction: “This sound is _____. What sound?” Sight Word Recognition: The student will verbally identify kindergarten sight words presented on flashcards using choral response. Error Correction: “This word is _____. What word?”	• Alphabet flashcards • Sight Word Cards (a, and, like, me, the, see, my, with, is, to, go, you, she, he)
10–12 min.	Phonics/Phonemic Awareness Objective: The student will identify individual sounds within consonant-vowel (c-v) and consonant-vowel-consent (c-v-c) words to spell and read. Student Objective: Listen for the sounds /a/ and /o/ to write and read words. Complete the following activities for each of these words: and, on, can, not, tap, top 1) Segmenting—“The first word is _____. What word?” Students respond chorally. “Yes, _____. Let’s count the sounds in the word _____.” Teacher models saying the words slowly while counting/tapping the sounds with fingers. “<i>How many sounds were in the word _____?</i>” 2) Spelling—“There were ____ sounds in the word _____, so I know it has at least ____ letters. I’m going to make a line on my dry erase board for each letter. Let’s figure out what letters are needed to spell this word. The first sound in _____ is _____. What letter makes that sound?” Students respond chorally. “Yes, the letter _____ says _____. That is the first letter in the word _____.”	• Word List • Dry erase boards, markers, & erasers • alphabet desk-top or alphabet linking chart • Dice with words—big, small, corners, capitals

	Teacher writes the letter on the first line. <i>“What’s the next sound in the word ____.”</i> Teacher guides students through identifying all of the sounds and letters in the word and writes them on the dry erase board. <i>“What word did we write?”</i> Students respond chorally. <i>“Yes, _____. Let’s spell _____ together.”</i> Students chorally spell the word while looking at the model on the dry erase board. 3) Writing—<i>“Now it’s your turn to write the word on your dry erase board. Let’s roll the die to see how we should write this word.”</i> Roll the die. <i>“The die says to write the word (<u>big, small, in capitals, on the corners</u>). Write the word. Make sure you listen for the sounds and put down the letters you hear. When you are done writing your word, check my dry erase board to see if you spelled it just right.”</i> Monitor student spelling and provide prompts as needed. Repeat steps 1 – 3 with different words until there are about 2 minutes remaining in the session. Ask students to erase the words on their boards before introducing a new word. Teacher will keep all of the words on her dry erase board. Closure/Wrap Up: <i>“Before we leave, I’d like each one of you to tell me one word that you learned how to read and spell today. You can erase the word from my dry erase board before lining up.”</i>	
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Appendix D. The Number Sense Brief
(Jordan et al., 2008, p. 57)

Name: _____ Date: _____

(Write 1 for correct, 0 for incorrect)

Give the child a picture with 5 stars in a line. Say: *'Here are some stars. I want you to count each star. Touch each star as you count.'* When the child is finished counting ask, *"How many stars are on the paper?"*

- 1) Enumerated 5 _____
- 2) Indicated that there were 5 stars on the paper _____

Say: *"I want you to count as high as you can. But I bet you're a very good counter, so I'll stop you after you've counted high enough, OK?"*

Allow children to count up to 10. If they don't make any mistakes, record "10."
Record the highest number they counted up to without error.

- 3) Write in the last correct number spoken _____
- Child counted up to 10 without error _____

Show the child a line of 5 alternating blue and yellow dots printed on a paper. Say:
"Here are some yellow and blue dots. This is Dino [show a finger puppet] and he would like you to help him play a game. Dino is going to count the dots on the paper, but he is just learning how to count and sometimes he makes mistakes. Sometimes he counts in ways that are OK but sometimes he counts in ways that are not OK and that are wrong. It is your job to tell him after he finishes if it was OK to count the way he did or not OK. So, remember you have to tell him if he counts in a way that is OK or in a way that is not OK and wrong. Do you have any questions?"

Trial Type	Response		
4) Left to right	OK	Not OK	_____
5) Right to left	OK	Not OK	_____
6) Yellow then blue	OK	Not OK	_____
7) Double first	OK	Not OK	_____

For items 8 through 11, point to each number that is printed on a separate card and say:
"What number is this?"

- 8) 13 _____
- 9) 37 _____
- 10) 82 _____
- 11) 124 _____
- 12) What number comes right after 7? _____

- 13) What number comes two numbers after 7? _____
- 14) Which is bigger: 5 or 4? _____
- 15) Which is bigger: 7 or 9? _____
- 16) Which is smaller: 8 or 6? _____
- 17) Which is smaller: 5 or 7? _____
- 18) Which number is closer to 5: 6 or 2? _____

Say: *"We are going to play a game with these chips. Watch carefully."* Place two chips on your mat. *"See these, there are two chips."* Cover the chips and put out another chip. *"Here is one more chip."* Before the transformation say, *"Watch what I do. Now make yours just like mine or tell me how many chips are hiding under the box."* Add/remove chips one at a time. Items 19 to 22 are the nonverbal calculations.

- 19) $2 + 1$ _____ (as described above)
- 20) $4 + 3$ _____
- 21) $3 + 2$ _____
- 22) $3 - 1$ _____

Say: *"I'm going to read you some number questions and you can do anything you want to help you find the answer. Some questions might be easy for you and others might be hard. Don't worry if you don't get them all right. Listen carefully to the question before you answer."*

- 23) Jill has 2 pennies. Jim gives her 1 more penny. How many pennies does Jill have now? _____
- 24) Sally has 4 crayons. Stan gives her 3 more crayons. How many crayons does Sally have now? _____
- 25) Jose has 3 cookies. Sarah gives him 2 more cookies. How many cookies does Jose have now? _____
- 26) Kisha has 6 pennies. Peter takes away 4 of her pennies. How many pennies does Kisha have now? _____
- 27) Paul has 5 oranges. Maria takes away 2 of his oranges. How many oranges does Paul have now? _____
- 28) How much is 2 and 1? _____
- 29) How much is 3 and 2? _____
- 30) How much is 4 and 3? _____
- 31) How much is 2 and 4? _____
- 32) How much is 7 take away 3? _____
- 33) How much is 6 take away 4? _____

Appendix E. Quantity Discrimination Measures

Quantity Discrimination Form A

10	2	7	17	2	0	9	15
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4	9	7	12	9	11	19	16
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11	0	5	14	16	19	8	12
----	---	---	----	----	----	---	----

3	9	5	6	5	19	18	10
---	---	---	---	---	----	----	----

2	2	3	1	19	9	13	11
---	---	---	---	----	---	----	----

10	2	7	17	2	0	9	15
----	---	---	----	---	---	---	----

4	9	7	12	9	11	19	16
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11	0	5	14	16	19	8	12
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3	9	5	6	5	19	18	10
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2	2	3	1	19	9	13	11
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Quantity Discrimination Form B

7	17
---	----

10	2
----	---

9	15
---	----

2	0
---	---

7	12
---	----

4	2
---	---

19	16
----	----

9	11
---	----

5	14
---	----

11	0
----	---

8	12
---	----

16	19
----	----

5	6
---	---

3	9
---	---

18	10
----	----

5	19
---	----

3	1
---	---

20	2
----	---

13	11
----	----

19	9
----	---

2	10	10	11	0	18	3	17
15	20	7	14	19	16	16	15
0	5	17	9	0	19	15	8
9	15	18	3	0	7	14	18
8	11	11	20	19	3	6	9

Quantity Discrimination Form C

0	2
---	---

15	9
----	---

2	10
---	----

17	7
----	---

11	9
----	---

16	19
----	----

2	4
---	---

12	7
----	---

19	16
----	----

12	8
----	---

0	11
---	----

14	5
----	---

19	5
----	---

10	18
----	----

9	3
---	---

6	5
---	---

9	19
---	----

11	13
----	----

2	20
---	----

1	3
---	---

17	3	18	0	11	10	10	2
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15	16	16	19	14	7	20	15
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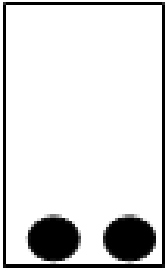
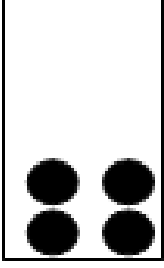
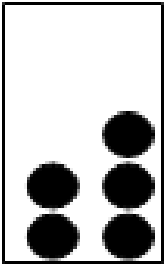
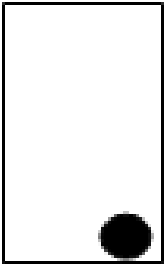
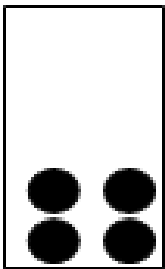
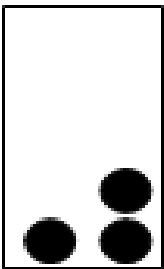
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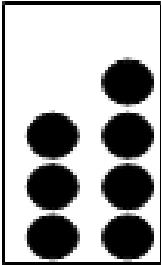
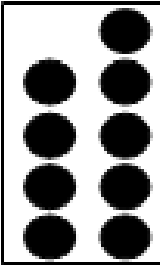
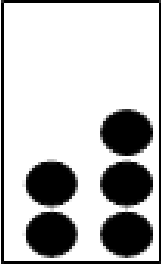
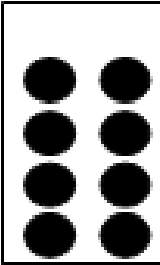
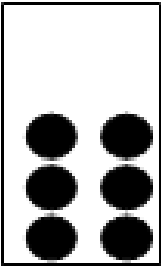
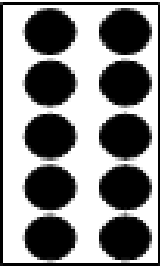
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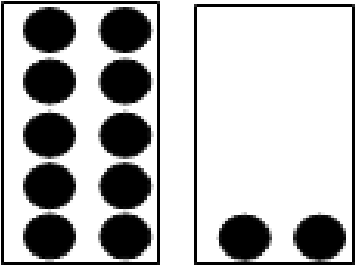
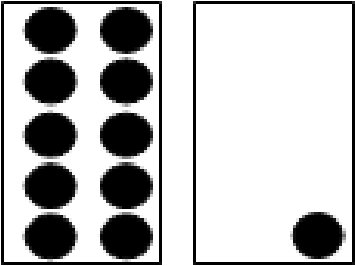
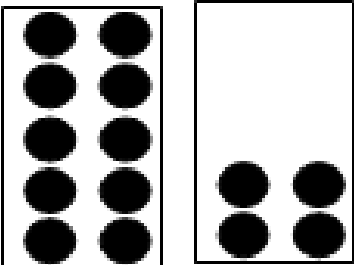
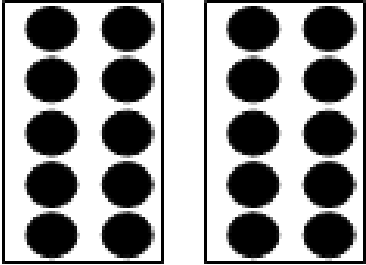
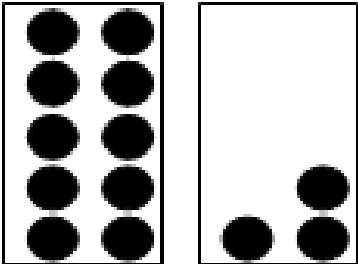
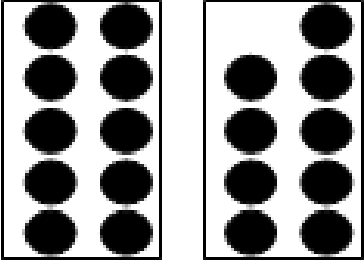
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Appendix F. Quantity–Numeral Correspondence Measures

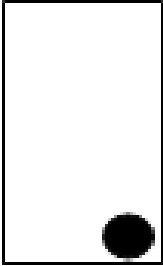
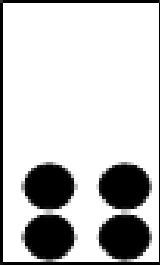
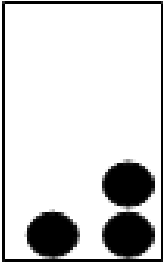
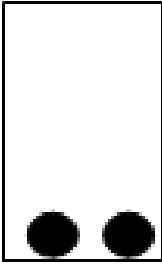
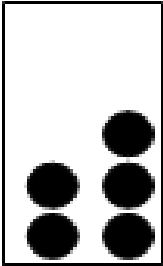
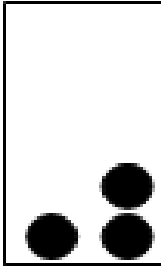
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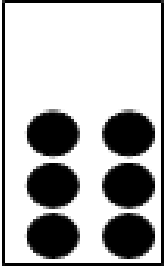
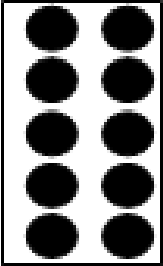
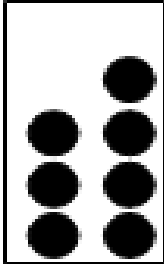
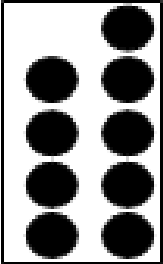
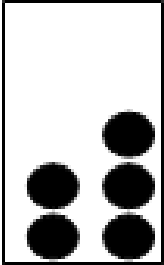
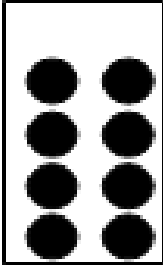
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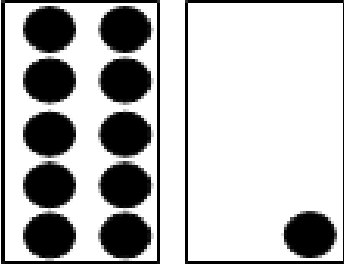
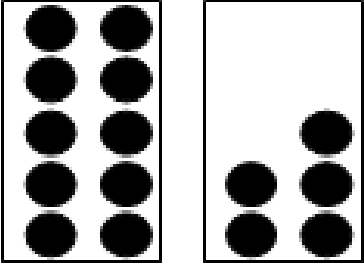
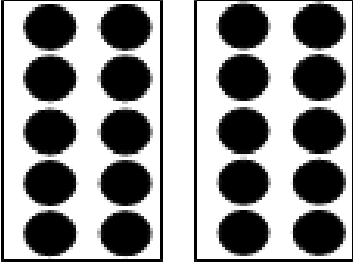
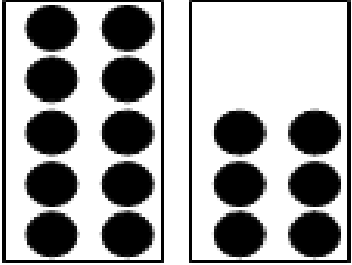
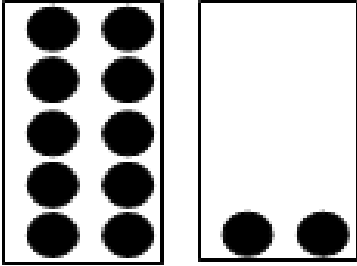
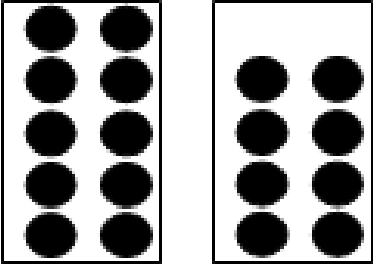
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Quantity–Numeral Correspondence Form B

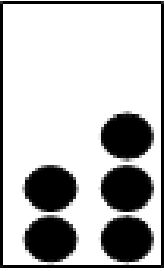
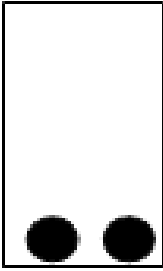
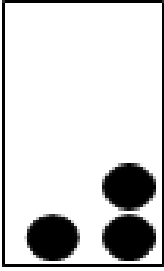
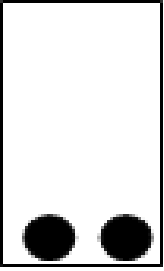
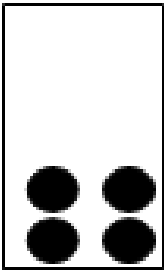
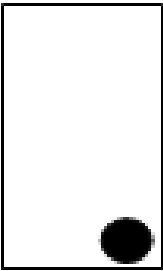
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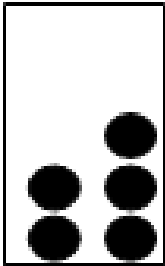
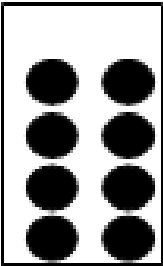
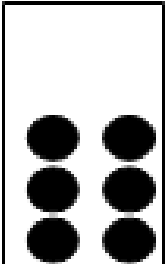
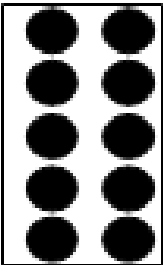
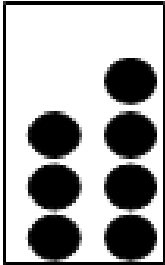
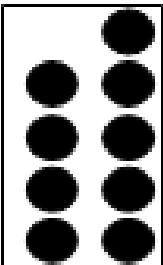
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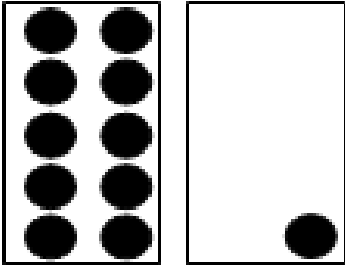
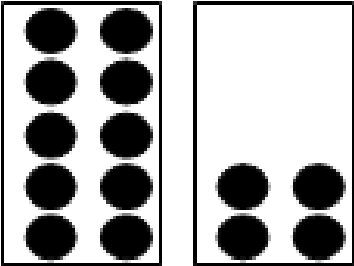
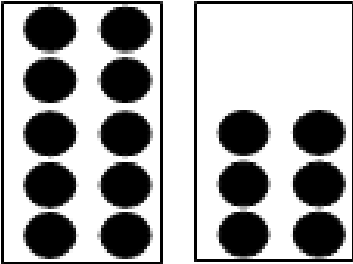
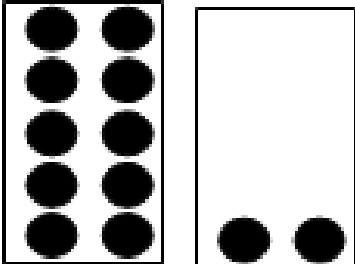
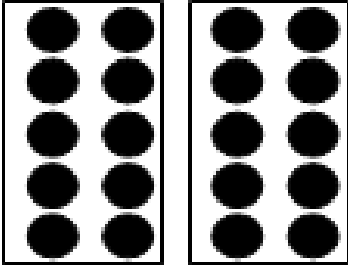
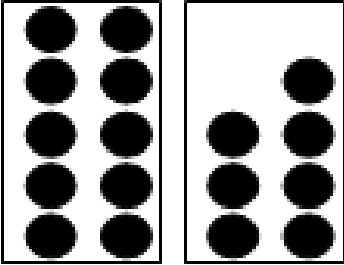
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Quantity–Numeral Correspondence Form C

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Appendix G. Intervention Schedule

Date	Group One Activities	Group Two Activities	Group Three Activities
1/2/13	Individual administration of the NSB	Individual administration of the NSB	Individual administration of the NSB
1/3/13	Baseline assessment 1 and introduction to “Ready to Learn” behaviors	Baseline assessment 1 and introduction to “Ready to Learn” behaviors	Baseline assessment 1 and introduction to “Ready to Learn” behaviors
1/4/13	Baseline assessment 2 and review of “Ready to Learn” behaviors	Baseline assessment 2 and review of “Ready to Learn” behaviors	Baseline assessment 2 and review of “Ready to Learn” behaviors
1/7/13	Baseline assessment 3 and review of “Ready to Learn” behaviors	Baseline assessment 3 and review of “Ready to Learn” behaviors	Baseline assessment 3 and review of “Ready to Learn” behaviors
1/8/13	Math intervention Lesson 1 and intervention assessments	Phonics intervention Lesson 1	Phonics intervention Lesson 1
1/9/13	Math intervention Lesson 2 and intervention assessments	Phonics intervention Lesson 2	Phonics intervention Lesson 2
1/10/13	Math intervention Lesson 3 and intervention assessments	Phonics intervention Lesson 3 and baseline assessment 4	Phonics intervention Lesson 3 and baseline assessment 4
1/11/13	Math intervention Lesson 4 and intervention assessments	Phonics intervention Lesson 4 and baseline assessment 5	Phonics intervention Lesson 4 and baseline assessment 5
1/14/13	Math intervention Lesson 5 and intervention assessments	Baseline assessment 6 and Math intervention Lesson 1	Phonics intervention Lesson 5
1/15/13 to 1/21/13	No research activities due to half days, teacher work days, and holiday	No research activities due to half days, teacher work days, and holiday	No research activities due to half days, teacher work days, and holiday
1/22/13	Math intervention Lesson 6 and intervention	Math intervention Lesson 2 and intervention	Phonics intervention Lesson 6

	assessments	assessments	
1/23/13	Math intervention Lesson 7 and intervention assessments	Math intervention Lesson 3 and intervention assessments	Phonics intervention Lesson 7 and baseline assessment 6
1/24/13 to 1/28/13	No research activities due to snow days	No research activities due to snow days	No research activities due to snow days
1/29/13	Math intervention Lesson 8 and intervention assessments	Math intervention Lesson 4 and intervention assessments	Phonics intervention Lesson 8 and baseline assessment 7
1/30/13	Math intervention Lesson 9 and intervention assessments	Math intervention Lesson 5 and intervention assessments	Math intervention Lesson 1 and intervention assessments
1/31/13	Math intervention Lesson 10 and intervention assessments	Math intervention Lesson 6 and intervention assessments	Math intervention Lesson 2 and intervention assessments
2/1/13	No research activities due to absences	Math intervention Lesson 7 and intervention assessments	Math intervention Lesson 3 and intervention assessments
2/4/13	Individual administration of the NSB and interview to Daniella	Math intervention Lesson 8 and intervention assessments	Math intervention Lesson 4 and intervention assessments
2/5/13	No research activities due to absences	Math intervention Lesson 9 and intervention assessments	Math intervention Lesson 5 and intervention assessments
2/6/13	No research activities due to absences	Math intervention Lesson 10 and intervention assessments	Math intervention Lesson 6 and intervention assessments
2/7/13	Maintenance assessments	Individual interviews with Charlie and Devin	Math intervention Lesson 7 and intervention assessments
2/11/13	Individual administration of the NSB and interviews to Adam and Dylan	Individual interview with Lacey	Math intervention Lesson 8 and intervention assessments
2/12/13	Phonics intervention Lesson 1	Individual administrations of the NSB with Charlie, Devin, and Lacey	Math intervention Lesson 9 and intervention assessments

2/13/13	Phonics intervention Lesson 2	Maintenance assessments	Math intervention Lesson 10 and intervention assessments
2/14/13 to 2/18/13	No research activities due to classroom events and weekend	No research activities due to classroom events and weekend	No research activities due to classroom events and weekend
2/19/13	Phonics intervention Lesson 3	Phonics intervention Lesson 5	Maintenance assessments
2/20/13	Phonics intervention Lesson 4	Phonics intervention Lesson 6	Individual administrations of the NSB and interviews to Aurora, Seth, and Steven

Appendix H. Student Interview

Introduction: Now that we have finished our extra math classes, I want to hear your ideas about how well they worked for you. I'm going to ask you some questions. You can answer honestly. It is okay to tell me what you liked or didn't like. My feelings won't be hurt and I won't get upset if you tell me you didn't like something. Your honest answers will help me find better ways to teach kids math.

I would like to use a video camera to record our talk. This will help me remember all of the important things that you say. Is that okay with you?

Do you have any questions for me before we get started?

Guiding Questions:

- 1) How were these math classes different from the way you usually learn math?
- 2) What was your favorite part of these math classes?
- 3) Was there anything you didn't like about these math classes? What was it? Why?
- 4) Did using the Numicon shapes help you learn? What did you learn? How did the shapes help you with that?
- 5) Would you like to use Numicon shapes in Mrs. Teacher's class? Why or why not?
- 6) Is there anything else that you want to tell me about our classes or the Numicon shapes?

The following probes will be used, as needed, to gain additional information or clarification:

Tell me more about_____.

Appendix I. Teacher Questionnaire

This survey is being conducted to gain a better understanding of how participation in the math intervention research impacted students' classroom performance. Please respond to the following statements in reference to Student Name's classroom performance after the intervention began.

1) After participating in the intervention, the student demonstrated improvement in rote counting skills.

Strongly Agree Agree Disagree Strongly Disagree Not Sure

2) The student demonstrated an improvement in his/her ability to count sets of objects.

Strongly Agree Agree Disagree Strongly Disagree Not Sure

3) The student demonstrated improvement in his/her ability to name written numerals.

Strongly Agree Agree Disagree Strongly Disagree Not Sure

4) The student demonstrated an improved ability to count backward from 10.

Strongly Agree Agree Disagree Strongly Disagree Not Sure

5) The student demonstrated a better understanding of the relationship between numerals and quantities (e.g., counted sets of objects more quickly; identified the quantity of a small set without counting).

Strongly Agree Agree Disagree Strongly Disagree Not Sure

6) The student used or expressed a desire to use Numicon shapes during regular class activities.

Strongly Agree Agree Disagree Strongly Disagree Not Sure

7) Are there any other observations that you would like to share regarding this student's participation in the math intervention classes?

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BIOGRAPHY

Melissa C. Jenkins graduated from Osbourn Park High School, Manassas, Virginia, in 1991. She received her Bachelor of Arts in Psychology from George Mason University in 1995 and her Master of Science in Special Education from Old Dominion University in 2004. She worked as an elementary school special education teacher for nine years serving students in kindergarten through fifth grade and was previously a therapeutic foster care social worker serving children from birth through age 18 in community settings.