DYNAMIC RESOURCE ALLOCATION IN OFDM-BASED COGNITIVE RADIO NETWORKS

by

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Dedication

This dissertation is dedicated to my parents, Goltaj Sourani and Farhad Baharlouei, for their unconditional love and support.

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I would like to thank my advisor, Dr. Bijan Jabbari, who has provided me with all kind of supports, without which the completion of my studies would have been impossible. With his stimulating insight, he has guided me through my research and pushed me forward to improve every aspect of my work.

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Abstract

DYNAMIC RESOURCE ALLOCATION IN OFDM-BASED COGNITIVE RADIO NETWORKS

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This dissertation addresses two important issues in cooperative cognitive radio networks: spectrum sensing in physical layer and dynamic resource allocation in MAC layer. The goal of spectrum sensing is to find the vacant spectrum bands opportunistically and have them accumulated in a spectrum pool. In the MAC layer the detected bands are allocated to the Secondary Users (SUs) dynamically to take the best advantage of the temporarily granted resources.

We first assume a general model of a multiuser OFDM cellular network with non-cognitive users. Then, a dynamic resource allocation scheme is proposed based on Nash Bargaining Solution (NBS) for the downlink of the network. Although NBS provides a fair and optimum approach in order to maximize the total rate of the network, there is no simple solution for the case of K users. Since NBS relates to maximizing the multiplication of the users utilities rather than the summation it makes the optimization problem harder to solve. Hence, by decomposing the NBS problem into two sub-problems, the power allocation reduces to the well-known water-filling algorithm and the subchannel assignment leads to a simple algorithm which takes the total channel gain of each user as the fairness factor. The proposed algorithm satisfies the minimum rate requirement of each user first

and allocates the excess subchannels by searching over the $K \times N$ subchannels to noise ratio matrix based on the bargaining process happening between the base station and the users. Hence, the complexity reduces from order $O(K^N)$ to $O(N \times K)$. Simulation results show that the proposed algorithm keeps a balance between the Max-Min approach and the Max-Sum where Max-Min aims at maximizing the worst user rate and Max-Sum maximizes the sum of the rates by blocking the users in the poor channel conditions. We demonstrate the application of this approach to LTE-Advanced systems as well.

In the ensuing work, we extend our system model to a cognitive radio network where secondary users need to monitor the spectrum of the primary for possible idle bands. As the sensing is the first step for the secondary operation we investigate the available centralized and distributed spectrum sensing schemes. We propose a collaborative spectrum sensing method based on Stackelberg game in order to improve the sensing performance of cognitive radio users, especially when they are operating under severe channel fading. In this scheme the users with acceptable received SNR are allowed to lead the network sensing process and share their observations with the ones experiencing weak channel conditions. By defining a new and more appropriate metric to evaluate the performance of the collaborative detection, the proposed method addresses the drawbacks of the conventional centralized or distributed spectrum sensing. Moreover, it does not require exchange of channel information among nodes and only minimum reporting of local observations is needed. The simulation results indicate that the proposed scheme improves the network sensing performance and reduces the overhead as compared to the non-cooperative case and the conventional collaborative schemes, respectively.

In order to allocate resources in a cognitive OFDM based network the availability of subchannels are needed to be considered. Hence, the imperfect sensing information will reflect in resource allocation procedure. We extend our proposed dynamic subchannel and power allocation scheme based on Nash Bargaining Game for an ad hoc network of Secondary Users (SU) coexisting opportunistically with a Primary base station. Each SU is equipped with an energy detector to sense the Primary User (PU) activity over N OFDM sub-channels. We deploy Nash Bargaining Solution to model the power and subchannel allocation of the SUs over the temporarily available transmission opportunities. Simulation results show that the proposed dynamic power and subchannel assignment is simple, effective and fair. Moreover, the total throughput of the network is highly dependent on the accuracy level of the sensing information and the percentage of PU silence. An acceptable secondary network throughput is achievable if the probability of PU presence is limited to less than 0.4.

Chapter 1: Introduction

The next generation wireless communication systems should be able to serve a large number of users with flexibility in their quality of service (QoS) requirements. The challenges to maintain such requirements originates from the limited availability of radio spectrum, the total transmit power and the nature of wireless channel. In broadband applications, the wireless channel suffers from frequency selective-multipath fading which results in severe inter-symbol interference (ISI) both in time and frequency which affects the service quality and data rates detrimentally. To overcome this issue, intelligent radio resource management algorithms operating in both the physical and the media access control (MAC) layers are needed to be designed.

Orthogonal Frequency Division Multiplexing (OFDM) is a multi carrier modulation which is developed into a popular scheme for wireless standards including 4G mobile communications, Wireless Local Area Networks (WLANS), and Wireless Metropolitan Area Networks (WMAN) which is later extended in IEEE802.16e (WiMAX). OFDM divides the available bandwidth into N orthogonal subchannels each with a bandwidth much smaller than the coherence bandwidth of the channel. The lower bandwidth of the subchannels converts the frequency selective fading into flat fading and the orthogonality of the subchannels makes multiple access possible.

In a single user system, the user can use the total power to transmit on all N subcarriers. However, in a multiuser OFDM system, there is a need for a multiple access scheme to allocate the subcarriers and the power to the users. Two classes of resource allocation schemes exist:

1. Fixed resource allocation

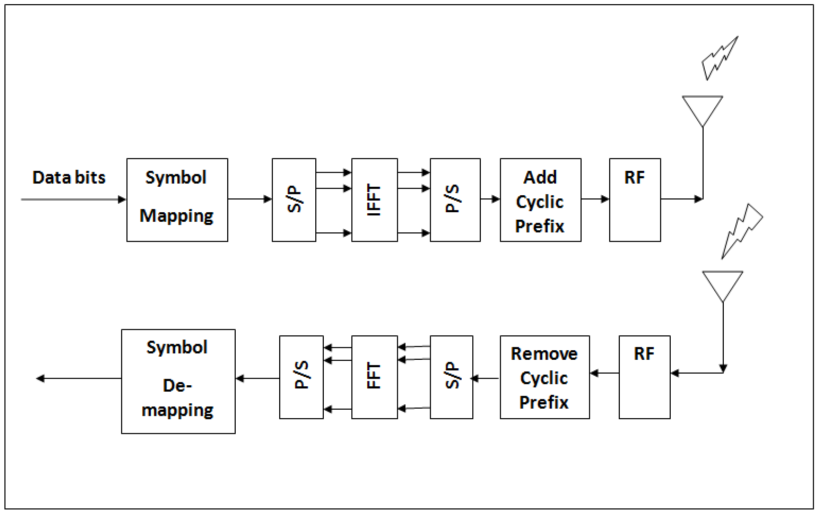

Figure 1.1: OFDM block diagram

2. Dynamic resource allocation

Fixed resource allocation schemes, such as time division multiple access (TDMA) and frequency division multiple access (FDMA), assign an independent dimension, e.g., time slot or subchannel, to each user. A fixed resource allocation scheme is not optimal, since the scheme is fixed regardless of the current channel condition. On the other hand, dynamic resource allocation allocates a dimension adaptively to the users based on their channel gains. Due to the time-varying nature of the wireless channel, dynamic resource allocation makes full use of multiuser diversity to achieve higher performance.

Two classes of optimization techniques have been proposed in the dynamic multiuser OFDM literature [1]:

- Margin adaptive (MA)
- Rate adaptive (RA)

The MA objective is to achieve the minimum overall transmit power given the constraints on the users data rates or bit error rates (BER) [2]. The RA objective is to maximize each users error-free capacity with a total transmit power constraint [3], [4]. These optimization problems are nonlinear and, hence, computationally intensive to solve. In [5], the nonlinear optimization problems were transformed into a linear optimization problem with integer variables. The optimal solution can be achieved by integer programming. However, even with integer programming, the complexity increases exponentially with the number of constraints and variables.

On the other hand, the growing demand for high data rate communications and wider bandwidth, spectrum scarcity is becoming an inevitable issue which reveals the inefficiency of the current static spectrum access techniques. Cognitive radio technology as a potential platform to implement the Dynamic Spectrum Access (DSA) has been captured the interest of researchers in recent years [6]. The basic idea of cognitive radio is to allow some unlicensed (Secondary) Users to utilize the spectrum of the licensed (Primary) users when it is idle. Figure (1.2) shows a cognitive radio network [7]. Dynamic spectrum access techniques allow the cognitive radio to operate in the best available channel. The main functions of a cognitive radio technology are ([7]):

- Spectrum sensing: determine which portions of the spectrum is available and detect the presence of licensed users when a user operates in a licensed band
- Spectrum management: select the best available channel
- Spectrum sharing: coordinate access to this channel with other users
- Spectrum mobility: vacate the channel when a licensed user is detected

As we can see spectrum sensing is the first and critical step for a cognitive radio device to operate. A secondary user senses the spectrum of the Primary periodically, and uses it when it is idle, and switch to other unused bands when primary system is present.

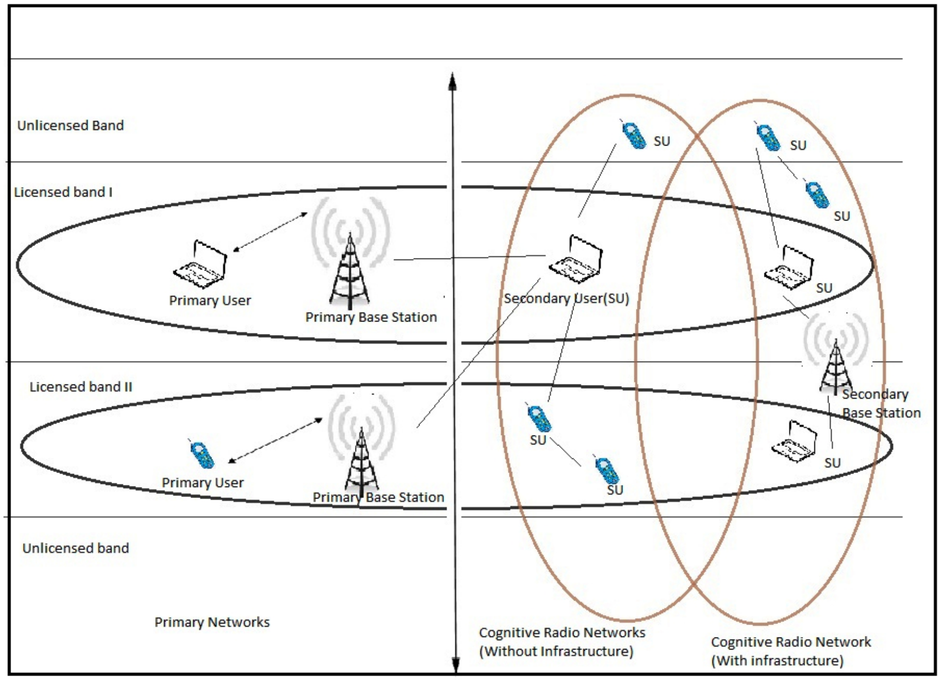

Figure 1.2: A cognitive radio network

1.1 Statement of the Problem

In this dissertation we consider a cellular network consists of a base station operating in downlink with N OFDM subchannels and K users. The bandwidth of each subchannel is w. The problem of resource allocation in this system is how to determine the elements of matrix $\mathbf{C} = [c_{i,n}]_{K\times N}$ specifying which subchannels should be assigned to which user and vector $\mathbf{P} = [p_{i,n}]_{K\times N}$ indicating how much power should be allocated to each subchannels. An overview of the problem is depicted in 1.3.

To solve the above resource problem of resource allocation the first question is how to define the objective function. There are three definition for objective function in the available literature as follows:

1. Maximize sum of the rate of each (Max-Sum)

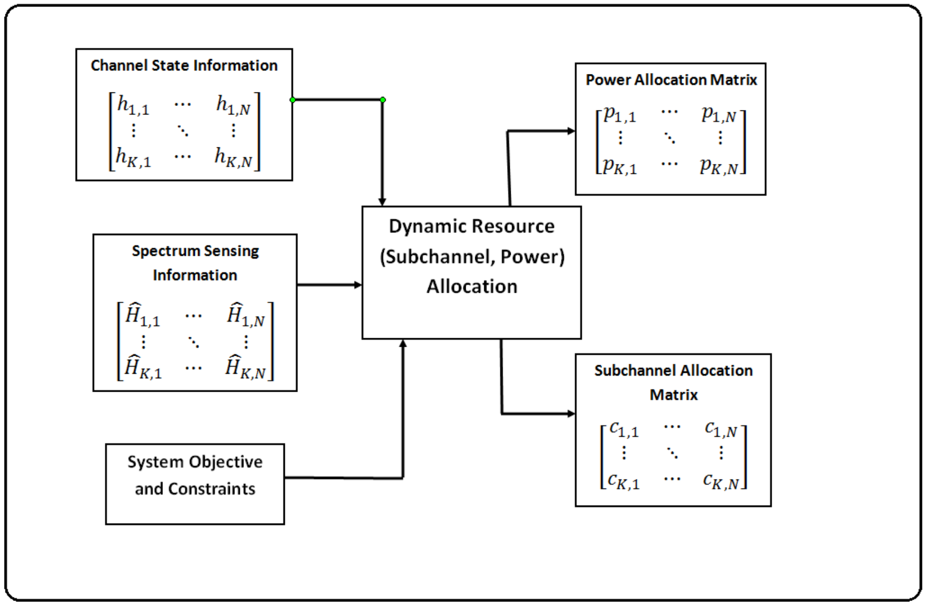

Figure 1.3: Overview of resource allocation problem in an OFDM system

2. Maximize the worst user rate (Max-Min)

3. Maximize product of the rates (Nash Bargaining Solution)

We borrow Nash Bargaining Solution from game theory. NBS is a cooperative game that provides optimality and fairness at the same time. The resulting optimization problem is an NP-hard problem and finding a closed form solution is not possible. Therefore, we decompose the problem into two subproblems of subchannel allocation and power allocation and propose a simple algorithm for resource allocation.

We further extend our system model to the case when an ad-hoc network of Secondary Users (SU) are coexisting opportunistically with the primary base station. Secondary users sense the spectrum of the primary periodically for the idle spectrum holes. In this case, the sensing parameters from physical layer such as the percentage of the PU activity and how accurately the sensing process is done are determining factors which indicates that a cross layer approach is needed to solve the resource allocation problem in MAC layer.

1.2 Related Work

1.2.1 Dynamic Resource Allocation in OFDM based Systems

The problem of dynamic spectrum allocation in OFDMA wireless networks has been widely studied in the literatures [1, 4, 8, 9]. In [4] the dynamic channel assignment is modeled as a convex optimization problem to maximize the minimum of the sum of all users rate with respect to some constraints and a simple suboptimal algorithm is proposed. However, maximizing the worst user (Max-Min) rate is at the cost of sacrificing the users with better channel conditions which leads to a lower total rate. Another adaptive resource allocation scheme is proposed in [1] with taking proportional fairness into account by adding a constraint on rate requirement of each user. Nevertheless, Proportional fairness is a special case of NBS when the disagreement point is zero. Generally allocating subchannels in order to maximize the sum throughput of the network is a linear optimization problem with integer variables which could be solved using integer programming but it is computationally complex. Hence, either suboptimum algorithms are proposed or the integer programming problem relaxed to a standard convex optimization.

By widespread use of wireless devices the available dynamic resource allocation schemes need to be even more flexible to be able to serve the unlicensed users while the primary users are in idle mode. This dissertation addresses this issue by using Nash Bargaining Solution (NBS) which provides optimality and fairness at the same time. NBS is a cooperative game which optimizes the multiplication of the objective function rather than the aggregate sum which results fair and optimal pay-off per user. In [10] a power control scheme is proposed based on NBS and KSBS for cognitive radio networks. In [11] also a resource allocation framework is presented for an ad hoc network of secondary users using two game theoretic methods. However, in modeling the system there is no constrain for subchannel sharing among users and each subchannel can be allocated to several users which is not practical for OFDM networks and lowers the total throughput [12]. Moreover, solving and formulating the Nash Bargaining game as a second approach is not complete. Another interesting approach for dynamic resource allocation using NBS approach is proposed in [9]. The problem is solved for two-user case in an iterative manner, but for the multiuser case the authors involve another game tool, called coalition games, which adds more iteration to the two-user case convergence. Furthermore, the convergence of the algorithm is not guaranteed.

The available literature applied NBS for only two user cases and the resulting optimization problem is not solved generally. This dissertation proposes an optimum and fair dynamic spectrum allocation schemes for the case of K users.

1.2.2 Cognitive Radio and Spectrum Sensing

There are several detectors to sense the spectrum of a primary user such as, matched filter, energy detector, and feature detection [13]. Feature detection is able to distinguish between the received signal energy and the noise energy but it requires long observation intervals of the received signal which leads to high computational complexity. Matched filter detector is the optimal option for the case of stationary Gaussian noise and when the primary signal is known. However, when the primary signal is unknown or complexity is an issue, matched filter and feature detection are ruled out, and energy detector appears to be the feasible choice.

1.2.3 Cross Layer Spectrum Sensing and Allocation

In [14] the dynamic resource allocation in cognitive radio networks with Imperfect Channel Sensing is investigated and is solved using a discrete stochastic optimization method. However, the imperfectness is assumed for the channel gain information. Basically, the goal of spectrum sensing is to monitor the activity of the PU which is the case in this dissertation. An interesting joint cross-layer scheduling and spectrum sensing for cognitive radio networks is proposed in [15] based on what is called 'Raw Sensing Information' and the power and subchannel allocation is solved using primal-dual decomposition approach. Their underlay spectrum sharing method with assuming some acceptable interference level differs from our work where we adopt overlay method which does not allocate an OFDM sub-channel to more than one user simultaneously. Moreover, our problem solving method is based on NBS which gives a fair and simple allocation scheme.

Following to our collaborative spectrum sensing work in [16], here, we use the sensing information of the energy detector of each SU to design an optimum power and subchannel allocation for SUs. First, we show that how the sensing bits of the PHY layer can affect the allocation process in MAC layer. Then, we propose a sub-optimum resource allocation algorithm in order to increase the network total throughput while maintaining fairness among users.

1.3 Outline and Contribution of Dissertation

In short, the contribution of this dissertation is as follows:

- We present an efficient and fair scheme for allocating resources in the downlink of multiuser OFDMA cellular networks based on Nash Bargaining. By decomposing the NBS problem into two sub-problems, the power allocation reduces to the well-known water-filling algorithm and the subchannel assignment leads to a simple algorithm which takes the total channel gain of each user as the fairness factor. The results are presented in [17] and [18] as well.

- A distributed spectrum sensing scheme is proposed based on Stackelberg game [16].

- A cross layer dynamic resource allocation scheme is presented for OFDM-based cognitive radio networks based on Nash bargaining game with the imperfect sensing information [19], [20] and [21].

The remaining of this dissertation is organized as follows. In Chapter 2 we discuss dynamic resource allocation for OFDM based networks and a dynamic power and subchannel allocation scheme is proposed. Chapter 3 addresses spectrum sensing in cognitive radio networks and a distributed spectrum sensing algorithm is proposed. In Chapter 4 we extend the proposed dynamic resource allocation for cognitive radio networks. In the end, Chapter

5 summarizes the contributions of this dissertation and some remarks are made for possible future research.

Chapter 2: Dynamic Resource Allocation

2.1 Introduction

With the growing demand for wireless services of higher Quality of Service (QOS) (data rate, latency, coverage, etc), the need for more efficient schemes to provide better utilization of the limited available resources such as spectrum becomes inevitable. The wireless service providers need to support large number of users with flexibility in their QOS requirements.

In an Orthogonal Frequency Devision Multiplexing (OFDM) Access the total bandwidth is divided into some orthogonal subchannels which converts frequency selective multipath fading into flat fading to combat ISI. In a static channel allocation scheme the predetermined subchannels are assigned to different users regardless of their channel gain, while in an adaptive scheme each subchannel is allocated to the user with the best channel gain on that subchannel which improves the network capacity significantly [4].

The problem of dynamic spectrum allocation in OFDMA wireless networks has been widely studied in the literatures [1, 4, 8, 9]. In [4] the dynamic channel assignment is modeled as a convex optimization problem to maximize the minimum of the sum of all users rate with respect to some constraint and a simple suboptimal algorithm is proposed. However, maximizing the worst user (Max-Min) rate is at the cost of sacrificing the users with better channel conditions which leads to a lower total rate. Another adaptive resource allocation scheme is proposed in [1] with taking proportional fairness into account by adding a constraint on rate requirement of each user. Nevertheless, Proportional fairness is a special case of NBS when the disagreement point is zero. Generally allocating subchannels in order to maximize the sum throughput of the network is a linear optimization problem with integer variables which could be solved using integer programming which is computationally complex. Hence, either sub-optimum algorithms are proposed or the integer programming problem relaxed to a standard convex optimization. A good survey of dynamic resource allocation schemes for OFDM networks is presented in [22].

This chapter addresses the issue of adaptive resource allocation by applying NBS which provides optimality while keeping fairness. NBS is a cooperative game where optimizes the multiplication of the objective functions rather than the aggregate sum which results in higher pay-off per user. In [10] a power control scheme is proposed based on NBS for cognitive radio networks. [11] also proposes a resource allocation framework for an ad hoc network of cognitive radio users using two game theoretic methods. However, in modeling the system there is no constrain for subchannel sharing among users and each subchannel can be allocated to several users which is not practical for OFDM networks and lowers the total throughput [12]. Moreover, NBS is used as a second approach to verify the performance of the Cognitive Radio Game for the case of two user. Another interesting approach for dynamic resource allocation using NBS approach is proposed in [9]. The problem is solved for two-user case in an iterative manner, but for the multiuser case the authors involve another game tool called coalition games which adds more iterations to the two-user case convergence. Furthermore, the convergence of the algorithm is not guaranteed.

The available literature applied NBS for only two user cases and the resulting optimization problem is not solved generally. This dissertation proposes an optimum and fair dynamic spectrum allocation schemes for the case of K users. First, we solve the problem for the case of K user. Then, we reduce the nonlinear logarithmic objective function to a linear Knapsack approach which is easier to handle. A direct benefit of applying NBS is that the fairness is guaranteed and the total achievable rate is higher than that of Max-Min approach.

The remaining of this chapter is organized as follows. Section II describes the system model. In section III the proposed method is elaborated. Simulation results are given in section IV and V concludes the chapter.

2.2 Wireless Channel Characteristics

In a wireless channel the transmitted radio waves arrive at the receiver after refection, diffraction and scattering from the natural and man-made objects. The incoming radio waves arriving from different paths have different propagation delays, amplitudes, and angles of arrival, causing the superposed received signal to distort or fade. As a result, the wireless channel is assumed to be wideband time-varying frequency-selective multipath fading. Figure (2.1) depicts the effect of a frequency selective channel on subcarriers of an OFDM signal. As seen some of the subcarriers are about to be completely nullified by the frequency selective fading. An overview on information-theoretic and communications features of fading channels has been studied in [23].

The coherence bandwidth measures the separation in frequency after which two signals will experience uncorrelated fading [24]. In flat fading, the coherence bandwidth of the channel is larger than the bandwidth of the signal. Therefore, all frequency components of the signal will experience the same magnitude of fading. In frequency-selective fading, the coherence bandwidth of the channel is smaller than the bandwidth of the signal. Different frequency components of the signal therefore experience uncorrelated fading. By choosing the bandwidth of the subchannels much smaller than the coherence bandwidth of the channel, each subchannel can be assumed to undergo flat fading. Figure (2.2) compares single wideband transmission versus multi-carrier signaling over a frequency selective fading channel.

One of the widely used models to explain the statistical nature of flat fading channels is Clarks model [24]. According to this model, the fading parameter of the channel is considered to be a random variable with Rayleigh distribution. In modeling the channel, it is also assumed that additive white Gaussian noise (AWGN) is present for all subcarriers of all users.

The advantages of adaptive resource allocation in multiuser OFDM systems are partially due to multiuser diversity which is based on assigning each subchannel to the user with good channel gain on it [22]. To this purpose, it is assumed that users perfectly estimate and

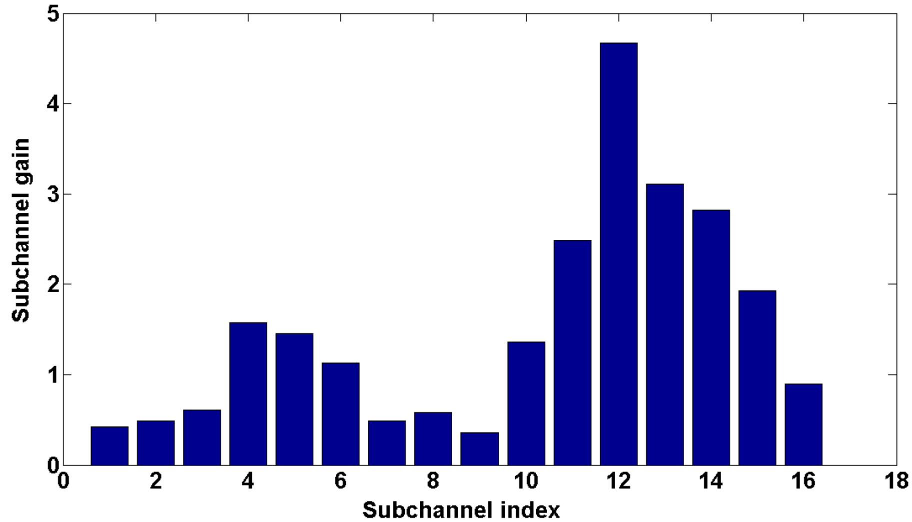

Figure 2.1: The effect of a frequency selective channel on subcarriers of an OFDM signal

feedback their channel information to the base station and the channel condition is always available to the base station in the beginning of each transmission frame. Also, it is assumed that the fading rate of the channel is slow enough such that the time-varying channel can be considered quasi-static where the channel condition does not change within each OFDM transmission frame.

In this dissertation we assume that channel information is perfect at both the transmitter and the receiver. While this is a reasonable assumption in wireline systems where the channel remains invariant, in wireless transmission, it is rarely possible for the transmitter to acquire perfect channel state information (CSI). This inaccuracy is due to channel estimation errors and channel feedback delay also referred to as channel mismatch errors. The latter is due to the variations of the wireless channel once it has been estimated.

Channel estimation methods for OFDM based systems have been studied [25], [26]. The effect of imperfect CSI on rate maximization of a single-user OFDM wireless system has been well studied in [27], [28]. Adaptive resource allocation in a multiuser system with imperfect or partial CSI was addressed in [29], [30].

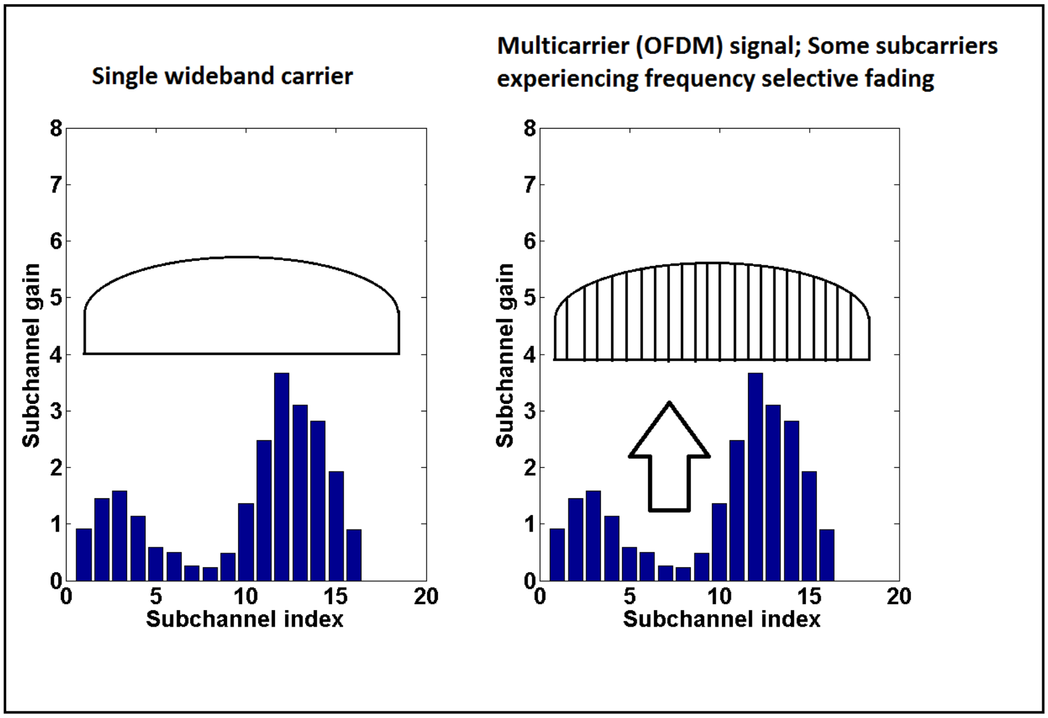

Figure 2.2: Transmission of a wideband single carrier signal vs multi-carrier (OFDM) over a frequency selective

2.3 System Model

Consider a single-cell multiuser OFDM network with K users and N OFDM sub-channels, each with bandwidth w. The rate for i-th user is expressed as:

$$R_i = \sum_{n=1}^{N} w b_{i,n} \tag{2.1}$$

where $b_{i,n}$ is the number of bits per symbol for the i-th user in subchannel n. Assuming $b_{i,n} \geq 2$ and $\mathrm{BER}_i \leq 10^{-3}$, the following approximation will hold [31]:

$$b_{i,n} \approx \log_2(1 + c_2 \gamma_{i,n} \ln(\frac{\mathrm{BER}_i}{c_1})) \tag{2.2}$$

where $c_1 = 0.2$, $c_2 = 1.5$. BER_i is the i-th user bit error rate, and $\gamma_{i,n}$ is the SNR in subchannel n. Hence, the rate for user i can be formulated as:

$$R_i = \sum_{n=1}^{N} c_{i,n} w \log_2(1 + c_3 \frac{p_{i,n} h_{i,n}^2}{\sigma_i^2}) \quad i = 1, ..., K; n = 1, ..., N. \tag{2.3}$$

where $c_3 = c_2 / \ln(c_1/\text{BER}_i)$, σ_i^2 is the noise power, and $h_{i,n}$ and $p_{i,n}$ are the channel gain and transmitted power of the i-th user on subchannel n respectively. Moreover, the subchannel assignment coefficient $c_{i,n}$ is given as:

$$c_{i,n} = \begin{cases} 1, & \text{subchannel } n \text{ is allocated to user } i \\ 0, & \text{Otherwise.} \end{cases} \tag{2.4}$$

A resource allocation problem is how to allocate the N subchannels and subsequently the transmitted power among K users so that the maximum throughput is achieved. To define the total throughput we take the product of the rates from Nash Bargaining game. We assume that channel state information of all users are known.

2.4 Resource Allocation for a single user OFDM

In this part we assume that there is only one user and of course all N subchannels are allocated to this user. The goal is to find out how much power the user should put in each subchannel so that the rate is maximized. The rate for the case of single user could be re-written as:

$$R = \sum_{n=1}^{N} w \log_2(1 + c_3 p_n G_n) \tag{2.5}$$

where $G_n = h_n^2/\sigma^2$, and p_n is the power amount in subchannel n. The optimization would be over power vector $(p_1, p_2, ..., p_N)$ as follows:

$$\arg \max_{p_1, p_2, ..., p_N} \sum_{n=1}^{N} w \log_2(1 + c_3 \frac{p_n h_n^2}{\sigma^2}) \tag{2.6}$$

subject to:

$$\sum_{n=1}^{N} p_n \leq P_{max} \tag{2.7}$$

Applying the method of Lagrange multiplier we obtain:

$$p_n + \frac{1}{G_n} = \frac{w}{\lambda} \tag{2.8}$$

where λ is the Lagrange multiplier, and $\frac{1}{G_n}$ is called noise to carrier ratio. looking at equation (2.8) we observe that the power plus noise to carrier (subchannel) ratio is a constant value. This is the well-known water filling algorithm [32] and has a unique optimal solution. $1/\lambda$ is called the water level and is obtained by solving the above optimization problem:

$$\frac{1}{\lambda} = \frac{P_{max} + \sum_{n=1}^{N} \frac{1}{G_n}}{Nw} \tag{2.9}$$

Substituting (2.9) in (2.8), the power per subchannel becomes:

$$p_n = \left[\frac{P_{max} + \sum_{n=1}^{N} \frac{1}{G_n}}{N} - \frac{1}{G_n} \right]^+ \tag{2.10}$$

where $x^+ = max(x, 0)$.

Let's illustrate water filling algorithm by an example. Assume that we are supposed

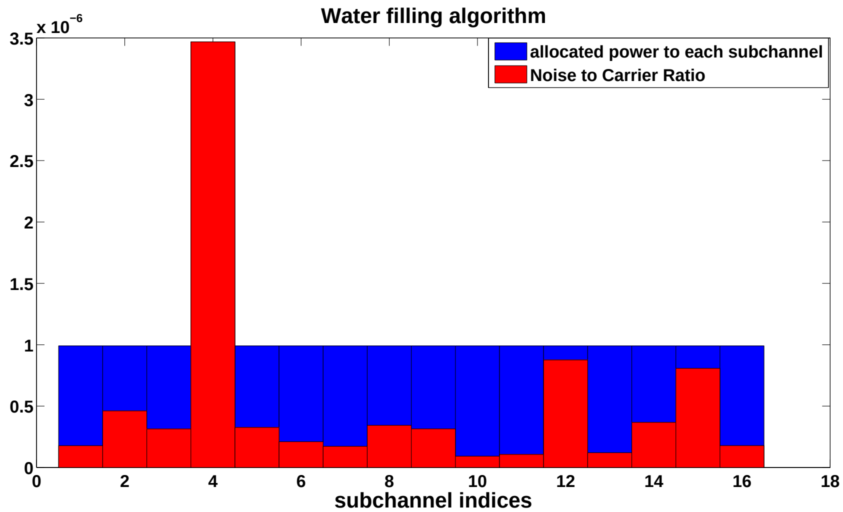

Figure 2.3: Power water filling for the case of single user, $N = 16$ subchannels, and $P_{max} = 0.01$ mW

to allocate a power amount of $P_{max} = 0.01$ mW among $N = 16$ OFDM subchannels. Subcchannel gains are assumed to be i.i.d Rayleigh random variables, and noise density is -80 dBm, and $w = 1$ MHz.

Figure (2.3) shows how water fills in each subchannel. The poured power into each subchannel is depicted by blue color and noise to carrier ratio is in red. We see that the lower the noise to carrier ratio the more power is assigned to that subchannel. Moreover, the power is allocated in a way that we have an even level of water for all subchannels. However, no power is poured into subchannel 4 since its noise to carrier ratio is very high.

The simulation is repeated for the same parameter except for a higher value of $P_{max} = 0.1$ mW. It is seen that the power level is increased compare to the case of $P_{max} = 0.01$ mW. Besides, power has filled all the subchannels.

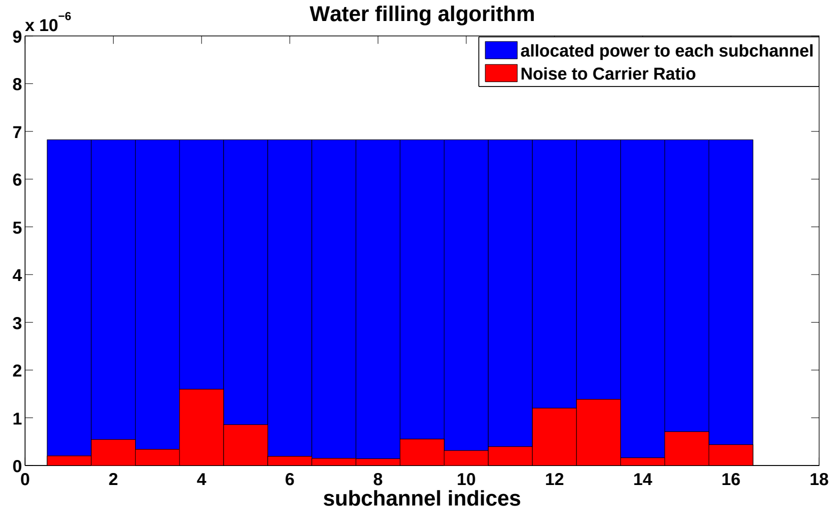

Figure 2.4: Power water filling for the case of single user, $N = 16$ subchannels, and $P_{max} =$ 0.1 mW

2.5 Resource Allocation for Multi-user OFDM

We perceived that water filling is the optimum way to allocate power for a single-user OFDM system. However, when multiple users are sharing OFDM subchannels, for example a base station that is transmitting to couple of mobile users in downlink, the optimization problem will not be simple as in (2.6). Other than power per subchannel per user, subchannel coefficients need to be determined. The first question is how we define the objective function which is the subject of the next subsection.

2.5.1 Objective Function Definitions

There are three ways in literature to define the objective function for the multi-user OFDM case as follows:

1. Maximize sum of the rate (Max-Sum)

   - $\max \sum_{i=1}^{K} R_i$

2. Maximize the worst user rate (Max-min)

- $\max \quad \min R_i$

3. Maximize product of rates (Nash Bargaining Solution)

   - $\max \prod_{i=1}^{K} R_i$

2.5.2 Max-Min Approach

This approach aims at finding the worst users ($\min_i$) and maximizing their capacity [4]:

$$\max_{\{\mathbf{P},\mathbf{C}\}} \min_i \quad R_i \tag{2.11}$$

where $\mathbf{P} = [p_{i,n}]_{K\times N}$ is power allocation matrix and $\mathbf{C} = [c_{i,n}]_{K\times N}$ is subchannel allocation matrix. Figure (2.5) illustrates how Max-Min approach allocates $N = 8$ subchannels among $K = 3$ users. From the top plot we know that User 2 (green) is the worst user with the lowest channel gain. As it is seen, Max-Min allocates three subchannels to this user. The main advantage of this scheme is its fairness to the users in poor channel conditions. However, the method penalizes users with better channels and reduces the total rate (lower efficiency).

The details of the Max-Min algorithm is given in table 2.1.

2.5.3 Max-Sum Approach

This approach finds the subchannels with maximum gain and allocates them [33] [1]:

$$\max_{\{\mathbf{P},\mathbf{C}\}} \sum_{i=1}^{K} R_i \tag{2.12}$$

where $\mathbf{P} = [p_{i,n}]_{K\times N}$ is power allocation matrix and $\mathbf{C} = [c_{i,n}]_{K\times N}$ is subchannel

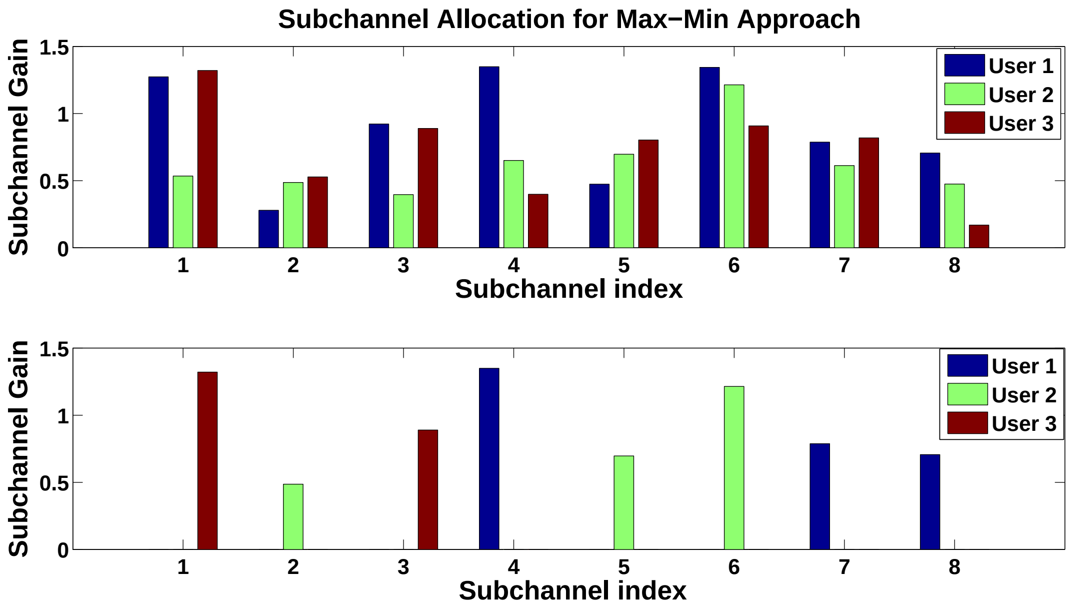

Figure 2.5: Allocation of $N = 8$ subchannels among $K = 3$ users

allocation matrix. An example of how Max-Sum works is depicted in Figure (2.6). It is seen that Max-Sum has done a good job in handpicking the best subchannels. However, User 2 (green) is the worst user and is totally ignored which increases user 2 blocking probability. Therefore, although Max-Sum maximizes the total network rate (Efficiency) it does not apply any fairness to the users far from base station or with lower power capability.

The details of the Max-Min algorithm is given in table 2.2.

In order to cover the drawbacks of Max-Sum and Max-Min we leverage Nash bargaining game for the power and subchannel allocation purpose since it provides efficiency and fairness at the same time.

Table 2.1: Subchannel allocation in Max-Min algorithm

1. Initialization: Set $R_i = 0$ for all $i = 1, 2, ..., K,\ A = \{1, 2, ..., N\}$.
2. For i = 1 to K a) Find n satisfying $\|G_{i,n}\| \geq G_{i,j}$ for all $j \in A$. b) Update R_i and A with n from a): $R_i = C(G_{i,n}), A = A - n$.
3. While $A \neq \emptyset$ a) Find i satisfying $R_i \leq R_k$ for all $k, 0 \leq k \leq K$. b) For the found k, find n satisfying $\|G_{i,n}\| \geq G_{i,j}$ for all $j \in A$. c) Update R_i and A with the i and n: $R_i = R_i + C(G_{i,n}), A = A - n$.

Table 2.2: Subchannel allocation in Max-Sum algorithm

1. Initialization: Set $R_i = 0$ for all $i = 1, 2, ..., K,\ A = \{1, 2, ..., N\}$.
2. While $A \neq \emptyset$ a) Find n satisfying $\|G_{i,n}\| \geq G_{i,j}$ for all $j \in A$. b) Update R_i and A with n from a): $R_i = R_i + C(G_{i,n}), A = A - n$.

2.6 Nash Bargaining Game

Nash Bargaining game [34] is a class of cooperative games where there is a mutual agreement among users for cooperation in order to achieve a higher utility comparing to the non-cooperative case. Let's define $\mathbf{u} = (u_1, u_2, ..., u_K)$ as the utility vector. The minimum attainable utility for the users without cooperation is called the disagreement point, and is expressed as $\mathbf{u^0} = (u_1^0, u_2^0, ..., u_K^0)$, and $U \subset \mathbb{R}^K$ is the feasible utility set.

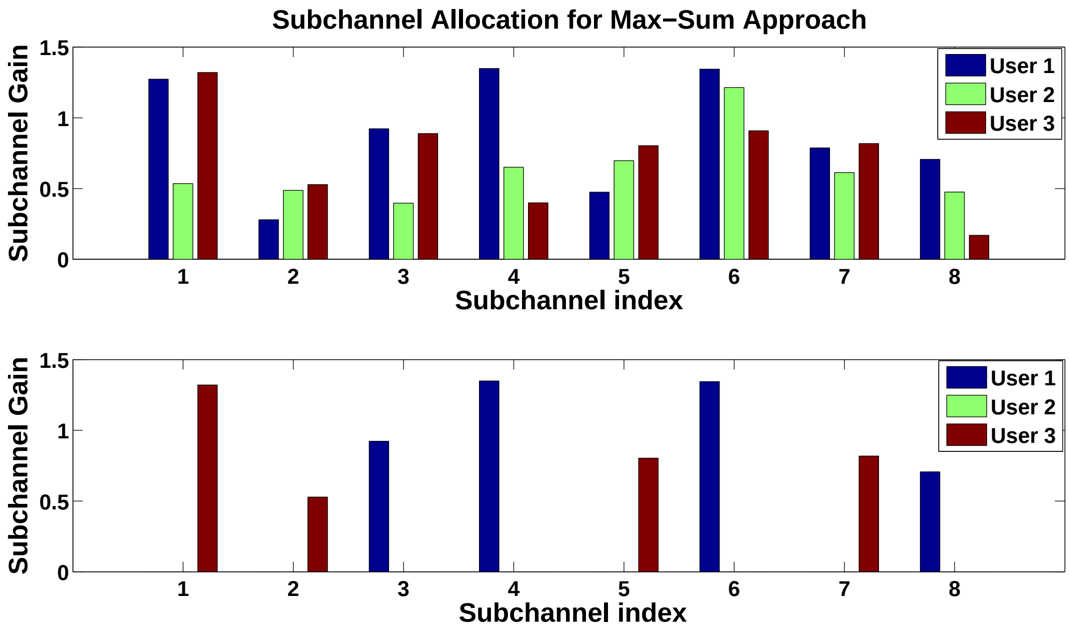

Figure 2.6: Allocation of $N = 8$ subchannels among $K = 3$ users

Definition 2.1. A K player bargaining problem is a pair $\langle U, \mathbf{u^0} \rangle$, where U is a compact, bounded, and convex set, and there exists at least one utility vector $(u_1, u_2, ..., u_K) \in U$ such that $u_1 \geq u_1^0, u_2 \geq u_2^0, ..., u_K \geq u_K^0$.

Theorem 2.1. A bargaining solution is a function $\mathbf{u}^* = (u_1^*, u_2^*, ..., u_K^*) = F(U, \mathbf{u^0})$ that assigns a unique element of U to the bargaining problem $\langle U, \mathbf{u^0} \rangle$. This solution is given by:

$$\mathbf{u}^* = \arg \max_{\mathbf{u} \in U, u_1 \geq u_1^0, u_2 \geq u_2^0, ..., u_K \geq u_K^0} \prod_{k=1}^{K} (\mathbf{u_k} - \mathbf{u_k^0}) \tag{2.13}$$

The NBS should satisfy the following set of axioms (for simplicity we assume the two user case) [35]:

- *Individual Rationality*: $u_1^* > u_1^0$ and $u_2^* > u_2^0$.

- *Feasibility*: $(u_1^*, u_2^*) \in U$.

- *Pareto Efficiency*: If $(u_1, u_2), (\acute{u}_1, \acute{u}_2) \in U, u_1 < \acute{u}_1$ and $u_2 < \acute{u}_2$, then $f(U, u_1^0, u_2^0) \neq (u_1, u_2)$.

- *Symmetry*: If $(u_1, u_2) \in U \Leftrightarrow (u_2, u_1) \in U$ and $u_1^0 = u_2^0$. Then, $u_1^* = u_2^*$.

- *Independence of Irrelevant Alternatives*: If $(u_1^*, u_2^*) \in \acute{U} \subset U$, then $f(\acute{U}, u_1^0, u_2^0) = f(U, u_1^0, u_2^0) = (u_1^*, u_2^*)$.

- *Independence of Linear Transformation*: Assume that $\acute{U}$ is obtained from U by linear transformation $\acute{u}_1 = a_1 u_1 + a_2$ and $\acute{u}_2 = a_3 u_2 + a_4$ with $a_1, a_3 > 0$. Then, $f(\acute{U}, a_1 u_1^0 + a_2, a_3 u_2^0 + a_4) = (a_1 u_1^* + a_2, a_3 u_2^* + a_4)$.

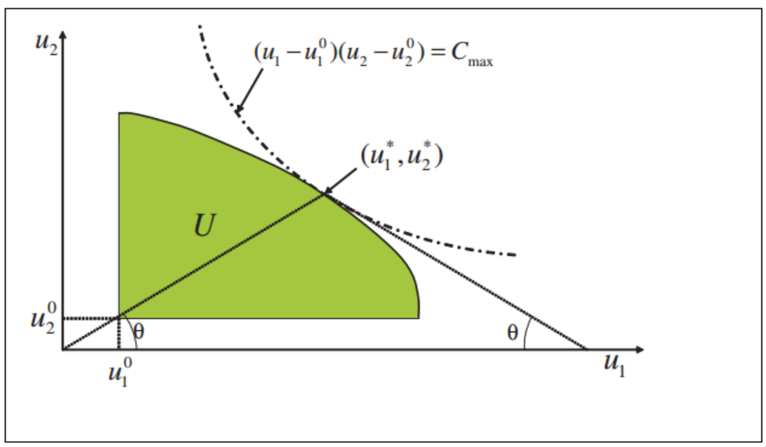

Figure 2.7: An illustration of a two-user bargaining problem

Figure 2.7 shows an illustration of bargaining problem for the case of two-user [36]. The shaded area in green is the feasible utility set, and C_{max} is the maximum value of the $(u_1 - u_1^0)(u_2 - u_2^0)$ product within the feasible utility set. As it is seen the NBS corresponds to the point (u_1^*, u_2^*).

In order to illustrate how the difference between the multiplication and summation as the utility function we consider a very simple example. We assume that $N = 4$ subchannels

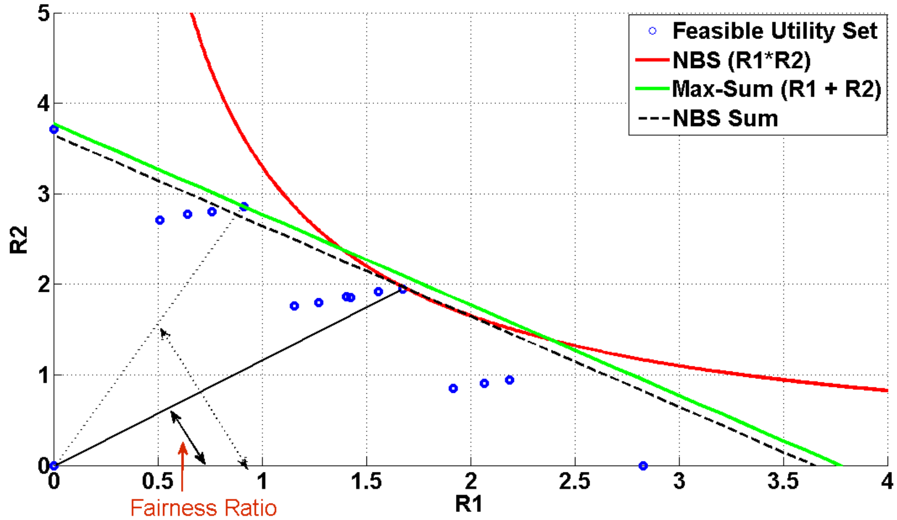

Figure 2.8: An illustration of a two-user and 4 subchannels problem

are to be allocated between two users while the power is constant. The disagreement point is set to zero for both users. The sample space, U consists of pair of (R_1, R_2) which is evaluated under all possible $2^4 = 16$ subchannel combinations and is depicted by blue points in Figure 2.8. Obviously the maximum rate for each user, $R_{1,max}$, and $R_{2,max}$ is achieve when all 4 subchannles are assigned to that particular user. The red curve is correspond to the multiplication of rates, i.e. $R_1 \times R_2$ and the green line shows the summation, $R_1 + R_2$. NBS is the intersection of the maximum value of $R_1 \times R_2$ curve with the sample space. The total rate of NBS is a little smaller than that of Max-Sum. However, the ratio of rates are smaller in NBS which indicates as the fairness ratio and will be explained in following sections.

2.7 Proposed resource allocation scheme using Nash Bargaining Game

In order to model the resource allocation problem we set each user rate R_i as its utility function and the minimum rate $R_{i,min}$ as the disagreement point.

Hence, $F(S, (R_{1,min}, R_{2,min}, ..., R_{K,min}))$ is a bargaining problem where the set S contains all the feasible rates, and its solution satisfies:

$$\arg \max_{\{\mathbf{P},\mathbf{C}\}, R_i \geq R_{i,min}} \prod_{i=1}^{K} (R_i - R_{i,min}) \tag{2.14}$$

subject to:

$$\begin{aligned} C_1 &: \quad \sum_{i=1}^{K} \sum_{n=1}^{N} p_{i,n} \leq P_{max} \\ C_2 &: \quad R_i \geq R_{i,min} \\ C_3 &: \quad c_{i,n} \in \{0, 1\} \\ C_4 &: \quad \sum_{i=1}^{K} c_{i,n} = 1 \\ C_5 &: \quad p_{i,n} \geq 0 \end{aligned}$$

where P_{max} is the maximum power budget of the base station. Constraint C_4 ensures that each subchannel is assigned to one user only. This optimization problem is hard to solve as it is dealt with both continuous and binary variables. Therefore, we relax the condition in C_4 by letting $c_{i,n}$ take values between $[0 \quad 1]$.

Lemma 1. The utility set S defined in (2.14) is bounded, nonempty, and convex.

Proof: Since the constraints in the above optimization problem are linear, the set is convex. The set is bounded because the maximum achievable rate is limited. As far as, $N \neq 0$, and $K \neq 0$, the set is nonempty.

Lemma 2. The utility function R_i is concave and injective.

Proof: For a two-variable function $R_i(c_{i,n}, p_{i,n})$ to be concave it suffices to show that its Hessian matrix $H(c_{i,n}, p_{i,n})$ is negative semidefinite.

Let's define $R_i = \sum_{n=1}^{N} r_{i,n}$ where:

$$r_{i,n} = c_{i,n} w \log_2(1 + c_3 \frac{p_{i,n} h_{i,n}^2}{\sigma_i^2}) \tag{2.15}$$

The Hessian matrix of $R_i(c_{i,n}, p_{i,n})$ becomes:

$$\begin{aligned} H(c_{i,n}, p_{i,n}) &= \begin{pmatrix} \frac{\partial^2 r_i}{\partial p_{i,n}^2} & \frac{\partial^2 r_i}{\partial p_{i,n} \partial c_{i,n}} \\ \frac{\partial^2 r_i}{\partial p_{i,n} \partial c_{i,n}} & \frac{\partial^2 r_i}{\partial c_{i,n}^2} \end{pmatrix} \\ &= \begin{pmatrix} \frac{-c_{i,n} w}{\ln 2 (1 + c_3 \frac{p_{i,n} h_{i,n}^2}{\sigma_i^2})} & 0 \\ 0 & 0 \end{pmatrix} \end{aligned} \tag{2.16}$$

As the first element of $H(c_{i,n}, p_{i,n})$ is non-positive, it is concluded that $H(c_{i,n}, p_{i,n})$ is negative semidefinite. Hence, the lemma is proved.

Lemmas 1 and 2 reduces the problem in (2.14) to a concave optimization over a convex set, and makes the NBS applicable to the problem as well.

2.7.1 KKT conditions

The optimization problem in (2.14) deals with both equality (C_4) and inequality (C_1, C_2, C_5) constraints. Hence, the method of Lagrange multipliers is not applicable. KarushKuhn-Tucker (KKT) conditions generalize the method of Lagrange multipliers to allow inequality constraints. KKT conditions are first order necessary conditions for a solution in nonlinear programming to be optimal, provided that some regularity conditions are satisfied.

In order to solve (2.14), we first form the Lagrangian and apply the KKT conditions as follows:

$$
\begin{aligned}
L(p_{i,n}, c_{i,n}) = (&\sum_{n=1}^{N} c_{1,n} w \log_2(1 + c_3 p_{1,n} G_{1,n}) - R_{1,min}) \\
&\ldots (\sum_{n=1}^{N} c_{i,n} w \log_2(1 + c_3 p_{i,n} G_{i,n}) - R_{i,min}) \\
&- \lambda \sum_{i=1}^{K} (\sum_{n=1}^{N} p_{i,n} - P_{max}) \\
&- \sum_{n=1}^{N} \mu_n (\sum_{i=1}^{K} c_{i,n} - 1) \\
&- \sum_{i=1}^{K} \sum_{n=1}^{N} \nu_{i,n} p_{i,n} \qquad (2.17)
\end{aligned}
$$

Hence, the KKT conditions are:

$$\frac{\partial L(p_{i,n}, p_{i,n})}{\partial p_{i,n}} = 0 \qquad (2.18)$$

$$\frac{\partial L(p_{i,n}, c_{i,n})}{\partial c_{i,n}} = 0 \qquad (2.19)$$

$$\lambda, \nu_{i,n}, \mu_n \geq 0 \qquad (2.20)$$

$$\lambda \sum_{i=1}^{K} (\sum_{n=1}^{N} p_{i,n} - P_{max}) = 0 \qquad (2.21)$$

$$\sum_{n=1}^{N} \mu_n (\sum_{i=1}^{K} c_{i,n} - 1) = 0 \qquad (2.22)$$

where $G_{i,n} = \frac{h_{i,n}^2}{\sigma^2}$.

From (2.18) we obtain the following power allocation equation:

$$p_{i,n} = \frac{\prod_{j=1,j\neq i}^{K} (R_j - R_{j,min})}{\lambda} c_{i,n} w - \frac{1}{c_3 G_{i,n}} \tag{2.23}$$

Assuming subchannel n is assigned to user i, i.e. $c_{i,n} = 1$, (2.23) has the familiar face of a water filling equation with slightly changes in the water level. Hence, more power will be allocated to the subchannels with higher gains. On the other hand, (2.19) and (2.22) yield to:

$$\frac{\log_2(1 + c_3 p_{1,n} G_{1,n})}{(R_1 - R_{1,min})} = \cdots = \frac{\log_2(1 + c_3 p_{K,n} G_{K,n})}{(R_K - R_{K,min})} \tag{2.24}$$

As it is seen, finding a closed form solution for $p_{i,n}$ and $c_{i,n}$ from (2.23) and (2.24) is an NP-hard. However, it casts light on the shape of the optimum solution. Looking at $c_{i,n}$ each fraction can be interpreted as the rate of each user in one subchannel to the total rate of all subchannels assigned to that user which illustrates the ratio of the rate in one subchannel to the total rate should be the same for all users. In other words, Rate in each subchannel is weighted by the total rate that allocated so far. This clue asserts the fairness of the optimal solution, and also, gives us a metric for subchannel allocation. Assuming that the subchannels are allocated,i.e. $c_{i,n}$ is known, from (2.23) and (2.21) we get the following familiar water filling equation:

$$p_{i,n} = \left[\frac{P_{max} + \frac{1}{c_3} \sum_{i=1}^{N} \frac{1}{G_{i,n}}}{N} - \frac{1}{c_3 G_{i,n}} \right]^+ \tag{2.25}$$

2.8 Simplifying NBS and the Proposed Sub-optimum Algorithm

As it is seen in previous section, Nash product terms are so hard to handle especially with the large number of users and subchannels that some simplifications are needed. On the other hand, all the available methods have ended up to some algorithms based on sorting the users subchannel gains which seems to be the only degree of freedom that we have handy in the rate equation in (2.3). Moreover, (2.3) is an increasing function of subchannel gain to noise ratio $G_{i,n}$. Therefore, instead of (2.14), we propose to solve the following linear problem:

$$\arg \max_{p_1,p_2,\ldots,p_K,c_{i,n}} \prod_{i=1}^{K} (\sum_{n=1}^{N} c_{i,n} G_{i,n}) \tag{2.26}$$

subject to:

$$C_1 : \quad c_{i,n} \in \{0,1\}$$

$$C_4 : \quad \sum_{i=1}^{K} c_{i,n} = 1$$

which is a binary integer programming problem and is simple to solve. Following the same approach as we did for (2.24), we obtain:

$$\frac{G_{1,n}}{\sum_{n=1}^{N} c_{1,n} G_{1,n}} = \cdots = \frac{G_{K,n}}{\sum_{n=1}^{N} c_{K,n} G_{K,n}} \tag{2.27}$$

Based on the above discussion we propose an algorithm for subchannel allocation while keeping the same power for each user by modifying the algorithm in [4] as follows:

Algorithm 1. 1. **Subchannel allocation**

(a) We assume a constant power for each user: $p_{i,n} = P_{max}/K$.

(b) Knowing power the allocation is done in three steps:

i. **Initialization:**

The initial rate for each user is set to zero : $R_i = 0$. Set subchannel set as $A = \{1, 2, ..., N\}$ and user set as $B = \{1, 2, ..., K\}$.

ii. **Meeting the minimum rate:**

The algorithm starts with finding the highest subchannel gain to noise ratio, $G_{i,n}$ and allocate it to the found user (i_{th}). The assigned subchannels will be removed: $A = A - \{n\}$. Once the minimum rate is met the user will be removed: $B = B - \{i\}$. This step repeats till B is empty, i.e., all users minimum are met.

iii. **Allocating the excess subchannels:**

For the remaining subchannels the allocation is done based on NBS metric found in equation (2.24): Subchannel n is allocated to the user with the highest value of $\frac{G_{i,n}}{\sum_{j \in \Omega_i} G_{i,j}}$. Ω_i is the set of subchannels allocated to user i so far. This step repeats till A is empty, i.e., all subchannels are gone.

2. **Power allocation**:

Power is water filled based on (2.7.1) for each $\Omega_i, i = 1, 2, ..., K$.

The proposed algorithm is summarized in table 2.3.

Table 2.3: The proposed subchannel and power allocation algorithm based on NBS

1. Initialization:
Set $R_i = 0, \Omega_i = \emptyset$ for all $i = 1, 2, ..., K$. Set $A = \{1, 2, ..., N\}$, and $B = \{1, 2, ..., K\}$.
2. Meeting the minimum rate requirement:
While $B \neq \emptyset$, a. Find (i, n) so that $\|G_{i,n}\|$ is maximum for all $i = 1, ..., K$, and $n = 1, ..., N$. b. For the found i if $R_i \leq R_{i,min}$ Let $\Omega_i = \Omega_i \cup \{n\}, A = A - \{n\}$ and update R_i according to (2.3). else $K = K - \{i\}$.
3. Allocating the excess subchannels:
While $A \neq \emptyset$, a. Find i satisfying $\frac{G_{i,n}}{\sum_{j \in \Omega_i} G_{i,j}} \geq \frac{G_{m,n}}{\sum_{j \in \Omega_m} G_{m,j}}$ for all m,$1 \leq m \leq K$. Ω_i and Ω_m are the subchannels allocated to user i and m respectively; b. For the found i, find n satisfying $\|G_{i,n}\| \geq \|G_{i,k}\|$ for all $k \in A$; c. For the found i and n, let $\Omega_i = \Omega_i \cup \{n\}, A = A - \{n\}$ and update R_i according to (2.3).
4. Power water filling:
Water fill power based on (2.7.1) for each $\Omega_i, i = 1, 2, ..., K$.

Bandwidth w	3.2 mbps
$R_{i,min}$	0
N_0	-110 dBm-Hz
channel gain	Rayliegh
P_{max}	0.3 Watt

Table 2.4: Simulation parameters

2.9 Numerical Results, Simulation and Discussion

In this section we evaluate the performance of the proposed resource allocation approach and compare it with the Max-Min and Max-Sum schemes. Channel gains are i.i.d. random variables with Rayleigh distribution. Noise spectral density is $N_0 = -110$ dBm-Hz and is the same for all K users. The total available bandwidth is $w = 3.2$ MHz, and the maximum allowable power per user is 0.3 W. The minimum rate requirement of each user is set to zero, i.e., $R_{i,min} = 0$.

In order to illustrate how the proposed algorithm work, we start with a simple example of allocating $N = 8$ subchannel among $K = 3$ users with $R_{min} = 0$ in Figure (2.9). The top plot shows the varying subchannel gain for all 3 users. In the bottom plot, however, we handpicked the best subchannels by applying the proposed algorithm. User 1 will fairly achieve the highest rate as it has the highest gain, and the worst user (User 2) gets two subchannels.

Figure 2.10 compares the fixed subchannel assignment where $N = 16$ subchannels are equally divided among $K = 6$ users with the proposed dynamic scheme. It is seen that by assigning the subchannels dynamically the higher rate per user is achieved.

In Figure 2.11 we compare fixed power and sub-channel assignment, waterfilled power but fixed subchannel assignment, and the proposed dynamic power and subchannel allocation. The first two bottom plots show the effect of water filling which improves the total rate. However, the top plot has a much better improvement indicating the effectiveness of

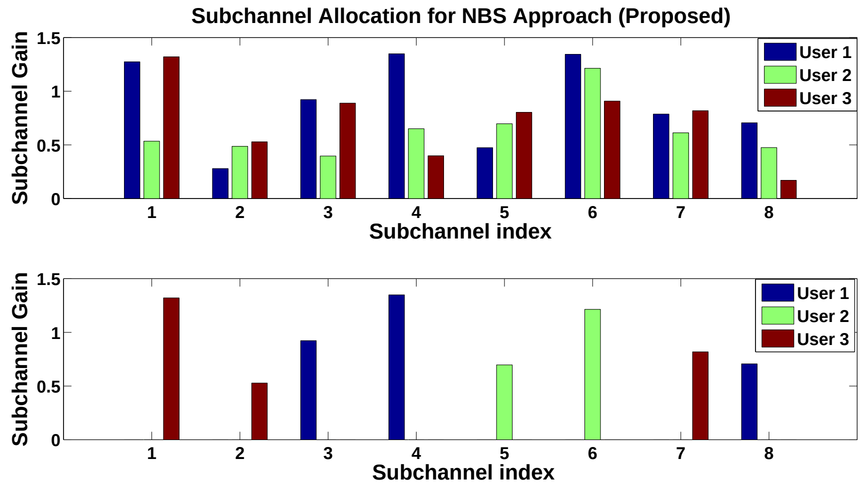

Figure 2.9: Allocation of $N = 8$ subchannels among $K = 3$ users

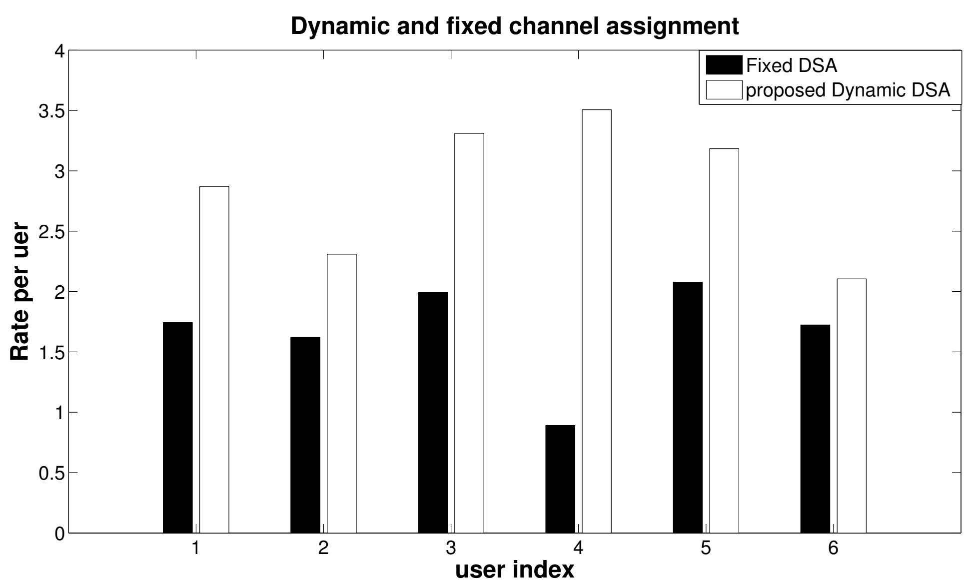

Figure 2.10: A comparison of fixed subchannel assignment and the proposed algorithm (N = 16, K = 6)

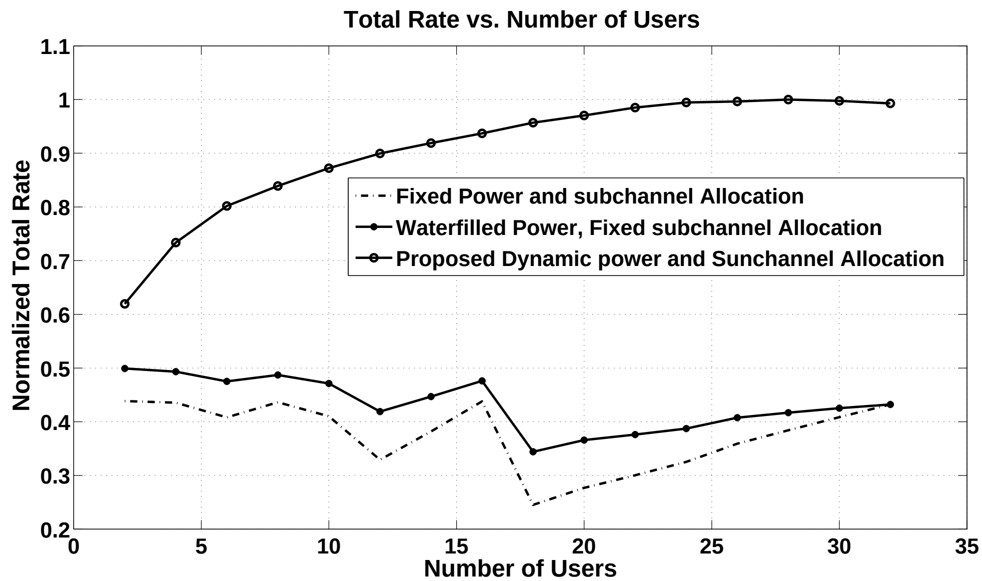

Figure 2.11: A comparison of fixed subchannel and power assignment, waterfilling power, and the proposed algorithm for N = 32 subchannels

the dynamic subchannel assignment.

Figure 2.12 shows the allocated rate per user per subchannel for $N = 16$ subchannels and $K = 6$ users for one channel realization with green bars for the proposed algorithm, blue bars for the Max-Sum rate and the brown bars for the Max-Min case. Let's consider the two extreme cases first. It is obvious that the second and third users have the worst channel conditions. As we see the Max-Sum algorithm does not allocate any subchannel to these two users since it lowers the total rate while the max-min approach treats these users almost the same as other users at the cost of reducing the total rate of the network. The proposed algorithm, however, aiming at balancing the optimality and the fairness, assigns the lowest rates to these users. Let's switch to the fifth user which is the best user with the highest channel gain. The Max-Sum allocates the highest rate to this user but the Max-Min assigns less rate which results in the underutilization of the fifth user good conditions. The proposed algorithm keeps the middle position. The same analysis applies for the rest of users which are between the two extreme cases. Generally, Max-Min tends to treat all users the same while the proposed method applies a proportional fairness depending on each user

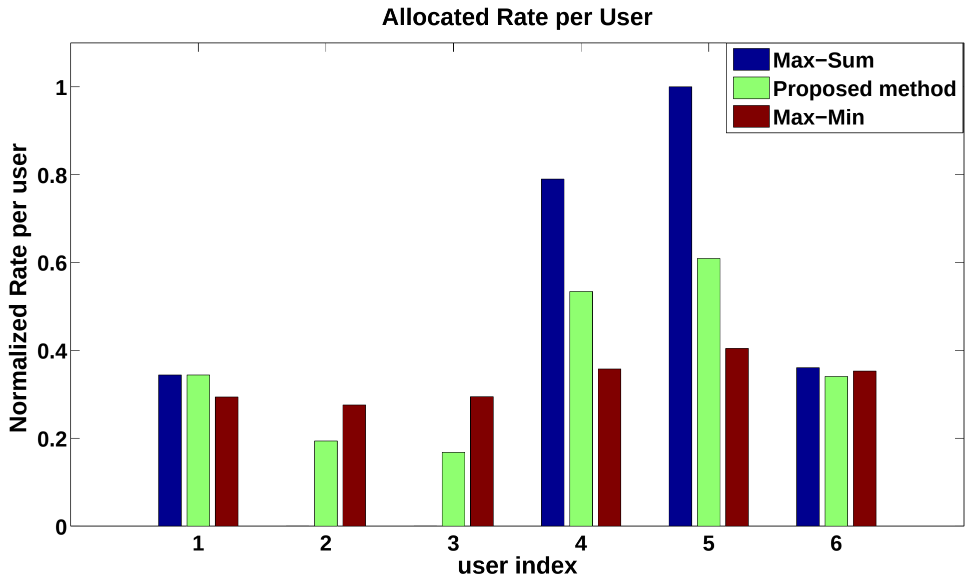

Figure 2.12: A comparison of the allocated rate per user for the Max-Min, Max-Sum, and the proposed algorithm (N = 16, K = 6)

channel condition.

Figure 2.13 shows the aggregate channel rate versus the number of users for the three algorithms with dashed line for the Max-Min case, '-O' line for the proposed algorithm, and the dash-point (-.) line for the Max-Sum method. As we expect from the above discussion the min-max has the lower rate while the Max-Sum takes the higher rate. The proposed algorithm lies in the middle so that combines the optimality of the Max-Sum and the fairness of the Max-Min. More specifically, the proposed algorithm performs up to %10 better than Max-Min.

The same comparison as Figure 2.13 is done in Figure 2.14 for the fixed number of users $K = 8$ but different subchannels $N = [8 : 2 : 32]$.

Figure (2.15) shows the allocated rate per user for two different values of minimum rate R_{min} but the same channel realization. For the left plot, it is assumed that $R_{min} = 0$ whereas for the right plot R_{min} is randomly generated with the mean of %10 of each user maximum rate. As we observe, according to NBS the minimum rate is filled first (the bars

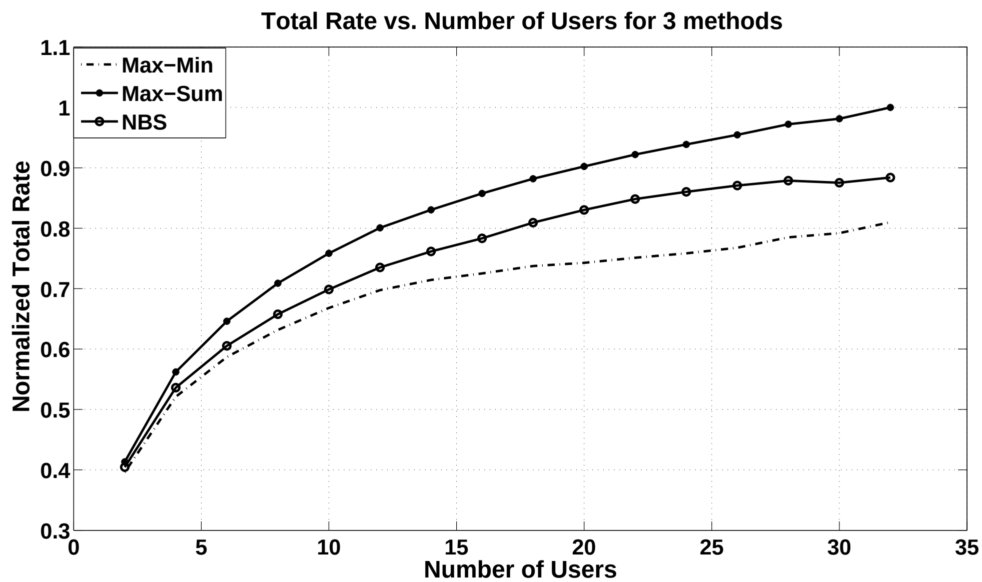

Figure 2.13: A comparison of the allocated rate per user for the Max-Min, Max-Sum, and the proposed algorithm (N = 32)

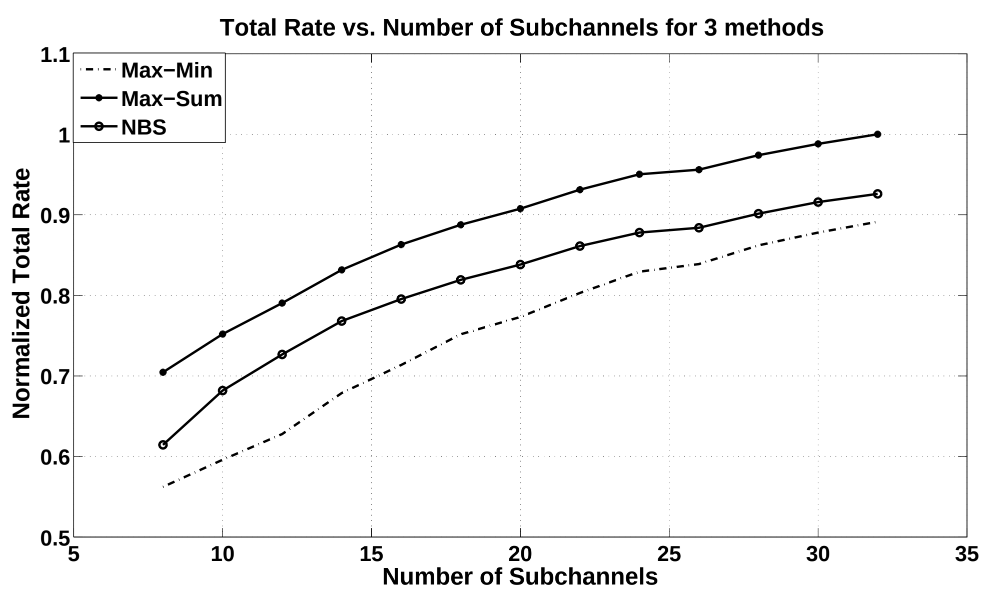

Figure 2.14: A comparison of the allocated rate per user for different number of subchannels the Max-Min, Max-Sum, and the proposed algorithm (K = 8)

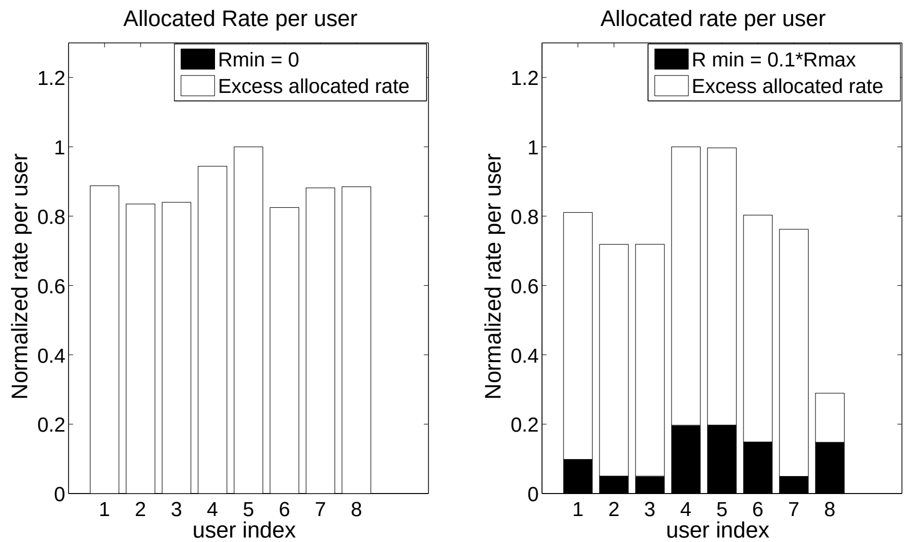

Figure 2.15: Allocation of minimum rate and the excess subchannels (N = 128, K = 8)

Carrier frequency	2.4 GHz
Sampling frequency, f_s	20 MHz
OFDM Symbol Duration	4 ms
FFT size, N	64
Subcarrier spacing, w	312.5 kHz
Max Delay Spread	250 ns
Doppler frequency $(3km/hr)$	13 Hz
Carriers used	52 (center + 11 sides not used)

Table 2.5: OFDM parameters for IEEE802.11g

in back) and then the excess subchannels are distributed. It is obvious that when there is no minimum rate constarint the rate is allocated more evenly as it is the case in the left plot. However, with the non-zero value of R_{min} the priority is given to meeting the mimum rate. It is worth to mention that if the minimum rate reqiurement (disagreement point) is set very high no excess subchannel will be left to allocate thourgh NBS. This is the mutual agreement that users make before entering the game so that they benefit from the cooperative game outcome.

Sampling frequency, f_s	192 MHz
FFT size, N	128
Subcarrier spacing, w	15 kHz
Noise power	-104.5 dBm
P_{max}	43 - 48 dBm

Table 2.6: OFDM parameters for LTE systems

2.9.1 Resource Allocation in MIMO-OFDM Systems

In this chapter we developed a dynamic subchannel and power allocation scheme for OFDM users assuming Single Input Single Output (SISO) systems. However, in all new generation wireless systems both base station and mobile users are equipped with multiple antennas. Therefore, we need to address the resource allocation problem for Multiple Input Multiple Output (MIMO)- OFDM systems. This will add the space as a new dimension to the power and subchannel allocation which will be the subject of possible future work for this dissertation. In the case of MIMO-OFDM systems since the number of antennas in base station side could be large, the load of the feedback required to report the channel information would be burdensome specially for Frequency Division Multiplexing (FDD) systems. Therefore, we may need to compress the feedback information using methods such as Compressive Sensing. A survey of compressive sensing and how to apply it to MIMO systems is presented in Appendix B for interested readers.

2.10 Conclusion

An effective and simple dynamic subchannel and power allocation algorithm proposed based on Nash bargaining Solution. A new fairness metric is introduced which leads to a simplified algorithm to allocate the subchannels while waterfilling the power. Comparing to the Max-Sum approach which totally ignores the users with weak channel conditions and the Max-Min scheme which maximizes the worst user rate at the cost of scarifying the good users, the proposed algorithm balances these two extreme cases by weighting each user according to its total channel gain. Provided simulation results shows that the proposed algorithm performs better than the Max-Min approach, and closely to the Max-Sum case especially when the number of subchannels is comparably larger than the number of users.

Chapter 3: Spectrum Sensing

With ever-increasing demand for high data rate communications and wider bandwidth, spectrum scarcity is becoming an inevitable issue which reveals the inefficiency of the current static spectrum access techniques. Cognitive radio technology as a potential platform to implement the Dynamic Spectrum Access (DSA) has been captured the interest of researchers in recent years [6]. Two major concerns in cognitive radio networks are spectrum sensing and spectrum allocation. A secondary user senses the spectrum of the Primary periodically, and uses it when it is idle, and switch to other unused bands when primary system is present. As we can see spectrum sensing is the first and critical step for a cognitive radio device to operate.

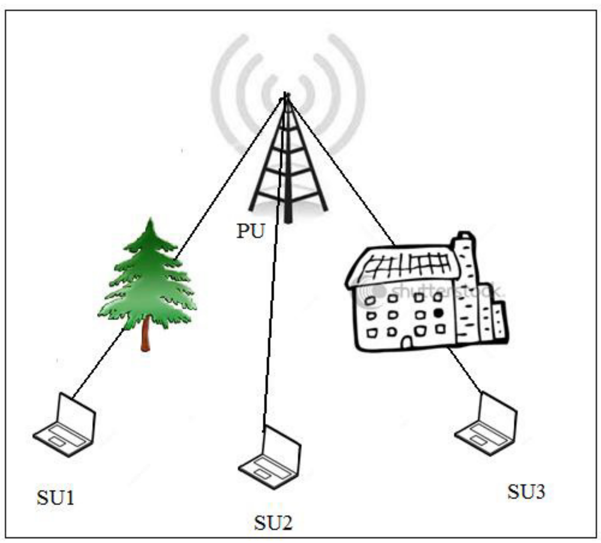

Figure 3.1: Hidden terminal problem

There are several detectors to sense the spectrum of a Primary User (PU) such as, matched filter, energy detector, and feature detection [13]. Feature detection is able to distinguish between the received signal energy and the noise energy but it requires long observation intervals of the received signal which leads to high computational complexity. Matched filter detector is the optimal option for the case of stationary Gaussian noise and when the primary signal is known. However, when the primary signal is unknown or complexity is an issue, matched filter and feature detection are ruled out, and energy detector appears to be the common and feasible choice. Nevertheless, the performance of an energy detector is deeply affected by hidden terminal nodes due to the path loss and shadowing (Figure 3.1). Indeed, while some secondary users are suffering from deep fading some others have a better reception of the primary signal. This problem brings up the idea of collaboration among secondary users in detecting the primary signal. Collaborative spectrum sensing has been studied in recent literatures [37–41], and two collaborative methods, centralized and distributed, are available. In the case of centralized spectrum sensing, Secondary Users (SU) report their observations of the PU signal to a central fusion center (FC) which combines the received information and broadcasts the final decision to SUs (Figure 3.2). Although collaboration enhances detection probability (P_d), it increases false alarm probability (P_f), at the same time. Moreover, having all SUs to report to a central entity is not scalable and may not be applicable where SUs belong to different service providers.

As opposed to the centralized approach, the distributed spectrum sensing partitions the secondary network to some groups or coalitions [41]. Within each coalition a SU with the best channel reception is selected as a head and other SUs report their sensing bits to the heads instead of a central entity. The head acts as a fusion center and makes the final decision within each coalition (Figure 3.3). The way that the network is partitioned is very important and it is done subject to keeping probability of false alarm under certain threshold, and improving probability of detection as much as possible per coalition. Despite the available centralized and distributed methods, an effective collaborative spectrum sensing is still lacking. The main drawback of current approaches is the metric that is applied to measure the collaborative detection probability in order to prove that the collaboration

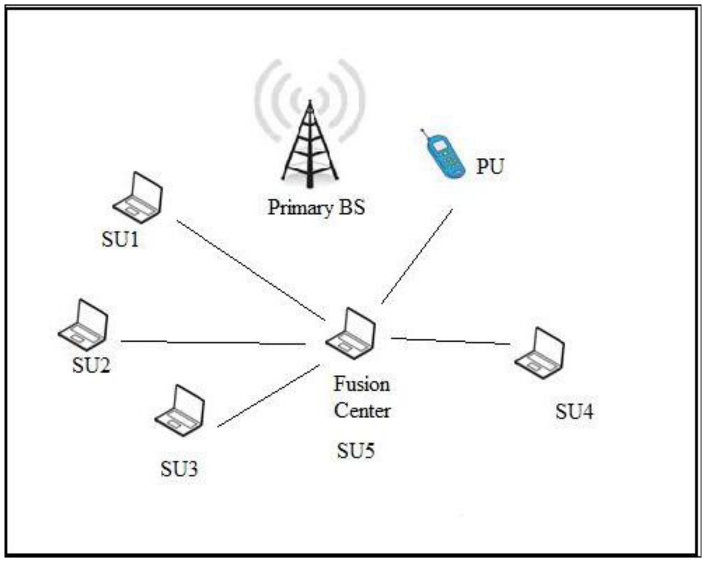

Figure 3.2: Centralized spectrum sensing

is beneficial [38, 41]. That is, no matter how accurately the SUs sense the channel, the resulted collaborative detection probability increases anyway especially with the number of SUs! Moreover, some selfish SUs may stop sensing the spectrum and just wait for the final decision made by FC, and save the power for their own transmission. On the other hand, the distributed approach enforces higher complexity and computations on the network by forming groups and selecting the "head". Besides, in order to form coalitions SUs need to know each other channel characteristics, which may not be favorable in sensitive applications. To address the above mentioned drawbacks, a Stackelberg game spectrum sensing method is proposed which works as follows: Based on the received SNR each SU is considered either as a leader or a follower. Leading SUs have higher detection probability as a result of good PU signal reception, so they will broadcast their sensing information. On the other hand, the follower SUs do not contribute to the sharing process while having doubtful observations, so they look for any announced information by a leading SU in order to discover the presence or absence of the PU. It is important to mention that the goal of collaboration is to detect the presence of PU. Hence, there is no point for SUs to share their observations when PU is not active. However, this point has fallen through the cracks of available collaborative spectrum sensing approaches which enforce unnecessary overhead on the nodes (SUs).

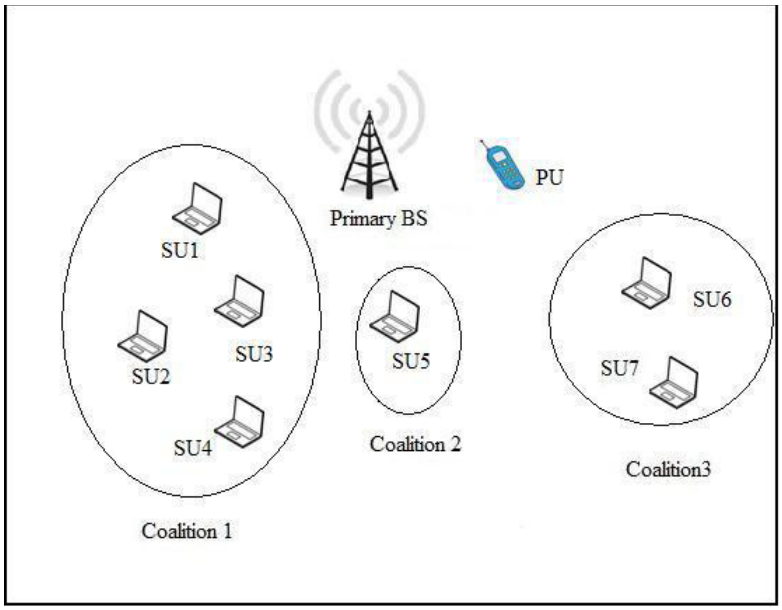

Figure 3.3: Distributed spectrum sensing

The application of game theory both in wireless and wired communications has attracted the attention of many researchers, and it includes many areas from resource allocation and power control purposes to cooperative communications and routing. For example, a thorough overview of game theory applications for cognitive radio networks is given in [36]. A Stackelberg game is a model for the case of hierarchical structure where a leader moves first and the follower moves sequentially. In [42], for example, this game is employed for the purpose of spectrum leasing.

3.1 System Model and Formulation

Consider a secondary ad hoc network including N nodes and the primary system as a cellular device operating in uplink which is the case for future wireless systems, e.g. Long Term Evolution (LTE). Assuming that all the SUs are using energy detector with the same parameters, the input signal to the detector is filtered by a band-pass filter with the bandwidth W, squared, and integrated over the observation time T (Figure 3.4). The output of the integrator Y , is compared with the threshold λ to make a decision out of these two hypotheses:

$$\begin{cases} Y \geq \lambda & H_1 \\ Y < \lambda & H_0 \end{cases} \tag{3.1}$$

where H_1 indicates the presence of the PU and H_0 denotes that PU is inactive.

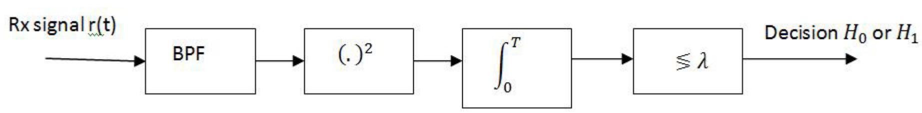

Figure 3.4: Block Diagram of an Energy detector

The probability that the i-th SU detects the PU signal for a non-fading channel given that PU is present is given by [37]:

$$P_{d,i} = P\{Y > \lambda | H_1\} = Q_m(\sqrt{2\gamma_i}, \sqrt{\lambda}) \tag{3.2}$$

where $m = TW$ is the time - bandwidth product, and $Q_m(.,.)$ is the generalized Marcum Q-function , and γ_i is the received SNR by i-th SU.

For the case of Rayleigh fading, γ_i follows exponential distribution with mean $\bar{\gamma}_i$. Therefore, averaging(3.2) over PDF of γ_i yields [37]:

$$P_{d,i} = e^{-\frac{\lambda}{2}} \sum_{n=0}^{m-2} \frac{1}{n!} \left(\frac{\lambda}{2}\right)^n + \left(\frac{1+\bar{\gamma_{i,j}}}{\bar{\gamma_{i,j}}}\right)^{m-1} \times \left(e^{-\frac{\lambda}{2(1+\bar{\gamma_{i,j}})}} - e^{-\frac{\lambda}{2}} \sum_{n=0}^{m-2} \frac{1}{n!} \left(\frac{\lambda \bar{\gamma_{i,j}}}{2\left(1+\bar{\gamma_{i,j}}\right)}\right)^n\right) \tag{3.3}$$

where $\bar{\gamma_i} = \frac{Ph_i}{\sigma^2}$ is the average SNR received by the i-th SU from PU, P is the PU transmit power, and σ^2 is the noise variance. Furthermore, $h_i = \frac{1}{d_i^\eta}$ is the path loss where d_i is the distance between i-th SU and PU, and η is the path loss coefficient.

Another parameter to evaluate the performance of the detector is false alarm probability and is given by [37]:

$$P_{f,i} = P_f = P\{Y > \lambda | H_0\} = \frac{\Gamma(m, \frac{\lambda}{2})}{\Gamma(m)} \tag{3.4}$$

where $\Gamma(.,.)$ is the incomplete gamma function and $\Gamma(.)$ is the gamma function. As it is seen from 3.4 P_f is just function of the energy detector parameters m and λ and not the location (d_i) of SUs. Hence, assuming that the detector parameters are the same for all SUs, subscript i in $P_{f,i}$ can be dropped.

A collaborative detection probability has been defined as follows in [38, 41]:

$$Q_{d,i} = 1 - \prod_{j=1}^{n} (1 - P_{d,j}) \tag{3.5}$$

Let's assume that all SUs have the same $P_d = 0.5$. A SU with this detection probability is unable to certainly determine if PU is present or not. However, the resulting collaborative detection probability for $n = 5$ would be $Q_{d,i} = 1 - (1 - 0.5)^5 = 0.9688$, which implies that the PU is there with almost certainty! Therefore, the applied $Q_{d,i}$ increases with n , no matter how poorly the SUs are sensing. Clearly, this cannot be true. Hence, we define another metric for each user to evaluate the collaborative probability. For this purpose, in the next subsection we derive an indicator for the level of received signal strength per SU.

3.2 Upper Bound and Lower Bound for P_d

Let's consider $P_{d,i}$ for two boundary cases. First, Assuming that $\bar{\gamma_i} >> 1$, then $\bar{\gamma_i} + 1 \approx \bar{\gamma_i}$. Hence, equation (3.3) can be re-written as:

$$\begin{aligned} P_{d,i} &\approx e^{-\frac{\lambda}{2}} \sum_{n=0}^{m-2} \frac{1}{n!} \left(\frac{\lambda}{2}\right)^n + 1 \times \left(e^{-\frac{\lambda}{2\bar{\gamma_i}}} - e^{-\frac{\lambda}{2}} \sum_{n=0}^{m-2} \frac{1}{n!} \left(\frac{\lambda}{2}\right)^n\right) \\ &= e^{-\frac{\lambda}{2\gamma_i}} \\ &= e^{-\frac{1}{2\alpha}} \end{aligned} \tag{3.6}$$

and $\alpha = \frac{\bar{\gamma_i}}{\lambda}$ which we use as a metric to distinguish SUs with strong signal reception from the ones with weak SNR reception. Figure 3.5 shows approximated $P_{d,i}$ versus α. As it is expected the better the $\bar{\gamma_i}$ is, the higher the detection probability is. For example, to get $P_{d,i} \geq 0.7$ the average SNR should be as big as 2λ , and for $P_{d,i} \geq 0.9$ we need to have $\bar{\gamma_i} \geq 5\lambda$. Moreover, it is seen that $P_{d,i}$ approaches its upper bound ($P_{d,i} = 1$) for quite large values of $\bar{\gamma_i}$.

In order to categorize SUs as leaders or followers each SU needs to evaluate its energy detector output (Y) which has a non-central chi-square distribution with $2m$ degrees of freedom and a non-centrality parameter 2γ under H_1 [37]. Therefore, the mean of Y, denoted by $\bar{Y}$, would be $2m + 2\gamma$. To keep $P_{d,i}$ above some threshold α_1, we need to have:

$$P_{d,i} \geq \alpha_1 \Rightarrow e^{-\frac{1}{2\alpha}} \geq \alpha_1 \tag{3.7}$$

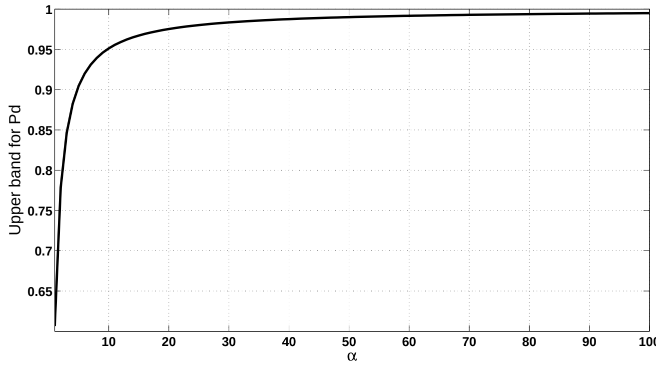

Figure 3.5: Approximation of $P_{d,i}$ versus α

After simplifications and noting that $\bar{Y} = 2m + 2\gamma$ we will get:

$$\bar{Y} \geq \frac{-\lambda}{\ln(\alpha_1)} + 2m = \beta \tag{3.8}$$

Hence, each SU having average Y greater than the above threshold will be considered as a leader, otherwise, as a follower. This point is of great importance since each SU just needs to monitor its own detector output, and makes autonomous decision without considering the other SUs channel information or average SNR which is the case in [41].

Now consider the case when $\bar{\gamma_i} \approx 0$, then from (3.2) we will have

$$Q_m(0, \sqrt{\lambda}) \approx 0 \Rightarrow P_{d,i} \approx 0 \tag{3.9}$$

The above is correspond to the case of hidden node. When a SU is hidden from PU path, then the received SNR will decrease sharply and $P_{d,i}$ gradually tends to zero.

3.3 Stackelberg Collaborative Spectrum Sensing Scheme

3.3.1 Stackelberg game formulation

In this subsection, a game theory tool is applied in order to design the collaborative spectrum sensing algorithm. Since SUs are considered either as leaders or followers based on their received SNR from the PU transmitter, the collaborative sensing process could be interpreted as a Stackelberg game [43]. Here, the leaders determine the sensing decision by which followers take actions. Therefore, the network sensing output is dominated by leaders who are the more reliable nodes which can benefit the SUs with low received SNR as well. Assuming that SUs are sharing their sensing result, the collaborative detection probability would be the average over the detection probability per SU ($P_{d,i}$) as follows:

$$
\begin{aligned}
Q_d &= \frac{\sum_{i=1}^{n} H_{k,i} P_{d,i}}{\sum_{i=}^{n} H_{k,i}} \\
Q_f &= \frac{\sum_{i=1}^{n} H_{k,i} P_{f,i}}{\sum_{i=}^{n} H_{k,i}}
\end{aligned}
\tag{3.10}
$$

where $H_{k,i} = H_{0,i}$ or $H_{1,i}$ is the sensing bit and also the action of the i-th SU, and n is the number of users collaborating with each other. As it is seen in 3.10, only having $H_{k,i} = H_{1,i}$ will affect the collaborative detection.

Furthermore, a possible concave utility function per user could be defined as:

$$
\begin{aligned}
u_i(H_i, H_{-i}) &= \log \left(\frac{Y_i}{\beta}\right)^{H_i + \frac{\left(\sum_{k=1,k\neq i}^{n} H_k\right)}{n}} \\
&= \begin{cases} 0 & H_k = 0 \\ \geq 0 & H_k = 1, n = 0, Y_i \geq \beta \\ < 0 & H_k = 1, n = 0, Y_i < \beta \end{cases}
\end{aligned}
\tag{3.11}
$$

where in $H_i + \frac{(\sum_{k=1,k\neq i}^{n} H_k)}{n}$ the first term determine the SU observation itself, and the second term is what SU i gets as the result of collaboration with n other SUs (H_{-i}). According to (3.11) each user would like to maximize its own utility which leads to increasing n. However, receiving more sensing bits requires more sensing time as well as increasing the false alarm probability.

Generally, the goal of collaborative sensing is to solve the following optimization problem:

$$\max_{n,H_k} \quad Q_d(n, H_k) \tag{3.12}$$

$$\text{Subject to} \quad Q_f \leq \alpha_2. \tag{3.13}$$

That is, how to arrange the network structure and consequently n so that a SU can detect the PU signal with the highest possible probability. On the other hand, increasing n as much as possible is not desirable since the sensing time will increase as well. However, the above optimization problem is an NP-complete problem which is not trivial to solve and obtain a closed-form result. Therefore, we propose a simple algorithm that enhances the whole network collaborative detection while maintain the false alarm probability under the given threshold.

The proposed algorithm for sensing process

In the proposed method based on the level of received signal at the energy detector output, the SUs are divided to leaders or followers as it was shown in (3.8). Furthermore, we assume that there is a signaling channel among SUs so that they can communicate with each other. Then, the collaboration is performed according to the following algorithm:

1. Only leaders are allowed to broadcast their sensing bits and only under hypothesis H_1. As it is mentioned the reporting of channel information such as h_i or γ_i are not required.

2. A follower SU needs to listen to the signaling channel in search of any decision H_1. If found any, the SU stops the search, and takes H_1 as its decision.

3. Information sharing happens only under H_1 when at least one user strongly detects the PU. There is no point in sharing information to detect the noise (H_0).

4. If there is no leader under H_1, the sensing would be done non-cooperatively since there is no point in sharing doubtful information.

5. The game reaches its equilibrium when each follower is able to pair with a leader.

6. Each leader can have more than one follower while each follower is matched with only one leader. The reason is that listening to more leaders will make the sensing process longer for a follower.

Figure 3.6 illustrates a scenario where SU6 is hidden from PU path, so it is considered as a follower while all other SUs are able to strongly receive PU signal. In this case, all SU1-SU5 will broadcast their decision H_1 whereas SU6 will do nothing but waiting for the observations by others, and it will probably catch the sensing bit sent by SU5 first since it is closer. Therefore, SU6 will get a true vision of the PU signal although it is hidden from PU path. Moreover, as soon as SU6 gets the first H_1 it will take it.

3.4 Simulation Results

For simulation, we assume N SUs are randomly distributed around a single PU within the radius of 3 km. We, furthermore, set $m = 5$, $P_f \leq 0.1$, and $\alpha_1 = 0.7$ as recommended by the IEEE 802.22 standards [44]. For the first case, let's have $N = 5$ with two weak SUs ($n_w = 2$) and the remaining three as leaders. That means that by setting $\alpha_1 = 0.7$ in (3.7) and (3.8) the detector output and average SNR for leading SUs are $\bar{Y} \geq 17$dB, and $\bar{\gamma}_i \geq 13.22$dB respectively. For the case of non-cooperative, the received $\bar{\gamma}_i$ is weakened by a factor 20% and 40% for the follower SUs because of the deep fading.

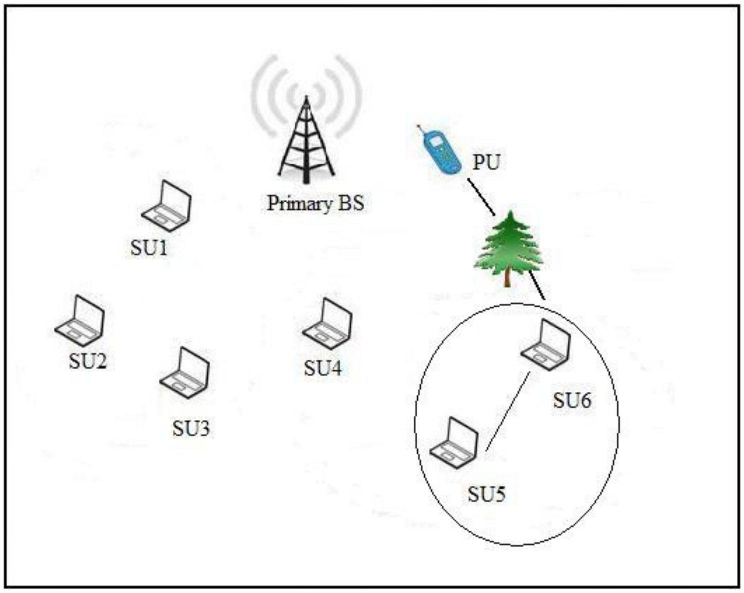

Figure 3.6: The proposed collaborative spectrum sensing scheme

Figure(3.7) shows the average collaborative detection probability versus average SNR ($\bar{\gamma}$). As is it seen, the proposed algorithm increases the collaborative probability by 35% for $\bar{\gamma}$ = 16dB as compared to the non-cooperative case. The reason is that the collaborative detection probability is dominated by leading users having an acceptably good SNR while the weak users lower the detection probability in non-cooperative case. In Figure (3.8), we keep the same parameters as Figure(3.7) except for $N = 7$ and $n_w = 1$. As we see the difference between collaborative and non-cooperative schemes decreases since the n_w is lower, but still collaborative scheme works 5.2% better for $\bar{\gamma} = 16$ dB. However, if we decrease n_w to zero then both schemes will work almost the same since there is not any weak SU, and the sensing is done non-cooperatively.

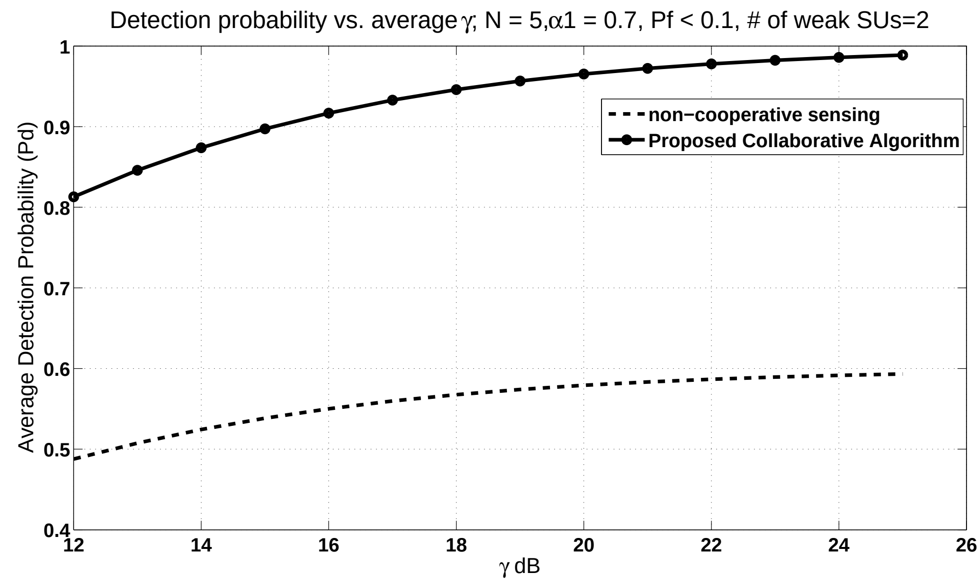

Figure 3.7: The average detection probability for the case of proposed collaborative algorithm (-o) and non-cooperative case(–). $N = 5$, $P_f = 0.1$, $\alpha_1 = 0.7$, $n_w = 2$

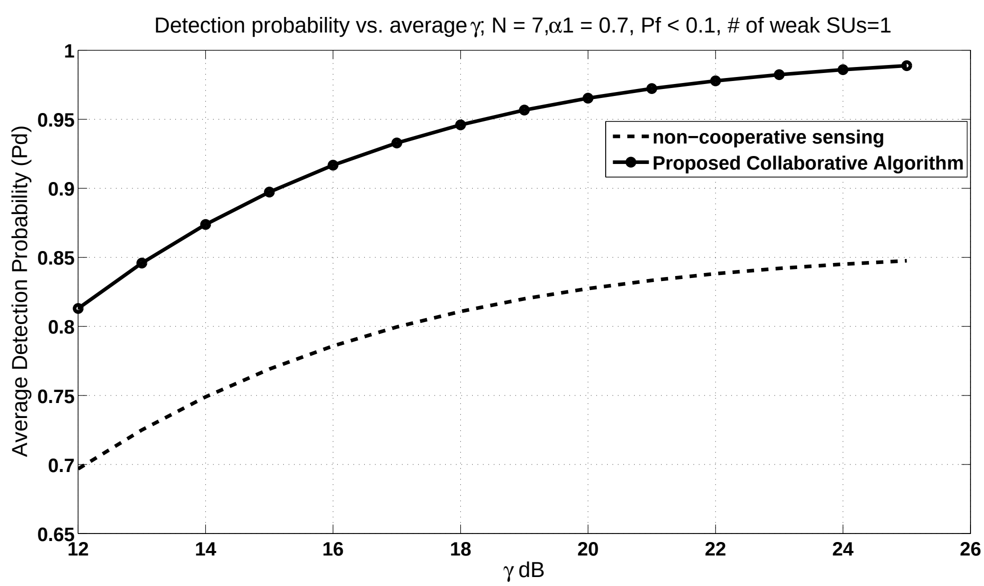

Figure 3.8: The average detection probability for the case of proposed collaborative algorithm (-o) and non-cooperative case(–). $N = 7$, $P_f = 0.1$, $\alpha_1 = 0.7$, $n_w = 1$

Chapter 4: Joint Spectrum Sensing and Allocation

4.1 Introduction

In Chapter 2 we studied dynamic resource allocation in non-cognitive based OFDM networks and a power and subchannel allocation scheme was proposed. We learned in Chapter 3 that in a cognitive radio network unlicensed users need to sense the spectrum in the hope for idle spectrum bands and a distributed spectrum sensing approach was presented. In this chapter we would like to generalize our resource allocation scheme to encompass cognitive users as well. In particular, since the spectrum sensing information in physical (PHY) layer will directly affect spectrum allocation in MAC layer, a cross layer a design is needed. Figure (4.1) shows the layer stack in a cognitive radio network along with brief description on each layer functionality [7].

In [14] the dynamic resource allocation in cognitive radio networks with Imperfect Channel Sensing is investigated and is solved using a discrete stochastic optimization method. However, the imperfectness is assumed for the channel gain information. Basically, the goal of spectrum sensing is to monitor the activity of the PU which is the case in this dissertation. An interesting joint cross-layer scheduling and spectrum sensing for cognitive radio networks is proposed in [15] based on what is called 'Raw Sensing Information' and the power and subchannel allocation is solved using primal-dual decomposition approach. Their underlay spectrum sharing method with assuming some acceptable interference level differs from our work where we adopt overlay method which does not allocate an OFDM sub-channel to more than one user simultaneously. Moreover, our problem solving method is based on NBS which gives a fair and simpler allocation scheme.

Here, we use the sensing information of the energy detector of each SU to design an optimum power and subchannel allocation for SUs. First, we show that how the sensing

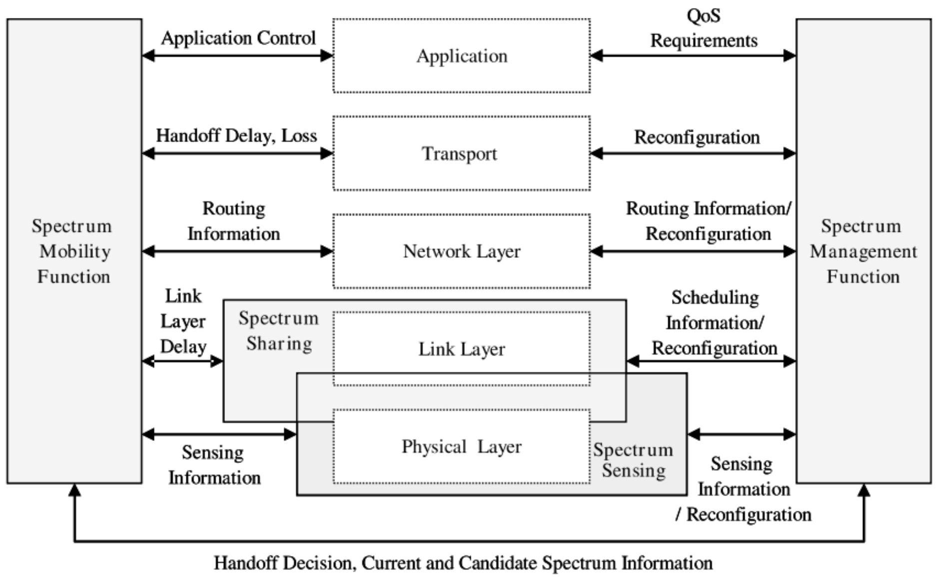

Figure 4.1: Layer functionalities in a cognitive radio network

bits of the PHY layer can affect the allocation process in MAC layer. Then, we propose a sub-optimum resource allocation algorithm in order to increase the network total throughput while maintaining fairness among users.

4.2 System Model

Consider an adhoc secondary network with K users competing over N OFDM sub-channels of a Primary base-station. Each subchannel has a bandwidth of w. The rate for i-th user is expressed as:

$$R_i = \sum_{n=1}^{N} w b_{i,n} \tag{4.1}$$

where $b_{i,n}$ is the number of bits per symbol for the i-th user in subchannel n. Assuming $b_{i,n} \geq 2$ and $\mathrm{BER}_i \leq 10^{-3}$, the following approximation will hold [31]:

$$b_{i,n} \approx \log_2(1 + c_2 \gamma_{i,n} \ln(\frac{\mathrm{BER}_i}{c_1})) \tag{4.2}$$

where $c_1 = 0.2$, $c_2 = 1.5$. BER_i is the i-th user bit error rate, and $\gamma_{i,n}$ is the SNR in subchannel n. Hence, the rate for user i can be formulated as:

$$R_i = \sum_{n=1}^{N} c_{i,n} w \log_2(1 + c_3 \frac{p_{i,n} h_{i,n}^2}{\sigma_i^2}) \tag{4.3}$$

where $c_3 = c_2 / \ln(c_1/\mathrm{BER}_i)$, σ_i^2 is the noise power, and $h_{i,n}$ and $p_{i,n}$ are the channel gain and transmitted power of the i-th user on subchannel n respectively. Moreover, the subchannel assignment coefficient $c_{i,n}$ is given as:

$$c_{i,n} = \begin{cases} 1, & \text{If subchannel } n \text{ is allocated to user } i \\ 0, & \text{Otherwise.} \end{cases} \tag{4.4}$$

On the other hand, we assume that each secondary user is equipped with an energy detector to sense the presence of PU on any of the N subchannels [16]. Recalling from Chapter 3, the output of the detector Y , is compared with the threshold λ to make a decision out of these two hypotheses (Figure 3.4):

$$\begin{cases} Y \geq \lambda & H_1 \\ Y < \lambda & H_0 \end{cases} \tag{4.5}$$

where H_1 indicates the presence of the PU and H_0 denotes that PU is inactive.

The probability that each SU correctly detects the presence of PU is $P_{d,i}$, and the probability that PU presence is falsely reported is $P_{f,i}$:

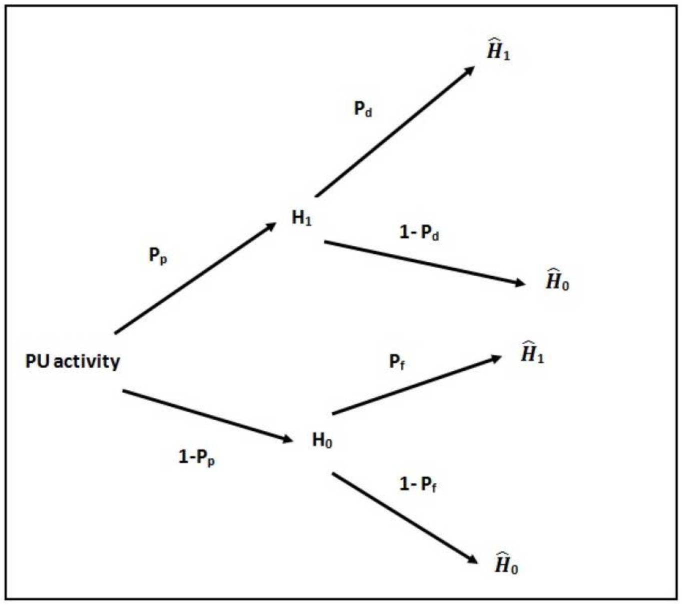

Figure 4.2: Sequential graph for SU sensing

$$P_{d,i} = P\{Y > \lambda | H_1\}$$

$$P_{f,i} = P\{Y > \lambda | H_0\} \tag{4.6}$$

Based on the above definition the following four cases could occur (Figure 4.2):

1. PU is present and SU detects correctly

2. PU is present and SU detects falsely

3. PU is inactive and SU detects the same

4. PU is inactive and SU detects active

Taking into account the sensing information, we define the utility per SU as follows:

$$\bar{U}_i = \sum_{n=1}^{N} \hat{H}_{i,n} r_{i,n}$$

$$= \sum_{n=1}^{N} \hat{H}_{i,n} c_{i,n} w \log_2(1 + c_3 p_{i,n} G_{i,n}) \tag{4.7}$$

where $\hat{H}_{i,n}$ is the sensing bit of the i-th user on subchannel n. $\hat{H}_{i,n} = 0$ means that the subchannel is occupied by the PU whereas $\hat{H}_{i,n} = 1$ indicates the availability of the the subchannel for SU usage.

The problem is how to allocate the N subchannels and subsequently the transmitted power among K SUs so that the maximum throughput is achieved. In order to define the total throughput we take the product of the rates from Nash Bargaining game. We assume that channel state information of all users are known. In order to model the resource allocation problem we set the utility function as follows:

$$\bar{U}_i = E\{U_i|H_0\}$$

$$= \sum_{n=1}^{N} E[\hat{H}_{i,n}|H_0] c_{i,n} w \log_2(1 + c_3 p_{i,n} G_{i,n}) \tag{4.8}$$

On the other hand, from Figure 4.2 it is obvious that:

$$P\{\hat{H}_{i,n}|H_0\} = \frac{(1-p_p)(1-p_f)}{(1-p_p)(1-p_f) + p_p(1-p_{d,i})}$$

$$= \beta_i \tag{4.9}$$

where p_p is the probability that PU is active on any of subchannels and β_i indicates how accurately a SU can sense the presence of PU as is defined in [15] as well. For example, the

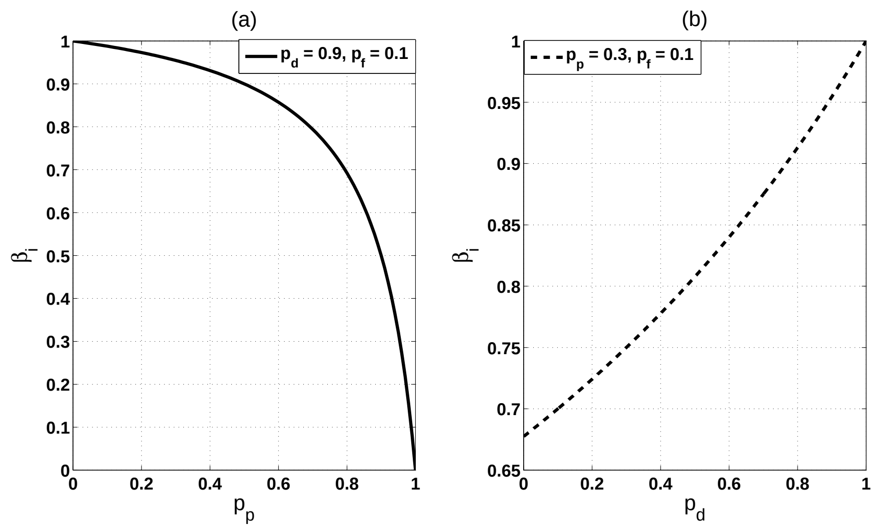

Figure 4.3: (a) β_i vs p_p, (b) β_i vs p_d

case of $p_{d,i} = 1$ results in $\beta_i = 1$ which states that detection is done with %100 certainty (see Figure (4.3)). Hence, (4.8) can be re-written as:

$$\bar{U}_i = \sum_{n=1}^{N} \beta_i c_{i,n} w \log_2(1 + c_3 p_{i,n} G_{i,n}) \tag{4.10}$$

Therefore, $F(S, (\bar{U}_{1,min}, \bar{U}_{2,min}, ..., \bar{U}_{K,min}))$ is a bargaining problem where the set S contains all the feasible rates, and its solution satisfies:

$$\arg \max_{\{\mathbf{P},\mathbf{C}\}, \bar{U}_i \geq \bar{U}_{i,min}} \prod_{i=1}^{K} (\bar{U}_i - \bar{U}_{i,min}) \tag{4.11}$$

subject to:

$$
\begin{aligned}
C_1 &: \quad \sum_{i=1}^{K}\sum_{n=1}^{N} p_{i,n} \leq P_{max} \\
C_2 &: \quad \bar{U}_i \geq \bar{U}_{i,min} \\
C_3 &: \quad c_{i,n} \in \{0,1\} \\
C_4 &: \quad \sum_{i=1}^{K} c_{i,n} = 1 \\
C_5 &: \quad p_{i,n} \geq 0 \\
C_6 &: \quad \beta_i \geq \beta_T
\end{aligned}
\tag{4.12}
$$

where $\mathbf{P} = [p_{i,n}]_{K\times N}$ is power allocation matrix and $\mathbf{C} = [c_{i,n}]_{K\times N}$ is subchannel allocation matrix. Moreover, P_{max} is the total power budget of all SUs. Constraint C_4 ensures that each subchannel is assigned to one user only. Constraint C_6 assures that sensing accuracy of the secondary network is greater that the acceptable threshold β_T.

4.2.1 Satisfying the Network Sensing Requirement

In order to solve the aforementioned resource allocation problem we start from constraint C_6 as it is independent from $p_{i,n}$ and $c_{i,n}$. For β_i to meet the minimum sensing accuracy requirement we need to have:

$$
\frac{(1-p_p)(1-p_f)}{(1-p_p)(1-p_f)+p_p(1-p_{d,i})} \geq \beta_T \tag{4.13}
$$

With further simplification we arrive at:

$$p_{d,i} \geq 1 - \left[(\frac{1-\beta_T}{\beta_T})(\frac{1-p_p}{p_p})(1-p_f) \right] \tag{4.14}$$

That is, in order to keep the constraint C_6, the detection probability should be greater than the threshold given in right-hand side of (4.14). On the other hand, the probability that the energy detector of i-th SU detects the PU signal for a Rayleigh fading channel, provided that PU is present, is given by [37]:

$$\begin{aligned} P_{d,i} &= e^{-\frac{\lambda}{2}} \sum_{n=0}^{m-2} \frac{1}{n!} \left(\frac{\lambda}{2}\right)^n + \left(\frac{1+\bar{\gamma_{i,j}}}{\bar{\gamma_{i,j}}}\right)^{m-1} \\ &\times \left(e^{-\frac{\lambda}{2(1+\bar{\gamma_{i,j}})}} - e^{-\frac{\lambda}{2}} \sum_{n=0}^{m-2} \frac{1}{n!} \left(\frac{\lambda\bar{\gamma_{i,j}}}{2\left(1+\bar{\gamma_{i,j}}\right)}\right)^n \right) \end{aligned} \tag{4.15}$$

where $m = TW$ is the energy detector time - bandwidth product, $\bar{\gamma_i} = \frac{Ph_i}{\sigma^2}$ is the average SNR received by the i-th SU from PU, P is the PU transmit power, and σ^2 is the noise variance. Furthermore, $h_i = \frac{1}{d_i^{\eta}}$ is the path loss where d_i is the distance between i-th SU and PU, and η is the path loss coefficient.

On the other hand, the false alarm probability is given by [37]:

$$p_{f,i} = p_f = P\{Y > \lambda | H_0\} = \frac{\Gamma(m, \frac{\lambda}{2})}{\Gamma(m)} \tag{4.16}$$

To obtain a better illustration of (4.16) we plot $p_{d,i}$ versus average SNR under different values of p_f in figure 4.4. For instance, to have $\beta_i \geq 0.9$, given $Pf = 0.1$ and $p_p = 0.3$, the detection probability should be greater than 0.76 according to (4.14) as is indicated in Figure 4.5 with gray shaded area. This means that the received average received SNR of the energy detector should be greater than 12 dB which is the shaded area in gray and

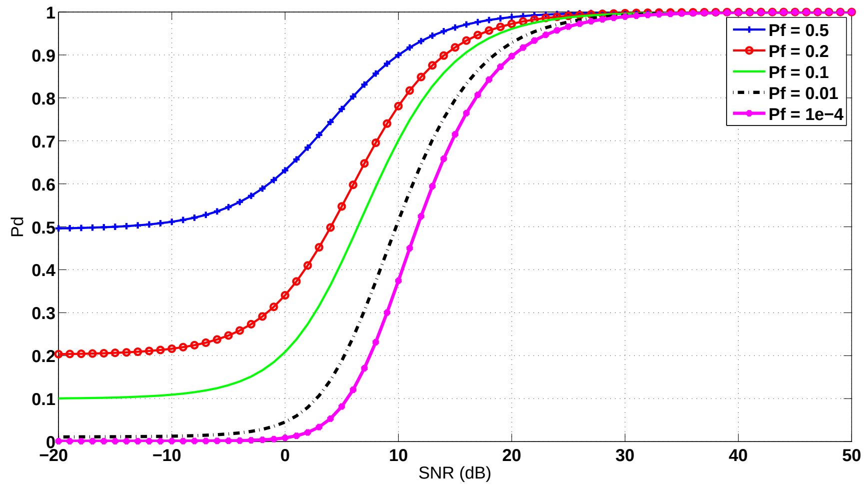

Figure 4.4: $p_{d,i}$ vs SNR for different values of P_f, $p_p = 0.3$

blue. Therefore, in addition to energy detector parameters (λ and m), the sensing accuracy of SUs is directly related to the received SNR of PU signal at the energy detector. In the case of deep fading or hidden terminal nodes the performance of energy detector reduces significantly which calls the need for collaborative sensing. A distributed collaborative sensing is presented in [16].

4.2.2 Resource Allocation Algorithm

We apply the same approach as in section 2.7.1 to solve the optimization problem in 4.11. By employing KKT conditions and some simplifications we obtain the following equations for power and subchannel allocation respectively:

$$p_{i,n} = \frac{\prod_{j=1,j\neq i}^{K} (\bar{U}_j - \bar{U}_{j,min})}{\lambda_i} \beta_i c_{i,n} w - \frac{1}{c_3 G_{i,n}} \tag{4.17}$$

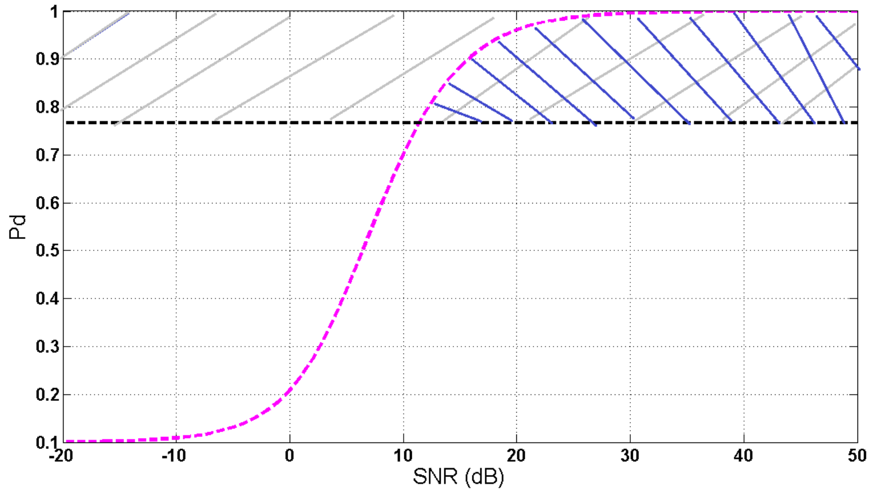

Figure 4.5: $p_{d,i}$ vs SNR, $Pf = 0.1$, $p_p = 0.3$, $\beta_T \geq 0.9$

$$\frac{\beta_1 \log 2(1 + c_3 p_{1,n} G_{1,n})}{(\bar{U}_1 - \bar{U}_{1,min})} = \cdots = \frac{\beta_i \log 2(1 + c_3 p_{i,n} G_{i,n})}{(\bar{U}_K - \bar{U}_{K,min})} \tag{4.18}$$

As it is seen, finding a closed form solution for $p_{i,n}$ and $c_{i,n}$ from (4.17) and (4.18) is an NP-hard problem. However, it casts light on the shape of the optimum solution. Looking at $c_{i,n}$ each fraction can be interpreted as the rate of each user in one subchannel to the total rate of all subchannels assigned to that user which illustrates the ratio of the rate in one subchannel to the total rate should be the same for all users. This clue asserts the fairness of the optimal solution and ,also, gives us a metric for subchannel allocation.

4.2.3 The Proposed Sub-optimum Power and Sub-channel Allocation Algorithm for a Cognitive ORDM Network

The proposed resource allocation scheme for cognitive uasers is detailed in Table 4.1. The approach is similar to non-cognitive radio case except that we need to check the accuracy

Table 4.1: The proposed subchannel and power allocation algorithm based on NBS

1. Meeting the network sensing requirement:
Check to see if $\beta_i \geq \beta_T$. If yes, start the allocation algorithm If no, disregards the sensing bits and wait for the new sensing bits..
2. Initialization for subchannel allocation:
Set $R_i = 0, \Omega_i = \emptyset$ for all $i = 1, 2, ..., K$. Set $A = \{1, 2, ..., N\}$, and $B = \{1, 2, ..., K\}$.
3. Meeting the minimum rate requirement:
While $B \neq \emptyset$, a. Find (i, n) so that $\lvert G_{i,n}\rvert$ is maximum for all $i = 1, ..., K$, and $n = 1, ..., N$. b. For the found i if $R_i \leq R_{i,min}$ Let $\Omega_i = \Omega_i \cup \{n\}, A = A - \{n\}$ and update R_i according to (4.3). else $K = K - \{i\}$.
4. Allocating the excess subchannels:
While $A \neq \emptyset$, a. Find i satisfying $\frac{G_{i,n}}{\sum_{j\in\Omega_i} G_{i,j}} \geq \frac{G_{m,n}}{\sum_{j\in\Omega_m} G_{m,j}}$ for all m,$1 \leq m \leq K$. Ω_i and Ω_m are the subchannels allocated to user i and m respectively; b. For the found i, find n satisfying $\lvert G_{i,n}\rvert \geq \lvert G_{i,k}\rvert$ for all $k \in A$; c. For the found i and n, let $\Omega_i = \Omega_i \cup \{n\}, A = A - \{n\}$ and update R_i according to (4.3).
4. Power water filling:
Water fill power based on (2.7.1) for each $\Omega_i, i = 1, 2, ..., K$.

of sensing bits.

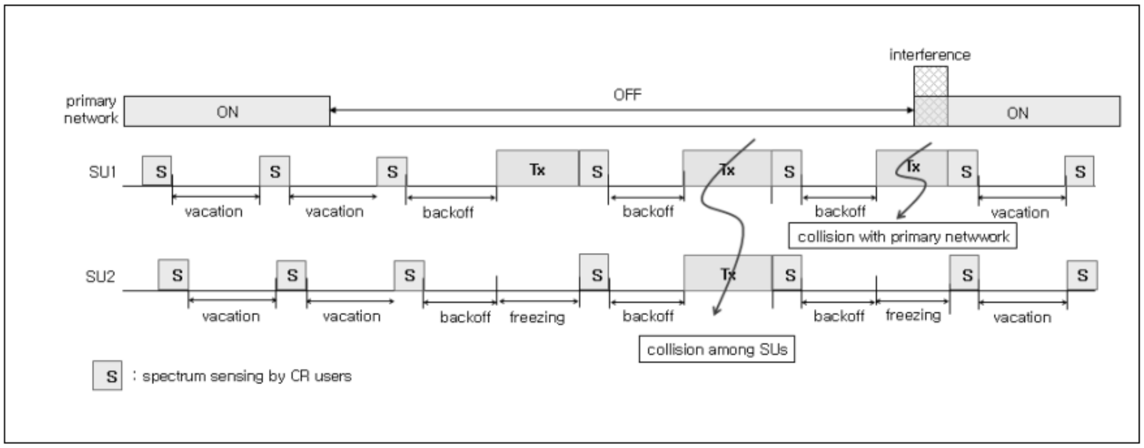

Figure 4.6: CSMA/CA protocol for cognitive radio networks

4.3 Medium Access Control in Cognitive Radios

The spectrum sensing of SUs can be further followed by collision avoidance techniques such as Carrier Sensing Multiple Access/ Carrier Avoidance (CSMA/CA) to prevent interference to both PU and other SUs. With successful employment of CSMA/CA in IEEE 802.11 standard, it can be applied to cognitive radio networks especially because of its listen-before-transmission nature. A cognitive-based CSMA/CA an operate as follows ([45]):

- An SU periodically senses the spectrum band with the structure of sensing and vacation during ON period when PU is active. If an SU finds the spectrum band to be in an ON state for a sensing time T_s, then the SU senses the spectrum band again after a fixed vacation time T_v. This process is repeated until it finds the spectrum band to be in an OFF state where PU is inactive.
- After the SU senses the spectrum band to be in OFF state in a sensing time, it operates the carrier sense multiple access/collision avoidance(CSMA/CA) with the binary exponential random backoff mechanism using IEEE 802.11 DCF as follows.

  1. The SU chooses a random number (so-called backoff counter) in the range $[0, CW-$

1] where CW is the contention window size and is initially set to W_0 at the backoff stage 0.

2. The SU counts down its backoff counter every time interval if the spectrum band is sensed to be idle.
3. While the backoff counter is decremented, the spectrum band might be occupied by either other SUs transmission or the transition from OFF period to ON period. If once the spectrum band is sensed to be busy by either one, the SU freezes its backoff counter for a SU packet transmission time T_p first. Then the SU senses the spectrum band for a sensing time T_s to check whether the spectrum band is in ON state or OFF state.
   - If the spectrum band is sensed to be idle in the sensing time, it means that the spectrum band is still in OFF state and so the SU resumes the backoff procedure from the frozen backoff counter point.
   - If the spectrum band is sensed to be busy in the sensing time, it means that the status of the spectrum band is changed to ON period and so the SU repeats alternating sensing and vacation, and then resumes the backoff procedure with the frozen backoff counter when the spectrum band is sensed to be in OFF state.
4. The SU transmits a packet when its backoff counter reaches zero.
5. If the packet transmission fails, the SU increases its backoff stage by one, doubles its CW and randomly chooses a backoff counter. Then SU proceeds the random backoff again with the newly chosen backoff counter.
6. If the backoff stage excess the maximum backoff stage, the SU discards its packet.

Figure (4.6) illustrates an example for operations of SUs described above [45]. To analyze the performance of a cognitive CSMA/CA SU's behavior could be modeled as a discrete time Markov Chain. The detail of this analysis is out of the scope of this dissertation and the interested readers are referred to [46] [47] [48].

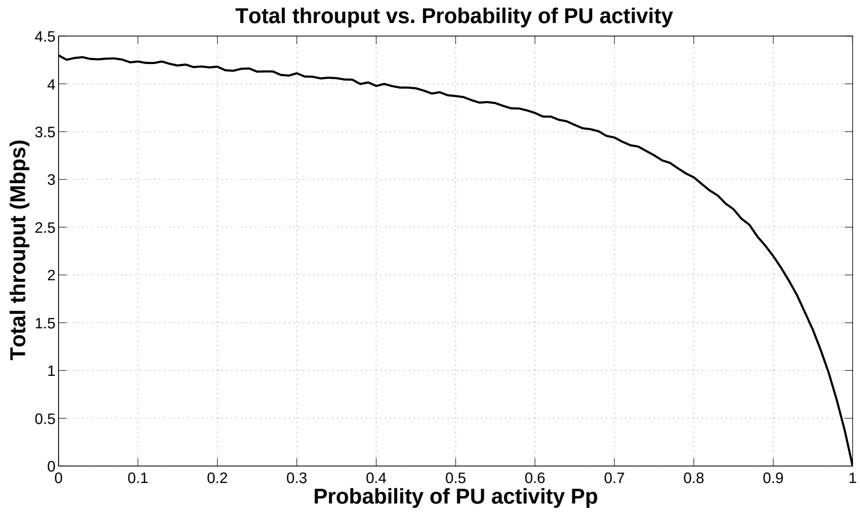

Figure 4.7: Total throughput vs. the probability of PU activity (K = 12, N = 32)

4.4 Simulation Results

In this section we evaluate the performance of the proposed subchannel and power allocation approach under different sensing parameters. It is assumed that the sub-channel gains are i.i.d. random variables with Rayleigh distribution. Noise spectral density is $N_0 = -110$ dBm-Hz and is the same for all K users. The total available bandwidth is $w = 3.2$ MHz, and the maximum allowable power per user is 0.3 W. Without loss of generality we set $\bar{U}_{i,min} = 0$.

Figure 4.7 illustrates the total throughput of the $K = 6$ users with $N = 32$ subchannels versus the probability of the PU activity. We observe that for the values of $P_p < 0.4$ the throughput does not drop that much. However, when P_p passes 0.5 the total throughput drops rapidly which can give us a threshold on where the co-existence of a PU and SUs are beneficial.

In Figure 4.8 we assess the total throughput of the secondary network under different values of β. We assume that all SUs are using an energy detector with the same parameters, therefore, the index i of β could be dropped. The very bottom plot is correspond to the

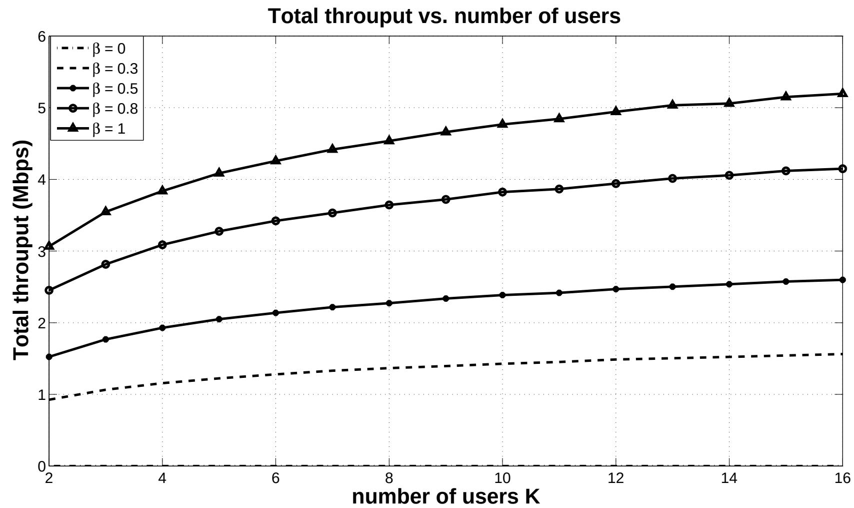

Figure 4.8: Total throughput vs. number of users (N = 32)

case of $\beta = 0$ which could be mapped to the case that PU is active all the time ($P_p = 1$) or the case that $P_f = 0$. In either case there would be no transmission opportunity for the SUs and no throughput will be achieved. As β increases the total throughput is growing accordingly which indicates that the more accurately the SUs sense the higher throughput is obtained. The maximum throughput is for the case of $\beta = 1$ which means either SUs are sensing perfectly $P_d = 1$ or PU is completely silent $P_p = 0$.

4.5 Conclusion

An effective dynamic subchannel and power allocation algorithm is proposed based on Nash bargaining Solution for an ad hoc network of secondary users with imperfectly sensing PU activity. It was shown that the total throughput of the secondary network is highly related to how accurately the sensing process is performing and how frequently the PU is present. The higher detection probability is desirable in order to take the best advantage of the temporarily available PU subchannels. Furthermore, a PU with more than %40 activity lowers the SU throughput sharply which could be an indicator for the SUs that the PU is not worth sensing.

Chapter 5: Summary, Contributions, and Future Work

5.1 Summary and Contributions

This dissertation addressed two important issues in cooperative cognitive radio networks: spectrum sensing in physical and dynamic spectrum allocation in MAC layer. First a dynamic subchannel and power allocation algorithm for the downlink of multiuser OFDM systems based on Nash Bargaining Solution was presented in Chapter 2. The NBS problem is decomposed into two sub-problems where the power allocation reduces to the well-known water-filling algorithm and the subchannel assignment leads to a simple algorithm which takes the total channel gain of each user as the fairness metric. Comparing to the Max-Sum approach which totally ignores the users with weak channel conditions and the Max-Min scheme which maximizes the worst user rate at the cost of scarifying the good users, the proposed algorithm balances these two extreme cases by weighting each user according to its total channel gain. Provided simulation results shows that the proposed algorithm performs better than the Max-Min approach, and closely to the Max-Sum case especially when the number of subchannels is comparably larger than the number of users. The proposed algorithm was implemented for the LTE systems as well which increased the total rate as much as %25 for the case of $K = 15$ users as compared to the fixed subchannel and power allocation schemes.

We further extend the dynamic resource allocation for the case of cognitive radio networks where some secondary users sense the spectrum of the primary system to obtain transmission opportunities. Since the sensing information in physical layer affects the resource allocation in MAC layer we first studied spectrum sensing methods in Chapter 3. Then, a collaborative and distributed spectrum sensing scheme was proposed to keep extra overhead as minimum as possible. We formulated the spectrum sensing problem as a

Stackelberg game and proposed a simple algorithm to solve the game. For this purpose, each SU in the network is considered either as a leader or follower. The leading SUs who are having the highest detection probabilities among the other users will broadcast their sensing information while the followers will take action sequentially based on what they receive from the leaders. As a result the whole network detection probability will increase since it is dominated by users having acceptable reception of the PU signal. The simulation result showed an improvement of %35 in average detection probability of the whole network for the case of two followers and three leaders scenario. Consequently, the proposed method significantly improves the average detection probability of the nodes as compare to the non-cooperative case especially when more SUs are suffering from fading.

Having spectrum sensing parameters from Chapter 3, we extended the proposed dynamic resource allocation scheme in Chapter 2 to encompass cognitive radio users as well. To this end, in Chapter 4 an effective dynamic subchannel and power allocation algorithm was proposed based on Nash bargaining Solution for an ad hoc network of secondary users with imperfectly sensing PU activity. It was shown that the total throughput of the secondary network is highly related to how accurately the sensing process is performing and how frequently the PU is present. The higher detection probability is desirable in order to take the best advantage of the temporarily available PU subchannels. Furthermore, a PU with more than %40 activity lowers the SU throughput sharply which could be an indicator for the SUs that the PU is not worth sensing.

5.2 Future Work

We can briefly list the possible extensions of this dissertation as follows:

- In chapter 2 we developed a dynamic subchannel and power allocation scheme for OFDM users assuming Single Input Single Output (SISO) systems. However, in all new generation wireless systems both base station and mobile users are equipped with multiple antennas. Therefore, we need to address the resource allocation problem for

Multiple Input Multiple Output (MIMO)- OFDM systems. This will add the space as a new dimension to the power and subchannel allocation.

- In the case of MIMO-OFDM systems since the number of antennas in base station side could be large, the load of the feedback required to report the channel information would be burdensome specially for Frequency Division Multiplexing (FDD) systems. Therefore, we may need to compress the feedback information using methods such as Compressive Sensing. A survey of compressive sensing and how to apply it to MIMO systems is presented in Appendix B.

- An interesting application of the resource allocation algorithm presented in Chapter 2 is developing a Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) based Mac protocol for cognitive radio networks. The listen-before-transmit nature of CSMA/CA makes it an ideal candidate for cognitive radio networks.

Appendix A: An Overview on Game Theory

Game theory is a mathematical tool to analyze the strategic interactions among multiple decision makers. While the goal in optimization theory is to optimize a single objective over a decision variable game theory is a solution for a multi-objective optimization problem. The major components in a strategic-form game are as follows:

- Finite set of players (N)
- A set of actions (x_i)
- Payoff/Utility function($u_i : X \to R$): Outcome of the game for player i determined by the actions of all players. For example let assume that $x_i \in R$, then:

$$u_i(x_i, x_{-i}), x_{-i} = (x_1, ..., x_{i-1}, x_{i+1}, ..., x_N), u_i \in R^N \tag{A.1}$$

  Here we have a multi-objective function. Given the notation above, a strategic game is denoted by $\langle N, (x_i), (u_i) \rangle$.

As a simple example let's assume that a group of players would like to make a pie together. Here Players have a common interest to make the pie as large as possible, but they have competing interests to maximize their own share of the pie.

A.1 Different Types of games

- Cooperative versus non-cooperative: there is no mutual agreement among players in non-cooperative game.
- Strategic- form or Static games versus Extensive-form or Multi-Stage or Dynamic Games: In strategic game players move simultaneously while in extensive form game players move sequentially. Moreover, the players are informed of the previous moves. For example, Chess is a strategic game.

Table A.1: Classifications of the games

Non-Cooperative	Cooperative
Static	Dynamic(Repeated)
Strategic form	Extensive form
Perfect information	Imperfect information
Complete information	Incomplete information

- Game with Perfect versus imperfect Information: in a perfect information game all information of history is perfectly known to all users. That is, each player knows the identity of other players and, also, the payoff resulting of each strategy per user. For instance, chess is a game with perfect information while poker is a game with imperfect information.

- Game with Complete versus incomplete information: each player can observe the action of each other player.

A.2 Nash equilibrium

In a non-cooperative game each player is only interested in his/her own benefit and takes action in order to maximize his/her utility function. The solution for such a game is the well-know Nash Equilibrium (NE)which is defined as follows:

Definition A.1. A Nash equilibrium of a strategic game $\langle N, (x_i, u_i)\rangle$ is the action set $x^* \in X$ such that for every player $i \in N$ we have

$$u_i(x_i^*, x_{-i}^*) \geq u_i(x_i, x_{-i}^*) \tag{A.2}$$

for all $x_i \in X_i$, where x_i indicates the action of player i and x_{-i} denotes the actions of all players other than player i. It is concluded from the definition that NE is a point that none of players tends to deviate from since it gives the best-response action for each player.

Theorem A.1. A strategic game $\langle N, (x_i), (u_i) \rangle$ has a Nash equilibrium if for all $i \in N$, the action set X_i of player i is a nonempty compact convex subset of a Euclidean space, and the utility function u_i is continuous and quasi-concave on X_i.

A.3 Stackelberg Games

Definition A.2. A Stackelberg game is a two-player extensive game with perfect information in which a "leader" chooses an action from a set X_1 and a "follower", informed of the leader's choice, chooses an action from a set X_2. A solution to a Stackelberg game is correspond to the following maximization problem:

$$\max_{(x_1,x_2) \in X_1 \times X_2} u_1(x_1, x_2) \tag{A.3}$$

subject to

$$x_2 \in \arg\max_{x_2' \in X_2} u_2(x_1, x_2')$$

It is seen that the action of the leader is given in the second equation.

[1] Appendix B: Compressive Sensing

B.1 Introduction

Compressive Sensing (CS), the idea of sampling below the well-know Nyquist rate, was first introduced in [49–51]. CS theory indicates that one can recover certain signals from far fewer samples than is needed in traditional methods. Although, CS gets its first applications in image processing, it quickly extends to telecommunication field as well. The problem of multiuser detection over a random access channel in Wireless sensor Networks seems to be a proper application for CS [52, 53]. Since the number of users are large with low activity ,then the received data at the aggregation point would be sparse. Hence, the fusion center can recover sensors data with less samples.Compressive Sensing has, also, attracted the interest of researchers in the field of Massive-MIMO [54]. Since the number of antennas in a Massive-MIMO system is very large, the load of the feedback required to report the channel information would be burdensome specially for Frequency Division Multiplexing(FDD) systems. Another application of Compressive Sensing in Cognitive Radio Networks for sensing a Primary User(PU) spectrum holes is proposed in [55]. It is shown that as the PU activity follows a sparse pattern, the spectrum usage information can be recovered from a small number of reports by Secondary Users. A quick start-up in Compressive sensing is given in [56].

In this chapter, we are interested in compressing channel information in a massive MIMO system and its direct effect on the channel rate. The rest of this report is as follows: a brief introduction on CS in given in Section B.2 . In Section B.3 we focus on modeling and simulation of aggregate channel rate with perfect and compressed information in a massive MIMO environment. In the end, Section B.4 concludes the chapter.

The results of this chapter is presented in [57] as well.

[1] This work was supported by Bell labs, Germany during my internship in summer 2012.

B.2 The basics of Compressive Sensing

Assume that we have a signal $f(t)$ which could be expressed in an orthonormal basis $\Psi = [\psi_1, \psi_2, ..., \psi_n]$ as follows:

$$f(t) = \sum_{i=1}^{n} x_i \psi_i(t), \qquad f \in \mathbb{R}^N \tag{B.1}$$

or equivalently, $x_i =< f, \psi_i >$, where $< . >$ denotes inner product operation. We measure f with another basis, Φ, as $y_k =< f, \phi_k >, k = 1, 2, ..., m$. Then, $\mathbf{x}$ can be fully recovered from $m < n$ samples by solving the following l_1 minimization problem:

$$min_{\tilde{x} \in \mathbb{R}^N} \left\| \tilde{x} \right\|_{l_1} \tag{B.2}$$

subject to $y_k =< \phi_k, \Psi\tilde{x} >, \quad \forall k \in M$.

Theorem B.1. If $\mathbf{x}$ is S- sparse in Ψ, then it can be recovered exactly with overwhelming probability by solving the above mentioned l_1 minimization given that the number of taken samples satisfies the following condition:

$$m \geq C\mu^2(\Phi, \Psi) S log n. \tag{B.3}$$

where C is some positive constant [49]. The probability of success exceeds $1 - \delta$ if

$$m \geq C\mu^2(\Phi, \Psi) S log(n/\delta). \tag{B.4}$$

where $\mu^2(\Phi, \Psi)$ is defined as the coherence between the sensing basis (Φ), and the representation basis(Ψ):

$$\mu(\Phi, \Psi) = \sqrt{n}.max \left|< \phi_k, \psi_j >\right|, \qquad 1 \leq k, j \leq n. \tag{B.5}$$

B.2.1 Robust Compressive Sensing

We saw that the exact reconstruction for a sparse signal is possible with much fewer samples, but in practice signals may be almost sparse and there is some noise. To count these two issues into account we are interested in recovering $x \in \mathbb{R}^n$ from the following data:

$$y = Ax + n \tag{B.6}$$

where A is an $m \times n$ "'sensing" or "measurement" matrix, $y \in \mathbb{R}^m$ is our observation vector, and z is the noise term. We need to solve an under-determined system as we have more unknowns(n) than equations(m). It is shown that if A meets RIP(Restricted Isometry Property), then x can be recovered from under-sampled data y.

Definition B.1. Restricted Isometry Property: For each integer $S = 1, 2, ...$ define the isometry constant δ of a matrix A as the smallest number such that

$$(1 - \delta_S) \|x\|_{l_2}^2 \leq \|Ax\|_{l_2}^2 \leq (1 + \delta_S) \|x\|_{l_2}^2 \tag{B.7}$$

holds for all S- sparse x [58].

That is, y tends to x in l_2 norm which implies that δ_S is not too close to one. Another interpretation is that a matrix obeys RIP of order δ_S if all subsets of S columns taken from A are almost orthogonal. The importance of above theorem is that if RIP holds then the l_1 minimization will give an accurate reconstruction of x from y.

Random Sensing: How to make RIP-met sensing matrices

In order to check whether a sensing matrix meets RIP, one has to evaluate the inequality in (B.7) which does not seem easy to handle. The following matrices will hold RIP condition:

- If the elements of A are random i.i.d Gaussian samples with zero mean and variance $1/m$.

- If A is formed by taking n random columns from unit sphere of $\mathbb{R}^m$.
- If the elements of A are random i.i.d entities of a symmetric Bernoulli distribution with $P(A_{i,j} = \pm 1/\sqrt{m} = 1/2)$ or other sub-Gaussian distribution.

All the above matrices obey RIP with overwhelming probability given that:

$$m \geq C.Slog(n/S) \tag{B.8}$$

where C is some constant depending on each application.

Sparsifying Matrix

In some practical cases x is not Sparse in its own domain and needs proper mapping which is called sparsifying transformation:

$$A = \Phi\Psi \tag{B.9}$$

where $\Phi_{m\times n}$ is the measurement matrix and $\Psi_{n\times n}$ is an arbitrary ortho-basis for sparsifying. Hence, x is not sparse by itself but Ψx is. If Ψ is arbitrary and Φ is formed based on the methods described earlier, then A still obeys RIP [56].

Example 1: A Sparse signal in frequency

This example shows a sparse signal in frequency. The input signal is a sinusoid with the frequency $f_1 = 20Hz$. Since $x(t)$ is not sparse by itself we define the sparsifying matrix Ψ as the 512×512 DFT matrix. The measurement matrix is $m = 124$ random columns taken from 512×512 IDFT matrix. Figure B.1 shows the time and frequency domain representation of $x(t)$, and figure B.2 represents the recovered $x(t)$ from m samples.

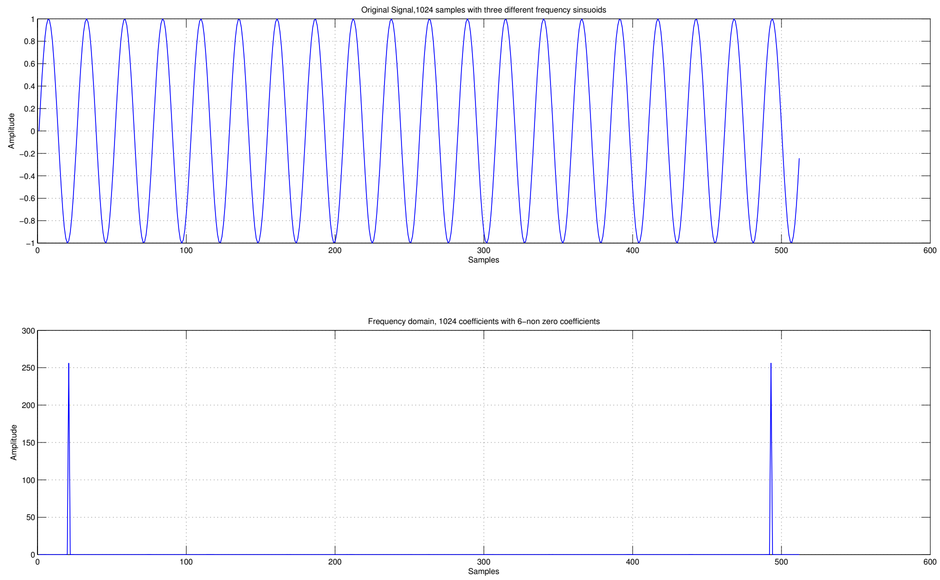

Figure B.1: A sinusoid in time and frequency

Example 2: A Sparse signal in time

In this section a sparse pulse with S non-zero samples is generated in time domain. The sensing matrix is random columns taken from a $N \times N$ DFT matrix. Moreover, the sparsity ratio is defined as S/N. The applied recovery algorithm is Orthogonal Match Pursuit(OMP) which is categorized as greedy algorithms and is faster and easier-to-implement than L_1 minimization. Then, the MSE error between the original signal and the recovered one is evaluated for different sparsity and SNR ratios and is demonstrated in Figure B.3. As we see, the more sparse the signal is, the lower MSE is achieved. And, the performance of the recovery algorithm is very sensitive to noise which makes the recovery almost impossible for the SNRs below $0dB$. For the SNR above $30dB$ the recovery algorithm performs the same as no noise case (Figure B.4)which is an interesting indicator for compression cost in terms of SNR.

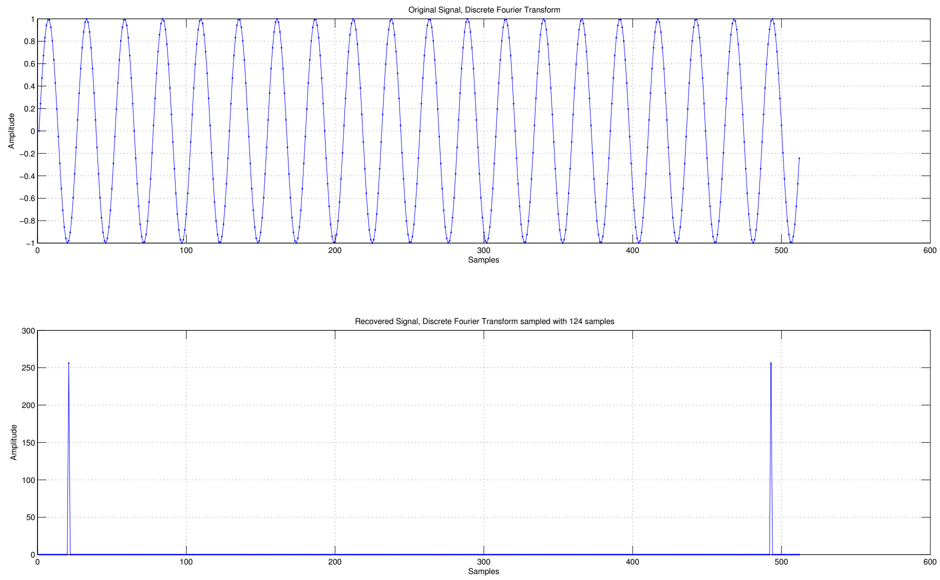

Figure B.2: Recovered sinusoid from compressed samples

Figure B.5 shows one snapshot of the above example for the case of $m = 100$, $N = 500$, $S = 10$, and no noise.

B.3 Channel Rate with "perfect" and "Compressed" Information

In this section we would like to see how much one can compress a wireless channel based on achievable rate.

B.3.1 System model

Consider a base station(BS) with N transmitting antennas operating in downlink, and K single antenna mobile users. We assume M is large enough to create some correlation among the signals reaching each element of base station array.Then, the $N \times 1$ channel vector$\mathbf{h_k}$

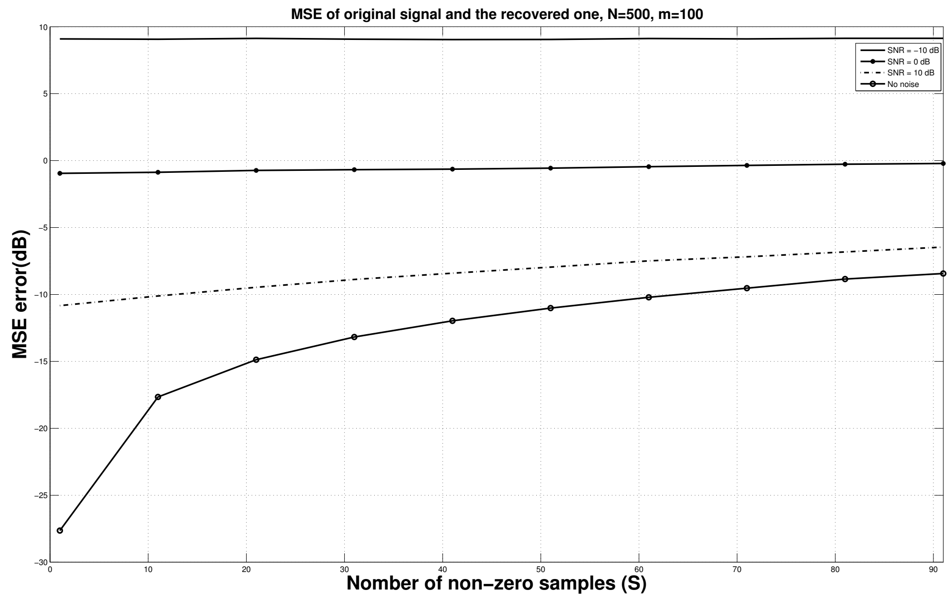

Figure B.3: MSE of Original and recovered time sparse pulse

between the k^{th} user and the base station can be expressed as:

$$\mathbf{h_k} = \mathbf{R}_{N\times N}^{1/2}\mathbf{z}_{N\times 1} \tag{B.10}$$

where $\mathbf{R}_{N\times N}^{1/2}$ is the correlation matrix of BS antennas,and $\mathbf{z}_{N\times 1}$ is the channel gain with i.i.d zero mean and unit variance complex Gaussian entities. The i^{th} row and the j^{th} column of $\mathbf{R}$ is defined as:

$$r_{ij} = J_0(\frac{2\pi\Delta d_{ij}}{\lambda}) \tag{B.11}$$

where $J_0(.)$ is the zeroth-order Bessel function of first kind, $d_{i,j}$ is the distance between the i^{th} and the j^{th} antennas of the BS, λ is the wavelength, and Δ is the angel spread. The

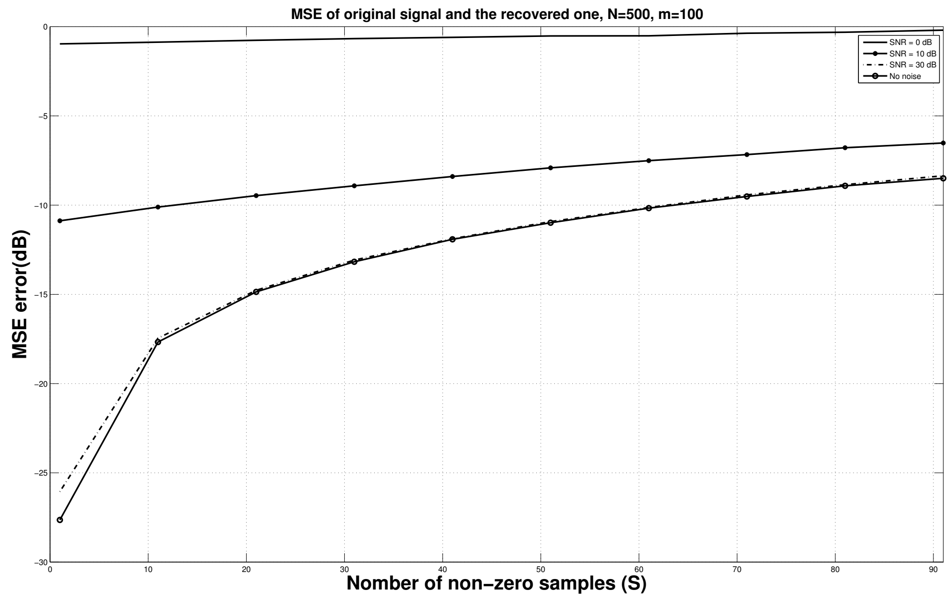

Figure B.4: MSE of Original and recovered time sparse pulse

received signal at the k^th mobile user can be written as:

$$y_k = \underbrace{\sqrt{\frac{P}{K}}\mathbf{h_k}^T\mathbf{w_k}s_k}_{desiredsignal} + \underbrace{\sqrt{\frac{P}{K}}\sum_{i\neq k}\mathbf{h_i}^T\mathbf{w_i}s_i + n_k}_{interference+noise} \tag{B.12}$$

where $\mathbf{w_k}$ is the channel precoder, s_k is the user message with unit power, and P is the BS transmitting power. Assuming a matched filter precoder for the channel, then $\mathbf{w_k} = \mathbf{h_k}^*$, and the SINR ratio for the k^{th} can be expressed as:

$$SINR_k = \frac{\frac{P}{\alpha K}\left|\mathbf{h_k}^T\mathbf{h_k}^*\right|^2}{1 + \frac{P}{\alpha K}\sum_{i\neq k}\left|\mathbf{h_i}^T\mathbf{h_i}^*\right|^2} \tag{B.13}$$

where α is the precoder normalizing factor. Hence,the achievable rate for all K users would be:

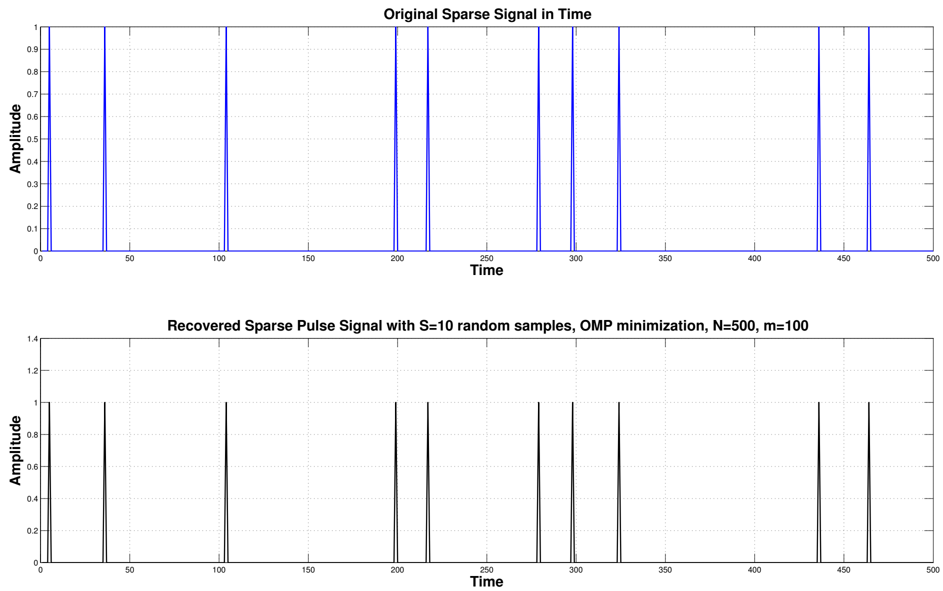

Figure B.5: Original and recovered time sparse pulse

$$R_{total} = \sum_{k=1}^{k=K} log_2(1 + SINR_k) \tag{B.14}$$

B.3.2 Channel rate with Compressed Channel Information

Having $\mathbf{h_k}$ as the channel vector per user, the channel matrix $\mathbf{H}$ can written as:

$$\mathbf{H}_{N\times K} = [\mathbf{h_1}\mathbf{h_2}...\mathbf{h_K}] \tag{B.15}$$

In order to compress $\mathbf{H}$ we first need to convert it to the vector format as follows:

$$\mathbf{h}_{1\times NK} = [\mathbf{h_1}^T\mathbf{h_2}^T...\mathbf{h_K}^T] \tag{B.16}$$

where $\mathbf{h}$ is the vectorized format of $\mathbf{H}$. The sensing matrix A is chosen as m random columns from $NK \times NK$ DFT matrix. Hence, the compressed $\mathbf{h}$ is

$$\tilde{\mathbf{h}} = A\mathbf{h} \tag{B.17}$$

Therefore, the compression ratio would be $\eta = m/NK$. The k^{th} user channel vector can obtain by converting the channel vector $\tilde{\mathbf{h}}$ to its matrix format $\tilde{\mathbf{H}}$.Then, the total rate with new channel vector, $\hat{\mathbf{h}}$, obtained from solving the L_1 minimization for (B.17) would be:

$$R_{\tilde{total}} = \sum_{k=1}^{k=K} log_2(1 + \frac{\frac{P}{\alpha K}\left|\mathbf{h_k}^T\hat{\mathbf{h_k}}^*\right|^2}{1 + \frac{P}{\alpha K}\sum_{i\neq k}\left|\mathbf{h_i}^T\hat{\mathbf{h_i}}^*\right|^2}) \tag{B.18}$$

B.3.3 Simulation results

In this part we will evaluate the performance of a channel with perfect information versus the imperfect one due to compression. To this end, we measure the total rate once when the channel vector $\mathbf{h}_k$ is known and when $\tilde{\mathbf{h}_k}$ is recovered using $m < N$ samples. we set $N = 100$, $K = 10$, $d = \lambda/2$, and the compression ratio as $\eta = \frac{m}{N}$. Figure B.6 shows the total rate versus SNR for different compression ratios. It is seen that the more we compress the channel, the more cost we need to pay in terms of rate.

In Figure B.7 the total rate is plotted versus SNR for several numbers of BS antennas N. The compression ratios is set to $\eta = 0.3$. It is obviously seen the total achievable rate is directly related to the number of BS antennas. This is not surprising as by increasing N the correlation among the received signal at BS is increased as well which lowers the effect of data loss due to compression.

Figure B.8, and B.9 show channel response in time and the reconstructed version of it. Considering theses two figures it is obvious that channel response is not sparse in time.

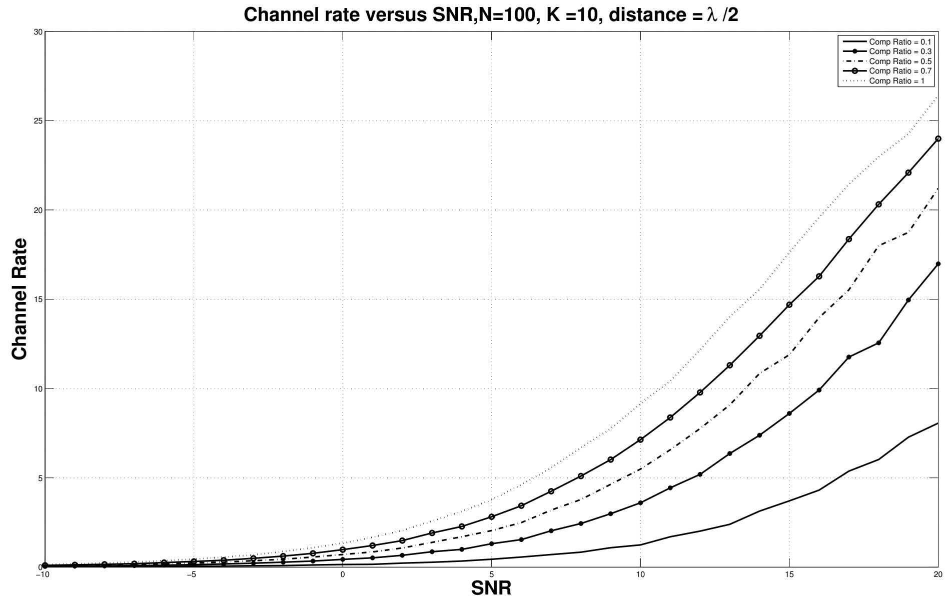

Figure B.6: Channel rate with perfect and compressed information for different compression ratios

That is the reason for the total rate loss when we compress the channel information. One idea is to map the channel information to a proper field to make it sparse first and then apply any compressing algorithm in order to keep the rate loss as low as possible. This issue will be taken into account in future work.

The channel rate with perfect and compressed channel information is simulated. As a future work, it would be interesting to evaluate this problem analytically. One possible option is to solve the following optimization problem:

$$min_{\mathbf{h}\in\mathbb{R}^{N}K}\,\|\mathbf{h}\|_{l_1} \tag{B.19}$$

subject to $\tilde{\mathbf{h}} = A\mathbf{h}$, and $R_{total} \leq R_{min}$.

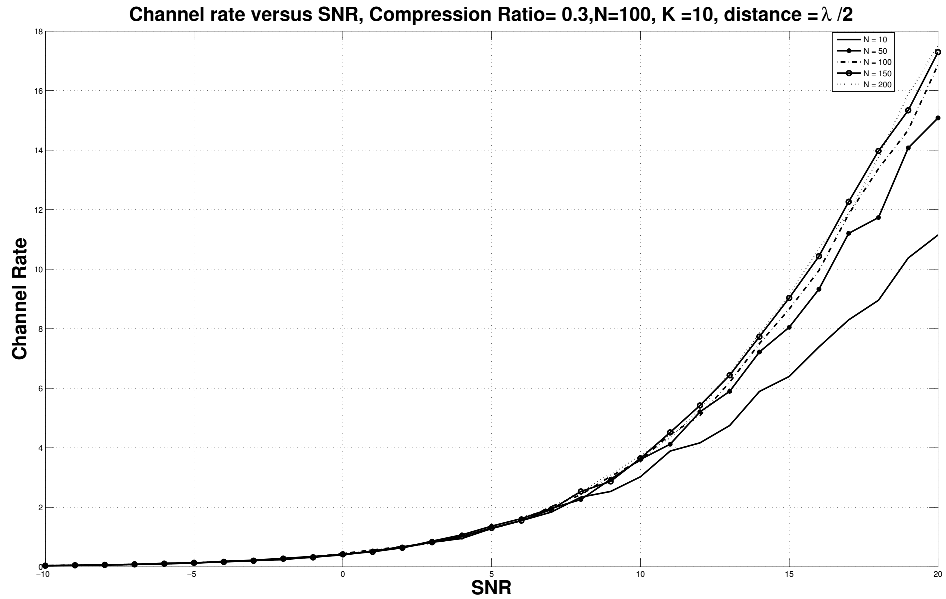

Figure B.7: Channel rate with compressed information for different numbers of BS antennas N

That is, how much we are allow to compress $\mathbf{h}$ so that the total achievable rate requirement is met.

Moreover, in order to minimize the rate loss a proper sparsifying basis is needed as we saw in previous section that channel response is not sparse in time domain. This would be another point to consider for the future work.

B.4 Conclusion

A brief introduction to Compressive Sensing was provided in this report. Then, it is applied to reduce the amount of channel information needed to be reported to the BS in a downlink scenario. Simulation results show that the total rate decreases as the compression ratio increases. Moreover, the compression ratio is directly proportional to the number of BS antennas as it spatially saprsifies the channel information in order to apply a CS algorithm.

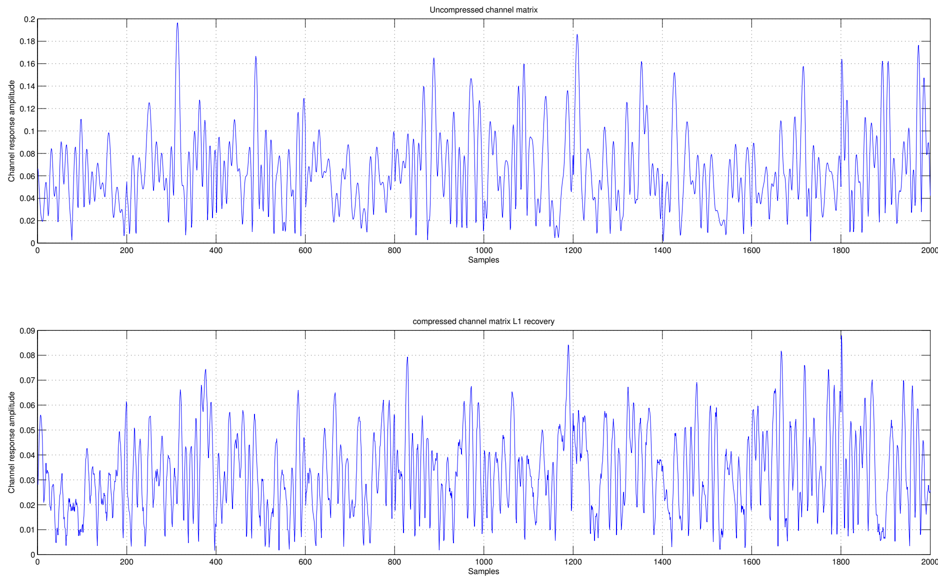

Figure B.8: Original and recovered channel response

However, according to simulation results a proper sparcifying basis is still needed as CS theory claims that it gives you the exact reconstruction under some condition, which would be the subject of the future work.

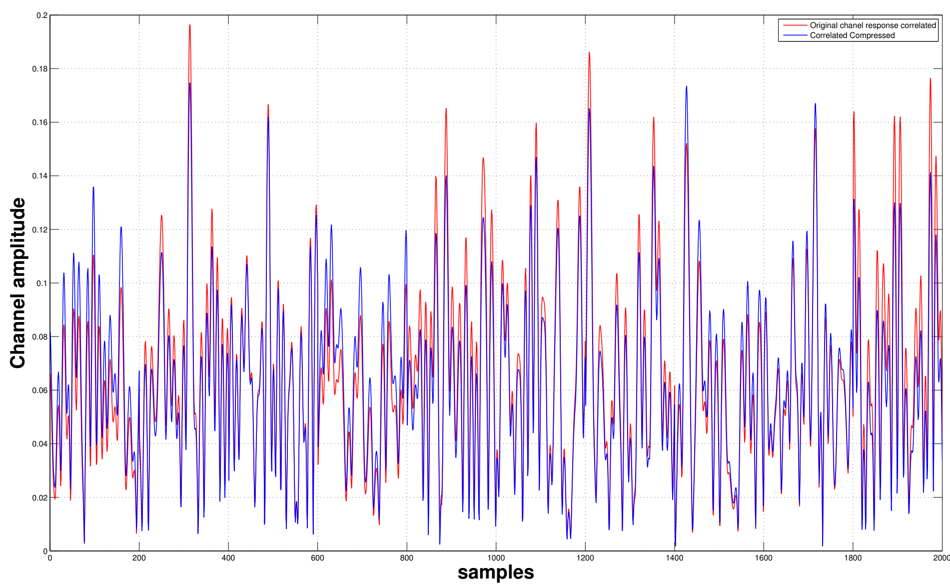

Figure B.9: Original and recovered channel response

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