THE RELATIONSHIP BETWEEN MATHEMATICS ACHIEVEMENT AND WORKING MEMORY ACROSS EDUCATION LEVEL

by

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Abstract

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The current study examined the relationship between mathematics achievement and working memory and whether this relationship changes across levels of math education. In addition to the effect of working memory on overall math achievement, its effect on three specific areas of math achievement were investigated, including knowledge of basic math facts, calculation skills and application of math concepts. Participants included 136 children and adolescents (age 6-16) who had undergone a comprehensive psychological evaluation. Results indicate that greater auditory working memory capacity predicts a higher level of math achievement in all areas. Auditory working memory explained unique variance, above and beyond the contributions of verbal and nonverbal reasoning and processing speed, in overall math achievement, fact fluency and applied problems, but not calculation skills. The variance in achievement in overall math, fact fluency and calculation skills explained by variance in working memory remained stable across two age groups representing elementary and secondary levels of education. The relationship between these two constructs increased across age for applied problem solving skills. These results provide evidence for the theory that both elementary and secondary level math achievement rely on auditory working memory. Theoretical and practical implications of these results, as well as directions for future research, are discussed.

Introduction

The level of mathematics and problem solving ability necessary in the workplace and in day-to-day living has increased dramatically (National Council of Teachers of Mathematics, 2000). Mathematics is a symbolic language that helps us to think about, record and communicate information and ideas. Mathematics is also a universal language, as it has meaning for all cultures. Given the importance of math skills, it is troubling to find that approximately 6% of students in general education classes show evidence of a serious mathematical difficulty, and about one quarter of students with diagnosed learning disabilities exhibit problems in mathematics (Cass et al., 2003).

There are many reasons children may fail to acquire math skills and develop understanding of math concepts, including math anxiety, lack of experience, poor motivation, reading difficulties and neuropsychological damage. A growing body of evidence indicates that math difficulties can be associated with various cognitive deficits. In fact, Geary (2004, 2005) has noted that between 5% and 8% of elementary school children have some form of specific memory or cognitive deficit that interferes with their ability to learn and understand numerical and arithmetical concepts or procedures involved in math domains.

Important progress has been made over the past several decades in the understanding of the cognitive deficits that contribute to difficulties in reading. Research in the area of math achievement has also advanced over the past couple decades, but more slowly than the study of reading (Cass et al., 2003). The complexity of the field of mathematics contributes to this delay. Difficulties in math can arise from deficient skills in one or more of the domains of mathematics, including arithmetic, algebra or geometry. These domains are also very complex, in and of themselves. All have many subdomains, and difficulties in any could cause problems with math achievement. Most research on math achievement has focused on basic math skills, and little is known about the cognitive processes that underlie more complex math skills, such as algebra and geometry.

Cognitive Processes

Researchers studying the difficulties experienced by children with math problems have investigated a number of cognitive mechanisms that may underlie these difficulties. Working memory is thought to play a central role in the acquisition and use of basic educational skills (Hitch & McAuley, 1991), and its role in the development of math abilities and disabilities has begun to be investigated (Geary, Hoard & Hamson, 1999; Hitch & McAuley, 1991; Floyd, Evans & McGrew, 2003; Jordan, Levine & Huttenlocher, 1995; Logie, Gilhooly & Wynn, 1994; McLean & Hitch, 1999; Passolunghi & Siegel, 2004; Swanson & Beebe-Frankenberger, 2004). Much of the research in this area has been inspired by ideas related to cognitive models of working memory and is related to general definitions of learning disabilities.

Learning disability refers to a neurobiological disorder in one or more of the basic processes involved in understanding spoken or written language (Lerner & Kline, 2006).

This variance in the brain may influence an individual's ability to speak, listen, spell, read, write, organize information, reason or do mathematical calculations (Lerner & Kline, 2006). The most widely used definition of learning disabilities is provided in the Individuals with Disabilities Education Improvement Act (IDEA-2004) (Public Law 108-446). This federal law provides the basis for most state definitions and is therefore used by many schools. The definition of learning disabilities provided by IDEA-2004 is:

> The term "specific learning disability" means a disorder in one or more of the basic psychological processes involved in understanding or in using language, spoken or written, which disorder may manifest itself in imperfect ability to listen, think, speak, read, write, spell or do mathematical calculations. Such term includes such conditions as perceptual disabilities, brain injury, minimal brain dysfunction, dyslexia, and developmental aphasia. Such term does not include a learning problem that is primarily the result of visual, hearing, or motor disabilities; of mental retardation; of emotional disturbance; or of environmental, cultural or economic disadvantage.

Among the elements the IDEA-2004 definition has in common with other significant definitions of learning disabilities (National Joint Committee on Learning Disabilties, NJCLD, and the Interagency Committee on Learning Disabilities, ICLD) are the implicit and explicit views that they are related to neurological factors and they involve disorder in one or more of the basic psychological processes. Because all learning originates within the brain, it follows that disorders in learning are caused by a dysfunction in the central nervous system. Mental ability is not a single entity, but is instead made up of many underlying abilities, including but not limited to memory, auditory, visual and tactile-kinesthetic perception, linguistic ability and thinking (Lerner & Kline, 2006). For a person with a learning disability, these component abilities do not develop in an even fashion (Lerner & Kline, 2006). While some may develop in the anticipated sequence or rate, others may lag in their development. One implication of this aspect of the definition is that each person with a learning disability will manifest strengths and weaknesses in different mental processes. Research supports the fact that learning disabilities in mathematics are not due to a general cognitive deficit or incapacity, and instead different patterns of cognitive functioning relate to different problems in math calculation (Jordan, Levine & Huttenlocher, 1995; Geary, Hoard & Hamson, 1999). In fact, Russell & Ginsburg (1984) found that children with math disabilities can possess many cognitive strengths.

Cognitive psychology, which focuses on the human processes of learning, thinking and knowing, and its theory of learning is most closely tied to the *disorders of psychological processes* definition of learning disabilities. The information processing model of learning traces the flow of information during the process of learning, from initial reception of information, through a processing function, and then to an action. Inputs are auditory, visual or tactile stimuli and can be either external or internal. Processing functions are cognitive processes such as associations, thinking, memory, and decision making. These processes involve storing and locating the information (memory systems), organizing the information and facilitating operations and decisions (executive functions). Outputs are actions or behaviors in response to the original stimuli (Mayer, 1996).

Important to the information-processing model is the multi-store memory system, which conceptualizes the flow of information through three types of memory. When the mind attends to selected stimuli, that information flows into the first memory system, the sensory register. This system holds the information long enough for it to be perceived, and perception gives meaning to stimuli.

In working memory, the pertinent information or current problem is receiving the person's conscious attention, and the individual can act on it. When a new problem begins receiving the person's conscious attention, and therefore replaces the old information in working memory, the old information either decays and is lost or is placed into long-term storage (Swanson, 1996). Students with learning disabilities often have problems with different types of working memory. Strategies that can extend the time information remains in working memory and facilitate moving it to long-term memory include rehearsal, chunking or organizing the information, and the use of key words (Mastropieri & Scruggs, 1998).

The difference between working memory and short-term memory must be made clear. While these terms were once used interchangeably, the distinction between the two concepts is important, as it has been shown that they relate differently to math achievement. The process of short-term memory relies on a passive storage system and involves the recall of information without changing it in any way. Working memory requires more active processes and involves temporarily held information being manipulated or transformed. Studies that compared the contributions of short-term memory and working memory to math achievement found that children with difficulties in math problem solving or arithmetic had impairments in working memory but not short-term memory (Passolunghi & Siegel, 2001; Passolunghi & Siegel, 2004; Geary, Hoard and Hamson, 1999). Further, studies that used short-term memory, or immediate rote recall, measures rather than working memory tasks found that short-term memory did not contribute unique variance to the prediction of math ability (Bull & Johnston, 1997; Butterworth, Cipolotti & Warrington, 1996).

Long-term memory is permanent memory storage. It is believed that once information is placed into long-term storage it remains there permanently. The difficulty people encounter in long-term memory is with retrieval, or how to recall or remember information stored in long-term memory. Before a person can think about a problem, the stored information must be retrieved from long-term memory and placed into working memory. The way information is stored in long-term memory helps with the process of retrieval (Masteropieri & Scruggs, 1998).

Executive control is the component of the information-processing model that refers to the ability to control and direct one's own learning, thinking and mental activity. Executive control directs the flow of thinking, manages the cognitive processes during learning, and keeps track of what information is being processed. It determines which mental activities occur and which processing components receive system attention resources. It involves planning, evaluating and regulating (Swanson, 1996). It seems clear from the research and theories in cognitive psychology that there are many cognitive processes involved in learning, and that the dysfunction or alternative functioning of any of these processes would lead to difficulties in learning.

Working Memory System

The structure and processes involved in working memory have been studied extensively. The working memory system is often thought of as a mental workbench, where conscious mental effort is applied (Baddeley, 1992; Baddeley & Hitch, 1974). When information is retrieved from long-term memory, for example, a basic math fact and arithmetic algorithm, and put together to solve a problem, working memory is where this combination occurs. Baddeley expanded on this definition by stating that major roles of working memory include retrieval of stored long-term knowledge relevant to tasks, the manipulation and recombination of material allowing for the interpretation of novel stimuli and the discovery of novel information or solutions to problems. Baddeley's theory of a working memory system provides a useful context for understanding recent studies on working memory and mathematics.

The working memory system Baddeley and Hitch (1974) hypothesized has three major components. This model distinguishes a central executive system from two "slave systems," the articulatory or phonological loop and the visuo-spatial sketchpad. The central executive is in charge of planning future actions, initiating retrieval and decision processes as necessary, and integrating information coming into the system. Consider the arithmetic problem [(5+3) x 2]/(4+2). The central executive triggers the retrieval of facts (5+3=8, 8x2=16) and invokes the problem-solving rules such as "how to add and multiply." It also decides that the intermediate value 16 must be held momentarily while further processing occurs. It activates the phonological loop and sends it the value 16 to rehearse until that value is needed again by the central executive.

Central Executive

In 1996, Baddeley proposed that the central executive is made up of separate but overlapping functions involving coordinating concurrent activities, switching retrieval plans, attending to inputs, and holding and manipulating information in long-term memory. The first executive function identified by Baddeley (1996), coordinating performance on two or more separate tasks, was described by the arithmetic example above. Switching retrieval strategies is the second executive function and is clearly necessary for problems such as multi-digit multiplication, which involves multiplying and adding. The third executive function involves attending selectively to different inputs, which is clearly a feature of multi-digit problems. The fourth executive function is activating and manipulating information stored in long-term memory, also illustrated in the example above. All of Baddeley's components of the central executive seem likely to be involved in arithmetical calculation.

Another cognitive activity that has been assigned to the central executive component of working memory is the suppression or inhibition of irrelevant information. Recent studies have suggested that activities related to the inhibition of irrelevant information are deficient in children with math disabilities (Passolunghi, Cordnoldi & De Liberto, 1999; Passolunghi & Siegel, 2004; Passolunghi & Siegel, 2001; Russell & Ginsburg, 1984). Passolunghi, Cordnoldi and De Liberto (1999) found that children who were poor mathematical problem solvers had low scores on working memory tasks that required the inhibition of irrelevant information. In a study conducted by Passolunghi and Siegel (2001) children who were poor in arithmetic problem solvers made a significantly higher number of intrusion errors on working memory tasks than did good problem solvers. These authors concluded that the poor problem solvers maintained information in memory that initially had to be processed, even when it was advantageous to suppress the information. Passolunghi and Siegel's (2004) study extended this body of research to include both computation and arithmetic word problem ability. Their intrusion error results provide further evidence that math achievement and math disabilities are related to the ability to inhibit the memory of irrelevant information.

Swanson, Cooney and Brock (1993), on the other hand, found significant correlations between working memory and recall of extraneous information but the correlations were not in the predicted direction; they found that the greater a child's working memory the more likely they will recall irrelevant information in a word problem. Finally, Swanson and Beebe-Frankenberger (2004) found a significant relationship between working memory and math problem solving even after the influence of inhibition (and other central executive processes) was partialed out of the analysis. This suggests that, while inhibition of irrelevant information may be related to math achievement, other aspects of working memory are also important and contribute unique variance in predicting solution accuracy. These aspects may be found in the other components of Baddeley's model of working memory, the slave systems.

Phonological Loop and Visuo-spatial Sketchpad

The slave systems each have a domain-specific task or set of responsibilities. They assist the central executive by being responsible for low-level processing involved in a task (Baddeley and Hitch, 1974, Baddeley, 1992). The phonological loop is the speech- and sound-related component responsible for rehearsal of verbal information and phonological processing. The visual spatial sketchpad specializes in visual and spatial information, holding or manipulating that information in a short-duration buffer. Most research supporting the relationship between working memory and mathematics achievement or disability has included only auditory working memory tasks (Passolunghi & Siegel, 2004; Swanson, Cooney and Brock, 1993; Floyd, Evans and McGrew, 2003; Fuchs, et al, 2005; Passolunghi & Siegel, 2001; Geary, Hoard and Hamson, 1999) or a combination of auditory and visual working memory tasks that are not separated during analysis (Swanson & Beebe-Frankenberger, 2004).

Several studies have attempted to determine the unique contributions of the phonological loop and visual spatial sketchpad to math achievement, with mixed results. Both verbal and visual-spatial working memory were found to be related to math achievement in some studies (Dark & Benbow, 1990; Swanson & Lee, 2001). Dark and Benbow (1990) studied mathematically talented middle school students. Mathematical talent was operationalized by high solution accuracy on word problems. When compared with peers of average math ability and college students, the mathematically talented middle school students performed significantly better on auditory working memory tasks. The talented youth also performed better than the other youth, but not college students, on tasks requiring the manipulation of spatial information. In a more recent study, Swanson and Lee (2001) investigated working memory abilities in children with and without math learning disabilities and found that both verbal and visual-spatial working memory contributed significant variance to children's math problem solving ability. They contend that no one process is more important than the other in predicting solution accuracy, indicating that both slave systems are necessary for math problem solving.

Other studies provide evidence that only one slave system is involved in the demonstration of math learning. McLean and Hitch (1999) have found evidence for the importance of visual-spatial sketchpad but not the phonological loop in children's arithmetical difficulties. However, the tasks used to measure the contribution of the phonological loop were indices of phonological short-term memory, specifically forward digit span and nonword repetition. These tasks do not involve the manipulation but rather the immediate recall of information. As noted earlier, previous studies have shown that working memory, and not short-term memory, is related to math achievement. McLean and Hitch (1999) did not include a true measure of auditory working memory, and therefore could not have ruled out the contribution of the phonological loop.

Further studies have found that auditory working memory, and not visual-spatial memory, contributes significantly to mathematics achievement and disability. Hitch and McAuley (1991) found that children with specific arithmetic difficulties (and without comorbid reading difficulties) were impaired on digit and counting span tasks but not on other complex span tasks. Wilson and Swanson's (2001) results indicated that mathematical computation was better predicted by verbal than by visual-spatial working memory. Logie, Gilhooly and Wynn (1994) found that when subjects were required to add two two-digit numbers under different types of dual-task conditions, performance was significantly disrupted by a concurrent oral task whether the addends were presented visually or auditorily. When the concurrent task was visual spatial, performance was only disrupted when the addends were presented visually. These results indicate that subvocal rehearsal seems to be implicated in mental calculation regardless of the modality of the stimuli, whereas the visuo-spatial sketchpad only plays a role when stimuli are presented visually. Logie, Gilhooly and Wynn (1994) suggested that the involvement of the visuo-spatial sketchpad may be restricted to the precalculation stage, when the visual problem is encoded. Noel, et al (2001) found that the speed and accuracy of complex mental calculation were significantly impacted when the problems presented were phonologically similar. When visual similarity was manipulated, a significant effect was not found. They concluded that the phonological loop is used for storing addends during mathematical calculation, rather than the visual-spatial sketchpad.

It is important to note a significant difference between the studies finding both verbal and visual-spatial working memory to be related to math achievement (Dark & Benbow, 1990; Swanson & Lee, 2001) and those finding a domain-specific working memory deficit, specifically verbal (Hitch & McAuley, 1991; Logie, Gilhooly and Wynn, 1994; Noel, et al, 2001). The first set of studies measured achievement in word problem solving, and the second set measured complex calculation and arithmetic skills. It is possible, therefore, that verbal working memory (or the phonological loop) is involved in the manipulation of numbers and the utilization of algorithms involved in calculation, and that visual-spatial working memory (or the sketchpad) is involved in the reading component of word problems. Further evidence for this is found in the research comparing those with learning disabilities in math and those with disabilities in both math and reading; children with only arithmetic disability show domain-specific working memory deficits, while children with comorbid reading and math disabilities have a general working memory impairment (Siegel & Ryan, 1989; Hitch & McAuley, 1991).

Specific Numerical Deficit

In addition to the involvement of the various components of Baddeley's working memory system, a specific numerical working memory deficit in children with math difficulties has been proposed. Studies involving only working memory tasks involving numbers have found significant relationships between working memory and math calculation difficulties and disabilities (Noel, et al, 2001; Geary, Hoard and Hamson, 1999). Siegel and Ryan (1989) found that children with a math learning disability performed similarly to children with normal achievement on a working memory task involving sentence processing, but their performance was impaired on a working memory task requiring the processing of numerical information. Hitch and McAuley (1991) also found that children with specific math disabilities were significantly impaired on counting and digit span tasks but not other working memory tasks. Passolunghi and Siegel (2001, 2004) found that poor problem solvers' working memory impairments were not specific to processing numerical information. It should be noted that all tasks in both Passolunghi and Siegel's studies (2001, 2004) were conducted in Italian. The theory has been suggested that digit spans differ in different languages due to the fact that digits in some languages are longer than digits in others (Geary, Bow-Thomas, Fan & Siegler, 1993). It is possible that a difference in digit length contributed to the difference between the outcomes of Passolunghi and Siegel's studies and others mentioned above. Regardless, in all studies including working memory tasks involving numbers, subjects with math difficulties showed significant deficits in numerical working memory. The theory of a specific numerical deficit is also consistent with the general finding that assessments that include numbers are most predictive of math outcomes and most useful for early screening of math disabilities (Fletcher, 2005; Gersten, Jordan and Flojo, 2005).

In summary, a review of the literature on working memory and math difficulties suggests that many components and processes that are encompassed by working memory are involved in math learning and achievement. Manipulation of information, including the use of algorithms to solve problems, retrieval of information from long term memory and inhibitory processes are all important activities conducted by the central executive. Although the visual-spatial sketchpad appears to be important when reading is involved, such as in word problems, or when the calculation problems are presented visually and must be encoded, the phonological loop and auditory working memory seem to be more closely related to most areas of math achievement. Finally, there is evidence to suggest that performance on working memory tasks involving numbers is related to math ability. The research on working memory components and modalities seems to indicate that the best measure of working memory for the present study would involve tasks that require the subject to manipulate numerical information presented in an auditory format.

Numerical and Arithmetical Cognition

One obstacle to the study of math achievement and disabilities is the large number and complexity of math domains, as noted by Geary, Hoard & Hamson (1999). Difficulties in math can be the result of deficits in the ability to process information in one or all of these domains (Russell & Ginsburg, 1984). In particular, the development of numerical and arithmetical cognition is important in understanding math achievement, and therefore disabilities in math (Geary, 1993; Geary, Hoard & Hamson, 1999; Geary, 2004; Geary, 2005).

Basic Numerical Competencies

Basic numerical competencies include number production and comprehension. These competencies require the ability to identify and process verbal and Arabic representations of numbers, to transcode numbers from one representation to another, and to compare the magnitudes of numbers. While there have been few studies conducted on the basic numerical competencies of children with math difficulties, the results suggest that the number production and comprehension of children with specific math disabilities, although often delayed, are largely intact for the processing of simple numbers (Geary, 1993; Geary, Hoard & Hamson, 1999; Geary, 2004).

Counting Knowledge

Children's counting is thought to be governed by five implicit principles (Gallistel & Gelman, 1992): one-to-one correspondence (one and only one word tag is assigned to each counted object), stable order (the order of the word tags must be the same across counted sets), cardinality (the value of the final word tag represents the quantity of items in the counted set), abstraction (objects of any type can be brought together and counted), and order irrelevance (items can be counted in any sequence). Briars and Seigler (1984) proposed essential features, similar to Gallistel & Gelman's principles, and unessential features of counting. The unessential features include start at an end, adjacency, pointing, and standard direction. As children's counting knowledge matures, they believe the essential features and principles and understand that the unessential features are not necessary for counting.

Research on the counting knowledge of children with math disabilities shows that children with comorbid math and reading disabilities understand most of the inherent counting principles, such as stable order and cardinality, but believe that order and adjacency are essential features of counting (Geary, 1993; Geary, 2004). However, Geary, Hoard and Hamson (1999) suggest that their results indicate that children with comorbid math and reading disabilities understand counting as a rote and mechanical activity, whereas children with only math disabilities show age-appropriate counting knowledge. One exception was noted: Math disabled children failed to detect a significant number of double-counting errors (when the first item in a set is counted twice). The children who performed poorly on these trials also demonstrated significantly lower performance on a working memory task than children who successfully detected the counting errors. Bull and Johnston (1997) also found that children with low math achievement were comparable to children with high math achievement on speed of counting. However, the low achievers made significantly more counting errors. The counting errors of children with math difficulties suggest another mechanism by which working memory could affect math achievement.

Arithmetic

Arithmetical competency is improved when there is a change in the distribution of strategies children use in problem solving. When first learning addition, children typically count both addends. Counting procedures can be done with the use of fingers (finger counting strategy) or without them (verbal counting strategy). In either case, the two most commonly used procedures are labeled counting-on and counting-all. Counting-all involves counting both addends, starting from 1, while counting-on involves stating the value of the larger addend and then counting up the value of the smaller addend. Children with math disabilities make more procedural errors and employ more developmentally immature procedures than average achieving peers (Geary, 1990; Geary, Hoard & Hamson, 1999; Geary, 2004; Gerston, 1999).

The frequent use of counting procedures eventually results in the development of long-term memory associations between problems and the answers generated by way of counting. The formation of these associations, in turn, leads to the use of memory-based procedures in problem solving. With each implementation of a computational strategy, the likelihood of direct retrieval increases for later solutions of the problem (Geary, 1993). Many children with math disabilities do not demonstrate the shift from procedural-based problem solving to direct retrieval (or memory-based) problem solving. Geary, Hoard and Hamson (1999) found that children with low math achievement made more memory-retrieval errors than normal achieving peers. This finding again points to the role of working memory in math achievement; in order for the use of a strategy to result in the development of a long-term representation between a problem and its answer, both the first number and second number, as well as the answer, must be active simultaneously in working memory.

Math Achievement

Math achievement can be, and has been, defined in many different ways. Definitions include fact fluency, calculation skills, and applications of math concepts such as in problem solving and word problems. These different aspects of math achievement have been found to rely on numerous underlying processes. In particular, performance on various measures of math knowledge and problem solving seem to be differentially related to working memory.

Fact Fluency

Fact fluency refers to how quickly and accurately a person can solve simple math problems. As noted above, much research has found that children with math difficulties or disabilities have particular problems in representation and retrieval of basic math facts (Jordan & Hanich, 2000; Jordan, Hanich & Kaplan, 2003; Joradan & Montani, 1997; Geary, 1993; Geary, Hoard, & Hamson, 1999; Gersten, Jordan & Flojo, 2005; Russell & Ginsburg, 1984; Bull & Johnston, 1997). In line with the research presented above, Russell & Ginsburg (1984) found that, while math disabled children had adequate knowledge of principles, they had significant difficulty with even simple addition facts. As noted above, poor working memory resources, together with immature counting strategies, have been implicated in the poor representation of arithmetic facts in long-term memory (Keeler & Swanson, 2001). Also, Swanson and Sachse-Lee (2001) found that working memory resources are used in the activation of relevant knowledge in long-term memory.

An alternative explanation for the relationship between working memory and the retrieval of math facts is suggested by Noel, et al (2001); that arithmetic facts have been taught as verbal routines and are stored in a verbal format. They posit that any calculations presented in an Arabic format have to be translated into the verbal format in order to retrieve the solution. This translation would occur within working memory.

Bull and Johnston (1997) and others suggested that an important factor that limits the retrieval of facts in children with low math achievement is slow speed of number identification, where representations of numbers are accessed from long-term memory. In this case, the difficulty falls with retrieving numerals, even before the retrieval of an entire math fact. Here working memory plays a role in fact fluency because resources are not available for storage when capacity is taken up by number identification, rather than due to a deficit in the ability to manipulate information. However, Geary, Hoard and Hamson (1999) also tested this hypothesis and found that children with math disabilities showed adequate number identification abilities.

Calculation Skills

Although fact retrieval and fluency is frequently weak in children with math difficulties, it is important to investigate weaknesses in other areas of mathematics, including calculation skills. In a large scale study, Bryant, Bryant and Hamilton (2000) found that the most common problem that differentiated children with math difficulties from children with other academic difficulties was in carrying out multi-step arithmetic. Logie, Gilhooly, and Wynn (1994) and Noel, et al (2001) found that verbal working memory is significantly related to complex mental calculation performance, whether the addends are presented visually or auditorily. This relationship between calculation skills and working memory may be associated with long-term memory of algorithms. Swanson and Sachse-Lee (2001) showed that, when compared to same-age peers, children with learning disabilities are deficient in retrieving knowledge related to algorithms. More research is needed to clarify the role of working memory systems in completing complex calculations.

Applications: Problem solving and word problems

Word problems are important means through which children learn to select and apply strategies needed for coping with problems in everyday life. Given the amount of incoming information that must be tracked in applied math and word problems, it seems intuitive that the maintenance and manipulation of information in working memory would be involved. In fact, the involvement of working memory may be even more complex than with fact fluency and calculation alone; not only is working memory used to access information from long-term memory (for example, algorithms to solve the problem or math facts) but a word problem also introduces a substantial amount of new information into working memory. The majority of research in this area supports the theory that word problem solution accuracy is related to working memory capacity (Passolunghi & Siegel, 2001; Swanson & Beebe-Frankenberger, 2004; Swanson & Sachse-Lee, 2001; Russell & Ginsburg, 1984).

Research also indicates that children with disabilities in learning have difficulty solving word and story problems. Swanson and Sachse-Lee (2001) noted the growing body of research finding that children with learning disabilities have significant difficulty constructing an adequate problem representation. Russell and Ginsburg (1984) found that children with math disabilities performed poorly on complex word problems, particularly those with irrelevant information.

Children's troubles in solving mathematical word problems have also been shown to be related to their deficient language and comprehension strategies. Swanson, Cooney and Brock (1993) found that working memory was not a major contributor to accuracy of word problem solutions above and beyond the contribution of reading comprehension. However, phonological processes are important in reading, and Swanson and Sachse-Lee (2001) results show that phonological processes do not mediate the contribution of working memory to word problem solution. Processes involved in verbal working memory play just as important a role as the phonological processes that are implicated in reading.

Secondary Level Math

Consistent with the natural progression and development of research in general, most math education research has been conducted around the earlier grades. Outside the areas of numbers, counting and simple arithmetic, theoretical models and research for mathematics abilities are not well developed. In most domains of mathematics, including geometry and algebra, very little is known about the normal development of related competencies to provide an organized framework for the study of math achievement and difficulties (Geary, 2005).

Given that very little formal research has been conducted on math at the secondary level, it is not surprising to find that only a handful of studies have investigated the relationship between working memory and math achievement in secondary students. Even fewer studies attempt to examine the effect of age on this relationship. Swanson and Beebe-Frankenberger's (2004) analysis found that the significant relationship between working memory and mathematical problem solving was stable across the age of their subjects. However, their subjects were elementary school students, and therefore, their results may not generalize to secondary level math domains. Wilson and Swanson (2001), on the other hand, found that mathematics calculation and working memory continued to be related across a broad age span. Dark and Benbow (1990) studied 12- and 13-year-olds, and their results suggest that working memory is related to algebra problem solving ability.

Evidence can also be found for a decline in the strength of the relationship between working memory and math achievement. Little & Widaman (1995) compared elementary and junior high school students with college students, and results indicated that a deficit in working memory resources places greater limits on the mathematical performance of children than on that of adults. While these results provide a useful addition to the sparse literature in this area, there are several limitations to the generalization of these findings. First, elementary and junior high school students were grouped together, but a review of mathematics curriculum suggests that the math knowledge and concepts required at these levels may differ significantly. Second, high school students were not included in the study at all. Moreover, the addition performance of subjects was the only math achievement measurement taken for all ages. Again, the knowledge and concepts required to perform successfully in math courses at different levels of education vary significantly. In order to determine whether or not the relationship between working memory and math achievement (as it would be helpful in their schoolwork) remains stable across age, measures of math achievement must test students' abilities to do math at all levels.

The Present Study

The present study was designed to extend and fill in a few of the gaps found in the literature on working memory and math achievement. In order to do so, answers to the three sets of questions below are explored. Rather than looking only at math disorders, the full range of math achievement was studied. Evidence has been found to suggest that reading disorders might represent the lower end of the distribution of reading ability (Shaywitz, Escobar, Shaywitz, Fletcher & Makuch, 1992). Along the same line, these findings indicate that the same may be true for math; that is, that disorders in math simply represent the low end of math achievement (Geary, 1993).

The first question investigated is: Is working memory related to math achievement? This question is intended to add to the research already conducted in this area and expand our knowledge of the relationship to include math achievement in elementary school through secondary school students. This study also explores whether or not there is a differential relationship between working memory and three areas of math achievement: fact fluency, calculation skills and applied math. It was expected that all three areas of math achievement would be related to working memory.

The second question investigated is: Is the relationship between working memory and math achievement stable across age? Based on what little is known about math achievement through the secondary level, working memory was expected to continue to be related to math achievement through the later grades; however, given the increased importance of abstract thinking and reasoning and the increased use of pencil and paper to solve complex problems in late elementary and secondary school math (Lerner & Kline, 2006), the relationship between these two constructs was expected to decline across age. The three areas of math achievement are explored separately. Similar results were expected for calculation and applied math as noted for overall math achievement, a continued but smaller relationship as age increases. Given the stable nature of student's difficulties in fact fluency, the relationship between working memory and achievement in this area was expected to remain stable across age.

Finally, the third question to be investigated is: Does working memory relate to math achievement above and beyond the effect of verbal and nonverbal reasoning and processing speed? Most studies of the relationship between working memory and math achievement or disability have not included the potential contributions of reasoning abilities or processing speed in their design. Studies that have included such measures have found that nonverbal reasoning (Swanson & Beebe-Frankenberger, 2004; Fuchs, et al, 2005) and processing speed (Swanson & Beebe-Frankenberger, 2004) are related to math ability. These studies also found that working memory contributes unique variance to math achievement or the prediction of math disabilities even after the effects of overall IQ (Geary, Hoard & Hamson, 1999; Russell & Ginsburg, 1984) or processing speed (Swanson & Beebe-Frankenberger, 2004) have been partialed out. It was predicted that similar results would be found in the present study for all areas of math achievement.

Method

Participants

The 136 participants in this study include children and adolescents who completed comprehensive psychological assessments at the Psychological Clinic of George Mason University in Fairfax, Virginia between 2003 and 2007. Seventy-one participants were between the ages of 6 and 10 years old (M = 8.42, SD = 1.16), and 65 participants were between the ages of 11 and 16 years old (M = 13.92, SD = 1.89). They included 84 males and 52 females. The race of 103 participants was not documented in their files and therefore was not able to be identified. Of those whose race was identifiable from their file, 26 were Caucasian, three were African-American, three were Asian, and one was Hispanic.

Procedure

Prior to data collection each student was given a standard battery of tests, including the Wechsler Intelligence Scale for Children – Fourth Edition (WISC-IV) and the Woodcock-Johnson Tests of Achievement – Third Edition (WJ Ach III). These tests are administered individually, with each participant working with the same examiner throughout testing. Each student’s parent signed a form giving their consent to have their child’s scores used for research purposes. The consent form clearly stated that no identifiable information would be used or connected to the research data. The standard scores of each index of the WISC-IV and each math cluster and subtest of the WJ Ach III were collected for each participant from archival data. Each index and subtest that was used in the analysis is described below.

Mathematics Measures

Three math measures were administered to each participant prior to data collection. These three measures compose the Broad Math cluster of the WJ Ach III. This cluster score provides a broad, comprehensive view of the child's math achievement level. The Broad Math cluster standard score is used in this study to represent the participant's overall math achievement in the analysis. The standard scores of each of the individual subtests making up this cluster represent the participant's achievement in each area of math: math fluency, calculation and applications.

Math fluency. WJ Ach III Math Fluency (Woodcock, et al., 2001) measures how quickly the participant can solve basic addition, subtraction, and multiplication facts. Items are presented in a visual format, and testing is discontinued after exactly three minutes. The score is based on the number of correct items and the normative data for age. As reported by McGrew and Woodcock (2001), one-year test-retest reliability was .87; the ratio of true score variance to observed variance was .87-.93. Coefficient alpha on their sample was .92.

Calculation. WJ Ach III Calculation (Woodcock, et al., 2001) is a paper-and-pencil task that requires the participant to perform a variety of math calculations, including basic addition, subtraction, multiplication and division, advanced calculations of each operation with regrouping and with negative numbers, fractions, percentages, algebra, trigonometry, logarithms, and calculus. All items are presented to the participant at once in a visual format. As reported by McGrew and Woodcock (2001), one-year test-retest reliability is .89; the ratio of true score variance to observed variance was .87-.96. Coefficient alpha on their sample was .93.

Applications. WJ Ach III Applied Problems (Woodcock, et al., 2001) measures the participant's skill in analyzing and solving practical math problems. Items are presented both orally and visually. Participants may choose to read along as word problems are presented orally by the tester, but no reading is required. As reported by McGrew and Woodcock (2001), one-year test-retest reliability is .85; the ratio of true score variance to observed variance was .88-.91. Coefficient alpha on their sample was .91.

Cognitive Measures

The Verbal Comprehension Index, Perceptual Reasoning Index, Working Memory Index, and Processing Speed Index scale scores of the Wechsler Intelligence Scale for Children – Fourth Edition (WISC-IV; The Psychological Corporation, 2003a) were used to measure aspects of participants' cognitive functioning.

Verbal Reasoning. The WISC-IV Verbal Comprehension Index (VCI) includes three subtests (Similarities, Vocabulary and Comprehension) and measures a participant's knowledge base, understanding and expression of verbal ideas, and verbal problem-solving ability. As reported by The Psychological Corporation (2003b), the average test-retest coefficient for the VCI is .93, and the average internal consistency coefficient is .94.

Nonverbal Reasoning. The WISC-IV Perceptual Reasoning Index (PRI) includes three subtests (Block Design, Matrix Reasoning and Picture Concepts) and measures a participant's nonverbal perception, visual-spatial analysis, and pictorial reasoning ability. As reported by The Psychological Corporation (2003b), the average test-retest coefficient for the PRI is .89, and the average internal consistency coefficient is .92.

Auditory Working Memory. The WISC-IV Working Memory Index (WMI) includes two subtests (Digit Span and Letter-Number Sequencing) and measures the participant's auditory working memory capacity, or the ability to temporarily keep spoken information in mind while performing some active transformation or manipulation of it. Items on both subtests are presented auditorily, cannot be repeated by the examiner, include numerical information and have no visual supplement. As reported by The Psychological Corporation (2003b), the average test-retest coefficient for the WMI is .89, and the average internal consistency coefficient is .92.

Processing Speed. The WISC-IV Processing Speed Index (PSI) includes two subtests (Coding and Symbol Search) and measures the rapidity with which a person can solve low difficulty problems over the span of a few minutes. As reported by The Psychological Corporation (2003b), the average test-retest coefficient for the PSI is .86, and the average internal consistency coefficient is .88.

Results

Working Memory and Math Achievement

In order to test the first hypothesis, that greater auditory working memory capacity predicts a higher level of math achievement, a simple regression was conducted of math achievement (dependent variable) on working memory (independent variable), using scores from all participants, regardless of age. Four separate simple regressions were run, each with a different measure of math achievement as the dependent variable and the participant's WISC-IV WMI standard score as the independent variable.

Overall Math Achievement. In order to test the hypothesis that greater auditory working memory will predict a higher level of overall math achievement, a simple regression was conducted of overall math achievement (WJ Ach Broad Math score) on auditory working memory capacity (WISC-IV WMI score). Analysis of the correlation between the two variables showed that 37.5% of the variance in the Broad Math scores is explained by variance in the WMI scores ($r = .61$, $p < .001$). The regression line crosses the Y-axis (B_0) at .75 ($p < .001$), and therefore when WMI = 0 the estimated mean of Broad Math is .75. Also, the slope of the regression line (B_1) is .61 ($p < .001$), and therefore for every 1 unit change in WMI, Broad Math is predicted to increase .61. The t-scores for B_0 ($t = 3.57$, $p < .01$) and B_1 ($t = 8.98$, $p < .001$) indicate that the intercept and slope fall within the critical region and are, therefore, significantly different from that of the general population. There was a reasonable amount of explained variance relative to unexplained variance, $r^2 = .38$, $F(1, 134) = 80.55$, $p < .001$. Therefore, the regression model explains a significant amount of variance in Broad Math scores.

Fact Fluency. In order to test the hypothesis that greater auditory working memory will predict a higher level of knowledge and fluency of basic math facts, a simple regression was conducted of knowledge and fluency of basic math facts (WJ Ach Fact Fluency standard score) on auditory working memory capacity (WISC-IV WMI score). Analysis of the correlation between the two variables showed that 24.7 % of the variance in the Fact Fluency scores is explained by variance in the WMI scores (r = .50, p < .001). The regression line crosses the Y-axis (B_0) at .65 (p < .001), and therefore when WMI = 0 the estimated mean of Fact Fluency is .65. Also, the slope of the regression line (B_1) is .50 (p < .001), and therefore for every 1 unit change in WMI, Fact Fluency is predicted to increase .50. The t-scores for B_0 (t = 2.96, p < .01) and B_1 (t = 6.64, p < .001) indicate that the intercept and slope fall within the critical region and are, therefore, significantly different from that of the general population. There was a reasonable amount of explained variance relative to unexplained variance, $r^2 = .25$, $F(1, 134) = 44.07$, $p < .001$. Therefore, the regression model explains a significant amount of variance in Fact Fluency scores.

Calculation Skills. In order to test the hypothesis that greater auditory working memory will predict a higher level of math calculation skill, a simple regression was conducted of math calculation skill (WJ Ach Calculation score) on auditory working memory capacity (WISC-IV WMI score). Analysis of the correlation between the two variables showed that 14.6% of the variance in the Calculation scores is explained by variance in the WMI scores ($r = .38$, $p < .001$). The regression line crosses the Y-axis (B_0) at .49 ($p < .001$), and therefore when WMI = 0 the estimated mean of Calculation is .49. Also, the slope of the regression line (B_1) is .38 ($p < .001$), and therefore for every 1 unit change in WMI, Calculation is predicted to increase .38. The t-scores for B_0 ($t = 5.42$, $p < .001$) and B_1 ($t = 4.79$, $p < .001$) indicate that the intercept and slope fall within the critical region and are, therefore, significantly different from that of the general population. There was a reasonable amount of explained variance relative to unexplained variance, $r^2 = .15$, $F(1, 134) = 22.98$, $p < .001$. Therefore, the regression model explains a significant amount of variance in Calculation scores.

Applied Problems. In order to test the hypothesis that greater auditory working memory will predict a higher level of skill in analyzing and solving practical math problems, a simple regression was conducted of skill in analyzing and solving practical math problems (WJ Ach Applied Problems score) on auditory working memory capacity (WISC-IV WMI score). Analysis of the correlation between the two variables showed that 27.9% of the variance in the Applied Problems scores is explained by variance in the WMI scores ($r = .53$, $p < .001$). The regression line crosses the Y-axis (B_0) at .65 ($p < .001$), and therefore when WMI = 0 the estimated mean of Applied Problems is .65. Also, the slope of the regression line (B_1) is .53 ($p < .001$), and therefore for every 1 unit change in WMI, Applied Problems is predicted to increase .53. The t-scores for B_0 ($t = 4.54$, $p < .001$) and B_1 ($t = 7.20$, $p < .001$) indicate that the intercept and slope fall within the critical region and are, therefore, significantly different from that of the general population. There was a reasonable amount of explained variance relative to unexplained variance, $r^2 = .28$, $F(1, 134) = 51.89$, $p < .001$. Therefore, the regression model explains a significant amount of variance in Applied Problems scores.

Gender Differences. When the first hypothesis was tested separately with male participants and with female participants, auditory working memory was found to predict overall math achievement, fact fluency, calculation skill and skill in solving practical problems in both groups. Among male participants, 39.7% of the variance in the Broad Math scores is explained by variance in the WMI scores ($r^2 = .40$, $F(1, 82) = 54.05$, $p < .001$), 25.7% of the variance in the Fact Fluency scores is explained by variance in the WMI scores ($r^2 = .26$, $F(1, 82) = 28.39$, $p < .001$), 11.0% of the variance in the Calculation scores is explained by variance in the WMI scores ($r^2 = .11$, $F(1, 82) = 10.18$, $p < .01$), and 23.7% of the variance in the Applied Problems scores is explained by variance in the WMI scores ($r^2 = .24$, $F(1, 82) = 25.41$, $p < .001$). Among the female participants, 35.7% of the variance in the Broad Math scores is explained by variance in the WMI scores ($r^2 = .36$, $F(1, 50) = 27.82$, $p < .001$), 23.6% of the variance in the Fact Fluency scores is explained by variance in the WMI scores ($r^2 = .24$, $F(1, 50) = 15.43$, $p < .001$), 20.5% of the variance in the Calculation scores is explained by variance in the WMI scores ($r^2 = .21$, $F(1, 50) = 12.91$, $p < .01$), and 35.0% of the variance in the Applied Problems scores is explained by variance in the WMI scores ($r^2 = .35$, $F(1, 50) = 26.88$, $p < .001$).

Math Achievement and Working Memory: Effect of Age

In order to test the second hypothesis, that the variance in math achievement explained by variance in working memory declines as age increases, the participants and their scores were first divided into two age groups (ages 6 - 10:11, and 11 - 16:11). These age groups are meant to represent approximate times in math education when changes may occur in the type or level of concepts students are expected to learn (elementary vs. secondary education). Descriptive statistics for the WISC-IV WMI scores and WJ Ach math scores of each age group are presented in Table 1 (ages 6 – 10:11) and Table 2 (ages 11 – 16:11). Each of the four regressions run above were conducted for each age group. For each measure of math achievement comparisons of the slopes of the regression lines across the two age groups were made using an analysis of covariance with indicator or “dummy” variables and an interaction term (WISC-IV WMI score x AgeGroup dummy variable). The primary hypothesis tested here was the hypothesis of coincidence. The secondary hypothesis tested was the hypothesis of parallelism.

Overall Math Achievement. In order to test the hypothesis that the variance in overall math achievement explained by variance in working memory declines as age increases, the regression lines of the two age groups (simple regression of the WJ Ach Broad Math score on the WISC-IV WMI score) were compared using an analysis of covariance with dummy variables. The t-scores for both the dummy variable (AgeGroup; $t = -6.11$, $p < .001$) and the interaction term (WMI x AgeGroup; $t = 5.63$, $p < .001$) were significant. Therefore the hypothesis of coincidence was not accepted (See Table 3). These results suggest that the relationship between math achievement and working memory is not the same for the two age groups.

In order to further test the hypothesis that the variance in overall math achievement explained by variance in working memory declines as age increases, the slopes of the regression lines of the two age groups were compared using an analysis of variance. The t-score for the interaction term (WMI x AgeGroup; $t = -1.93$, $p = .06$) was not significant. Therefore the slopes of the regression lines were not statistically different, and the hypothesis of parallelism was accepted (See Table 4). These results do not support the hypothesis that the variance in overall math achievement explained by variance in working memory changes with age.

Fact Fluency. In order to test the hypothesis that the variance in fluency of basic math facts explained by variance in working memory declines as age increases, the regression lines of the two age groups (simple regression of the WJ Ach Fact Fluency score on the WISC-IV WMI score) were compared using an analysis of covariance with dummy variables. The t-scores for both the dummy variable (AgeGroup; $t = -5.44$, $p < .001$) and the interaction term (WMI x AgeGroup; $t = 5.04$, $p < .001$) were significant. Therefore the hypothesis of coincidence was not accepted (See Table 5). These results support the hypothesis that the variance in fluency of basic math facts explained by variance in working memory changes with age.

In order to further test the hypothesis that the variance in fact fluency explained by variance in working memory declines as age increases, the slopes of the regression lines of the two age groups were compared using an analysis of variance. The t-score for the interaction term (WMI x AgeGroup; $t = -1.61$, $p = .11$) was not significant. Therefore the slopes of the regression lines were not statistically different, and the hypothesis of parallelism was accepted (See Table 6). These results do not support the hypothesis that the variance in fact fluency explained by variance in working memory changes with age.

Calculation Skills. In order to test the hypothesis that the variance in math calculation skill explained by variance in working memory declines as age increases, the regression lines of the two age groups (simple regression of the WJ Ach Calculation score on the WISC-IV WMI score) were compared using an analysis of covariance with dummy variables. The t-scores for both the dummy variable (AgeGroup; $t = -3.11$, $p < .01$) and the interaction term (WMI x AgeGroup; $t = 3.00$, $p < .01$) were significant. Therefore the hypothesis of coincidence was not accepted (See Table 7). These results support the hypothesis that the variance in math calculation skill explained by variance in working memory changes with age.

In order to further test the hypothesis that the variance in calculation skills explained by variance in working memory declines as age increases, the slopes of the regression lines of the two age groups were compared using an analysis of variance. The t-score for the interaction term (WMI x AgeGroup; $t = -.31$, $p = .75$) was not significant. Therefore the slopes of the regression lines were not statistically different, and the hypothesis of parallelism was accepted (See Table 8). These results do not support the hypothesis that the variance in overall math achievement explained by variance in working memory changes with age.

Applied Problems. In order to test the hypothesis that the variance in solving practical math problems explained by variance in working memory declines as age increases, the regression lines of the two age groups (simple regression of the WJ Ach

Applied Problems score on the WISC-IV WMI score) were compared using an analysis of covariance with dummy variables. The t-scores for both the dummy variable (AgeGroup; $t = -5.00$, $p < .001$) and the interaction term (WMI x AgeGroup; $t = 4.35$, $p < .001$) were significant. Therefore the hypothesis of coincidence was not accepted (See Table 9). These results support the hypothesis that the variance in skill in solving practical math problems explained by variance in working memory changes with age.

In order to further test the hypothesis that the variance in solving practical math problems explained by variance in working memory declines as age increases, the slopes of the regression lines of the two age groups were compared using an analysis of variance. The t-score for the interaction term (WMI x AgeGroup; $t = -2.96$, $p < .05$) was significant. Therefore the slopes of the regression lines were statistically different, and the hypothesis of parallelism was not accepted (See Table 10). These results support the hypothesis that the variance in applied problem solving skill explained by variance in working memory changes with age.

Among participants aged six to ten years old, 22.6% of the variance in the Applied Problems scores is explained by variance in the WMI scores ($r^2 = .23$, $F(1, 69) = 20.17$, $p < .001$). Among participants aged eleven to sixteen years old, 27.9% of the variance in the Applied Problems scores is explained by variance in the WMI scores ($r^2 = .28$, $F(1, 63) = 24.42$, $p < .001$). These results indicate that the variance in applied problem solving skill explained by variance in working memory increases with age.

Unique Contribution of Working Memory on Math Achievement

In order to test the third hypothesis, that auditory working memory explains variance in math achievement above and beyond the contributions of verbal and nonverbal reasoning and processing speed, a sequential regression was conducted of math achievement (dependent variable) on verbal and nonverbal reasoning, processing speed and working memory (independent variables). Four separate sequential regressions were run, each with a different measure of math achievement as the dependent variable and the participant's WISC-IV VCI, PRI, PSI and WMI standard scores as the independent variables. For each regression, the VCI, PRI and PSI standard scores were entered into the equation first, followed by the WMI standard score.

Overall Math Achievement. It was hypothesized that auditory working memory would predict overall math achievement beyond verbal and nonverbal reasoning and processing speed. Sequential regression supports this hypothesis; an additional 9.0% of the variance in overall math achievement is explained by working memory beyond the variance explained by the other factors, $\Delta R^2 = .09$, $Fchange(1,131) = 25.54$, $p < .001$ (See Table 11). Based on the data presented here, auditory working memory explains unique variance in overall math achievement above and beyond the contributions of reasoning ability and processing speed.

Fact Fluency. It was hypothesized that auditory working memory would predict fluency of basic math facts beyond verbal and nonverbal reasoning and processing speed. Sequential regression supports this hypothesis; an additional 9.3% of the variance in basic fact fluency is explained by working memory beyond the variance explained by the other factors, $\Delta R^2 = .09$, $Fchange(1,131) = 18.56$, $p < .001$ (See Table 12). Based on the data presented here, auditory working memory explains unique variance in fluency of basic math facts above and beyond the contributions of reasoning ability and processing speed.

Calculation Skills. It was hypothesized that auditory working memory would predict math calculation skill beyond verbal and nonverbal reasoning and processing speed. Sequential regression does not support this hypothesis, $\Delta R^2 = .02$, $Fchange(1,131) = 3.57$, $p = .06$ (See Table 13). Based on the data presented here, auditory working memory does not explain unique variance in math calculation skill above and beyond the contributions of reasoning ability and processing speed.

Applied Problems. It was hypothesized that auditory working memory would predict skill in solving practical math problems beyond verbal and nonverbal reasoning and processing speed. Sequential regression supports this hypothesis; an additional 6.2% of the variance in solving practical math problems is explained by working memory beyond the variance explained by the other factors, $\Delta R^2 = .06$, $Fchange(1,131) = 16.35$, $p < .001$ (See Table 14). Based on the data presented here, auditory working memory explains unique variance in skill in analyzing and solving practical math problems above and beyond the contributions of reasoning ability and processing speed.

Discussion

The results of this study replicate and extend findings from the cognitive abilities-organized mathematics achievement research. When integrated with prior research, the current study contributes to an emerging body of knowledge regarding the relationship between working memory and math achievement. While limitations to this study exist, the results have potentially important implications for research and theory examining math achievement as well as related education practices.

The results demonstrate that children's auditory working memory abilities are related to their overall mathematics achievement, calculation skills, fact fluency and applied math skills throughout both elementary and secondary math education. Prior research has shown the importance of other cognitive variables in math achievement, including verbal reasoning (Floyd, Evans & McGrew, 2003), fluid reasoning (Floyd, Evans & McGrew, 2003; Swanson & Beebe-Frankenberger, 2004; Fuchs, et al, 2005) and processing speed (Bull & Johnston, 1997; Floyd, Evans & McGrew, 2003; Swanson & Beebe-Frankenberger, 2004). The results presented here extend this body of research by showing that auditory working memory also makes its own unique contribution to a student's math achievement in the areas of fact fluency and applied math, but not in the area of calculation skills.

Fact Fluency and Working Memory

The speed and accuracy with which a person can solve simple math problems is clearly affected by a student's auditory working memory capacity. This finding is consistent with previous research implicating the dysfunction or alternative functioning of working memory processes in poor fluency of basic math facts (Keeler & Swanson, 2001; Swanson & Sachse-Lee, 2001).

These results are not surprising given the strong link between math difficulties and problems in representing and retrieving math facts from long-term memory (Jordan & Hanich, 2000; Jordan, Hanich & Kaplan, 2003; Joradan & Montani, 1997; Geary, 1993; Geary, Hoard, & Hamson, 1999; Gersten, Jordan & Flojo, 2005; Russell & Ginsburg, 1984; Bull & Johnston, 1997). Long before the math facts can be retrieved from long-term store the associations must be created, a process that is dependent on working memory resources. Before a student has stored a basic math fact, he relies on counting strategies. Each time he counts and adds two numbers together, an association is made between the two addends and the result (one math fact). However, in order for this association to be made, both addends and the result must all be present in working memory simultaneously. If the first and/or second addend fade from working memory as the student is counting, and therefore before the result is known, the association cannot be made or strengthened in long-term memory. This proposed mechanism for the relationship between fact fluency and working memory is consistent with the research showing that the more information is manipulated and organized before storage in long-term memory the easier the information is to retrieve (Mastropieri & Scruggs, 1998).

At first glance, the mechanism described above may seem sufficient in explaining the importance of working memory on fact fluency. However, given that working memory has been shown to be related to both the acquisition and use of basic educational skills (Hitch & McAuley, 1991), it is possible that working memory is also important during the demonstration of fact fluency and the use of basic facts while solving problems. Simply demonstrating knowledge of basic facts may not seem to require substantial manipulation of information retrieved from long-term store, and short-term memory or rote recall has not been found to be related to math achievement. However, arithmetic facts are often presented in an Arabic format but are stored in a verbal format (Noel, et al., 2001). The translation from Arabic to verbal representation, and back again, occurs in working memory. Other functions of working memory may also be important in utilizing math facts. For example, working memory resources are used to activate relevant knowledge in long-term memory, including basic facts (Swanson & Sachse-Lee, 2001).

Most previous studies of math achievement have assessed only one or two cognitive variables, making it difficult to estimate the unique contribution of working memory. However, in order to utilize our findings to generate interventions for poor math achievement and to accurately inform future theory and research, it is necessary to discover which cognitive variables are important enough that a deficit or strengthening in that area will significantly impact achievement independent of other factors. Given the possible explanations for the role of working memory in the development and use of basic math facts described above, it is not surprising that the current study supports the hypothesis that auditory working memory contributes a unique 9.3% of the variance in fact fluency even after the effects of verbal and nonverbal reasoning and processing speed are partialed out. This means that, regardless of a student's reasoning ability or processing speed, the speed and accuracy with which she can solve basic math facts could be improved by either increasing her auditory working memory capacity or making accommodations specific to her working memory difficulty.

Calculation Skills and Working Memory

Given the relationship described above between working memory and fact fluency and that fluent knowledge of basic math facts is essential to reaching a correct solution during complex math calculations, it would be expected that working memory would also be important in the development of calculation skills. In line with previous research (Logie, Gilhooly, and Wynn, 1994; Noel, et al., 2001), the current study found that auditory working memory does explain 14.6% of the variance in math calculation performance. However, when previous research was expanded to determine the unique contribution of working memory, the current results indicate that auditory working memory is not important to math calculation above and beyond the contributions of reasoning abilities and processing speed.

One explanation for these results could be that visual-spatial working memory is more involved in calculation skills. The WISC-IV Working Memory Index very clearly measures auditory working memory; the results of this study can only point to the contributions of auditory or verbal working memory to math achievement. Therefore, while auditory working memory may not be uniquely important, visual working memory or the visuo-spatial sketchpad of Baddeley and Hitch's (1974) model of working memory could be necessary in the development of calculation skills. Studies investigating the contribution of visual working memory to math achievement have been mixed; several studies provide evidence for the importance of the visual spatial sketchpad in children's arithmetical computations (Dark & Benlow, 1990; Swanson & Lee, 2001; McLean & Hitch, 1999), while others suggest that visual working memory does not contribute to calculation skills above verbal working memory (Hitch & McAuley, 1991; Logie, Gilhooly, and Wynn, 1994; Noel, et al., 2001; Wilson & Swanson, 2001). None of the previous research, however, has investigated the unique contribution of visual-spatial working memory on math calculation skills above and beyond reasoning ability and processing speed. Clearly further investigation is needed in this area.

Discovering the cognitive variables underlying calculation skills is especially important because the most common problem that differentiates children with math difficulties from children with other academic difficulties is their ability to solve multi-step problems. The results of the current study seem to suggest that higher reasoning ability or faster processing speed can, in essence, make up for poor auditory working memory when it comes to performing complex calculations. Along the same line, poor reasoning ability or slow speed of processing may overtax adequate working memory resources.

The results of Bull and Johnston (1997) suggest an explanation; they found that math achievement is limited by slow speed of number identification or retrieval of number representations from long-term memory. Given that number retrieval is slow, the same could be true for symbol identification and retrieval of learned procedures needed for completing calculations. In fact, Swanson & Sachse-Lee (2001) showed that children with learning disabilities are deficient in retrieving knowledge related to algorithms. If during a novel calculation an inappropriately large amount of cognitive resources are spent accessing representations of numbers, symbols and procedures from long-term store or determining which procedure is needed, fewer resources are available for storage of new information. Therefore, during novel calculations designed to reinforce a new concept being taught, the new procedure is not learned or strengthened in long-term memory. This explanation suggests that difficulties with calculation skills are not due to a deficit in the ability to manipulate information in working memory, but rather to difficulties encountered while trying to encode the procedure.

Applied Problems and Working Memory

Results of the current study are consistent with previous research indicating that working memory is important in the solution of word problems (Passolunghi & Siegel, 2001; Swanson & Beebe-Frankenberger, 2004; Swanson & Sachse-Lee, 2001; Russell & Ginsburg, 1984). The current results extend the relationship between working memory and applied problem solving to include utilizing graphs, making measurements, and time and money concepts. Also, auditory working memory was found to make a unique contribution to the solution of everyday math problems; working memory explained a unique 6.2% of the variance in applied problem solving above the contributions of reasoning abilities and processing speed.

The involvement of verbal or auditory working memory in the solution of applied math problems may be complex, involving many of the theorized functions of working memory. As mentioned above, working memory facilitates the access of needed information from long-term memory. In addition, the problem itself introduces a substantial amount of new information that must be held, interpreted and manipulated in working memory. The relationship between reading and working memory has been well-established (Lerner & Kline, 2006), and therefore problems that require reading, such as word problems and, to a lesser extent, problems involving the interpretation of graphs, will rely on the working memory system.

Applied problems often contain extraneous information and the correct algorithm or procedure is not always readily apparent. One function of working memory is the suppression of irrelevant information, which has been found to be related to math achievement and disabilities (Passolunghi, Cordnoldi & De Liberto, 1999; Passolunghi & Siegel, 2004; Passolunghi & Siegel, 2001; Russell & Ginsburg, 1984). Children with high math achievement have also been found to utilize more diverse strategies when problem-solving. However, in order to actively choose from multiple strategies, weighing the pros and cons of each, and select the most appropriate procedure, working memory resources are needed. Given all the functions of working memory and the multiple facets of applied problems it is not surprising that working memory would play such an important role.

Math Achievement and Working Memory: Effect of Age

The variance in overall math achievement, fact fluency, and calculation skills was not significantly different between the two age groups studied here. These results are consistent with and expand on studies that have also found a stable relationship between working memory and math achievement across age but have either limited their age range to elementary (Swanson & Beebe-Frankenberger, 2004) or secondary school children (Dark & Benbow, 1990), or to a specific area of math achievement, such as calculation (Wilson & Swanson, 2001). The current study provides evidence for the theory that both elementary and secondary level math achievement rely on auditory working memory to the same degree.

An alternative explanation for these results could be that the tasks used to represent math achievement did not adequately test the range from elementary to secondary level math. For example, the nature of basic math facts and fluency do not change as students progress through grade levels; rather, students are simply expected to complete a greater number of problems accurately in a given time period. While the calculation tasks tapped into secondary level math concepts, such as algebra and geometry, performance on these tasks continued to be dependent on early math skills.

Students' achievement at a secondary level is affected by prior achievement (Jones, 1997). Therefore, a student's working memory impairment could relate to secondary math achievement through the earlier relationship between working memory and basic math achievement. That is, given the results of the current study, a student with low working memory abilities would be expected to demonstrate low math achievement in the early grades; this low achievement in the early grades would then impact math achievement in later grades, regardless of working memory capacity at that time.

One final explanation for these results must be posited: that visual-spatial working memory becomes more important in completing and understanding higher level, and more complex, math problems. Some aspects of secondary math, such as algebraic word problems, are more reading intensive than elementary level math problems. Others are more visually complex and may require the manipulation of spatial features, such as understanding fractions and geometry. Therefore, while auditory working memory seems to be equally important in most areas of elementary and secondary level math, the contribution of visual-spatial working memory remains unknown.

The results of the current study indicate that working memory also continues to be related to applied problem solving skills through the later grades; however, the relationship between these two constructs increased across age. Therefore, auditory working memory may play a greater role in a student's ability to solve applied problems in the later grades. This finding is consistent with the increased complexity of problems in late elementary and secondary level math. For example, as noted above, word problems may become more reading intensive in secondary level math.

Also, secondary applied math problems may require the simultaneous use of a larger number of skills than would be needed for elementary level problems. Elementary level applied problems may involve choosing the correct operation out of four (addition, subtraction, multiplication and division), knowledge of basic math facts and completing multi-digit calculations. Secondary applied math problems often involve more numbers, terms and irrelevant information, and require the student to sort through a greater number of possible algorithms (including algebraic equations and geometry postulates) in addition to retrieving basic math facts and completing the complex calculations. Therefore, secondary level applied problems may require the student to hold and manipulate a significantly greater amount of information in working memory than elementary level problems.

Additional Theoretical Implications

The results of this study have clearly shown the importance of auditory working memory as an underlying process involved in the learning and/or demonstration and use of math skills. However, the results further support previous research and theory in cognitive psychology indicating that there are many cognitive processes involved in learning, and that the dysfunction or alternative functioning of any of these processes would lead to difficulties in learning. Many diagnosticians have focused on students' performance patterns on cognitive tests in order to diagnose learning disabilities (Bannatyne, 1974; Kaufman & Lichtenberger, 2002; Kamphaus, 2001). These models assume that particular learning disabilities (i.e., reading, mathematics or written language) are the result of a specific pattern of neuropsychological deficits. However, research has repeatedly shown that these patterns do not consistently identify children with specific learning disabilities. D'Angiulli and Siegel (2003) showed that some children with learning disabilities demonstrated the predicted patterns, but at least 65% of the diagnosed children did not. Watkins, Kush and Schaefer (2002) showed that the WISC-III Learning Disability Index (LDI) exhibited low diagnostic accuracy, resulting in a correct diagnostic decision only 55 to 64% of the time. In addition, the ACID profile and SCAD profile have also been shown to have little or no diagnostic utility (Watkins, Kush, and Glutting, 1997a; Watkins, Kush and Glutting, 1997b). This research indicates that, while children with learning disabilities do have cognitive deficits, no one pattern is enough to account for all learning difficulties, even when separated into disorders of reading, written expression and arithmetic.

Results of the current study have shown that not all math difficulties are created equal. Calculation skills, and fact fluency and applied problem solving are differentially affected by auditory working memory abilities. This supports the theory that different underlying cognitive processes are at work in developing different math skills. The results add to the evidence that learning disabilities in general, and math disabilities specifically, are heterogeneous; one child with a math disorder may have a different pattern of cognitive strengths and weaknesses than another child with a math disorder. Therefore, while math learning disabilities may represent an extreme end of the continuum of math achievement (Geary, 1993), current results suggest, instead, that disorders in math represent the extreme ends of several continuums of math achievement, one for each math skill or domain.

Furthermore, while differences in auditory working memory uniquely explained some of the variance in fact fluency and applied problem-solving, working memory did not come close to explaining all of the variance in these skills. Therefore, even in the areas of fluency and applied problems, other cognitive processes likely contribute to math achievement. Given that there were decreases in the variance explained by working memory when verbal and nonverbal reasoning and processing speed were included in the regression equations, some combination of these cognitive variables are likely important in the three areas of math achievement studied here. This is consistent with prior research (Bull & Johnston, 1997; Floyd, Evans & McGrew, 2003; Swanson & Beebe-Frankenberger, 2004; Fuchs, et al, 2005). Many other elements of information processing are also linked to mathematics, including attention, visual-spatial perception and processing, auditory processing, memory and retrieval, and motor skills (Wilson & Swanson, 2001; Gersten, Jordan, and Flojo, 2005; Dowker, 2005; Kroesbergen, Van Luit, and Naglieri, 2003). Poor understanding of language and low reading abilities can also negatively influence math learning (Fletcher, 2005; Lerner & Kline, 2006).

Practical Implications

Results of the current study have several implications for practices in math education and in diagnosis of math learning disabilities. Given the vast number of cognitive processes and other variables that are important in learning math, it seems clear that one or two general interventions are not adequate to help overcome or compensate for all math difficulties. In order to better alleviate a student's unique difficulties in learning, interventions must be tailored to his/her combination of cognitive strengths and weaknesses.

However, the current research suggests that strategies that boost or facilitate the use of auditory working memory resources would be helpful in improving math achievement for some students, particularly in the areas of basic fact fluency and applied problems. It may be helpful to explicitly teach the student specific memory strategies and how to recognize the most useful strategy to use in a variety of situations. Examples include using verbal rehearsal, chunking, making ridiculous visual images composed of items that one has to remember, and creating first-letter mnemonic strategies. Understanding also facilitates memory, and ensuring that a student has a strong understanding of underlying concepts before learning new math skills is important. The stronger understanding a student has of these related concepts, the more working memory resources he will have available to make associations, thereby facilitating the encoding of new information in long-term memory.

Other strategies can be used to accommodate for weaknesses in auditory working memory. When learning basic math facts to automaticity, for example, encoding can be facilitated by providing both visual and verbal representations of the problem as the student is counting. In this way, early in the learning process the student would not have to utilize working memory resources translating the problem from Arabic to verbal representation. Providing visual and verbal representations of applied or word problems would also be beneficial. This may include verbal descriptions of graphs and charts or pictures that depict word problems. When learning new math procedures and solving novel problems, students with weaknesses in auditory working memory may also benefit from having a visual or written description of the necessary steps (examples, lists or flow charts) available in front of them.

Results of the current study indicate that strategies for facilitating weaknesses in auditory working memory would also be beneficial for students learning math at the secondary level. In order to reduce the amount of working memory resources needed, the use of a calculator or fact chart would be helpful when the focus of instruction is on learning higher level procedures and operations. When a student has difficulty organizing applied problems that are complex or involve many variables, she may reduce working memory demands by constructing a similar problem with fewer variables or substituting lower values and solving it. She may then solve the more complex problem by adapting the solution or procedure she used for the simpler problem.

These strategies may be helpful for improving students' calculation skills. However, given that the results of the current study show that auditory working memory explains negligible variance in calculation skills above and beyond other cognitive variables, time and effort would most likely be better spent employing strategies that improve or accommodate for relative weaknesses in verbal or nonverbal reasoning, processing speed, or other variables found to be related to these skills.

Finally, a relative weakness in auditory working memory may be one example of a cognitive process deficit that might be used to identify a student as having a math learning disability. However, given that working memory does not explain all variance in math achievement and other cognitive processes are necessary, adequate or above average working memory abilities should not be used to rule out a math learning disability diagnosis.

Limitations

The research based on archival data presented here has several limitations, most related to the lack of information collected and recorded in client files or the power restrictions of a small sample size. Participants were not a random sample of the population. Instead, all participants were originally referred for cognitive and academic testing by their parents. An assumption can be made that, according to their parents, the children included in this study were all struggling, either academically or emotionally. These difficulties could have included math achievement, but may have also involved reading or writing difficulties, attention or impulsivity, or emotional issues. The inability to control for these individual differences that were unrelated to the purpose of the current study may have confounded the results in several ways.

Math difficulties and learning disabilities in general have been found to be significantly comorbid with Attention-Deficit Hyperactivity Disorder (ADHD) (Semrud-Clikeman, Biederman & Sprich-Buckminster, 1992; Lerner & Kline, 2006). Significant memory differences have been found between students with ADHD and comorbid ADHD and LD; specifically, there is evidence that memory deficits in children with both ADHD and learning difficulties are significantly greater than those found in children with only ADHD (Jakobson & Kikas, 2007). Math difficulties are also often correlated with reading difficulties (Lerner & Kline, 2006; Mazzocco & Myers, 2003). Swanson & Jerman (2006) found that children with only a math disability demonstrated greater verbal working memory than children with both math and reading disabilities.

A full scale or overall IQ of participants was not included in the current analysis, and therefore it is possible that the clinic sample used in this study may have also included children with a mild impairment in overall intellectual functioning. Several studies have provided evidence that children with general intellectual impairments demonstrate relative weaknesses in verbal working memory (Rosenquist, Conners & Roskos-Ewoldsen, 2003; Lanfranchi, Cornoldi & Vianello, 2004; Purser & Jarrold, 2005; Silverman, 2007). In addition, math anxiety also has a significant impact on math achievement for some children (Miller & Bichsel, 2004; Lerner & Kline, 2006). Given that the tasks used to measure working memory in this study involved memory for numbers, these participants could have performed more poorly on the WISC-IV WMI due to their anxiety, rather than a true cognitive deficit. These math anxious participants would then have low performance on both achievement and working memory measures, possibly inflating the relationship between the two constructs. Therefore, the possible inclusion of children with various attention, learning and anxiety problems or mild intellectual impairments may have influenced the results of the current study.

While results of the current study provide preliminary evidence that the relationship between working memory and math achievement may be similar between males and females, too few female participants were included to reach a power level necessary to make a clinically significant statement about gender. Results of prior research investigating the difference between males' and females' performances on working memory tasks have been mixed. Robert and Savoie (2006) and Rucklidge and Tannock (2002) found no significant gender differences in verbal or visual-spatial working memory. In contrast, Sutcliffe, Marshall & Neill (2007) showed that female rats and rats with higher levels of female hormones performed better on a measure of general working memory ability. Still others have found that males perform better on separate visual and spatial working memory tasks (Geiger & Litwiller, 2005; Cattaneo, Postma & Vecchi, 2006; Sutcliffe, Marshall & Neill, 2007) or verbal working memory tasks (Geiger & Litwiller, 2005).

Due to the inability to identify the race of 103 out of 136 participants, analysis of ethnic differences in working memory, math achievement and the relationship between the two constructs was not possible. Also, the inability to make statements regarding whether or not the participants adequately mirrored the ethnic distribution of the general population compromised the generalizability of the results of this study. This is particularly important in light of research that suggests that ethnicity significantly predicts variance in performance on some working memory tasks (Diehr, Heaton & Miller, 1998).

Another confounding variable in the current study may have been the use of medications by the participants. This aspect may have been influenced by the examiners at the time of testing based on the referral question but was not able to be controlled by the current experimenters. For example, if the assessment was being conducted to test for ADHD a client may have been asked to refrain from taking any medication prescribed for inattention or hyperactivity but if the referral question involved a possible learning disability the client may have been asked to remain on medication in order to control for symptoms related to ADHD or other medical disorders. Research has found that when children with ADHD are taking stimulant medication they show improvement in spatial (Kempton, Vance & Maruff, 1999; Barnett, Maruff & Vance, 2001) and general working memory abilities (Frank, Santamaria & O'Reilly). Arnsten (2006) found that the relationship is more complicated; she found that working memory in children with ADHD is improved with low doses of a stimulant but is impaired with high doses of the same medication. To complicate the possible effects of medication even more, medications used to treat other medical conditions may have differential effects; for example, Lee, Jung and Suh (2006) found that the working memory of epilepsy patients was impaired with the use of seizure medication.

One final confounding variable in the current study may be in the tasks used to measure working memory. The tasks making up the WISC-IV WMI involve the manipulation of digits (Digit Span) or digits and letters (Letter Number Sequencing). It may be that some other cognitive factor involved in processing numerals or numeric information is affecting performance on both the working memory tasks and the math tasks. A deficit in this other area of processing would result in lower scores on both the WMI and the math achievement tasks, thereby inflating the variances in math achievement explained by working memory found in the current study. Future studies could explore this possibility by including measures of auditory working memory that involve words rather than numbers. The results of the current study should be thought of as describing the relationship between math achievement and numerical auditory working memory.

Future Directions

While the possibly confounding variables presented above limit the generalizability of the current findings, given the emerging nature of research in this area, they do not reduce the importance and utility of the results in guiding future research in the area of math achievement and working memory. The current results, together with previous research, indicate the importance of continuing to study the relationship between working memory and math achievement, specifically basic fact fluency and applications.

In particular, it will be important to identify the contribution that auditory working memory makes to the learning of math concepts compared with the contribution it makes to the demonstration or use of these skills. Making this distinction and clarifying the role of working memory will help to determine which strategies are most useful and efficient in math education, further narrowing the search for interventions before students find success.

As noted above, the involvement of verbal or auditory working memory in the solution of applied problems may be complex and involve many of the theorized functions of working memory, including the manipulation of information, activation of information in long-term store, inhibition of irrelevant information and maintenance of new information. This, together with the fact that there are many different types of applied problems, including word problems and problems involving charts, graphs, and time and money concepts, suggests that further study is needed in this area. Investigation into which functions of auditory working memory are most important for which types of applied problems would again help to further refine the search for interventions.

The fact that auditory working memory does not seem to underlie achievement in math calculation indicates the need for additional research. What underlying cognitive processes are important in the development of calculation skills? In the current study the use of verbal and nonverbal reasoning abilities and processing speed to determine that verbal working memory does not contribute unique variance suggests a few starting points. Given the inconsistent results of previous research, the role of visual and spatial working memory in math calculation should continue to be investigated. Evidence of a relationship between general math achievement and other factors, including attention, visual-spatial perception and processing, auditory processing, retrieval from long-term memory, and motor skills, implicates these areas as additional possibilities for underlying processes in the development of math calculation skills.

The investigation into the relationship between working memory and secondary level math education is still new. While the results of the current study suggest that the contribution of auditory working memory to math achievement continues from elementary to secondary level math, this relationship requires further dissection. First, prior achievement affects achievement at a secondary level. Therefore, further research is needed to determine whether the continued relationship between auditory working memory and secondary level math found in the current study is moderated by the earlier relationship between working memory and basic math achievement. This could be examined by controlling for the impact of early math achievement while studying the relationship between working memory and math in a longitudinal study. Second, visual-spatial working memory may become more important as math concepts become more complex. Therefore, while there is preliminary evidence that visual-spatial working memory does not contribute to math achievement at an elementary level, the contribution to secondary math achievement should be explored.

Finally, the limitations of the current study mentioned above suggest directions for future research in this area. First, to improve the generalizability of the current results, comparisons of the relationship between working memory and math achievement in different ethnic groups and genders should be investigated. Second, in future studies, the effects of medications on this relationship should not only be controlled for but also examined.

Final Thoughts

Mathematics and problem-solving ability is becoming more important in our society. Yet many of our students struggle with these concepts from elementary through secondary school because of some form of specific memory or cognitive deficit. Math achievement, and in particular its relationship to cognitive processing, has only just begun to be examined. The results of this study shed light on one area of processing by providing evidence for the differential effects auditory working memory has on basic fact fluency, applied math, and calculation skills. The relationship found between auditory working memory and math achievement continued from the elementary to the secondary level, further highlighting the need for additional research into the processes underlying the development of more complex math skills. Continued research in this area will benefit not only the individual children struggling to learn and utilize math skills but society, as more people will be able to use this symbolic language that helps us to record and communicate information and ideas.

Appendix

Table 1: *Descriptive Statistics of WMI and WJ Ach measures (ages 6 – 10:11)*

Variable	Minimum	Maximum	Mean (SD)
WMI	71	129	102.39 (12.06)
Broad Math	65	148	109.06 (14.91)
Calculation	55	148	104.83 (16.84)
Fact Fluency	49	129	97.10 (16.12)
Applied Problems	70	148	111.41 (15.31)

Table 2: *Descriptive Statistics of WMI and WJ Ach measures (ages 11 – 16:11)*

Variable	Minimum	Maximum	Mean (SD)
WMI	62	123	97.29 (12.76)
Broad Math	72	138	101.52 (15.31)
Calculation	69	146	102.51 (15.06)
Fact Fluency	57	134	90.22 (16.11)
Applied Problems	62	136	101.71 (14.39)

Table 3: *Significance Testing of the Test of Coincidence (Broad Math)*

Source	SS	Df	MS	*F*
Regression	7799.40	1	3899.70	21.01*
Error	24682.33	133	185.58	
Total	32481.74	135		

*$p < .001$

Table 4: *Significance Testing of the Test of Parallelism (Broad Math)*

Source	SS	Df	MS	*F*
Regression	876.05	1	876.05	3.71
Error	31605.68	134	235.86	
Total	32481.74	135		

Table 5: *Significance Testing of the Test of Coincidence (Fact Fluency)*

Source	SS	Df	MS	*F*
Regression	7181.87	2	3590.93	16.35*
Error	29215.16	133	219.66	
Total	36397.03	135		

*$p < .001$

Table 6: *Significance Testing of the Test of Parallelism (Fact Fluency)*

Source	SS	Df	MS	*F*
Regression	687.62	1	687.62	2.58
Error	35709.41	134	266.49	
Total	36397.03	135		

Table 7: *Significance Testing of the Test of Coincidence (Calculation)*

Source	SS	Df	MS	*F*
Regression	2360.02	2	1180.01	4.88*
Error	32185.36	133	242.00	
Total	34545.38	135		

*$p < .01$

Table 8: *Significance Testing of the Test of Parallelism (Calculation)*

Source	SS	Df	MS	*F*
Regression	25.46	1	25.46	0.10
Error	34519.92	134	257.61	
Total	34545.38	135		

Table 9: *Significance Testing of the Test of Coincidence (Applied Problems)*

Source	SS	Df	MS	*F*
Regression	6892.32	2	3446.16	17.65*
Error	25961.62	133	195.20	
Total	32853.93	135		

*$p < .001$

Table 10: *Significance Testing of the Test of Parallelism (Applied Problems)*

Source	SS	Df	MS	*F*
Regression	2009.70	1	2009.70	8.73*
Error	30844.24	134	230.18	
Total	32853.93	135		

* $p < 0.05$

Table 11: *Significance Testing of the Overall Model (Broad Math)*

Variable	β	sR^2
Step 1		
VCI	.54***	.29
R^2	.29	
F(1,134)	54.17***	
Step 2		
VCI	.26**	.04
PRI	.41***	.09
R^2	.38	
F(2,133)	40.93***	
ΔR^2	.09	
Fchange(1,133)	20.01***	
Step 3		
VCI	.25**	.03
PRI	.33***	.06
PSI	.27***	.07
R^2	.45	
F(3,132)	35.46***	
ΔR^2	.07	
Fchange(1,132)	15.56***	
Step 4		
VCI	.17*	.02
PRI	.26**	.03
PSI	.16*	.02
WMI	.36***	.09
R^2	.54	
F(4,131)	37.93***	
ΔR^2	.09	
Fchange(1,131)	25.54***	

* $p < .05$, ** $p < .01$, *** $p < .001$

Table 12: *Significance Testing of the Overall Model (Fact Fluency)*

Variable	β	sR^2
Step 1		
VCI	.21*	.04
R^2	.04	
F(1,134)	5.96*	
Step 2		
VCI	.08	.003
PRI	.19	.02
R^2	.06	
F(2,133)	4.51*	
ΔR^2	.02	
Fchange(1,133)	2.98	
Step 3		
VCI	.05	.001
PRI	.06	.002
PSI	.46***	.19
R^2	.25	
F(3,132)	14.62***	
ΔR^2	.19	
Fchange(1,132)	32.69***	
Step 4		
VCI	-.03	.0005
PRI	-.01	.00006
PSI	.35***	.09
WMI	.37***	.09
R^2	.34	
F(4,131)	17.07***	
ΔR^2	.09	
Fchange(1,131)	18.56***	

* $p < .05$, ** $p < .01$, *** $p < .001$

Table 13: *Significance Testing of the Overall Model (Calculation)*

Variable	β		sR^2
Step 1			
VCI	.45***		.20
R^2		.21	
$F(1,134)$		34.49***	
Step 2			
VCI	.23*		.03
PRI	.33**		.06
R^2		.27	
$F(2,133)$		23.93***	
ΔR^2		.06	
Fchange(1,133)		10.85**	
Step 3			
VCI	.23*		.03
PRI	.30**		.05
PSI	.10		.01
R^2		.27	
$F(3,132)$		16.56***	
ΔR^2		.01	
Fchange(1,132)		1.60	
Step 4			
VCI	.19		.02
PRI	.27*		.04
PSI	.05		.002
WMI	.17		.02
R^2		.29	
$F(4,131)$		13.55***	
ΔR^2		.02	
Fchange(1,131)		3.57	

* $p < .05$, ** $p < .01$, *** $p < .001$

Table 14: *Significance Testing of the Overall Model (Applied Problems)*

Variable	β		sR^2
Step 1			
VCI	.48***		.23
R^2		.23	
$F(1,134)$		39.03***	
Step 2			
VCI	.07		.003
PRI	.61***		.20
R^2		.43	
$F(2,133)$		50.09***	
ΔR^2		.20	
Fchange(1,133)		47.59***	
Step 3			
VCI	.07		.002
PRI	.57***		.17
PSI	.10		.01
R^2		.44	
$F(3,132)$		34.46***	
ΔR^2		.01	
Fchange(1,132)		2.26	
Step 4			
VCI	.001		.000001
PRI	.52***		.14
PSI	.01		.0001
WMI	.30***		.06
R^2		.50	
$F(4,131)$		32.94***	
ΔR^2		.06	
Fchange(1,131)		16.35***	

* $p < .05$, ** $p < .01$, *** $p < .001$

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