Handheld Computer Algebra Systems in the Pre-Algebra Classroom

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DEDICATION

This is dedicated to my loving husband Peter, and my two wonderful children--Mikaela and Carter.

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ABSTRACT

HANDHELD COMPUTER ALGEBRA SYSTEMS IN THE PRE- ALGEBRA CLASSROOM

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This mixed method analysis sought to investigate several aspects of student learning in pre-algebra through the use of computer algebra systems (CAS) as opposed to non-CAS learning. This research was broken into two main parts, one which compared results from both the experimental group (instruction using CAS, $N = 18$) and the control group (traditional instruction without CAS, $N = 14$), and another which looked more in-depth at eight students' ability to answer questions following instruction using CAS. The first purpose of this study was to evaluate the impact of using CAS on student learning and the second was to explore students' attitudes towards mathematics and whether certain aspects of a student's attitude could be linked to their achievement. This research did show significant difference in gain scores for the experimental group over the control group, $F(1, 32) = 12.368$, $p = 0.003$. However, triangulation between the different

measures used to support increased procedural and conceptual understanding proved inconclusive. The current results do not predict future trends on the effectiveness of CAS; however, these findings suggest that CAS could play a role in student retention and understanding of procedures as well as improved attitudes towards mathematics. Future studies on CAS should look to disaggregate student performance by Adequate Yearly Progress (AYP) subgroups.

1. Introduction

Introduction

In 2000, the National Council of Teachers of Mathematics (NCTM) released the *Principles and Standards for School Mathematics*. Technology was one of six major principles addressed by NCTM in the document. "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning." (NCTM, 2000, p. 524) A typical mathematics classroom in the United States features the use of technology.

However, hand held technology is constantly growing and changing, so much so that the ability to manipulate algebraic symbols is now readily available. Systems that can manipulate algebraically are called Computer Algebra Systems (CAS). In March 2008, NCTM released an amendment to their position statement, which noted, "Calculators and other technological tools, such as computer algebra systems, interactive geometry software, applets, spreadsheets, and interactive presentation devices, are vital components of a high-quality mathematics education." Although CAS technology has been available for over two decades, it has not been being widely utilized in the United States with students at the pre-college level, even though it is being used at the secondary level in countries such as Austria, Belgium, Denmark, Germany, The Netherlands, Scotland, Switzerland, and Australia (Bohm, Forbes, Herweyers, Hugelshofer, & Schomacker, 2004; Fey, Cuoco, Kieran, McMullin, & Zbiek, 2003).

Studies of the usage of CAS at the college level have shown increased conceptual understanding of mathematical concepts (Connors, 2000; Heid, 1988; Leinbach, 2001; Nunes-Harwitt, 2004). In a study which compared results from nine previous studies on the use of CAS, eight of the nine studies showed that students who learned using CAS did just as well or better on assessments which were non-calculator and procedurally oriented as students who did not learn with this technology (Heid, Blume, Hollebrands, & Piez, 2002).

Current trends in mathematics encourage algebra for all, following the belief that anyone can learn algebra, particularly if given appropriate experiences that provide motivation to explore main concepts, processes, and algebraic skills. In the era of No Child Left Behind the nation is trying to raise the bar on what is expected of students, and algebra is referred to as the "gatekeeper" subject (Jacobs, Loef Franke, Carpenter, Levi, & Bettey, 2007). In the United States, algebra I is the prerequisite for all higher-level math: geometry, algebra II, trigonometry, precalculus, and calculus. However, many students do not get a solid foundation in mathematics and therefore have to take remedial math in college. Even with the use of graphing calculators and a heightened focus on modeling, students are still struggling with algebra (Baldi, Jin, Skemer, Green, & Herget, 2007; Gonzales et al., 2008; Kaput, 1995; Pierce & Stacey, 2001a).

Although the number of studies on high school algebra is limited, there have been studies that have focused on the use of CAS to teach remedial algebra courses at the college level. Students taking remedial algebra courses in college are those who were not successful at algebra in high school or tend to be students who were not successful in mathematics previously (Harper, 2007). There is limited research on the use of CAS at the pre-algebra level. The demographics of students enrolled in 8th grade pre-algebra consist of greater percentages of students with Individual Education Plans (IEPs), and students who are English Language Learners (ELL), than fellow schoolmates enrolled in other courses during their 8th grade year. Therefore, I looked at CAS as a tool that may help students experience success learning algebraic concepts the first time around. Reform-based curriculums, such as the Connected Mathematics Project (CMP), were developed on the belief that observations of patterns and relationships are the key to deep understanding in mathematics. This standards-based curriculum is organized so students solve problems by observing patterns and relationships, engaging in conjecture, testing their conjecture, and discussing and verbalizing their findings in order to generalize the patterns observed.

I wanted to test whether a similar design could work in a classroom which utilized traditional textbooks as opposed to a reform-curriculum. For this study, I designed an eighth grade pre-algebra classroom where students worked in cooperative groups. The activities they worked on I will refer to as *generative learning activities*. Specifically, generative learning activities are activities where students were asked to complete problems, make observations, make conjectures, and discuss results. This type of activity emulates the type of practice a student gets with the CMP, although the problems I used were not necessarily real world. While completing the generative activities, students either worked in peer groups with CAS (experimental group) or without CAS (control group). I looked to see what effect the use of the CAS technology had on eighth grade pre-algebra students' achievement in the areas of integers, variable expressions, equations, and inequalities. I also looked for differences in achievement between the experimental and control groups and tried to determine whether students who develop the rules and procedures for algebraic (symbolic) manipulation using CAS accurately learn, use, and retain the rules and procedures as well as students who learn the same material without the technology.

Studies on attitudes about mathematics (Hannula, 2002; Pierce, Stacey, & Barkatsas, 2007; Schreiber, 2002; Utley, 2007; Wilkins & Ma, 2003) have shown that attitudes do have an effect on students' ability to learn. I decided that it would be important to make sure that students' attitudes about mathematics were taken into account to see if perhaps there was any correlation between the eighth-grade pre algebra students' attitudes towards mathematics and their achievement and gain in knowledge.

Background of the Problem

Conceptual Framework

I will explain the direction of this study with respect to the conceptual framework (Figure 1) that I developed through my initial exposure to CAS research. Over the last two decades, technology has had a major impact on the way we learn and teach mathematics. Rose Zbiek (2003) summarized research on the use of CAS, writing about both the context and characteristics of many studies and making suggestions for future research based on these studies. She noted that comparison studies tended to involve two or more different curricula and that studies which utilized identical teaching materials in comparing CAS-use and non-CAS-use were rare. Also noted was that many supposedly CAS-focused research reports also included graphing tasks. "This graphing presence raises a question as to whether some of the research studies that seem to be CAS studies are actually graphing-utility studies" (Zbiek, 2003, p. 209). Zbiek also encouraged the exploration of the use of CAS in developing by-hand manipulation skills, which I will refer to as procedural skills, as well as investigating the relationship between skill acquisition and conceptual understanding.

In considering a study that would incorporate many of these suggestions, I started to consider what a classroom might look like where the symbolic manipulation features of CAS were stressed. One of the main ways CAS has been used in algebra classrooms emphasizes looking at patterns and making and testing conjectures. Therefore, the design of the classroom I intended to study involved having students work in cooperative learning groups using generative learning activities to encourage student exploration of patterns. Using this as the classroom design I tested procedural and conceptual understanding of mathematical concepts through pre- and post-test comparisons as well as by looking at a variety of other data sources, such as teacher logs, video taped lessons, audio taped communication, student activities, and student task-based interviews. There are mixed views on student attitudes when using CAS (Pierce & Stacey, 2001b), indicating this is also an important aspect to consider, especially since I was working with 8th grade pre-algebra students who had not had the opportunity to use graphing calculators or other advanced handheld technology in their previous mathematics classes.

With the push for algebra for all, the students who are not placed in algebra in eighth grade tend to be those that have difficulty in mathematics. Many students with difficulty in mathematics at the secondary level are required to take remedial math courses when they enter university. In the fall of 2000 approximately 22% of freshmen beginning at an institution which offered remedial classes were enrolled in remedial mathematics (Parsad & Lewis, 2003). Since studies using CAS in college algebra classes tend to show heightened conceptual understanding (Heid et al., 2002), I felt it was important to see if this would be true at the eighth grade level when students began learning algebraic concepts. As there is a push for most eighth graders to be in Algebra, the students in this 8th grade pre-algebra who did not place into the Algebra class would be among those most likely to need to take a remedial mathematics course in college. Thus, I decided to look at eighth graders in pre-algebra and look at subgroups of these eighth graders which might have heighten risk factors, such as ELL students or students with an IEP. I disaggregated the data to look at achievement differences of these subgroups.

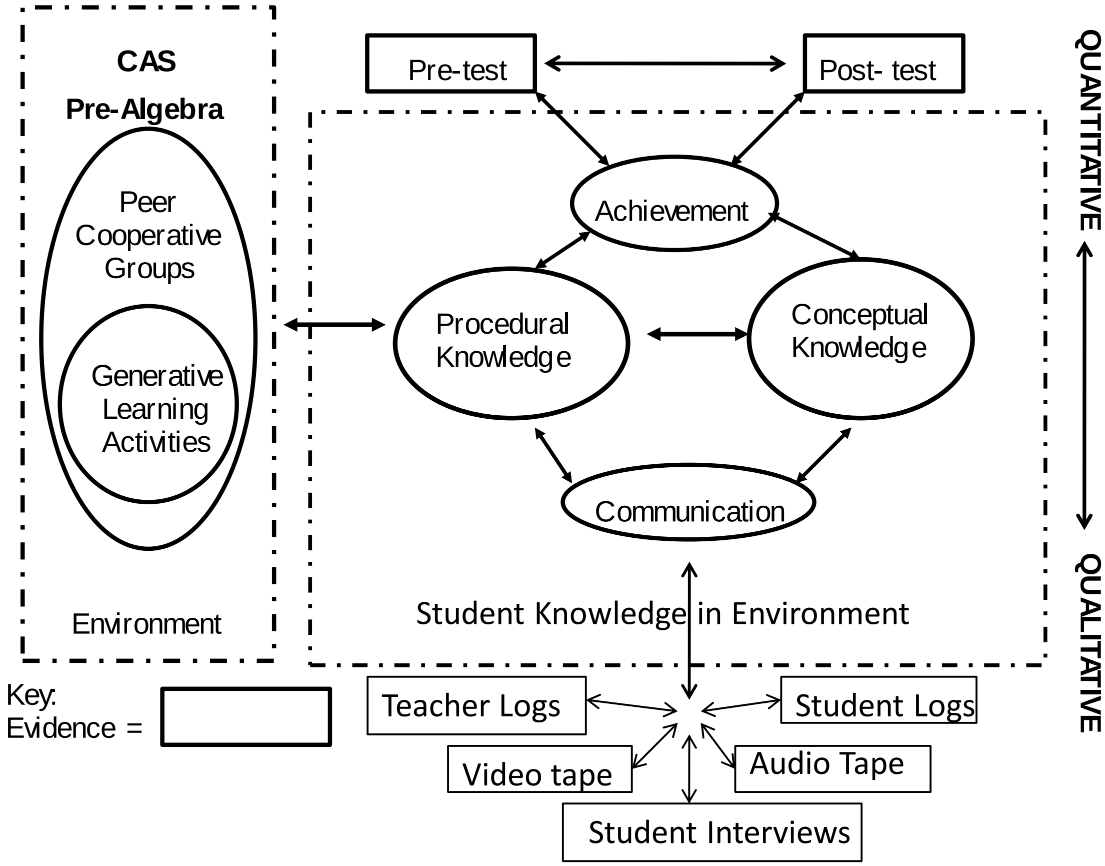


Figure 1. Researcher's conceptual framework

Statement of Problem

The development of graphing calculators in the 1980's changed the way mathematics was taught. The use of CAS adds a new dimension to the use of handheld technology. A CAS handheld can do most of what is taught in a traditional algebra course for the student, thus creating the need to address fundamental questions about what school mathematics really is and what skills will be important for students.

CAS technology is currently used as a means to teach mathematics in several countries; however, its use in pre-college education in the United States has been predominantly limited to Calculus classes. In recent years with CAS readily available in handheld calculators, there have been some studies on its use in algebra instruction (Drijvers, 2003, 2004; Kieran & Drijvers, 2006; Pierce & Stacey, 2001b). However, research on its use in algebra is also linked with changes in the curriculum or the use of new curriculum. Although CAS technology has not become commonplace in middle and high school classrooms in the United States, CAS technology has been accepted for use on Advanced Placement Exams for several years. In fact, the College Board accepts the use of CAS on all of its exams.

There are several issues surrounding the use of CAS. One of the major issues is the lack of acceptance of CAS in the 6-12 mathematics community in the United States. The mathematics community I refer to includes the researchers, decision makers, and the stakeholders, thus I am primarily referring to 6-12 mathematics teachers, coordinators, and supervisors, but also to other stakeholders such as administration and parents. Some of the opposition to CAS centers on the view that it is not a tool to help students work out problems, but a black box that spits out answers with little understanding on the part of the students as to how the answer was determined (Drijvers, 2000). Many students accept the answers provided by the calculator with little hesitation. Students need to be trained to use CAS technology properly, including how to input expressions and interpret results. However, with advances in technology, specifically with the ability to symbolically manipulate expressions now readily available in user-friendly handhelds, it becomes more critical that teachers have a strong understanding of the pedagogical content knowledge needed and are able to integrate subject matter knowledge and pedagogy in creating activities and assessments for their students.

Another point of opposition to CAS is the impact on students' performance on standardized tests. Using CAS as a tool for instruction is hindered by the fact that most standardized exams will not allow its use. Schools are held accountable if students do not successfully complete course end of year tests and achieve Adequate Yearly Progress (AYP) under NCLB regulations. Thus, teachers want to make sure students are proficient in the technology they will be able to use on exams. During the process of getting permission to use the TI-Nspire CAS with my students for a pilot study my school superintendant wanted to know if the technology could be used on our state exams, and if not, would this put students at a disadvantage? These types of questions and the concerns new technologies bring to the discussion table are exactly those that need to be addressed in research.

Now that CAS is available in a handheld calculator, further research at all levels of secondary education must be conducted to determine where and how CAS can and possibly should be used. Technology does not tend to bring us backwards. Once we can do something using technology the goal then is to concentrate on understanding the processes and how we can use the tool to advance our own understanding of mathematical concepts. We cannot wish away the invention of CAS technology; therefore we should figure out how it can be used to help students learn mathematics. As Kutzler said, “teachers have the pedagogical duty to use all available resources to facilitate the learning process of their students” (2003, p. 57).

Allowing CAS at the secondary level will entail changing the way topics are taught as well as considering with which topics it should be used. There is already much debate over the use of CAS over pencil and paper. It needs to be determined what algebraic concepts will be emphasized and decide whether pencil and paper or CAS would be better at achieving the goals set with respect to these concepts. Allowing students to use this tool will require both teachers and test writers to redesign instructional materials as well as tests. We must once again redesign activities based around a technology and envision how this technology alters the way we teach and test in mathematics. If the use of technology--including computer algebra systems--is the change being sought by NCTM, which sets the bar for mathematics education in our country, it seems feasible that our state end of course mathematics exams can be modified to incorporate classrooms which have the opportunity to utilize CAS technology.

In this study I am looking at the use of technology as a symbolic manipulator. To more accurately determine how having and using this tool would affect student learning would have required studying the instrumental genesis of how and whether the TI-Nspire CAS as an artifact actually became an instrument that enabled students to learn mathematics and develop conceptual understanding. “Instrumental genesis includes both the user shaping the tool for her or his purposes (instrumentalization) and the user’s understanding being shaped by the tool (instrumentation) (Zbiek, Heid, Blume, & Dick, 2007).”

In terms of the student-tool relationship, the activities students completed in cooperative groups were designed primarily as generative activities. Initially I contemplated trying to follow students’ use of the TI-Nspire CAS as a tool and looking for individual cases of students’ use extending from merely exploratory to expressive. By expressive activity, I refer to the variety of activities and approaches that students produce when left to independently solve a problem (Zbiek et al., 2007). As the generative activities were completed within cooperative groups the teachers communicated how they saw students using the TI-Nspire CAS. In student focus group interviews I asked questions about how students learned to use the TI-Nspire CAS over time; however I was not able to acquire significant data that I could link to student procedural or conceptual understanding of mathematics. In order to have truly tracked student instrumental genesis I would have needed to keep track of whether students appeared to be acquiring conceptual understanding through use of the TI-Nspire CAS as a tool (Artigue, 2002; Trouche, 2004) on a daily basis. Instrumentalization is more of a psychological process which leads to the internalization of processes and uses of the CAS (Guin & Trouche, 1999) and hence would have required more time and my constant presence in the classroom.

In CAS research much has been written about instrumentation, which is a process directed towards the subject (Drijvers, 2000; Guin & Trouche, 1999). This is where students learn not only the capabilities of the tool, but also its constraints. There are not only constraints to the internal operation of the TI-Nspire CAS which limit what it can do, but there are external constraints such as the user interface and the syntax students must learn in order to utilize the tool (Trouche, 2004). Through the clinical task-based interviews, I asked students questions and asked them to explain syntax from the TI-Nspire CAS. Thus I am able to speak to some degree about the difficulties my subjects faced with aspects of instrumentation, but I do not have sufficient data to truly document the instrumentation process for each student.

I asked questions about technology during group interviews and on the attitude survey to see if there was any correlation between attitudes about technology and achievement using technology. In instrumentation the student-tool relationship is not necessarily a direct correlation--meaning that the instrumental genesis is not necessarily greater in students who are avid users/adopters. According to research (Guin & Trouche, 1999), the transformation of a tool into a mathematical instrument does not necessarily lead to greater mathematical understanding.

Purpose of the Study

This study has two main purposes concerning the use of CAS in the pre-algebra classroom. The first purpose of this study is to evaluate the impact of using CAS on student learning. The second purpose of this study is to explore students' attitudes towards mathematics and whether certain aspects of a student's attitudes could be linked to their achievement.

Research Questions

This research project originated from suggestions made by Rose Zbiek (2003) in Chapter 12 of *Computer Algebra Systems in Secondary School Mathematics Education*. Additional ideas came from the work on student attitudes by Pierce, Stacey, and Barkatsas (2007) and a dissertation by Harper (2007) on the use of CAS in introductory algebra. I decided to synthesize aspects of the preceding research by asking the following research questions:

1. What is the effect of the use of the TI-Nspire CAS technology on eighth-grade Pre-algebra students' performance in the areas of integers, variable expressions, equations, and inequalities?

- Are there differences in achievement between experimental and control groups?
    - Are there differences in performance between experimental and control groups due to gender, students with an Individual Education Plan (IEP), or students who are English Language Learners (ELL)?

2. Do students who develop the rules and procedures for algebraic (symbolic) manipulation using CAS accurately learn, use, and retain these rules/procedures as well as students who learned the same rules/procedures traditionally?

    - Are there differences in gain scores of experimental group over the control group on algebraic expectation?
    - Are there differences in gain scores of experimental group over the control group on algebraic expectation for subgroups (sex, students with IEP, ELL students)?

- What are common student procedural/conceptual misconceptions with respect to algebraic insight?

3. Is there any correlation between eighth-grade pre-algebra students' attitudes toward mathematics and their achievement and gain in knowledge?

  - Is there a difference in this correlation with respect to students who did or did not use the TI-Nspire CAS?

Significance of the Study

The lack of success of students taking algebra calls for a new approach to the learning and teaching of the "big ideas" in algebra. Much of the research on CAS handheld technology does not always specifically consider the CAS part of the technology. These handheld calculators can do everything their graphing calculator counterparts can and more. CAS calculators enable students to approach problem solving graphically, numerically, and algebraically (symbolically). There have been numerous studies that have looked at the positive benefits of multiple representations. However, in research it is not clear the actual affect of the CAS (symbolic manipulation) on achievement as the multiple representational aspects, such as graphs and tables, often were not controlled for in studies. Thus, I wanted to target the symbolic manipulation aspects of the CAS handheld which had not been studied significantly in the existing research. This study also targets a population not often exposed to CAS - middle school students.

2. Literature Review

The review of literature begins with the concerns of algebra readiness and key algebraic concepts. This is followed by examples of CAS research which discuss its use as a procedural tool as well its use in pre-college level instruction. Next is a discussion about the debate over procedural knowledge versus conceptual knowledge. I conclude this section with a discussion and support for my use of clinical task-based interviews.

Algebra Readiness

This section of Chapter 2 will discuss research on the algebra we teach students as well as the influence of technology on algebra instruction. Research on the importance of understanding concepts of operations, the equal sign, and equivalence will be reviewed.

Algebraic Insight

When it comes to international discussions about algebra it is quite difficult for researchers to come to agreement given that how and what we teach in algebra is quite different. Even within the United States, algebra courses can vary significantly with respect to the content and focus. It is common in traditional settings in the United States for algebra to be taught as a separate course, whereas it is often integrated throughout other curriculum in other countries.

Pierce and Stacey (2001a) designed a framework in order to reflect on and assess students' algebraic insight (Table 1). In this study I will discuss results from student individual work- including class activities and tests as well as student interviews in relation to the elements from certain aspects of Pierce and Stacey's framework.

Algebra for all! This is the belief that all students can and should learn algebra, especially when given appropriate experiences that motivate them to explore the "big ideas" of algebra, which includes both the processes and the skills of algebra. A study carried out in Australia surveyed teachers who used CAS to compare what different educators thought was a reasonable future goal for students to complete particular items by-CAS or by-hand (Flynn, Berenson, & Stacey, 2002). In fact several studies concluded, "simplifying expressions by-hand could perhaps gain added importance for a fuller comprehension of CAS output" (p.11). Some of the "big ideas" found in research on success in algebra are number sense, understanding operations--including the order of operations, understanding the equal sign, and understanding equivalence (Artigue, 2002; Jacobs et al., 2007; Pierce & Stacey, 2001a; Schneider & Peschek, 2002).

Table 1

Algebraic Insight Framework from Pierce and Stacey, 2001, p. 3.

Aspects		Elements	Common Instances
1	Algebraic Expectation	1.1 recognition of conventions and basic properties	1.1.1 Know meaning of symbols 1.1.2 Knowing order of operations 1.1.3 Knowing properties of operations
		1.2 Identification of structure	1.2.1 Identify objects 1.2.2 Identify strategic groups of components 1.2.3 Recognize simple factors
		1.3 Identification of key features	1.3.1 Identify form 1.3.2 Identify dominant term 1.3.3 Link form to solution type
2	Ability to link representations	2.2 Linking of symbolic and numeric representations	2.2.1 Link number patterns or type to form 2.2.2 Link key features to suitable increment for table 2.2.3 Link key features to critical intervals in table

Number Sense

Number sense involves understanding the meaning of numbers, the magnitude of a number, ways to represent a number, as well as the relationships between numbers and the ability or skill to work with them. Number sense is something that students learn over time through use of numbers in their everyday life. One of the obstacles to algebra is students' ability with numbers as well as their number sense. In algebra students are often introduced to variables, but still do not understand that a variable represents a number. Building students' relational thinking by helping them to see the relations among numbers as well as understand the fundamental properties of number operations will help to develop students' ability to reason algebraically (Jacobs et al., 2007).

Algebraic Reasoning

The transition from arithmetic to algebra is an obstacle for many students. Work by the NCTM Algebra Working Group as well as the Early Algebra Group demonstrate combined efforts on integrating algebraic reasoning into the K-12 curriculum (Jacobs et al., 2007). There are many ways algebraic reasoning presents itself. Kaput (1998) identifies five;

- Algebra as generalizing and formalizing patterns and regularities, in particular, algebra as generalized arithmetic;
- Algebra as syntactically guided manipulations and symbols;

- Algebra as a study of structure and systems abstracted from computations and relations;
- Algebra as the study of functions, relations, and joint variation; and
- Algebra as modeling. (p.26)

In this research the content used focused on the first three.

Whether it is PEMDA (parentheses, exponents, multiplication or division, addition or subtraction) or BIMDAS (brackets, indices, multiplication or division, addition or subtraction) the question remains whether students retain procedures or ever internalize and conceptually understand the order necessary when working with and combining numbers or numerical expressions. Even if calculators with CAS capabilities could do all of the computations and algebraic manipulations, it is still important for users to be able to input correct operations and interpret the results (Fey, 1990).

Studies have shown that many students see the equal sign as an indicator to carry out a calculation (Kieran, 1992; Knuth, Stephens, McNeil, & Alibali, 2006) as opposed to a symbol indicating a relation. Thus, developing a relational understanding of the equal sign is critical for the learning of algebra. The process of solving an equation or transforming an equation requires students to understand that adding or subtracting the same number from both sides of the equal sign leaves the relationship between the expressions on both sides unchanged (Jacobs et al., 2007; Kieran, 1992). Without this understanding these transformations become memorized procedures which make little sense to students.

Students using CAS need to be able to understand its output and be able to recognize equivalent answers. For example if a student was asked to add 2 + 4x + 5x, would a student who got 2 + 9x be able to determine when checking answers in their book that their answer is the same as 9x +2. In addition, if a student typed in $\frac{1}{2}(a+b)$ and once they had entered this expression the calculator changed it to $\frac{a+b}{2}$ on the display screen, would students think they typed the expression incorrectly, will they think the calculator is broken, or will they come to understand that these two expressions are equivalent? Students will need to understand the CAS output and be able to recognize equivalent expressions.

Procedural Skills, Procedural Understanding, and Conceptual Understanding

This section will discuss mathematical pedagogy as well as the acquisition of procedural skills toward procedural understanding and conceptual understanding and will compare and contrast views from researchers on the link between procedural and conceptual understanding.

There has been an increasing amount of research on the role technology plays in mathematics education. However, it is becoming increasingly important to distinguish between two constructs which are often described in mathematics educational settings: procedural and conceptual understanding.

Although there are differing definitions of these terms, I use the term procedural skills to refer to the ability to perform accurate operations in problem solving. Procedural understanding in this study is knowledge that is acquired as the frequency of incorrect procedures decreases and the frequency of accurate procedures leading to correct solutions increases. Thus it is an understanding which comes through knowledge which is gained through mechanical or technical mathematical activity. Procedural understanding is acquired when one has a strong number sense and has internalized procedures for numeric operations and can transfer that understanding to symbols. I use the term conceptual understanding to refer to knowledge gained through making connections and the ability to manipulate algebraic symbols through generalization of arithmetic operations. For example one can have a conceptual understanding of an equal sign. This is when one does not just see the equal sign as symbol or as a means to carry out an operation, but rather understands that in an equation it represents a means to equate two expressions. There is a growing trend against taking a dualistic epistemological stance between conceptual and technical dimensions of mathematical activity (Star, 2007; Zbiek et al., 2007). Although I would agree with Star (2005) that there must be levels of procedural knowledge(understanding) that can be achieved, the research I have conducted demonstrates the importance of the interconnection of procedural and conceptual understanding and supports further research into alternative categories that are less restrictive or perhaps determining how one supports or encourages the other.

I do believe that procedures and knowledge of procedures that govern mathematical calculations are richly connected. For example, in solving an equation such as $2(x + 5) = 4x - 8$, a student may arrive at a solution by a series of “procedures” governed by that student’s procedural knowledge. Knowledge of procedures is merely having a set of procedures to carry out, but not necessarily understanding when and why to carry out a particular procedure, or whether the procedure is executed correctly. When assessing understanding by merely evaluating the Boolean nature of the solution, right or wrong, we are not truly assessing the student’s procedural understanding or conceptual understanding. I often say to students the work is much more important than the answers. For example, a student may have arrived at an incorrect or correct answer in the manner shown in Table 2.

Table 2

Comparisons of Possible Student Solutions

Solve: 2(x + 5) = 4x – 8	
Student 1	Student 2
2(x+5) = 2(2x – 4)	2(x+5) = 4x -8
x+5 = 2x – 4	2x + 10 = 4x – 8
3x = 9	2x = 18
x = 3	x = 9

Once again, from looking only at the answers, a conclusion might be drawn that student 1 does not understand this problem procedurally or conceptually as he/ she was not able to arrive at the correct answer. However, from looking at the work, one could possibly argue that student 1 has a strong conceptual understanding of equations and balancing equations as well as number sense, and quite possibly made a careless error. In order to know the true level of understanding a student needs to explain his or her process for solving the equation step-by-step with explanations for choices at each step.

Student 2 accurately solved the same equation, but the work shown is that of a standard algorithm. Does this student understand why he/she distributed the 2 across the parenthesis, or is this merely a memorized procedure? Does this student understand why the 10 and -8 on opposite sides combined to 18 on one side? Was this student just lucky on this one, or can he or she explain each step and give reasons why each step was taken? These examples are given to stress the importance of the use of clinical task-based interviews in determining a student's procedural and or conceptual understanding, as a student's work does not necessarily reveal his/her procedural or conceptual understanding of a problem.

One of the criticisms of mathematics education suggests teachers in a traditional classroom merely tell students what to do and have them memorize rules to follow without understanding why. However, what if students were encouraged to use technology to understand and learn procedures by looking for patterns and making conjectures and not by merely being told what they had to do? I wanted students to use these kinds of generative activities to better compare the procedural and conceptual understanding gained by students who did or did not use CAS. Although I do not think that this technology was designed explicitly for this purpose, in a Vygotskian way the CAS activities that are included in this research were designed to promote pattern exploration and thus provided the arena for student conversations which were mediated by the use of the TI-Nspire CAS as a tool (Roschelle & Jackiw, 2000; Vygotsky, 1986).

Central to the use of the technology was the type of activities used as well as the expectation that students were to write their results and discuss their findings with their group.

Computer Algebra Systems

This section will discuss the research on computer algebra systems (CAS) in teaching mathematics. First is a discussion of the history of the technology and use of CAS followed by research on its effect on procedural and conceptual knowledge. I will then discuss research on the effective uses of CAS and the obstacles to using CAS in the United States. Lastly I will examine different roles of the symbolic manipulation capabilities of CAS.

History of CAS

Computer Algebra Systems have been being used in the United States since the 1970's. CAS software programs such as Derive and Maple became popular in the late 1980's for their ability to perform symbolic manipulation. I recall learning to use Derive during my teacher training courses at Rutgers University in 1989. Initially I saw it as software that would do the "hard" math for me. I recall not being sure how I could use it other than to possibly check answers or do the work for me. This is exactly the sentiment that leads to much of the opposition to the use of CAS (Nunes-Harwitt, 2004). In reading research on CAS, and attending conferences specifically on CAS and CAS research I have learned and read about several ways that this technology can be used to improve and transform instruction in the mathematics classroom. I am still skeptical at times, not about its potential to aid instruction, but regarding its possible misuse. For example, the goal is to use the technology to come to a stronger conceptual understanding. However, the role of the teacher in a CAS environment should not be to teach students to merely be button pushers. The debate over CAS is strongly polarized. The opposition is concerned about dependency and over-reliance on the technology (Macintyre & Forbes, 2002). However, even many advocates of CAS would still caution against giving students unrestricted access.

One of the major benefits of CAS is that it allows students to analyze and see a problem through multiple perspectives. "Being able to visualize what is happening in a mathematical setting is often a crucial prerequisite for understanding" (Nunes-Harwitt, 2004, p. 158). Being able to see graphical, numeric, algebraic, and even written representations strengthens a student's conceptual understanding. In a study performed in an introductory calculus class, Heid found that students in her experimental group which used CAS showed greater understanding of concepts and also a greater ability to use different representations (1988).

Although the technology has been available in handheld form for several years, the use of CAS is not widespread in the United States. Most current secondary school

CAS users are students in Calculus courses. On the contrary, in Europe CAS has been used at the secondary level for several years in courses other than Calculus. In the US there are several barriers that potentially constrain the use of CAS in secondary classrooms; one is cost. Handheld versions of CAS provide a much more convenient and personal way for students to interact with and use the technology, but up until recently individual handheld CAS technology was quite expensive compared to typical graphing calculators. However, the price of CAS technology is decreasing, which will probably affect willingness to use CAS. Another barrier is convenience and ease of use, but newer CAS models are becoming more user friendly than earlier versions with menus and submenus which are easier to navigate.

Another barrier to the use of CAS is individual state assessments. In the age of No Child Left Behind, these assessments frame our educational system as well as policy. Many states assessments do not allow the use of CAS, and when this is the case, it can be difficult to "sell" the idea CAS should be used or obtain funding for its use in school districts. There also may be a realization within state agencies that the inclusion of CAS would drastically affect the format of the test and would call for an overall change in the way assessment questions are asked. I would also caution state agencies which allow CAS on their assessments to closely examine tested items and make adjustments in their questioning to ensure that they are "testing" understanding and not just button pushing using CAS.

Understanding and Communication

A Scottish study showed that a class which used CAS improved achievement scores 7% over a control group (Macintyre & Forbes, 2002). However, there are mixed results on improved achievement levels in a CAS rich environment as compared to a control; one difference appears to be in the improved conceptual understanding of the mathematics that is witnessed through the use of CAS (Foletta, 2002; Macintyre & Forbes, 2002; Tonisson, 2002). In a study where the integration of paper-and-pencil techniques were used alongside computer algebra techniques, the link between these techniques as well as student conceptual understanding was strengthened (vanHerwaarden & Gielen, 2002).

In an atmosphere where students are asked to explore and make detailed observations, students are also expected to write observations in a clear and understandable manner. Students in this environment become increasingly aware of communicating through written language and their written comments through these activities helps to develop their ability to transmit their understanding to others (Schneider & Peschek, 2002). In a study of junior high school students, Brown also found that, "The students' written comments throughout their class work also indicated an understanding of the algebraic concepts that had been introduced" (1998, p. 8). In addition, in a CAS environment, students are encouraged to discuss the answers received using the technology as well as negotiate their own interpretations and justify their conclusions to their peers. This helps students develop their ability to communicate mathematically through oral communication.

Changing Pedagogy with CAS

The use of CAS requires changes in organization, teaching materials, and assessment (Pierce & Stacey, 2002). With this new technology comes a change in expectations about the learning goals and ways in which students will be assessed. However, the goal of using CAS is not to replace learning, but rather facilitate it. Ultimately, students are still expected to perform algorithms without the aid of the CAS; however, the manner in which they learn to do this may be altered. Many rules and algorithms in a CAS environment can be developed through a classroom environment where students use exploration to make detailed observations, make conjectures, test their conjectures, and use inductive reasoning to develop rules (Heid, 1988; Heid & Zbiek, 1995). Students who used CAS saw the technology not as a tool to 'do the math,' but rather a tool which helped them to do and explore the math (Pierce & Stacey, 2001c).

The learning in a CAS rich environment is one where active exploration and conjecture-making is commonplace. There is an increased need for the teacher to monitor students progress through the use of the technology to ensure that it is being used effectively as a tool (Pierce & Stacey, 2002). As teachers begin to use this technology their understanding of how to use it grows and decisions about classroom activities and expectations also begin to alter their classroom environment (Zbiek, 1995).

Positive results on attitudes about CAS are seen in research with respect to both teacher and student. Teachers found that the time saved in calculations was spent either discussing results or teaching efficient calculator procedures (Stacey, Kendal, & Pierce, 2002). Students in a CAS environment were open to the use of the new technology and felt it helped them better understand. Although initially students used CAS for functional purposes and were quite passive about its use, Pierce and Stacey found over the course of their study that students' began to interact with the technology and become responsive to the answers they were receiving. This led students to a strategic manner of approach where they began making their own conjectures and testing their own hypotheses (Pierce & Stacey, 2002). In a study on 10th grade Algebra students, all students had reported that using CAS had a positive change on their attitudes towards mathematics. The use of this technology helped students to be less anxious about mathematics (Noguera, 2001).

Gaps in Research

A significant portion of CAS research is on its use in Calculus and College Algebra. Little research has been conducted at the lower secondary level. If CAS can help improve algebraic thinking it should be evident with any level of student. Thus I have decided to focus my study on 8th grade math students in pre-algebra. I will specifically look at the symbolic manipulation features of the technology.

Much of the current research on CAS merely demonstrates a way of varying heuristics used in solving standard word problems. CAS can be viewed as a cognitive technology, which means it allows the student to do or understand something they otherwise would not have been able to. CAS as a cognitive technology can play two roles--that of a reorganizer or that of an amplifier (Heid, 2001). There have been several studies on CAS as a reorganizer (Harper, 2007; Heid, 1988; Matras, 1988; O'Callaghan, 1994, 1998; Palmiter, 1991). In this capacity, the CAS is used in a manner that changes the fundamental nature and progression of the curriculum. In my study, however, I was looking at CAS as an amplifier. As I used a pre-existing traditional curriculum, I was able to look at CAS as a means to extend the traditional curriculum. I analyzed the use of the TI-Nspire CAS on facilitating student learning of mathematics while keeping the basic goals and sequence of the existing curriculum intact.

Many current studies on CAS include the use of graphical representations. Therefore results do not necessarily accurately show how CAS uniquely contributed to student improvement in achievement or understanding as graphing ability is available in non CAS handheld technology. Although CAS may be used as a graphical tool, it is its algebraic/symbolic manipulation ability that was the focus of this study.

Roles of CAS

Heid and Edwards (2001) described four possible roles of CAS. In this study I looked at CAS in three of the four roles described. The first role, which I did not use, entails using the CAS as a white box in order to allow students to focus on a more conceptual understanding of mathematics. There are two reasons why I chose not to focus on this role. The current curriculum is traditional, and my intent is not to create an entirely new way of designing the class, but rather find a way that CAS can be used as a tool in the existing confines of the classroom. Consequently, I am trying to compare a traditional pre-algebra class to one where the only difference is the use of CAS. The second reason I am not investigating this role is that I am focusing specifically on the use of CAS to develop a students' ability to recognize patterns and formulate rules.

Therefore, in some ways I am using it just as a black box (a machine that just gives answers), but my intent is to have students decode the answers it gives. My goal is to look at the expansion of procedural as well as conceptual knowledge in the development of algebraic thinking.

The second possible role Heid and Edwards discussed is using CAS to "create and generate symbolic procedures" (2001, p. 131). This includes activities where students enter steps one by one into the CAS in order to transform an equation until it is solved (see Figures 2- 4). This allows students to practice appropriate sequences that are needed when solving an equation. Figure 2 is what the screen would look like if a student typed in an equation and then pressed "-5." This would be the equivalent of subtracting 5 from both sides. In Figure 3 students who arrived at an answer which was not "simplified" could delete their attempt and try something new. In this manner students will be using the CAS as a pedagogical tool, which assists them in constructing their conceptual understanding of symbolic manipulation. In Figure 4 the previous problem is completed when a student divides by 3. The display shows 3 being divided on both sides and the final answer where the variable is isolated.

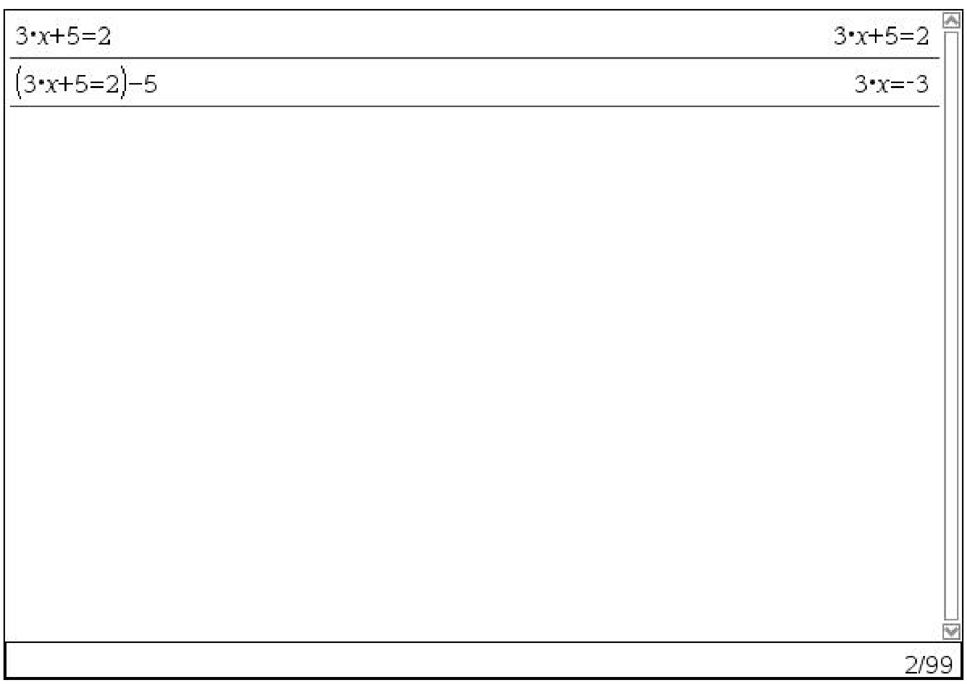


Figure 2. Subtracting 5 from both sides of an equation on the TI-Nspire CAS

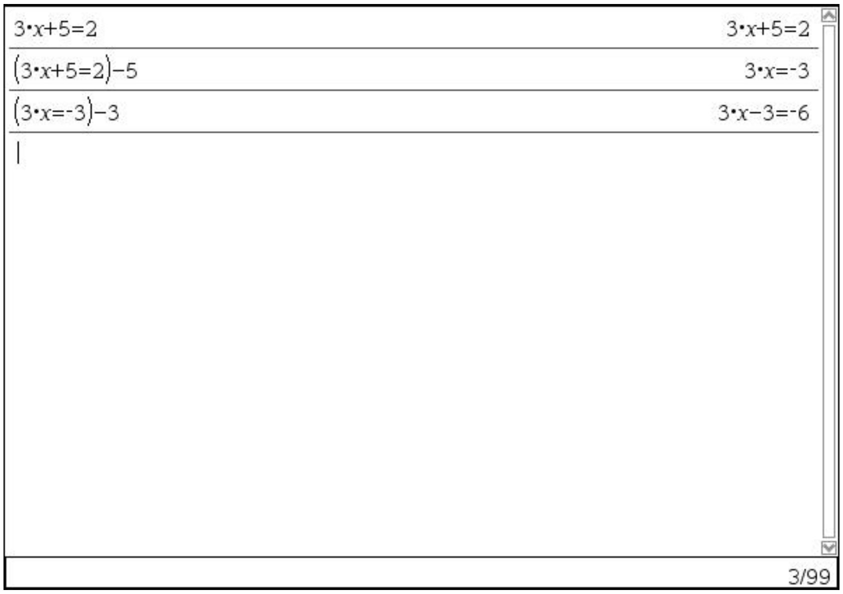


Figure 3. Subtracting 3 from both sides of the equation 3x=-3 using the TI-Nspire CAS

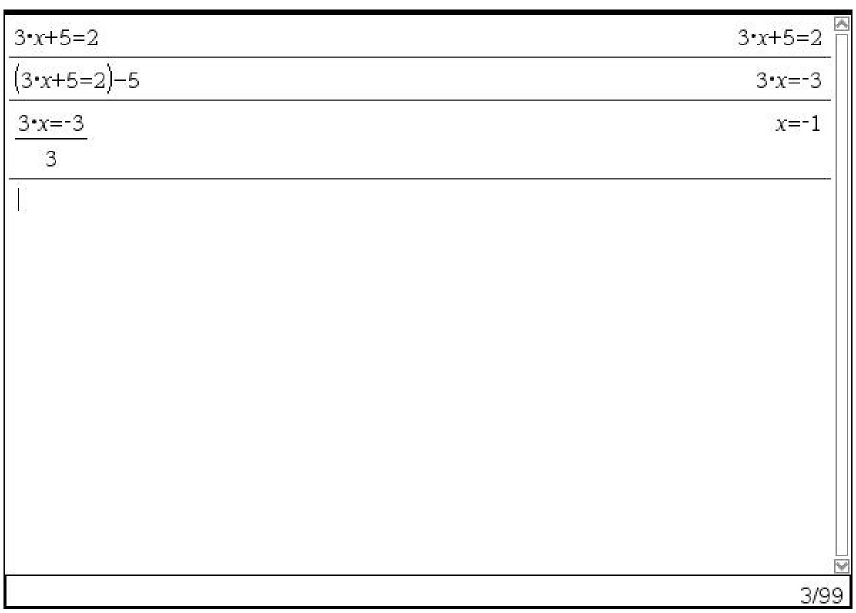


Figure 4. Dividing both sides of the equation 3x=-3 by 3 on the TI-Nspire CAS

The third role described by Heid and Edwards is that CAS enables students to generate many examples from which they can seek symbolic patterns. Figure 5 provides an example of how CAS could be used to help students generate a rule for multiplying terms by formulating conjectures based on answers provided by the CAS.

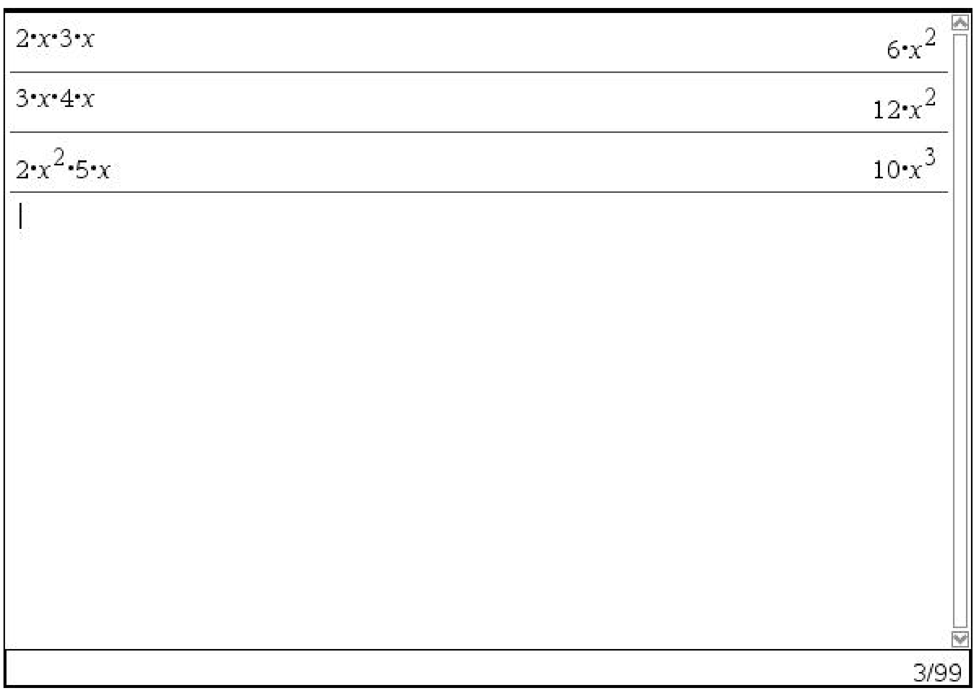


Figure 5. Multiplying like terms on the TI-Nspire CAS

Obstacles for CAS

There are many obstacles to CAS, especially as it pertains to its adoption and acceptance in the mathematics education arena in the United States. However, in this section I plan on discussing those that are related to the student and teacher use of CAS. One of the disadvantages of learning with CAS is that students have to learn how to use the device. Some students are apprehensive about learning a new technology and this could have an impact on their ability to learn (Artigue, 2002; Lagrange, 1999). In addition, studies have shown that students have different adoption levels of technology and that some students prefer to use pencil and paper (Lagrange, 1999; Pierce & Stacey, 2001b, 2002).

Pierce and Stacey (2002, p. 3) developed a framework for the effective use of CAS (Table 3). In this framework, effective use is divided into both a technical and personal aspect. The technical use of CAS deals with being able to use it as a tool, whereas the personal aspect takes into account student attitudes as well as how they used the CAS.

Table 3

Framework for Effective Use of CAS (Pierce & Stacey, 2002, p. 3)

Aspects	Elements	Common Instances
1. Technical	1.1 Fluent use of program syntax	1.1.1 Enter syntax correctly 1.1.2 Use a sequence of commands and menus proficiently
	1.2 Ability to systematically change representation.	1.2.1 CAS plot a graph from a rule and visa versa 1.2.2 CAS plot a graph from a table and visa versa 1.2.3 Create table from a rule or visa versa
	1.3 Ability to interpret CAS output	1.3.1 Locate required results 1.3.2 Interpret symbolic CAS output as conventional mathematics 1.3.3 Sketch graphs from CAS plots
2. Personal	2.1 Positive attitude	2.1.1 Value CAS availability for doing mathematics 2.1.2 Value CAS availability for learning mathematics
	2.2 Judicious Use of CAS	2.2.1 Use CAS in a strategic manner 2.2.2 Discriminate in function use of CAS 2.2.3 Undertake pedagogical use of CAS

For my study I examined several of the technical as well as personal aspects of effective uses of CAS. On the technical side, through task-based interviews I was able to look at students' ability to enter syntax correctly as well as use sequences of commands and menus proficiently. Through the task-based interviews and review of the generative activities, I investigate the students' functional use of CAS. By functional use I refer to students' ability to use the CAS to arrive at an answer.

In terms of personal aspects I decided to look at students' attitudes towards mathematics to determine if those attitudes differed in the control and experimental groups and whether their attitudes correlated with their achievement. Finally, with regard to 'judicious use of CAS' from the preceding framework presented in Table 3, I looked at the pedagogical use of CAS. I did this through having students explore patterns as well as make and test conjectures while completing the generative activities. As my study is restricted to using the symbolic manipulative aspect of CAS, I did not study students' ability to systematically change representations. Rather, I looked only at the student's use of the CAS to make and test conjectures.

3. Methodology

Introduction

I separated the data collection and analysis into two phases. For the first phase, I analyzed the items I intended on quantifying in order to answer my research questions. In the second phase, I used qualitative methods to analyze the clinical task-based interviews, focus group interviews, as well as teacher interviews. I used this qualitative data to not only help answer my research questions, but also to support results from the quantitative aspects of the analysis. I have included a detailed description of the research design process that I used including instruments, data collection sources, and methods of data analysis.

Research Questions

This research project originated from suggestions made by Rose Zbiek (2003) in Chapter 12 of *Computer Algebra Systems in Secondary School Mathematics Education*. Additional ideas came from the work on student attitudes by Pierce, Stacey, and

Barkatsas (2007) as well as Harper (2007). I synthesized aspects of the preceding research by asking the following research questions:

1. What is the effect of the use of the TI-Nspire CAS technology on eighth-grade Pre-algebra students' performance in the areas of integers, variable expressions, equations, and inequalities?

- Are there differences in performance between experimental and control groups?
  - Are there differences in performance between experimental and control groups due to gender, students with an Individualized Education Plan (IEP), students who are English Language Learners (ELL)?

2. Do students who develop the rules and procedures for algebraic (symbolic) manipulation using CAS accurately learn, use, and retain these rules/procedures as well as students who learned the same rules/procedures traditionally?

  - Are there differences in gain scores of experimental group over the control group on algebraic expectation?
  - Are there differences in gain scores of the experimental group over the control group on algebraic expectation for subgroups (sex, students with IEP, ELL students)?

- What are common student procedural/conceptual misconceptions with respect to algebraic insight?

3. Is there any correlation between eighth-grade pre-algebra students' attitudes toward mathematics and their achievement and gain in knowledge?

    - Is there a difference in this correlation with respect to students who did or did not use the TI-Nspire CAS?

Overview of Methodology

I used both qualitative and quantitative methods to address the research questions. An eight week quasi-experimental design experiment using four classes of 8th grade pre-algebra students was used. Scores from pre- and post-number expectation quizzes and algebraic expectation quizzes were used to answer the first research question. I received permission from Dr. Robyn Pierce of the Real World Problems and Information Technology Enhancing Mathematics (RITEMATHS) project in Australia to use two assessments, Numeric Expectation (Appendix D) and Algebraic Expectation (Appendix F), which were developed through the RITEMATHS program. Scores on test items from students' Chapter 3 (equations and inequalities) end of chapter test were also used to answer the first research question. In addition, data from student activities, teacher logs, as well as the teacher feedback were analyzed. I conducted clinical task-based interviews with selected students from the experimental group in order to investigate the mathematical understanding of concepts as well as the students' use of the TI-Nspire CAS. Student Mathematics and Technology attitude surveys (Appendix B and C) combined with student interviews, Chapter 3 test scores, and gain scores were used to address the second question. The pre and post attitude surveys have been modified from the original survey that was also developed for the RITEMATHS project. I received permission to use/modify this survey.

The Research Design Process

As part of my theoretical framework, using CAS to look at patterns, make conjectures, and test conjectures is best achieved in a classroom where students work in collaborative learning groups (Grouws, 2003; Heid & Blume, 2008; Kieran, 1992; Schoenfeld, 1992; Zbiek & Hollebrands, 2008). Encouraging discourse and communication of the patterns and discussion of rules through generative activities will also help students understand the mathematics and communicate their understanding (Ball, 2008; Cass, 2009; Gronewold, 2009; Kazemi, 2008). Therefore I created generative activities for both the control and experimental groups, so that students were learning the same material and concepts were reinforced through activities where all students were encouraged to explore patterns, make conjectures, and communicate with their peers in cooperative learning groups. The major difference was that the experimental group during the 8 week period was using the TI-Nspire CAS and the control group was not. This study did not look at a new curriculum, but rather assessed how students would perform if CAS was used as an amplifier in an existing curriculum, which was the 8th grade pre-algebra curriculum (Appendix A) (Chapin, Illingworth, Landau, Masingila, & McCracken, 2001).

Choosing Content for Instruction

As the intention was to look solely at the symbolic manipulation capabilities of the TI-Nspire CAS I used research on CAS in learning algebra in order to determine topics of the existing 8th grade pre-algebra curriculum where understanding symbolic manipulation was taught. I looked at research on teaching algebra to lower secondary students as well as research on effective uses of CAS. I combined the CAS research with research on student success in algebra to gain insight into key objectives to include in my study.

Pilot Study

Prior to this study I carried out a pilot study in order to investigate the effects of the use of TI-Nspire Computer Algebra System (CAS) on student learning in mathematics classes. All students in an integrated algebra geometry course used the TI-Nspire CAS. Qualitative data were collected and analyzed. This included student journals, teacher logs, video taped class lessons, and audio taped student interviews as well as actual student work.

The goal was for the data collected and analyzed to provide useful information about student use of this technology and its effects on at least some aspect of students' learning of mathematics. The intent of the pilot study was to gain insight into student use of the technology. The results of my pilot study helped to inform me about topics which could be studied using CAS technology as well as methods appropriate for relevant data collection. Overall, most students were able to use the technology to complete generative activities in their groups. Although some students were hesitant about using the technology, when they would get stuck using paper-and-pencil methods, they would ask group members who were more comfortable with the TI-Nspire CAS for help. Most students did like the way the CAS technology made things look. In fact during one group interview students commented that using the calculator to solve geometry problems involving surface area and volume actually helped them to memorize the formulas. When asked how, Joe replied, "when you type it in again and again, it keeps it the way you plugged it in, so you can see it that way and remember it" (Gantz, 2008, p. 16).

From the data I analyzed during my pilot study, I found supporting evidence that using the TI-Nspire CAS had a positive effect on my students' understanding of solving equations, using parentheses, and understanding equivalent operations (e.g., dividing by 2 and multiplying by ½). Therefore, CAS does have a positive effect on student learning of algebraic rules and symbolic manipulation for some students for at least some topics. Several students did comment on how using the TI-Nspire CAS helped them to understand the procedures used in solving an equation, or solving for a specific variable in an equation. However, I did not test retention of these skills and procedures without a calculator. The data I collected did not clearly demonstrate to what extent CAS had helped and whether students' knowledge had reached that of a conceptual level. During coding I took note of which students appeared to be using the TI-Nspire CAS regularly versus not. From comparing student work and video analysis I realized that there were distinct levels of student adoption of the TI-Nspire CAS; however there was not enough consistent data to compare student adoption levels to achievement.

Design of the Study

This study employed a quasi-experimental design using as independent variables the forms of instruction. An experimental group (instruction using TI-Nspire CAS technology) versus control group (traditional instruction) using a pre-test/post-test assessment design was utilized. A mathematics and technology pre-attitude survey (Appendix B) was given to gather baseline data on student attitudes about mathematics. A post-attitude survey (Appendix C), with additional questions specifically on the use of the TI-Nspire CAS was given to the experimental group at the end of the 8 week period.

There was a pre-quiz on number expectation and algebraic expectation in order to compare pre-test to post-test scores. The same number expectation quiz was re-administered at the end of Chapter 2, and the algebraic expectation quiz was re-administered at the end of Chapter 3. There was also a comprehensive post-test at the end of Chapter 3: Equations and inequalities that had questions where students wrote out steps as well as problems set in a contextual setting.

Student participants were 8th graders, thus approximately 13-14 years of age. All students who were enrolled in pre-algebra at a suburban public school in northern Virginia were approached to participate in this study. There were a total of 4 classes able to participate in the study and two of the classes were co-taught. Three teachers--Annabree, Brenda, and Cathy--took part in the study. The same co-teacher (Cathy) worked with the other two teachers. I could not randomly select control and experimental groups due to teacher schedules as well as which sections were to be co-taught. Thus, Annabree taught two experimental groups and one control group. Brenda taught one control group, and Cathy co-taught with one of Annabree's experimental groups as well as Brenda's control group. The content for both the control and experimental groups was Chapter 2: Integers and variable expressions and Chapter 3: Equations and inequalities from Middle Grades Math Course 3 (Chapin et al., 2001).

Procedure

I submitted my proposal to obtain permission from both George Mason University's (GMU) Human Subjects Review Board (HSRB) and from the superintendant of the public school district where the research was to be conducted.

Once permission had been granted by both GMU HSRB and the intended research site, I met individually with teachers to discuss the research project and receive teacher consent to take part in my study. I also discussed the purpose of my study and the design of the classroom I envisioned for the use of the generative activities. I asked the teachers to discuss the purpose of this study and what was involved with their classes and had them distribute and collect informed consent and assent forms (Appendix M). I already had a close working relationship with all three teachers and I had already spoken with them about my research to make sure I had initial interest. I asked to be introduced to each of the classes so students knew who was conducting this research and to expect to see me in their classroom from time to time, and I explained why I was conducting this research. In the experimental group I handed out the technology to the students.

Population and Sample

This study included 36 eighth grade students (26 female, 10 male) whose ages range from 13-14. Approximately 11% of students are black and 17% Hispanic. There are 10 students (28%) who have an Individualized Education Plan (IEP) and 7 students (19%) in the English Language Learners (ELL) program. Students were chosen based on their enrollment in 8th grade pre-algebra.

Three teachers participated in this study. All are Caucasian females between the ages of 24 and 35. Teachers were chosen because they taught 8th grade pre-algebra.

Annabree and Brenda both were full time math teachers with Master's degrees in Mathematics Education. Cathy co-taught with both Annabree and Brenda. Students from all four sections of 8th grade pre-algebra participated in this research. The curriculum map (Appendix A) lists the course outline for the material of the two chapters used in this study; Chapter 2: Integers and variable expressions and Chapter 3: Equations and inequalities of Middle Grades Math Course 3 (Chapin et al., 2001)

Instructional Methods and Materials

Experimental group. The experimental group used generative activities that were developed and modified in collaboration with the teacher participants which utilized the capabilities of the TI-Nspire CAS. These generative activities were intended to encourage students to find patterns as well as make conjectures and communicate ideas mathematically in cooperative learning groups. Upon completion of the 8 weeks, students in the experimental group had the opportunity to participate in clinical task-based interviews as well as focus group interviews.

The experimental teacher was given a TI-Nspire CAS handheld as well an overhead screen to use for modeling in the classroom. Common free time as well as e-mail communication was utilized to discuss student progress on the activities as well as student use of the technology and suggestions for modifications of activities. All teachers were able to preview and request changes to activities prior to instruction. Teachers of experimental groups were given a log book to record observations.

Each student was given a TI-Nspire CAS calculator to take home and use in the classroom. As this was a new technology, students required instruction on a few keystrokes they would need to know in order to complete tasks, so the teacher went through limited instruction with students. The instruction included setting up and using the calculator screen and inputting expressions and equations. Students were given a generative lesson in the form of a handout. The students in the experimental group utilized the TI-Nspire CAS to answer questions. They were then asked a question where they had to explain in words any patterns that they noticed and generalize rules for that instructional unit. Students worked in small groups and discussed their answers with the other members of their cooperative group. The groups discussed their responses together and as a class.

Control group. The control group received traditional instruction, which varied slightly between teachers, but generally consisted of teacher notes which included worked examples and encouraged student participation. The control group also used generative activities that were developed and modified in collaboration between the researcher and the teacher participants. These activities were intended to develop the skill of finding patterns as well as making conjectures and communicating ideas mathematically in cooperative learning groups. These activities were completed without the use of TI-Nspire CAS technology. Students from both the experimental and the control groups took the same quizzes and tests. Teachers of control groups were also given a log book to record observations.

Data Collection

Table 4

Data Sources

Independent Variable	1. Instruction using CAS	2. Instruction using Traditional Instruction
Dependent Measure 1	Number Expectation Quiz	Number Expectation Quiz
Dependent Measure 2	Algebraic Expectation Quiz	Algebraic Expectation Quiz
Dependent Measure 3	Chapter 3 Test	Chapter 3 Test
Dependent Measure 4	Mathematics and Technology pre-attitudes survey	Mathematics and Technology pre-attitudes survey
Data Source	Teacher interview, logs, observations	Teacher interview, logs, observations
Data Source	Student interviews	Students interviews

In Table 4 there is a list of the data sources collected and analyzed followed by a brief description of each source.

Quantitative Assessments

Mathematics and technology attitude survey. A student pre-attitude survey (Appendix B) was distributed at the beginning of the school year to determine student's attitudes and comfort with mathematics, pattern finding, writing, and technology. A similar survey (Appendix C) with four additional questions specifically pertaining to the use of CAS was distributed at the end of the eight week period to the experimental group in order to determine if there were any differences noted, and whether there were differences in achievement that could be related to attitude.

Number expectation quiz. A student number expectation quiz (Appendix D) was administered to both experimental and control groups at the beginning of the school year in order to get baseline data on student number sense pertaining to equivalence. The number expectation quiz consisted of twenty-two questions containing problems from topics in Middle Grades Math Course 3 (Chapin et al., 2001) Chapter 2 (including questions on the order of operations and rules of exponents). The quiz was given as a timed Power Point presentation. Students were not allowed to use calculators and were given approximately 10 seconds to circle an answer (Appendix E). The number expectation quiz was not to be shown to or returned to students. The same quiz was re-administered after the completion of Chapter 2 in order to compare gain scores.

Algebraic expectation quiz. A student Algebra Expectation Quiz (Appendix F) was administered to both experimental and control groups in order to get baseline data on student number sense pertaining to equivalence. The Algebraic Expectation Quiz consisted of twenty-five questions containing problems from topics in Middle Grades Math Course 3 (Chapin et al., 2001) Chapter 3 - including questions on combining like terms and rules of exponents. The quiz was given as a timed Power Point presentation. Students were not allowed to use calculators and were given approximately 10 seconds to circle an answer (Appendix G). The Algebraic Expectation Quiz was not to be shown to or returned to students. After the Chapter 3 test was administered, the same Algebraic Expectation Quiz was re-administered in order to compare gain scores.

Chapter 3 comprehensive test. This test (Appendix H) was administered to both the experimental and control groups. Test items included free response items where students had to show their work as well as problems which were more conceptual in nature. This assessment allowed for a comparison of achievement scores and particular test items were analyzed in order to gain understanding of student procedural and conceptual understanding.

Qualitative Assessments

Clinical task-based interviews. Clinical content or task-based interviews provide a structured mathematical environment that can to some extent be controlled (Goldin, 2000). I also realized that every student's understanding is unique. The best way for me to get at a particular student's understanding was not merely by a test score, but by their responses to clinical content task-based questioning which involved open-ended questioning and probing of their understanding. Although the teachers in both the experimental and control groups would perform some form of instruction on the content, ultimately each student would be constructing his or her own mathematical knowledge. In addition, I knew I could not control for how a particular problem would be interpreted by a student. Therefore, the only way I could begin to appreciate the depth of understanding a student had was to have them explain their work during clinical task-based interviews.

My goal in the task-based interviews was to get at an understanding of what the student was thinking and understanding about problem solving. I was also looking to see whether students from the experimental group made use of the TI-Nspire CAS technology--as I was hoping to see if and how the use of the TI-Nspire CAS influenced students' understanding. I built questions into my clinical task-based interviews as well as my focus group interviews that would address this (Goldin, 2000; Heid, Glendon, Zbiek, & Edwards, 1998).

The data gathered from these clinical task-based interviews provided a detailed qualitative report of students' work as well as their algebraic insight and understanding. Although this data does not provide results which can be generalized, the scripts of the interviews themselves are "sufficiently detailed to enable other researchers to conduct "the same" or structurally similar interviews with other subjects" (Goldin, 2000, p. 524).

I used Goldin's guidelines in creating my interview protocol (see Appendix I) (1997; Goldin, 2000). The first step in a task-based interview is to pose the question. I gave the student time to work the example or examine and explain the example. If the student did not know how to begin, I used heuristic suggestions to prompt them to think independently about the problem. For example, some prompts used were; "What does solving mean?", "What should your problem look like when you are done?" , "How do you know when you are done?" I made use of guided heuristic suggestions to ask about the student's process and reasons for choices if not initially provided. For example; "Can you tell me what you did?", "Why did you do that?", "What were you trying to do?" Lastly I asked exploratory (metacognitive) questions to determine why the student did what he/she did or why he/she selected a student's work as correct and ask if there were other ways he/she could have solved the problem. For example, I asked "How else could you have solved this equation?"

I used the clinical task-based interviews (Appendix I) not only as a research instrument, but also as an assessment tool. In order to design this research instrument, I had to consider the types of problems that students had been working on during their generative activities. I needed to create problems which would be similar or equal to those students would see on their test. I gave each student a group of three problems of differing difficulty that provided students with the opportunity to demonstrate their ability and understanding of how to solve both equations and inequalities. This enabled a heightened understanding of students' number sense, understanding of operations, understanding of the equal sign, and understanding of equivalence. I then had to create my line of questioning so that I could keep my interview to my script as much as possible. However, I had to build in contingencies for students who were not able to complete a task or asked for assistance in problem solving.

I also decided to give students a task where they were given three worked problems and they had to decide which were correct and which were not. This task would enable me to analyze student understanding of a "correct method" as well as their understanding of a solution. I had to be cautious during the interview process to allow the student ample time to explain his or her reasoning and not make the student feel there was any one right method. I created my interview protocol to get at the students' thinking with the awareness that I wanted to avoid leading questioning (Heid et al., 1998). As students compared and contrasted worked examples their explanations allowed me to see signs of student conceptual understanding. These additional tasks also provided the opportunity for students to communicate their understanding of operations, the order of operations, inverse operations, equivalent expressions, the distributive property, and the equal sign.

The next task was designed to examine whether students understood the syntax of the CAS calculator. As students in the experimental group were encouraged to use the TI-Nspire CAS to practice and check their solutions, I wanted to give an example (a screen capture) of a worked example from the TI-Nspire CAS and have the students interpret the display. This allowed me to understand to what extent the student was familiar with and understood the display and symbols used on the TI-Nspire CAS.

For the last task, I wanted to address the fact that students often use the phrase "isolate the variable" when asked to solve an equation with one variable for the value of the variable. I introduced a second variable into an equation to determine whether students could use what they knew about "isolating the variable" in an equation with one variable to isolating a specific variable if there was more than one variable in the equation. Students were asked to isolate the y in the linear equation $6x + 3y = 12$. They had not yet been introduced to equations with two variables, so I was curious as to whether students would be able to do this type of problem with or without the use of the CAS.

Using task-based interviews has several limitations. The biggest limitation in my case is student participation. Although I had 36 students participating in my study, I was only focusing on the nineteen students in the experimental group for the task-based interviews. The only time I was able to meet with students for interviews was before or after school. The students in this study were eighth grade pre-algebra students. Many of these students have difficulties in mathematics, thus being asked questions in a small setting was uncomfortable to many. For this reason I allowed up to three students at a time for the clinical task-based interviews. In addition, some students in the study were involved with extra-curricular activities, so scheduling an interview outside school hours was also difficult. Thus, I realized I would most likely be limited in the number of participants for the task-based interviews.

In addition, although I had only been in each classroom a couple of times, students did not really know me very well. Therefore, although I had a good working relationship with the teachers, it was difficult to “convince” students to participate in the interviews. This was also an issue with participation in my study. Although in the four classes there were a total of 70 students, only 36 volunteered to take part in the study.

Two of the teachers made comments about the females being more organized about getting their informed consent forms turned in. Although the population of males to females in the classes was roughly 50-50, in this study 72% of the participants were female.

The teachers also said that the population of students was one where few take advantage of after school opportunities for extra help or even opportunities to get bonus points for test corrections. These are social conditions which impacted my data that I would have to take into consideration for my analysis. In order to encourage student participation, as clinical task based interviews were conducted on a volunteer basis, I did offer small prizes (approximate value $1.00) as a token of appreciation for their time.

Although I was not the students' math teacher, students did know I was a teacher in the school and did speak with their teachers. Some students may have seen these interviews as a school/classroom activity and answered questions the way they knew their math teacher would have wanted it answered. Some students may have chosen to use the CAS or felt they needed to use this technology because they were aware that I was doing research on the use of the TI-Nspire CAS and thought that I expected them to use the TI-Nspire CAS calculator to solve problems.

During the clinical task-based interviews I attempted to position the camera so that it could capture the student/ students as closely as possible. I also made a digital audio recording of the interviews as a reassurance that the audio quality was clear. In transcribing the audio in conjunction with the video, I added observations about facial expressions or gestures made by students during their session. I also looked at what the student showed in terms of work (without prompting) and made note of the confidence with which the student answered the questions. I noted if the student decided to work out the problem solely by hand or whether they chose to use technology to aid in problem solving. Students had a 4 function calculator as well as a TI-Nspire CAS available. I assumed all students had some understanding of how to solve an equation, so I looked at each student's work and asked questions to determine to what degree the student understood his/her steps. I tried to use scaffolding in my questioning as I looked at different aspects of the students thinking as it pertained to understanding operations and inverse operations, the equal sign, as well as equivalent expressions.

I video taped the clinical task-based interviews with the experimental group students. During the interview students explained their understanding of how to solve problems. During these interviews I allowed them the use of a four function calculator as well as the TI-Nspire CAS at their discretion in order to assist them in problem solving. Some of the problems I used were similar to those on the students' quizzes and test from Chapter 3. However there were also items which were meant to challenge student procedural understanding, conceptual understanding, and understanding of the CAS display output. Students explained how to solve problems. In accordance with typical clinical task-based interviews, once the students explained their understanding of what they did to solve a problem, if a student indicated an incorrect method, I sometimes attempted to elicit understanding through probing and scaffolding (Appendix I).

Focus groups. I conducted two focus group interviews. The feedback I received from students during video-taped focus group interviews was used to better understand how the technology was utilized, determine how students collaborated and used the technology as a pedagogical tool while completing the generative activities, and gather general attitudes about the technology. The students volunteered to take part in the interviews. Pizza was provided as a snack during these interviews which took place after school hours. The two focus group interviews were transcribed. The data were coded and analyzed for common themes.

Teacher feedback. Teachers used logs, electronic mail, and verbal communication to inform me about observations they had made. The teachers involved in the study were asked to keep a log and share information with me about their lessons with regard to student involvement and understanding of the material as well as recommendations to improve further research on the use of CAS technology.

I made notes and wrote down questions which arose over the duration of the study. For fidelity of instruction I spoke with Annabree, Brenda, and Cathy. I also interviewed Annabree and I spoke with Cathy who was a special education teacher who co-taught with both Annabree and Brenda in order to determine whether teachers were teaching in similar manners and were implementing and using the generative activities in a similar manner.

Data Analysis

Quantitative Assessments

The pre- and post-number expectation quiz and algebraic expectation quiz for each student were compared. I calculated the Hake gain score from the students' pre-number expectation to their post-number expectation quiz as well as their gain from the pre-algebraic to post-algebraic expectation quiz. A Hake gain score is calculated as follows: (post test score – pre test score) / (total possible points – pre test score). I also compared student achievement scores from their Chapter 3 test by calculating the overall mean and compared the means and standard deviations of both the experimental and control groups. I also compared results by disaggregating the data by several factors; gender, students with Individualized Educational Plans (IEP), students who are English Language Learners (ELL).

An analysis of variance (ANOVA) for Chapter 3 Test for Between Subject Effects on Group and Baseline Score was used to determine whether there was a significant increase in achievement for those in the experimental group over the control group. I conducted a regression analysis to further test for differences between classes with respect to instructional methods.

The video taped clinical task-based interviews transcriptions helped triangulate data. The student pre- and post-attitude surveys were compared to see if attitudes changed with respect to mathematics, pattern finding, writing, and technology after the use of CAS technology as well as to see if there were differing attitudes with respect to the control and experimental groups and if this could have affected achievement.

Qualitative Assessments

Clinical interviews. Individual clinical task-based interviews with students where they explained how to solve problems were used to determine procedural and conceptual understanding. In order to assess procedural knowledge skills as well as student conceptual understanding I took specific items from their Chapter test and compared them to the answers students provided during these clinical task-based interviews as a form of cross checking for student understanding. I looked for evidence of student retention and correct application of procedural skills, as well as their ability to complete test items which required conceptual understanding of the concepts studied.

I looked qualitatively at the written responses to test items and reviewed the work (steps) they showed in problem solving in order to assess a students procedural and conceptual understanding. I made use of a scoring rubric (Table 5) to determine a students' score based on the Chapter 3 test and re-assessed the score based on the clinical-task based interview data. Specifically, in addition to the student overall

Table 5

Scoring Rubric

Score	Description
0	The student showed no understanding of procedural skills.
1	The student showed some understanding of the procedural skills.
2	The student appeared to understand the correct procedure, but made an arithmetic error.
3	Student showed understanding and accuracy of procedural skills.
4	Student showed understanding and accuracy of procedural skills as well as procedural understanding
5	Student showed understanding and accuracy of procedural skills as well as procedural and conceptual understanding

percent achievement on their Chapter 3 test, I looked at how students fared on particular test items. A subset of five procedural understanding questions (problems 3, 7, 12, 16, and 20), from the Chapter 3 comprehensive test were used to assess students' procedural understanding. Scores from each were compared to determine accuracy of scoring on the basis of the test alone. Individual clinical task-based interviews with students where they explained how to problem solve were used to determine conceptual understanding.

The conclusions I reached were based on the responses to the questions that I asked. Some students may have been able to answer the questions I posed, but have some other misunderstanding that was not noticed through my questioning and the responses they gave. I tried to design tasks that all students should have been able to attempt and to solve to some degree. Although I used an interview protocol, I will include suggestions as to how I could have improved my line of questioning in the next chapter.

Teacher feedback. Teacher feedback was used to determine methods for improving this research design as well as any evidenced accounts of support for or against the use of CAS technology in instruction. Teacher and researcher comments from interviews and logs were also used to triangulate data with respect to students' procedural and conceptual understanding through the use of the TI-Nspire CAS.

Reliability or Fidelity of Treatment

I communicated with the teachers involved in instruction a minimum of two days prior to instruction using a generative activity in order to discuss any concerns. We communicated on at least a bi-weekly basis to discuss instructional activities and give feedback. Teachers were provided with the generative activities for both the CAS and non-CAS groups. I worked with teachers to modify the generative activities that students would use prior to its use in class. Teachers kept logs or communicated with me in order to share their results on each activity as well as things they noticed. This procedure helped ensure the instruction of both groups was implemented faithfully. It also allowed me to emphasize that I was trying to control for the TI-Nspire CAS so the expectation of students working cooperatively with one another and communicating their answers was achieved by all students.

Scoring Procedures and Reliability of Scoring

Number expectation and algebraic expectation quizzes. On these quizzes students were asked to compare a student response to a textbook answer and determine the equivalence of the two responses. Students were to circle one of the following choices; definitely right (dr), probably right (pr), no idea (ni), probably wrong (pw), and definitely wrong (dw). A question was considered correct if the question was a true equivalence and the student circled dr or pr, or if the question was false and the student circled dw or pw. I used a weighted score [dr (+2), pr (+1) , ni (0), pw (-1), dw (-2)] that accumulated to a certainty index for each question.

Mathematics and Technology Attitude Survey. Scores from pre-attitude survey and post-attitude surveys from students in the experimental group were compared to note any changes in students' attitude with respect to their affective engagement (AE), behavioral engagement (BE), confidence with technology (TC), and mathematics confidence (MC). A category mathematical technology (MT) was also created for the post-test attitude survey which specifically referred to using the TI-Nspire CAS. Survey results were evaluated on pre and post measures by looking at the distribution of the responses across items as well as the average score per item. A Likert-type scale was to be used for each subcategory: AE, BE, TC, MC, and MT. Four questions from each category were scored for a total of 20 points possible per category. Students earning above a 17 were considered to have a very positive attitude, between 13 and 16 had a positive attitude, and below 12 had a neutral to low attitude.

Table 6

Timeline

October	Activity
Week 1	Researcher is introduced to classes Distribution of consent forms Collect consent forms Create name code Distribute TI-Nspire CAS Generative activity #1 (3.1) Generative activity #2 (3.2) Numeric Expectation Post-Test
Week 2	Communicate with teachers of experimental and control groups. Generative Activity #3 (3.3) Generative Activity #4 (3.4)

Week 3	Generative Activity #5 (3.6) Communicate with teachers of experimental and control groups.
Week 4	Generative Activity #6 (3.9) Communicate with teachers of experimental and control groups.
November Week 5	Generative Activity #7 (3.10) Communicate with teachers of experimental and control groups.
Week 6	Chapter 3 Comprehensive Test Algebraic expectation quiz (post) Communicate with teachers of experimental and control groups
Week 7	Algebraic Expectation Post Test Generative Activity #8 & 9 (Review #1 & 2) Communicate with teachers of experimental and control groups.
December Week 8	Generative Activity #10 &11 (Review 3 & 4) Communicate with teachers of experimental and control groups.
Week 9	Post Attitude Survey for Experimental Group
Week 10-11	Student focus group interviews for experimental group Clinical task-based interviews
January	Transcribe focus group interviews
February	Transcribe task-based interviews Begin initial coding of interviews for themes Code, score, and input data on Mathematics and Technology post-attitude survey, number expectation quiz, and algebraic expectation quiz into spreadsheet.
March -June	Work on data analysis
July - December	Complete data analysis and writing results

Conclusion

The ability to obtain baseline data through the use of instruments which have been researched and tested through the University of Melbourne's RITEMATHS project allowed me to quantify levels of achievement in my experimental and control groups as well as compare gain scores. Comparing the baseline data from the attitude surveys coupled with the results from the focus group interviews did not necessarily allow me to determine the effect of using the TI-Nspire CAS on students' attitudes towards mathematics and technology, but it allowed me to see if there were any perceived or real differences. By comparing the data transcribed from video taped clinical task-based interviews combined with teacher observations and copies of actual student work, I was able to gain some insight into how students use the technology and its possible effects on their understanding of concepts as well as their ability to perform and retain procedures.

4. Results and Limitations

Introduction

This chapter contains the results of the data analysis--both qualitative and quantitative--conducted during this study. I start by discussing to what extent the research study went as originally conceived. I discuss the themes which arose through the analysis of the clinical task-based interviews and provide examples of student work as well as justification for an algebraic insight score based on their responses to task items, their Chapter 3 test, and generative activities. I then give an overall statement of the findings pertaining to the use of CAS with respect to procedural and conceptual understanding, and finally provide the results of the quantitative analysis as I discuss how those results relate to my initial research questions.

Faithfulness to the Intended Use of CAS

Interviews with students and teachers support the contention that students completed generative activities in class and that students in the experimental group were asked and reminded they were to be using the CAS calculator to complete the generative class activities. Meetings with students and teachers also corroborate that students did discuss their answers with classmates.

Initially students had difficulty inputting data on the technology and the teacher had to do a lot of scaffolding to get students to understand that they were supposed to be coming up with the rules themselves. The intent was for students to use the technology to develop the procedural rules by themselves. However, since students were having some difficulty, all teachers started their lessons with traditional instruction and then provided the generative activities for students to complete as an extension or amplifier of the traditional class lesson content. The generative activities provided students with the opportunity to look at patterns and make conjectures based on the patterns. However, if they understood the traditional lesson they may have "memorized" a procedure and used that instead. The examples in the generative activity nevertheless enabled students to experiment with different example types and provided a visual reinforcement to scaffold the procedural "rule" to the results of actual examples, thus amplifying or reinforcing student understanding. Consequently, the CAS was used as an extension/amplifier to the actual traditional lesson. I analyzed the use of the TI-Nspire CAS on facilitating student learning of mathematics while keeping the basic goals and sequence of the existing curriculum intact.

I had given all students their own handheld CAS calculator so they would have access to it whenever they wanted. My idea was that some students would play around with the CAS and I hoped to make it easier for them to learn to use the device. However, there were more problems with the physical use of the CAS than I had initially expected. I will speak in detail about some of these issues in the student comments section and I will discuss the results they could have had on this study in the conclusions section.

Teachers had administered a numeric expectation pre-test at the beginning of the year, and a numeric expectation post-test as students were beginning to work with the technology, to see what gains students had made with the instruction they had received prior to working with the technology. I was going to give another post-number expectation test after students completed the eight weeks to see if their number expectation increased with algebraic instruction, but unfortunately, time did not allow for this last test.

Conclusions: Research Question 1

Although there were originally 36 research subjects only 32 students were used in calculating the descriptive statistics for the Chapter 3 test. Two students' scores were outliers, both of whom belonged to the 'no accommodations' subgroup of the control group. Two other students' scores also had to be removed due to an incomplete data set. These students were both in the 'no accommodations' subgroup, however one was from the experimental group and one was from the control group.

When comparing the pre-algebraic expectation to the post-algebraic expectation for both the experimental and control groups, there was a positive correlation ($R^2 = 0.996$)

for both groups. I compared the achievement scores on the Chapter 3 test between the groups and there was little difference in the mean scores. Therefore, in order to determine whether students improved, I realized I would have to compare each student's Chapter 3 score to a baseline score. In order to provide basic baseline data to compare students in the control and experimental groups, I used the Algebraic Expectation pre-test. Instead of using the original -2 to +2 scale, which takes confidence into account, I used a more basic method to calculate the baseline data to compare the Chapter 3 test results. A point was given any time a student indicated "definitely right" or "probably right" for an item that was correct or "definitely wrong" or "probably wrong" for an that was not correct. I will refer to this as the baseline. The raw scores in Table 7 below indicate the average number of problems out of 25 that the students in each group got correct. Therefore, the gains in Table 7 represent the average number of problems by which each group improved. I calculated the gains by first looking at each student individually to determine the student's individual gain, and then I summed the gains by subgroups and divided by the total number of students in the subgroup to arrive at an average gain per subgroup.

A one way analysis of variance test was conducted to compare the Chapter 3 test scores for the experimental and control groups. There is no interaction between the baseline data and whether students were in the experimental or control groups, $F(1, 32) = 1.515$, $p > .05$.

Table 7

Descriptive Statistics for the Algebraic Expectation Pre and Post Test by Subgroups

Accommodations	Group	Raw Score Pre-test	Raw Score Post-test	Gain	*N*
No Accommodations	Control Group	8.625	10.250	1.625	8
	Experimental Group	6.778	8.556	1.778	9
	Total	7.647	9.353	1.706	17
ELL Students	Control Group	10.500	14.000	3.500	2
	Experimental Group	9.000	9.250	0.250	4
	Total	9.500	10.833	1.333	6
ELL Students (Hispanic)	Control Group	10.000	10.000	0.000	1
	Experimental Group	9.000	9.250	0.250	4
	Total	9.200	9.400	0.200	5
Students with an IEP	Control Group	7.750	8.500	0.750	4
	Experimental Group	7.200	9.600	2.400	5
	Total	7.444	9.111	1.667	10
Total	Control Group	8.643	10.286	1.643	14
	Experimental Group	7.389	9.000	1.611	18
	Total	7.938	9.563	1.625	32

Although the means for the experimental (M = 83.833) and control groups (M = 81.071) on the Chapter 3 test were relatively equal. I disaggregated the data into accommodation subgroups to answer my research question about subgroup performance (Table 9). Table 9 shows the descriptive statistics used to analyze whether there were differences in achievement on the Chapter 3 test between experimental and control

Table 8

ANOVA for Chapter 3 Test for Between Subject Effects on Group and Baseline Score

Source	*df*	*F*	η^2	*p*
Between Subjects				
Group (G)	1	0.201	27.706	0.661
Baseline(B)	12	1.515	209.274	0.227
G x B	1	.0.746	103.025	0.577
Error	14		138.119	
Total	32			

Table 9

Descriptive Statistics for the Chapter 3 Test by Subgroups (Outliers Omitted)

Accommodations	Group	Mean	*SD*	*N*
No Accommodations	Control Group	79.25	11.508	8
	Experimental Group	88.80	8.670	9
	Total	84.56	11.097	17
ELL Students	Control Group	87.50	9.500	2
	Experimental Group	73.50	5.679	4
	Total	78.17	9.754	6
Students with an IEP	Control Group	81.50	12.913	4
	Experimental Group	80.00	15.130	5
	Total	80.67	14.204	9
Total	Control Group	81.071	12.003	14
	Experimental Group	83.833	12.144	18
	Total	82.625	12.160	32

groups due to students with an Individualized Education Plan (IEP) or students who are English Language Learners (ELL).

In the ‘no accommodations’ subgroup, the experimental group had a higher mean score (*M* = 88.80, *SD* = 8.67) than the control group (*M* = 79.25, *SD* = 11.51). However, the two ELL students in the control group (*M* = 87.50, *SD* = 9.50) had higher scores overall than their four counterparts in the experimental group (*M* = 73.50, *SD* = 5.68). These subgroups were quite small.

To see any trends from the Algebraic Expectation pre-test data to the Chapter 3 test I graphed a scatter diagram, and ran a linear regression, however no relationship was found (*R* = -0.03). However, this baseline data did give me a point of comparison when discussing the results of the Chapter 3 test. Overall, the control group started with a higher average raw score on the Algebraic Expectation pre-test (*M* = 8.733) than the experimental group (*M* = 7.389). However, it was the experimental group which had the higher mean (*M* = 83.83) than the control group (*M* = 81.07) on the Chapter 3 test.

Therefore, although some differences in student performance were noted, there were no significant differences found in scores between the experimental and control groups with respect to achievement as based on the Chapter 3 test.

Conclusions: Research Question 2

The results from the Chapter 3 test and the Hake gain scores for algebraic expectation were used to determine whether students in one group learned and retained the rules/procedures better than another group. According to the results from an analysis of variance (ANOVA) test of between subject effects for the Hake gain score on Algebraic Expectation pre and post test (Table 10), there was a significant difference in gain scores on Algebraic Expectation between the experimental and control groups, $F(1, 32) = 12.368$, $p = 0.003$. The experimental group demonstrated a positive gain overall compared to the control group. This is also significant as a Hake gain score is a measure

Table 10

ANOVA for Hake Gains in Algebraic Expectation Pre and Post Tests

Source	*df*	*F*	η^2	*p*
Between Subjects				
Group (G)	1	12.368	0.126	.003
Baseline (B)	12	4.438	0.045	.005
G x B	4	5.670	0.058	.006
Error	14	0.010		
Total	32			

of student gain in knowledge and therefore accounts for pre-existing differences in students. Thus, students in the classroom which used CAS as an amplifier while learning algebraic concepts showed the potential for positive gains at the 95% confidence level.

There were significant differences in Hake gain scores in Algebraic Expectation with regards to the baseline score, $F(12, 32) = 4.438$, $p = .005$. There were also between subject effects for the baseline score and group with, $F(4, 32) = 5.670$, $p = 0.006$. Recall the baseline score was a pre-test used to establish pre-existing differences in the control and experimental group and was calculated using the 0 or 1 point scoring system, as opposed to the Hake gain which compared scores using the -2 to 2 point system.

In trying to address my research questions about effects on subgroups, when looking at Algebraic Expectation gain scores with respect to subgroups I realized that the 3.5 gain by ELL students (Table 7) in the control group was the sole contribution of one student. Upon further investigation, I noted that this student was the only non-Hispanic ELL student in this research project, therefore I decided to look at the data specifically with ELL Hispanic students.

Figure 6 shows the total number of students who made gains in the control versus experimental group. For example, the first column indicates that 14 students out of the 18 students in the experimental group (78%) did better on the post-test, whereas only 9 out of the 15 students in the control group (60%) showed improved scores.

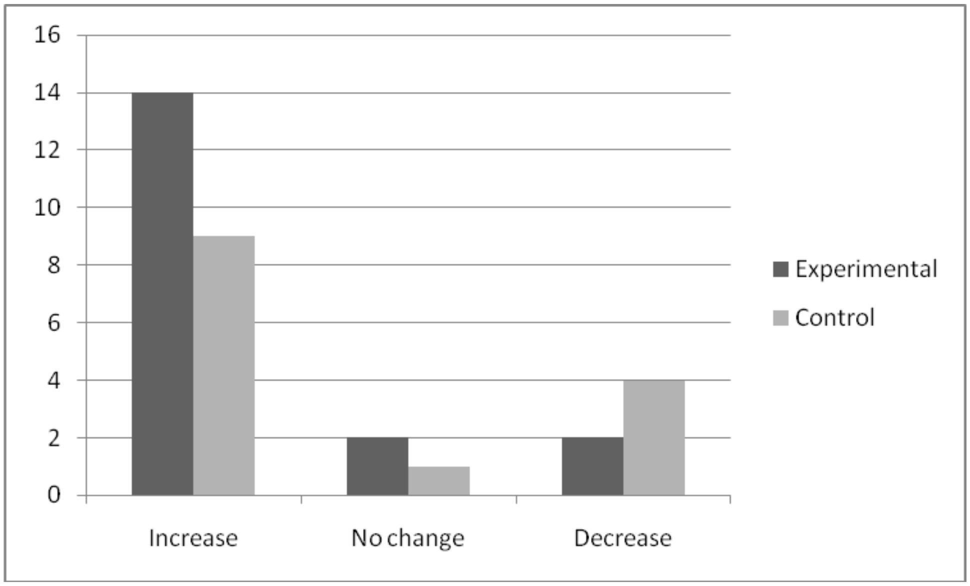


Figure 6. Total numbers of students with gains from Algebraic Expectation pre-test to post-test.

In order to answer my research question, I disaggregated the data with respect to each subgroup in order to see whether there were differences in gains with respect to student accomodation subgroups. This data is provided in Table 11. I also tried to determine if the gains appeared to be made equally by males and females. The figure below relates student gains based on gender. This chart indicates that approximately 82% of the females in the experimental group had gains as compared to only 62% of the females in the control group. In the experimental group, approximately 71% (5 out of 7) of the males in the experimental group had gains as compared to 50% (1 out of 2) in the control group.

Table 11

Number of Students Who Made Gains by Subgroups on Algebraic Expectation Raw Pre – Post Test

Accommodations	Group	Increase	No change	Decrease	*N*
No Accommodations	Control Group	5	0	3	8
	Experimental Group	8	1	0	9
	Total	13	1	4	17
ELL Students	Control Group	1	1	0	2
	Experimental Group	2	0	2	4
	Total	3	1	2	6
Students with an IEP	Control Group	3	0	1	4
	Experimental Group	4	1	0	5
	Total	7	1	1	9
Total	Control Group	9	1	4	14
	Experimental Group	14	2	2	18
	Total	23	3	7	32

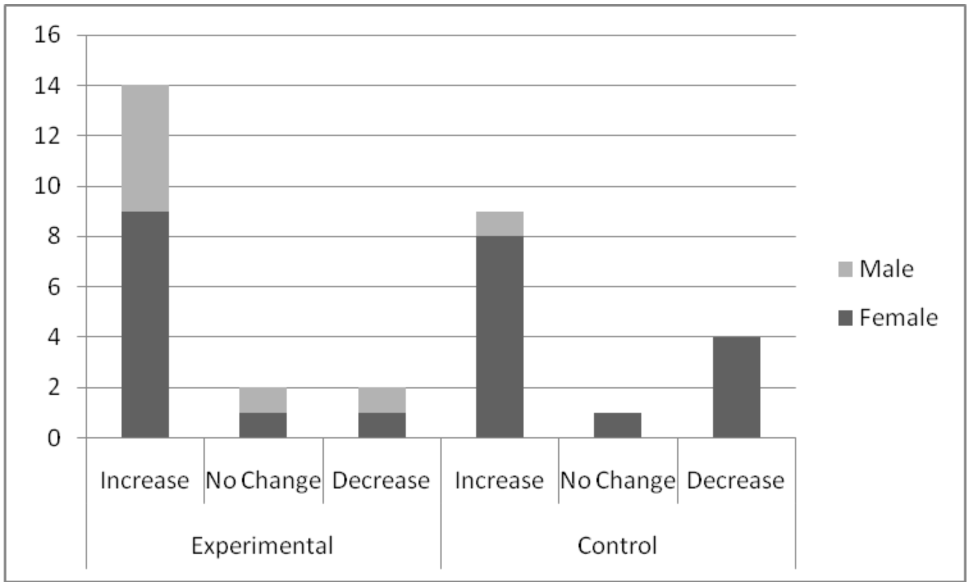


Figure 7. Total numbers of females and males with gains from Algebraic Expectation pre-test to post- test.

I conducted a chi-squared test on the number of boys and girls who had pre-test to post-test gains. I tested whether H_o: Gain scores of males and females are independent of whether the students were in the experimental or control group. I found χ^2 (1, N=23) = 1.7198, p = 0.189717. Therefore, since χ^2 (1, N=23) = 1.7198 < 3.841 the null hypothesis was accepted. Hence, gain scores of males and females are independent of whether the students were in the experimental or control group.

Observed

	Female	Male	
Experimental	9	5	14
Control	8	1	9
	17	6	23

Expected

	Female	Male	
Experimental	10.348	3.652	14
Control	6.652	2.348	9
	17	6	23

Figure 8. Observed and expected Chi-squared data for males versus female gains in algebraic expectation.

I also conducted a chi-squared test to compare the number of students who had pre test to post test increase, decrease, or no change with respect to whether students were in the control or experimental group. I tested whether H_o: Students having an increase, no change, or decrease in problems correct were independent of whether the students were in the experimental or control group. I found χ^2 (2, N=32) = 1.612, p = 0.4466.

Therefore, since χ^2 (2, N=32) = 1.612 < 5.991 the null hypothesis is accepted.

Accordingly, whether students increased their score, stayed the same, or decreased their score from the pre-test to the post-test, the results are independent of whether the students were in the experimental or control group.

Observed

	Experimental	Control	
Increase	14	9	23
No change	2	1	3
Decrease	2	4	6
	18	14	32

Expected

	Experimental	Control	
Increase	12.938	10.063	23
No change	1.688	1.313	3
Decrease	3.375	2.625	6
	18	14	32

Figure 9. Observed and expected Chi-squared data for pre test to post test changes in algebraic expectation based on whether students were in the experimental or control group.

One final chi-squared test was used to compare the number of students who had pre-test to post-test gains with respect to the student's subgroup. I tested whether H_o: Gain scores of subgroups are independent of whether the students were in the experimental or control group. I found χ^2 (2, N=23) = 0.0856 , p = 0.9581. Once again, since χ^2 (2, N=23) = 0.0856 < 5.991 the the null hypothesis is accepted. Therefore, student average gains from the Algebraic Expectation pre-test to the post-test by subgroups are independent of whether the students were in the experimental or control

Observed

	Experimental	Control	
No Accomodation	8	5	13
ELL	2	1	3
IEP	4	3	7
	14	9	

Expected

	Experimental	Control	
No Accomodation	7.913	5.087	13
ELL	1.826	1.174	3
IEP	4.261	2.739	7
	14	9	23

Figure 10. Observed and expected Chi-squared data for pre test to post test changes in algebraic expectation based on student accommodations.

group. Overall, there does not appear to be any evidence that suggests there are significant gains in the experimental group over the control group with respect to student accommodations.

Conclusions: Research Question 3

The results from the Mathematics Attitude Survey Pre-Test are plotted in the box plot in Figure 11. This box plot shows the differences in attitudes of the students in the experimental and control groups prior to use of CAS. The categories show the differences in the groups with respect to their affective engagement (AE), behavioral engagement (BE), confidence with technology (TC), and mathematics confidence (MC).

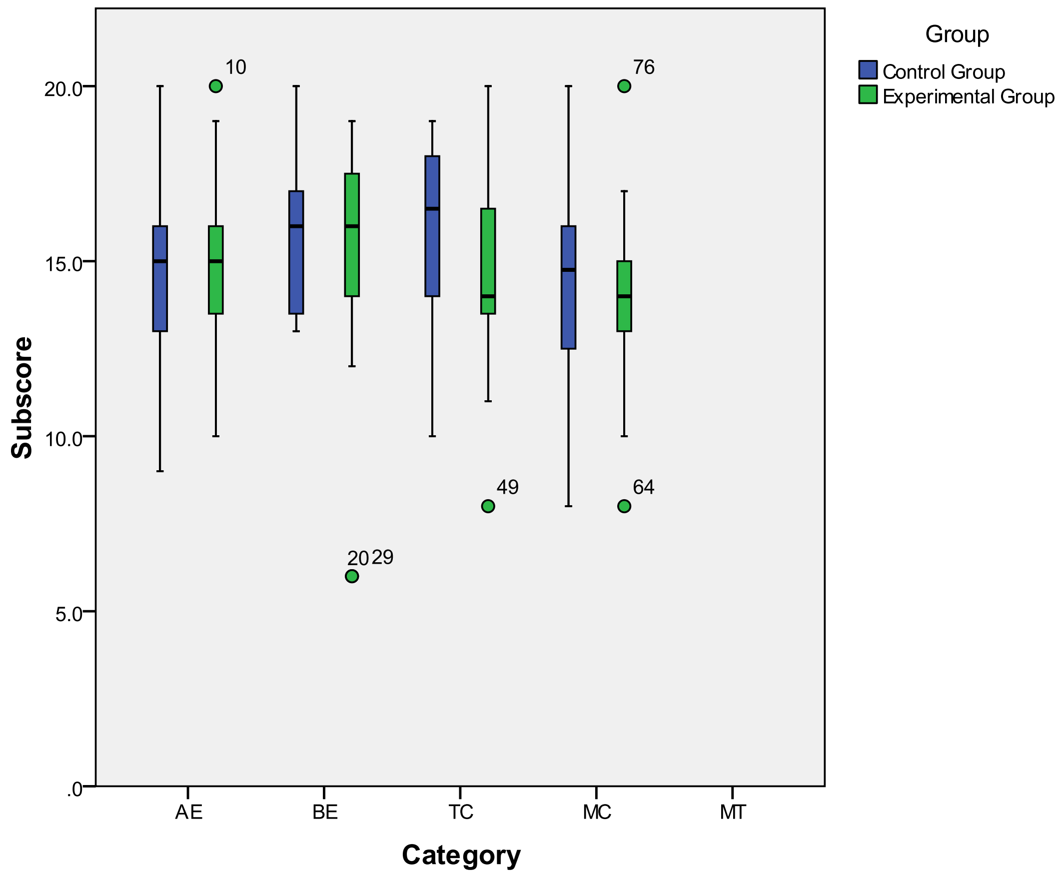


Figure 11. Mathematics and Technology Pre-Attitude Survey

The category MT was only used as a post-test item with the experimental group as it dealt specifically with attitudes pertaining to the TI-Nspire CAS. Four questions from each category were scored for a total of 20 points possible per category. Students earning above a 17 were considered to have a very positive attitude, between 13 and 16 had a positive attitude, and below 12 had a neutral to low attitude. The bar graph shows that the students' averages for both affective (AE) and behavioral (BE) engagement are relatively equal. This means that, in terms of the types of students in each group, the groups are fairly equal with regards to student attitudes and behavior such as perseverance and general interest in mathematics and its perceived benefits. It is important that the groups are similar with respect to their efforts and attitude towards math so that this could be seen as a controlled factor.

When comparing the mathematics pre-attitude surveys of the control and experimental groups, the mean scores for AE and BE were the same, and the experimental group had a 5% lower average than the experimental group for MC and a 10% lower average for TC (Figure 11). Perhaps the fact that these students showed a lower confidence level in both mathematics and using technology suggests that there was a confounding effect on the results of this study. However, as this research used a new technology and the experimental group did have a lower attitude about their confidence with math technology prior to the introduction of the CAS technology, the results indicate that increases in confidence with technology could be attributed to the use of the CAS.

Therefore, looking at the differences from the pre-test to the post-test (Figure 12) of the experimental group I found there was an increase of 10% in students' confidence

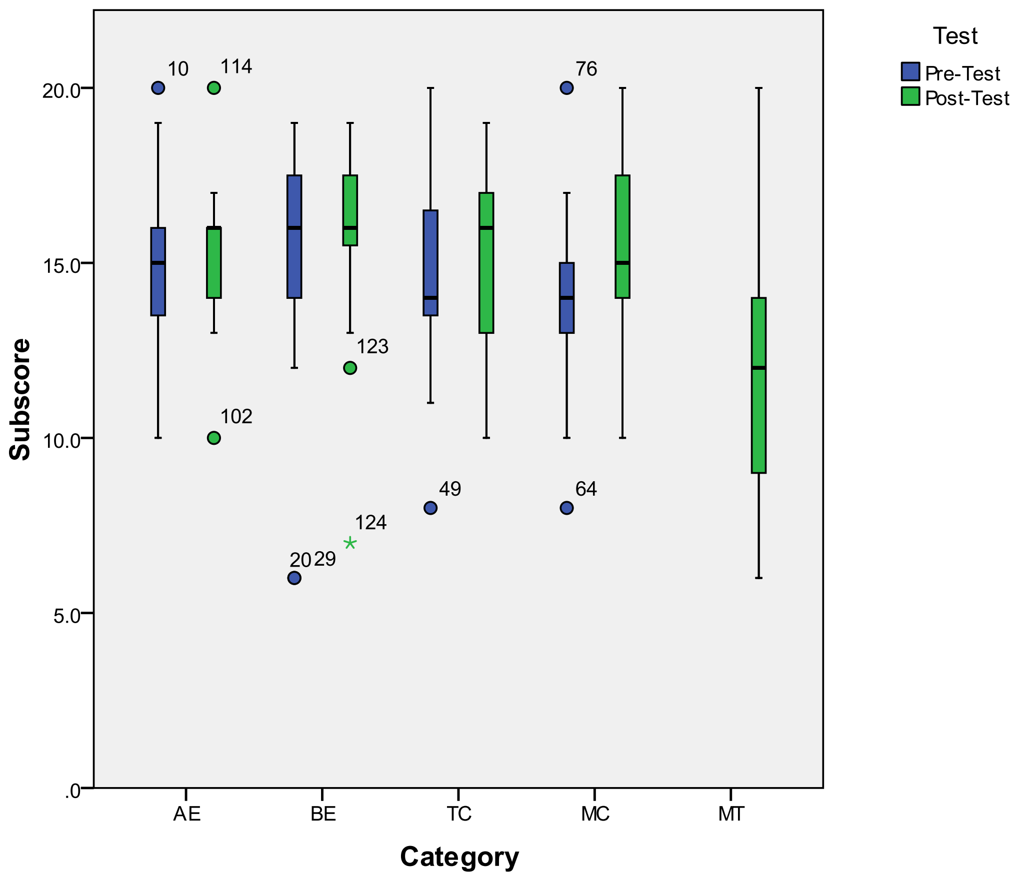


Figure 12. Experimental group's pre-attitudes survey versus post- attitudes survey.

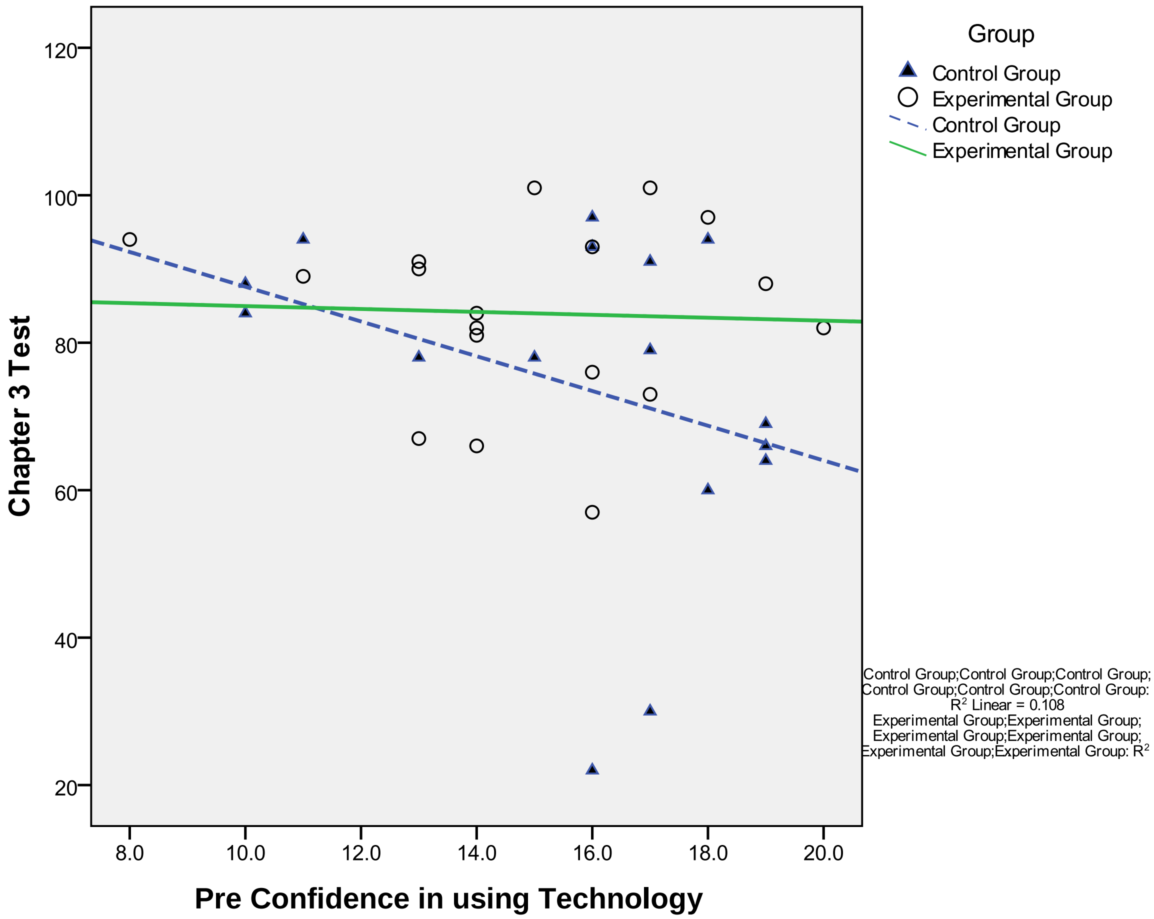


Figure 13. Chapter 3 test versus pre confidence using technology.

with technology scores. I analyzed these data using linear regressions. However, before I calculated the regression I excluded the following outliers (AE 10, BE 20, 29, TC 49, MC 64,76) from each category from Figure 8.

The students' pre-confidence in using technology was the strongest unique contribution to explaining achievement scores on the Chapter 3 test when the variance in AE, BE, and MC are controlled. However it did not make a significant unique contribution (p = .172).

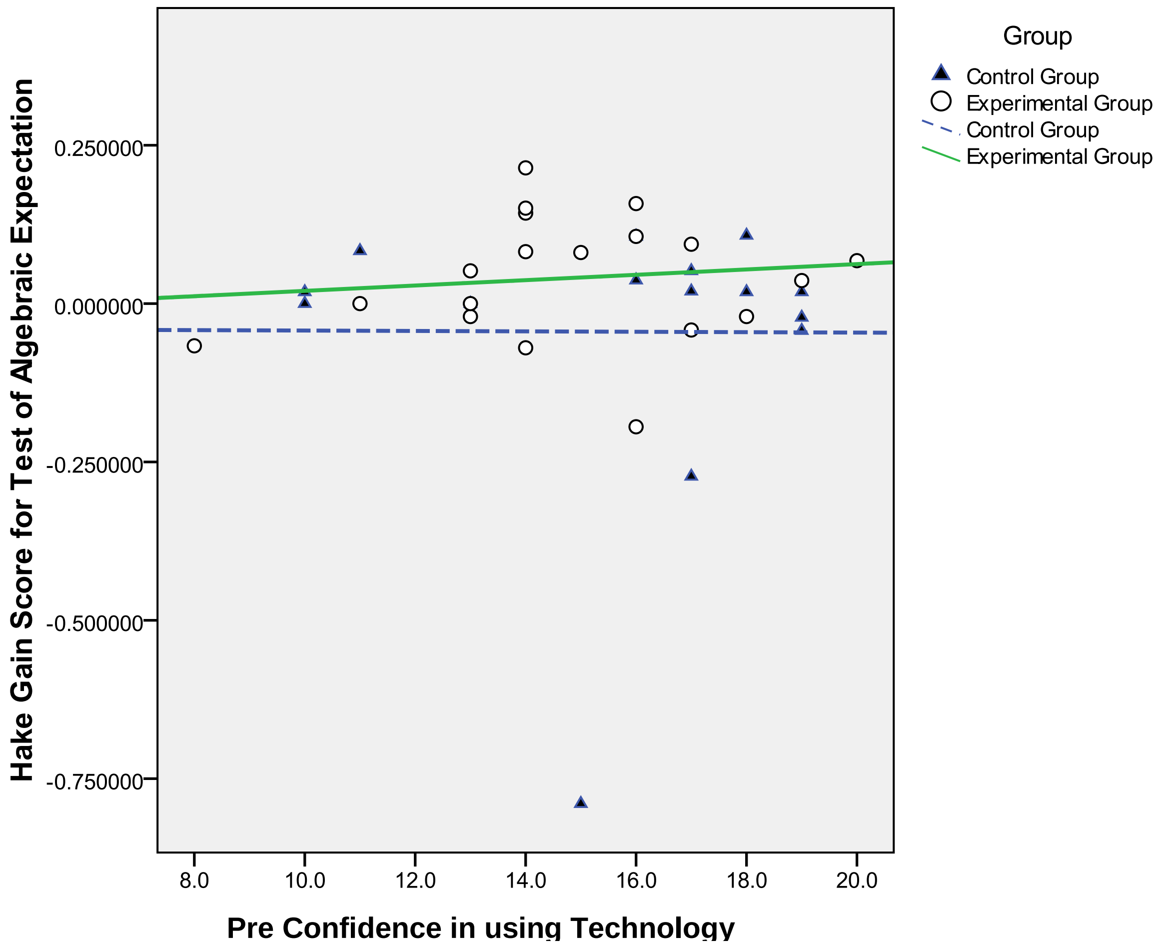


Figure 14. Hake gain score for test of algebraic expectation versus pre confidence in using technology.

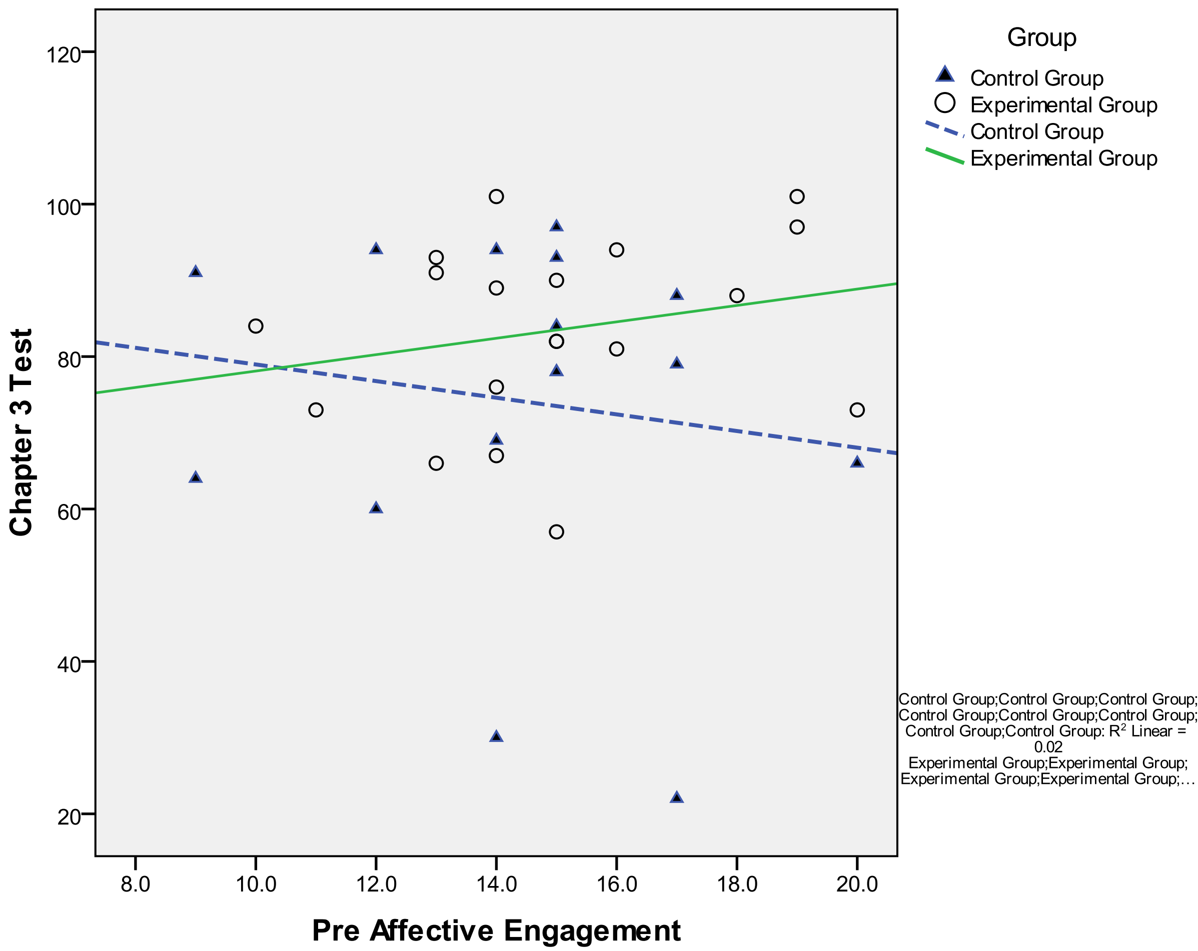


Figure 15. Chapter 3 test versus pre-affective engagement for experimental and control groups.

When looking at achievement as determined by the students' score on their Chapter 3 test or their Hake Gain Score in Algebraic Expectation (Figures 13 -15) there did not appear to be any correlation between these scores and students' attitudes (AE, BE, TC, MC) based on their grouping.

Clinical Task-Based Interview Analysis

Clinical task-based interviews were conducted to gain deeper understanding of students' mathematical understanding that could not be determined by a paper and pencil test alone. By mathematical understanding I am referring to both procedural and conceptual understanding as well as procedural skills that can demonstrate procedural knowledge. Clinical task-based interviews were conducted with the following students: Brian (B), Stephan (P), Rebecca (R), Melanie (M), Allie (A), Samantha (S), Jim (J), and Henry (H). From the transcribed interviews ten themes emerged as I analyzed student work. Each theme is named by a reference to a description of what had transpired which indicated a strength and or weakness in the students' mathematical skills. The themes are: take away versus divide; understanding the equal sign; understanding the distributive property; combining like terms; order of operations; solving; understanding a variable; verifying results; understanding CAS; and understanding symbols.

As I wanted to relate student understanding back to algebraic insight, for each theme I reviewed each students responses to clinical-task based interview questions as well as their performance on the Chapter 3 test to establish what level of algebraic insight a student was demonstrating (low (L), medium (M), or high (H)) with respect to the theme. The descriptions for the levels are found in Table 12. In Table 13, I provide in table format the results from my analysis. The written evidence along with student examples and comments follow. These examples provide justification and support for

Table 12

Levels of Algebraic Insight

Score	Description	Level of Algebraic Insight
0	Student showed no understanding of procedural skills	Low
1	Student showed some understanding of the procedural skills	Low
2	Student appeared to understand the correct procedure, but made an arithmetic error	Medium
3	Student showed understanding and accuracy of procedural skills.	Medium
4	Student showed understanding and accuracy of procedural skills as well as procedural understanding	High
5	Student showed understanding and accuracy of procedural skills as well as procedural and conceptual understanding	High *

** It is the researcher's view that one can have high conceptual understanding without high procedural understanding.*

their level classification. I then quantify the results with respect to the categories low (1), medium (2), and high (3), and provide an algebraic insight index for each student and compare this to their achievement on their Chapter 3 test.

Table 13

Algebraic Insight by Theme and Student

Category \ Student	**A**	**B**	**H**	**J**	**M**	**R**	**S**	**P**
Take away versus divide	H	M	H	L	H	L	M	M
Understanding the equal sign	L	L	L	L	M	H*[C] L*[P]	M	M
Understanding the distributive property	L	M	H	H	L	M	L	H
Combining like terms	H	H	L	L	M	L	M	L
Order of operations	M	M	L	L	H	L	H	L
Solving	L	L	L	M	L	M	M	M

Understanding a Variable	M	L	L	L	L	H*[C] L*[P]	L	M
Verifying Results	M	M	L	L	M	H	L	L
Understanding CAS	L	L	M	L	L	L	L	M
Understanding symbols	M	H	L	M	L	M	L	L
Total Algebraic Insight index	18	18	15	14	17	17	16	17
Chapter 3 Test Score (%)	90	101	57	81	89	93	82	73

Take Away Versus Divide

Although half of the students interviewed made errors in swapping terms or symbols when referring to division and subtraction, most still demonstrated correct procedures with respect to these operations. However, this is still a high number of students who verbally or symbolically mix up two of the basic operations.

Table 14

Take Away Versus Divide

Student	Level	Rationale
Allie (A)	H	No evidence of misuse of term, notation, or operation Correct procedures
Brian (B)	M	Misused term - “take away” to refer to the operation of division Correct procedures
Henry (H)	H	No evidence of misuse of term, notation, or operation Correct procedures
Jim(J)	L	Misused operation-- used division to “undo” addition of a negative Error(s) in procedures
Melanie(M)	H	No evidence of misuse of term, notation, or operation Correct procedures
Rebecca(R)	L	Misused term-- said “divide” instead of “subtract” Error(s) in procedures
Samantha(S)	M	Misused term-- wrote “÷” when she meant “–” Correct procedures
Stephan(P)	M	Arithmetic error using inverse operation

There were several occasions where students reversed the words and the meaning between subtraction and division, thus in terms of algebraic expectation they demonstrated a lack of ability to recognize and identify operations accurately. For example, on clinical interview task 6 Brian used the words “I took away the 6 from both sides” and “I got 2, then I took away 3.” This did not agree with his work (Figure 16) so I asked, “Are you taking it away or dividing by it?” and he changed what he said to “dividing by it.” Here Brian mistakenly used the term “take away” to refer to the operation of division. Misinterpreting these symbols or the words which relate to these operations corresponds to a student’s ability to recognize conventions and basic properties as outlined in Table 1. Here, although he has “said” the wrong thing, his work (Figure 16) does show that he was using division. Thus he did understand the correct procedure (division) as opposed to subtraction. The ability to solve this problem is not addressed here, only Brian’s use of incorrect terminology.

$$\boxed{\begin{array}{l} 6x+3y=12 \\ \dfrac{6\mathrm{x}}{6}+3y=\dfrac{12}{6} \\ x+\dfrac{3y}{3}=\dfrac{2}{3} \\ x+y=\dfrac{2}{3} \end{array}}$$

Figure 16. Brian task 6

Rebecca also made a similar error on task 6. For this task I asked Rebecca to describe her work (Figure 17) step by step. She said “divide by 3” and then immediately said “no, subtract 3” and “then I divided by 6, then I had x and y.” Although she did immediately self correct, she looked at her work and her initial statement referred to the operation as division not subtraction. In reality if she had divided all terms by 3 she may have been able to isolate the y. Thus this example, although an example where the student improperly identified an operation, also demonstrates the lack of ability to understand the structure of the problem. Rebecca failed to recognize that the 6x and 3y are separate unlike terms and cannot be combined. She also failed to recognize that the terms6x, 3y, and 12 all have a common factor. Thus Rebecca is showing weakness on

$$\boxed{\begin{array}{c} 6x+3y=12 \\ \quad -3 \quad\quad -3 \\ \dfrac{6xy}{6} = \dfrac{9}{6} \\ \dfrac{xy}{x} = \dfrac{1.5}{x} \\ y = \dfrac{1.5}{x} \end{array}}$$

Figure 17. Rebecca task 6

several levels of the Algebraic Insight Framework (Table 1). She initially said division and her work showed subtraction. However, she did more than just "take away" a 3, she also "took away" the + sign. Her work demonstrates no real understanding of the procedural skills or rules needed to "subtract," thus with respect to the work related to the recognition of conventions that relates not only to divide versus take away, but to the actual application of those skills, she received a low score.

The last example in this category demonstrates a lack of or misunderstanding of the operation "$\div 2$". In this example, Samantha started task 4 by working out the problem and got x = 9. In her work (Figure 18) she wrote $\div 2x$, but she did not divide the terms with an x by 2x, but instead it appears that she actually subtracted 2x. Here her error is in a misinterpretation of a symbol. Her actual work leading to her answer does demonstrate strengths with regards to algebraic insight, with the exception of the misuse of the "$\div$" symbol. She does demonstrate understanding as well as accuracy of procedural skills. However, in task 2 Samantha completed the work and got 10a=28 as her final answer. I asked her what a final answer should look like. "It is usually a letter equals a number" she said. "I don't see this as solving I see it as simplifying." I asked if she could get the "a" by itself and she said yes if she subtracted 10 from both sides. I went through step by step asking her to repeat why she did what she did and she referred to "doing the opposite" I asked her what 10a meant and she said 10 times a, then she realized the opposite would be dividing by 10 and was able to complete the problem.

Thus Samantha scored a medium for algebraic insight with regards to take away versus divide.

$$
\begin{aligned}
6x + 3y &= 12 \\
-3 \quad & \quad -3 \\
\frac{6xy}{6} &= \frac{9}{6} \\
\frac{xy}{x} &= \frac{1.5}{x} \\
y &= \frac{1.5}{x}
\end{aligned}
$$

Figure 18. Samantha task 4

All three examples demonstrate that according to the Algebraic Insight Framework (Table 1), the three students did not recognize conventions and basic properties – specifically they confused the concepts of "take away" and "divide" either in the translation of word to symbol, or in the misuse of symbol to application. This led to the identification of differing problem solving abilities which were related to their levels of algebraic insight.

On task 3 (Figure 19), Jim got stuck when he got to 17 > 5x-3 and was not sure what to do. I asked him “Why isn’t the 5x by itself right now” he said because it is multiplied by 5. I said you want to get x by itself, but first you must get 5x by itself. Why is the 5x not by itself. He said there was a negative 3 (as opposed to minus 3). I asked him how he would get rid of it/undo it and he said divide. His work (Figures 19 and 20) shows how he divided the - 3 by “-3” as opposed to adding three. Even on his second attempt he is using the incorrect inverse operation. His Chapter 3 test also revealed similar errors in mixing up division and subtraction when performing inverse operations.

$$\boxed{\begin{array}{l} 17 > 5x - 3 \\ \dfrac{17}{-3} > 5x - \dfrac{3}{-3} \\ \dfrac{-5.\overline{6}}{5} > \dfrac{5x}{5} \end{array}}$$

Figure 19. Jim task 3 first attempt

Thus, although he does show some understanding of the difference in these procedural skills, he does not always appear to understand the correct procedure, and thus earned a low score for algebraic insight with respect to the difference between take away and divide.

$$
\boxed{\begin{array}{l}
17 > 5x - 3 \\
\dfrac{17}{3} > 5x - \dfrac{3}{3} \\
\dfrac{5.6}{5} > \dfrac{5x}{5} \\
x = 1.12
\end{array}}
$$

Figure 20. Jim task 3 second attempt

Understanding the Equal Sign

Under this category I include examples where students show understanding or misunderstanding of the equal sign as a symbol of equivalence. I also include examples which relate to the understanding of an inequality. The examples are then matched to the level of algebraic insight framework (Table 1). Below is a brief summary of the rationale for the students' level of algebraic insight. Typically it appears that although students

Table 15

Understanding the Equal Sign

Student	Level	Rationale
Allie (A)	L	Neglected to divide all terms by a given number Did not “do the same thing on both sides” Only divided terms which yielded integers Procedural error(s)
Brian (B)	L	Neglected to divide all terms by a given number Did not “do the same thing on both sides” Does not appear to see each side of equation as one expression Only divided terms which yielded integers
Henry (H)	L	Moved terms to other side of equal sign and did not change signs Procedural error(s)
Jim(J)	L	Neglected to divide all terms by a given number Did not “do the same thing on both sides” Mirrored operations to solve
Melanie(M)	M	Arithmetic error(s)
Rebecca(R)	H^{*C}	Understands “balancing the equation”
	L^{*P}	Procedural errors
Samantha(S)	M	Arithmetic error(s)
Stephan(P)	M	Neglected to divide all terms by a given number Did not “do the same thing on both sides” Only divided terms which yielded integers

often referred to “do the same thing to both sides” there appears to be several interpretations of what that actually means.

An examination of four students’ work illustrated a common error. Brian, Stephan, Allie, and Jim all showed instances where they neglected to divide all terms by a given number. In Brian’s work on task 6 (Figures 16) he divided only the 6x and 12 by 6. This leads me to question his understanding of the equal sign and looking at an equation as a balancing scale. However, he could be under the misconception that he actually did do the same thing to both sides without realizing that the 6x + 3y on the right represents one expression and therefore it must in its entirety be divided by 6. In this case I showed Brian a numerical example showing why when you divide by a number each term must be divided by that number. He then redid the problem (Figure 21). However, although in the first step he divided every term by 3, once he simplified to 2x + y = 4, he did not divide each term by the 2, which supports his lack of understanding of

$$6x+3y=12$$
$$\frac{6x}{3}+\frac{3y}{3}=\frac{12}{3}$$
$$\frac{2x}{2}+y=\frac{4}{2}$$
$$x+y=2$$

Figure 21. Brian task 6 second attempt

the equal sign equating two expressions. He also did not seem to comprehend that when viewing the structure one side of the equal sign represents one strategic component. Although Brian shows some understanding of procedural skills, he does not appear to see each side as one expression; therefore he performs operations to one term in a binomial expression and not both. He does not fully understand this procedure, thus has received a low score with regard to algebraic insight through the lens of "understanding the equal sign."

For task 6, Stephan also shows similar errors in terms of performing the same operations to both terms, the one on the right and the one on the left (Figure 22). He appears not to understand the structure and that the 6x + 3y is one expression. He also

$$\frac{6x}{6} + 3y = \frac{12}{6}$$

$$x + \frac{3y}{3} = \frac{2}{3}$$

$$x + y = .\overline{6}$$

$$-x \qquad -x$$

$$y = -.\overline{6}\,x$$

$$y = .\overline{6} - x$$

Figure 22. Stephan task 6

only divides the terms which are divisible by 6 and 3 and appears to disregard terms which would result in a non-integer result as Brian had done in the previous question. These errors are similar and point to a common procedural misunderstanding. However, his ability to relocate the x and "solve for y" does demonstrate his understanding of what the key feature "solving for y" means. He has received a medium score as he does demonstrate understanding the correct procedures for isolating the variable even though there is some question as to why he did not divide the x by 3.

Allie also demonstrated lack of understanding of structure on task 3 (Figure 23). Allie's work below shows that she started out in the first two steps using correct procedures, but in the 3rd step she did not divide all terms by 5. Although this is an inequality, she demonstrates here that she does not truly understand the idea of the equal

$$17 > 3(2x-1) - x$$
$$17 > 3 \bullet 2x - 3 - x$$
$$17 > 6x - 3 - x$$
$$\frac{17}{5} > \frac{5x}{5} - 3$$
$$3.2 > x - 3$$
$$+3 \qquad +3$$
$$6.2$$

Figure 23. Allie task 3

sign signifying equivalence, or similar to Brian and Stephan she possibly does not understand that 5x-3 represents one strategic component. In her work she also loses the variable x and the inequality symbol. Her work showed 6.2 as her final answer with no reference to the variable.

Allie again performs a partial division on task 6 (Figure 24). She only divided two of the three terms by six. My questioning did not elicit why she only divided these terms. Similar to Stephan and Brian's work, perhaps she only divided terms that gave whole number answers, which may stem from a lack of conceptual understanding that each side of the equal sign represents an expression and that the expressions are equal.

$$\frac{6x}{6} + 3y = \frac{12}{6}$$
$$\frac{3y}{3} = \frac{2}{3}$$
$$y = \frac{2}{3}$$

Figure 24. Allie task 6

Allie ended up losing the x after dividing by six which does suggest that perhaps, conceptually, she does not understand what the solution to an inequality represents. She also did not check her final answer or attempt to check if it made sense. These checks would have indicated at least some conceptual understanding of the task. The number of mistakes that she does make in these examples demonstrates an overall low level of algebraic insight.

It took Jim over ten minutes to come to an answer on task 1(Figure25). Initially he started by dividing by 2, but he rewrote and started the problem over. When I asked him what he did he said he divided by -4.5, however he only divided the -7 and the -4.5 by -4.5. The fact that he did not perform the same operation to each term on both sides of the equation indicates a lack of understanding of the equal sign and its relation to equivalence. His final answer was $x = 1.\bar{5}$. He did not explain how he got this answer.

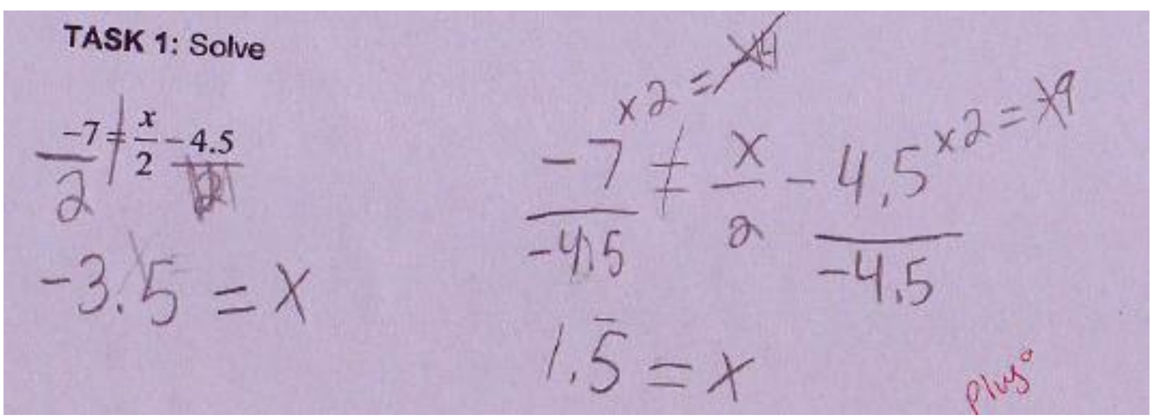


Figure 25. Jim task 1

In looking at his work in Figure 25, perhaps he did not consider the right side of the equation in completing the problem and solely divided the -7.5 by -4.5 or perhaps since the x was already divided by 2 he multiplied each term by 2. However, either way, this work leads to an incorrect answer.

On task 3, Jim made two attempts (Figures 19 and 20), and in both he did the same thing as in task 1, he did not divide the entire expression (5x-3) by the same number. With Jim's work, there was also a bit of inconsistency when looking at one worked example to another. In addition to the example in Figure 25, which demonstrated a lack of understanding of the equal sign, Jim may have an additional misconception. If you compare the answer from problem 12 (Figure 26) with his work from task 1(Figure 25), it appears from these two examples that he may have a different misconception about "doing the same thing to both sides." When he sees an operation being performed on the

$$\begin{array}{l} -7 = \frac{x}{2} - 4.5 \\ +4.5 \qquad\quad +4.5 \\ \frac{-2.5}{2} = \frac{x}{2} \\ x = -1.25 \end{array}$$

Figure 26. Jim problem 12 Chapter 3 test

x (at least when it is division) and the variable term is not already isolated, it appears that he understands that to mean to do the same thing to the other side -- as opposed to performing the inverse operation. However, once the variable is isolated, he appears to use the inverse operation correctly.

In problem 16 (Figure 27), Jim performed the correct steps necessary to complete the problem. However, in a problem using inequalities (Figure 28), he proceeds by adding a constant term with a linear term. He then divides both sides by 8, but from here he still did not solve the inequality correctly. This makes it appear as if he has some procedural rules memorized, but has very little procedural understanding as to when to use certain rules or perhaps even why to use them conceptually. Jim does not meet the requirement for a medium level of algebraic insight due to the inconsistency in his procedures from one problem to the next.

$$\boxed{\begin{array}{l} 17 = 3(2k-1)-k \\ 17 = 3\bullet 2k - 3\bullet 1 - k \\ 17 = 5k-3 \\ +3 \qquad +3 \\ \dfrac{20}{5} = \dfrac{5k}{5} \\ k = 4 \end{array}}$$

Figure 27. Jim problem 16 Chapter 3 test

$$-6b \le b - 14$$

$$+14 \qquad +14$$

$$\frac{8b}{8} \le \frac{b}{8}$$

$$1 \ge b$$

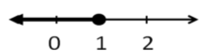


Figure 28. Jim problem 20 Chapter 3 test

The interviews for Brian, Stephan, Allie, and Jim unfortunately did not provide sufficient time to question them deeply enough on what they did and why to determine exactly what their individual misconceptions were. All appeared to be related to structure and the understanding of the equal sign equating two expressions. However, it was not clear whether their neglect to not divide all terms by a divisor was due to not understanding the one strategic component concept, or their misunderstanding of the convention "do the same thing on both sides." This is a topic in need of further research.

When it came to a strong conceptual understanding of structure and the equal sign, Rebecca demonstrated on several occasions a high algebraic insight. For example, on task 1, most students used a standard algorithm to solve the equation; however Rebecca started the first problem quite differently than other students. "I plugged in different numbers to see what worked and I tried to see when I got that number" (pointing to -7). She used a four-function calculator to guess and check. Thus, although Rebecca did not demonstrate procedural knowledge in solving this problem in a traditional way, her method does demonstrate her understanding of the equal sign and that she was equating two expressions. She understood that the number which made both sides the same value was the answer, which shows a high conceptual level of algebraic insight.

However, in task 3 (Figure 29), it becomes obvious that her procedural understanding (low) is in contrast to her conceptual understanding (high). Initially Rebecca started task 3 by subtracting the x from the 2x in the parentheses, but then stopped and started over using the distributive property, but got stuck. Then she said "Oh I understand this now" and she started plugging in values for x. She said "I got x = 2" so I asked her how and she said she plugged in 2 for x and got $5 \bullet 2 - 3$ and this was less than 17. I felt that perhaps her initial struggle with working procedurally had made her overlook the problem structure. She did say her answer gave her a result "less than 17" so I decided to probe further and I asked her to try x = 3 and see if it would work. She said yes, then I asked about 1 and 2.5. I asked her to generalize her answer and she

$$17 > 3(2x-1)-x$$

17 > 3(2x - 1) - x

-x +x

17 > 3 • 1x - 1

Figure 29. Rebecca task 3

said it would work for all numbers and she paused and said “wait I’ll find it” and she came up with $x < 4$. Thus, although Rebecca does not appear to have any memorized set of rules that she methodologically follows, she does understand the structure and sees the problems as two sides she is trying to equate. She also appears to understand conceptually what the solution to an inequality represents.

In this case Rebecca initially gave me one answer. Although her answer was a correct solution and her explanation why it was a correct answer showed some understanding of the inequality symbol she did not appear to be able to logically solve the inequality using operations or inverse operations in a traditional way. Once again, her ability to understand the structure is high, yet her ability to be able to perform necessary operations in an attempt to maintain balance in the equation is very low. Rebecca is an example of a student who demonstrates high algebraic insight with respect to conceptual understanding of problem solving, but with regard to performing necessary procedural skills is unable to demonstrate understanding of the traditional procedures used in problem solving.

Henry demonstrated a very low algebraic insight with respect to his understanding of the equal sign as based on his work from task 2 (Figure 30). He moved all like terms with a variable to the left side of the equal sign and he put all constants on the right side. He did not change signs, he just moved them. He was not able to explain what he was doing using vocabulary such as simplify or combining like terms. Henry's work demonstrates that he does not understand the equal sign in the manner his test appeared to demonstrate. He is stringing the equal sign as if he is continually simplifying, not as if he has two sides he is trying to maintain equivalent. I found nothing that indicated this misconception on his Chapter 3 test, which was given prior to completing this task.

$$\boxed{\begin{array}{l} 4a-6-a=22-1+5a \\ 4a+5a+a=22-1-6=\dfrac{10a}{10}=\dfrac{15}{10} \\ a=\dfrac{2}{3} \end{array}}$$

Figure 30. Henry task 2

Review of Melanie's clinical task-based interview, Chapter 3 test, and written activity answers shows consistency in balancing equations by applying correct procedures to balance the equation. However, her work did indicate a few arithmetic errors, thus Melanie received a medium score.

For task 2 (Figure 31), Samantha's work demonstrates that she does not necessarily see the equal sign as a balance. Although most of her work looks as if she

$$\boxed{\begin{array}{lr} \multicolumn{2}{c}{4a-6+a=22-1+5a} \\ +5a & -5a \\ \multicolumn{2}{c}{9a-6+a=22-1} \\ \multicolumn{2}{c}{+6 \qquad +6} \\ \multicolumn{2}{c}{9a+a=28} \\ \multicolumn{2}{c}{10a=28} \end{array}}$$

Figure 31. Samantha task 2

understands that in order to balance an equation one must perform the same operations to both sides, this example may have been an arithmetic or annotative error. Due to this error, Samantha received a medium score for her algebraic insight with regards to understanding the equal sign.

Understanding the Distributive Property

The distributive property was one task that most students seemed to understand and execute correctly on a consistent basis. However, there was almost a false sense of security with regards to this property. All students were readily able to recognize errors in the distributive property when presented with someone else's work; however, there were three students who did not recognize errors in their own execution of the distributive property.

Many students referred to the distributive property of multiplication over addition and subtracting as "bombing." For example if given the expression 3(x+2) students would say you had to "bomb" the 3, which implied it had to be multiplied through the terms in the parentheses. However, there were some students who did not distribute or who combined terms from outside and inside of the parentheses.

Stephan had scored a 73% on his Chapter 3 test. This initially indicated that he did not have a very strong understanding the concepts, but does not give any indication as to his procedural versus conceptual understanding of the concepts. When I looked at the test items from Chapter 3 as well as the tasks which incorporated the distributive property, he executed this property correctly every time. Jim scored an 81% and Henry scored a 57% on their Chapter 3 tests, but both consistently were able to perform the distributive property correctly on test and task items. Because they were able to accurately carry out the procedures of distribution regularly and did not attempt to combine terms within the parentheses, they all received a high score for algebraic insight with regard to knowing and understanding the procedures in executing the distributive property.

Table 16

Understanding the Distributive Property

Student	Level	Rationale
Allie (A)	L	Recognized error of distributed property in others work Error(s) in distributing
Brian (B)	M	Recognized error of distributed property in others work Arithmetic errors
Henry (H)	H	Accurately carried out the procedures of distribution regularly
Jim(J)	H	Accurately carried out the procedures of distribution regularly
Melanie(M)	L	Recognized error of distributed property in others work Error(s) in distributing
Rebecca(R)	M	Recognized error of distributed property in others work Arithmetic errors
Samantha(S)	L	Recognized error of distributed property in others work Error(s) in distributing
Stephan(P)	H	Accurately carry out the procedures of distribution regularly

For task 4 (Figure 32), Melanie, Allie, Samantha, Jim, and Rebecca had similar answers. In this activity, students were asked to tell which student's work was correct and explain why. Melanie and Allie started by actually doing the problem themselves (no calculator). Allie initially got x = 6 (Figure 33). However, when she checked her answer the CAS said false. “Then it’s not right” she said and went back and found her

Solve: $2(x + 5) = 4x – 8$

Student 1	Student 2	Student 3
$2(x + 5) = 4x – 8$	$2(x+5) = 4x – 8$	$2(x+5) = 4x – 8$
$2(x+5) = 2(2x – 4)$	$2x + 10 = 4x – 8$	$2x + 5 = 4x – 8$
$x+5 = 2x – 4$	$2x = 18$	$13 = 2x$
$3x = 9$	$x = 9$	$x = 6.5$
$x = 3$		

Figure 32. Task 4 from clinical task-based interview

mistake and changed her answer to x = 9. Melanie said student 1 was wrong because “You always have to bomb” and Allie and Samantha also said student 1 did not “bomb.”

$$\begin{array}{l}
2(x+5)=4x-8 \\
2x+10=4x-8 \\
\quad +8 \qquad\quad +8 \\
2x+18=4x \\
-2x \qquad -2x \\
\qquad \dfrac{18}{2}=\dfrac{2x}{2} \\
\qquad 6
\end{array}$$

Figure 33. Allie task 4

Allie looked at step 3 and initially did not even consider whether step 2 was right or not. Melanie and Allie did not necessarily recognize step 2 to be equivalent to the original problem, but it was the ‘bombing’ in step 3 that made Melanie realize the work was incorrect. Allie said the student “did not bring down the 4x and -8.” She did not realize that what was written 2(2x-4) was equivalent to 4x - 8. Samantha said it looked like the student “broke the 4x-8 in half.” She was not able to explain why, but she did not like this step. Jim did not mention bombing for student 1, and in fact he apparently did not understand the student’s work.

Each of these students stated that student 2 was correct. Melanie said student 2 was "True because in the beginning on this equation you have to bomb the parentheses." Rebecca also said student 2 was right because he "bombed" through both terms in the parentheses. Melanie, Allie, Jim, and Rebecca also each immediately dismissed the answer of student 3 due to the bombing error. Allie said student 3 "didn't bomb right. Then they got everything else right, but because they didn't bomb right so it messed him up." However, she did state that all of the other work looked good and that student 3 was "almost there." Melanie also was able to correctly perform the distributive property and recognize errors in work involving the distributive property. However, in task 4, distributing correctly appears to be the only thing Melanie, Rebecca, and Jim considered in determining which student's work was correct versus incorrect. None of these students appeared to take the rest of the work into consideration and did not verify the answer was correct.

Both Melanie and Samantha seemed sure of the distributive property when explaining task 4, but their work shown below on task 3 indicates that perhaps they possess a false sense of confidence when it comes to this property. Melanie used the CAS for task 3 (Figure 34) and typed the equation in the calculator. She solved it using the steps below. She actually drew arrows on her paper indicating that she had to distribute, but never did. Samantha referred to bombing whenever there were parentheses. Perhaps the inequality symbol threw her off, but she not only disregarded the inequality symbol, she also did not appear to physically attempt to distribute the 3. Her work (Figure 35) appears to be randomly combining terms. When asked what she did to solve the problem, she was not able to explain the reasons for what she did to get to the next step.

$$
\boxed{
\begin{array}{l}
17 > 3(2x-1) - x \\
17 > 6x - 1 - x \\
\quad\ +x \quad\ +x \\
17 > 7x - 1 \\
\frac{18}{7} = \frac{7x}{7} \\
x = 2.5
\end{array}
}
$$

Figure 34. Melanie task 3

$$
\boxed{
\begin{array}{l}
17 > 3(2x-1) - x \\
34x - 17 - x \\
33x - 17 \\
\div 33 \div 33 \\
x = 0.51
\end{array}
}
$$

Figure 35. Samantha task 3

Thus with respect to algebraic insight and understanding the distributive property most of the students were able to recognize the distributive property and whether it was carried out correctly. However Melanie, Allie, and Samantha, contradicted their apparent understanding. Although they did demonstrate some understanding of the procedural skills, they did not consistently perform the distributive property with accuracy. They therefore were given a low score for algebraic insight with respect to understanding the distributive property.

Although Henry did not go into the detail the others did in his explanation of task 4, his work from the clinical task-based interview along with his Chapter 3 test and class activities all show Henry was able to accurately carry out the procedures of the distributive property with consistency. He showed understanding with regards to how and when to use the distributive property. His score was high for algebraic insight with regards to understanding the distributive property.

Brian appeared to understand the procedure for distributing, but on his work for task 3 he did make arithmetic errors which resulted in a score of medium for algebraic insight with regards to the distributive property (Figures 36 and 37).

$$
\begin{array}{l}
17 > 3(2x-1) - x \\
\quad\quad 3\cdot 2x - 3\cdot 1 \\
17 > 5x - 31 - x \\
17 > 4x - 31 \\
+31 \quad\quad +31 \\
\frac{48}{4} > \frac{4x}{4} \\
12 > x
\end{array}
$$

Figure 36. Brian task 3 first attempt

$$
\begin{array}{l}
\quad\quad 3\cdot 2x - 3\cdot 1 \\
17 > 5x - 3 - x \\
17 > 4x - 3 \\
+3 \quad\quad +3 \\
\frac{20}{4} > \frac{4x}{4} \\
5 > x
\end{array}
$$

Figure 37. Brian task 3 second attempt

Combining Like Terms

Students with a low algebraic insight tended to either lack procedural skills or face difficulty actually identifying like terms. In addition, three of the eight students did not use procedures consistently when combining like terms. Instead, they used inverse

Table 17

Combining Like Terms

Student	Level	Rationale
Allie (A)	H	Consistently combined like terms accurately
Brian (B)	H	Consistently combined like terms accurately
Henry (H)	L	Combined terms on both sides of equation without regard to sign Inconsistent use of procedures in combining terms Errors in identifying like terms
Jim(J)	L	Combined terms on both sides of equation without regard to sign Inconsistent use of procedures in combining terms Errors in identifying like terms
Melanie(M)	M	Recognized like terms and how to combine them Arithmetic error(s)
Rebecca(R)	L	Lacked procedural skills to combine like terms Errors in identifying like terms
Samantha(S)	M	Recognized like terms and how to combine them Arithmetic error(s)
Stephan(P)	L	Combined terms on both sides of equation without regard to sign

operations to combine terms with like variables that were on opposite sides of the equal sign. The following table briefly shows the rationale for the algebraic insight level afforded each student, and a more detailed explanation with student worked examples follows.

Rebecca's work on task 6 (Figure 17) shows a low algebraic insight due to a lack of understanding of the procedural skills necessary to combine like terms. She showed she was subtracting 3 from both sides and subtracted 3 from 3y. She then combined the 6x and the remaining y through multiplication. Thus, this work demonstrated at least twice her lack of understanding or ability to combine like terms.

It appeared that Stephan did have a strong understanding of the distributive property, however, his work showed less certainty on how to combine like terms and when to add terms or subtract them. For example, on problem 16 from his Chapter 3 test (Figure 38) he took 2k – k and got 3k instead of k. Although it may appear that Stephan merely added the 2k and the k, the interview transcript unfortunately did not reveal how he got the 3k.

On task 2, Stephan combined the 4a on the left with the 5a on the right (Figure 39). Here he appears to know they are "like terms," but he does not appear to know that in order to combine like terms from opposite sides of an equation you must add the inverse of one to the other. In fact, for task 4, he started by working the problem out by himself, and he once again combined variables on two sides of an equal sign without

$$
\begin{aligned}
&17 = 3(2k-1)-k\\
&17 = 3\cdot 2k-3-k\\
&+3 \qquad\qquad +3\\
&20 = 3\cdot 2k-k\\
&\frac{20}{3} = \frac{3}{3}\bullet 3k\\
&\frac{6.\overline{6}}{3} = \frac{3k}{3}\\
&k = 2.\overline{2}
\end{aligned}
$$

Figure 38. Stephan problem 16 Chapter 3 test

$$
\begin{aligned}
&4a-6+a = 22-1+5a\\
&9a-6+a = 22-1\\
&10a-6 = 22-1\\
&\quad +6 \qquad +6\\
&10a = 22+5\\
&10a = 27\\
&\frac{10a}{10} = \frac{27}{10}\\
&a = 2.7
\end{aligned}
$$

Figure 39. Stephan task 2

regard to their sign (Figure 40). Initially in Stephan's work, it appeared as if this error occurred only when combining linear terms and not when combining constants; however, when reviewing the work of student 2 in task 4, Stephan said this student was wrong because the 10 and -8 should be 2 not 18. He also said that student 3 combined the 5 and -8 wrong. Thus, he continued to demonstrate a lack of strong procedural knowledge with regards to how to combine like terms when solving. His inability to perform the necessary procedural skills to combine like terms earned him a low score for algebraic insight for this category.

$$\begin{aligned}
&2(x+5) = 4x-8\\
&2x+10 = 4x-8\\
&6x+10 = -8\\
&\quad -10 \quad -10\\
&\quad \frac{6x}{6} = \frac{-18}{6}\\
&\quad x = 3
\end{aligned}$$

Figure 40. Stephan task 4

For task 2, both Jim and Henry demonstrated difficulty with combining like terms (Figures 41 and 30). Jim wrote 4a - 6 + a = -2a (because 4 – 6 is negative 2). I asked where the “+ a” part went and he said “I made it so it didn’t change.” He also did not attempt to check it. He once again combined terms on the left with those on the right without consideration of the need for a sign change. Henry likewise moved terms from one side to the other with no regard to what their sign was. Although both Jim and Henry show a low level of algebraic insight with regards to combining like terms, their misconceptions appear to be different. From the work in task 2 it appears that Henry at

$$\boxed{\begin{array}{c} 4a-6+a=22-1+5a \\ 4a-6a=22-1+5a \\ -2a=21+5a \\ \frac{3a}{3}=\frac{21}{3} \\ a=7 \end{array}}$$

Figure 41. Jim task 2

least could identify "like terms," but perhaps does not understand that "combine like terms" does not mean just to put these "like terms" together. Henry failed to understand the structure and how to use inverse operations to combine terms. Jim and Henry have demonstrated an inability to perform the necessary procedural skills to combine like terms, thus also receiving a low score for algebraic insight for their ability to combine like terms.

Melanie's work indicated a consistent ability to combine like terms. However, on a few occasions she made errors combining variable terms due to arithmetic errors. Therefore, Melanie received a medium score for algebraic insight in terms of combining like terms. Brian and Allie's work on the clinical task-based interview as well as on their Chapter 3 tests and activities showed both understanding and accuracy of procedural skills. Both Brian and Allie scored high with regard to algebraic insight for combining like terms. Samantha's work (Figure 42) also revealed an understanding of how to combine like terms, but there were arithmetic errors noted in her work. Thus, Samantha received a medium score for algebraic insight with regard to combining like terms.

$$\boxed{\begin{array}{l} -6b \le b - 14 \\ -b \quad\ \ -b \\ -5b \le -14 \\ \div 5 \quad\ \ \div 5 \\ \quad b \le 2.8 \end{array}}$$

Figure 42. Samantha problem 20 Chapter 3 test

Order of Operations

The link from number sense to algebraic insight is dependent upon students

Table 18

Order of Operations

Student	Level	Rationale
Allie (A)	M	Arithmetic errors
Brian (B)	M	Most of the time demonstrated accurate of procedures with regards to performing the order of operations Made an isolated procedural error
Henry (H)	L	Inconsistent use of procedures in order of operations Arithmetic errors
Jim(J)	L	Used division to "undo" subtraction
Melanie(M)	H	Consistently showed correct and accurate procedures with regards to performing the order of operations
Rebecca(R)	L	Combined terms outside parentheses with those inside
Samantha(S)	H	Consistently showed correct and accurate procedures with regards to performing the order of operations
Stephan(P)	L	Use of incorrect procedures Inconsistent use of procedures in order of operations

understanding the basics of carrying out arithmetic operations--the order of operations. Considering the order of operations represents procedures for working with numbers used since elementary school, many students remain unsure of how to combine terms and in what order to perform the procedures. Several students showed low algebraic insight with respect to understanding of the order of operations necessary for solving equations. For example, in task 3 Rebecca started by subtracting the x from the 2x that was in the parentheses. She tried to combine terms with no regard to the parentheses and the order in which operations need to be performed (Figure 29).

Stephan's work from problem 16 of his Chapter 3 test (Figure 38) is an example of his misunderstanding of the order of operations. He combined, although incorrectly, the 2k and –k before he multiplied the 3 times the 2k. He perhaps did not realize that he had to multiply before he could add or subtract to this situation, or perhaps he has not mastered the order of operations. Task 3 from the clinical task based interview was basically the same problem as problem 16 on the Chapter 3 test, but I changed it to an inequality. During his explanation of how he solved it, I asked Stephan what happened to the $3 \bullet 2x$ and he said he combined the 2x and the x. I then asked what happened to the negative sign in front of the x and he revised his work (Figures 43 and 44). However, he still combined the 2x and –x without considering the fact that the 2x was being multiplied by 3, which supports the fact that he has a low level of algebraic insight for the order of operations.

$$
\begin{aligned}
&17 > 3(2x-1)-x\\
&17 > 3\bullet 2x-3-x\\
&17 > 3\bullet 3x-3\\
&+3 \qquad\qquad +3\\
&\frac{20}{3} > \frac{3}{3}\bullet\frac{3x}{}\\
&\frac{6.\overline{6}}{3} > \frac{3x}{3}
\end{aligned}
$$

Figure 43. Stephan task 3 first attempt

$$
\begin{aligned}
&17 > 3(2x-1)-x\\
&17 > 3\bullet 2x-3-x\\
&17 > 3\bullet 1x-3\\
&+3 \qquad\qquad +3\\
&\frac{20}{3} > \frac{3}{3}\bullet\frac{1x}{}\\
&\frac{6.\overline{6}}{1} > \frac{1x}{1}\\
&6.\overline{6} > x\\
&6.\overline{6} = x
\end{aligned}
$$

Figure 44. Stephan task 3 second attempt

Jim also made mistakes on task 3 with regard to the order of operations (Figure 19). Initially, he used division to get rid of a -3 as opposed to using the inverse operation. Although I classified this example as an "order of operations" error, this example also supports his lack of understanding of structure and properties of operations. Perhaps he did not see the "-3" as a "minus 3" that he needed to eliminate.

Overall, Melanie's work indicates a strong understanding of the order of operations. She made consistent and accurate use of these procedures, which resulted in a high score for algebraic insight with regards to order of operations. As for Brian, although most of his work did appear to demonstrate his understanding of the order of operations, he did have an error in these procedures on his Chapter 3 test. On question 20 he divided to get rid of a coefficient before combining all variable terms (Figure 45). The combined errors in this problem indicate that procedural understanding may not been fully developed. Brian scored a medium for algebraic insight with regards to order of operations.

$$\boxed{\begin{aligned} &-6b \le b - 14 \\ &\frac{-6b}{-6} \le b - \frac{14}{-6} \\ &\quad b \ge b - -2.34 \\ &\quad b \ge -2.34 \end{aligned}}$$

Figure 45. Brian problem 20 Chapter 3 test

Samantha scored high on algebraic insight with regards to order of operations as her work consistently showed correct and accurate procedures with regard to performing the order of operations. Allie's work demonstrated that she understood the correct procedure; however she did have a couple of problems where she made arithmetic errors, so her score for algebraic insight is a medium for order of operations. Henry's work showed several errors, which indicate not only arithmetic mistakes, but inconsistencies in how to combine terms in an equation. Therefore, his score for algebraic insight for this category is low.

Solving

For solving, I looked at students who showed consistency or inconsistency between their definitions of what solving was and what their work demonstrated. For example, on task 6 students were asked to "solve for y." Brian's final work on task 6 provides an end result of $x + y = 2$ (Figure 21). I asked if this was solved for y and he said he could not because he could not get rid of the x. He said he was finished even though he admitted he did not get the y by itself.

Table 19

Solving

Student	Level	Rationale
Allie (A)	L	Neglected to divide all terms by the same number Did not recognize what a solution represented (inequality)
Brian (B)	L	Disconnect between "solving" (meaning getting the variable by itself) and understanding what that means and how to do it Neglected to divide all terms by the same number
Henry (H)	L	Inconsistent use of inverse operations to solve
Jim(J)	M	Disconnect between "solving" (meaning getting the variable by itself) and understanding what that means with two variables Able to use inverse operations to isolate the variable
Melanie(M)	L	Errors in using inverse operations Did not recognize what a solution represents (inequality)
Rebecca (R)	M	Demonstrated understanding the correct procedures for solving an equation Arithmetic errors
Samantha(S)	M	Demonstrated correct procedures for solving Unable to perform error analysis on others' work Confusion between terms simplifying and solving
Stephan(P)	M	Disconnect between "solving" (meaning getting the variable by itself) and understanding what that means with two variables Able to use inverse operations to isolate the variable Error combining terms

In the previous example, Brian demonstrated some understanding of what "solve" meant, but he was unable to consistently demonstrate answers which indicated he could follow the correct procedures to arrive at the solution. Therefore, he received a low score for algebraic insight with respect for solving. Another example which shows low algebraic insight with regard to solving equations and inequalities is Melanie's work on task 3 (Figure 34). In her work, Melanie should have combined the 6x and the –x, however she added an x to both, which changed the problem instead of keeping it balanced.

One aspect of solving is being able to identify key features. Thus for task 3, a student who had a strong algebraic insight with regards to key features would have recognized that he/she was solving an inequality and would recognize what the final solution would look like. In task 3 Allie appears to have lost sight of the fact she was solving an inequality (Figure 23). She started out in the first two steps using correct procedures, but in the third step she did not divide all terms by 5. She then lost the x and the inequality symbol. Her work showed 6.2 as her final answer with no reference to the variable. She also did not appear to realize that the 5x-3 acts as one term and if she needed to divide by 5 she needed to either first isolate the 5x or divide both the 5x and the -3 by 5.

On task 6 Allie once again demonstrates that she is not certain of the procedures necessary to solve the equation (Figure 24). She divided only 2 out of the 3 terms by 6, which was discussed in reference to the equal sign and "doing the same thing to both sides," but in addition, she then "lost" the x term. On this task I did not further question her as to where the $\frac{6x}{6}$ term went, but this along with the previous examples of her work are used to demonstrate her low algebraic insight with respect to procedural knowledge for solving equations.

On task 6, Stephan did tell me that to solve meant to "get the variable all by itself." He first divided the x term by 6 to "isolate the x" without dividing each term by the same value (Figure 22). He arrived at $x + y = .\overline{6}$ and said he was done. Thus, although he could state a definition for what solve meant, when the question asked him to "solve for y" he did not initially provide an answer, which demonstrated the variable y by itself. I restated the question where he was asked to "solve for y." He looked at the problem and continued to work to isolate the y. When attempting to isolate the y he brought the x to the other side as –x. Initially it appeared he multiplied to get $y = -.\overline{6}\,x$. When I asked him to explain his work he realized that he should not have multiplied the two terms and wrote $y = .\overline{6} - x$.

There were students who did demonstrate correct procedures for solving, but appeared to be lacking in confidence. For example, on task 1 I asked Samantha if she was sure she was right. She said she was pretty sure because she was “used to doing this type of problem.” She said to solve for x means “to figure out the missing number.” She used a standard algorithm and got the correct answer.

For task 2 Samantha completed the work below (Figure 31). I asked if 10a = 28 was her final answer and she said yes. I asked her what a final answer should look like. “It is usually a letter equals a number,” she said. “I don’t see this as solving I see it as simplifying.” I asked if she could get the “a” by itself and she said yes if she subtracted 10 from both sides. I went through step by step asking her to repeat why she did what she did and she referred to “doing the opposite.” I asked her what 10a meant and she said “10 times a,” then she realized the opposite would be dividing by 10 and was able to complete the problem. She did not realize, however, that she had made errors in combining like terms.

When asking about task 4 (Figure 32), for student 3, Samantha said student 3 “Did the bombing well, he put 2x+5. He chose a variable side and a number side.” I asked whether the student’s answer was right or wrong and she replied “I don’t know--wrong.” I asked why it was wrong and she asked me to hold on as she worked it out. She then worked it out herself --her steps are shown in Figure 46.

$$
\begin{array}{l}
2x+5=4x-8 \\
\quad +8 \qquad +8 \\
2x+13=4x \\
\qquad x=6.5
\end{array}
$$

Figure 46. Samantha task 4 second attempt

Researcher: “Was student 3 right?”

Samantha: “I am not sure anymore, I got this (x = 6.5) answer for this and this answer (x = 9) for that one.”

Researcher: “Wasn’t it the same question?”

Samantha: (nodded up and down) “Uh huh.”

Researcher: “Could you get two different answers?

Samantha: “Maybe”

Researcher: “Is there something you can do to check it?”

Samantha: “ Plug in” (She did some work on a calculator).

Samantha: “Student 1 is wrong, 2 was right, and 3 --I am not sure.”

This dialog between myself and Samantha shows a disconnect between what she says and what it means to her. She said the problems were the same and when I had previously asked her what a final answer to a problem she was solving should look like, she said, “It is usually a letter equals a number.” She says she could plug numbers in to check if her answer was correct, yet she still remained uncertain about an answer that student 3 produced. Thus, whereas other students dismissed student 3’s answer due to an error in the distributive property without considering the answer, Samantha did the problem herself and could not see an error in her work or the student’s.

Another example of errors in solving was Jim’s work on task 6 (Figure 47). I started by asking Jim about solving and what it meant to solve. He said that solving for x was “getting the x by itself.” Thus I asked him what “solving for y” would mean. Jim said getting y by itself. However, if you look at his work (Figure 47), what is interesting is that he did actually solve for one solution to the equation. Perhaps “getting the x by itself” to Jim actually translates to getting a value for the variable(s). Therefore, he did

$$6x + 3y = 12$$
$$6 + 6 = 12$$
$$6 \bullet 1 + 3 \bullet 2 = 12$$
$$x = 1; y = 2$$

Figure 47. Jim task 6

solve for a value of y, but did not solve for y (in terms of x). Jim said that the “goal” of solving is to get the variable by itself, and he knows there are procedures he must do in order to get there. However, he uses the word simplify to combine terms, but does not appear to understand the rules governing what terms he can combine or that he needs to use the “reverse” order of operations when isolating the variable.

Henry also was not able to translate his understanding that to “solve for y” meant to get the y by itself into proper results. He did not use inverse operations to get the y by itself. However, on some problems, such as task 3 (Figure 48), he was able to identify that he was solving an inequality and carry out steps accurately.

$$17 > 3(2x-1) - x$$
$$17 > 3 \bullet 2x - 3 \bullet 1 - x$$
$$17 > 5x - 3$$
$$17 + 3 > 5x$$
$$\frac{20}{5} > \frac{5x}{5}$$
$$4 > x$$

Figure 48. Henry task 3

Looking at Rebecca's work it is not always obvious that one thing she does very well is verify her results. Often she came up with the answers and then would work through the steps to show the work that the teacher expected. Her work does indicate that she appears to understand the correct procedures for solving an equation or inequality, even if it is not her first instinct to start solving a problem traditionally. However, her work also indicates arithmetic errors; therefore Rebecca received a medium for algebraic insight with respect to solving.

Understanding a Variable

In this theme I have included examples which demonstrate a student's ability to use symbols as numbers and perform indicated operations or inverse operations. For example, if Brian in task 6 (Figure 16) understood that the x represented a numerical value and he wanted to get rid of it he would have realized he needed to subtract it. Instead he was not able to isolate the y as he did not understand that the expression x + y = 2 meant some number plus y equaled 2.

In Stephan's work to task 6 (Figure 22), initially he thought he was finished with the task when he arrived at $x + y = .\overline{6}$. Once the directions to "solve for y" were repeated, he performed the necessary operations to isolate the y. Thus he, unlike Brian, understood that x represented a number and to isolate the y he had to perform the necessary inverse operation.

Table 20

Understanding a Variable

Student	Level	Rationale
Allie (A)	M	Able to consistently apply inverse operations Overlooked/disregarded variable terms when solving
Brian (B)	L	Did not understand that the x represented a numerical value Procedural errors isolating the variable
Henry (H)	L	Moved variables from one side of an equation to another without changing the sign
Jim(J)	L	Inconsistencies with regard to understanding of variables and using inverse operations Errors simplifying expressions Lack of understanding of variables representing exact values as opposed to estimates
Melanie(M)	L	Overlooked/disregarded variable terms when solving
Rebecca(R)	H*^{C} L*^{P}	Understood that the x represented a numerical value Able to identify strategic groups of components Procedural errors isolating the variable
Samantha(S)	L	Did not recognize that in examples the variable represented only one number
Stephan(P)	M	Able to isolate the variable using inverse operations

On task 1, Rebecca's ability to use a four function calculator to guess and check to solve the equation $-7 = \frac{x}{2} - 4.5$ demonstrated her understanding of the equal sign, but when I asked how she knew where to start picking numbers, she looked at the equation and said she knew it had to be negative because the 7 was negative. From looking at her Chapter 3 test alone, the work she showed demonstrated understanding and accuracy of procedural skills, however knowing how she started this problem also demonstrated her algebraic insight (Table 1) in terms of being able to identify strategic groups of components. She was able to see the $\frac{x}{2}$ as a number that needed to be added to -4.5 to get -7 and in doing so demonstrated a strong algebraic insight with regard to a conceptual understanding of what a variable represents. However, in general the steps in her written work showing her procedural understanding indicate a contrasting procedural understanding of how to work with variables (Figures 17 and 29).

In Jim's work on problem 12 from the Chapter 3 test (Figure 26), when he was at the penultimate step, if he understood equivalence or had a strong number sense or sense of what the variable represented, he might have noticed that the only way for these ratios to be equal, since their denominators were already equal, would be for the numerators to also be equal, and thus conclude x = -2.5. My questioning did not elicit why he was dividing -2.5 by 2 as opposed to multiplying by 2, but his work does show inconsistencies with regard to understanding of variables.

Jim’s work on task 3 (Figure 19) also demonstrates his lack of understanding of inverse operations as well as difficulty simplifying expressions. For example, in his initial attempt, he divided the -3 from 5x-3 by -3 and the term disappeared. In his second attempt he did basically the same thing again, crossed out his last two steps and reworked the problem (Figure 20). What is interesting is that for his second attempt he rounded the 5.66666 to just 5.6, which shows disregard to understanding variables representing exact values.

Some students showed blatant disregard to variables and ‘made them disappear’ if they did not understand what to do. For example, on task 6, I asked Melanie what happened to the x and she said “x and y don’t mix” so she just ignored the x term (Figure 49). If she understood that the x and y did represent actual numbers than she perhaps would not have just dismissed x as if it made no difference.

$$\boxed{\begin{aligned} &6x+3y=12\\ &\frac{3y}{3}=\frac{12}{3}\\ &y=4 \end{aligned}}$$

Figure 49. Melanie task 6

Henry's work shows instances of moving variables from one side of an equation to another without changing the sign; however, when he had to get rid of an actual number he always performed the inverse operation. This indicates he did not have a strong understanding that the variables represented numbers. His score is low for algebraic insight with regards to understanding a variable. In terms of understanding the variable, most all of Allie's work appeared to demonstrate her use of variables as numbers. However, on task 6 (Figure 24) she divided 6x by 6 and the x variable was eliminated. I coded this as an arithmetic error thus giving her a medium for algebraic insight with respect to understanding a variable.

Samantha's explanation for task 4 indicated she did not have a strong understanding of what a variable represented (Figures 18 and 46). She initially stated that student 2 was correct (answer x = 9). I asked her to explain what the three students did right and what they did wrong. She said student 1 did not bomb and "it looks like he broke the 4x-8 in half." She did not know why, but she did not like the step. She said student #3 "Did the bombing well, he put 2x+5. He chose a variable side and a number side." I asked whether the student was right or wrong and she replied "I don't know--wrong." I asked why and she asked me to hold on as she worked it out. She then worked it out herself and got x = 6.5. I asked again if this student was right. "I am not sure anymore, I got this (x = 6.5) answer for this and this answer (x = 9) for that one," Allie said. I asked, "Wasn't it the same question?," to which she affirmatively nodded and said "uh huh." "Could you get two different answers," I asked. She replied, "Maybe." This indicates that she has a low algebraic insight with respect to understanding a variable.

Verifying Results

Many students made reference to knowing they were right and could verify their results. Most students referred to this as checking, but not all students appeared to understand what "checking" meant from a conceptual standpoint.

Table 21

Verifying Results

Student	Level	Rationale
Allie (A)	M	Misunderstood what "to check" meant Able to explain why a solution is correct Used technology to verify a result (true/false)
Brian (B)	M	Knew to "to plug his answer in," and verify he got "same number on both sides" for his own work Unable to readily apply the above knowledge to verify results of others
Henry (H)	L	Misunderstood what "get the same number on both sides" meant and how to do that Misunderstood what "to check" meant Knew "to plug his answer in," but misunderstood how to verify he got "same number on both sides"

Jim(J)	L	Misunderstood what “to check” meant Unable to explain why his solution was correct Unable to readily apply the above knowledge to verify results of others
Melanie(M)	M	Used technology to verify a result (true/false) Understood the concept of “plugging answer back in” by hand when one side of equal sign contained a constant Unable to “plug in” to verify when variables were on both sides of equation or when solving an inequality by hand
Rebecca(R)	H	Misunderstood what “to check” meant Used “guess and check” method to solve and verify solution
Samantha(S)	L	Knew “ to plug his answer in,” but misunderstood how to verify he got “same number on both sides” Unable to readily apply the above knowledge to verify results of others
Stephan(P)	L	Misunderstood what “to check” meant Unable to explain why his solution is correct

Rebecca did not specifically solve and check in a traditional way; however, her ability to guess and check demonstrated her understanding of what checking means, to plug in a number for the variable and see if both sides of the equation give you the same value. Nevertheless, on task 4, when her task was to tell me which students’ work was correct or incorrect she did not use “checking” to verify the answers. When I asked how she knew the answer was right she replied “I checked it.” When I asked what she meant by checking she had a hard time telling me. She said she looked at it and it was multiplied right. This leads me to believe that Rebecca, although she knows she is supposed to check her answers, does not fully know what this means. However, she does understand the big picture and could explain why an answer was right.

For task 1 Henry started by adding 4.5 to the left side, and then added the numbers on the left without regard for their sign. The interesting thing is he was certain he was right because he had "checked" it (Figures 50 and 51). He said to check meant to plug in and make sure it works. He said he had to "get the same number on both sides." His verbal answers made me feel as if he knew what he was doing, and in looking at his work he did get the same number on both sides, but what I needed to ask was *how* he got

$$-7 = \frac{x}{2} - 4.5$$
$$-7 + 4.5 = \frac{x}{2}$$
$$11.5 = \frac{x}{2}$$
$$11.5 \cdot 2 = x$$
$$x = 23$$

Figure 50. Henry task 1

$$-7 = \frac{23}{2} - 4.5$$
$$11.5 = 11.5$$

Figure 51. Henry task 1 check

the same number on both sides. For this same problem (#12) on his Chapter 3 test his work and check were different. On this problem he combined the like terms incorrectly and his check makes me think that Henry may know he needs to have the same number on both sides, but does not appear to have the conceptual understanding for why that must be so. Procedurally he knows he has to plug his answer in and that he needs to get the same number on both sides, but he does not appear to understand how to do that using his operations.

I asked Melanie if her task 2 was correct. She said, "I would make sure it is right on the calculator, but I can't do that because you don't really know what the right number is." She seems to understand plugging back in to verify a specific number. However, when a problem had variables on both sides she was not sure how to verify whether the solution was correct as there was no given number with which to make the other side equal.

Once Melanie completed task 3 (Figure 34) I asked if she was right and she went to type it on the calculator and asked if the “<” sign was on it. This time when she typed the equation she replaced x with 2.5 she said “I got true! Yes - I knew it was right! That does mean its right, right?” I explained to her that ‘true’ meant that whatever number she plugged in worked because it was a solution. I then asked her to go back and plug in 3. She said “It says true--does it always say true?” I told her if the number is a solution it will. She did not do any further exploration to see what other numbers worked or to generalize the answer. Thus although she did understand that she had to substitute her answer back in, she did not necessarily understand what a solution to an inequality looked like nor how checking through substitution worked for inequalities.

Allie completed the first task and said she was done. I asked her how she knew if it was right and she said “plug it in.” “Did you do that?” I asked, and she said no. I asked her to explain her steps. She said she added 4.5 then multiplied by 2. She appeared to understand her method and it was valid, but she did not demonstrate what she meant by “plug it in.” Allie worked task 4 out by herself (Figure 33) before looking at the other students’ work. Initially she got x = 6. However, when she checked her answer the CAS said false. “Then it’s not right,” she said and went back and found her mistake. Allie checked by going back through her work and found a mistake. She apparently understood that to check she had to plug her answer back into the equation as she used her calculator to do just that. Unfortunately, she did not know exactly how to interpret that answer.

Samantha had some difficulty figuring out which students had correct answers for task 4. I asked her "Is there something you can do to check it?" She said she could "plug it in." I assumed 'it' referred to the answer. She entered something into the calculator and she said her final answer was student 1 was wrong, student 2 was right, and for student 3 she was not sure. Had she understood specifically what steps were required for plugging in and checking, there should be no reason for her answer to be "not sure."

Brian stated that to "check" meant to "put the x back in and see that the sides come out to be the same number." When asked he would verify his answers. However, on task 4, where he was asked to assess which of the three students, if any, were correct, he initially said that none of them were correct. He never even attempted to use "verifying" to check the answers to see if they worked. Because he did show understanding and accuracy when checking his answers he scored a medium on algebraic insight with regards to verifying results. A score of high would have meant he had a strong procedural understanding in which case I would have expected to see him verifying answers more regularly, especially on task 4.

When asked if he checked his answers Jim said, “I check with the answer I get and then I use it in the problem and kinda reverse it.” He later said to check it meant “to do it again to see if you get the answer right.” However, when I reviewed his work, he had several answers that were incorrect and no evidence was given to support that he actually understood how to check and answer. Stephan also had several errors with no support as to why his answer was right. On task 4, Jim chose student 2 as correct because the distributive property was performed correctly without regard to whether the other aspects of the work were correct. Stephan actually chose student 1 as correct because, “I did the problem myself and that’s what I got.” Thus both Jim and Stephan demonstrated very little understanding of the procedures for verifying results and therefore scored a low for algebraic insight in this category.

Understanding CAS

Unfortunately, I was not able to collect specific data on the ways CAS may have been used by students. For example, I was not able to see every step they input to see if they were using it as a basic calculator, or if they were using more of its symbolic manipulative capabilities. The clinical task-based interview asked students to interpret a view screen where a student solved an equation using the calculator. Although students did solve equations, it would have been interesting to have included a problem set like those used in class and take note as to how students used the CAS to investigate patterns and make conjectures.

Table 22

Understanding CAS

Student	Level	Rationale
Allie (A)	L	Used CAS to verify answers- true/false Did not understand CAS solution output
Brian (B)	L	Did not understand CAS solution output
Henry (H)	M	Did not understand CAS solution output Used CAS regularly
Jim(J)	L	Did not understand CAS solution output
Melanie(M)	L	Did not understand CAS solution output Used CAS to verify answers- true/false
Rebecca(R)	L	Did not understand CAS solution output
Samantha(S)	L	Did not understand CAS solution output
Stephan(P)	M	Some understanding of the procedures used on the CAS Arithmetic error

Algebraic insight with regard to understanding CAS is whether or not a student is able to use the CAS and recognize the value of using CAS to perform procedures on variables. In a previous example (Figure 33), Allie's work on task 4 indicated that she understood how to check, however when she executed her equation with the value plugged in, the CAS gave her the answer 'false' and she did not appear confident with what that meant. Melanie also used the CAS to check answers. On task 3 (Figure 34) she rewrote the inequality and replaced the x with the value 2.5 and said "I got true! Yes - I knew it was right!" then she paused and asked, "that does mean its right, right?" An important concept to understand when using CAS is what is meant when you get a statement such as true or false. True indicates that substituting in a particular value makes the equation a true statement or a false statement. If it yields a true statement than the answer you got was correct.

Task 5 was specifically written to see if students understood the display the CAS provided when a student solved an equation step by step using CAS. On task 5 students had to interpret a screen shot from the CAS. The original problem was to solve 3x + 5 = 2. Students were asked to describe what the student did to solve the problem. Allie was quickly able to say they subtracted 5 from both sides then divided by 3. However, she wrote that the person "put parentheses around the equation and added -5 to the end." She--like many of the students--physically described the line on the output without realizing that that is what the calculator would give if you entered the equation and then typed -5. The students could have just looked at the right column, which was the output. The output shows the final steps someone using a traditional algorithm would have gotten. However, no students were able to explain that the parentheses came when you enter 3x + 5 = 2 and press "-5." The display (3x + 5 = 2) - 5 means to take 5 away from both sides of the equation.

When questioned about task 5, Melanie's first question was " I'm not sure - where did the 2 come from? Because when I put in 3x + 5 I got 11." She must have used the store feature at some point and stored x = 2 and did not realize that by typing 3x + 5 she was really evaluating 3x + 5 when x = 2. Melanie could have looked at the right side of the screen and understood step by step what to do, but she did not recognize the notation on the left and therefore was unable to relate the screen capture to the procedures executed by the student.

Out of all students who participated in the clinical task-based interviews, Henry was the student who most often chose to use the CAS when asked to complete a task. In task 5 when he was supposed to describe what the output meant and how the student got the display, Henry was able to say that $\frac{3 \cdot x = -3}{3}$ meant "they divided both sides by three." He appeared to understand the procedure being followed and hence received a medium score for algebraic insight on this category.

Samantha tried to recreate the calculator screen for task 5, but was unable to. She did realize what the problem was and what the final answer was. However, she was unable to indicate what was done to obtain the output provided. For her inability to recognize the procedures from the display she received a low score for algebraic insight for understanding CAS.

Both Brian and Jim were also not able to understand the CAS display. Both did most of their work without the calculator, so using the data from task 5, each would score low for algebraic insight with respect to understanding CAS. Rachel used the calculator frequently to check numbers (guess and check), but when asked to interpret the screen capture for task 5, she could not explain what the capture represented. From the data available her score is low for algebraic insight with regards to understanding CAS.

For task 5, Stephan input the equation into the CAS himself and started to do the problem himself. He was able to tell me the problems the student had as well as the solution they got, but he did say that $(3 \cdot x + 5 = 2) - 5$ meant that both sides were being multiplied by negative 5. He said the result would be $3 \cdot x = 3$ and then they divided the whole thing by 3. He does appear to have some understanding of the procedures used on the CAS, but made an error (arithmetic) in his explanation. Stephan scored medium for algebraic insight with regard to understanding CAS.

Understanding Symbols

The symbols--other than division and subtraction that were most commonly misinterpreted or misunderstood were inequality symbols. Most students when solving inequalities replaced the inequality symbol with an equal sign and most made no attempt to express their final solutions in terms of the inequality.

For task 3 Jim started the problem and then redid it a second time (Figures 19 and 20). Jim wrote his final answer using an equal sign as opposed to an inequality sign. This indicated perhaps he did not understand what the solution of an inequality represented. However, on problem 20 from his Chapter 3 test (Figure 28), he maintained the inequality symbol throughout the problem and he even graphed his solution correctly. Allie was similar to Jim in that her in work on task 3 (Figure 23), she too dropped the inequality symbol, yet her other work using inequalities on her Chapter 3 test indicated solutions which were accurate and graphed correctly. Therefore, both Jim and Allie received a medium score for algebraic insight for understanding symbols.

Table 23

Understanding Symbols

Student	Level	Rationale
Allie (A)	M	Replaced inequality symbol with equal sign Inconsistent understanding of what the solution to an inequality represents
Brian (B)	H	Correct use of symbols
Henry (H)	L	Confused symbols “<” and “>”
Jim(J)	M	Replaced inequality symbol with equal sign Inconsistent understanding of what the solution to an inequality represents
Melanie(M)	L	Replaced inequality symbol with equal sign Does not understand what the solution to an inequality represents
Rebecca(R)	M	Correct use of symbols Arithmetic error(s)
Samantha(S)	L	Did not understand what “ >” meant Replaced inequality symbol with equal sign Did not understand what the solution to an inequality represented
Stephan(P)	L	Replaced inequality symbol with equal sign Did not understand what the solution to an inequality represented

Henry consistently demonstrated that he did not understand the difference between the symbols “<” and “>”. I asked Henry to read his answer to task 3 (Figure 48) and he had difficulty as to whether to read the inequality as “x is less than 4” or “x is greater than 4.” In addition, for a graph whose solution was $20 \geq b$, he shaded the line to the right of -20. Henry received a low score for algebraic insight with respect to understanding symbols.

Once Melanie completed task 3 (Figure 34) I asked if she was right and she went to type it on the calculator and asked if the “<” sign was on it. This time when she typed the equation she replaced x with 2.5 she said “I got true! Yes - I knew it was right! That does mean its right, right?” Although I used scaffolding to have her discover other numbers worked as well, she did not do any further exploration to see what other numbers worked or to generalize the answer. She stayed with her original answer x = 1.12, which indicates she did not understand what the solution to an inequality represented.

For Rebecca, although she did appear to understand the correct procedures for using the symbols, her work exhibited arithmetic errors. Therefore, she scored medium for algebraic insight with respect to understanding symbols. On task 3, Stephan started out solving an inequality and ended up with the solution x = $6.\bar{6}$ (Figure 43). On his Chapter 3 test, he wrote the solution to a problem was $b \leq b + 2.\bar{3}$ and went on to graph the solution by shading as if his solution was $b \leq 2.\bar{3}$. His work along with his solutions demonstrated some understanding of procedural skills as they related to understanding symbols. Stephan earned a low score for algebraic insight for this category.

Procedural versus Conceptual Understanding

Brian's Chapter 3 test documented accurate step-by-step procedures culminating in the correct solution, and he was also able to interpret problems in context and demonstrate accurate work. He knew that his final answer was supposed to yield a value for x. However, when I looked at his task-based interview to delve further into his procedural and conceptual understanding, on task two he was not sure how to interpret his result. In task two the x terms were eliminated and he was not sure what this meant. He thought he did something wrong. This was understandable, as students apparently only had experience with problems yielding one unique solution (one value for x) rather than problems with 'no solution' (no value of x would work) or all real numbers (any value of x would work) as solutions. In order to help him understand the solution I used scaffolding and spoke about lines and intersections and he appeared to follow a discussion of, when each side represented the equation of a line, the solution would be the point where they intersected. Only after scaffolding was he able to interpret his answer as a place where there would be no intersection and he was able to say, "the lines are parallel." His ability to do this with only mild scaffolding supports my belief he had a strong facility for conceptual understanding. Brian showed strong procedural skills, but did not practice procedures with consistency when faced with problems that looked a bit different than he was used to. Brian also demonstrated a basic conceptual understanding and a strong procedural understanding, but I am not sure that he has bridged his procedural knowledge with conceptual understanding.

Stephan's initial score of 73% and a review of his Chapter 3 test indicated he did not have a very strong understanding of procedural or conceptual skills. Investigating test items showed he did have a strong understanding of the distributive property, but was less certain on how to combine like terms and when to add terms or subtract them. From his work, it was not clear that he understood the order of operations or how to combine like terms. He was also unable to interpret accurate use of an inequality in a problem-solving context. In replaying his clinical task-based interview, he continued for the most part to demonstrate a lack of strong procedural knowledge, including understanding of an equal sign.

When assessing Rebecca's Chapter 3 test by itself, her work demonstrated understanding and accuracy of procedural skills. Furthermore, knowing that she often used guess and check based on her clinical task-based interview, I felt she did have a conceptual understanding of what she was supposed to do when she asked to solve both equations and inequalities. However, Rebecca's work on her clinical task-based interviews indicated she does not appear to have any memorized set of rules that she methodologically follows when solving. Thus, although she does have an apparent conceptual understanding, she does not have a very strong procedural understanding of the steps involved to solve a problem.

Melanie, Allie, and Samantha appeared to understand procedurally the order in which to perform the steps to solve an equation. However, none had a strong conceptual understanding of what it meant to solve an inequality. Samantha also demonstrated a lack of conceptual understanding of solving equations in that she lacked the ability to explain what her answer represented or determine if a solution was correct.

Jim demonstrated a consistent lack of procedural knowledge and understanding when it came to solving both equations and inequalities. He consistently did not perform the same operation to each term on both sides of the equation. This work from his Chapter 3 test compared with his answers from the clinical task-based interview supports the fact that Jim does not have a strong sense of the equal sign or in looking at the equivalence nature. He appears to have some procedural skills or perhaps just procedural rules memorized, but he did not appear to understand the order in which to perform operations. Jim demonstrated little procedural understanding as to when to use certain rules or perhaps even conceptually why to use them. When reviewing Jim's Chapter 3 test, he appeared to have at least a conceptual understanding of inequalities, as he was able to show what a solution to an inequality would look like using symbols and he was able to graph the solution on a number line. However, his work on his clinical task based interview contradicted this observation, thus supporting an assessment of weak procedural understanding coupled with memorized procedures.

Henry was able to answer questions about vocabulary with confidence. He said to check meant he had to "get the same number on both sides." His verbal answers made me feel he knew what he was doing, and in reviewing his work, he did get the same number on both sides, but what I needed to ask was how he got the same number on both sides. It was as if he conceptually knew what was supposed to happen, but did not know procedurally how to make it happen. His work showed that he often combined like terms incorrectly. Procedurally he knows he has to plug his answer in and that he needs to get the same number on both sides, but he does not appear to have the procedural understanding of how to do that using his operations. However, when actually problem solving on his test, he did appear to have an understanding of the equal sign in that he consistently performed the same operation to both sides of the equation--understanding the fact that the two sides must remain equivalent. However, Henry was the only student who ever demonstrated the error of stringing the equal sign as if he was continually simplifying. This example contradicts prior interpretation of Henry's ability, as there was nothing that indicated this misconception on his Chapter 3 test.

Summary

In summarizing the results from the themes that emerged from the qualitative analysis, I also drew upon and revisited examples of how student work did or did not show procedural or conceptual understanding. Three out of the eight students interviewed showed some error with interpreting or expressing the difference between 'take away' referring to subtraction rather than dividing. It is difficult to know if this error is caused by the switch from the use of the division symbol "$\div$" to the use of a fraction bar $\frac{3}{5}$, which we use to express division, or whether there is a deeper conceptual misunderstanding of what it means to divide and how that is interpreted by students. However, with regard to the basic tenants of algebraic insight, knowing and understanding the meaning of the basic operations is essential.

Although students were able to recognize the distributive property, some did not recognize the name of this property and referred to it as "bombing." This was a procedural skill which most students interviewed had mastered, although some students did show misunderstanding when there was a complex expression requiring the use of the distributive property and combining like terms. Some students did attempt to combine terms without distributing, which demonstrated a lack of procedural understanding-specifically a lack of understanding the order of operations in problem solving.

Concerning solving equations and inequalities, there was a lot of inconsistency in student work. For example, Jim solved an inequality with 100% accuracy on his test, yet on a similar problem on the clinical task-based interview he made several errors. Some students appeared to forget the "process" from one problem to the next. This indicates most of the students interviewed are still developing algebraic insight with regards to the procedures for solving equations.

Understanding what a variable represents and being able to link that to how you would work with numbers is also a common weakness among students. In this study, I found it interesting that Rebecca had a high level of understanding of structure and was able to see a "variable" or "variable expression" as a number, but at the same time had a low understanding of the procedures used to solve an equation by hand.

The vocabulary we use in mathematics is very important. Consequently, assessing the student's ability to define terms has become a very important check for teachers. However, the examples from my research indicate how important it is to make sure students fully understand the terms we (the educators) use. For example, several students claimed that they could verify their results by "checking." However, when I asked what it meant to "check" I had some students who referred to "looking it over" and others who referred to "plugging in." Those who referred to "looking it over" literally did just that. They looked at the work and could not see anything wrong with it, and that to them was verifying their answer. Out of those who said "plugging in" some students were not able to substitute their x value into the original equation to verify their result. Henry knew both sides had to be equal, so his work showed him replacing the x with his "answer" and then he made the two sides equal rather than verifying they were equal. Another student, Samantha, did not know what to do when there were variables on both sides of the equation. She did know she was supposed to get "the same thing on both sides," but she wanted one side provided so she had a numerical value against which to check. She did not realize she just needed to replace all x values with her answer and check that the values of both sides were equal.

Although two students appeared to understand the use of CAS to verify results, none of the students interviewed were able to explain the screen capture provided in task 5. In the clinical task based interview, I did not provide other questions to test other aspects of the CAS display that students used during the generative activities. Some students, such as Henry, routinely used the CAS during the task-based interview to work out problems, whereas other students rarely used it at all.

The symbols we use in mathematics are key elements that students need to understand. Identification of these elements enables students to make conjectures about solutions prior to starting a problem. It is akin to estimating an answer before beginning a problem--if you know something about the answer you will most likely arrive at the answer more quickly, or at least know if you did something wrong.

Student Feedback

Although at first many students were interested in the new CAS calculator, students lost interest in it as it was “too hard” to use, especially as they had not used even four function calculators regularly in previous math classes. “It was kind of complicated for the math we were doing, like it would probably be a lot better for a higher level math, but it just made everything we could do on a normal calculator a lot more complicated…I think it’s more for older people.” These and similar statements were made by students in both focus group interviews as well as by the experimental group instructor, Annabree.

One student referred to an example where he used the calculator to divide 5 by 2; he said “it just gave me a fraction instead of giving me the answer and I was like - I just didn’t understand. I just got frustrated sometimes because something so big can’t do something like that.” Both the students and Annabree noted this and other syntax and usage problems. For example, Annabree noted problems students had with the difference between the minus sign and the negative sign.

The consensus appeared to be that students felt they were not ready for such advanced technology. “Maybe it would help people in our grade - maybe in honors algebra.” Overall students did not feel that the TI-Nspire CAS either helped them or hurt them with respect to learning algebraic concepts.

Experimental Group Instructor Feedback

Regarding the experimental group's understanding of the math in relation to the calculator, Annabree noted, "I think that for some aspects they really like it. I mean they really did, especially when they first got them they were like 'oh this is cool and it's new technology' and they were excited about it…what I found which I think led them to eventually not use it as much as we were hoping they would is it's such a jump from the fact that they only used at highest a scientific calculator…I think that part of it was that they wanted it to be fun and they wanted it to be exciting, but they didn't know how to use it well enough to make it fun and exciting." Initially a major issue was "they weren't even used to the standard features so they didn't know how to make it fun to play around with."

For the experimental group Annabree noted that "the difference in the technology level was such a high jump that even though they had instructions, there were some things that were hard" and she listed several examples where, although students had step by step instructions, they still made errors inputting data. She also stated "a lot of their number sense is not strong enough to know if they typed in an error." For example, students "weren't realizing that minus and negative on a graphing calculator are two different buttons." In addition, when it came to using the calculator to simplify expressions, Annabree felt that students did not understand what the calculator was doing and therefore would think an output was incorrect. She noted that on one example they were asked to add terms x^2+y^2. Previously, they had completed examples with $x^2 + 2x^2$ and received $3x^2$, but on this example the calculator provided the answer x^2+y^2. She said “90% of the kids raised their hand and said it’s broken.” Annabree’s reasoning was that “because they were so unsure about their own abilities with the CAS that instead of realizing, ‘Oh this is simplified’, the response was ‘the calculator is broken’.”

For students who felt intimidated using the calculator, Annabree felt that even during the generative activities some “were more likely to do it by hand or do the method that we had written on the board.” She mentioned a couple students who she felt were comfortable using the CAS, but did not always use it. She mentioned that a particular student was “very comfortable with math so he would do it the first two times he’d see the pattern and he didn’t use it because he felt it took him longer.” The specific students mentioned by this teacher who felt comfortable with the CAS, were those who caught on quickly, were borderline concerning placement, and possibly could have gone into algebra. She noted that students tended to use the calculator when “it’s faster to do it on the calculator.” Therefore, once they saw what the rule was, they tended to start writing answers without using the technology – not even to verify answers.

Annabree also mentioned students’ attitudes toward completing class work. She said students sometimes did not use the technology once they thought they knew the rule because they “were trying to get through the questions.” Part of the ‘buy-in’ problem was that “they weren’t willing to invest the time.” She said her 8^{th} grade pre-algebra students were grade driven. Their first questions when she introduced anything new were; “Is this on the test...Is this on the SOL...Is this…?” They wanted to know how it applied immediately to their percentage – their grade. So when the question was “Oh, I can use it on the quiz” and the response is “No, you cannot use that on the quiz, you can not use it on the SOL,” then at least half of them responded “well then why are we using it?”

Annabree felt they were collectively not able to look at the benefits of anything new and understand how a tool used in class could benefit them even if they could not use it on the test. Thus, she felt that many students possibly did not make full use of the CAS technology as “they still saw it as an investment in another educational tool that didn’t really matter because they weren’t being graded on it - and they’re so driven by their grade.” They “weren’t really willing to put in the time to make themselves comfortable with it.” She noted that, in general, the 8^{th} grade pre-algebra students “all think they’re behind and because of it whether they are getting an A in my class or a D in my class they think that they’re bad at math. And so they don’t invest the time to go above and beyond in math or to spend any more time in math because for a lot of them…I mean - it’s a dreaded subject.”

When working on the generative activities, Annabree related, “It was useful when they saw the lesson and they saw that the calculator could do that and it was confirming that x squared times x to the third is x to the fifth.” She felt it was helpful for students to see the pattern repeated when they were required to work through the steps of the generative activities on the calculator. She suggested the technology provided good reinforcement when students worked through problems using the CAS and noted patterns.

Annabree said she did monitor students to make sure that they were using the CAS. “They’d have the answer down and I would walk by and ask if they had worked it out and ask to see their calculator and if they hadn’t done it I would say ‘Go through and double check it in the calculator’.” She stated that even if a student had previously just put down an answer, making them go back and verify their answers using the CAS was reinforcing their understanding by using the technology.

This teacher did feel that when the generative activities were used as a discovery method with no initial instruction, most of her students had difficulty completing the activities. She stated, “when it was a discovery activity it was a hindrance…they could not make those connections.” She felt their number sense and background knowledge as well as self-confidence were not strong enough for them to complete this as an unscaffolded activity. She noted she had to do more explanation of how to use the calculator than she had thought she would have to, stating “I think that that jump of technology was a bigger hurdle than I thought it would be too.” She also suggested the activities could have been better integrated into her lessons.

Conclusions

Quantitative

These conclusions are based on the analysis of the quantitative aspects of the data collected from all students in this study. Although there were no significant differences overall in student achievement (as based on the Chapter 3 test), this study did show a significant difference in gain scores for the experimental group over the control group. There were also differences between the experimental and control groups with regards to the subgroups (No accommodation, ELL, IEP); student achievement by the no accommodations subgroup in the experimental group was higher than the same subgroup in the control group.

Students in the experimental and control groups scored relatively equal for affective engagement and behavioral engagement; however, students in the experimental group scored 5% lower for mathematics confidence and 10% lower in their confidence with technology. Although there was no correlation found between eighth-grade pre-algebra students’ attitudes toward mathematics in terms of their achievement and gain in knowledge, it is important to note that the highest gains (10%) were by the experimental group in the “confidence with technology” category. Thus, although the students may have not felt very comfortable using the CAS, using it showed a positive effect on their confidence with technology.

Qualitative

These conclusions are based on the analysis of the eight students who participated in the clinical task-based interviews. In comparing the algebraic insight index to the students’ chapter test scores, there does not appear to be any correlation between a student’s algebraic insight and his/her test score. Achievement scores on tests and quizzes measure student proficiency in a traditional course. However, although the objective of traditional tests may claim to be a check for understanding, many merely assess a student’s ability to produce a correct solution. In the age of differentiation, teachers often allow students to solve problems by their own methods. Thus, it matters whether teachers verify that the student’s method was correct, or merely grade the student on the correct solution. If students are to understand procedures then educators must start analyzing the errors students make when carrying out procedures.

Perhaps specific aspects of number sense and algebraic insight are closely linked to student achievement in a traditional classroom. Therefore, to comprehend how well a student understands the mathematics, both procedurally and conceptually, educators also have to look beyond just the solution to problems. Although incorrect procedures typically lead to incorrect solutions, a series of mistakes or incorrect procedures could also inadvertently provide a correct solution. To be sure of a student's procedural understanding educators need to look at the consistency with which students use correct procedures to solve problems accurately. Sometimes understanding what a student did to solve a problem is more than just looking at his/her work.

The findings from the clinical task-based interviews in this study suggest students' algebraic insight did not have any correlation to their Chapter 3 test, although there were a few weeks between the students' test on Chapter 3 and their clinical task-based interview. There were cases where students demonstrated correct procedures on the test and then incorrect procedures on the same or similar problems on the task based interview. There were also students whose algebraic insight index based on the task-based interviews were the same, but whose scores on the Chapter 3 test differed by at least 10%. This suggests one key question for the pre-algebra classroom is determining the best way to prepare students for algebra and test for understanding. Are traditional tests, where students are asked to produce solutions, and possibly required to show steps, sufficient? Tests designed to check for procedural understanding and require students to demonstrate understanding of the relationship between numbers and variables will help students develop and strengthen algebraic insight. Activities where students are asked to prove their solutions and support their results in a step-by-step fashion will help strengthen students' deductive reason skills and understanding of the properties, theorems, definitions, and symbols used in mathematics.

The study showed misconceptions with respect to students' ability to use the correct vocabulary and to demonstrate mathematically what terms meant. All students in the study referred to terms such as "checking." However, not all were able to recognize whether student work was correct using this method. Terms such as 'plugging in', 'get the same number on both sides', and 'do the same thing to both sides' were often said with confidence by students, but analysis of student work showed a medium to low understanding of these terms.

Students in the study showed the strongest algebraic insight with respect to the distributive property and combining like terms. However, only one out of eight students in this part of the study showed a high understanding of the variable and this student was one who at the same time showed little understanding of procedures.

Limitations

I would have liked to spend time observing both the experimental and control groups, but as I was conducting this research while teaching full-time, I was unable to do so. Unfortunately, this also meant I was unable to see how students interacted and whether the interaction met my expectation. Observation would also have enabled me to see which groups tended to better grasp the concepts taught.

The population for this study was limited. I realize that 36 is a small sample size, but I did ask all students in pre-algebra to participate. There were only 70 students in the entire 8th grade enrolled in pre-algebra so I did get over 50% participation. Another limitation was the number of students I was able to get to come in for the task-based interviews. There were 19 students in the experimental group. Out of the 19, I was only able to get eight students to come in for a clinical-task based interview.

In discussions about the role that CAS can play in the classroom, Heid and Edwards (2001) describe the white-box versus black-box idea. In my study, I was trying to isolate more of the black-box nature of CAS. My intention was to see how the CAS ability to perform symbolic manipulation could help enhance student understanding. Thus, the intent of the generative activities was to develop a students' ability to recognize patterns and formulate rules. The initial intent was for students to develop these rules prior to formal traditional instruction. However, as many of the students initially had difficulty with the syntax and were unsure of their results, Annabree, the teacher who taught two experimental sections and one control section, improvised and started teaching traditional lessons to introduce the concepts prior to them actually working through the generative activities using the TI-Nspire CAS. According to Annabree, "The good part- I mean what I thought was at least the positive in that is it was consistent teaching—I mean I taught it in the same way to all blocks, and then the activity was different. You know—the way that their extension was, was a little bit different."

Thus, one major limitation is that the students did not use the technology to look at patterns and describe patterns in the way I had initially planned. However, as the teacher explained, they still worked through the generative activities and wrote and discussed the patterns, thus they were using the CAS as a form of enrichment or an amplifier of the traditional concepts that were taught that day. The drawback is that even if students were asked to complete the generative activity using CAS, once students were shown a rule, some of them may have completed the generative activity based on what was taught in class and not use the CAS to reinforce the concepts.

One of the major limitations was student "buy in." According to the student and teacher accounts, students did use the CAS calculators for the generative activities, but the extent to which students used them varied. Each student in the experimental group was provided his/her own calculator to keep for the duration of the research project. Only a few students said they used it outside of class. Of those that did, many used it as a four-function calculator. The focus group interviews showed that one reason for this was the lack of prior experience using technology in math class. There was a huge learning curve from a four-function calculator to the TI-Nspire CAS.

When I designed the activities, I should have taken into consideration the types of things students did regularly on their calculator and ensured they were taught how to do those same things on the new calculator. This would have developed a familiarity with the new technology so students would at least be able to use the CAS calculator for the “easy calculations.” I also should have modeled how to evaluate entire expressions at once. For example, if I had done one order of operation problem with students, they may have seen how they could have done the whole problem at once instead of in pieces. This would have made computation quicker and maybe would have led to some students becoming interested in seeing what else the new technology could do. However, as I had not familiarized students with the CAS calculator in terms of the way they normally used their own technology, the CAS technology became less user friendly for simple tasks and thus less likely to be used with frequency outside of the classroom.

I could have asked students to write about what they used their four-function calculators for and how they helped them in math, and to provide examples. One idea was to ask students to design a calculator to help them in math and ask them what some of the things it could do that their current calculators could not do. I might even ask students to describe what it might look like and come up with a sketch of a prototype.

5. Implications and Further Research

Introduction

After running the analyses of the data, my assessment is that the results neither strongly support nor hinder a case for CAS at the pre-algebra level. The post-study analysis has illustrated things I could have changed if I was able to redo my research. As a teacher, I find myself considering the professional development that mathematics educators would have to undertake in order for the use of CAS to get off the ground and how a form of CAS that was more user-friendly would have affected this study. As a researcher I have considered what other researchers might get out of this piece of the puzzle regarding the case for CAS, as well as what types of future studies might be influenced by my work.

Changes to My Study

Looking at the task-based interviews, most students were strong in the use of the distributive property. In task 4 (Figure 32) of the clinical task-based interviews, I should have rewritten the responses so student number 1's problem worked out correctly. Many students felt there was an error in the student's method and seemed to dismiss the rest of the problem. For student number 2, I should have shown the work with correct distribution of multiplication over addition (or "bombing" as Annabree referred to it) and then presented a mistake in the next step. This would have enabled me to see if students could recognize the correct answer and not base right or wrong on one aspect of the work. This would possibly also have led to some students realizing that the first step student number 1 did was valid, enabling the students in the study to provide more in-depth explanations for different solution methods and assess the validity of the response provided.

One suggestion made by both teachers and students related to their ability to use the CAS. One major change for future research would be to make sure that both students and teachers learn how to use the basic features on the CAS with ease (such as those on a four-function calculator) before trying to show them how to use it to do anything else. Perhaps the CAS I used for this research project was too complicated for the level of students in the study, suggesting a more basic CAS that was more user friendly would have yielded different results.

This study did provide ecological validity. This means that the CAS was introduced by adding it into the natural environment (the pre-existing curriculum) and students in the control and experimental group both received practically the same instruction and activities. In addition, the teachers were not especially familiar or skilled with the particular technology similar to most teachers who might begin teaching with a CAS or other type of technology. Therefore, to test a “basic CAS” a researcher would not necessarily need to reproduce the conditions of this study, but rather introduce CAS to one of several sections taught by a teacher and create generative activities that could be done with or without CAS. The key is to use the CAS with a traditional curriculum. Many of the studies that reported significant differences using CAS did not use traditional curriculum (Geddings, 2003).

Design of CAS Technology

In this study, following the use of CAS by a set of 8^{th} grade pre-algebra students, results demonstrated no major differences in achievement between students who did or did not use CAS. The results from the Chapter 3 test did not show any differences between the experimental and control groups. The Hake gain score for Algebraic Expectation was the only positive trend noted, where the students in the experimental group showed higher gains than the control group. However, the qualitative data collected through the clinical task-based interviews and focus group interviews indicated students still did not fully understand the concepts. Therefore, in this study there is no clear link between the use of CAS and improved conceptual and procedural understanding. The key question thus becomes determining what about the design of the study did not work?

The interviews (including the clinical task-based, focus group, and teacher) all clearly indicate that with this group of students the difference in the technology level was perhaps too big a jump. One student said, “It was kind of complicated for the math we were doing. It would probably be a lot better for a higher level math, but it just made everything we could do on a normal calculator a lot more complicated.” Statements from both students and teachers refer to the technology as too complicated. Geddings likewise concluded that “had the CAS group been more comfortable with the computer algebra system…, they would have been more willing to experiment and use the computer algebra system” (2003, p.112). Research has supported that the CAS ease of use--including syntax and output--are often noted as weaknesses with the technology (Artigue & Lagrange, 1997; Drijvers, 2004; Edwards, 2003; Geddings, 2003; Jakucyn & Kerr, 2002).

Frequently within the results of research on CAS is an emphasis on the findings, but one major underlying difference is the type of CAS used. Often studies researching CAS underplay the actual CAS being used, such as in this study, as if the results would be generalizable to other forms of CAS such as seen in the meta-analysis by Tokpah (2008). Thus, one factor requiring further research is what a user friendly CAS would look like for students with limited mathematics technology experience beyond that of a four-function calculator. Another factor requiring further research is how even subtle differences in design could have significant impacts on students' learning.

One major concern illustrated by this study was the inability of students to use the CAS as easily as a four-function calculator. One potential solution could be a "basic CAS" calculator, or even a CAS application for an iPod touch/ iPhone/smart phone, based on a four-function calculator platform. Additional buttons would be added to a four-function calculator platform, which would allow students to easily input and solve equations. Such buttons would include an "x" and a "y" key as well as parentheses "(" and ")". In order to allow students to replicate problems visually, the addition of a fraction key would be useful to facilitate the creation of a fraction so that problems could look like those presented in text. The "basic CAS" would have a large enough screen display that students could see or easily scroll back to the last data input. This "basic CAS" could simplify algebraic expressions as well as solve algebraic equations step-by-step without being overcomplicated or visually intimidating. In fact, with so many students now familiar with touch screens, adding the ability to touch the screen to move the cursor would also be useful.

From what student and teacher interviews revealed, using a version of CAS that was less intimidating and more user-friendly to students could have a positive effect on student use of the technology--yielding increased motivation to use it. Whether the increased student use of CAS technology in this study would have had an affect on student procedural and conceptual understanding is for future research to determine.

A CAS requiring less instructional time would also require less teacher preparation time as well as student time, and time is one of the three independent variables found to significantly moderate the effect of CAS in Tokpah's meta-analysis (2008). This study lasted only eight weeks and perhaps was not long enough to yield results, as students were trying to learn how to use the technology during the study. A longer study may have yielded different results, as students would have had more time to learn the technology. However, apparently time is not always the issue. Edwards did a yearlong research project with a control and experimental group and actually found that the control group outperformed the experimental CAS group on the end of year algebra exam (2001).

An interesting finding from the Edwards' study was that the low-performing non-CAS students significantly outperformed the CAS counterparts, ($p=0.029$). This is also supported by the teachers in a study who "believed that CAS is of most benefit to their high ability students, and may present an obstacle to their low ability students' learning of mathematics" (Pierce, Ball, & Stacey, 2009, p. 1149). With the push for algebra for all, the 8^{th} grade mathematics (pre-algebra) course contains students who were not ready for algebra in 8th grade--typically lower ability students. Therefore, students in this study have similar characteristics to those students in the low-ability groups in the Edwards and Pierce, Ball, and Stacey studies. Perhaps a similar study conducted on pre-algebra students in 6th or 7th grade, typically a higher performing group, would yield different results.

In this study students first conducted the generative activity independently then discussed their findings within cooperative groups. However, low-ability students perhaps mathematics technology are too intimidated to come up with results on their own and less likely to share what they noticed with their group for fear of being wrong. Cognitive research shows that the computer tool (CAS) can serve as a means to reorganize and give structure to students' new knowledge (Cooper, 2000). Thus, perhaps the procedure would benefit from changes so that students start by working together on the generative activity. In this way, the generative activity becomes the platform for group learning and exploration. The generative activity becomes a learning task that provides the opportunity for peer interaction and discussion, helping students broaden and extend their understanding through the thoughts of others. Perhaps looking at the research from this perspective would help to understand whether meaning (in this case mathematical understanding) is socially constructed.

The other aspect not previously considered was whether students and teachers in this study were ready for the design of the activities being used in this study in the first place. The basis for the generative activity has roots in constructivism. Students were expected to construct their own knowledge (or reinforce concepts) by completing the generative activity and discussing their results with their cooperative group. The comfort students had with this discovery method is unclear. If students were familiar with a process where they were given the rules and then applied mathematics technology them, in this study they first would have needed to get used to a new way of learning, which could have had a negative impact on their attitude and motivation to use the CAS.

Motivation to use or want to use the CAS was an additional concern. The students knew early on in the study that the CAS could be taken home and used in class, but also knew they would not be able to use it on tests and quizzes nor on the end of the year state exam. The teacher, Annabree, felt students were collectively not able to look at the benefits of anything new and understand how a tool used in class could benefit them even if they could not use it on the test. Thus, she felt that many students possibly did not make full use of the CAS technology as “they still saw it as an investment in another educational tool that didn’t really matter because they weren’t being graded on it - and they’re so driven by their grade.”

The teachers' familiarity with the technology as well as the evaluation methods is critical. Studies that have looked at the teacher's role in implementing a new technology show that the teacher has a substantial effect. "Larry Cuban (1995), who has chronicled the history of technology in the American classroom, suggests that the success or demise of the innovation rests ultimately with the individual instructor" (Cooper, 2000, p. 871). Consequently, an important point to note from my pilot study is that students' procedural and conceptual understanding improved after using the CAS. This led me to start the primary research project envisioning there would be positive gains by the experimental group, based on my experience in the pilot study. However, in retrospect, it is quite possible that my knowledge of the CAS technology as well as my enthusiasm for its use made a substantial difference when the research subjects were my own students. According to Cooper "commitment to all of the aspects of the design is not likely to be as strong in secondary implementers because they have not been a part of the research that established the design" (2000, p. 871). This is not saying there was a lack of faithfulness to the design, nor any lack of ability of the teachers who carried out its implementation—they just were not intimately familiar with and enthused by the CAS technology as I was.

There is a framework known as Technical Pedagogical Content Knowledge (TPACK), which describes the kinds of knowledge needed by a teacher for effective technology integration (Mishra & Koehler, 2006). Effective technology integration for pedagogy around specific subject matter requires developing sensitivity to the dynamic, transactional relationship between the three primary forms of knowledge: content, pedagogy, and technology. As a National Board Certified teacher and a veteran teacher of over eighteen years, I had both a strong mathematical content knowledge as well as a strong pedagogical content knowledge for teaching mathematics which are considered critical for the success of reform in the classroom (Ball & Bass, 2000). The teacher participants in this study also had these two forms of knowledge. However, as I had been using the technology and attending conferences where researchers presented on the use of the CAS technology, I had significantly more expertise. "A teacher capable of negotiating these relationships represents a form of expertise different from, and greater than, the knowledge of a disciplinary expert (say a mathematician or a historian), a technology expert (a computer scientist) and a pedagogical expert (an experienced educator)" (Mishra, 2008, p. 1).

Research Implications

The results of this study indicate there are design features to consider when developing, implementing, and researching the effectiveness of CAS for learning and teaching. Perhaps existing studies need to be examined to look for possible connections between student age level, ability level, and the specific CAS that was used as well as whether the researcher or teachers involved with the studies would have been classified as having technical pedagogical content knowledge. Also, looking specifically how results were determined and what type(s) of assessments were used would be critical. For example, in this study, even though there were signs of an increase in gain scores for the experimental group over the control group on the Algebraic Expectation pre- to post-test, the qualitative data indicated that students still did not have strong conceptual and procedural understanding. I was only able to compare test and achievement scores to the clinical task-based interviews of the eight interviewees, but the data from this small sample indicated a lack of algebraic insight. Without speaking with all students so they could explain what they did to solve problems, it is difficult to understand comprehensively their level of algebraic insight. Although I had quantitative data in the form of achievement and gain scores, the clinical task-based interviews provided me a clearer picture of where student misconceptions were and allowed me to intervene to address and better understand specific misconceptions.

If it were possible to meet with and observe students regularly, a researcher could study the instrumentation of the CAS and possibly take a closer look at how knowledge develops over time with the regular use of CAS technology and how the use of CAS could influence algebraic insight.

Future Research

As algebra is considered the gateway to further mathematics and number sense has been linked to success in algebra, perhaps CAS's symbolic manipulative capabilities could be used to help strengthen a student's number sense and help them to create and generate their own procedures, thus enhancing their procedural knowledge and conceptual knowledge through integration of increased number sense and algebraic reasoning. However, verification of this will take substantial careful and deliberate monitoring. Research on CAS remains in need of case studies featuring regular monitoring of individual student progress, particularly at the pre-algebra level. This would aid in the understanding of the link between procedural skills, procedural knowledge, and conceptual knowledge and how they are connected or interconnected as a student learns algebraic concepts using CAS. This might also allow researchers to determine whether the categories of procedural and conceptual knowledge are too restrictive and whether case studies might lead to alternative categories that are less restrictive and can more accurately explain the interconnection in the development of algebraic insight.

The clinical task-based interviews revealed ten common themes used to describe student misconceptions with respect to algebraic insight. Studies concentrating on only one of the themes may help to provide a more focused understanding of the specific aspects of number sense and algebraic insight more closely linked to student achievement in a traditional classroom as well as student success in algebra and in higher-level mathematics.

If a student receives an A in a pre-algebra class, one might assume that student has the knowledge necessary for success in algebra. However, when students' grades are based exclusively from tests and quizzes, how well do those tests and quizzes in a traditional math class identify a student's procedural or conceptual knowledge? If students are to be successful in algebra, there is a need to further assess students on their understanding of each objective to identify where misconceptions or misunderstandings arise. Identification of common misconceptions should be an integral part of pre-service mathematics teacher education. Research on just what a grade in a mathematics course means and how it relates to student test and quiz scores in that course as well as achievement on other assessments at the state and national level might suggest a need to examine the traditional way we assess students.

This research involved eighth graders in pre-algebra. None of the students in the clinical task-based interviews showed high overall algebraic insight. All students showed at least several areas of weakness. Identification of those weaknesses in the classroom, which would enable teachers to take correctional steps, will be required for these students to understand algebra. Research on error analysis could perhaps help students identify mistakes and help students better understand the terms and expressions used in mathematics. This could take place in a course where students regularly look at work done by others and are asked to tell whether the answer provided is correct or not and explain why. Perhaps, as with repetition, the more students correct the mistakes someone else has made, the less likely they will be to repeat it. Targeting specific areas of misconception such as "checking," "plugging in," "get the same thing on both sides," and "do the same thing to both sides" would likely be beneficial.

As the CAS provides a quick way to perform and check algebraic procedures, it might be a good way to teach students, such as Rebecca, who have a strong conceptual understanding, but who do not yet understand the procedures to solving an equation. Perhaps a study targeted at students who appear to have strong number sense, but who do not yet understand how to work with variables could examine the possible benefits of CAS for such students. Another potential target group for future CAS research is ELL students. Although the ELL subgroup for this study was small, the results of Hispanic ELL students indicated differences in achievement based on accommodations subgroup. Although not significant, this indicates that data based on accommodations, especially ELL students, should be disaggregated in future research on CAS.

In this study, I investigated mathematical attitudes; and in focus group interviews, I asked questions about the students' comfort level with the TI-Nspire CAS technology as well as how often they used the technology. I did not track their usage or attitudes over time. It would be interesting to track the level to which a student adopted the technology and determine, based on their rate of use, whether a students' adoption rate influences their overall achievement in gaining algebraic insight. It would also be helpful to track usage to determine whether students develop the ability to use the CAS strategically as opposed to just trying things out randomly. Tracking this over time could show the transition of the use of the TI-Nspire CAS from artifact to instrument as well as examining such instrumental genesis and its role in student conceptual understanding. The use of a more “basic CAS” would facilitate this process.

Professional Development

Teachers are like students in many regards. The students want to know if what they are learning is going to be on a test or the state end-of-year exam. When the students found out they would not be able to use the TI-Nspire CAS on class tests or the state exam, they questioned why they should put the time and effort into learning it. Thus, there was a general lack of motivation to use the technology. I think teachers also often feel this way. If we want to change the culture and methods by which we teach mathematics we need to start with the teachers. As with any new technology, if the TI-Nspire or any CAS technology is to be adopted, teachers will need to attend professional development training not only on how to use the specific CAS technology, such as the TI-Nspire CAS, but how to teach a student to navigate through and use its interface. Students and teachers have to become comfortable with the CAS technology as the instrumentalization process helps them develop their own insight and understanding of the tool--they need to possess technical pedagogical content knowledge. The introduction of CAS at a younger grade level could be a vehicle that drives the change towards alternative teaching and learning style in the classroom. In a CAS classroom, students can start to answer questions on their own, so teacher training would have to encourage teachers to use scaffolding to support student investigation and learning as opposed to telling students the rules. This change may be difficult for teachers to embrace as using CAS puts greater demand on teachers to learn the new technology, develop lessons and activities, and redesign assessments.

Concluding Comments

The future role of CAS in United States secondary schools is not nearly as questionable as its role in middle schools. Researchers are always looking for new instructional methods to help improve student learning of algebra—including new ways to deliver the mathematics. Technology is often the venue used in change. With the emergence of handheld CAS and the greater availability and affordability of handheld systems, secondary mathematics curricula developers are now including CAS in instructional materials (Heid, 2008). Although the data from the student focus group interviews as well as the teacher interviews reflect the perceptions of those interviewed, perception will play a large role in whether CAS is adopted. Technology for the classroom is often purchased because a teacher or administrator perceived benefits from its use. Thus, the perception of handheld CAS by teachers and students would likely strengthen or weaken the potential for its adoption.

The results of this study provide information to middle and high school educators and mathematics leaders as well as policy makers seeking to integrate CAS technology into their middle school classrooms. According to teacher and student perceptions in this study, as well as the actual test scores of the participants, the use of CAS neither advanced nor hindered a child's ability to solve problems by hand. Thus, although there is evidence in the research that suggests that the use of CAS technology in mathematics may help to deepen a students' procedural knowledge (Heid, 2008; Pierce et al., 2009), in this study few significant differences were found when comparing the students who used CAS to those who did not. Is the fact that the CAS caused no harm a sufficient reason to adopt CAS? Perhaps there were differences that were not noted due to the testing methods used. Repeated tests at differing ability levels of students taking pre-algebra which yield consistent positive results will most likely be needed prior to CAS implementation at the pre-algebra level. However, this study supports others in that CAS has little affect on increasing procedural or conceptual understanding in mathematics for low-ability students.

This study also points to the need for researchers not to equate achievement scores directly with algebraic understanding. If number sense and algebraic insight are important for future success in mathematics, perhaps CAS can be used as a vehicle to help to strengthen specific misconceptions that have been identified in other studies. I would be interested to see what case studies that targeted specific types of students/learners would reveal. In the age of No Child Left Behind and Adequate Yearly Progress, I also think about how CAS has affected certain subgroups--specifically ELL students. Perhaps CAS technology is a tool that can traverse language barriers to aid students in learning how to apply correct mathematical procedures as well as assist in the development of conceptual understanding of mathematics.

APPENDIX A: 8th GRADE CURRICULUM

Middle Grades Math Course 3

Prentice Hall, 2001

List of topics covered in 8th grade pre-algebra in Chapters 2 and 3.

Chapter 2: Integers and Variable Expressions

- 2.1 Integers and absolute value
- 2.2 Writing and evaluating variable expressions
- 2.3 Adding integers
- 2.4 Subtracting integers
- 2.5 Multiplying and dividing integers
- 2.6 Exponents and multiplication
- 2.7 Evaluating expressions with exponents
- 2.8 Mental Math and properties of numbers
- 2.9 Guess and test
- 2.10 Exponents and division
- 2.11 Scientific notation

Chapter 3: Equations and Inequalities

- 3.1 Simplifying variable expressions
- 3.2 Solving equations by subtracting or adding
- 3.3 Solving equations by dividing or multiplying
- 3.4 Solving two-step equations
- 3.5 Writing an equation
- 3.6 Simplifying and solving equations
- 3.7 Formulas
- 3.8 Inequalities
- 3.9 Solving inequalities by subtracting or adding
- 3.10 Solving inequalities by dividing or multiplying

APPENDIX B: MATHEMATICS AND TECHNOLOGY PRE-ATTITUDES SURVEY

MATHEMATICS AND TECHNOLOGY PRE-ATTITUDES SURVEY

Name..	*Teacher*...
Date	*Block* ....

Please circle your responses to the following:

		Hardly Ever	Occasionally	About Half the time	Usually	Nearly Always
1.	I concentrate hard in math	HE	Oc	Ha	U	NA
2.	I try to answer questions the teacher asks	HE	Oc	Ha	U	NA
3.	If I make mistakes, I work until I have corrected them.	HE	Oc	Ha	U	NA
4.	If I can't do a problem, I keep trying different ideas.	HE	Oc	Ha	U	NA
		Strongly Disagree	**Disagree**	**Not Sure**	**Agree**	**Strongly Agree**
5.	I am good at using computers	SD	D	NS	A	SA
6.	I am good at using things like VCRs, DVDs, MP3s and mobile phones	SD	D	NS	A	SA
7.	I can fix a lot of computer problems	SD	D	NS	A	SA
8.	I am quick to learn new computer software needed for school	SD	D	NS	A	SA
9.	I have a mathematical mind	SD	D	NS	A	SA
10.	I can get good results in math	SD	D	NS	A	SA
11.	I know I can handle difficulties in math	SD	D	NS	A	SA
12.	I am confident with math	SD	D	NS	A	SA
13.	I am interested to learn new things in math	SD	D	NS	A	SA
14.	In math I get rewards for my effort	SD	D	NS	A	SA
15.	Learning math is enjoyable	SD	D	NS	A	SA
16.	I get a sense of satisfaction when I solve maths problems	SD	D	NS	A	SA
17.	Mathematics makes me feel uneasy and confused.	SD	D	NS	A	SA
18.	Mathematics is important to me for my future goals.	SD	D	NS	A	SA
19.	Talking with other students helps me to learn mathematics.	SD	D	NS	A	SA
20.	There is nothing creative about mathematics; it is just a lot of memorization of rules.	SD	D	NS	A	SA

Please turn over

MATHEMATICS AND TECHNOLOGY PRE-ATTITUDES SURVEY

21. I expect the following grade in this class.

1. F 2. D 3. C 4. B 5. A

22. Compared to other 8th grade students in mathematics ability, I am…

1. In the top 10%
2. Above average
3. About average
4. Below average
5. In the bottom 10%

23. During this year, I plan to do the work assigned in the class…

1. Always
2. Most of the time
3. About half the time
4. Once in a while
5. Almost never
6. Never

24. How important is it for you to do well in math?

1. Very important
2. Sort of important
3. Not very important
4. Not important at all

THANK YOU FOR YOUR TIME

APPENDIX C: MATHEMATICS AND TECHNOLOGY POST-ATTIUDES SURVEY

MATHEMATICS AND TECHNOLOGY POST-ATTITUDES SURVEY

Name.. *Teacher*...

Date .... *Block* ....

Please circle your responses to the following:

		Hardly Ever	Occasionally	About Half the time	Usually	Nearly Always
1.	I concentrate hard in math	HE	Oc	Ha	U	NA
2.	I try to answer questions the teacher asks	HE	Oc	Ha	U	NA
3.	If I make mistakes, I work until I have corrected them.	HE	Oc	Ha	U	NA
4.	If I can't do a problem, I keep trying different ideas.	HE	Oc	Ha	U	NA
		Strongly Disagree	Disagree	Not Sure	Agree	Strongly Agree
5.	I am good at using computers	SD	D	NS	A	SA
6.	I am good at using things like VCRs, DVDs, MP3s and mobile phones	SD	D	NS	A	SA
7.	I can fix a lot of computer problems	SD	D	NS	A	SA
8.	I am quick to learn new computer software needed for school	SD	D	NS	A	SA
9.	I have a mathematical mind	SD	D	NS	A	SA
10.	I can get good results in math	SD	D	NS	A	SA
11.	I know I can handle difficulties in math	SD	D	NS	A	SA
12.	I am confident with math	SD	D	NS	A	SA
13.	I am interested to learn new things in math	SD	D	NS	A	SA
14.	In math you get rewards for your effort	SD	D	NS	A	SA
15.	Learning math is enjoyable	SD	D	NS	A	SA
16.	I get a sense of satisfaction when I solve maths problems	SD	D	NS	A	SA
17.	Mathematics makes me feel uneasy and confused.	SD	D	NS	A	SA
18.	Mathematics is important to me for my future goals.	SD	D	NS	A	SA
19.	Talking with other students helps me to learn mathematics.	SD	D	NS	A	SA
20.	There is nothing creative about mathematics; it is just a lot of memorization of rules.	SD	D	NS	A	SA
21.	I like using the TI-Nspire CAS calculators for math	SD	D	NS	A	SA
22.	Using the TI-Nspire in math is worth the extra effort	SD	D	NS	A	SA
23.	Maths is more interesting when using the TI-Nspire CAS calculators	SD	D	NS	A	SA
24.	The TI-Nspire CAS calculators help me learn maths better	SD	D	NS	A	SA

MATHEMATICS AND TECHNOLOGY POST-ATTITUDES SURVEY

25. I expect the following grade in this class.

1. F 2. D 3. C 4. B 5. A

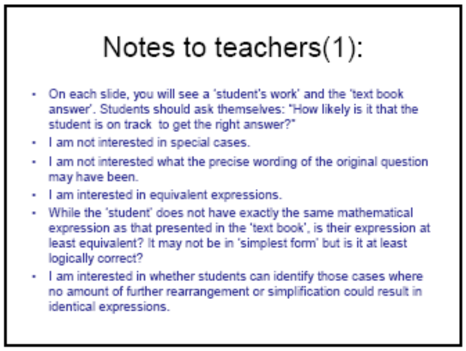
26. Compared to other 8th grade students in mathematics ability, I am…
   1. In the top 10%
   2. Above average
   3. About average
   4. Below average
   5. In the bottom 10%

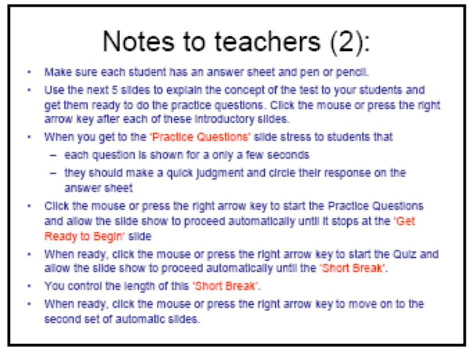
27. During this year, I plan to do the work assigned in the class…
   1. Always
   2. Most of the time
   3. About half the time
   4. Once in a while
   5. Almost never
   6. Never

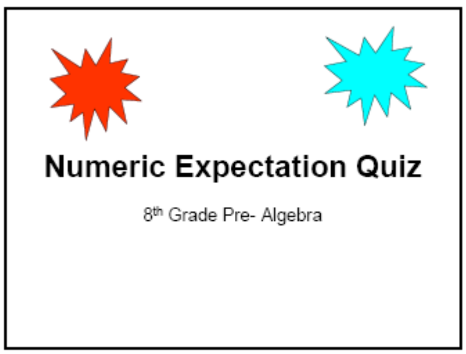
28. How important is it for you to do well in math?
   1. Very important
   2. Sort of important
   3. Not very important
   4. Not important at all

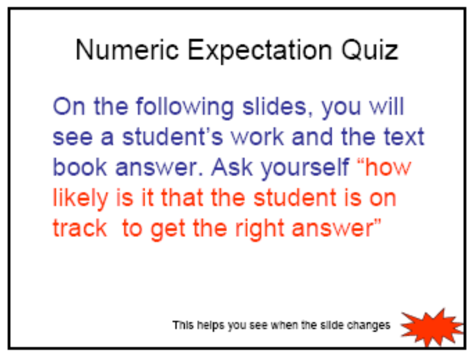
THANK YOU FOR YOUR TIME

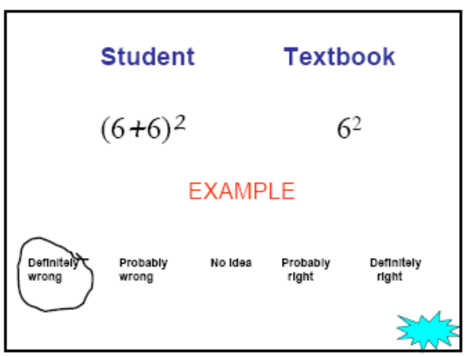
APPENDIX D: NUMERIC EXPECTATION QUIZ

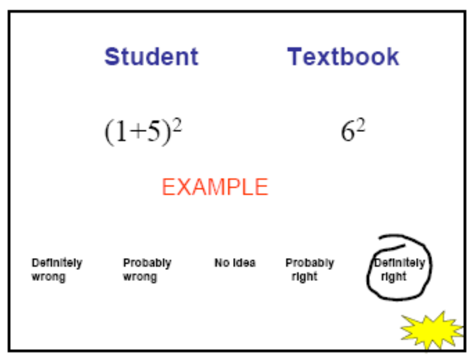












Practice Questions

- There are 3 practice questions.
- You will see each question for only a few seconds.
- Make a quick judgment about the student's work

Definitely wrong | Probably wrong | No Idea | Probably right | Definitely right

- Circle your response for the question on the answer sheet.

P1. Student | Textbook

$$(3-2) \qquad 1$$

P2. Student | Textbook

$$12-4\times 2 \qquad 16$$

P3. Student | Textbook

$$\frac{1}{2}\times\frac{4}{3} \qquad \frac{5}{3}$$

Get Ready to Begin!

Numeric Expectation Quiz

1. Student | Textbook

$$\frac{12\times 7}{6\times 7} \qquad 2$$

2. **Student** **Textbook**

$$\frac{12\times100}{6\times100} \qquad 200$$

3. **Student** **Textbook**

$$5\times9 \qquad 5^9$$

4. **Student** **Textbook**

$$\frac{5-0}{7-2} \qquad \frac{5}{7}$$

5. **Student** **Textbook**

$$\frac{1}{3}+\frac{1}{4} \qquad \frac{2}{7}$$

6. **Student** **Textbook**

$$\frac{1+100}{100+4} \qquad \frac{1}{4}$$

7. **Student** **Textbook**

$$\frac{2}{5}+\frac{1}{3} \qquad \frac{11}{15}$$

8. **Student** **Textbook**

$18+2\div 5$ $\quad$ 4

9. **Student** **Textbook**

$\frac{6}{15}+\frac{5}{15}$ $\quad$ $\frac{2}{5}+\frac{1}{3}$

10. **Student** **Textbook**

$\sqrt{3}+\sqrt{2}$ $\quad$ $\sqrt{5}$

A Short Break !

11. **Student** **Textbook**

$20+10\div 5$ $\quad$ 22

12. **Student** **Textbook**

$\sqrt{7}\times\sqrt{7}$ $\quad$ 7

13.	Student	Textbook
	$\sqrt{32}$	$4\sqrt{2}$

14.	Student	Textbook
	$\frac{3611}{1000}$	3.611

15.	Student	Textbook
	2π	6.28

16.	Student	Textbook
	$\sqrt{-17}$	$-\sqrt{17}$

17.	Student	Textbook
	$45 \div 23$	$23 \div 45$

18.	Student	Textbook
	5×9	9^5

19. **Student** **Textbook**

$$\frac{80+3}{80} \qquad 1\frac{3}{80}$$

20. **Student** **Textbook**

$$3.612 \qquad \frac{903}{250}$$

21. **Student** **Textbook**

$$\frac{-4-\sqrt{7}}{2} \qquad \frac{-\sqrt{7}}{2}-2$$

22. **Student** **Textbook**

$$\frac{-5}{-3} \qquad \frac{5}{3}$$

Now a well deserved rest!

Thank you!

APPENDIX E: NUMERIC EXPECTATION QUIZ STUDENT ANSWER SHEET

Numeric Expectation Quiz

Numeric Expectation Quiz

Name: **Date:**

Teacher: **Block:**

Question	Response (circle your response to each question)				
P1	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
P2	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
P3	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right

Question	Response (circle your response to each question)				
1	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
2	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
3	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
4	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
5	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
6	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
7	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
8	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
9	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
10	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right

SHORT BREAK - Please Turn over

Numeric Expectation Quiz

11	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
12	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
13	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
14	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
15	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
16	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
17	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
18	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
19	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
20	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
21	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
22	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right

APPENDIX F: ALGEBRAIC EXPECTATION QUIZ

Notes to teachers(1):

- On each slide, you will see a 'student's work' and the 'text book answer'. Students should ask themselves: "How likely is it that the student is on track to get the right answer?"
- I am not interested in special cases.
- I am not interested what the precise wording of the original question may have been.
- I am interested in equivalent expressions.
- While the 'student' does not have exactly the same mathematical expression as that presented in the 'text book', is their expression at least equivalent? It may not be in 'simplest form' but is it at least logically correct?
- I am interested in whether students can identify those cases where no amount of further rearrangement or simplification could result in identical expressions.

Notes to teachers (2):

- Make sure each student has an answer sheet and pen or pencil.
- Use the next 3 slides to explain the concept of the test to your students and get them ready to do the practice questions. Click the mouse or press the right arrow key after each of these introductory slides.
- When you get to the 'Practice Questions' slide stress to students that
  - each question is shown for a only a few seconds
  - they should make a quick judgment and circle their response on the answer sheet
- Click the mouse or press the right arrow key to start the Practice Questions and allow the slide show to proceed automatically until it stops at the 'Get Ready to Begin' slide
- When ready, click the mouse or press the right arrow key to start the Quiz and allow the slide show to proceed automatically until a 'Short Break' slide.
- You control the length of each 'Short Break'.
- When ready, click the mouse or press the right arrow key to move on to the next set of automatic slides.

Algebraic Expectation Quiz

8th Grade Pre-Algebra

For each question in the Quiz you will see a student's answer and the text book answer. Ask yourself

'How likely do you think it is that the student's answer is right?'

Example:

Student	Text
$3+2x$	$2x+3$

Definitely wrong | Probably wrong | No idea | Probably right | Definitely right

This helps you see when the slide changes

Example

Student	Text
$(x+x)^2$	x^2

Definitely wrong | Probably wrong | No idea | Probably right | Definitely right

Practice Questions

- There are 3 practice questions.
- You will see each question for only a few seconds.
- Make a quick judgment about the student's answer

Definitely wrong | Probably wrong | No idea | Probably right | Definitely right

- Circle your response on the answer sheet.

3. **Student** **Textbook**

$5m$ m^5

4. **Student** **Textbook**

$\frac{1}{3}+\frac{1}{y}$ $\frac{2}{3+y}$

5. **Student** **Textbook**

$(a+p)\div q$ $\frac{a+p}{q}$

6. **Student** **Textbook**

$\frac{s}{t}+\frac{p}{t}$ $\frac{s+p}{t}$

7. **Student** **Textbook**

$2f-g+3f-g$ $5f-2g$

8. **Student** **Textbook**

$\frac{4+b}{4}$ $1+\frac{b}{4}$

9. **Student** **Textbook**

Student	Textbook
$6+(4a+2b)\div 2$	$6+2a+b$

A Short Break !

10.

Student	Textbook
$2x+3=y$	$x=\dfrac{y-3}{2}$

11.

Student	Textbook
$-2y+6$	$-2(y-3)$

12.

Student	Textbook
$a(a+1)$	$1+a^2$

13.

Student	Textbook
2s + 3	2t + 3

14. **Student** **Textbook**

$$\frac{2+3x}{x} \qquad 5$$

15. **Student** **Textbook**

$$\frac{1}{3}(2x-5y) \qquad \frac{2x-5y}{3}$$

16. **Student** **Textbook**

$$ax^2+bx+c \qquad a(x+b)(x+c)$$

17. **Student** **Textbook**

$$(-x)^2 \qquad -x^2$$

A Short Break !

18. **Student** **Textbook**

$$a-(b-c) \qquad a-b+c$$

19. **Student** **Textbook**

$(b+a)^2$ $a^2 + b^2 + 2ab$

20. **Student** **Textbook**

$\dfrac{-j+(-k)}{-m}$ $\dfrac{j+k}{m}$

21. **Student** **Textbook**

$(x+1)^3$ $(y+2)^3$

when

$x = y+1$

22. **Student** **Textbook**

$n^2(n^3+1)$ n^5+n^2

23. **Student** **Textbook**

$(x+2)(x+1)$ $2x+3$

24. **Student** **Textbook**

x^2+6 $(x+2)(x+3)$

25.	Student	Textbook
	$-p+q$	$-(p-q)$

Now a well earned rest!

Thank you!

APPENDIX G: ALGEBRAIC EXPETATION QUIZ STUDENT ANSWER SHEET

Algebraic Expectation Quiz

Algebra Expectation Quiz

Name: **Date:**

Teacher: **Block:**

Question	Response (circle your response to each question)				
P1	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
P2	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
P3	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right

Question	Response (circle your response to each question)				
1	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
2	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
3	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
4	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
5	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
6	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
7	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
8	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
9	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right

SHORT BREAK - Please Turn over

10	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
11	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
12	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
13	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
14	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
15	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
16	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
17	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
SHORT BREAK					
18	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
19	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
20	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
21	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
22	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
23	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
24	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right
25	Definitely wrong	Probably wrong	No idea	Probably right	Definitely right

APPENDIX H: CHAPTER 3 COMPREHENSIVE TEST

Name (print) ______________________________

8th Grade Math Chapter 3 Test

SHOW ALL OF YOUR WORK WHEN POSSIBLE. YOUR WORK IS WORTH CREDIT!

Simplify each expression. (3 points each)

1. $3(7-x)$ 2. $-3c+12-4c-4$ 3. $2(g-4)+6g$ 4. $7y+6x-2-4y$

Solve each equation (2-4 points each).

5. $x-2.4=1.7$ 6. $15+x=-12.1$ 7. $-4+x=-12.5$

________________ ________________ ________________

8. $2.5x=-12$ 9. $\frac{w}{7}=-2.4$ 10. $\frac{x}{-3.7}=4$

________________ ________________ ________________

11. $4x - 6 = 22$

12. $-7 = \frac{x}{2} - 4.5$

13. $-2.8 + \frac{c}{-2.1} = 1$

______________ ______________ ______________

14. $4x - 6 + a = 22 - 1 + 5a$

15. $4x - 9 = -2x + 9$

16. $17 = 3(2k - 1) - k$

______________ ______________ ______________

Write an equation or an inequality for each problem. Define the variable you use and then solve each problem.

17. Yesterday Krista sold some boxes of Halloween candy. Today she sold 10 boxes, how many did she sell yesterday? (3 points)

18. Twelve costumes cost $136 in all. If all the costumes were the same price, what was the cost of one costume? (3 points)

Solve each inequality and graph the solution (2-4 points each).

19. $x - 12 < 18$

20. $-6b \leq b - 14$

21. $6 \geq 5w - 6$

22. $x < 5$

23. $-4r \leq 12$

24. $3 - c \leq 2c$

Write an equation or an inequality for each problem. Define the variable you use and then solve each problem.

25. Charlie was setting up for a dinner at his house. He knew that no more than 10 people were coming to eat. How many plates will Charlie need to set out? (3 points)

BONUS-
Shawn received $150 for his birthday. He then decided to save and additional $20 per week until he had enough money to buy a computer. If a computer costs at least $750, at least how many weeks will Shawn need to save money? (4 points)

HONOR PLEDGE & SIGNATURE-

APPENDIX I: CLINICAL INTERVIEW PROTOCOL

Name: Date:

Block:

Questioning for TASK #1-3:

Introduction: Thank you for coming. I really appreciate your time. I will be giving you questions to solve and explain. My goal is to see how you think about and solve the problems. I will ask questions to try and better understand what you understand.

When you are trying to solve or answer any questions you may use scratch paper or a calculator at any time.

*If the interviewee is a student in the experimental group-- I will add the TI-Nspire CAS to the list of tools the student can use.

1. "I am going to give you a series 3 questions that you will have to solve. I will stop after each question to ask or answer questions."

2. Give the task and ask students to solve it.

3. If a student says that he/she does not know how to solve the equation, or asks for help, I will give minimal heuristic suggestions by using the following prompts.

- What does solving mean?

- What is your goal?
- What should your problem look like when you are done?
- How do you know when you are done?
- If the students says “you get x by itself” I may ask how do you get the x by itself, or why isn’t the x by itself. If more prompting is necessary, I might ask how to you “undo” what is being done to the x.
- If a student is having difficulty adding like terms I will give an expression and ask he or she to simplify the expression either by hand or using the TI-Nspire CAS. I will follow up with questions on what they did and how they know they cannot simplify any more.

3. Once the student appears to be finished, I will ask the following questions

If the student did not check his/her work I will begin by asking

- How do you know whether or not you are right?
- What can you do when you solve a problem to make sure you solved it correctly?

I will continue with the following guided heuristic questions

- Can you tell me what you did?
- Why did you do that?
- What are you trying to do?

4. Last, I will ask exploratory or metacognitive questions.

- How else could you have solved this equation?

Questioning for Task #4

In the next task, three students were asked to solve the same problem. I am going to show you the answers that were given by the three students. Your job is to determine which, if any, of the students are correct. I will ask you to explain your reasoning and attempt to explain where the students may have done something incorrect. You may write on this to help show or explain why you agree/disagree with the student's work.

If the student selects an answer as correct, I will ask the student:

- How do you know this is correct?
- How would you solve this problem?
- Can you explain what the student did? Why do you think student #1 did that? Is that incorrect? Why? Why not?
- Can there be more than one way to solve a problem?
- Would everyone who solved a problem have to have the same steps?

Questioning for Task #5

In this task a students was asked to solve an equation using the TI-Npire CAS. I will show you the screen capture after the student finished the problem. I will ask you to explain what the screen capture shows and what it means.

If the students seam unsure where to start his/her explanation I will ask.

- What was the problem the student was solving?
- What were the student's steps in solving it?
- Was the student correct?
- How do you know whether or not the student was correct?
- Could you solve this equation another way?
- Can you show me how to solve this another way?

Questioning for Task #6

We have gone over problems where you have been asked to solve for a variable. In this task, I will give you a problem and I will ask you to use what you know about solving to solve the equation I give you for the variable y.

If a student is not sure how to start, I may re ask the question:

- What does it mean to solve?

Based on the answer to this question I may as follow up guided heuristic questions.

- How can you get the y by itself?
- Why isn't the y by itself?
- What is being done to the y?
- How do you undo what is being done?

APPENDIX J: CLINICAL TASK-BASED INTERVIEW QUESTIONS

Name: __________________________ Block:___________ Date:___________

TASK 1: Solve

$$-7 = \frac{x}{2} - 4.5$$

TASK 2: Solve

$$4a - 6 + a = 22 - 1 + 5a$$

TASK 3: Solve

$$17 > 3(2x - 1) - x$$

TASK 4

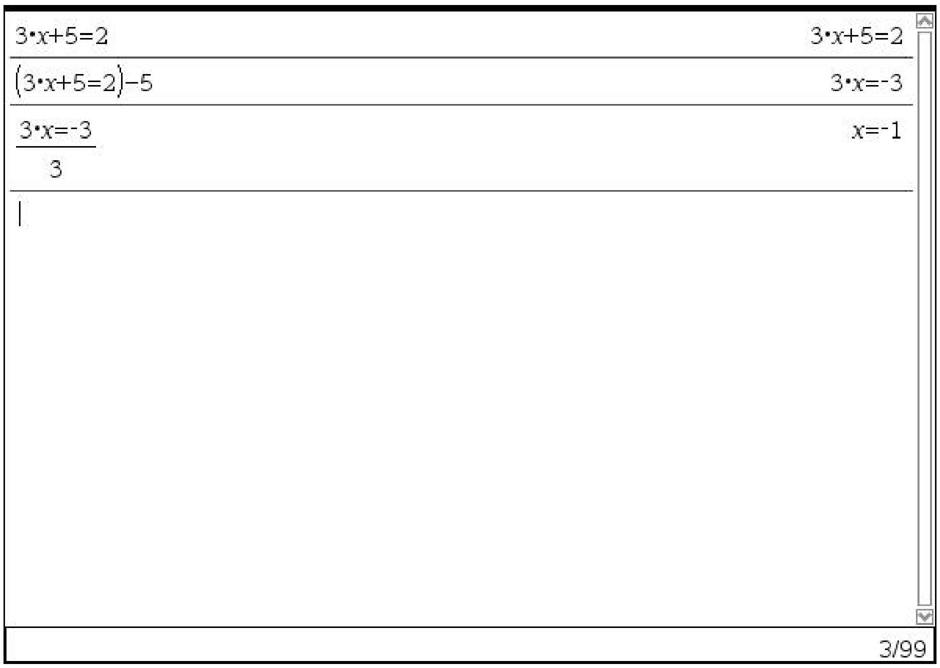
Three students were asked to solve the same problem. Below are the three students' solutions to the problem. Which, if any, are correct? Explain your reasoning. You may write on this to help show or explain why you agree/disagree with the student's work.

Solve: 2(x + 5) = 4x – 8

Student 1	Student 2	Student 3
2(x + 5) = 4x – 8	2(x+5) = 4x – 8	2(x+5) = 4x – 8
2(x+5) = 2(2x – 4)	2x + 10 = 4x – 8	2x + 5 = 4x – 8
x+5 = 2x – 4	2x = 18	13 = 2x
3x = 9	x = 9	x = 6.5
x = 3		

TASK 5

A student solved an equation using the TI-Nspire CAS. Look at the screen capture below and decide what it is showing, then explain what the screen is showing in your own words.



TASK 6

Solve for y in the equation below:

6x + 3y = 12

APPENDIX K: STUDENT FOCUS GROUP QUESTIONS

Focus Group Interview Questions
The following is a preliminary list of interview questions: "The purpose of this interview, is to get feedback from you as to how using the TI-Nspire CAS helped or did not help you understand mathematics."

First, I want to talk about technology…..

1. What types of technology have you used in previous math classes?
2. How many of you play or like playing video games?
3. What other types of technology do you use on a regular basis?
4. What has been your experience with learning new technology?
    a. Is it easy or difficult?

Now I would like to talk specifically about the TI-Nspire CAS calculator….

5. What do you think of the TI-Nspire CAS?
6. Was the TI-Nspire CAS easy to learn how to use?
7. How comfortable do you feel using the TI-Nspire CAS?
8. How often did you use the TI-Nspire CAS to complete your homework assignments?
    a. What types of things would you use it for?
    b. Can you describe some of the activities you have used the TI-Nspire CAS for?
9. Did you ever use the TI-Nspire CAS outside of math class or completing math homework?
    a. When?
    b. For what?
10. How useful do you feel the TI-Nspire CAS was in helping you understand the material you were studying?
11. What are some of the things you were able to do using the TI-Nspire CAS that helped you to remember a particular concept?
12. Overall, would you recommend using the TI-Nspire CAS with other classes?
13. What suggestions would you give to students learning to use the calculator?
14. Is there anything I have not asked about the TI-Nspire you would like to add?
15. Is there anything about math class that you would like to add?

APPENDIX L: GENERATIVE ACTIVITIES

E-Lab 3.1 Name:_______________________________

Block ____ Date:_______________________________

1. a) Fill in the Answer column in the chart below

Problem	Answer
3 - 3	
5 - 5	
7 - 7	

b) Explain how could you express the answers above as an expression using the original number ?

____

____

c) If this was true for all numbers, how could you write this as a rule?

d) Discuss your rule with your group.

2. a) Fill in the Answer column in the chart below

Problem	Answer
3 + 3	
7 + 7	
11 + 11	

b) Explain how could you express the answers above as an expression using the original number?

c) If this was true for all numbers, how could you write this as a rule?

d) Discuss your rule with your group.

3. a) Fill in the Answer column in the chart below

Problem	Answer
5 + 5 +5	
7 + 7 + 7	
9 + 9 +9	

b) Explain how could you express the answers above as an expression using the original number?

c) If this was true for all numbers, how could you write this as a rule?

d) Discuss your rule with your group.

4. a) Fill in the Answer column in the chart below

Problem	Answer
2·**4** + 3·**4**	
2·**5** + 3·**5**	
2·**7** + 3·**7**	

b) Explain how could you express the answers above as an expression using the number which is in **bold** print ?

c) If this was true for all numbers, how could you write this as a rule?

d) Discuss your rule with your group.

5.a) Use your TI-Nspire CAS to simplify the expressions below

Expression	Simplified form
2x + 3x	
5m + 7m	
9p – 8p	
3x + 3y	
2x + 6 + 8x	
2m +5m – 3m	
8t – 5 +4t	
6p +1 – 2p – 2	
9y – y + 8	
5b +7c + 9	
2x – 6x + 3y – 7y	

b) Do all of the expressions above simplify? If not explain why.

c**) Explain** how you combine the terms in the expressions above?

d) Check your explanation with your group. Do they understand it?

e) Try the following on your own without the calculator, then once you have an answer, use the CAS to check your work.

Expression	**You try** Simplified form	Check using CAS
2x + 5x – x – 5		
3m – 7m + 4		
5t – 5 + 8m – m + 13		

C- Lab 3.1 Name:________________________________
Block _____ Date:_________________________________

1. a) Fill in the Answer column in the chart below

Problem	Answer
3 - 3	
5 - 5	
7 - 7	

b) Explain how could you express the answers above as an expression using the original number ?

____
____

c) If this was true for all numbers, how could you write this as a rule?

d) Discuss your rule with your group.

2. a) Fill in the Answer column in the chart below

Problem	Answer
3 + 3	
7 + 7	
11 + 11	

b) Explain how could you express the answers above as an expression using the original number?

____
____

c) If this was true for all numbers, how could you write this as a rule?

d) Discuss your rule with your group.

3. a) Fill in the Answer column in the chart below

Problem (**expanded form**)	Answer (**simplified form)**
5 + 5 +5	
7 + 7 + 7	
9 + 9 +9	

b) Explain how could you express the answers above, as an expression using the original number?

____

____

c) If this was true for all numbers, how could you write this as a rule?

d) Discuss your rule with your group.

4. a) Fill in the Answer column in the chart below

Problem	Answer
2·**4** + 3·**4**	
2·**5** + 3·**5**	
2·**7** + 3·**7**	

b) Explain how could you express the answers above as an expression using the number which is in **bold** print ?

____

____

c) If this was true for all numbers, how could you write this as a rule?

d) Discuss your rule with your group.

5. Complete the chart below

Term form	**Expanded form**
3n	n + n + n
4w	
	b + b + b + b + b + b
	w + w
2t	
	z + z + z
2n + 4n	n + n + n + n + n + n
3w + w	
2t + 2t	
4m + 3m	
3b + 5	b + b + b + 5
2b + b + 5	

When we have an expression with “like terms”, we can simplify it by combining any terms that have the same variable:

Like terms	**expanded form**	**Simplified form**
2m + 2m	m + m + m + m	4m
p + 3p		
n + n + 32n		
4t + 3t		
2x + 2x + 3	x + x + x + x + 3	4x + 3
2h + 7 + 3h	h + h + 7 + h + h + h	
3p + p + 4t		
r + 2w		
2b + b + n + 2n		
2w + 5 + 3w		

Check your answers with your group

E-Lab Section 3.2 Name:______________________________
Block ____ Date:_______________________________

Objective: Students will be able to solve one-step equations by adding or subtracting.

What is an equation? ____

What is the solution to an equation? ____

How would you know if a solution to an equation is correct?
For example, if said the solution to x + 4 = -8 was -4. Is this true or false? How do you know?

The goal is to get the variable by itself (**isolate the variable**).
We **isolate the variable** by "undoing" what is being done to the variable. To do this we use inverse or opposite operations.

Opposite operations

What is the opposite of adding 3? ________________

What is the opposite of subtracting 8? ________________

What is the opposite of adding -3? ___________________or _______________

For each example:

- **show what you did**
- **write your step in words next to it**
- **check your answer**
- *** You may use the TI-Nspire CAS to double check your work**

Example: Solve x + 3 = 7

$$\begin{array}{rcl} x + 3 & = & 7 \\ \underline{\quad -3} & \underline{=} & \underline{-3} \\ x & = & 4 \end{array}$$ Subtract 3 from both sides

√Check x + 3 = 7
4 + 3 = 7 Substitute 4 for x
7 = 7 True

When you are done, you can also check your work on the TI-Nspire CAS.

Type the equation on the screen.

Press enter

If you want to subtract 3 from both sides, just type - 3 , then enter.

Remember to follow the directions and show your work!

Example 2: Solve $a + 13 = -7$

Example 3: Solve $m - 9 = -7$

Example 4: Solve $b + 4.5 = -6.7$

Example 5: Solve $38 + x = 21$

Example 6: Solve $-87 = p + (-12)$

Example 7: Solve $n - \frac{1}{3} = 5$

Example 8: Solve $y - 8 = \frac{1}{5}$

☺ *Check your answers with your group*

C-Lab Section 3.2 Name:______________________________
Block ____ Date:_______________________________

Objective: Students will be able to solve one-step equations by adding or subtracting.

What is an equation? ____

What is the solution to an equation? ____

How would you know if a solution to an equation is correct?
For example, if said the solution to $x + 4 = -8$ was -4. Is this true or false? How do you know?

The goal is to get the variable by itself (**isolate the variable**).
We **isolate the variable** by "undoing" what is being done to the variable. To do this we use inverse or opposite operations.

Opposite operations

What is the opposite of adding 3? ________________

What is the opposite of subtracting 8? ________________

What is the opposite of adding -3? ___________________or _____________

For each example:

- **show what you did**
- **write your step in words next to it**
- **check your answer**

Example: Solve $x + 3 = 7$

$$\begin{array}{rcl} x + 3 & = & 7 \\ \underline{-3} & \underline{=} & \underline{-3} \\ x & = & 4 \end{array}$$ Subtract 3 from both sides

√Check $x + 3 = 7$
$4 + 3 = 7$ Substitute 4 for x
$7 = 7$ True

Remember to follow the directions and show your work!

Example 2: Solve $a + 13 = -7$

Example 3: Solve $m - 9 = -7$

Example 4: Solve $b + 4.5 = -6.7$

Example 5: Solve $38 + x = 21$

Example 6: Solve $-87 = p + (-12)$

Example 7: Solve $n-\frac{1}{3}=5$

Example 8: Solve $y-8=\frac{1}{5}$

☺ *Check your answers with your group*

E-Lab Section 3.3 Name:_______________________________
Block_____ Date:________________________________

<u>Objective:</u> Students will be able to solve equations by division or multiplication.

What is an equation? ____

What is the solution to an equation? ____

How would you know if a solution to an equation is correct?
For example, if said the solution to 4x = -8 was -4. Is this true or false? How do you know?

The goal is to get the variable by itself (**isolate the variable**).
We **isolate the variable** by "undoing" what is being done to the variable. To do this we use inverse operation.

<u>Inverse operations</u>

What is the inverse of multiplying by 3? ________________

What is the inverse of dividing by -4 ? ________________

For each example:
- **show what you did**
- **write your step in words next to it**
- **check your answer**
- *** You may use the TI-Nspire CAS to double check your work**

Example: Solve 3x = -18

$$\frac{3x}{3} = \frac{-18}{3}$$ divide both sides by 3

x = -6

√Check 3x = -18
3 (-6) = -18 Substitute -6 for x
-18 = -18 True

When you are done, you can also check your work on the TI-Nspire CAS.

Type the equation on the screen.
Press enter
If you want to divide both sides by 3, just type ÷ 3 , and then enter.

Remember to follow the directions and show your work!

Your answers should be exact-do not round. You may keep your answers as a simplified fraction or decimal.

Example 2: Solve $13a = -5$

Example 3: Solve $-9.5m = -117.8$

Example 4: Solve $-3.2b = -28.8$

Example 5: Solve $-30 = 180p$

Example 6: Solve $5 = \frac{x}{-13}$

Example 7: Solve $8 = \frac{-n}{2}$

Example 8: Solve $\frac{y}{6} = -3$

☺ *Check your answers with your group*

C-Lab Section 3.3 Name:________________________________
Block_____ Date:_________________________________

Objective: Students will be able to solve equations by division or multiplication.

What is an equation? ____

What is the solution to an equation? ____

How would you know if a solution to an equation is correct?
For example, if said the solution to $4x = -8$ was -4. Is this true or false? How do you know?

The goal is to get the variable by itself (**isolate the variable**).
We **isolate the variable** by "undoing" what is being done to the variable. To do this we use inverse operation.

Inverse operations

What is the inverse of multiplying by 3? _________________

What is the inverse of dividing by -4 ? _________________

For each example:

- **show what you did**
- **write your step in words next to it**
- **check your answer**

Example: Solve $3x = -18$

$\frac{3x}{3} = \frac{-18}{3}$ divide both sides by 3

$x = -6$

√Check $3x = -18$

$3(-6) = -18$ Substitute -6 for x

$-18 = -18$ True

Remember to follow the directions and show your work!

Your answers should be exact-do not round. You may keep your answers as a simplified fraction or decimal.

Example 2: Solve $13a = -5$

Example 3: Solve $-9.5m = -117.8$

Example 4: Solve $-3.2b = -28.8$

Example 5: Solve $-30 = 180p$

Example 6: Solve $5 = \frac{x}{-13}$

Example 7: Solve $8 = \frac{-n}{2}$

Example 8: Solve $\frac{y}{6} = -3$

☺ *Check your answers with your group*

E-Lab Name:______________________________
Section 3.4 Date:_______________________________

<u>Objective:</u> Students will be able to solve two-step equations.

Example 1: Using your TI-Nspire CAS

1. Type x onto your calculator screen.
2. Multiply it by 3 (Press x 3)
3. Now subtract 5 (Press - 5)

You should have 3x -5 on your screen.

Your goal is now to use your calculator to try to **undo** what you just did until you get x by itself.

Write down the operations you need to do in order to get from 3x – 5 back to x.

Your steps to get from 3x-5 to x

1.____

2.____

Compare your steps with your group. Make sure the other members of your group have gotten x by itself. Did you come up with the same steps?

Example 2: Using your TI-Nspire CAS

1. Type x onto your calculator screen.
2. Divide it by 5 (Press ÷ 5)
3. Now add 7 (Press + 7)

You should have $\frac{x}{5}+7$ on your screen.

Your goal is now to try to **undo** what you just did until you get x by itself.

Write down the operations you need to do in order to get from $\frac{x}{5}+7$ back to x.

Your steps to get from $\frac{x}{5}+7$ to x

1.____

2.____

Compare your steps with your group. Make sure the other members of your group have gotten x by itself. Did you come up with the same steps?

Compare what you did to x (the steps given to you) to what you had to do to undo those steps (your steps) in examples 1 and 2. How would you explain how to isolate the x? What operations would you have to do in order to undo something (be specific)? What operations would you have to do first? Does the order you do the operations matter?

Your idea:____

____

____

____

Compare *your idea* with the ideas of *your group* members. Do you agree? Try to come to an agreement on what you would have to do to isolate x, and write your group's idea below.

Your group's idea:____

____

____

Your group should be prepared to share your answers with your class.

In order to isolate the variable you must ____

____

____

Practice: Remember…

For each example:

- **show what you did**
- **write your step in words next to it**
- **check your answer**

Example: Solve $8x - 5 = -19$

$$\begin{aligned} 8x - 5 &= -19 && \text{add 5 to both sides} \\ +5 &= +5 \\ 8x &= -14 && \text{divide both sides by 8} \\ \frac{8x}{8} &= \frac{-14}{8} && \text{simplify} \\ \boxed{x = \frac{-7}{4}} \end{aligned}$$

√Check $8x - 5 = -19$

$8\left(-\frac{7}{4}\right) - 5 = -19$ Substitute $-\frac{7}{4}$ for x

$-14 - 5 = -19$

$-19 = -19$ True

You try:

Your answers should be exact-do not round. You may keep your answers as a simplified fraction or decimal.

Example 2: Solve $11m - 12 = -10$

Example 3: Solve -12t - 19 = -25

Example 4: Solve -3.5m +7. 6 = 15.3

Example 5: Solve $-35 = 18p + 46$

Example 6: Solve $5 = \frac{x}{-9} + 7$

Example 7: Solve $-8 = \frac{-y}{2} + 5$

Example 8: Solve $\frac{k}{6} - 17 = -3$

C-Lab
Section 3.4

Name:______________________________
Block:_______Date:___________________

Objective: Students will be able to solve two-step equations.

Example 1: Creating an expression

In the box to the right try to create the algebraic expression described below.

4. Start by writing the number x.
5. Multiply it by 3
6. Now subtract 5

Did you fill in the box? What did you get?

You should have 3x – 5.

Your goal is now to try to **undo** what you just did until you get x by itself.

Write down the operations you need to do in order to get from 3x – 5 back to x.

Your steps to get from 3x-5 to x

1.____

2.____

Compare your steps with your group. Make sure the other members of your group have gotten x by itself. Did you come up with the same steps?

Example 2: Create and expression

In the box to the right try to create the algebraic expression described below.

4. Start by writing the number x
5. Divide it by 5
6. Now add 7

You should have $\frac{x}{5}+7$ on your screen.

Your goal is now to try to **undo** what you just did until you get x by itself.

Write down the operations you need to do in order to get from $\frac{x}{5}+7$ back to x.

Your steps to get from $\frac{x}{5}+7$ to x

1.____

2.____

Compare your steps with your group. Make sure the other members of your group have gotten x by itself. Did you come up with the same steps?

Compare what you did to x (the steps given to you) to what you had to do to undo those steps (your steps) in examples 1 and 2. How would you explain how to isolate the x? What operations would you have to do in order to undo something (be specific)? What operations would you have to do first? Does the order you do the operations matter?

Your idea:____

Compare *your idea* with the ideas of *your group* members. Do you agree? Try to come to an agreement on what you would have to do to isolate x, and write your group's idea below.

Your group's idea:___________________ ___________________________________

____

Your group should be prepared to share your answers with your class.

In order to isolate the variable you must ______________________________________

____

____

Practice: Remember…

For each example:

- **show what you did**
- **write your step in words next to it**
- **check your answer**

Example: Solve $8x - 5 = -19$

$$\begin{aligned}
8x - 5 &= -19 && \text{add 5 to both sides}\\
\underline{\quad +5} &= \underline{+5} &&\\
8x &= -14 && \text{divide both sides by 8}\\
\frac{8x}{8} &= \frac{-14}{8} && \text{simplify}\\
\boxed{x = \frac{-7}{4}} &&&
\end{aligned}$$

√Check $8x - 5 = -19$

$8\left(-\frac{7}{4}\right) - 5 = -19$ Substitute $-\frac{7}{4}$ for x

$-14 - 5 = -19$

$-19 = -19$ True

You try:

Your answers should be exact-do not round. You may keep your answers as a simplified fraction or decimal.

Example 2: Solve $11m - 12 = -10$

Example 3: Solve $-12t - 19 = -25$

Example 4: Solve $-3.5m + 7.6 = 15.3$

Example 5: Solve $-35 = 18p + 46$

Example 6: Solve $5 = \frac{x}{-9} + 7$

Example 7: Solve $-8 = \frac{-y}{2} + 5$

Example 8: Solve $\frac{k}{6} - 17 = -3$

E-Lab
Section 3.6

Name:______________________________
Date:_______________________________

<u>Objective:</u> Students will be able to solve equations involving the distributive property and variables on both sides of an equation.

Example 1: You will use the TI-Nspire CAS to expand each of the expressions below.

On the TI-Nspire CAS:

7. Press menu →4:Algebra→3:Expand
8. Type the expression below into the parenthesis on your screen and close the parenthesis.
9. Enter the result in the chart below.

- *Although we often do not write the multiplication symbol "·" when writing a number multiplied by a parenthesis, we must type the expression this way on the calculator for it to understand what we want it to do.*

$3\cdot(x-5)$	
$2\cdot(p-8)$	
$-6\cdot(m-5)$	
$a\cdot(b+c)$	
$(9-x)\cdot 5$	
$(b+4)\cdot 7$	
$(b+c)\cdot a$	

<u>Your rule</u>: ***Explain*** *in your own words what the calculator does when you tell it to expand the expressions you gave it.*

____

Share & compare *your rule with your group. Work together with your group to write concisely in words what is happening when you expand the expressions above.*

Your group's rule:

*Be prepared to share by **reading** your group's answer to your class.

Remember, the goal of solving an equation is to isolate the variable.
What does it mean to isolate a variable?

Practice: Remember…

For each example:

- show what you did

***You may use the TI-Npire CAS to check your steps along the way to see if what you are doing is helping you to isolate the variable before you write the step down.*

- write your step in words next to it

- check your answer

**You may use the TI-Nspire CAS to double check your work*

Example: Solve $7x - 6 = -5(x+2)$

$7x - 6 = -5x - 10$	distributive property of multiplication over addition
$\underline{+5x} = \underline{+5x}$	add 5x to both sides of the equation
$12x - 6 = -10$	add 6 to both sides of the equation
$\underline{+6} = \underline{+6}$	
$12x = -4$	

$$\frac{12x}{12} = \frac{-4}{12}$$

$x = \frac{-1}{3}$

√Check Solve 7x – 6 = -5(x+2)

$$7\left(\frac{-1}{3}\right)-6=-5\left(\frac{-1}{3}+2\right)$$
$$\frac{-25}{3}=\frac{-25}{3}$$

You try:

Your answers should be exact-do not round. You may keep your answers as a simplified fraction or exact decimal. Remember, your goal is to isolate the variable.

Example 2: Solve 9x = 5x + 8

Example 3: Solve 13m – 2 = -11m +10

Example 4: Solve 3x + 5x = 6x + 28 – 2x

Example 5: Solve $4m + 20 = 7(m + 3)$

Example 6: Solve $9(2 - x) = -5(x + 8)$

Example 7: Solve $3(2y + 5) + 4y = 6(y + 2) - 2y$

C-Lab
Section 3.6

Name:_____________________________
Date:______________________________

<u>Objective:</u> Students will be able to solve equations involving the distributive property and variables on both sides of an equation.

Example 1: Expand each of the following expressions using the distributive property.

$3\cdot(x-5)$	
$2\cdot(p-8)$	
$-6\cdot(m-5)$	
$a\cdot(b+c)$	
$(9-x)\cdot 5$	
$(b+4)\cdot 7$	
$(b+c)\cdot a$	

***<u>Your rule</u>: Explain** in your own words what using the distributive property means.*

____

____

____

***Share & compare** your rule with your group. Work together with your group to write concisely in words what is happening when you expand the expressions above.*

Your group's rule:

____

____

____

*Be prepared to share by **reading** your group's answer to your class.

Remember, the goal of solving an equation is to isolate the variable.
What does it mean to isolate a variable?

____

____

Practice: Remember…

For each example:
- **show what you did**
- **write your step in words next to it**
- **check your answer**

Example: Solve $7x – 6 = -5(x+2)$

$7x - 6 = -5x - 10$	distributive property of multiplication over addition
$\underline{+5x \quad = +5x}$	add 5x to both sides of the equation
$12x - 6 = -10$	add 6 to both sides of the equation
$\underline{\quad +6 = +6}$	
$12x = -4$	

$$\frac{12x}{12} = \frac{-4}{12}$$

$$x = \frac{-1}{3}$$

√Check Solve $7x – 6 = -5(x+2)$

$$7\left(\frac{-1}{3}\right) - 6 = -5\left(\frac{-1}{3} + 2\right)$$

$$\frac{-25}{3} = \frac{-25}{3}$$

You try:

Your answers should be exact-do not round. You may keep your answers as a simplified fraction or exact decimal. Remember, your goal is to isolate the variable.

Example 2: Solve $9x = 5x + 8$

Example 3: Solve $13m - 2 = -11m + 10$

Example 4: Solve $3x + 5x = 6x + 28 - 2x$

Example 5: Solve $4m + 20 = 7(m + 3)$

Example 6: Solve $9(2 - x) = -5(x + 8)$

Example 7: Solve $3(2y + 5) + 4y = 6(y + 2) - 2y$

E-Lab Name:______________________________
Section 3.9 Block:________Date:_________________

Objective: Students will be able to solve inequalities by adding or subtracting.

Write the following number in words

$2\frac{3}{4}$

Answer the following by circling your choice:

1. Do you have the word '**and**' in your written answer? YES NO
2. What does and mean to do in mathematics? ADD SUBTRACT MULTIPLY DIVIDE

So Two and three fourths (or two and three quarters) is really

$2 + \frac{3}{4}$

Which of the following is the correct way to add these numbers?
Check if you are correct by using your TI-Nspire CAS. Type 2 + 3/4 and press enter.

A $2 + \frac{3}{4}$	B $2 + \frac{3}{4}$	C $2 + \frac{3}{4}$	D $2 + \frac{3}{4}$
$\frac{2+3}{4} = \frac{5}{4}$	$2 = \frac{8}{4}$ so	$2 = \frac{8}{4}$ so	$2 = \frac{8}{4}$ so
	$\frac{8}{4} + \frac{3}{4} = \frac{11}{8}$	$\frac{8}{4} + \frac{3}{4} = \frac{11}{4}$	$\frac{8}{4} + \frac{3}{4} = \frac{21}{16}$

Did you notice the answer is an improper fraction? (the numerator is larger than the denominator)

How else have you learned to change a mixed number into and improper fraction? Write what you would do for this example in words. Is the answer the same as above?

$2\frac{3}{4}$

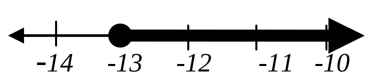
Inequalities show a relationship between two values.

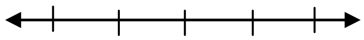
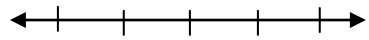
For example $3 < 5$

If I add the same number to both sides, the left side will still be smaller than the right side. $3 + 7 < 5 + 7$ $10 < 12$	If I subtract the same number from both sides, the left side will still be smaller than the right side. $3 – 6 < 5 – 6$ $-3 < -1$ ** On a number line, -3 is farther to the left (smaller) than -1*

As long as you are only adding or subtracting the same numbers to both sides of an inequality, the relationship/inequality ($<$, $>$, $\leq$, $\geq$) will not change. So it will look just like solving an equation, but the symbol is not an = sign, and your answer is not just one number.

For each of the following. Solve the inequality by showing your step(s) and graph your final answer on the number line provided.
** You may check your answer by typing these same steps on the TI-Nspire CAS.*
Below is what this would look like.

Example: $x + 8 \geq -5$ $\underline{-8} \geq \underline{-8}$ Subtract 11 from both sides of the equation $x \geq -13$ 	*Using CAS to check*

<table>
<tr><td>Example 2: p – 1.5 > 3.2
</td><td>Example 3: 4.8 < m + 8
</td></tr>
<tr><td colspan="2">Example 4: $-3\frac{2}{5} \le y - \frac{1}{5}$ * Hint- Change mixed numbers to improper fractions.
</td></tr>
</table>

C-Lab
Section 3.9

Name:______________________________
Block:_________Date:_________________

<u>Objective:</u> Students will be able to solve inequalities by adding or subtracting.

Write the following number in words

$2\frac{3}{4}$

Answer the following by circling your choice:

1. Do you have the word ‘**and’** in your written answer? YES NO
2. What does and mean to do in mathematics? ADD SUBTRACT MULTIPLY DIVIDE

So Two and three fourths (or two and three quarters) is really

$2 + \frac{3}{4}$

Which of the following is the correct way to add these numbers?

A $2 + \frac{3}{4}$ $\frac{2+3}{4} = \frac{5}{4}$	B $2 + \frac{3}{4}$ $2 = \frac{8}{4}$ so $\frac{8}{4} + \frac{3}{4} = \frac{11}{8}$	C $2 + \frac{3}{4}$ $2 = \frac{8}{4}$ so $\frac{8}{4} + \frac{3}{4} = \frac{11}{4}$	D $2 + \frac{3}{4}$ $2 = \frac{8}{4}$ so $\frac{8}{4} + \frac{3}{4} = \frac{21}{16}$

Compare your answers with your group.

Did you notice your answer is an improper fraction? (the numerator is larger than the denominator)

How else have you learned to change a mixed number into and improper fraction? Write what you would do for this example in words. Is the answer the same as above?

$2\frac{3}{4}$

Inequalities show a relationship between two values.

For example   3 < 5

If I add the same number to both sides, the left side will still be smaller than the right side. 3 + 7 < 5 + 7 10 < 12	If I subtract the same number from both sides, the left side will still be smaller than the right side. 3 – 6 < 5 – 6 -3 < -1 ** On a number line, -3 is farther to the left (smaller) than -1*

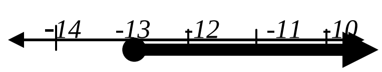
As long as you are only adding or subtracting the same numbers to both sides of an inequality, the relationship/inequality (<, >, ≤, ≥) will not change.   So it will look just like solving an equation, but the symbol is not an = sign, and your answer is not just one number.

For each of the following.   Solve the inequality by showing your step(s) and graph your final answer on the number line provided.

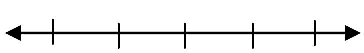
Example: $x + 8 \geq -5$

$\underline{-8} \geq \underline{-8}$ Subtract 11 from both sides of the equation

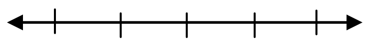
$x \geq -13$



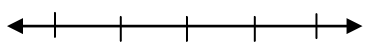
Example 2:   p – 1.5 > 3.2



Example 3: $4.8 < m + 8$



Example 4: $-3\frac{2}{5} \leq y - \frac{1}{5}$ * Hint- Change mixed numbers to improper fractions.



E-Lab Name:______________________________
Section 3.10 Block:________Date:_________________

Objective: Students will be able to solve inequalities by multiplying or dividing.

I. Follow the directions and fill in the chart below.
Then, use your TI-Nspire CAS to re-do each problem…If you pay attention you will see that some of your answers have a surprising twist!
The directions of how to use the TI-Nspire CAS are written in italics for the 1st example.

	Multiply both sides by 2	Multiply both sides by -2	Divide both sides by 2	Divide both sides by -2
$6 < 8$	$6 \cdot 2 < 8 \cdot 2$ $12 < 16$			
Is your inequality true/false?	T F	T F	T F	T F
Type the inequality above into the TI-Nspire CAS and press enter. It should say … true	*Type in (like above) Write result*	*Type in Write result*	*Type in Write result*	*Type in Write result*

What caused the inequality you created to be false?

II. Complete the following using the TI-Nspire CAS.

	Multiply both sides by 3	Multiply both sides by -3	Divide both sides by 3	Divide both sides by -3
Type this into the calculator press enter $x \geq 9$	*Press X 2 Write result* $2x \geq 18$	*Press x -2 Write result*	*Press ÷ 2 Write result*	*Press ÷ -2 Write result*
Did the inequality change Yes/No?	Y N	Y N	Y N	Y N

When does the relationship/inequality symbol (<, >, ≤, ≥) change?

The inequality changes so that the statement remains true.
How does the inequality change (fill in chart on right)?

Original inequality	New inequality
≥	
>	
≤	
<	

Check your answers with your group.

Therefore, solving an inequality will look just like solving an equation, but the symbol is not an = sign, and your answer is not just one number **AND** whenever you multiply or divide by a negative number you must *reverse the direction of the inequality symbol*.

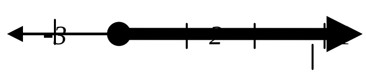
For each of the following. Solve the inequality by showing your step(s) and graph your final answer on the number line provided.

Example: 2x ≥ -5

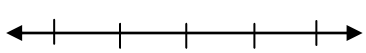
$$\frac{2x}{2} > \frac{-5}{2}$$ Divide both sides by 2.

$$x > -2.5$$

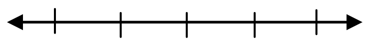
We did not reverse the direction of the inequality symbol as we did not



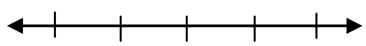
Example 2: – 1.4p > 3.5



Example 3: - 4.8 < 2m



Example 4: $-3\frac{2}{5}x \le \frac{1}{5}$ * Hint- Change mixed numbers to improper fractions.



C-Lab
Section 3.10

Name:______________________________
Block:________Date:_________________

<u>Objective:</u> Students will be able to solve inequalities by multiplying or dividing.

Fill in the chart below.

	Multiply both sides by 2	Multiply both sides by -2	Divide both sides by 2	Divide both sides by -2
$6 < 8$	$6(2) < 8(2)$ $12 < 16$			
Is the statement true/false	T F	T F	T F	T F
	Multiply both sides by 3	Multiply both sides by -3	Divide both sides by 3	Divide both sides by -3
$12 > 9$				
Is the statement true/false	T F	T F	T F	T F

When does the relationship/inequality symbol ($<$, $>$, $\leq$, $\geq$) change?

Therefore, solving an inequality will look just like solving an equation, but the symbol is not an = sign, and your answer is not just one number **AND** whenever you multiply or divide by a negative number you must *reverse the direction of the inequality symbol*.

Original inequality	If it was reversed
$\geq$	
$>$	

$\leq$	
$<$	

Check your answers with your group.

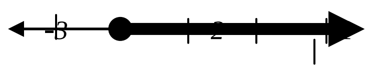
For each of the following. Solve the inequality by showing your step(s) and graph your final answer on the number line provided.

Example: 2x $\geq$ -5

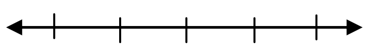
$$\frac{2x}{2} > \frac{-5}{2}$$ Divide both sides by 2.

$$x > -2.5$$

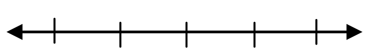
We did not reverse the direction of the inequality symbol as we did not



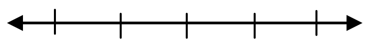
Example 2: – 1.4p > 3.5



Example 3: - 4.8 < 2m



Example 4: $-3\frac{2}{5}x \leq \frac{1}{5}$ * Hint- Change mixed numbers to improper fractions.



Algebra Review
Warm Up E1

Name:________________________
Block:___________Date:_________

2.10 Review

Zero Exponent Property & Negative Exponent Property

Fill in the answers below.
You may use the TI-Nspire CAS to help you fill in the answer.
To type an exponent use the ^ symbol. Your calculator should be in exact mode.

2^3 =

3^3 =

2^2 =

3^2 =

2^1 =

3^1 =

2^0 =

3^0 =

Based on your values above.

1. a) How do you get from one term to the next on after it?

   b) What can you say about the value of 4^0 ?

   c) What is the value of x^0?

   d) Is there any exception as to what x can be?

Algebra Review
Warm Up C1

Name:________________________
Block:___________ Date:_________

2.10 Review

Zero Exponent Property & Negative Exponent Property

Evaluate the first three in each column below.

$2^3 =$

$2^2 =$

$2^1 =$

$2^0 =$

How do you get from the first answer to the next one?

Do the same thing again to get from the 2nd to the 3rd and the 3rd to the 4th answer?

$3^3 =$

$3^2 =$

$3^1 =$

$3^0 =$

Based on the answers above…

1. a) What can you say about the value of 4^0 ?

b) What is the value of x^0?

c) Is there any exception as to what x can be?

Algebra Review Name:________________________

Warm Up E2 Block:___________Date:_________

2.10 Review

Zero Exponent Property & Negative Exponent Property

Fill in the answers below.

You may use the TI-Nspire CAS to help you fill in the answer.

To type an exponent use the ^ symbol. *Your calculator should be in exact mode.*

2^3 =	3^3 =
2^2 =	3^2 =
2^1 =	3^1 =
2^0 =	3^0 =
2^{-1} =	3^{-1} =
2^{-2} =	3^{-2} =
2^{-3} =	3^{-3} =

Based on your values above.

1.a) What is the value of 4^{-2} ?

b) In general explain what happens when you put a number to a negative power? *(You may use an example to help your explanation)*

Algebra Review Name:________________________

Warm Up C2 Block:___________Date:_________

2.10 Review

Zero Exponent Property & Negative Exponent Property

Evaluate the first three below.

2^3 =

How do you get from the first answer to the next one?

2^2 =

You should be able to do the same thing again to get from the 2nd to the 3rd answer?

2^1 =

2^0 =

Continue with this pattern and complete the rest. Make sure to leave your final answers as

2^{-1} =

2^{-2} =

2^{-3} =

Based on the answers above…

1. a) What is the value of 3^{-2} ?

   b) In general explain what happens when you put a number to a negative power? *(You may use an example to help your explanation)*

Algebra Review
Warm Up E3

Name:________________________
Block:___________Date:_________

2.6 review

Product of Powers Property

A. Use the TI-Nspire CAS to simplify the following expressions:
Make sure to use a multiplication sign x between the terms.

For example #1 you would type: x ^ 3 x x ^ 4

1. $x^3 \cdot x^4 =$	3. $y^{-3} \cdot y^{-4} =$
2. $x \cdot x^7 =$	4. $x^5 \bullet x^5 =$

Using your answers above, explain a rule for simplifying exponential expressions.

B. NO CALCULATOR! Test your conjecture by simplifying the following expressions **<u>without</u>** using the calculator.

5. $y^{65} \cdot y^{-13} =$	6. $y^2 \cdot y^{-73} =$

Algebra Review — Name:________________________

Warm Up C3 — Block:___________Date:_________

2.6 review

Product of Powers Property

Express each of the following as a power of a single term.

Reminder: 3^4 in expanded form is $3 \cdot 3 \cdot 3 \cdot 3$

Expression	Expanded form	Simplified form
1. $x^3 \cdot x^4 =$		
2. $x \cdot x^7 =$		
3. $x^5 \bullet x^5$		

Using your answers above, explain a rule you could use for simplifying exponential expressions.

Discuss your rule with your group.

What would you get using your rule for these two?

4. $y^{65} \cdot y^{-13} =$	5. $y^2 \cdot y^{-73} =$

Algebra Review — Name:________________________

Warm Up E4 — Block:___________Date:_________

2.10 Review

Quotient of Powers Property

A. Use the TI-Nspire CAS to simplify the following expressions:

1. $\frac{2x^{12}}{x^3} =$	3. $\frac{9x^{-3}}{x^5}$ _____________
2. $\frac{5y^{11}}{y^2} =$	4. $\frac{3y^5}{y^{-7}}$ _____________

Using your answers above, find a rule for simplifying exponential expressions that are expressed as quotients of powers.

B. NO CALCULATOR. Test your conjecture by simplifying the following expressions **<u>without</u>** using the calculator.

5. $\frac{2x^3}{x^{-7}} =$	6. $\frac{x^9}{x^9} =$

Algebra Review Name:_______________________
Warm Up C4 Block:__________Date:_________

2.10 Review
Quotient of Powers Property

Simplify the following

Reminder: 3^4 in expanded form is $3 \cdot 3 \cdot 3 \cdot 3$

Expression	Expanded form Expand the numerator & denominator	Simplified
1. $\frac{2x^{12}}{x^3} =$		
2. $\frac{5y^{11}}{y^2} =$		
3. $\frac{x^9}{x^9} =$		

Using your answers above, explain a rule for simplifying exponential expressions in the form of a quotient.

Discuss your rule with your group.

What would you get using your rule for these two?

5. $\dfrac{2x^{3}}{x^{-7}}$ ________________	6. $\dfrac{9x^{-3}}{x^{5}}$ ________________

APPENDIX M: HSRB CONSENT AND ASSENT FORMS

The Instrumental Genesis of CAS as an Amplifier in a Generative Pre-Algebra Classroom.

STUDENT INFORMED ASSENT FORM

RESEARCH PROCEDURES

This research project is being conducted to determine the effects of the TI-Nspire Computer Algebra System (CAS) on student learning. The TI-Nspire CAS is a calculator which has the ability to manipulate algebraic symbols. If you agree to participate, you will be giving permission for Mrs. Gantz to collect and use the following data for analysis; pre and post attitude surveys, pre and post number and algebraic expectation quizzes, classroom assessments, video taped lessons, and journal entries. The purpose of using this data is to analyze the effects of this technology on student learning. You may also agree to participate in a group interview, which will take approximately 40 minutes, by checking the appropriate box below. You may be selected to be in the experimental or control classroom. The experimental classrooms will start using the TI-Nspire CAS approximately 8 weeks before the control classrooms will. The data for this project will be collected over approximately an 8 week period.

RISKS

There are no foreseeable risks for participating in this research.

BENEFITS

There are no direct benefits to you as a participant other than to further research on this technology.

CONFIDENTIALITY

The data in this study will be confidential. The student journal entries, the video taped lessons, pre and post attitude surveys as well as test scores, and the audio taped group interviews will be coded so that names will not be made public. Only the teacher and researcher will have access to the identification key.

PARTICIPATION

Your participation is voluntary, and you may withdraw from the study at any time and for any reason. If you decide not to participate or if you withdraw from the study, there is no penalty or loss of benefits to which you are otherwise entitled and your grade will not be affected. However, you will still be required to participate in all classroom activities. There are no costs to you or any other party.

CONTACT

[redacted]
Research Subject Protections at 703-993-4121 if you have questions or comments regarding your rights as a participant in the research.
This research has been reviewed according to George Mason University procedures governing your participation in this research.

ASSENT

☐ I have read this form and **agree** to participate in this study.

☐ Yes, I WILL participate in a group interview.

☐ No, I will not participate in a group interview.

____
Name (Student)

____
Signature (Student)

____
Date of Signature

Version date:9/14/08

The Instrumental Genesis of CAS as an Amplifier in a Generative Pre-Algebra Classroom.

PARENTAL INFORMED CONSENT FORM

RESEARCH PROCEDURES

This research project is being conducted to determine the effects of the TI-Nspire Computer Algebra System (CAS) on student learning. The TI-Nspire CAS is a calculator which has the ability to manipulate algebraic symbols. If you agree to allow your child to participate, you will be giving permission for Mrs. Gantz to collect and use the following data for analysis; pre and post attitude surveys, pre and post number and algebraic expectation quizzes, classroom assessments, video taped lessons, and journal entries. The purpose of using this data is to analyze the effects of this technology on student learning. Your child may also agree to participate in a group interview, which will take approximately 40 minutes, by checking the appropriate box below. You may be selected to be in the experimental or control classroom. The experimental classrooms will start using the TI-Nspire CAS approximately 8 weeks before the control classrooms will. The data for this project will be collected over approximately an 8 week period.

RISKS

There are no foreseeable risks for participating in this research.

BENEFITS

There are no direct benefits to your child as a participant other than to further research on this technology.

CONFIDENTIALITY

The data in this study will be confidential. The student journal entries, the video taped lessons, pre and post attitude surveys as well as test scores, and the audio taped group interviews will be coded so that names will not be made public. Only the teacher and researcher will have access to the identification key.

PARTICIPATION

Your child's participation is voluntary, and your child may withdraw from the study at any time and for any reason. If you or your child decide not to participate or if your child withdraws from the study, there is no penalty or loss of benefits to which you or your child are otherwise entitled and your child's grade will not be affected. However, you will still be required to participate in all classroom activities.

CONTACT

[redacted]

Research Subject Protections at 703-993-4121 if you have questions or comments regarding your rights as a participant in the research.

This research has been reviewed according to George Mason University procedures governing your participation in this research.

CONSENT

☐ I have read this form and **agree** to allow my child to participate in this study.

☐ Yes, my child WILL participate in a group interview.

☐ No, my child will not participate in a group interview.

______________________________ of ______________________________

Name (Parent) Name of Child

____

Signature (Parent)

______________________________ Version date: 9/14/08

Date of Signature

The Instrumental Genesis of CAS as an Amplifier in a Generative Pre-Algebra Classroom.

TEACHER CONSENT FORM

RESEARCH PROCEDURES

This research project is being conducted to determine the effects of the TI-Nspire Computer Algebra System (CAS) on student learning. The TI-Nspire CAS is a calculator which has the ability to manipulate algebraic symbols. If you agree to participate, you will be asked to brief your students on this study, as well as distribute and collect attitude surveys to your students. You will be asked to teach students using guided learning activities where students work in cooperative learning groups for topics in Chapter 2 and Chapter 3. You may be asked to teach your students using the TI-Nspire CAS. You will be asked to work with the researcher as well as other teachers involved in this study on the development of some of these activities. You will be asked to keep a teacher log to document the success of activities, suggestions for future implementation, and changes noted in student learning. Your 8th grade pre-algebra classrooms will be selected to be either experimental or control classrooms. The experimental classrooms will start using the TI-Nspire CAS approximately 8 weeks before the control classrooms will. The data for this project will be collected over approximately an 8 week period.

RISKS

There are no foreseeable risks for participating in this research.

BENEFITS

There are no direct benefits to you as a participant other than to further research on this technology.

CONFIDENTIALITY

The data in this study will be confidential. The teacher log entries and the video taped lessons will be coded so that names will not be made public. Only the researcher will have access to the identification key.

PARTICIPATION

Your participation is voluntary, and you may withdraw from the study at any time and for any reason. If you decide not to participate or if you withdraw from the study, there is no penalty or loss of benefits to which you are otherwise entitled. There are no costs to you or any other party.

CONTACT

[redacted] contact the George Mason University Office of Research Subject Protections at 703-993-4121 if you have questions or comments regarding your rights as a participant in the research.
This research has been reviewed according to George Mason University procedures governing your participation in this research.

CONSENT

☐ I have read this form and **agree** to participate in this study.

____
Name (Student)

____
Signature (Student)

____
Date of Signature

Version date:9/14/08

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