#### WIDEBAND AND MULTIBAND TEMPORAL SENSING FOR OPPORTUNISTIC SPECTRUM ACCESS

by

Joseph M. Bruno A Dissertation Submitted to the Graduate Faculty of George Mason University In Partial fulfillment of The Requirements for the Degree of Doctor of Philosophy Electrical Engineering

Committee:

	Dr. Brian L. Mark, Dissertation Co-Director
	Dr. Yariv Ephraim, Dissertation Co-Director
	Dr. Zhi Tian, Committee Member
	Dr. Chun-Hung Chen, Committee Member
	Dr. Monson H. Hayes, Chair, Department of Electrical and Computer Engineering
	Dr. Kenneth S. Ball, Dean, Volgenau School of Engineering
Date:	Spring Semester 2017 George Mason University Fairfax, VA

Wideband and Multiband Temporal Sensing for Opportunistic Spectrum Access

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy at George Mason University

By

Joseph M. Bruno Master of Science The Johns Hopkins University, 2013 Bachelor of Science University of Delaware, 2011

Co-Directors: Dr. Brian L. Mark, Professor and Dr. Yariv Ephraim, Professor Department of Electrical and Computer Engineering

> Spring Semester 2017 George Mason University Fairfax, VA

Copyright © 2017 by Joseph M. Bruno All Rights Reserved

## Dedication

I dedicate this dissertation to my family. My wife Stephanie, my parents Mark and Irene, and my sisters Anna and Maria have given me so much love and support, for which I am eternally grateful.

### Acknowledgments

I would like to thank my advisors, Dr. Brian Mark and Dr. Yariv Ephraim. They have invested many hours into my knowledge and abilities, and I have benefited immensely from their hard work. Furthermore, they have held me to the highest standard, which I absolutely appreciate. Thank you very much for your support.

I would like to thank my committee members, Dr. Zhi Tian and Dr. Chun-Hung Chen. I have had the pleasure of working with them on portions of this dissertation, and I value their expertise and willingness to provide quality input and feedback.

I would like to thank the National Science Foundation, who financially supported my research under grants 1421869 and 1205453.

I would like to thank my colleagues at the JHU Applied Physics Lab, where I have many wonderful coworkers, too numerous to name. Without such bright, hard-working, and reliable people around me, I would never have found the balance required to complete my dissertation research.

I would like to thank my family, who have always been there for me. Thank you for bringing stability to my life when I was neck-deep in my studies and research. I would like to thank my parents for teaching me the importance of academic achievement and providing me with the opportunities to do well in school. I would like to thank Stephanie for her endless support, and somehow knowing when I needed a break or when I needed to be encouraged to get back to work. I could not have done this alone.

## Table of Contents

			I	Page	
Lis	t of Л	Tables		ix	
Lis	t of F	Figures		х	
Ab	Abstract				
1	Introduction				
	1.1	Motiv	ation	1	
	1.2	Proble	em statement	2	
	1.3	Contri	ibutions and outline of dissertation	3	
2	Bac	kgroun	d and Literature Review	5	
	2.1	Narro	wband Sensing with Hidden Markov Models	5	
		2.1.1	Accurate Parameter Estimation and User Detection	5	
		2.1.2	User Occupancy Prediction	6	
		2.1.3	Markov Modulated Gaussian Process	7	
		2.1.4	Score Function and Fisher Information Matrix	7	
	2.2	Wideb	band Spectrum Sensing	7	
		2.2.1	Wideband Energy Detection	8	
		2.2.2	Wideband Edge Detection	9	
		2.2.3	Compressed Sensing	10	
	2.3	Multil	pand Spectrum Sensing	11	
	2.4	Optim	al Compute Budget Allocation	12	
3	Multiband Spectrum Sensing Based on Markov Modulated Gaussian Processes $.$ 1				
	3.1	Introd	luction	13	
	3.2	Syster	n Model	14	
		3.2.1	Multiband Channel Model	14	
		3.2.2	PU Traffic Model	15	
		3.2.3	Cognitive Receiver Model	15	
		3.2.4	Markov-Modulated Gaussian Process	16	
	3.3	Multil	oand Spectrum Sensing From Observed Continuous-time Markov Model	17	
		3.3.1	Moment Estimator	17	
		3.3.2	Allocation of Sensing Effort	18	

		3.3.3	Bias of Moment Estimator in Noise	19
	3.4	Multik	band Spectrum Sensing based on a Markov-Modulated Gaussian Process	20
		3.4.1	MMGP Parameter Estimation	20
		3.4.2	Allocation of Sensing Effort	24
		3.4.3	Averaging of Parameter Estimates	26
		3.4.4	MAP Decision Rule	27
	3.5	Simula	ation and Numerical Results	27
	3.6	Conclu	usion	30
4	A C	Computi	ing Budget Allocation Approach to Multiband Spectrum Sensing	32
	4.1	Introd	uction	32
	4.2	System	n Model	34
	4.3	MMSI	E Multichannel Estimation	37
	4.4	OCBA	Multichannel Parameter Estimation	39
		4.4.1	OCBA Sensing Allocations	39
		4.4.2	Channel Elimination	40
	4.5	Nume	rical Examples	42
		4.5.1	Example 1	42
		4.5.2	Example 2	44
		4.5.3	Example 3	45
		4.5.4	Discussion	47
	4.6	Conclu	usion	47
5	A R	lecursiv	e Algorithm for Wideband Temporal Spectrum Sensing	49
	5.1	Comp	arison of Wideband Spectrum Sensing Techniques	51
		5.1.1	Wideband Energy Detector	52
		5.1.2	Wideband Edge Detector	53
		5.1.3	Compressive Sensing	55
	5.2	System	n Model	56
		5.2.1	PU Traffic Model	57
		5.2.2	Cognitive Receiver Model	57
	5.3	Recurs	sive Algorithm for Wideband Temporal Sensing	58
		5.3.1	Wideband Tree Search	59
		5.3.2	Channel Selection	60
		5.3.3	Hidden Markov Model for Narrowband Sensing	61
		5.3.4	Baum-Welch Algorithm and MAP Detector	62
		5.3.5	Channel Usability and Channel Capacity	64
		5.3.6	Channel Aggregation	66

		5.3.7	Algorithm Descriptions	
	5.4	Simula	ation and Numerical Results	
		5.4.1	Simulation 1: Comparison of Techniques	
		5.4.2	Simulation 2: Performance at Varying SNR	
		5.4.3	Simulation 1 Results: Qualitative Comparison of Techniques 71	
		5.4.4	Simulation 1 Results: Quantitative Comparison of Techniques 72	
		5.4.5	Simulation 2 Results	
	5.5	Concle	usion $\ldots$ $\ldots$ $\ldots$ $\ldots$ $74$	
6	An	Edge D	Detection Approach to Wideband Temporal Spectrum Sensing 79	
	6.1	Introd	$uction \dots \dots$	
	6.2	Syster	n Model	
		6.2.1	Wideband Channel Model    80	
		6.2.2	PU Traffic Model	
		6.2.3	Cognitive Receiver Model	
	6.3	Comp	arison of Wideband Spectrum Sensing Techniques	
	6.4	Propo	sed Algorithm	
		6.4.1	Channelization of Received Wideband Signal	
		6.4.2	Sensing of Narrowband Subchannels	
		6.4.3	Edge Detection	
	6.5	5 Simulation and Results		
		6.5.1	Simulation Setup	
		6.5.2	Qualitative Results	
		6.5.3	Numerical Results	
	6.6	Conclu	usion $\ldots$ $\ldots$ $\ldots$ $\ldots$ $\ldots$ $$ $91$	
7	Con	clusion	s	
	7.1	Multil	band Spectrum Sensing	
	7.2	Wideb	band Spectrum Sensing	
	7.3	Future	e Work	
		7.3.1	Multiband Spectrum Sensing	
		7.3.2	Wideband Spectrum Sensing 95	
		7.3.3	Implementation	
А	Der	ivation	of Conditional Distribution of a Rayleigh Channel Observed through	
	an E	nergy I	Detector	
	A.1	Narro	wband Channel Model	
	A.2	PU Tr	raffic Model	
	A.3	Cogni	tive Receiver Model	

В	Derivation of the Fisher Information Matrix for a 2-State Markov-Modulated			
	Gaussian Process			
	B.1	Definitions	99	
	B.2	Two-State Log Likelihood	100	
	B.3	Important Expected Values	100	
		B.3.1 Expected Number of Jumps Between States	101	
		B.3.2 Expected Number of Samples in a State	106	
B.4 Score Function			108	
		B.4.1 Score for Transition Rates	108	
		B.4.2 Score for Gaussian Process Means	108	
		B.4.3 Score for Gaussian Process Variances	109	
	B.5	Fisher Information Matrix	109	
	B.6	Asymptotic Approximation	113	
С	Deri	vation of the Fisher Information Matrix for a CTMC	114	
	C.1	Definitions	114	
	C.2	Log Likelihood	114	
	C.3	Log Likelihood In Terms of Mean Dwell Time and Transition Probability $\ .$	115	
	C.4	Score Function	116	
	C.5	Fisher Information Matrix	116	
	C.6	Expected Number of Jumps Between States	117	
	C.7	Expected Time in States	118	
	C.8	Asymptotic Approximations	118	
	C.9	Asymptotic MMSE Allocations	119	
Bib	oliogra	aphy	122	

## List of Tables

Table		Page
3.1	Continuous-time Markov process channel parameters.	28
3.2	Simulation parameters	28
5.1	Algorithm complexity parameters.	68
6.1	Simulation parameters	89

## List of Figures

Figure		Page
3.1	Direct observation of PU state	19
3.2	Effect of a decision error on sojourn time estimates	20
3.3	Multiband sensing performance at 10 dB SNR	29
3.4	Multiband sensing performance at 0 dB SNR	30
3.5	Multiband sensing performance at -5 dB SNR	30
4.1	Continuous-time Markov chain model for PU state of a single channel	35
4.2	MSE for all channels using Equal, MMSE, and OCBA allocations	43
4.3	MSE for best channel using Equal, MMSE, and OCBA allocations $\ . \ . \ .$	43
4.4	MSE for all channels using Equal, MMSE, and OCBA allocations	44
4.5	MSE for optimal subset using Equal, MMSE, and OCBA allocations $\ . \ . \ .$	45
4.6	Probability of correct selection using Equal, MMSE, and OCBA allocations	46
4.7	Expected opportunity cost using Equal, MMSE, and OCBA allocations $~$ .	46
5.1	Results of a wideband energy detector for OFDM signals with 10 dB SNR $$	
	and 100%, 50%, 25%, and 12.5% duty cycles. $\ldots$ $\ldots$ $\ldots$ $\ldots$ $\ldots$	53
5.2	Results of a wideband energy detector for GMSK signals with 10 dB SNR $$	
	and 100%, 50%, 25%, and 12.5% duty cycles. $\ldots$ $\ldots$ $\ldots$ $\ldots$	54
5.3	Results of a wideband edge detector for OFDM signals with 10 dB SNR and	
	100%, 50%, 25%, and 12.5% duty cycles	54
5.4	Results of a wideband edge detector for GMSK signals with 10 dB SNR and	
	100%, $50%$ , $25%$ , and $12.5%$ duty cycles	55
5.5	A wideband channel, i.e., a spectrum band with bandwidth $W_0$ , organized	
	into a balanced binary tree	59
5.6	A simple digital downconvertor for signal channelization	60
5.7	Results of wideband temporal spectrum detector for OFDM signals with	
	10 dB SNR and 100%, 50%, 25%, and 12.5% duty cycles. $\ldots$ .	72
5.8	Results of wideband temporal spectrum detector for GMSK signals with	
	10 dB SNR and 100%, 50%, 25%, and 12.5% duty cycles	73

5.9	ROC curve for wideband energy detector for OFDM signals with 10 dB SNR $$	
	and 100%, 50%, 25%, and 12.5% duty cycles	73
5.10	ROC curve for wideband energy detector for GMSK signals with 10 dB SNR $$	
	and 100%, 50%, 25%, and 12.5% duty cycles.	74
5.11	ROC curve for wideband edge detector for OFDM signals with 10 dB SNR $$	
	and 100%, 50%, 25%, and 12.5% duty cycles.	75
5.12	ROC curve for wideband edge detector for GMSK signals with 10 dB SNR $$	
	and 100%, 50%, 25%, and 12.5% duty cycles.	75
5.13	ROC curve for wideband temporal spectrum detector for OFDM signals with	
	10 dB SNR and 100%, 50%, 25%, and 12.5% duty cycles. $\ldots$	76
5.14	ROC curve for wideband temporal spectrum detector for GMSK signals with	
	10 dB SNR and 100%, 50%, 25%, and 12.5% duty cycles. $\ldots$	76
5.15		77
5.16		77
6.1	Results of a wideband edge detector for OFDM signals with 10 dB SNR and $$	
	100%, 50%, 25%, and 12.5% duty cycles [12]	83
6.2	Results of a wideband temporal energy detector for OFDM signals with 10 dB $$	
	SNR and 100%, 50%, 25%, and 12.5% duty cycles [12]	84
6.3	Results of wideband temporal energy detector for OFDM signals with 5 dB $$	
	SNR and 100%, 50%, 25%, and 12.5% duty cycles [12]	85
6.4	Results of the proposed wideband temporal edge detector for OFDM signals	
	with 10 dB SNR and 100%, 50%, 25%, and 12.5% duty cycles [12]	89
6.5	Results of wideband temporal detectors for OFDM signals with 10 dB SNR.	90
6.6	Results of wideband temporal detectors for OFDM signals with 5 dB SNR.	90
6.7	Results of wideband temporal detectors for OFDM signals with 0 dB SNR.	91

## Abstract

# WIDEBAND AND MULTIBAND TEMPORAL SENSING FOR OPPORTUNISTIC SPECTRUM ACCESS

Joseph M. Bruno, PhD

George Mason University, 2017

Dissertation Co-Directors: Dr. Brian L. Mark, Dr. Yariv Ephraim

Opportunistic spectrum access is a proposed solution to the problem of increasing scarcity of radio resources. In certain bands, spectrum is utilized extremely inefficiently by the licensed, or primary, users. Opportunistic spectrum access would allow a secondary user to utilize spectrum when the primary user is idle while not causing harmful interference when the primary user is active. Spectrum sensing techniques determine portions of the spectrum that are occupied by primary user signals at a given time and location. Temporal sensing of a known narrowband channel involves modeling the temporal dynamics of the primary user signal and performing estimation and prediction of the primary user state. Wideband sensing involves determining which parts of a given wide spectrum are occupied or unoccupied at a given point in time. Both temporal and wideband sensing have been studied extensively in the literature. There has been relatively little work on temporal sensing over a wide spectrum band with either well-defined or unknown channels.

In this dissertation, novel approaches to wideband and multiband temporal sensing are developed. A class of hidden Markov models is proposed to jointly model time dynamics of the primary system and channel impairments between the primary user and the secondary user over a wide spectrum band. Methods to segment a wide spectrum band into individual channels and to optimize parameter estimation over the channels are proposed. Simulation results are presented to evaluate the effectiveness of the proposed wideband and multiband temporal sensing schemes. Some comparisons to performance bounds are provided.

## Chapter 1: Introduction

### 1.1 Motivation

As demand for radio resources continues to grow, novel approaches to spectrum usage must be developed to meet demands. Many of these innovations have included enhancement to physical layers, development of better communications hardware, and development of more efficient communications protocols. While these improvements are welcome, they do not solve the problem of inefficiency in spectrum allocations. Today, radio frequency (RF) bands are statically allocated to a licensed user. Some bands, such as the allocated cellular bands and the industrial, scientific, and mechanical (ISM) bands are heavily utilized, but other bands are woefully underutilized by their primary users [24]. Opportunistic spectrum access (OSA), known also as dynamic spectrum access (DSA), is a proposed solution to the problem of inefficient spectrum allocations. In OSA, the band is shared by the licensed user, referred to as the primary user (PU), and an unlicensed secondary user (SU). The SU will be responsible for detecting the presence of the PU and only utilizing the channel when the PU is inactive. A perfect OSA system would allow a SU to utilize all idle periods, where the PU is not in use, without causing harmful interference to the PU during its active periods.

For a SU to efficiently operate, it can attempt to predict changes in PU state and act accordingly. Because of this, parametric modeling of PU activity is desired. A popular model for prediction is the use of a Markov process to model the PU activity [3, 43, 55]. Some work has been done to extend this model to the multiband case, where many channels are sensed jointly [55], but the theory is relatively undeveloped and many considerations such as noise and fading between the PU and SU have not yet been investigated. To the author's knowledge, no research has been done to model the wideband case, where channel boundaries are unknown, using Markov processes.

## 1.2 Problem statement

In this dissertation, the approach of modeling the PU as a Markov process is extended to the wideband and multiband case, specifically using Hidden Markov Models (HMMs). Hidden Markov models allow the SU to account for varying signal-to-noise ratio (SNR) and other channel conditions. HMMs are applied to correct specific shortfalls in the current state-of-the-art for wideband and multiband sensing.

In wideband spectrum sensing, the SU is missing external knowledge of channel allocations, and must perform segmentation to jointly detect channel boundaries and PU activity. Current wideband spectrum sensing research has focused on segmenting the spectrum in one of two ways: wideband energy detection [13, 29], and wideband edge detection [56]. The wideband edge detector, although it can perform better than the energy detector on certain signals, is a signal-dependent detector. Signals with sharp band edges will be detected accurately, but signals with sloping band edges will not be detected reliably, even in very high SNR. Both detectors have a shortfall in that they require a time-average power spectral density (PSD) for accurate detection. The time averaged PSD estimate is not a problem if the PU is always either busy or idle over a long time window, but if the PU has rapid changes over time, the detector reliability can be degraded. Depending on the PU duty cycle, some amount of noise will be averaged into the PSD estimate. If the PU duty cycle is low enough, a SU using traditional wideband sensing algorithms will not reliably detect the PU and possibly cause unwanted interference.

In multiband spectrum sensing, channels have already been defined, but a SU must scan over many channels to detect PU activity. Rather than spending fixed durations on each channel, it is desirable to optimize sensing time allocations for each channel. In [55], a multiband system is modeled such that there are many channels, each with an independent PU. The PU is modeled as a continuous time Markov process, and the transition rates,  $\lambda_1$ and  $\lambda_2$  are estimated. Sensing periods are allocated such that the Cramér Rao bound (CRB) of the transition rate estimates for all PUs is minimized. While this approach is generally promising, several simplifications were made to achieve nice theoretical results. These simplifications rendered the multiband algorithm proposed in [55] very impractical. First, the PU was modeled as a continuous-time Markov process (CTMP), where the PU state is directly observed. This assumption is impractical because the PU state typically must be inferred in a noisy environment. Another shortcoming is that the optimal sensing periods rely on a sensing scheme where sensing durations increase without bound. In a practical OSA system, sensing durations remain finite. IEEE 802.22 [2] and IEEE 802.11af [1] may be referenced as examples of OSA designs that include finite-length sensing intervals. Finally, the objective of minimizing MSE across all channels allowed for the derivation of nice closedform solutions, but it does not closely match what a practical SU would do. A practical SU would want to consider the usability of a channel while allocating resources. Suppose one channel is determined to be useless due to having few spectrum opportunities, and a second channel is determined to be promising, having many spectrum holes, a SU would want to focus on estimation of the promising channel, such that it can use the channel more effectively. The objective of minimizing MSE does not necessarily motivate the SU to allocate more sensing time to promising channels.

#### **1.3** Contributions and outline of dissertation

The contributions of this document are outlined as follows.

In Chapter 2, background and literature review are given. Research on wideband sensing, multiband sensing and hidden Markov models is discussed.

In Chapter 3, the problem of multiband spectrum sensing is considered. A type of HMM is proposed as a solution to multiband optimization, where sensing durations are assigned to each channel in a set based on previously-estimated parameters. The objective of variance minimization for all sojourn time estimates is discussed.

In Chapter 4, a new method of multiband spectrum sensing which allows for maximization of many different benefit functions is introduced. Benefit functions may be chosen by an SU to achieve a specific objective. Results from simulation optimization are leveraged to facilitate fast selection of the best channels, based on a given criterion. In Chapter 5, the application of hidden Markov models to wideband spectrum sensing is developed in detail. To address the shortfall of existing wideband sensing algorithms, that PU bursting is not properly modeled, an HMM-based sensing algorithm is used in a recursive tree search for channel boundaries. By modeling the PU as an HMM, the SU can obtain accurate PU power level estimates in the idle and busy states. Using this pair of estimates, noise is not averaged into any PSD estimate. Based on Neyman-Pearson hard decisions from the PU parameter estimation, the locations of channel boundaries are detected.

In Chapter 6, the technique developed in Chapter 5 is extended. The result of the recursive channel search is generalized as a conditional form of the PSD. Multi-resolution edge detection is applied to this conditional PSD, allowing for more accurate spectrum sensing at lower SNR.

In Chapter 7, results are summarized, and proposed follow-on research is discussed.

## Chapter 2: Background and Literature Review

Spectrum sensing for opportunistic spectrum access has been well-studied, and a relatively rich body of research is available for narrowband temporal spectrum sensing. Although some research into wideband and multiband spectrum sensing has been published, these more general spectrum sensing problems still have plenty of room for innovation and theoretical investigation. In particular, many of the innovations of narrowband temporal spectrum sensing may be applied to wideband and multiband sensing problems. In this chapter, background research on wideband and multiband spectrum sensing is discussed. Additionally, we discuss narrowband spectrum sensing research that uses hidden Markov models (HMMs). A motif of this dissertation is that HMMs may be used to simplify difficult multiband and narrowband sensing problems, and that research into HMMs in the narrowband case may serve as the foundation for the wideband and multiband cases.

## 2.1 Narrowband Sensing with Hidden Markov Models

For narrowband spectrum sensing, Hidden Markov Models (HMMs) have been proposed for estimation of bursty signals and prediction of channel occupancy [3,43]. Ephraim and Merhav give a thorough review on HMMs in [21].

#### 2.1.1 Accurate Parameter Estimation and User Detection

Many narrowband sensing methods, such as energy detection and cyclostationary detection, require integration over long windows for reliable detection [65]. If the PU is bursting, cycling between busy periods and idle periods, detector performance can be degraded. If the PU changes state during an estimation window, energy from busy and idle periods will be averaged together, which will increase the probability of a detection error. By modeling the PU as a 2-state HMM, the Baum-Welch algorithm [9], or equivalently, the Expectation Maximization (EM) algorithm, can be used to estimate the PU parameter. During the expectation phase of the Baum-Welch algorithm, an auxiliary function is calculated as the conditional mean of the logarithm of the complete statistics given the observations and a current estimate of the parameter. During the maximization phase, the auxiliary function is maximized over the parameter and thus provides a new update of the current estimate of the parameter.

#### 2.1.2 User Occupancy Prediction

Besides improving detector reliability, HMMs provide additional power in that they enable prediction of future PU states [3, 43]. When the parameter of the HMM is accurately estimated and a record of recent data has been obtained, the SU may predict future states of the PU. This prediction power can allow a SU to overcome hardware latency and stop transmitting or change channels if the PU is likely to come online. Use of an HMM for modeling PU dwell times is limited in the sense that discrete-time Markov processes always have geometric dwell time distributions, which is rarely a realistic model. Semi-Markov models, Markov models whose dwell times may be from any distribution are proposed in [3]. In general, semi-Markov processes are impractical from an implementation standpoint because, although an EM algorithm has been proposed, the computational complexity is relatively high [64]. Nguyen et al. proposed a hidden bivariate Markov model (HBMM) for more accurate modeling of PU dwell distributions [43]. An HBMM is a hidden semi-Markov model, but with the additional advantage in that it may be treated like a standard HMM. The ability to treat a bivariate Markov process as a standard Markov process allows us to use the Baum-Welch algorithm for efficient parameter estimation and other results on HMMs. A bivariate discrete-time Markov process has dwell times that come from a discrete phase-type distribution, of which the geometric distribution is a special case [22]. The ability of an SU to make predictions is substantially enhanced by using an HBMM, especially when looking many samples in the future [43].

#### 2.1.3 Markov Modulated Gaussian Process

Similar to the HMM is the Markov-modulated Gaussian process (MMGP). The primary difference between the HMM and the MMGP is that the underlying Markov process is discrete-time for the HMM and continuous-time for the MMGP. An EM algorithm for parameter estimation of a MMGP is derived in [47]. The MMGP is interesting because the the continuous-time nature of the underlying Markov process does not require selection of a sample rate as part of the model. The parameter estimator may sample the data at whatever rate is deemed appropriate for the application. To our knowledge, no research has previously been done on spectrum sensing using MMGPs.

#### 2.1.4 Score Function and Fisher Information Matrix

The score function and Fisher information matrix (FIM) are used for a variety of statistical applications, but are generally difficult to calculate for stochastic processes with recursive definitions such as Markov processes [22]. In [39], Lystig and Hughes derived exact algorithms for the score function and FIM based on the forward recursion calculated during the expectation phase of the Baum-Welch algorithm [9]. The calculated score function can provide statistical insight such as confidence intervals, and is used in Rydén's recursive online HMM parameter estimator [49, 54]. The inverse of the FIM gives the Cramer-Rao lower bound on the covariance matrix of any unbiased estimator of the parameter. [32]. The FIM can therefore be used to test model design. The FIM is used in [55] to determine sensing time allocations across multiple channels.

### 2.2 Wideband Spectrum Sensing

The problem of wideband spectrum sensing revolves around channel segmentation, the process of dividing a band into component channels. These channels may be heterogeneous in bandwidth and waveform. Wideband spectrum sensing is required as an initialization phase if no external channel information is given. In many use cases, wideband spectrum sensing may not be strictly required due to externally defined channels. One such use case is TV whitespace applications, where 6 MHz channels are given. An opportunistic spectrum access system that is operating without externally-defined channels will need a way to find channel boundaries for subsequent sensing applications. Most of the wideband spectrum sensing literature is divided into energy detection and edge detection techniques.

#### 2.2.1 Wideband Energy Detection

The simplest and most pervasive wideband sensing method is wideband energy detection, a wideband extension from the classic narrowband energy detector proposed in [58]. Wideband energy detection involves estimation of the power spectral density (PSD) and using thresholds to determine subband occupancy. A variety of PSD estimators is available, including parametric autoregressive modeling, averaged periodograms, and multitaper estimation [29]. Many papers, including [37,46], have leveraged averaged periodogram methods, such as the Bartlett [8] or Welch [62] methods. In [23], the multitaper method was used to estimate the PSD, showing detector performance improvement over averaged periodograms.

A drawback to wideband energy detection is that to reliably detect low-energy signals, many estimates must be averaged together over time [60]. This is handled naturally when averaged periodograms are used, but multitaper estimation must also average over time if detector sensitivity is to be improved. Averaging over time will increase detector reliability if the PU signal is continuous in nature. However, if the signal is bursting, noise energy will be averaged into the estimator, reducing reliability. The performance degradation from averaging over time is based on the duty cycle. High duty cycle signals will have little degradation in detector sensitivity due to averaging, but detection of low duty cycle signals will be severely degraded. In [46], use of a maximum hold instead of an average was proposed to combat this degradation. Although the maximum hold would prevent averaging of noise into the estimator, it would increase the false alarm rate, especially at low SNR [60].

#### 2.2.2 Wideband Edge Detection

Another common wideband spectrum sensing method is edge detection, leveraging work in image processing to find discontinuities in the PSD. A fundamental treatment of edge detection is given by Canny in [14]. In [63], PSD discontinuities are found using the first derivative of the PSD.

The multiscale wavelet product, a wideband edge detection method proposed by Tian et al. has been widely cited in the cognitive radio literature [56]. To perform the proposed wideband edge detection, we start by by estimating the PSD,  $S_r(f)$ , of received signal r(t). We then decompose  $S_r(f)$  into a set of resolutions using the continuous wavelet transform (CWT). The CWT of  $S_r(f)$  for a resolution s is given by

$$\mathcal{W}_s\left\{S_r(f)\right\} = S_r(f) * \phi_s(f) \quad , \tag{2.1}$$

where \* is the discrete-time convolution operator and  $\phi_s(f)$  is a Gaussian wavelet of scale s, given by:

$$\phi_s(f) = \frac{1}{s}\phi\left(\frac{f}{s}\right) \quad . \tag{2.2}$$

The  $j^{th}$  resolution of the PSD has scale s where  $s = 2^j$  and j = 1, 2, ..., J. Once the PSD is decomposed into component resolutions using the CWT, edge detection is performed by taking the first derivative of each component resolution:

$$\mathcal{W}'_s\{S_r(f)\} = s \frac{d}{df} \left(S_r(f) * \phi_s(f)\right) \quad . \tag{2.3}$$

Finally, we compute the multiscale wavelet product from the resulting gradient estimates:

$$\mathcal{U}_J\{S_r(f)\} = \prod_{j=1}^j \mathcal{W}'_{s=2^j}\{S_r(f)\} \quad .$$
(2.4)

The rationale behind computing the multiscale product is that the noise at the various resolutions is uncorrelated, while signal at the various resolutions is correlated. By multiplying the component resolutions together, the signal is amplified, while the noise is not, resulting in noise suppression [50].

#### 2.2.3 Compressed Sensing

A class of sensing algorithms known as compressive sensing (CS) has been proposed for surveying very wide bandwidths with sub-Nyquist sampling rates. Because much of the radio spectrum is underutilized, available bands may be represented as a sparse dataset, and depending on the sparsity order of the dataset, the wideband signal may be sensed at a fraction of the Nyquist rate [53]. To perform sub-Nyquist sampling, the signal time series is divided into length-M blocks of Nyquist-rate samples, of which K samples are kept, giving an undersampling fraction of  $\frac{N}{K}$ . Reconstruction of the sparse PSD from the undersampled data is accomplished by solving for a linear inverse, which in the sparse case, requires a numerical solution [57]. To select an appropriate under sampling fraction, the cognitive receiver must have prior knowledge of the PU sparsity order. An online sparsity estimator has been proposed in [52] that can be used to quickly determine an undersampling ratio.

Although CS can be utilized to sense much wider bandwidths than can be done with traditional analog to digital conversion hardware, the result of CS typically involves a static PSD estimate. For example, in [57], the estimated sparse PSD is analyzed with the wavelet-based edge detector proposed in [56]. Because current CS methods rely on a static PSD estimate, they suffer from a similar shortfall where low duty-cycle PUs can drastically reduce the detector sensitivity. In [53] it is stated that the state of the art CS methods are inadequate to properly handle sparsity in time and space.

## 2.3 Multiband Spectrum Sensing

Multiband spectrum sensing involves accurate estimation of PU parameters across multiple channels. Given a finite observation time, a SU should develop a sensing plan, where the times spent on each channel are chosen to minimize some objective function.

A method for allocating sensing times for multiband sensing was proposed by Tehrani et al. in [55] where the objective function is the CRB of the PU parameters. Tehrani *et al.* modeled the PU of each channel as a continuous-time Markov process. The PU state was directly observed by the SU, and the transition probabilities were estimated using the maximum likelihood parameter estimator from [4]. The set of channels was sensed sequentially, and after the entire set was sensed, the sensing duration for each channel was reallocated, with the objective of minimizing the MSE of the transition rates for all channels. A closed form solution for the asymptotically optimal sensing durations was derived and shown to minimize the CRB and be strongly consistent given that the length of the sensing interval approaches infinity.

Some papers on multiband sensing have been published which leverage energy detection, such as [25,36]. In [36], multiband energy detection is proposed in a system where an optimal subset of channels is selected such that the combined capacity of the subset is maximized given that the size of the subset does not surpass a certain number. The capacity metric in [36] considers, among other parameters, the proportion of time that the channel would need to be sensed in order to keep the probability of interference to the PU below a threshold. Also considered is the proportion of time that the PU is idle, which is not estimated in [36]. The capacity in [36] for channel i is formally defined as:

$$C_i^{\rm op} = \eta_i \cdot \rho_i \cdot W_i \cdot P_{off,i} \quad , \tag{2.5}$$

where  $\eta_i$  is the sensing efficiency for channel *i*, or the proportion of time that is reserved for sensing,  $\rho_i$  is the spectral efficiency of channel *i* in (bits/s/Hz),  $W_i$  is the bandwidth of channel *i*, and  $P_{off,i}$  is the probability that the PU is idle, allowing for opportunistic access. In [25], an adaptive sensing method is employed which selects a subset of channels that minimizes the probability of PU detection error over a fixed sensing budget. As probability of false detect is estimated, candidate channels are ruled out until the list of candidate channels is shorter than a given threshold.

In [33], a Bayesian approach to multiband spectrum sensing is proposed. In this approach, it is assumed that the channel gain between the PU and SU has already been accurately estimated. The SU optimizes channel utilization rules such that the throughput is maximized and the probability of interference with the PU is constrained.

There is plenty of room for innovation in multiband spectrum sensing, and as such, the majority of this proposal is devoted to furthering the state of the art. There are obvious practical concerns in [55] to address, such as the continuous-time Markov Chain (CTMC) model when sampling is required, the direct observation of the PU state when there will be a noisy channel between the PU state and observations, and the requirement for infinite sensing durations. Improvements to multiband sensing that are more theoretical in nature may be made as well, such as deriving optimal sensing allocations for a more general set of objective functions and modeling the PU as a bivariate Markov process for better fitting of the dwell time distributions.

### 2.4 Optimal Compute Budget Allocation

In the unrelated field of simulation optimization, optimal compute budget allocation (OCBA) has been developed to minimize the simulation time required to find the optimal design [16]. OCBA employs a generic cost function minimization that can be easily applied to other fields. In this proposal, OCBA will be leveraged for multiband sensing, selecting the best channel with high reliability and a minimum amount of sensing time. OCBA may be applied more generally to find the optimal subset of designs, where the N best designs of a set are chosen and ranked [17].

## Chapter 3: Multiband Spectrum Sensing Based on Markov Modulated Gaussian Processes

## **3.1** Introduction

In this chapter, we focus on the problem of spectrum sensing for multiple PU channels, which we refer to as multiband spectrum sensing. In multiband sensing, an SU performs spectrum sensing on multiple independent channels in parallel. Multiband sensing has applications in scenarios in which the SU has the option of exploiting spectrum hole opportunities in multiple channels. An example of such a scenario is spectrum sensing for white space on broadcast TV channels. In [12], a recursive algorithm for wideband spectrum sensing was developed, which identifies the set of channels occupied by independent PUs in a given spectrum band. Multiband sensing can then be applied to the set of identified channels. Thus, multiband sensing can also be an integral component in a scheme for wideband sensing, wherein the PU channel boundaries are not known a priori.

The spectrum access activity of a PU on a given narrowband channel may be modeled as a 2-state process; i.e., the process alternates between an active state, in which the PU transmits data, and an idle state, in which the PU does not transmit. In the literature, the PU state process is often modeled as a Markov chain, either in continuous-time or discretetime. In practice, however, the PU state cannot be observed directly by a SU, but can only be inferred from received signal measurements taken by the SU. Thus, hidden Markov models (HMMs) have been found to be a useful class of models for temporal spectrum sensing [3]. An HMM can take into account the noise and channel impairments introduced by the wireless channel. In [43], the application of HMMs for spectrum sensing was extended to the more general hidden bivariate Markov model (HBMM). The main advantage of the HBMM is that it can model non-geometric sojourn time distributions for the state process, whereas the HMM restricts the state sojourn times to be geometrically distributed. In particular, the state sojourn time distribution of an HBMM belongs to the class of discrete phase-type distributions, which is dense in the class of all possible sojourn time distributions [43].

In [55], a multiband sensing policy was derived which allocates sensing resources such that the Cramér Rao bound (CRB) for estimators of the channel parameters is minimized across all channels under constrained sensing time. In that work, the PU signal associated with each channel is modeled as a continuous-time Markov process and it is assumed that the PU state is directly observable. In practical scenarios, however, the PU state can only be inferred from discrete-time measurements taken by the SU, so a more robust system model must be employed to handle sampling and channel impairments such as noise and fading. In this chapter, we propose the use of a Markov-modulated Gaussian process (MMGP), a type of hidden Markov process to more accurately model PU traffic as observed by the SU. An MMGP is a continuous-time finite-state homogeneous Markov chain observed through a discrete-time memoryless Gaussian channel [47]. We will use the expectation-maximization algorithm for MMGPs derived in [47] to estimate the PU parameter for each channel and use an extended version of the sensing allocation updates from [55] to allocate optimal sensing times for each channel such that the CRB for estimators of the MMGP parameter is minimized.

### 3.2 System Model

#### 3.2.1 Multiband Channel Model

A set of M channels with known center frequency and bandwidth is observed, each with a single independent PU. It is assumed that PU channels are not overlapping in frequency. The channel over which the  $i^{th}$  PU is observed is assumed to be flat Rayleigh fading with parameter  $\sigma_{f,i}$  combined with zero mean additive white Gaussian noise (AWGN), defined by the circularly symmetric complex normal distribution  $C\left(0, \sigma_{n,i}^2\right)$ . The mean SNR of the

received signal on channel i, given that the PU is transmitting is

$$\overline{\text{SNR}}_i = \frac{\sigma_{f,i}^2}{\sigma_{n,i}^2},\tag{3.1}$$

at the input to the energy detector.

#### 3.2.2 PU Traffic Model

A PU may be transmitting or idle at any given time. The state of the  $i^{th}$  PU is denoted by the random variable  $X_i$ , where  $X_i = 0$ , when the PU is not transmitting, and t  $X_i = 1$ , when the PU is transmitting. The  $k^{th}$  state of the PU is denoted by  $X_{i,k}$ . The sequence of active/idle states from each PU is modeled by a continuous-time homogeneous Markov chain with generator matrix  $Q_i$  and initial distribution  $\pi_i$  defined, respectively, as

$$Q_{i} = \begin{bmatrix} -\lambda_{0,i} & \lambda_{0,i} \\ \lambda_{1,i} & -\lambda_{1,i} \end{bmatrix},$$
(3.2)

$$\pi_{0,i} = P(X_1 = 0), \pi_i = P(X_{i,1} = 1), \qquad (3.3)$$

where  $\lambda_{j,i}$  is the rate of the exponential sojourn time distribution in state j for PU i.

#### 3.2.3 Cognitive Receiver Model

Let  $Y_{i,k}$  denote the average energy at time k = 0, 1, ... of the narrowband signal from channel *i* over *N* samples. Let the sequence of energy estimates for channel *i* be denoted  $Y_i^n = \{Y_{i,1}, ..., Y_{i,n}\}$ . The  $k^{th}$  sample in the energy detection sequence,  $Y_{i,k}$ , is defined as

$$Y_{i,k} = \frac{1}{N} \sum_{j=1}^{N} |Z_{i,(k-1)N+j}|^2.$$
(3.4)

Assuming that N is sufficiently large,  $y_{i,k}$  will approximately be conditionally normal

with distribution

$$y_{i,k} \sim \begin{cases} \mathcal{N}\left(2\sigma_{n,i}^{2}, \frac{4\sigma_{n,i}^{4}}{N}\right), & X_{i,k} = 0, \\ \mathcal{N}\left(2\sigma_{f,i}^{2} + 2\sigma_{n,i}^{2}, \frac{4\left(\sigma_{f,i}^{2} + \sigma_{n,i}^{2}\right)^{2}}{N}\right), & X_{i,k} = 1, \end{cases}$$

$$(3.5)$$

This approximate conditional distribution is derived in Appendix A.

#### 3.2.4 Markov-Modulated Gaussian Process

The continuous-time Markov chain representing the active/idle states of the PU, observed through a sampled Gaussian channel, yields an MMGP. The parameter of the MMGP is

$$\phi = \left(\mathbf{Q}, \boldsymbol{\pi}, \boldsymbol{\mu}, \boldsymbol{\sigma}^2\right), \tag{3.6}$$

where  $\mathbf{Q}$  is the generator matrix for the CTMC,  $\pi$  is the initial state distribution, and  $(\mu, \sigma^2)$  are the mean and variance respectively of the conditional Gaussian distribution. In the case of spectrum sensing, the underlying CTMC will have two states, off and on, resulting in the generator defined in Eq. (3.2). In the case of Rayleigh fading and additive white Gaussian noise observed through an energy detector, the conditional Gaussian distribution is defined in Eq.(3.5). Other fading channels may be modeled using MMGPs as well, under the assumption that the output of the energy detector is approximately Gaussian due to the Central Limit Theorem, but closed-form expressions for the conditional Gaussian distribution may not exist.

## 3.3 Multiband Spectrum Sensing From Observed Continuoustime Markov Model

In this section, we review the main results from [55], based on a continuous-time Markov process model, which will be used to develop an MMGP-based multiband spectrum sensing scheme in Section 3.4.

#### 3.3.1 Moment Estimator

We assume that time is divided into a sequence of sensing intervals. Each sensing interval is in turn subdivided into M sensing subintervals, one for each of the M channels. Let  $T_n$ denote the duration of the nth sensing interval, and let  $T_{i,n}$  denote the duration of the nth sensing subinterval devoted to channel i. To perform the first step of multiband sensing, all M channels are sensed for exactly the same amount of time. In other words, the duration of the initial sensing interval  $T_0$  is divided into M equal subintervals. Each channel is sensed in sequence. For channel i, the sojourn times in the on and off state are calculated, based on direct observation of the PU state. The parameter of the continuous-time Markov process model of the ith channel, i.e., the pair of transition rates  $(\lambda_{0,i}, \lambda_{1,i})$ , is then estimated.

Since the PU state could extend beyond the beginning or end of the sensing interval, the first and last sojourn time measurements are not used in the estimator. During a sensing subinterval for channel *i*, the number of sojourns in each state is counted, and the counts are denoted  $n_i^{\text{off}}$  and  $n_i^{\text{on}}$  for the number of sojourns in the off and on states, respectively. The *j*th recorded sojourn time for channel *i* during a sensing subinterval for the off and on states are denoted by  $z_{i,j}^{\text{off}}$  and  $z_{i,j}^{\text{on}}$  respectively. The estimator for the transition rates proposed in [55], referred to in that chapter as the "moment estimator," is given by:

$$\hat{\lambda}_{0,i} = \frac{n_i^{\text{off}} - 1}{\sum_{j=1}^{n_i^{\text{off}}} z_{i,j}^{\text{off}}}, \quad \hat{\lambda}_{1,i} = \frac{n_i^{\text{on}} - 1}{\sum_{j=1}^{n_i^{\text{on}}} z_{i,j}^{\text{on}}}.$$
(3.7)

This estimator is the well-known maximum likelihood estimator (MLE), with consistency

proved by Albert [4, Theorem 6.10]. Asymptotic efficiency of this estimator was also proved by Albert in section 7 of [4]. Efficiency of the moment estimator was demonstrated in [55] through simulation.

#### 3.3.2 Allocation of Sensing Effort

After all of the channels have been sensed for an interval, an allocation of sensing time for each channel *i*, as a fraction,  $\alpha_i$ , of the total sensing time in the next sensing interval is determined. In [55],  $\alpha_i$  is derived by minimizing the CRB for all transition rates, and is given as follows:

$$\alpha_i = \frac{\sqrt{\left(\lambda_{0,i}^2 + \lambda_{1,i}^2\right)\left(\frac{1}{\lambda_{0,i}} + \frac{1}{\lambda_{1,i}}\right)}}{\sum_{j=1}^M \sqrt{\left(\lambda_{0,j}^2 + \lambda_{1,j}^2\right)\left(\frac{1}{\lambda_{0,j}} + \frac{1}{\lambda_{1,j}}\right)}}.$$
(3.8)

The sensing duration allocated to channel i for the nth sensing interval, is given by  $T_{i,n} = \alpha_i T_n$ 

The observed sojourn times used to compute the moment estimator (3.7) are saved for the next sensing interval. Using the entire record of sojourn time measurements will improve estimator accuracy. This allocation strategy was shown in [55] to approach, as  $n \to \infty$ , the (CRB) for the joint estimation of all M independent channels, which is given as follows:

$$\sigma^2 \ge \sum_{i=1}^M \frac{\left(\lambda_{0,i}^2 + \lambda_{1,i}^2\right) \left(\frac{1}{\lambda_{0,i}} + \frac{1}{\lambda_{1,i}}\right)}{T_i}.$$
(3.9)

In [55], these results are proven to minimize the asymptotic CRB, as  $T \to \infty$ . A more practical scenario is when a finite sensing interval is considered. CR media access control (MAC) protocols such as 802.22 [2] and 802.11af [1], rely on finite-duration quiet periods for



Figure 3.1: Direct observation of PU state

all nodes to search for PU activity and enable coexistence with incumbent systems. With this motivation, we will develop sensing allocations for finite interval durations in the next section.

#### 3.3.3 Bias of Moment Estimator in Noise

In a practical scenario, the PU state can only be observed through a noisy, possibly fading channel. The simplest method of PU state estimation in noise is energy detection. With an energy detector (cf. [58]), the state of the PU is determined to be on if the received power surpasses a threshold, and the PU is otherwise determined to be off.

In Figure 3.1, the sojourn times from directly observing the PU state are shown. In Figure 3.2, the effect of a decision error on the received PU signal is shown. The average sojourn time during this sensing interval will be shorter, due to the detection error. The parameter estimates  $\hat{\lambda}_{0,i}$  and  $\hat{\lambda}_{1,i}$  will be greater than the true parameter values. As the SNR decreases, probability of detection error will increase, and the mean error of the moment estimator will increase. Therefore, decreasing SNR will result in bias in the moment estimator.

Even with moderate SNR, estimation of the PU parameter using an energy detector and moment estimator may be significantly impaired. In this chapter, we propose a sensing algorithm which overcomes this deficiency.



Figure 3.2: Effect of a decision error on sojourn time estimates

## 3.4 Multiband Spectrum Sensing based on a Markov-Modulated Gaussian Process

In Section 3.2, we proposed a CTMC observed through a sampled Gaussian process to model a PU on a single channel. In this section, we detail the process of spectrum sensing by training an MMGP and propose a multiband optimization algorithm based on minimizing the CRB of the parameter estimates across multiple MMGP channels.

#### 3.4.1 MMGP Parameter Estimation

In [47], an algorithm for estimating the parameter of an MMGP was developed based on the Expectation Maximization (EM) algorithm. We shall apply this algorithm to multiband spectrum sensing through fading and additive noise.

Given a received energy sequence measurements for channel  $i, Y_i^n$ , as defined in Eq. (3.4), we want to derive the MMGP parameter for channel  $i: \phi_i = (\mathbf{Q}_i, \pi_i, \mu_i, \sigma^2_i)$ , given in Eq. (3.6).

We start with an initial MMGP parameter estimate for channel *i*:  $\hat{\phi}_{i,0} = \left(\hat{\mathbf{Q}}_{i,0}, \hat{\pi}_{i,0}, \hat{\mu}_{i,0}, \hat{\sigma}_{i,0}^2\right)$ . The *j*th iteration of the EM algorithm for channel *i* produces parameter estimate  $\hat{\phi}_{i,j}$  with likelihood greater than or equal to that of  $\hat{\phi}_{i,j-1}$ . Each iteration of the algorithm involves the computation of forward and backward recursions [47, Section III].

Let  $\hat{\phi}_{i,j-1}$  denote the current parameter estimate at the start of the *j*th iteration of the EM algorithm for channel *i*. Define the probability densities

$$b_{i,0}(Y_k) = f_{i,0}(Y_k),$$
  

$$b_{i,1}(Y_k) = f_{i,1}(Y_k),$$
(3.10)

as the conditional probability densities given that the underlying Markov chain for channel  $i, X_i$ , resides in state 0 or 1, respectively. The Gaussian density functions are defined as

$$f_{i,0}(Y_k) \sim \mathcal{N}\left(\mu_{i,0}, \sigma_{i,0}^2\right),$$
  
$$f_{i,1}(Y_k) \sim \mathcal{N}\left(\mu_{i,1}, \sigma_{i,1}^2\right),$$
(3.11)

which in the case of Rayleigh fading are given by Eq. (3.5). Define a diagonal matrix

$$\mathbf{B}_{i}(Y_{k}) = \operatorname{diag}\left\{b_{i,0}, b_{i,1}\right\}.$$
(3.12)

Define the transition density matrix for sample k of channel i as

$$\mathbf{f}_{i}\left(Y_{k}\right) = e^{\mathbf{Q}_{i}h}\mathbf{B}_{i}\left(Y_{k}\right),\tag{3.13}$$

where  $\hat{\mathbf{Q}}_i$  is the most recent estimate of the generator matrix for channel *i*, and *h* is the sampling period. We denote the scaled forward and backward recursion vectors for sample *k* of channel *i* by  $\mathbf{L}_i(k)$  and  $\mathbf{R}_i(k)$  respectively. The forward recursion vector is defined as a row vector

$$\mathbf{L}_{i}(1) = \frac{\hat{\boldsymbol{\pi}}_{i} \mathbf{B}_{i}(Y_{1})}{c_{i,1}},$$
$$\mathbf{L}_{i}(k) = \frac{\mathbf{L}_{i}(k-1) \mathbf{f}_{i}(Y_{k})}{c_{i,k}}, \qquad k = 2, \dots, n,$$
(3.14)

where  $\hat{\pi}_i$  is the most recent estimate for the initial distribution of channel *i* and  $c_{i,k}$  is a scaling constant defined as

$$c_{i,1} = \hat{\pi}_{i} \mathbf{B}_{i} (Y_{1}) \mathbf{1},$$
  

$$c_{i,k} = \mathbf{L}_{i} (k-1) \mathbf{f}_{i} (Y_{k}) \mathbf{1}, \qquad k = 2, \dots, n,$$
(3.15)

where **1** is a column vector of all ones. The backward recursion vector is defined as a column vector

$$\mathbf{R}_{i}(n+1) = \mathbf{1},$$
  
$$\mathbf{R}_{i}(k) = \frac{\mathbf{f}_{i}(Y_{k+1}) \mathbf{R}_{i}(k+1)}{c_{i,k+1}}, \qquad k = n, \dots, 1.$$
 (3.16)

The forward and backward recursion vectors are used to estimate  $m_{a,b}$  and  $T_a$ , the number of state transitions from state a to b, and the amount of time spent in each state a, respectively for all states. Define a  $(2 \times 2)$  matrix

$$\mathbf{N}_{i} = \sum_{k=2}^{n} \frac{1}{c_{i,k}} \mathbf{B}_{i}(Y_{k}) \mathbf{R}_{i}(k) \mathbf{L}_{i}(k-1).$$
(3.17)

Note that  $\mathbf{R}_{i}(k)$  is a column vector and  $\mathbf{L}_{i}(k-1)$  is a row vector, so the product in 3.17 is an outer product, resulting in a  $(2 \times 2)$  matrix. It is shown in [48] that the matrix of
estimated transition counts can be computed as

$$\hat{\mathbf{m}}_i = \hat{\mathbf{Q}}_i \odot \mathcal{I}' \tag{3.18}$$

where  $\mathbf{X}'$  denotes the transpose of matrix  $\mathbf{X}$ ,  $\odot$  denotes element-wise matrix multiplication, and the matrix  $\mathcal{I}$  is defined as

$$\mathcal{I} = \int_0^h e^{\hat{\mathbf{Q}}_i(h-t)} \mathbf{N}_i e^{\hat{\mathbf{Q}}_i t} dt.$$
(3.19)

The integral 3.19 can be evaluated efficiently using the approach in [59]. Define a  $(4 \times 4)$  matrix

$$\mathbf{C}_{i} = \begin{bmatrix} \hat{\mathbf{Q}}_{i} & \mathbf{N}_{i} \\ 0 & \hat{\mathbf{Q}}_{i} \end{bmatrix}.$$
 (3.20)

 $\mathcal{I}$  is given by the upper-right (2 × 2) block of the matrix  $e^{\mathbf{C}_i h}$ . The diagonal elements of  $\hat{\mathbf{m}}_i$ and  $\hat{\mathbf{Q}}_i$  are used to compute the estimated time in each state

$$\hat{T}_{i,1} = \frac{\hat{m}_{i,0,0}}{\hat{q}_{i,0,0}},$$

$$\hat{T}_{i,2} = \frac{\hat{m}_{i,1,1}}{\hat{q}_{i,1,1}}.$$
(3.21)

The final calculation of the Expectation phase is to compute the *a posteriori* probability of each state. Define  $\boldsymbol{\xi}_i(k)$ , the vector of *a posteriori* probabilities for each state at sample k on channel *i*, as

$$\boldsymbol{\xi}_{i}\left(k\right) = \frac{\mathbf{L}_{i}\left(k\right) \odot \mathbf{R}_{i}\left(k\right)'}{\mathbf{L}_{i}\left(k\right) \odot \mathbf{R}_{i}\left(k\right)' \mathbf{1}}.$$
(3.22)

For the Maximization phase, the parameter is re-estimated based on the results of the

Expectation phase. The elements of the generator matrix are estimated using the result of Eqs. (3.18) and (3.21)

$$\hat{q}_{i,a,b} = \frac{\hat{m}_{i,a,b}}{\hat{T}_{i,a}}, \qquad a \neq b$$
$$\hat{q}_{i,a,a} = -\sum_{b \neq a} \hat{q}_{i,a,b}.$$
(3.23)

The *a posteriori* probabilities from Eq. (3.22) are used to re-estimate  $\mu_{i,a}$  and  $\sigma_{i,a}^2$ , which are respectively the conditional mean and variance for state *a* on channel *i*. The conditional distributions are estimated as

$$\hat{\mu}_{i,a} = \frac{1}{\hat{n}_{i,a}} \sum_{k=1}^{n} \hat{\xi}_{i,a} Y_i(k) , \qquad (3.24)$$

$$\hat{\sigma}_{i,a}^{2} = \frac{1}{\hat{n}_{i,a}} \sum_{k=1}^{n} \hat{\xi}_{i,a} \left( Y_{i}\left(k\right) - \hat{\mu}_{i,a} \right)^{2}, \qquad (3.25)$$

where  $\hat{n}_{i,a}$  is the estimated number of samples in state *a* for channel *i*, computed as

$$\hat{n}_{i,a} = \sum_{k=1}^{n} \hat{\xi}_{i,a}.$$
(3.26)

#### 3.4.2 Allocation of Sensing Effort

In our approach towards allocation spectrum sensing time between channels, we will take a similar approach to that proposed in [55] of minimizing the CRB. Such an optimization requires derivation of the Fisher information matrix (FIM) of an MMGP, which is done in Appendix B. The FIM for a two-state MMGP is given as

$$I(\mathcal{L}^{c}) = \operatorname{diag}\left\{\frac{\mathrm{E}(m_{01})}{\lambda_{0}^{2}}, \frac{\mathrm{E}(m_{10})}{\lambda_{1}^{2}}, \frac{\mathrm{E}(n_{0})}{\sigma_{0}^{2}}, \frac{\mathrm{E}(n_{1})}{\sigma_{1}^{2}}, \frac{\mathrm{E}(n_{0})}{2\sigma_{0}^{4}}, \frac{\mathrm{E}(n_{1})}{2\sigma_{1}^{4}}\right\}$$
(3.27)

where  $E(m_{01})$  and  $E(m_{10})$  are the expected number of jumps between the two states of the underlying Markov chain, and  $E(n_0)$  and  $E(n_1)$  are the expected number of samples in each state. Closed-form expressions for these expected values are given

$$E(m_{01}) = \frac{\lambda_0 \lambda_1}{\lambda_0 + \lambda_1} t + \left(\pi_0 - \frac{\lambda_1}{\lambda_0 + \lambda_1}\right) \frac{\lambda_0}{\lambda_0 + \lambda_1} \left(1 - e^{-(\lambda_0 + \lambda_1)t}\right),$$

$$E(m_{10}) = \frac{\lambda_0 \lambda_1}{\lambda_0 + \lambda_1} t + \left(\pi_1 - \frac{\lambda_0}{\lambda_0 + \lambda_1}\right) \frac{\lambda_1}{\lambda_0 + \lambda_1} \left(1 - e^{-(\lambda_0 + \lambda_1)t}\right),$$

$$E(n_0) = \frac{\lambda_1}{h(\lambda_0 + \lambda_1)} t + \frac{\pi_0 \lambda_0 - \pi_1 \lambda_1}{h(\lambda_0 + \lambda_1)^2} \left(1 - e^{-(\lambda_0 + \lambda_1)t}\right),$$

$$E(n_1) = \frac{\lambda_0}{h(\lambda_0 + \lambda_1)} t + \frac{\pi_1 \lambda_1 - \pi_0 \lambda_0}{h(\lambda_0 + \lambda_1)^2} \left(1 - e^{-(\lambda_0 + \lambda_1)t}\right),$$
(3.28)

where  $\pi_0$  and  $\pi_1$  are the initial probabilities, and h is the sampling period. In the case of long sensing durations, a greatly simplified asymptotic approximation may be used

$$E(m_{01}) = \frac{\lambda_0 \lambda_1}{\lambda_0 + \lambda_1} t, \ t \to \infty,$$
  

$$E(m_{10}) = \frac{\lambda_0 \lambda_1}{\lambda_0 + \lambda_1} t, \ t \to \infty,$$
  

$$E(n_0) = \frac{\lambda_1}{h(\lambda_0 + \lambda_1)} t, \ t \to \infty,$$
  

$$E(n_1) = \frac{\lambda_0}{h(\lambda_0 + \lambda_1)} t, \ t \to \infty.$$
(3.29)

The CRB on the total mean squared error in estimating the parameter for a single channel is the trace of the inverse FIM, which in the case of a diagonal matrix is simply the element-wise inverse. Because the channels are independent, the multichannel CRB is simply the sum of the single channel CRB for each of the M channels

$$\sigma^{2} \geq \sum_{i=1}^{M} \left( \frac{\lambda_{i,0}^{2}}{\operatorname{E}(m_{i,01})} + \frac{\lambda_{i,1}^{2}}{\operatorname{E}(m_{i,10})} + \frac{\sigma_{i,0}^{2} + 2\sigma_{i,0}^{4}}{\operatorname{E}(n_{i,0})} + \frac{\sigma_{i,1}^{2} + 2\sigma_{i,1}^{4}}{\operatorname{E}(n_{i,1})} \right).$$
(3.30)

In the case of longer sensing times, the asymptotic approximation in (3.29) results in a CRB of

$$\sigma^2 \ge \sum_{i=1}^M \frac{\lambda_{i,0} + \lambda_{i,1}}{T_i} \left( \frac{\lambda_{i,0}}{\lambda_{i,1}} + \frac{\lambda_{i,1}}{\lambda_{i,0}} + \frac{h\sigma_{i,0}^2}{\lambda_{i,1}} + \frac{h\sigma_{i,1}^2}{\lambda_{i,0}} + \frac{2h\sigma_{i,1}^4}{\lambda_{i,1}} + \frac{2h\sigma_{i,1}^4}{\lambda_{i,0}} \right), \qquad t \to \infty, \quad (3.31)$$

where  $T_i$  is the time spent sensing channel *i*.

To minimize the CRB in (3.30), numerical optimization must be used. For (3.31), a closed-form optimization may be found using Lagrange multipliers. Minimizing (3.31) under the constraint that the sum of all  $T_i$  equals T yields

$$T_{i} = \frac{\sqrt{\alpha_{i}}}{\sum_{i=1}^{M} \sqrt{\alpha_{i}}} T,$$
  

$$\alpha_{i} = (\lambda_{i,0} + \lambda_{i,1}) \left( \frac{\lambda_{i,0}}{\lambda_{i,1}} + \frac{\lambda_{i,1}}{\lambda_{i,0}} + \frac{h\sigma_{i,0}^{2}}{\lambda_{i,1}} + \frac{h\sigma_{i,1}^{2}}{\lambda_{i,0}} + \frac{2h\sigma_{i,0}^{4}}{\lambda_{i,1}} + \frac{2h\sigma_{i,1}^{4}}{\lambda_{i,0}} \right).$$
(3.32)

Minimization of (3.30) requires numerical methods. In our simulations, we used Sequential Quadratic Programming [44] to find the optimal sensing allocations. The approximate solution in Eq. 3.32 should be used as the initial time durations for the numerical optimization, which should reduce time to convergence and increase the likelihood that a global optimum is reached.

#### 3.4.3 Averaging of Parameter Estimates

Between sensing intervals, the MMGP parameter estimates are averaged to take advantage of all previous sensing intervals. During subsequent sensing intervals, the averaged parameter from the previous sensing interval is used as the initial parameter for the EM algorithm applied to the current sensing interval.

A simple averaging scheme is the infinite impulse response (IIR) average or exponential average. Let  $\tilde{\phi}_{i,n}$  denote the averaged MMGP parameter for channel *i*, for sensing intervals up to and including the *n*th, and let  $\phi_{i,n}$  denote the (unaveraged) parameter estimate for channel *i*, corresponding with the *n*th sensing interval. An exponential averaging scheme with weight  $\omega$  is given as

$$\tilde{\phi}_{i,n} = \omega \tilde{\phi}_{i,n} + (1-\omega) \hat{\phi}_{i,n}, \qquad 0 < \omega < 1.$$
(3.33)

An exponential average of the parameter estimates is useful when the MMGP parameter is changing over time. In the static case where the MMGP parameter never changes, a block average will be better. In the case of the block average, the parameter estimate  $\hat{\phi}_{i,n}$ is weighted by the sensing times allocated to channel *i* 

$$\tilde{\phi}_{i,n} = \frac{\sum_{j=1}^{n-1} T_{i,j}}{\sum_{j=1}^{n} T_{i,j}} \tilde{\phi}_{i,n-1} + \frac{T_{i,n}}{\sum_{j=1}^{n} T_{i,j}} \hat{\phi}_{i,n}.$$
(3.34)

In the case where the MMGP parameter does not change over time, block averages will outperform any exponential average. Because the MMGP in this chapter is modeled as static, block averaging is used.

#### 3.4.4 MAP Decision Rule

Although not specifically used in the estimation of multiple PU statistics, maximum *a* posteriori probability (MAP) decisions may be determined using the results of MMGP estimation. The most likely state for channel *i* at time *k* is denoted  $\hat{x}_i(k)$ , defined as

$$\hat{x}_{i}(k) = \underset{x_{k} \in \{0,1\}}{\arg \max} \left[ \boldsymbol{\xi}_{i}(k) \right].$$
(3.35)

## 3.5 Simulation and Numerical Results

To evaluate the proposed algorithms, a simulation was performed in which multiband spectrum sensing was performed and estimator errors were compared to the theoretical minimum in (3.30). Three multiband data sets were generated, and three methods for spectrum sensing were tested: energy detection, MMGP parameter estimation with IIR averaging between intervals, and MMGP parameter estimation with block averaging between intervals. In this simulation, the energy detector used perfect knowledge of signal and noise statistics to select a threshold which jointly minimized false alarm and false positive probability. The MMGP parameter estimators were seeded with random values, and therefore performance is shown without any prior knowledge of signal and noise parameters.

The simulated multiband scenario contained 4 channels as was done in [55] with transition rates given in Table 3.5.

	Ch 1	Ch 2	Ch 3	Ch 4
$\lambda_0$	0.1	0.6	0.2	0.5
$\lambda_1$	0.7	0.1	0.8	0.9

Table 3.1: Continuous-time Markov process channel parameters.

Simulations were performed against channels with SNR of 10, 0, and -5 dB. The other simulation parameters are given in Table 3.5.

Parameter	Value	Description	
M	4	Number of channels	
$N_{sim}$	200	Number of simulation itera- tions	
$N_{sense}$	20	Number of sensing intervals	
$T_0$	100	Initial sensing duration for the first interval	
$N_{ed}$	100	Number of samples to aver- age for energy detector	
h	10	Sampling rate after energy detection	
$T_n$	$1.25T_{n-1}$	Sensing duration for the $n$ th interval	

Table 3.2: Simulation parameters.

The simulation results for SNR of 10, 0, and -5 dB are shown in Figures 3.3, 3.4, and 3.5,



Figure 3.3: Multiband sensing performance at 10 dB SNR

respectively. It is apparent that MMGP parameter estimation with block averaging is the best option in all cases in terms of sensitivity. At the higher SNR of 10 dB, the energy detector performed comparably, but its performance was inadequate at lower SNR. Use of fixed-gain IIR averaging between intervals never reaches the efficiency of block averaging at any SNR, but as discussed previously, IIR averaging enables tracking of PUs with statistics that change over time. The simulation results also highlight the trade-off between acquisition time and tracking performance of fixed-gain IIR estimates. Smaller gain values will have reduced tracking errors but will take longer to converge. This can be seen in the simulation results where the IIR estimator with feedback gain of 0.5 had lower estimation error early on, but as more estimates were accumulated, the IIR estimator with the lower feedback gain of 0.2 had smaller error. An IIR estimator with variable gain may be used to remedy this trade-off, where the initial feedback gain is large to enable fast acquisition, and the feedback gain is systematically decreased to enable accurate tracking. In the case where the system parameter has absolutely no change over time, the proposed block averaging estimator is the ideal variable-gain IIR estimator.



Figure 3.4: Multiband sensing performance at 0 dB SNR



Figure 3.5: Multiband sensing performance at -5 dB SNR

# 3.6 Conclusion

In this chapter, we proposed an MMGP as an accurate model for spectrum sensing in noise and fading, and we have extended the work of [55] to perform efficient spectrum sensing based on this model. The FIM of a two-state MMGP was derived for finite sensing intervals, and subsequently a numerical optimization problem was formulated to select sensing allocations that minimize the CRB. The closed-form FIM was used to derive sensing intervals to minimize the CRB on the variance of parameter estimation across multiple channels. For asymptotically long sensing subinterval durations, closed-form allocations were derived that minimize the CRB. A numerical method was proposed for determining optimal sensing allocations in the general case. In simulation, performance of the MMGP parameter estimator was compared to that of an energy detector. It was shown that MMGP parameter estimation is efficient even in low SNR. A suggested extension to this work is consideration of objective functions which may be more directly applicable to spectrum sensing, such as minimization of estimator error on channels determined to be most usable by an SU.

# Chapter 4: A Computing Budget Allocation Approach to Multiband Spectrum Sensing

## 4.1 Introduction

In multiband spectrum sensing, the SU tracks states of PUs operating on a given set of channels to determine spectrum access opportunities. We extend the work in [55], where the active/idle state process of each PU is modeled as a two-state homogeneous continuous-time Markov chain, and the Markov chains corresponding to different PUs are assumed statistically independent. We assume M channels, each having the same bandwidth. The parameter of each Markov chain is not known in advance and hence is estimated from observations of the state processes. We assume, as in [55], that the PU state processes are observed directly, and thus we ignore channel impairments. Our analysis is suitable for channels with very high signal-to-noise ratio. Subsequent work should address the adverse affects of the channels.

In the proposed approach, given a sensing interval of length T seconds, the SU senses each channel *i* for  $T_i$  seconds such that  $\sum_{i=1}^{M} T_i = T$ . We address the problem of allocating the sensing times  $T_i$  among the M channels such that a subset of  $N \leq M$  channels with the largest mean idle times can be selected. As an additional objective, the total parameter estimation error for channels in the selected set should be minimized. In practice,  $N \ll M$ , i.e., the number of channels for spectrum sensing is much smaller than the total number of channels in a given spectrum band. When N = 1, the problem reduces to allocating the sensing time budget to determine, the channel with the largest mean idle time, and minimizing the estimation error for the associated parameter. When N = M, our approach defaults to the framework used in [55], where minimum mean squared error (MMSE) parameter estimation is performed over all M channels. To address the multichannel estimation problem described above, we adapt the optimal computing budget allocation (OCBA) methodology [16] from the field of simulation optimization. The OCBA framework was developed as a means of testing multiple designs through simulation by allocating simulation time to the designs with the objective of maximizing the probability that the best design is selected according to a given cost function [18]. The technique was subsequently extended to determine the best N > 1 designs among a given set of M designs [17]. In the context of multichannel parameter estimation, instead of allocating simulation time we allocate sensing times, and instead of simulating multiple designs, we perform parameter estimation of multiple channels.

The OCBA approach is generally applied iteratively to a sequence of simulation time intervals. Likewise, our proposed algorithm for multichannel parameter estimation iterates over a sequence of sensing intervals. In a departure from the traditional OCBA, we have as an additional objective, minimizing the estimation error for the parameters of the channels in the selected subset. During each iteration of the algorithm, we employ the Bhattacharyya distance metric [31] to eliminate from consideration channels that are unlikely to belong to the selected subset. This approach allows the sensing resources to be concentrated, in subsequent iterations, on estimation of the channels that are more likely to belong to the selected subset.

The work in [55] allocates the sensing times  $\{T_i\}$  by minimizing the Cramer-Rao lower bound on the minimum mean squared error in estimating the parameters of *all M* channels. Our proposed approach focuses the estimation effort on a much smaller subset of the *N* most promising channels with respect to mean idle time. In [55], an asymptotic expression for large observation time of the inverse Fisher information matrix (FIM) is used. We refine this result to apply to any finite time interval. By using the asymptotic expression, closedform formulas for the MMSE sensing time allocations were obtained in [55], as shown in (4.9). In this chapter, we use an exact expression for the FIM, but resort to a numerical optimization approach to solve for the sensing time allocations.

A number of articles on multiband spectrum sensing have approached the problem

as a type of multi-armed bandit problem [6, 45, 61, 66] or the related partially observable Markov decision process (POMDP) [67]. Several assume knowledge of the parameters of the underlying Markov chains, but do not address the important issue of parameter estimation [6, 61, 67]. Our proposed multichannel parameter estimation algorithm obtains estimates of this parameter, and thus could, in principle, be used in conjunction with these approaches. Moreover, knowledge of the parameter can be used to improve spectrum detection performance and allows the prediction of future PU state, which provides clear advantages for spectrum sensing [43, 61]. In [43], for example, a likelihood ratio detector for PU state on a given channel is proposed based on an estimate of the associated parameter.

The rest of the chapter is organized as follows. In Section 4.2, we present the system model assumed in the chapter. In Section 4.3, we summarize the multichannel estimation algorithm in [55], which is based on minimizing the mean squared error over all channels. In Section 4.4, we develop the proposed algorithm for multichannel estimation based on OCBA and the Bhattacharyya distance measure. In Section 4.5, we present simulation results that demonstrate the performance of the algorithm. The chapter is concluded in Section 4.6 with additional comments. A portion of the work in this chapter has been published in [10].

## 4.2 System Model

Consider a multiband spectrum sensing scenario consisting of M channels, which an SU may leverage for opportunistic spectrum access. In each band, an independent PU is operating. Each PU is modeled by a two-state continuous-time Markov chain as depicted in Fig. 4.1, where state 0 represents an idle PU and state 1 represents a busy PU. An SU may only use the band when the PU is in the idle state. For a given PU, let  $\{X_t, t \ge 0\}$  denote the Markov chain associated with the state process. The transition rate from state 0 to 1 is denoted  $\lambda_0$ , and the transition rate from state 1 to 0 is denoted  $\lambda_1$ . The parameter of the Markov chain is given by  $\theta = (\mu_0, \mu_1)$ , where  $\mu_j$  is the mean sojourn time in state j,  $\mu_j = 1/\lambda_j$ . For simplicity, we assume, as in [55], that the SU directly observes the PU state



Figure 4.1: Continuous-time Markov chain model for PU state of a single channel.

process  $\{X_t, t \ge 0\}$ . The model could be extended to incorporate channel impairments, as was done in [43], for example.

Let  $N_t(j, k)$  denote the number of jumps of the PU state from state j to state k over the time interval [0, t), and denote its expected value by  $\overline{N}_t(j, k)$ , where  $j, k \in \{0, 1\}$ . Let  $T_j$ denote the total PU time spent in state j over the time interval [0, t). We assume that the Markov chain  $\{X_t\}$  has initial state probabilities  $\{\pi_0, \pi_1\}$ , where  $\pi_j = P(X_0 = j), j = 0, 1$ . The Fisher information matrix (FIM) for  $\{X_t\}$  is derived in Appendix C as follows:

$$I(t) = \begin{bmatrix} \frac{2\overline{T}_0}{\mu_0^3} - \frac{\overline{N}_t(0,1)}{\mu_0^2} & 0\\ 0 & \frac{2\overline{T}_1}{\mu_1^3} - \frac{\overline{N}_t(1,0)}{\mu_1^2} \end{bmatrix}.$$
 (4.1)

The derivation follows directly from the definition of the FIM and an expression for the likelihood function of a continuous-time Markov chain given in [4, Sec. 4]. Let  $\lambda = \lambda_0 + \lambda_1$ . The expected number of jumps from state j to state k can be expressed as follows (see Appendix C for the derivation):

$$\overline{N}_{t}(j,k) = \frac{1}{\mu_{j} + \mu_{k}} t + \left(\pi_{j} - \frac{\mu_{j}}{\mu_{j} + \mu_{k}}\right) \frac{\mu_{k}}{\mu_{j} + \mu_{k}} \left(1 - e^{-\left(\frac{\mu_{j} + \mu_{k}}{\mu_{j}\mu_{k}}\right)t}\right).$$
(4.2)

The expected total time in state j, fully derived in Appendix C, can be expressed as follows:

$$\overline{T}_{t}(j,k) = \frac{\mu_{j}}{\mu_{j} + \mu_{k}} t + \frac{\mu_{j}\mu_{k} (\pi_{j}\mu_{k} - \pi_{k}\mu_{j})}{(\mu_{j} + \mu_{k})^{2}} \left(1 - e^{-\left(\frac{\mu_{j} + \mu_{k}}{\mu_{j}\mu_{k}}\right)t}\right).$$
(4.3)

An expression for the inverse FIM in the nonstationary case follows from (4.1) and (4.2). An asymptotic expression for the FIM of a stationary two-state Markov chain, valid in the regime of large t, was derived in [55, Theorem 1]. The asymptotic FIM leads to a closedform solution for the sensing time allocations, see [55, Eq. (17)], but incurs non-negligible approximation error for smaller values of t.

We use a subscript *i* to denote the *i*th channel, e.g., we denote the PU Markov process for the *i*th channel as  $\{X_{i,t}, t \ge 0\}$  and its associated parameter by  $\theta_i = (\mu_{0,i}, \mu_{1,i})$ . Let  $\sigma_i^2(t)$  denote the sum of the variances in estimating the two components of  $\theta_i$  by an unbiased estimator over a sensing interval of length *t* seconds. Let  $I_i(t)$  denote the FIM for the *i*th channel over the same sensing interval. The Cramér-Rao Bound (CRB) for a single channel *i* over time *t* is given by [55]

$$\sigma_i^2(t) \ge \operatorname{trace}[I^{-1}(t)]. \tag{4.4}$$

Applying (4.1), the CRB for a 2-state CTMC is given by

$$\sigma_i^2(t) \ge \frac{\mu_0^3}{\overline{T}_t(0) - \mu_0 \overline{N}_t(0, 1)} + \frac{\mu_1^3}{\overline{T}_t(1) - \mu_1 \overline{N}_t(1, 0)}.$$
(4.5)

The sensing interval of length t is partitioned into subintervals of length  $t_i$ , i = 1, ..., M, where  $t_i$  is the time spent estimating channel i, with  $t = \sum_{i=1}^{M} t_i$ . The SU observes  $\{X_{i,t}\}$ for an interval of length  $t_i$  in the sequence i = 1, ..., M. The CRB for estimation of all M channels over a sensing interval of length t is given by

$$\sigma^{2}(t) \geq \sum_{i=1}^{M} \left( \frac{\mu_{0,i}^{3}}{\overline{T}_{t_{i}}(0) - \mu_{0,i}\overline{N}_{t_{i}}(0,1)} + \frac{\mu_{1,i}^{3}}{\overline{T}_{t_{i}}(1) - \mu_{1,i}\overline{N}_{t_{i}}(1,0)} \right).$$
(4.6)

## 4.3 MMSE Multichannel Estimation

In this section, we adapt the multichannel parameter estimation algorithm in [55], which iteratively determines the sensing intervals and parameter estimates, to minimize sensing error based on Eq. (4.6). In [55], a multiband sensing algorithm was developed to minimize the CRB of the estimates of the transition rates  $\{\lambda_0, \lambda_1\}$ . We have adapted this work to minimize the CRB of the estimates of the mean sojourn times  $\{\mu_0, \mu_1\}$ . As shown in this section, the estimates of the mean sojourn times are asymptotically normal, a property that will strengthen the OCBA methods developed in Section 4.4.

We assume that time is divided into a sequence of sensing intervals,  $\{T_n\}_{n=0}^{\infty}$ . Each sensing interval is in turn subdivided into M sensing subintervals, one for each of the Mchannels. Let  $T_n$  denote the duration of the *n*th sensing interval, and let  $T_{i,n}$  denote the duration of the *n*th sensing interval that is devoted to channel *i*, such that  $T_n = \sum_{i=1}^{M} T_{i,n}$ . To perform the initial iteration of multiband sensing, all M channels are sensed for exactly the same amount of time, i.e., we set

$$T_{i,0} = \frac{T_0}{M}, \qquad i = 1, \dots, M.$$
 (4.7)

During each sensing interval, each channel is sensed in sequence.

For channel *i*, the number of sojourns in each state is counted, and the counts recorded up to and including time *t* are denoted  $n_i^{\text{off}}(t)$  and  $n_i^{\text{on}}(t)$  for the off and on states, respectively. The *j*th recorded sojourn times for channel *i* are denoted by  $z_{i,j}^{\text{off}}$  and  $z_{i,j}^{\text{on}}$ , respectively. The estimator for the mean sojourn times is based on the estimator for the transition rates proposed in [55], referred to in that chapter as the "moment estimator":

$$\hat{\mu}_{0,i}(t) = \frac{1}{\hat{\lambda}_{0,i}(t)} = \frac{\sum_{j=1}^{n_i^{\text{off}}(t)} z_{i,j}^{\text{off}}}{n_i^{\text{off}}(t) - 1}, \quad \hat{\mu}_{1,i}(t) = \frac{1}{\hat{\lambda}_{1,i}(t)} = \frac{\sum_{j=1}^{n_i^{\text{on}}(t)} z_{i,j}^{\text{on}}}{n_i^{\text{on}}(t) - 1}, \quad (4.8)$$

Because the sojourn times are IID, it is clear that the estimates for the mean sojourn times are asymptotically normal due to the Central Limit Theorem. The parameter estimate  $\hat{\theta}_i$ obtained at the end of the *n*th sensing interval is given by (4.8), and is used to calculate the sensing time allocations for the next sensing interval. This estimator is the well-known maximum likelihood estimator (MLE), with consistency normality proved by Albert [4, Theorem 6.10]. Asymptotic efficiency of this estimator was also proved by Albert [4, Section 7]. Efficiency of the moment estimator was demonstrated in [55] through simulations.

Multichannel parameter estimation should be designed in such a way that the sensing intervals are used most effectively. In Appendix C.9, sensing time allocations are derived such that the right-hand side of the multichannel CRB in Eq. (4.6) is minimized. In this derivation, similar to that performed in [55], an asymptotic approximation for inverse FIM was used, which led to closed-form expressions for the proportion  $\alpha_i$  of the sensing interval  $T_n$  that should be allocated to channel *i*, given as follows:

$$\alpha_{i} = \frac{\sqrt{(\hat{\mu}_{0,i} + \hat{\mu}_{1,i}) \left(\hat{\mu}_{0,i}^{2} + \hat{\mu}_{0,i}^{2}\right)}}{\sum_{i=1}^{M} \sqrt{(\hat{\mu}_{0,i} + \hat{\mu}_{1,i}) \left(\hat{\mu}_{0,i}^{2} + \hat{\mu}_{1,i}^{2}\right)}}$$
(4.9)

where the estimates,  $\hat{\mu}_{0,i}$  and  $\hat{\mu}_{1,i}$ , are computed by applying (4.8) and  $T_{i,n} = \alpha_i T_n$ ,  $i = 1, \ldots, M$ . The allocation strategy based on (4.9) was shown in [55] to approach, as  $T_n \to \infty$ , the CRB for the joint estimation of all M independent channels.

In this section, we focus on an approach for selecting a smaller subset of the most promising channels for opportunistic spectrum access, while estimating their associated parameters. We adapt OCBA to determine the appropriate sensing time allocations to achieve this objective.

#### 4.4.1 OCBA Sensing Allocations

Our goal is to determine the N channels with largest mean dwell time in the idle state, where ideally  $N \ll M$ . Equivalently, we seek the N channels with minimum cost, where the cost function for channel *i* is defined by

$$J_i = -\mu_{0,i}.$$
 (4.10)

We assume that the initial sensing interval  $T_0$  is allocated according to (4.7). At the end of the *n*th sensing interval of length  $T_n$ , the PU parameter is re-estimated using Eq. (4.8) and we apply OCBA [16] to determine the channels with the lowest cost functions. In the context of simulation optimization, given a fixed total computing budget and M alternative designs, OCBA determines the computing budget allocation for simulating the M designs that maximizes the probability of selecting the subset of N designs out of M with minimum cost. The OCBA methodology requires knowledge of the standard deviation of the cost function, which we denote by  $s_i$  for channel i. A lower bound on the standard deviation of the cost function (4.10) follows from the CRB for estimating  $\mu_0$ , which can be derived from the FIM in Eq. (4.1):

$$s_i(t) \ge \sqrt{\frac{\mu_{0,i}^3}{\overline{T}_{t_i}(0) - \mu_{0,i}\overline{N}_{t_i}(0,1)}}.$$
(4.11)

To find the subset of the N best channels, we first sort the estimated values of the cost function J in (4.10), denoted by  $\hat{J}_i = -\hat{\mu}_{0,i}(t)$ , such that  $\hat{J}_1 \leq \hat{J}_2 \leq \ldots \leq \hat{J}_{M-1} \leq \hat{J}_M$ . We then compute a reference constant c, which in [17] is the midpoint between the highest cost value of the selected subset and the next highest cost value among the M channels, i.e.,

$$c = \frac{\hat{J}_N + \hat{J}_{N+1}}{2}.$$
(4.12)

We denote the total sensing time on channel *i* up to and including iteration *n* as  $\Sigma_{T,i,n}$ . Applying OCBA, we must next find sensing intervals such that

$$\frac{\Sigma_{T,1,n}}{\left(s_1/(\hat{J}_1 - c)\right)^2} = \dots = \frac{\Sigma_{T,M,n}}{\left(s_M/(\hat{J}_M - c)\right)^2}$$
(4.13)

and

$$\sum_{i=1}^{M} \left( \Sigma_{T,i,n} - \Sigma_{T,i,n-1} \right) = T_n.$$
(4.14)

The resulting sensing time allocations are then given by

$$T_{i,n} = \Sigma_{T,i,n} - \Sigma_{T,i,n-1}.$$
(4.15)

#### 4.4.2 Channel Elimination

Once a desired level of certainty that a channel is not a member of the selected subset has been reached, our algorithm ceases allocating sensing time to that channel for the current and future iterations. This results in a smaller parameter estimation error for the channels in the eventual selected subset compared to the standard OCBA. To determine whether a channel *i* is unlikely to be part of the eventually selected subset, we compare it to the member of the current selected subset with the highest cost,  $\overline{J}_N$ . We make use of the Bhattacharyya distance for this purpose. The Bhattacharyya distance between a pair of normal random variables  $U \sim \mathcal{N}(\mu_u, \sigma_u^2)$  and  $V \sim \mathcal{N}(\mu_v, \sigma_v^2)$  is given by [19, p. 777]

$$D_B(U,V) = \frac{(\mu_u - \mu_v)^2}{4(\sigma_u^2 + \sigma_v^2)} + \frac{1}{4} \log\left[\frac{1}{4}\left(\frac{\sigma_u^2}{\sigma_v^2} + \frac{\sigma_v^2}{\sigma_u^2} + 2\right)\right].$$
(4.16)

The Bhattacharyya distance is a useful metric for classification between a pair of normal random variables, because it is related directly to the Chernoff upper bound on the probability of classification error [31]. The probability of classification error between distributions i and j, denoted  $p_e(i, j)$  is determined in [31] to be bounded by

$$\frac{1}{4}e^{-2D_B(i,j)} \le p_e(i,j) \le \frac{1}{2}e^{-D_B(i,j)},\tag{4.17}$$

given that the two random variables have equal prior probabilities. Therefore, a distance threshold  $\gamma$  may be selected such that, within a certain probability of error, we can ensure that a channel is not a true member of the optimal subset. A larger minimum distance will allow for increased certainty of correct decision, but will increase convergence time. Channel *i* is eliminated if

$$D_B(\hat{J}_N, \hat{J}_i) > \gamma, \quad \hat{J}_i > \hat{J}_N, \tag{4.18}$$

where  $\gamma$  is a threshold chosen by the system designer. A larger value of  $\gamma$  will allow for increased certainty of correct decision at the expense of longer time required to obtain the final selected subset of N channels.

When we are left with the selected subset of N channels, subsequent estimation effort can be applied to these channels. In the case of N = 1, the optimal sensing strategy is simply to allocate all sensing time to the selected channel. More generally, MMSE allocations are applied to the N selected channels, and the other M - N channels receive no sensing time allocation. Thus, parameter estimation proceeds along the lines of [55] for the selected set of N channels, except that we use the closed-form expression (4.2) to calculate the FIM. Consequently, we resort to sequential quadratic programming [44] to numerically solve for the sensing time allocations. Nevertheless, the closed-form sensing time allocations given in (4.9) may serve as the initial values to speed up convergence and increase likelihood of a global optimum.

## 4.5 Numerical Examples

To test the proposed algorithm, we present numerical results of three example scenarios based on examples that were considered in [55]. Results were obtained using the Python packages SciPy, NumPy, and Matplotlib.

#### 4.5.1 Example 1

In the first example, four channels were defined with parameter values given as follows:

$$\{\lambda_{0,i}\} = \{0.1, 0.6, 0.2, 0.5\},\$$
$$\{\lambda_{1,i}\} = \{0.7, 0.1, 0.8, 0.9\}.$$

We used an initial allocation of  $T_0 = 1000$  samples, 250 samples per channel. We increased sensing time by  $T_n = 2T_{n-1}$  for each sensing iteration and performed 15 sensing iterations. As a baseline, we performed the MMSE allocations as proposed in [55], as well as equal allocations where the sensing duration is divided evenly among all tested channels. We compared the MSE from the MMSE and equal allocations to OCBA allocations as proposed in this chapter. For the fist simulation, we searched for a selected subset of size N = 1, i.e., we only searched for a single best channel. We recorded the MSE for the known best channel as well as the system MSE for all 4 channels. A total of 200 simulations were performed, and a minimum Bhattacharyya distance of  $\gamma = 18$  was used.

The results of this example are plotted in Figures 4.2 and 4.3. Figure 4.2 shows the total MSE in estimating the parameters of all channels compared to the associated CRB. After the best channel is determined, i.e., channel i = 1 with  $\lambda_{0,i} = 0.1$ , the MSE for all



Figure 4.2: MSE for all channels using Equal, MMSE, and OCBA allocations



Figure 4.3: MSE for best channel using Equal, MMSE, and OCBA allocations

channels in the case of OCBA allocation diverges from the CRB. This is because all of the sensing time is devoted to estimation of the parameter of the selected channel, while the MSE for the other channels remains constant. Fig. 4.3 shows the MSE for only the best channel in terms of the longest mean dwell time in the idle state. The MSE resulting from both allocation strategies is compared to the CRB under the assumption that all sensing



Figure 4.4: MSE for all channels using Equal, MMSE, and OCBA allocations

time is given to the best channel.

#### 4.5.2 Example 2

Here, 10 channels were defined with the following parameter values:

 $\{\lambda_{0,i}\} = \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0\},\$  $\{\lambda_{1,i}\} = \{0.2, 0.7, 0.1, 0.1, 0.4, 0.8, 0.4, 0.9, 0.1, 0.4\}.$ 

We used an initial allocation of  $T_0 = 2500$  samples, 250 samples per channel. We increased sensing time by  $T_n = 2T_{n-1}$  for each sensing iteration and performed 15 sensing iterations. Again, the resulting MSE in estimating the parameters of the channels from MMSE allocations and equal allocations was compared to that obtained from the OCBA allocations. For the OCBA allocations, we set N = 3. We recorded the MSE for the set consisting of the best three channels, i.e.,  $\{1, 2, 3\}$ , as well as the system MSE for all 10 channels. In total, 200 simulation runs were performed, and a minimum Bhattacharyya distance of  $\gamma = 18$  was used.



Figure 4.5: MSE for optimal subset using Equal, MMSE, and OCBA allocations

The results for Example 2 are plotted in Figs. 4.4 and 4.5. Fig. 4.4 shows the total MSE for estimation of all channels, compared to the CRB for all channels. As in Fig. 4.2, the MSE for the OCBA allocation in Fig. 4.4, diverges from the CRB after the best subset is selected. Fig. 4.5 shows the MSE for only the selected subset. The MSE resulting from both allocation strategies is compared to the CRB under the assumption that all sensing time is given to the selected subset.

#### 4.5.3 Example 3

We performed a set of randomized trials to observe the convergence rates of the proposed OCBA allocations in comparison to MMSE or equal allocations. For this example, 1000 simulations were performed. For each simulation, we generated a random set of 25 Markov transition rates from the range [0.25, 0.75]. We used an initial allocation of  $T_0 = 6250$  samples, 250 samples per channel. To observe convergence over a finer time scale, we did not increase sensing time between iterations, i.e.  $T_n = T_{n-1}$ . A total of 20 sensing iterations was performed for each simulation. The probability of correct selection, the probability that the selected best channel based on the channel estimates is the true best channel, was



Figure 4.6: Probability of correct selection using Equal, MMSE, and OCBA allocations



Figure 4.7: Expected opportunity cost using Equal, MMSE, and OCBA allocations

calculated at each iteration and averaged across simulations. The expected opportunity cost, the mean difference between the selected best channel and the actual best channel, was also computed at each iteration.

The results for Example 3 are plotted in Figs. 4.6 and 4.7. Fig. 4.6 shows the probability of correct selection over time. In our tests, we observed that the OCBA allocations converged

onto the correct selection much faster than MMSE or equal allocations. Fig. 4.7 shows the expected opportunity cost over time. In our tests, we observed that OCBA allocations also has reduced opportunity cost compared to MMSE and equal allocations.

#### 4.5.4 Discussion

The numerical examples demonstrate that when total system MSE is to be minimized for all channels, the MMSE approach proposed in [55] achieves the CRB. However, when the performance of a smaller subset of channels is more important, the proposed algorithm based on OCBA may be used to quickly determine the selected subset and to focus subsequently on the most promising channels. This is an important feature of our approach, as the total number of channels M may be significantly larger than the eventual number of channels of interest, i.e.,  $M \gg N$ . In such a scenario, diluting the spectrum sensing effort among M channels is heavily resource-intensive, and ultimately is likely to result in degraded opportunistic spectrum access. Thus, focusing the parameter estimation on a much smaller set of N candidate channels is more efficient both computationally and in terms of exploiting the spectrum access opportunities available among the original set of M channels.

## 4.6 Conclusion

We proposed a multichannel parameter estimation algorithm for multiband spectrum sensing based on OCBA and compared it with an earlier algorithm of [55] which relies on an MMSE approach. Over a sequence of sensing intervals, the algorithm iteratively allocates sensing time among a set of channels that have the largest mean idle sojourn times under a Markov model. The algorithm was verified through simulation and shown to approach the CRB on the variance of the parameter estimator for channels in the selected set. Spectrum sensing on the channels in the selected subset can be performed using an approach along the lines of [43].

In this chapter, we have assumed that the PU state is directly observable. In principle, the Markov chain model could be extended to take into account channel impairments as in Chapter 3 and to accommodate non-exponential PU state sojourn time distributions, as was done in [43]. We have also assumed a simple cost function, i.e., Eq. (4.10), based on the mean idle time of the PU. Alternative cost functions may be considered depending on the spectrum access needs of the SU. For example, in addition to the mean idle time, the variance of the idle time and the channel bandwidth may be incorporated into the cost function.

# Chapter 5: A Recursive Algorithm for Wideband Temporal Spectrum Sensing

Spectrum sensing techniques can be organized into three basic categories [53]:

- 1. *Narrowband:* A single channel is clearly defined, and the SU will only sense that channel.
- 2. *Multiband:* Multiple narrowband channels, assumed to be independent, have been defined, and the SU must sense each channel. Multiband techniques are useful for applications such as TV whitespace where multiple independent PUs operate on clearly-defined channels.
- 3. *Wideband:* The SU must sense over a wide bandwidth which may contain multiple narrowband channels with unknown boundaries. In wideband sensing, no extrinsic information on channel boundaries or occupancy can be leveraged to simplify the sensing task.

Of the three classes, narrowband techniques have been studied most extensively. Wellknown detection algorithms for narrowband sensing include energy detection, cyclostationary feature detection, and matched filter detection [65]. The energy detector is the simplest of the narrowband detectors and requires no a priori knowledge of the channel, but it performs poorly in the case of low signal-to-noise ratio (SNR). The most sensitive of the listed sensing algorithms is the matched filter, which requires a priori knowledge of the the PU waveform, but can detect PU activity at extremely low SNR. Cyclostationary feature detection lies between the matched filter and energy detector in terms of performance at low SNR, but requires significant computation times and long integration windows in the case of low SNR. Cyclostationary detector performance is degraded in the case of low PU duty cycle [26].

Additional complexity is added when dynamic behavior of the PU is considered in the system model. Many narrowband spectrum sensing algorithms assume that the PU state is constant over long periods of time. Given that many modern waveforms employ some sort of time division multiple access (TDMA), Spectrum sensing algorithms such as [42], which incorporate a dynamic PU model are highly desirable.

Research has also been performed on narrowband spectrum sensing algorithms which use hidden Markov models (HMMs) to characterize dynamic behavior of the PU and predict future spectrum holes on a narrowband channel [3]. The more general hidden bivariate Markov model has been applied to model and predict the occurrence of temporal spectrum holes with a high degree of accuracy [43]. Modeling PU activity as a Markov process has been extended to the multiband case, where the allocation of total sensing time among bands has been studied [55]. A multichannel MAC is proposed in [67] where the PU channels are modeled as Markov on-off processes.

Wideband spectrum sensing, which is discussed in greater detail in Section 5.1, requires algorithms which segment a band into independent channels. PU signals may be heterogeneous in frequency, bandwidth, and power, so robust algorithms must be developed to detect all PU activity in the spectrum band. Incumbent wideband sensing algorithms, the wideband energy detector and the wideband edge detector are considered in Section 5.1. We show that these algorithms are inadequate for detection of PUs which dynamically change states, and are especially inadequate for PUs with low duty cycle.

Many advanced wideband spectrum sensing methods have been proposed which offer various improvements over standard wideband energy or edge detection but all model the PU state as either on or off, not changing over time [5,7,38,41,51,64]. To our knowledge, no wideband spectrum sensing research has been published where the system model includes a PU which switches state and spectrum segmentation is performed.

In this chapter, we focus on the problem of wideband spectrum sensing, and we propose

a framework for wideband temporal sensing that can leverage the large set of narrowband temporal sensing techniques that have already been developed. Specifically, we leverage a hidden Markov model temporal sensing algorithm, allowing for detection of PUs with low duty cycle in the wideband regime. We model the channel as a balanced binary tree and perform a recursive search for spectrum holes. If any holes are detected that are adjacent in frequency, they can be merged into a single spectrum hole with the objective of maximizing PU independence between bands. We present experimental results obtained by simulation.

The remainder of the chapter is organized as follows. In Section 5.1, we discuss and evaluate the performance of two existing wideband spectrum sensing techniques. In Section 5.2, we introduce a system model for a dynamic PU in fading and noise. In Section 5.3, we develop a recursive tree search algorithm to perform temporal sensing in the wideband regime. In Section 5.4 we describe the simulation that was used to compare the proposed algorithm to existing algorithms and present numerical results. Concluding remarks are given in Section 5.5.

## 5.1 Comparison of Wideband Spectrum Sensing Techniques

In the wideband spectrum sensing scenario, a SU must sense an entire band and determine channel boundaries. The bandwidth that must be sensed can vary from the order of 1 MHz to 1 GHz. Wideband spectrum sensing is required if the SU can not leverage any external information about channel allocation. An example of external channel information is the television bands in North America where 6 MHz channels have been clearly defined by national regulatory bodies. It is possible for a SU to only perform wideband sensing during initialization and then revert to multiband or narrowband sensing during normal operation.

To evaluate the incumbent wideband sensing techniques, orthogonal frequency division multiplexing (OFDM) and Gaussian minimum shift keying (GMSK) are used. OFDM signals exhibit sharp rectangular band edges, and GMSK signals exhibit gradual sloping band edges. Because these signals represent extremes in the boundaries between signals, the performance of the evaluated detectors with other modulation schemes should fall somewhere between that of OFDM and GMSK. Not only do they have drastically different band edges, but GMSK and OFDM are pervasive in modern wireless standards such as GSM (GMSK), WiFi (OFDM), and LTE (OFDM).

We assume that a channel can take on one of two states: an idle state, in which the PU does not transmit, and an active state, in which the PU transmits. We denote idle and active states by 0 and 1, respectively. For a given channel, the steady-state probabilities that the PU is idle and active are denoted, respectively, by  $\pi_0$  and  $\pi_1$ . The duty-cycle of the channel corresponds to  $\pi_1$  stated as a percent value.

#### 5.1.1 Wideband Energy Detector

A very simple wideband sensing technique is a wideband energy detector [29], [13] where the SU estimates the power spectral density (PSD) over the entire band and employs an energy threshold to determine PU activity. Many PSD frames may be averaged to increase reliability. This simple algorithm has many limitations. Like all energy detectors in additive white Gaussian noise (AWGN), this technique has limited sensitivity, and performance is severely degraded at low SNR. Furthermore, the sensitivity of the averaged PSD estimate will be degraded in the case where the PU exhibits dynamic behavior. If the PU employs a bursting signal or frequency hopping, idle periods may be averaged together with active periods, which compromises the estimator's accuracy.

Figures 5.1 and 5.2 qualitatively show the sensing results of a frequency-domain energy detector for OFDM and GMSK signals, respectively. Shaded areas represent detected spectrum holes. All of the signals shown have an SNR of 10 dB, but for the bursting signals, the magnitude of the PSD estimate decreases with the duty cycle. This decreased PSD magnitude degrades the performance of the energy detector for both modulation schemes.

Performing a maximum hold operation rather than averaging PSD frames has been proposed for detecting dynamic PUs [46]. However, maximum hold energy detectors are outperformed by averaging detectors in low SNR [46]. Furthermore, maximum hold energy detectors can actually cause an increased probability of false alarm as observation lengths



Figure 5.1: Results of a wideband energy detector for OFDM signals with 10 dB SNR and 100%, 50%, 25%, and 12.5% duty cycles.

are increased due to increased likelihood of an abnormally high noise power during the sensing interval. These two shortfalls make maximum hold energy detectors inadequate for cognitive radio applications and motivate the need for a wideband sensing algorithm that adequately detects dynamic PU activity.

#### 5.1.2 Wideband Edge Detector

A popular approach to wideband spectrum sensing involves performing frequency-domain edge detection to determine channel boundaries. The edge detector proposed in [56] uses the continuous wavelet transform to decompose edge detection into multiple resolutions and then multiplies the resolutions together, which has a beneficial effect of reducing the noise. While edge detectors do offer an improvement over energy detectors in terms of performance at low SNR, they come with several limitations. Most importantly, edge detectors require that PU signals have sharp transitions in the frequency domain. This allows them to work well with the rectangular spectra of signals like OFDM (see Fig. 5.3) and quadrature amplitude modulation (QAM) with low excess bandwidth, but edge detectors tend to perform



Figure 5.2: Results of a wideband energy detector for GMSK signals with 10 dB SNR and 100%, 50%, 25%, and 12.5% duty cycles.



Figure 5.3: Results of a wideband edge detector for OFDM signals with 10 dB SNR and 100%, 50%, 25%, and 12.5% duty cycles.

poorly on signals with gradual slopes on their band edges, such as QAM with large excess bandwidth and GMSK.

The performance of an edge detector using the multi-resolution enhancements from [56]



Figure 5.4: Results of a wideband edge detector for GMSK signals with 10 dB SNR and 100%, 50%, 25%, and 12.5% duty cycles.

is shown for GMSK in Fig. 5.4. The figure shows that wideband edge detectors suffer from the same shortfall as wideband energy detectors in that they are also degraded by dynamic behavior of the PU. Because received signal samples from both idle and active cycles are averaged into the PU detector, the performance of the detector deteriorates with decreasing duty cycle of the PU.

#### 5.1.3 Compressive Sensing

A class of sensing algorithms known as compressive sensing has been proposed for surveying very wide bandwidths with sub-Nyquist sampling rates. Because much of the radio spectrum is underutilized, available bands may be represented as a sparse dataset, and depending on the sparsity order of the dataset, the wideband signal may be sensed at a fraction of the Nyquist rate [53]. To perform sub-Nyquist sampling, the signal time series is divided into length-M blocks of Nyquist-rate samples, of which K samples are kept, giving an undersampling fraction of K/M. Reconstruction of the sparse PSD from the undersampled data is accomplished by solving for a linear inverse, which in the sparse case requires a numerical solution [57]. To select an appropriate undersampling fraction, the cognitive receiver must have prior knowledge of the PU sparsity order. An online sparsity estimator has been proposed in [52] that can quickly determine an undersampling ratio.

Although compressive sensing can be utilized to sense much wider bandwidths than can be done with traditional analog to digital conversion hardware, the result of compressive sensing typically involves a static PSD estimate. For example, in [57], the estimated sparse PSD is analyzed with the wavelet-based edge detector proposed in [56]. Since current compressive sensing methods rely on a static PSD estimate, the presence of low duty-cycle PU signals can drastically reduce the detector sensitivity. In [53] it is stated that current compressive sensing can not be used to properly handle sparsity in time and space. Although our proposed sensing algorithm requires sampling at the Nyquist rate and can therefore not be used for ultra wideband sensing, its success does not rely on signal sparsity in any domain, and it more flexibly detects bursting signals by leveraging time-domain sensing methods.

## 5.2 System Model

Over a frequency band B, an unknown quantity of independent PUs is operating. Each PU has an unknown center frequency,  $f_c$  and bandwidth, W. It is assumed that PU channels are not overlapping. The channel over which the  $i^{th}$  PU is observed is assumed to be flat Rayleigh fading with parameter  $\sigma_{f,i}$  combined with additive white Gaussian noise (AWGN), defined by the circularly symmetric complex normal distribution  $\mathcal{C}\left(0, \sigma_{n,i}^2\right)$ . The mean SNR of the received signal on channel i, given that the PU is transmitting is

$$\overline{\mathrm{SNR}}_i = \frac{\sigma_{f,i}^2}{\sigma_{n,i}^2} \tag{5.1}$$

at the input to the energy detector.

#### 5.2.1 PU Traffic Model

A PU may be transmitting or idle at any given time. The state of the  $i^{th}$  PU, denoted by random variable  $X_i$ , may alternate between the idle state  $X_i = 0$ , where the PU is not transmitting, and the busy state  $X_i = 1$ , where the PU is transmitting. The  $k^{th}$  PU state is denoted  $X_{i,k}$ . Each PU is modeled by a discrete-time Markov chain with transition matrix  $G_i$  and initial distribution  $\nu_i$ , defined as

$$G_i = [g_{i,ab} : a, b \in \{0, 1\}], \tag{5.2}$$

$$g_{i,ab} = P(X_{i,k} = a, X_{i,k+1} = b),$$
(5.3)

$$\nu_{i,0} = P(X_{i,1} = 0), v_{i,1} = P(X_{i,1} = 1).$$
(5.4)

#### 5.2.2 Cognitive Receiver Model

#### **Received Wideband Signal**

A transmitting PU will generate a bandpass signal  $\tilde{t}_{i,k}$ . The transmitted signal for PU *i* at any time k is

$$t_{i,k} = \tilde{t}_{i,k} \cdot 1_{\{X_{i,k}=2\}},\tag{5.5}$$

where  $1_{\{A\}}$  is the indicator function on the set or condition A. The  $i^{th}$  PU signal is multiplied at time k by fading signal  $f_{i,k} \sim \mathcal{C}\left(0, \sigma_{f,i}^2\right)$ . All M PU signals are received simultaneously and added to noise signal  $n_k \sim \mathcal{C}\left(0, \sigma_{n,i}^2\right)$ . The received wideband signal is represented by a sequence of samples  $z_{wb}^n = \{z_{wb,1}, \ldots, z_{wb,n}\}$ , where  $z_{wb,k}$ , the  $k^{th}$  I-Q sample from the wideband channel, is defined as

$$z_{wb,k} = \sum_{i=1}^{M} t_{i,k} f_{i,k} + n_k.$$
(5.6)

#### **Channelized Received Signal**

The SU will divide the wideband received signal into J narrowband subchannels. Initially this division must be done done arbitrarily, but after wideband sensing, the set of subchannels should describe all PU statistics as well as the statistics of the spectrum holes between PU signals. The sequence of observation samples in the  $j^{th}$  subband is denoted  $z_j$ .

#### **Energy Detected Signal**

For spectrum sensing, the channelized narrowband signals are processed with an averaging energy detector, which estimates the power of each sample and averages N samples together. The resulting random variable for the received energy in in subchannel j is denoted by  $Y_j$ , and the sequence of energy estimates for subchannel j is denoted  $y_j^n = \{y_{j,1}, \ldots, y_{j,n}\}$ . The  $k^{th}$  sample in the energy detection sequence,  $y_{j,k}$ , is defined as

$$y_{j,k} = \frac{1}{N} \sum_{i=1}^{N} |z_{j,(k-1)N+i}|^2.$$
(5.7)

Assuming that N is sufficiently large,  $y_j$ , k will be conditionally normal with distribution

$$y_{j,k} \sim \begin{cases} \mathcal{N}\left(2\sigma_{n,i}^{2}, \frac{4\sigma_{n,i}^{4}}{N}\right), & X_{i,k} = 0, \\ \mathcal{N}\left(2\sigma_{f,i}^{2} + 2\sigma_{n,i}^{2}, \frac{4\left(\sigma_{f,i}^{2} + \sigma_{n,i}^{2}\right)^{2}}{N}\right), & X_{i,k} = 1, \end{cases}$$

$$(5.8)$$

This conditional distribution is derived in Appendix A.

## 5.3 Recursive Algorithm for Wideband Temporal Sensing

In this section, we propose an approach that extends narrowband temporal sensing techniques to the wideband scenario. Narrowband techniques that use HMMs [21] to model


Figure 5.5: A wideband channel, i.e., a spectrum band with bandwidth  $W_0$ , organized into a balanced binary tree.

the dynamic behavior of the PU [3] are leveraged to overcome the limitations of current wideband spectrum detectors. The proposed wideband search algorithm may be adapted to leverage other narrowband sensing techniques for various special purposes. For example, for channels with high duty cycle but very low SNR, the proposed wideband algorithm could be adapted to work with a cyclostationary detector.

#### 5.3.1 Wideband Tree Search

In our proposed algorithm for wideband temporal sensing, the spectrum band is organized as a balanced binary tree, where each node has two child nodes representing the upper and lower halves of the band. The band is recursively divided into smaller pieces as depth increases [34]. A maximum depth is selected based on a desired resolution for the wideband sensing algorithm. The depth of the tree is given by  $d = \lceil \log_2(W_0/W_r) \rceil$ , where  $W_0$  is the bandwidth, and  $W_r$  is the maximum frequency resolution. The division of a band into subbands using a balanced binary tree is shown in Fig 5.5.

The algorithm recursively divides a given channel in half until the desired resolution is reached. An inorder traversal, a recursive search where child nodes are visited before parent nodes [34] is performed on the balanced binary tree that is used to model the spectrum band.



Figure 5.6: A simple digital downconvertor for signal channelization.

At the highest resolution, each subband or channel is sensed using a narrowband temporal spectrum sensing technique.

#### 5.3.2 Channel Selection

A channelizer must be employed to divide the wideband channel into  $2^d$  subbands, where d is the search tree depth. A conceptually simple channelizer is a bank of digital downconverters (DDCs), with one DDC for each subband. A diagram for a simple DDC is shown in Fig. 5.6. Given a sequence  $\{a_k\}_{k=1}^{\infty}$ , we use the convenient notations  $a_k^n = \{a_k, \ldots, a_n\}$  and  $a^n = \{a_1, \ldots, a_n\}$ . The received wideband signal can then be represented by a sequence of samples  $z_{wb}^n = \{z_{wb,1}, \ldots, z_{wb,n}\}$ , where  $z_{wb,k}$  denotes the kth I-Q sample from the wideband channel. When the received wideband signal  $z_{wb}^n$  is passed into the DDC, it will first be mixed down by center frequency  $f_c$ , such that the center of the band of interest is now at baseband. The baseband signal is next lowpass filtered with FIR taps h(n) to isolate the band of interest. Finally, the signal is decimated by rate dec, keeping 1 sample out of every dec. The channelized narrowband signal is denoted  $z^n$ .

Because all subbands are eventually channelized by the recursive search, a frequencydomain channelizer using the fast Fourier transform (FFT) [27,28] can substantially reduce the computational cost of the channel selection. Frequency-domain channelizers have been studied in detail [27], and while faster computationally, use of a frequency-domain channelizer would not alter the outcome of the proposed algorithm. Therefore, for the sake of simpler algorithm description, a simple filter-and-decimate channelizer was discussed above. For spectrum sensing, the channelized narrowband signals are processed with an averaging energy detector, which estimates the power of each sample and averages  $N_{\text{avg}}$  samples together. The received power estimate of the sample  $z_k$  in linear units, e.g., mW, is denoted  $y_k$ , and is calculated as follows:

$$y_k = \frac{1}{N_{\text{avg}}} \sum_{i=1}^{N_{\text{avg}}} |z_{k+i}|^2 \,.$$
(5.9)

#### 5.3.3 Hidden Markov Model for Narrowband Sensing

Although the recursive tree search that we propose can leverage a variety of narrowband techniques, we are addressing the specific issue of PU dynamics such as bursting and frequency hopping. An HMM is used to model the channel dynamics, assuming a lognormal shadowing model. In [43,54], a more general form of HMM referred to as a hidden bivariate Markov model (HBMM) is applied to narrowband temporal spectrum sensing. An extension of the Baum-Welch algorithm was developed in [43] for estimating the parameter of a HBMM. The Baum-Welch algorithm is an offline algorithm, which iteratively produces a sequence of parameter estimates with increasing likelihood, based on a given an observation sequence. An online parameter estimation algorithm for the HBMM was developed in [54]. Since the focus of the present chapter is on wideband sensing, we will restrict ourselves to the simpler HMM and the standard Baum-Welch algorithm for parameter estimation.

We use P to denote a generic probability measure and  $P_{\phi}$  to denote a probability measure that depends on a parameter  $\phi$ . Similarly, we use p and  $p_{\phi}$  to denote a probability density function or probability mass function as appropriate. In the notation  $p(x_k) = P(X_k = x_k)$ , the lowercase symbol  $x_k$  on the left-hand side implicitly implies the associated random variable represented by the uppercase symbol  $X_k$ . The HMM, denoted by (Y, X), consists of an observable sequence of received signal strengths,  $Y = \{Y_k\}_{k=1}^{\infty}$ , and an underlying or hidden state sequence  $X = \{X_k\}_{k=1}^{\infty}$ , which is assumed to be a discrete-time Markov chain. At time k,  $Y_k$  represents the averaged received signal power, after processing, in linear units (mW) and  $X_k$  represents the state of the PU, i.e.,  $X_k = 0$  when the PU is idle and  $X_k = 1$  when the PU is active. Assuming a standard path loss plus Rayleigh fading model, the received signal power  $Y_k$  can be expressed as follows (cf. [54]):

$$Y_{k} = \begin{cases} \mu_{1} + \epsilon_{1}, & X_{k} = 0, \\ \mu_{2} + \epsilon_{2}, & X_{k} = 1, \end{cases}$$
(5.10)

where  $\mu_a$  represents the mean received signal power when the PU is in state  $a \in \{0, 1\}$ , and  $\epsilon_a$  is a zero-mean Gaussian random variable with standard deviation  $\sigma_a$ , which may represent impairments such as receiver noise, fading, or shadowing. This model was validated empirically in the context of temporal spectrum sensing of a narrowband channel in [43]. In this chapter, Rayleigh fading was simulated, resulting in Eq. (5.8) for  $Y_k$ .

Let  $G = [g_{ab}: a, b \in \{0, 1\}]$  denote the transition matrix of the underlying Markov chain X, where  $g_{ab}$  denotes the transition probability from state a to state b. Let  $\nu = [\nu_0, \nu_1]$  denote the initial state probability distribution, where

$$\nu_1 = P(X_1 = 0), \quad \nu_2 = P(X_1 = 1).$$

The parameter of the HMM is given by  $\phi = (\nu, G, \mu, R)$ , where  $\mu = [\mu_0, \mu_1]$  and  $R = [\sigma_0^2, \sigma_1^2]$ .

#### 5.3.4 Baum-Welch Algorithm and MAP Detector

The Baum-Welch algorithm [9] is applied to obtain an estimate of the HMM parameter for a given channel, as part of the recursive tree search. The input to the algorithm is an initial parameter estimate  $\phi^s$  and an observed sequence  $y^n$  obtained from the channel. Starting with the initial estimate,  $\hat{\phi}_0 = \phi^s$ , the  $\iota$ th iteration ( $\iota \ge 1$ ) of the algorithm produces a new estimate  $\hat{\phi}_{\iota}$  with likelihood greater than or equal to that of  $\hat{\phi}_{\iota-1}$ . Each iteration of the algorithm involves the computation of forward and backward recursions [21, Section V.A]). Let  $\phi$  denote the current parameter estimate at the start of an iteration of the Baum-Welch algorithm. Define a diagonal matrix

$$B(y_k) = \text{diag}\{p_{\phi}(y_k \mid x_k = 0), p_{\phi}(y_k \mid x_k = 1)\}.$$

We denote the (scaled) forward and backward variables by  $\bar{\alpha}(x_k, y^k)$  and  $\bar{\beta}(y_{k+1}^n \mid x_k)$ , respectively. The forward vector is defined as a row vector

$$\bar{\alpha}_k = [\bar{\alpha}(x_k = 0, y^k), \bar{\alpha}(x_k = 1, y^k)],$$

while the backward vector is defined as a column vector

$$\bar{\beta}_k = [\bar{\beta}(y_{k+1}^n \mid x_k = 0), \bar{\beta}(y_{k+1}^n \mid x_k = 1)]',$$

where ' denotes matrix transpose. Let **1** denote a column vector of all ones, of appropriate dimension depending on the context. The forward recursion is given by

$$\bar{\alpha}_1 = \frac{\nu B(y_1)}{c_1}, \quad \bar{\alpha}_k = \frac{\bar{\alpha}_{k-1} G B(y_k)}{c_k}, \quad k = 2, \dots, n,$$
 (5.11)

where  $c_1 = \pi B(y_1)\mathbf{1}$ , and  $c_k = \bar{\alpha}_{k-1}GB(y_k)\mathbf{1}$  for k = 1, ..., n. The forward variables have the following interpretation:  $\bar{\alpha}(x_k, y^k) = p(x_k \mid y^k)$ . The backward recursion is given by

$$\bar{\beta}_n = \mathbf{1}; \quad \bar{\beta}_n = GB(y_{n+1})\frac{\bar{\beta}_{n+1}}{c_n}, \quad k = n - 1, \dots, 1.$$
 (5.12)

The state conditional probability can be obtained from

$$p_{\phi}(x_k \mid y^n) = \bar{\alpha}(x_k, y^k) \bar{\beta}(y_{k+1}^n \mid x_k).$$
(5.13)

The joint state conditional probability can be calculated as follows:

$$p_{\phi}(x_{k-1}, x_k \mid y^n) = \frac{\bar{\alpha}(x_{k-1}, y^{k-1})\bar{\beta}(y_{k+1}^n \mid x_k)g_{x_{k-1}, x_k}p_{\phi}(y_k \mid x_k)}{\sum_{x_{k-1}, x_k} \bar{\alpha}(x_{k-1}, y^{k-1})\bar{\beta}(y_{k+1}^n \mid x_k)g_{x_{k-1}, x_k}p_{\phi}(y_k \mid x_k)}.$$
(5.14)

The re-estimation formulas for the new parameter estimate are given in terms of (5.13) and (5.14) as follows:

$$\hat{g}_{ab} = \frac{\sum_{k=2}^{n} p_{\phi}(x_{k-1} = a, x_k = b \mid y^n)}{\sum_{k=2}^{n} p_{\phi}(x_{k-1} = a \mid y^n)},$$

$$\hat{\mu}_a = \frac{\sum_{k=1}^{n} p_{\phi}(x_k = a \mid y^n) y_n}{\sum_{k=1}^{n} p_{\phi}(x_k = a \mid y^n)},$$
(5.15)

$$\hat{\sigma}_a^2 = \frac{\sum_{k=1}^n p_\phi(x_k = a \mid y^n) \ (y_k - \hat{\mu}_a)^2}{\sum_{k=1}^n p_\phi(x_k = a \mid y^n)},\tag{5.16}$$

where  $a, b \in \{0, 1\}$ .

After the Baum-Welch algorithm converges to a final parameter estimate  $\phi$ , the maximum a posteriori (MAP) decisions may be obtained from the a posteriori state probabilities, as given in (5.13), as follows:

$$\hat{x}_{k} = \arg\max_{x_{k} \in \{0,1\}} p_{\phi}(x_{k} \mid y^{n}).$$
(5.17)

Since the MAP decisions take into account the temporal dynamics of the PU signal, the MAP detector can be significantly more accurate than a standard energy detector (cf. [43]). The MAP detector (5.17) can be used for online spectrum sensing of the given channel.

#### 5.3.5 Channel Usability and Channel Capacity

A heuristic test based on the HMM parameter estimate for a channel is performed to determine whether the channel can be used by the SU. Given the transition matrix G, the

stationary state distribution  $\pi = [\pi_0, \pi_1]$  can be computed from the equations

$$\pi = \pi G, \quad \pi \mathbf{1} = 1. \tag{5.18}$$

The channel is deemed to be a *hole* if the probability that the PU is idle,  $\pi_0$ , exceeds a threshold  $\pi_{\min,0}$  (see Algorithm 1, line 13). Note that  $\pi_1$  represents the duty cycle of the channel. If the sensed channel is determined to be a hole, the center frequency, bandwidth, MAP decisions on the PU state, and filtered decimated samples of the channel are passed to the parent node in the tree.

Given an estimate of the HMM parameter for a channel, an estimate of the SNR for the channel can be obtained. Let  $\mu_a$  denote the mean received signal strength in linear units, e.g., mW, for a = 0, 1. The SNR estimate is computed as

$$\frac{S}{N} = \frac{\mu_1 - \mu_0}{\mu_0}.$$
(5.19)

The capacity of the channel can then be estimated using the sensed bandwidth, the estimated SNR, and the stationary distribution of the HMM. The capacity is derived from the capacity for a single user with availability  $\pi_0$  in a TDMA system [20, Eq. 15.150]. We have defined  $\pi_0$  as the stationary probability that the PU is not using a given band. With these considerations, the capacity in (bits/s/Hz) is computed as follows:

$$C = \pi_0 \log_2 \left( 1 + \frac{S}{N} \right). \tag{5.20}$$

The proposed estimate for channel capacity does not play a direct role in our algorithm for wideband temporal sensing, but is useful for assessing the potential capacity gains achievable through spectrum sensing.

#### 5.3.6 Channel Aggregation

As the algorithm recurses upward, the parent nodes combine two lists of spectrum holes: one from the lower half of the band, and the other from the upper half of the band. If the highest-frequency hole from the lower band and the lowest-frequency hole from the upper band are adjacent, the two holes can possibly be combined. The objective of wideband sensing is to determine a set of narrowband channels that can be sensed independently and shared by the SU. To achieve this objective, the adjacent holes will only be combined if they are sufficiently correlated. The channel aggregation scheme proposed in this chapter is based on the time-domain cross-correlation. The rationale behind doing so is that the resulting narrowband channels will be uncorrelated and may therefore be treated as independent. This enables multiband spectrum sensing techniques to be applied to the set of aggregated channels.

The proposed channel aggregation function, while based on time-domain correlation, must account for dynamic signals. Two perfectly-correlated bursting signals will appear uncorrelated during idle periods, since white noise signals are by nature uncorrelated. The MAP detector in (5.17) can be used to determine the periods during which the PUs are most likely idle for both adjacent channels. Based on the MAP decisions, a correlation metric between two adjacent channels can be computed. Let  $Z_{lo} = \{Z_{lo,k}\}_{k=1}^{\infty}$  and  $Z_{hi} = \{Z_{hi,k}\}_{k=1}^{\infty}$ denote the observation sequences for the lower and higher frequency channels, respectively. The observed sequences from *n*-sample realizations are denoted by  $z_{lo}^n$  and  $z_{hi}^n$ , respectively. The HMM parameter estimates  $\phi_{lo}$  and  $\phi_{hi}$  are obtained for the two channels using the Baum-Welch algorithm. Let  $\hat{x}_{lo}^n = \{\hat{x}_{lo,1}, \ldots, \hat{x}_{lo,n}\}$  and  $\hat{x}_{hi}^n = \{\hat{x}_{hi,1}, \ldots, \hat{x}_{hi,n}\}$  denote the corresponding decision sequences determined according to (5.17).

The normalized crosscorrelation at zero lag between the sequences  $z_{lo}^n$  and  $z_{hi}^n$  is given by

$$\rho\left(z_{\rm lo}^{n}, z_{\rm hi}^{n}\right) = \frac{|\langle z_{\rm lo}^{n}, z_{\rm hi}^{n}\rangle|}{\left\|z_{\rm lo}^{n}\right\| \left\|z_{\rm hi}^{n}\right\|},\tag{5.21}$$

where

$$\langle z_{\rm lo}^n, z_{\rm hi}^n \rangle = \sum_{k=1}^n z_{{\rm lo},k} z_{{\rm hi},k}^*, \qquad (5.22)$$

denotes the Hermitian inner product between  $z_{lo}^n$  and  $z_{hi}^n$ ,  $z^*$  denotes the complex conjugate of z, and  $\|\cdot\|$  denotes the standard  $\ell^2$ -norm. However, for the purpose of channel aggregation, we require a correlation metric that takes into account the idle periods that coincide for the two channels. We denote the indicator function on the set or condition A by  $1_A$ , and the indicator function for the complement of A by  $1_{A^c}$ . Using this notation, we define modified observation sequences for the two channels by zeroing out the samples for which the PU is detected to be idle on both channels, i.e.,

$$\tilde{z}_{\text{lo},k} = z_{\text{lo},k} \cdot \mathbf{1}_{\{\hat{x}_{\text{lo},k} = \hat{x}_{\text{hi},k} = 0\}^c},$$

$$\tilde{z}_{\text{lo},k} = z_{\text{lo},k} \cdot \mathbf{1}_{\{\hat{x}_{\text{lo},k} = \hat{x}_{\text{hi},k} = 0\}^c},$$
(5.23)

for k = 1, ..., n. The fraction of observation samples for which the PU is detected to be idle on both channels is given by

$$\gamma = \frac{1}{n} \sum_{k=1}^{n} \mathbb{1}_{\{\hat{x}_{\text{lo},k} = \hat{x}_{\text{hi},k} = 0\}}.$$
(5.24)

For such samples, the correlation should be assigned the value 1, indicating perfect correlation. We then define a modified correlation metric as follows:

$$\tilde{\rho} = \gamma + (1 - \gamma)\rho\left(\tilde{z}_{\rm lo}^n, \tilde{z}_{\rm hi}^n\right).$$
(5.25)

It is easy to see that  $0 \leq \tilde{\rho} \leq 1$ . In our channel aggregation algorithm, two channels are merged if their correlation  $\tilde{\rho}$ , computed using (5.25), exceeds a threshold  $\tilde{\rho}_{\min}$  (see Algorithm 2, line 7). When holes are combined, their MAP decisions must be combined as well. This combination of decisions is given by

$$\hat{x}_{k} = \begin{cases}
0, & \text{if } \hat{x}_{\text{lo},k} = \hat{x}_{\text{hi},k} = 0 \\
1, & \text{otherwise.} 
\end{cases}$$
(5.26)

The PU in the combined channel is determined to be idle at time k if the PU in both subbands is determined to be idle at time k. Otherwise, the PU is determined to be active at time k.

### 5.3.7 Algorithm Descriptions

Table 5.1	Algorithm complexity parameters.

Parameter	Description
$N_c$	Number of channels at the finest sens- ing resolution
$N_t$	Number of filter taps for the channel selecting LPF
$N_s$	Number of samples in the sensing duration
$N_i$	Number of Baum-Welch iterations

A formal description of the proposed recursive wideband temporal sensing framework is given in Algorithm 1. The computational complexity is given by

$$O\left(N_c \log_2 N_c \cdot N_t N_s + N_c N_i N_s\right), \tag{5.27}$$

where the various parameters involved are shown in Table 5.3.7. The terms of the complexity equation are derived as follows:  $N_c \log_2 N_c$  is the number of nodes in the binary tree [34] and is therefore the maximum number of narrowband channels that can be sensed;  $N_t N_s$  is the complexity of the filtering operation used to select a narrowband channel for sensing. The term  $N_i N_s$  represents the per-channel complexity of the Baum-Welch algorithm.

Algorithm 1 Wideband temporal sensing algorithm.

1: function RSense $(f_c, W, W_r, z_{wb}^n)$ if  $W > W_r$  then 2:  $L_{\text{lo}} = \text{RSense}(f_c - W/2, W/2, W_r, z_{\text{wb}}^n);$ 3:  $L_{\rm hi} = \operatorname{RSense}(f_c + W/2, W/2, W_r, z_{\rm wb}^n);$ 4: if  $L_{\rm hi}$  and  $L_{\rm lo}$  are not empty then 5:  $L = \text{AggregateCh}(L_{\text{hi}}, L_{\text{lo}}, z_{\text{wb}}^n);$ 6: 7: else L =empty list; 8: else 9:  $h(n) = \text{LPF}(W, N_t);$ 10: $\operatorname{dec} = \operatorname{Floor}(W_0/W);$ 11:  $z^n = \text{DDC}(z^n_{wb}, f_c, h(n), \text{dec});$ 12:13: $y^n = \text{EnergyDet}(z^n);$  $(\nu, G, \mu, R, \hat{x}^n) = \text{BaumEst}(y^n);$ 14: $\pi = \operatorname{StatDistr}(G);$ 15:16:if  $\pi_1 > \pi_{\min,1}$  then  $L = \text{list with single entry } (f_c, W, z^n, \hat{x}^n);$ 17:18: else L = empty list;19:20:return L;

We shall not formally describe any of the other functions used in Algorithms 1 and 2, but basic descriptions are given. The function AggregateCh $(L_{\rm hi}, L_{\rm lo}, z_{\rm wb}^n)$ , as specified in Algorithm 2, determines whether two adjacent holes should be combined. The function Correlate $(n, z_{\rm lo}^n, \hat{x}_{\rm lo}^n, z_{\rm hi}^n, \hat{x}_{\rm hi}^n)$  computes the modified correlation metric given by (5.25). The function LPF(W) designs a finite impulse response (FIR) lowpass filter with bandwidth W. The function DDC $(z_{\rm wb}^n, f_c, h(n), \text{dec})$  performs channelization as discussed in Section 5.3.2. The wideband signal  $z_{wb}^n$  is mixed down by center frequency  $f_c$ , lowpass filtered by a FIR filter with discrete taps h(n), and decimated by dec. The function EnergyDet $(z^n)$  performs energy detection based on the processed received power samples given in (5.9).

The function  $\operatorname{BaumEst}(y^n)$  estimates the parameter of the PU in the selected narrowband channel with processed received power samples,  $y^n$ , using the Baum-Welch algorithm as summarized in Section 5.3.4. The function  $\operatorname{StatDistr}(G)$  computes the stationary state distribution corresponding to the transition matrix G using (5.18). The functions HighestCh(L) and LowestCh(L) select the highest-frequency narrowband channel and the

Algorithm 2 Aggregate channels.

1:	function AggregateCh $(L_{\rm hi}, L_{\rm lo}, z_{\rm wb}^n)$
2:	$(f_{\mathrm{lo},c}, W_{\mathrm{lo}}, z_{\mathrm{lo}}^n, \hat{x}_{\mathrm{lo}}^n) = \mathrm{LowestCh}(L_{\mathrm{hi}});$
3:	$(f_{\mathrm{hi},c}, W_{\mathrm{hi}}, z_h^n, \hat{x}_h^n) = \mathrm{HighestCh}(L_{\mathrm{lo}});$
4:	$L = \text{CombineLists}(L_{\text{hi}}, L_{\text{lo}});$
5:	if $f_{hi,c} - W_{hi}/2 == f_{lo,c} + W_l/2$ then
6:	$\rho = \text{Correlate}(n, z_{\text{lo}}^n, \hat{x}_{\text{lo}}^n, z_{\text{hi}}^n, \hat{x}_{\text{hi}}^n);$
7:	${\bf if}\tilde{\rho}>\tilde{\rho}_{\rm min}{\bf then}$
8:	Remove $(f_{lo,c}, W_{lo}, z_{lo}^n, \hat{x}_{lo}^n)$ and
9:	$(f_{\mathrm{hi},c}, W_{\mathrm{hi}}, z_{\mathrm{hi}}^n, \hat{x}_{\mathrm{hi}}^n)$ from L;
10:	$h(n) = \operatorname{LPF}(W_{\mathrm{lo}} + W_{\mathrm{hi}}, N_t);$
11:	$dec = Floor(W_0/(W_{lo} + W_{hi}));$
12:	$f_c = f_{lo,c} + W_{lo}/4 + W_{hi}/4;$
13:	$z^n = \text{DDC}(z^n_{wb}, f_c, h(n), \text{dec});$
14:	$\hat{x}^n = \text{Merge}(\hat{x}^n_{\text{lo}}, \hat{x}^n_{\text{hi}});$
15:	Add $(f_c, W_{\rm lo} + W_{\rm hi}, z^n, \hat{x}^n)$
16:	to $L$ ;
17:	$\mathbf{return} \ L;$

lowest-frequency narrowband channel, respectively, from a list of estimated channel parameters L. The function CombineLists $(L_1, L_2)$  merges two lists of estimated channel parameters into a single list and sorts the list in decreasing order of center frequency. The function  $Merge(\hat{x}_{lo}^n, \hat{x}_{hi}^n)$  combines the MAP decisions from the two channels as in (5.26).

### 5.4 Simulation and Numerical Results

#### 5.4.1 Simulation 1: Comparison of Techniques

We tested the wideband energy detector, the wideband edge detector, and the proposed wideband temporal spectrum detector against OFDM and GMSK signals with duty cycles varying among 1.0, 0.5, 0.25, and 0.125. We used an energy detection average of N = 1. We assumed a minimum duty cycle  $\pi_{\min,1} = 0.1$  and a minimum modified correlation threshold for combining channels of  $\tilde{\rho}_{\min} = 0.7$ . For each modulation scheme and duty cycle tested, a wideband capture was generated with signals of random center frequency and baud rate. The modulated data on the signals was generated by a uniform random number generator. All of the signals were received through a simulated AWGN and Rayleigh fading channel with 10 dB SNR and used the currently tested modulation and duty cycle. A total of 10,000 simulation iterations were performed for each modulation and duty cycle pair.

Wideband signals were also generated specifically for plotting qualitative results. These wideband signals contained 4 narrowband signals with 1 MHz bandwidth and carrier spacing of 2 MHz. The four signals have duty cycles of 1.0, 0.5, 0.25, and 0.125 from lowest-frequency to highest-frequency. All highlighted PSD plots in this chapter show the results of applying a wideband sensing algorithm to one of these wideband signals, where the shaded areas are the detected holes and the white areas are the detected signals.

#### 5.4.2 Simulation 2: Performance at Varying SNR

To test the performance of the proposed wideband temporal detector, we tested the detector against OFDM and GMSK signals with duty cycle of 0.125 and SNR ranging between -20 and 20 dB. We varied the energy detection window, N, between 1, 10, 100, and 1000. We assumed a minimum duty cycle  $\pi_{\min,1} = 0.1$  and a minimum modified correlation threshold for combining channels of  $\tilde{\rho}_{\min} = 0.7$ . For each modulation scheme, a wideband capture was generated with signals of random center frequency and baud rate. The modulated data on the signals was generated by a uniform random number generator. All of the signals were received through a simulated AWGN and Rayleigh fading channel. A total of 10,000 simulation iterations were performed for each modulation and energy detection window.

#### 5.4.3 Simulation 1 Results: Qualitative Comparison of Techniques

Qualitative results of the proposed wideband temporal sensing algorithm are depicted in Fig. 5.7 for OFDM and Fig. 5.8 for GMSK. Shaded areas represent detected spectrum holes. It can be seen that the proposed wideband temporal spectrum detector performed well for all tested duty cycles and both simulated modulation schemes. The qualitative simulation results of the proposed spectrum detector can be compared to the qualitative results from Section 5.1. Comparing Fig. 5.7 to Figs. 5.1 and 5.3 shows that reducing the duty cycle does not degrade the performance of the proposed detector for OFDM like it does



Figure 5.7: Results of wideband temporal spectrum detector for OFDM signals with 10 dB SNR and 100%, 50%, 25%, and 12.5% duty cycles.

for wideband energy detection. Similarly, comparing Fig. 5.8 to Figs. 5.2 and 5.4 shows that the proposed detector is also not degraded by reduced duty cycles for GMSK. Furthermore, comparing Fig. 5.8 to Fig. 5.4 shows that the smooth band edges of GMSK do not degrade the performance of the proposed detector like they do for the wideband energy detector.

#### 5.4.4 Simulation 1 Results: Quantitative Comparison of Techniques

Quantitative sensing results are depicted by ROC (receiver operating characteristic) curves generated by the simulation. The ROC curves represent the average detector performance over many random wideband captures using the same modulation, duty cycle, and SNR. Performance of the wideband energy detector is shown in Fig. 5.9 for OFDM and Fig. 5.10 for GMSK. Performance of the wideband edge detector is shown in Fig. 5.11 for OFDM and Fig. 5.12 for GMSK. In the wideband energy and edge detector results, it can clearly be observed that detector performance degrades as PU duty cycle decreases. In the case of GMSK, the performance of the wideband edge detector is substantially degraded, due to the edge detector's hindered ability to detect gradual changes. Performance of the wideband



Figure 5.8: Results of wideband temporal spectrum detector for GMSK signals with 10 dB SNR and 100%, 50%, 25%, and 12.5% duty cycles.



Figure 5.9: ROC curve for wideband energy detector for OFDM signals with 10 dB SNR and 100\%, 50\%, 25\%, and 12.5% duty cycles.

temporal spectrum detector is shown in Fig. 5.13 for OFDM and Fig. 5.14 for GMSK. It is clear from these results that the proposed detector's performance was not significantly degraded by reduced duty cycles.



Figure 5.10: ROC curve for wideband energy detector for GMSK signals with 10 dB SNR and 100%, 50%, 25%, and 12.5% duty cycles.

#### 5.4.5 Simulation 2 Results

For simulation 2, false alarm rate and true positive rate were collected for a variety of thresholds at all tested SNR and energy detection windows. To impose many ROC curves onto a single plot, true positive rate at a constant false alarm rate (CFAR) of 0.01 is shown. In Fig. 5.15, true positive rate for a CFAR of 0.01 is shown for the proposed detector against OFDM signals with duty cycle of 0.125 and varying SNR. In Fig. 5.16, true positive rate is shown for GMSK signals. As the energy detection window increases, the sensitivity of the detector increases. The drawback to using too large of a detection window, however, is that increasing energy detector length increases the likelihood that samples from idle cycles and busy cycles are averaged together, degrading detector performance.

# 5.5 Conclusion

The proposed wideband temporal spectrum sensing framework performed comparably for bursting signals with various duty cycles to the wideband energy detector applied to signals with 100% duty cycle. In the case of bursting signals, the recursive wideband temporal



Figure 5.11: ROC curve for wideband edge detector for OFDM signals with 10 dB SNR and 100\%, 50\%, 25\%, and 12.5% duty cycles.



Figure 5.12: ROC curve for wideband edge detector for GMSK signals with 10 dB SNR and 100%, 50%, 25%, and 12.5% duty cycles.

spectrum sensing algorithm proved to be much more robust than the frequency-only sensing algorithms. The power of the proposed sensing algorithm comes at the cost of computation time:  $O(N_c \log_2 N_c)$  narrowband sensing operations must be performed, as well as FIR



Figure 5.13: ROC curve for wideband temporal spectrum detector for OFDM signals with 10 dB SNR and 100%, 50%, 25%, and 12.5% duty cycles.



Figure 5.14: ROC curve for wideband temporal spectrum detector for GMSK signals with 10 dB SNR and 100%, 50%, 25%, and 12.5% duty cycles.

filtering for channel selection. We suggest that a cognitive radio would use this wideband sensing algorithm during initialization and revert to narrowband or multiband sensing once the set of independent channels has been determined.



Figure 5.15:



Figure 5.16:

Several extensions of the proposed wideband temporal spectrum sensing algorithm could be explored further. To reduce overall computation, use of a frequency-domain channelizer that allows the channel selection operators to share filter computations and leverages heavily optimized implementations for the FFT could be investigated. To improve detection accuracy for a wider range of PU behaviors, the HMM could be extended to a hidden bivariate Markov model [43], which has phase-type, rather than geometric state sojourn time distributions. In the present chapter, a simple energy detector was used as a front-end for the HMM-based parameter estimator and state detector. Even with the performance gain that could be achieved by extending our scheme using a hidden bivariate Markov model, detection of a PU at very low SNR using an energy detector may perform poorly. For such low SNR scenarios, better performance could be achieved by means of a matched filter or cyclostationary detector in conjunction with the recursive channel search.

# Chapter 6: An Edge Detection Approach to Wideband Temporal Spectrum Sensing

### 6.1 Introduction

In [12], a sensing framework for reliable wideband detection of PUs with low duty cycle was developed. The approach, referred to as wideband temporal sensing, involves partitioning the given spectrum band into smaller subchannels. The energy in each subchannel is measured and an HMM-based spectrum sensing approach is applied to each subchannel. A recursive tree search is performed to aggregate correlated subchannels into a set of independent narrowband channels, which effectively reduces the sensing task to the multiband case. The wideband temporal sensing approach developed in [12] allows PU signals with low duty cycle to be detected accurately at high to moderate SNR.

The main contribution of this chapter is to apply an edge detection algorithm to wideband temporal sensing, which allows for more reliable detection at low SNR compared to the wideband temporal energy detector of [12]. Moreover, the use of edge detection avoids the need for the recursive tree search used in the wideband temporal energy detector, resulting in a computationally more efficient spectrum sensing scheme. Our approach incorporates the wavelet-based edge detection algorithm of [56] into the wideband temporal sensing framework proposed in [12]. We present experimental results obtained through simulation.

The remainder of the chapter is organized as follows. In Section 6.2, we define the system model for wideband spectrum sensing. In Section 6.3, we discuss and evaluate the performance of two existing wideband spectrum sensing techniques. In Section 6.4, we develop the proposed edge detection approach to wideband temporal spectrum sensing. In Section 6.5, we describe the simulation that was used to compare the proposed algorithm

to existing algorithms and present numerical results. Concluding remarks are given in Section 6.6. A portion of the work in this chapter has been published in [11].

### 6.2 System Model

#### 6.2.1 Wideband Channel Model

Over a given wideband spectrum band, we assume that an unknown number of independent PUs are operating. Each PU has an unknown center frequency and bandwidth. It is assumed that PU channels do not overlap in frequency. The channel over which a given PU is observed is assumed to be flat Rayleigh fading with parameter  $\sigma_f$  combined with additive white Gaussian noise (AWGN), defined by the circularly symmetric complex normal distribution  $C(0, \sigma_n^2)$ . The mean SNR of the received signal on the PU channel, given that the PU is transmitting, is given by

$$\overline{\text{SNR}} = \frac{\sigma_f^2}{\sigma_n^2},\tag{6.1}$$

at the input to the energy detector.

#### 6.2.2 PU Traffic Model

A given PU may be transmitting or idle at any given time. The state of the PU is denoted by a discrete-time random process  $X = \{X_k\}_{k=1}^{\infty}$ , where  $X_k = 1$  if the PU is idle or  $X_k = 2$  if the PU is active at time k. We shall assume that the PU state process X is characterized by an ergodic time-homogeneous discrete-time Markov chain with transition matrix  $G = [g_{ab} : a, b \in \{1, 2\}]$ , where

$$g_{ab} = P(X_2 = b \mid X_1 = a),$$
 (6.2)

and initial distribution  $\nu = [\nu_a : a = 1, 2]$ , where

$$\nu_1 = P(X_1 = 1), \quad v_2 = P(X_1 = 2).$$
 (6.3)

The equilibrium state distribution, denoted by  $\pi = [\pi_1, \pi_2]$ , satisfies the following equations:

$$\pi = \pi G, \quad \pi_1 + \pi_2 = 1. \tag{6.4}$$

The value  $\pi_2$  corresponds to the duty cycle of the PU in steady-state.

#### 6.2.3 Cognitive Receiver Model

#### **Received Wideband Signal**

A transmitting PU will generate a bandpass signal  $\tilde{t}_{i,k}$ . The transmitted signal for PU *i* at any time *k* is

$$t_{i,k} = \tilde{t}_{i,k} \cdot 1_{\{X_{i,k}=2\}},\tag{6.5}$$

where  $1_{\{A\}}$  is the indicator function on the set or condition A. The  $i^{th}$  PU signal is multiplied at time k by fading signal  $f_{i,k} \sim \mathcal{C}\left(0, \sigma_{f,i}^2\right)$ . All M PU signals are received simultaneously and added to noise signal  $n_k \sim \mathcal{C}\left(0, \sigma_{n,i}^2\right)$ . The received wideband signal is represented by a sequence of samples  $z_{wb}^n = \{z_{wb,1}, \ldots, z_{wb,n}\}$ , where  $z_{wb,k}$ , the  $k^{th}$  I-Q sample from the wideband channel, is defined as

$$z_{wb,k} = \sum_{i=1}^{M} t_{i,k} f_{i,k} + n_k.$$
(6.6)

#### **Channelized Received Signal**

The SU will divide the wideband received signal into J narrowband subchannels. Initially this division must be done done arbitrarily, but after wideband sensing, the set of subchannels should describe all PU statistics as well as the statistics of the spectrum holes between PU signals. The sequence of observation samples in the  $j^{th}$  subband is denoted  $z_j$ .

#### **Energy Detected Signal**

For spectrum sensing, the channelized narrowband signals are processed with an averaging energy detector, which estimates the power of each sample and averages N samples together. The resulting random variable for the received energy in in subchannel j is denoted by  $Y_j$ , and the sequence of energy estimates for subchannel j is denoted  $y_j^n = \{y_{j,1}, \ldots, y_{j,n}\}$ . The  $k^{th}$  sample in the energy detection sequence,  $y_{j,k}$ , is defined as

$$y_{j,k} = \frac{1}{N} \sum_{i=1}^{N} |z_{j,(k-1)N+i}|^2.$$
(6.7)

Assuming that N is sufficiently large,  $y_j$ , k will be conditionally normal with distribution

$$y_{j,k} \sim \begin{cases} \mathcal{N}\left(2\sigma_{n,i}^{2}, \frac{4\sigma_{n,i}^{4}}{N}\right), & X_{i,k} = 0, \\ \mathcal{N}\left(2\sigma_{f,i}^{2} + 2\sigma_{n,i}^{2}, \frac{4\left(\sigma_{f,i}^{2} + \sigma_{n,i}^{2}\right)^{2}}{N}\right), & X_{i,k} = 1, \end{cases}$$

$$(6.8)$$

This conditional distribution is derived in Appendix A.

### 6.3 Comparison of Wideband Spectrum Sensing Techniques

It was shown in Chapter 5 and [12] that standard wideband detection methods are inadequate for PUs with low duty cycle. The edge detection algorithm from [56] was performed



Figure 6.1: Results of a wideband edge detector for OFDM signals with 10 dB SNR and 100%, 50%, 25%, and 12.5% duty cycles [12].

on a wideband signal with four orthogonal frequency division multiplexing (OFDM) carriers of varying duty cycle. It was demonstrated that as the duty cycle was reduced, the detector sensitivity was also reduced. Performance of the standard edge detector is shown in Fig. 6.1 for OFDM signals with SNR of 10 dB and varying duty cycles, where the shaded areas indicate detected spectrum holes. Clearly, the edge detector fails to correctly detect the two rightmost OFDM signals.

Wideband temporal spectrum sensing was introduced in Chapter 5 and [12] to more reliably detect a PU with low duty cycles, where the proposed algorithm performed comparably to energy detection for duty cycles of 1.0 and did not degrade substantially for lower duty cycles. Because we are extending the wideband temporal spectrum sensing algorithm from [12], we shall refer to the incumbent algorithm as *wideband temporal energy detection*. Performance of the wideband temporal energy detector is shown in Fig. 6.2 for OFDM signals with SNR of 10 dB and varying duty cycles.

Although wideband temporal energy detection has been shown to reliably detect PU signals at 10 dB SNR for a variety of duty cycles, spectrum sensing applications may demand accurate PU detection at substantially lower SNR. It can be seen in Fig. 6.3 that at higher



Figure 6.2: Results of a wideband temporal energy detector for OFDM signals with 10 dB SNR and 100%, 50%, 25%, and 12.5% duty cycles [12].

noise levels, the wideband temporal energy detection algorithm proposed in [12] begins to experience detection errors. Note that the wideband temporal energy detector incorrectly characterizes the rightmost spectrum hole. The edge detection algorithm proposed in [56] allows for accurate detection of high duty cycle signals in high noise levels, which motivates the development of an algorithm that extends wideband temporal spectrum sensing with edge detection.

### 6.4 Proposed Algorithm

In this section, we extend the wideband temporal sensing algorithm from [12]. Since the proposed algorithm uses edge detection to determine channel boundaries, the recursive tree search used in [12] for channel aggregation is not necessary.

#### 6.4.1 Channelization of Received Wideband Signal

First, the received wideband signal is divided into J narrowband signals of equal bandwidth. The narrowband signals are spaced such that they are non-overlapping and cover the entire



Figure 6.3: Results of wideband temporal energy detector for OFDM signals with 5 dB SNR and 100%, 50%, 25%, and 12.5% duty cycles [12].

band. Selection of J depends on the desired sensing resolution,  $W_r$ . Signals narrower than  $W_r$  may not be reliably detected, and detected channel boundary locations may have a frequency error as large as  $\frac{W_r}{2}$ . The number of subchannels required to achieve sensing resolution  $W_r$  is given by

$$J = \left\lceil \frac{W_0}{W_r} \right\rceil,\tag{6.9}$$

where  $W_0$  is the width of the entire band. Channelization may be accomplished in a conceptually simple fashion using a bank of digital downconverters or more efficiently using a frequency-domain channelizer, as described in [12, Sec. III-b]. If a frequency-domain channelizer is used, J from Eq. (6.9) should be rounded up to the next power of 2 for efficient FFT computation. The resulting set of all J narrowband received signals is denoted by  $\mathbf{Z} = \{Z^{(1)}, \ldots, Z^{(J)}\}.$ 

#### 6.4.2 Sensing of Narrowband Subchannels

The observation of PU traffic through a noisy channel can be accurately modeled using a hidden Markov model (HMM), denoted by (Y, X), where X is an underlying discrete-time Markov chain and Y is a random sequence of observations, conditionally dependent on X. The transition matrix and initial distribution of the HMM are given in (6.2) and (6.3), respectively. The noisy samples are modeled by normal distributions. The parameter of the HMM for a PU is given by  $\phi = (\nu, G, \mu, \Sigma)$ , where  $\mu = [\mu_1, \mu_2]$  and  $\Sigma = [\sigma_1^2, \sigma_2^2]$  are, respectively, the sets of conditional means and conditional variances for subchannel j given by (6.8).

For a set of J subchannels that partition a spectrum band evenly, the conditional means

$$\boldsymbol{\mu}_{a} = \left\{ \mu_{a}^{(1)}, \dots, \mu_{a}^{(J)} \right\}, \quad a = 1, 2,$$
(6.10)

determine the conditional power spectral density of the received signals on the subchannels. If the J uniformly distributed channels which cover the band have frequencies  $\{f_1, \ldots, f_J\}$ , the conditional power spectral densities are defined as

$$\mu_a(f) = \sum_{j=1}^J \frac{\mu_a^{(j)}}{\Delta_f} \operatorname{rect}\left(\frac{f-f_j}{\Delta_f}\right), \quad a = 1, 2,$$
(6.11)

where rect(·) denotes the unit rectangular function and  $\Delta_f$  is the frequency spacing between subchannels. Here,  $\mu_1(f)$  is the power spectral density of the received signal given that all PUs are idle, and  $\mu_2(f)$  is the power spectral density of the received signal given that all PUs are transmitting.

For each of the J narrowband subchannels, energy detection and HMM parameter estimation is performed. The set of observed energy sequences is denoted  $\mathbf{Y} = \{Y^{(1)}, \ldots, Y^{(J)}\}$ and is given by Eq. (6.7). Subchannel j is characterized by an HMM parameter,  $\phi^{(j)} =$   $(\nu^{(j)}, G^{(j)}, \mu^{(j)}, \Sigma^{(j)})$ , which is estimated using the Baum-Welch algorithm [9]. In the wideband temporal energy detector proposed in [12], the set of HMM parameters for the entire band is used directly.

Our performance baseline will be the wideband temporal energy detector from [12], which directly computes  $\mu_2(f)$  from the conditional power spectral density, defined in Eq. (6.11), with a threshold  $\lambda$  to determine which subchannels contain an active PU. The adjacent active subchannels which are determined to be correlated are combined into a single channel.

#### 6.4.3 Edge Detection

In our proposed algorithm, we apply the wideband edge detection algorithm from [56] to the conditional power spectral density of the received signal. We first decompose the conditional power spectral density into a set of resolutions using the continuous wavelet transform (CWT). The CWT of  $\mu_2(f)$  for a resolution  $\gamma$  is given as

$$\mathcal{W}_{\gamma} \{ \mu_2(f) \} = \mu_2(f) * \psi_{\gamma}(f) ,$$
 (6.12)

where \* denotes convolution and  $\psi_{\gamma}(f)$  is a wavelet of scale  $\gamma$ , given by

$$\psi_{\gamma}(f) = \frac{1}{\gamma}\psi\left(\frac{f}{\gamma}\right).$$
 (6.13)

The mother wavelet,  $\psi(t)$ , is the Ricker wavelet, defined in [40, Eq. (4.34)] as

$$\psi(t) = \frac{2}{\pi^{1/4}\sqrt{3\sigma}} \left(\frac{t^2}{\sigma^2} - 1\right) \exp\left(\frac{-t^2}{2\sigma^2}\right).$$
(6.14)

The Ricker wavelet is the second derivative of a Gaussian function, and a standard Ricker wavelet, where  $\sigma = 1$ , is particularly useful for edge detection [40]. The  $r^{\text{th}}$  resolution of the conditional power spectral density has scale  $\gamma$  where  $\gamma = 2^r$  and  $r \in \{1, 2, ..., R\}$ , where R

is the number of CWT resolutions.

Once the conditional power spectral density is decomposed into component resolutions using the CWT, edge detection is performed by taking the first derivative of each component resolution:

$$\mathcal{W}_{\gamma}'\left\{\mu_{2}(f)\right\} = \gamma \frac{d}{df}\left(\mu_{2}(f) * \psi_{\gamma}\left(f\right)\right).$$
(6.15)

We then compute the multiscale wavelet product from the resulting gradient estimates:

$$\mathcal{U}_{R}\left\{\mu_{2}(f)\right\} = \prod_{r=1}^{R} \mathcal{W}_{\gamma}'\left\{\mu_{2}(f)\right\}\Big|_{\gamma=2^{r}}.$$
(6.16)

By multiplying the component resolutions together, the signal is amplified, while the noise is not, resulting in noise suppression [50]. The resulting peaks in  $\mathcal{U}_R \{\mu_2(f)\}$  are determined to be channel boundaries.

### 6.5 Simulation and Results

#### 6.5.1 Simulation Setup

We tested the proposed wideband temporal edge detector against the wideband temporal energy detector from [12]. Wideband signals with OFDM carriers were tested. A duty cycle of  $\pi_2 = 0.1$  was used, and SNR values of 0, 5, and 10 dB were tested. A total bandwidth of 10 MHz with four randomly placed PU signals was tested. The bandwidth and center frequency of each non-overlapping PU carrier were randomly generated each iteration, and the PU signal bandwidths were drawn randomly within the range 0.5 to 2.0 MHz. The full set of simulation parameters is enumerated in Table 6.5.1. Note that the number of CWT resolutions (*R*) applies only to the edge detector.

Simulation Parameter	Value
Modulation	OFDM
Total bandwidth $(W)$	10 MHz
Duty cycle $(\pi_2)$	0.1
SNR	$\{0, 5, 10\} dB$
Number of PU carriers	4
Number of narrowband subchannels $(J)$	1024
Energy detector average length $(N)$	10
Number of CWT resolutions $(R)$	4
Sensing duration per iteration	0.01 s
Number of simulation iterations	10000

Table 6.1: Simulation parameters.



Figure 6.4: Results of the proposed wideband temporal edge detector for OFDM signals with 10 dB SNR and 100%, 50%, 25%, and 12.5% duty cycles [12].

#### 6.5.2 Qualitative Results

The visual output of the proposed wideband temporal edge detector is shown in Fig. 6.4, where the proposed detector was tested against a wideband signal with PUs of varying duty cycle with 5 dB SNR. This performance may be contrasted with the wideband temporal energy detector proposed in [12] in 5 dB SNR plotted in Fig. 6.3. Visually, it can be seen



Figure 6.5: Results of wideband temporal detectors for OFDM signals with 10 dB SNR.



Figure 6.6: Results of wideband temporal detectors for OFDM signals with 5 dB SNR.

that the proposed wideband temporal edge detector performs accurately in relatively low SNR.

#### 6.5.3 Numerical Results

Next, we present numerical results demonstrating the performance improvement achieved by wideband temporal edge detection for a variety of medium to low SNR signals. In each



Figure 6.7: Results of wideband temporal detectors for OFDM signals with 0 dB SNR.

simulation iteration, every narrowband channel was recorded as either a true detect, a true positive, a false detect, or a false positive, depending on the known PU signal locations and the detector results. A variety of thresholds were tested so that the relationship between detection rates can be observed. Averaged detection characteristics are plotted as receiver operating characteristic (ROC) curves. The resulting plots are shown in Fig. 6.5, 6.6, and 6.7 for SNRs of 10, 5, and 0 dB, respectively. From these results, it is apparent that the wideband temporal edge detector performs favorably compared to the wideband temporal energy detector from [12] for all simulated SNR values. The performance benefit of edge detection is especially pronounced at the low SNR of 0 dB. The simulated duty cycle of  $\pi_2 = 0.1$  was lower than any duty cycle simulated in [12], and the wideband temporal energy detector produced similar results to those in [12] at 10 dB SNR.

### 6.6 Conclusion

We have proposed a wideband spectrum sensing algorithm that is capable of detecting PU signals at low duty cycles and relatively low SNR. We leveraged the wideband temporal sensing framework introduced in [12], which had been shown to perform well for low duty

cycle PU signals at moderate SNR. We enhanced the wideband temporal sensing framework with the edge detection algorithm from [56]. This enhancement was shown to perform substantially better at lower SNR, making the proposed algorithm more suitable for cognitive radio tasks that require highly reliable detection at low to moderate SNR.

Several extensions to the proposed algorithm are being investigated in our ongoing work. One such extension is failure detection, where the SU would detect that no signal edges are present and default to wideband temporal energy detection, which would allow for sensing of PU signals without sharp band edges, as discussed in [12]. Another extension to the proposed sensing algorithm involves smoothing of the conditional power spectral density in Eq. (6.11) using the maximum a posteriori (MAP) decisions produced by the Baum-Welch algorithm to more reliably estimate average received signal power.

# Chapter 7: Conclusions

This dissertation presented a set of spectrum sensing algorithms which can enable wideband or multiband opportunistic spectrum access. Although each of the sensing schemes developed in Chapters 3, 4, 5, and 6 may stand alone as a useful spectrum sensing algorithm, the set of proposed algorithms builds a picture of how a standalone secondary user can characterize its environment.

Hidden Markov processes are used throughout this thesis to model primary user activity and channel impairments jointly. By modeling the primary user traffic as a Markov process, prediction of future transmit states may be performed, and use of the hidden Markov process allows for inference of the primary user state through noisy observations.

### 7.1 Multiband Spectrum Sensing

In Chapters 3 and 4, we considered multiband spectrum sensing, the case in which frequency channels have already been identified, and the secondary user must observe many to determine a plan for whitespace exploitation. Although a baseline for multichannel estimation exists, this dissertation addresses many of the practical concerns mentioned in Chapter 2. In Chapter 3, we propose the Markov-modulated Gaussian process, a continuous-time version of a hidden Markov process, to model a discrete-time noisy channel. Multichannel estimation is performed against noisy data, and we compare the simulated estimator variance to a theoretical lower bound. In moderate SNR or higher, the variance of the proposed multichannel estimation algorithm approaches the theoretical lower limit. In Chapter 4, we present optimal compute budget allocation (OCBA) as a framework for multichannel optimization. We use OCBA to select the "best" channel, or subset of channels, with the goal of maximizing a certain objective function. Under the condition that the objective function is normal, OCBA minimizes the sensing time required to select the optimal subset of channels.

# 7.2 Wideband Spectrum Sensing

In Chapters 5 and 6, we consider wideband spectrum sensing, the case in which a band may contain many channels, but the number of channels, center frequencies, and bandwidths are unknown. We develop a wideband spectrum sensing algorithm called *wideband temporal spectrum sensing*, which performs accurate detection of bursting primary users and performs parameter estimation on the primary user, which has been modeled as a hidden Markov process, allowing for time series prediction during spectrum exploitation. In Chapter 5, the wideband temporal spectrum sensing is introduced and shown to work on a variety of signals with different modulations and duty cycles. In Chapter 6, the wideband temporal spectrum sensing algorithm is extended to perform multi-resolution edge detection, allowing for accurate detection of certain signals at much lower SNR.

### 7.3 Future Work

The work in this dissertation may be extended in a number of directions. Many opportunities exist for implementation of cognitive radio systems based on this work. There are also some excellent opportunities for original research based on this work. Some of these opportunities are enumerated in this section.

#### 7.3.1 Multiband Spectrum Sensing

Multiband spectrum sensing using hidden *bivariate* Markov processes would be extremely beneficial. Hidden bivariate Markov processes have substantially improved predictive power (c.f. [43]), and have elegant parameter estimators based on those developed for standard hidden Markov processes. The expectation maximization algorithm from [47] can be extended to the bivariate case and used for multiband optimizations.
Additional research into OCBA for multiband spectrum sensing is recommended. First, the two-part algorithm proposed in Chapter 4 should be reconsidered as a single-part estimation algorithm, where minimization of the variance of the "best" channel is attempted.

Furthermore, OCBA only minimizes search time in the case where the objective function is normal. We proposed an objective function that is asymptotically normal, but other more complicated functions may not work as well with the proposed algorithm. Creating versions of OCBA where the objective functions map to other distributions would give implementers more freedom in selecting objective functions while maintaining the guarantees of OCBA.

#### 7.3.2 Wideband Spectrum Sensing

While wideband temporal spectrum sensing allows for spectrum sensing in a much greater range of environments, many deficiencies still exist. First, wideband temporal spectrum sensing should be considered in very low SNR, in which some cognitive radios will be required to operate. Detectors with increased processing gain like the cyclostationary feature detector should be considered as a front-end to wideband temporal spectrum sensing, which currently relies on an energy detector.

Furthermore, wideband temporal spectrum sensing does not support the case where primary user channels overlap in frequency. Extensions should be considered in which overlapping channels can be correctly identified.

#### 7.3.3 Implementation

Implementers of opportunistic spectrum sensing systems should consider how to use the multiband and wideband sensing algorithms in this dissertation in a coordinated fashion. One example of such an implementation would be use of wideband temporal spectrum sensing as an initialization stage to identify the channels, and then multiband spectrum sensing to track primary users. In more dynamic environments, wideband spectrum sensing could be repeated periodically.

# Appendix A: Derivation of Conditional Distribution of a Rayleigh Channel Observed through an Energy Detector

In this appendix, we derive the conditional normal distribution of a PU signal received over a Rayleigh fading channel with additive white Gaussian noise (AWGN) and observed through an energy detector.

### A.1 Narrowband Channel Model

A channel with known center frequency and bandwidth is and a single PU is observed. The channel over which the PU is observed is assumed to be flat Rayleigh fading with parameter  $\sigma_f$  combined with zero mean AWGN, defined by the circularly symmetric complex normal distribution  $C(0, \sigma_n^2)$ . The mean SNR of the received signal on the channel, given that the PU is transmitting is

$$\overline{\text{SNR}} = \frac{\sigma_f^2}{\sigma_n^2},\tag{A.1}$$

at the input to the energy detector.

### A.2 PU Traffic Model

A PU may be transmitting or idle at any given time. The state of the PU is denoted by the random variable X, where X = 0, when the PU is not transmitting, and t X = 1, when the PU is transmitting. The  $k^{th}$  state of the PU is denoted by  $X_k$ . The sequence of active/idle states from the PU is modeled by a continuous-time homogeneous Markov chain with generator matrix Q and initial distribution  $\pi$  defined, respectively, as

$$Q = \begin{bmatrix} -\lambda_0 & \lambda_0 \\ \lambda_1 & -\lambda_1 \end{bmatrix}, \tag{A.2}$$

$$\pi_0 = P(X_1 = 0), \pi_1 = P(X_1 = 1), \qquad (A.3)$$

where  $\lambda j$  is the rate of the exponential sojourn time distribution in state j.

### A.3 Cognitive Receiver Model

Let  $Y_k$  denote the average energy at time k = 0, 1, ... of the narrowband signal over N samples. Let the sequence of energy estimates be denoted  $Y^n = \{Y_1, ..., Y_n\}$ . The  $k^{th}$  sample in the energy detection sequence,  $Y_k$ , is defined as

$$Y_k = \frac{1}{N} \sum_{j=1}^N |Z_{(k-1)N+j}|^2.$$
(A.4)

An SU will need to detect slow changes in the PU state to properly leverage spectrum holes. We therefore assume that N is sufficiently small such that no state changes occur within the N samples. We further assume that  $\{Z_k\}$  are iid Gaussian with conditional distribution

$$Z_k \sim \begin{cases} \mathcal{C}\left(0, \sigma_n^2\right), & X_k = 0, \\ \mathcal{C}\left(0, \sigma_f^2 + \sigma_n^2\right), & X_k = 1, \end{cases}$$
(A.5)

The resulting energy estimates,  $y_k$ , will be scaled chi-squared random variables with 2N degrees of freedom. We will denote a chi-squared distribution with D degrees of freedom

 $\mathcal{X}^{2}(D)$ . The conditional distribution of the energy detector is therefore

$$y_k \sim \begin{cases} \frac{\sigma_n^2}{N} \mathcal{X}^2(2N), & X_k = 0, \\ \frac{\sigma_f^2 + \sigma_n^2}{N} \mathcal{X}^2(2N), & X_k = 1, \end{cases}$$
(A.6)

The mean and variance of a chi-squared distribution with D degrees of freedom are D and 2D respectively. Assuming that N is sufficiently large,  $y_k$  will be conditionally normal with distribution

$$y_k \sim \begin{cases} \mathcal{N}\left(2\sigma_n^2, \frac{4\sigma_n^4}{N}\right), & X_k = 0, \\ \mathcal{N}\left(2\sigma_f^2 + 2\sigma_n^2, \frac{4\left(\sigma_f^2 + \sigma_n^2\right)^2}{N}\right), & X_k = 1, \end{cases}$$
(A.7)

# Appendix B: Derivation of the Fisher Information Matrix for a 2-State Markov-Modulated Gaussian Process

## **B.1** Definitions

In [47], a Markov-Modulated Gaussian Process (MMGP) is defined as a continuous-time finite-state homogeneous Markov chain observed through a discrete-time memoryless Gaussian channel. The log likelihood function for an MMGP [47, Eq. 5] is given:

$$\log \mathcal{L}^{c} = \sum_{i=1}^{r} \mathbb{1}_{\{X(0)=i\}} \log \pi_{i} - \sum_{i=1}^{r} T_{i}q_{i} + \sum_{i\neq j} m_{ij} \log q_{ij}$$
$$-\frac{1}{2}n \log 2\pi - \frac{1}{2}\sum_{i=1}^{r} n_{i} \log \sigma_{i}^{2} - \frac{1}{2}\sum_{i=1}^{r} \sum_{k=1}^{n} \xi_{k}(i) \frac{(y_{k} - \mu_{i})^{2}}{\sigma_{i}^{2}} \quad , \tag{B.1}$$

where  $1_{\{\cdot\}}$  is the indicator function, X(0) denotes the initial state of the underlying Markov process,

$$T_i = \int_0^T \mathbf{1}_{\{X(t)=i\}} \,\mathrm{d}t \tag{B.2}$$

denotes the total time that the underlying Markov process spent in state i during the interval [0, T],

$$m_{ij} = \sum_{k=1}^{m} \mathbb{1}_{\{s_k = i, s_{k+1} = j\}}$$
(B.3)

denotes the number of jumps from state i to state j, where  $i \neq j$ , the state sequence over the interval [0,T] has m jumps, and  $s_k$  is the state of the Markov chain immediately after its kth jump.

$$\xi_k(i) = \mathbf{1}_{\{X(t_k)=i\}} \tag{B.4}$$

indicates whether or not the underlying Markov process is in state i for sample k,

$$n_i = \sum_{k=1}^{m+1} \xi_k(i)$$
(B.5)

denotes the number of samples of the observed signal while the Markov chain is in state i, and  $y_k$  denotes the  $k^{th}$  sample of received noisy data.

### B.2 Two-State Log Likelihood

For spectrum sensing applications, 2-state Markov processes are commonly used to model the signal from the PU. This allows us to make some important simplifications to the log likelihood function. The likelihood function of the Markov chain and observed sequence is given by

 $\log \mathcal{L}^{c} = 1_{\{X(0)=0\}} \log \pi_{0} + 1_{\{X(0)=1\}} \log \pi_{1}$  $-T_{0}\lambda_{0} - T_{1}\lambda_{1} + m_{01} \log \lambda_{0} + m_{10} \log \lambda_{1}$  $-\frac{1}{2}n \log 2\pi - \frac{1}{2}n_{0} \log \sigma_{0}^{2} - \frac{1}{2}n_{1} \log \sigma_{1}^{2}$  $-\frac{1}{2}\sum_{k=1}^{n} \xi_{k}(0) \frac{(y_{k} - \mu_{0})^{2}}{\sigma_{0}^{2}} - \frac{1}{2}\sum_{k=1}^{n} \xi_{k}(1) \frac{(y_{k} - \mu_{1})^{2}}{\sigma_{1}^{2}} , \qquad (B.6)$ 

where  $\lambda_i$  is the transition rate out of state *i*. For a two-state underlying Markov process:

$$\lambda_0 = q_0 = q_{01} \tag{B.7}$$

$$\lambda_1 = q_1 = q_{10} \tag{B.8}$$

## **B.3** Important Expected Values

Certain expected values are required to derive the Fisher Information Matrix.

#### **B.3.1** Expected Number of Jumps Between States

For a two-state CTMC, the expected number of jumps between states may be derived using renewal theory.

#### **Renewal Events**

Consider a sample function of the process such that X(0) = 0. Then the state of X alternates from 0 to 1, and the cycle of a sojourn in state 0 followed by a sojourn in state 1 repeats. Alternatively, if X(0) = 1, a cycle consists of a sojourn in state 1 followed by a sojourn in state 0. For our purposes, the sequence of states within a cycle is immaterial. Let us call the completion of each cycle a renewal event. Let M(t) denote the number of renewal events in (0, t]. Then, the number of transitions from state 1 to two in (0, t] can be related to M(t) as follows:

$$N_t(0,1) = M(t) + \mathbb{1}_{\{X(0)=0,X(t)=1\}},$$
(B.9)

where  $1_{\{A\}}$  is the indicator function of event A. This additional term is necessary to account for the event of an additional transition from state 0 to state 1 without the completion of an entire renewal, i.e. a transition back to state 0. Similarly, the number of transitions from state 1 to state 0 in (0, t] can be written as

$$N_t(1,0) = M(t) + \mathbb{1}_{\{X(0)=1,X(t)=0\}\}}.$$
(B.10)

Letting R(t) = E(M(t)), the expected number of transitions from 0 to 1 and 1 to 0, respectively can be written as

$$E(N_t(0,1)) = R(t) + \pi_0 P_{01}(t), \qquad (B.11)$$

$$E(N_t(1,0)) = R(t) + \pi_1 P_{10}(t), \qquad (B.12)$$

where  $P_{01}(t)$  and  $P_{10}(t)$  are respectively defined as:

$$P_{01}(t) = P(X(0) = 0, X(t) = 1), \qquad (B.13)$$

$$P_{10}(t) = P(X(0) = 1, X(t) = 0).$$
(B.14)

#### Transition probabilities for a finite-state Markov process

Kolmogorov's forward and backward equations are given respectively:

$$\frac{dP(t)}{dt} = P(t)Q = QP(t), \tag{B.15}$$

where  $P(t) = [P_{ij}(t)i, j \in X]$  denotes the transition probability matrix at time t [35, Eq. 16.35, 16.38]. Note that

$$P(0) = I, (B.16)$$

where I is the identity matrix. The unique solution to the forward and backward equations,

$$P(t) = e^{tQ} = \sum_{n=0}^{\infty} \frac{(tQ)^n}{n!},$$
(B.17)

applies when all entries of Q are bounded [35, Eq. 16.39].

#### Transition probabilities for a 2-state Markov process

For the 2-state case, the forward and backward equations (B.15) can be solved explicitly. From Eq. (B.7, B.8), the generator for a 2-state CTMC is

$$Q = \begin{bmatrix} -\lambda_0 & \lambda_0 \\ \lambda_1 & -\lambda_1 \end{bmatrix}.$$
 (B.18)

Because the rows of a transition probability matrix must sum to 1, it is clear that

$$P_{00}(t) = 1 - P_{01}(t),$$
  

$$P_{11}(t) = 1 - P_{10}(t).$$
(B.19)

The transition probabilities for a 2-state CTMC are given [30, Ch. 3, Eqs. 28-29]:

$$P_{01}(t) = \frac{\lambda_0}{\lambda_0 + \lambda_1} \left( 1 - e^{-(\lambda_0 + \lambda_1)t} \right),$$
  

$$P_{10}(t) = \frac{\lambda_1}{\lambda_0 + \lambda_1} \left( 1 - e^{-(\lambda_0 + \lambda_1)t} \right).$$
(B.20)

#### **Renewal function**

An expression for R(t) can be obtained using some results from renewal theory, see, e.g., Cinlar [15]. A renewal process can be defined as follows. Let

$$S_0 = 0; \quad S_{n+1} = S_n + W_n, \quad n = 0, 1, 2, \dots$$
 (B.21)

The sequence  $S = \{S_n; n = 0, 1, ...\}$  is called a renewal process if  $W_1, W_2, ...$  are independent and identically distributed random variables. The  $S_n$  are called renewal times. Let F denote the distribution function of the interrenewal times  $W_n$  and let M(t) denote the number of renewals in (0, t]. Then we can write

$$M(t) = \sum_{n=1}^{\infty} \mathbb{1}_{\{S_n \le t\}}.$$
(B.22)

Therefore,

$$R(t) = E[M(t)] = \sum_{n=1}^{\infty} P[S_n \le t] = \sum_{n=1}^{\infty} F^{(n)}(t),$$
(B.23)

where  $F^{(n)}$  denotes the *n*-fold convolution of F with itself, defined by

$$F^{(1)} = F(t),$$
 (B.24)

$$F^{(n)} = F^{(n-1)} * f(t).$$
(B.25)

Let  $\tilde{R}(s)$  denote the Laplace-Stieltjes transform of R(t) and let  $\tilde{F}(s)$  denote the Laplace-Stieltjes transform of F(t). From (B.23), we obtain (cf. [35, Eq. (14.79)])

$$\tilde{R}(s) = \sum_{n=1}^{\infty} \tilde{F}^n(s) = \frac{\tilde{F}(s)}{1 - \tilde{F}(s)}.$$
(B.26)

Returning to the 2-state Markov process, the interrenewal time distribution is the convolution of the distributions of two exponential random variables, one with parameter  $\lambda_0$ and the other with parameter  $\lambda_1$ . Thus, the Laplace-Stieltjes transform of the interrenewal time is given by

$$\tilde{F}(s) = \frac{\lambda_0}{s + \lambda_0} \cdot \frac{\lambda_1}{s + \lambda_1} = \frac{\lambda_0 \lambda_1}{s^2 + s\lambda_0 + s\lambda_1 + \lambda_0 \lambda_1}.$$
(B.27)

Applying (B.27) into (B.26), we obtain

$$\tilde{R}(s) = \frac{\frac{\lambda_0 \lambda_1}{s^2 + s\lambda_0 + s\lambda_1 + \lambda_0 \lambda_1}}{1 - \frac{\lambda_0 \lambda_1}{s^2 + s\lambda_0 + s\lambda_1 + \lambda_0 \lambda_1}} = \frac{\lambda_0 \lambda_1}{s^2 + s\lambda_0 + s\lambda_1}.$$
(B.28)

Using partial fraction expansion:

$$\tilde{R}(s) = \frac{\lambda_0 \lambda_1}{\lambda_0 + \lambda_1} \left( \frac{1}{s} - \frac{1}{s + \lambda_0 + \lambda_1} \right), \tag{B.29}$$

from which we obtain

$$r(t) = \frac{\lambda_0 \lambda_1}{\lambda_0 + \lambda_1} \left( 1 - e^{-(\lambda_0 + \lambda_1)t} \right).$$
(B.30)

Finally, we derive the renewal function:

$$R(t) = \int_0^t r(\tau) d\tau = \left[ \frac{\lambda_0 \lambda_1}{\lambda_0 + \lambda_1} \tau + \frac{\lambda_0 \lambda_1}{(\lambda_0 + \lambda_1)^2} e^{-(\lambda_0 + \lambda_1)\tau} \right]_0^t$$
(B.31)

$$= \frac{\lambda_0 \lambda_1}{\lambda_0 + \lambda_1} t - \frac{\lambda_0 \lambda_1}{(\lambda_0 + \lambda_1)^2} \left( 1 - e^{-(\lambda_0 + \lambda_1)t} \right).$$
(B.32)

Applying (B.32) and (B.20) in (B.11), we have:

$$E(N_t(0,1)) = R(t) + \pi_0 P_{01}(t)$$
$$= \frac{\lambda_0 \lambda_1}{\lambda_0 + \lambda_1} t + \left(\pi_0 - \frac{\lambda_1}{\lambda_0 + \lambda_1}\right) \frac{\lambda_0}{\lambda_0 + \lambda_1} \left(1 - e^{-(\lambda_0 + \lambda_1)t}\right).$$
(B.33)

Similarly, we have:

$$E(N_t(1,0)) = \frac{\lambda_0 \lambda_1}{\lambda_0 + \lambda_1} t + \left(\pi_1 - \frac{\lambda_0}{\lambda_0 + \lambda_1}\right) \frac{\lambda_1}{\lambda_0 + \lambda_1} \left(1 - e^{-(\lambda_0 + \lambda_1)t}\right).$$
(B.34)

As t approaches  $\infty$ , the constant term becomes negligible, and we have the asymptotic approximation:

$$\mathbf{E}\left(N_t(0,1)\right) = \mathbf{E}\left(N_t(1,0)\right) = \frac{\lambda_0\lambda_1}{\lambda_0 + \lambda_1}t = \frac{t}{\frac{1}{\lambda_0} + \frac{1}{\lambda_1}}, \text{ as } t \to \infty.$$
 (B.35)

## B.3.2 Expected Number of Samples in a State

The expected number of samples in state i is related to the expected amount of time in state i and the sampling interval h:

$$\mathbf{E}(n_i) = \frac{1}{h} \mathbf{E}(T_i).$$
(B.36)

The expected amount of time in state i is related to the transition probability:

$$E(T_0) = \pi_0 \int_0^t P_{00}(\tau) d\tau + \pi_1 \int_0^t P_{10}(\tau) d\tau,$$
  

$$E(T_1) = \pi_1 \int_0^t P_{11}(\tau) d\tau + \pi_0 \int_0^t P_{01}(\tau) d\tau.$$
(B.37)

Applying (B.19):

$$E(T_0) = \pi_0 \int_0^t (1 - P_{01}(\tau)) d\tau + \pi_1 \int_0^t P_{10}(\tau) d\tau,$$
  
$$E(T_1) = \pi_1 \int_0^t (1 - P_{10}(\tau)) d\tau + \pi_0 \int_0^t P_{01}(\tau) d\tau.$$
 (B.38)

Applying (B.20), the above integrals are solved as:

$$\int_{0}^{t} P_{01}(\tau) d\tau = \frac{\lambda_0}{\lambda_0 + \lambda_1} \int_{0}^{t} \left( 1 - e^{-(\lambda_0 + \lambda_1)\tau} \right) d\tau,$$
$$= \frac{\lambda_0}{\lambda_0 + \lambda_1} t - \frac{\lambda_0}{(\lambda_0 + \lambda_1)^2} \left( 1 - e^{-(\lambda_0 + \lambda_1)t} \right). \tag{B.39}$$

$$\int_0^t (1 - P_{01}(\tau)) d\tau = \int_0^t \left( \frac{\lambda_0 + \lambda_1}{\lambda_0 + \lambda_1} - \frac{\lambda_0}{\lambda_0 + \lambda_1} + \frac{\lambda_0}{\lambda_0 + \lambda_1} e^{-(\lambda_0 + \lambda_1)\tau} \right) d\tau,$$
$$= \frac{\lambda_1}{\lambda_0 + \lambda_1} t + \frac{\lambda_0}{(\lambda_1 + \lambda_0)^2} \left( 1 - e^{-(\lambda_0 + \lambda_1)t} \right).$$
(B.40)

Similarly,

$$\int_0^t P_{10}(\tau) d\tau = \frac{\lambda_1}{\lambda_0 + \lambda_1} t - \frac{\lambda_1}{(\lambda_0 + \lambda_1)^2} \left( 1 - e^{-(\lambda_0 + \lambda_1)t} \right), \tag{B.41}$$

$$\int_0^t (1 - P_{10}(\tau))(\tau)d\tau = \frac{\lambda_0}{\lambda_0 + \lambda_1}t + \frac{\lambda_1}{(\lambda_0 + \lambda_1)^2} \left(1 - e^{-(\lambda_0 + \lambda_1)t}\right).$$
(B.42)

Applying (B.40) and (B.41) into (B.38),

$$\mathbf{E}(T_0) = \frac{\lambda_1}{\lambda_0 + \lambda_1} t + \frac{\pi_0 \lambda_0 - \pi_1 \lambda_1}{(\lambda_0 + \lambda_1)^2} \left(1 - e^{-(\lambda_0 + \lambda_1)t}\right).$$
(B.43)

Similarly,

$$\mathbf{E}(T_1) = \frac{\lambda_0}{\lambda_0 + \lambda_1} t + \frac{\pi_1 \lambda_1 - \pi_0 \lambda_0}{(\lambda_0 + \lambda_1)^2} \left(1 - e^{-(\lambda_0 + \lambda_1)t}\right).$$
(B.44)

Applying (B.43) and (B.44) into (B.36):

$$\mathbf{E}(n_0) = \frac{\lambda_1}{h(\lambda_0 + \lambda_1)} t + \frac{\pi_0 \lambda_0 - \pi_1 \lambda_1}{h(\lambda_0 + \lambda_1)^2} \left(1 - e^{-(\lambda_0 + \lambda_1)t}\right),\tag{B.45}$$

$$\mathbf{E}(n_1) = \frac{\lambda_0}{h(\lambda_0 + \lambda_1)} t + \frac{\pi_1 \lambda_1 - \pi_0 \lambda_0}{h(\lambda_0 + \lambda_0)^2} \left(1 - e^{-(\lambda_0 + \lambda_1)t}\right).$$
(B.46)

As t approaches  $\infty$ ,

$$\mathbf{E}(n_0) = \frac{\lambda_1}{\lambda_0 + \lambda_1} \frac{t}{h}, \text{ as } t \to \infty,$$
(B.47)

$$E(n_1) = \frac{\lambda_0}{\lambda_0 + \lambda_1} \frac{t}{h}, \text{ as } t \to \infty.$$
(B.48)

# **B.4** Score Function

The score function is defined as:

$$\chi = \nabla \log \mathcal{L}^c. \tag{B.49}$$

In the case of the two-state MMGP, the score function is given:

$$\chi = \left[\frac{\partial}{\partial\lambda_0}, \frac{\partial}{\partial\lambda_1}, \frac{\partial}{\partial\mu_0}, \frac{\partial}{\partial\mu_1}, \frac{\partial}{\partial v_0}, \frac{\partial}{\partial v_1}\right] \log \mathcal{L}^c, \tag{B.50}$$

where  $v_i = \sigma_i^2$  is used to denote the variance, in the interest of simpler notation.

#### B.4.1 Score for Transition Rates

The score for  $\lambda_0$  is given:

$$\frac{\partial}{\partial \lambda_0} \log \mathcal{L}^c = -T_0 + \frac{m_{01}}{\lambda_0}.$$
 (B.51)

Similarly, the score for  $\lambda_1$  is given:

$$\frac{\partial}{\partial \lambda_1} \log \mathcal{L}^c = -T_1 + \frac{m_{10}}{\lambda_1}.$$
(B.52)

#### B.4.2 Score for Gaussian Process Means

The score for  $\mu_0$  is given:

$$\frac{\partial}{\partial\mu_0}\log\mathcal{L}^c = \frac{\partial}{\partial\mu_0} \left[ -\frac{1}{2} \sum_{k=1}^n \xi_k(0) \frac{(y_k - \mu_0)^2}{v_0} \right],$$
$$= \sum_{k=1}^n \xi_k(0) \frac{y_k - \mu_0}{v_0}.$$
(B.53)

Similarly, the score for  $\mu_1$  is given:

$$\frac{\partial}{\partial \mu_1} \log \mathcal{L}^c = \sum_{k=1}^n \xi_k(1) \frac{y_k - \mu_1}{v_1}.$$
 (B.54)

#### B.4.3 Score for Gaussian Process Variances

The score for  $v_0$  is given:

$$\frac{\partial}{\partial v_0} \log \mathcal{L}^c = \frac{\partial}{\partial v_0} \left[ -\frac{1}{2} n_0 \log v_0 - \frac{1}{2} \sum_{k=1}^n \xi_k(0) \frac{(y_k - \mu_0)^2}{v_0} \right],$$
$$= -\frac{n_0}{2v_0} + \frac{1}{2v_0^2} \sum_{k=1}^n \xi_k(0) \left(y_k - \mu_0\right)^2. \tag{B.55}$$

Similarly, the score for  $v_1$  is given:

$$\frac{\partial}{\partial v_1} \log \mathcal{L}^c = -\frac{n_1}{2v_1} + \frac{1}{2v_1^2} \sum_{k=1}^n \xi_k(1) \left(y_k - \mu_1\right)^2.$$
(B.56)

## **B.5** Fisher Information Matrix

To compute the Fisher Information Matrix (FIM), we must first compute the Hessian of the log likelihood function. From the score functions derived in the previous sections, it is apparent that many of the parameters are orthogonal, corresponding to zero elements in the FIM. The Hessian may be substantially simplified:

$$\nabla^2 \log \mathcal{L}^c =$$

$$\begin{bmatrix} \frac{\partial^2}{\partial \lambda_0^2} & 0 & 0 & 0 & 0 & 0\\ 0 & \frac{\partial^2}{\partial \lambda_1^2} & 0 & 0 & 0 & 0\\ 0 & 0 & \frac{\partial^2}{\partial \mu_0^2} & 0 & \frac{\partial^2}{\partial \mu_0 \partial v_0} & 0\\ 0 & 0 & 0 & \frac{\partial^2}{\partial \mu_1^2} & 0 & \frac{\partial^2}{\partial \mu_1 \partial v_1}\\ 0 & 0 & \frac{\partial^2}{\partial v_0 \partial \mu_0} & 0 & \frac{\partial^2}{\partial v_0^2} & 0\\ 0 & 0 & 0 & \frac{\partial^2}{\partial v_1 \partial \mu_1} & 0 & \frac{\partial^2}{\partial v_1^2} \end{bmatrix} \log \mathcal{L}^c$$
(B.57)

The individual partial derivatives are given:

$$\frac{\partial^2}{\partial \lambda_0^2} = -\frac{m_{01}}{\lambda_0^2} \tag{B.58}$$

$$\frac{\partial^2}{\partial\lambda_1^2} = -\frac{m_{10}}{\lambda_1^2} \tag{B.59}$$

$$\frac{\partial^2}{\partial \mu_0^2} = \sum_{k=1}^n \xi_k(0) \frac{-1}{v_0}$$
(B.60)

$$\frac{\partial^2}{\partial \mu_1^2} = \sum_{k=1}^n \xi_k(1) \frac{-1}{v_1}$$
(B.61)

$$\frac{\partial^2}{\partial v_0^2} = \frac{n_0}{2v_0^2} - \frac{1}{v_0^3} \sum_{k=1}^n \xi_k(0) \left(y_k - \mu_0\right)^2 \tag{B.62}$$

$$\frac{\partial^2}{\partial v_1^2} = \frac{n_1}{2v_1^2} - \frac{1}{v_1^3} \sum_{k=1}^n \xi_k(1) \left(y_k - \mu_1\right)^2 \tag{B.63}$$

$$\frac{\partial^2}{\partial v_0 \mu_0} = \frac{\partial^2}{\partial \mu_0 v_0} = -\frac{1}{v_0^2} \sum_{k=1}^n \xi_k(0) \left(y_k - \mu_0\right)$$
(B.64)

$$\frac{\partial^2}{\partial v_1 \mu_1} = \frac{\partial^2}{\partial \mu_1 v_1} = -\frac{1}{v_1^2} \sum_{k=1}^n \xi_k(1) \left( y_k - \mu_1 \right)$$
(B.65)

The FIM is defined as:

$$I(\mathcal{L}^c) = \mathcal{E}\left(-\nabla^2 \log \mathcal{L}^c\right) \tag{B.66}$$

The individual components of the FIM are given:

$$\mathbf{E}\left(-\frac{\partial^2}{\partial\lambda_0^2}\right) = \mathbf{E}\left(\frac{m_{01}}{\lambda_0^2}\right) = \frac{1}{\lambda_0^2}\mathbf{E}\left(m_{01}\right) \tag{B.67}$$

$$\mathbf{E}\left(-\frac{\partial^2}{\partial\lambda_1^2}\right) = \mathbf{E}\left(\frac{m_{10}}{\lambda_1^2}\right) = \frac{1}{\lambda_1^2}\mathbf{E}\left(m_{10}\right) \tag{B.68}$$

$$E\left(-\frac{\partial^2}{\partial\mu_0^2}\right) = E\left(\sum_{k=1}^n \xi_k(0)\frac{1}{v_0}\right) = \frac{1}{v_0}\sum_{k=1}^n E\left(\xi_k(0)\right) = \frac{1}{v_0}E\left(n_0\right)$$
(B.69)

$$E\left(-\frac{\partial^2}{\partial\mu_1^2}\right) = E\left(\sum_{k=1}^n \xi_k(1)\frac{1}{v_1}\right) = \frac{1}{v_1}\sum_{k=1}^n E\left(\xi_k(1)\right) = \frac{1}{v_1}E\left(n_1\right)$$
(B.70)  
$$E\left(-\frac{\partial^2}{\partial v_0^2}\right) = E\left(-\frac{n_0}{2v_0^2} + \frac{1}{v_0^3}\sum_{k=1}^n \xi_k(0)\left(y_k - \mu_0\right)^2\right)$$
$$= -\frac{1}{2v_0^2}E\left(n_0\right) + \frac{1}{v_0^3}\sum_{k=1}^n E\left(\xi_k(0)\left(y_k - \mu_0\right)^2\right)$$
$$= -\frac{1}{2v_0^2}E\left(n_0\right) + \frac{1}{v_0^2}E\left(n_0\right) = \frac{1}{2v_0^2}E\left(n_0\right)$$
(B.71)

Similarly,

$$E\left(-\frac{\partial^{2}}{\partial v_{1}^{2}}\right) = E\left(-\frac{n_{1}}{2v_{1}^{2}} + \frac{1}{v_{1}^{3}}\sum_{k=1}^{n}\xi_{k}(1)(y_{k} - \mu_{1})^{2}\right)$$

$$= \frac{1}{2v_{1}^{2}}E(n_{1}) \qquad (B.72)$$

$$E\left(-\frac{\partial^{2}}{\partial v_{0}\mu_{0}}\right) = E\left(-\frac{\partial^{2}}{\partial \mu_{0}v_{0}}\right) = E\left(\frac{1}{v_{0}^{2}}\sum_{k=1}^{n}\xi_{k}(0)(y_{k} - \mu_{0})\right)$$

$$= \frac{1}{v_{0}^{2}}\sum_{k=1}^{n}E\left((y_{k} - \mu_{0})I(X(t_{k}) = 0)\right)$$

$$= \frac{1}{v_{0}^{2}}\sum_{k=1}^{n}P\left(X(t_{k}) = 1\right)E\left((y_{k} - \mu_{0})|X(t_{k}) = 0\right) = 0 \qquad (B.73)$$

Similarly,

$$\mathbf{E}\left(-\frac{\partial^2}{\partial v_1\mu_1}\right) = \mathbf{E}\left(-\frac{\partial^2}{\partial \mu_1 v_1}\right) = \mathbf{E}\left(\frac{1}{v_1^2}\sum_{k=2}^n \xi_k(1)\left(y_k - \mu_1\right)\right) = 0 \tag{B.74}$$

The FIM is given:

$$I(\mathcal{L}^{c}) = \operatorname{diag}\left\{\frac{\mathrm{E}(m_{01})}{\lambda_{0}^{2}}, \frac{\mathrm{E}(m_{10})}{\lambda_{1}^{2}}, \frac{\mathrm{E}(n_{0})}{\sigma_{0}^{2}}, \frac{\mathrm{E}(n_{1})}{\sigma_{1}^{2}}, \frac{\mathrm{E}(n_{0})}{2\sigma_{0}^{4}}, \frac{\mathrm{E}(n_{1})}{2\sigma_{1}^{4}}\right\}$$
(B.75)

Because the FIM is a diagonal matrix, inversion is trivial. The inverse FIM is given:

$$I^{-1}(\mathcal{L}^{c}) = \operatorname{diag}\left\{\frac{\lambda_{0}^{2}}{\mathrm{E}(m_{01})}, \frac{\lambda_{1}^{2}}{\mathrm{E}(m_{10})}, \frac{\sigma_{0}^{2}}{\mathrm{E}(n_{0})}, \frac{\sigma_{1}^{2}}{\mathrm{E}(n_{1})}, \frac{2\sigma_{0}^{4}}{\mathrm{E}(n_{0})}, \frac{2\sigma_{1}^{4}}{\mathrm{E}(n_{1})}\right\}$$
(B.76)

# B.6 Asymptotic Approximation

As t increases, the FIM is simplified:

$$I(\mathcal{L}^c) = \frac{t}{\lambda_0 + \lambda_1} \operatorname{diag}\left\{\frac{\lambda_1}{\lambda_0}, \frac{\lambda_0}{\lambda_1}, \frac{\lambda_1}{h\sigma_0^2}, \frac{\lambda_0}{h\sigma_1^2}, \frac{\lambda_1}{2h\sigma_0^4}, \frac{\lambda_0}{2h\sigma_1^4}\right\}, \text{ as } t \to \infty.$$
(B.77)

$$I^{-1}\left(\mathcal{L}^{c}\right) = \frac{\lambda_{0} + \lambda_{1}}{t} \operatorname{diag}\left\{\frac{\lambda_{0}}{\lambda_{1}}, \frac{\lambda_{1}}{\lambda_{0}}, \frac{h\sigma_{0}^{2}}{\lambda_{1}}, \frac{h\sigma_{1}^{2}}{\lambda_{0}}, \frac{2h\sigma_{0}^{4}}{\lambda_{1}}, \frac{2h\sigma_{1}^{4}}{\lambda_{0}}\right\}, \text{ as } t \to \infty.$$
(B.78)

# Appendix C: Derivation of the Fisher Information Matrix for a CTMC

In this appendix, we derive the Fisher information matrix for a continuous-time Markov chain (CTMC) in terms of the mean sojourn times and transition probabilities. A closedform expression for the 2-state case is given.

## C.1 Definitions

For a continuous-time finite-state homogeneous Markov chain, we define the generator matrix as  $\mathbf{Q}$ , where  $q_{ij}$  is the transition rate from state i to state j ( $i \neq j$ ), and the diagonal terms  $q_i$  are defined as  $q_i = \sum_{i \neq j} q_{ij}$ . We define the state at time t as X(t). When observing a continuous-time Markov chain (CTMC) over the closed time interval  $[0 \leq t \leq T]$ , we denote the time spent in state i as  $T_i$ . We define the initial distribution  $\boldsymbol{\pi}$  as  $\pi_i = P(X(0) = i)$ .

## C.2 Log Likelihood

The log likelihood function for a CTMC is given in [4, Sec. 4] as

$$\log \mathcal{L}^{c} = \sum_{i=1}^{r} \mathbb{1}_{\{X(0)=i\}} \log \pi_{i} - \sum_{i=1}^{r} T_{i}q_{i} + \sum_{i \neq j} m_{ij} \log q_{ij},$$
(C.1)

where  $1_{\{\cdot\}}$  is the indicator function, X(0) denotes the initial state of the underlying Markov process,

$$T_i = \int_0^T \mathbf{1}_{\{X(t)=i\}} \,\mathrm{d}t \tag{C.2}$$

denotes the total time that the underlying Markov process spent in state i during the interval [0, T], and

$$m_{ij} = \sum_{k=1}^{m} \mathbb{1}_{\{s_k = i, s_{k+1} = j\}}$$
(C.3)

denotes the number of jumps from state i to state j, where  $i \neq j$ , the state sequence over the interval [0, T] has m jumps, and  $s_k$  is the  $k^{th}$  state in the sequence.

# C.3 Log Likelihood In Terms of Mean Dwell Time and Transition Probability

The log likelihood function may be expressed in terms of the mean dwell times  $\mu_i$  and the transition probabilities,  $p_{ij}$ , given by

$$\mu_i = \frac{1}{q_i},\tag{C.4}$$

$$p_{ij} = \begin{cases} 0, & i = j \\ \frac{q_{ij}}{q_i}, & i \neq j \end{cases}$$
(C.5)

Applying Eqs. (C.4) and (C.5) into Eq. (C.1), we get a new log likelihood function in terms of the mean dwell times and transition probabilities

$$\log \mathcal{L}^{c} = \sum_{i=1}^{r} \mathbb{1}_{\{X(0)=i\}} \log \pi_{i} - \sum_{i=1}^{r} \frac{T_{i}}{\mu_{i}} + \sum_{i \neq j} m_{ij} \log \left(\frac{p_{ij}}{\mu_{i}}\right)$$
$$= \sum_{i=1}^{r} \mathbb{1}_{\{X(0)=i\}} \log \pi_{i} - \sum_{i=1}^{r} \frac{T_{i}}{\mu_{i}} + \sum_{i \neq j} m_{ij} \log p_{ij} - \sum_{i \neq j} m_{ij} \log \mu_{i}.$$
(C.6)

# C.4 Score Function

The score function for a random process is defined as:

$$\chi = \nabla \log \mathcal{L}^c. \tag{C.7}$$

The score functions with respect to the mean dwell times and transition probabilities are given

$$\frac{\partial}{\partial \mu_i} \log \mathcal{L}^c = \frac{T_i}{\mu_i^2} - \sum_{i \neq j} \frac{m_{ij}}{\mu_i}$$
(C.8)

$$\frac{\partial}{\partial p_{ij}} \log \mathcal{L}^c = \frac{m_{ij}}{p_{ij}} , \quad i \neq j .$$
 (C.9)

# C.5 Fisher Information Matrix

To compute the Fisher Information Matrix (FIM), we must first compute the Hessian of the log likelihood function. It is apparent from Eqs. (C.8) and (C.9) that the off-diagonal elements of the Hessian are zero. The diagonal elements of the Hessian are given

$$\frac{\partial^2}{\partial \mu_i^2} \log \mathcal{L}^c = \sum_{i \neq j} \frac{m_{ij}}{\mu_i^2} - \frac{2T_i}{\mu_i^3} , \qquad (C.10)$$

$$\frac{\partial^2}{\partial p_{ij}^2} \log \mathcal{L}^c = -\frac{m_{ij}}{p_{ij}^2} , \quad i \neq j .$$
 (C.11)

The FIM is defined as the expected value of the negative Hessian matrix of the log likelihood function. For a CTMC with two states, the FIM is given as

$$\mathcal{I}(\mathcal{L}^c) = \mathcal{E}\left(-\nabla^2 \log \mathcal{L}^c\right). \tag{C.12}$$

The diagonal terms of the FIM are given

$$\operatorname{E}\left(-\frac{\partial^2}{\partial\mu_i^2}\log\mathcal{L}^c\right) = \frac{2\operatorname{E}\left(T_i\right)}{\mu_i^3} - \sum_{i\neq j}\frac{\operatorname{E}\left(m_{ij}\right)}{\mu_i^2} , \qquad (C.13)$$

$$\operatorname{E}\left(-\frac{\partial^2}{\partial p_{ij}^2}\log\mathcal{L}^c\right) = \frac{\operatorname{E}(m_{ij})}{p_{ij}^2} , \quad i \neq j .$$
(C.14)

In the special case of a 2-state CTMC, the transition probabilities are known, and the transition rates or the mean dwell times are a sufficient statistic. The resulting FIM for a 2-state CTMC is given

$$\mathcal{I}(\mathcal{L}^{c}) = \mathbf{E}\left(-\nabla^{2}\log\mathcal{L}^{c}\right)$$
$$= \begin{bmatrix} \frac{2\mathbf{E}(T_{1})}{\mu_{1}^{3}} - \frac{\mathbf{E}(m_{12})}{\mu_{1}^{2}} & 0\\ 0 & \frac{2\mathbf{E}(T_{2})}{\mu_{2}^{3}} - \frac{\mathbf{E}(m_{21})}{\mu_{2}^{2}} \end{bmatrix}.$$
(C.15)

## C.6 Expected Number of Jumps Between States

The expected number of jumps from state j to k for a 2-state CTMC is given in B as

$$E\left(N_t(j,k)\right) = \frac{\lambda_j \lambda_k}{\lambda_j + \lambda_k} t + \left(\pi_j - \frac{\lambda_k}{\lambda_j + \lambda_k}\right) \frac{\lambda_j}{\lambda_j + \lambda_k} \left(1 - e^{-(\lambda_j + \lambda_k)t}\right).$$
(C.16)

In terms of the mean dwell times in states j and k, the expected number of jumps is expressed as

$$E(N_t(j,k)) = \frac{1/(\mu_j\mu_k)}{1/\mu_j + 1/\mu_k} t + \left(\pi_j - \frac{1/\mu_k}{1/\mu_j + 1/\mu_k}\right) \frac{1/\mu_j}{1/\mu_j + 1/\mu_k} \left(1 - e^{-(1/\mu_j + 1/\mu_k)t}\right)$$
$$= \frac{1}{\mu_j + \mu_k} t + \left(\pi_j - \frac{\mu_j}{\mu_j + \mu_k}\right) \frac{\mu_k}{\mu_j + \mu_k} \left(1 - e^{-\left(\frac{\mu_j + \mu_k}{\mu_j + \mu_k}\right)t}\right).$$
(C.17)

# C.7 Expected Time in States

The expected time spent in state j for a 2-state CTMC, where state k = -j + 1, is given in B as

$$E(T_t(j)) = \frac{\lambda_k}{(\lambda_j + \lambda_k)} t + \frac{\pi_j \lambda_j - \pi_k \lambda_k}{(\lambda_j + \lambda_k)^2} \left(1 - e^{-(\lambda_j + \lambda_k)t}\right).$$
(C.18)

In terms of the mean dwell times in states j and k, the expected time in state j is expressed as

$$E(T_t(j)) = \frac{1/\mu_k}{(1/\mu_j + 1/\mu_k)} t + \frac{\pi_j/\mu_j - \pi_k/\mu_k}{(1/\mu_j + 1/\mu_k)^2} \left(1 - e^{-(1/\mu_j + 1/\mu_k)t}\right)$$
$$= \frac{\mu_j}{\mu_j + \mu_k} t + \frac{\mu_j\mu_k(\pi_j\mu_k - \pi_k\mu_j)}{(\mu_j + \mu_k)^2} \left(1 - e^{-\left(\frac{\mu_j + \mu_k}{\mu_j\mu_k}\right)t}\right).$$
(C.19)

# C.8 Asymptotic Approximations

The expressions for the expected number of jumps and the expected time in a state may be simplified in the asymptotic regime. Approximate values from Eqs. (C.17) and (C.19) are given

$$\mathbf{E}\left(N_t(j,k)\right) = \frac{1}{\mu_j + \mu_k}t , \quad t \to \infty, \tag{C.20}$$

$$\mathcal{E}\left(T_t(j)\right) = \frac{\mu_j}{\mu_j + \mu_k} t , \quad t \to \infty.$$
(C.21)

Applying Eqs. (C.20) and (C.21) to Eq. (C.15), we derive the Asymptotic FIM

$$\mathcal{I}(\mathcal{L}^{c}) = \frac{t}{\mu_{0} + \mu_{1}} \begin{bmatrix} \frac{1}{\mu_{0}^{2}} & 0\\ 0 & \frac{1}{\mu_{1}^{2}} \end{bmatrix}, \quad t \to \infty.$$
(C.22)

The resulting Cramér Rao bound for a single channel is therefore

$$\sigma^{2}(t) \ge \frac{\mu_{0} + \mu_{1}}{t} \left(\mu_{0}^{2} + \mu_{1}^{2}\right) , \quad t \to \infty,$$
(C.23)

and the multi-channel CRB is

$$\sigma^{2}(t) \geq \sum_{i=1}^{M} \frac{\mu_{0,i} + \mu_{1,i}}{t} \left( \mu_{0,i}^{2} + \mu_{1,i}^{2} \right) , \quad t \to \infty,$$
(C.24)

## C.9 Asymptotic MMSE Allocations

In [55], per-channel sensing allocations were derived such that the multi-channel CRB is minimized. Because our multi-channel CRB in Eq. (C.24) is different than that in [55, Eq. 14], we must derive new sensing durations which minimize our new CRB. We use Lagrange multipliers to derive the optimal time durations which minimize Eq. (C.24) under the constraint

$$\sum_{i=1}^{M} T_i = T. \tag{C.25}$$

Define the vector  $\mathbf{t} = [T_1, \dots, T_M]$  as the vector of time allocations for all M channels. We define our optimization functions as

$$f(\boldsymbol{t}) = \sum_{i=1}^{M} \frac{\beta_i}{T_i} ,$$

$$g(\boldsymbol{t}) = T - \sum_{i=1}^{M} T_i , \qquad (C.26)$$

where  $\beta_i = (\mu_{0,i} + \mu_{1,i}) \left( \mu_{0,i}^2 + \mu_{1,i}^2 \right)$ . We define our auxiliary function as

$$\mathcal{L}(\boldsymbol{t},\lambda) = f(\boldsymbol{t}) + \lambda g(\boldsymbol{t}) . \qquad (C.27)$$

We compute the gradient with respect to  $\boldsymbol{t}$ 

$$\nabla \mathcal{L}(\boldsymbol{t}, \lambda) = 0 ,$$
  

$$\nabla f(\boldsymbol{t}) + \nabla \lambda g(\boldsymbol{t}) = 0 ,$$
  

$$\frac{-\boldsymbol{\beta}}{\boldsymbol{t}^2} - \lambda \mathbf{1} = 0 ,$$
(C.28)

where  $\boldsymbol{\beta} = [\beta_1, \dots, \beta_M]$ ,  $t^2$  is the component-wise square of the vector t, and  $\mathbf{1}$  is a row vector where every element is a 1. Solving for t, we get

$$\boldsymbol{t} = \sqrt{\frac{-\boldsymbol{\beta}}{\lambda}} , \qquad (C.29)$$

where  $\sqrt{\mathbf{x}}$  is the element-wise square root of a vector  $\mathbf{x}$ . Applying the solution Eq. (C.29) to the constraint Eq. (C.25), we can solve for  $\lambda$ .

$$\sum_{i=1}^{M} \sqrt{\frac{-\beta_i}{\lambda}} = T .$$

$$\sqrt{\frac{-1}{\lambda}} \sum_{i=1}^{M} \sqrt{\beta_i} = T .$$

$$\frac{-1}{\lambda} \left( \sum_{i=1}^{M} \sqrt{\beta_i} \right)^2 = T^2 .$$

$$\frac{-\left( \sum_{i=1}^{M} \sqrt{\beta_i} \right)^2}{T^2} = \lambda . \qquad (C.30)$$

Applying Eq. (C.30) into Eq. (C.29), we solve for the optimal time durations

$$t = \sqrt{\frac{-\beta}{\frac{-(\sum_{i=1}^{M}\sqrt{\beta_{i}})^{2}}{T^{2}}}},$$

$$= T \frac{\sqrt{\beta}}{\sum_{i=1}^{M}\sqrt{\beta_{i}}},$$

$$T_{i} = T \frac{\sqrt{\beta_{i}}}{\sum_{i=1}^{M}\sqrt{\beta_{i}}},$$

$$= T \frac{\sqrt{(\mu_{0,i} + \mu_{1,i})(\mu_{0,i}^{2} + \mu_{1,i}^{2})}}{\sum_{i=1}^{M}\sqrt{(\mu_{0,i} + \mu_{1,i})(\mu_{0,i}^{2} + \mu_{1,i}^{2})}}$$
(C.31)

## Bibliography

- "IEEE draft standard for information technology telecommunications and information exchange between systems - local and metropolitan area networks - specific requirements - part 11: Wireless LAN medium access control (MAC) and physical layer (PHY) specifications amendment 5: TV white spaces operation," *IEEE P802.11af/D4.0, April* 2013, pp. 1–326, Oct 2013.
- [2] "ISO/IEC/IEEE international standard information technology telecommunications and information exchange between systems – local and metropolitan area networks– specific requirements – part 22: Cognitive wireless RAN medium access control (MAC) and physical layer (PHY) specifications: Policies and procedures for operation in the TV band," ISO/IEC/IEEE 8802-22:2015, pp. 1–678, May 2015.
- [3] I. Akbar and W. Tranter, "Dynamic spectrum allocation in cognitive radio using hidden Markov models: Poisson distributed case," in *SoutheastCon*, 2007. Proceedings. IEEE, March 2007, pp. 196–201.
- [4] A. Albert, "Estimating the infinitesimal generator of a continuous time, finite state Markov process," Annals of Mathematical Statistics, vol. 23, no. 2, pp. 727–753, Jun. 1962.
- [5] L. Angrisani, G. Betta, D. Capriglione, G. Cerro, L. Ferrigno, and G. Miele, "Proposal and analysis of new algorithms for wideband spectrum sensing in cognitive radio," in 2014 IEEE International Instrumentation and Measurement Technology Conference (I2MTC) Proceedings, May 2014, pp. 701–706.
- [6] S. Bagheri and A. Scaglione, "The restless multi-armed bandit formulation of the cognitive compressive sensing problem," *IEEE Transactions on Signal Processing*, vol. 63, no. 5, pp. 1183–1198, March 2015.
- [7] D. Bao, L. D. Vito, and S. Rapuano, "A histogram-based segmentation method for wideband spectrum sensing in cognitive radios," *IEEE Transactions on Instrumenta*tion and Measurement, vol. 62, no. 7, pp. 1900–1908, July 2013.
- [8] M. S. Bartlett, "Smoothing periodograms from time series with continuous spectra," *Nature*, vol. 161, no. 4096, pp. 686–687, 1948.
- [9] L. E. Baum and T. Petrie, "Statistical inference for probabilistic functions of finite state Markov chains," Annals of Mathematical Statistics, vol. 37, no. 6, pp. 1554–1563, Apr. 1966.

- [10] J. M. Bruno, B. L. Mark, Y. Ephraim, and C. Chen, "A computing budget allocation approach to multiband spectrum sensing," in Wireless Communications and Networking Conference (WCNC), 2017 IEEE, March 2017, pp. 1–6.
- [11] J. M. Bruno, B. L. Mark, and Z. Tian, "An edge detection approach to wideband temporal spectrum sensing," in 2016 IEEE Global Communications Conference (GLOBE-COM), Dec 2016, pp. 1–6.
- [12] J. Bruno and B. Mark, "A recursive algorithm for joint time-frequency wideband spectrum sensing," in Wireless Communications and Networking Conference Workshops (WCNCW), 2015 IEEE, March 2015, pp. 235–240.
- [13] D. Cabric, S. Mishra, and R. Brodersen, "Implementation issues in spectrum sensing for cognitive radios," in Signals, Systems and Computers, 2004. Conference Record of the Thirty-Eighth Asilomar Conference on, vol. 1, Nov 2004, pp. 772–776 Vol.1.
- [14] J. Canny, "A computational approach to edge detection," Pattern Analysis and Machine Intelligence, IEEE Transactions on, vol. PAMI-8, no. 6, pp. 679–698, Nov 1986.
- [15] E. Çinlar, "Markov Renewal Theory: A Survey," Management Science, vol. 21, no. 7, pp. 727–752, Dec. 1975.
- [16] C.-H. Chen, Stochastic simulation optimization: an optimal computing budget allocation. World scientific, 2010, vol. 1.
- [17] C.-H. Chen, D. He, M. Fu, and L. H. Lee, "Efficient simulation budget allocation for selecting an optimal subset," *INFORMS Journal on Computing*, vol. 20, no. 4, pp. 579–595, 2008.
- [18] C.-H. Chen, J. Lin, E. Yücesan, and S. E. Chick, "Simulation budget allocation for further enhancing the efficiency of ordinal optimization," *Discrete Event Dynamic Systems*, vol. 10, no. 3, pp. 251–270, 2000.
- [19] G. B. Coleman and H. C. Andrews, "Image segmentation by clustering," Proceedings of IEEE, vol. 67, no. 5, pp. 773–785, May 1969.
- [20] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. Wiley-Interscience, 2006.
- [21] Y. Ephraim and N. Merhav, "Hidden Markov processes," *IEEE Transactions on In*formation Theory, vol. 48, no. 6, pp. 1518–1569, June 2002.
- [22] Y. Ephraim and B. L. Mark, "Bivariate Markov processes and their estimation," Foundations and Trends in Signal Processing, vol. 6, no. 1, pp. 1–95, 2012.
- [23] T. Erpek, A. Leu, and B. Mark, "Spectrum sensing performance in TV bands using the multitaper method," in *Signal Processing and Communications Applications*, 2007. *SIU 2007. IEEE 15th*, June 2007, pp. 1–4.
- [24] FCC, "Spectrum policy task force," Rep. ET Docket, Federal Communications Commission, Tech. Rep. 02-135, Nov. 2002.

- [25] Y. Feng and X. Wang, "Adaptive multiband spectrum sensing," Wireless Communications Letters, IEEE, vol. 1, no. 2, pp. 121–124, April 2012.
- [26] W. A. Gardner, "Exploitation of spectral redundancy in cyclostationary signals," Signal Processing Magazine, IEEE, vol. 8, no. 2, pp. 14–36, April 1991.
- [27] F. Harris and C. Dick, "Performing simultaneous arbitrary spectral translation, and sample rate change in polyphase interpolating or decimating filters in transmitters and receivers," in *Software defined radio conference*, 2002, San Diego, California, 2002, pp. 11–12.
- [28] F. J. Harris, Multirate Signal Processing for Communication Systems. Upper Saddle River, NJ, USA: Prentice Hall PTR, 2004.
- [29] M. H. Hayes, Statistical Digital Signal Processing and Modeling, 1st ed. New York, NY, USA: John Wiley & Sons, Inc., 1996.
- [30] P. G. Hoel, S. C. Port, and C. J. Stone, *Introduction to stochastic processes*. Waveland Press, 1986.
- [31] T. Kailath, "The divergence and Bhattacharyya distance measures in signal selection," *IEEE Transactions on Communication Technology*, vol. 15, no. 1, pp. 52–60, 1967.
- [32] S. M. Kay, Fundamentals of Statistical Signal Processing: Estimation Theory. Upper Saddle River, NJ, USA: Prentice-Hall, Inc., 1993.
- [33] S.-J. Kim and G. Giannakis, "Sequential and cooperative sensing for multi-channel cognitive radios," *Signal Processing*, *IEEE Transactions on*, vol. 58, no. 8, pp. 4239– 4253, Aug 2010.
- [34] D. Knuth, The Art of Computer Programming, Vol. 1: Fundamental Algorithms, 3rd ed. Addison-Wesley, 1997.
- [35] H. Kobayashi, B. L. Mark, and W. Turin, Probability, Random Processes, and Statistical Analysis. Cambridge University Press, 2012.
- [36] W.-Y. Lee and I. Akyildiz, "Optimal spectrum sensing framework for cognitive radio networks," Wireless Communications, IEEE Transactions on, vol. 7, no. 10, pp. 3845– 3857, October 2008.
- [37] A. E. Leu, M. McHenry, and B. L. Mark, "Modeling and analysis of interference in listen-before-talk spectrum access schemes," *International Journal of Network Man*agement, vol. 16, no. 2, pp. 131–147, 2006.
- [38] R. Lopez-Valcarce and G. Vazquez-Vilar, "Wideband spectrum sensing in cognitive radio: Joint estimation of noise variance and multiple signal levels," in 2009 IEEE 10th Workshop on Signal Processing Advances in Wireless Communications, June 2009, pp. 96–100.
- [39] T. C. Lystig and J. P. Hughes, "Exact computation of the observed information matrix for hidden Markov models," *Journal of Computational and Graphical Statistics*, vol. 11, no. 3, pp. 678–689, 2002.

- [40] S. Mallat, A wavelet tour of signal processing: the sparse way, 3rd ed. Academic Press, 2009.
- [41] A. Mariani, A. Giorgetti, and M. Chiani, "Wideband spectrum sensing by model order selection," *IEEE Transactions on Wireless Communications*, vol. 14, no. 12, pp. 6710– 6721, Dec 2015.
- [42] A. Mariani, S. Kandeepan, and A. Giorgetti, "Periodic spectrum sensing with noncontinuous primary user transmissions," *IEEE Transactions on Wireless Communications*, vol. 14, no. 3, pp. 1636–1649, March 2015.
- [43] T. Nguyen, B. L. Mark, and Y. Ephraim, "Spectrum sensing using a hidden bivariate Markov model," *IEEE Transactions on Wireless Communications*, vol. 12, no. 9, pp. 4582–4591, Aug. 2013.
- [44] J. Nocedal and S. Wright, Numerical optimization. Springer Science & Business Media, 2006.
- [45] J. Oksanen, V. Koivunen, and H. V. Poor, "A sensing policy based on confidence bounds and a restless multi-armed bandit model," in 2012 Conference Record of the Forty Sixth Asilomar Conference on Signals, Systems and Computers (ASILOMAR), Nov 2012, pp. 318–323.
- [46] O. Olabiyi and A. Annamalai, "Extending the capability of energy detector for sensing of heterogeneous wideband spectrum," in *IEEE Consumer Comm. and Net. Conf.* (CCNC), Jan 2012, pp. 454–458.
- [47] W. J. Roberts and Y. Ephraim, "An EM algorithm for ion-channel current estimation," Signal Processing, IEEE Transactions on, vol. 56, no. 1, pp. 26–33, 2008.
- [48] W. J. Roberts, Y. Ephraim, and E. Dieguez, "On rydén's em algorithm for estimating mmpps," *IEEE Signal Processing Letters*, vol. 13, no. 6, p. 373, 2006.
- [49] T. Rydén, "On recursive estimation for hidden Markov models," Stochastic Processes and their Applications, vol. 66, no. 1, pp. 79–96, 1997.
- [50] B. M. Sadler and A. Swami, "Analysis of multiscale products for step detection and estimation," *Information Theory, IEEE Transactions on*, vol. 45, no. 3, pp. 1043–1051, 1999.
- [51] M. Sanna and M. Murroni, "Opportunistic wideband spectrum sensing for cognitive radios with genetic optimization," in *Communications (ICC)*, 2010 IEEE International Conference on, May 2010, pp. 1–5.
- [52] S. Sharma, S. Chatzinotas, and B. Ottersten, "Compressive sparsity order estimation for wideband cognitive radio receiver," *Signal Processing, IEEE Transactions on*, vol. 62, no. 19, pp. 4984–4996, Oct 2014.
- [53] H. Sun, A. Nallanathan, C.-X. Wang, and Y. Chen, "Wideband spectrum sensing for cognitive radio networks: a survey," *Wireless Communications, IEEE*, vol. 20, no. 2, pp. 74–81, April 2013.

- [54] Y. Sun, B. L. Mark, and Y. Ephraim, "Online parameter estimation for temporal spectrum sensing," *IEEE Transactions on Wireless Communications*, vol. 14, no. 8, pp. 4105–4114, Aug. 2015.
- [55] P. Tehrani, L. Tong, and Q. Zhao, "Asymptotically efficient multi-channel estimation for opportunistic spectrum access," *IEEE Transactions on Signal Processing*, vol. 60, no. 10, pp. 5347–5360, Oct. 2012.
- [56] Z. Tian and G. B. Giannakis, "A wavelet approach to wideband spectrum sensing for cognitive radios," in Proc. 1st Int. Conf. on Cog. Radio Oriented Wireless Nets. and Comms. (CROWNCOM), June 2006, pp. 1–5.
- [57] Z. Tian and G. Giannakis, "Compressed sensing for wideband cognitive radios," in Acoustics, Speech and Signal Processing, 2007. ICASSP 2007. IEEE International Conference on, vol. 4, April 2007, pp. IV-1357–IV-1360.
- [58] H. Urkowitz, "Energy detection of unknown deterministic signals," Proceedings of the IEEE, vol. 55, no. 4, pp. 523–531, 1967.
- [59] C. Van Loan, "Computing integrals involving the matrix exponential," Cornell University, Tech. Rep., 1977.
- [60] H. L. Van Trees, *Detection, estimation, and modulation theory*. John Wiley & Sons, 2004.
- [61] K. Wang, L. Chen, Q. Liu, W. Wang, and F. Li, "One step beyond myopic probing policy: A heuristic lookahead policy for multi-channel opportunistic access," *IEEE Transactions on Wireless Communications*, vol. 14, no. 2, pp. 759–769, Feb 2015.
- [62] P. D. Welch, "The use of fast Fourier transform for the estimation of power spectra: A method based on time averaging over short, modified periodograms," Audio and Electroacoustics, IEEE Transactions on, vol. 15, no. 2, pp. 70–73, Jun 1967.
- [63] L. Yang, W. Hou, L. Cao, B. Y. Zhao, and H. Zheng, "Supporting demanding wireless applications with frequency-agile radios." in NSDI, 2010, pp. 65–80.
- [64] S.-Z. Yu, "Hidden semi-Markov models," Artificial Intelligence, vol. 174, no. 2, pp. 215–243, 2010.
- [65] T. Yucek and H. Arslan, "A survey of spectrum sensing algorithms for cognitive radio applications," *IEEE Communications Surveys and Tutorials*, vol. 11, no. 1, pp. 116– 130, March. 2009.
- [66] Q. Zhao, B. Krishnamachari, and K. Liu, "On myopic sensing for multi-channel opportunistic access: Structure, optimality, and performance," *IEEE Transactions on Wireless Communications*, vol. 12, no. 7, pp. 5431–5440, Dec. 2008.
- [67] Q. Zhao, L. Tong, A. Swami, and Y. Chen, "Decentralized cognitive MAC for opportunistic spectrum access in ad hoc networks: A POMDP framework," *IEEE Journal* on Selected Areas in Communications, vol. 25, no. 3, pp. 589–600, April 2007.

## Curriculum Vitae

Joseph Michael Bruno received his B.S. in Electrical Engineering from the University of Delaware in 2011, where he was a student member of the IEEE Communications Society and a member of the Eta Kappa Nu honor society. He completed his M.S in Electrical Engineering from the Johns Hopkins University in 2013, focusing on digital signal processing and wireless communications. He has been a student at George Mason University, in the PhD in Electrical and Computer Engineering program, since 2013, where he has advanced his coursework in statistics, digital signal processing, and wireless communications. Under the tutelage of Dr. Brian Mark and Dr. Yariv Ephraim, Joe has completed extensive research in cognitive radio for opportunistic spectrum access.

Joe has been a full-time communications researcher at the Johns Hopkins University Applied Physics Lab since 2011, where he has extensively designed, developed, and tested a wide variety of wireless systems. His specialties include software-defined radio and digital receivers.