

Who Goes First?  
An Examination of the Impact of Activation on Outcome  
Behavior in Agent-based Models

A dissertation submitted in partial fulfillment of the requirements for the degree of  
Doctor of philosophy at George Mason University

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Spring Semester 2014  
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## **Dedication**

This work is dedicated to wife and my life's partner, Robbie Comer, without whom my list of accomplishments would be meager, my horizons bounded, my vision clouded, and the dissertation that follows would not be possible.

# Acknowledgments

The author would like to acknowledge the generous and unflinching support of the faculty of the Systems Engineering Operations Research department, and especially my advisor, counselor, fellow veteran, and friend, Professor Andrew Loerch. Equally important to the success of this multidisciplinary effort has been the guidance and encouragement of the Department of Computational Social Sciences and its chairman, Dr. Robert Axtell

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# ABSTRACT

WHO GOES FIRST?

AN EXAMINATION OF THE IMPACT OF ACTIVATION ON OUTCOME  
BEHAVIOR OF APPLIED AGENT-BASED MODELS

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George Mason University, 2014

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In the design process of an agent-based model the pattern chosen for the activation of the agents is an important choice. Every model design must include – either explicitly or implicitly – the conditions under which each agent will call its methods and update its state. Often, however, this is not described in literature and some model designers do not even make this design decision explicitly. Three agent-based models described in the literature in three separate domains were replicated and the impact of various activation schemes on the emergent population patterns and dynamics was analyzed. It was demonstrated that the choice of activation type is important for the outcome behavior of the model and should be stipulated in any published description of an agent-based model. In some experiments the differences noted, while significant, were only statistical. In others they led to substantial differences in either outcomes or model behavior. Further investigation showed that sophisticated activation schemes can become powerful tools to

produce unexpected or unpredicted behavior of multi-agent systems. Thus, activation becomes more than an inconvenient detail to be dealt with during design, and is shown to be a source of exploratory variation as modelers of self-organizing social systems seek to match the behavior of natural systems.

# **Background and Introduction**

Complexity – the science of complex adaptive systems – has emerged as a dynamic new approach to managing systems and making rational, theory-based decisions in the face of natural, grown-in-place systems. Complexity theory has, over the past two decades, developed a theoretical understanding and mathematical deconstruction of emergent behavior in a broad range of sciences, including biology, neurology, anthropology, epidemiology, sociology, and physics (Mitchell, 2009). In addition, complexity theory has helped develop an understanding of the weaknesses of traditional, equation-based approaches and has begun to fill a role as a complementary decision aid in a wide range of business and government applications such as marketing, finance, economics, organizational design, social policy (Joshua M. Epstein & Axtell, 1996; Gilbert & Troitzsch, 2005; Johnson, 2009; Miller & Page, 2007), and military operations (Henscheid, Koehler, Mulutzie, Tivnan, & Turnley, 2010; Ilachinski, 2004; North & Macal, 2005).

To date, however, complexity theory has focused on observational science, mostly cataloguing relationships between individual-scale rules, patterns, and behavior and large-scale emergent patterns, structures, and entities. To become useful, to establish the type of value that operations research has provided to decision-makers, for example,

complexity needs to be able to move from the theoretical and descriptive to the practical and actionable, generating statements about entities in the real world. As complexity has been most effective at bridging the conceptual gap between small and large scale, the logical place for complexity science would be recommendations for interventions at the individual scale that have the best likelihood for generating favorable outcomes at larger scales.

Simulation is the primary tool of understanding complex adaptive systems – to explore their behavior, to observe correlations and cause-and-effect relationships (or show they do not exist!), to test hypotheses and assumptions. Complex systems – economies, societies, wars – normally exist on such a large scale that repeated experimentation on real-world systems is rarely possible. When more traditional methods of describing system behavior such as system dynamics (based on differential equations) fail, then complexity theory can help understand the impact of neighborhoods and underlying connections (social networks, for example) on overall system evolution. And, agent-based simulation is the tool used to conduct this exploration. In the words of one pioneer in the field, “If you didn’t grow it, you didn’t explain it.” (J. M. Epstein, 2006, p. xii).

With the advent of object-oriented programming in the 1990s, it has become easier to create simulations that treat the individual agents as objects, and to infuse them with this adaptivity. Such simulations have been called “agent-based models” (ABMs). Just as the definition of complex systems is difficult to establish, so is it difficult to determine the boundaries that distinguish an agent-based model from other object-oriented programs

and simulations. A clear trend in the literature and the practice, however, is to closely connect agent-based models and complexity. That is, when the behavior of the system, as expressed in its macro-level parameters, depends strongly on the interactions among and local environment of the individual components, then the science under study is ‘complexity’ and the tool is called ‘agent-based’.

As a young field, complexity science has not clearly established its definitions, and many of the terms used above are not part of a widely accepted taxonomy. It is important to examine the implications of proceeding without established definitions. The ‘essence’ issue has been the topic of philosophers for centuries. John Locke’s *An Essay Concerning Human Understanding* elaborates on the question of whether one must define things to understand them. Locke even delved into the question of complex ideas, wandering notably close to the questions addressed in modern complexity theory (Locke, 2007). Here, however, I pass over all the elegant philosophy to simply state that human history is filled with examples where *engineering* preceded *science*. We have worked substantial improvements and scored stunning advances before we could understand the underlying science or even define the phenomena. The Romans built beautiful archways long before the science of structural mechanics was developed. Edison and Westinghouse electrified entire cities in the late 19<sup>th</sup> century, decades before the discovery of the electron. Biologists have created libraries of textbooks and a titanic multi-branched field of science without an accepted working definition of life itself.

Similarly, much of the exploration of complex adaptive systems *management* is empirical. Relationships, boundaries, and scope of various phenomena are under exploration. A recent article in the Journal of Artificial Societies bemoans the fact that researchers rarely attempt to replicate one another's models and simulations (Wilensky & Rand, 2007). They note that such practices are critical to the advancement of science and represent core practices of any field.

Agent-based models have successfully modeled the behavior and structure of a wide range of systems. An early example was the successful simulating of the spread of disease – particularly AIDs. A wide range of successful applications followed during the 1990s, including anthropology, stock market behavior, crowd dynamics, traffic patterns, technology adaptation, tax evasion, crime proliferation, revolt and insurgency, management of ecosystems, public policy, and economics. A search of Amazon reveals over 2,000 books that have been published that refer to “agent-based models”, and over 1,000 articles can be found in Google Scholar published since 2009 with “agent-based model” in the title.

Agent-based models have become compelling tools for decision sciences because they (alone, in many instances) have been able to replicate the qualitative behavior of real-world complex systems (S. E. Page, 2005), and at times they can match quantitative outcomes as well (R. L. Axtell, 2001). That is, the patterns and relationships that are observed in the real world can be replicated in the agent-based system. In some cases, this similarity can be shown to pass traditional statistical tests, such as rejecting the

hypothesis that the ABM data were the product of random fluctuations. Additionally, this is often a result that cannot be achieved using traditional models where, for example, populations are treated parametrically and not as populations.

Despite their verisimilitude, it is difficult to prove that agent-based models are valid (Gilbert & Troitzsch, 2005, p. 22). That is, the similarity between the model behavior and the real world could be artificial. The model might not respond to stimuli, to change, or to intentional intervention in the same way that the real world would. Thus, an extensive discussion has taken place in the literature of artificial societies on the question of *epistemology*. That is, how do we know we have found the truth? (Raubal, 2001; Banks, Lempert, & Popper, 2002; Guzy, Smith, Bolte, Hulse, & Gregory, 2008)

Agent-based models are not the only simulations that must deal with difficult validation issues. This has happened to computational approaches for a wide range of reasons. Many models, however, encompass a domain so large that running experiments to collect data is not a feasible solution. Other models envision systems that defy experimentation, such as combat or crime. Still other modeling efforts address issues in the domain of observational science, such as astronomy.

Even though the validation process is unclear and undeveloped for agent-based models, there are still some facets of that process that are certain to emerge as constituents of an overall validation formalism. One of these is the concept of a *taxonomy* of agent-based models. This would be more than just a classification scheme: A working taxonomy would have to accommodate the broad scope of agent-based models across the

## NetLogo

NetLogo is an agent-based modeling environment developed by Northwestern University's Center for Connected Learning and Computer-based Modeling. It incorporates weak-typed variables in a high-level language that operates on the Java virtual machine. NetLogo features a robust body of documentation and tutorials, and includes a large library of agent-based models across a broad spectrum of domains (biology, chemistry, earth science, social science, etc.). A large user community has grown up over the past decade in both academia and industry. NetLogo is often the entry-point for those interested in modeling complexity.

While many researchers conduct all of their investigations in NetLogo, it is rarely cited in the literature. Opinions differ as to the reason for this; it may simply be a matter of perception – some refer to NetLogo models as “toy models”. The NetLogo architecture does result in slow execution when the number of agents gets large.

wide range of domains. Moreover, it would have to include a rule set for such classifications. On the whole, such a system would require an extensive and detailed review of the entire model life cycle, from conceptualization to validation.

One of the first decisions that agent-based model builders must make, and a key component of such a taxonomy, is the process by which agents are *activated*. This means, in the implementing computer code, at what point in the algorithm (and at what step in the program) will the agents' internal

methods be called? All builders of agent-based models must make a decision about this process. In many instances, this decision is not made explicitly. Some environments – NetLogo, for example – establish an activation pattern within a single high-level command (“ask”) (Wilensky, 2013). Alternative activation patterns can be programmed manually, but the model developer must make a conscious effort to do so.



It is important to focus on early design decisions in the modeling process. It has been long recognized in the operations research community that the greatest learning and discovery takes place during this phase of model-building (Simon, 1996, p. 13). Unfortunately, in the long journey from design concept to published research these early decisions are often long forgotten. Thus, even if the researcher did tradeoff studies about different activation patterns; such information is lost -- possibly existing only in the researcher's research notes, documentation, or early code.

There are several activation schemes. Initially, only *synchronous* or *asynchronous* activation schemes (defined below) were considered. For programming applications before the advent of object-oriented languages, synchronous activation was the easiest to develop and the most straightforward algorithm to code. To achieve it, an array of entities would be interrogated one at a time in a loop, where the loop index is the array element number. Once the decisions have been recorded, the entire population would undergo a simultaneous state-change.

Many early models, and, in fact, some that are broadly accepted today, use an unstated and simplistic activation process. NetLogo, as noted above, uses a turn-based activation in which the order of agent activation is shuffled in each turn. This can be changed in NetLogo, but it requires intentional modification of the code. Moreover, the default activation process has changed as NetLogo has been modified and updated since it was first deployed in 1999. Most important, however, is that the documentation of the NetLogo model library – those models submitted to the NetLogo sponsor by researchers

across the domains – shows that most model builders do not explore different activation schemes. In fact, it is most likely that a NetLogo builder is unaware that he or she has chosen a specific activation scheme by using the command structure of the NetLogo program.

Activation, sometimes referred to as ‘updating’, was identified as early as 1993 as an important determinant of model behavior and emergent patterns. A 1992 article on “Evolutionary Games and Spatial Chaos” showed that elaborate patterns emerged in an agent-based model where the agents played the Prisoner’s Dilemma (PD) game with one another on a two-dimensional grid (Nowak & May, 1992). The agent’s score was calculated based on the results of play with each of the other agents in their Von Neumann (eight-cell) neighborhood. The agents would choose the strategy of the neighbor with the highest score. This last feature led to an “evolutionary” process in strategy selection. Depending on the payoff table, the authors were able to either create stable patterns of interlaced networks or a constantly-changing kaleidoscope that was not quite chaos but never reached equilibrium.

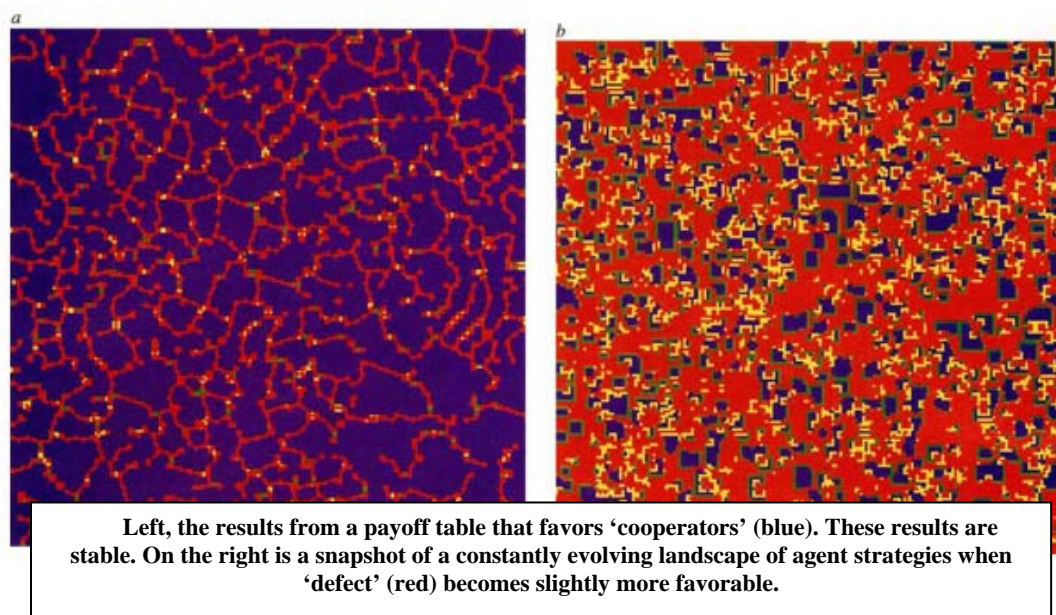


Figure 1. NetLogo Replication of Prisoners' Dilemma Models

But, a year later, researchers Huberman and Glance noticed that this elaborate pattern depended on exactly how the agents were programmed. Nowak and May had established a turn-based algorithm (Huberman & Glance, 1993). All the agents in figure 1 chose their strategies at the same time, based on the strategies and resulting scores of their neighbors on the previous turn. This is deemed 'synchronous' activation. If the agents were assigned to select and execute their strategies one-at-a-time and at random ('asynchronous' activation), the elaborate patterns disappeared. Of note, in synchronous activation, the 'landscape' is not adjusted until all of the agents have rendered their PD decision (cooperate or defect). Their selections therefore are set aside or placed in a 'buffer' until the end of the turn. For that reason, synchronous is sometimes called 'buffered'.

In 2001, Axtell identified a robust classification scheme for the different activation processes (R. Axtell, 2001). He began with a subtle redefinition of *asynchronous* activation processes. Axtell notes that very few natural systems exhibit synchronous processes. But, even in asynchronous activation, where agents update completely one at a time, the question of order must still be delineated. Thus, Axtell goes on to define *uniform* and *random* types of asynchronous processes. In uniform processes, agents are activated exactly once per turn, but their order is randomized (shuffled). In random activation, agents have equal probabilities of being activated in a turn and they can be activated more than once per turn. A further complication is mentioned in the Axtell paper (but not given a specific name), whereby agents can be activated with different probabilities.

The Axtell paper did not explore, however, the boundary where the patterned behavior observed by Nowak and May gives way to the pattern breakdown observed by Huberman and Glance. In Axtell's scheme, there are three different conditions for homogeneous activation rules. This would mean there are two 'boundaries' among the three rules. These are the boundaries between synchronous (simultaneous) and asynchronous-uniform (turn-based, shuffled), and between asynchronous-uniform (turn-based, shuffled) and asynchronous-random (no turns, agents sampled with replacement). Huberman and Glance note that the behavior difference occurs across the second boundary, the asynchronous uniform-random boundary. Of note, in my replications of the Prisoners' Dilemma model described below, I was able to validate that this is exactly where the departure occurs.

An additional option is available from the design of simulation systems. The above activation schemes are all part of a discrete event simulation design method known as ‘event-driven’. An alternative design would be ‘time-driven’ system in which activation times are established (using one of a variety of methods), and added to an event table. The most straightforward method would be a Poisson activation process in which agents have activation times taken as arrival times from an exponential distribution with the arrival rate parameter,  $\lambda$ . In simulation design the exponential interarrival times for agents’ activation are generated by the equation  $t_i = \frac{\log(U)}{\lambda}$  where  $U$  is a uniformly distributed random variable in the interval (0, 1). It is deemed Poisson because the number of activations in a given period (or ‘turn’) is a discrete Poisson random variable. Homogeneous activation methods would have all agents share the same value of  $\lambda$ . Of note, however, is the fact that homogeneous Poisson activation (all agents have the same  $\lambda$ ) will deliver an activation sequence that is just as random as Axtell’s random activation pattern. Turn length, however, would be defined in terms of the clock in a Poisson activation process and would not be uniform. This asymmetry can be minimized by renormalizing the lambdas for all the agents in such a way that the average lambda was 1.0. In that way, a Poisson activation scheme would deliver about one population’s worth of activations per turn. (In the event-driven activation schemes, a turn is typically defined as  $N$  activations, where  $N$  is the total number of agents.) I will use this renormalization process throughout this research as it allows side-by-side comparisons of Poisson activation with the other activation schemes.

The primary motivation for introducing the increased model complexity for Poisson activation (that is, moving from an event-driven to a clock-driven model), is the ability to vary the agents rate of activation. At least one researcher (Fernández-Gracia, Eguíluz, & San Miguel, 2011) has noted that varying activation rates based on the current state of the agent, a style of activation called *endogenous* activation, can provide different model behavior where all the other schemes do not. Poisson activation will allow the introduction of agent heterogeneity, which has deep motivation in observations in the real world.

## **Structure of the Paper**

This dissertation will answer the question of the impact of various activation schemes on the outcome of a number of important models. First, a literature review will present the twenty-year evolution of various expectations and beliefs – far too disparate to be called a body of theory – about activation. The literature research will show that

- Many believe activation to be unimportant and ignore it.
- Those who explore activation have done so on abstract models.
- Those who call for more standardization of agent-based models ignore activation in their standard-building.
- Generally the more policy-centric or consequential models do not explore activation, and many make no mention of it.
- The recent discovery of endogenous activation opens new possibilities in model design

After this, the methodology will be discussed. In short, three models in three different domains were selected, and the methodology will discuss the motivation for this selection. The three models were examined as “cases”, and several activation schemes were explored. Conclusions for each individual case are presented and the overall conclusions follow the final case. The contributions are documented in the overall conclusions. Finally, a section describes the numerous future research paths that may flow from these findings.

## Literature Review

There are three relevant threads of research and discussion that followed the early developments discussed above. First, along with the Axtell articles cited, there is a growing body of literature calling for, discussing, and –at times – proposing a systematic taxonomy of the agent-based modeling field. This literature review will present several of these articles, but it appears as if none of them propose to make activation part of the classification scheme. Second, there is a large and growing literature on agent-based models that are *purposeful*. That is, many such simulations are proposed to model real-world system for some purpose beyond studying the model itself. Many models cited in this literature search are intended to support decisions. As such, they are proposed to occupy the same niche as models and simulations in engineering and operations research. Others are used to support research in traditional sciences such as medicine, epidemiology, or economics. This section will show the rich diversity of applied agent-based modeling research. Finally, there is a substantial literature that analyzes activation (or ‘updating’). Much research simply takes the Huberman and Glance result, and assumes that random asynchronous activation is the most realistic and appropriate. Thus, no side-by-side comparison is explored. A handful of articles, however, appear to focus directly on the differences in population behavior that result from different activation schemes. This analysis, however, is conducted only for the most abstract computer



simulations: cellular automata (CA) or, as in the 1990s, grids of agents playing prisoners' dilemma.

The net result of this literature survey will show that there is a great need for analytic work at the intersection of these three research threads:

- Agent-based models must be systematized, but those who propose schemes for systematizing ABMs appear not to have considered activation as one of the classification boundaries
- Applied ABMs normally choose one activation scheme, and do not explore the impact of varying that treatment
- Those who explore variations in activation schemes do so only with abstract models. This exploration needs to be migrated into ABMs that would be used by non-modelers, and perhaps even non-academics.

## **Calls for Standardization**

For more than a decade, researchers in several fields have been calling for standard protocols in agent-based models with the intention of making them more transparent and easier to replicate. The obvious motive for this is the need to show that the findings in a single study are universal – at least within a given domain. In addition, standardization will make the sharing of knowledge and insights more efficient. Researchers can also rely on a standard modeling paradigm to ensure their research is sufficiently broad in scope, and that key issues (such as a dependency on activation scheme) are not overlooked.

A 2000 paper entitled “Towards a Standardization of Multi-Agent Systems Framework” is one of the earliest discussions on this topic (Flores-Mendez, 1999). It

came after agent-based models had been in use broadly for about a decade (three of the 39 references came from before 1990, and the oldest is 1987). The author proposes specific definitions of ‘agent’ and works to differentiate terms such as ‘architectures’, ‘frameworks’, and ‘infrastructures’. Discussion draws heavily from the artificial intelligence and distributed computing research communities, and summarizes the earlier standardization work of four computer science groups. But, despite the breadth of issues the author attempts to define and formalize, there is no real mention of activation or updating sequences. In fact, the author does not reference the literature on the subject (*e.g.* Nowak and May; Huberman and Glance). Subsequent research that emerged from this paper focused primarily on the computer science and artificial intelligence aspects of modeling, and, in general, did not link back to the main body of agent-based computing. Thus, despite the promising title, this paper appears to have had very little impact on the issue of ABM standardization.

In a 2003 paper, Suematshu, et. al. claimed to propose the “first attempt” to systematize the design of agent-based models (Suematshu, Takadama, Nawa, Shimohara, & Katai, 2003). One of their most important conceptual breakthroughs is to divide the ABM design decisions into those that apply at the agent scale and those that would be considered at the global or environmental scale. Their discussion and research evaluates both design areas equally, and they conclude with experimentally-justified recommendations for designers of agent-based models. Except for a mention of activation as an issue when models are scaled up, however, they do not include activation among

those things that should be standardized or systematized. Their experimental investigations do not vary or even mention the activation scheme.

In 2006, a group of 28 biologists and other life scientists proposed a “standard protocol for describing” agent-based models (Grimm et al., 2006). Their framework was more comprehensive than the 2003 paper, and undoubtedly benefitted from the larger number of models. They first break down model design issues into three descriptive areas: overview, design concepts, and details – the ‘ODD’ framework. These are further subdivided into concept areas. In the overview of each ABM, the authors propose a segment on “process overview and scheduling” in which activation should be treated. They do not use the term ‘activation’. In fact, the idea is further deconstructed into the scheduling of model processes and updating agents’ state variables. The scheduling, they note, can be in continuous time or discrete, and the updating, which is a function of the scheduling, can be synchronous or asynchronous. They make no recommendations about examining these design choices for validity.

Grimm, et. al. are strong advocates, however, of explicit descriptions of activation schemes (scheduling and updating) as ABM research is presented. And, they elaborate on the most effective ways to present the scheduling of agents. In this, they mention (briefly), the use of flow charts and pseudo-code. Each has its advantages, but they expect individual researchers to choose the most appropriate. They claim that flow charts have a weakness in which they must ‘correspond literally to the flow processes of the model’ – with the potential of reducing clarity rather than improving it. More elaboration

of this problem would help if the aim is to get all researchers on the same page. It is noteworthy, however, that these authors seem to encourage more close examination of the different ways to *present* activation than of the different ways to *implement* activation. Still, of all the standardization protocol proposals, this has the most explicit and detailed discussion of the concept of activation.

The most important aspect of the Grimm, et. al. article is its goal of facilitating replication in model-building. At the outset, the 28 authors state the purpose of developing protocols and a standard framework for model description is to enable subsequent researchers to extend investigations on different programming platforms (operating systems and modeling languages). The need for replication is so great that, in 2010, the European Social Simulation Association announced (in conjunction with Volterra Consulting) an annual prize of 500 Euros for the best example of rebuilding a published model with different technology. Very little has been written about this since 2010, however, and subsequent winners have not been announced on the Volterra Consulting website.

No proposal for ABM standardization has been widely accepted, even within a single discipline. The ODD framework discussed above probably has the largest following. In Google Scholar, 622 citations refer to the original paper and mention ODD. It is a small number, however, when compared with the over 40,000 articles in Google scholar since 2008 that mention the phrase “agent-based model”.

This review of the standardization literature, while certainly not exhaustive, demonstrates that it treats the activation question very lightly. And, where it does discuss updating or activation, it does not recommend a robust subdivision of the set, nor does it create well-defined terms and common practices. Thus, while standardization and replication are recognized by numerous researchers as valuable elements in building a community of practice, a literature review shows that activation – a key factor in model building – is not treated explicitly.

## **The Growth in Applied Agent-Based Modeling**

Since the early efforts, the applications of agent-based models have exploded. In part this has been due to the close connection between agent-based computing concepts and modern object-oriented programming languages. The OOP concepts of inheritance and encapsulation are excellent tools in designing agent-based models. In fact, the two are so intertwined that it is likely that the programming design patterns form the basis for the other.

For whatever reason, agent-based modeling research literature spans a wide range of disciplines. There are three international professional societies dedicated to the computational social sciences. In Europe, the European Social Science Association holds an annual conference and publishes the online *Journal of Artificial Societies*. Its North American counterpart is the North American Association for Computational, Social, and Organizational Sciences. NAACSOS began as a series of workshops offered by the Institute for Operations Research and the Management Sciences. NAASCOS publishes a

journal, *Computational and Mathematical Organization Theory*, is published four times a year. The Asian counterpart is the Pacific-Asian Association for Agent-based Approach in Social System Sciences. All three organizations convened at the Second World Congress at George Mason University in July, 2008.

One of the most prominent applications has been in the field of epidemiology. A 2005 article in *Scientific American* (Barrett, Eubank, & Smith, 2005) described an agent-based model approach to generating inoculation policy during a hypothetical smallpox epidemic. What's more, this article demonstrates several of the key reasons why agent-based models provide analytic capabilities that are not available in traditional simulations. Traditional simulations of epidemics have been based on differential equation models which have as an assumption (normally unstated) that each individual has an equal likelihood of infecting every other individual. This allows for an elegant mathematical model of disease spread, which has been well established and accepted in the field. In fact, epidemiologists capture the virulence of a disease with the "reproductive number", the average number of individuals infected by a single contagious person or site.

But such an approach misses many real-world issues. Individuals are not equally susceptible to being infected, and infectious individuals do not spread the disease at equal rates. The heterogeneity of the environment, the connectedness of individuals, the incubation period and the period of infectiousness all differ from individual to individual. There had been a growing desire to evaluate the impact of these and other parameters on

the spread of disease. More importantly, models of disease spread are used to determine appropriate government policy. But, the policy recommendations from a differential-equation approach to epidemic modeling may differ from those of agent-based modeling.

In the *Scientific American* article, the authors used the computing power resident at Los Alamos National Laboratory to substantially increase the scale of an agent-based simulation. They worked with an existing transportation model, TRANSIMS, to create a simulation of movement and interaction of a simulated city, Portland, OR. Prior to this, agent-based models of disease spread were only able to deal with problems of smaller scale such as office buildings or schools. EPISIMS, the epidemic modeling of TRANSIMS, also created a social network that imitated the connectedness of individuals in a population.

Axtell has used an agent-based model to help develop a “theory of the firm”, which explains, among other things, why salaries *increase* by 10% with every tenfold increase in firm size (R. Axtell, 1999). Traditional economic theory dictates that larger firms should pay less because employment there is less risky. This phenomenon is known as ‘local increasing returns.’ Axtell’s model consisted of agents interacting in a non-spatial economy. They may join firms or leave firms. Each agent has a certain productivity factor (small values indicate ‘lazy’ agents). And, each firm produces based on the overall productivity of the agents in the firm. Agents will leave a firm when they assess that their reward is insufficient in relation to what they might expect elsewhere.

There are other important results that cannot be obtained by traditional models. These include the distribution of firm sizes in an economy. In a real economy, firm size is power law distributed, with a scaling constant of approximately two (Acs & Audretsch, 1990; R. L. Axtell, 2001). Thus, firms of size 500 are about four times as common as firms of size 1000. Axtell's data, when analyzed on a log-log scale, fits an ordinary least squares estimate of the power law at  $k = 2.28$ . ( $R^2 = 0.99$ ). No macroeconomic model has been able to achieve results that are power-law distributed, much less that match the actual shaping constant.

Another key example has been in the analysis and even prediction of crime behavior and interventions. A recent compilation of this research, *Artificial Crime Analysis Systems: Using Computer Simulations and Geographic Information Systems*, describes the generation of crime patterns that match real-world patterns using agent-based models (Liu, 2008). The objective is to identify hidden processes that cause specific types of criminal behavior. These models have been used to develop patrol patterns to optimize the deterrence of crime. Some researchers have even adopted the lessons learned from crime pattern analysis (but not the simulations that helped develop them) to understand patterns of insurgent activity in Iraq (Townsley, Johnson, & Ratcliffe, 2008).

Agent-based models have been used to analyze the adoption of new ideas and products. This has been extensively used by marketing analysts. Researchers, for example, have used agent-based models to simulate the efficiency of different promotional strategies that support the launch of a new product (Delre, Jager, Bijmolt, &



Janssen, 2007). The authors use diffusion theory, analyzed using agent-based models, to help identify the optimal target groups and optimal timing for different types of products. They conclude with recommendations about the proper timing for the employment of a mass market campaign, and discuss the trade space for targeting large group clusters versus a broad seeding of the overall population (strategies they refer to as “throwing rocks” and “throwing gravel”.) This article is typical of many in the literature. Not only is there no citation as to the type of activation used in the model, there is no indication of what language or environment it was coded in. Thus, it would be impossible to replicate these results based on the model (without further information from the authors).

Changes to the rules of trading systems have been evaluated by agent-based models, including auctions and even the stock market. The most prominent example of this was the analysis undertaken by NASDAQ on the advent of decimalization. In 1998, NASDAQ partnered with an agent-based modeling consulting firm, Bios Group, to explore the impact of changes to market rules (especially changing the price tick from sixteenths to hundredths) (Darley & Outkin, 2007). The extensive preparatory research, working with data that is normally closely held – mostly because of its value to day traders – Bios Group was able to develop and validate an agent-based model of NASDAQ pricing. This model allowed them to predict a number of post-decimalization behaviors. They examined “parasitic” trading, or trading that takes place within the spread of the bid and asked price for a market-traded security. One of the questions was whether decimalization would reduce or increase overall investors’ wealth. Bios Group found that without parasites investors’ overall wealth would increase, but with parasites

the effect would be the opposite: the overall wealth of investors would go down. Agent-based model analysis allowed them to examine the explanations for these results, and to suggest ways to mitigate these and other problems.

Despite the widely accepted success of the NASDAQ model, and the subsequent use of agent-based models in many investment planning processes, the question of agent activation was left unspecified in published model descriptions. In Darley and Outkin's appendix, "A.6 Basic Description of Object Simulation Framework", there is an abbreviated discussion of the object classes involved in the model. Activation would be defined in the actual methods and design patterns in the model, details that were not published. It might be suggested that providing too many model details would expose proprietary design details. Merely knowing the activation pattern, however, does not allow an imitator to replicate the code of a model. Conversely, without published details of agent activation, the research model is more like a "black box". Subsequent researchers would be prevented from attempting to replicate the published results or to explore the reasons for different results.

In anthropology, agent-based models have been combined with climatological data to help explain the rise and fall of small civilizations – in particular, the Anasazi tribes of the US Four Corners region (J. M. Epstein, 2006, pp. 88–89). A working group of Santa Fe Institute researchers, originally tasked with guiding modern development toward more sustainable processes, observed similarities with their Sugarscape 'artificial society' models and the growth and disappearance of the Anasazi culture in the period AD 800 to

AD 1350. They deemed their project the ‘1050 project’, a play on words that referred to their original sustainable development project, the ‘2050 project’.

The 1050 researchers discovered that there was significant data on the Anasazi, especially on the yearly changes in population. In addition, they discovered substantial data on climate, much of it from dendrochronology (the science of studying tree rings). After an extended process of model design and redesign, the 1050 team was able to generate results that mimicked the historical Anasazi demographics. They published these results in three papers (others were to follow) that have received substantial acclaim. To date the 1050 project forms one of the most prominent and compelling cases for the use of agent-based models of *any* application.

But, even in this, the seminal achievement of the early decades of generative social science, the details of the models in the published works are obscure. It would be nearly impossible to replicate the models from the published literature (certainly not from the updated research Epstein presents in his 2006 book). And, of course, the authors did not state for future researchers what form of activation they used in the successful model, nor did they say whether they experimented with different activation schemes.

Modeling and simulation have been long accepted tools of military modelers. At the campaign level, however, much of the simulation activity has been focused on modeling traditional military combat – the analysis of force-on-force combat (Loerch & Rainey, 2007; Maxwell, 2000; Steeb, 2004). There is a rich literature on such modeling, sometimes called ‘attrition modeling’ or ‘Lanchesterian’ because it is derived from an

early treatment of this process by F. W. Lanchester in 1914. Lanchester treated the change in two opposing military forces as a set of differential equations. These equations can be solved to determine the winner and loser, the relative casualty rates, the amount of time before the battle is determined and other parameters. This research formed the basis for extensive work in military operations research.

Such combat models, however, have limited applicability in many elements of military. The Global War on Terror is just such a conflict that defies analysis by Lanchestrian models. Terrorists in Iraq, for example, are not an organized force that can be defeated through attrition. They are often analyzed as a loosely-connected set of self-organizing networks that share resources, expertise, and personnel. There are no command centers and there is no hierarchy. Models that depend on these features are inadequate for analysis. This problem becomes even more acute because the military depends heavily on such modeling as a tool for evaluating potential acquisitions, new courses of action, or proposed restructuring and reorganizing of forces.

There is another motivation to move beyond attrition modeling: the influence of civilians on military outcomes. In an inspiring dissertation, Yuna Wong has examined a wide range of US-involved military conflicts in the post-Cold War era to determine the impact of civilians on the course of military operations (Wong, 2006). Dr. Wong clearly demonstrates that the civil population, while disorganized, heterogeneous, and purposeless, can influence events in ways that are far beyond the vision of military

models. Moreover, to ignore the impact of civilian actions and attitudes is to invite mission failure or other high-impact undesirable outcomes.

As noted in the introduction, instances of analytic inadequacy and failure can become fertile ground for complexity theory and agent-based models. Dr. Wong, in fact, examines several such models in her dissertation. The two current conflicts, Operation Iraqi Freedom and Operation Enduring Freedom (Afghanistan), are laden with all the challenges of a complex adaptive system:

At least one strategist has observed that the insurgency we face is a self-organizing network that does not always have a clear mission or strategic plan (Kilcullen, 2005). This means that, to defeat the insurgency we must be careful not to mistake it for something it is not – an admonition that Kilcullen draws directly from Clausewitz in his first paragraph. Thus, the insurgency that we face in the Global War on Terror is unlike the unitary and localized (at least within one country) adversaries in past conflicts. While classical counterinsurgency analysis uses systems theory, this is probably not a good tool for understanding the new *jihadi* threat.

Kilcullen argues aggressively that complexity and complex adaptive systems theory should be used to understand insurgency. In fact, he insists that the ‘organic’ theory of insurgency is not merely a metaphor, but that insurgencies operate as organic systems. In his development of the analytical construct, Kilcullen notes that in modeling insurgent networks, understanding, characterizing, and interdicting links is more important than analyzing nodes.

Kilcullen stops short of recommending the use of agent-based models to implement this analytic process, but the link between complex adaptive systems and agent-based models was sufficiently strong that the US Department of Defense commissioned such a model to implement his ideas. It was sponsored by the Joint IED Defeat Organization (JIEDDO) and conducted by the MITRE Corporation. Version 1.0 of the model was developed to analyze the conflict in Iraq, and version 2.0 for Afghanistan (Henscheid et al., 2010).

Agent-based model developers from academia have also attempted to model insurgencies or civil unrest. An early and often-quoted example was developed by Epstein (J. M. Epstein, 2002). Epstein's model examined the impact of government policy – in particular the deployment of police and the length of detention – on different kinds of civil violence. The model observed a number of different emergent behaviors, including the existence of deceptive behavior for insurgents in the vicinity of police. The Epstein article also provides enough information to attempt a replication of his results (although it requires some unpublished knowledge of which of his many cases were chosen to produce his graphical results).

Epstein's model demonstrates many of the contributions to this field can be examined in a generative model. For example, the US military's counterinsurgency guidebook (United States Department of Defense, 2007) provides planning guidelines for the force levels necessary to ensure victory in a counterinsurgency. The manual first notes that, rather than measuring troop ratios to the number of insurgents, it is more appropriate to

measure the ratio to inhabitants. Moreover, the manual declares that 20 counterinsurgents per 1000 residents is the minimum necessary to suppress an insurgency. The Epstein model is uniquely well-designed to test these guidelines, determine their sensitivity to other factors, and evaluate the parameters (government legitimacy and length of jail term, for example) under which these guidelines are valid.

A slightly different approach was suggested by Carley (2006). Envisioning an insurgency as a self-organizing network, Carley proposed the creation of a multi-agent modeling tool that would help understand the network's dynamism and test methods of inhibiting its adaptation. After discussing the difficulties imposed by the covert nature of insurgent networks, Carley suggests that a network simulator – a synthetic 'red team' – might help to understand the impact of different government interventions. Moreover, her research team has created a modeling tool that can be used in this endeavor. It is, of course, agent-based.

Networks can also be analyzed by agent-based models as part of a counterinsurgency campaign. First, however, an accurate understanding of the underlying target network must be developed. One tool that has been used to achieve information about covert networks is the wiretap. Tsvetovat and Carley (2007) examined the types of errors that would be generated when a wiretap-based sampling procedure (a so-called 'snowball sampling' procedure) is used to map a network. To synthesize the actual network – the 'ground truth' – against which this sampling procedure would be measured, Tsvetovat and Carley used an agent-based model, NetWatch.

A more aggressive approach to countering terrorist networks was developed by Argonne National Laboratory. Deemed *NetBreaker*, the model simulates the formation of covert networks and allows for experimentation with methods that might prevent or slow the genesis of the insurgent networks (North & Macal, 2005). The model allows the analyst to determine the capacity of the (synthetic) networks to learn, earn and spend money, and develop and emplace weapons. It further allows the exploration of the impact of key individuals on these performance parameters, identifying which types of networks are vulnerable to ‘reshaping’ activities and which are not.

Agent-based models have been used to evaluate the impact of extraneous factors on the formation and evolution of extremist groups. For example, Butler and Bryson built an agent-based model to observe the impact of local opinion exchange and the polarizing effect of mass media on small extremist groups (Butler & Bryson, 2007). They also examined the impact of policing, and of high turnover in societal membership, as would be observed near a university.

Economists have been long troubled with an emergent pattern called the “tragedy of the commons”. This arises under systems of self-government where individuals will over-use a resource that is a common good. Developing effective policies to interdict this emergent behavior, and testing these policies, has been the subject of a number of agent-based modeling efforts (Schuster, 2005). The actions of individual agents who behave with ‘bounded rationality’ can replicate the problem. Moreover, this problem has a much wider application than just sharing of common land in a village. The tragedy of the



commons appears often as challenge for economic policy and has been described as a problem for which there is no technical solution (Hardin, 1968).

Agent-based modeling was recently helpful in evaluating policies to interdict global warming. An agent-based model helped to formulate more effective interventions in Indonesia to limit deforestation. Numerous actors with a broad spectrum of motivations were modeled to determine the expected outcomes of an array of policy choices. (Purnomo, Suyamto, Akiefnawati, Abdullah, & Harini, 2011).

Another ‘niche’ that traditional models have a hard time simulating is stock market behavior. In particular, rational actor models do not replicate the behavior of stock markets. Stock markets often experience ‘bubbles’ and ‘crashes’ that are caused more by the emotion of traders than by rational choices. In one example, an agent based model of a stock market imitated the different stock-trading strategies (Lux, 1998). These would include ‘chartist’ strategies (where traders make decision solely on the price activity of a stock) and ‘fundamentalist’ strategies (where traders examine the non-market parameters represented by the actual corporation). Lux’s agent-based model was able to replicate much of the real-world behavior. He reported that his agents experienced waves of pessimism and optimism, his ‘chartist’ agents would chase trends, all agents would switch strategies (resulting in high volatility), returns showed a power-law (fat-tailed or leptokurtotic) distribution.

Agent-based models have been used to examine a closely-related phenomenon in markets, clustered volatility. Clustered volatility is a widely-observed trend in market

prices across a broad range of markets (Kuhlmann, 2006). It does not appear to matter which type of market (Moss reports results for alcoholic beverages, tea, biscuits, and shampoo), the phenomenon is apparent (Moss, 2002). A number of agent-based models have been built that replicate this process, including models using the NetLogo environment.

In every one of the cases of ABM application proposed above – many of them in use by decision-makers – the activation scheme is not clearly described in the published description of the model. Some models such as Axtell's firm-size model provide pseudocode to present an elaboration of some design patterns. Epstein's model of civil unrest clearly defines the agent's characteristics.

Moreover, there appears to be no evidence that ABM designers explore different activation schemes as they make the critical early design choices. Page (1997) noted that economic models' activation processes can be redesigned to change the model results. This is what Page deems the *irony of robustness* in that agent based models can be adjusted to 'dock' with any reality. Page cites agent activation as a key design pattern that would allow for this 'robustness', but notes that researchers rarely discuss the activation scheme they have chosen. At the same time, Page suggests that sequential activation schemes, which appear (in context) to include both synchronous and asynchronous, may not be adequate to describe many economic situations.

Page (1997) is also one of the few researchers who have explored activation processes that depend on the actual state of the agents. He created two different cellular automata

(CA) models – the ‘game of life’ and a game involving agent conformity. He also examined three different activation schemes: synchronous, random asynchronous and incentive-based asynchronous. He observed that there were a number of differences among the outcomes. In both his CA models, Page found that incentive-based asynchronous updating create outcomes that were more sensitive to initial conditions than synchronous or random asynchronous schemes. In one of his models, the conformity model, incentive-based asynchronous updated led to global steady states more often than other activation schemes. This difference was not observed in the case of the ‘game of life’ CA model.

Page concluded in 1997 that activation schemes are both important and overlooked by researchers. He suggested that other results, such as Schelling’s segregation model, be revisited using the incentive-based activation schemes. The differences he observed after making subtle changes in the model algorithm suggests to him that researchers should be careful drawing inferences about social phenomena from these models. Moreover, he suggests that some of the effects induced by incentive-based activation might be misinterpreted as neighborhood economic effects (Scott E. Page, 1997, p. 85).

To what extent did the agent-based model development community act on the recommendations inherent in Page’s research? According to this literature review, there is very little to indicate Dr. Page’s challenge was accepted. Moreover, personal conversation by this author with analysts both in the US and Europe has shown that there

does not appear to be an active community examining the different design patterns for agent-based models.

The explorations have continued since the beginning of this research project. Recently published articles that received multiple (greater than five) citations in their first year of publication include studies on UK demographics(Wu & Birkin, 2012); cancer spread (Wang, Bordas, Sagotsky, & Deisboeck, 2012); the US housing market loan crisis (Geanakoplos et al., 2012); and the reliability of the scientific peer-review process (Squazzoni & Gandelli, 2012). None of these papers include an investigation of activation, nor do the authors indicate how they activate their agents.

## **Explorations of Activation**

In order to demonstrate that this research and dissertation will explore unexamined questions in complexity research and the body of literature on agent-based models, I reviewed the literature that referenced the original articles on the importance of activation. According to the search tool, Web of Science®, 201 articles cited the Huberman and Glance article. I have reviewed the body of this research. I have also reviewed the 138 articles written since 2008 that Google Scholar links to Huberman and Glance. In addition, I have examined the abstracts of the 27 articles written since 2008 that have referenced the 2000 Axtell article on Google Scholar. Together, these articles (many of which are included multiple times) form the body of activation literature. It is assumed that, if a researcher has not cited Huberman and Glance or Axtell, the chances

are that this researcher has chosen to investigate issues outside the topic of this dissertation.

This review showed important insights from a small number of articles, revealing several research ‘threads’ on activation. Most of the literature, however, makes only passing reference to the activation question. Among the researchers that provide a more robust treatment of the subject, most have conducted no comparison studies. Many researchers select one activation scheme and report that choice. The most common activation scheme is what was earlier deemed asynchronous – random. In fact, one researcher coins the acronym RAU for ‘random, asynchronous updating’ (Fernández-Gracia et al., 2011). It is noteworthy, however, that no common lexicon has emerged across the domains of agent-based modelers.

Curiously, a small subset of authors in the activation literature, upon reviewing the same studies that are cited here, have concluded that – at least for their purposes – activation does *not* make a difference. In 2009, Roca, et. al. (Roca, Cuesta, & Sánchez, 2009) claimed to have conducted “a systematic and exhaustive simulation” of the space of 2 X 2 games, networks, and update rules. They claim that the Huberman and Glance result represented an anomaly, and that, for the vast majority of cases the choice of synchronous vs. asynchronous made no difference. At about the same time, Lozano et. al. claimed that asynchronous activation was too difficult to implement in evolutionary games, and varied based on the diverse choices of the individual researcher (Lozano, Arenas, & Sánchez, 2008). They use this to rationalize their decision to evaluate only

synchronous activation in their research. Follow-on researchers have used these findings as justification to ignore the activation question as well. A 2011 study of evolutionary games on networks (Buesser, Peña, Pestelacci, & Tomassini, 2011), for example, claim that the literature shows activation “does not change the main qualitative aspects of the dynamics of games on networks.” They chose, therefore, to use synchronous activation because of its simplicity in coding.

These articles, however, represent a minor strain in an otherwise rich ecosystem of ABM research that examines activation in a wide range of contexts across many domains – including 2 X 2 PD games – and finds that activation choices drive important differences in model results. Researchers working with PD games at the same time as Roca, et. al. examined not just synchronous vs. asynchronous activation, but the full spectrum of asynchronous updating schemes described by Axtell (uniform, random, and Poisson) (Newth & Cornforth, 2009). They found substantial differences that have important implications for modeling efforts across the domains. Wardil and DiSilva also showed differences in a structured PD game model when activation was varied (Wardil & da Silva, 2010). Yamauchi, et. al. conducted a full factorial analysis of synchronous vs. asynchronous update dynamics across a wide range of other ABM design parameters (number of agents, average degree, update rules, and network type) for a PD grid (Yamauchi, Tanimoto, & Hagishima, 2010). They showed that activation ("update dynamics") to be one of the most influential factors in overall results. They recommended researchers choose asynchronous random updating.

Researchers working in similar domains (game theory) also report that activation makes a difference. Mosetti, et. al. showed important differences when they varied synchronicity in the Minority Game (Mosetti, Challet, & Solomon, 2009).

And, in domains beyond game theory researchers have also demonstrated the importance of varying activation schemes. Shrimali et al. (2007) examined models of threshold-activated coupling on a lattice of chaotic elements. (As a motivation, they cite a wide range of physical systems.) They demonstrate the intriguing result that asynchronous activation (either uniform or random) produces *more* order in the emergent patterns than synchronous activation. A study by Caron-Lormier et al. (2008) compares synchronous and asynchronous-random activation in the context of a wide range of parameters in an abstract biological agent-based model. They show that the differences in results from varying activation become more pronounced as the population density increases or as the interactions become more complex.

Probably the most rigorous examination of the influence of activation on outcomes in recent literature has come from those working with cellular automata (CA). CA models were originally conceived as discrete spatial grids with synchronous updating. But, if the latter condition is relaxed, a rich texture of results is made available to researchers. In 2010 Baetens et al. (2012) studied varying activation in CA and evaluating the Lyapunov exponent in the output to quantify the different results. They explored much more than the synchronous-asynchronous differences. They examined four different asynchronous update methods, which they term random-independent, random-order, cyclic-order, and

exponential clock. (These appear to be analogous to the three Axtell asynchronous methods: uniform, random, and Poisson.) Using the Lyapunov exponent as an output metric, they show that there is substantial difference between synchronous and asynchronous, but little differences among the asynchronous methods.

The most important insight from the literature research came from a recent paper that varied the update rules in an agent-based model of voters (Fernández-Gracia et al., 2011). They showed that an important difference in outcome is generated when the activation rate is varied based on the state of the agent. They distinguish between update rules that are imposed by the system (or the model, or the clock), i.e. *exogenous* update rules, and those that are generated by the agent itself (*endogenous*). For a specific endogenous rule, they established that the update probability becomes a function of the time spent since the last change of state of the agent. They examine these two methods in the model of voters' opinions across a wide range of network topologies and model dynamics. They conclude that endogenous vs. exogenous activation makes a difference in some cases, and does not in others. But, the differences are significant enough to motivate further exploration of this additional activation scheme.

The common thread for the in-depth activation research in this body of literature is its focus on abstract models (*e.g.* Cellular Automata, Prisoners Dilemma, and other stylized games). The previous section demonstrated that there is a broad spectrum of applied agent-based models in which there is little or no examination of the impact of activation.



This demonstrates that there is an important gap to be filled. This dissertation will make a contribution to the advancement of modeling science to fill this gap.

## **Exploring Agent Activation Processes in Policy-relevant Models**

The Axtell paper called for expansion of agent activation research into more natural (i.e. random) patterns of activation. What's more, this research needs to help develop a better *taxonomy* of activation schemes. And that taxonomy needs to be based on a rational classification scheme. Plato implores us to 'carve nature at its joints' (Plato, n.d., sec. 265e). Here, it means that a series of explorations must be conducted on applied ABMs to determine where the boundaries are. These boundaries should form the basis for a set of definitions of activation schemes. The explorations should include those that have proven fruitful in the literature, and possibly some others. Once a clear set of definitions for agent activation can be identified, further research can show the impact of activation design decisions on model behavior under other circumstances, such as increases in scale.

This boundary will probably be different depending on the class of agent-based model. Classes are not clearly defined, but models fall in a number of broad categories. They may use networks, or not, for example. Or, they may be geospatial (a specialized type of network.) Axtell analyzed only a single class of models, and did not attempt to generalize the results.

The research in the third thread, which investigated the impact of varying activation methods on abstract models, provides a useful catalog of metrics that could be tried in these explorations. Some of the metrics might have to be modified, and not all may be

useful. It appears likely, however that this thread of the literature will be very fruitful in ideas as to measuring and validating differences that might appear. Moreover, if the metrics applied to the abstract models show *no* difference between activation methods, then we can be reasonably certain we have exhausted the set of tests that could be applied. This would strengthen the negative conclusion, which would likewise be valuable to the builders of applied ABMs.

## Methodology

After a more extensive survey of the spectrum of applied agent-based models (beginning with the candidates above), I selected a number of representative models for further examination. I worked on a broad spectrum of model designs, and thus examined the impact of activation across a diverse design space. Many models have small-scale counterparts in the NetLogo library, and I considered examining the impact of changing activation schemes in NetLogo as an important indicator that there may be promising results at larger scales. This was useful in Case I, but proved ineffective when I worked on financial models as the NetLogo library in that domain is sparse.

I identified three ABMs that have impact—one on military affairs, one on finance, and one in physics (but with broad application to social system dynamics). And all three have implications for government policy. I examined a much larger set of potential models, discerned from the mature literature (such as *Scientific American*) or by personal contacts with decision-makers, scientists, and policy-makers – the user community for the ABM tool. Many of these may form the basis for further research. The three models selected here were chosen specifically because they form the basis for significant follow-on research or they have broad applicability:

- The Epstein model was useful as a template for irregular warfare analysis.

- The Zero Intelligence Traders (ZIT) model was a replication of a 1993 study that had over 1100 references from follow-on researchers
- The interacting particle system (IPS) "averaging" model is proposed to influence at least five social system dynamics research domains.

Once a subset of existing models was selected, I attempted to analyze and classify the activation scheme the designers chose. In two cases -- the civil revolt model and the interacting particle IPS swap model -- this scheme was described in the model documentation. In the case of the ZIT model, the activation scheme had to be deduced from the narrative. The ability to replicate a model from the actual published literature – a common characteristic of mature branches of science such as chemistry or physics – was a factor in this selection process. At any rate, the taxonomy of activation schemes described above appeared adequate, but the code necessary to implement them in NetLogo or Python had to be written for this project. While it would be preferable to keep the software constant across the investigations, this was not possible.

The replication process is a critical part of this research. To examine if results in published articles are dependent on activation schemes I must work with the published research and not off-line or informal contact with the authors. Once I created replications as described above, I sought to investigate the impact of alternative activation schemes. Most models in the literature are complex, and there are a wide number of different modeling languages in use. Additionally, two cases were based on articles published some two decades ago. Thankfully, software has evolved to become much more efficient

and easier to code. My simulation languages, NetLogo and Python, were not available when these articles were published.

NetLogo also presented some disadvantages in creating alternate activation schemes. Literature research indicated that.

Once alternative schemes were created and coded, I ran models side by side to determine if the different activations schemes created any differences that could be measured with commonly acceptable statistical techniques. That answered the question of whether there was *any* difference. If a distinction in behavior exists, it was judged whether this would lead to different policy recommendations. This demonstrates the importance of selecting models that are used for more than just academic research. By choosing only those ABMs that influence an actual decision, it will be much easier to distinguish important differences from minor output anomalies. In examining the parameter space of these side-by-side models, I will use a design of experiments that will provide the most contrasts across the widest range of input variables and variable types. In at least two or three such models, I will conduct an analysis of the impact of different activation schemes in the presence of multiple nuisance and decision variables. An example of such an experiment on Epstein's Revolt Model is described in the Preliminary

Results section.

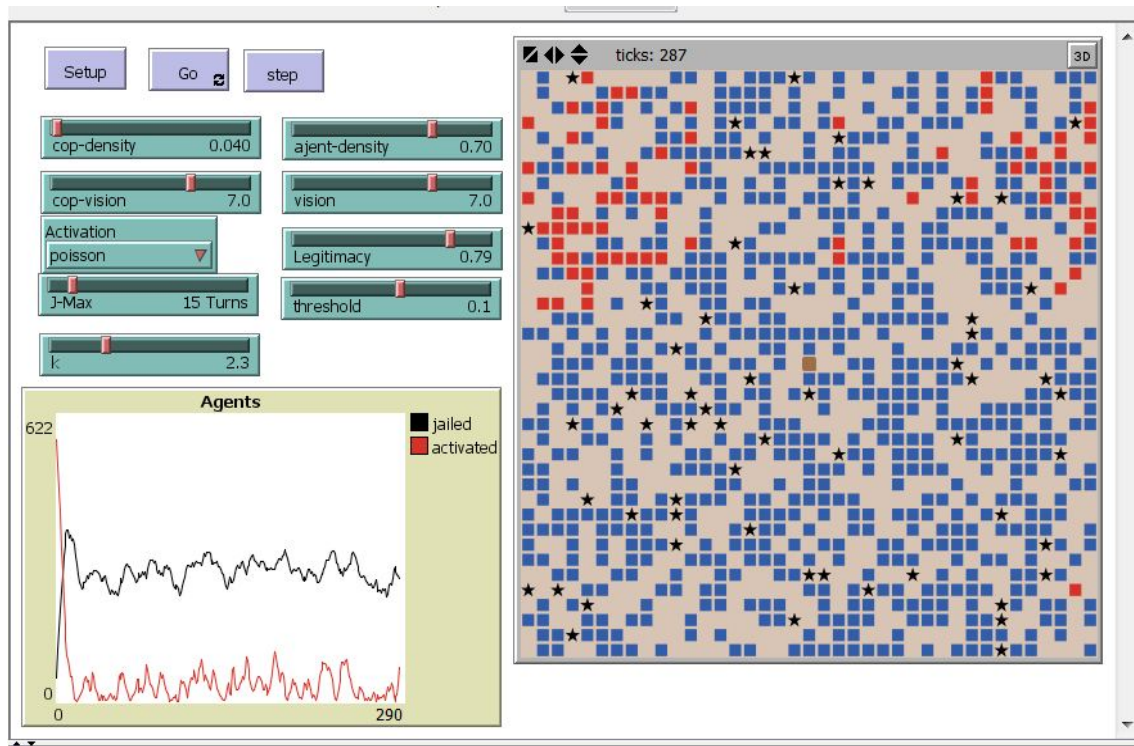


Figure 2. Civil Revolt Model Replication (in NetLogo)

## Design of Experiments

For each of the models, I conduct a full factorial experiment to determine the robustness of the activation conclusion. For example, in examining convergence in the Case III ‘averaging’ model, I ran 100 trials at each of the activation schemes. And, the Case II market model experiments collected thousands of points at the published parameters using the full range of activation treatments.

On a broader scale, this effort conducted observations across a range of agent-based models and application domains. It has focused on the question of whether different

policy recommendations appear in the face of different activation schemes. Even the mixed results (it makes a difference, but only sometimes) show that researchers need to discuss and evaluate activation as a key element of design. It is apparent that this sample is broad enough to refute the null hypothesis that activation *never* makes a difference.

In the process of conducting the background research the concept of activation was expanded beyond the early literature concepts that included synchronous, uniform, and random. It was originally conceived that this project would examine only those few activation schemes across a larger number of domains. In reviewing the most recent research, however, the addition of endogenous activation increased the complexity of each agent-based model. At the same time, however, endogenous activation appeared to more reliably generate differential results. Thus, while it was originally conceived that the research would have to explore much more of the context of activation, such as determining if activation is only important in models that involve agent movement, the exploration of activation types themselves proved a highly fertile area for this and further research.

While there were several important new discoveries, the methodology proved to be more than sufficient to answer the initial question: As users further develop ABMs, is important that they report what activation scheme they chose? Even better, however, this research provided ABM designers with a much broader range of activation schemes that could be used during the design process.

## **Verification and Validation of Results**

This project produces three versions of executable models. These purport to duplicate results of published models in prominent literature articles. As many of these published results are only exemplary, it is difficult to establish that the replication is exact in accordance with accepted statistical principles. But, if the qualitative behavior observed in the published model is replicated in the research model, this can be sufficient to claim the results as verified. For example, the literature may refer to an outcome as a pattern of pulses. All that would be necessary to verify the research code, for example, would be to show an output time series had a similar trend. In instances where the literature models show degenerate behavior – where all the agents adopt the same state, for example – it would be sufficient to establish that the model arrives at the same end-state.

Results of any model of a self-organizing system could demonstrate emergent behavior. This would give a wide range of outcome patterns, and possibly additional contradictions or paradoxes. It is still necessary to ensure that these results are valid – that they reflect real model output and not an artifact of the random number sequence. To ensure this additional runs with a different random seed were used to ensure the stability of each significant result. A factorial design examined some of the models' other input parameters was also completed. Results that did not prove useful or probative were not subject to a large number of runs.

It is important to note, however, that the underlying purpose of agent-based models is not to drive toward a verified replication of reality. Real world complex adaptive systems



are normally beyond the reach of simulation methods. A real world insurgency for example, can only be partly represented in a model. By contrast, a real world naval battle can be replicated with much closer to a one-for-one concordance. To Epstein (2006, pp. i – ii), the actual contribution of agent-based models is the ability to *explain* the relationship between inputs and outputs in complex adaptive systems. He repeats an earlier claim that, in order to explain a thing, you must be able to grow it. Only when you have discovered the most important relationships, characteristics, and system parameters can you build a model that imitates the *behavior* of the real world. Validation, in this context, has little meaning and less utility. At any rate, the crucial research question for this dissertation is to determine if it makes a difference what path Epstein and his colleagues take as they grow social structures *in silico*.

## Results

Before beginning with new cases, it is important to demonstrate that the original discovery and exploration of activation can be replicated. NetLogo 4.0 was used to recreate the Nowak and May synchronous activation model, replicating all the patterns that were reported in the original model.

**Figure 3. Literature Results vs. Replication Results in Prisoners' Dilemma Model**

First, the interlaced pattern of defection networks on a backdrop of cooperators was recreated (Figure 3). Nowak and May did not report what value of  $b$  (the payoff for defecting when your partner cooperates) that they used to create the fine-grain pattern. It was discovered that a rather precise value of  $b$  (1.799) generates this exact pattern, and small changes to  $b$  cause somewhat different results. Next, the chaos that occurs for values of  $1.8 < b < 2.0$  was replicated in the same model (not shown because of its dynamic character). The patterns above were initiated with an initial random distribution of 10% defectors.

Nowak and May experimented with the patterns that evolved when a single defector ‘seed’ began in the center of a field of cooperators. In starting with a central “seed”, Nowak and May demonstrated that the model was actually *deterministic*. Absent the random placement of agents, the evolution of strategy had no stochastic character at all. As Figure 4 shows, the NetLogo model can precisely replicate the Nowak and May central-seed pattern.

**Figure 4. Replication of Deterministic Patterns**

This replication of the Nowak and May result (for which the researchers in the 1990s did not use any modern agent-based modeling language) demonstrates the ability of the modern NetLogo language to create reproducible results (an important validation result).

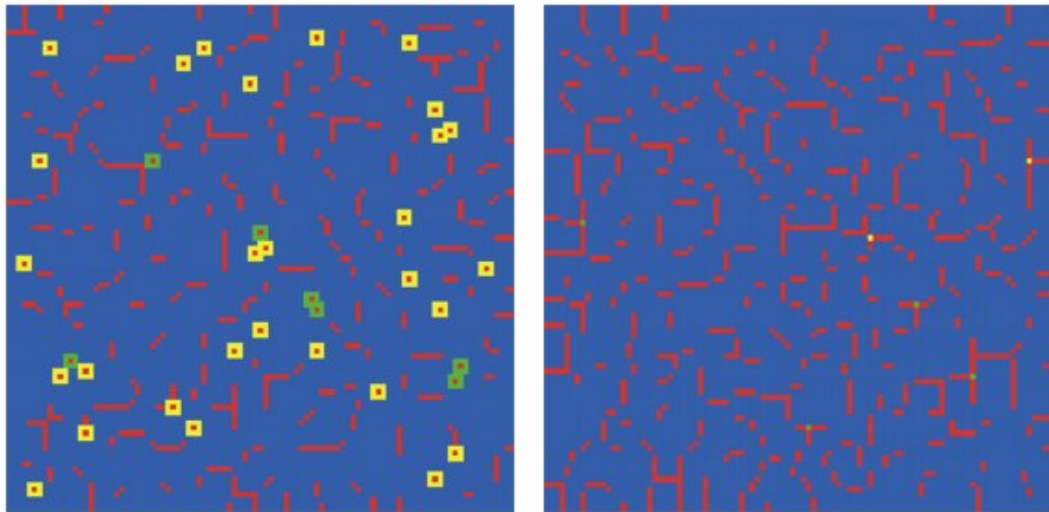
Once it was possible to replicate the 1992 demonstration of the evolution of cooperation, NetLogo was used to explore asynchronous activation. Huberman and Glance reported that they used an activation pattern they referred to as *asynchronous*. But, it is not certain whether they used an *asynchronous – uniform* (agents take turns, but the order is shuffled) or *asynchronous – random* (agents are selected at random from the population and activated). Thus, the question becomes: where is the boundary where the synchronous patterns break down?

NetLogo is a scripting language that creates Java code from the higher-level NetLogo command set. The command that creates the agent activations in NetLogo is ‘ask’. If the algorithm is proceeding iteratively, each time the ‘ask’ command is reached, the agents are activated in random order. This is the asynchronous – uniform method referred to above. In order to activate an agent at random, the NetLogo code is ‘ask one-of’. This will create an asynchronous – random activation process.

The patterns above, where NetLogo matched the synchronous (Nowak and May) result, were achieved with the ‘ask’ command. Thus, they were achieved using asynchronous – uniform activation. The breakdown in this pattern (and devolution of all agent strategies to ‘defect’) was only replicated when the ‘ask one-of’ command was used. This shows that, contrary to the Huberman and Glance claim, it is not sufficient to use just any asynchronous activation scheme to get the degenerate result. If the scheme is uniform – if all agents take a turn – results similar to the synchronous pattern appear. In order to achieve a more ‘realistic’ result (the Nash equilibrium for the Prisoners’

Dilemma game is ‘always defect’), it appears necessary to use the more natural asynchronous – random activation pattern. This is an important result, which demonstrates NetLogo can not only replicate a simple and well-understood model, but it can explore various activation methods.

Follow-on experiments further explored the insights that could be achieved from NetLogo. Several different values of  $b$  for asynchronous and synchronous activation were evaluated. In the following depictions, the left hand pattern represents the asynchronous - uniform pattern (the same algorithms depicted above) and the right hand pattern is the asynchronous – random outcome.

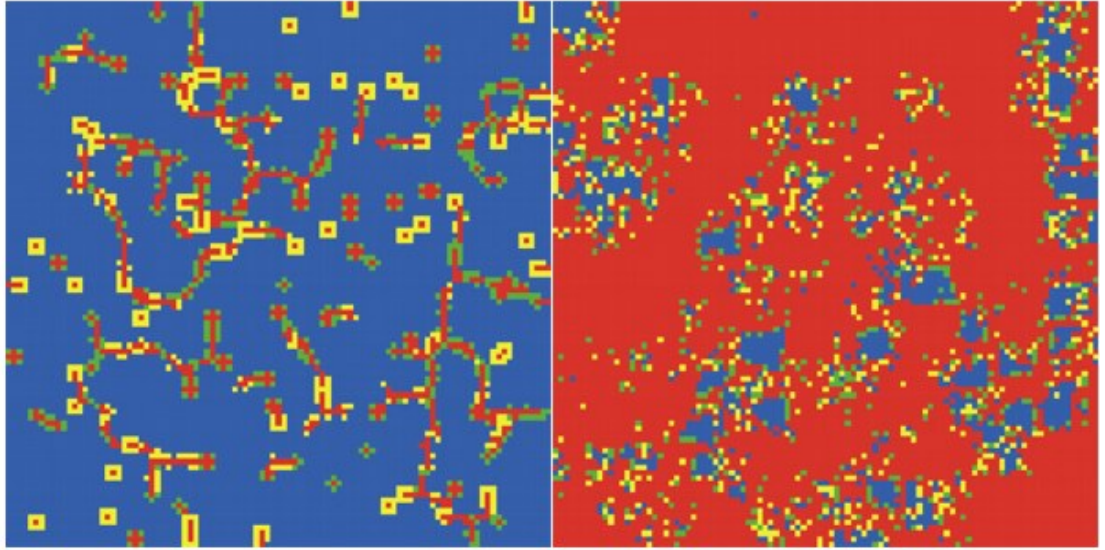


**Figure 5. Synchronous and Asynchronous Patterns with  $b = 1.2$**

The first such exploration involved the parametric region where most of the agents would be expected to adopt a cooperation strategy. This is the region where  $b$  lies between 1.0 and 1.8. The patterns generated by the agents typically did not degenerate for smaller values of  $b$ . That is, at least some agents continued to exhibit each of the two strategies indefinitely.

But, as  $b$  is increased, the two methods begin to diverge significantly. By the time  $b$  reached 1.5, the two activation methods resulted in completely different macro-scale patterns. The uniform activation scheme at  $b = 1.5$  continued to display end-state behavior that included agents with both strategies. The interlaced networks were a finer grain than in the  $b = 1.2$  case, and the progression toward the fine-grain structure that emerges with  $b = 1.799$  is evident. As the figure above shows, with  $b = 1.2$ , the synchronous and asynchronous patterns look mostly alike.

With  $b = 1.5$ , the synchronous result resembles a network of defectors on a background of cooperators. This (l) is the end-state. If asynchronous activation (r) is used, cooperators devolve into pockets, and eventually disappear. The result shown (Figure 6) is in turn 100. The end state is an all-red field of defectors.



**Figure 6. Synchronous and Asynchronous Patterns with  $b = 1.5$**

The random activation scheme, however, caused a complete collapse of all cooperating agent patterns. The agents would evolve into smaller and smaller pockets of cooperators (blue), and eventually these pockets would disappear completely. This is much closer to the degenerate result reported by Huberman and Glance.

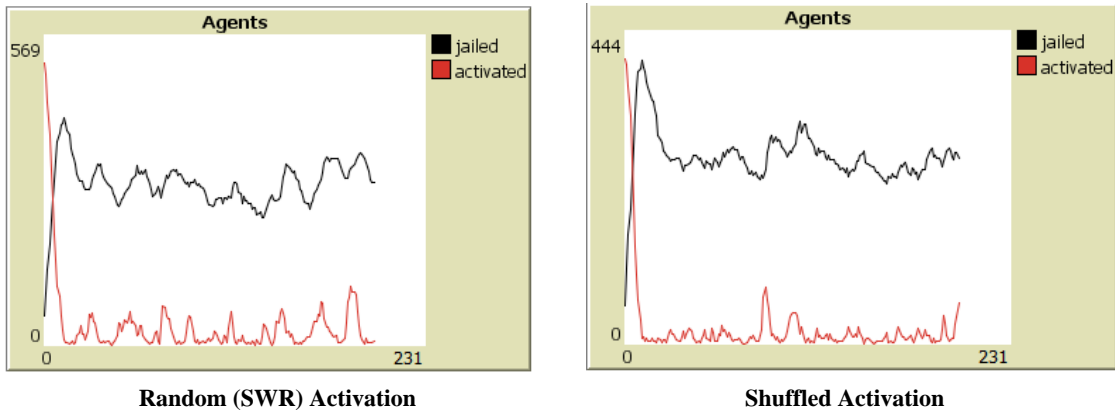
These additional results confirm that Huberman and Glance used a random activation scheme and not a uniform scheme. Many researchers since have related the Huberman and Glance discovery to the *asynchronicity* of their activation scheme. In reality, it is not sufficient that agents activate asynchronously to achieve these results. In order to establish a different outcome scheme it is apparently necessary to eliminate the turn-based activation process. It also argues for a precise statement of the activation process,



as it was not apparent from the Huberman and Glance narrative what asynchronous scheme they used.

These initial results demonstrated that NetLogo can be used to explore the impact of activation on qualitative behavior. This may be sufficient to motivate researchers to publish full details of activation. Often, agent-based models are used to prevent or induce macro-scale system behavior such as extinction, epidemics, or market crashes. The case research below will evaluate whether and when a *quantitative* (or statistically significant and scientifically testable) difference can be demonstrated in outcome behavior as activation is varied. The first case study, for example, shows what can be done with a replicated model in NetLogo.

In examining this model with uniform (shuffled) vs. random activation, the results in Figure 7 were achieved.



**Figure 7. Revolt Model Time Series - Shuffled vs. Random Activation**

Behavior for the two patterns appear slightly different (for the same input parameters), but the differences are subtle. The average number in jail is clearly lower for the random activation scheme (note the difference in scales). A full examination of this model will take an exploration across the parameters using properly designed experiments, and a careful choice of output statistics to determine if there are testable differences between the activation schemes. Further, this examination will force me to apply rigorous systems analysis methods to deal with start-up or run-in issues in accordance with common practice – another important contribution to this field.

As part of developing a model of the labor market, three activation schemes were examined. The same model was evaluated with uniform activation, asynchronous – turn-based (shuffled), and asynchronous – random (non-turn-based, sampled-with-

replacement). In preliminary results at a small scale, a wide range of results (unemployment rate, firm size, job-change rate, Philips Curve, etc.) are insensitive to the activation pattern. The results are not included in the final dissertation as the model itself has not become the basis for published research. While the explorations were abandoned and subsequent evaluations might show some differences, the early explorations of the labor market model show that activation may not make a significant difference in every model.

### **Case I: Activation in a Model of Civil Revolt**

As noted earlier, the field of unconventional or irregular warfare appears to have great potential for agent-based models. They are especially promising tools for analysis of insurgencies and terrorism because of the self-organizing nature of terrorist organizations (Sageman, 2008). The primary research model used by JIEDDO to analyze insurgent emplacement networks is the COIN model, which was based on an earlier model published by Epstein (J. M. Epstein, 2002; Henscheid et al., 2010). It would make sense, therefore, in attempting to demonstrate that activation is a key element in the process of model replication to begin with the Epstein model. This is particularly attractive because Epstein had so explicitly described not only the activation process, but provided extensive details about all the agent parameters, the agent rule set, and the model behavior.

### ***On the Choice of Epstein's Model***

Epstein (2002) proposed a model of civil unrest based on a relatively simple set of rules and behaviors. His model is one of the most cited in this domain, referred to by over 290 other published articles. Many of these subsequent efforts created more complicated agent populations or rule sets. It is telling that these subsequent efforts are nearly all terminal research threads, with no complex model creating a critical mass of research explorations and replications. For example, one model (RebeLand), proposed eight years later had only a handful of references in the literature (Cioffi-Revilla & Rouleau, 2010). RebeLand was written in MASON, an agent-based modeling environment that depends heavily on Java. Thus, it has a much less forgiving learning curve (than NetLogo or Python), and cannot provide the programming efficiency that drives a broad user community.

So it appears that the Epstein model is a form of ‘root’ model and exploring the Epstein model would have implications for subsequent (and future) research into civil violence. Not only does it spawn a US Government model (Henscheid et al., 2010), but it also forms the basis of numerous excursions in related domains.

### ***Simple Rules Create Realistic Outcomes***

Epstein begins with a theory that each individual is ‘born’ with two characteristics. These do not change for each individual. During the course of model runs (agent movement and state-changing) these inherent characteristics form the basis for

modification of derivative parameters based on the environment and the experience of the agent.

The first characteristic is the level of hardship for the individual. As Epstein develops the idea, this does not appear to actually mean hardship, but more the perception of hardship. Agents are assigned their hardship ‘endowment’ over a random, uniform interval  $U(0, 1)$ . Note that this is not a realistic reflection of actual hardship of individuals in an economy. The distribution of wealth normally follows a Pareto distribution, with a small percentage of individuals holding most of the wealth. Thus, in natural populations, far more than a half of all individuals would have below average wealth. If hardship were a measure of perception and not reality, however, this might be closer to reality. In many societies people who are satisfied with their circumstances can be found at all income levels.

The second constant, intrinsic characteristic in the Epstein model is the belief in the legitimacy of government. In his discussion of the model, Epstein notes that this can be a surrogate for the level of government corruption (or perceived corruption), support of the people, or possibly just effectiveness. Legitimacy is similarly distributed among individuals as a uniform random variable,  $U(0, 1)$ . Legitimacy’s random selection is made independently of hardship’s random selection. It may seem a bit unnatural that this parameter would remain fixed for an individual agent regardless of circumstances or history, but this is one of the simplifications that all model designers much choose.

Each agent would derive a value for their grievance (G) dependent on their hardship (H) and their sense of the government's legitimacy (L). Epstein chose the formula:

$$G = H ( 1 - L )$$

This will create a distribution of grievance that grows with hardship and shrinks with perceived legitimacy of the regime. Epstein uses the example of the British population in World War II, which endured extreme privation, but remained non-rebellious because the belief in the legitimacy of the government was high. Corrupt governments often fail to survive much less amount of popular hardship.

In addition to the citizen-agents the model included cop-agents. (Hereafter they are referred to as agents and cops for clarity.) Cops normally appear with much less density than agents, with cop population of only about five percent of agent population. Both cops and agents have a 'vision' and they are placed randomly on a 40 by 40 grid. Agents' and cops' positions are exclusive and no two can occupy the same grid location. Agents and cops move by selecting another grid square within their vision (including their current square) in which there are no other agents or cops. Agents can have two states, passive or active. These represent normal, peaceful behavior or violent rock-throwing type behavior.

The cops' algorithm is quite simple. They will arrest one active agent, if there is one within their vision, per turn and send that agent to jail.

Upon activation, an agent will decide to change its state based on a risk-analysis algorithm. Each agent is endowed with a sense of risk aversion,  $R$ , which is uniformly distributed by choosing an independent random variate from the interval  $(0, 1)$ . Upon activation, the agent observes the number of cops,  $C$ , within its vision and the number of active agents,  $A$ . It then calculates the probability that it will be arrested as a function of the ratio of cops to agents:

$$P = f(C/A)$$

This fraction will always be finite because an agent always counts itself as active in making this calculation. In order to create a realistic decay in probability as  $C/A$  drops, the function:

$$P = 1 - \exp(-k C/A) \tag{1}$$

was chosen, where  $k$  was set such that  $P = 0.9$  when  $C=1$  and  $A = 1$ . This was viewed as a “realistic” assessment. If I’m the only rebellious agent and there’s at least one cop in the vicinity, it’s pretty likely that he will arrest me. (It’s not certain, because there may be other rebellious agents within the cop’s vision but outside of mine.)

One anomaly has been discovered since the Epstein paper was published. In evaluating equation(1), Epstein inadvertently introduced a conversion of the factor  $C/A$  to an integer, truncating the fraction. Thus, he uses the modulus of  $C/A$  instead of the real number  $C/A$ . Oddly, this programming anomaly appears to be necessary to develop the realistic emergent behavior, the ‘punctuated equilibrium’ observed below. That is,

without this truncation, bursts of agent revolt do not appear. Apparently, this has not been documented in the original literature but it is common knowledge in the agent-based modeling community. It has been described to me several times, and the NetLogo design team has documented it in their instantiation of the civil violence model. This documentation is included in the NetLogo model library (Wilenski, 2004). I have, in my implementation of this model, included this modification as well.

Next the agent calculates a net risk based on the probability of arrest and the risk aversion. This was set to be a simple product of the two variables, so that net risk,  $N$ , is defined as:

$$N = RP$$

With these parameters set (many of which are determined by the local environment and the state of other agents), an agent calculates the difference between grievance and net risk, or  $G - N$ . If this value exceeds some threshold  $T$ , then the agent changes to an active agent. If not, the agent becomes passive (or quiescent in the terminology of the published article). The threshold  $T$  is a system-wide variable that is constant for all agents.  $G$  and  $N$  are heterogeneous and dynamic.

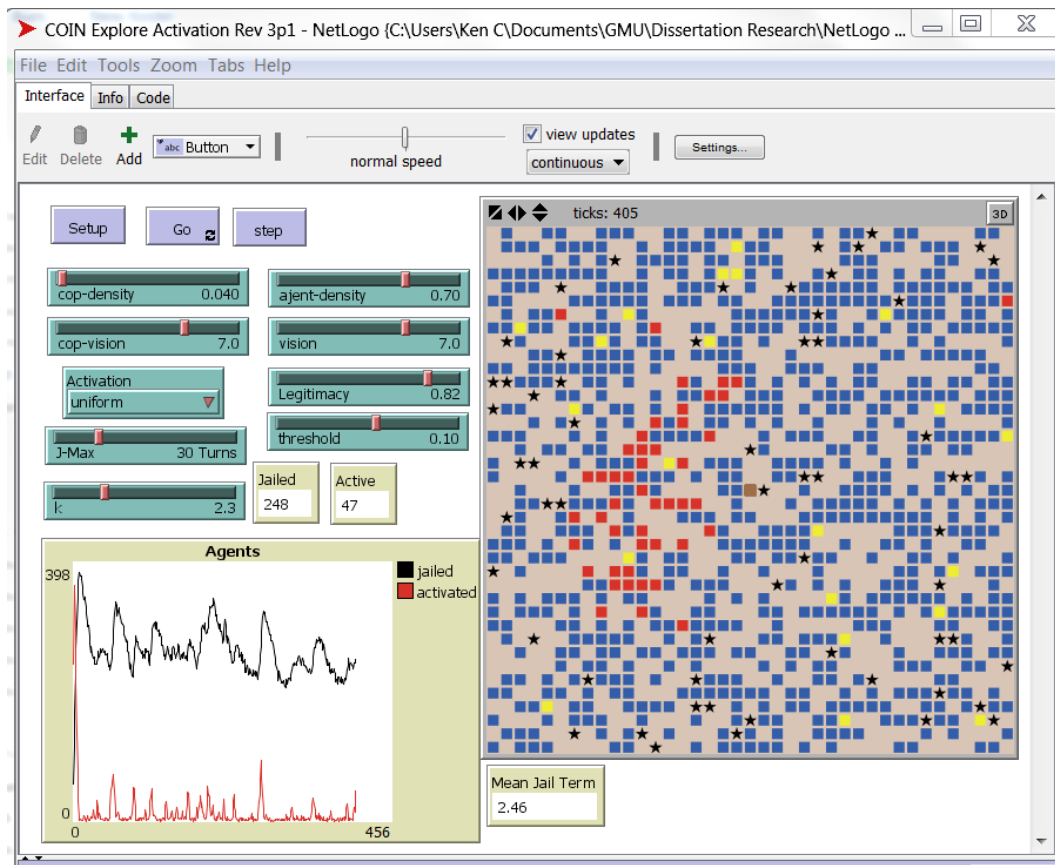
Jail terms are determined as part of the arrest routine. There is a global maximum jail term, J-Max. When an agent is sent to jail, the term is sent as a random variable in the interval  $(0, \text{J-Max})$ . Once the end of jail term is reached, the agent is returned to its original position as a passive agent. It then executes the activation algorithm above just like any other agent.



Epstein addressed the question of the validity of these input distributions (and, in fact, all the rules). He “makes no pretense” of basing these rules and parameters on real-world values, and he does not reference any actual published research on the measurements. Epstein’s stated goal is to determine if the emergent outcome represents “recognizable macroscopic revolutionary dynamics of fundamental interest.”

Epstein's simulation, despite its simple rule set, demonstrated exactly the complex emergent behavior he was seeking. His model showed ‘bursts’ of violence that appear as localized outbreaks. In addition, the model showed that these bursts were heavily dependent on the jail term once arrested and the density of police officers.

I recoded the model described by the Epstein paper in NetLogo, and observed that the general behavior was as described in the original article. That is, emergent behavior curves were demonstrated that were similar to the published curves in the Epstein model. Revolts appear as localized bursts of activity that occur when a large concentration of potential extremists (those with high grievance and who believe the government is illegitimate) appear in a location where there are (momentarily) few cops.



**Figure 8. Model Visualization During a Revolt Outburst**

Figure 8 shows the NetLogo instantiation of Epstein's model. On the left hand side the global parameters are shown, and they can be varied across a wide range. On the right the population is shown, with the color scheme as follows: passive agents are blue, active agents are red, jailed agents are grey, and recently released agents are yellow. Cops are depicted as black stars. (These mimic the color scheme shown in the referenced article itself.) At the moment of the snapshot, at turn 405, a revolt has burst in the northwest quadrant.

This figure also better shows the global parameters. The value of  $k = 2.3$  in the probability of arrest equation is the same for all agents and was chosen in all of Epstein's published runs. While it can be explored, it was not. The selection box for activation, of course, is unique to this instantiation of the model. Epstein used only one activation scheme: asynchronous "once per period, random order" (*i.e.* uniform).

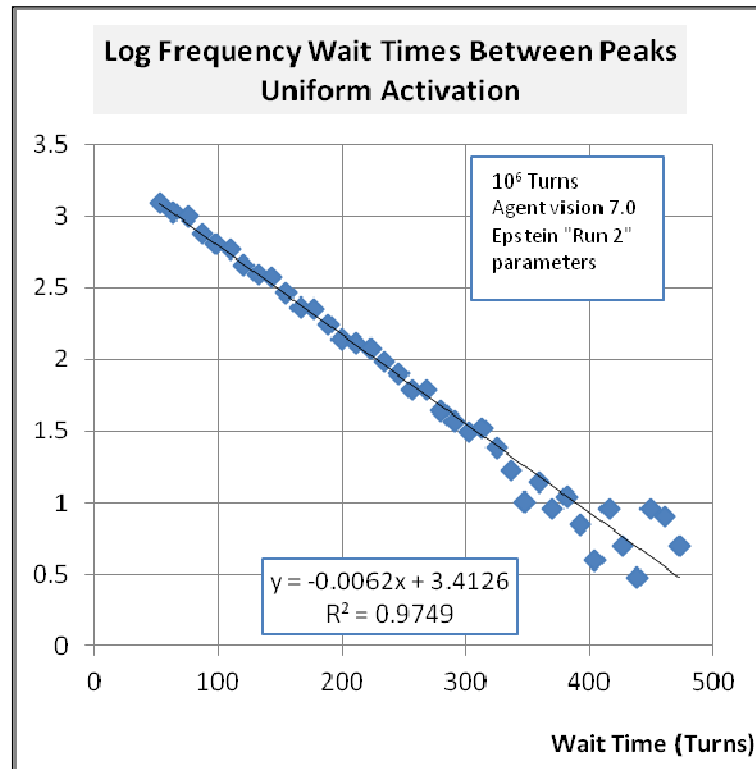
It is important to note that the Epstein model was chosen carefully. It is a rare instance in which the activation scheme is described explicitly, as well as a wide number of other model features and parameters. Thus, while I was successful in replicating Epstein's results, replicating a model in which these parameters were not published would be much more challenging. Just testing all combinations of known activation schemes described above could potentially overwhelm the research time available. And, it is possible that model builders could create, unwittingly, a novel activation method unlike any other tried earlier. If this were undocumented, replication would be nearly impossible.

### ***Alternative Activation Schema for the Civil Revolt Model***

According to the original article, the only activation scheme examined was uniform. Agents would update asynchronously, activating once per turn in random order. In the repeat instantiation described here, uniform activation was first examined. NetLogo creates straightforward commands such that agents are given a turn to move and choose revolt once per 'tick', but the order in which these agents execute these actions is shuffled. For this model, the details of activation are likely to be important because

movement and, especially, the decision to change state depend on the movement and states of all the agents in the vicinity.

After the results were obtained with uniform activation, the model was re-evaluated using a synchronous and a random activation scheme. In the synchronous case, agents would move first and then sense their environment. They would count the cops and the active agents in their vision radius. At that point, they would make the active-passive decision and store it in a buffer. Once all the agents have made their decisions, the buffered decisions were applied. In random activation, agents chosen at random were moved and then they would decide whether to activate based on their new neighborhood. A ‘tick’ or turn was defined as complete when a full population-worth of agents had activated.



**Figure 9. Linear Pattern of Log Frequency Wait Times**

### ***Outcome Behavior Metrics***

There are numerous ways to describe the outcome qualitatively, but in the original article (J. M. Epstein, 2002, p. 7275), Epstein has chosen a few quantitative measures: “waiting time” or the time between the arrivals of peaks and the height of individual peaks. Epstein defined a peak as an episode in which the number of ‘active’ citizens exceeds 50. That is, a peak begins on the turn when the active count exceeds 50 and ends

when the active count falls below 50. (Note how valuable this precise description of his methods is in defining the quantitative outcome of his model.)

In replicating Epstein's behavior, I have chosen to redefine a peak threshold slightly to be three standard deviations above the mean value of the active population. This, of course, necessitates collecting the data for a complete run before the peak threshold can be calculated. I chose this redefinition to allow for a more general analysis of models at a wide range of scales. This allows us to examine, for example, model behavior if total agent population were twice or ten times the size of the published experiments.

Epstein reported that the frequency distribution of wait times was exponential. He showed this by conducting a linear regression of the logarithm of the frequency of wait times in each bin of a histogram versus the wait times. He reported an  $r^2$  for this regression of 0.98. In our analysis of  $10^6$  runs, I achieved nearly-identical results, with an  $r^2$  of 0.97. The difference in the slope and  $y$ -intercept can be explained by the fact that I adopted a different threshold ( $3\sigma$ ) for the definition of a peak. That is, with a different threshold, there will be a slightly different number of 'peaks' and a different time between these peaks. The figure shows, however, that the statistical behavior is identical to the Epstein model.

In addition to providing confidence that I have effectively replicated the original model, the two measures – wait time and peak height – provide valuable quantitative measures of model behavior. Our goal to demonstrate whether activation changed model behavior could be put to a statistical test with these measures.

### ***Experimental Method: Examine Two Additional Activation Schema***

Epstein examined only uniform activation. In such a scheme all citizens and cops are activated once per turn, and the order in which they are activated is shuffled before they are activated. Activation consists of movement, arrest (in the case of cop-agents), and for non-cop, non-jailed citizens, deciding what state they will change to (active or non-active). In order to make the latter decision, citizens must sense their environment, which consists of counting the currently active citizens and the cops within their vision.

Needless to say, if these actions were performed in a different order, the results might be different. To examine this prospect, I re-arranged the model to evaluate two different activations schema, *synchronous* and *random*.

In synchronous activation, all agents undergo a movement phase at the beginning of the turn. Citizens and cops move in this model by selecting a vacant position within their vision and moving to it. Once they move, citizens sense the environment (count active citizens and cops) and make a decision whether they will activate in the next turn. This decision is placed in a 'buffer'. Of note, however, their count of active citizens only includes those that were active at the end of the last turn. Cops will arrest an active citizen once they move, but only a citizen whose state is already active.

Random activation is conducted similar to uniform activation (agents move, decide, and change state once they are activated, and they are inert at all other times), but there is no enforcement of the once-per-turn rule. Thus, activation consists of selecting an agent (cop or citizen) at random and executing all the activation functions (move, then sense,

then act – arrest or chose new state). The random activation process eliminates the concept of a ‘turn’, which opens a question of how to do side-by-side comparisons with the two other schema. To support these comparisons, a ‘turn’ is defined as the number of random activations equal to one complete population of agents. This will maintain the same ‘scale’ when model output is compared.

At least one researcher has used this definition of a turn in random activation (Fernández-Gracia et al., 2011), deeming a full turn as a “Monte Carlo update”. The motivation is straightforward: on average each agent will be activated once per turn. About half of the agents will not be activated at all, and it will be extremely rare that an agent will be activated more than five times. (This is based on simple rules of Bernoulli trials.) As I noted in other matters, the definition of a turn in models that use random activation is a key specification required for replication of results and comparison of output.

### ***Behavior Space Explorations***

#### Phase 1 – Validating the Activation Impact

The behavior of the model was explored for the three activation schema at the initial, “run 2” parameters from the Epstein article. These are:

**Table 1. Revolt Model Baseline Parameters**

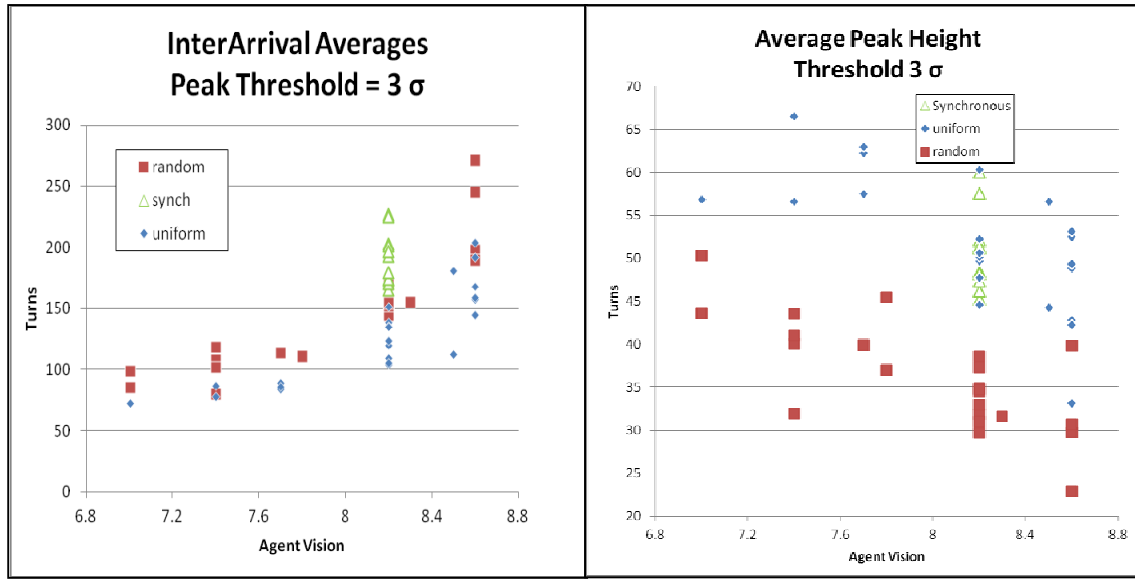
Parameter	Value	Parameter	Value
Landscape	40 X 40 Torus	Citizen Density	0.7
Maximum Jail Term	30	Cop Density	0.04
Legitimacy	0.82	Citizen Vision	7
Arrest Probability Constant, $k$	2.3	Cop Vision	7



As the behavior was explored, some differences appeared between the activation schemes as observed in the waiting times and the peak heights. It was discovered that, as the citizen vision increases, the differences become more pronounced. As the purpose of this experiment was to determine if there were *any* differences as a result of activation, the slightly larger citizen vision of 8.2 was chosen for further exploration. The value 8.2 was chosen because at this level differences in impact from changing activation became the most pronounced. All of the subsequent synchronous activation trials were conducted at the value of 8.2.

In evaluating these sample average wait times, it became apparent that the model was not producing behavior consistent with the Central Limit Theorem. That is, as the sample size (the number of peaks) was increased, the variance of the sample average did not go down. This suggests that a non-parametric test is appropriate, as the population average and variance may be undefined.

The appropriate non-parametric test to determine if multiple samples are generated from the same population is the Kurskal-Wallis test. Using a K-W test of both waiting times and peak heights, I can reject the null hypothesis that all three were drawn from the same population with a  $p$ -value of  $2.8 \times 10^{-6}$  and  $7.6 \times 10^{-5}$  respectively. This allows us to reject the null hypothesis that all three were drawn from the same population, but as there are only three samples (for each of the two output variables), I can do a pair-wise test, the Fisher Exact Test. The results are shown in Table 2.



**Figure 10. Results for Civil Revolt Model Varying Agent-vision and Activation**

Thus, for five out of the six pairwise comparisons, I can reject the null hypothesis that the samples were drawn from the same population and activation made no difference. Only in the uniform-synchronous comparison for revolt peak heights results are data so intermingled that no significance can be attached to a hypothesis.

**Table 2. Phase 1 Civil Revolt Model Results:  $p$ -values for Fisher Exact Test - Cop Density 0.04, Agent Vision 7.0**

<i>Waiting Times</i>		<i>Peak Heights</i>	
Comparison	$p$ -value (Fisher exact test)	Comparison	$p$ -value (Fisher exact test)
Random-Uniform	0.00021**	Random-Uniform	$4.1 \times 10^{-3}$ **
Random-Synchronous	0.00011**	Random-Synchronous	0.00011**
Uniform-Synchronous	0.00011**	Uniform-Synchronous	0.315

## Phase 2 – Cop Density Explorations

While Phase 1 demonstrated that activation would have an impact, only one variable was explored (agent vision). In fact, that variation was only explored in order to show that the differences seen at the baseline vision of seven grid squares was robust. That determination is critical for all potential explorations of this type of model, and represents a key consideration in creating replicable models.

One of the principles I used in choosing this model, however, was its impact on policy recommendations. Nothing that was varied in Phase 1 would be under the control of the decision-makers on the government side of the equation. (These would be the ostensible customers for such a simulation, even though the author did not say so explicitly.) Of the left-hand side parameters, the ‘sliders’ that could be varied at the beginning of the situation, the only one that would be under the control of the on-scene commander would be cop density.

The density of counter-insurgency or security forces during stability operations is a subject of active policy debate, and may represent one of the most important quantitative planning parameters for a future conflict. The Counterinsurgency Manual, Field Manual 3-24 (United States et al., 2007, sec. 1–67) has an extended discussion of this question. In traditional force planning guidelines, the adequacy of forces for a particular situation has been historically based on force-on-force ratios. Further, the ratios necessary to maintain stability have long been recognized to be different than those required for combat. Historically it was believed that a force ratio of 15:1 for security forces to insurgents

would be necessary to ensure victory. FM 3-24, however, notes that this does not appear to be an adequate guideline. In the revolt model, it's easy to see why this rule of thumb breaks down. At any given time, the number of active insurgents can vary widely. If the measurement is done during a quiet period the force size recommendation would be much lower than if it were made during a rebellious outbreak.

The manual goes on to suggest that a more appropriate ratio – one that is much more stable in its computation – is the ratio of counterinsurgency forces to the resident population. It suggests that security forces (which would include police) should be established to be about 20 to 25 per 1000 civil residents. If population were to fall below 20 per 1000, this would be inadequate. This is backed by a rich literature of studies of a wide range of insurgencies (Goode, 2010). Thus, it is appropriate to observe the sensitivity of the two key outcome metrics – TDOA and height for the peaks of violence – to different values of cop density. It is also important, as in phase 1, to explore this sensitivity to the troop-to-populace ratio by conducting a parametric analysis of cop density around the published value of 0.040.

An initial series of experiments were run with the original parameters (agent vision set back to 7.0). The first chart, Figure 11, shows the time difference of arrival (TDOA) for various densities of security forces. And, Figure 12 shows the average peak heights for the same input parameters. A polynomial trend line is inserted into both graphs, more for comparison purposes than as a proposed output model. Still, the fit of these polynomials is quite good for the Average Peak Height plot, with all three having an  $R^2$  of 0.98.

The regression model for Figure 11 has data that are far less easily explained by polynomial equations than that, and there are no models that can be said to explain the TDOA data. It appears that the model behavior begins to become random across a broad range as cop density moves too far from 0.04. Either revolts become near-constant at low cop density or they become rare and very small events as cop density moves above 0.05. The left-hand side is a region in which there is a great deal of revolt activity is far from desirable. The revolt peaks appear far away only because there are so many that the chances of one reaching three standard deviations is highly unlikely. Figure 13 shows the actual time series of the number of active insurgents in the model at the extremes of cop density 0.028 and 0.052.

This provides a valuable caution to military planners and other decision-makers. If the decision variable is set to be the frequency of ‘violent incidents’, the guidance can fail to capture a real failure situation. In the data, there is a high frequency of violent events in this model somewhere around 0.04. For all activations and all runs, the arrival rate for peaks of violence is greatest (and the inter-arrival time is smallest) around 0.04. But, this is an anomaly of the definition of a peak, which is three standard deviations above the mean number of active agents. Thus, in the model runs with smaller cop density, the populace is in revolt most of the time – certainly a less desirable situation.

For the second metric – the average height of each revolt – there appears to be a simple relationship: the more security forces, the smaller the outbursts of violence. Thus, given that large TDOA and small revolts are the desired outcome, the model appears to

show that the left hand side of the cop density region is to be avoided. Interestingly, the FM 3-24 manual recommends 20 to 25 counterinsurgency forces per thousand. In this model, the agent density is 0.7 throughout, so a cop density of 0.04 would give a value of about 57 per thousand. The model, therefore, ‘recommends’ a far greater density than has been established in US doctrine and provided in the literature. Of course, this model is quite abstract and, as many models before it, useful only for sensitivity analysis.

The insight that the rate of arrival of ‘three-sigma’ violent events is a poor measure of counterinsurgency performance is one of the myriad benefits that accrue to the model designer and executor. It demonstrates the need for close cooperation between the ‘decision consumer’ and the modeling team.

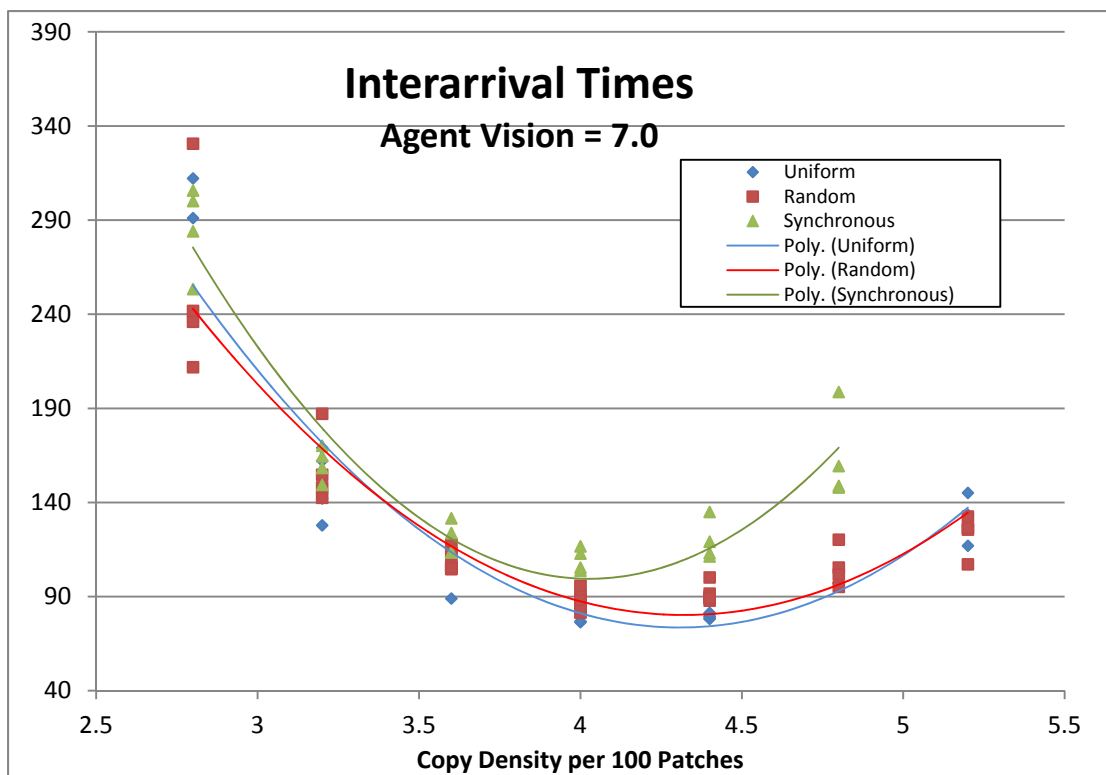
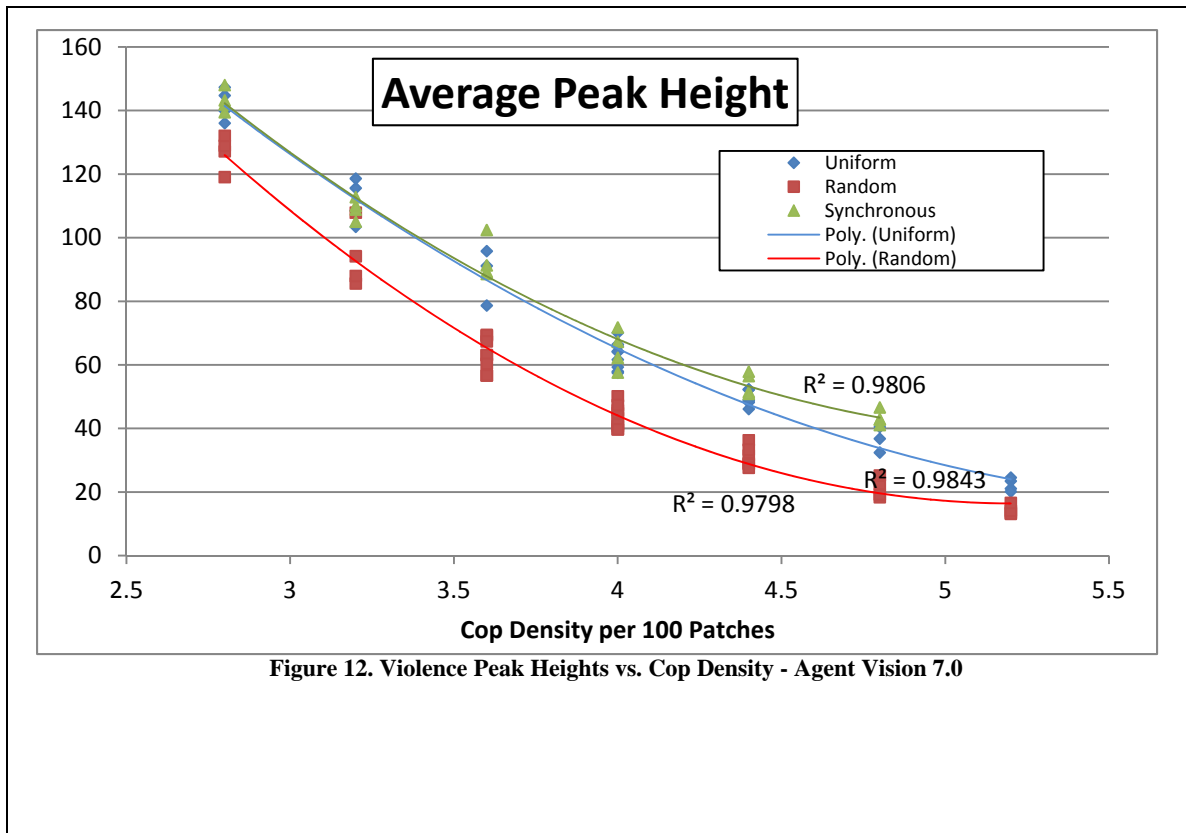
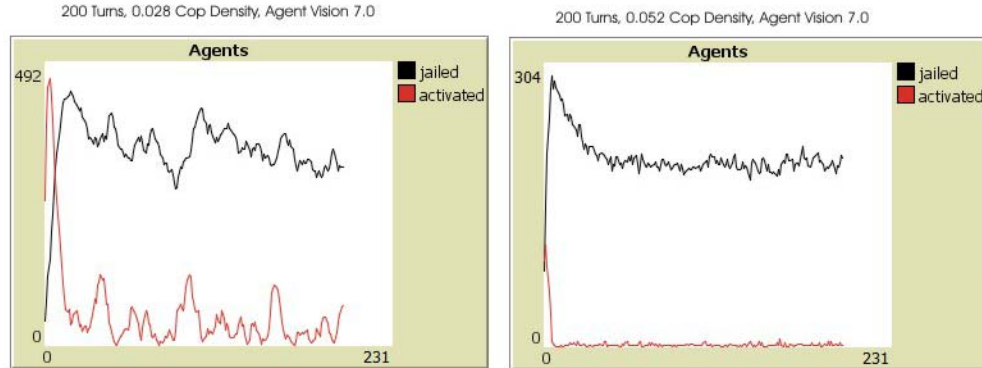


Figure 11. TDOA for Peaks vs. Cop Density - Agent Vision = 7.0



But, are there any differences in the activations schemes? As in the phase 1 explorations, the different activation methods display significantly different results at nearly every level of cop density. If minimum TDOA are to be avoided, the synchronous scheme shows a minimum at 0.04, while the two asynchronous schemes – random and uniform – show a somewhat higher “danger minimum” at 0.044.





**Figure 13. Revolt Time Series at Extremes of Cop Density**

The earlier explorations have shown that an agent vision of 8.2 patches (vice the published level of 7.0) provides a more stark contrast among the agent activation schemes. It was for that reason that these cop density explorations were extended to that part of the parameter space in order to conduct some statistical tests. The results for the agent vision of 8.2 are shown in Figure 14 and Figure 15.

Again, the TDOA outcomes appear to have a minimum in the vicinity of 0.04 (Figure 14). On first glance, this would represent a region to avoid, as it means violent events happen most frequently here. As in the case above, however, this is an artifact of the definition of a peak and not a real consideration for force planning. The curve minima all appear to happen at a slightly lower value of cop density, but this is probably the result of the many interactions in the model and does not seem to have significance. Note also, that it is equally difficult to find a regression model that explains this data. The second-

order polynomials provided the best, and their  $R^2$  values (shown on the figure) were quite low.

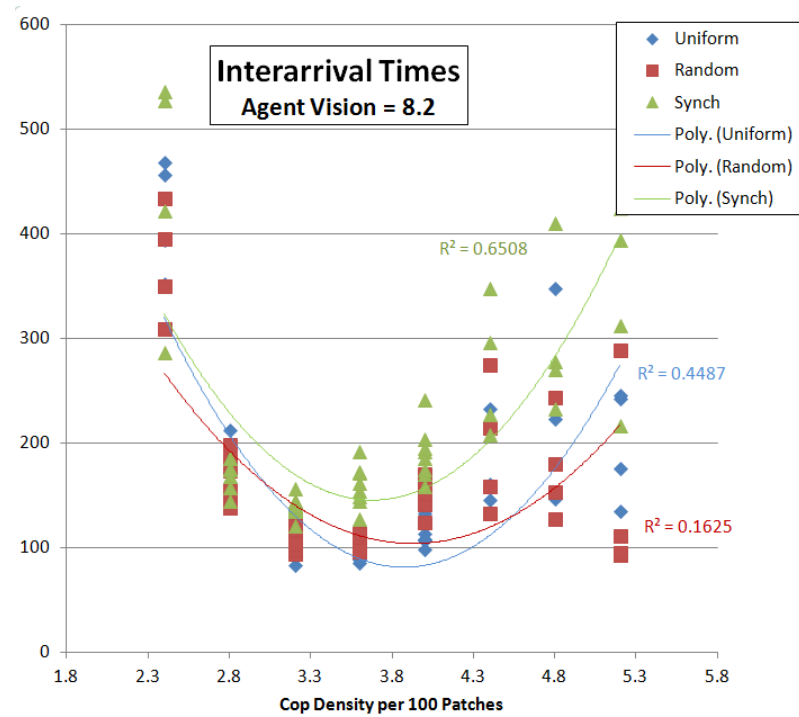
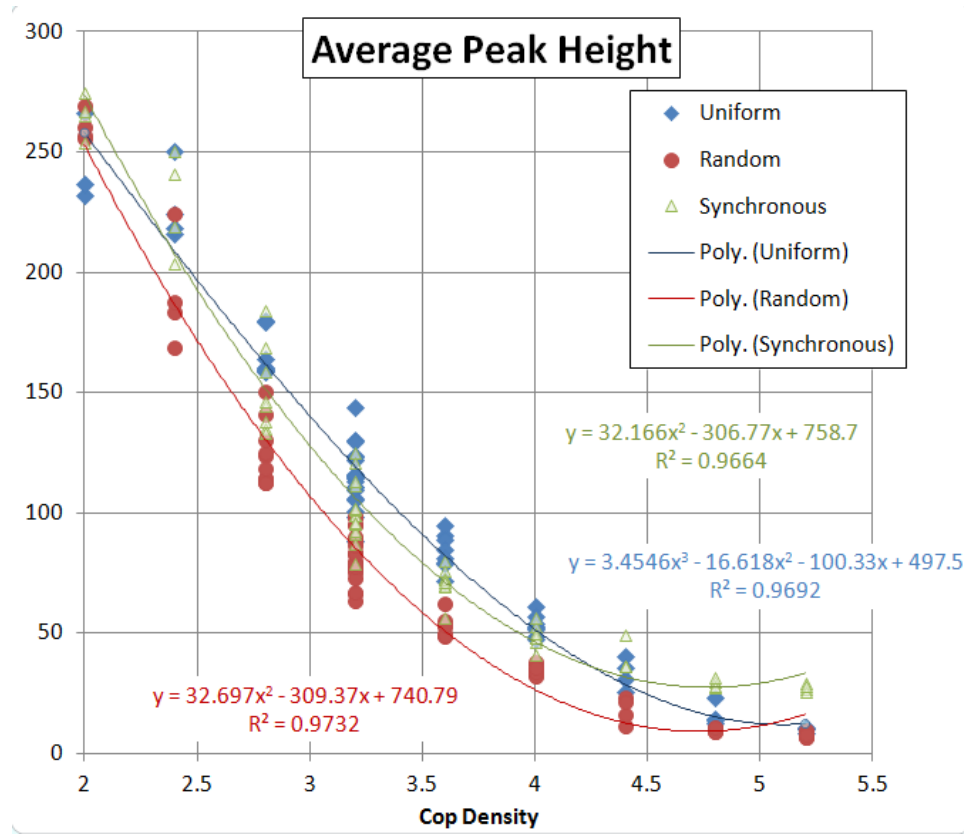


Figure 14. TDOA for Peaks vs. Cop Density - Agent Vision 8.2

Figure 15 gives a much more stable picture of the cause-and-effect relationship between cop density and civil violence. For one thing, the regression  $R^2$  values are much better, suggesting the second order polynomials do explain much of the variance in the data. Additionally, the data depict the generally direct relationship between higher density security forces and a better overall security situation.



**Figure 15. Violence Peak Heights vs. Cop Density - Agent Vision of 8.2**

Finally, a statistical test was performed on these second data sets (those with agent vision of 8.2) to determine if there is a statistically significant difference among the activation patterns. The cop density setting of 0.032 was chosen for extended investigation, and one can see in Figure 15 the larger number of runs at this value.

I again used the Fisher Exact Test as the three runs are manageable. The statistical significance for the first set of data, the inter-arrival times, shows significant results in

comparing the synchronous with the two asynchronous activation types. But, between the random and the uniform activation types, the  $p$ -value was 0.044. This suggests we would reject the null hypothesis that the data were chosen from identical populations and that their sequential order (smallest to largest) would be this extreme (or more) based only on randomness at the 95% level, but not at the 99% level.

**Table 3. Phase 2 Civil Revolt Model Results:  $p$ -values for Fisher Exact Test - Cop Density 0.032, Agent Vision 8.2**

<i>Waiting Times</i>		<i>Peak Heights</i>	
Comparison	$p$ -value (Fisher exact test)	Comparison	$p$ -value (Fisher exact test)
Random-Uniform	0.044*	Random-Uniform	$1.6 \times 10^{-7} **$
Random-Synchronous	$1.7 \times 10^{-6} **$	Random-Synchronous	$4.2 \times 10^{-3} **$
Uniform-Synchronous	$2.9 \times 10^{-9} **$	Uniform-Synchronous	0.016*

For the second set of data, the peak height data, the pair-wise comparison showed that there was also significant difference between the random and synchronous activation types, and between the random and uniform. But, between the uniform and synchronous activation types the separation was much weaker.

### ***Case I Conclusions***

The data above establish that activation makes a clear difference in at least one policy-centric model. This suggests that, at the very least, ABM researchers should fully describe their activation scheme if they are to guide other researchers in proper

replication of their models. This means that the sequence of events in the code (movement, environment-sensing, and state-change) should be explained explicitly in order that subsequent researchers can replicate the experiment. The Epstein model (2002) was chosen specifically because it is an excellent example of such specification. It can serve as a template for models in other domains.

A second goal was the determination if varying activation changed the outcome in this specific model so much that it would affect a policy decision. For this question, the evidence is much less clear. The level of violence at various counterinsurgency force densities is statistically different between at least two of the activation schemes. But, the difference would not appear to be enough that it would force a different policy decision. FM 3-24 gives a range of viable security force densities for planning purposes (between 2 and 2.5%). The model shows that violence can vary significantly within a range about that large, but this sensitivity is present in all activation schemes.

As this model was programmed in NetLogo, it was difficult to include activation schemes that depend on the state of the individual agent – so-called Poisson or endogenous activation schemes. In Case II these will be examined using the more flexible Python programming language. On the question of policy-feeding counterinsurgency models, however, it is sufficient to note that without proper documented activation design it would be difficult and potentially impossible to replicate the model. Given that questions of national security are often emergent, concern matters of life and death, and can effect global events for years and potentially decades, the need for model replications

across different computer languages and with different input parameter sets would be critical.

## **Case II: Activation in a Zero Intelligence Trader Double Auction Market**

Finance is an area of high activity for complexity science and agent-based models. It was one of the primary motivations behind the founding of the Santa Fe institute (Waldrop, 1992). Agent-based models, with their many independent decision-makers, seem to be excellent surrogates for traders in a securities market. Agents can be infused with a number of different strategies, and global information can be made available either market-wide or differentially to only select traders.

One of the simplest market models is called a "zero-intelligence trader" or ZIT model. In such a model, a large body of traders is chosen in pairs. They are unaware of market-wide parameters such as the last trade price or the trade price history or even the details of their counterparty's financial position. In the most straightforward ZIT models, traders trade a single commodity. In order to make some sense, the traders are not completely devoid of knowledge: the sellers know their own cost of acquisition, and the buyers know the future price at which they can expect to liquidate the asset. (The latter might seem a bit artificial, but becomes somewhat more realistic if one considers book value of assets or the surrender value of a bond.) Thus, its simplicity makes the ZIT model an excellent baseline case for studying the impact of activation on financial market models.

### ***On the Choice of the Gode and Sunder Model***

The most referenced ZIT model was introduced by Gode and Sunder (Gode & Sunder, 1993) in an article entitled “Allocative Efficiency of Markets with Zero-Intelligence Traders: Market as a Partial Substitute for Individual Rationality.” According to Google Scholar, nearly 1200 scholarly articles have referenced Gode and Sunder over the past two decades. They were initially researching whether a rule-based double auction market simulation would show the same market success as an experimental market of actual individuals. They used graduate students incentivized by academic grade credits to simulate profit-motivated traders. They then simulated two double auction markets to compare with the real-world experiment.

### ***Bounded ZIT Model Description***

Both simulations began with a small number of traders: six buyers and six sellers. These would trade one ‘share’ at a time. One simulation was unbounded, with the buyers and sellers making offers randomly selected between 0 and 200. The more rational simulation was termed a ‘bounded’ simulation. In this, the buyers would have a ‘supply’ curve in which their cost for their next share to be sold is determined by an escalating price curve. The sellers would likewise have a redemption price, at which they may liquidate any item they buy. This redemption price was a decreasing curve that depended upon how many shares they possess at the end of the trading day. For each trade, buyers and sellers calculate their profit. Buyers would subtract the cost from the trade price, and sellers would subtract the trade price from the redemption price. Buyers and sellers were bounded in that they were not allowed to make an offer that would lose money.

Gode and Sunder made three simplifications to a double auction model:

- Only one unit was traded at a time.
- Once a trade took place, all outstanding offers were canceled.
- If bid and ask offers crossed (seller asked more than the buyer bid or vice versa), the price was set by that of the earliest offer.

Buyers are informed ‘privately’ of the redemption value of each share. This value,  $v_i$ , depends on the number of shares the individual buyer has already bought. The buyer knows his own demand curve, but the market demand curve is not available to any trader. Similarly, sellers are endowed with a supply curve that represents the cost,  $c_i$ , of the  $i$ th unit sold. This supply curve applies to each individual seller and the market supply curve is also not known to any trader. Each trade, therefore, created a profit. For the seller the profit is the net of the price and the cost,  $p - c_i$ . Similarly, the buyer’s profit is the net of the redemption value and the price,  $v_i - p$ . Buyers and sellers form offers at a rate and in a sequence determined by the activation scheme. All buyers have the same individual demand curve, and all sellers have the same individual supply curve. The offer for buyers is a random value between 0 and their current redemption value,  $v_i$ . The offer for sellers is a random value between their cost,  $c_i$ , and 200. This is what was meant by the bounded market. The unbounded market was also examined, but that is not considered here. (Nor is the experiment using graduate students.)

Gode and Sunder conducted six runs of the bounded market, with all values reset at the beginning of each run. The runs were terminated after 30 seconds. Beyond this specification, they did not state the rate at which offers were formed. They also did not



state what computer language or hardware was used. Undoubtedly, more efficient code in a faster language on a faster processor would form more offers in the 30 seconds than others.

Gode and Sunder examined five markets, or five sets of supply and demand curves.

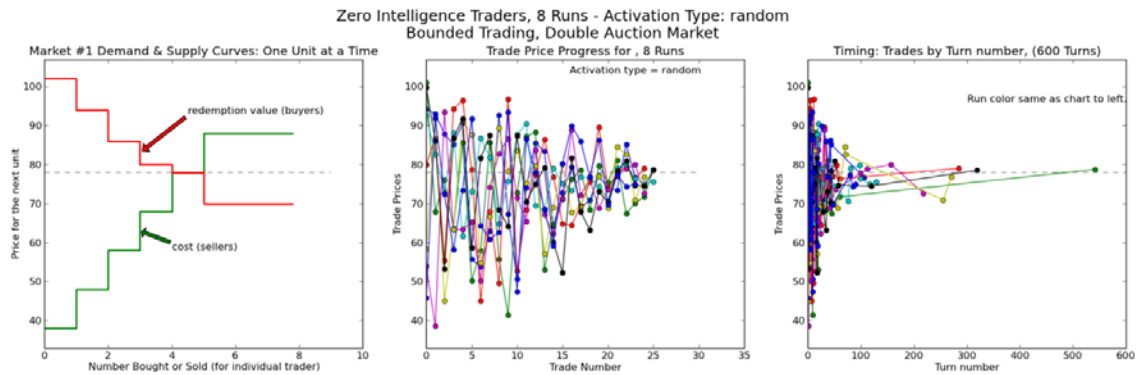


Figure 16. Market 1 Random Trade Price vs. Trades and vs. Turns

The only description of these curves was in the market-by-market graphs in which they are shown without annotation and with no labels. For the first four markets it was possible to estimate these values by inspection, but the fifth market had supply and demand curves with a structure with too fine a grain to reliably estimate. Only markets one through four were replicated here.

### ***Model Replication***

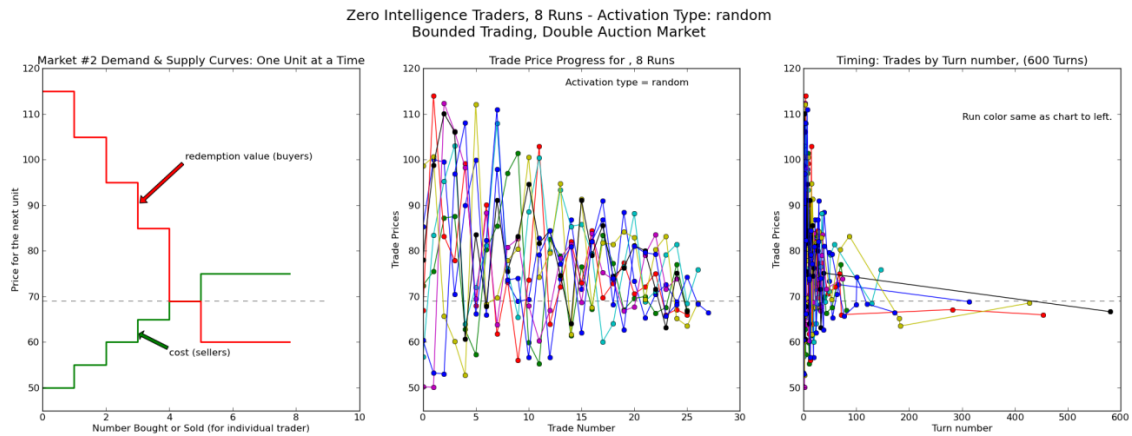
Working in Python, I was able to create a double-auction model in which the traders behave in the manner described in the source article. In order to perform diagnostics, it was necessary to impose some metrics on the dynamic processes of the model. I

introduced the concept of a *turn*, which I define lasting as long as one full population of traders have generated offers. A turn, therefore, is driven by events and not by time. This deviates somewhat from the source article, but allows side-by-side comparison of a variety of activation schemes (see below).

**Once the turn in which trades take place is measured, a price series of trades can be observed in market time instead of trade time. The Gode and Sunder paper plotted trade price per trade number. Thus, they did not observe the fact that later trades occurred much later in a run, after many, many offers had been made. See**

Figure 16 for a depiction of this dynamic behavior for Market 1.

Figure 16 also shows a number of other aspects of my market model. Instead of stopping after 30 seconds of execution, I have chosen to stop after a constant number of turns. For this graphic, I chose 600 turns, but in the full experiments I ran the market out to 5000. Even with these extended runs there are still trades taking place. That is, even after many turns and many offers are generated there is still one buyer or seller who has redemption or cost set just above or below the market-clearing price.



**Figure 17. Market 2 Random Trade Price vs. Trades and vs. Turns**

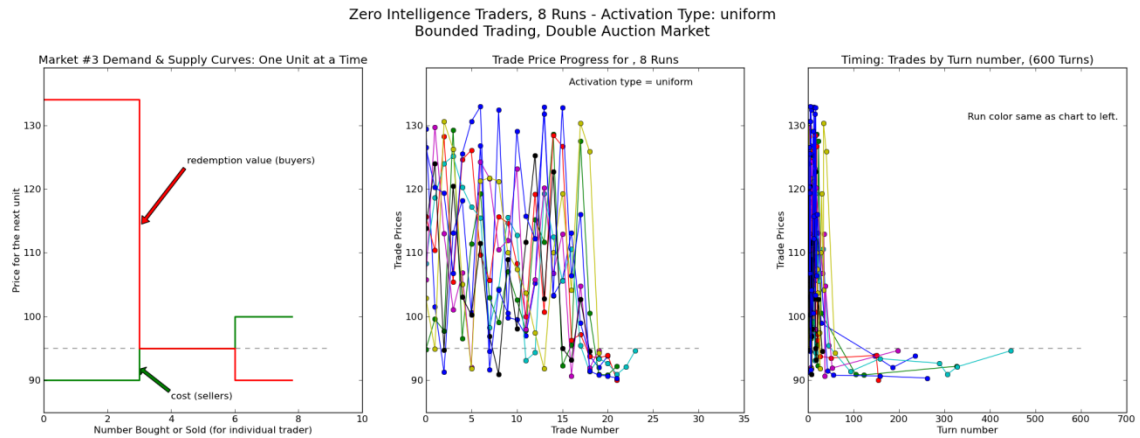
#### Another model behavior apparent in

Figure 16 is the direction of convergence. Market 1 is a market in which the sellers approach the market clearing price from farther away (*i.e.* below) that value. That is, seller's costs are much further from the equilibrium value than the buyers' redemption values. As a result, for any given point in the inventory, a seller will be willing to sell for a price further from the market clearing price than an equivalent buyer would.

Figure 17 shows a market with the asymmetry in the opposite direction – a steeper demand curve and a shallower supply curve. In both cases the trades approach the market clearing price from the direction of the steepest curve. In Market 1, they approach from below because the supply curve is steeper. In Market 2, trades arrive at the market clearing price from above because the demand curve is steeper.

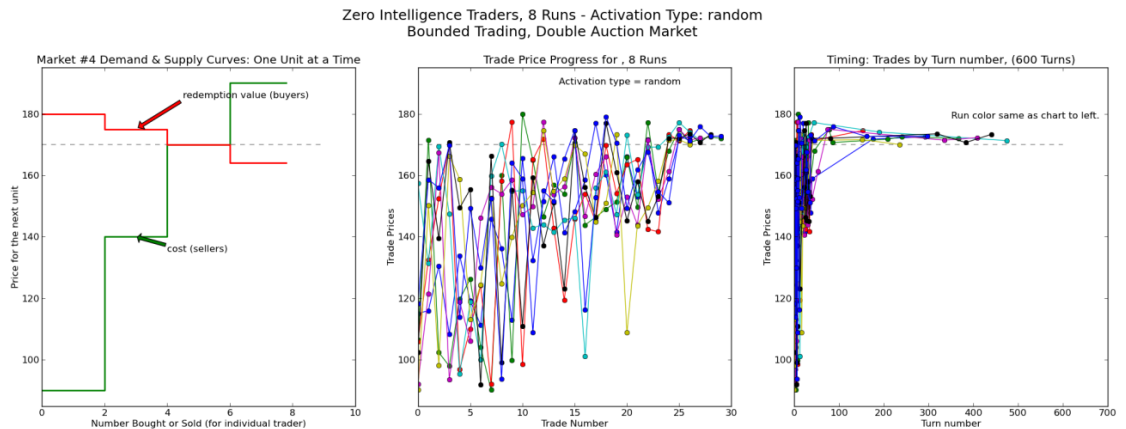
Gode and Sunder indicate that the main question addressed in their study was how much of the rationality associated with human traders (vice purely random, unbounded

traders) was attributable to human decision-making motivated by profit and intelligence and how much is due to simple market discipline – the requirement that a seller can't sell below cost and a buyer can't buy above redemption value. While the bounded market's appears to be in between the random and the human market (by inspection), and the bounded market appears to converge to the same equilibrium price as the human market (determined by a regression of the bounded market curves, averaged over five runs), Gode and Sunder used two rigorous measures to answer the question: efficiency and wealth distribution. These will be discussed in the Model Behavior Results section.



**Figure 18. Market 3 Uniform Trade Price vs. Trade and vs. Turn**

Markets 3 and 4 were intended to explore supply and demand curves of different shapes. It was possible that these might stress markets more, and make convergence slower or, perhaps, eliminate it entirely. As seen in Figure 18 and Figure 19, the trading model behaved appropriately, converging on the market clearing price from the direction favored by the furthest of the supply or demand curves.



**Figure 19. Market 4 Uniform Trade Price vs. Trade and vs. Turn**

It is important to note that in the above figures the supply and demand curves for each market were determined from the reference paper, but the price time series results were from my own replication of this double-auction model coded in Python.

### ***Alternative Activation Schemes***

Given the context of this model, it was possible to postulate a broad spectrum of different activation schemes, but not all.

#### Synchronous Activation

Synchronous activation was not instituted. In the 1990s literature on Prisoners' Dilemma models, synchronous activation separates and buffers the agents' decision (choice of 'defect' or 'cooperate') from the agents' actual change of state. Thus, agents made an internal selection, and did not manifest that selection until all agents had chosen. Then state change occurred across the landscape simultaneously.

There does not appear to be an analogous process among the traders in a ZIT model. Traders already keep their inventories ‘private’, and thus their demand or supply signal is not observable by other agents. The model might make use of the turn structure, storing offers in a buffer and adjudicating them at the end of a turn. But, it is likely that several offers will cross – the sellers offering to sell for more than the buyers are offering to buy. Adjudicating these to determine who trades with whom at the end of the turn would increase complexity and do serious violence to the concept of ‘zero intelligence’ trading.

It might be possible to mimic the behavior of a ‘closed’ market in which trades arrive before trading hours, but the simple structure of this model would mean a trivial result. In real world markets, traders are always allowed to bid and to see others bids before trading begins. As no trades take place, cross-orders are common. In fact, NASDAQ has an elaborate opening procedure to deal with this, called the “opening cross”. The market authority seeks to avoid heavy volatility at the opening by promulgating the “would trade for” price to a large population of traders. This is a weighted average of the volume and price of all outstanding offers, and it’s declared about two minutes before a market open. This initiates a flurry of modified offers ending with the market opening.

To simulate this procedure, the ‘market maker’ in the ZIT model would observe the buffered bids and calculate an average bid. Procedures would have to be written to adjudicate all initial trades at this price. As above, this moves somewhat beyond the scope of ‘zero intelligence’ traders. It is clear why Gode and Sunder did not include this complication in their model (if they even considered it).

### Random Activation

There are several suggestions in the original paper that the authors chose asynchronous random activation. The initial papers on activation were published in the same year (1992), so it is not unexpected that Gode and Sunder would not consider elaborating on the issue.

In my instantiation, random activation merely means that traders are chosen at random from the set of all traders. These traders form an offer. A turn is defined as complete when a number of traders equal to the total number of traders have made an offer. No data points are collected at the end of one turn, and no values are reset. All offers to sell or buy that are in the auction at the end of a turn continue in force at the beginning of the next turn. In fact, these offers are frequently canceled. The original model design had all offers canceled once a trade was complete.

*Initialization and reinitialization:* On the first activation, and every time the offers have been canceled, the first trader's offer will establish the new "best offer" of that type. Thus, if a seller is chosen first, he will choose a proposed sell price that is a uniform random variable between zero and his cost (for this item in his inventory sequence). A buyer will, likewise, establish the new "best buy" offer. Trading can commence as early as the second offer.

### Uniform Activation

Asynchronous uniform activation is executed in a manner similar to random activation. At the beginning of each turn, the array of traders is shuffled. In one turn of



uniform activation, all traders will be activated. Otherwise, the trade rules are the same: offers are carried over from turn to turn, but are canceled once a trade is complete. Initialization and reinitialization are conducted in the same manner.

The trade timing plots for markets 3 and 4 above are shown for the uniform activation scheme. There does not appear to be any significant difference in trade timing behavior between random and uniform.

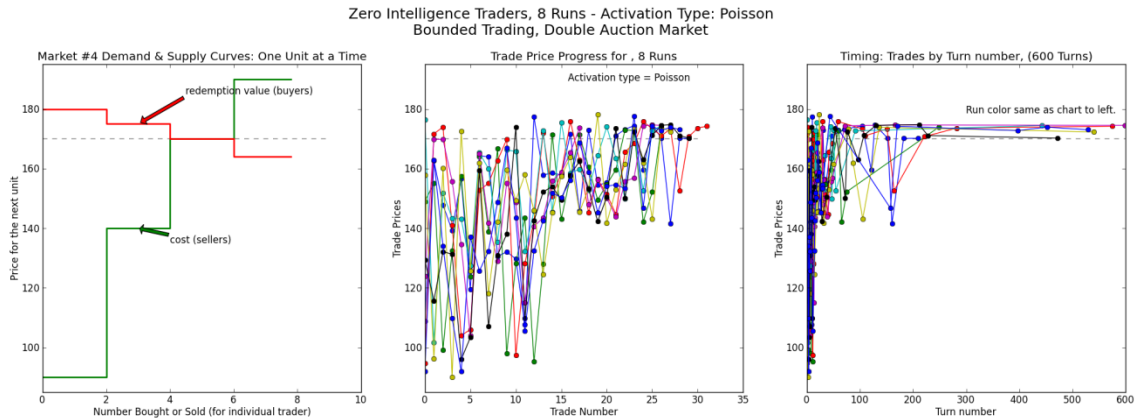
### Poisson Activation

Poisson activation is a process in which agents are activated according to an exponential distribution with an arrival rate,  $\lambda_A$ . This will mean that activations for any given agent are a Poisson process. In its simplest form, a Poisson activation scheme would have all agents activated with the same  $\lambda$ . This, however, would merely replicate the random selection method so I explore only the case of heterogeneous values for  $\lambda_A$ .

Poisson activation differs from other asynchronous methods in that this variation among the agents can be based on the state of each agent or some internal parameter value. For my explorations, I chose agent wealth, which was calculated at the beginning of each turn. Thus, agent activation rates are made proportional to agent wealth values. In order to investigate the ‘leveling’ nature of these computer-based trading markets – a key question for the original researchers – I chose to make activation rates proportional to the absolute distance between the agent’s wealth and the average wealth of the population of agents. In that way, agents that are at the extremes (rich or poor) will likely trade more often.

In order to make appropriate comparisons between Poisson activation and other activation methods, it is necessary to re-normalize all of the values of  $\lambda_A$  so that, on average, each turn there will be one full population of traders' activations. I accomplish this by building activation time for each agent and adding it to an 'event list'. Trader-agent activation times are drawn sequentially from an exponential distribution and each added to the previous until the times exceed 1.0. These times are then all sorted and the trader agent sequence that results from that is passed to the program as a list of activations. Offer-making proceeds in accordance with this list for a given turn. At the beginning of the next turn the values of  $\lambda_A$  are again calculated and another sequence is generated. The order of each turn's sequence is dependent on the current values of trader wealth and on a random draw.

This process works well once the model is established, but at the beginning of the model no trades have taken place and, thus, traders have no wealth. In these cases the values of  $\lambda_A$  are merely assigned randomly (and normalized as above). Once one trader has acquired some wealth the process can proceed as designed.



**Figure 20. Market 4 Inverse Poisson Trade Price vs. Trades and vs. Turns**

The Poisson process takes advantage of the ‘memoryless’ feature of the underlying exponential distribution. Thus, for every trader at the beginning of each turn can treat the ‘wait time’ as starting anew. It does not matter, given the waiting time is exponentially distributed, how long each trader has been waiting since the last activation.

### Inverse Poisson Activation

The process of activating agents faster if they are further from the average has an interesting counterpart: activation rates that favor proximity to the average. Thus, it is interesting to examine a  $\lambda$ -setting process that slows down agent activations when the trader wealth is farther from the mean wealth. This inverse Poisson activation rate is the fourth activation scheme to be examined in the four markets.

It is important to note that the two Poisson schemes represent a conceptual departure from the other two asynchronous schemes. In varying the activation rate based on the agent state, I am examining *endogenous* activation. At least one article (Fernández-

Gracia et al., 2011) has found that this can show differences in outcome behavior when compared with the more normal exogenous activation.

### ***Outcome Behavior Metrics***

Gode and Sunder do not rely heavily on precise quantification of the market results. This is consistent with their goal of measuring the performance of an automated market against that of a human market. They are trying to determine how much market efficiency (in profit creation and distribution) is due to the constraints of profit and loss rules and how much is due to human trading. Thus, they take the unconstrained automated market and the human market as two extremes and see where the bounded ZIT market falls. In most cases they judge that it falls much closer to the human market, but this is generally a qualitative judgment.

I chose to measure three aspects of the constrained ZIT market: its efficiency in generating wealth (or profits), its effectiveness in evenly allocating wealth among the traders, and the time it takes to reach equilibrium. Gode and Sunder used the first two measures in their paper, but the third left unexamined.

### **Wealth Generation**

It is a straightforward matter to measure total wealth at the end of a run. One of the key (and unstated) influences on this total is the length of a run. Gode and Sunder ran a trading ‘day’ for 30 seconds. In my runs, I made use of the turn structure to better standardize the runs, choosing 5000 turns as a standard run.

The total wealth in the market is compared with the total *theoretical* wealth. Smith's definition of market efficiency was used (Smith, 1962). Thus, the allocative efficiency of a market is expressed as the total profits earned in one run (added across all traders at the end of the run) divided by the maximum profits available. Actual human markets quickly converge to 99% efficiency. Markets only vary from this, the authors noted in 1992, when typographic errors in market orders create a distortion in the price time series. (Considering the events of the past two decades, the Gode and Sunder paper should have been seen as an important early warning of such market 'errors'.)

### Profit Allocation

The second metric chosen by Gode and Sunder was the profit allocation among the traders. To determine this, they calculated the cross-sectional root mean squared difference between the actual and the equilibrium profits across the traders. They defined the value  $a_i$  as the profits (or total wealth) acquired by trader  $i$ . They also calculated the theoretical profits for this trader as  $\pi_i$ . Thus, the dispersion across all traders becomes

$$D = \sqrt{\frac{1}{n} \sum_i (a_i - \pi_i)^2} \quad (2)$$

They left unstated how they calculated the equilibrium values. I divided equilibrium profits into those for buyers and those for sellers. I assumed buyers' equilibrium profits as the profits they could earn if they traded all the shares they could at the market clearing price. This, of course, would only include those shares with a redemption value above the market clearing price. Similarly, the sellers values of  $\pi_i$  was determined as the profits a seller would earn if all those shares held with costs below the market clearing

price were sold at the market clearing price. Thus, to calculate  $D$ , it is necessary to separate the calculation of the sum into two parts. More correctly, it should be:

$$D = \sqrt{\frac{1}{n} \left[ \sum_s (a_s - \pi_s)^2 + \sum_b (a_b - \pi_b)^2 \right]} \quad (3)$$

Where  $s$  = seller  $s \in S$  and  $b$  = buyer  $b \in B$ . Also,  $n$  = the total number of traders. This separation is necessary because the supply and demand curves are not symmetrical. Sellers' equilibrium profits differ from those of buyers in essentially all markets.

#### Time to Last Trade

Gode and Sunder did not examine the model behavior over the long term for a variety of reasons. They were comparing simulated markets with actual human experiments. The human experiments had a finite duration because they were limited by many factors that are not present in simulations. Thus, the simulated markets were truncated and the long-term data are missing (or, in the terminology of statistics, the data were 'censored').

In my examination of the markets, I expected to run the markets to exhaustion. That is, I experimented with a number of lengths of runs in the random and uniform activation types to find a reasonable point at which trading ended. I chose what I believed was a conservative length of 5000 turns, believing this would encompass all trades for all markets and all activations. As noted in the result section, there was still censored data even at these extended runs. In fact, this represents a major difference among the activation schemes. Thus, while I didn't collect a comprehensive set of data, analysis of

the turn at which the ‘last trade’ took place certainly achieved one of the key goals of this project – differentiating among activation schemes.

### ***Model Results***

A full spectrum of experiments was run: four activation schemes across four markets. Each experiment consisted of 2000 runs of the market and activation, with each run including 5000 turns. At the end of each run, total wealth, wealth dispersion, and the turn of the last trade were collected.

Histograms of the first two measures (total wealth and wealth dispersion) for the 2000 runs are shown in the following figures for Market 1. Similar histograms are available for the other three markets, and are included in the appendix.

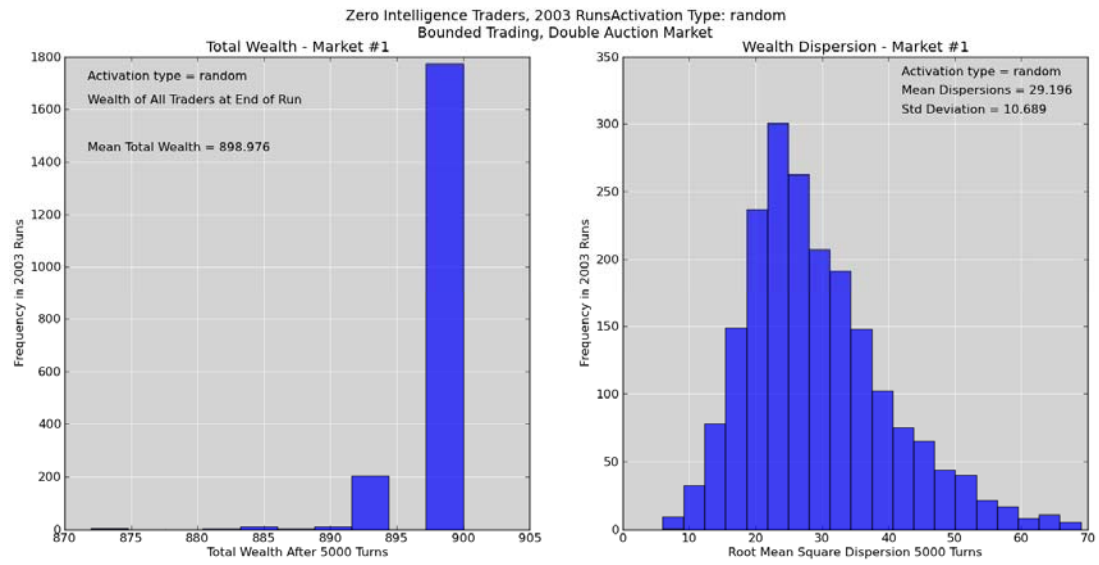


Figure 21. Market 1 Random Results (Wealth)

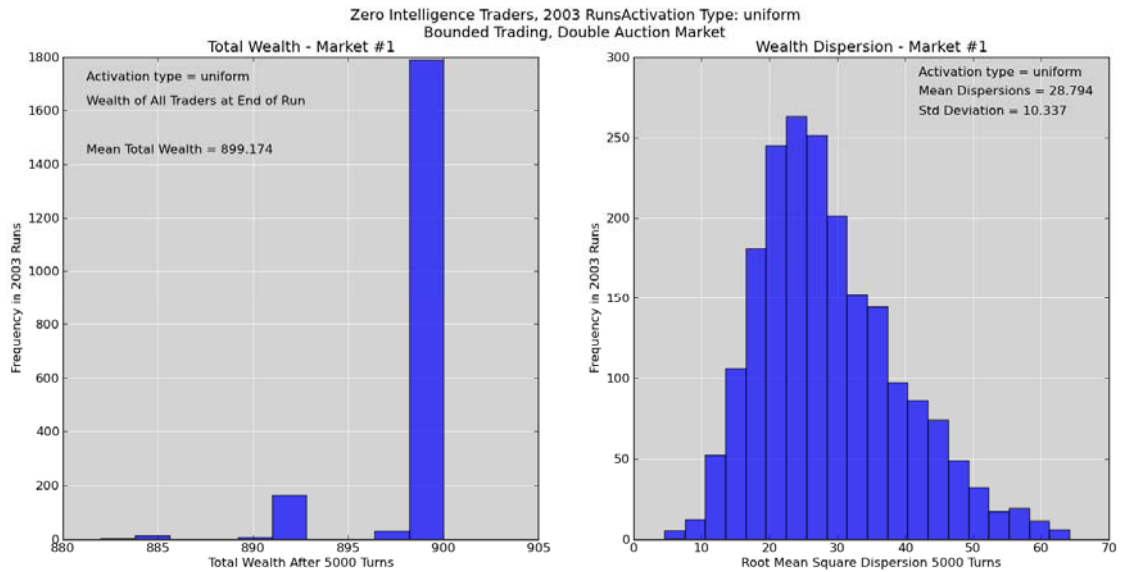
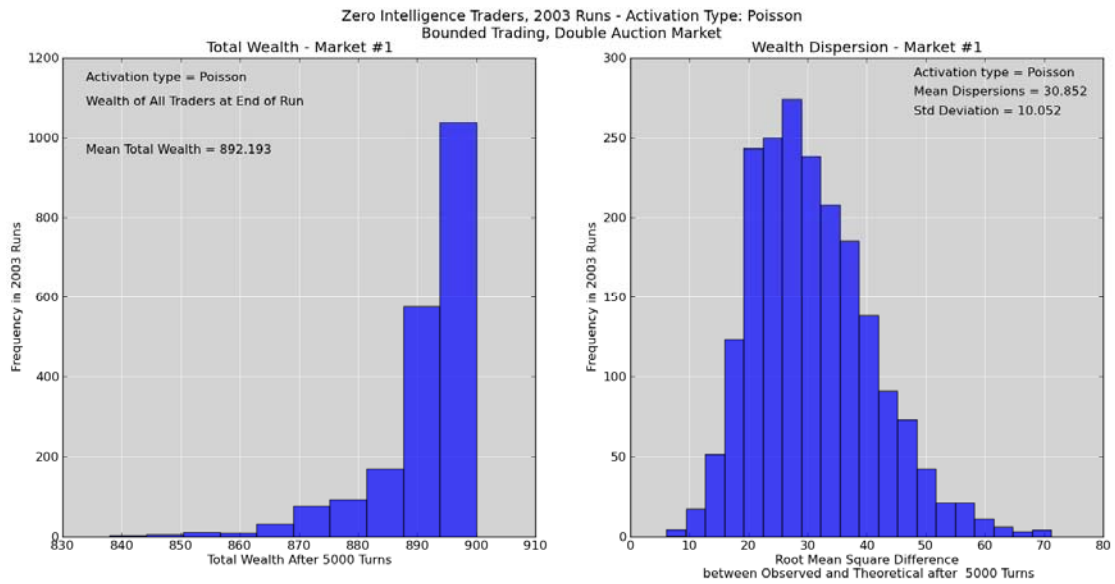
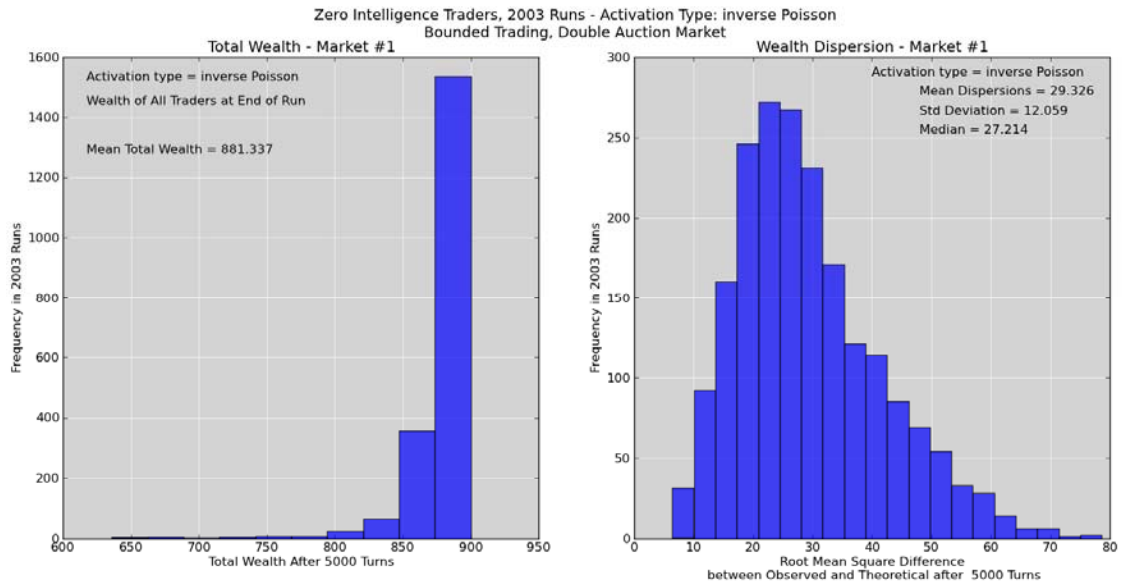


Figure 22. Market 1 Uniform Results (Wealth)





**Figure 23. Market 1 Poisson Results (Wealth)**



**Figure 24. Market 1 Inverse Poisson Results (Wealth)**

It should first be noted that the scales of all four histograms are not equal. Unequal scales were chosen for display so that the shapes of the histograms could be fully examined and compared. The histograms associated with random and uniform activation schemes are far less spread out than those with Poisson activations. As will be observed in the last-trade analysis below, this is because the random and uniform markets normally run to completion. For Market 1, this means a total wealth of 900. After the traders have achieved that there are no more profitable trades remaining. In a few runs of the random and uniform markets there are still trades left to be made, and each of these falls a set distance from the maximum profit. This means that these markets end in one of a small number of configurations.

The endogenous markets, however, have total wealth outcomes that are spread much lower from the maximum. This is mostly due to the fact that there are remaining trades available to these markets, even after 5000 turns. Thus, the average total wealth among the four activation schemes varies significantly.

**Table 4. Average Total Wealth - All Markets, All Activations**

<i>Average Total Wealth</i>		<i>Market</i>		
Activation	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
Random	899.0	1016.6	791.6	1497.7
Uniform	899.2	1017.2	791.7	1498.2
Poisson	892.2	1003.2	785.4	1480.5
Inverse Poisson	881.3	999.6	785.1	1488.8
Max Wealth	900	1020	792	1500

Market 4 is an interesting case in which the average total wealth for inverse Poisson

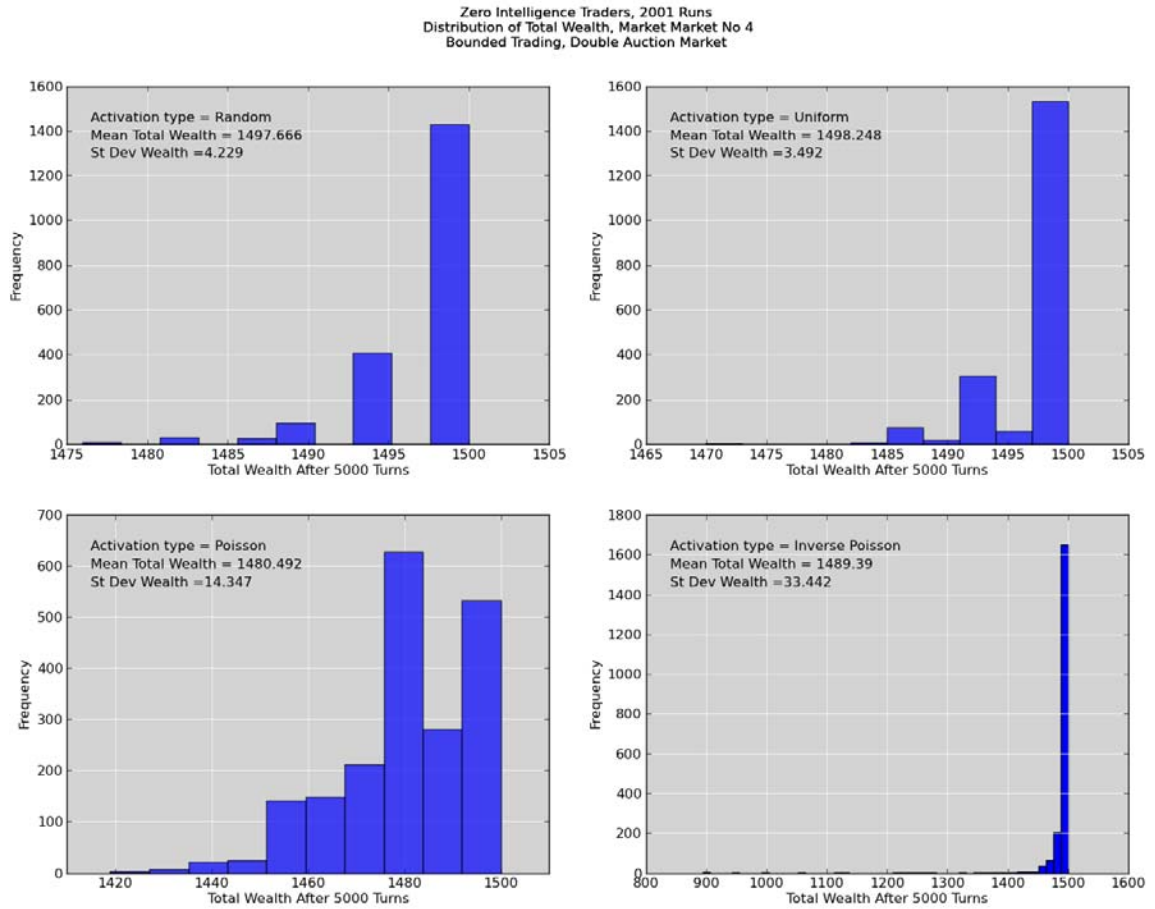


Figure 25. Market 4 Total Wealth Histograms

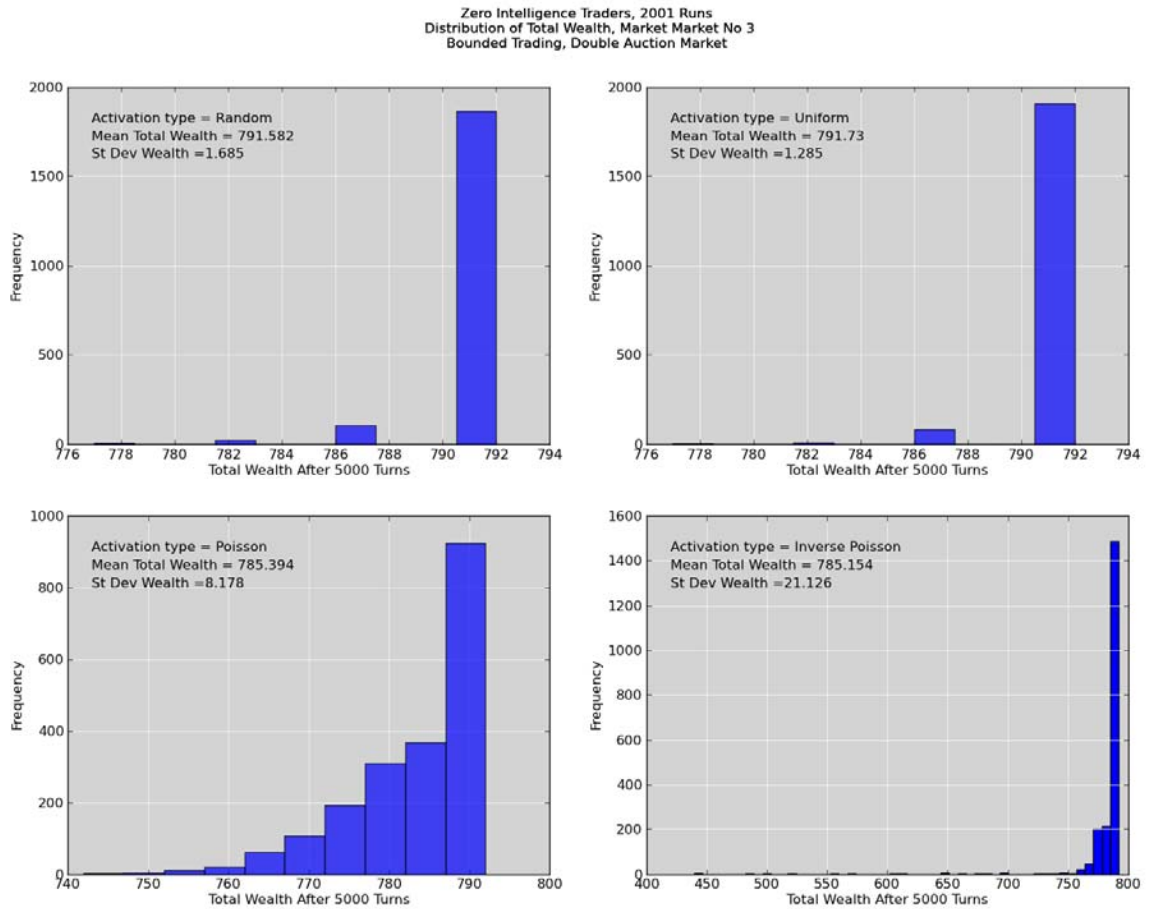
over the 2000 runs is larger than the average total wealth for the Poisson. Figure 25 shows the four histograms of total wealth for Market 4. The inverse Poisson activation does have an extreme value of an outlier below 400, but it actually bunches much of the wealth closer to the maximum value (1500) than its Poisson companion.

With 2000 runs, it is possible to test the hypothesis that these means are drawn from different populations against the null hypothesis that the variation is simply due to random errors (and that the random errors are normally distributed).

**Table 5.  $p$ -values for Average Total Wealth**

<i>p-values</i>	<i>Market</i>			
Comparison	1	2	3	4
<i>Random - Uniform</i>	0.021	<u><b><math>1.3 \times 10^{-05}</math></b></u>	0.035	<u><b><math>4.09 \times 10^{-05}</math></b></u>
<i>Random - Poisson</i>	<u><b><math>7 \times 10^{-200}</math></b></u>	<u><b>0</b></u>	<u><b><math>8.9 \times 10^{-214}</math></b></u>	<u><b>0</b></u>
<i>Random - Inverse Poisson</i>	<u><b><math>2.3 \times 10^{-242}</math></b></u>	<u><b><math>1.5 \times 10^{-124}</math></b></u>	<u><b><math>7.1 \times 10^{-42}</math></b></u>	<u><b><math>1.2 \times 10^{-31}</math></b></u>
<i>Poisson - Inverse Poisson</i>	<u><b><math>9.8 \times 10^{-103}</math></b></u>	<u><b><math>1.47 \times 10^{-07}</math></b></u>	0.525422	<u><b><math>3.2 \times 10^{-28}</math></b></u>

With four activation schemes there would be sixteen pairwise comparisons. It is not necessary to examine these exhaustively to see differences among the activation types. As Table 5 shows, most of these comparisons are highly significant. Even the random-uniform comparisons – the closest averages for all the markets – allow the rejection of the null hypothesis for markets 2 and 4. (While the averages are close, the power of the test is derived from the  $n = 4000$  combined data points for the pair.) Figure 26 depicts the four total wealth histograms (and the output data used to calculate the  $p$ -values) for Market 3. Again, note that the histograms are on a different scale, which I denote with a grey background.



**Figure 26. Market 3 Total Wealth Histogram**

Gode and Sunder compared the total wealth in the simulated markets to the maximum total wealth possible. This is shown on the final row of the wealth table for each of the four markets. Their objective was to compare how close the simulation came to maximum wealth with the proximity of the human markets. They deemed that their simulations across the four markets achieved essentially the same results as the human market, with efficiency percentages between 96 and 98%. These results were replicated

in all markets by all activation types. The lowest percentage was 97.9% in the case of the inverse Poisson in Market 1.

Similar analysis can be conducted on the much more bell-shaped wealth dispersion. They are shown (for Market 1) in the right-hand histograms in Figure 21 through Figure 24. The original researchers, in discussing the results of market dispersion, noted only that profits are dispersed among individual traders with a slightly larger spread (larger root mean square value) than for flesh and bone traders. Also, while human traders showed signs of learning, memory and adaptation were not part of the ZIT simulation. Each run was independent.

**Table 6. Mean Wealth Dispersion, All Markets, All Models**

<i>Average Wealth Dispersion</i>	<i>Market</i>			
Activation	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
Random	29.2	51.2	54.9	110.3
Uniform	28.8	50.5	53.2	111.0
Poisson	31.6	51.9	57.0	102.9
Inverse Poisson	28.7	51.8	56.8	110.9

While the wealth dispersion appeared to vary little across the runs, the large number of runs allowed me to determine that many of these differences were statistically significant. Using similar calculations to the averages of the wealth, we can develop another table of  $p$ -values. In this case, somewhat fewer of the pairings show differences that are significant. Market 3 shows some interesting behavior in that even the random – uniform comparison results in a difference that is significant at the 99% confidence level. For that reason, Market 3 is chosen for more in-depth analysis. Still, I reject the null hypothesis

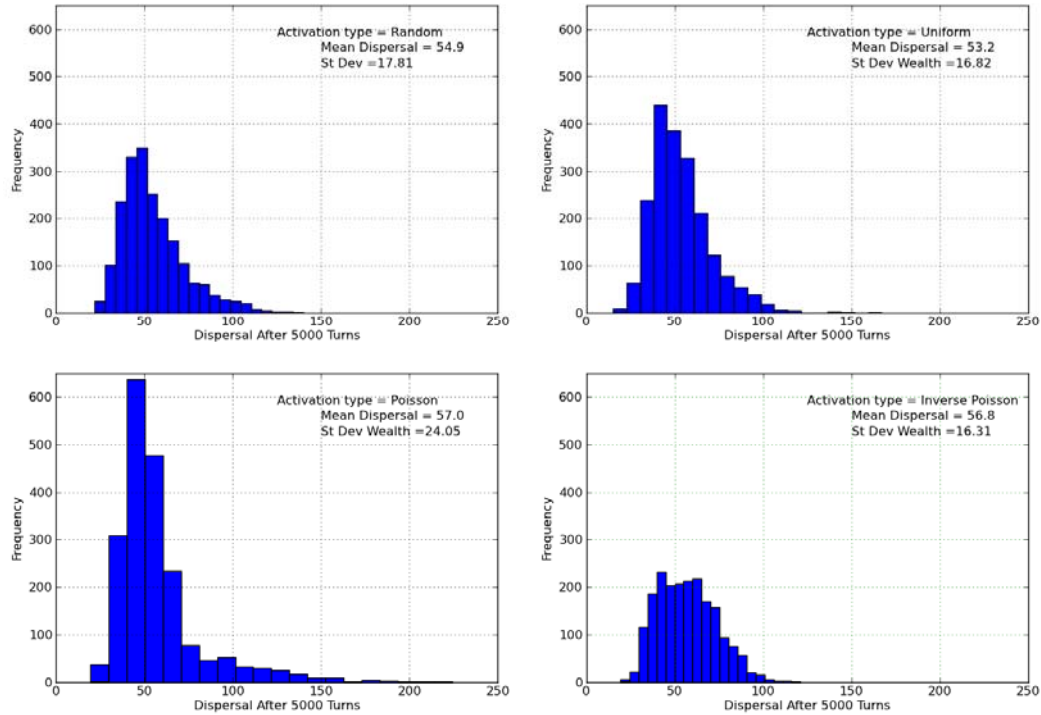
that the differences between these sample means is a product of random fluctuations in seven of the 16 cases examined. Activation type makes a difference, at least statistically.

**Table 7. *p*-values for Average Profit Dispersal**

<i>p-values</i>	<i>Market</i>			
Comparison	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
<i>Random - Uniform</i>	0.23	0.20	<b><u>0.002</u></b>	0.36
<i>Random - Poisson</i>	<b><u>1.7 X 10<sup>-12</sup></u></b>	0.278	<b><u>0.002</u></b>	<b><u>4.6 X 10<sup>-27</sup></u></b>
<i>Random - Inverse Poisson</i>	0.14	0.28	<b><u>4.0 X 10<sup>-4</sup></u></b>	0.457
<i>Poisson - Inverse Poisson</i>	<b><u>1.2 X 10<sup>-25</sup></u></b>	0.59	0.67	<b><u>1.3 X 10<sup>-49</sup></u></b>

First, I examine the histograms for Market 3 (Figure 27). These have all been adjusted so that they appear on the same *x*- and *y*-axis scales, which I designate with a white background. With the scales adjusted, it's clear that the histograms appear significantly different. The Poisson activation histogram shows a significantly larger tail than the others. This may not be apparent from the small size of the bars on the far right hand side of that plot, but the automatic adjustment of the graphing program clearly adjusts for larger bins for the Poisson case to accommodate the larger range of data.

Zero Intelligence Traders, 2001 Runs  
Histogram of RMS Wealth Distributions, Market 3  
Bounded Trading, Double Auction Market



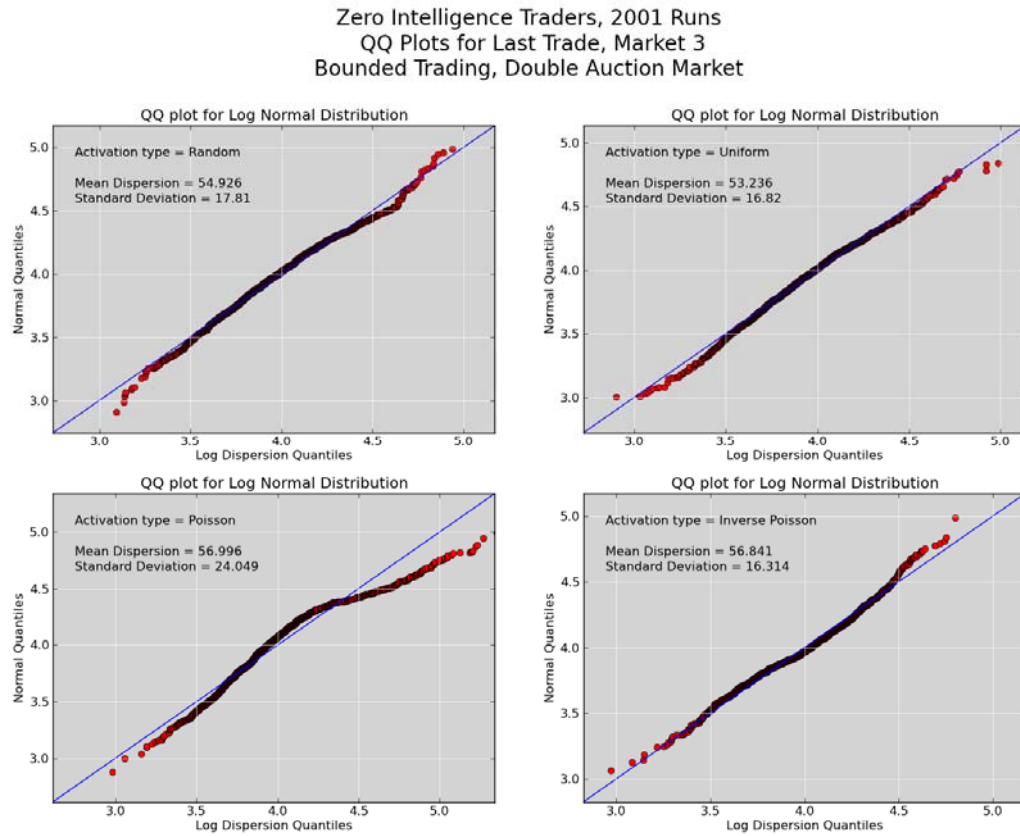
**Figure 27. Histograms of Market 3 Average Profit Dispersal**

In addition to the odd shape of the Poisson activation histogram, it's also clear that the inverse Poisson activation type has a much tighter bunch of averages. The means between the two are quite similar (57 and 56.8), but the standard deviation is substantially larger for the Poisson activation scheme. In fact, the inverse Poisson standard deviation is nearly equal to that of the exogenous activation types (random and uniform).

Figure 28 shows the QQ-plots for the same four activation schemes in the same market (3). The heavy right tail of the Poisson activation scheme is readily apparent. Also, the unusual shape of the inverse Poisson – especially its light right tail – stands out. And, it is clear that the log-normal distribution is a good representation for the random



and the uniform activation types. Even so, at the extremes the random exhibits light tailed behavior and the uniform exhibits heavy tailed.



**Figure 28. Market 3 QQ-Plots for Wealth Dispersion Averages**

The third and final result that I analyzed was an evaluation of these markets and activation schemes over the long term. Gode and Sunder did not consider the dynamics of their simulation during extended runs because they were comparing them with human traders in finite-time markets. I chose to record the turn at which the last trade took place before the end of run and use this as a metric for market closure. My initial expectation was that 5000 turns was more than adequate to capture all the trading that might be done for any activation scheme. In evaluating the results, it appears that 5000 turns was more

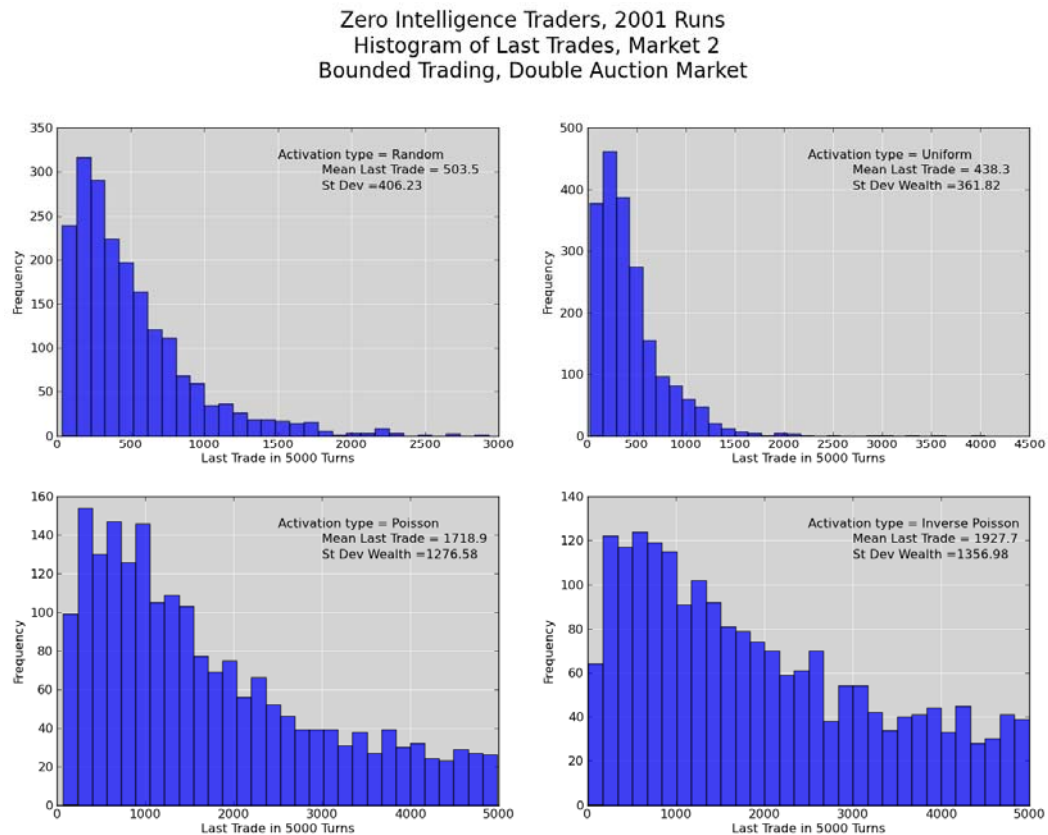
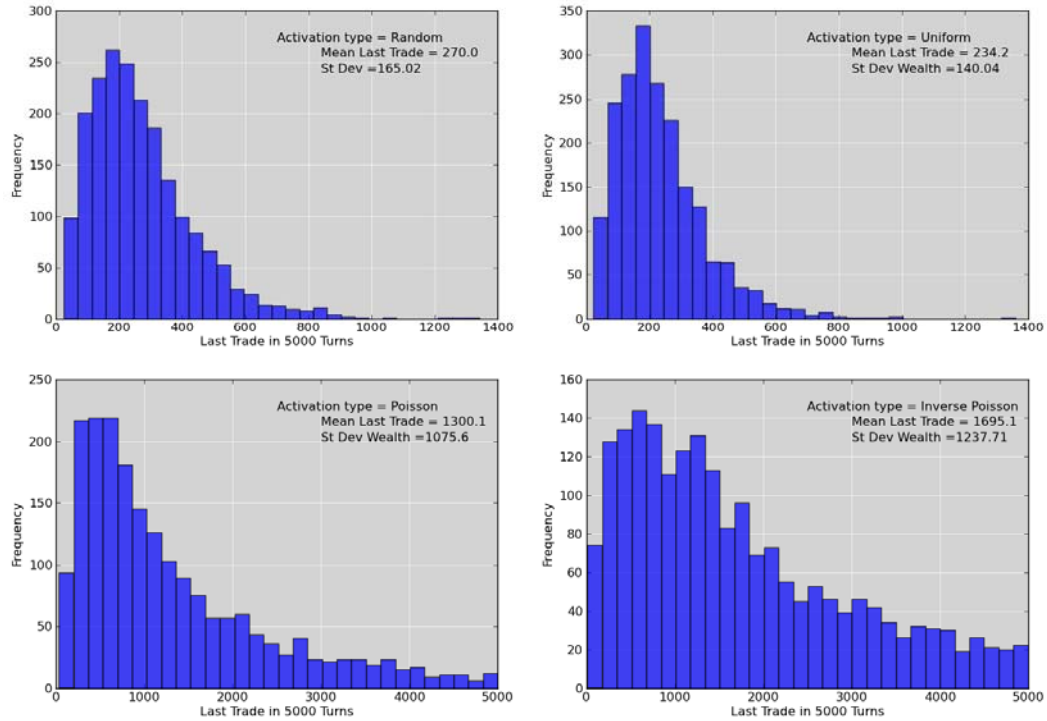


Figure 29. Last Trade Behavior Market 2 - All Activations

than adequate for the random and uniform activation methods, but that Poisson and inverse Poisson were still exhibiting trading behavior late during a 5000-turn run (!).

Figure 29 shows the behavior of all four last trades for the four activation schemes for Market 2, and Figure 30 shows the same for Market 4. Clearly the extent of the trading varies substantially as the activation type is changed. Not only are the histograms of somewhat different shape, the Poisson and inverse Poisson clearly have censored trading activity, especially in Market 2.

Zero Intelligence Traders, 2001 Runs  
Histogram of Last Trades, Market 4  
Bounded Trading, Double Auction Market



**Figure 30. Last Trade Behavior Market 4 - All Activations**

In Market 4, it still appears that trading activity would continue beyond the 5000<sup>th</sup> turn for Poisson and inverse Poisson, but the trend is somewhat less pronounced. It is important to note that this phenomenon would somewhat affect the analysis of such ZIT models, especially if trading were cut off after a few hundred turns. It is uncertain where Gode and Sunder stopped trading. They set their cutoff at thirty seconds of computer time, which itself might be a different measure for endogenous than for exogenous activation. In executing my simulations, the random and uniform experiments would take about half the time as the two Poisson activation experiments.

**Table 8. Mean Turn of Last Trade Over 2000 Runs**

<i>Mean Turn of Last Trade 5000- Turn Experiment</i>	<i>Market</i>			
Activation	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
Random	1377.2	503.5	415.1	270.0
Uniform	1273.4	438.3	357.6	234.2
Poisson	1919.3	1718.9	947.4	1300.1
Inverse Poisson	2124.9	1927.7	2240.6	1695.1

Table 8 shows a full factorial analysis of the actual values of the mean (over the 2000 runs for each condition) . The sizeable difference can be observed by inspection, but a complete analysis of the  $p$ -values confirms the statistical significance of the result. There is no pairing that has a  $p$ -value larger than  $5 \times 10^{-11}$ . Thus, it can be concluded that activation makes a potent difference in the later stages of the ZIT model.

### ***Case II Conclusions***

There are several motivations behind the basic research question – does activation change the outcome of agent-based models. The Case II excursions of the bounded ZIT model appear to answer different questions in different ways.

For the simple issue of analyzing statistical results, the analysis shows that for all three metrics (total wealth, wealth dispersion, and the last-trade parameter), there are statistically significant differences between at least some of the activation schemes, and for one metric there are significant differences among all of them.

The purpose of choosing a ‘real world’ model – one that moves beyond simple abstract agents engaged in mathematical game theory – is to observe the impact of differences on policy recommendations. The underlying purpose of the Gode and Sunder paper was to determine to what extent markets are made efficient by structural features (such as the requirement to make profitable trades) as opposed to the rational decisions of human traders. They determined, using qualitative (but quite reasonable) analysis, that the constrained ZIT simulation essentially replicated the efficiency of the human traders in achieving the total theoretical wealth. They also concluded that the simulated traders distributed the wealth close to but a little more than the human traders, at least in the early stages of trading. After a time, the human traders dispersed their profits more evenly, but this was undoubtedly due to the memory effect.

Would Gode and Sunder’s conclusions have been different if they used different activation schemes? Probably not:

- All activation schemes and all markets ended with a total wealth that was between 97.92 and 99.96% of maximum wealth.
- Profit dispersion has a somewhat higher variance for the endogenous activation patterns, so it is possible that, given that they only did six runs, the authors might have generated outlier results. If they increased the number of runs, however, they would have returned to their original conclusion (simulated ZIT traders produce slightly larger dispersion, but far closer to human traders than unconstrained trading).

Gode and Sunder did not examine the question of model convergence or trade evolution. Thus, they would not have noticed the significant differences that appear in the last-trade statistics among the different activation schemes.

A third motivation for evaluating the importance of activations schemes is in establishing a proper standard for research in which the agent-based models of one scientific team are replicated by subsequent researchers. The Gode and Sunder article was chosen because it appeared as a reference in 1171 subsequent articles. Clearly, many other researchers are at least working with the concept of simulating markets, and many are actually building agent-based models using the zero-intelligence trading paradigm. (None of those 1171 use the words “Updating” or “Activation” – or their derivatives – in the title, so activation is not a major research focus in this domain.) In the research reported above, the differential results from last trade analysis alone (if not all the results) show that if a replication of ZIT model is expanded beyond the work of Gode and Sunder, the results must be shown to be robust over different activation schemes. Thus, if agent-based researchers are to meet the standard of other sciences and work on replicating one another’s experimental results, then reports of their results must include the activation scheme used in the model.

### **Case III: Activation in an Interacting Particle System Swap Model**

The earlier cases evaluated the activation question either as a nuisance, hindering the accurate construction of replications of published models, or as a potential tool to replicate real-world behavior. It was also shown (in the ZIT model replication) that

activation may change the dynamics of a model so much that the design of the experiment needs to be reconsidered.

### ***The Theory – Simulation Partnership***

Varying activation can have another impact on modeling and system analysis: simulations with different activation schemes can allow researchers to examine system behavior that is more complex, variable, or heterogeneous than that postulated in a theoretical construct. In other words, activation can be one of the research variables that can be changed as we harness the power of simulation and move beyond equation-based descriptions of system behavior.

In this context, I examine theoretical models of system behavior that have their root in the domain of physics. Physics is a branch of science substantially different from that of civil revolt or financial markets considered above. At first glance it might appear to have little in common with these social systems. Complex adaptive systems researchers, however, have strongly benefitted from the theoretical approaches that are common in modern physics. The Santa Fe Institute, for example, was partly founded by physicists. Nobel Prize-winning physicist Murray Gell-Mann was the co-founder of SFI, and remains a distinguished fellow.

There is actually a rich collaboration between physics research and the analysis of complex adaptive systems. A number of summaries, perspectives, and histories of the field of econophysics have established the origins in the mid-1990s (Carbone, Kaniadakis, & Scarfone, 2007; Gingras & Schinckus, 2012; Roehner, 2010). In general,



econophysics looks to overcome the flaws of traditional macroeconomics by relaxing many of the assumptions that fuel mainstream economic analysis. In particular, econophysics moves beyond the assumptions of normal distributions in nature and identical or ‘representative’ individuals whose average can be determined and, through aggregation, determine the system-wide behavior (Cho, 2009). Some important examples of the application of physics to economics include

- The observation that non-equilibrium price time series (such as seen in most markets) can be replicated by a simple Ising model of a one-dimensional array of particles (Sznajd-Weron & Weron, 2002).
- Percolation theory has been broadly applied to help understand consumer adoption behavior and social influence on economic trends (Kiesling, Günther, Stummer, & Wakolbinger, 2012). They have also been important in depicting the influence of herd behavior on market fluctuations (bubbles and crashes in particular) (Cont & Bouchaud, 2000).
- In analyzing price time series in markets, Benoit Mandelbrot noted that there exist extremely long memory effects. To measure these, he drew from the geophysical field of hydrology, which measures long memory in time series using the Hurst exponent (Hurst was a civil engineer working on flood control in the Nile River valley) (Mandelbrot & Hudson, 2004).

### ***Interacting Particle Systems as Social Models***

In this chapter I will focus on interacting particle systems and their analogs within the social science world. This relationship has stirred considerable interest among advanced researchers who seek to combine the mathematics with complexity theory. One team, for example, claims to demonstrate that the field of ‘out-of-equilibrium’ statistical physics is uniquely appropriate for understanding complex system dynamics. Such a fusion of the fields can help to explain the ubiquitous appearance of non-stationary and non-ergodic statistical processes and inverse power-law statistical distributions (West, Geneston, & Grigolini, 2008). Markov processes have proven useful in understanding and modeling the negotiation process (Weingart, Prietula, Hyder, & Genovese, 1999). More afield (and, perhaps less directly related), the broad field of Markov Chain Monte Carlo methods, and its combination with evolutionary algorithms, has also been applied to the information exchange process. While the models bear little resemblance to agent-based models, the extensive application of ideas first developed for physics and chemistry to search algorithms (among other problems) (Laskey & Myers, 2003) show that this is a fecund combination of disciplines. Another large field that uses concepts of particle physics and recently employs the insights of Markov chains is the question in signal processing of finding the best filter (Lee & West, 2013).

The fusion of stochastic particle physics and social systems analysis is also taking place in the opposite direction: social scientists are finding new applications that bring the rigor and mature theory to their emergent problems. For example, Cai and Ishii have started with straightforward social science questions – the formation of a consensus and

the distribution of wealth – and solved the question of convergence using defined and quantized Markov chains (Cai & Ishii, 2012). In their conclusions, however, the authors point to a major issue in applying this extensive mathematical treatment to real world situations. In their final remark (Remark 16), Cai and Ishii note that extending their results becomes difficult if the topologies of agent interactions are less well defined. They don't mention this, but if the agent interaction topologies are inconstant in time, extension of this mathematical approach may be unachievable.

### ***Theoretical Baseline: Assumptions, Derivations, Predictions***

Interacting Particle Systems have been well-defined mathematically. An elegant theory based on the statistics of continuous time Markov chains provides mathematical solutions (once the system parameters are known) for convergence rates, steady-state distributions, mean arrival times (for a given state), and other outcome behaviors of interest.

Aldous, in exploring this theory, blends interacting particle systems with social systems analysis, and draws an analogy from game theory (Aldous & Lanoue, 2012). Game theory has its origins among physicists, but is now broadly applied to social science issues. Moreover, the field is characterized by a small number of simple games which have an abiding importance across a broad range of domains: Prisoner's Dilemma, Tragedy of the Commons, Battle of the Sexes, etc.

Aldous has noted that the extension of interacting particle systems is also based on the application of a small number of straightforward models. IPS are characterized by a common structure:

- A large population of agents – normally taken to represent individuals.
- A network or graph that defines the connections among these individuals.

Commonly, the edges of this graph are weighted.

- A meeting model that interprets the weights of the edges as the frequency of meeting
- A meeting algorithm in which the agents exchange information, possibly changing their state in the process.

The last point has motivated Aldous to coin the name Finite Markov Information Exchanges to describe the specialized application of IPS to social science. He notes that the lack of a common name has probably limited the impact of such research, and hindered the formation of a community of practice similar to those who work in the area of game theory.

Aldous (2013) defines interaction rates as the symmetric matrix  $\mathcal{N}$ , which has zeroes in the diagonal (agents don't have meetings with themselves). The non-diagonal elements are defined in terms of their meeting rates,  $v_{ij} \geq 0$ . In order to use the same techniques used to characterize Markov chains,  $\mathcal{N}$  is assumed to be a stochastic matrix, with normalized rates of interaction, so that:

$$\nu_i := \sum_j \nu_{ij} = 1 \text{ for all } i.$$

Note that, while Aldous defines the diagonal elements equal to zero as part of the structure of his problem, this is not part of the definition of a stochastic matrix.

$\mathcal{N}$  also defines a geometric substructure for the interactions. It can take on any form, but Aldous limits his analysis to the most common form. Here I consider only his first topology, which he terms the complete graph or mean field model. In this case, every node or agent has an equal likelihood of interacting with every other. Thus,  $\mathcal{N}$  is defined by:

$$\nu_{ij} = 1 / (n - 1), \quad j \neq i$$

Aldous also considers other, more complicated topologies including small worlds or random graphs, but my analysis is limited to this straightforward case.

First consider an explicit description of how continuous time Markov chains are expressed mathematically. The objective is to define a method for stating the Markov chain transition probability matrix for a continuous-time Markov chain. Starting with an analogy that the eigenvalues of an invertible square matrix  $A$  are those values of  $\lambda_i$  that solve the equation:  $\lambda_i A = \lambda_i v_i$  where  $v_i$  = the associated eigenvector. Now, consider a Markov transition process (and associated probability matrix) in which the system operates in continuous time (but still with a finite, countable state space). Thus, the transition matrix would not be a matrix of discrete probabilities – the probabilities of

moving from one state to another in one time step. It would, rather, be a continuous function of time,  $P(t)$  such that the probability the system is in state  $j$  after time  $t$ , given that it is in state  $i$  at time 0, is  $p_{ij}(t)$ .

In order to analyze  $P(t)$ , a matrix  $Q$  is defined such that  $P(t) = e^{tQ}$ . This notation, raising a matrix to a power, is a shorthand for an infinite series on the exponential of  $Q$  that is analogous to Euler's formula:

$$e^Q = \lim_{x \rightarrow \infty} \left( \sum_{k=0}^x \frac{Q^k}{k!} \right)$$

We also know the following:

$$e^{nQ} = (e^Q)^n = P^n$$

and

$$\frac{d}{dt} P(t) = P(t)Q \tag{4}$$

So, the  $Q$ -matrix for a complete graph pattern, in which an agent has equal probability of interacting with each of its partners, is given by:

$$\begin{bmatrix} -1 & 1/n-1 & \cdots & 1/n-1 \\ 1/n-1 & -1 & 1/n-1 & \cdots \\ \cdots & 1/n-1 & \cdots & \cdots \\ 1/n-1 & \cdots & 1/n-1 & -1 \end{bmatrix}$$

Solving for the eigenvalues of this matrix are 0 and  $-\left(n + \frac{1}{n}\right)$ . It is important to note that the non-zero eigenvalues approach unity as  $n$  becomes large.

This exposition is important for follow-on analysis. The Markov chain generating matrix,  $Q$ , can be interpreted as a rate-flow matrix in a continuous time Markov chain. It is also the matrix that generates the Markov chain transition matrix,  $P$ . Note that, while  $P$  is a stochastic matrix,  $Q$  is not. (Norris, 1998, p. 64)

### ***The “Leveler Model” – Theoretical Development***

Aldous used the structure of continuous-time Markov chains to complete his understanding of convergence in a social problem he deemed the “averaging process” (Aldous & Lanoue, 2012) or the “Leveller” problem (Aldous, 2013).

In Leveler, each member the population is endowed with an account of ‘wealth’, which normally begins as differentiated. At each meeting, the two interacting agents reset their individual wealth to the average of their two accounts. Clearly, over time, the population will converge to the point where every agent has the same wealth, especially if all  $v_{ij} > 0$  if  $i \neq j$ . In fact, the population will converge to the average wealth in any case where all states communicate in the Markov Chain transition matrix. Additionally, with this rule all wealth in the system will remain constant, as will the mean wealth.

This leads to a theoretical result in which, given an unchanging meeting matrix,  $\mathcal{N}$ , the population’s convergence – measured as the decay of the standard deviation of wealth to zero – is defined by the Markov chain processes. To begin, Aldous rewrites the

definition of  $\mathcal{N}$  such that it is a matrix of transformation *rates* in which the rows sum to zero. Thus, he revises the definition, defining the transition rate from  $i$  to  $j$  as  $v_{(i,j)}$ . From this, he establishes the matrix as:

$$v_{ij} = v_{(i,j)}, i \neq j; \quad v_{ii} = -\sum_{j \neq i} v_{ij} \quad (5)$$

This is, of course, no longer a stochastic matrix. In fact, from the theoretical development of Markov chain analysis, this is equivalent to the generating matrix,  $Q$  (Norris, 1998). Aldous goes on to develop a theory of convergence rates that depend upon this new  $\mathcal{N}$ , which will be here denoted as  $Q$ . Aldous shows that the convergence rate (under all the previously stated conditions of stationary transition probabilities and finite, countable states), that, if the convergence is measured in terms of the standard deviation of wealth, it is bounded in its convergence to zero.

The notation used in Aldous is a bit different from that used in normal statistical treatments. In his initial conditions, Aldous assumes that the average wealth is zero. This will mean, of course, that the average wealth at all times is zero as the Leveler process does not change the mean wealth. This simplifies Aldous's mathematical notation to the more familiar statistical notation. Given a vector in which the mean value is 0, that is:

$$\frac{1}{n} \sum_{i=1}^n x_i = 0 \quad (6)$$

Aldous defines the “norm”, which is equivalent to the standard deviation.



$$\|\mathbf{x}\|_2 := \sqrt{\frac{1}{n} \sum_i x_i^2} = \sigma_x \quad (7)$$

Thus, where  $\sigma_w$  is the standard deviation of the wealth at time  $t$  and  $\sigma_0$  is the standard deviation of the wealth at time  $t = 0$ , Aldous shows that the convergence is determined by:

$$E[\sigma_w(t)] \leq \sigma_0 e^{-\lambda t/4} \quad (8)$$

where  $\lambda$  = the spectral gap of  $Q$ . Finally, as noted above, as  $n$  becomes large, the non-zero eigenvalues of  $Q$  approach unity. Thus, the exponential decay rate in Equation 8 will be approximately -1/4.

It is important to consider whether this convergence rate, dependent on  $\lambda$  is a function of the number of agents. This depends on how  $Q$  is defined, and, thus depends on the definition of the meeting rates. Moreover, the meeting rates are determined by the definition of the unit of time. If the rates are set as above, a unit of time is defined as that amount of time such that, on average, one interaction will take place among the all the agents in a single unit time. If time were defined in such a way that *each* agent would initiate a meeting once per unit time,  $Q$  would be a matrix with all non-diagonal elements equal to one, and the diagonal elements equal to  $n - 1$ . The non-zero eigenvalues of such a matrix would equal  $-n$ , and the spectral gap and the convergence rate would certainly

vary with the scale of the system. Thus, the definition of time units becomes a key constituent in moving from the mathematical definition of the system to its simulation.

### ***Extending Theory Through Simulation***

A commonly-used technique in operations research and systems engineering is to start with a well-developed mathematically-defined system and build a simulation. The simulation will allow the researcher to relax the assumptions of the model, through the design of the code, and examine system behavior. In the general case this allows the operations research analyst to leverage mathematical prediction and extend the range of quantitative analysis. (Simulation is also used to extend the insights gained from physical experimentation, further adding to the utility to decision-makers and the broad confidence non-academic professionals place on simulation.)

Agent-based models also have been used extensively to evaluate the diffusion of information in a population. Herrmann, et. al., have recently modeled the diffusion of urgent information (weather warnings or high-profile news events) on a network using an agent-based model (Herrmann, Rand, Schein, & Vodopivec, 2013). Hui, et. al. simulated the diffusion of evacuation warnings to a population of agents connected via a network. A simulation approach was necessary because, as agents evacuated, the network topology would change (Hui, Goldberg, Magdon-Ismail, & Wallace, 2010). Rosvall and Sneppen explored the exchange of information in an agent-based model of a dynamic network (Rosvall & Sneppen, 2003). And, Cui and Potok, using an agent-based swarm-type model of insurgency showed that information exchange among disparate, self-organized groups

can be just as efficient as in a hierarchical insurgency with unified leadership and strategic planning (Cui & Potok, 2007).

### ***Agent-based Model and Varying Activation***

The Leveler theoretical model assumes that the meeting matrix or transition matrix  $\mathcal{N}$  (or its generating matrix  $Q$ ) remains unchanged during the course of the model. It has no concept of ‘turn’ in which a full population of agents are activated. The model evolves in ‘secular’ time, and all  $n$  agents activate in accordance with their own Poisson process. Most of these assumptions are made in order to make this elegant derivation of the convergence rate as a closed-form inequality possible.

Do these conditions exist in the real world? Aldous cautions researchers who extrapolate these abstract models to real-world movement of knowledge in a population. Information does not take on well-defined scalar values (such as wealth in the Leveler model), and individuals find many ways to move information beyond a simple meeting protocol (Aldous & Lanoue, 2012).

To capture some of the non-abstract real-world behavior, a Leveler model was created in Python. The convergence of wealth – as measured by  $\sigma_w$  -- was examined using different activation schemes:

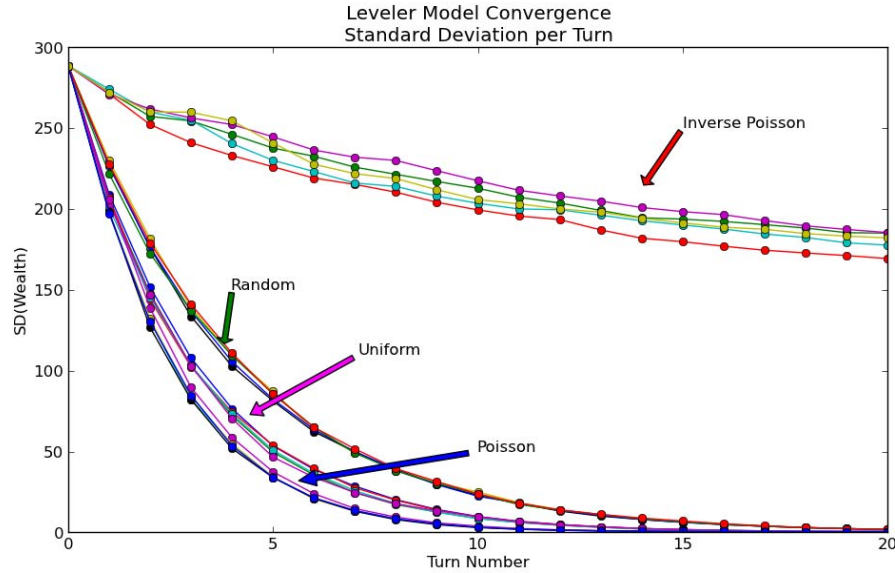
- Uniform activation creates a sequence of pairs from the population through sampling without replacement. The pairs leveled their wealth when they were activated. One turn is defined as activating the entire population (in pairs)

exactly once. (Odd-numbered populations will have one inactive agent in each turn, randomly assigned.)

- Random activation involves selecting pairs of agents from the population *with* replacement. A turn is defined as complete when a full population has been activated, or after  $n/2$  pairs have been selected.
- Poisson required the determination of the activation rate,  $\lambda_i$ , for each individual agent. These rates were normalized at the beginning of a turn so that, on average, one population's worth of agents would be activated on each turn. Thus, the mean  $\lambda$  would be  $1/n$ . Also at the beginning of a turn, a Poisson process was populated for each agent in accordance with the individual arrival rate,  $\lambda_i$ . These arrival times were placed in sequence on an 'activation table'. By design, the average number of agents on each turn's activation table was one population's worth of agents. The leveling process took place by selecting the agents from the table two at a time. At the beginning of the next turn, agents' values of  $\lambda_i$  were recomputed. Several rules are possible to determine this  $\lambda_i$ . I chose to make  $\lambda_i$  proportional to the distance between the individual agent's wealth and the mean wealth. Those furthest from the mean wealth would activate more frequently, those closest to the mean would activate at a slower value of  $\lambda$ .
- Inverse Poisson activation merely reversed the above rule. Those agents closest to the mean would activate fastest while those furthest from the mean (the richest and the poorest agents) would be the least likely to 'share the wealth'.

It should be noted that there are many other options for establishing the values of  $\lambda_i$ . In order to create a distribution of wealth, each agent was endowed with a wealth ‘account’ equal to his index value. Thus, the first agent started with a wealth of 1 and the 1000<sup>th</sup> agent began with a wealth of 1000. Thus, the average wealth was 500.5, and the initial standard deviation was 288.675. Five runs were conducted for each activation scheme.

## Results



**Figure 31. Convergence in the Leveler Model**

As shown in Figure 31 the four activation schemes resulted in markedly different convergence rates. As the exponent of decay was the most important for these time series, shows the average coefficient of the time variable. Note that, for this simulation, time is defined in turns. In the theoretical construct, turns do not exist and time is defined in terms of the individual agents Poisson process.

**Table 9. Decay Rates for Leveler Model by Activation Type**

<i>Decay Rate</i>	<i>Experimental Run</i>				
Activation	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>
Random	-0.2511	-0.2550	-0.2506	-0.2535	-0.2534
Uniform	-0.3455	-0.3520	-0.3399	-0.3467	-0.3472
Poisson	-0.4566	-0.4597	-0.4597	-0.4543	-0.4566
Inverse Poisson	-0.0190	-0.0196	-0.0168	-0.0195	-0.0203

Thus, it is clear that these average decay rates differ consistently, and that they are quite stable once activation has been set. Further, the decay rate from the random activation process closely tracks the theoretical rate of  $-1/4$ . The obvious conclusion that the runs are different can be confirmed by a Fisher Exact test of any of the five runs compared with any other of the five runs would give a  $p$ -value of 0.004. (This could have easily been driven smaller with more runs, but the outcome is rather obvious from Table 9 and Figure 31.)

### ***Conclusions***

The most important result was the replication – through simulation – of the theoretical result. The alignment of the random activation scheme decay rate with the theoretically-predicted value of  $-1/4$  implies that random activation most closely represents the natural process described in the theoretical model. Agents interact in accordance with their own, internal “clocks”, unaware of the actions of other agents. It also validates the conventional definition of a ‘turn’ as a population’s worth of agent activations. While that definition might have seemed contrived, it does appear to align with the system behavior predicted by the MC model.

This result is important because it allows the use of the theory-simulation analytic paradigm. The theoretical development led to the conclusion that, given the appropriate definitions of time and standard deviation, the mean convergence rate for a wealth-averaging system should be ‘no greater than’  $-1/4$ . Thus, as the assumptions about homogeneous, constant activation are relaxed, the impact on convergence can be observed through simulation. Simulation, therefore, can be used to extend the analytic reach of theory in such models. And, it has been shown, changing activation does impact the outcome patterns of this simple model. It is reasonable to assume that more complex models might see similar differences and experiments should be conducted to investigate such differences.

While it is important merely to show that there are differences, it is also interesting to note that the differences are not of uniform magnitude. Clearly the inverse Poisson convergence rate is very much less (in absolute value) than the other convergence rates. Inverse Poisson activation was based on the assumption that the agents with the most extreme wealth would enter the wealth-swapping process the slowest.

These results suggest that the choice of activation pattern can become an important tool for researchers attempting to simulate real-world self-organizing systems. That is, rather than treating activation as an arbitrary and confounding choice, it can become a treatment parameter for exploring various emergence phenomena. Often agent-based models are built in an attempt to mimic real-world behavior. It would not be unusual for the model-builder to grow acquire data or insights into the real-world activation patterns

of individuals. This might come from theory or there may actually be empirical data. If, in the real world this data is not stationary, then the researcher would have a tool to adjust the model structure to match behavior. This is especially true in the case of endogenous or state-based activation. In fact, most intuitive expectations for real-world systems would assume that activation would be based on state: diseased individuals will interact more rarely than healthy people; wealthier people normally trade stock with greater frequency, etc. If these differences can be parameterized, the activations schemes denoted here can help create a better model.

The clustering of the convergence rates within the activation types is somewhat of a surprise, varying much less among runs with the same activation than between runs with different activation. Clearly activation has a dominant effect on this model's variation. Thus, if one were to choose to create a model based on this construct (Leveler has been proposed as a real-world scenario for the movement of gossip across a population), then various activation schemes should be explored and reported.

The original authors did not propose the leveler model as a potential input to policy of any sort. Thus, unlike the two earlier cases, there is no straightforward connection between these differences and policy recommendations. Even in this abstract case, however, there are hints toward a real-world impact on decisions and policies. Often tax or other financial policies have the goal – stated or unstated – to redistribute wealth among the population. In fact, the leveler model can be seen as a simple – and perhaps the simplest – redistribution model. It converges at the theoretical rate if the individuals



in the society are simply mechanical agents, each obeying the swap rules without adaptation, taking their place in line to swap their wealth at the appropriate but externally determined time. A more natural behavior, however would be to allow wealthier agents to reduce their rate or probability of doing a swap. This would cause the redistribution to take place at a different and possibly slower rate. In fact, this is shown on Figure 32. The curve marked “Natural” is the convergence rate if the activation rate were inversely proportional to the agent’s wealth. Thus, the wealthiest would resist the most and the poorest would enter the swap process the fastest. In fact, this result falls half way between the original Poisson scheme (agents furthest from mean wealth activate faster) and the inverse Poisson scheme (agents nearest to the mean wealth activate faster).

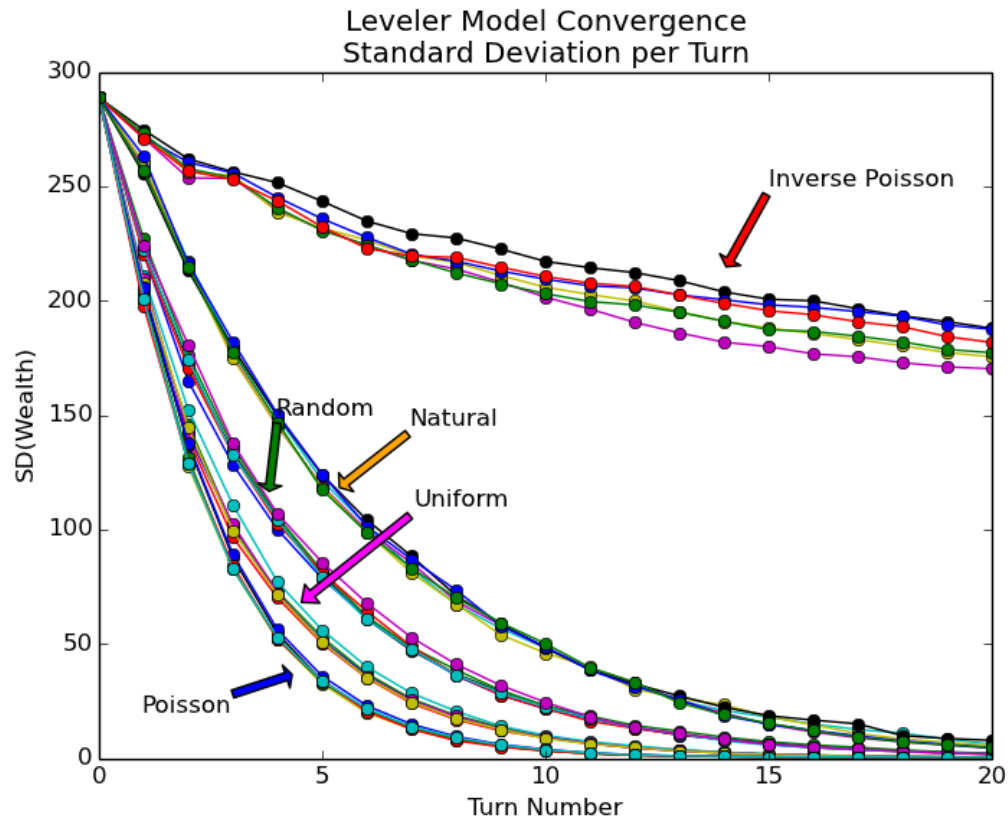


Figure 32. Leveler with Natural Behavior Included

The conclusion for policy, therefore would be that in real world situations where individuals can avoid redistribution efforts, the redistribution rate would fall somewhat short of what one would expect based on mathematical equations alone.

# Conclusions

## Replication

In all three cases simulations that were described in earlier research were replicated. In Cases I and II these earlier models were explicitly described, while in Case III they were implied by the theoretical structure of the problem. This research, therefore, had to confront the issue of how to implement and replicate influential research. Failure to adequately replicate the published model would likely make research that extends the published model at the least unconvincing and probably impossible. As noted in the earlier chapter on methodology, many agent-based models were considered before these cases were selected. The Case I and Case II models were among the few that provided enough information on the algorithms used and the parameters and starting conditions that an adequate model can be examined. And, activation was clearly one of the critical specifications.

The detail on how the original model's agents were activated in the reference models ranged from highly specified (in Case I, the civil revolt model) to strongly implied (in Case II, the ZIT model), and finally to unspecified and left to the simulation designer (Case III). Simulations in which there are extensive reported results but with little information on how the agents were updated are among the most challenging to replicate. This places severe restrictions on the process of extending and exploring existing models,

and imbues the whole body of complex systems research with a disconnected, ‘stovepipe’ character. Many model results are reported, but very few are verified through replication. But, as one professor at George Mason has noted, “modelers are more likely to use your toothbrush than they are to use your model” (Kennedy, 2013).

## **Generalized Results – Activation Makes a Difference**

All three cases have demonstrated that changing activation causes statistically significant differences in the quantified output parameters. This creates a replicable ‘prediction’ ability in that, if the input parameters and the output data point were available for a given model (without knowing the activation), one could reliably distinguish which activation scheme was used. In some cases this distinction could be drawn with certainty, and in others it would be with high probability. While this is surely not sufficient to prove the general case that activation *always* makes a difference, it does eliminate the ‘null hypothesis’ that activation makes no difference and can be safely ignored. Thus, this research goes a long way to establishing that the potential importance of activation is a universal, necessary and certain fact of nature: an extension of knowledge.

Sometimes activation only appears to make a statistical difference. In Case I the policy difference would be to generate a different recommendation for ‘cop density’ or, by implication, troop strength necessary to sustain a ‘stability’ mission. This is listed as a key parameter in US military counterinsurgency doctrine and a subject of active research. The three activation schemes did not appear to substantially alter the recommendations of

the model, at least in the range of the parameters examined here. But, there are many instabilities and ‘tipping points’ in the behavior of complex systems and if the model were explored across a wider parameter space it’s possible that the statistical differences would grow to become policy differences.

In Case II the policy question at hand centered on the efficiency and performance of a double-auction market. The original researchers measured whether the double-auction market delivered returns that were closest to the theoretical maximum and that wealth was not distributed disproportionately at the end of trading. Gode and Sunder, however, did not contemplate another important policy question, which was the rate of convergence or the length of time necessary to achieve this success. If activation is varied, it was shown, it was possible to envision a market in which there were many trades left to execute and many returns ‘on the table’. It is not difficult to envision how this can be extended to become an important policy question.

Case III contemplated a simple model, but demonstrated the most distinct difference among activation schemes. The difference appeared across all activation schemes, and created a much greater variation than the variation observed through execution of different runs of the model. The Leveler model, however, was quite abstract, and no policy implications flow from this finding. As the brief literature survey showed and Aldous stated specifically (Aldous, 2013), the body of work that applies the physics of interacting particle systems to social systems is as rich and diversified as the corpus of research that falls under the umbrella term “game theory”. Aldous postulates that only a

lack of a common name has prevented the recognition of the large impact of IPS models on social theory and, by implication, on policy. Thus, the demonstration that activation choice has a strong influence on whether an IPS model follows the theoretical predictions has policy implication, albeit much more indirect than in Cases I and II.

Activation can also be important to the effective operation of the simulation. Among the many decisions that must be made as simulations are executed is: when do we stop the model? In several of these cases, particularly in Case II and III endogenous activation, the models were far from running to completion when they were stopped. As these models were executed, moreover, it was noticed that some activation schemes took a long time to execute while others executed very quickly. And, the differences were not intuitive. For the Leveler model, it took 17 minutes to do 100 runs of uniform activation, but only 2 minutes to complete 100 runs of all the other activation types *combined*, including the much more complex inverse Poisson.

Another important result is the confirmation that state-based or endogenous activation has the most reliable and substantial effect of all. It also allows a wide variety of activation schemes, even though only two were tried in Case II and III. This confirms earlier (but still recent) research (Fernández-Gracia et al., 2011), and applies it to the more policy-centric models. It takes a bit more coding to incorporate endogenous activation into a model, and perhaps a bit more experimentation to determine its impact, but the results above show that the benefits in diversity of model outcome are potentially worth it.

## **Activation is Also a Tool**

Case III emphasizes a more general outcome of this research: activation is more than a nuisance variable that must be determined, encoded, explained and reported. It also can be an important tool for examining natural systems, and extending theoretical models through simulation. In designing a simulation to implement a theory, activation should be matched to nature. It is not beyond reason that the activation process can be observed in the real-world system, and that data can be collected about this process. In such cases, matching activation to the known process can help build confidence that the model is effectively mimicking its real-world counterpart.

It is not uncommon that a simulation, when first built, would deviate substantially from the natural system it seeks to model. Simulation designers often make adjustments to the model to determine if they have ‘left out’ some critical real-world parameter or behavior. In fact, this adjustment to experiment may represent the restatement of the hypothesis about what parts of the object system are important. Alternative activation schemes can and should be examined in this process.

## **Contribution**

There are four main contributions associated with this work:

- In the replication of models in three different domains, it was established that when the activation scheme was varied, there was variation of the outcome. This did not occur in all instances and in all models, but it certainly occurred enough to reject the null hypothesis that activation is always insignificant.

Because of this finding, published research in agent-based models must adequately describe the activation scheme if they are to be thorough and if they lay claim to be ‘replicable science’.

- It was established that the replication process itself, if conducted on well-documented models, reveals important details that may have been missed by the original researchers. For example, while Huberman and Glance claim to show that asynchronous activation causes the elimination of patterns of cooperation in a Prisoners’ Dilemma landscape model, it was determined that this loss only occurs in asynchronous *random* activation. Asynchronous uniform activation delivers the same patterns as synchronous activation.
- The new, recently proposed *endogenous* (or state-based) activation scheme was shown to be a significant source of variation in outcome across two model cases. And, a new algorithm for creating and exploring endogenous activation (in Python) was proposed and implemented.
- It was demonstrated that variation in the design of agent activation can extend the theoretical and equation-based models of interacting agents to real-world situations in which the assumptions of theory are violated. Thus, activation can be an instrument for extending the partnership between mathematical theory and computational science.



## **Future Research**

This project provided important insights into various models, as well as illuminating a number of issues important to all scientists who seek to apply agent-based modeling technology. Possibly more importantly, it opened up a fertile research field of replicating published models and extending peer-reviewed research.

As a general rule, once a model code has been developed and the output aligned to natural systems or to published, widely accepted models of natural systems, a tool is available to examine the boundaries and the idiosyncrasies of the original results. For example, in each of the model replications discussed below, the question of scalability can be examined. Do the published results (not only the activation results, but the general outcomes of the model) obtain if the number of agents is increased tenfold? What about other orders of magnitude?

### **Extending Analysis of Civil Revolt**

The key purpose of modeling self-organizing civil violence is to help with the decisions that security and military planners must make. Chief among these are the commitment of troops, which is represented by the ‘cop density’ parameter. The activation research reported here was limited to determining the impact of activation on an exact replication of the published model. Model explorations at different cop densities,

however, revealed important variation in emergent violent ‘outbreaks’. Further insights can surely be developed as other parameters are varied along with activation.

Epstein examined civil violence on a relatively small scale. Personal conversations with the authors of Henscheid, et al. (2010) show that many emergent patterns disappear at larger scale. The civil violence model would be a particularly important case for examination of scalability. Of course, the time involved in running this model at larger scales would be substantial, and NetLogo does not create the most efficient code.

Epstein extended his analysis of civil unrest to other situations in his paper (2002). In particular, he changed the conditions of the agents and the rules to replicate systems of ethnic cleansing, similar to events in former Yugoslavia. Follow-on work can examine if these results are independent of activation.

The civil revolt model is the only one in which endogenous activation was not explored. Implementing state-based activation schemes in NetLogo is awkward, and may well slow down the model even further. But, the Case I results fall in the category of statistical differences that do not appear to affect policy considerations. If the more influential changes apparent in other models when endogenous activation is used apply to the civil revolt ABM, it may be that activation would impact cop density (and, through that, policy recommendations).

Finally, the NetLogo library also has a replication of the Epstein model. It produces similar results to the model built for this project, but there appear to be subtle differences. The model uses the ‘out of the box’ activation scheme available in NetLogo, uniform

activation. (Every ‘turtle’ gets activated once per turn, and the order is shuffled between turns.) This creates an interesting case in which two models were written in the *same* language. Further research could implement other activation schemes in that model and compare the results side-by-side with the model reported here. Does it give the same results? Do all these results scale? And, if patterns disappear, do those patterns disappear at the same point when the scale increases?

### **Further Investigations with the Zero-Intelligence Traders (Constrained) Double Auction Market Model**

The ZIT market simulation discussed here was tied very closely to the original research article (Gode & Sunder, 1993). The mere fact that this model was referred to by over 1100 other peer-reviewed reports demonstrates that this is a productive field for research into the design of markets. Extensions can be worked in two directions: the four activation schemes can be examined in more complicated market models, or the activation schemes themselves can be made more complicated.

A good example of the latter expansion would be an exploration of different Poisson probabilities. What if the richest activated fastest? Or the poorest? What if those with the history of the most trades traded more often (simulating individuals ‘addicted’ to trading). What about those with the fewest trades (Simulating a diminishing motivation to trade.)?

Scalability is the simplest expansion, and could be quite interesting. What happens to this model if it is expanded from twelve traders to 1200? (The execution penalty is probably much less than Case I because this model was created in Python.)

With a large trading population, an examine the wealth distribution with different activation schemes might create interesting patterns. A common measure of the distribution of wealth is the “Genie” coefficient. It was not calculated in the original research because the numbers of agents were small and their individual wealth accounts did not appear to vary much. But what would happen in a larger population?

The Gode and Sunder paper was a straightforward representation of a single market structure, the double auction market. They were trying to determine what kind of efficiencies market, in its simplest form, would provide. Other researchers have compared the double auction market with other market structures. They generated findings on the relative strengths and weaknesses of different markets. Thus, a valuable extension of this research would be to determine if the choice of activation scheme changed that description of strengths and weaknesses. This would move the ZIT model further in the direction of policy recommendations, especially as financial regulation policy is in flux and may require the support of simulations to validate proposed new regulations.

Even working within the framework of the zero-intelligence trader model,, there are many ways to make the market more complex: traders might be allowed to trade in multiple shares, the same trader might allowed to both buy and sell, etc. All these

refinements, however, would come at the cost of simplicity. Model simplicity probably is what intrigued the many follow-on researchers, and deviating from that would likely reduce the apparent generality of the ZIT simulation.

## **Going where No Markov Chain Model Has Gone Before**

I found that computational analysis can reach areas of the behavior space that are not accessible to equation models. If there are real world systems that fit this description, computational approaches – specifically ABMs – might expand the toolkit for those who would optimize, forecast, and determine risk.

## **General Expansion of Activation Analysis**

This has been an examination of three of the many agent-based models described in the literature. As noted above, hundreds of new ABMs are created and used in research every year. Many of these could be productively re-engineered to examine the impact of activation. In the process of preparing this dissertation, for example, two published results were investigated to the point of actually writing the model: an Ising model of stock price formation that replicates the real-world statistics of a price time series (Sznajd-Weron & Weron, 2002), and a large model of the labor market. In addition, a model of the propagation of fraud in a company had an excellent description, and the authors invited follow-on researchers to investigate their results for different activation schemes.

As noted in Cases II and III, the recent discoveries about the strong dependence of disparate results on state-based or endogenous activation provides motivation to examine

this further. Endogenous activation conditions provide nearly an endless potential for varying the rules of activation. The earliest proposals suggested varying activation rates depending upon the agents' time since the last activation. (This, of course, would eliminate the possibility for replicating an exponential distribution and its accompanying 'memoryless' property.) Agents activation rates could be varied based upon their cumulative characteristics (wealth, for example), or based upon some quantitative value from their last activation (profit from the last trade, for example). Finally, activation rate might even depend on some parameter associated with their neighbors, which would have echoes to the original prisoner's dilemma research (Nowak & May, 1992; Huberman & Glance, 1993). In that model structure, agents might activate – reassess their prisoner's dilemma strategy – if their score is much less than their neighbors or much better. Only the researcher's imagination – perhaps spurred by observations on natural world systems – would limit the possibilities under endogenous activation rates. And, it is hoped that progress and insights produced by such explorations would motivate more replications of agent-based models and break down the barriers that exist among ABM research communities.

# **Appendix A**

## **Supplementary Graphics for Zero Intelligence Trader Model**

Numerous graphic explorations were made with the ZIT model that would have interrupted the text. They are presented here to document the full scope of model behavior explorations. These graphics are organized market by the three output measures: total wealth, dispersion of wealth, and timing of last trade. Within each market, the graphics for the four activation schemes are depicted. A small number of these were also used in the main text.

## Total Wealth

Presented here (Figure 33) are the full results of runs that show total wealth. All four markets are shown; each market is shown in one row. Each column represents one of the four activation patterns, random, uniform, Poisson, and inverse Poisson.

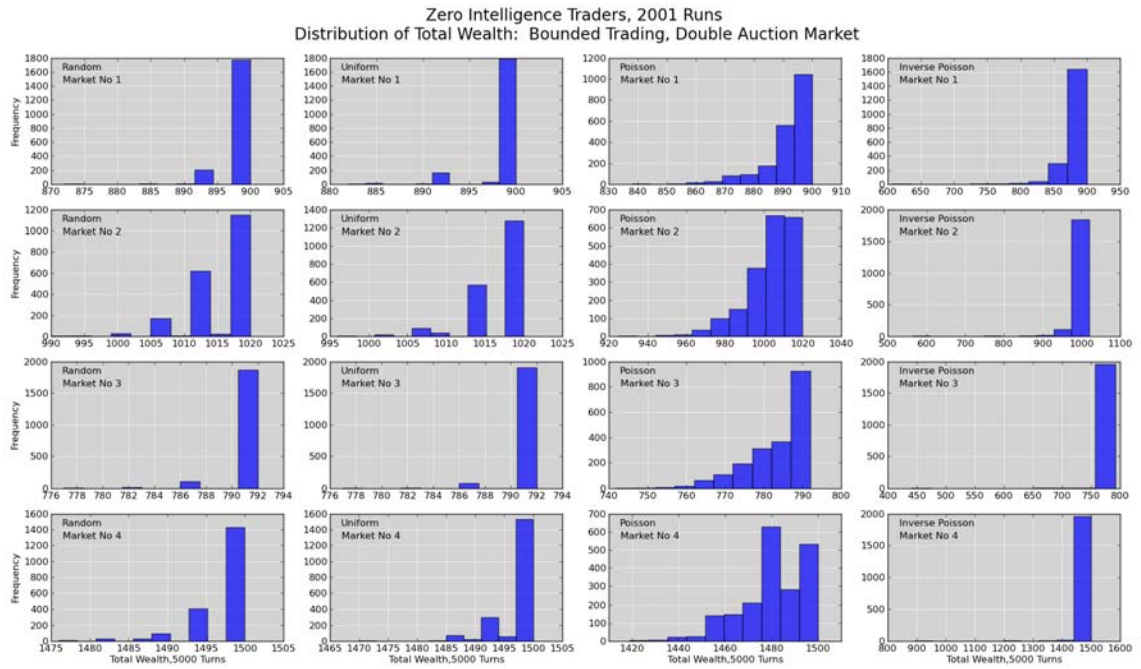


Figure 33. Total Wealth: All Markets, All Activation Patterns



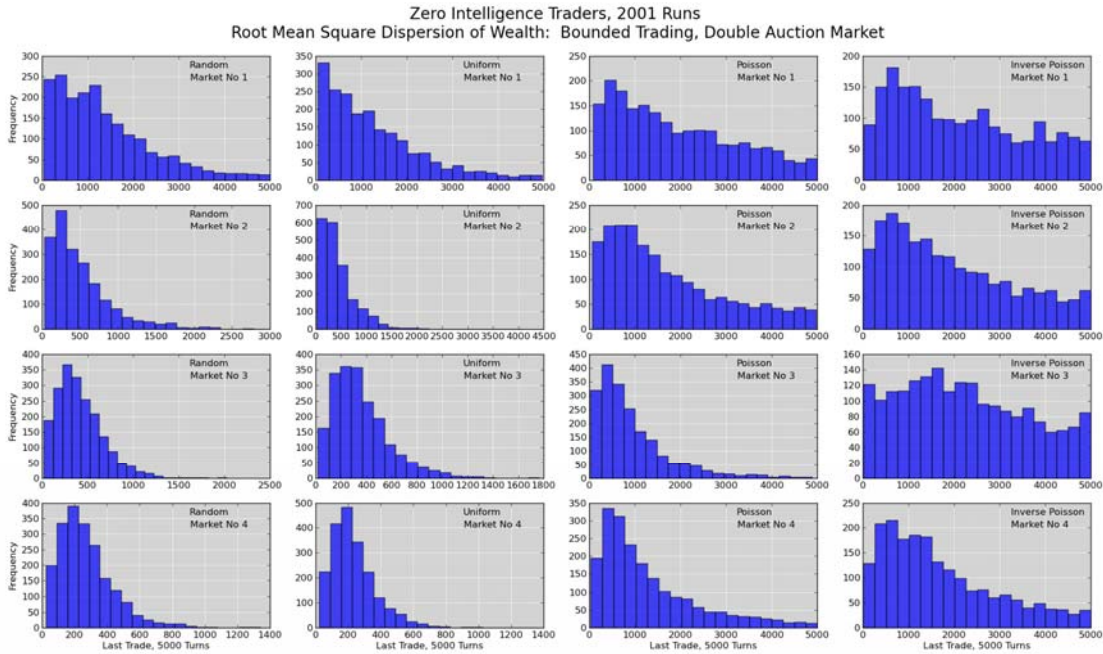
Note that both the vertical and horizontal scales differ from one plot to another. This figure is actually provided to allow comparisons among the shapes of the histograms. Clearly these differ substantially across the activation patterns.

## Wealth Dispersal



**Figure 34. Dispersion of Wealth: All Markets, All Activation Patterns**

Figure 34 shows the histograms of the dispersion of wealth for all sixteen cases. Like its predecessor, it depicts markets in the different rows, and activation schemes in different columns. Again, the scales are not constant across all cases.



**Figure 35. Last Trade Timing: All Markets, All Activation Patterns**

### ***Last Trade Timing***

Similar results are shown for the timing (in turn number) of the last trade. As in the other output graphics, it should be noted that the scales vary and these plots are shown to compare shapes rather than absolute values.

Many statisticians believe that Q-Q plots are much more probative than histograms. Included below are the Q-Q plots for all the markets and all the activations for the last trade over 2000 runs. The QQ plots of the same data show differences as well.

To build the proper QQ plots, it must be noted that the above times are clearly not normally distributed (there are no times below zero, for example), but log-normally

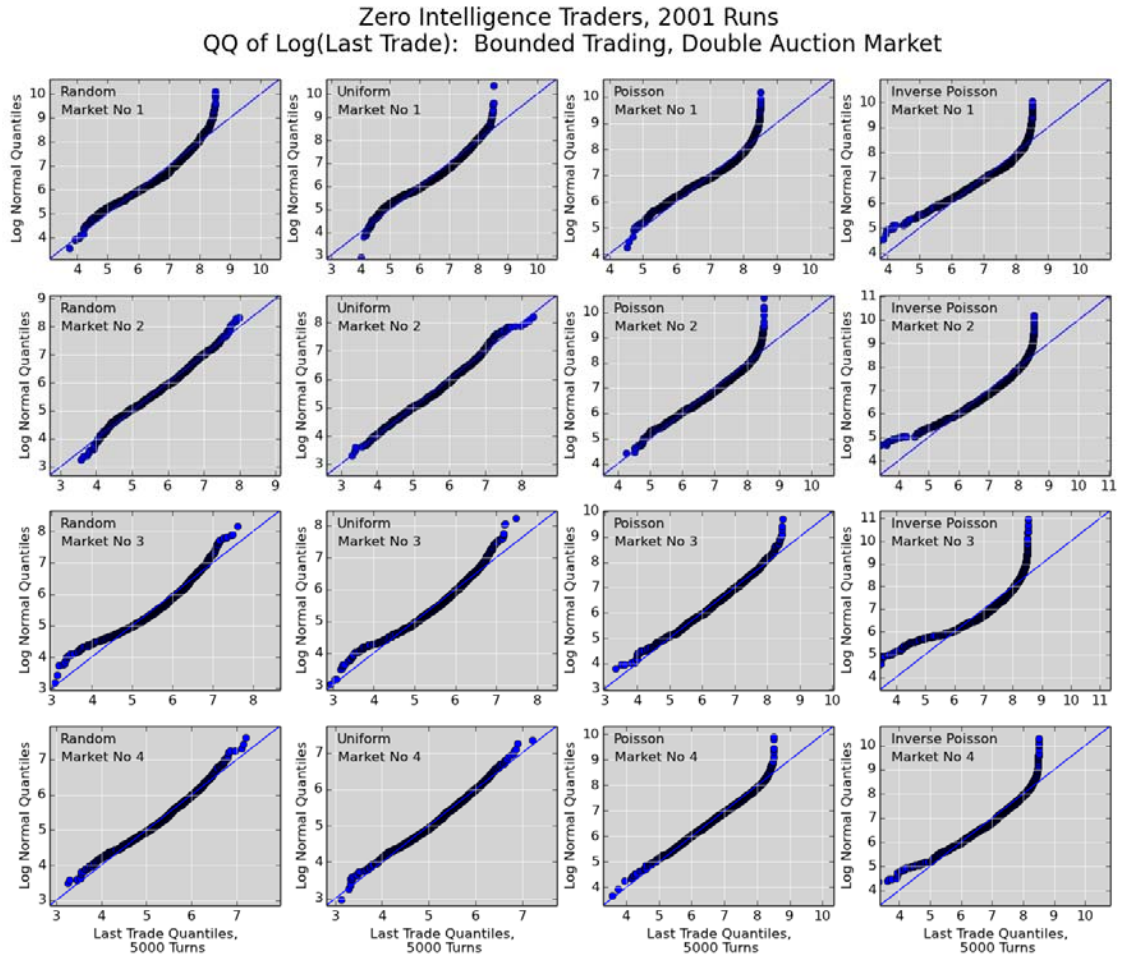


Figure 36. QQ Plots of All Last Turn, All Activations

distributed. Thus, the QQ plots depicted in Figure 36 test the experimental distributions against a log-normal distribution.

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## Biography

Kenneth Comer is a father of three, grandfather of three, husband of Roberta Comer for 38 years, and a veteran of the United States Navy. He retired in 2012 as a member of the US Senior Executive Service. From 2010, Mr. Comer was Executive Manager of the Terrorist Device Analytic Center, a laboratory in Quantico, Virginia. He earlier served as the Deputy Director for Intelligence and Analysis in the Joint IED Defeat Organization (JIEDDO) and founded the Operations Research Systems Analysis (ORSA) Division.

Mr. Comer is a 1974 graduate of Cornell University, majoring in chemistry and biochemistry. He served five years on active duty as a nuclear submarine officer, earning his submariner dolphins on the *USS Archerfish* (SSN-678). Subsequently, Mr. Comer earned a MA from Georgetown University in national security studies and an MS from The George Washington University in operations research.

Mr. Comer completed a 22 year career as an analyst at the Central Intelligence Agency in July, 2003. From 2004 to 2007, He consulted with Office of the Secretary of Defense Program Analysis and Evaluation and the Chief of Naval Operations' Campaign Analysis Staff (N81). His work helped to develop a new generation of non-traditional modeling methods, to include agent-based models of social systems (such as insurgencies and terrorist networks). Mr. Comer was able to demonstrate that computational approaches can help understand and prepare for difficult-to-forecast risks and low-probability, high-impact events.

As a senior executive, Mr. Comer enabled state-of-the-art research into the patterns of enemy terrorist networks. He has been quoted in prestigious publications such as *Nature*, *Science*, and *The Economist*. Mr. Comer sponsored the first conference on "The Mathematics of Terrorism" at the renowned Santa Fe Institute. One of his most long-lasting achievements is to encourage and promote a team of advanced systems scientists in support of the counter-IED and counter-terrorism mission, which has led to important results reported in *Science* and *The Small Wars Journal*. This analytic capability will pay dividends in the understanding of the risk from the acts of self-organizing violent extremist groups well beyond the post-9/11 conflicts.

For his service in JIEDDO, in 2012 Mr. Comer was awarded the highest civilian award, the Exceptional Service Medal.

Mr. Comer is a long distance bicyclist. In January 2013, he completed his lifelong goal of riding a century (100 miles) in every state by circumnavigating the island of Hawai'i.