CLUSTER-LEVEL CORRELATED ERROR VARIANCE AND THE ESTIMATION OF PARAMETERS IN LINEAR MIXED MODELS

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ABSTRACT

CLUSTER-LEVEL CORRELATED ERROR VARIANCE AND THE ESTIMATION OF PARAMETERS IN LINEAR MIXED MODELS

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Multilevel theory is extended primarily through the evaluation of cross-level effects, or how some between-cluster predictor explains a within-cluster outcome. Cross-level effects are often estimated using linear mixed models (LMMs). LMMs are susceptible to a bias from correlated error variance, resulting from omitted predictors and correlated error variance or common method variance. The effects of correlated error variance are well known in linear regression, but are relatively less understood in LMMs, an extension of LMM. The current study extends previous research on correlated error variance on cross-level effect LMM parameter estimation by applying a tracing rule methodology to demonstrate the mathematical structure of the bias produced by correlated error variance. The current study shows that bias is mainly produced by omitted variable-between-cluster predictor relationships paired with common method variance in the between-cluster predictor. In particular, both parameters can produce attenuation or accentuation of parameter estimates, depending on the magnitude and direction of the effects. The

study concludes by outlining remedial and preventative measures practicing researchers can take to remove correlated error from parameter estimates and, therefore, produce unbiased cross-level effect estimates.

CHAPTER ONE – INTRODUCTION

Understanding the behavior of people, work teams, and organizations requires integration of theory explaining behavior at each level. Organizational science benefits from theory using concepts at the *between-cluster* or *macro-level* (i.e., attributes of collections of units) and at the within-cluster or *micro-level* (i.e., attributes of individual units). Integrating macro- *and* micro-level theory results in *multilevel* theory (House, Rousseau, & Thomas-Hunt, 1995). However important, multilevel theorizing is less common, and arguably more difficult, than theory building at a single level (i.e., macro *or* micro). Consequently, many have decried the underdeveloped state of multilevel theory in several fields such as sociology (Wiley, 1988), economics (Weintraub, 1979), and organizational behavior (Klein & Kozlowski, 2000).

In recent years, organizational science has begun to devote more attention to developing multilevel theory. Such multilevel theory development usually occurs through studies examining whether between-cluster attributes affect attributes within-clusters. For instance, some multilevel theory involves hypotheses positing that between-cluster predictors affect the cluster means of within-cluster dependent variables—hence, between-cluster predictors affect between-cluster outcome means. Such hypotheses are known as *cross-level effect hypotheses*. Because cross-level hypotheses integrate the effects of macro-level (i.e., between-cluster) and micro-level (i.e., within-cluster) theory, cross-level hypotheses represent an important and commonly used method for multilevel theory development and testing.

Tests of multilevel theory often use survey or questionnaire designs, measuring between- and within-cluster attributes at single point in time (e.g., Ostroff, Kinicki, & Clark, 2002). Single-source, cross-sectional designs are especially susceptible to measurement-related *correlated error variance* (e.g., Campbell & Fiske, 1959; Evans, 1985). Correlated error variance is also produced through the omission of relevant causal variables in a predictive model, when the omitted predictor is related to other predictors; a process that produces biased parameter estimates and statistical inference (Kim & Frees, 2006; Mauro, 1990). Irrespective of its origin, correlated error variance is a threat to the validity of cross-level research.

Research shows that parameter estimates from *linear mixed models* (LMM; Raudenbush & Bryk, 2002)—arguably the most common analysis tool for cross-level hypothesis tests—are biased when correlated error variance is present (e.g., Ebbes, Böckenholt, & Wedel, 2004; Lai, Li, & Leung, 2013). Research on correlated error in LMMs is fragmented however, with much research in education, marketing, and economics focusing on omitted variables-related bias (e.g., Kim & Frees, 2006), whereas research in organizational science and psychology focuses on systematic measurement error (Lai et al., 2013). Unfortunately, the overall effects of correlated error variance in LMMs in the measurement and predictive model are, currently, unknown and potentially greater than the effects of LMMs in the measurement or predictive model alone.

Additionally, research to date on bias in LMMs has relied on the use of computer simulation. Although computer simulation can discern the effect of correlated error variance on parameter estimation, the conclusions from simulations are restricted to the conditions included in the simulation. By contrast, analytic methods such as *tracing rules* (Curran & Bauer, 2007; Duncan, 1966) provide more general conclusions regarding the effect of correlated error variance given a plausible causal model across many more possible LMMs – and can extend to evaluating extant LMM estimates when contaminated by correlated error variance. Taken together, the present work contributes to the literature by demonstrating that correlated error variance is a substantial validity threat which can affect parameter estimates through many more mechanisms than previously thought which threatens the development of multilevel theory testing research that organizational researchers cannot afford to ignore.

The purpose of the present work is to examine the effect of between-cluster correlated error variance—owing to both omitted causal effects and systematic measurement error—on cross-level parameter estimates in LMMs using tracing rules. To begin, I briefly review the conceptualization of the LMM model and provide a substantive example for use throughout the remainder of the paper. Following the LMM review, I outline research examining how correlated error variance affects parameter estimates in both linear regression and LMMs. I then outline a population causal model as well as the assumptions under which I apply tracing rules to the evaluation of correlated error in LMMs and examine the impact of the equation derived from the tracing rules on LMM estimates. I conclude the present work by providing an example derived from the extant literature in which correlated error could pose a substantial threat to the conclusions derived from the study. Finally, I provide researchers and reviewers a comprehensive outline of issues to look for in evaluating the quality of research using LMMs as well as for use in planning studies which intend to utilize LMMs as an analytic tool.

LMM: A Brief Review and Substantive Example

In order to evaluate how correlated error variance affects cross-level parameter estimates, consider first the conceptual model represented in Figure 1. Figure 1 contains 3 variables of substantive interest. The organizational climate variable is at the organizational or between-cluster level of analysis, whereas positive mood and job satisfaction are at the individual or within-cluster level of analysis.

Figure 1 shows that we are interested in using positive mood (i.e., $Mood_{ij}$) to predict job satisfaction (i.e., $Satisfaction_{ij}$). Moreover, we are interested in using organizational climate (i.e., $Climate_j$) for two purposes: to predict the organization (i.e., between-cluster) means of job satisfaction directly and also as a moderator of the positive mood - job satisfaction relationship. Using Figure 1, we can set up the series of equations to be estimated using a LMM. The model in Figure 1 suggests the following within-cluster equation:

$$Satisfaction_{ij} = \beta_{1j} Mood_{ij} + \beta_{0j} + r_{ij} \quad (1)$$

In Equation 1, β_{1j} is the regression weight for organization j predicting $Satisfaction_{ij}$ from $Mood_{ij}$. β_{0j} is the intercept for the organization j $Mood_{ij} - Satisfaction_{ij}$

relationship. r_{ij} is a random error of prediction or disturbance in the organization *j* $Mood_{ij} - Satisfaction_{ij}$ equation. Importantly, the LMM treats Equation 1 as having one or more random coefficients. That is, not only is the $Satisfaction_{ij}$ variable represented as a random variable (as in linear regression), but in the present case the β_{0j} coefficients are also treated as random, normally distributed variables, at the between-cluster level. Hence, the LMM acknowledges that there are 2 sources of variability in the job satisfaction outcome: within-cluster person-to-person differences and between-cluster mean/intercept differences (i.e., β_{0j} variance). In some cases, researchers are also interested in explaining and between-cluster predictive equation differences (i.e., β_{1j} variance) through *cross-level interaction* estimates. In the present study, I do not consider cross-level interactions, but rather assume that the $Mood_{ij} - Satisfaction_{ij}$ relationship is stable between-clusters. Although I do not consider cross-level interactions in the present study, I will return to address the issue of cross-level interactions in the discussion—noting their similarity to cross-level effects for estimation purposes and the results derived from the present study.

Figure 1 also implies that between-cluster effects are to be estimated in the LMM. The relationships implied by Figure 1 are captured in the following between-cluster equations:

$$\beta_{0j} = \gamma_{01} Climate_j + \gamma_{00} + \upsilon_{0j} \qquad (2)$$

In Equation 2, γ_{00} represents the average within-cluster $Mood_{ij} - Satisfaction_{ij}$ intercept, γ_{01} represents the regression weight predicting the within-cluster $Mood_{ij} - Satisfaction_{ij}$ intercepts from $Climate_j$, and υ_{0j} represents random errors of prediction for the between-cluster $Mood_{ij} - Satisfaction_{ij}$ intercept equation. The LMM estimation equation integrates Equations 1 and 2 by substituting Equation 2 into Equation 1 to produce:

$$Satisfaction_{ij} = \beta_{1j} Mood_{ij} + (\gamma_{01} Climate_j + \gamma_{00} + \upsilon_{0j}) + r_{ij} \quad (3)$$

Equation 3 shows more clearly the role of the within- and between-cluster predictors as they predict $Satisfaction_{ij}$. Expanding Equation 3 I obtain:

$$Satisfaction_{ij} = \beta_{1j} Mood_{ij} + \gamma_{01} Climate_j + \gamma_{00} + \upsilon_{0j} + r_{ij} \quad (4)$$

The coefficient γ_{01} carries the effect of the between-cluster $Climate_j$ predictor on the outcome $Satisfaction_{ij}$. As can be seen from Equation 3, the intercepts of the $Mood_{ij} - Satisfaction_{ij}$ relationship are a function of $Climate_j$, mean intercept γ_{00}, and the intercept error term υ_{0j}.

Equation 4 represents the equation to be estimated to discern the magnitude of the effects implied in Figure 1. In the section to come, I begin to discuss correlated error variance by reviewing previous research on the impact of correlated error variance on parameter estimates in linear regression and LMMs as well as provide an underlying causal model—including sources of correlated error—that could affect the estimation of parameters of Equation 4.

Cluster-level Correlated Error Variance as Left Out Variables Error and Common Method Variance

Omitted, yet relevant predictors are known to have an important influence on parameter estimation. Omitted predictors in a predictive model create what Mauro

(1990) refers to as *left out variables error* (Mauro, 1990). By contrast, omitted measurement parameters (i.e., multidimensional constructs in which some constructs are not modeled) produces what is known as *common method variance* (Podsakoff, MacKenzie, Lee, & Podsakoff, 2003). Both left out variables error and common method variance are (some of the possible; see Ebbes et al., 2004 for others) sources of correlated error variance that could affect LMMs. Correlated error variance is a non-0 correlation between outcome prediction error and a predictor in a statistical model—a violation of the strict exogeneity assumption common to most linear regression models (e.g., Berry, 1993)

Traditionally, correlated error variance sources in the organizational sciences have been examined in linear regression models. Whereas both sources of correlated error have been the focus of study, past research has tended to focus on *either* left out variables error *or* common method variance sources of correlated error variance and not integrate both. The first stream of research on left out variables error focuses on correlated error variance owing to the omission of relevant "third-variables" which are related to both the predictor and the outcome. Such left out variables produce a non-0 relationship between the prediction error of the outcome and the predictor that results in the magnitude of the omitted variable's effects being "absorbed" by the included predictor, thereby biasing parameter estimates (e.g., Mauro, 1990; Meade, Behrend, & Lance, 2009). In particular, research on left out variables error demonstrates that, depending on the direction and magnitude of the omitted effects, as well as intercorrelations between the omitted predictors, the resulting bias can be positive or negative and, under certain circumstances, can change the interpretation of an effect (Mauro, 1990, p. 316).

By contrast, the second stream of research, focusing on measurement error, examines the effect of common method variance on measurement model coefficients. Common method variance is a form of systematic measurement error that can be conceptualized more broadly as correlated measurement error between two variables with methodological similarities (e.g., Bagozzi & Yi, 1991), though the term almost always refers to variables measured using self-report methods. Common method variance produces correlated error variance as the omitted variable has a causal effect on the measure or the observed response (i.e., whether a respondent indicates they *strongly disagree* or *disagree* on a Likert-type opinion or attitude scale), but not the latent construct reflected by the observed response (i.e., whether a respondent's opinion or attitude actually *strongly disagrees* or *disagrees* with a statement). For instance, self-presentation concerns could compel a respondent to report their task performance on their job as being "exceptional," and their job satisfaction as "extremely satisfied," even when their task performance is less-than exceptional and they are less-than satisfied. Whereas such self-presentation concerns affect the observed response or measure of task performance and satisfaction (i.e., impacting the apparent predictive validity of the measure), self-presentation concerns may not have a causal effect on how a respondent actually accomplishes their task work or on how they actually feel about their job. Therefore, method variance driven by self-presentation has a causal effect on how an individual responds to measures (i.e., their response behavior), but not on their standing on the constructs of interest (e.g., how their satisfaction cognitions impact their job performance behavior). Consequently, common method variance produces a non-0 correlation between measurement error in the predictor and measurement error in the outcome variable, i.e., correlated error variance.

Organizational science has produced much research on the effects of common method variance and demonstrates that its effects on parameter estimates are complex. In fact, studies show that, depending on the data analytic conditions (e.g., construct interrelationships, percentage of observed variance attributable to measurement method), common method variance, like left out variables error, can positively or negatively bias parameter estimates (e.g., Podsakoff et al., 2003; Williams & Brown, 1994). Siemsen et al. (2010; Equation 9) show that the direction of the bias depends on the ratio of the common method variance in the outcome to that in the predictor. When the ratio is greater than the true correlation between the outcome and the predictor, bias is positive (in this instance, referring to positive multiplicative bias or accentuation; effect strengthened by multiplying by some value greater than 1 in absolute value). By contrast, when the ratio is lower than the true correlation between the outcome and predictor, bias is negative (i.e., negative multiplicative bias or attenuation; effect weakened by multiplying by some value less than 1 in absolute value).

Taken together, research on correlated error variance from a left out variables error as well as a common method variance demonstrate that correlated error variance is an important issue for parameter estimation and can produce substantial bias in situations where omitted variables have causal effects on the latent variables of interest (i.e., left out variables error) or the measures used to assess the latent variables of interest (i.e., common method variance).

Correlated Error Variance in Multilevel Research

Although most research on correlated error variance in the organizational sciences has evaluated its effects in linear regression without considering between-cluster effects, the biasing effects of correlated error variance are not restricted to single-level situations. In fact, situations where omitted between-cluster predictors might be relevant are not difficult to generate. Imagine, for instance, for the *actual* underlying causal model in Equation 4 that a policy-related variable, such as flextime benefits or teleworking capabilities, has a causal effect on both positive organizational climate as well as the mean of job satisfaction for each organization. Because flextime/teleworking benefits are not modeled in Equation 4, yet is a component of the true causal model for satisfaction, the effect of flextime/telework benefits will result in correlated error variance owing to left out variables error and will bias the positive organizational climate parameter estimate. The effects of left out variables on LMM estimation has been the subject of past research. In fact, Raudenbush and Bryk (2002; Chapter 9) demonstrate that misspecification, in terms of omitted, relevant predictors at the within- and between-cluster level do produce inaccurate coefficients (see also Ebbes et al., 2004; Kim & Frees, 2006, 2007). In fact, previous research finds that even with modest predictor between-cluster predictor-error variance correlations (i.e., .3) that fixed effect estimates, such as cross-level effect parameters, can be severely biased (i.e., by about 25%; Ebbes et al., 2004).

Additionally, Ostroff, Kinicki, and Clark (2002), have shown that common method variance from aggregated survey responses (e.g., composition models; Chan, 1998) appear to produce a positive bias in cross-level correlations similar to the bias found in correlations of variables within-cluster. Other research agrees that common method variance biases parameter estimates and also, consequently, increases Type I error rates. Situations in which common method variance could also have an impact on between-cluster measurement models such as composition models, are also not hard to imagine. For example, a situation in which between-organization differences in response style to a survey effort occurs owing to different levels of familiarity with surveys—and thus use of the response options—across the two organizations.

Although research on the correlated error variance-producing factors of left out variables error and common method variance have historically been separate concepts in organizational science, both causes of correlated error variance are similar conceptually (i.e., related to omitted causal effects) and functionally (i.e., parameter bias producing). Additionally, both sources of correlated error could operate in tandem, either independently of one another or interacting with one another to produce magnitudes of bias in LMM parameter estimates that have been, to this point, ignored. In the current study, I explore the implications of *both* sources of correlated error variance on parameter estimates in a LMM in an effort to more fully outline the potential bias such correlated error can produce. To explore the implications of correlated error variance on the estimation of parameters in Equation 4, I construct an underlying causal model to represent the causal effects between the latent variables as well as the measurement models—represented in Figure 2. The situation represented in Figure 2 is intended to be a worst case scenario in which an omitted, between-cluster variable not only affects all of the measures involved in Equation 4 (i.e., produces common method variance), but also affects the latent variables involved in Equation 4 (i.e., produces left out variables error). The depiction of the Figure 2 structural and measurement model aspects follow from suggestions offered by Curran and Bauer (2007) for depicting multilevel models graphically.

In Figure 2, each of the solid lines (i.e., "—") represent the effects of interest that are represented in Equation 4. Figure 2 also shows all the ways in which correlated error variance is introduced into the model to estimate in Equation 4. Specifically, the paths from the omitted variable V_j to each of the other latent variables are represented using double-dotted and dashed lines (i.e., "··-··-") and are the left out variables error effects that produce correlated error variance. The paths from the omitted variable V_j to each of the other measured variables are represented using dotted lines (i.e., "……") and are the common method variance factors that produce correlated error variance. Intercepts at the within- and between-cluster level are indicated in Figure 2 by triangles. Triangle 1_1 represents within-cluster intercepts and triangle 1_2 represents between-cluster intercepts. The randomly varying between-cluster intercepts are represented by latent variable situated in the middle of the path from triangle 1_1 to S_{ij} (represented by S_j). The meaning for each of the variables included in Figure 2 is discussed in the following section.

Introducing sources of correlated error variance into a LMM. The model implied by Figure 2 has many components dealing with both underlying causal relationships between latent variables and measurement models of those latent variables. I begin by outlining the measurement models for each of the observed variables $Satisfaction_{ij}$, $Climate_j$, and $Mood_{ij}$, separating or decomposing each observed variable into between- and within-cluster components and, subsequently, into focal construct-related and error-related variance. My presentation of the decomposition of each of the measured variables follows loosely from previous research on left out variables error (e.g., Mauro, 1990; Siemsen et al., 2010), but is primarily derived through the application of tracing rules (e.g., Curran & Bauer, 2007) applied to Figure 2. In Figure 2, and thus all the equations to come, I define all the exogenous latent variables to be distributed unit multivariate normally (i.e., mean of 0 and variance of 1). Therefore, all of the between-cluster exogenous variables in Figure 2 V_j, ε_{sj}, ε_{cj} , ε_{mj}, and c_j each have a mean of 0 and variance of 1 across all *j* organizations and *i* individuals. In addition, each exogenous within-cluster variable in Figure 2 is distributed unit multivariate normally. Thus, ε_{sij}, M_{ij}, s_{ij}, and m_{ij} have also have a mean of 0 and variance of 1 across all *i* individuals—however, because of the relationship of V_j with S_j and M_j, both $Satisfaction_{ij}$ and $Mood_{ij}$ are endogenous.

The measurement model decompositions of each of the observed variables are as follows:

$$Satisfaction_{ij} = Btw\_Satis_j + W/in\_Satis_{ij} = \left(b_{sj}\mathrm{S}_j + b_{sv}\mathrm{V}_j\right) + \left(b_{si}\mathrm{S}_{ij} + s_{ij}\right) \quad (5)$$

$$Climate_j = Btw\_Clmt_j = b_{cj}\mathrm{C}_j + b_{cv}\mathrm{V}_j + c_j \quad (6)$$

$$Mood_{ij} = Btw\_Mood_j + W/in\_Mood_{ij} = \left(b_{mj}\mathrm{M}_j + b_{mv}\mathrm{V}_j\right) + \left(b_{mi}\mathrm{M}_{ij} + m_{ij}\right) \quad (7)$$

Where $Satisfaction_{ij}$ is the satisfaction score for person *i* in organization j, $Mood_{ij}$ is the mood score for person *i* in organization j, and $Climate_j$ is the climate score for organization j.

Equations 5-7 all begin by breaking each observed variable into components or parts that are attributable to different, independent variables at the between- (i.e., "Btw_") vs. within-cluster (i.e., "W/in_") level. Focusing on Equation 5, notice first that 2 components comprise Btw_Satis_j, specifically $b_{sj}\mathrm{S}_j$ and $b_{sv}\mathrm{V}_j$. $b_{sj}\mathrm{S}_j$ represent the component of Btw_Satis_j that derives from focal construct-related between-cluster differences in satisfaction (e.g., differences that might be due to different organizational policies that foster satisfaction), with b_{sj} representing the factor loading of S_j onto the observed variable $Satisfaction_{ij}$. By contrast, $b_{sv}\mathrm{V}_j$ represents the component of Btw_Satis_j that corresponds to measurement contamination or common method variance with the extent of contamination by the omitted factor represented by the factor loading b_{sv}. If $b_{sv} = 0$, then Btw_Satis_j reduces to only include focal construct-related between-cluster differences in satisfaction

Equation 5 also shows that $Satisfaction_{ij}$ has 2 components that comprise W/in_Satis_j, specifically $b_{si}\mathrm{S}_{ij}$ and s_{ij}. $b_{si}\mathrm{S}_{ij}$ represents the component of W/in_Satis_j that derives from focal construct-related within-cluster differences in satisfaction (e.g., differences due to variability in mood states that foster satisfaction), with b_{si} representing the factor loading of S_{ij} onto the observed variable $Satisfaction_{ij}$. Finally, s_{ij} represents random measurement error in the observed $Satisfaction_{ij}$

measure. Equation 7 follows the exact same structure as does Equation 5 save for the components of Equation 7 are related to the components of $Mood_{ij}$.

Moving to Equation 6 I begin by noting that $Climate_j$ is an observed variable that only varies at the between-cluster level, thus $Climate_j$ produces no within-cluster variance component. Unlike $Satisfaction_{ij}$, $Climate_j$ has 3 components that comprise its between-cluster component Btw_Clmt_j: $b_{cj}\mathrm{C}_j$, $b_{cv}\mathrm{V}_j$ and c_j. $b_{cj}\mathrm{C}_j$ represents the component of Btw_Clmt_j that derives from focal construct-related between-cluster differences in organizational climate (e.g., different organizational policies that impact shared sense of climate), with b_{cj} representing the factor loading of C_j onto the observed variable $Climate_j$ Similar to $Satisfaction_{ij}$, $b_{cv}\mathrm{V}_j$ represents the component of Btw_Clmt_j that corresponds to common method variance with the extent of contamination represented by the factor loading b_{cv}. The third and final component of Btw_Clmt_j is c_j, or random measurement error at the between-cluster level (e.g., disagreement between employees about organizational climate owing to transient factors).

Separate from the measurement models outlined above, the model in Figure 2 also implies a set of structural relationships between each of the latent constructs. I begin by considering the relationships between the between-cluster constructs and, similar to the decomposition of the measured variables above, I decompose each of the endogenous variables in terms of their causal model. I begin by decomposing the endogenous latent organizational climate factor, C_j.

$$\mathrm{C}_j = \delta_{\mathrm{vc}}\mathrm{V}_j + \varepsilon_{cj} \quad (8)$$

Equation 8 shows that C_j is a function of 2 components, $\delta_{\mathrm{vc}}\mathrm{V}_j$ and ε_{cj}. The first component $\delta_{\mathrm{vc}}\mathrm{V}_j$, represents the causal effect, carried by δ_{vc}, of the exogenous, yet omitted, contaminating variable V_j, on C_j. The second component ε_{cj}, represents independent or exogenous variance in C_j produced through other means, but not explainable by or related to variables in Figure 2.

Figure 2 shows that C_j has a causal effect on M_j, the between-cluster mood construct, which can be decomposed as follows:

$$\mathrm{M}_j = \delta_{\mathrm{vm}}\mathrm{V}_j + \delta_{\mathrm{cm}}\mathrm{C}_j + \varepsilon_{mj} = \delta_{\mathrm{vm}}\mathrm{V}_j + \delta_{\mathrm{cm}}\left(\delta_{\mathrm{vc}}\mathrm{V}_j + \varepsilon_{cj}\right) + \varepsilon_{mj} \quad (9)$$

Equation 9 shows that M_j is a function of 3 components $\delta_{\mathrm{vm}}\mathrm{V}_j$, $\delta_{\mathrm{cm}}\mathrm{C}_{j,}$ and ε_{mj}. The first component $\delta_{\mathrm{vm}}\mathrm{V}_j$, produces the causal effect, δ_{vm}, of the contaminating variable V_j, on M_j. The second component is more complex, as it is a function of a function. To be precise, $\delta_{\mathrm{cm}}\mathrm{C}_j$ represents the causal effect, δ_{cm}, of both components of C_j on M_j. As the far right hand side of Equation 9 shows, $\delta_{\mathrm{cm}}\delta_{\mathrm{vc}}\mathrm{V}_j$ is the indirect causal effect of the contaminating variable V_j, through C_j, whereas $\delta_{\mathrm{cm}}\varepsilon_{cj}$ represents the direct causal effect of C_j on M_j. The third, and final, component ε_{mj}, like the similar component in C_j, represents independent or exogenous variance in M_j produced through other means, but not explainable by or related to variables in Figure 2.

Figure 2 also shows that both C_j and M_j have a causal effect on S_j, the between-cluster satisfaction construct, which can be decomposed as follows:

$$S_j = \delta_{vs}V_j + \delta_{cs}C_j + \delta_{ms}M_j + \varepsilon_{sj}$$

$$= \delta_{vs}V_j + \delta_{cs}(\delta_{vc}V_j + \varepsilon_{cj}) + \delta_{ms}(\delta_{vm}V_j + \delta_{cm}[\delta_{vc}V_j + \varepsilon_{cj}] + \varepsilon_{mj}) + \varepsilon_{sj} \quad (10)$$

Equation 10 shows that S_j is a function of 4 components $\delta_{vs}V_j$, $\delta_{cs}C$, $\delta_{ms}M_j$, and ε_{sj}. The first component $\delta_{vs}V$, produces the causal effect, δ_{vs}, of the contaminating variable V_j, on S_j. The second component, as with the decomposition of M_j, is more complex. Here, as in M_j, $\delta_{cs}C_j$ represents the causal effect, δ_{cs}, of both components of C_j on S_j—$\delta_{cs}\delta_{vc}V_j$ as the indirect causal effect of V_j, through C_j, and $\delta_{cs}\varepsilon_{cj}$ represents the direct causal effect of C_j on S_j. The third component of S_j is even more complex, containing several indirect causal effects. Specifically, $\delta_{ms}M_j$ can be divided into 4 separate effects: i] $\delta_{ms}\delta_{vm}V_j$ is the indirect causal effect of the contaminant V_j through M_j, ii] $\delta_{ms}\delta_{cm}\delta_{vc}V_j$ is the "once-removed" indirect effect of the contaminant V_j through both C_j and M_j, iii] $\delta_{ms}\delta_{cm}\varepsilon_{cj}$ is the indirect causal effect of C_j on S_j, and iv] $\delta_{ms}\varepsilon_{mj}$ is the direct causal effect of M_j on S_j. The final component ε_{sj}, like the other latent constructs, represents independent or exogenous variance in S_j produced through other means, but not explainable by or related to variables in Figure 2.

In addition to relationships between between-cluster latent constructs, Figure 2 also shows relationships between within-cluster constructs. Most importantly, Figure 2 shows that the latent within-cluster satisfaction construct S_{ij}, independent of the between-cluster construct S_j, is endogenous. The within-cluster satisfaction construct S_{ij} is decomposed as follows:

$$\mathrm{S}_{ij} = \mathrm{M}_{ij}\beta_{1j} + \varepsilon_{sij} \quad (11)$$

Equation 11 shows that S_{ij} is a function of 2 components $\mathrm{M}_{ij}\beta_{1j}$ and ε_{sij}. The first component $\mathrm{M}_{ij}\beta_{1j}$, represents the causal effects, β_{1j}, of M_{ij} on S_{ij}. The final component of the within-cluster satisfaction construct, ε_{sij}, is the independent or exogenous variance in S_{ij} produced through other means, but not explainable by or related to variables in Figure 2.

The measurement and structural decompositions in Equations 5-11 allow for a more thorough delineation of how and where correlated error variance (i.e., any component of Equations 7-13 containing V_j), affects the observed measure of the outcome, $Satisfaction_{ij}$. Most importantly, the decompositions contained in Equations 5-11 can show more clearly how and why correlated error variance affects LMM parameter estimates. Going forward, I assume that the factor loadings for all components of the observed variable, save for the omitted, contaminating factor V_j, are 1—dropping out of the equation. Focal construct-related variable factor loadings are set to 1 as an important aspect of evaluating common method variance is the proportion of the measure comprised by method variance, which can be most easily manipulated through a single factor loading, the factor loading with the contaminant variable $\mathrm{V}_{j.}$ Additionally, I rename the coefficients attached to the omitted factor from b_{cv} and b_{mv} to b_m and b_c to simplify subscripting in the equations to come. I will also note that the same simplification applies to the outcome (i.e., b_{sv} will be renamed b_s). In the coming section, I outline how Equation 4 will be evaluated in order to extract how and why correlated error affects parameter estimation in LMMs.

CHAPTER TWO - EXAMINING THE EFFECT OF CLUSTER-LEVEL CORRELATED ERROR VARIANCE ON PARAMETER ESTIMATES

LMM is an extension of traditional linear regression and, under certain conditions, the LMM estimation process simplifies to a set of 2 linear regressions. More generally, the process through which LMMs operate is in generating a set of parameters to optimally explain variance at both the between- and within-cluster level, of an outcome variable using a set of predictors at both the within- and between-cluster level. LMMs then estimate parameters based on variances of and covariances among the predictor and outcome variables (Raudenbush & Bryk, 2002).

Most parameter estimates in an LMM approach estimation by weighting the contribution of each cluster to the parameter estimate by the between- and within-cluster variance for that cluster (Raudenbush & Bryk, 2002, pp. 42–43)—clusters with smaller within-cluster variance contribute relatively more to the parameter estimate than clusters with greater within-cluster variance. Although an LMM tends to be estimated using numerical methods such as maximum likelihood, the LMM model has, historically, not used numerical methods but a strong set of assumptions to derive parameter estimates (i.e., the "between-effects" estimator: Cameron & Trivedi, 2005). In particular, under the assumption that each cluster is the same size and that the distribution of the within-cluster predictors is equal across each cluster, the within-cluster residuals will be equal (or balanced) across all clusters. Because obtaining an estimate of within-cluster residual variance is one of the most problematic aspects of LMM estimation, the LMM estimation process simplifies greatly when balanced into a “2-step” ordinary least squares (OLS) linear regression. The first step constitutes a series of separate regressions for each cluster using the within-cluster predictors to predict the outcome. The second step then involves using the obtained parameter estimates from the first step (i.e., regression slopes, intercepts) as dependent variables in a between-cluster regression in which the between-cluster predictors are used to explain between-cluster variance in the first-step’s estimated parameters.

Because LMMs can be estimated using a 2-step OLS regression when balanced, LMMs do not require the use of simulations in order to derive the impact of different input population causal models on parameter estimation (see Raudenbush & Bryk, 2002, p. 43). In fact, given that OLS can be applied to LMM estimation, many useful methods applied to linear models also apply to LMMs such as tracing rules (Duncan, 1966; Equation 5; Wright, 1934). Tracing rules are derived from path analysis and involve partitioning the covariance between two variables into components. Specifically, there are 5 main rules for partitioning a covariance: a] the researcher can trace backward up an arrow and then forward along the next, or forwards from one variable to the other, but never forward and then back; b] the researcher can pass through each variable only once in a given chain of paths; c] no more than one bi-directional arrow (i.e., unanalyzed path/covariance) can be included in each path-chain, d] at any change of direction (i.e., exogenous variable) in a tracing route which is not a bi-directional arrow connecting different variables in the chain (i.e., unanalyzed path/covariance), the variance of the variable at the point of change is included in the product of path coefficients; and e] when deriving variances, the path from a dependent variable to an independent variable and back to itself is only counted once. Covariances are then computed by multiplying all the coefficients in a chain and summing over all possible chains. Chains are considered independent/different if: i] they don't have the same coefficients, or ii] the coefficients are in a different order. Residual variances are included as unanalyzed paths/covariances in the tracing rules.

Because Figure 2 outlines a full path model depicting the underlying population causal model, the paths in Figure 2 can be used in the tracing rules analysis to derive variances and covariances to be "tracing rule" decomposed. Tracing rules can then extend past research on correlated error in LMM estimation by providing information about the mathematical form of the parameter estimate result—as opposed to just the end value result that computer simulations provide. Knowing of the mathematical form of a result has been helpful in situations such as Siemsen et al. (2010) who show that the effect of having common method variance in only the outcome on a linear regression coefficient is multiplicative—always attenuating the magnitude of the estimated coefficient effect toward 0 (whether positive or negative). Knowing the form of the effect is important both for remedial action for the analyst as well as an informed evaluation of the conclusions that can be drawn from a parameter estimate. In the section to come I evaluate LMM parameter estimates, under the assumption of balanced clusters, using tracing rules.

CHAPTER THREE - BETWEEN-CLUSTER CORRELATED ERROR VARIANCE AND CROSS-LEVEL EFFECT ESTIMATION[i]

Because the clusters in the present work are assumed to be balanced, estimating the effect of the between-cluster predictor on within-cluster intercepts can be conducted using the OLS estimator. The OLS estimator for a parameter is the covariance between the predictor and outcome divided by the variance of the predictor (see Cohen, Cohen, West, & Aiken, 2003). Because the cross-level effect parameter estimate is a ratio of a covariance to a variance, the tracing rule methodology can be applied to decompose the covariance and variance separately. In particular, the cross-level effect parameter estimate γ_{10} will be a ratio of the covariance between the within-cluster intercepts of the $Mood_{ij} - Satisfaction_{ij}$ relationship and the between-cluster predictor, $Climate_j$, over the variance of $Climate_j$. The tracing rule paths derived for the decomposition of the OLS estimator of the cross-level effect parameter are outlined in Table 1. Representing the tracing rules derived in Table 1 as the covariance to variance ratio:

$$\gamma_{10} = \frac{\mathrm{cov}\left[Climate_j, \beta_{0j}\right]}{\mathrm{var}\left[Climate_j\right]}$$

$$= \frac{\mathrm{var}\left[\varepsilon_{cj}\right](\delta_{\mathrm{cs}} + \delta_{\mathrm{ms}}\delta_{\mathrm{cm}})}{\mathrm{var}\left[\mathrm{V}_j\right](\delta_{\mathrm{vc}}^2 + b_c\{2\delta_{\mathrm{vc}} + b_c\}) + \mathrm{var}\left[\varepsilon_{cj}\right] + \mathrm{var}\left[c_j\right]}$$

$$+ \frac{\delta_{\mathrm{vc}}\mathrm{var}\left[\mathrm{V}_j\right](\delta_{\mathrm{vs}} + \delta_{\mathrm{ms}}\delta_{\mathrm{vm}} + \delta_{\mathrm{vc}}\{\delta_{\mathrm{cs}} + \delta_{\mathrm{ms}}\delta_{\mathrm{cm}}\} + b_s)}{\mathrm{var}\left[\mathrm{V}_j\right](\delta_{\mathrm{vc}}^2 + b_c\{2\delta_{\mathrm{vc}} + b_c\}) + \mathrm{var}\left[\varepsilon_{cj}\right] + \mathrm{var}\left[c_j\right]}$$

$$+ \frac{b_c\mathrm{var}\left[\mathrm{V}_j\right](\delta_{\mathrm{vs}} + \delta_{\mathrm{ms}}\delta_{\mathrm{vm}} + \delta_{\mathrm{vc}}\{\delta_{\mathrm{cs}} + \delta_{\mathrm{ms}}\delta_{\mathrm{cm}}\} + b_s)}{\mathrm{var}\left[\mathrm{V}_j\right](\delta_{\mathrm{vc}}^2 + b_c\{2\delta_{\mathrm{vc}} + b_c\}) + \mathrm{var}\left[\varepsilon_{cj}\right] + \mathrm{var}\left[c_j\right]} \quad (12)$$

Equation 12 is the most basic result for the effect of correlated error variance on the estimation of cross-level effects parameters. Equation 12 is subdivided into the sum of 3 fractions, each of which represents conceptually different contributions to the cross-level effect. The first summand of Equation 12 represents the effect of latent $Climate_j$-related variance components on the cross-level effect. The second summand represents the effects related to left out variables error. Finally, the third summand represents common method variance.

The estimate for the cross-level effect has a great many ways in which to be biased (i.e., all of the second and third summands in Equation 12), and does not lend itself to a simple evaluation of trends. Hence, to elucidate the effect of correlated error variance on cross-level effect estimation, I graph Equation 12 in Figure 3 to more easily discern trends in Equation 12 based on several parameters which are likely to have important effects. Figure 3 depicts the relationship between the magnitude of the common method variance effect on the x-axis and the difference between the estimated parameter and the actual parameter for the cross-level effect on the y-axis. The latent variances in Figure 3 (i.e., $\text{var}[\text{V}_j]$ and $\text{var}[\varepsilon_{cj}]$) were set at 1 (i.e., the latent variances were standardized), the δ_{cs} parameter was set to .1 and the $\text{var}[c_j]$, δ_{cm}, b_s, δ_{vm} and δ_{ms} parameters were set to 0. Figure 3 is divided into 4 lines by crossing levels of the δ_{vc} parameter at 2 settings: present at .1 and absent at 0, with levels of the δ_{vs} parameter at 2 settings: present at.1 and absent at 0—as the δ_{vc} (relationship between omitted variable on latent $Climate_j$) and δ_{vs} (causal effect of omitted variable on latent, between-cluster $Satisfaction_{ij}$) parameters are likely to be important contributors to bias.

The trends outlined in Figure 3 show that additive accentuation or inflation only occurs primarily under 2 conditions, both of which include situations where there is an omitted cross-level effect: a] when the omitted variable has a relationship with latent $Climate_j$, and b] when common method variance is present in the $Satisfaction_{ij}$ measure. In fact, the magnitude of the accentuation is in upwards of 25% the magnitude of the actual latent $Climate_j$ effect (when the proportion of the variance in the predictor is near 25% common method variance). Perhaps most notable about the findings outlined in Figure 3 is how it reflects the second or left out variables error summand of Equation 12 in that even in the absence of common method variance. Specifically, an omitted effect with omitted variable-latent $Climate_j$ variable relationship produces a baseline 10% accentuation bias for the cross-level effect. Therefore, Figure 3 shows that even with relatively modest correlated error, the bias in LMM parameter estimation can be quite large—consistent with the finding by Ebbes et al. (2004) and Lai et al. (2013).

By contrast, Figure 3 shows that, in the absence of an omitted cross-level effect, the general trend is toward an attenuated estimate of the cross-level effect. The level of attenuation of the cross-level effect is rather severe at high levels of common method variance, nearing a 25% underestimate of the cross-level effect near 35% common method variance contamination in the $Climate_j$ measure. Importantly, across all conditions, Figure 3 shows that common method variance produces a non-linear or multiplicative trend on the estimation of the cross-level effect. To be specific, when there is an omitted variable effect, introducing common method variance—and thus, increasing the proportion of common method variance in the $Climate_j$ measure—the accentuation effect reaches maximum at around 25% common method variance contamination and then begins to drop. In fact, the attenuation effect when an omitted variable effect is absent is similar, save that under such conditions the attenuation increases at a faster rate as common method variance increases. The nature of the non-linear common method variance effect occurs as a result of the denominators of the 3 summands in Equation 12—attributable to the "extra" variance produced in the $Climate_j$ measure that doesn't correlate with within-cluster intercept variance and, therefore, doesn't contribute to the covariance between the $Climate_j$ measure and within-cluster intercepts. With increasing common method variance contamination, the size of the denominator (which contains several b_c terms) begins to outpace the magnitude of the parameters in the numerators of Equation 12, producing a net attenuation effect in the cross-level effect estimate. Hence, Figure 3 also shows that in the absence of any left out variables error, common method variance in the predictor alone can produce substantial bias which, on top of random measurement error, can severely underestimate the size of a cross-level effect and potentially affect the conclusions from hypothesis tests.

Equation 12 reveals many other potential situations in which attenuation, or accentuation, of the cross-level parameter could occur as well. Although not depicted in Figure 3, 2 potentially important ones are the $\delta_{\mathrm{vc}} b_s$ effect as well as $b_c b_s$ effect. Both effects stem from between-cluster common method variance in the $Satisfaction_{ij}$ measure—an effect which could occur for reasons similar to those that could produce between-cluster common method variance in the $Climate_j$ measure. Whereas $b_c b_s$ is a situation outlined by Lai et al. (2013), $\delta_{\mathrm{vc}} b_s$ is a very new concept in which the omitted factor has a relationship with the latent predictor and also produces common method variance in the outcome. The potential for such effects in LMM parameter estimation have not been appreciated in past research but constitute a plausible and potentially important source of bias for cross-level effects.

Whereas other parameter products are clearly present in Equation 12, their likely influence on the final parameter estimate are likely to be relatively smaller as all other effects are triple or quadruple products, that are likely to diminish the overall magnitude of the impact on the parameter estimate. To conclude, the results obtained in Equation 12 shows that the effect of correlated error variance on the magnitude of the obtained cross-level effect is can be complex and can, as is predicted in Figure 3, produce both potentially accentuating or attenuating (depending on the sign of the parameters) additive and attenuating, multiplicative bias. In the section to come, I move beyond Figure 3 to apply the results of Equation 12 to real examples found in the literature in which correlated error is likely to have affected the results of studies utilizing LMM. In addition, I provide both reviewers and researchers a set of suggestions for planning and evaluating LMM-based research in light of the findings outlined in Equation 12.

The Real Impact of Between-cluster Correlated Error: Illustrations and Suggestions

An Empirical Example

The results from Equation 12 provide a mechanism for examining the effect of correlated error on parameter estimates in LMM analysis which can and should be evaluated in terms of how correlated error affect the final parameter estimate of research study. Hence, the results of Equation 12 can be used to examine whether the results from articles published in well-respected journals, with many citations, are trustworthy. In the paragraphs to follow, I select the influential Ilies and Judge (2002) study from *Organizational Behavior and Human Decision Processes* which has a high number of citations (i.e., 99 according to PSYCinfo as of 2/3/2014). Ilies and Judge focus on the examination of between-person positive affect and between-person job satisfaction in an LMM for their Hypothesis 1a. Their Hypothesis 1a states that average positive affect will result in higher for positive average job satisfaction. For the present work, I focus on the estimates provided in Table 2, Model 1, which shows a statistically significant estimate of .37 of average positive affect on average job satisfaction.

One important point to note about their Model 1 is that it is a very simple predictive model – job satisfaction is a function of average positive (and negative) affect only. Such a small model is highly restrictive and makes the strong assumption that no other variables are relevant to the prediction of average, or between-person, job satisfaction, for their estimate to be unbiased. However, as Ilies and Judge note, such average positive and negative affect are thought to reflect trait positive affectivity and negative affectivity. A great deal of research exists which links positive affectivity to myriad other variables—many of which are also considered to be predictors of between-person job satisfaction. For example, meta-analytic research shows that supervisor support has a .22 correlation with positive affectivity, as well as many cogent theoretical reasons for being related to supervisor support (Ng & Sorensen, 2009). Additionally, supervisor support has been shown to have a relationship with job satisfaction (i.e., r = .48; Ng & Sorensen, 2008). Therefore, supervisor support is a strong candidate for an omitted variable which is likely to bias the positive affectivity coefficient.

Consequently, Ilies and Judge's obtained estimate of .37 can, assuming for the current illustration that no common method variance is present in their measure and ignoring that negative affectivity was included in the LMM, as Equation 13.

$$.37 = \frac{\mathrm{var}[\varepsilon_{cj}]\delta_{\mathrm{cs}} + \delta_{\mathrm{vc}}\delta_{\mathrm{vs}}\mathrm{var}[\mathrm{V}_j] + \delta_{\mathrm{vc}}^2\delta_{\mathrm{cs}}\mathrm{var}[\mathrm{V}_j]}{\mathrm{var}[\mathrm{V}_j]\delta_{\mathrm{vc}}^2 + \mathrm{var}[\varepsilon_{cj}] + \mathrm{var}[c_j]} \quad (13)$$

As is outlined above, it is very likely the case that δ_{vc} (in this instance, the relationship between positive affect and supervisor support) and δ_{vs} (in this instance, the relationship between job satisfaction and supervisor support) are non-0. Moreover, it is very likely that both that δ_{vc} and δ_{vs} are positive based on extant research and theorizing. Therefore, the obtained estimate of .37 is very likely to be an overestimate of δ_{cs}(i.e., the relationship between positive affectivity and job satisfaction), assuming that random measurement error $\mathrm{var}[c_j]$ is not substantially downwardly biasing the estimate (which the coefficient alpha reported of .93 reveals it not likely to be). Although the data are not available to estimate actual magnitude of the overestimation, it is clear that between the $\delta_{vc}\delta_{vs}\mathrm{var}[V_j]$ and $\delta_{vc}^2\delta_{cs}\mathrm{var}[V_j]$ terms that the magnitude could be substantial *simply from leaving out the single supervisor support variable*. However unlikely, as positive affectivity does have strong conceptual reasons to be related to job satisfaction, depending on how upwardly biased the estimate of .37 is, the estimate could have produced a spuriously statistically significant estimate. Taken together, Equation 13 shows that the potential for Ilies and Judge's estimate of the effects of positive affectivity to be contaminated by correlated error, and thus be a—possibly substantial—overestimate, is likely.

What to Look for in LMM Research

As the previous example reveals, there are important issues reviewers and researchers planning a study should take into account when considering the quality of an LMM estimate presented by a research study.

Left out variables. As was outlined in the above example, variables that are known to share a strong relationship with both a predictor as well as the outcome in a study, but that are omitted in an LMM, are likely to produce parameter estimate bias. Left out variables error is not a common criticism of research studies in the organizational sciences (see Antonakis et al., 2012 for a similar view), yet occurs whenever a known predictor is excluded without a good conceptual reason. As is demonstrated in Equation 12, cross-level effects can be strongly biased by omitted predictors, therefore I urge reviewers evaluating LMM-based research to utilize their knowledge of a domain to ensure that important effects that are likely to produce correlated error are addressed either through a compelling theoretical argument that it should not be impactful, or methodologically through experimental design or statistical control or treatment. Left out variables error, left undetected or corrected, could be highly misleading changing the conclusions drawn by the study. In fact, given some known omitted variable, I recommend setting up an equation similar to Equation 13 from which the plausible effect of correlated error could be estimated, provided trustworthy data such as meta-analytic or other high quality parameter estimates are available to provide at least a likely direction for the bias.

In addition to known omitted variables, models involving a small number of predictors are likely to be too simple to accurately represent reality, especially for outcomes such as job performance, satisfaction, or intentions to quit which have many known predictors and extensive research literatures. In particular, very simple statistical models are likely to be affected by correlated error variance as it is almost undoubtedly the case some predictors of the outcome is omitted, and that at least some subset of the omitted predictors correlate with one or more of the included predictors. Whereas I recognize that any one model cannot possibly represent reality fully, reality still produced the data. As a consequence, without some experimental or statistical control or remediation, the parameter estimates from smaller predictive models could very well be so biased as to be completely uninformative. Hence, I urge reviewers evaluating LMM research in which there are relatively few predictors to request the authors make a strong case that the few predictors included are not subject to between-cluster correlated error—in essence, small predictive models should be subject to extra scrutiny.

***Common measurement sources*.** In situations where a between-cluster predictor and the outcome are measured from the same source, the likelihood of common method variance-related bias is higher than when measured from separate sources. In the present study, the mechanisms that produce common method variance are identical to those that produce left out variables error (i.e., they're the same variable/their correlation is 1). Although in reality left out variables error and common method variance may be produced through different mechanisms, if these mechanisms are at all correlated, many of the paths that link left out variables to common method variance will produce additional correlated error variance beyond that of left out variables error and common method variance alone.

Whereas many reviewers look for what they might call "very high" correlations as an indication of common method variance (see Pace, 2010), the current work as well as Siemsen et al. (2010) demonstrate that common method variance is more complicated than simply inflating relationships. Unfortunately, there is no simple way to discern whether a study has or has not been affected by common method variance-related bias and, consequently, it may be safest to assume that any same-source study *has* to some degree been affected. Hence, authors of studies utilizing same source, between-cluster predictors must make *strong arguments* as to how or why their measure is free of or, alternatively, that their measure is contaminated to a negligible extent by common method variance. Moreover, if the author does not provide a convincing rationale or methodology for why the estimate is not also free of left out variables error, the possibility of both sources of correlated error variance combining in LMM parameter estimates increases the likelihood of severe bias as is observed in the results outlined in Figure 3.

I mention above that small predictive models tend to have a higher likelihood of being biased due to left out variables error. Owing to the findings by Siemsen et al. (2010) as well as Lai et al. (2013) that when fewer common method variance contaminated variables are included in a predictive model that each variable contaminated is biased to a greater extent. Therefore, the advantages of building a larger predictive model not only for left out variables error reasons, but also for common method variance reasons, are compelling. In sum, small predictive LMMs are parsimonious, yet without adequate experimental or statistical control, both left out variables error as well as common method variance are could produce severely misleading estimates.

CHAPTER FOUR – DISCUSSION AND CONCLUSIONS

The purpose of the present work was to evaluate the effects of between-cluster correlated error variance in the estimation of cross-level effect parameters in LMMs. To do so, I evaluated the tracing rules implied by Figure 2 for the regression of a between-cluster predictor (i.e., $Climate_j$) on the within-cluster intercept of the $Mood_{ij} - Satisfaction_{ij}$ relationship. The current study shows that LMM parameter estimates are a complicated combination of the parameters outlined in Figure 2. My findings show that correlated error variance can produce substantial bias in terms of cross-level effects parameter estimates, which depend primarily on the magnitude of the common method variance contamination of the $Climate_j$ measure and the relationship between the latent $Climate_j$ and the omitted variable.

Additionally, I use an empirical example, published in a well-respected journal with an impressive citation count, is potentially contaminated with correlated error variance and could very well be a substantial overestimate of the effect of positive affectivity on job satisfaction. Deriving from the empirical example, I also demonstrate how to evaluate cross-level effect research in terms of examining the likely effects of left out variables error-related bias and provide several suggestions for reviewers of studies using LMMs to look for in terms of evaluating the trustworthiness of the cross-level effect estimates provided.

The findings from my analysis contribute to the literature in 4 ways. First, my findings unequivocally demonstrate the importance of preventing and controlling for correlated error variance in cross-level estimates using LMMs. Correlated error variance, when ignored, can produce substantial bias in parameter estimates (i.e., 30%; see Figure 3). Thus, the present work joins that of Lai et al. (2013) as well as Ostroff et al. (2002) in showing that reviewers and researchers can simply afford to ignore correlated error from left out variables or common method variance if the organizational sciences are to have a cumulative knowledge base.

Second, the present work provides reviewers several suggestions related to how to evaluate cross-level effect research using LMMs in order to gauge the likelihood of their having correlated error variance-related bias. Missing known predictors, small predictive models, and same source data are all issues that are likely to produce correlated error variance. Thus, a reviewer can and should use all 3 criteria to request an author provide evidence or a conceptual argument that their cross-level estimates are not (substantially) biased. Additionally, researchers estimating cross-level effects should use these points as important issues to address in their manuscript and methodology in order to demonstrate the validity of their estimates.

Third, the present study generalizes mathematical results obtained for the linear, additive effect results of Siemsen et al. (2010) to the case in which a the researcher is examining mixed effects—allowing the relationship between 2 variables to vary randomly within a clustering variable (i.e., the within-cluster intercepts for the $Mood_{ij} - Satisfaction_{ij}$ relationship within an organization) and estimating the extent to which a between-cluster predictor explains variation in the within-cluster intercepts. Therefore, given a plausible causal model, any researcher could use Equation 12 to evaluate the likely effect left out variables error or common method variance would have on estimates and, moreover, provides important information on the trustworthiness of estimates from LMMs suspected of having correlated error variance.

Finally, my findings extend other research on linear regression as well as LMMs by integrating two forms of correlated error variance, left out variables error-related effects, and common method variance-related effects, into a single set of findings to show how both combine in a LMM analysis to affect parameter estimation. Correlated error variance is a single problem and, consequently, the conclusions derived from LMM analysis should be evaluated considering both simultaneously. Specifically, my analysis reveals that under certain conditions in a LMM, correlated error variance effects both cross-level effects in a way similar to that of a traditional linear regression—by changing the magnitude of main effect estimates and, when common method variance is substantial, attenuating estimates toward 0 (see Figure 3).

Limitations and Future Directions

The present work suffers from several limitations that warrant mention. Firstly, the focus of the present study centers on the estimation of cross-level effects only without consideration of cross-level interactions. Although cross-level interactions differ conceptually from cross-level effects, how they are treated in estimation is actually rather similar and many of the conclusions reached regarding bias related to left out variables error especially, but many related to common method variance as well, apply to cross-level interaction estimation as well. Generally speaking, explaining variance in random effects is agnostic to the source of the random effect—that is, the estimator is fairly agnostic as to between-cluster variability being explained stems from slopes or intercepts (see Raudenbush & Bryk, 2002, chap. 3). Hence, given the same causal structure as applied to cross-level interaction estimation, similar conclusions will hold[ii].

Additionally, the present study is restricted to the examination of LMMs without examining the role of correlated error variance in parameter estimation in multilevel structural equation modeling (MSEM)—an increasingly influential analysis for behavioral and organizational science data (e.g., Rabe-Hesketh, Skrondal, & Pickles, 2004). As previously noted LMMs are more commonly used and, arguably, better understood analysis procedures in the organizational sciences, a growing number of studies are showing advantages that MSEM has over LMMs for the estimation of more complex statistical models such as mediation effects (Preacher, Zyphur, & Zhang, 2010; Zhang, Zyphur, & Preacher, 2009). Importantly however, MSEMs are a flexible analysis framework of which LMMs are a special case (Muthén, 2002; Rabe-Hesketh et al., 2004), thus I expect that the findings obtained in the present work should generalize to the MSEM broadly.

In addition, I make several assumptions about the nature of the measurement model. Whereas random measurement error is modeled and shows an attenuating effect on parameter estimation, as expected (i.e., random measurement error in Equation 12 is only in the denominators), some methodologists may disagree with the treatment of correlated measurement error or common method variance. In fact, some research shows that common method variance is, fundamentally a multiplicative effect on correlations between measures (Campbell & O'Connell, 1967). In particular, the multiplicative common method variance effect produces a higher correlation between measures of two constructs when the true relationship between two constructs is high than when the true relationship between the constructs is low—hence, a multiplicative "common method" effect. Although most studies assume linear common method variance effects, and empirical evidence suggests many studies are accurately characterized by linear effects, there is evidence for multiplicative effects in some studies (Bagozzi & Yi, 1990). Incorporation of the multiplicative common method variance effects in future research on correlated error variance could be quite informative and would certainly affect parameter estimation for cross-level (and single-level) interactions.

The analysis conducted in the present study did make other important assumptions about the effect of correlated error variance across levels. Specifically, I assume that correlated error variance is present only between- and not at within-cluster. I acknowledge that examining the role of correlated error variance at both levels is important and is a potential future direction for the present research, however the only instance in which correlated error variance would affect cross-level effects would be when between-cluster and within-cluster correlated error variance are correlated. Whereas such cross-level correlated error variance effects do have additional, complex biasing effects on LMMs (e.g., Ebbes et al., 2004), cross-level correlated error variance suggests that the variances of each of the *j* groups depends on the Level of the between-cluster correlated error variance factor and thus closed form solutions using OLS estimation will not produce best linear unbiased estimates of population parameters. Rather, in such instances iterative estimation methods such as iterated generalized least squares or maximum likelihood must be employed which are more appropriately addressed using simulation methods.

Conclusion

Interest in correlated error variance as a source of bias in parameter estimation is increasing in recent years as is evidenced by the coverage the topic has received both in major academic journals (i.e., volume 13[3] of *Organizational Research Methods*) as well as in academic conferences (i.e., *Society for Industrial and Organizational Psychology*, *Academy of Management*). Most scholars agree that correlated error variance is a substantive problem, stemming from omission of relevant causal factors or other methodological artifacts (e.g., Podsakoff et al., 2003). Whereas scholars can agree on the nature of the correlated error variance issue, there is relatively less agreement over the extent to which correlated error variance (and common method variance in particular) is an issue in research practice (Pace, 2010; Spector, 2006). In the present study I build on prior research evaluating the extent and magnitude of correlated error variance-related effects in LMM when the correlated error variance factor exists only at the cluster-level. Through my analysis I show that both LMM can produce biased estimates in the presence of correlated error variance. Moreover, my analysis provides insight as to the mathematical structure of the bias imparted by correlated error variance. Understanding such structure can provide insight to practicing researchers as to how to avoid correlated error variance related bias through experimental control or statistical remediation.

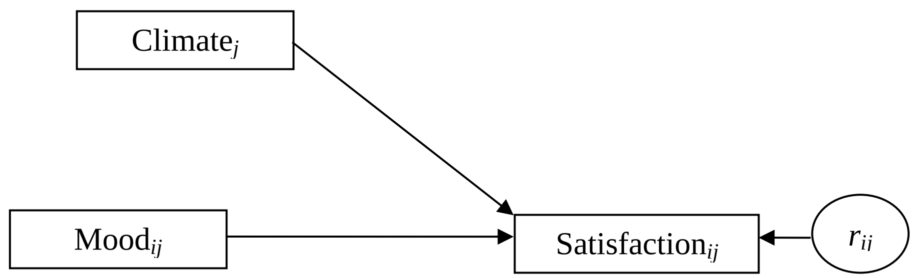

Figure 1. *Estimated Linear Mixed Model*

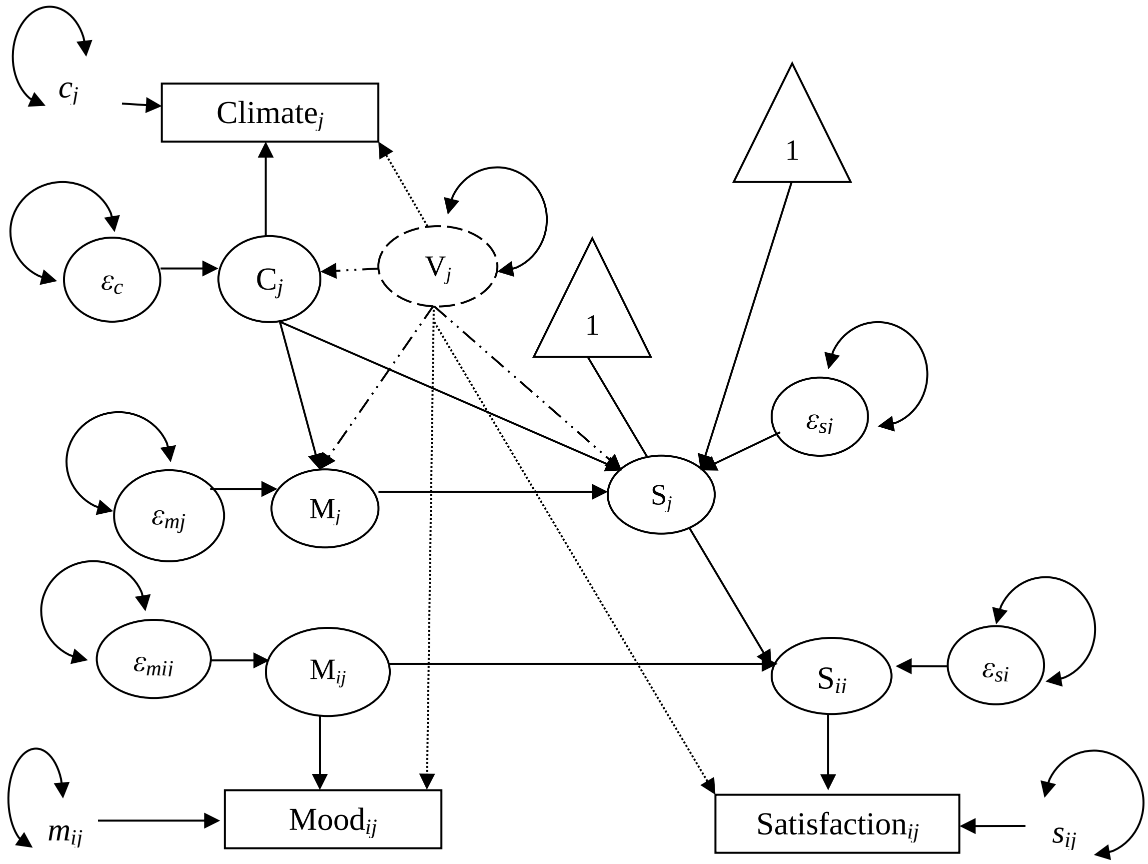

Figure 2. *Underlying Causal Model for Linear Mixed Model*

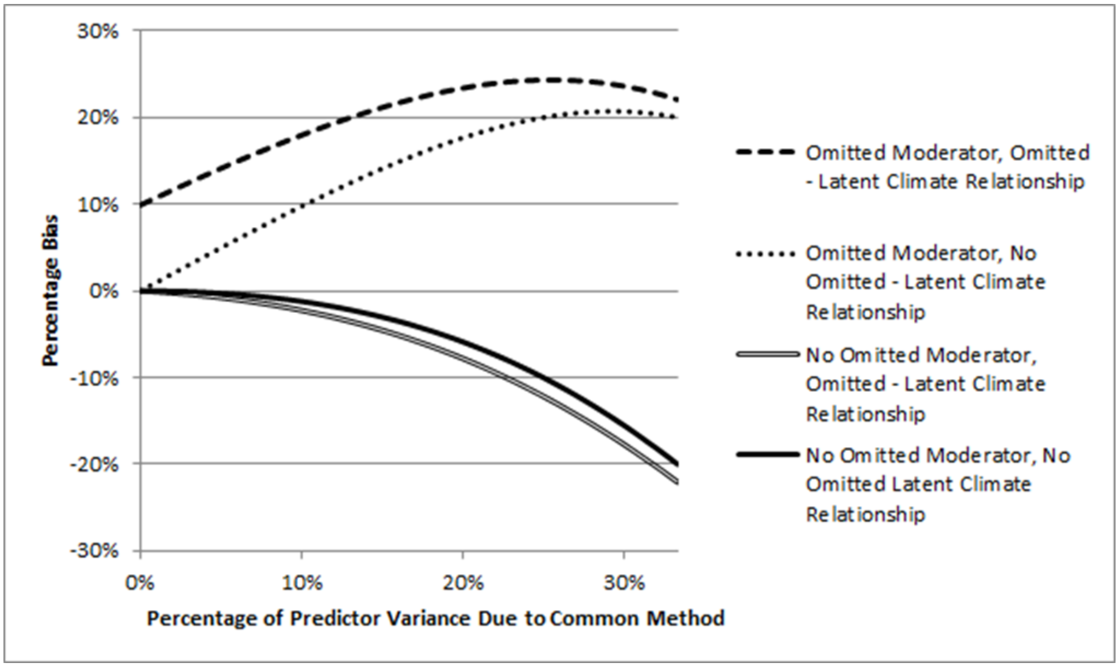

Figure 3. *Effect of Omitted Variables and Common Method on Cross-level Effect Estimate Bias*

Table 1. *Tracing Rules for Paths Comprising Cross-level Effect (γ_{10})*

	Pathways	Summand	Designation	Effect Type
1	$\delta_{\mathrm{cs}}\mathrm{var}[\varepsilon_{cj}] \rightarrow \beta_{0j}$	$\delta_{\mathrm{cs}}\mathrm{var}[\varepsilon_{cj}]$	Construct	Direct
2	$\delta_{\mathrm{cm}}\mathrm{var}[\varepsilon_{cj}] \rightarrow \delta_{\mathrm{ms}} \rightarrow \beta_{0j}$	$\delta_{\mathrm{ms}}\delta_{\mathrm{cm}}\mathrm{var}[\varepsilon_{cj}]$	Construct	Indirect
3	$\delta_{\mathrm{vc}}\mathrm{var}[\mathrm{V}_j] \rightarrow \delta_{\mathrm{vs}} \rightarrow \beta_{0j}$	$\delta_{\mathrm{vc}}\delta_{\mathrm{vs}}\mathrm{var}[\mathrm{V}_j]$	Omitted	Spurious-indirect
4	$\delta_{\mathrm{vc}}\mathrm{var}[\mathrm{V}_j] \rightarrow \delta_{\mathrm{vm}} \rightarrow \delta_{\mathrm{ms}} \rightarrow \beta_{0j}$	$\delta_{\mathrm{vc}}\delta_{\mathrm{ms}}\delta_{\mathrm{vm}}\mathrm{var}[\mathrm{V}_j]$	Omitted	Spurious-indirect
5	$\delta_{\mathrm{vc}}\mathrm{var}[\mathrm{V}_j] \rightarrow \delta_{\mathrm{vc}} \rightarrow \delta_{\mathrm{cs}} \rightarrow \beta_{0j}$	$\delta_{\mathrm{vc}}^2\delta_{\mathrm{cs}}\mathrm{var}[\mathrm{V}_j]$	Omitted	Spurious-indirect
6	$\delta_{\mathrm{vc}}\mathrm{var}[\mathrm{V}_j] \rightarrow \delta_{\mathrm{vc}} \rightarrow \delta_{\mathrm{cm}} \rightarrow \delta_{\mathrm{ms}} \rightarrow \beta_{0j}$	$\delta_{\mathrm{vc}}^2\delta_{\mathrm{cm}}\delta_{\mathrm{ms}}\mathrm{var}[\mathrm{V}_j]$	Omitted	Spurious-indirect
7	$\delta_{\mathrm{vc}}\mathrm{var}[\mathrm{V}_j] \rightarrow b_s \rightarrow \beta_{0j}$	$\delta_{\mathrm{vc}}b_s\mathrm{var}[\mathrm{V}_j]$	Omitted & Method	Spurious-indirect
8	$b_c\mathrm{var}[\mathrm{V}_j] \rightarrow \delta_{\mathrm{vs}} \rightarrow \beta_{0j}$	$b_c\delta_{\mathrm{vs}}\mathrm{var}[\mathrm{V}_j]$	Omitted & Method	Spurious-indirect
9	$b_c\mathrm{var}[\mathrm{V}_j] \rightarrow \delta_{\mathrm{vm}} \rightarrow \delta_{\mathrm{ms}} \rightarrow \beta_{0j}$	$b_c\delta_{\mathrm{ms}}\delta_{\mathrm{vm}}\mathrm{var}[\mathrm{V}_j]$	Omitted & Method	Spurious-indirect
10	$b_c\mathrm{var}[\mathrm{V}_j] \rightarrow \delta_{\mathrm{vc}} \rightarrow \delta_{\mathrm{cs}} \rightarrow \beta_{0j}$	$b_c\delta_{\mathrm{vc}}\delta_{\mathrm{cs}}\mathrm{var}[\mathrm{V}_j]$	Omitted & Method	Spurious-indirect
11	$b_c\mathrm{var}[\mathrm{V}_j] \rightarrow \delta_{\mathrm{vc}} \rightarrow \delta_{\mathrm{cm}} \rightarrow$	$b_c\delta_{\mathrm{vc}}\delta_{\mathrm{cm}}\delta_{\mathrm{ms}}\mathrm{var}[\mathrm{V}_j]$	Omitted &	Spurious-

	$\delta_{\mathrm{ms}} \rightarrow \beta_{0j}$		Method	indirect
12	$b_c\mathrm{var}[\mathrm{V}_j] \rightarrow b_s \rightarrow \beta_{0j}$	$b_c b_s\mathrm{var}[\mathrm{V}_j]$	Method	Spurious-indirect
13	$\updownarrow \mathrm{var}[\varepsilon_{cj}] \rightarrow Clim_j$	$\mathrm{var}[\varepsilon_{cj}]$	Construct	Direct
14	$\updownarrow \delta_{\mathrm{vc}}\mathrm{var}[\mathrm{V}_j] \rightarrow Clim_j$	$\delta_{\mathrm{vc}}^2\mathrm{var}[\mathrm{V}_j]$	Omitted	Spurious
15	$Clim_j \leftarrow \delta_{\mathrm{vc}} \leftarrow b_c\mathrm{var}[\mathrm{V}_j] \rightarrow Clim_j$ & $Clim_j \leftarrow \delta_{\mathrm{vc}}\mathrm{var}[\mathrm{V}_j] \rightarrow b_c \rightarrow Clim_j$	$2\delta_{\mathrm{vc}} b_c\mathrm{var}[\mathrm{V}_j]$	Omitted & Method	Spurious-indirect
16	$\updownarrow b_c\mathrm{var}[\mathrm{V}_j] \rightarrow Clim_j$	$b_c^2\mathrm{var}[\mathrm{V}_j]$	Method	Spurious
17	$\updownarrow \mathrm{var}[c_j] \rightarrow Clim_j$	$\mathrm{var}[c_j]$	Measurement Error	Direct

Note: $Clim_j$= *Climate$_j$*. $\updownarrow$ = Unanalyzed path applied to exogenous variances (i.e., "self"-causation path).

ENDNOTES

[i] All tracing rules results were confirmed using *asymptotic theory* (Cameron & Trivedi, 2005; Appendix A). Asymptotic theory results are available upon request from the author.
[ii] Derivations regarding cross-level interactions are available upon request from the author.

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