THE ROLE OF MATHEMATICS ENGAGEMENT AS A MEDIATOR IN THE RELATIONSHIP AMONG ATTRIBUTION FOR FAILURE, MATHEMATICS SELF-EFFICACY, AND MATHEMATICS PERFORMANCE

by

Silvia E. Moore
A Dissertation
Submitted to the
Graduate Faculty
of
George Mason University
in Partial Fulfillment of
The Requirements for the Degree
of
Doctor of Philosophy
Education

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Date: July 14, 2016 Summer Semester 2016
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Dedication

I dedicate my dissertation to my husband, Thomas, whose support and encouragement made this work possible. Thank you for allowing me to lock myself away to write for hours on end. Thank you for reassuring me that there was an end to the madness that is writing, revising, editing, and writing again. Most of all, thank you for believing in me and insisting that my work was important.

Acknowledgements

I would be remiss if I did not thank my committee first and foremost. I would like to thank my Ph.D. chair and cheerleader, Dr. Anastasia Kitsantas, who has been with me since the beginning. As a mentor, she has been generous with her time and experience, and I have benefitted from her guidance throughout my time at George Mason University. I would also like to thank Dr. Ellen Rodgers who spent many hours providing meticulous feedback on early drafts, and whose enthusiastic encouragement has been absolutely invaluable. Last, but definitely not least, I thank Dr. Jahanzeb Cheema who generously shared with me his expertise in statistical analysis of large datasets. His help was instrumental for completing this research.

I am indebted to Flint Hill School, Oakton, Virginia, for its financial support and encouragement. My Flint Hill colleagues inspire me every day with their extraordinary efforts in the classroom.

Thank you also to my children and grandchildren who kept me going and understood when I occasionally had to cancel Sunday dinners.

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Abstract

THE ROLE OF MATHEMATICS ENGAGEMENT AS A MEDIATOR IN THE RELATIONSHIP AMONG ATTRIBUTION FOR FAILURE, MATHEMATICS SELF-EFFICACY, AND MATHEMATICS PERFORMANCE

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The purpose of this study was to validate the existence of a three-dimensional mathematics engagement model, and to examine the mediating effects of mathematics engagement on the relationships among attribution for failure, mathematics self-efficacy, and mathematics performance, while controlling for gender, ethnicity, and economic, social, and cultural status (ESCS). The study used survey items from the 2012 Program for International Student Assessment (PISA) comprising responses from a sample of 4,978 fifteen-year-old students. The results revealed that the three-dimensional mathematics engagement model was confirmed: Mathematics engagement was shown to have a marginal mediating role in the relationships among attribution for failure, mathematics self-efficacy, and mathematics performance. Supplementary analysis revealed differences in attribution for failure, mathematics self-efficacy, mathematics engagement, and mathematics performance exist by gender, ethnicity, and ESCS. Overall, this study showed support for conceptualizing mathematics engagement as a

three-dimensional construct and its role as a mediating variable. This line of research has the potential to provide compelling data to understand how student engagement—and the differing patterns of student engagement—are associated with outcomes and have implications for future research and practice.

Chapter One

Mathematics Performance

Mathematics performance is essential for students' success in school, and increasingly for future growth careers (Brusi, Portnoy, & Toro, 2013; National Center for Education Statistics [NCES], 2012a, 2015; Stinson, 2004). There is widespread concern in the United States over students' mathematics performance (President's Council of Advisors on Science and Technology [PCAST], 2012). Student scores on standardized tests have failed to improve since 2004, with U.S. students ranking 24th among 33 developed countries (Organization for Economic Cooperation and Development [OECD], 2012). Evidence suggests that students who complete a mathematics sequence from Algebra II to Precalculus or Calculus by 12th grade are better critical thinkers and more likely to successfully complete college (Achieve, Inc., 2008, 2011; Adelman, 2006; Klepfer & Hull, 2012). Yet, students are still not learning the mathematicsthey need for general success in their education or career (Achieve, Inc., 2008; PCAST, 2012). Individuals with at least some college are more likely be employed, as 19 of the projected 30 fastest growing occupations between 2012 and 2022 (e.g., occupational/physical therapy, nurse practitioner, information security analyst) will require some level of postsecondary education for entry (Bureau of Labor Statistics [BLS], 2015).

Several factors have been identified as determinants of mathematics achievement, such as valuing learning, teacher support, student motivation, and mathematics engagement (Martin, Way, Bobis, & Anderson, 2014). The degree to which students value learning has been shown to significantly impact their engagement in mathematics across grade levels: When students feel that what they are learning is instrumental, they are more likely to meaningfully engage in the process (Green, Miller, Crowson, Duke, & Akey, 2004; Martin et al., 2014). Likewise, student perception of teacher support has been shown to positively influence students' level of mathematics engagement and performance (Rimm-Kaufmann, Baroody, Larsen, Curby, & Abry, 2015). There is consensus among educators that increasing student engagement in mathematics is key to improving mathematics performance (National Assessment of Education Progress [NAEP], 2015; OECD, 2013; PCAST, 2012). Therefore, this study is focused on mathematics engagement as a predictor of mathematics performance, and secondarily engagement as a mediator of the relationships among attribution for failure, mathematics self-efficacy, and mathematics performance, while accounting for the effects of gender, ethnicity, and economic, social, and cultural status (ESCS, a Program for International Student Assessment (PISA)-derived index and a proxy for socioeconomic status SES).

Engagement is widely defined as effortful investment in learning (Appleton, Christenson, & Furlong, 2008; Christenson, Reschly, & Wilie, 2012; Fredricks, Blumenfeld, & Paris, 2004). Students who are engaged are active participants in the learning process: They use deep strategies to make meaning of ideas and concepts, turn homework in on time, and hold positive feelings about their learning experiences. This study draws from the Fredricks et al. (2004) conceptualization of engagement as a three-dimensional construct comprised of students' cognition, behaviors, and emotions. Engagement is separate from, but nonetheless related to, motivation: Engagement represents the action that follows a student's motivational intent.

The cognitive dimension of engagement is defined as being personally invested and having a willingness to exert effort to master ideas and skills, for example, using deep strategies and creating more connections among ideas (Fredricks et al., 2004 Fredricks & McColskey, 2012). Cognitive engagement is less observable and more indicative of the internal processes taking place during learning.

The behavioral dimension of engagement is defined as participation. More specifically it is units of actions specific to learning (Christenson, Reschly, & Wylie, 2012; Fredricks et al., 2008), characterized as involvement in actions directly connected to learning such as completing homework on time, listening in class, or keeping work organized.

The emotional dimension of engagement is defined as student feelings about learning and is linked to a willingness to do work. It is characterized as enjoyment in the learning or activity at hand. It can also reflect a student's feelings of anticipation, as in looking forward to lessons.

These engagement dimensions are assumed to be interconnected. Ladd and Dinella (2009), in a study of students in Grades 1 to 8, demonstrated the interactive and additive features of students' school engagement and long-term academic outcomes. Chase, Hilliard, Geldhof, Warren, and Lerner (2014) confirmed the reciprocal relationship amongst the dimensions of engagement for students in Grades 10 to 12 using a structural equation model that showed the mutually predictive value of engagement's dimensions and achievement over time.

Motivation and engagement are distinct constructs, yet the literature does not always distinguish them as such (Appleton, Christenson, Kim, & Reschly, 2006; Appleton et al., 2008). Motivation is the intent to learn; engagement is the action of learning (Christenson et al., 2012; Lam, Wong, Yang, & Liu, 2012). That is, engagement is preceded by motivational factors (e.g., attribution for failure and self-efficacy). Self-efficacy is a student's belief in their ability to organize and execute action for learning (Lam et al., 2012; Linnenbrink & Pintrich, 2003). Mathematics self-efficacy, or the confidence a student has in his or her ability to do math, plays an important motivational role in students' decisions about whether or not to engage in mathematics. Attribution for failure is the explanation students give for why they were unsuccessful. Attributions for failure—why students believe their performance falls short—have implications over their subsequent engagement and outcomes in mathematics. Attribution for failure is an aspect of attribution theory that encompasses beliefs surrounding competence and control.

This study draws on the concept of engagement proposed by Fredricks et al. (2004) and frames it using a social cognitive perspective that acknowledges the influential role played by students' personal factors. Therefore, this study of students' mathematics engagement examines the interrelationships amongst mathematics engagement, attribution for failure, mathematics self-efficacy, mathematics performance, gender, ethnicity, and ESCS. Specifically, this study posits a three-dimensional model in which engagement is comprised of students' cognition, behaviors, and emotions. Additionally, this study hypothesizes engagement mediates the relationships between students' personal factors (e.g., attribution for failure and mathematics self-efficacy) and mathematics performance, while controlling for the impact of gender, ethnicity, and ESCS.

Historical Overview of Mathematics Performance

The waning mathematics performance of U.S. students is better understood in the historical context of mathematics education in the United States. In the period from 1933 to 1954 only 39% of all U.S. high school students were enrolled in mathematics courses beyond Algebra. Mathematics performance continued to decline throughout the 20th century's progressive education period (1890-1960s), a period defined by experiential learning and strongly differentiated by social class (Klein, 2003). Progressive education proponents maintained at the time that only practical mathematics should be taught to the general population; advanced mathematics, they argued, was a luxury that few could or should undertake (National Education Association [NEA], 1920). By 1983, enrollment in courses at or beyond Algebra had decreased to 31%, a trend finally recognized as detrimental to the nation's global competitiveness. The Presidential Commission report entitled *A Nation at Risk: The Imperative for Educational Reform* found that mathematics education from 1975 to 1983 reflected low expectations at the high school level, low enrollment in advanced mathematics courses, and low college entrance requirements (National Commission on Excellence in Education [NCEE], 1983).

The 1990s were marked by policy encouraging equality in education (e.g., The National Council of Teachers of Mathematics [NCTM] professional standards for curriculum teaching and assessment; American Mathematical Society [AMS]). One aim of the NCTM standards was to equalize the quality of mathematics education for students from all socioeconomic status (SES) levels and ethnic backgrounds. However, opposition groups argued that the standards did little to develop students' arithmetic and algebra skills (Klein, 2003). This was the beginning of the "math wars," fueled by a report from The National Assessment of Educational Progress (NAEP, 2015) showing only 17% of 12th graders in 1996 performing at or above the national proficiency level. By 2005 this level had climbed only modestly to 23% and by 2012 to 26%, prompting the nation's leading scientists to call on President Obama to address the mathematics preparation gap or risk increasing the income inequality of American workers.

The high percentage of students underperforming in mathematics is worrisome as it is an essential skill needed to enter and complete college, attain higher earnings, and achieve upward mobility (Stinson, 2004). According to projected job market figures, an estimated 50% of the jobs in the future will necessitate adequate preparation in science, technology, engineering, and mathematics (STEM) (Langdon, McKittrick, Beede, Khan, & Doms, 2011); yet 40% of students who intend to pursue a degree in a STEM field fail to complete the degree (PCAST, 2012). Factors affecting student persistence in STEM fields include lack of confidence in their ability; dissatisfaction with instructional delivery (e.g. lecture versus active exploration); and lack of identification with a group or community that promotes engagement. Among these factors, engagement in mathematics is recognized as a significant issue (Appleton & Lawrenz, 2011; Chase et al., 2014; National Mathematics Advisory Panel [NMAP], 2008; PCAST, 2012). Student must be engaged in mathematics to excel. More studies are needed that investigate mathematics performance as a function of student engagement to better understand the complexity and intricacies of the learning process and what it takes to achieve.

A Historical Overview of Student Engagement

Early research situated engagement as a protective factor inside the school dropout literature, which defined it as participation in learning activities (Brophy, 1983; Mosher & McGowan, 1985). This literature found that contextual factors (e.g., task variables and situational factors) strongly influenced students' behavioral engagement, which in turn impacted school dropout rates. For the most part, researchers approached engagement from a developmental perspective focusing on several observable behaviors of engagement: absenteeism, participation, and delinquency. Engagement was conceptualized as having multiple determinants that were interactive rather than additive or mediatory, and included psychological factors such as intelligence, self-concept, aspirations, attitudinal aspects, valuing, and attributional beliefs (Mosher & McGowan, 1985).

The view of engagement as a viable construct for predicting achievement expanded with research grounded in achievement-motivation theory which focused on engagement as an outcome of school processes (Finn & Voelkl, 1993; Skinner & Belmont, 1993). Finn and Voelkl (1993) used hierarchical linear modeling to show that school engagement predicted the successful achievement outcomes of 6,488 eighth-grade students. In turn, school engagement was linked to environmental factors such as enrollment and the racial/ethnic composition of schools. This study conceptualized student engagement as a two-dimensional construct with psychological and behavioral components, as did Skinner and Belmont (1993) in their investigation of engagement as an outcome of environmental factors. Skinner and Belmont found that engagement level (i.e., the intensity and emotional quality in initiating and carrying out learning activities) of 144 students from Grades 3 to 5 were positively associated with a teacher's classroom structure, level of autonomy support, and level of involvement with students. Engagement was conceptualized as a two-dimensional construct including the components of behavior and emotion. This line of research led researchers to further explore the associations surrounding student engagement and achievement (i.e., personal, social, and environmental student factors).

Subsequent research has demonstrated the link between performance and student engagement and the additive properties of engagement (Chase et al., 2014; Galla et al., 2014). For instance, Chase et al. (2014) explored the strength and directional relationship between the engagement and achievement of 710 fifteen-year-old high school students. These authors identified an auto-regressive relationship between engagement and achievement across grade levels (specifically, Grade 10 behavioral engagement predicted Grade 12 grade point average [GPA]; Grade 10 GPA predicted engagement at Grades 11 and 12). Galla et al. (2014) using longitudinal data from 135 students between the ages of 5 and 12 provided support for the notion that student engagement levels have important consequences for outcomes. Chase et al. took this to mean that engagement is a "rich get richer" factor, where students who exhibit engagement early on will exhibit engagement again at a later period. In sum, these studies support the notion that engagement and achievement are linked. This is just the sort of important information to be gained from examining the role of engagement in student performance, results that can have an immediate practical application in designing academic programs and interventions.

There is an additive effect of engagement and it is positively associated with personal factors (e.g., self-efficacy, valuing mathematics, school SES) (Greene et al., 2004; Ladd & Dinella, 2009; Martin et al., 2014). Among the most compelling evidence in support of the important link between engagement and achievement is that showing its predictive value for all students. Using path analysis, Sciarra and Seirup (2008) revealed that the engagement of 13,420 high school seniors from five racial groups (i.e., Indian, Asian, Black, Latino, and White) significantly predicted group mathematics achievement. In a similar vein, using a multilevel analysis, Rimm-Kaufmann et al. (2015) found that gender significantly predicted mathematics engagement levels, and in turn mathematics achievement, of 387 fifth-grade students (203 females). These studies affirm the premise that engagement is a robust construct for research, capable of identifying factors to enhance the performance of all students.

Engagement and motivational beliefs. Engagement and motivation are related but distinct constructs (Reschly & Christenson, 2012). Some engagement researchers subsume motivation into the engagement construct, characterizing it as having a sense of belonging, valuing learning/school, having an interest in the learning/school, and/or setting goals (Appleton et al., 2008; Betts, Appleton, Reschly, Christenson, & Huebner, 2010; Martin, 2006; Martin et al., 2014); others argue that an overlap between the motivation and engagement constructs confounds the interpretation of results, as well as confuses the pathways to achieving outcomes (Skinner & Pitzer, 2012). Therefore, in order to further the study of engagement, it is important to recognize motivation as a precursor rather than a part of engagement. Bandura (1997) assigned motivation variables (e.g., self-efficacy, goal setting, causal attributions) a determinant role in initiating human behaviors such as engagement.

Maintaining the distinction between the indicators and facilitators of engagement is important to understand its relation to achievement (Dettmers et al., 2011; Komarraju & Nadler, 2013; Lam et al., 2012; Rimm-Kaufmann et al., 2015; Wang & Eccles, 2011). Lam et al. (2012) examined the achievement of 822 secondary Chinese students as a function of their engagement, instructional context, and self-efficacy. The results showed that high self-efficacy predicted engagement, which in turn predicted better academic performance. Similarly, Komarraju and Nadler (2013) found engagement played a central role between motivational beliefs and achievement: Engagement, in the form of effort, of 407 first-year university students mediated the relationship between self-efficacy and students' grade point average (GPA). Perry, Stupnisky, Hall, Chipperfield, and Weiner (2010) offer more evidence of the important mediating role of engagement in a study that included 457 university students whose attribution beliefs were linked to GPA. In this intervention study, attribution retraining was shown to promote engagement. This study demonstrated that engagement is a malleable construct; therefore, studies that help identify factors to target in order to reengage students could be valuable.

Some researchers have called engagement the "glue" linking students' personal, environmental and social factors to achievement outcomes (Rimm-Kaufmann et al., 2015). Indeed, investigations of the mediating effect of engagement are worthwhile given its responsiveness to manipulation. Further research exploring the mediating role of engagement is needed to fully understand the robustness of a three-dimensional model.

Theoretical Framework

This study of engagement is centered on mathematics and grounded in the Fredricks et al. (2004) multidimensional conceptualization of engagement framed within social cognitive theory (Figure 1). Social cognitive theory (SCT) compliments Fredricks et al. insofar as it distinguishes the factors of motivation from engagement by separating student intent from student actions, while acknowledging that a reciprocal relationship exists with the personal and social factors present in students' lives. Further, SCT recognizes the importance of students' self-efficacy for initiating (self-agency) engagement actions to effect desired results.

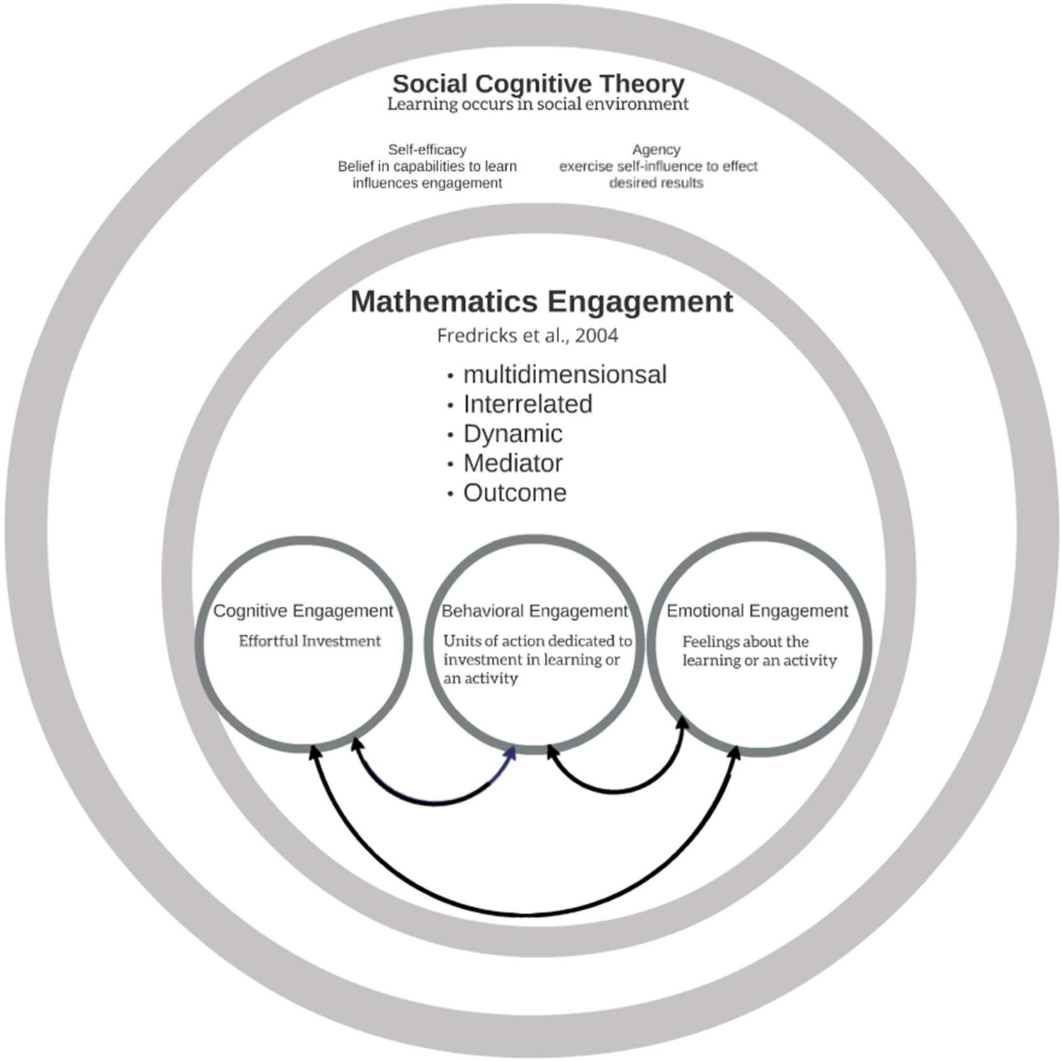


Figure 1. Theoretical framework. Adapted from Fredricks et al.'s (2004) conceptualization of engagement and situated in the social cognitive perspective (Bandura, 1977).

The Fredricks et al. (2004) conceptualization of the interrelated and dynamic three-dimensional engagement construct includes cognition, behavior and emotion. The cognitive dimension is defined as showing personal investment in learning and having a willingness to exert effort to learn mathematics (e.g., self-regulate, create goals)

(Appleton et al., 2006; Fredricks et al., 2004; Fredricks & McColskey, 2012). The behavioral dimension is described as units of action specific to learning or an activity (Christenson, Reschly & Wiley, 2012; Fredricks et al., 2008). The emotional dimension is defined as student's feelings in reaction to learning or activities.

Within this framework, the engagement dimensions are perceived to be interrelated and dynamic. They function in unison to influence achievement or interact reciprocally with antecedents, that is, facilitators of engagement. According to Fredricks et al. (2004), the examination of cognition, behaviors, and emotions together "provides a richer characterization" of students (p. 61). This complex system of observable processes offers a robust construct for investigating how successful academic performance is achieved.

Several researchers have sought to capture and explain the multiple aspects of engagement (Li & Lerner, 2011; Wang, Willett, & Eccles, 2011). For instance, Wang et al. (2011) examined the dimensionality and invariance of gender and ethnicity in school engagement, and provided statistical data showing that engagement's dimensions do not operate in isolation. They stated covaried latent factors of cognitive, behavioral, and emotional engagement significantly predicted academic achievement of 1,103 adolescents. Likewise, Li and Lerner (2011) used longitudinal data to examine the links and interactive effects of each dimension on the assessment outcomes of 1,029 youths from Grades 9 to 11. This study confirmed the existence of reciprocal influencing effects amongst the dimensions of cognition, behavior, and emotion. For instance, emotional engagement at Grade 9 predicted behavioral engagement at Grade 10 and emotional engagement at Grade 11. Additionally, emotional engagement at Grade 10 predicted cognitive engagement at Grade 11. In sum, the Fredricks et al. conceptualization of engagement as a dynamic, interrelated multidimensional construct is worthy of further research for understanding student outcomes.

Framing this study within social cognitive theory (SCT) is crucial as SCT places an emphasis on the central roles that self-efficacy and agency play in student motivation, and they are prerequisites of engagement. SCT acknowledges learners as active seekers and processors of knowledge (i.e., learners are motivated and distinguish themselves by their capabilities in vicarious, symbolic, and self-regulated learning). Vicarious learning occurs through observation and imitation, and also influences beliefs, cognition, emotions, skills, strategies, and behaviors. Symbolic learning takes place through language, mathematics, science, reading, and writing; it also influences learners' capabilities to adapt with and change the environment. Self-regulatory learning processes emerge as individuals strive toward goals by choosing strategies, monitoring progress, and evaluating performance against goals. These processes are influenced by self-efficacy, a belief in one's capabilities to perform the appropriate actions to achieve a desired goal. Bandura assigned self-efficacy a primary role in social cognitive theory, as it is recognized to be a potent determinant of motivation and learning, as well as human agency (Bandura, 1977; Schunk, 2012). From the social cognitive view, in order to understand engagement, researchers must consider the importance of self-efficacy and student agency.

Consistent with the Fredricks et al. conceptualization of engagement's interacting and influential dimensions, social cognitive theory is premised on Bandura's (1982) reciprocal determinism theory which states that human behaviors result from a triadic reciprocal interaction among personal, behavioral, and social/environmental factors. In this view individuals are recognized to be proactively engaged in their own learning and performance in accordance with contextual influences, their beliefs, and outcome expectations (Bandura, 1977; Schunk, 2012); in other words, the context in which students reside influences their actions; students' self-influence, or sense of agency, also brings about desired results by promoting motivation (Bandura, 1997). Engagement is the manifestation of motivation (Appleton et al., 2008).

As for refining the definition of engagement and distinguishing the factors of engagement from motivation, social cognitive research is replete with findings demonstrating the differences between the factors of intent (motivation) and action (engagement) (Schunk, Pintrich, & Meece, 2008; Walker & Greene, 2009; Zimmerman, 2000). Cleary and Zimmerman (2012) define motivation as the desire to engage or as a goal-directed process that leads students to initiate actions for learning and to sustain those actions to complete a chosen task. Other studies grounded in SCT define engagement as a function of purposeful or goal-directed efforts (Walker & Greene, 2009; Zimmerman, 2000). Reporting on relationships of students' motivational beliefs and engagement, Walker and Greene (2009) found that the cognitive engagement of 249 high school students, measured as use of strategies, was significantly predicted by the motivational variables of perceived instrumentality and sense of belonging.

Viewing engagement through a social cognitive framework takes into account that learning occurs in a reciprocal process within the student's social environments. Therefore, this present study seeks to further examine the links that exist among engagement and students' personal beliefs and their social/environmental milieu. Several studies have shown that the interactions between the personal beliefs of self-efficacy and attributions influence and are influenced by engagement, as characterized by learning strategies, persistence, and taking enjoyment from participation in learning or an activity (Ainley & Ainley, 2011; Hampden-Thompson & Bennett, 2013; Martin et al., 2014; Wolters, Fan, & Daugherty, 2013). In examining how social/environmental factors interact with the personal factors of gender, socioeconomic status (SES), and ethnicity, other studies have shown these variables to be highly predictive of students' levels of self-efficacy, attribution, and engagement (Hampden-Thompson & Bennett, 2013; Kitsantas, Cheema & Ware, 2011; Ladd & Dinella, 2009; Rimm-Kaufman et al., 2015; Sciarra & Seirup, 2008). Thus, it follows that this present study is framed in social cognitive theory, which delineates the conceptual relationships among motivation and engagement and clearly differentiates the measures that form intent from those that constitute engagement.

Purpose

The purpose of the study is twofold. First, it seeks to establish the structure of a three-dimensional model of mathematics engagement (Figure 2). Second, this investigation examines the mediating role of mathematics engagement between the student-level factors of self-efficacy and attribution for failure on mathematics performance, while controlling for the demographic variables of gender, ethnicity, and ESCS (Figure 3).

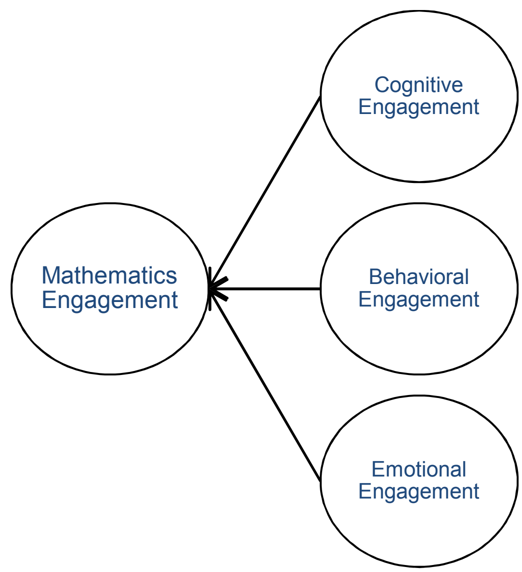


Figure 2. Conceptualization of three-dimensional construct of mathematics engagement.

This undertaking is in response to a call by Christenson et al. (2012) to provide better "conceptual clarity and methodological rigor" (p. vii) in the study of engagement in order to advance the construct and inform educators of its usefulness for intervention programs.

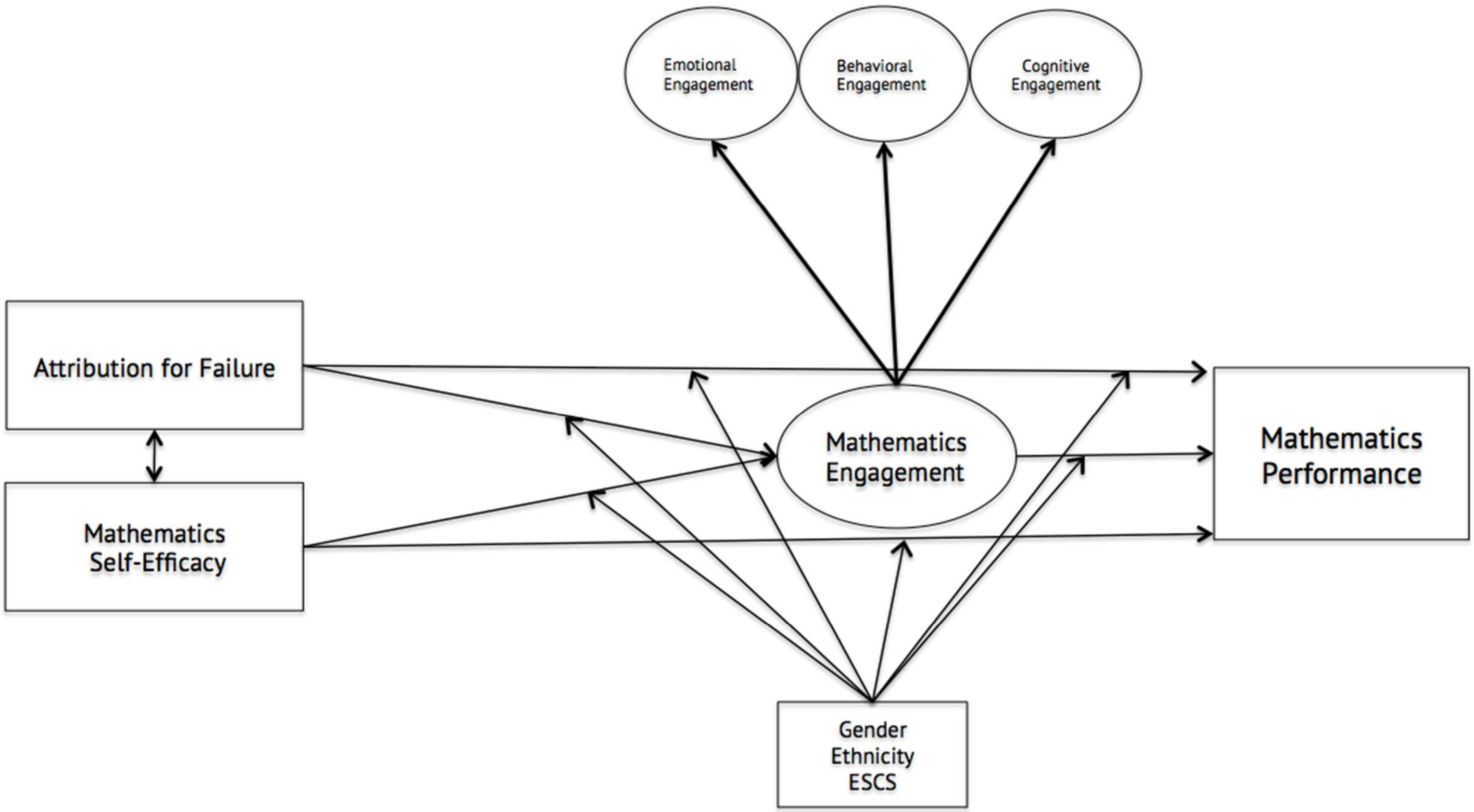


Figure 3. Hypothetical model. a) Direct effects: Attribution for failure and mathematics self-efficacy on mathematics engagement and mathematics performance; and mathematics engagement on mathematics performance. b) Mediation effect of mathematics engagement between attribution for failure and mathematics self-efficacy on mathematics performance. c) Moderating effect of gender, ethnicity, and economic, social, and cultural status (ESCS) on attribution for failure and self-efficacy for mathematics engagement; and on attribution for failure, self-efficacy, and mathematics engagement for mathematics performance.

Significance

This study will advance understanding in a number of ways. First, it will advance engagement research by confirming the existence of a structural model of engagement with three dimensions. The reliability of the model is strengthened by the use of clearly defined factors that are operationalized using a social cognitive perspective. According to social cognitive theory, engagement follows motivation. Therefore, the variables used to measure engagement reflect actions such as listening in class, homework completion, and enjoyment of learning activity; while its facilitators are measured with variables reflecting intent, self-efficacy, and attributions for failure.

Second, this investigation contributes to an understanding of the relevant factors to target for mathematics engagement and mathematics performance. It explores the role played by beliefs such as self-efficacy and attribution for failure in students' mathematics engagement and achievement, and it tests the causal ordering of the relationships amongst mathematics performance and the personal (self-efficacy and attribution for failure), behavioral (engagement: effort, strategy use, and enjoyment of learning), and social/environmental (gender, ESCS, and ethnicity) factors. To my knowledge, this model of engagement has not been tested in previous research. Although previous studies have presented three-dimensional models of engagement, the specific item ordering in this study has not been tested.

This study of mathematics engagement seeks to explain what matters most and for whom. This is explored by testing what differences exist by gender, ethnicity, and ESCS in the interrelationships amongst mathematics performance, mathematics engagement, self-efficacy, and attribution for failure. In sum, the findings of this study will enhance our understanding of motivational beliefs, mathematics engagement, and mathematics performance.

Research Questions

The intent of this study is to affirm the existence of a three-dimensional model of engagement. Further, this investigations seeks to explain the relationship of mathematics engagement with attribution for failure and mathematics self-efficacy on mathematics performance, and the effects of gender, ethnicity, and ESCS. Although previous studies have shown that an association among these variables exists, the causal ordering of a full model with three-dimensions of engagement has not been tested previously. The findings of this study may be used to enhance our understanding of students' mathematics engagement and performance.

1. Can mathematics engagement be modeled as a three-dimensional construct, and what are the components?
2. Are the effects of attribution for failure and mathematics self-efficacy on mathematics performance mediated by mathematics engagement while controlling for gender, ethnicity, and ESCS?

Definitions

The following terms are used in specific ways in this research.

Attribution for failure. Attribution for failure is a concept within attribution theory that is concerned with how learners explain the causes of failure represented by three dimensions: internal or external to the individual, stable or unstable over time, and controllable or uncontrollable by the individual (Weiner, 1979).

Mathematics egagement. Mathematics engagement is the willingness to exert cognitive effort to master learning, enact units of actions specific to learning and achievement, and having negative and positive feelings toward learning and achievement (Fredricks et al., 2004). It is comprised of the three dimensions of cognitition, behavior, and emotions.

Cognitive dimension. Cognitive engagement is defined as students' investment in the learning process that includes a willingness to exert effort toward comprehension of ideas and skills (Fredricks et al., 2004).

Behavioral dimension. Behavioral engagement is defined as active participation in the learning process and environment with units of actions specific to learning (Christenson et al., 2012; Fredricks et al., 2004).

Emotional dimension. The emotional dimension of engagement is defined as student feelings about learning. It is characterized as the positive and negative reactions to learning or activity at hand (Fredricks et al., 2004).

Economic, social, and cultural status. The economic, social, and cultural status (ESCS) is an index comprised of responses to questions about parents' occupation, education level, and household possessions (e.g., wealth: computers, own room, Internet; cultural possessions: books; works of art; and home educational resources: a desk to study at, reference books, educational software) (OECD, 2012).

Ethnicity. Ethnicity is a self-identification based on the race or races with which students most closely identify (NCES, 2015).

Mathematics performance. Mathematics performance is a proxy for the PISA 2012 mathematics literacy measure of 15-year-olds reflecting their ability to use, describe, predict, and explain mathematics. The proficiency levels range from 1 to 6. The baseline proficiency level is 2 (below average), while top performers are at levels 5 and 6. Scale scores range from 0 to 1000; a score of 407 is baseline proficiency (OECD, 2012).

Mathematics self-efficacy. Mathematics self-efficacy is the belief that one has the capability to execute the appropriate actions to succeed in mathematics without serious doubts (Bandura, 1977).

Chapter Two

Review of the Literature

This study was primarily intended to investigate the level of student mathematics performance as a function of student engagement and student-level factors (attribution for failure, self-efficacy, gender, ethnicity, ESCS). Mathematics has long been recognized as the gatekeeper to college entrance, college graduation, higher earnings, and upward mobility (Stinson, 2004). Yet, approximately three-fourths of students in the United States fail to reach the basic baseline level of 2, demonstrating weak mathematics performance where mathematics is interpreted and linked to real world problems or translated into models (OECD, 2013). Prior research has identified several factors positively correlated with students' mathematics performance: instructional strategies (Boaler & Foster, 2014; Klem & Connell, 2004), parental involvement (Martin et al., 2014) student valuing or interest in school or the learning process (Appleton & Lawrenz, 2011; Rimm-Kaufmann et al., 2015) and student engagement. Engaged students are characterized as doing more than attending class: They pay attention to the teacher, seek help when needed; and look forward to lessons as they strive to succeed. Studies have found that students who are engaged earn better grades and are more likely to graduate (Lam et al., 2012, Martin et al., 2014; Ouweneel, Schaufeli, & Blanc, 2013; Schelenker, Schelenker, & Schelenker, 2013; Wolters et al., 2013). This study focuses on student engagement given the malleable contextual factors that can be targeted, through intervention programs, to enhance students' cognitive, behavioral, and emotional participation in their own learning,

Engagement is widely conceptualized and defined as a three-dimensional construct, characterized as the effortful cognitive, behavioral, and emotional investment in learning activities directed toward achieving an outcome (Appleton et al., 2008; Fredricks et al., 2004; Reschly & Christenson, 2012). This is the definition used in this study, which uses data from the Program for International Student Assessment (PISA) to examine the measurement and structural model of mathematics engagement and its relationship between students' beliefs (attribution for failure and self-efficacy) and mathematics performance, and tests the mediators of this relation. Gender, ethnicity, and ESCS differences were also explored. The majority of prior research has relied on models of engagement limited to one or two dimensions when examining the link between achievement outcomes and engagement. This investigation examined these relationships utilizing a unified three-dimensional model of engagement that includes the cognitive, behavioral, and emotional dimensions.

Within this chapter is an integrated review of relevant literature. While some of the literature is focused on mathematics engagement, the majority examines engagement as a general phenomenon. This review is divided into four sections. The first section puts forth this study's conceptualization and definition of engagement using researchers' findings. Sections two through four consider related studies that have focused on engagement and achievement, attribution for failure, and self-efficacy separately, or that have included two or three of the variables of interest together. The relationships of gender, ethnicity, and ESCS with engagement are discussed in section five.

Engagement

Engagement is a “theoretically messy” construct that integrates important aspects of learning (cognition, behavior, and emotion) and can significantly increase our understanding of student performance (Appleton et al., 2008; Fredricks et al., 2004, p. 84). Relative to other constructs on student learning and achievement supported by decades of research, there is no yet a universal agreement about how engagement is conceptualized or defined. For the purpose of this study, engagement is a three-dimensional construct defined as students’ effortful investment in learning, active participation in learning or the classroom, and having positive or negative reaction to learning-related activities (Fredricks et al., 2004). For instance, while the predominant conceptualization of engagement is that of a three-dimensional construct (Appleton et al., 2008; Fredricks et al., 2004), others see it as having two dimensions (cognitive and behavioral, or behavioral and emotional) or four-dimensions that can include either agentic, academic, social, or psychological engagement (Appleton et al., 2006; Finn, 1989; Reeve, 2012).

Adding to the “haziness” of the engagement construct are the myriad definitions applied by researchers to explain its dimensions (Blumenfeld & Meece, 1988; Brophy, Rashid, Rohrkemper & Goldberger, 1983; Finn 1989; Reeve, 2012; Reschly & Christenson, 2012, p. 3). In the school dropout literature, for instance, engagement is seen as a protective factor and defined as students’ deep processing of information and their intrinsic motivation (Brophy et al., 1983). Other researchers define engagement as students' active participation in school and classroom activities, and their feelings of identification with school (Chase et al., 2014; Finn, 1989; Finn & Voekle, 1993; Sciarra & Seirup, 2008). Reeve (2012) define engagement as a student's attention, effort, level of interest, choice of strategies, and the extent to which he or she actively enriches the learning environment. There is an important discussion in the engagement community about how the overlap between motivation and engagement confounds engagement findings (Christenson et al., 2012). Motivation and engagement are sometimes used interchangeably, leading to debates about the usefulness of the engagement construct if there is no distinction from motivation. This present study holds that motivation and engagement are distinct constructs. Engagement is considered to be dependent on motivational factors such as students' beliefs about their capability to learn (Martin & Rimm-Kaufmann, 2015) and the types of reasons they give for their success or failure (Wolters et al., 2013).

Regardless of the differences in conceptualization and definitions that exist in the current literature, engagement is recognized as vital for student performance and achievement (Appleton et al., 2006; Boaler & Foster, 2014; Brophy et al., 1983; Finn, 1989; Holt, Bry, & Johnson, 2008). In a three-year intervention study conducted by Boaler and Foster (2014) with 2,489 middle school students whose teachers committed to teach high-level mathematics, all students outperformed a comparison group (6,378) whose school district continued to offer some students low-level mathematics classes. The findings revealed that students from eight districts whose teachers were taught to have high expectations for all students and who crafted and conducted challenging and engaging mathematics lessons achieved at higher levels on narrow state mathematics tests and broader conceptual tests than the comparison group. In mathematics achievement the intervention group score was 48%, compared to 38% for the nonintervention group. Likewise, the intervention group score for mathematics concepts was 38%, versus only 18% for the comparison group. These results suggest that engagement is responsive to students' learning environments, which can be manipulated to change students' active and effortful participation levels and to attain successful outcomes (Appleton et al., 2008). Clearly, there is value in pursuing research that aims to understand the contextual factors that can enhance student engagement and lead to higher achievement.

Therefore, this present study sought to more clearly conceptualize the engagement construct based on the seminal review of the literature by Fredricks et al. (2004), and other research findings that differentiate engagement and motivation (Appleton et al., 2008; Greene et al., 2004). Fredricks et al. conceptualized engagement as a three-dimensional construct (cognition, behavior, and emotion) and generally defined as students' effortful investment in learning, active participation in learning or the classroom, and having positive or negative reaction to learning-related activities. The cognitive dimension is defined as effortful investment in learning and school. Indicators of this dimension are students' willingness to go beyond the requirements of the class and/or to be open to problem solving and challenges (Diseth, 2011; Greene et al., 2004; Lam et al., 2012). The behavioral dimension is defined as students' active participation in school and learning. Indicators of this dimension are observable and include paying attention and listening in class, homework quality, and keeping schoolwork organized (Chase et al., 2014; Rimm-Kaufmann et al., 2015). Behavioral engagement is a crucial variable for successful outcomes. The final component of engagement has to do with students' emotions about their learning and school. Emotional engagement is defined as students' feelings about learning, classes, or school. Indicators of students' emotional engagement include taking joy from learning or looking forward to lessons (Hampden-Thompson & Bennett, 2013 Ladd & Dinella, 2009: Lam et al., 2012). The multidimensional engagement construct is valuable because it provides "a richer characterization of children than is possible in research on a single component… [because] in reality these factors [dimensions] are dynamically interrelated within the individual. They are not isolated processes" (Fredricks et al., 2004, p. 61).

Engagement and Achievement

A multidimensional engagement construct has the potential to capture and explain what it takes for students to achieve, yet little research exists testing the impact of a unified three-dimensional model on achievement outcomes (Chase et al., 2014; Ladd & Dinella, 2009; Martin et al., 2014; Rimm-Kauffman et al., 2015; Sciarra & Seirup, 2008). Sciarra and Seirup (2008) conducted an investigation examining the relationship between school engagement and school achievement across five racial groups (*N* = 11,388; Indian, Asian, Black, Latino, and White). The study conceptualized engagement as incorporating three dimensions (cognition, behavior, and emotions). It characterized engagement as related to investment in learning, participation in school and learning activities, feelings about school, and caring about school. Findings showed that the three-dimensional engagement model effectively predicted the mathematics outcomes for all racial groups in this sample. Differences in the level at which each dimensions contributed to outcomes were reported. For instance, behavioral engagement and cognitive engagement emerged as significant predictors for Asian (R^2 = .11), Black (R^2 = .07) and American Indian (R^2 = .21) students. Emotional engagement contributed to the variance at a nonsignificant level. Conversely, emotional engagement significantly predicted mathematics scores for Latino (R^2 = .07) and White (R^2 = .14) students. However, for the White students the emotional engagement dimension had less impact than the cognitive or behavioral dimensions.

Sciarra and Seirup (2008) were able to show how a three-dimensional model of engagement offers a holistic understanding of students' engagement characteristics, including those with diverse backgrounds. From this data it is reasonable to surmise that an intervention for Latino students could focus on enhancing their emotional engagement by manipulating variables such as sense of belonging or perceived teacher caring. Further research is needed using a three-dimensional model of engagement with the potential to inform individualized or group-centered intervention programs to understand what matters most for closing the achievement gap for students from diverse backgrounds.

In another study incorporating the three dimensions of engagement simultaneously, Chase et al. (2014) report findings showing the relationship between school engagement and academic success, as well as the additive effects of engagement over time for 710 high school students. In this multilevel modeling analysis looking at engagement across Grades 10 to 12, engagement—conceptualized as three-dimensional—was found to be a strong predictor of achievement. The cognitive dimension was defined as students' goal orientation, identification with school, and perceptions of the link between students' lives and school; it was measured with four items reflecting the value of education/learning and students' beliefs about learning. The behavioral dimension was defined as students' shallow engagement (attendance) and deeper engagement (effort) and was measured with five items representing skipping class and contribution to class discussions. The emotional dimension was defined as students' sense of belonging; affect toward school and was measured with five items, including "I care about the school I go to" and others that related to connectedness, belonging, and bonding. The data from this full three-dimensional model suggest that in addition to engagement's ability to predict achievement, its three dimensions also have an auto-regressive relationship across grade levels. The strongest links appeared between students' behavioral school engagement at Grade 10 and their GPA at Grade 12 (ß = .19); and, students' GPA at Grade 10 on their behavioral school engagement at Grade 12 (ß = .21). These findings suggest there is import in understanding student engagement and addressing low or full disengagement early in students' academic careers.

In further support of a multidimensional model, Martin et al. (2014) found a three-dimensional model of engagement to be positively associated with the academic performance of 1,601 Australian students. Engagement was defined as students' willingness to invest effort in academics, as their active participation in academics, and as students' positive and negative reactions to the academic environment. Using multilevel modeling, Martin et al. sought to explain student engagement shifts at key transition points across their academic lives (Grades 5 to 8). The investigation revealed engagement's strong auto-regressive effect, meaning that prior engagement predicted future engagement in the middle years. They also found that mathematics self-efficacy, mathematics valuing, and prior achievement explained the bulk of the variance in student achievement, and to a lesser extent the contextual factors of parental valuing and classroom climate. Further studies are suggested to address the variance inside engagement, but which also separate the motivation and engagement variables.

Other studies have sought to understand not only the contextual factors that influence engagement, but also the gender differences that may exist. Rimm-Kaufman et al. (2015) found that students who perceived teachers' emotional support as high were more likely to be cognitively, behaviorally, emotionally, and socially engaged in their mathematics learning. These researchers also conceptualized engagement as a four-dimensional construct. Cognitive engagement was defined as students' willingness to exert effort to understand content, work through difficult problems, and manage and direct their attention toward the task at hand. The behavioral dimension referred to students' attention in class, completing assigned work, participating in learning opportunities, and nondisruptive behaviors. Emotional engagement consisted of students' feelings of connection to content, interest in learning, and enjoyment of solving problems and thinking about content. Social engagement was defined as daily peer interactions connected to the instructional content.

Using data from 387 fifth-grade students in 63 classrooms, these authors reported students worked harder, enjoyed learning math, and shared their ideas and materials with others when environmental and social influences like teacher-related actions and organization of the classroom were present. They also reported finding gender differences in the engagement variable as a function of the environment. For instance, higher levels of organization improved the behavioral engagement of boys, that is, absence of disruptive behavior. Girls, on the other hand, were less socially engaged in classrooms with higher instructional support. Consistent with other findings (Wang et al., 2011), girls were more cognitively and behaviorally engaged in general than boys, although their mathematics self-efficacy and performance were significantly lower than their male peers. More studies are needed that investigate how achievement hinges on engagement and the impact of student-level factors such as gender and beliefs.

Studying engagement to understand the complexity and intricacy of factors involved in the learning process is promising. Hampden-Thompson and Bennett (2013), for instance, used a multilevel analysis (student, class, and school) to investigate the association between the science engagement of 11,775 British 15-year old students and their social/environmental context. The authors conceptualized engagement as a multidimensional construct that includes aspects of cognition, behavior, and emotion. In this study only cognitive and emotional engagement were analyzed. Engagement was defined as students' psychological investment and their affective reactions to science. Cognitive engagement was measured with instrumental motivation and future orientation items such as "making an effort in my science subject(s) is worth it because this will help me in the work I want to do later on," and "I would like to work in a career involving science." Emotional engagement was measured with items representing enjoyment in science, for example, “I generally have fun when I’m learning science topics,” motivation in science, enjoyment of science, and future orientation. The findings from this study indicate students were more engaged in science when higher frequencies of debates/discussions, hands-on investigations, and application experiences were present. School factors that negatively influenced engagement included a shortage of teachers and large class size. Students of higher economic status were more likely to report being highly engaged in science learning than their peers of lower economic status. Difference by gender in students’ science engagement revealed that males were more engaged in science learning than females. It is unclear what these associations mean; more studies are needed to investigate the link between engagement, SES, and gender. Similarly, more studies are needed investigating how the different dimensions of engagement are influenced by students’ personal and contextual factors can better inform educators about the best practices to apply to raise students’ performance and encourage them to participate in mathematics courses beyond the basic requirements.

Taken together, these studies demonstrate that the more holistic multidimensional model of engagement as the three dimensions of cognition, behaviors, and emotions warrants further investigation (Chase et al., 2014; Ladd & Dinella, 2009; Martin et al., 2014; Rimm-Kauffman et al., 2015; Sciarra & Seirup, 2008). These studies are valuable because they can provide a richer student profile reflecting on the intricate and complex aspects of learning. What is left to explore is a measurement model of engagement with indicators that do not overlap with motivational variables such as valuing learning, feeling a sense of belonging, having interest in a subject or school, or future orientation

(Chase et al., 2014; Rimm-Kaufman et al., 2015; Sciarra & Seirup, 2008). Clearly, more research is needed to understand how engagement functions. An important aim of the present study was to use clearly defined and psychometrically sound measures of engagement with variables that reflect the actions of effortful investment in learning rather than students' intent to learn.

Attribution for Failure and Engagement

Attribution theory is concerned with the causal reasons learners give for academic successes or failures (Weiner, 1979). Within the motivation literature, attribution is focused on students' beliefs about competence and control (Schunk & Zimmerman, 2006; Weiner, 1979, 2008, 2010; Wolters et al., 2013). Attribution theory is important to the study of engagement because of the influence that personal judgments about prior experiences exert on personal, behavioral, and social/environmental factors (Schunk, 2012; Weiner, 2008). According to Weiner (1979), attribution is initiated when learners search for an understanding of "why" they succeeded or failed. This process is more likely to occur in learners after failure (Stupnisky, Stewart, Daniels, & Perry, 2011; Weiner, 1979).

Weiner (1979, 2008, 2010) notes that the causes for success and failure are represented along three dimensions: internal or external to the individual, stable or unstable over time, and controllable or uncontrollable by the individual. These dimensions are hypothesized to influence the cognitive, behavioral, and emotional engagement of students (Weiner, 1985). The types of attributions applied to failures influence learners' beliefs and perceptions, in addition to their emotions and behavioral engagement. For instance, a learner's affect (e.g., pride or shame) is influenced by the attribution of failure to internal (e.g., effort) or external (e.g., luck) causes. The choices of tasks, level of effort, persistence, and level of achievement are a function of learners' perception of competence and control, and are linked to students' self-efficacy (Wolters et al., 2013). Learners who attribute their success or failure to internal and controllable causes are more inclined to feel efficacious about school, and are more likely to use self-regulatory processes (e.g., goal setting, strategy choices, progress monitoring, and self-evaluation) than their peers who claim internal and uncontrollable causes (e.g., ability as stable, luck or difficulty of subject) for their success or failure (Schunk, 1994; Weiner, 1979). When learners recognize, appreciate, and understand their academic ability as malleable, they are able to engage in appropriate actions (e.g., goal setting), and assume a greater sense of control over academic behavior and motivation (McCombs & Marzano, 1990; Stewart, Latu, & Myers, 2010).

In the achievement domain, the learners' search for causes of success or failure is associated with learning outcomes (Blackwell, Trzesniewski. & Dweck, 2007; McClure et al., 2011; Weiner, 2008). The literature suggests that attributions for failure to uncontrollable, stable, or internal causes is linked to dysfunctional attributional thinking that erodes motivation and in turn impacts engagement. Findings from an intervention study by Blackwell et al. (2007) indicate that responses to failure establish a pattern of responses that shape students' motivation, engagement, and in turn affects their academic trajectories. Blackwell et al. present findings from two studies investigating the role played by the implicit theories of intelligence. In their first study, these authors submit data from 373 seventh-grade students showing how their responses for failures are predicted by their theories of intelligence (malleable or fixed). Further, they showed that students' response to failure mediates the relationship between their effort beliefs and positive strategy use, an indicator of cognitive engagement. These findings suggest that students' beliefs are strong determinants of students' decisions to engage in effortful investment in their own learning. In a follow-up study with 91 seventh graders, Blackwell et al. (2007) sought to test the malleability of students' beliefs about failure. Students were placed into an experimental or comparison group, and both groups participated in eight sessions consisting of 25-minute workshops about the physiology of the brain, study skills, and antistereotypic thinking. In addition, the experimental group was taught that intelligence is malleable, learning changes the brain, and students have control of this process. The results showed that the training had positive effects. Students' declining academic trajectories were reversed. An analysis of data from teacher questionnaires showed that of the 48 students in the intervention group, 27% had positive changes in their mathematics learning behavior, for example, "voluntarily seeks help" at the end of the semester. In contrast, only 9% of the 43 students in the control group had positive changes in their mathematics learning behavior. The Blackwell et al. findings suggest that attribution for failure remains an important belief to include in studies examining the engagement and achievement patterns of students. Although not found to directly influence achievement, students' attribution for failure was shown to mediate the relationship between students' intent and engagement toward achievement.

In support of the link between students attributional thinking and engagement, Stupnisky et al. (2011) report findings showing that students who experienced negative outcomes engaged in higher levels of causal search and were more prone to attribute their failure to lack of ability, test difficulty, and luck. Using data for 371 Canadian university students enrolled in a two-semester introductory psychology course, these authors sought to investigate students' causal search precursors, and its association with students' achievement outcomes. Causal search was found to be activated more strongly by unexpected negative outcomes, as well as the level of importance. Higher levels of causal search were associated with detrimental emotions and performance outcomes. Specifically, higher causal search levels after failure correlated with feelings of shame, anger, guilt, and regret. These results are in keeping with Weiner's (1985) theory suggesting attribution for failure to uncontrollable factors impacts student motivation, decisions to engage in learning, and performance outcomes.

Social cognitive theorists see attribution for failure as being affected by experiences, social norms, and causal reasoning (Weiner, 2008). It is well known that prior academic experience shapes students' future striving (Bandura, 1977; Schunk, 1994; Zimmerman, Bandura, Martinez-Pons, 2008). Therefore, students prone to failure are vulnerable, often displaying low self-efficacy, low motivation to engage meaningfully in their studies, and low achievement (Lam et al., 2012; McClure et al., 2011; Perry et al., 2010; Wolters et al., 2013). McClure et al. showed that attribution for failure was related to the effort levels of 5,333 New Zealand students in Grades 10 and 11. In this research, students' attributions for best and worst marks were found to be associated with their motivation orientation and achievement. Students were asked to rate their best and worst grades using the seven causal attributions: ability, effort, luck, task difficulty, teacher, peers, and family. Attribution was then associated with four motivation orientations: (a) “doing my best” reflected students’ intention to do as well as they can in their schoolwork, for example, “I will strive for merit or excellence even when I don’t need this to achieve my goals”; (b) “doing just enough” measured students’ intention to do the minimum to achieve their goal, for example, “I will work for the number of credits I need for each level, no more”; and two social motivation measures of (c) teacher caring, that is, “I’ll learn more in subjects when the teacher cares how well I do,” and (d) peer collaboration, that is, “I do best in classes where students are allowed to work together.” Achievement was measured with GPA scores. The results indicate a negative correlation between students “doing just enough” (effort level) and their attribution to luck, friends, and teacher.

McClure et al. revealed that students’ adaptive attribution for low marks (quasifailure) is associated with achievement. Students who attributed failure to effort and ability performed better than students who attributed their worst marks to ability and effort, as well as luck, task difficulty, and teacher. In addition, a difference in attributional thinking by gender was found. Females tended to attribute their worst marks to ability (stable) and teacher (external/uncontrollable), while males tended to attribute their worst marks to luck (external/uncontrollable). These causal factors (e.g., ability, luck, task difficulty) are described in the attribution literature as maladaptive learning behaviors that incite feelings of having no control, which have been shown to affect students' efficacy to engage in learning.

The relationship between attribution and engagement toward achievement is best examined by attributional retraining studies. These studies focus on shifting attribution for failure from the mindset that outcomes are uncontrollable, stable (not malleable), and externally determined to a mindset that one is capable of organizing and taking appropriate actions to attain a goal. Perry et al. (2010) applied attributional retraining with 457 students enrolled in an entry-level psychology course. The study attempted to change students' dysfunctional attributional thinking and motivational deficit, fostered by failure, to controllable attributions, positive affect, and academic performance. Attribution was conceptualized as a causal explanation for failure and was measured by asking students to what extent each of four factors (strategy, effort, professor quality, and helplessness) influenced their performance in an introductory psychology course. Emotions were conceptualized as positive and negative effects resulting from attribution and achievement. Emotions were measured by asking students to rate from 1 to 10 seven emotions related to their performance in an introductory psychology course so far in the year: hope, pride, shame, guilt, helpless, anger, and worry (an achievement-related emotion). Participants were categorized into three initial groups according to GPA (low, average, and high). Participants from each of the groupings were selected and placed in either a no-attribution retraining group (No-AR) or an attribution retraining group (AR). At Time 1, the AR group received feedback on the first test of the semester and was encouraged to view failure with the mindset that it was within students' control to change future outcomes. The No-AR group received no feedback or training at any time. At Time 2, test scores, attribution, and emotion measures were collected for all participants. At Time 3, class grade and GPA were collected for the course and semester.

The results revealed a difference by attribution, emotion, and achievement between the low and average students in the AR group, compared to their counterparts in the No-AR group. The low and average students received higher test and course grades, as well as higher GPAs than their peers in the No-AR group. No difference was found between high average students in the AR and No-AR groups. The link between attribution for failure and achievement-related emotion was inconclusive, possibly because it was measured with only one item. A large portion of the variance for emotion was accounted for by the worry variable linked to the distal outcome of achievement. The study suggested the efficacy of persuading students that attribution for failure is internal, controllable, and malleable (the AR treatment) to increase their motivation to use strategies and invest more effort in coursework. The present study further examines this link between attribution for failure and emotions with the more proximal outcome of engagement. As an intermediary construct, engagement is likely to provide better data for examining the importance of students' attribution for failure as it is more closely related to achievement.

Previous research links engagement and attribution to learning outcomes. In a 12-country study examining the antecedents and facilitators of student engagement, Lam et al. (2012) examined the link between student engagement and contextual factors (e.g., instruction type and social support), personal factors (i.e., self-efficacy and attribution) and student outcomes. Focusing on the engagement of 822 Chinese junior high school students, engagement was conceptualized as a metaconstruct with three dimensions: cognition, behavior, and affect. The engagement construct was measured with a scale of 33 items consisting of three subscales. The cognitive dimension (12 items) was defined as students' meaningful information processing strategies for learning and was measured with items such as "When I study, I try to connect what I am learning with my own experience." The behavioral dimension (12 items) was conceptualized as students' persistence and effort in learning and was captured with questions such as "I try hard to do well." Emotional engagement (9 items) was conceptualized as having a penchant for learning and interest in school and was measured with statements such as "I like what I'm learning in school." Facilitators, or sources of engagement, included attribution, which was measured with questions asking students to state by percentage how much their performance was influenced by their ability, effort, luck, or the situation (e.g., being sick). Among the four attributions in this study, effort attribution had a positive relationship with student engagement, while situation attribution (teacher, difficulty of task, sickness) had a negative correlation. Further, this study confirmed there is a positive association between student engagement and performance, as measured by teacher reports about schoolwork, tests, and assignment outcomes. The proposed study is predicated on this to further examine the latent construct of engagement as a fully unified three-dimensional model.

Wolters et al. (2013) lend further support to the link between engagement and attribution. Using hierarchical linear modeling, these authors showed that certain types of attributions were associated with a more adaptive pattern of behavioral and cognitive engagement of 224 high school students from Grades 9 to 12 enrolled in Algebra II. Engagement was conceptualized as academic functioning: cognitive and behavioral. The cognitive aspect was defined as strategies students used to complete daily assignments and study for tests in mathematics and was measured with items asking students about their metacognitive strategy use (planning, monitoring, regulation) and the strategy methods used to encode: rehearsal, elaboration, and organization of material. Behavior was defined as students' attitudes, beliefs, and behaviors surrounding mathematics tasks and was measured with items reflecting choice, effort, persistence, and procrastination. Attribution was measured with 32 items reflecting attribution for success to failure. These were combined into four scales: success-ability and failure-ability, success-effort and failure-effort, success-environment and failure-environment, and success-task and failure-task.

Attribution for failure to effort predicted student engagement in terms of behavioral and strategy use (cognitive strategies and metacognition). Attribution for failure to task predicted engagement and metacognition, but did not predict cognitive strategies. Attribution for failure to ability was not predictive of engagement or strategy use. Attribution to success or failure did not predict performance outcomes. As explained in Wolters et al. (2013), this is consistent with Weiner's (2008) view that attribution is more closely tied to psychological and behavioral variables than to distant outcomes.

An interesting pattern emerged in the results, suggesting a possible mediating effect occurring with self-efficacy and attribution for failure. In the three-step multiple regression analysis, self-efficacy emerged as a significant predictor of cognitive engagement (strategy use) at Step 2, but failed to predict cognitive engagement at a significant level after the addition of attribution beliefs in Step 3. Also, contrary to findings in Step 2, self-efficacy predicted engagement at Step 3 when attribution was entered into the analysis. This pattern may suggest possible reciprocal relationships occurring between attribution beliefs and self-efficacy. According to Wolters et al. (2013), this is a reasonable contention given the findings; however, further investigation is needed. In sum, Wolters et al. revealed a statistically significant relationship between causal attributions, self-efficacy, engagement, and student performance. What is left to explore is the association of attributions to a fully specified three-dimensional model of student engagement, and how this relationship is mediated with other factors such as self-efficacy and performance, while controlling for gender, ethnicity, and SES.

Based on the aforementioned research there is sufficient support for considering a causal model of engagement that includes attribution for failure (Lam et al., 2012; McClure et al., 2011; Perry et al., 2010; Wolters et al., 2013). Attribution for failure has also been shown to be an important factor in an examination including student engagement and performance. Further research is needed to better understand the degree of influence attribution for failure exerts on each dimension of engagement directly and indirectly through self-efficacy.

Self-Efficacy and Engagement

Self-efficacy is the belief that one is capable of executing the appropriate actions toward an outcome without serious doubts. Bandura (1977), a seminal figure in social cognitive literature, posited that self-efficacy determines the level of effort and the degree to which strategies will be employed. Self-efficacy plays a pivotal role within social cognitive theory; it is a “key determinant of individuals’ motivation, learning and self-regulation” (Schunk, 2012, p. 109). Sources of self-efficacy include mastery experience, vicarious experience, verbal persuasion, and physiological states (Bandura, 1997).

Mastery experience is believed to be most influential on self-efficacy (Bandura, 1986, 1997). Students evaluate and interpret results of their actions through processes inside the self-regulation model of learning (Zimmerman et al., 1992). In particular, mastery experience that results from success or failure in challenging tasks is the most powerful predictor of self-efficacy (Bandura, 1997). Observing others also influences self-efficacy in that individuals gauge their capabilities in comparisons to others performing like tasks (Schunk, 1994). Another source of self-efficacy is verbal or social persuasion. The best type of persuasion encourages individuals that success is measured by personal growth rather than a comparison of scores with others (Ryan, Gheen & Midgley, 1998). The fourth source of self-efficacy is individuals’ emotional and physiological states. Individuals’ reactions to tasks form their precepts of self-efficacy: feelings of anger, anxiety, dread, or apprehension reduce self-efficacy; while the state of well-being increases self-efficacy (Usher & Pajares, 2008). High self-efficacy results in improved homework behaviors, effective learning approaches, the ability to set appropriately challenging goals, and academic engagement (Kitsantas et al., 2011; Martin, 2012).

Numerous studies have found self-efficacy to determine students' level of engagement and successful outcomes (Diseth, 2011; Greene et al., 2004; Komarraju & Nadler, 2013; Martin et al., 2014; Ouweneel et al., 2013; Wolters et al., 2013). For instance, using path analysis, Greene et al. tested the causal ordering of mastery focus, autonomy support, noncompetitive evaluation, instrumental motivation, and achievement in predicting cognitive engagement and achievement in English courses for 220 high school students. The results showed that self-efficacy predicted cognitive engagement (ß = .14); and that together self-efficacy and engagement positively predicted achievement (grade percentage) (ß = .38, ß=. 15 respectively), and were the only variables with direct paths to achievement. This study supports the notion of a causal model that includes self-efficacy predicting engagement and achievement. What remains to be explored is a causal model that includes self-efficacy with more than one engagement dimension.

Confirming the strong link between self-efficacy and student engagement, Diseth (2011) used confirmatory factor analysis and path analysis to examine motives and strategies as mediators between prior achievement and subsequent achievement of 177 first-year psychology students at a Norwegian university. High school grade point average (GPA) was investigated as a measure of prior achievement; self-efficacy and mastery, performance, and avoidance goal orientation as motives; deep learning strategies (a proxy for cognitive engagement) and surface learning strategies; and the final course exam measured subsequent achievement. Like Greene et al. (2004), Diseth focused only on cognitive engagement, defined as deep strategy use. This study found that self-efficacy positively predicted cognitive engagement and achievement. Moreover, self-efficacy mediated the relationship between prior academic performance and engagement. In addition, Diseth reported finding a partial mediation effect, in which self-efficacy's influence on achievement was mediated by engagement. The findings confirmed prior research showing the strong relationship between self-efficacy and engagement, and lend further support to the present study's hypothesized causal ordering that attribution for failure (judgments about "prior experience") and mathematics self-efficacy predict mathematics engagement, and consequently mathematics achievement.

In support of the existence of the relationship between self-efficacy and outcomes, Wolters et al. (2013) conducted a multiple regression analysis examining the theoretical and empirical link between achievement goals (mastery, performance-avoidance, performance-approach) and self-efficacy beliefs, attribution beliefs, and engagement for achievement. Not surprisingly, self-efficacy emerged as the strongest predictor of mathematics achievement for the 224 high school students enrolled in a high school Algebra II course. Further, Wolters et al. reported that self-efficacy positively predicted both cognitive and behavioral engagement. Worthy of note was the relationship that surfaced between self-efficacy and attribution for failure in mathematics. Specifically, when achievement goals and self-efficacy were together in Step 2 of the analysis, self-efficacy predicted cognitive engagement. However, self-efficacy did not emerge as a significant predictor after attribution for failure was entered into the model. Likewise, this pattern of change was noted in the model predicting metacognitive strategies. In Steps 1 and 2, self-efficacy did not predict engagement, but became a significant predictor when attribution for failure was entered into the model at Step 3. This pattern suggests further investigations are needed to understand the link between students' self-efficacy and attribution as it relates to student engagement and performance.

Komarraju and Nadler (2013) offer more insight into the potential of self-efficacy for predicting engagement and achievement in a study of 257 college students enrolled in an introductory psychology course. Drawing from two studies using multiple analysis of covariance correlation, and regression analyses to examine the importance of theories of intelligence and achievement goals for academic achievement, the authors reported self-efficacy and engagement predicted 18% of the variance in GPA. Cognitive and metacognitive learning strategies (proxy for cognitive engagement) were measured with items reflecting strategies such as rehearsal, elaboration, organization, critical thinking, and self-regulation; resource management (a proxy for behavioral engagement) was measured with time and study environment, effort regulation, peer learning, and help-seeking items. The correlation analyses indicated GPA was positively correlated with self-efficacy and variables indicating cognitive engagement (time management and effort regulation), but not with behavioral indicators such as help seeking. The former was a surprising result as others have reported positive correlation between behavioral indicators of engagement and achievement (Chase et al., 2014; Ladd & Dinella, 2009; Lam et al., 2012; Martin et al., 2014). In block 1 of the step-wise regression procedure, self-efficacy emerged as the major predictor of GPA (ß =. 30) from the following factors: intrinsic and extrinsic motivation, value, control of learning, and low-test anxiety. In Block 2, effort regulation emerged as a significant contributor to the variance of GPA (ß =.32). Block 3 revealed effort regulation as the most significant contributor of GPA, followed by self-efficacy (ß = .17) and help-seeking (ß = -.15). The mediation effect of effort regulation between self-efficacy and GPA is also worth noting. This study supports the existing evidence that self-efficacy plays a substantial role in explaining achievement outcomes and is essential in facilitating cognitive engagement for academic performance. More research is needed, however, to explore the interplay between self-efficacy, engagement, and achievement; specifically, using the three dimensions of engagement in unison to explore the importance of students' theories of intelligence may yield a more robust profile of what matters most for students' academic success.

In support of the link that exists among self-efficacy, engagement, and achievement, Ouweneel et al. (2013) presented results of two studies exploring how self-efficacy levels covary with engagement and performance. In these studies, engagement was conceptualized as a three-dimensional construct with an emphasis on the study engagement dimension. Study engagement was defined by measures reflecting cognitive engagement, which asked students about their effort; For example, "When I'm doing my work as a student, I feel bursting with energy." Study 1 included a sample of 335 university students in their 20s who were asked about their self-efficacy levels over a period of three months, while controlling the study criteria in terms of age, years of study, and gender. Participants were categorized into four groups according to their self-reported self-efficacy at Time 1 (T1) and Time 2 (T2): (a) the low stability group, which started with a low self-efficacy at T1 and ended with a low self-efficacy at T2; (b) the increase group, which started with a low self-efficacy at T1 that increased at T2; (c) the decrease group, whose self-efficacy decreased at T2 from their already low self-efficacy levels at T1; and (d) the high stability group, starting and ending with high self-efficacy. No significant difference was found for self-efficacy and engagement levels in terms of age, years of study, or gender. Other findings revealed that changes in self-efficacy aligned with changes in engagement, but not with changes in performance outcomes (GPA). Ouweneel et al. concluded that a causal path placing self-efficacy as primarily related with engagement and secondarily with achievement could explain the relationship link between changes in engagement and self-efficacy. This notion seems justified as no link was found between the levels of change in self-efficacy and performance. However, another explanation could be that self-efficacy was not domain specific: These measures might not apply to mathematics. Self-efficacy has been shown to be a better predictor within specified domains rather than in general settings (Bandura, 1986).

To document the causal assumption of Study 1, Ouweneel et al. conducted a second study to explore the relationships among self-efficacy, engagement, and performance. Study 2 reported on data collected from 91 participants with a mean age of 20 years placed in one of three conditions: positive feedback, negative feedback, and no feedback (control group). This study sought to test changes in self-efficacy, engagement, and performance, while controlling age, years of study, and gender. In this study, groups were given IQ tasks related to spatial awareness, consisting of 15 questions at T1 and T2. In between T1 and T2, participants were presented with “bogus” positive, “bogus” negative, or “no” feedback with the aim of manipulating their self-efficacy. Positive feedback included a phrase at the end of T1 that stated, “Congratulations! Your IQ score belongs to the top 10% of participants so far.” The negative feedback statement included, "Unfortunately, your IQ score belongs to the worst 10% of participants so far." The control group was given no influencing statements.

In this second study, the engagement measure was changed to reflect a change in task specificity, which was expressed as, "I felt energetic when I carried out the task." The results revealed no difference by age, years of study, or gender. The results of the MANOVA and posthoc tests were interesting: in all three conditions, a main effect was found for time on engagement and performance. Follow-up analysis indicated that the performance increase for the positive group was statistically significant, except for engagement. In the negative group, the test showed that both engagement and achievement decreased at a statistically significant level. The control group showed a significant decrease in engagement, yet performance remained stable. These results demonstrated how the manipulation of self-efficacy changes performance outcomes and supports the importance of including self-efficacy in the present study.

In a more recent longitudinal multilevel analysis, Martin et al. (2014) supported the assumption that self-efficacy is an important predictor of engagement and achievement. Specifically, Martin et al. sought to explain how student variables (self-efficacy and valuing), home variables (parental valuing and computer accessibility), classroom variables (average achievement and perceived climate), and school factors (socioeconomic status and ethnic composition) influence mathematics engagement. The authors studied changes occurring at three key school-year transitions, using a study group of 1,601 Australian Catholic school students in Grades 5 through 8.

Conceptualized as a three-dimensional construct including cognitive, behavioral, and emotional engagement, self-efficacy was found to predict mathematics engagement shifts and achievement across Grades 5 to 8. Cognitive engagement was defined as a willingness to invest effort and was measured with variables such as planning, task management, persistence, self-handicapping, and disengagement. Behavioral engagement was defined by active participation, and was measured with variables such as class participation, homework completion, and effort (the extent and quality of homework). Emotional engagement was termed “affect” due to the positive and negative emotional reactions to the academic environment. It was also measured as enjoyment of mathematics. Consistent with other studies, Martin et al. recognized self-efficacy as a relevant influencer of students’ engagement in mathematics. Further, key school-year transition effects on mathematics engagement were only found to be significant when the student, home, class, and school variables were not entered into the analysis. However, when the latter factors were subsequently entered, self-efficacy predicted the major share of the variance for all measures of mathematics engagement, except homework completion; here, self-efficacy was the second best predictor after the class average for homework. While the sample in this study was large enough to provide sufficient power, it is important to test these results utilizing a U.S. sample. Further, it is important to account for SES, as this has been shown to account for differences in achievement (Martin, Liem, Mok, & Xu, 2012). Because achievement is bound to the social, cultural, and economic levels of individuals, students benefit from belonging to a high SES group as they are more inclined to have access to resources that give them an advantage over their lower SES peers.

Gender, Ethnicity, Economic, Social, and Cultural Status and Engagement

Research has shown that gender, ethnicity, and SES are relevant factors in student engagement and achievement (Hampden-Thompson & Bennett, 2013 Ladd & Dinella, 2009; Rimm-Kaufman et al., 2015; Sciarra & Seirup, 2008). With regard to gender, studies consistently confirm the existence of gender differences in student engagement and achievement (Rimm-Kaufman et al., 2015; Wang & Eccles, 2011). Specifically, girls are more likely to be engaged at a higher level than boys. Ladd and Dinella (2009), reporting from longitudinal data on 189 boys and 194 girls followed from kindergarten to eighth grade, found mean differences in their engagement. Engagement was conceptualized as a three-dimensional construct that included cognition, behavior, and emotion. Cognitive engagement was defined as students' linguistic maturity. Behavioral engagement was defined by students' cooperative-resistant behaviors such as responding promptly to the teacher. Emotional engagement was defined as liking school and measured with positive and negative indicators such as students' enjoyment of classroom activities or with agreeing to statements like "makes up reasons to go home from school." ANOVA analysis (2 X 3) revealed that in the early grades (1 to 3) girls are more behaviorally engaged than boys. Using growth data analysis, Ladd and Dinella found no difference by gender, ethnicity, or SES in student engagement or achievement across Grades 1 to 8. More research explaining the structure and function of engagement is needed to better understand the how engagement develops and the influential factors involved.

Rimm-Kaufman et al. (2015) found that in a sample of 203 fifth-grade girls and 184 boys, girls were more engaged than boys. Tests revealed that when it comes to the learning environment, girls and boys react differently. For instance, as classroom organization increases, boys' cognitive and emotional engagement increases. In contrast, as instructional support increases, the social engagement (collaborative work) of girls decreases, while the slope for boys shows a modest positive rise. From this it can be inferred that while there seems to be a general difference in the level of engagement by gender, engagement is context bound and therefore a malleable.

Like gender, research has shown a link between engagement and ethnicity. Sciarra and Seirup (2008) found that while overall engagement predicted achievement for all ethnic groups, a statistically significant difference existed in the engagement of American Indians, Asians, Blacks, Latinos, and Whites. In this three-year longitudinal study that included 11,388 students from Grades 9 to 12, different dimensions of engagement were found to predict differently for different racial groups. Emotional engagement predicted achievement for Latinos and Whites; while the behavioral engagement dimension contributed the most to the variance in the achievement of Blacks, Asians, and Whites. Likewise, the cognitive engagement dimension contributed more to the variance of Latinos and American Indians. These results denote the importance of considering the relationship between the ethnicity of students and engagement needs when building programs aimed at increasing student achievement.

Ladd and Dinella (2009), seeking to determine the continuing contribution of engagement on the achievement of 383 students followed from kindergarten to eighth grade, provided correlational data that showed nonminority students and those from high SES were more likely to be behaviorally engaged in primary grades than minority students or those from low SES backgrounds. Moreover, these researchers found that students from high SES were more likely to be emotionally engaged, although it did not predict future achievement. Behavioral was the best predictor of future achievement. From this study, it can be surmised that early engagement is key to continued engagement and long-term achievement. While gender, ethnicity, and SES predicted students' initial engagement and achievement levels, they did not significantly predict scholastic growth across grades. These results suggest that engagement and achievement trajectories are not stable and that with focused interventions student factors that cannot be changed (gender, ethnicity, and SES) are less important than other more malleable factors like students' self-efficacy and attributional thinking or the instructional approaches and environment of the schools they attend.

Wang and Eccles (2011), looking at the trajectory of engagement and achievement across Grades 7 to 11, agree that school context is easier to change in support of student engagement and achievement than demographic factors. This study of 1,148 students sought: (a) to determine if the three dimensions of engagement have different trajectories throughout the development of students from Grades 7 to 11; and, (b) to examine whether changes in engagement are a function of changes in students' educational achievement and/or their aspirations. The findings showed that changes in engagement from Grades 7 to 11 corresponded to changes in GPA and students' future aspirations (i.e., declines in engagement were linked to declines in GPA and educational aspirations). The growth model analysis data reported that an initial difference by gender, ethnicity, and SES in the engagement and GPA of students exists, but only socioeconomic status predicted GPA and student aspirations across time. Martin et al. (2015) found these same results in a sample of 1,601 Australian students in Grades 5 to 8.

Reporting from multilevel analysis data, Martin et al. (2014) found SES was consistently associated with students' mathematics engagement shifts and mathematics achievement levels. Measuring SES with resource factors, this study showed that students' socioeconomic status can affect their ability to effectively engage in the learning process, and therefore schools need more information about how engagement functions and what is needed to differentiate the approach to address student engagement and mathematics achievement.

Altogether, this literature review has shown the value in examining the association amongst student beliefs, engagement, and achievement. Engagement is recognized as an outcome of student beliefs such as self-efficacy and attribution for failure and success, as well as a mediator for achievement between attribution for failure and self-efficacy. What remains to be explored is the interrelatedness of students' performance, attribution for failure, and self-efficacy with a unified three-dimensional model of engagement. Some studies have looked at attribution for failure and self-efficacy with engagement separately (De Castella, Byrne, Covington, 2013; Galla et al., 2014; Walker & Greene, 2009); others have combined attribution and self-efficacy with engagement (Wolters, 2013), but no study to date has tested a full model of engagement that specifies attribution for failure and self-efficacy as sources of mathematics engagement, which in turn lead to mathematics performance. Therefore, after confirming the existence of a three-dimensional model, this study tested a causal model that explored the variance explained by attribution for failure, self-efficacy, gender, ethnicity, ESCS, and a three-dimensional model of engagement as it relates to mathematics performance.

Chapter Three

Methods

This study used the statistical data from a national database to examine the measurement and structural model of a three-dimensional mathematics engagement construct comprising cognition, behavior , and emotion as framed by Fredricks et al. (2014). The Program for International Student Assessment (PISA) dataset represents a substantial cache of information on the mathematics performance of U.S. students and is considered representative of the population, and consequently generalizable. This study is situated within a social cognitive perspective that acknowledges the influential role played by students' personal factors.

Study design. After the university Institutional Review Board determined this research to be exempt (Appendix A), the researcher analyzed the data in two phases. Phase one sought to validate a three-dimensional model of engagement using confirmatory factor analysis (CFA). In Phase 2, structural equation modeling (SEM) was conducted in order to examine the mediating effects of mathematics engagement on the relationships among students' attribution for failure and mathematics self-efficacy on mathematics performance. The methods of analysis are detailed in the sections that follow.

Data source. The data from the 2012 Program for International Student Assessment (PISA) allows countries to compare learning outcomes of high school students. In the United States the National Center for Education Statistics (NCES) conducts the surveys and makes the data freely accessible to the public on the NCES website (NCES, 2012b). PISA 2012 constitutes the PISA's fifth cycle of data gathering. Each PISA data collection cycle assesses the domains of mathematics, science, and reading every three years. One domain rotates as the central focus in the PISA data collection. For 2012, mathematics was the expanded focus.

The 2012 PISA U.S. national school sample consisted of 162 schools. The PISA requires a national sample standard of 35 to 50 participating students per school. The survey lasted a total of two hours for each student, with two-thirds of the testing time being dedicated to mathematics, the major focus for 2012. PISA utilizes a questionnaire rotation method whereby not all students are administered the attitudinal and noncognitive construct questions, and two-thirds of students respond to the same construct. This approach limits the testing time to decrease response burden. Rotation design distributes items across several different booklets so that each student is only asked to respond to a limited number of questions (OECD, 2016). As a result of this approach, 37% of the data is missing by design.

The U.S. PISA sample was stratified into eight groups by school (public or private) and region (Northeast, Central, West, Southeast). This frame was further stratified by categorical variables such as (a) grade range of the school (five categories), (b) location (city, suburb, town, rural), (c) ethnic diversity of above or below 15% (Black, Hispanic, Asian, Native Hawaiian/Pacific Islander, and American Indian/Alaska Native students), (d) gender (mostly female [percentage female >= 95%], mostly male [percentage female < 5%]), and (5) state.

The assessment instrument was developed by international experts and was reviewed by representatives from each of the education systems expected to participate in the PISA. All members of the education systems field-tested the 222 assessment items (85 mathematics, 44 reading, 53 science, and 40 financial literacy). The PISA consisted of a paper-based assessment with a two-hour administration time that provided performance scores for students' mathematics, science, and reading literacy (mathematics performance is a proxy for mathematics literacy in this study). An additional student questionnaire was completed in approximately 30 minutes comprised of items asking about student background, attitudes toward mathematics, and learning strategies. Finally, school principals were asked to complete a 30-minute school questionnaire describing the school's demographics and learning environment. This present study only focuses on student responses about their engagement in mathematics and mathematics performance scores.

Participants. The 2012 U.S. Program for International Student Assessment's (PISA) national student sample consisted of 4,978 high school students in Grades 8 to 12 from 162 schools. Respondents included 2,453 females and 2,525 males. There were 6 students in eighth grade, 538 freshman, 3,633 sophomores, 794 juniors, and 7 seniors. The ethnic composition of participants was 2,553 White, 641 Black or African American, 1,176 Hispanic, 227 Asian, and 213 Multiracial. A small group of students chose not to report ethnicity (other = 101). The final sample consisted of 3,007 respondents with 1,515 females and 1,492 males (1,593 White, 382 Black/African American, 713 Hispanic, 143 Asian, and 176 Multiracial/Other [collapsed]).

Data preparation. An initial examination of the PISA (2012) dataset was conducted. The indices for attribution for failure and mathematics self-efficacy were PISA 2012-provided measures. The controlling variables of gender and ethnicity were retained as provided by the PISA dataset. The economic, social, and cultural status (ESCS) continuous variable was converted into a categorical variable using the median to create low and high ESCS groups.

Preliminary analysis included variable distribution, assumptions underlying parametric statistics (normality, linearity, and heteroscedasticity), and adjustment for missing data. Data were examined through analysis of residual plots, histograms, and scatterplots. Listwise deletion was considered an acceptable approach to deal with missing data; as discussed earlier this missing data (40%) was by design.

Assumptions of multicollinearity and homogeneity of variance were met. A curve estimation for all relationships in the model determined that all relationships were sufficiently linear to be tested using covariance-based structural equation modeling such as the one used in SPSS AMOS software.

Measures. Measures in this analysis assessed relationships among attribution for failure in mathematics, mathematics self-efficacy, mathematics engagement, mathematics performance, gender, ethnicity, and economic, social, and cultural status (ESCS).

Mathematics performance. Mathematics performance is a proxy for the PISA 2012 mathematics literacy measure indicated by students' ability to formulate situations mathematically; employ mathematics facts, procedures, and reasoning; and interpret, apply, and evaluate mathematics outcomes (OECD, 2016). PISA uses the imputation method of plausible values to report mathematics performance. Plausible values represent the range of abilities that may exist in the general population. Plausible values are used to achieve a broad coverage by dividing the total pool of items consisting of 270 minutes into 9 clusters of 30 minutes each. Under this design, each student responds to only a fraction of the entire assessment in the form of a booklet. This is done to minimize school disruption and limit the testing time. PISA provides five plausible values for mathematics performance with a theoretical range from 220 to 784 with a mean score of 482 and standard deviation of 89.24 for the 4,978 U.S. respondents (for OECD, $M = 500$). For this present study only one plausible value was used, as working with one plausible value has been shown to adequately provide an unbiased estimate of population parameters (OECD, 2013). As designed, there is very little variation across the five plausible values provided by PISA: The difference between any two plausible values is no more than 1.32 points (OECD, 2016). After dealing with missing data the mean mathematics performance score for the sample consisting of 3,007 fifteen-year-olds was 486.

Test questions represented a wide range of difficulty. Three types of question formats were used to assess mathematics performance: multiple choice, closed short answer, and open constructed-response. An example of a closed short answer question is:

> The Gotemba walking trail up Mount Fuji is about 9 kilometers (km) long. Walkers need to return from the 18 km walk by 8 pm. Toshi estimates that he can walk up the mountain at 1.5 kilometers per hour on average, and down at twice that speed. These speeds take into account meal breaks and rest times. Using Toshi's estimated speeds, what is the latest time he can begin his walk so that he can return by 8 pm? (OECD, 2013)

Engagement. Mathematics engagement is conceptualized as a three-dimensional construct that includes the cognitive, behavioral, and emotional aspects involved in learning mathematics for 15-year-old high school students. The initial mathematics engagement construct was comprised of 14 items guided by a review of empirical research along with a comparison and analysis of the availability of items in the PISA 2012 dataset. Item inclusion in the construction of the mathematics engagement index was made in consideration of the conceptualization of mathematics engagement as a three-dimensional construct (Fredricks et al. 2004) and as the actions that follow motivation (Appleton et al., 2006, 2008), as well as the logical validity that items were indeed indicators of mathematics engagement. A similar approach for creating indices of engagement was conducted by Hampden-Thompson and Bennett (2013). The reliability for this 14 item scale was $\alpha = .84$.

Cognitive engagement. Cognitive engagement is defined as students' effortful investment in mathematics learning and characterized by their openness to and persistence in problem solving. Six items from students' level of agreement with the following statements: (a) I seek explanations, (b) I can easily link facts together, (c) I can handle a lot of information, (d) I remain interested, (e) I continue to perfection, and (f) I exceed expectations. A five-point scale was used with response categories of "not at all like me," "not much like me," "somewhat like me," "mostly like me," and "very much like me." The theoretical score range is from 5 to 30. The reliability for this scale was α = .81.

Behavioral engagement. Behavioral engagement is defined as purposeful actions for mathematics learning and characterized by students' mathematics work ethic. It includes five items from students' level of agreement with the following statements: (a) I complete homework in time for class, (b) I work hard on my mathematics homework, (c) I pay attention in my mathematics class, (d) I keep my mathematics well organized, and (e) I listen in mathematics class. A four-point scale was used with response categories of "strongly agree," "agree," "disagree," and "strongly disagree" with a point value from 1 to 4. The theoretical score range is from 4 to 20. The reliability for this scale was α = .84.

Emotional engagement. Emotional engagement is defined as students' positive feelings about learning and is characterized as enjoyment of learning, and can also reflect students' feelings of anticipation, as in looking forward to lessons (Fredricks et al., 2004; Ladd & Dinella, 2009; Sciarra & Seirup, 2008). It includes four items assessing students' level of agreement with the following statements: (a) I enjoy reading about mathematics, (b) I look forward to my mathematics lessons, (c) I do mathematics because I enjoy it, and (d) I am interested in the things I learn in mathematics. A four-point scale was used with response categories of "strongly disagree," "disagree," "agree," and "strongly agree" with a point value from 1 to 4. The theoretical score range is from 4 to 16. The reliability for this scale was α = .83.

Attribution for failure in mathematics. The attribution for failure in mathematics is a PISA scaled index using a weighted likelihood estimate (WLE). Attribution for failure in mathematics was constructed using student responses from six questions on a 4-point scale with a response category of “very likely,” “likely,” “slightly likely,” and “not at all likely.” The scale was reverse coded for ease of analysis; higher scores represent higher attribution for failure. This variable measured students’ perceived self-responsibility for failing in mathematics using the following scenario, “Each week, your mathematics teacher gives a short quiz. Recently you have done badly on these quizzes. Today you are trying to figure out why.” Responses about their level of agreement were measured with the following statements: (a) I’m not very good at solving mathematics problems, (b) My teacher did not explain the concepts well this week, (c) This week I made bad guesses on the quiz, (d) Sometimes the course material is too hard, (e) The teacher did not get students interested in the material, and (f) Sometimes I am just unlucky. The theoretical score range is 4 to 24. The reliability for this scale was α = .73.

Mathematics self-efficacy. Mathematics self-efficacy is an 8-item PISA scaled index using a weighted likelihood estimate (WLE) that asked student to indicate their confidence level for mathematics tasks with “very confident,” “confident,” “not very confident,” “not at all confident.” Responses about their level of confidence were measured with: (a) “Using a train schedule to figure out how long it would take to get from one place to another,” (b) “Calculating how much cheaper a TV would be after a

30% discount," (c) "Calculating how many square meters of tiles you need to cover a floor," (d) "Understanding graphs presented in newspapers," (e) "Solving an equation like 3x + 5 = 17," (f) "Finding the actual distance between two places on a map with a 1:10,000 scale," (g) "Solving an equation like 2(x+3) = (x+3)(x-3)," and (h) "Calculating the petrol consumption rate of a car." The theoretical score range is 4 to 32. The reliability for this scale was $\alpha = .85$.

Moderators. The analysis included gender, ethnicity, and economic, social, and cultural status (ESCS) as controlling variables.

Gender. Gender is based on student responses to the question: Are you female or male? Data indicated 49% female and 51% male.

Ethnicity. Ethnicity is based on student responses identifying themselves as White (*n* = 1,593), Black or African American (*n* = 382), Hispanic (*n* = 713), Asian (*n* = 143), or Multiracial/Other (*n* = 176) (NCES, 2014). Multiracial and Other were collapsed.

Economic, social, and cultural status. The PISA index for economic, social, and cultural status ESCS was captured with three factors: parental occupation , parent level of education, and home possessions related to family wealth and home educational resources (OECD, 2012). Question format included open-ended questions about their mother and father's occupation and educational level, for example, "What is your mother/father's main job?," "What is the highest level of schooling completed by your mother/father?," and a question about their possessions at home, which asked students to respond yes or no to a list of items that are in their home (e.g., a desk to study at, a room of your own, a quiet place to study). In order to fully understand the impact of ESCS on the variables of interest in this study, this continuous variable with a range from -3.80 to 3.12 was transformed into a dichotomous variable by splitting the ESCS variable by its median value of .31. The lowest values through .31 were recoded as 1 to represent the low ESCS group. All values above .31 were recoded as 2 to represent students in the high ESCS group. The reliability for this scale was $\alpha = .70$.

Data Analysis Approach

A two-phase approach was adopted to examine the tenability of the measurement and structural model of the latent mathematics engagement factor. Phase 1 was conceptual and included using confirmatory factor analysis to assess the validity of a three-dimensional model of engagement. A set of variables was chosen and tested according to procedures detailed by Kline (2011) to test whether a distinct construct emerged using convergent and discriminant validity statistics. In Phase 2, path analysis was used to test a fully articulated model framing the impact of students' attribution for failure and mathematics self-efficacy on the three-dimensional engagement model and mathematics performance, while controlling for gender, ethnicity, and SES. This procedure was guided by the analytic functions of mediators offered by Baron & Kenny (1986) and Little, Card, Bovaird, Preacher, and Crandall (2007) and methodology detailed by Mueller and Hancock (2007).

In Phase 1, confirmatory factor analysis (CFA) was conducted using AMOS 22.0 software with maximum likelihood as the method of model estimation. CFA was used to obtain a three-dimensional measurement model of engagement with convergent and discriminant validity, and reliable measures. The CFA was deemed the most powerful approach for assessing the measurement validity of a theoretical structural model. CFA accounts for measurement error and incremental assessment of how the data fits the model in accordance with fit indices for absolute fit such as Discrepancy Chi-square (Wheaton et al., 1977) and Root Mean Square of Error Approximation (RMSEA; Browne & Cudeck, 1993); incremental fit is assessed with Comparative Fit Index (CFI; Bentler, 1990), Tucker-Lewis Index (TLI; Bentler & Bonett, 1980); and Chi Square/Degrees of Freedom (χ^2/df; Marsh & Hocevar, 1985) used to identify a parsimonious fit. Configural and metric invariance tests were conducted to ensure the latent mathematics factor produced in CFA was a comparable measure across gender, ethnic, and ESCS groups equally (Mueller & Hancock, 2007). Configural invariance is attained if model fit is achieved when groups are tested together and freely. Metric invariance was assessed with a multigroup moderation test using Stat Tools Package.xlsm (Gaskin, 2012b) to compare individual group regression weights with critical ratio for differences. At least one standardized regression weight for each factor should be nonsignificant to achieve metric invariance.

Phase 2 of the data analysis involved using structural equation modeling conducted using AMOS 22.0 with maximum likelihood as the method of model estimation. Maximum likelihood estimation was used as assumptions of a sufficient sample size (100 to 200), multivariate normality, and a correctly specified model were met. In order to determine parameter interpretability acceptable model fit is required (Hu & Bentler, 1999). Acceptable model fit is determined by comparing the incremental, absolute, and parsimonious indices that assess the relationships among attribution for failure, mathematics self-efficacy, mathematics engagement, and mathematics performance, while controlling for gender, ethnicity, and ESCS on mathematics performance scores ($\chi^2/df \leq 5$; RMSEA $\leq .06$; CFI $\geq .95$ and NFI $\geq .90$) (Chou & Bentler, 1995; Marsh, Hau, & Wen, 2004; Mueller & Hancock, 2007).

In sum, the methodology used in this study focused on analysis conducted to assess that the contribution of each item to the engagement construct was valid and statistically significant in phase 1. In phase 2, the focus was on understanding the relationship of mathematics engagement to mathematics performance, attribution for failure and mathematics self-efficacy. This phase provided information about mediation effects and estimations of the proportion of variances explained, which included gender, ethnicity, and economic, social and cultural status as control variables. For completeness, supplementary analysis were conducted to explore group differences, direct and indirect analysis of mediation, and the relationship of the control variables with attribution for failure, mathematics self-efficacy, mathematics engagement and performance.

Chapter Four

Results

The purpose of this study was to investigate the existence of a three-dimensional model of engagement and its mediating effect on the relationships among attribution for failure, mathematics self-efficacy, and mathematics performance. The investigation will answer the following questions:

1. Can mathematics engagement be modeled as a three-dimensional construct, and what are the components?
2. Are the effects of attribution for failure and mathematics self-efficacy on mathematics performance mediated by mathematics engagement while controlling for gender, ethnicity, and ESCS?

The following section contains a description of the preliminary analyses undertaken, including data cleaning and how missing data were addressed. In the second section, the descriptive and correlational analyses for the initial and final measurement and structural models are presented. The final two sections present the results for each research question.

Preliminary analysis. A detailed list of the measures used in this study can be found in Appendix B. An initial examination of the 2012 PISA dataset identified 14 items consistent with the conceptualization and definition of the three-dimensional latent engagement construct adopted from Fredricks et al. (2004). Two PISA-derived scales were retained to investigate the mediating effect of a three-dimensional engagement construct for the relationships among attribution for failure, mathematics self-efficacy, and mathematics performance. In addition, three demographic variables were used as controls: gender, ethnicity, and economic, social and cultural status (ESCS).

A preliminary missing value assessment revealed 40% missing data. As discussed earlier, the PISA is a large survey whose systematic pattern of missingness is most likely missing at random due to the rotating design of the survey. Therefore, listwise deletion, using SPSS 22.0, was considered appropriate given the sample size was sufficiently large (N = 3,007) that there is little to gain from reduced standard errors and statistical power (Gagné & Hancock, 2006; Marsh & Hau, 1999).

An examination of histograms and boxplots indicated that scores on the variables of interest were normally distributed, with two outliers appearing in the Continue to Perfection variable (case numbers 2,880 and 2,930). Because these two outliers were at the extreme (3 standard deviations), they were excluded. Univariate screening of all independent variables was conducted, yielding no issues of normality. *Z*-scores for individual items were within the expected range (-1.96 to 1.96), values of skewness were greater than 2, and values for Kurtosis were within the acceptable range of -1.96 to 1.96 (Curran, West, & Finch, 1997). Multivariate assumption of linearity and collinearity were investigated. A curve estimation determined that all relationships in the model were sufficiently linear, and thus could be tested using covariance-based structural equation modeling as used in the AMOS 22 statistical software package. No issues with collinearity or multicollinearity were observed: (a) tolerance levels were greater than .10 (Tabachnick & Fidell, 2001); (b) the variance inflation factors (VIF) were all less than 4 (O'Brien, 2007) for all independent variables (see Table 1), and (c) the statistically significant Pearson's *R* correlation values ranged from -.32 to .84 (see Table 2).

Table 1

Descriptive Statistics for All Variables in Initial Confirmatory Factor Analysis (CFA) Model of Mathematics Engagement

Variable	*M*	*SD*	Min	Max	Collinearity Tolerance	Collinearity *VIF*
Mathematics Performance	486.42	88.28	224.73	783.70	.91	1.10
Gender	–	–	1.00	2.00	–	–
ESCS	–	–	1.00	6.00	–	–
Ethnicity	–	–	1.00	5.00	.82	1.22
Attribution for Failure	-.35	1.16	-3.77	3.91	.85	1.18
Self-Efficacy	.15	1.00	-3.75	2.27	.61	1.63
R_Interested	3.60	.98	1.00	5.00	.62	1.61
To Perfection	3.61	1.07	1.00	5.00	.51	1.97
Exceed Expectations	3.38	1.09	1.00	5.00	.59	1.71
Handle Information	3.67	.99	1.00	5.00	.58	1.74
Seek Explanations	3.88	.98	1.00	5.00	.64	1.57
Can Link Info	3.73	1.00	1.00	5.00	.50	2.01
HW On Time	3.12	.76	1.00	4.00	.54	1.84
Work Hard on HW	3.00	.74	1.00	4.00	.48	2.09
Attention	3.22	.65	1.00	4.00	.28	3.63
Listen	3.26	.62	1.00	4.00	.27	3.70
HW Organized	3.01	.80	1.00	4.00	.65	1.55
Enjoy R_Math	2.20	.83	1.00	4.00	.47	2.14
Look Forward	2.43	.89	1.00	4.00	.35	2.87
Enjoy Math	2.27	.93	1.00	4.00	.32	3.12
Mathematics Engagement	.01	.49	.90	3.90	.76	1.31

Note. *N* = 3,007. Ethnicity = White, Black/African American, Asian, Hispanic, Multiracial/ Other. For gender, females = 1, males = 2; HW = Homework.

Table 2

Correlation Analysis and Reliability for Variables in CFA and SEM Model of Mathematics Engagement

Variables	α	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1 Mathematics Performance		–																			
2 Gender		-.07**	–																		
3 ESCS	.70	.40**	.01	–																	
4 Attribution for Failure	.73	-.16**	.08**	-.05**	–																
5 Self-Efficacy	.85	.57**	-.14**	..30**	-.25**	–															
6 CD: R_Interested	.81	.12**	.05**	.09**	-.13**	.25**	–														
7 CD: To Perfection		.18**	-.06**	.15**	-.12**	.31**	.57**	–													
8 CD: Exceed Exp		.12**	.02	.08**	-.14**	.29**	.48**	.59**	–												
9 CD: Handle Information		.32**	-.09**	.19**	-.21**	.44**	.31**	.35**	.35**	–											
10 CD: Seek Explanations		.27**	-.02	.18**	-.09**	.34**	.26**	.33**	.33**	.44**	–										
11 CD: Can Link Info		.32**	.10**	.22**	-.17**	.46**	.31**	.35**	.37**	.59**	.56**	–									
12 BD: HW On Time	.84	.17**	.04*	.21**	-.19**	.29**	.22**	.29**	.25**	.24**	.22**	.20**	–								
13 BD: Work Hard on HW		.05**	.08**	.08**	-.19**	.21**	.23**	.31**	.27**	.22**	.20**	.20**	.63**	–							
14 BD: Attention		.08**	.06**	.07*	-.19**	.21**	.24**	.26**	.24**	.20**	.22**	.21**	.42**	.52**	–						
15 BD: Listen		.13**	.07**	.09**	-.18**	.24**	.24**	.28**	.25**	.21**	.24**	.22**	.43**	.52**	.84**	–					
16 BD: HW Organized		.04*	.15**	.08**	-.15**	.19**	.23**	.31**	.26**	.19**	.19**	.18**	.42**	.49**	.47**	.48**	–				
17 ED: Enjoy R_Math	.83	.13**	-.13**	.01	-.27**	.32**	.20**	.22**	.21**	.24**	.13**	.19**	.19**	.24**	.21**	.23**	.20**	–			
18 ED: Look Forward		.09**	-.06**	-.02	-.30**	.34**	.27**	.27**	.25**	.23**	.15**	.19**	.25**	.35**	.32**	.34**	.29**	.66**	–		
19 ED: Enjoy Math		.15**	-.05**	-.01	-.32**	.37**	.25**	.26**	.24**	.23**	.14**	.20**	.25**	.32**	.30**	.31**	.27**	.70**	.78**	–	
20 MathematicsEngagement	.83	.19**	.01	.13**	-.29 **	.43**	.65**	.71**	.67**	.42**	.37**	.40**	.60**	.66**	.63**	.65**	.52**	.55**	.64**	.59**	–

Note. ESCS = Economic, social, and cultural status. CD = Cognitive dimension of engagement; BD = dimension of engagement; EE = Emotional dimension of engagement, HW = Homework.
$^{**}p < .01$. $^{*}p < .05$ (2-tailed).

Descriptive and correlation analyses. Table 3 provides means and standard deviations of variables in the final measurement and structural models for the 3,007 participants in this study. The internal consistency reliability estimates were acceptable for all scales.

Table 3

Mean and Standard Deviations of Variables in Final Measurement and Structural Model

	Min	Max	*M*	*SE*	*SD*	α
Mathematics Performance	224.73	783.70	486.42	1.61	88.28	
Attribution for Failure	-3.77	3.91	-.36	.02	1.16	.72
Self-Efficacy	-3.75	2.27	.16	.02	1.00	.85
Gender	1.00	2.00	–	–	–	
Ethnicity	1.00	5.00	–	–	–	
ESCS	1.00	2.00	–	–	–	.70
CD: R_Interested	1.00	5.00	3.60	.02	.99	.78
CD: To Perfection	1.00	5.00	3.38	.02	1.08	
CD: Exceed Exp	1.00	5.00	3.37	.02	1.09	
BD: HW on Time	1.00	4.00	3.12	.01	.76	.83
BD: Work Hard HW	1.00	4.00	3.00	.01	.74	
BD: Attention	1.00	4.00	3.22	.01	.65	
BD: Listen	1.00	4.00	3.27	.01	.62	
ED: Enjoy R_Math	1.00	4.00	2.20	.02	.83	.88
ED: Look Forward	1.00	4.00	2.43	.02	.89	
ED: Enjoy Math	1.00	4.00	2.27	.02	.93	
Mathematics Engagement	.90	3.90	2.78	.01	.49	.83

Note. *N* = 3,007. Ethnicity = White, Black/African American, Asian, Hispanic, Multiracial, and Other (multiracial and other were collapsed). ESCS = Economic, social, and cultural status. CD = Cognitive dimension of engagement; BD = Behavioral dimension of engagement; ED = Emotional dimension of engagement, HW = Homework.

An analysis of means and standard deviation values listed in Table 3 for the sample of 3,007 fifteen-year-old U.S. students indicated that mathematics performance scores were at the upper limits of the level-2 baseline proficiency (*M* = 486.42, *SD* = 88.28). Overall, students reported average levels of mathematics engagement (*M* = 2.78, *SD* = .49), adaptive attribution for failure beliefs (*M* = -.02, *SD* = 1.16), and mathematics self-efficacy (*M* = .16, *SD* = 1).

Bivariate correlation patterns among the variables in the final CFA and SEM analyses (attribution for failure, mathematics self-efficacy, mathematics engagement [9 items], gender, ethnicity, and ESCS) were all in the expected direction (Table 2). The statistically significant association strength among the independent and dependent variables ranged from *r*= -32 to *r* = .78. Consistent with this study's theory, mathematics performance is correlated with attribution for failure (*r* = -.16) and mathematics self-efficacy (*r* = .57), all the engagement items, mathematics engagement (*r* = .19), gender (*r* = -.07), and ESCS (*r* = .40); the strongest correlation occurred with mathematics self-efficacy (*r* = .57). Attribution for failure was more highly correlated with mathematics engagement (*r* = .-29) followed by mathematics self-efficacy (*r* = -.25), mathematics performance (*r* = -.16), gender (*r* = .08), and ESCS (*r* = -.06). As for correlations involving the demographic variables, gender was not associated with ESCS at a significant level, but it was associated with mathematics self-efficacy (*r* = -.14).

Supplementary analysis of groups. In order to examine the mean differences for attribution for failure, mathematics self-efficacy, mathematics engagement, and mathematics performance by gender, ethnicity, and ESCS, independent sample *t*-tests and ANOVA analyses were carried out. Affect sizes for statistical differences were calculated using Cohen's *d*: small effect size = .20, medium effect = .50, large effect = .80. When the sample was split by gender, the mean scores of male students (*M* = 492.68, *SD* = 90.37) tended to be slightly higher than the scores of female students (*M* = 480.26, *SD* = 85.75) for mathematics performance. The independent sample t-test showed this difference (Δ = 12.42) was statistically significant, *t*(2991) = 3.865, *p* = .001 [Levene's assumption of homogeneity of variance was violated, *F*(2, 3005) = 8.93, *p* = .003]. Likewise, males had slightly higher scores for adaptive attribution for failure (*M* = - .44, *SD* = 1.18); and mathematics self-efficacy beliefs (*M* = .29, *SD* = 1.06) than female students (attribution for failure *M* = -.27, *SD* = 1.13; self-efficacy *M* = .02, *SD* = .91). These differences were also statistically significant [attribution for failure: *t*(3005) = -4.14, *p* = .001; self-efficacy, equal variances not assumed *F*(2, 3005) = 1.77, *p* = .001; *t*(3005) = 7.53, *p* = .001. Gender differences in mathematics engagement were not statistically different *t*(3005) = -.59, *p* = .56. The mean differences uncovered were examined using Cohen's *d* statistics and ranged from .10 to .26, which are a small effects based on Cohen's (1992) guidelines (see Table 4).

Table 4

Descriptive Statistics by Gender for Variables in Final CFA and SEM

	Females		Males			
	M	*SD*	*M*	*SD*	*t*-value	Cohen's *d*
Mathematics Performance	480.26	85.75	492.68	90.37	3.87***	.14
Attribution for Failure	-.27	1.13	-.44	1.18	-4.14***	.16
Self-Efficacy	.02	.91	.29	1.06	7.54***	.09
CD: Interested	3.55	.99	3.65	.97	2.78*	.10
CD: to Perfection	3.67	1.08	3.55	1.06	-3.13*	.11
CD: Exceed Exp	3.40	1.08	3.35	1.10	-1.13*	.16
BD: HW On Time	3.15	.76	3.09	.75	-2.34	—
BD: Work Hard HW	3.06	.70	2.94	.76	-4.50***	.17
BD: Attention	3.26	.64	3.19	.65	-3.29*	.11
BD: Listen	3.30	.61	3.22	.63	-3.70***	.13
ED: Enjoy R_Reading	2.09	.80	2.30	.84	7.03***	.26
ED: Look Forward	2.38	.90	2.47	.88	3.01*	.10
ED: Enjoy Math	2.22	.94	2.32	.93	2.74*	.11
MathematicsEngagement	2.79	.48	2.78	.50	-.59	—

Note. *N* = 3,007 (Females = 1,515; Males = 1,492). CE = Cognitive dimension of engagement; BD = Behavioral dimension of engagement; ED = Emotional dimension of engagement. Cohen's *d*: Small effect size = .20. Medium effect = .50. Large effect = .80.
* $p < .05$. *** $p < .001$.

Table 5 displays results from ANOVA analysis comparing the means for all variables by the five ethnic groups in this study. Overall there were statistical differences in mathematics performance by ethnicity: $F(4, 3006) = 117.41$, $p < .001$. Students who identified as Asian reported slightly higher scores for mathematics performance (*M* = 543, *SD* = 85.79) than White (*M* = 508.31, *SD* = 82.81), Multiracial/Other (*M* = 487.22, *SD* = 90.31), Hispanic (*M* = 458.07, *SD* = 82.30), and students who identified as Black/African American (*M* = 426.48, *SD* = 73.28).

Table 5

Descriptive Statistics by Ethnicity for Variables in Final CFA and SEM

	White Students		Black/African American		Hispanic Students		Asian Students		Multiracial/ Other		
Variables	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*F*-Value
MathematicsPerformance	508.31	82.81	426.48	73.28	458.07	82.30	543.15	85.79	487.22	90.31	117.41 ***
FAILMAT	-.33	1.13	-.45	1.30	-.34	1.11	-.40	1.15	-.36	1.24	.90
MATHEFF	.23	1.01	-.04	.91	.01	.91	.57	1.11	.20	1.09	16.06 ***
CD: Interested	3.60	.97	3.68	1.04	3.55	.98	3.70	.94	3.51	1.06	1.89
CD: to Perfection	3.64	1.08	3.64	1.06	3.51	1.06	3.78	.97	3.52	1.09	2.91 *
CD: Exceed Exp	3.39	1.09	3.31	1.13	3.40	1.06	3.48	.98	3.25	1.16	1.33
BD: HW On Time	3.17	.74	3.15	.76	2.98	.77	3.36	.67	3.03	.83	12.30 ***
BD: Work Hard HW	2.97	.75	3.10	.72	2.96	.72	3.14	.68	3.07	.73	4.79 *
BD: Attention	3.20	.65	3.26	.67	3.24	.63	3.33	.64	3.24	.63	1.98
BD: Listen	3.24	.63	3.28	.63	3.26	.60	3.38	.62	3.27	.63	1.85
ED: Enjoy R_Reading	2.11	.81	2.26	.89	2.29	.80	2.52	.84	2.23	.86	13.46 ***
ED: Look Forward	2.30	.87	2.57	.95	2.58	.85	2.67	.85	2.41	.95	19.23 ***
ED: Enjoy Math	2.16	.90	2.41	.99	2.39	.93	2.55	.89	2.31	1	14.18 ***
Mathematics Engagement	-.08	.58	.10	.62	.08	.56	.10	.55	.06	.60	5.22 ***

Note. *N* = 3,007. † = Reference group. White = 1,593; Black = 382; Hispanic = 713; Asian = 143; Multiracial/Other = 176. FAILMAT = attribution for failure; MATHEFF = mathematics self-efficacy; CD = Cognitive dimension of engagement; BD = Behavioral dimension of engagement; ED = Emotional dimension of engagement; HW = Homework.
**p* < .05. ****p* < .001.

There were no statistically significant differences for attribution for failure by ethnicity, *F*(4, 3006) = .90, *p* = .46. Regarding mathematics self-efficacy, the results showed a statistically significant difference existed by ethnicity: *F*(4, 3006) = 16.06, *p* = .001. The mean for students who identified as Asian was higher (*M* = 57, *SD* = 1.11) than White (*M* = .23, *SD* = 1.01), Multiracial/Other (*M* = .20, *SD* = 1.09), Hispanic (*M* = .01, *SD* = 91), and Black/African American (*M* = -.04, *SD* = .91). The mathematics engagement mean values were also statistically different by ethnicity: *F* (4, 3006) = 15.38, *p* = .001. Asians (*M* = 2.94, *SD* = .46) and Blacks/African Americans (*M* = 2.83, *SD* = .50) reported being slightly more engaged in mathematics learning than Hispanics (*M* = 2.78; *SD* = .48), Whites (*M* = 2.76; *SD* = .49), or students who identified as Multiracial/Other (*M* = 2.75, *SD* = .54). Although statistically significant differences were found, for most relationships, the effect sizes were negligible (see Cohen's *d* statistics in Table 6), except in the case of mathematics performance. The differences for mathematics performance ranged from .04 to 1.64, while the range for mathematics self-efficacy was .03 to .60.

Table 6

Cohen's d Statistics for Groups by Ethnicity

	Black/ African American	Hispanic	Asian	Multiracial/ Other
	Effect Size for Mathematics Performance			
White	1.05	.61	.41	.24
Black/African American		.41	1.46	.74
Hispanic			1.01	.34
Asian				.63
	Effect Size for Mathematics Self-Efficacy			
	Black/ African American	Hispanic	Asian	Multiracial/ Other
White	.27	.23	.32	.03
Black/African American		.05	.60	.23
Hispanic			.55	.19
Asian				.34
	Effect Size for Mathematics Engagement			
	Black/ African American	Hispanic	Asian	Multiracial/ Other
White	.13	.04	.39	.01
Black/African American		.10	.38	.14
Hispanic			.39	.05
Asian				.37

Note. Mathematics engagement is a computed latent factor using AMOS.
Cohen's *d*: Small effect size = .20. Medium effect = .50. Large effect = .80.

The mean scores and effect size of the differences between economic, social, and cultural status groups (low versus high) are displayed in Table 7. The independent sample *t*-tests showed that students from high ESCS reported higher scores in mathematics performance (M = 517.90; SD = 86.19, $F(2, 3005) = 15.95$, p = .001, Levene's test for violation of homogeneity of variance; $t(2971) = -20.71$, p = .001) and mathematics self-efficacy (M = 2.73; SD = .50; $F(2, 3005) = 15.95$, p = 001, Levene's test for violation of homogeneity of variance; *t*(2970) = -.14.11, *p* = .001) and mathematics engagement with equal variance assumed (*M* = 2.83, *SD* = .48; *t*(2970) = -5.95, *p* < .001) than their same-age peers who belong to the low ESCS group (*M* = 455.47, *SD* = 78.88; *M* = -.09, *SD* =.92; *M* = 2.73, *SD* = .48). Levene's test indicated unequal variances, so degrees of freedom were adjusted for all variables, except mathematics engagement . No statistical differences were noted on the attribution for failure variable by ESCS.

Table 7

Descriptive Statistics by ESCS for Variables in Final CFA and SEM

	Low ESCS		High ESCS				
Variables	Mean	*SD*	Mean	*SD*	*t*-statistic		Cohen's *d*
Mathematics Performance	455.47	78.88	517.90	86.19	-20.71	***	.76
Attribution for Failure	-.31	1.22	-.39	1.09	1.95		—
Mathematics Self-Efficacy	-.09	.92	.40	1.01	-14.12	***	.52
R_Interested	3.52	1.01	3.82	.95	-4.59	***	.17
To Perfection	3.47	1.09	3.75	1.03	-7.29	***	.27
Exceed Exp	3.31	1.12	3.45	1.05	-3.52	***	.13
HW on Time	3.01	.78	3.23	.72	-8.30	***	.30
Work on HW	2.95	.74	3.04	.73	-3.35	*	.12
Attention	3.20	.64	3.25	.66	-2.35	*	.09
Listen	3.22	.61	3.30	.63	-3.37	*	.12
Enjoy R-Math	2.18	.84	2.21	.82	-1.05		.—
Look Forward	2.43	.89	2.42	.89	.29		—
Enjoy Math	2.26	.93	2.28	.94	-5.95		.—
Mathematics Engagement	2.73	.50	2.83	.48	1.13	***	.13

Note. *N* = 3,007. Low ESCS = 1,516; High ESCS = 1,491. HW = homework. Cohen's *d*: Small effect size = .20. Medium effect = .50. Large effect = .80.
**p* < .05, ****p* < .001.

Research Question 1: Can Mathematics Engagement Be Modeled as a Three-Dimensional Construct, and What Are the Components?

In order to answer question one of this study, a confirmatory factor analysis was conducted with AMOS 22.0 to assess how well the hypothetical model in Figure 4 fits the PISA 2012 data. This initial hypothetical measurement model with 14 items proved inadequate (Figure 4). The low loading on factors and nonoptimal fit indices (X^2/df (2664.51/74) = 36.01, $p < .001$; CFI = .87; TLI = .84; SRMR = .07; RMSEA = .11 with 90% CI lower bound = .10 and upper bound = .11) confirmed the data-model misfit. This initial model was trimmed guided by a priori estimate of factor loadings (loading ≥ .60 were retained) and moderate correlation between factors (.30-.50), as well as established cut-off values of fit statistics set by Hu and Bentler (1999) assessing: (a) absolute fit of the model to the data (standardized root mean square residual [SRMR ≤ .08] and chi-square test [χ^2/df < 3]); (b) comparison of the parsimony of parameters fitting the population covariance matrix (root mean square error of approximation [RMSEA ≤ .06]); and (c) a comparison of the chi-square value to a baseline model (the comparative fit index [CFI ≥ .95] and Tucker Lewis Coefficient [TLI ≥ .95]).

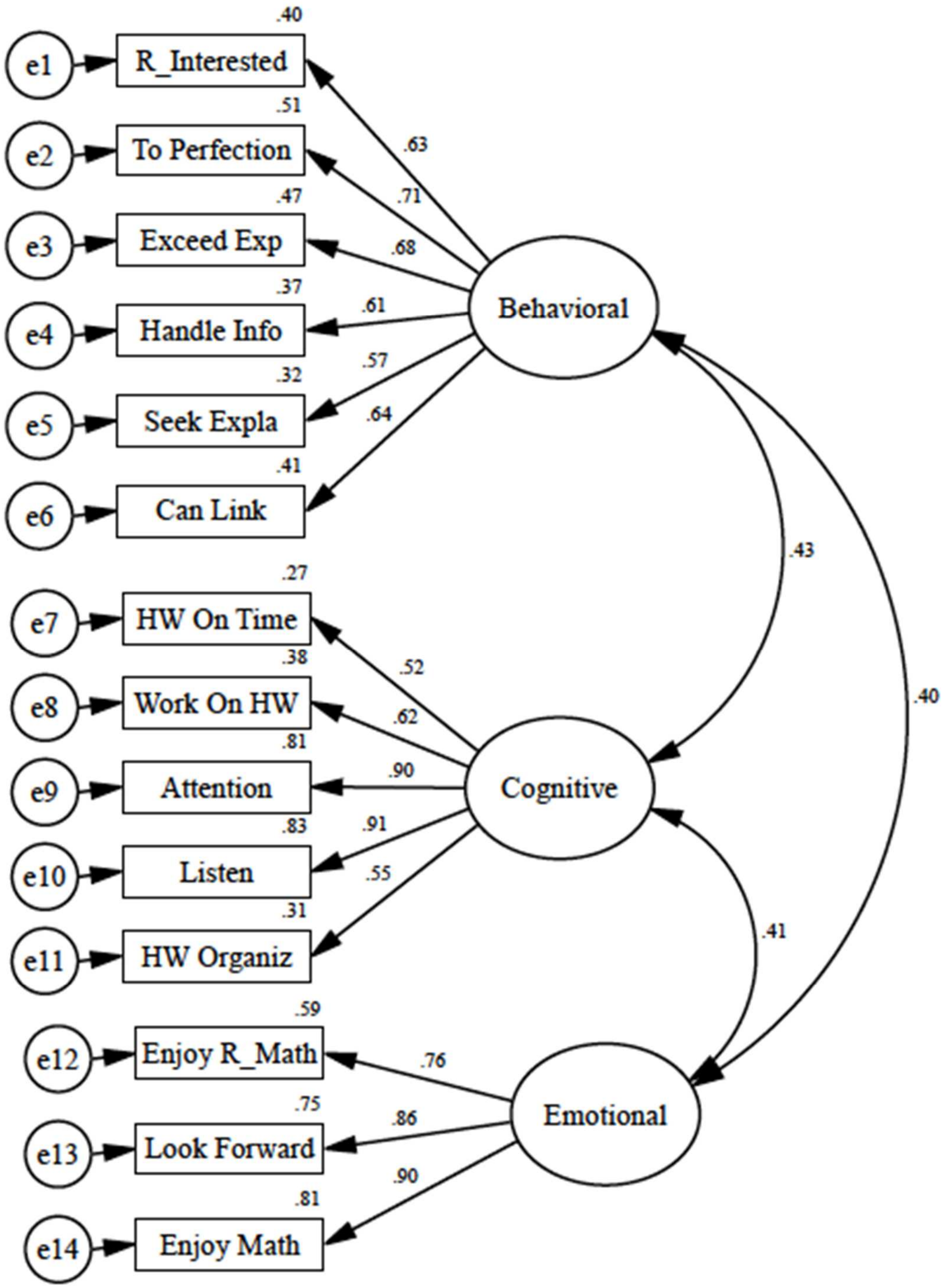


Figure 4. Initial confirmatory factor analysis (CFA) with 14 items: Theoretical three-dimensional measurement model of mathematics engagement.

Specification of the initial hypothetical measurement model was also driven by theoretically justifiable and rational considerations that included the conceptualization and definition of the engagement dimensions. Therefore, in order to arrive at the final measurement model of mathematics engagement four paths with inadequate loadings (<.6) were removed from the model (Exceed Exp, Seek Explan, Can Link, and HW Organiz), which confirmed convergent and discriminant validity measures (loadings and covariance between latent variables). Error variance correlations between some items in the behavioral and cognitive engagement factors were applied (see Figure 5), consistent with the theoretical conceptualization of mathematics engagement in this study.

The final measurement model supported the existence of a three-dimensional mathematics engagement construct. This model showed an excellent fit to the data χ^2/df (72.59/29) = 2.50, $p < .001$; CFI= 1, TLI= 1; SRMR = .02; and RMSEA = .02 with 90% CI lower bound = .02 and upper bound= .03). Mathematics engagement was identified with 10 items (cognitive = 3 items, behavioral = 4 items, and emotional = 3 items). The three dimensions of engagement were correlated according to the conceptualization of this study (dimensions are interrelated). Correlation between the three factors was moderate (.38 to .50) indicating good discriminant validity and that a second-order model is plausible (Glanville & Wildhagen, 2007; Wang et al., 2011).

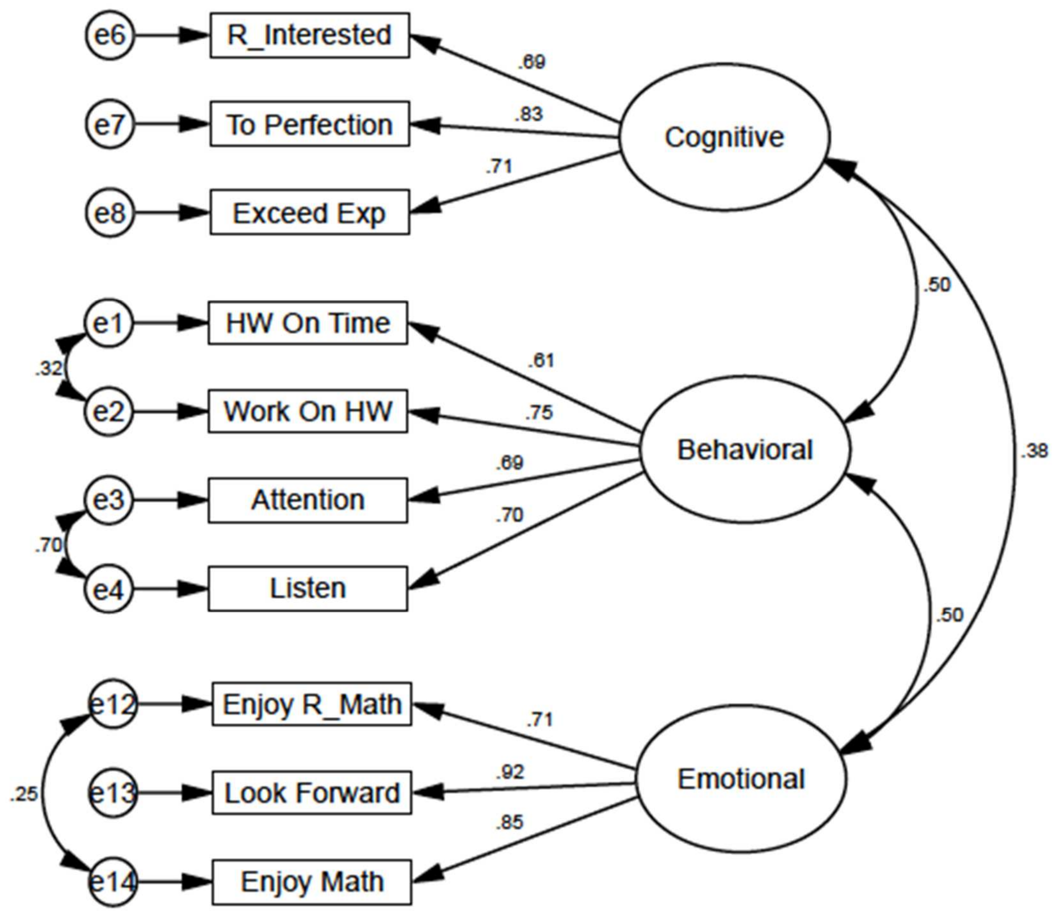


Figure 5. Final measurement model of three-dimensional mathematics engagement construct.

The standardized regression loadings of the final model ranged from .61 to .92 and were significant at $p < .001$, providing evidence of convergent validity. Table 8 displays the standardized and unstandardized coefficients and correlation estimates for the three-dimensional measurement model of engagement.

Table 8

Unstandardized and Standardized Coefficients for CFA Mathematics Engagement

Observed Variable		Latent Construct	*B*	*SE*	β
Correlation					
Interested	←	Cognitive	1	.03	.61 ***
To Perfection	←	Cognitive	1.19	.04	.75 ***
Exceed Expectations	←	Cognitive	.96	.05	.69 ***
HW on Time	←	Behavioral	.93	.05	.70 ***
Work on HW	←	Behavioral	1	.03	.69 ***
Attention	←	Behavioral	1.31	.04	.83 ***
Listen	←	Behavioral	1.14	.04	.71 ***
Enjoy Reading Math	←	Emotional	1	.03	.71 ***
Look Forward	←	Emotional	1.39	.05	.92 ***
Enjoy Math	←	Emotional	1.34	.03	.85 ***
Covariance					
Cognitive	↔	Emotional	.14	.01	.50 ***
Behavioral	↔	Emotional	.15	.01	.38 ***
Cognitive	↔	Behavioral	.16	.01	.50 ***

Note. CFA = Confirmatory Factor Analysis, HW = Homework.
*** $p < .001$

Subsidiary analyses. Invariance tests were performed to ensure the measurement model of mathematics engagement derived from the CFA analysis was equivalent across groups (9): gender (females and males), ethnicity (White, Black/African American, Hispanic, Asian, and Multiracial/Other), and ESCS (low and high). This was also done to ensure the SEM model, that included the CFA-constructed mathematics engagement construct, meaningfully predicted mathematicsperformance for all groups. The three-dimensional mathematics engagement model was found invariant. Configural invariance involved splitting the measurement model along gender, ethnicity, and ESCS attending to model fit and significant paths. The resultant models all achieved good fit confirming the configural invariance of the measurement model for mathematics engagement. A chi-square difference test was performed using the Stats Tool Package.xlsm χ^2 difference test (Gaskin, 2012b) to further verify the configural invariance of the final measurement model. The chi-square test involved a comparison of the chi-square value and degrees of freedom for the unconstrained measurement model with nine groups and a fully constrained measurement model with nine groups. The results verified that the groups were invariant (χ^2 (9) = 94.07, p = .14). See Table 9 for more details.

Table 9

Chi-square Difference Test of Invariance for Mathematics Engagement Measurement

					Chi-square Thresholds		
Overall Model	Unconstrained	Fully constrained	Number of groups	Difference	90%	95%	99%
χ^2	439.02	533.09		94.07	452.38	454.53	459.11
df	261	341	9	80	269	269	269
p-value				.14	.10	.05	.01
Threshold Difference					13.36(8)	15.51(8)	20.09(8)

Metric invariance was performed as a multigroup moderation test using critical ratios for differences. This test compared the regression weights for each group and the critical ratios for differences using Stats Tool Package.xlsm (Gaskin, 2012a). Invariance was determined by a *z*-score comparison of each measure of the latent variables (Aiken, Stein & Bentler, 1994; Byrne, 2004; Schmitt & Kuljanin, 2008). The results confirmed the final measurement model reached metric invariance. See Appendices C through H for more details regarding the metric invariance test scores. Standardized weight scores for ethnicity are found in Appendices I through M.

Validity and reliability tests were performed to ensure the factors derived from CFA would adequately perform in SEM. This entailed comparing the correlations and the standardized regression weights using Stats Tool Package.xlsm for validity and reliability (Gaskin, 2012b). The results indicated the CFA-derived model is a useful model for conducting causal model testing. See Table 10 for validity and reliability measures demonstrating adequate thresholds were met as established by Hair, Black, Babin, and Anderson (2010), as well as Malhotra and Dash (2011).

Table 10

Validity and Reliability of the Engagement Construct with Three Dimensions

	Reliability CR	Convergent Validity AVE	Discriminant Validity MSV	ASV
Behavior	.79	.55	.25	.20
Cognitive	.75	.51	.25	.13
Emotion	.87	.69	.15	.07

Note. *N* = 3,007. CR = Composite validity > .7; AVE = Average Variance Extracted > .5; MSV = Maximum shared variance < AVE; ASV = Average Shared Variance < AVE.

A common latent factor (CLF) was created to test for common method bias (MacKenzie & Podsakoff, 2012; Peterson & Kim, 2012; Podsakoff, MacKenzie, Lee, & Podsakoff, 2003; Podsakoff, MacKenzie & Podsakoff, 2012). Because the standardized regression weights are different when the CLF is added to the model, common method bias is suspected in the cognitive dimension factor. Therefore, the CLF was retained and common method bias adjusted variables were created. These computed composite factors (collapsed latent factors into single-weighted averages based on their factor weights) were used to build the structural model of engagement in SEM to address question 2 of this study. In sum, the answer to question one is yes. The existence of a three-dimensional engagement construct was found and validated with convergent and discriminant statistics, model fit data, invariance tests, and reliability statistics such as composite validity.

Research Question 2: Are the Effects of Attribution for Failure and Mathematics Self-Efficacy on Mathematics Performance Mediated by Mathematics Engagement while Controlling for Gender, Ethnicity, and ESCS?

Having shown that the psychometrics of the measures for mathematics engagement were sound and robust, structural equation modeling was performed with AMOS 22.0 to examine the mediating effect of mathematics engagement for the relationships of attribution for failure and mathematics self-efficacy on mathematics performance. Means and standard deviations for the measures used in the final CFA and SEM analyses are presented in Table 11.

Table 11

Descriptive Statistics for Final Structural Model of Mathematics Engagement

Variables	Min	Max	*M*	*SE*	*SD*
Mathematics Performance	224.73	783.70	486.42	1.61	88.28
Gender	1.00	2.00	—	—	—
Ethnicity	1.00	5.00	—	—	—
ESCS	1.00	2.00	—	—	—
Attribution for Failure	-3.77	3.91	-.35	.02	1.16
Mathematics Self-Efficacy	-3.75	2.27	.15	.02	1.00
Mathematics Engagement	-2.48	1.89	0	.01	.59

Note. *N* = 3,007. Mathematics engagement = Common Latent Factor using AMOS 22.0; ESCS = Economic, social, and cultural status.

Because the sample size was sufficient (3,007) and met multivariate normality assumptions, the maximum likelihood parameter estimation was chosen (Kline, 2000). Model 1 (Figure 6) depicts the correlated observable factors of attribution for failure and mathematics self-efficacy as predicting mathematics performance directly and indirectly through mathematics engagement, while controlling for gender, ethnicity, and ESCS. Ethnicity and ESCS were allowed to correlate. This was a reasonable specification as studies have shown that there is a strong association between student ethnicity and their economic, social, and cultural status levels (Costello, Keeler, & Angold, 2001; National Center for Education Statistics, 2010). This model fit the data well (NFI = 1; CFI = .99; TLI: 1; SRMR = .001; RMSEA = .001 with 90% Confidence Interval (CI) lower bounds = .001 and upper bounds = .001), however, the chi-square statistic showed the model was not statistically significant (χ^2 /df (.986/5) = .197, p = .96).

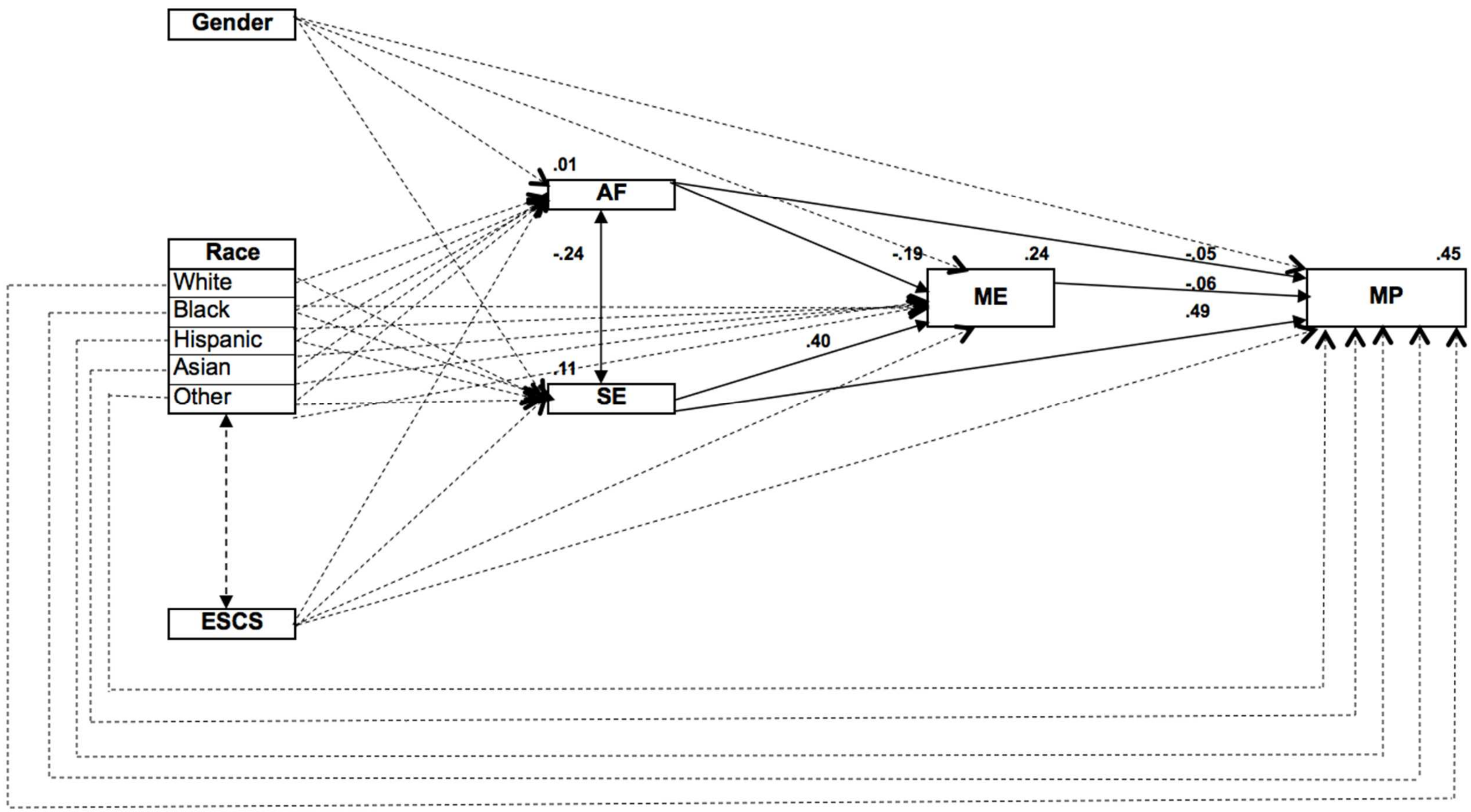


Figure 6. Model 1 is an initial structural model testing the mediating effect of mathematics engagement.
Note. Solid lines: Primary Variables of interest. Coefficients shown are significant at the $p < .001$ level of significance. Dash lines: Control variables. AF = Attribution for Failure, SE = Mathematics self-efficacy, MP = Mathematics Performance, ESCS = Economic, social, and Cultural Status; ME = Mathematics Engagement.

Several hypothesized paths in this model were nonsignificant at the more stringent $p < .001$ level used to reduce the probability of incorrectly rejecting the null hypothesis (Fisher, 1925; Kirk, 1996; Kline, 2004). Therefore, respecification was necessary to delete these paths. For instance, attribution for failure was predicted by gender and ESCS only; while self-efficacy was predicted by gender, ESCS, and ethnicity (Asian and Black/African Americas). Mathematics engagement was predicted by attribution for failure, mathematics self-efficacy, gender, and ethnicity (Asians, Black/African Americans and Hispanics); while mathematics performance was predicted by mathematics self-efficacy and engagement, as well as ethnicity (except Multiracial/other) and ESCS.

Figure 7 displays the respecified model, which provided an excellent fit to the data (χ^2 / df (30.90/16) = 1.93, $p = .01$; NFI = 1; CFI = 1; TLI: 99; SRMR = .01; RMSEA = .02 with 90% Confidence Interval (CI) lower bounds = .01 and upper bounds = .03). The predicted paths were generally supported (See Table 12 for unstandardized and standardized statistics). Attribution for failure was negatively associated with mathematics engagement (ß = -.19, $p < .001$), meaning that students who endorsed attributions for failure to teacher or lessons or luck also reported having lower mathematics engagement. The relationship between attribution for failure and mathematics performance occurred only when mediated, while holding mathematics self-efficacy constant and controlling for gender, ethnicity, and ESCS constant. The paths from mathematics self-efficacy to mathematics engagement (ß = .40) and mathematics performance (ß = .50) were positively associated. Students who reported higher levels of

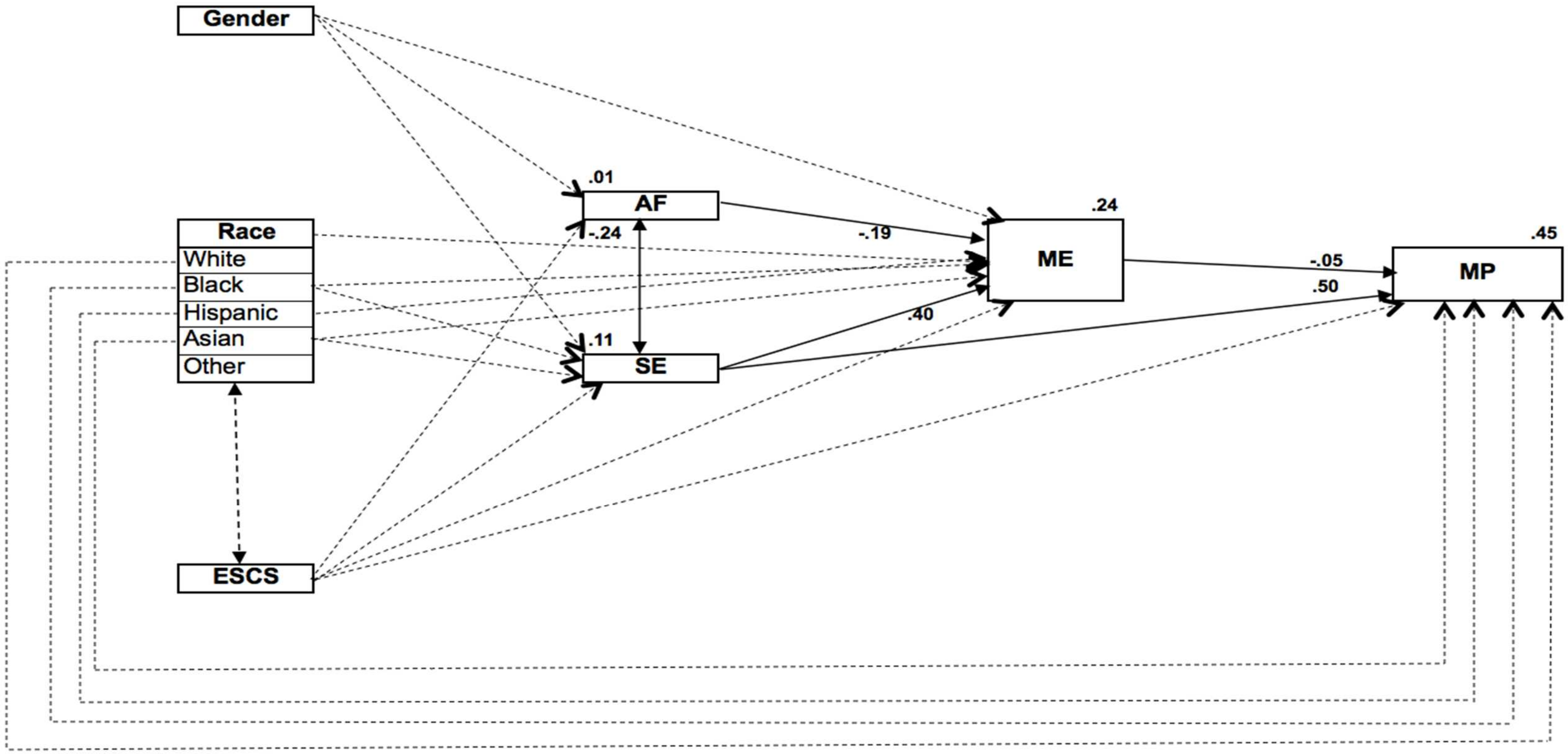


Figure 7. Final structural equation modeling (SEM) model 2 testing the mediating effects of mathematics engagement. *Note*. Solid lines: Primary variables of interest, all paths are significant at the $p < .001$ level of significance. Dashed line: Control variables. AF = Attribution for Failure, SE = Mathematics self-efficacy, ME = Mathematics Engagement, MP = Mathematics Performance, ESCS = Economic, social, and cultural status.

self-efficacy also reported higher levels of mathematics engagement and performance, while holding attribution for failure constant and controlling for gender, ethnicity, and ESCS.

Table 12

Unstandardized and Standardized Statistics for Structural Model of Mathematics Engagement

Variables			ß	B	*SE*	Critical Ratio
Attribution for Failure (AF)	←	Gender	.08	.18***	.04	4.15
Attribution for Failure (AF)	←	ESCS	-.06	-.07***	.02	-3.64
Mathematics Self-Efficacy (SE)	←	Gender	-.14	-.27***	.03	-7.99
Mathematics Self-Efficacy (SE)	←	ESCS	.29	.29***	.02	16.63
Mathematics Self-Efficacy (SE)	←	Black	-.07	-.20***	.05	-3.94
Mathematics Self-Efficacy (SE)	←	Asian	.07	.33***	.08	4.23
Mathematics Engagement (ME)	←	Gender	.08	.08***	.02	4.94
Mathematics Engagement (ME)	←	AF	-.19	-.08***	.01	-11.74
Mathematics Engagement (ME)	←	SE	.40	.20***	.01	23.94
Mathematics Engagement (ME)	←	Black	.07	.11***	.02	4.41
Mathematics Engagement (ME)	←	Hispanic	.05	.06*	.02	3.08
Mathematics Engagement (ME)	←	Asian	.04	.10*	.02	2.69
Mathematics Performance (MP)	←	ESCS	.21	19.24***	1.39	13.90
Mathematics Performance (MP)	←	SE	.50	44.14***	1.40	31.57
Mathematics Performance (MP)	←	ME	-.05	-9.44***	2.73	-3.45
Mathematics Performance (MP)	←	Black	-.23	-60.65***	3.77	-16.11
Mathematics Performance (MP)	←	Hispanic	-.10	-20.25***	3.19	-6.39
Mathematics Performance (MP)	←	Asian	.06	23.80***	5.74	4.14

Note. Paths for demographic variables included regardless of significance. Ethnicity variable is dummy coded with White students as the reference.
* $p < .05$,*** $p < .001$.

On the other hand, the path from mathematics self-efficacy to mathematics achievement, mediated through mathematics engagement, was significant. Although this relationship was statistically significant, the effect size was negligible; see Table 13 for more details about the mediation effects of mathematics engagement uncovered. Overall, this model explained 45% of the variance in mathematics achievement. Of note, the path from engagement to mathematics performance was negatively associated. This negative relationship was not anticipated and may point to suppression effects. As this was not the main focus of this study, no follow-up analyses were conducted; however, this result will be addressed further in the discussion section of this paper.

Table 13

Standardized Estimates of the Mediation Effects of Mathematics Engagement

Relationship	Direct effect without mediator	Direct effect with mediator	CI	Mediation Type
MathematicsPerformance ← SE	.47*	.50*	-.04 to -.02	Partial
MathematicsPerformance ← AF	-.03	-.05*	.01 to .02	Full

Note. SE = Mathematics Self-Efficacy, AF = Attribution for Failure, CI = Confidence Interval.
*$p < .001$

Direct and indirect effects. Table 13 displays the standardized estimates of the direct and indirect effects of the predictor variables on mathematics performance. A mediation analysis was performed using the Baron and Kenny (1986) causal steps approach; in addition, a bootstrapped confidence interval (CI) for the indirect paths was obtained using procedures described by Preacher and Hayes (2008). The initial causal variables were attribution for failure and mathematics self-efficacy, the outcome variable was mathematics performance, and the proposed mediating variable was mathematics engagement. Refer to Figure 7 that corresponds to this mediation hypothesis.

The direct parameter estimate between attribution for failure and mathematics performance (ß =-.03) was not statistically significant at the $p < .001$ level. However, the indirect relationship between these variables through mathematics engagement was significant (ß = -.05). These results indicated that the relationship between attribution for failure and mathematics performance was fully mediated by mathematics engagement. The Sobel Test (1982) was used to asses mathematics engagement as a mediator of the relationship between attribution for failure and mathematics performance. The relationship was found to be fully mediated at a statistically significant level, [$t(3,007) = 17.62$, $p < .001$], while controlling for gender, ethnicity, and ESCS. Several criteria can be used to judge the significance of the indirect paths. In this case, the a and b coefficients were statistically significant, and the Sobel Test for the *ab* product was significant. Moreover, the bootstrapped (resampling = 2,000) CI did not include zero (lower limit was -.04 and the upper limit was -.02).

As for the mediation effects of mathematics engagement for the relationship from mathematics self-efficacy and performance, a partially mediated relationship was established, $t(3,007) = 3.81$, $p < .001$; a bootstrapped CI for the product of the ab paths did not include zero (lower limit was .01 and the upper limit was .02). It should be noted that the direct path from mathematics self-efficacy to mathematics performance was the strongest in the model (ß = .50).

In sum, a comparison of the coefficients for the direct versus indirect paths ($c = c'$ + ab) for mathematics self-efficacy (.48 = .50 + -.02) suggests that a relatively small part of the effect of mathematics self-efficacy on mathematics performance is mediated by mathematics engagement. Conversely, attribution for failure (-.04 = -.05 + .01) is fully mediated by mathematics engagement. It should be noted that the effects of mediation discovered are negligible. Regardless, the answer to research question 2 is yes: The effects of attribution for failure and mathematics self-efficacy on mathematics performance are mediated by mathematics engagement while controlling for gender, ethnicity, and ESCS, albeit to a negligible level.

Relationships of controlling variables in the model. As for the controlling variables, gender (reference group = female) was positively associated with attribution for failure (β = .08) and mathematics engagement (β = .08), but negatively associated with mathematics self-efficacy (β = -.14). In other words, females engaged in and felt more capable of having the tools to do well in mathematics, but were also more likely to attribute failure to external and uncontrollable factors (e.g., teachers, difficulty of test) than their male peers. Gender did not contribute significantly to mathematics performance.

Ethnicity was not statically significantly associated to attribution for failure. Because this variable was dummy coded, it was possible to detect differences amongst the groups. For instance, among the five ethnic groups, Asian students reported higher mathematics performance than any of the other groups. These results will be discussed further in the discussion section, as they are not the focus of the present study.

As for the influence that economic, social, and cultural status exerts, ESCS was not shown to be associated to mathematics engagement. However, it was positively associated with mathematics self-efficacy (β = .29) and mathematics performance (β = .21), but negatively associated with attribution for failure (β = -.06). These results show that students from a higher ESCS group are more likely to report having a high degree of belief about their capabilities to perform in mathematicsand lower maladaptive attributional beliefs about why they fail.

Supplementary analyses. The above analyses have provided empirical support for a three-dimensional construct of engagement and a causal model that situates mathematics engagement as a mediating factor between students' attribution for failure and self-efficacy beliefs and their mathematics performance (Figure 7; Table 13). Hence, a further examination of a modified model was performed by testing the salience of attribution for failure in predicting mathematics self-efficacy and mathematics engagement (Figure 8). This model fit the data tolerably (χ^2/df (30.90/16) = 1.93, p = .01; NFI = .99; CFI = 1; TLI = .99; SRMR = .02; RMSEA = .02 with 90% Confidence Interval (CI) lower bounds = .01 and upper bounds = .03). Overall this model did not increase the variance accounted for in mathematics engagement or performance. However, attribution for failure increased the variance explained in mathematics self-efficacy by β = .05.

Of the three models tested in this study, the significant χ^2 statistics suggest that models 2 and 3 were a better fit to the data than model 1. As discussed earlier, although model 3 showed good fit statistics, it did not add variance to the predictive power of the model. Taken together, although the supplementary analysis provided support for attribution for failure as a predictor of mathematics self-efficacy, it underscored the heuristic superiority of model 2 (Figure 7). See Table 14 for model comparisons.

Table 14

Structural Model Comparison for Mathematics Engagement

	Fit Statistics								
Subscales	χ^2	*df*	χ^2/df	CFI	TLI	SRMR	RMSEA	LO	HI
Model 1	.99	5	.197	1	1	.01	.00	.00	.00
Model 2	30.90	16	1.93*	1	.99	.01	.02	.01	.03
Model 3	30.90	16	1.93*	1	.99	.01	.02	.01	.03

Note. *$p < .05$

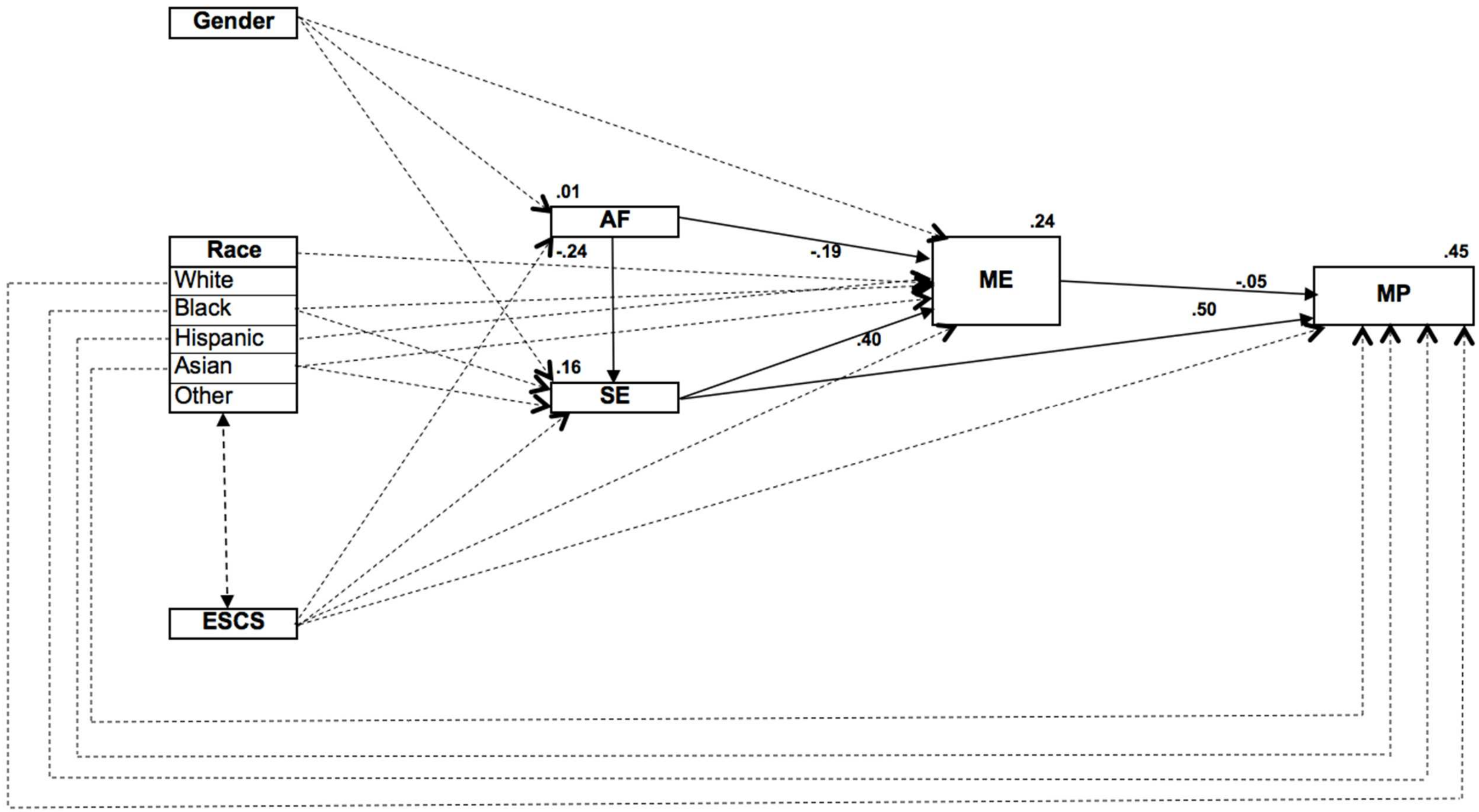


Figure 8. Model 3 is an alternative model testing the predictive path from attribution for failure to mathematics self-efficacy. *Note*. Solid lines: Primary variables of interest, all paths are significant at the $p < .001$ level of significance. Dash line: Control variables. AF = Attribution for Failure, SE = Mathematics self-efficacy, ME = Mathematics Engagement, MP = Mathematics Performance, ESCS = Economic, social, and cultural status.

Chapter Five

Discussion Overview

Student engagement has been theorized to be an important ingredient for learning and achievement since its emergence in the literature in the 1980s (Brophy, 1983; Chase et al., 2014; Finn & Voelkl, 1993; Fredricks et al., 2004; Mosher & McGowan, 1985; Skinner & Belmont, 1993). Despite decades of study, researchers have yet to agree on one conceptualization or definition of engagement. While most agree that engagement is a multidimensional construct (Appleton et al., 2008; Burch, Heller, Burch, Freed, & Steed, 2015; Fredricks et al., 2004; Li, Turner, & Turner 2010; Liem & Martin, 2012), it has been conceptualized as having two (Finn, 1989), three (Fredricks et al., 2004), and four dimensions (Appleton et al., 2006; Reeve & Lee, 2013). Two-dimensional models usually incorporate cognition and behavior, while four-dimensional models include agentic engagement (Reeve, 2012; Reeve & Lee, 2013) or bifurcate behavioral engagement into academic and behavior subtypes. How engagement is defined depends on how each dimension is operationalized. Some researchers overlap engagement with motivation, defining the cognitive dimension as control and relevance (Appleton, et al. 2008; Reschly & Christenson, 2012) and the emotional dimension as interest or sense of belonging (Finn & Zimmer 2012; Pekrun & Linnenbrink-Garcia, 2012; Voelkl, 2012), this present study adheres to the more prevalent three-dimensional model which incorporates three interrelated and dynamically connected processes for learning: cognition, behavior, and emotions, and defines engagement as the actions that follow motivation. Cognitive engagement is defined as students' deep-processing strategies for learning. Students who utilize cognitive engagement approach tasks with the intent to master the material. Behavioral engagement is defined as active participation, which can be characterized as paying attention in class and completing work on time and at a high quality. Emotional engagement are feelings about learning and is characterized by students liking or enjoying learning. Therefore, one objective of this study was to evaluate the existence of a three-dimensional model of mathematics engagement conceptualized as a multidimensional construct that is separate from, but nonetheless related to, motivation. In this study, engagement represents the action that follows students' motivational intent, defined as the effortful and meaningful investment in learning, and characterized by students' vigorous and determined attempts to learn, that is, remaining interested in lessons, working to perfection, exceeding expectations, completing homework on time and at a high quality, paying attention and listening in mathematics classes, and taking joy in learning.

The second objective of this study was to examine the mediation effects of mathematics engagement on the relationships between mathematics performance and two student beliefs, attribution for failure and mathematics self-efficacy, while controlling for gender, ethnicity, and economic, social, and cultural status. The potent effect of self-efficacy is well known. When students believe that they have the tools needed to be successful, self-efficacy can explain up to 20% or more of student achievement (Bandura, 1989; Galla et al., 2014; Kitsantas et al., 2011). Attribution for failure is a belief associated with the concept of ability (Weiner, 1979). Failures can be attributed to either internal or external factors. External factors include blaming the teacher, luck, or difficulty of the material; internal factors include lack of effort or ability (Dweck & Leggett,1988). Attribution for failure affects self-efficacy and has been shown to be distally associated to outcomes (Schunk, 1994 Wolters, 2013). The discussion that follows considers the relevant findings.

Finding One: Affirmation of a Three-Dimensional Model of Engagement

Overall, the results of the confirmatory factor analysis (CFA) supported the existence of a three-dimensional model of engagement demonstrating that students can be cognitively, behaviorally, and emotionally engaged with mathematics. Others have used PISA survey items to examine one or two of the dimensions of engagement (Dettmers et al., 2011; Hampden-Thompson & Bennett, 2013; Martin et al., 2012); this study used the PISA 2012 dataset to examine the existence of the more prevalent three-dimensional model of engagement shown in Figure 2. Specifically, the final measurement model, with 10 items grouped into 3 factors, provided empirical evidence supporting that the three-dimensional measurement model of mathematics engagement was invariant for gender, ethnicity, and economic, social, and cultural status. That is, the model was shown to be valid for comparing and explaining variations of mathematics engagement across groups.

This study supports prior research findings showing an excellent fit of data for a three-dimensional model of engagement (Burch et al., 2015; Martin et al., 2014; Wang et al., 2011). Wang et al. (2011), for instance, reported findings from a CFA that showed school engagement to be a multidimensional construct. In this study, Wang and colleagues used a large-scale dataset to confirm that cognition (regulation and strategy), behavior (attention and compliance), and emotion (belonging and valuing) comprised the engagement construct. Likewise, Burch et al. (2015) used exploratory factor analysis to show the existence of a three-dimensional model of academic engagement. Three dimensions—cognition, behavior, and emotion—emerged from 24 items reflecting students' attention in class, working hard on homework and turning in assignments on time, and positive feelings about learning. In another multilevel study supporting the existence of a three-dimensional mathematics engagement construct, Martin et al. (2014) operationalized engagement with nine items: planning, task management, persistence, self-handicapping, disengagement, class participation, effort, homework completion, as well as the nature and extent of mathematics enjoyment.

A major contribution of the present study is the examination of the psychometric properties of the three-dimensional mathematics engagement model constructed by testing for measurement invariance, validity, and reliability. The traditional CFA approach of analyzing the worthiness of a model was extended in several ways. First, by computing the variance shared by the measurement items of mathematics engagement (average variance extracted [AVE]), discriminant validity was further confirmed. Second, this study tested the model's invariance for gender, ethnicity, and ESCS using the multigroup moderation procedure and a comparison of the regression weights and critical ratios (Lowry & Gaskin, 2014).

Finally, this study examined and accounted for common method bias (CMB), such as possible data distortion due to collecting all exogenous and endogenous variables using the same source. For example, the Program for International Student Assessment is a self-reporting survey that captures the criterion and predictor measures from the same person (15-year-old mathematics students). Common method bias effects are produced by subjects' use of consistent motif, implicit theories, and illusionary correlations, and social desirability (Podsakoff et al., 2003). *Consistent motif* is characterized by a participant's desire to remain consistent when answering survey questions. Choosing to remain consistent can distort relationship existence or the degree of association between variables. *Implicit theories* and *illusionary correlations* can bias the results when respondents possess assumptions about the co-occurrence of items and therefore seek to answer questions in the same manner. This leads to a misrepresentation of the covariance of variables. Another common method bias source is *social desirability* (Crowne & Marlowe, 1964). Spurious relationships (suppression or moderation of relationships between variables) occur when participants answer items in a way that presents them favorably. Overall, the CFA analysis confirmed mathematics engagement was psychometrically sound and an invariant three-dimensional construct. This is consistent with the conceptualization by Fredricks et al. (2004).

Subsidiary analysis. Having confirmed the metric invariance of the measurement model of mathematics engagement using multigroup differences, differences at the dimensional level were explored. This is reasonable, given that the engagement construct is a composite of formative indicators which can vary independently or even inversely

(Bollen & Bauldry, 2014; Grace & Bollen, 2008). Regarding gender, females reported being more emotionally and cognitively engaged than male students, but less behaviorally engaged. Given that these dimensions are interrelated and dynamic, these differences could suggest that optimal engagement occurs when all the dimensions are engaged at similar levels. Despite the small differences observed between the dimensions of mathematics engagement for females and males, these differences may be relevant given that females lag behind males in mathematics performance and continue to be underrepresented in STEM fields (Eccles, 2009).

One possible explanation for the difference in mathematics engagement by gender could be that females participate at lower levels because they are less confident and receive less encouragement than their male peers. In a recent study exploring mathematics anxiety, Stoet, Bailey, Moore, and Geary (2016), reporting from PISA 2003 and 2012 data, explained that 15-year old females from developed and more gender-equal countries experienced higher levels of mathematics anxiety than their male peers. Females were less likely to receive parental valuation for mathematics engagement than their male siblings, even when parents were employed in STEM-related fields. A closer look within the behavioral dimension factor suggested that paying attention and listening in mathematics classrooms may be important factors to consider in discussions about the mathematics performance of females, given that males—who traditionally score better in mathematics performance—reported higher levels in these two indicators than females. The difference in the beta loadings between females and males ranged from 0 to 5 standardized points (favoring males).

With regard to ethnicity, all groups reported lower levels of cognitive and behavioral engagement in mathematics than emotional engagement. At the cognitive dimension, the differences by ethnicity were relatively small (ß = .72 to .75). While the differences were minor, this information is useful for creating engagement profiles to assist educators in identifying strengths and weaknesses in students' engagement in mathematics. For instance, at the cognitive dimension of engagement Black/African American students report that they remain interested (cognitive engagement) more than other students, but are less likely to continue with mathematics to perfection or attempt to exceed expectations, factors known to be important indicators of mathematics performance (Bandura, 1997; Martin, 2012; Zimmerman, 1995). When comparing the cognitive engagement of the groups in relation to mathematics performance, it is noteworthy that the profile of the highest mathematics performers (Asian students) had the lowest scores for remaining interested in mathematics, but reported higher scores for working to perfection and exceeding expectations. In contrast, the lowest performers (Black/African American students) report they are more interested in mathematics and working to perfection than they are in attempting to exceed expectations. One possible explanation for how these differences in engagement affect outcomes could be that students who complete work to perfection and exceed expectations are more successful because they dedicate more time and energy to learning mathematics concepts than students who meet the minimum expectations by completing their homework on time and ensuring that it is correct (working to perfection).

The behavioral engagement data showed that Black/African American students reported being overall more behaviorally engaged followed by Multiracial/Other, White, Hispanic, and Asian students. This behavioral engagement pattern was surprising as others have found Asian students to be more behaviorally engaged than Whites, Black/African Americans, Hispanics, and Multiracial/Others, respectively (Hsin & Xie, 2014; Sciarra & Seirup, 2008). However, this may be a reasonable difference in results given that others have measured behavioral engagement with a mix of performance and mastery-oriented actions, that is, attendance and attentiveness (Chase et al., 2014; Sciarra & Seirup, 2008). The present study measured behavioral engagement with items reflecting mastery actions, for example, two in-class behaviors (attending and listening in class) and two out-of-class behaviors (working hard on homework and completing it on time).

A comparison of the loading on the emotional engagement dimension for the high and low mathematics performers indicated the largest difference occurred in the "I enjoy reading about mathematics" indicator ($\Delta\beta = 10$). Overall, Black/African American students reported having higher levels of looking forward to mathematics classes ($\Delta\beta = 6$) and lower levels of enjoying mathematics ($\Delta\beta = 2$) or enjoying reading about mathematics ($\Delta\beta = 10$) than Asian students. Although these differences were not statistically significant, understanding the emotional engagement profiles of students could provide information to better understand how to address achievement gaps by ethnicity. For instance, the profiles of the highest performers may indicate that enjoyment of mathematics and reading about mathematics are important factors to target for lower performing students.

One unexpected result from this study was the finding that Multiracial/Other, White, and Hispanic students reported being more emotionally engaged in mathematics than Black/African American students, as prior research has shown Black/African American and Hispanic students as having relatively higher emotional engagement than other forms of engagement (Sciarra & Seirup, 2008). This may be due to the difference in how the emotional engagement dimension was measured. This study measured emotional engagement with items representing reactions to learning such as enjoyment of learning, whereas others have defined emotional engagement with motivational variables such as sense of belonging (Wang et al., 2011).

When the engagement dimensions are compared by ESCS, the results are consistent with prior research findings that show students from low ESCS reporting lower cognitive, behavioral, and emotional engagement than their high ESCS peers (Ainley & Ainley, 2011; Martin et al., 2014). Between-group comparisons show that both groups report being more emotionally engaged than either cognitively or behaviorally, respectively. Statistically significant differences within the engagement construct indicated that students from high ESCS had higher values on variables such as "I work to perfection," "I exceed expectations," "I turn homework in on time," "I pay attention and listen in class," and "I enjoy mathematics." As discussed earlier, because the higher ESCS group reported higher mathematics performance, focusing on one or a combination of the indicators that seem to make a difference for students in the high ESCS group may be a worthwhile approach for improving the scores of students in the low ESCS group. In total, the results of this study suggest that differences by gender, ethnicity, and economic, social, and cultural status of 15-year-olds should be taken into account in studies on mathematics engagement and performance.

Finding Two: Mathematics Engagement Mediates Attribution for Failure and Mathematics Self-Efficacy on Mathematics Performance

The results of structural equation modeling supported the notion that engagement mediates the relationship between attribution for failure and mathematics self-efficacy on mathematics performance. Consistent with others who have shown attributions to be distantly associated with outcomes (Weiner, 2008; Wolters et al., 2013), the results of the current study demonstrate attribution for failure exerted its influence on mathematics performance solely through mathematics engagement. With respect to mathematics self-efficacy, its effect on mathematics performance proved to be partially mediated by mathematics engagement. However, contrary to previous studies that established the association of engagement with self-efficacy and outcomes (Greene et al., 2004; Kitsantas et al., 2011; Komarraju & Nadler, 2013; Martin et al., 2014; Ouweneel et al., 2013), the mediation effects found in this present study were negligible, as they added little to explaining the variance found in mathematics performance. One possible reason may be that engagement and self-efficacy work in tandem. There is support for this idea in the literature that claims self-efficacy is shaped by experiences of effortful engagement (Bandura, 1997), and that as self-efficacy beliefs increase so does engagement (Galla et al., 2014; Zimmerman et al., 1992).

An unexpected insight gained from this study involved the negative path coefficient between mathematics engagement and mathematics performance. Despite the sensibleness and literature supporting a positive relationship between mathematics engagement and performance (Chase et al., 2014; Ladd & Dinella, 2009; Martin et al., 2014; Rimm-Kauffman et al., 2015; Sciarra & Seirup, 2008), the results indicated mathematics engagement had no practical impact on mathematics performance. When considering the results of the mediating effects of engagement in the present study, it is reasonable to suppose that the controlling variables of gender, ethnicity, and ESCS conditioned the relationship between mathematics engagement and mathematics performance, such as the possible existence of suppression effects exerted by the demographic variables in the study. According to Baron and Kenny (1986), moderators are variables affecting the relationship, in strength or directionality, of independent and dependent variables. The directionality change in the zero-order correlation and path coefficient between mathematics engagement and performance supports a moderation effect supposition. A comparison between the zero-order correlation statistics and the path coefficients, derived from structural equation modeling (SEM), of the demographic variables suggests they may have played an important role in the relationship findings between mathematics engagement and performance. For example, the relationship between gender and mathematics performance was nonsignificant in the SEM analysis, but in the zero-order correlation results this relationship was strong and significant. Similarly, the relationship between gender and mathematics engagement changed in directionality and significance from the zero-order correlation statistics to the path analysis, respectively. More support for the possible presence of suppression was the directionality change that occurred in the relationship between mathematics engagement and ethnicity.

Overall, this study has shown that a three-dimensional engagement construct exists. Despite the small effect size, this investigation has also established that mathematics engagement is a mediating factor for attribution for failure and mathematics self-efficacy on mathematics performance. Further, this study showed that mathematics engagement differences by gender, ethnicity, and ESCS exist. Finally, the results of students' attribution for failure and mathematics self-efficacy on mathematics engagement are consistent with prior research findings suggesting self-efficacy is a potent predictor of mathematics engagement, while attributions for failure to uncontrollable and external factors reduce mathematics engagement (Martin et al., 2014; Wolters et al., 2013). These results underscore the importance of supporting students' beliefs about their ability to do mathematics and helping them to revise maladaptive attributions for failure in order to positively impact their cognitive, behavioral, and emotional energies toward mathematics learning.

Implications for Practice

This study has broad implications for educators and researchers. For instance, when designing curricula teachers can infuse lessons with strategies tailored to encourage the three dimensions of engagement: cognition, behavior, and emotion. In particular, providing students with interesting school assignments that connect what they are learning to the real world (i.e., project-based learning) can increase cognitive engagement. Likewise, behavioral and emotional engagement can be improved with instructional practices that promote persistence in mathematics. Results from a study by Klem and Connell (2004), with longitudinal data from 1,886 students followed from elementary to middle school, showed that teacher support is positively associated with student engagement and academic performance. Therefore, educators should make an effort to create safe classrooms where students learn from each other in a collaborative and well-structured, judgment-free environments, where the teacher sets high expectations while ensuring that students feel safe to learn from mistakes. Several studies have shown that students who learn in classrooms where exploration and active participation are encouraged show high levels of problem-solving skills, hold positive attitudes about mathematics, and are more likely to engage in mathematics (Kauffman et al., 2015; Saritas & Akdemir, 2009; Wang & Eccles, 2011). Therefore, educators should eschew the traditional lectures and ask more questions (inquiry-based learning), connect learning to real world situations (project-based learning), flip their classrooms to individualize instruction and encourage collaboration, and design and assign homework that students feel is meaningful and relevant (Dettmers et al., 2011).

The findings from this study also suggest that teachers should take students' self-efficacy and attributional beliefs into account when formulating intervention plans. Students with patterns of low self-efficacy or maladaptive attributional thinking are more likely to struggle academically absent effective intervention to revise maladaptive attributional thinking and build self-efficacy. Curriculum designs should incorporate strategies for developing self-efficacy using Bandura's (1989) four sources of self-efficacy: mastery experience, modeling, social persuasion, and managing physiological arousal. Teacher training should include education about the importance of positive student beliefs for increasing students' self-regulatory processes of goal setting, choosing effective strategies, progress monitoring, and evaluating outcomes to revise goals or to set new goals (Zimmerman, 2002). The challenge for educators is to develop educational experiences that account for students' unique personalities, beliefs, and learning needs, thereby supporting the dynamically interrelated three-dimensional processes of mathematics engagement. This is an especially difficult challenge in the current high-stakes testing era, but it is imperative.

For researchers, this study offers support for conceptualizing engagement as a three-dimensional construct and its applicability for different learners (different gender, ethnicity, and/or ESCS). The importance of the engagement construct is the capacity it offers for investigating three interrelated and meaningful aspects of students' learning process: cognition, behavior, and emotions. Second, the results of this study support the notion that different learners (gender, ethnicity, and/or ESCS) will tend to engage in varying degrees. This line of research has the potential to provide compelling data to understand how differing patterns of student engagement are associated with outcomes. Intervention researchers can apply these results to further our understanding of student persistence and resilience for mathematics learning.

Limitations

As with any study, there are limitations that should be taken into consideration. First, survey data restricts the depth of responses or clarifications from students as to how they interpreted the meaning of questions. For instance, the item “I pay attention in mathematics class” was used to indicate behavioral engagement as it unexpectedly loaded on that factor rather than cognitive engagement as expected. The decision to retain this item was made because it proved an essential element in holding the behavioral factor together, confirmed by fit statistics. Second, although the mathematics engagement measurement model was a good fit to the data and provided reliable information about the relationships under investigation in this study, the indicators included in this study of engagement were restricted to the 10 items available in the PISA dataset. Although these items fit the mathematics engagement model, items specific to mathematics engagement are needed. For instance, the cognitive dimension items weakly establish the characterization of students’ cognitive engagement. Altogether the six initial items were expected to fit the model well, however, items such as “I can easily link facts together” and “I seek help when I don’t understand,” and “I can handle a lot of information” were dropped from the analysis due to statistical misfit. This was surprising as these items were strongly expected to fit securely in the three-dimensional model of engagement. Thirdly, PISA is cross-sectional data, meaning it provides only a snapshot of students in 2012 and does not allow longitudinal or causal analysis.

Future Research

Although a three-dimensional construct of engagement was established in the present study, future research should further investigate this construct using items developed specifically for engagement. Further, it may be useful to view engagement as an academic subject-matter domain specificity construct. Like self-efficacy, it can be argued that student engagement is context bound (Bandura, 1997).

While this study found engagement mediated the relationships of attribution for failure and self-efficacy on mathematics performance, the effect was weak. Therefore, future research should seek to further establish the mediational pathway of engagement between self-efficacy and performance. In addition, studies should explore the moderation effects of gender, ethnicity, and ESCS in unison. Using contextual moderators may not only better explain the mediation effects of the engagement construct, but can also add richness to the portrait of an engaged student and enrich teacher knowledge of how to individualize the curriculum to reach every student.

This study recognizes that the learning environment is seminal in the study of student engagement (Fredricks et al., 2004). Therefore, future research should focus on triangulation of data to account for the perspectives of students, teachers, parents, and administrators. Finally, there is a need for longitudinal studies that allow researchers and educators to evaluate how student engagement profiles develop over time.

Finally, researchers should consider using the PISA dataset to shed light on important aspects of student learning. This international test offers many worthwhile research topics for exploration including comparison studies across nations about the indicators of equity by gender, ethnicity, social, and/or economic contexts, or the extent to which high standards are achieved.

In sum, the findings from this study support the proposed three-dimensional model of engagement by Fredricks et al. (2004), where the different dimensions of engagement include student cognition, behavior, and emotion. The analyses conducted establish construct validity and reliability using sound statistical methods (Baron & Kenny, 1986; Hu & Bentler, 1999; Kline, 2011; Marsh, Balla, & McDonald, 1988). It is important to reiterate that although a mediation effect on mathematics performance by attribution for failure and self-efficacy was established, this relationship was negligible. Among possible explanations for this outcome is the fact that the questions asked in the PISA 2012 (the study instrument) were not crafted specifically to measure engagement. More items are needed to accurately assess the cognitive, behavioral, and emotional aspects of engagement; further, including more contextual items may lift the importance of the mediation effect of engagement, for example, perceived teacher help, and sense of belonging. Future research should include more contextual motivation variables to further assess the nature and magnitude of the mediational role of engagement.

This study has shown the crucial role that mathematics engagement, mathematics self-efficacy, and adaptive attributional thinking play in the mathematics scores of U.S. 15-year-olds. Therefore, in order for students to excel in mathematics, educators must design curricula that promote persistence in mathematics engagement by undergirding students' self-efficacy for mathematics and providing students opportunities to learn from their failures. Moreover, educators must take into account the differing patterns of mathematics engagement and seek to individualize instruction that assures all students, regardless of gender, ethnicity, social, or economic status, are capable of unlocking the gateway to higher education corresponding to better paying jobs.

Appendix A

IRB Exemption Form

Office of Research Integrity and Assurance

Research Hall, 4400 University Drive, MS 6D5, Fairfax, Virginia 22030
Phone: 703-993-5445; Fax: 703-993-9590

DATE:	February 11, 2016
TO:	Silvia Moore, PhD
FROM:	George Mason University IRB
Project Title:	[814052-1] THE ROLE OF MATHEMATICS ENGAGEMENT AS A MEDIATOR IN THE RELATIONSHIP AMONGST ATTRIBUTION FOR FAILURE, MATHEMATICS SELF-EFFECACY, AND MATHEMATICS PERFORMANCE
SUBMISSION TYPE:	New Project
ACTION:	DETERMINATION OF NOT HUMAN SUBJECT RESEARCH
DECISION DATE:	February 11, 2016

Thank you for your submission of New Project materials for this project. The Office of Research Integrity & Assurance (ORIA) has determined this project does not meet the definition of human subject research under the purview of the IRB according to federal regulations.

Please remember that if you modify this project to include human subjects research activities, you are required to submit revisions to the ORIA prior to initiation.

If you have any questions, please contact Katherine Brooks at (703) 993-4121 or kbrook14@gmu.edu. Please include your project title and reference number in all correspondence with this committee.

This letter has been electronically signed in accordance with all applicable regulations, and a copy is retained within George Mason University IRB's records.

Appendix B

Item Development Guidelines

Construct	α	Operationalized Definition	Measurement/ items	Scale	Coding
Mathematics Performance	.85	Plausible values: Estimate of students' mathematics proficiency	1. Mathematics performance	Multiple choice; Closed/short response; Open constructed responses	$1 \leq 420.02$
			2. Formulating situations mathematically		2 = 420.07 - 482.38
			3. Concepts, facts, and reasoning		3 = 482.38 - 544.68
			4. Interpreting, applying, evaluating		4 = 544.58 - 606.99
			5. Change and relationships		5 = 606.99 - 669.30
			6. Space and shapes		6 = 669.30 - 1000
			7. Quantity		
Mathematics Engagement (Fredricks et al., 2004)	.78	Cognitive Engagement	1. I seek explanation	1-5 Likert scale	1 = Not at all like me
			2. I can link facts		2 = Not much like me

Construct	α	Operationalized Definition	Measurement/ items	Scale	Coding
			3. I can handle a lot of information		3 = Somewhat like me
			4. I remain interested		4 = Mostly like me
			5. I continue to perfection		5 = Very much like me
			6. I exceed expectations		
		Behavioral Dimension “purposeful amount and quantity” (Appleton et al., 2008; Fredrick et al., 2004;	1. I complete homework in time for class		1 = Strongly disagree
	0.82	Voelkl, 2012)	2. I work hard on homework	1-4 Likert Scale	2 = Disagree
			3. I attend in class		3 = Agree
			4. I keep work organized for Math classes		4 = Strongly agree
			5. I listen in class		
		Emotional Engagement	1. I enjoy mathematics	1-4 Likert Scale	1 = Strongly disagree
	0.85	Affect and Feelings	2. I enjoy reading about mathematics		2 = Disagree

Construct	α	Operationalized Definition	Measurement/ items	Scale	Coding
		(Fredrick et al., 2004)	3. I look forward to Lesson		3 = Agree 4 = Strongly agree
Self-efficacy (Bandura, 1977; Schunk, 2012; Zimmerman 2000)	.85	A belief in one's capabilities to undertake mathematics	"How confident do you feel about having to do the following mathematics?" 1. Train timetable 2. TV discount 3. Spare meters tiles 4. Graphs in newspapers 5. Solving equations 6. Distance 7. Solving Equation 8. Rate	1- 4 Likert scale	1 = Not at all confident 2 = Not very confident 3 = Confident 4 = Very Confident
Attribution	.80	Attribution for Failure, Locus of control	"Each week your mathematics teacher gives a short quiz. Recently you have done badly on these quizzes. Today you are trying to figure out why.	1-4 Likert scale	1 = Not at all likely

Construct	α	Operationalized Definition	Measurement/ items	Scale	Coding
(Graham & Weiner, 2012; Wiener, 2011)		(skill vs. luck/other's actions)	1. Not good at math 2. Teacher fault no explaining 3. Bad guesses 4. Material too hard 5. Teacher fault no interest 6. Sometimes I'm just lucky		2 = Slightly likely 3 = Likely 4 = Very likely
Gender		Gender		Categorical	1 = female 2 = male
Ethnicity		Group self-identification		Categorical	1 = White 2 = Black 3 = Hispanic 4 = Asian 5 = other
Economic, social, and Cultural Status (ESCS)		Parent occupation, educational level, and household possessions		Continuous	

Appendix C

Metric Invariance Tests by Gender

Observed Variable		Latent Construct	Females *B*	Males *B*	z-score
R_Interested	<---	Cognitive	1.00 ***	1.00 ***	–
To Perfection	<---	Cognitive	1.33 ***	1.28 ***	-.72
Exceed Exp	<---	Cognitive	1.11 ***	1.16 ***	.66
HW On Time	<–	Behavioral	1.00 ***	1.00 ***	–
Work On HW	<–	Behavioral	1.17 ***	1.20 ***	.43
Attention	<–	Behavioral	.91 ***	1.00 ***	.96
Listen	<–	Behavioral	.87 ***	1.00 ***	.96
Enjoy R_Math	<–	Emotional	1.00 ***	1.00 ***	–
Look Forward	<–	Emotional	1.46 ***	1.32 ***	-1.64
Enjoy Math	<–	Emotional	1.37 ***	1.33 ***	-.75

Note. Invariance test conducted by comparison of unstandardized regression weights and critical ratio table. Invariant = At least 1 z-score in each individual latent factor is nonsignificant at the .05 level. † = Reference Group
*** *p*-value < 0.01; ** *p*-value < 0.05; * *p*-value < 0.10

Appendix D

Metric Invariance Tests by Ethnicity with Unstandardized Estimate

			White†	Black		Hispanic		Asian		Multiracial/Other	
Observed Variable		Latent Factor	*B*	*B*	*z*-score	*B*	*z*-score	*B*	*z*-score	*B*	*z*-score
R_Interested	<—	Cognitive	1 ***	1 ***	–	1 ***	–	1 ***	–	1 ***	–
To Perfection	<—	Cognitive	1.39 ***	1.09 ***	-2.74 ***	1.32 ***	-0.71	1.51 ***	0.50	1.17 ***	-1.39
Exceed Exp	<—	Cognitive	1.19 ***	0.93 ***	-2.54 **	1.11 ***	-0.92	1.41 ***	0.96	1.33 ***	.81
HW On Time	<—	Behavioral	1 ***	1 ***	–	1 ***	–	1 ***	–	1 ***	–
Work Hard HW	<—	Behavioral	1.18 ***	1.16 ***	-.23	1.17 ***	-0.14	1.35 ***	0.72	1.16 ***	-.15
Attention	<—	Behavioral	.93 ***	.98 ***	.44	.98 ***	0.47	.95 ***	0.09	.93 ***	-.01
Listen	<—	Behavioral	.88 ***	.99 ***	.87	.99 ***	.93	.80 ***	-.35	.96 ***	.45
Enjoy R_Math	<—	Emotional	1 ***	1 ***	–	1 ***	–	1 ***	–	1 ***	–
Look Forward	<—	Emotional	1.42 ***	1.57 ***	.91	1.24 ***	-1.61	1.21 ***	-0.12	1.49 ***	.40
Enjoy Math	<—	Emotional	1.32 ***	1.40 ***	.82	1.40 ***	1.12	1.18 ***	-1.20	1.39 ***	.62

Note. † = Reference Group. Invariance test conducted by comparison of unstandardized regression weights and critical ratio table.
Invariant = At least 1 z-score in each individual latent factor is nonsignificant at the .05 level.
*** $p < 0.01$; ** $p < 0.05$; * $p < 0.10$.

Appendix E

Metric Invariance Tests by Ethnicity

Invariance Test by Ethnicity with unstandardized Estimate Continued

			White†	Black		Hispanic		Asian	
Observed Variable		Latent Factor	*B*	*B*	*z*-score	*B*	*z*-score	*B*	*z*-score
R_Interested	<—	Cognitive	1 ***	1 ***	–	1 ***	–	1 ***	–
To Perfection	<—	Cognitive	1.39 ***	1.09 ***	-2.74 ***	1.32 ***	-.71	1.51 ***	.5
Exceed Exp	<—	Cognitive	1.19 ***	.93 ***	-2.54 **	1.11 ***	-.92	1.41 ***	.96
HW On Time	<—	Behavioral	1 ***	1 ***	–	1 ***	–	1 ***	–
Work Hard HW	<—	Behavioral	1.18 ***	1.16 ***	-.23	1.17 ***	-.14	1.35 ***	.72
Attention	<—	Behavioral	.93 ***	.98 ***	.44	.98 ***	.47	.95 ***	.09
Listen	<—	Behavioral	.88 ***	.99 ***	.87	.99 ***	.93	.8 ***	-.35
Enjoy R_Math	<—	Emotional	1 ***	1 ***	–	1 ***	–	1 ***	–
Look Forward	<—	Emotional	1.42 ***	1.57 ***	.91	1.24 ***	-1.61	1.21 ***	-.12
Enjoy Math	<—	Emotional	1.32 ***	1.4 ***	.82	1.4 ***	1.12	1.18 ***	-1.2

Note. † = Reference Group. Invariance test conducted by comparison of unstandardized regression weights and critical ratio table.

Invariant = At least 1 z-score in each individual latent factor is nonsignificant at the .05 level.

*** $p < 0.01$; ** $p < 0.05$; * $p < 0.10$.

Appendix F

Metric Invariance Tests by Ethnicity

Invariance Test by Ethnicity with Unstandardized Estimate Continued

			Hispanic†	Asian		Multiracial/ Other	
Observable Variables		Latent Factor	*B*	*B*	*z*-score	*B*	*z*-score
R_Interested	<—	Cognitive	1***	1***	–	1***	–
To Perfection	<—	Cognitive	1.32***	1.51***	-.76	1.17***	-.87
Exceed Exp	<—	Cognitive	1.11***	1.41***	-1.28	1.33***	1.24
HW On Time	<—	Behavioral	1***	1***	–	1***	–
Work Hard HW	<—	Behavioral	1.17***	1.35***	-.75	1.16***	-.06
Attention	<—	Behavioral	.98***	.95***	.12	0.93***	-.3
Listen	<—	Behavioral	.99***	.8***	.77	0.96***	-.15
Enjoy R_Math	<—	Emotional	1***	1***	–	1***	–
Look Forward	<—	Emotional	1.24***	1.21***	.18	1.49***	1.29
Enjoy Math	<—	Emotional	1.4***	1.18***	1.73*	1.39***	-.08

Note. † = Reference Group. Invariance test conducted by comparison of unstandardized regression weights and critical ratio table. Invariant = At least 1 z-score in each individual latent factor is nonsignificant at the .05 level.

Appendix G

Metric Invariance Tests by Ethnicity

Invariance Test by Economic, Social, and Cultural Status

Observed Variables		Latent Factor	Low *B*	High *B*	*z-scores*
R_Interested	←	Cognitive	1 ***	1 ***	–
To Perfection	←	Cognitive	1.21 ***	1.42 ***	2.54 **
Exceed Exp	←	Cognitive	1.09 ***	1.23 ***	1.96 *
HW on Time	←	Behavioral	1	1 ***	–
Work On HW	←	Behavioral	1.16 ***	1.26 ***	1.27
Attention	←	Behavioral	.89 ***	1.08 ***	1.97 *
Listen	←	Behavioral	.87 ***	1.04 ***	1.85 *
Enjoy R_Math	←	Emotional	1	1 ***	–
Look Forward	←	Emotional	1.34 ***	1.46 ***	1.35
Enjoy Math	←	Emotional	1.28 ***	1.4 ***	2.09 **

Note. Invariance test conducted by comparison of unstandardized regression weights and critical ratio table. Invariant = At least 1 z-score in each individual latent factor is nonsignificant at the .05 level.

† = Reference Group

*** *p*-value < 0.01; ** *p*-value < 0.05; * *p*-value < 0.10

Appendix H

Metric Invariance Tests by ESCS

Invariance Test by Economic, Social, and Cultural Status

Observed Variables		Latent Factor	Low *B*	High *B*	*z-scores*
R_Interested	←	Cognitive	1 ***	1 ***	–
To Perfection	←	Cognitive	1.21 ***	1.42 ***	2.54 **
Exceed Exp	←	Cognitive	1.09 ***	1.23 ***	1.96 *
HW on Time	←	Behavioral	1	1 ***	–
Work On HW	←	Behavioral	1.16 ***	1.26 ***	1.27
Attention	←	Behavioral	.89 ***	1.08 ***	1.97 *
Listen	←	Behavioral	.87 ***	1.04 ***	1.85 *
Enjoy R_Math	←	Emotional	1	1 ***	–
Look Forward	←	Emotional	1.34 ***	1.46 ***	1.35
Enjoy Math	←	Emotional	1.28 ***	1.4 ***	2.09 **

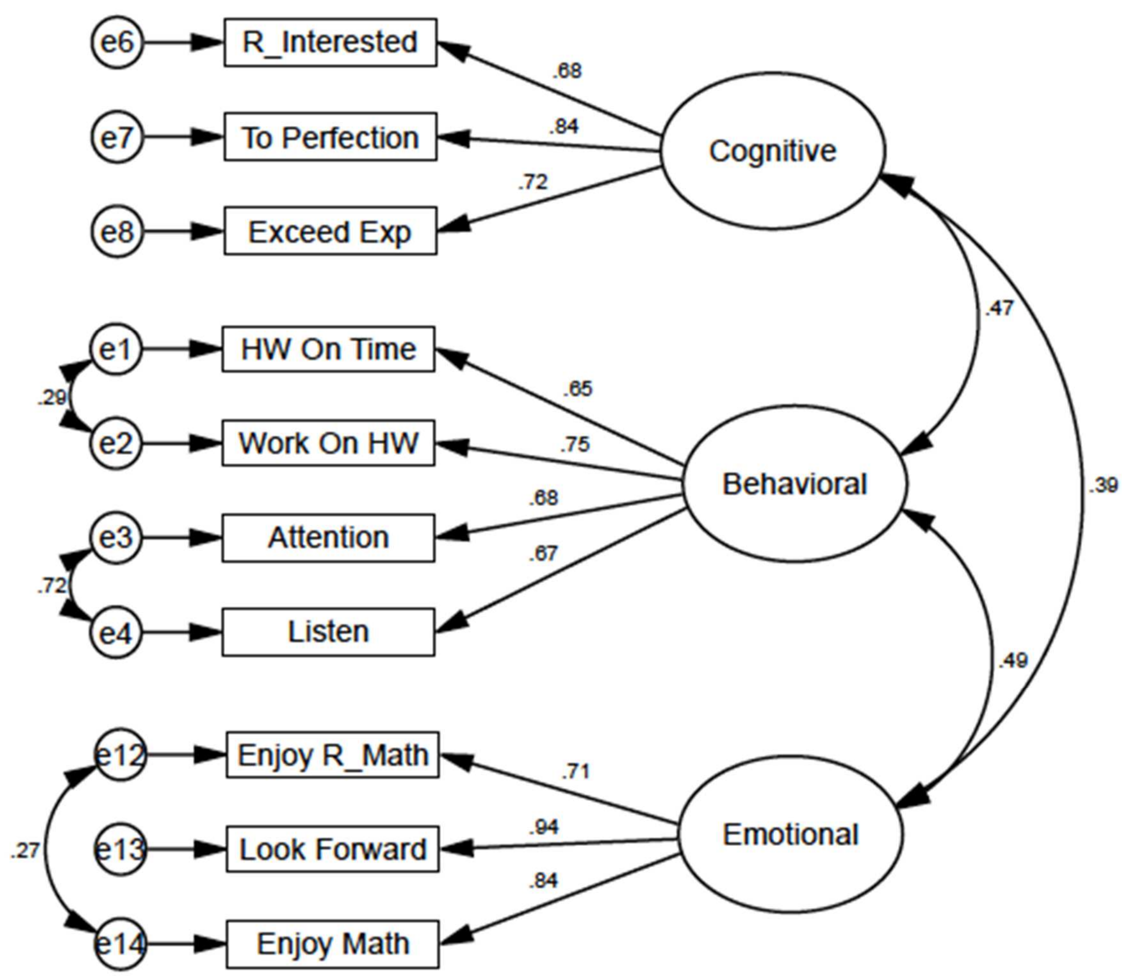
Note. Invariance test conducted by comparison of unstandardized regression weights and critical ratio table. Invariant = At least 1 z-score in each individual latent factor is nonsignificant at the .05 level.

† = Reference Group

*** *p*-value < 0.01; ** *p*-value < 0.05; * *p*-value < 0.10

Appendix I

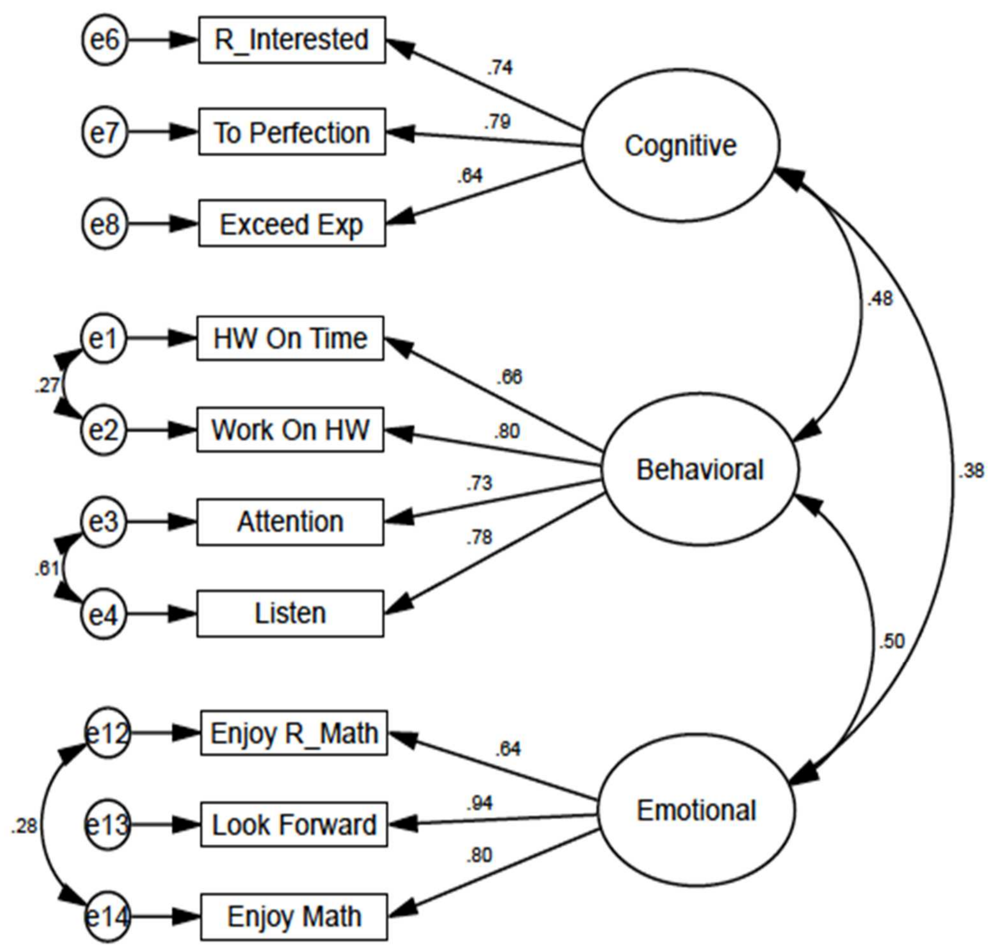
Mathematics Engagement Measurement Model for White Students



Final Measurement Model with statistically significant ($p < .05$) standardized loadings standardized loadings for White Students.

Appendix J

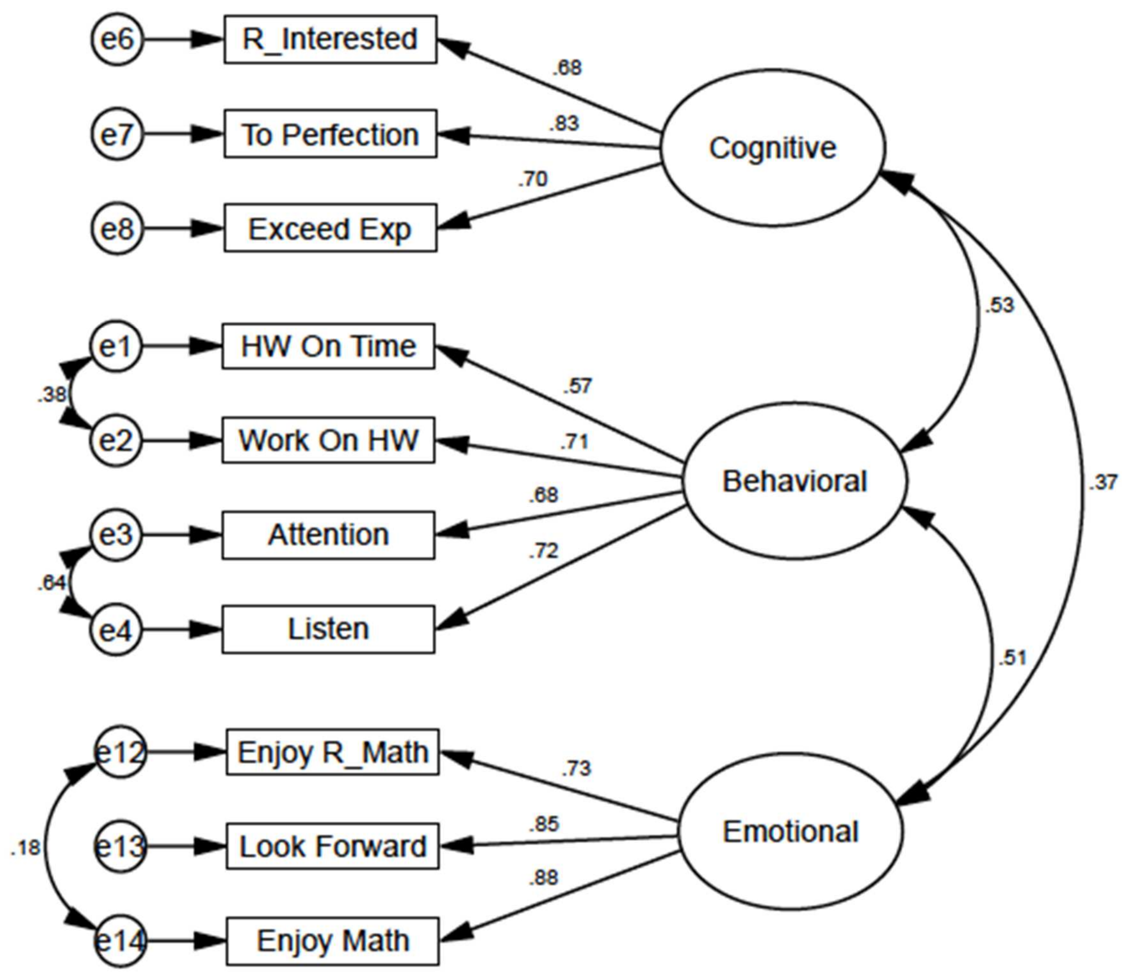
Mathematics Engagement Measurement Model for Black/African American Students



Final Measurement Model with statistically significant ($p < .05$) standardized loadings for Black/African American Students.

Appendix K

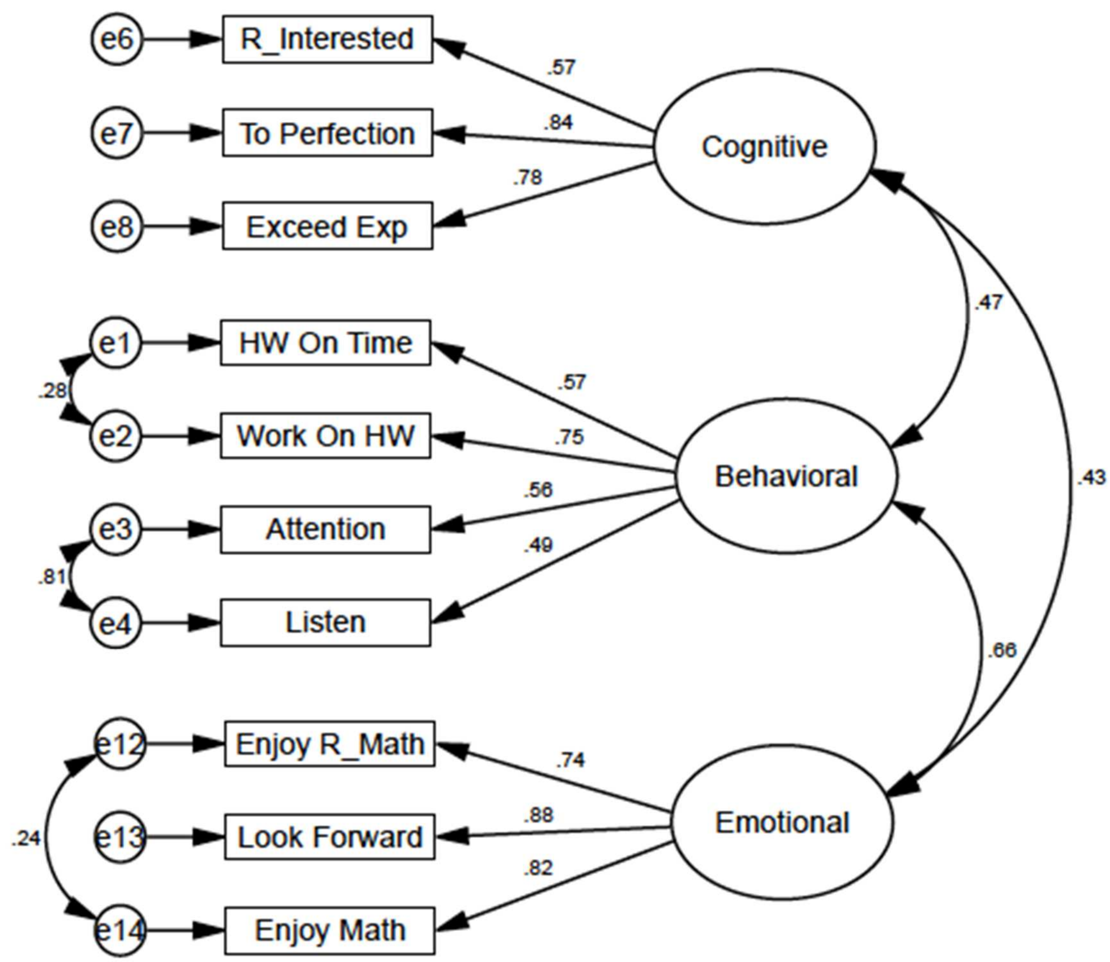
Mathematics Engagement Measurement Model for Hispanic Students



Final Measurement Model with statistically significant ($p < .05$) standardized loadings for Hispanic Students.

Appendix L

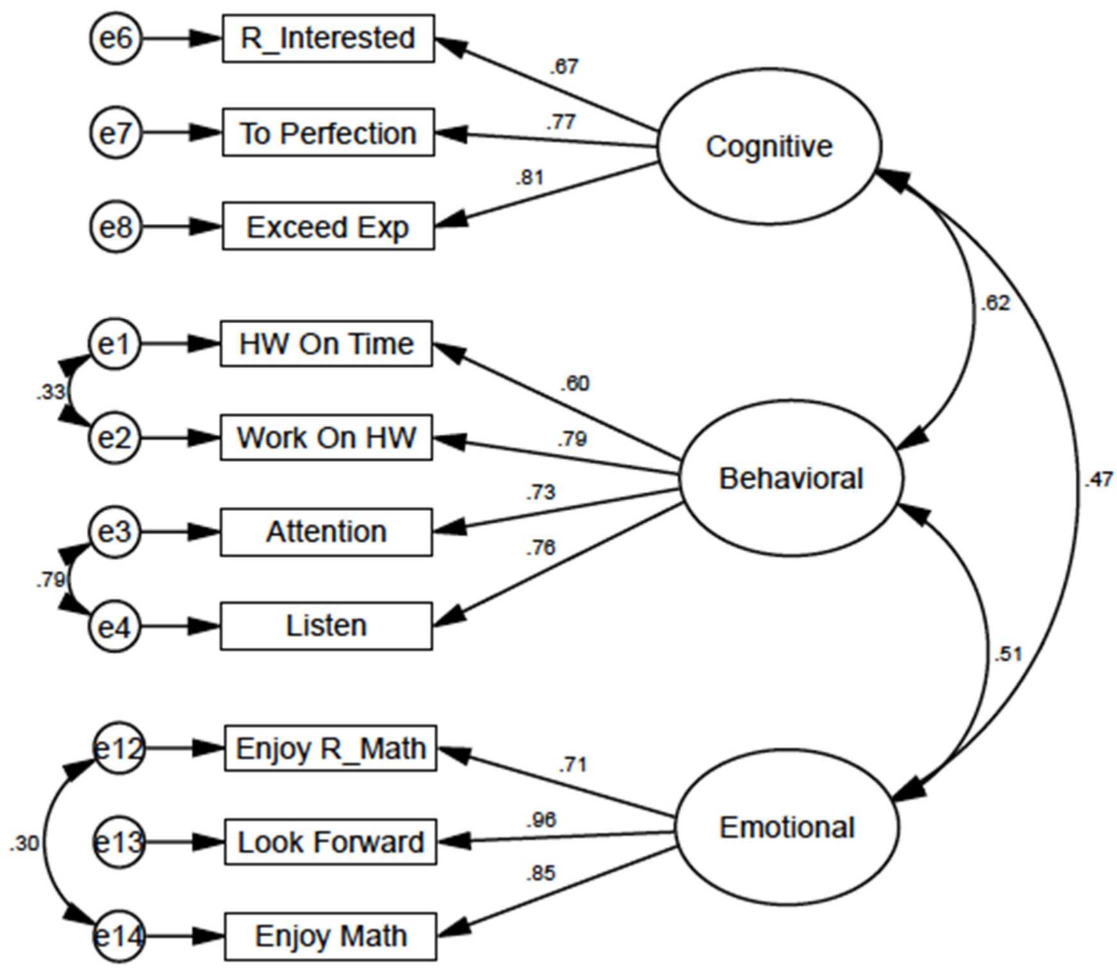
Mathematics Engagement Measurement Model for Asian Students



Final Measurement Model with statistically significant ($p < .05$) standardized loadings standardized loadings for Asian Students.

Appendix M

Mathematics Engagement Measurement Model for Multiracial/Other Students



Final Measurement Model with statistically significant ($p < .05$) standardized loadings standardized loadings for Multiracial/Other Students.

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Biography

Silvia E. Moore graduated from Chula Vista High School, Chula Vista, California, in 1983. She received her Bachelor of Arts from Alliant University in 2005 and her Master of Arts in Education from George Mason University in 2008. She was employed as a teacher in Rome, Italy and Fairfax County, Virginia.