<u>A MODEL OF DIFFUSION OF DIGITAL INFORMATION GOODS: ROLES OF A KEY AGENT</u>

by

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DEDICATION

This is dedicated to my parents and my elder sister.

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TABLE OF CONTENTS

Page

List of Tables xii

List of Figures xvi

List of Equations xviii

List of Abbreviations xix

Abstract xx

Chapter 1: Introduction 1

1.1 Diffusion of innovations 3

1.2 Digital piracy 5

1.3 Complex adaptive systems (CAS) 8

1.4 Classical mathematical approaches vs. CAS 9

1.5 Agent-based modeling 11

1.6 Dissertation layout 11

Chapter 2: Background 13

2.1 Reviews of diffusion of innovations 13

2.1.1 Diffusion of digital information goods 14

2.1.2 Bass diffusion model 15

2.1.3 Social network topologies 16

2.1.4 Impacts of social network topologies 18

2.1.5 Modeling consumer adoption patterns 20

2.1.6 Impacts of negative word-of-mouth (WOM) 21

2.2 Reviews of marketing strategies in the diffusion of innovations 23

2.2.1 Impacts of pricing and promotion strategies – mathematical models 23

2.2.2 Impacts of promotion strategies – agent-based models 24

2.2.3 Impacts of opinion leaders (influentials) vs. average individuals 26

2.3 Reviews of marketing literature 30

2.4 Reviews of digital piracy 33

2.4.1 Motivations of digital piracy 33
2.4.2 Reviews of digital piracy models 33
2.4.2.1 Eliminate digital piracy 34
2.4.2.2 Survive under the influences of digital piracy 35
2.4.2.2.1 Factors influencing consumers' piracy decision-making 36
2.4.2.2.2 Pricing strategies of digital content providers 36
2.4.2.2.3 Other marketing strategies of digital content providers 39
2.4.2.2.4 Influence of government policies 41
2.4.2.3 Positive influences of digital piracy 42
2.4.2.4 Utilizing positive influences of digital piracy 44
2.5 Reviews of impacts of competition 45
2.5.1 Impacts of competition on diffusion of innovations 45
2.5.2 Impacts of competition on digital piracy 47
2.6 Limitations of existing research on diffusion of digital information goods 50
2.6.1 Insufficient research on effective marketing strategies 50
2.6.2 Insufficient research on the diffusion of digital information goods 51
2.6.3 Lack of explicit use of social networks 51
2.6.4 Insufficient research on competitive environment 52
2.6.5 Limitations of classical mathematical approaches 52
2.7 Focus of my dissertation 54
2.8 Modeling assumptions 55
Chapter 3: Methodology 56
3.1. Modeling of marketing strategies 56
3.1.1 Evolutionary algorithms (EA) 57
3.1.1.1 Representation 58
3.1.1.2 Reproductive operators 58
3.1.1.3 Selections 60
3.1.1.4 Exploration vs. exploitation 61
3.1.2 Learning classifier systems (LCS) 61
3.2 Modeling of consumers and social networks 64
3.3 Applying learning classifiers to agent-based models 67
3.4 Applying evolutionary algorithms to a duopoly marketing environment 68

3.4.1 Coevolutionary algorithms (CoEA) 68
Chapter 4: Monopoly – System confgurations 71
4.1 Model description 71
4.1.1 Design of consumers – Agent-based model 72
4.1.2 Design of a digital content provider – EA 73
4.1.3 Workflow 75
4.2. Experiment configurations 77
4.2.1 Parameters configuration of EA 78
4.2.2 Parameters configuration of the agent-based model 80
4.2.3 Configurations of network structures 84
4.2.3.1 Configurations of small-world networks 84
4.2.3.2 Configurations of scale-free networks and random networks 86
Chapter 5: Monopoly – Dynamic and self-learning marketing strategy 87
5.1 Small-world network 87
5.1.1 Model 1-1: a key agent is an average consumer 88
5.1.2 Model 1-2: a key agent is a persuasive consumer 89
5.1.3 Model 2-1: a key agent is an average pirate 92
5.1.4 Model 2-2: a key agent is a persuasive pirate 93
5.2 A scale-free network and a random network 96
Chapter 6: Monopoly – Effects of variations of network topology on the diffusion speeds and profits 97
6.1 Effects of network rewiring probability on diffusion speeds 97
6.2 Effects of network rewiring probability on profits 102
6.3 Conclusions concerning the impacts of rewiring probabilities on diffusion speeds and profits 107
Chapter 7: Monopoly – Impact of a connector 108
7.1 Homogeneous consumers 109
7.1.1 Small-world network 109
7.1.2 Scale-free network 110
7.1.3 Random network 110
7.1.4 Experimental results and analysis 112
7.1.4.1 Comparison of run 45 among NP, P83, and P11 114
7.2 Heterogeneous consumers 117

7.2.1 Comparison of run 10 among NP, P83, and P11 119
7.3 Conclusions concerning a connector 120
Chapter 8: Monopoly – Impact of an opinion leader 122
8.1 Homogeneous consumers 123
8.1.4 Experimental results and analysis 123
8.2 Heterogeneous consumers 126
8.3 Conclusions concerning an opinion leader 127
Chapter 9: Monopoly – position of a key agent 129
9.1 Homogeneous consumers in small-world networks 130
9.1.1 Model 2-1 (an average pirate) 131
9.1.1.1 Why does only the pirate in the first level (L1) make a difference on profits and diffusion speeds? 132
9.1.1.2 Does positioning a pirate in the first level (L1) guarantee a higher profit and faster diffusion? 135
9.1.2 Model 2-2 (a persuasive pirate) 138
9.1.3 Model 1-1 (an average consumer) 139
9.1.4 Model 1-2 (a persuasive consumer) 139
9.2 Heterogeneous consumers in small-world networks 140
9.2.1 Model 2-1 (an average pirate) 140
9.2.2 Model 2-2 (a persuasive pirate) 141
9.2.3 Model 1-1 (an average consumer) 142
9.2.4 Model 1-2 (a persuasive consumer) 143
9.3. Homogeneous consumers in scale-free networks 144
9.3.1 Model 2-1 (an average pirate) 145
9.3.2 Model 2-2 (a persuasive pirate) 145
9.4 Homogeneous consumers in random networks 147
9.4.1 Model 2-1 (an average pirate) 147
9.4.2 Model 2-2 (a persuasive pirate) 148
9.5 Conclusions concerning positions of the key agent 149
9.6 Timing of a key agent 150
Chapter 10: Competitions among digital content providers 151
10.1 Applying CoEA to a duopoly marketing environment 151
10.1.1. Design and workflow of digital content providers 151

10.1.2. Workflow of a consumer 153
10.2. Experiments and configurations of conservative strategies on Model 2-1 154
10.2.1 Configurations of conservative strategies 154
10.2.2 Experiments and Results 157
10.2.2.1 Homogenous consumers 158
10.2.2.1.1 Experiments on non-piracy 158
10.2.2.1.2 Experiments on a pirate in L1 167
10.2.2.1.3 Experiments on a pirate in L2 171
10.2.2.2 Heterogeneous consumers 172
10.2.2.2.1 Experiments on non-piracy 173
10.2.2.2.2 Experiments on a pirate in L1 179
10.2.2.2.3 Experiments on a pirate in L2 180
10.3 Experiments and configurations of audacious strategies on Model 2-1 182
10.3.1 Configurations of audacious marketing strategies 182
10.4 Comparisons between conservative and audacious strategies 184
10.5 Experiments of audacious marketing strategies on Model 2-2 184
10.6 Experiments of audacious marketing strategies on scale-free networks 185
10.7 Conclusions 185
Chapter 11: Sensitivity analysis 187
11.1 Scalability of the model 187
11.2 Variations of marketing strategies 190
11.2.1 More conservative variations of prices and costs. 190
11.2.2 Unit of variation from 5% to 1% 192
11.3 Variations of EA parameters 193
11.3.1 Population size 400 and generation count 600 193
11.4 Models design justifications 195
Chapter 12: Conclusions 196
12.1 Contributions 196
12.1.1 Impacts of a key agent in the diffusion of digital information goods 197
12.1.1.1 Impacts of a connector and an opinion leader 197
12.1.1.2 Position of a key agent 197
12.1.1.3 Timing of a key agent 199

12.1.2 Dynamic marketing strategies ... 199
12.1.3 Impacts of variations of networks on profits and diffusion speeds ... 200
12.1.4 Relationship between marketing strategies and network topologies ... 201
12.1.5 Competition between digital content providers ... 201
12.2. Future work ... 202
Appendices ... 203
Appendix A: Network topologies of 15 small-world networks ... 204
Appendix B: Initiator, L1, L2, and L3 of 15 small-world networks ... 219
Appendix C: Network topologies of three scale-free networks ... 249
Appendix D: Network topologies of three random networks ... 252
Appendix E: Flow of consumer adoption decisions in duopoly models ... 255
Appendix F: Values of D, p, and mean for Model 2-1 ... 262
Homogeneous consumers ... 262
Heterogeneous consumers ... 264
Appendix G: Values of D, p, and mean for Model 2-2 ... 267
Homogeneous consumers ... 267
Heterogeneous consumers ... 269
Appendix H: Values of D, p, and mean for Model 1-1 ... 273
Homogeneous consumers ... 273
Heterogeneous consumers ... 274
Appendix I: Values of D, p, and mean for Model 1-2 ... 277
Homogeneous consumers ... 277
Heterogeneous consumers ... 280
Appendix J: Values of D, p, and mean for scale-free networks ... 285
Model 2-1 ... 285
Model 2-2 ... 285
Appendix K: Values of D, p, and mean for random networks ... 287
Model 2-1 ... 287
Model 2-2 ... 287
Appendix L: Results of audacious marketing strategies in Model 2-1 ... 289
L-1. Homogeneous consumers ... 289
L-1-1. Experiments on non-piracy ... 289

L-1-2. Experiments on a pirate in L1 ... 290
L-1-3. Experiments on a pirate in L2 ... 291
L-2. Heterogeneous consumers ... 293
L-2-1 Experiments on non-piracy ... 293
L-2-1 Experiments on a pirate in L1 ... 294
L-2-3 Experiments on a pirate in L2 ... 295
Appendix M: Comparisons of number of "lock-in" between conservative and audacious strategies ... 297
Appendix N: Results of audacious marketing strategies in Model 2-2 ... 300
N-1 Homogeneous consumers ... 300
N-1-1 Experiments on non-piracy ... 300
N-1-2 Experiments on a pirate in L1 ... 300
N-1-3 Experiments on a pirate in L2 ... 301
N-2 Heterogeneous consumers ... 303
N-2-1 Experiments on non-piracy ... 303
N-2-2 Experiments on a pirate in L1 ... 303
N-2-3 Experiments on a pirate in L2 ... 304
References ... 306
Biography ... 316

LIST OF TABLES

Table Page
Table 1: Parameters for EA of monopoly models 78
Table 2: Parameters for homogeneous consumers.. 81
Table 3: Parameters for heterogeneous consumers... 82
Table 4: Five small-world networks using random seed 1 85
Table 5: Five small-world networks using random seed 2 85
Table 6: Five small-world networks using random seed 3 85
Table 7: Three random networks using three random seeds.. 86
Table 8: Comparison of profits for three types of networks 112
Table 9: Comparison of diffusion speeds for three types of networks 112
Table 10: Comparison of amount of adoptions among NP, P83, and P11 115
Table 11: Comparison of profits among NP, P83, and P11.. 116
Table 12: Comparison of profits for three types of networks...................................... 117
Table 13: Comparison of diffusion speeds for three types of networks 117
Table 14: Comparison of adoptions among NP, P11, and P83..................................... 120
Table 15: Comparison of profits for three types of networks...................................... 123
Table 16: Comparison of diffusion speeds for three types of networks 123
Table 17: Comparison of profits for three types of networks...................................... 127
Table 18: Comparison of diffusion speeds for three types of networks 127
Table 19: Comparison of profits and diffusion speeds (Homo, Model 2-1).................. 131
Table 20: Comparison of amount of adoptions between NP and P7 133
Table 21: Comparison of profits between NP and P7 134
Table 22: Comparison of total profits between NP and P7 134
Table 23: Comparison of amount of adoptions between NP and P7 136
Table 24: Comparison of profits between NP and P7 137
Table 25: Comparison of total profits between NP and P7 137
Table 26: Comparison of profits and diffusion speeds (Homo, Model 2-2).................. 138
Table 27: Comparison of profits and diffusion speeds (Homo, Model 1-1).................. 139
Table 28: Comparison of profits and diffusion speeds (Homo, Model 1-2).................. 140
Table 29: Comparison of profits and diffusion speeds (Hetero, Model 2-1)................. 141
Table 30: Comparison of profits and diffusion speeds (Hetero, Model 2-2)................. 142
Table 31: Comparison of profits and diffusion speeds (Hetero, Model 1-1)................. 143
Table 32: Comparison of profits and diffusion speeds (Hetero, Model 1-2)................. 144
Table 33: Comparison of profits and diffusion speeds on scale-free networks (Homo, Model 2-1) 145

Table 34: Comparison of profits and diffusion speeds on scale-free networks (Homo, Model 2-2) 146
Table 35: Comparison of profits and diffusion speeds on random networks (Homo, Model 2-1) 148
Table 36: Comparison of profits and diffusion speeds on random networks (Homo, Model 2-2) 148
Table 37: Impacts of the position of a key agent, a persuader, and an opinion leader ... 150
Table 38: Parameters for EA of duopoly model 155
Table 39: Parameters for conservative marketing strategies (Homo) 155
Table 40: Parameters for conservative marketing strategies (Hetero) 156
Table 41: Competition results of networks of seed 1 (NP, Homo, conservative) 159
Table 42: Competition results of networks of seed 2 (NP, Homo, conservative) 159
Table 43: Competition results of networks of seed 3 (NP, Homo, conservative) 159
Table 44: Analysis of "lock-in" (NP, Homo) 160
Table 45: Analysis of "winning" (NP, Homo) 164
Table 46: Competition results of networks of seed 1 (P7, Homo, conservative) 167
Table 47: Competition results of networks of seed 2 (P7, Homo, conservative) 167
Table 48: Competition results of networks of seed 3 (P7, Homo, conservative) 168
Table 49: Analysis of "winning" (L1, Homo) 168
Table 50: Competition results of networks of seed 1 (P8, Homo, conservative) 171
Table 51: Competition results of networks of seed 2 (P8, Homo, conservative) 171
Table 52: Competition results of networks of seed 3 (P8, Homo, conservative) 172
Table 53: Competition results of networks of seed 1 (NP, Hetero, conservative) 173
Table 54; Competition results of networks of seed 2 (NP, Hetero, conservative) 174
Table 55: Competition results of networks of seed 3 (NP, Hetero, conservative) 174
Table 56: Analysis of "lock-in" (NP, Hetero) 175
Table 57: Attributes of heterogeneous consumers 3, 4, 6, and 7 in L1 176
Table 58: Analysis of "winning" (NP, Hetero) 177
Table 59: Attributes of heterogeneous consumers 4, 7, and 64 in L1 178
Table 60: Competition results of networks of seed 1 (P7, Hetero, conservative) 179
Table 61: Competition results of networks of seed 2 (P7, Hetero, conservative) 179
Table 62: Competition results of networks of seed 3 (P7, Hetero, conservative) 180
Table 63: Competition results of networks of seed 1 (P8, Hetero, conservative) 180
Table 64: Competition results of networks of seed 2 (P8, Hetero, conservative) 181
Table 65: Competition results of networks of seed 3 (P8, Hetero, conservative) 181
Table 66: Parameters for EA of duopoly model 182
Table 67: Parameters for audacious marketing strategies (Homo) 183
Table 68: Parameters for audacious marketing strategies (Hetero) 183
Table 69: Processing time of different consumer size 187
Table 70: Comparison of profits and diffusion speeds using conservative strategies (Homo, Model 2-1) 191
Table 71: Comparison of profits and diffusion speeds using conservative strategies (Hetero, Model 2-1) 191

Table 72: Comparison of profits and diffusion speeds using variation unit 1% (Homo, Model 2-1) 192
Table 73: Comparison of profits and diffusion speeds using variation unit 1% (Hetero, Model 2-1) 193
Table 74: Comparison of profits and diffusion speeds using population size 400 and generation count 600 (Homo, Model 2-1) 194
Table 75: Comparison of profits and diffusion speeds using population size 400 and generation count 600 (Hetero, Model 2-1) 194
Table 76: Impacts of a connector, an opinion leader, and position of a key agent 198
Table 77: Competition results of networks of seed 1 (NP, Homo, Model 2-1) 289
Table 78: Competition results of networks of seed 2 (NP, Homo, Model 2-1) 289
Table 79: Competition results of networks of seed 3 (NP, Homo, Model 2-1) 290
Table 80: Competition results of networks of seed 1 (P7, Homo, Model 2-1) 290
Table 81: Competition results of networks of seed 2 (P7, Homo, Model 2-1) 291
Table 82: Competition results of networks of seed 3 (P7, Homo, Model 2-1) 291
Table 83: Competition results of networks of seed 1 (P8, Homo, Model 2-1) 292
Table 84: Competition results of networks of seed 2 (P8, Homo, Model 2-1) 292
Table 85: Competition results of networks of seed 3 (P8, Homo, Model 2-1) 292
Table 86: Competition results of networks of seed 1 (NP, Hetero, Model 2-1) 293
Table 87: Competition results of networks of seed 2 (NP, Hetero, Model 2-1) 293
Table 88: Competition results of networks of seed 3 (NP, Hetero, Model 2-1) 294
Table 89: Competition results of networks of seed 1 (P7, Hetero, Model 2-1) 294
Table 90: Competition results of networks of seed 2 (P7, Hetero, Model 2-1) 295
Table 91: Competition results of networks of seed 3 (P7, Hetero, Model 2-1) 295
Table 92: Competition results of networks of seed 1 (P8, Hetero, Model 2-1) 296
Table 93: Competition results of networks of seed 2 (P8, Hetero, Model 2-1) 296
Table 94: Competition results of networks of seed 3 (P8, Hetero, Model 2-1) 296
Table 95: Comparisons of "lock-in" between conservative and audacious strategies (NP, Homo) 297
Table 96: Comparisons of "lock-in" between conservative and audacious strategies (P7, Homo) 298
Table 97: Comparisons of "lock-in" between conservative and audacious strategies (P8, Homo) 298
Table 98: Comparisons of "lock-in" between conservative and audacious strategies (NP, Hetero) 298
Table 99: Comparisons of "lock-in" between conservative and audacious strategies (P7, Hetero) 299
Table 100: Comparisons of "lock-in" between conservative and audacious strategies (P8, Hetero) 299
Table 101: Competition results of networks of seed 1 (P7, Homo, Model 2-2) 300
Table 102: Competition results of networks of seed 2 (P7, Homo, Model 2-2) 301
Table 103: Competition results of networks of seed 3 (P7, Homo, Model 2-2) 301
Table 104: Competition results of networks of seed 1 (P8, Homo, Model 2-2) 302
Table 105: Competition results of networks of seed 2 (P8, Homo, Model 2-2) 302

Table 106: Competition results of networks of seed 3 (P8, Homo, Model 2-2)............ 302
Table 107: Competition results of networks of seed 1 (P7, Hetero, Model 2-2)........... 303
Table 108: Competition results of networks of seed 2 (P7, Hetero, Model 2-2)........... 303
Table 109: Competition results of networks of seed 3 (P7, Hetero, Model 2-2)........... 304
Table 110: Competition results of networks of seed 1 (P8, Hetero, Model 2-2)........... 304
Table 111: Competition results of networks of seed 2 (P8, Hetero, Model 2-2)........... 305
Table 112: Competition results of networks of seed 3 (P8, Hetero, Model 2-2)........... 305

LIST OF FIGURES

Figure Page
Figure 1: Diffusion of innovations (Rogers, 1962).. 5
Figure 2: Measure of piracy concerns in 2011... 7
Figure 3: Demonstration of 2-point crossover 59
Figure 4: Regular, small-world, and random networks (Watts and Strogatz, 1998) 66
Figure 5: Structure of a single rule 74
Figure 6: Structure of four rules associated with market share 5% 75
Figure 7: Workflow of one simulation run 77
Figure 8: Crossover of rule of market share 5% 80
Figure 9: Distribution of reservation prices of 100 consumers... 83
Figure 10: Distribution of promotion cost thresholds of 100 consumers 83
Figure 11: Distribution of piracy detection cost thresholds of 100 consumers 84
Figure 12: Initiator, L1, and L2 of network seed 1, rewiring probability 0.1................... 88
Figure 13: Best of best-so-far strategies on a small-world network (Model 1-1)............ 89
Figure 14: Best of best-so-far strategies on a small-world network (Model 1-2)............ 90
Figure 15: Comparison of total adoptions between Model 1-1 and Model 1-2................ 91
Figure 16: Comparison of total profits between Model 1-1 and Model 1-2..................... 91
Figure 17: Best of best-so-far strategies on a small-world network (Model 2-1)............ 93
Figure 18: Best of best-so-far strategies on a small-world network (Model 2-2)............ 94
Figure 19: Comparison of total adoptions between Model 2-1 and Model 2-2................ 95
Figure 20: Comparison of total profits between Model 2-1 and Model 2-2..................... 95
Figure 21: Diffusion speeds of homogeneous consumers on networks of seed 1. 98
Figure 22: Diffusion speeds of homogeneous consumers on networks of seed 2 99
Figure 23: Diffusion speeds of homogeneous consumers on networks of seed 3 99
Figure 24: Diffusion speeds of heterogeneous consumers on networks of seed 1. 101
Figure 25: Diffusion speeds of heterogeneous consumers on networks of seed 2 101
Figure 26: Diffusion speeds of heterogeneous consumers on networks of seed 3. 102
Figure 27: Profits of homogeneous consumers on networks of seed 1.......................... 103
Figure 28: Profits of homogeneous consumers on networks of seed 2.......................... 104
Figure 29: Profits of homogeneous consumers on networks of seed 3.......................... 104
Figure 30: Profits of heterogeneous consumers on networks of seed 1......................... 106
Figure 31: Profits of heterogeneous consumers on networks of seed 2.......................... 106
Figure 32: Profits of heterogeneous consumers on networks of seed 3.......................... 107
Figure 33: Network of seed 3, rewiring probability 0.01 109
Figure 34: Node 11 and its direct neighbors... 109
Figure 35: Node 83 and its direct neighbors... 110

Figure 36: A scale-free network of seed 1 111
Figure 37: A random network of seed 3 111
Figure 38: Best of best-so-far strategies of P2 (Model 2-2) 124
Figure 39: Initiator, L1, and L2 of network of seed 1, rewiring probability 0.01 133
Figure 40: Initiator, L1, and L2 of network of seed 1, rewiring probability 0.05 135
Figure 41: Initiator, L1, and L2 of network of seed 1, rewiring probability 0.01 162
Figure 42: Initiator, L1, and L2 of network of seed 1, rewiring probability 0.03 164
Figure 43: Initiator, L1, and L2 of network of seed 1, rewiring probability 0.05 177
Figure 44: Processing time for different consumer sizes.. 188
Figure 45: Variations of total diffusion time steps for 1000 consumers......................... 189
Figure 46: Variations of total time steps for 100 consumers.. 189

LIST OF EQUATIONS

Equation Page
Equation 1: Bass diffusion model 15
Equation 2: Profit of a strategy 76
Equation 3: Diffusion speed of a strategy 76

LIST OF ABBREVIATIONS

Complex Adaptive System ....CAS
Word-of-mouth .... WOM
Evolutionary algorithm .... EA
Coevolutionary algorithm ....CoEA
Bucket brigade algorithm... BBA
Learning classifier system.. LCS
Homogeneous consumers ....Homo
Heterogeneous consumers ....Hetero
Generalized Bass model.. GBM
Reinforcement learning algorithm .... RL

ABSTRACT

A MODEL OF DIFFUSION OF DIGITAL INFORMATION GOODS: ROLES OF A KEY AGENT

Jue Wang, Ph.D.

George Mason University, 2017

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Digital information goods include computer software, music, movies, electronic version of newspapers and magazines, electronic books, video games, and other multimedia products. These goods are characterized by a relatively high cost of initial creation, a low cost in distribution through Internet, possible rapid updates, and a low cost (near $0) of duplication. The digital information goods play important roles in everyone's daily life and work, restructure industries, and more importantly, provide great opportunities and potentials for digital content providers. On the other hand, the near-zero duplication cost also makes digital information goods vulnerable to digital piracy which results in huge loss in profits. As a result, developing effective marketing strategies which are able to manage the diffusion process, increase profits, and accelerate diffusion speeds becomes an important task for the digital content provider today.

In addition, there is another important factor in the research of diffusion of innovations, roles of so-called “key players”. Marketing professionals and scholars have believed that opinion leaders are key players, and winning over the opinion leaders was crucial to the successful diffusion of new products. Then, Watts (2007) demonstrated that instead of opinion leaders, average individuals can trigger a “tipping point” in the diffusion of new products. According to his results, the triggering of the tipping point is the accidental outcome of many factors coming together at just the right moment, and can be initiated by almost any individual. The heated debate is ensued.

This dissertation attempts to study and explore marketing strategies that enable providers to leverage the “key agent” in the diffusion of digital information goods. By a “key agent” I mean any individual of the type described in the literature under the names connector, maven, persuader, opinion leader, influencer, innovator, and early adopter. Additionally, a digital pirate has much in common with a key agent. My approach is to explicitly represent diffusion processes between individual consumers using social networks and agent-based models, and my approach determines effective marketing strategies for digital content providers via learning classifier systems.

In the existing research on the diffusion of innovations there are several areas that are not sufficiently studied and which this dissertation addresses. First, little extant work has been conducted on developing effective marketing strategies which actively manage how new products diffuse. Specifically, while a variety of promotion strategies, pricing strategies, and product feature choices have been investigated, these are typically looked at individually. My approach permits the synthesis of effective strategies in an integrated fashion that considers these several dimensions simultaneously.

Second, while processes associated with the diffusion of generic innovations have been extensively studied, there has been much less focus on digital information goods. Digital information goods are unmaterialized, requires high cost in creation, distribution cost is variable, and most importantly, the near $0 marginal cost of duplication of such good gives them special properties, such as making them vulnerable to various kinds of illegal activities, including digital piracy. Although there exists some research on digital piracy, few of them pay attention to the impacts of piracy on the diffusion process. In this dissertation, I will focus exclusively on such goods.

Third, although the agent-based diffusion model on generic products utilize various network topologies, the existing research on digital information goods does not use social networks explicitly. In this dissertation, I study diffusion processes in realistic social networks – small-world networks which explicitly represent each potential adopter and their social connections.

Fourth, most research in this area neglects the role of competition between digital content providers. My dissertation provides effective marketing strategies for providers competing in the same markets.

Lastly, classical mathematical approaches hold the advantages of simplicity and tractability. However, their failure to consider non-linear interactions among individuals, social networks, as well as heterogeneity and adaptations of individuals, make them special cases of the more general computational formalism that I have adopted and prevents them from generating realistic results.

In this dissertation, I build models to simulate the market of digital information goods as a complex adaptive system. The models develop dynamic and self-learning marketing strategies in the context of potential adopters connected via social networks. Those marketing strategies are able to dynamically adjust various marketing factors, including prices, promotion costs, and/or piracy detection costs through an entire diffusion process. The resulting strategies would enable digital content providers to leverage a key agent to speed up new product diffusion and increase profits in both monopoly and duopoly environments. Also, the models demonstrate that the profits and the diffusion speed are also influenced by the topologies of social networks and location of the key agent in networks.

This dissertation attempts to answer the following research questions: What are the characteristics of successful marketing strategies? What is the impact of network topologies? Does the position of a key agent in a network matter? How do a monopoly and a duopoly marketing environment differ with respect to above questions?

This is a rich area of research and I have used modern computational methodologies to investigate it broadly. I believe that my results are interesting and the methodology I have employed will be useful down the road as successive generations of scholars investigate problems of the type investigated here.

CHAPTER 1: INTRODUCTION

Digital information goods include computer software, music, movies, electronic version of newspapers and magazines, electronic books, video games, and other multimedia products. These goods are characterized by a relatively high cost of initial creation, a low cost in distribution through Internet, possible rapid updates, and a low cost (near $0) of duplication. The digital information goods not only change everyone's daily life and work tremendously, but they also present massive opportunities and potential for digital content providers. Developing effective marketing strategies that actively manage how the new products diffuse is an important task. However, the majority of research on marketing strategies on diffusion of innovations uses the classical mathematical models (Ruiz-Conda, Leeflag & Wieringa, 2006). The classical mathematical models can not handle the heterogeneity of consumers, a dynamic marketing environment, non-linear interactions among consumers, and social communication networks. Such limitations reduce the applicability and values of these models and their results. So far agent-based models of diffusion of innovations study exclusively promotion strategies and ignore other marketing factors, such as prices and product features (Kiesling, Gunther, Stummer, and Wakolbinger, 2011).

Great opportunities also bring challenges, the near-zero duplication cost also makes digital information goods vulnerable to digital piracy which results in huge loss in profits. However, piracy could also bring benefits to providers, that is, it may accelerate the diffusion of innovations. So far, only a few scholars have realized the positive influence of piracy, and even fewer scholars have tried to develop strategies to control and manipulate piracy in order to utilize its positive influence.

This dissertation attempts to study and explore marketing strategies that enable providers to leverage the “key agent” in the diffusion of digital information goods. By a “key agent”, I mean any person of the type described in the literature under the names connector, maven, persuader, opinion leader, influencer, innovator, and early adopter. Additionally, a digital pirate has much in common with the key agent.

In this dissertation, I build models to simulate the market of digital information goods as a complex adaptive system. In a complex adaptive system, there are many adaptive actors or agents, each pursuing their own goals and objectives. The coevolution of these agents may lead the overall system to have complex behaviors. The models develop dynamic and self-learning marketing strategies in the context of potential adopters connected via social networks. Those marketing strategies are able to adjust dynamically various marketing factors, including prices, promotion costs, and/or piracy detection costs over the course of an entire diffusion process. The resulting strategies enable digital content providers (providers, musical artists and producers, game makers, publishers, etc.) to leverage the key agent to speed up new product diffusion and increase profits in both monopoly and duopoly environments. In addition, the models demonstrate that the profits and the diffusion speeds are also influenced by the topologies of social networks and positions of the key agent within its networks.

In order for a reader to have a thorough understanding of the dissertation, I need to explain several basic concepts before proceeding. What is the diffusion of innovations? What is a tipping point? Section 1.1 explains the history and basic concepts of this theory. I also propose to build models to simulate a market of digital information goods as a complex adaptive system. What is a complex adaptive system? Due to the near zero duplication cost, digital information goods are vulnerable to digital piracy. Section 1.2 provides an overview on different types of digital piracy and its loss in revenue. Section 1.3 introduces a complex adaptive system and its features. What are the proper approaches for building a complex adaptive system? Classical mathematical approaches have a long history in classical economics research and may seem like good candidate approaches for solving this problem. Section 1.4, however, discusses limitations of classical mathematical approaches. Section 1.5 introduces an alternative out-of-equilibrium approach, agent-based modeling, which compensates for the shortcomings of classical mathematical approaches.

1.1 Diffusion of innovations

Everett Rogers first systematically studied the diffusion of innovations in 1962. In his book, *Diffusion of Innovations*, Rogers defines the concept as "the process by which an innovation is communicated through certain channels over time among the members of the social system. It is a special type of communication in that the messages are concerned with the new ideas" (Rogers, 1983). Research on diffusion of innovations focuses on identifying key factors and exploring how those factors affect the diffusion patterns and the diffusion speed. Rogers points out four important elements affecting the spread of diffusion. These are innovation itself, communication channels, time, and social systems (Rogers, 1983).

Rogers refines the concept of an “opinion leader”. Opinion leaders are adopters whose opinions towards new technologies are more influential than average adopters are and have greater exposure to mass media. Rogers also writes extensively about “critical mass” (also known as a tipping point). Critical mass refers to a point that indicates that the number of adopters has reached a certain critical level, and from that point, a new technology grows exponentially and spreads like wildfire (Rogers, 2003).

Rogers divides consumers into five categories based on their degree of openness towards new technologies. These categories are innovators, early adopters, early majority, late majority, and laggards. Consumers in each category possess different attitudes towards new technologies, and those attitudes in turn affect their adoption rate. The innovators are more adventurous. Their unique view and their ability to understand complex technologies separates them from their peers. The early adopters are more likely to become opinion leaders. The early majority may deliberate for some time before making the adoption decision. The late majority are skeptical. Their adoptions are the result of economic necessity and peer pressures. The laggards are traditional and resistant towards innovations (Rogers, 2003). In Figure 1, the curve represents the percentage of adoption by adopters in each category.

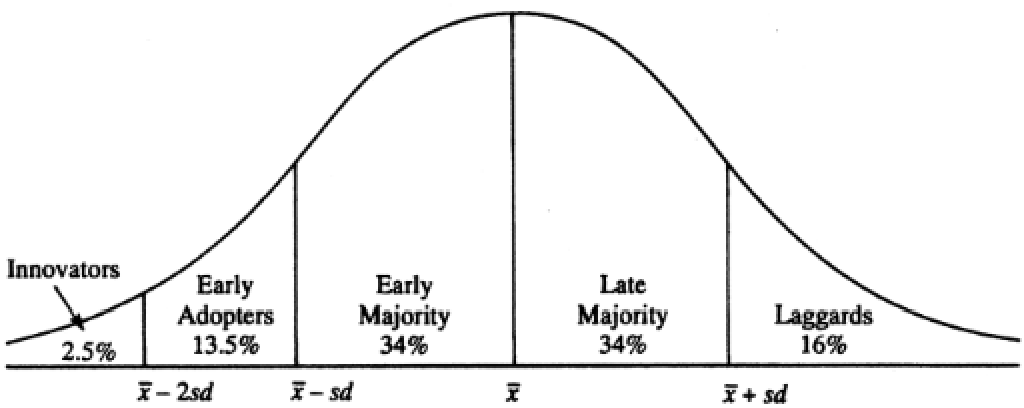


Figure 1: Diffusion of innovations (Rogers, 1962)

1.2 Digital piracy

Digital piracy is the copying and/or distribution of digital information goods for personal and/or business use without the authorization of the copyright holder (Symantec, 2017). Digital piracy is constantly evolving as technology changes. Today digital piracy concentrates on computer software, music, movies, video games, and eBooks.

A global survey conducted by Business Software Alliance reveals that 39% of all software products installed on PCs worldwide during the year 2015 were pirated copies. The loss to the IT industries was $52.2 billion ("BSA Global Software Survey," 2016). International Federation of the Phonographic Industry (IFPI) reports that there were four billion illegal music downloads via BitTorrent alone in the year 2014 ("IFPI Digital Music Report 2015," 2015).

The most prevalent types of digital piracy for individual consumers are counterfeiting and internet piracy. Counterfeiting is illegal copying and distribution of copyrighted materials. Internet piracy refers to copyrighted software that is downloaded free through internet auction sites or peer-to-peer networks. To conduct counterfeiting, the Digital Rights Management (DRM) of the digital goods has to be cracked first. DRM refers to a wide range of access control techniques to prevent digital goods from unauthorized use. Past research demonstrates that it only takes a small number of technology savvy individuals to break the protection. Once the DRM is cracked, it is distributed to a wide audience (Smallridge & Roberts, 2013). In addition, a pirated copy can be made not only from an original legal copy, but also from a pirated copy, and there is no deterioration in quality. Besides counterfeiting, internet piracy is a fast-growing type of digital piracy with a growing number of online consumers and a dramatic increase of internet connection speed in recent years.

Sudler (2013) indicated that according to the 2011 Anti-Piracy and Content Protection Summit survey, piracy concerns are concentrated on two types of piracy: illegal physical piracy and online piracy (see Figure 2).

For software piracy, besides counterfeiting and internet piracy, there are also end user piracy, client-server overuse, and hard disk loading. End user piracy and client-server overuse involve violation of software license agreements. For example, consumers use one software license to install the same software on multiple computers, or many users use the same software installed on a central server without the proper license agreement. Hard disk loading refers to when a business sells computers with pirated software installed to make the purchase of machines more attractive (Symantec & McAfee, 2017).

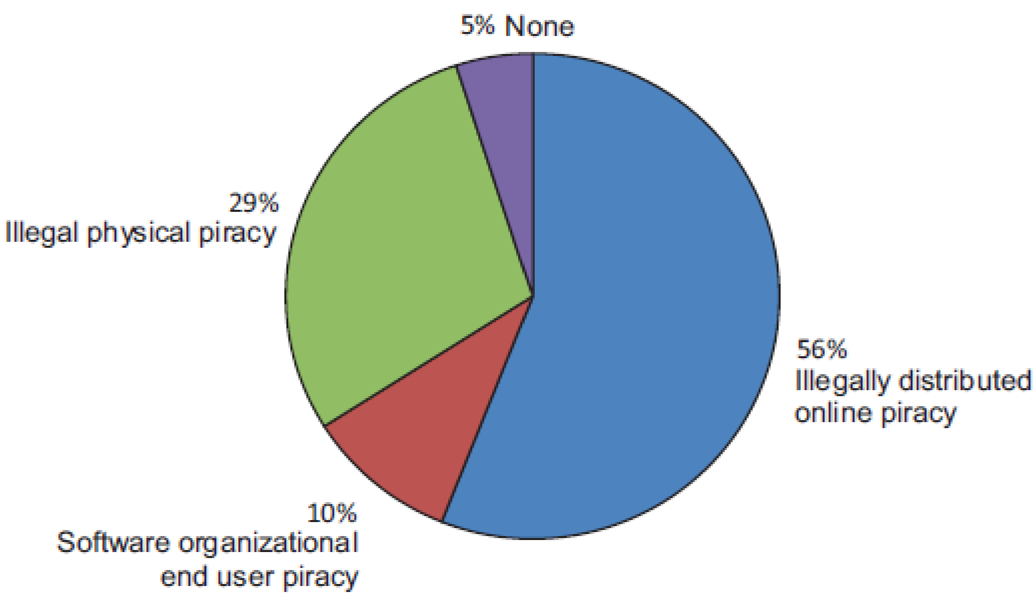


Figure 2: Measure of piracy concerns in 2011

There are two types of hardware platforms for playing video games: computers and gaming consoles. Video games designed for computers unavoidably suffer loss due to piracy. Video games designed for gaming consoles do not suffer huge loss (Depooter, 2014). Unlike copy-protection technology of software and music, in order for gaming consoles to play pirated video games, the chip inside the gaming console has to be modified. Not many people have the skills to modify the hardware, and such modifications have the risk of damaging the gaming console.

How to succeed under rampant digital piracy is a critical task for digital content providers and scholars today. Besides investing in various anti-piracy technologies and marketing strategies, scholars have begun to realize the positive influence of piracy (Conner & Rumelt, 1991; Givon, Mahajan, & Muller 1995, 1997). For example, piracy may speed up the diffusion process of a new product. Proper control of piracy helps providers achieve ideal market shares and profits in a dynamic marketing environment. In summary, digital piracy could either harm or benefit providers, but outcomes totally depend on how providers manage and control piracy.

1.3 Complex adaptive systems (CAS)

Complex adaptive systems, hereafter CAS, is a system in which numerous components, called agents, interact with each other in many ways. These agents may be anything from cells to individual people to nations. In a marketing environment, for example, an agent is either a consumer or a digital content provider.

A CAS possesses several important features. The first feature is simple rules. An agent continuously interacts with other agents and its environment by following very simple rules. For example, as an agent in a marketing environment, a consumer purchases a product as long as the price is below the consumer's reservation price. The second feature is adaptations. An agent does not respond to events passively; instead, it actively adapts and modifies its behaviors through interactions with other agents and its environment. The third feature is non-linear interactions. Interactions among agents do not follow a linear pattern. Therefore, the emerged pattern of behaviors through interactions is also non-linear, and it is more complex than a sum of individual phenomena. The fourth feature is coevolution. Adaptations of an agent influence both other agents and the environment in which it lives. Other agents in turn evolve to ensure their best interests in the new environment. The fifth feature is self-organization. There is no central control among the interactions of agents. Agents only try to satisfy their own interests through interactions with their neighbors. Unconsciously those agents organize themselves into a system that is in a constant self-organizing process to find the best fit within the environment (Waldrop, 1992).

1.4 Classical mathematical approaches vs. CAS

Mathematical approaches have a long history in classical economics research. Such approaches formulate mathematical models by taking into account relevant factors of problems. However, to maintain tractability, simplifying assumptions have to be made with respect to agents' attributes and behaviors. These assumptions cause many of these models to neglect essential aspects of problems, thereby detracting from the models' usefulness.

Microeconomics focuses on interactions between people using mathematical approaches founded on the rational choice theory. This theory states that agents make rational and logical decisions providing them with high welfare. In macroeconomics, mathematical models are founded on the rational expectation theory. This theory states that "agents deduce their optimum behavior by logical processes from the circumstances of any situation, assuming others will do likewise" (Palmer, Arthur, Holland, LeBaron, and Tayler, 1994). These two theories are widely used due to their tractability and simplicity. However, such simplicity comes with a price. These two theories heavily depend on several unrealistic assumptions: 1) perfect rationality, 2) complete information, and 3) common expectation (strategic behaviors). Perfect rationality assumes that agents are able to deduce rational and logical decisions no matter how complex an environment is. Complete information assumes that agents possess all information to make the rational choice, which ensures that agents have no incentive to change. Common expectations (strategic behaviors) assume that agents know that all other agents are making the same decisions as them, conducting the same actions based on the same information (Green, 2002; Palmer, Arthur, Holland, LeBaron, and Tayler, 1994). In this way, an agent's behavior can be captured by mathematical equations.

A typical example of the classical mathematical model is the Bass diffusion model. In the Bass diffusion model, adopters are homogeneous: innovators and imitators. The value estimation of external and internal influences is drawn from the past sales periods, which assumes that the marketing environment will never change. The details of the Bass diffusion model are explained in chapter 2.1.2.

Unfortunately, those assumptions are rarely satisfied in a real marketing environment for several reasons. First, people are not perfectly rational. Human make judgments based on instincts, peer pressures, or emotions, instead of logic. Firms are not clever enough nor do they have enough computing power to make the right decisions in every instance. Second, there is incomplete information. In a marketing environment that consists of a number of consumers and firms that are changing simultaneously, it is impossible to possess the complete information describing every agent at any given moment. Third, there are no common expectations. Every agent is unique, possesses different information, and makes different decisions.

In a CAS, agents are heterogeneous; an agent does not possess complete information and does not always make a rational decision, because an agent continuously acquires more information and adapts through interactions with other agents.

Furthermore, a CAS constantly evolves to new states due to the continuous adaptations of agents. Therefore, the unrealistic assumptions of the rational choice theory and the rational expectation theory make classical mathematical approaches inappropriate for building a CAS. There is an alternative approach, however, which is free of the restrictions of classical mathematical approaches. This approach is agent-based modeling.

1.5 Agent-based modeling

An agent-based model consists of a large number of agents who interact with each other. Each agent imitates a corresponding real-world counterpart; for example, an agent represents either a consumer or a provider in a marketing environment. Agent-based modeling provides a more realistic way to model a complex and dynamic environment. First, it allows a natural representation of an agent by incorporating rich details of the agent's real-world counterpart. Second, it explicitly models non-linear interactions among agents. Third, it is a bottom-up approach that contrasts with classical mathematical approaches that start at highly aggregated levels. Outcomes emerging from micro-levels reveal rich information about both agents and their environment. In summary, agent-based modeling is the appropriate approach for building a CAS.

1.6 Dissertation layout

The dissertation is organized into the following chapters. After this introductory chapter, chapter 2 provides reviews of existing research on diffusion of innovations, marketing, marketing strategies in diffusion of innovations, digital piracy, and influences of competition on digital piracy and diffusion of innovations. It also identifies and discusses limitations of existing research on diffusion of digital information goods in detail, addresses the focus of my dissertation, and discusses the design of the system. Chapter 3 explains the modeling methodologies. Chapter 4 introduces the model and configurations of the experiments. Chapter 5 presents the self-learning and dynamic marketing strategies. Chapter 6 analyses the effects of variations of networks on profits and diffusion speeds. Chapter 7 discusses the impact of a connector. Chapter 8 discusses the impact of an opinion leader. Chapter 9 analyzes the importance of network topologies and impact of the key agent's position through monopoly models. Chapter 10 presents duopoly models and provides marketing strategies for the provider to defeat his opponent. Chapter 11 provides a sensitivity analysis on the performance of the models. Chapter 12 presents the conclusions, contributions of my dissertation, and suggestions of future work.

CHAPTER 2: BACKGROUND

In this chapter I present reviews of existing research on diffusion of innovations, marketing strategies in diffusion of innovations, CAS on marketing, digital piracy, and competition between firms. Research in those areas covers a broad range of topics, including marketing, economics, financial investment, and business management. I am only presenting works that relate to my research topics.

2.1 Reviews of diffusion of innovations

While some innovative products took over a market like a storm, for example, the Sony Walkman (Clarke, 1999), others took a century to penetrate a market, for example, fax machines (Choi, Kim & Lee, 2010), and many others failed to win over consumers and eventually disappeared from a market. The models of diffusion of innovations start from classical mathematical models and gradually adopt agent-based modeling techniques in recent years. Among the research on diffusion of innovations, there is very little focusing on digital information goods. However, the diffusion of other types of products also offers valuable insights to the research of diffusion of innovations. In this chapter, I start with the classical diffusion model – Bass model, and then review existing research on agent-based modeling of diffusion of innovations that include impacts of social network topologies, consumer adoption patterns, and positive/negative word-of-mouth.

2.1.1 Diffusion of digital information goods

There is very little research studying the ways that digital information goods change the diffusion process. Inceoglu and Park (2011) studied the interdependence between the hardware demand and software supply – indirect network effect. They found that the increase of sales in DVD titles increase the sales of DVD players.

Nylén and Holmström (2015) presented a new framework for managing the digital innovations. They pointed out that digital innovation restructures industries and bring tremendous challenges to businesses. Their framework suggests that managers focus on user experience, value positions, digital scanning for identifying new opportunities, and training employees for new skills.

Liu, Wang, Chen, and Jiang (2014) analyzed the diffusion of mobile digital content in China. They identified that population, education level, and mobile technology usage are determining factors in the diffusion of mobile content.

Digital information goods are characterized by their high cost in creation and low cost of duplication. The near-zero cost of duplication makes digital information goods vulnerable to digital piracy, and piracy could change the diffusion patterns of digital information goods. Although there is some research on digital piracy, few scholars paid attention to the impacts of digital piracy on the diffusion of innovations. Those research efforts are presented in chapter 2.4.2.2.

Some scholars, including Conner and Rumelt (1991), Givon, Mahajan, and Muller (1995), Givon, Mahajan, and Muller (1997), have realized that piracy could accelerate the diffusion of innovations. Prasad and Mahajan (2003) and Haruvy, Mahajan, and Prasad

(2004) developed marketing strategies to utilize this feature of digital piracy to speed up diffusion and increase profits. The details of their research work are explained in sections 2.4.2.3 and 2.4.2.4.

2.1.2 Bass diffusion model

A famous model describing the process of innovation diffusion is the Bass model. Developed by Frank Bass in 1969 (Bass, 1969), the Bass model (as shown in Eq. 1) is a mathematical model and it is widely used in industry as the fundamental model of diffusion. It provides an elegant mathematical equation to forecast the sales and adoption of a new product. It describes the adoption process of a new technology by incorporating the interactions between promotion campaigns and consumers, and interactions (also known as word-of-mouth) among those consumers who have adopted the technology and who have not.

Equation 1: Bass diffusion model

$$n(t) = \left(p + q\frac{N(t)}{M}\right)[M - N(t)].$$

p: coefficient of innovation (external influence)

q: coefficient of imitation (internal influence)

M: total number of adopters

N(t): number of adopters already adopted the product at time t.

(Ruiz-Conda et al., 2006)

Some argue that the Bass model relies on several unrealistic assumptions. The first assumption is that the coefficients of internal and external influences are constant through the entire adoption process. These two coefficients are calculated from data in past sales periods. Applying such coefficients assumes a market is static and will never change in the future. The second assumption is that the consumer decision process in the Bass model is binary (adopt or do not adopt). An actual consumer purchasing decision is a multi-stage process: awareness → interest → adoption → word-of-mouth. The third assumption is that word-of-mouth always has positive impacts which is not always true. The fourth assumption is that every consumer can contact all other consumers (Lilien, Rangaswamy, & Bruyn, 2007).

Rand and Rust (2011) developed an agent-based model based on the classical Bass diffusion model. Their model has the same settings as the classical Bass model and incorporates all its limitations. For example, coefficients of internal and external influences are constant. Each agent makes an adoption decision by comparing two coefficients with two randomly generated numbers respectively. Rand and Rust were able to validate their agent-based model by replicating outputs of the classical Bass model.

2.1.3 Social network topologies

Diffusion of innovations involves communications among individuals. Scholars often use hypothetical networks to imitate a social network in reality. A vertex represents an individual, and a connection between individuals is represented by edges. A network topology refers to the structure of a social network, through which awareness, information, and opinions about an innovation are spread (Bohlmann, Calantone, and Zhao (2010)**.**

Bohlmann et al. (2010), quoting works from Newman (2000), discussed the three critical network properties. The first property is the average path length. It is an average of the shortest paths between all pairs of vertices. The second property is the clustering coefficient, which measures the cliquishness of a network. The third property is the degree distribution. The number of edges connected to a vertex is defined as the degree of a vertex. The degree distribution measures the probability distribution of the degree of every vertex over the whole network. A vertex that has the highest degree is called a hub.

Bohlmann et al. (2010) also summarized those properties of every network topology. Four network topologies, a regular network, a random network, a small-world network, and a scale-free network, demonstrate those properties in various ways. A regular network is a simple representation of a spatial structure for interacting agents. It has a high clustering property. However, its network diameter (longest shortest path) is large and proportional to the size of the network. It does not have hubs due to the constant number of edges between vertices.

A random network was proposed by Erdos and Renyi (1959). It is a network in which vertices are randomly connected to each other. The diameter of a random network is small, and it does not exhibit clustering feature when the network is large.

A random network and a regular network are two extremes of network topologies. A small-world network (Watts & Strogatz, 1998) interpolates between these two extremes by randomly rewiring any two edges of a regular network. The resulting network incorporates a higher degree of clustering and a shorter average path length. In a small-world network, most nodes are not neighbors of one another, but most nodes can be reached from each other by a small number of steps.

None of the three network topologies have hubs. Albert and Barabási (2002) propose a scale-free network whose degree distribution follows a power law. During network growth of a scale-free network, a new vertex may exhibit the property of preferential attachment; that is, a new vertex has a higher probability to connect to a vertex that already has a larger number of edges. An example of a scale-free network is the World Wide Web.

2.1.4 Impacts of social network topologies

Delre, Jager, and Janssen (2007) demonstrated how network structure and consumer heterogeneity affect the diffusion speed. In their agent-based model, a consumer's adoption depends on both the consumer's own preference and neighbors' influences. Results suggest that the speed of diffusion is influenced by the randomness of the network; for example, the diffusion speed is low at a regular network (lattice), increases at a small-world network, and is very slow at a random network. In addition, results show that the more heterogeneous consumers are, the faster the speed of diffusion.

Choi et al. (2010) investigated why some diffusion fails and others succeed by varying the network cliquishness (close and extensive connections) and bridges (random connections). Their findings indicate that a new product is more likely to succeed in a cliquish network than a random network. However, once the diffusion goes beyond a certain critical mass, the diffusion speed accelerates in a network that has more random bridges.

Bohlmann et al. (2010) analyzed the influence of locations of initial innovators, network topologies, and acceptance thresholds on diffusion of innovations. Their model is evaluated using innovators at various locations and acceptance thresholds on a regular network, a small-world network, a scale-free network, and a random network. They concluded that the locations of initial innovators, the network topology, and the acceptance threshold affect diffusion patterns differently among various network structures. The acceptance threshold negatively affects the likelihood of diffusion, diffusion speed, and the number of new adopters. In other words, the increasing acceptance threshold delays the peak adoption time and reduces the number of new adopters. A more clustered network is more likely to diffuse under high adoption thresholds (conservative consumers). The network topology is the key factor among all factors that influence diffusion patterns. The locations of initial innovators, the network topology, and the acceptance threshold have less impact on diffusion patterns in a random network due to the relative absence of clustering.

Instead of studying the impacts on diffusion among different types of networks, Peres (2014) utilizes an agent-based model to study the impacts of three network metrics: the average degree, the relative degree of social hubs, and the clustering coefficient, on the diffusion of innovations. The model indicates that the average degree and the relative degree of social hubs have positive influences on diffusion, and the clustering coefficient has negative impact on diffusion. Higher average degree and lower clustering coefficient indicate the network is highly connected, therefore, the positive impacts of average degree and negative impact of clustering coefficient on diffusion are obvious.

2.1.5 Modeling consumer adoption patterns

A consumer's purchasing behavior is complicated and influenced by many factors. Modeling a consumer purchasing decision in a computational model is a difficult task. Scholars have utilized several modeling methodologies so far. Kiesling et al. (2011) summarized the adoption patterns in existing research, including threshold approaches (simple decision rules), utilitarian approaches, psychological approaches, and state-transition approaches.

The first methodology is a threshold approach. A consumer adopts a product as long as the number of neighbors who adopted the product is above the consumer's threshold. The threshold is either deterministic or probabilistic. In the deterministic threshold approach, an agent adopts the product after the threshold is reached (Goldenberg, Libai, Solomon, Jan, & Stauffer, 2000; Alkemade & Castaldi, 2005). In the probabilistic threshold approach, after the adoption threshold is reached, an agent becomes an adopter with certain probability (Bohlmann et al. 2010).

The second methodology is a utilitarian approach. A consumer makes an adoption decision through a utility function. Delre, Jager, Bijmolt, and Janssen (2007) and Choi et al. (2010) utilize a utility function that incorporates both a consumer's own preference for a product and the influence of the consumer's neighbors.

The third methodology is a psychological approach. The decision rules incorporate the richness of human psychological behaviors. Jager, Janssen, De Vries, De

Greef, and Vlek (2000) developed a social psychological framework called “consumat”. In this framework, a consumer switches among various purchasing strategies including repetition, deliberation, imitation, and social comparison based on the satisfaction levels of different needs and degrees of uncertainty regarding the outcomes of behaviors.

The fourth one is a state-transition approach. The decision process of an individual is modeled as a state machine and an individual makes adoption decisions by going through a sequence of transitions among different states. Deffuant, Huet, and Amblard (2005) molded an individual’s adoption decision with combinations of three interest states (no, maybe, yes) and five information states (not concerned, information request, no adoption, pre-adoption, adoption).

2.1.6 Impacts of negative word-of-mouth (WOM)

Bad news travels fast**.** According Kotler (2004), unsatisfied customers may take public action by complaining to the seller, taking a legal action, or complaining to other groups, including government or private agencies. They may also take private actions by switching brands or spreading bad words about the product. Among those actions, negative WOM is most damaging because it requires little effort from a consumer. To make it worse, as Day, Grabicke, Schaetzle and Staubach (1981) and Kotler (2004) pointed out, it is hard to trace the impacts of negative WOM in a market because only a small percentage of unsatisfied consumers complain to the sellers.

In addition, unsatisfied customers spread their opinions to more acquaintances than the satisfied consumers do (Kotler, 1991). Research conducted by Herr, Kardes, and Kim (1991) also demonstrate that consumers tend to put more weight on negative opinions in such communications. Thus, an unsatisfied consumer influences other consumers more than a satisfied one.

Goldenberg, Libai, Moldovan, and Muller (2007) analyzed the influence of negative WOM by building an agent-based model. They concluded that the influence of negative WOM is substantial even when the number of unsatisfied consumers is small. The network topology they use is a dynamic small-world network. Permanent connections within the cluster are called strong ties, and randomly generated connections with consumers outside the cluster are called weak ties. No individual evaluation is considered for an agent; only network externalities influence adoption decision. The random weak ties work as bridges among different clusters and expose the consumers in the cluster to the information from other parts of the network. Thus, with the presence of negative WOM, the random weak ties facilitate the spreading of negative WOM even when the number of unsatisfied consumers is small.

Deffuant et al. (2005) simulated the formation and spread of both positive and negative WOM in their model. In addition, they investigated the impacts of "extremists" by incorporating a small number of individuals with definite opinions in the model. Deffuant et al. (2005) did not explain the concept of "social opinion". I interpret the "social opinion" as the opinion towards the social influence of the innovation. An individual makes adoption decision by evaluating the trade-off between social influence and personal preference. Using a small-world network, they experimented with varying factors, including 1) initial distributions of social opinions, 2) individual benefit, 3) average size of the individual's social network, and 4) the frequency of mass media messages. Results show that individuals who have high opinions of an innovation tend to communicate with others more often in order to have a more accurate evaluation of individual benefits. Therefore, innovations with high social opinions and low individual benefits have a greater chance of succeeding than innovations with low social opinions and high individual benefits. For many individuals, the high social opinions compensate the low individual benefits. In addition, a small group of extremists can strongly affect the levels of adoption when both the density of social networks and the frequency of discussion are high.

2.2 Reviews of marketing strategies in the diffusion of innovations

The majority of research on pricing strategies uses classical mathematical models. Agent-based diffusion models by far concentrate exclusively on promotion strategies and neglect other marketing factor, such as prices and product attributes (Kiesling et al., 2011).

2.2.1 Impacts of pricing and promotion strategies – mathematical models

Ruiz-Conda et al. (2006) provided a comprehensive review on marketing strategies in the diffusion of innovations using classical mathematical models. The traditional mathematical diffusion models, such as the Bass model, are criticized for the implicit consideration of the marketing factors, such as price and promotion. As indicated by Ruiz-Conda et al. (2006), Robinson and Lakhani (1975) are the first to include price in the mathematical diffusion model. More scholars follow his example by adding the price parameter to the model and generate a mix of conclusions on the impacts of prices on the diffusions. Bass, Krishnan and Jain (1994) proposed the Generalized Bass model (GBM) that extends the Bass model by incorporating price and advertising affects.

Some research has been conducted on the pricing and promotion strategies using classical mathematical models. The classical mathematical models are not able to handle the heterogeneity of consumers, non-linear interactions among consumers, dynamic marketing environment, and the social communication networks. Besides the limitations of mathematical models, Ruiz-Conde et al. (2006) also pointed out that there is no agreement on what marketing factors should be included and how they should be applied to the diffusion model.

2.2.2 Impacts of promotion strategies – agent-based models

Promotion is one of four key elements in a marketing mix (product, price, promotion, distribution) (McCarthy, 1960). It includes various aspects of marketing communications that effectively deliver the information of a product to consumers. Promotion strategies are crucial to the diffusion of a new product.

Alkemade et al. (2005) developed an agent-based model to help firms evolve successful directed-advertising strategies under different network topologies. A consumer makes his adoption decision based on the number of adopters in his neighborhood and his exposure threshold. Firms utilize genetic algorithms to evolve their own advertising strategies. Each bit in the strategy represents one consumer. The agent-based model functions as a fitness evaluation function in the genetic algorithm. Experiments were conducted on a regular network (lattice), a random network, and a small-world network. Various scenarios, such as homogeneous/heterogeneous consumers and positive/negative network externalities, were included in the experiments. They compared diffusion results obtained with a dynamic directed-advertising strategy (adapts after each period) to random advertising results, and concluded that the evolved directed-advertising strategies outperform random advertising strategies. They also concluded that successful advertising strategies should include the following features according to different types of networks. In a regular network, the strategies should target consumers with a high exposure threshold, that is, the conservative consumers. In a small-world network, the strategies should target consumers with a large number of neighbors. In a random network, the strategies should target isolated consumers.

Moldovan and Goldenberg (2004) studied the influence of promotion strategies in a market in which both positive WOM and negative WOM exist. They concluded that both positive leaders (opinion leaders who hold the positive opinion of a particular product) and promotion strategies have little impact on the size of the market growth once the resistance leaders (opinion leaders who hold negative opinions of a particular product) have enrolled. Therefore, a better promotion strategy is to undermine the impact of resistance leaders by activating positive leaders before the promotion effort activates resistance leaders. The network topology adopted by the model is a regular network.

Delre, Jager, Bijmolt, and Janssen (2007) examined the timing and the targeting of promotion strategies. They concluded that promotion should not be too strong at the early stage of product diffusion. If promoters advertise a product too soon and too strongly, the diffusion does take off but results in a low final market penetration later on. This is because consumers decide not to adopt a product as the result of not enough consumers having done so yet. Such negative social influence may cause market penetration to remain low.

When choosing a targeted marketing segmentation, Delre et al. (2007) suggested maintaining a balance between targeting a small number of big and highly connected groups and a large number of smaller groups according to the levels of diffusion. When market penetration becomes higher, the better targeting strategy is to select more and smaller groups. On the other hand, when the market penetration is lower, the better targeting strategy is to aim at fewer but bigger groups. The network topology utilized in the model is a small-world network.

2.2.3 Impacts of opinion leaders (influentials) vs. average individuals

When mentioning key players in the diffusion of innovations, people usually think of opinion leaders. An opinion leader is a person who is socially very well connected and whose opinions are much more influential compared to regular individuals. In marketing research, it has long been believed by scholars that an opinion leader could influence the customers' attitudes and purchasing behaviors for a product. In other words, winning over the opinion leader is crucial to the success of the marketing campaign. According to Weimann (1994), there are around 3,900 studies conducted on opinion leaders and personal influences since 1955. Here I only cover the most influential research work.

The concept of opinion leader was first presented by Katz and Lazarsfeld in 1955. In their book, *Personal Influence: the Part Played by People in the Flow of Mass Communications*, they introduced a "two-step model". In this model, a small group of opinion leaders acts as the intermediaries between mass media and majority of the society. The information flows from the mass media to opinion leaders, and then opinion leaders pass the message together with their own interpretations to his followers.

Valente and Davis (1999) pointed out that few have attempted to use the lessons learned from diffusion of innovations to speed up a diffusion process. They built a model to demonstrate that opinion leaders are able to accelerate the diffusion of innovations. The model uses homogeneous agents. The agent adopts the innovation once at least 15% of his neighbors have adopted the product. The model utilizes a random graph by allowing every agent to randomly selects seven agents as his neighbors. Valente et al. (1999) compared the experimental results based on whether the first 10 adopters are opinion leaders, random individuals, and the individuals who are least influential. The results of the model indicate that the adoption diffuses faster when opinion leaders initiate it.

Everett Rogers, in his third edition (1983), *Diffusion of Innovations*, pointed out that the information flow is too complicated to be adequately represented by the two-step flow model. The impacts of innovation on an individual go through five stages: 1) knowledge of the innovation, 2) persuasion, 3) decision to adopt or reject, 4) implementation, and 5) confirmation. In every stage, the opinion leaders are not the only individuals that are exposed to the mass media. Rogers (2003) further summarized six characteristics of opinion leaders: 1) they have great exposure to mass media, 2) they are more cosmopolitan, 3) they have greater contact with a change agent who bring innovations to the community, 4) they have greater social participation, 5) they have higher socioeconomic status, and 6) they are more innovative. Based on the work from

Valente et al. (1999), Rogers (2003) further proves the importance of opinion leaders by citing eight completed experiments, which were performed from year 1995 to year 2002. Results from all the experiments give significant evidence that opinion leadership intervention was effective in the behavior changes and the acceleration of the diffusion process. In addition, Rogers contributes the typical S-shaped adoption curve as the outcome of opinion leadership intervention.

In his book, *The Tipping Point*, Gladwell (2000) extends Roger's work with his "The Law of the Few". As Gladwell states: "The success of any kind of social epidemic is heavily dependent on the involvement of people with a particular and rare set of social gifts". Those people are divided into three types: a connector, a maven, and a persuader. A connector knows many people, and, more important, knows the right kind of people. A maven is an information expert. He has rich specific knowledge in a specific field, and more importantly, he actively communicates and shares his knowledge with others. A persuader is someone with extraordinary personal charm and skills of persuasion. Connectors, mavens, and persuaders are excellent candidates for starting diffusion of innovations efficiently.

Delre, Jager, Bijmolt, and Janssen (2010) built an agent-based model of diffusion of innovations on a scale-free network. In their model, a consumer's adoption decision relies on both his own preference and his neighbors' influences. The model tests how the number of contacts and degree of social influence of a consumer affects the diffusion of innovations. Results indicate that a market in which a consumer's adoption decision relies more on decisions of his neighbors leads to a lower diffusion of innovations.

Because the social influence only has effect when there are many neighbors that have already adopted the product. A consumer refuses to adopt the product because there are not enough adopters in his neighborhood at the initial stage of adoptions, which hampers the diffusion of innovation. Delre et al. (2010) also investigated the impacts of hubs on the diffusion of innovations. In this model, a hub has no influence on the adoption decisions of other consumers. Results indicate that a hub indeed accelerates the diffusion process but the improvements are very small.

Watts (2007) presented a different idea on the impacts of opinion leaders. Through experiments and models, he demonstrates that in the majority of the situations, the people who trigger "tipping points" in diffusion processes are average individuals, instead of opinion leaders. In his model, only personal influence is studied, and media influences, for example, mass media, web-loggers, social web sites, online forums, etc., are not included. In Watts's model, there is one trendsetter and 10,000 heterogeneous agents. Everyone is able to communicate with anyone nearby, and each had a probability of influencing another. Each agent is assigned a list of neighbors randomly. The top 10% most connected agents are selected as opinion leaders. In another experiment, he made the opinion leader 10 times more connected than average agents, the model produced the similar results. Watts concludes that every average individual could trigger a trend if the society is ready to embrace it. He modified the adoption thresholds of agents to make them more easily influenced, and then the number of trends skyrocketed.

In his experiments to prove the "six degree of separations", Watts recruited around 61,000 people and asked them to deliver messages to 18 targets worldwide. When he examined these pathways of message delivery, he found that only 5% of the email messages passed through “hubs”– highly connected people (Thompson, 2008).

In Watts’s opinion, the trend is an accidental outcome when all the right factors come together at the right moment. For example, as he explained, a forest fire requires a combination of low humidity, wind, temperature, dry woods, and poorly equipped fire departments, not just the contributions of the exceptional abilities of a few sparks.

Not surprisingly, Watts’s opinion encountered lots of criticism from the scholars and marketing professionals. However, he pointed out several flaws in the existing research on opinion leaders. First, those marketing professionals do not analyze the personal influences between opinion leaders and their followers. In his words, an opinion leader could influence the mass in many ways, and different influence mechanisms could generate different results. Second, viral thinkers analyzed the trend after it has started, and then went back to the few consumers who first used it, and identified them as opinion leaders. However, there is nothing special about those so-called opinion leaders, either with regard to their personalities or with regard to their abilities to influence others (Watts, 2007; Thompson, 2008). Third, Watts pointed out a flaw in Roger’s theory, that is, the S-shaped diffusion curve does not require an opinion leader. For example, the Bass diffusion model described before can generate an S-shaped diffusion curve without an opinion leader (Watts, 2007).

2.3 Reviews of marketing literature

One category of CAS research in marketing describes and models consumers’ behaviors and social interactions in order to identify a pattern of their decision-making process. By entering corresponding marketing stimuli into a model, a firm is able to observe the responses from consumers, and predict the fate of a new marketing campaign. In such models, consumers are modeled as a large number of agents that interact with each other and adapt through interactions over time. A firm acts as a passive observer. Besides providing necessary marketing stimuli, a firm does not actively learn and interact with consumers and its competitors in the market.

Said, Bouron, and Drogoul (2002) proposed an agent-based model to simulate and analyze consumer response to markets. The model consists of a consumer population. Every consumer is modeled as an agent with behavioral attitudes (imitation, opportunism, innovativeness, opportunism, mistrust) and social factors (age, education, social class). Three firms provide marketing stimuli (promotion, brand loyalty actions) to the consumer population. The model demonstrates consumers' responses to different stimuli from firms. Results indicate that average intensities of behavioral attitudes of older consumers are more stable than those of younger consumers. Results also show that a firm starting with a large market share tends to keep a lock-in status forever. However, if all competing firms start with an equal market share, none of them will dominate the market.

This model is not tailor-made for any specific product market. In reality, each product market has its own specific features. Therefore, building a model that tries to capture every aspect of a general market is not practical; and not surprisingly, outcomes of the model only replicated some classical marketing theories and did not reveal any innovative information about the market.

The second category of CAS research in marketing is the opposite of the first one. A firm is modeled as an adaptive agent that continually refines its marketing strategies through interactions with consumers and competitors. Consumers are considered simple and follow very simple rules. For example, when facing products from multiple firms, a consumer chooses the one with the lowest price. Consumers repeat the same action and never learn or interact with other consumers.

Tesauro and Kephart (2002) utilized a Q-learning algorithm to help firms determine prices in a duopoly marketplace. Their model is designed for a general product market. In the model, there are two types of agents: firms and consumers. Two firms sell identical products and only differ in price. These two firms take turns setting up prices. In addition, each firm has full knowledge of the consumer demand for any price pair and profit functions at all times. Firms are adaptive agents that learn over time by the Q-learning algorithm. Consumers do not have strategic roles and do not learn over time. Consumers buy a product with the lowest price at every turn. Results show that 1) price wars occur when both firms use a myopic pricing strategy, 2) one Q-learner firm opposing its myopic opponent yields great profits for both firms, and 3) simultaneous Q-learning for both firms also generates promising results, although there is no theoretical proof of convergence under the condition of simultaneous Q-learning by multiple agents.

This model indeed brings up one interesting marketing strategy. If one firm adopts the Q-Learning algorithm to modify its pricing strategy, but its competitor utilizes a myopic pricing strategy, both firms benefit from the competition.

In summary, learning processes in existing marketing models concentrate on consumer side or firm side. In a real marketing environment, both consumers and firms are dynamic and adaptive agents that are learning and changing simultaneously

2.4 Reviews of digital piracy

The research on digital piracy concentrates on two types of digital information goods: software and music. Scholars today utilize either classical mathematical approaches or agent-based modeling to build models to study digital piracy. While classical mathematical approaches account for a large portion of existing research, both types of approaches offer valuable insights into the domain of digital piracy.

2.4.1 Motivations of digital piracy

The factors motivating software piracy by consumers are prices, restricted access to legitimate software, low learning costs, and cultural views of piracy as technology sharing (Nill and Shultz, 2009). Rajput (2013) summarized motivations for digital piracy as follows: the consumer believes that an immaterial product does not have ownership; the consumer believes that “everyone else does it”; copying is easy; and legitimate digital content has a high cost.

2.4.2 Reviews of digital piracy models

Scholars generally have four different views regarding what digital content providers should do about piracy. One view is that piracy is pure evil, and providers should develop various technologies to eliminate it. Another view is that we should acknowledge the fact that piracy cannot be eliminated, and try to survive under its influences. A third view is that we should realize the positive influences of piracy. Finally, a fourth view is that we should propose marketing strategies to utilize positive influence of piracy. I will now discuss each of them in turn.

2.4.2.1 Eliminate digital piracy

This view holds that piracy is a purely destructive force, and the best way to deal with piracy is to prevent it from happening. Rajput (2013) investigated contemporary technologies for preventing piracy of digital discs. These are (1) non-standard formatting, (2) incorrect table of contents (TOC) which provides an index of the starting position of the tracks on the disc, (3) adding a fictitious track in the genuine track, (4) invalidating the track number, (5) data track disguised as audio, (6) introduction of key mark, (7) file encryption, (8) secuRom, a protection mechanism developed by Sony, and (9) digital watermarking.

Jakobsson and Reiter (2001) suggested using frequent and random software updates (software aging) to discourage piracy. An obsolete version ceases functioning after an updated version is released, thereby forcing users of illegitimate copies to contact pirates again for the latest version. This is extremely inconvenient for users because most pirates try to keep their communication channels and identification hidden. For pirates, frequent updates increase production costs because of obligations to break a defense mechanism on every update. In addition, the risk of being caught increases with the frequency of communication. Gopal and Sanders (2000) pointed out that the rate of piracy is related to the gross domestic product (GDP) of every country instead of personal incomes. A global piracy investigation conducted by SPA (Software Provider

Association) and BSA (Business Software Alliance) indicates that the lower the GDP, the higher the piracy rate a country has. The cause is that prices of legal software are set at a U.S. level, which is much higher than most users could afford in developing countries. The recommended strategy is to apply global price discrimination according to the GDP of every country. Anckaert, Sutter, and Bosschere (2004) introduced a dynamic protection mechanism. Existing protection mechanisms built into software are static; once they are broken, nothing can be done to prevent the spreading of piracy. A dynamic protection mechanism guarantees that every legal copy is unique and installation is machine-dependent. Therefore, a pirated copy only works for a specific machine, thereby preventing pirated copies from having any value. Other common anti-piracy technologies include utilizing tailor-made software CDs or encoding encryption code into software.

2.4.2.2 Survive under the influences of digital piracy

Under this view, scholars still consider piracy to be a destructive force. However, instead of working to eliminate it, scholars try to survive under its influences. The majority of research on piracy concentrates in this area. Research in this category focuses on studying the relationship among consumers (pirates and legal buyers), marketing strategies of providers, and government copyright protection policies. Common parameters that are considered are: prices, promotion, and detection cost in the marketing strategies of providers; piracy cost, prices, the risk of being caught, and social influences in purchasing (or pirating) decisions of consumers; the penalty of piracy; and subsidies for legal purchase in government copyright protection policies. Tuning those parameters to generate ideal profits for providers or governments, or observing how variation of parameter values affects the outcome of markets, are two important research topics.

2.4.2.2.1 Factors influencing consumers' piracy decision-making

Several factors influence a consumer's decision-making process for conducting piracy. These include software price (Nill et al., 2009), levels of self-control, perceived punishment severity, perceived punishment certainty (probability of being caught), self-efficacy (the difficulty level of pirating the software) (Zhang, Smith, & McDowell, 2009; Limayem, Khalifa & Chin, 2004), social influences (Haque, Khatibi & Rahman, 2009), and piracy cost (Bae & Choi, 2006). Bae and Choi identified the piracy cost as reproduction cost and degradation cost. The reproduction cost refers to the price of pirated software and the searching cost; the degradation cost refers to the cost incurred due to lack of manual and customer service and the difficulty of a future upgrade.

2.4.2.2.2 Pricing strategies of digital content providers

Many consumers resort to piracy because they are not willing to pay the high price of the software (Nill et al., 2009). Therefore, proper pricing strategies are essential to providers. Khouja and Smith (2007) built a mathematical model to study the effectiveness of skimming pricing strategies in the music industry under the influences of piracy and saturation effects. In their model, one firm tries to maximize profits by changing prices at specific points in a product life cycle. A consumer purchases one unit when the price is below the consumer's reservation price. The condition under which consumers conduct piracy is not clearly specified. However, the model indicates that purchase only happens when the seller's price is beneath a consumer's reservation price. Therefore, it may imply that the piracy happens when the price is above a consumer's reservation price. The consumers' demand linearly decreases with the price. Results indicate that with piracy and rapid saturation effects, it is better for a monopoly provider to abandon a skimming pricing strategy and use a single price.

Khouja, Hadzikadic, Rajagopalan, and Tsay (2007) built an agent-based model to explore the ideal pricing strategy on music industry under the influences of piracy. There is a seller who controls prices and advertising cost, and the performance of his pricing strategy is evaluated based on profits. There are 10,000 consumers with heterogeneous reservation prices, risk of piracy, probabilities of pirating, costs of piracy, and number of neighbors. The network used is a lattice. The results indicate that piracy makes a skimming strategy the least favored pricing strategy because it introduces more pirates when the initial price is high. In addition, the network externalities also reduce the effects of skimming strategies.

Based on the research of Khouja and Smith (2007), Khouja and Rajagopalan (2008) further studied monopolist pricing strategies in the music/motion picture industry. In this improved model, consumers conduct piracy under two conditions: 1) the price is above the consumer's reservation price, and 2) a consumer has the possibility to conduct piracy even when the price is below the consumer's reservation price. The amount of piracy is proportional to the amount of legal copies at each time period. Khouja and Rajagopalan estimated consumers' demand through an approximating function of the cumulative demand over several time periods. Results conclude that the existence of piracy causes the monopolist to charge a higher price to obtain optimal profits. The price increases with the speed of piracy and the length of the product life cycle. Khouja and Rajagopalan also suggested that a two-price strategy and dual distribution channels might reduce the negative effects of piracy. The two-price strategy attracts more price-sensitive consumers with an initial low price, and shifts demands to earlier periods of product life cycle. The dual distribution channels are brick-and-mortar and online retailers. Khouja and Rajagopalan assumed that products sold through online retailers have a higher risk of piracy because the products have been digitized and consumers are technologically savvy. Therefore, products distributed through online channels are set with a lower price to encourage consumers to purchase instead of pirate. Less technologically savvy consumers purchase products from brick-and-mortar retailers with higher prices.

Jeong, Khouja, and Zhao (2012) built a mathematical model to study the impacts of music piracy on the profits of the entire supply chain, including the musician, the record label company, and the online retailer. A consumer faces two types of piracy cost: a variable cost that increases linearly with the number of songs pirated, and a fixed cost that is incurred regardless of the number of songs pirated. There are two types of contracts between the online retailer and the record company: per song contract, which the retailer pays the record company for each song downloaded, and fixed fee contract, which the retailer pays the record company a fixed fee for the entire album. The results indicate that the optimal price in the presence of piracy is always lower than the price without piracy. In addition, the magnitude of the loss of profits depends on the types of piracy costs. The supply chain generates higher profits when the customer adopts linear piracy cost instead of fixed piracy cost.

Liu, Cheng, Tang, and Eryarsoy (2011) extended the Bass diffusion model to study the optimal pricing of software under the influence of piracy. The results indicate that if the demand from innovators is high, a skimming pricing strategy generates ideal profits; otherwise, a penetration pricing strategy is recommended. In other words, the demand from innovators is a determining factor of the proper pricing strategy. In addition, Liu et al. (2011) also claimed that the rate of conversion from imitators to final adopters has little impact on the pricing strategy. The model also demonstrates some unusual results. For example, it claims that multi-price strategies have no significant advantages over a one price strategy.

2.4.2.2.3 Other marketing strategies of digital content providers

Besides pricing strategies, a variety of marketing strategies have been investigated and tested by scholars. Kwan, Jaisingh, and Tam (2008) conducted research on proper protection strategies of providers. The model establishes a duopoly marketing environment with two types of product differentiation: vertical (firms compete in product quality) and horizontal (firms compete in product features). Results indicate that the cost of developing protection mechanisms is the determining factor of protection strategies. In a vertically differentiated market, the lower-quality product should adopt non-protection strategies. In all other cases, protection strategies should be adopted only when the cost is small. No network externalities are included in the model.

Nill et al. (2009) demonstrated that firms take actions against piracy based on the strategic importance of the software and the tendency to pirate. For example, if the software plays an important strategic role for the firm, and the tendency of piracy is high, the firm needs to take serious actions against piracy. On the other hand, if the strategic importance of the software is high, but piracy tendency is low, then the firm should monitor the market and take precautions to preempt piracy. If the strategic role is low, and the piracy tendency is high, the firm should apply a cost-effective measure to fight piracy. It is not advisable to allocate too many resources to such a software product. If strategic importance is low, and piracy tendency is low, then the firm should just ignore the piracy issue.

Sudler (2013) pointed out that the supply chains of the music and motion picture industries have changed with the arrival of digital media and the World Wide Web. Traditional DRM (digital rights management) not only fails to prevent piracy, but also discourages legitimate buyers and increases the cost of management. Sudler (2013) investigated present marketing strategies that encourage consumers to consume legally. The first strategy is providing free online videos with embedded advertisements. It is more convenient and less risky for consumers to watch videos from a reliable host service than to download them from illegitimate websites. The latter usually comes with various viruses and ad-ware together with the videos. The second strategy is providing consumers with the values and convenience they want. For example, “iTunes” provides access to a vast music repository with a small annual fee. The third strategy is utilizing state-of-the-art piracy detection tools to scan videos online and take down pirated content. For example, a software tool developed by NEC, known as Media Serpia, can scan 1,000 hours of videos in a second.

Jeong and Khouja (2013) built an agent-based model to study impacts of various piracy control strategies on consumers, retailers, record companies, and musicians. Legal strategies, education strategies, low-price strategies, and value-added strategies were experimented with. The results show that education strategies are effective to deter piracy. The low-price and value-added strategies attract more legal buyers. The results suggest that ideal piracy control strategies are a combination of a legal or education strategy with a value-added or low-price strategy.

2.4.2.2.4 Influence of government policies

In addition to the various marketing strategies of providers, government copyright protection policies also play an important role in digital piracy. Chen and Png (2003) developed a model that is used to study the relationship between software pricing and copyright enforcement policies. The model incorporates a provider, a government, and consumers. The monopoly provider determines the prices of software and the costs of detecting piracy. The government sets fines for piracy, taxes on copy media, and subsidies for legitimate purchases. Consumers are divided into ethical and unethical groups. Consumers in the ethical group never pirate the software, and consumers in the unethical group pirate the software if the benefit of doing so outweighs the benefit of purchasing. From this model, Chen and Png drew two conclusions: 1) For a provider, reducing prices is better than increasing detection costs, and 2) for a government, taxing copy media is more effective than fining piracy.

Banerjee (2003) explored the government's role in restricting commercial piracy in a software market. Banerjee examined optimal piracy monitoring rates and the penalty of government policies, and ideal pricing strategies of providers. In his model, there is a provider, a pirate who sells illegal copies with deteriorated quality, consumers with heterogeneous evaluation of the software, and a government agency to monitor and fine pirates once detected. The government does not punish consumers who use pirated copies. The provider, in order to compete against the pirate, utilizes three pricing strategies, Bertrand, leader-followers, and monopoly pricing. Models with or without network effects are also analyzed. Inspired by Banerjee's model, Jaisingh (2009) built a model with similar settings. However, Jaisingh's model explored the impact of government policies on both the quality and the prices of software, whereas Banerjee only examined impacts of government policies on software prices.

2.4.2.3 Positive influences of digital piracy

Some scholars have recognized positive influences of piracy, such as benefiting a new product's diffusion process, increasing market share, and finally helping providers establish a dominate position and high profits. Conner and Rumelt (1991) pointed out that piracy is a more efficient "gift-giving" method than mailings of free software to consumers because consumers, not the producer, carry the costs of pirated gifts. Givon, Mahajan, and Muller (1995) analyzed positive impacts of piracy by extending the Bass diffusion model. The model tracks the diffusion due to both piracy and legal sales. Their results indicate that six out of seven consumers use pirated software, but these pirates are responsible for generating 80% of new software buyers. On the other hand, Givon et al.

only addressed how piracy affects the size of a consumer base, not other marketing factors, such as prices and costs. An increasing market share is not necessarily equal to increasing profits. Givon, Mahajan, and Muller (1997) extended the Bass diffusion model by incorporating competing software brands: Lotus 1-2-3 versus other spreadsheet products, including Microsoft Excel. The model indicates that in the presence of piracy and possibilities of brand switching, firms that can take advantage of the user base generated by piracy is the winner. In other words, it implies that the failure of Lotus 1-2-3 was caused by its lower tolerance of piracy.

Besides accelerating new product diffusion, Choi and Perez (2007) claimed that online piracy benefits the invention of new technologies and creation of new legitimate businesses. Choi and Perez reviewed the history of online piracy, focusing on the development of Napster and BitTorrent. They concluded that contributions of online piracy include: 1) pioneering in the use of new technology, for example, peer-to-peer file sharing technology; 2) providing marketing insights into the needs of consumers; 3) creating new markets, for example, previous Napster customers become new customers of iTunes; and 4) spurring the creation of new legitimate business models. Peitz and Waelbroeck (2006) and Gopal, Bhattacharjee, and Sanders (2006) developed models to study the piracy of online music. Results indicate that online music sampling leads to piracy, but at the same time it encourages more consumers to explore unknown songs and artists, and increases the sales of legal music copies.

2.4.2.4 Utilizing positive influences of digital piracy

Scholars not only realize the positive impacts of piracy, but also consider how to control properly the amount of piracy in order to maximize benefits. Prasad and Mahajan (2003) developed another extended Bass diffusion model. It examines the relationship between the speed of a diffusion process and other marketing factors, including prices, degrees of tolerance of piracy, protection costs, and market entrance time of each version for multiple-generation software. The model demonstrates that proper controls of piracy at the different stages of a product life cycle are able to maximize both market shares and profits of a provider. The model analyzes three cases: monopoly, multiple generations under a monopoly, and duopoly market conditions. From this analysis, Prasad and Mahajan drew the following conclusions. First, under the monopoly scenario, a provider should start with minimum protection. After the diffusion reaches the peak, the provider should impose a maximum protection for the rest of the diffusion cycle. Secondly, under the multiple generation scenarios, the tolerance level of the previous generation is closely related to the expected profit margins of the following generation. Thirdly, under the competitive scenario, results were briefly analyzed, and Prasad and Mahajan only suggested that the providers should set the protection level lower versus the protection level in the monopoly environment.

Haruvy, Mahajan, and Prasad (2004) examined how piracy affects the diffusion process of subscription software. In this model, the subscription software still functions with certain disabled features after an expiration date. Consumers could choose to pay to renew the subscription, pirate, or not renew at the end of each time period. The model demonstrates that the optimal combination of protection levels and prices facilitates faster adoptions and higher prices for the subscription software. It also points out that tolerance of piracy will be less profitable when piracy control is costly, information is precise, penetration is quick, externalities are low, future profits are greatly discounted, customer inertia is low, or its product life is short.

2.5 Reviews of impacts of competition

So far there are not much research done on studying the competition involved with diffusions of innovations and digital piracy.

2.5.1 Impacts of competition on diffusion of innovations

Kim and Hur (2013) built an agent-based model to study the competing innovations in a small-world network. Two new products are introduced to the market at the same time and compete for adopters. There are two types of consumers: influentials and followers. An influential makes purchasing decisions based on his own preference, and a follower makes a decision according to opinions of other followers and influentials. Kim et al. (2013) experimented with different proportion of influentials and followers to explore the impacts on adoptions. The results revealed that proportion of influentials and followers is crucial to the adoptions of two products. If both the amount of influentials and followers are small, two products take almost equal market shares. If the amount of influentials is small, but followers is large, the diffusion presents a lock-in status, which means one product take over the entire market. If the amount of influentials and followers are both large, the diffusion process is random, and the results of competition is inconclusive.

Pegoretti, Rentocchini, and Marzetti (2012) analyzed how the different information regime and network structures affect the diffusion of innovations in both a monopoly and a duopoly market. The model is an agent-based model in a small-world network. There are two types of information regimes: perfect information and imperfect information. In the case of perfect information, a consumer is aware of all existing brands and make purchasing decision based on the choices of his neighbors. In the case of imperfect information, a consumer only realizes the existence of other brands through his neighbors' choices. In a monopoly market, the simulation results revealed that, with perfect information, the innovation diffuses faster in a random network; with imperfect information, the innovation diffuses faster in a small-world network. In a duopoly market, with imperfect information, the winner take-it-all (one innovation takes over the entire market) happens in a small-world network; with imperfect information, the winner take-it-all happens in a random network.

Stummer, Kiesling, Gunther, and Vetschera (2015) built an agent-based model to study the diffusion of four competing brands of fuels. The model incorporates rich details of products, points of sales, consumers, and social networks. The market starts with conventional fuels without brands and conventional fuels with brands, then introduced the conventional premium at later stage, and introduced the biofuel at last. The adoption curve of each brand is plotted to study the brand switching behaviors of consumers. The results revealed that adoption curves of three conventional fuel are declining when biofuel are introduced to the market. The biofuel takes away consumers of conventional fuel. The model predicts that the biofuel will replace the traditional conventional fossil fuels.

Gruber (2001) adopted the epidemic diffusion model to study the competition and innovations of mobile telecommunication in the European market. The model concludes that increasing the number of firms accelerates diffusions. The simultaneous entries of multiple firms diffuse faster than the sequential entry of firms. However, in this paper, Gruber did not discuss the competition among multiple firms. His definition of competition is equivalent to multiple firms co-existing in the same market.

2.5.2 Impacts of competition on digital piracy

Most research on positive impacts of piracy indicates that strong network effects are required. Does piracy still pose positive impacts without network effects? Gu and Mahajan (2004) studied the impacts of piracy in a duopoly market without network impacts. There are two types of consumers: poor consumers and rich consumers. The model revealed that piracy could be beneficial even without network impacts if there is a significant difference between the marginal utility of money of the two types of consumers. Poor consumers are price sensitive, and therefore piracy is very attractive to them. Remaining consumers in the market are not price-sensitive, and without a price war, a firm’s profit increase despite of the total consumers in the market is decreasing. Piracy removes poor consumers from the market; therefore, it reduces the possibility for the firm to involve with the self-destructing price war. A price war is “commercial competition characterized by the repeated cutting of prices below those of competitors" ("Price war," 2017).

Shy (1999) built a classical mathematical model to study the price competition in a duopoly market with network effects. The network-externality is modeled using a classical Hotelling spatial model. Results show that if the network effect is weak, a high-price equilibrium emerges; if the network effect is strong, a low-price equilibrium emerges. Therefore, for weak network effects, both firms choose to protect their product; for strong network effects, both firms choose non-protection.

Most times the retailer faces the competition not only from piracy services, but also from peer retailers simultaneously. Geng and Lee (2013) built a model in which consumers could obtain digital products from a legitimate channel that contains many retailers, or a piracy channel that contains many piracy services. The model revealed that the strategies for retailers to deal with piracy services depends on the level of competitions among retailers. If the competition among retailers is strong, it implies that the retailers are already involved in pricing wars (lowering the price to attract consumers). Under such condition, the legal products are almost no different from pirated products with regard to price. Therefore, the piracy has no impact on profits of retailers. If the competition among retailers is medium, it is beneficial for retailers to give away some consumers to piracy. By keeping the consumers who are less likely to conduct piracy, a retailer is able to charge a higher price. If the competition among retailers is weak, the retailer also loses customers to piracy service and, therefore, a retailer has to lower the price to fight the piracy service.

Chang and Walter (2015) built a classical mathematical model to study the price-quality competition between a legal retailer and P2P network that offer free download of pirated product. In their model, the legal retailer determines the price of the product, and then the P2P host determines the investment for improving the quality of the download of the free product. The profit of P2P networks come from advertisement. The analysis concludes that whether a retailer is able to gain profits and market share depends on the degrees of horizontal differentiation (differentiation of product features) of the product. A retailer is not able to defeat P2P network based on product vertical differentiation (product quality).

Martínez-Sánchez (2011) utilized a classical mathematical model to study the possibility of price collusion between competing software firms under the influences of piracy. The model concludes that firms only collude on prices if they value future profits sufficiently, that is, there is a high discount factor. The model also concludes that the low cost of making pirated copies facilitates collusion, and high quality of pirated copies hinders collusion. Hence the cost of making a pirated copy is almost zero and the quality of pirated copy is not deteriorated, the overall effects of piracy on price collusion is uncertain. The model does not make meaningful contribution largely because price collusion is a criminal offence. The definition of the price collusion defined by the Business Dictionary online is “Criminal offense where numerous companies work together to keep the price of a product or service at an elevated level with the goal of receiving large profits or cornering the market” (“What is price collusion,” 2017).

2.6 Limitations of existing research on diffusion of digital information goods

Some issues are not sufficiently studied in the existing research on diffusion of digital information goods.

2.6.1 Insufficient research on effective marketing strategies

There has been little research conducted on developing effective marketing strategies to actively manage new products diffusion process. While a variety of promotion strategies, pricing strategies, and product feature choices have been investigated, they are typically looked at individually. The majority of research on pricing strategies of diffusion utilize mathematical models, for example, the Bass diffusion model. Mathematical models are not able to handle the heterogeneity of consumers, non-linear interactions among consumers, dynamic marketing environment, and the social network. Therefore, its applicability and values of results are reduced. Agent-based diffusion models concentrate exclusively on promotion strategies and neglect other marketing factor, such as prices and product attributes (Kiesling et al., 2011). For example, they consider finding the right consumers for effective advertising campaign.

There are few research efforts that develop classical mathematical models to explore effective marketing strategies that use digital piracy to accelerate diffusion speeds and increase profits. No scholars have considered developing dynamic and self-learning marketing strategies to effectively manage the diffusion process and leverage the key agent to increase profits and speed up the diffusion process.

2.6.2 Insufficient research on the diffusion of digital information goods

Although digital information goods possess certain unique features, for example, the low duplication cost, low distribution cost through the internet, and no warehouse storage required if distributed through internet, there is very little research conducted on how those features change the diffusion patterns or speed. The near zero duplication cost makes digital information goods vulnerable to digital piracy. Piracy could change the diffusion patterns of digital information goods. Although there is some research on digital piracy, only a few research efforts indicate that piracy could accelerate the diffusion of innovations. That research work utilizes classical mathematical models which reduces the generality of the model and the values of the its output. The majority of research on digital piracy focuses on helping providers survive under the influence of piracy by tuning critical parameters of purchasing behaviors of consumers, marketing strategies of providers, and policies of governments. Digital piracy is a problem that every provider has to face when launching a new product. However, most scholars have failed to relate piracy to the diffusion of a new digital information goods.

2.6.3 Lack of explicit use of social networks

Although there is extensive research on the diffusion of generic products using agent-based model and various social networks, the few existing research efforts on digital piracy utilizes classical mathematical models that do not make explicit use of social networks. It is common for mathematical models to represent potential adopters as a large and homogeneous pool, for example, so-called "mean field" models with "well-mixed" populations. As mentioned in section 2.6.2, the majority of the research on digital piracy failed to relate the piracy with the diffusion of new products, and consequently, they do not realize the impacts of social network topologies on the spreading of piracy and the diffusion of the new products.

2.6.4 Insufficient research on competitive environment

The majority of research on digital piracy and diffusion of innovations concentrates on monopoly industries. Very little research analyzes the impacts of digital piracy in a duopoly market. However, in reality, there are usually several competing digital products with similar functionalities that co-exist in the market.

In addition, as mentioned in 2.6.3, the research on the diffusion of digital information goods does not use a social network. The same problem also exists in the research on studying competition.

2.6.5 Limitations of classical mathematical approaches

As discussed in chapter 1.4, classical mathematical approaches are founded on the rational choice theory. This theory relies on several assumptions: perfect rationality, complete information, and common expectation. The rational choice theory is widely used due to its tractability and simplicity. However, such simplicity comes with a price, that is, it is not able to handle heterogeneity of individuals, non-linear interactions, social networks, and dynamic environment.

First, classical mathematical approaches ignore the fact that every consumer in the market is unique. Each consumer possesses different attributes and takes different actions according to the same marketing stimuli. In the Bass diffusion model, adopters are divided into two groups, innovators and imitators. The model implies that 1) there are only two types of consumers, and 2) inside each group, every consumer is the same as other consumers. Khouja and Smith (2007) utilized a simple linear demand function, which implies that all consumers will behave the same way when prices change. Chen and Png (2003) divided consumers into ethical and unethical groups based on the same assumption. Furthermore, valuable information is lost during the process of segmentation. In fact, there is no clear boundary between segments. Consumers' ethical values and preferences lie on a continuum. Many consumers may purchase some legitimate software and make pirated copies for some other software. This big portion of the market is not included in the existing model.

Second, classical mathematical approaches can not incorporate a social network explicitly in the model. A typical example is the Bass diffusion model. In the Bass model, every consumer can contact all other consumers, which implies the social network is a complete network. A complete network is not a realistic social network.

Third, classical mathematical approaches assume that a marketing environment is static. A typical example is the Bass diffusion model. The value estimation of external and internal coefficients is critical for the accuracy of the Bass model's predictability. However, these values are estimated using data from several sales periods in the past (Lilien, Rangaswamy, & Bruyn, 2007). Using such values to predict the sales and diffusions of a new product in the future depends on the assumption that the marketing environment will never change. Classical mathematical approaches work well in a relatively homogeneous and static environment. They can not handle the heterogeneity of individuals and a constant changing marketing environment.

2.7 Focus of my dissertation

Section 2.6 addresses limitations of existing research on digital piracy, including 1) insufficient research on the diffusion of digital information goods, 2) insufficient research on effective marketing strategies, 3) lack of explicit use of social networks, 4) neglecting a duopoly marketing environment, and 5) limitations of classical mathematical approaches.

This dissertation eliminates those limitations and accomplishes the following: First, I build models to simulate the market of digital information goods as a complex adaptive system. Second, I develop dynamic and self-learning marketing strategies in the context of potential adopters connected via social networks. Those marketing strategies are able to adjust dynamically to various marketing factors, including prices, promotion costs, and/or piracy detection costs through an entire diffusion process. The resulting strategies would enable digital content providers to leverage the key agent to speed up new product diffusion and increase profits in both monopoly and duopoly environments. Third, I model diffusion processes in realistic social networks – small-world network – explicitly representing each consumer as a vertex in the network. Fourth, I analyze the impacts of the duopoly marketing environment on providers' strategy-making. Fifth, I explore other modeling methodologies that overcome the shortcomings of classical mathematical approaches.

In summary, my models explicitly represent diffusion processes between individual consumers using social networks and agent-based models, and determine effective marketing strategies for digital content providers via learning classifier systems. I have not found any research work that considers all those factors together.

This dissertation answers the following research questions: What is the impact of network topologies? Does the position of a key agent in a network matter? What are the characteristics of successful marketing strategies? How do a monopoly and duopoly marketing environments differ with respect to above questions?

2.8 Modeling assumptions

I built two groups of models, monopoly models and duopoly models, to simulate a market of digital information goods. The models include the following features. First, the provider gains revenue from selling its product. Second, the digital information goods provided by each provider have similar functionality, but they are not compatible with each other. This guarantees that providers compete in the same market. To study the competitive marketing environment, my dissertation starts with two competitors, but is open to more competitors in further research beyond this dissertation. Last, the digital good is a brand-new product, thereby preventing bias from the existing consumer base.

CHAPTER 3: METHODOLOGY

There are two important components that need to be modeled: marketing strategies and consumers. How can these components be modeled?

3.1. Modeling of marketing strategies

The dissertation proposes to develop effective marketing strategies which will utilize various marketing factors, including prices, promotion costs, and the piracy detection cost to achieve ideal profits and market shares. What is a marketing strategy? A marketing strategy is making the judgement of the current marketing situation through feedback, and taking action by adjusting all the available marketing factors to achieve the goal. In other words, a strategy helps the provider respond correctly to various "what if" situations in the market. The learning classifier system (LCS) provides an ideal modeling technique to develop such strategy. LCS is a rule-based machine learning algorithm which adopts learning algorithms to learn and evolve good rules. A rule in the LCS is in the form of "IF condition THEN action". Evolutionary algorithms (EA) are often used as the learning algorithms in the LCS because of the ability of EA to perform "rule discovery". That is, it is able to evolve new rules based on the existing rules. No other learning algorithm possesses such features. In order for a reader to have a good understanding of the LCS, I need to introduce evolutionary algorithms before providing the details of LCS.

3.1.1 Evolutionary algorithms (EA)

Evolutionary algorithms are machine learning algorithms. EA adopt ideas from Darwin's natural selection theories. EA contain these basic elements: a parent population, an offspring population, a parent selection method, an offspring survival selection method, reproductive operators, and methods of representing individuals (De Jong, 2006).

EA start with an initial population of individuals. Each individual represents one possible solution. The individuals in the initial population are created randomly. Individuals interact with each other and their environment. The performance of an individual is evaluated through a fitness function, and the result of the evaluation (fitness value) is assigned to each individual. A selection method is utilized to choose individuals to become parents based on their fitness values. The role of parent selection is to allow better individuals to reproduce offspring. Those chosen parents create an offspring population using reproductive operators. The offspring bear close resemblance to their parents, but also have their own variations. After the offspring population is evaluated, the offspring survival selection is utilized. The selected offspring population becomes the next population. During such evolution process, individuals with higher fitness will have better chances of mating and reproducing offspring, but individuals with lower fitness will eventually become extinct. The above process repeats until certain number of generations reached or when there is no significant improvement on fitness values of the population. In his book, *Evolutionary Computations*, Kenneth De Jong gives a comprehensive review on important subjects in EA. I will now discuss each of them in turn.

3.1.1.1 Representation

In order to solve a problem in the real world, we need to map the real world into the EA world first, that is, information in the real world need to be encoded into a solution. A solution is usually a string of values, or more complicated data structures, for example, a tree or a graph. A solution needs to be encoded using either genotypic or phenotypic representations. Solution in the original problem context is referred as a phenotype, and its binary encoding is referred a genotype. A mapping between genotypic and phenotypic representation is required. For a phenotypic representation, the reproductive operator has to be problem-specific. The mutation and crossover operator has to be designed to fit the problem-specific requirement. For a genotypic representation, the productive operator does not have to be problem-specific. However, the reproductive operator may produce invalid values which do not fit the problem domain.

3.1.1.2 Reproductive operators

There are two types of reproductive operators: mutation and crossover (recombination). A mutation is a stochastic one-parent reproductive operator. An offspring is produced by cloning the parent first and then making some small variations by modifying certain genes in the parent. For a genotypic representation, the mutation operator switches the values between 0 and 1 for selected genes. For the phenotypic representation, small random changes are made to the selected genes. A crossover is a stochastic two-parent reproductive operator. Two parents create an offspring by copying and recombining certain subcomponents of their own. There are different types of crossovers which vary on crossover cutting points and the amount of cutting points. Through mutation and cross-over, new solutions are created and those new solutions bear resemblance to their own parents with their own variations. The “rule-discovery” mentioned in the LCS comes from the reproductive operators. It is not necessary to use both crossover and mutation operators. The selection of the reproductive is problem specific. Figure 3 demonstrates an example of two-point crossover. The arrows point at the cutting points.

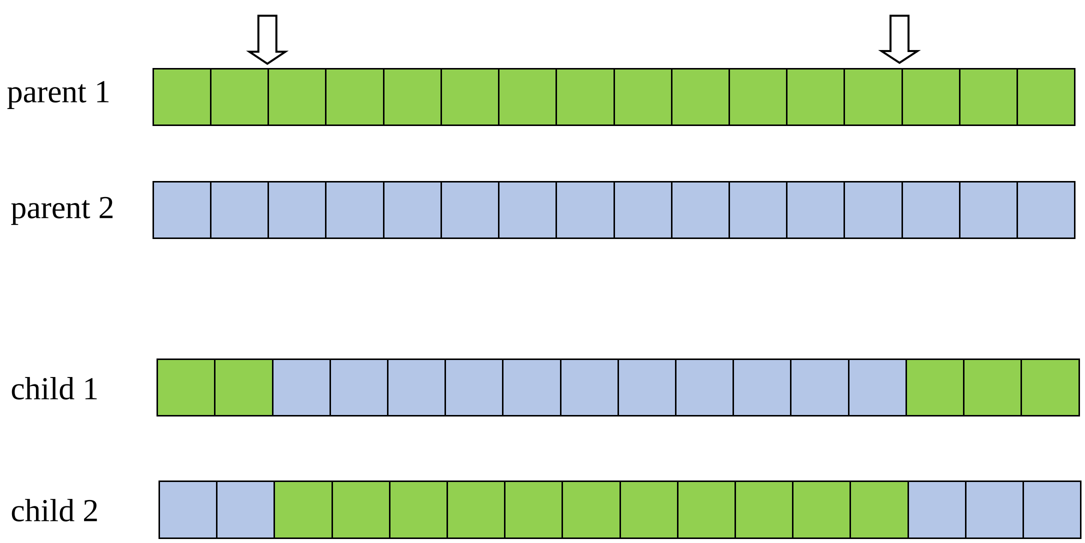


Figure 3: Demonstration of 2-point crossover

3.1.1.3 Selections

The parent and offspring survival selections guides the evolution process to gradually filter out individuals with bad fitness values. The most well-known selection algorithms are: uniform, fitness proportional, ranking, tournament, and truncation. Uniform selection selects individuals randomly. In a fitness-proportional selection, each individual is assigned a probability, $fitness_i / fitness_{sum}$. The individual which has a higher fitness value has a better chance to be selected. In a ranking selection algorithm, the individuals are ranked based on their fitness values. The individual with the lowest fitness is ranked first, and the individual with the best fitness is ranked last. The ranking selection resolves a problem in the fitness proportional selection, that is, the fitness value of individuals differs too much. In a tournament selection, k individuals are selected randomly for competition, and the winner is selected. In a truncation selection, the individuals are sorted based on their fitness values, and the top N are selected. The selection pressure of each algorithm is different. The ranking from weakest to strongest is (De Jong, 2006):

- uniform
- fitness-proportional
- linear ranking and binary tournament
- nonlinear ranking and tournament with $k > 2$
- truncation

The selection pressure is the pressure to survive. For the truncation selection, the less fit individuals have less chance to survive, thus the selection pressure is strong. On the other hand, for the uniform selection, the less fit individuals have much better chance to survive due to the random selection. So uniform selection has weak selection pressure.

If the selection pressure is too strong, it is likely that the EA converges too fast to a sub-optimal solution. On the other hand, a selection with weaker selection pressure is able to add diversity to the population of individuals. When choosing selection algorithms, the selection pressures of the parent and offspring need to be counterbalanced. For example, if the parent selection uses a uniform selection, then the offspring survival selection could use a truncation selection.

3.1.1.4 Exploration vs. exploitation

In EA, the exploitation searches for good solutions among existing solutions, for example, the parent and offspring selections. The exploration looks for new solutions, for example, the new solutions created through mutation and crossover. Similar to the issue of keeping the balance of selection pressures, it is important to keep a balance between exploration and exploitation, otherwise, the it is likely the EA will converge to a sub-optimal solution. For example, a strong selection pressure is counterbalanced with more explorative reproductive operators.

3.1.2 Learning classifier systems (LCS)

There are two main types of learning classifier systems (LCS). One is the Michigan classifier system (Holland and Reitman, 1977), and the other one is the Pittsburgh classifier system (Smith, 1980). These two algorithms are the archetypes of LCS. Many LCS today are the variations or customizations of these two algorithms which try to resolve domain-specific problems. Most variations focus on the Michigan LCS because it is considered as a standard LCS.

In the Michigan classifier system, each individual classifier in the population represents one single rule (IF condition THEN action). The EA operates at the level of an individual rule. The final solution is a population of rules which work collaboratively. For a set of cooperative rules, how to assigning the credit to an individual rule is problematic. In 1985, Holland introduced a credit-assignment algorithm, the bucket brigade algorithm (BBA) (Holland, 1985). BBA determines which rule to select and how the payoff is distributed among rules. The algorithm works as below.

The Michigan LCS contains a message list which is updated with new messages. The classifier system will be notified by the arrival of new messages, and then the rules whose condition matches the messages are triggered. Usually there more than one rule is triggered. If there are multiple rules fired, a bid is posted by each rule. The bid is proportional to the fitness value of the rule, therefore, a rule with higher fitness has a better chance of being selected. The selected rule posted his message on the message list, and pay his bid as a reward to those rules responsible for sending messages that matched the condition of the bidding rule. After each rule acquired the credit (fitness value), the evolution of rule set starts (Holland, 1985).

In the Pittsburgh classifier system, each individual classifier in the population is a rule-set, that is, a collection of "IF condition THEN action". Each individual rule-set is a potential solution. One advantage of the Pittsburgh classifier system is its simplicity of credit assignment. The credit is assigned to the entire rule-set as opposed to an individual rule, therefore, no bucket brigade algorithm is needed.

Which approach is better? There is rarely any discussion on this topic. Based on the comment from De Jong (1988), “It is too early to answer this question or even to determine if the question is valid. A popular view is that Michigan classifier system is more useful for an online, real-time environment, and radical changes in behaviors could not be tolerated. The Pittsburgh classifier system is more useful for offline environment and radical changes in behaviors are acceptable.” In addition, the Michigan LCS is complicated due to BBA, and the Pittsburgh LCS is computationally expensive due to the long classifier. Only Wilson and Goldberg (1989) pointed out two problems in the BBA. One is that it is hard to maintain a long bucket brigade chain. In a BBA, a rule in the later stage could be trigger by many sequence of rules, and BBA needs to update credits of all of them. As pointed out by Wilson, the amount of updates for a bucket of n members need to be in the order of 10n. The other problem is that both earlier rules and later rules in the same rule chain get involved in the evolutionary competition for survival. Wilson et al. (1989) indicated that the earlier rule tends to be weaker due the payoff reaching earlier rule less than the later rule, as a result, the earlier rule tends to be removed from the population. However, the later rule replies on the earlier rule. Their conclusion is that Michigan LCS works better for a short chain of rules.

There are many variations of Michigan classifiers which were developed in later years which either try to simplify the bucket brigade algorithms or make the classifier tailor-made for more specific problems. With the development of Reinforcement Leaning

(RL) algorithms in 90’s, researchers realized that BBA is essentially same as the Q-learning algorithm. Wilson (1994) developed a zeroth-level classifier (ZCS) by introducing the Q-learning into the Michigan LCS. ZCS removes the message list and the bidding process in BBA, and replaces BBA with a Q-learning algorithm (QBB). Based on ZCS, Wilson (1995) introduced XCS that measure the fitness of a rule based on its prediction accuracy. Urbanowics and Moore (2009) provided a list of all the LCS which are the variations of Michigan LCS.

I chose to use the Pittsburgh LCS for my models due to its simplicity and also due to the fact my research evolves strategies offline. In addition, Wilson et al. (1989) pointed out two problems of BBA. Q-learning is essentially same as BBA, so even the new Michigan style LCS who adopted Q-learning may have the same problems as mentioned by Wilson et al. (1989).

3.2 Modeling of consumers and social networks

Due to the heterogeneity of consumers, the non-linear interactions among consumers, and the dynamic marketing environment, as discussed in chapter 1.4, 1.5 and chapter 2.6.5, the ideal modeling methodology for consumers is agent-based modeling. Inside the agent-based model, each consumer is represented by an agent. Agent-based modeling allows a natural representation of an agent by incorporating rich details of the agent’s real-world counterpart; it explicitly models non-linear interactions among agents; it is a bottom-up approach that contrasts with classical mathematical approaches that start at aggregation levels. Outcomes emerging from micro-levels reveal rich information of both agents and their environment.

Chapter 2.1.3 introduced four different types of social network topologies: a regular network, a random network, a small-world network, and a scale-free network. Which network topology should be used to represent the social network of my model? Before I answer this question, I need to introduce a concept: a small-world phenomenon or six degrees of separation. The six degrees of separation indicates that a person is able to connect to any other persons on earth with no more than six intermediaries. Stanley Milgram in the 1960s conducted an experiment. He asked participants to forward a letter to a target person – a stranger, near Boston. Each participant, however, is required to forward the letter to only one acquaintance. The results show that the median of the length of the delivery chain is six. Some criticize that the that the result is inconclusive because sample size used by Milgram is too small. In 2001, Duncan Watts, a professor at Columbia University, conducted the same experiment on the Internet. He asked the 48,000 participants to forward an email to 19 targets in 157 countries. The data collected showed that the median of the number of intermediaries are indeed six ("Six degrees of separations," 2017).

Watts et al. (1998) proposed a network with this small-world property. A rewiring probability is an important attribute of the social network. It controls the randomness of the network. The randomness of the network increases as the rewiring probability increases. If the rewiring probability is zero, then the network is a ring lattice (regular network). If the rewiring probability is one, then the network is a random network. The rewiring probability of a small-world network is between 0.01 to 0.1. As shown in Figure

4, if the rewiring probability is above 0.1, a random network is created. If the rewiring probability is below 0.01, a regular network is created.

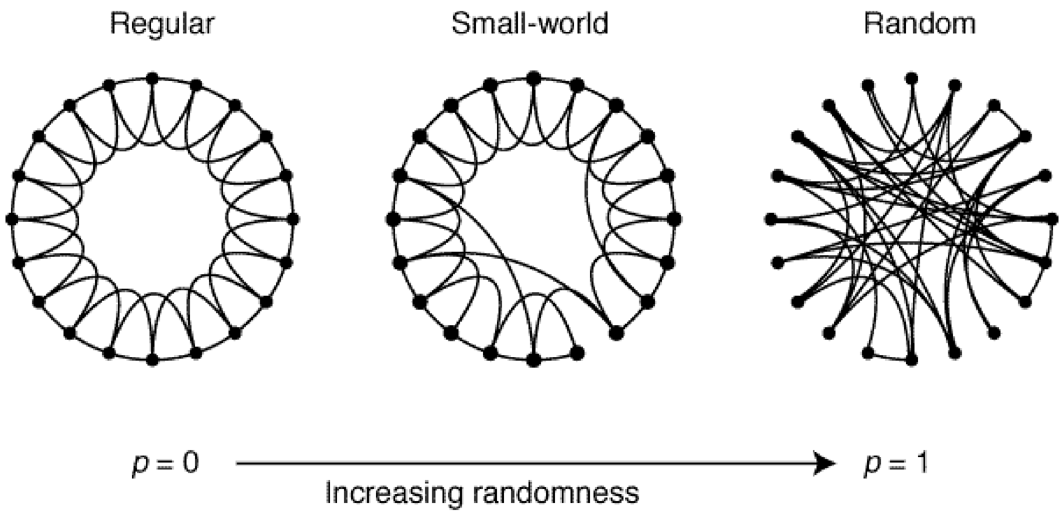


Figure 4: Regular, small-world, and random networks (Watts and Strogatz, 1998)

The small-world network is created by following simple steps: Start with a ring lattice (regular network), if the random number generated for a node is smaller than the rewiring probability, then the rewiring happens, and destination node is chosen randomly from nodes other than the starting node. The same task is repeated for every node in the network. The final network is a highly clustered subnetwork of acquaintances and a collection of random long-range shortcuts. In this network, most nodes are not neighbors of one another, but most nodes can be reached from each other by a small number of steps. The small-world network is the closest representation of the social interactions. As a result, a small-world network is utilized as the social network in my model.

The degree in a scale-free network follows the power-law distribution, which indicates that a few nodes in the network are hubs which have a large number of connections. Watts (2007) defines those hubs as hyperinfluentials and indicates that a person who is able to influence such a large number of people is not realistic in personal communication. The closest counterpart to a hub in life is the public media, for example, a web site or a video broadcast. The random network is modeled after biological contagion, for example, the spreading of an epidemic. The difference between biological contagion and social contagion is the process of “infection”, as indicated by Kleinberg (2010). In the social contagion, the person makes decision to adopt the new product or not. However, in the biological contagion, there is no adoption decision involved, and the process of infection is very complicated. The randomness models, in an “abstract way”, the complex process of infection (Easley & Kleinberg, 2010).

Although a scale-free network and a random network are not suited for the personal communication in my model. I still decide to explore and experiment my models on these two types of networks. However, those are not the focus in this dissertation. The real focus is still the small-world networks.

3.3 Applying learning classifiers to agent-based models

An agent-based modeling is the appropriate approach for modeling heterogeneous consumers. I also decided to use evolutionary algorithms to model the dynamic and self-learning marketing strategies. How does one combine evolutionary algorithms with agent-based modeling? To do so, three methodologies are generally utilized.

The first methodology is introduced in the Embodied Evolution model by Watson, Ficici, and Pollack (2001). The model consists of a population of agents without central control. A population of agents is equal to a population in EA. The parent selection, reproduction, and survival selection are carried out among agents. The second methodology is introduced by Duong and Grefenstette (2005). In this model, each agent is represented by a population in EA. The parent selection, reproduction, and survival selection are carried out inside the population of each agent. In other words, each agent has an EA running inside his or her head. The third methodology is to evolve EA offline. There is an EA population, and there is also an agent-based model. The strategy in the EA population is injected into the agent-based model, and the agent-based model functions as the fitness evaluation function. As discussed in chapter 2.2.2, Alkemade et al. (2005) utilize the same methodology to evolve effective promotion strategies. My research utilizes the third methodology to build models.

3.4 Applying evolutionary algorithms to a duopoly marketing environment

3.4.1 Coevolutionary algorithms (CoEA)

In biology, “Coevolution is the process of reciprocal evolutionary change that occurs between pairs of species or among groups of species as they interact with one another” (Rafferty & Thompson, 2014). Species involved within the coevolution apply selection pressure on the other species and force each other to adapt and evolve. A typical example is the arm race between predator and its prey.

Coevolutionary algorithms (CoEA) are inspired by the coevolution concept in biology. In CoEA, the fitness of an individual in a population is affected and evaluated by the presence of individuals in the same or another population. The CoEA is either competitive coevolution and or cooperative coevolution.

Luke (2013) summarized and explained three modeling techniques of CoEA:

- 1-population competitive coevolution
- 2-population competitive coevolution
- N-population cooperative coevolution

In a 1-population competitive coevolution, each individual competes against the remaining individuals in the same population, and his fitness value is evaluated by the result of the competition. An individual could compete against only one other individual, but the results is probably very noisy. For the sake of robustness, an individual could compete against every other individual in the population. However, if the same test is conducted for every individual in the population, there are way too many tests that need to be conducted. So a k-fold algorithm is proposed as a middle ground. An individual competes against randomly selected k individuals in the same population. The fitness value could be either the score of the game or the count of the winning game.

In a 2-population competitive coevolution, one population, P, is the primary one, which contains the individuals we are trying to optimize, and the other population, Q, works as a test case, referred as an alternative population. The coevolution of P and Q could be either in sequential or in parallel. In the sequential algorithm, first, every individual in P competes against every (or K-fold) individual in Q, then P breeds;

secondly, every individual in Q competes against the new P, then Q breeds, and so forth. The problem of this algorithm is that Q competes against new P, always one step behind. The fitness evaluations of P and Q are not synchronized. A better alternative is the 2-population parallel competitive coevolution. In the parallel coevolution, P competes against Q, and Q already acquired fitness through competition with Q, then both P and Q breed and evolve independently in parallel.

An example of a robotic soccer team is introduced to explain the N-population cooperative coevolution. Each soccer player is represented as a population of behaviors, therefore, there are 11 populations of behaviors. An individual from one population is tested by playing a match with randomly selected individuals from other 10 populations as a team. The result of the match is the fitness value of the individual. The same operation is conducted for each individual in the 11 populations. And then each of the 11 population breeds and evolves independently in parallel.

In my dissertation, two providers compete against each other in the same market. Therefore, the 2-population competitive coevolution is an ideal methodology to model two competing providers.

CHAPTER 4: MONOPOLY – SYSTEM CONFGURATIONS

This chapter gives a detailed description of the monopoly model, including the design and configurations of the consumer and the digital content provider.

4.1 Model description

In this model, there is one provider and a large number of consumers. The provider is in the form of a population of marketing strategies. A consumer is modeled as an agent in the agent-based model. Among consumers, one is an innovation initiator (trendsetter) who possesses the product originally. The agent-based model acts as a fitness evaluation function for the marketing strategies. The social networks considered are a small-world network, a scale-free networks, or a random network in which each node represents a consumer in the agent-based model. The focus is the small-world network.

There are four types of monopoly models. In the first type of model (Model 1-1), there is no piracy. The provider "bribes" a consumer – a key agent, with a free digital information good. The key agent is an average consumer who has no influence on the adoptions decisions of other consumers. In second type of model (Model 1-2), there is still no piracy. The key agent appointed by the provider is a "persuader" (Gladwell, 2000) who is not necessarily highly connected in the social network, but is able to impact the adoption behaviors of other consumers.

In the third type of model (Model 2-1), digital piracy exists. The key agent is a pirate, and remaining consumers are legal buyers who do not conduct piracy. The provider does not provide a free product to any consumer, instead the gift-giving activity is replaced by the piracy. The pirate gets the free digital information goods through piracy, as indicated by Conner et al. (1991). They pointed out that piracy is a more efficient “gift-giving” method because the costs of gifts are carried by consumers, not providers. In Model 2-1, the key agent is an average pirate who has no influence on the adoption decision of other consumers. In the fourth type of model (Model 2-2), the digital piracy exists and the key agent is a persuasive pirate. In Model 2-1 and Model 2-2, the piracy detection cost is an important factor in the marketing strategy of a provider. There is no piracy detection cost in the marketing strategy in Model 1-1 and Model 1-2.

4.1.1 Design of consumers – Agent-based model

A consumer is represented as an agent in the agent-based model. A consumer has a homogeneous/heterogeneous reservation price, promotion acceptance threshold, or/and piracy detection threshold. Only one consumer - innovation initiator, possesses the product initially. A consumer only interacts with directly connected neighboring consumers. A consumer is allowed to possess at most one product.

A consumer is only aware of the product through interactions with neighboring consumers, referred to as imperfect information by Pegoretti et al. (2012). A business owner likes to appoint a consumer who has already heard of or is interested in his product as a key agent. The advantages of approaching such consumer, as pointed out as Barker (2016): 1) It takes less effort to convince him to work with you, and 2) your partnership will be more authentic. Barker identifies the key agent from the followers of his small business in his Instagram profile. Mention.com also provides tools to identify people who are already talking about your business on Twitter or web (“Find Influencers – Mention”, 2017).

In Model 1-1 and Model 1-2, if a consumer finds out that at least one of his direct neighbors possesses the product, he makes a purchasing decision by evaluating the influence of his neighbors first. If the consumer’s neighborhood contains a persuader, the consumer adopts the product immediately regardless of the promotion cost and the price charged by the provider. Otherwise, the consumer evaluates the influences of promotion campaign and the price of the provider, for example, if the promotion cost of the provider is above his promotion cost acceptance threshold, and the price of the provider is beneath his reservation price, he purchases the product. Otherwise, the consumer does nothing.

In Model 2-1 and Model 2-2, the key agent is a pirate. He only needs to consider the piracy detection cost of the provider. If the piracy detection cost is beneath his piracy detection threshold, he pirates the product. Otherwise, the pirate does nothing. The remaining consumers are legal buyers who conduct the same evaluation on neighbor’s influence, prices, and promotion costs, as the consumers in Model 1-1 and Model 1-2.

For all four types of models, the agent-based model terminates when every consumer possesses one product or the specified time steps is reached.

4.1.2 Design of a digital content provider – EA

The provider is a population of marketing strategies. The design employs the Pittsburgh learning classifier (Smith, 1980). As introduced in chapter 3, the Pittsburgh learning classifier is a rule-based machine learning algorithm in which each individual in the population consists of a rule set. In my model, each marketing strategy is a rule set. Each rule contains conditions and corresponding actions. The conditions are the amount of market share, status of the current sales, and status of the current profit of the provider. The corresponding actions are various adjustments of the price, the promotion cost, and the piracy detection cost. Only a strategy in Model 2 includes the piracy detection cost.

The structure of a single rule is demonstrated in Figure 5. Conditions include M, S, and P, and its corresponding actions include Pa, Pp, Aa, Ap, Da, and Dp.

M = market share: accumulative amount of legal buyers / total consumers / 5.

S = status of current sales: increase, or stay unchanged (0/1)

P = status of current profit: increase/unchanged, or decrease (0/1)

Pa = price action: increase or decrease (0/1)

Pp = amount of price change: unit of 5%

Aa = promotion cost action: increase or decrease (0/1)

Ap = amount of promotion cost change: unit of 5%

Da = piracy detection cost action: increase or decrease (0/1)

Dp = amount of piracy detection cost change: unit of 5%

M	S	P	Pa	Pp	Aa	Ap	Da	Dp

Figure 5: Structure of a single rule

The market share is divided into 20 sections, as shown in the formula of M. Therefore, value of M is 0%, 5%, 10%, 15%, up to 100% of the total market. For each value of M, the combination of values of S and P is four, so there are four rules associated with each M value. For example, for the market share 5%, four rules are demonstrated as below:

5	0	0	action	5	0	1	action	5	1	0	action	5	1	1	action

Figure 6: Structure of four rules associated with market share 5%

In summary, there are 80 rules in one marketing strategy. In the actions of each rule, the percent of change is in the unit of 5%. For example, for the actions of price, if Pa is 1 (increase), and the current price is P, then the new price is: P + P * Pp * 5%. In my dissertation, for the conservative strategies, Pp, Ap, and Dp range from 0 to 4, which indicates the maximum 20% variation of the current price and cost. For the audacious strategies, Pp, Ap, and Dp range from 0 to 20, which indicates that maximum 100% variation of the current price and cost.

4.1.3 Workflow

The workflow of one simulation run is described in Figure 7. In this model, every marketing strategy is injected into the agent-based model, and the agent-based model functions as a fitness evaluation for the strategy.

Strategies are selected sequentially from a population of size N. Each strategy is injected into the agent-based model. The strategy guides the agent-based model to go through a diffusion process until each consumer possesses the product or specified time steps reached. A strategy has two fitness values. One is the accumulated profit through the entire diffusion process, and the other one is the diffusion speed. The diffusion speed is defined as the actual number of time steps taken to finish the diffusion process. Among these two fitness values, the profit has a higher priority. In other words, when comparing the fitness values of two strategies, the strategy with a higher profit is selected. Diffusion speeds are compared only when profits from two strategies are equal.

Equation 2: Profit of a strategy

$$profit = \sum_{t=0}^{n} (price_t * \ buyers_t - \ promotion\_cost_t \ - \ detection\_cost_t)$$

n: time steps consumed by the diffusion process

Equation 3: Diffusion speed of a strategy

Diffusion speed = n

n: time steps consumed by the diffusion process

When every strategy of the population has executed the same task and acquired its fitness values, the reproduction process starts. Offspring are selected through binary tournament selection. Like their parents, every strategy in the offspring population is injected into the agent-based model to acquire its corresponding fitness values. Finally, the parents and offspring are combined into one population of size 2N, and only N are selected as the new strategy population.

The new strategy population repeats the same operation as above until a specified number of generations is reached. The simulation is repeated 50 times on the same network with same consumers. Each run uses a different random seed to control the evolution process. During each run, the best-so-far strategy is recorded.

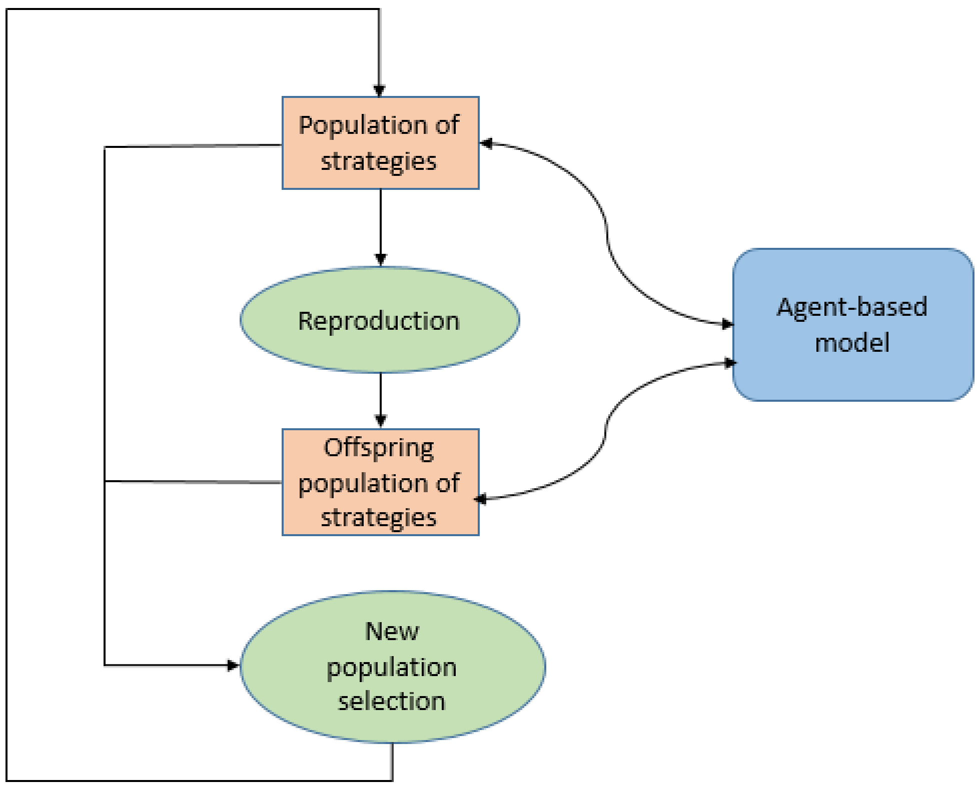


Figure 7: Workflow of one simulation run

4.2. Experiment configurations

For each model, several experiments under different conditions are presented. The base case does not have a key agent. The experiments range from adaptive marketing strategies, to experiments concerning the impacts of a connector, an opinion leader, and the position of the key agent within the social network.

Each experiment consists of 50 runs. The profit and diffusion speed of the best-so-far strategy of each run is recorded. The profits and diffusion speed of the 50 runs are compared among different scenarios statistically at 95% confidence interval using the Kolmogorov–Smirnov (K-S) test. The Apache Common Math java library is used for the calculation of value of D and p in the K-S test.

4.2.1 Parameters configuration of EA

Table 1: Parameters for EA of monopoly models

Parameter	**Value**
EA population size (number of strategies)	200
EA generation count	300
Parent selection	Binary tournament
Offspring selection	Truncation
Crossover operator	10-point
Mutation operator	Bit-flip / random value
Mutation probability	0.2F
Simulation runs per evaluation	50

The size of the population in the EA is 200, which indicates that one population contains 200 strategies. The population evolves 300 generations. In order to keep a balance of the selection pressure between the parent and the offspring population, I choose an algorithm of medium selection pressure for the parent selection and an algorithm of strong selection pressure for the survival selection. The parent selection algorithm is the binary tournament whose selection pressure is medium. The offspring selection algorithm is truncation whose selection pressure is strong.

It is not necessary to use both crossover and mutation operators. However, the parent and offspring selection algorithms utilized have relative strong selection pressures, in order to keep a balance between exploration and exploitation, the reproductive operators need to be more explorative. Therefore, I utilized both crossover and mutation operators. The crossover is a 10-point crossover which is more explorative than the traditional 1 point or 2 points crossover but less explorative than the uniform crossover. The mutation probability is 0.2, which indicates generally 20% of genes will be changed through mutations.

There are 80 conditions in one strategy. Those 80 conditions list all possible conditions which a provider will encounter. As a result, the mutation operator does not touch the condition of the rule. The mutation operator only mutates the action of each rule. The mutation probability is 0.2, and the mutation operator is either bit-flip or a random value based on the position in the strategy. For every position in the action, a random number is generated. If the random number is less than 0.2, there are two situations to consider: if the position is Pa, Aa, or Da, then the value 1 is changed into 0, and 0 is changed into 1 (bit-flip); if the position is Pp, Ap, or Dp, then a random value is generated between [0, 4] for conservative strategies, and [0, 20] for audacious strategies. The crossover utilizes a 10-point crossover algorithm, which indicates that 10 random cutting points are generated, and two strategies swapped sections between these 10 points. Cutting points in conditions do not change the value of conditions, see the below example, so the crossover does not need avoid cutting the condition of the rule. The arrows point at the cutting points of crossover. Figure 8 demonstrates the situation in which the cutting points are located inside the condition of a rule. As shown in Figure 8, the conditions do not change in the children after swapping sections of parents along the cutting points.

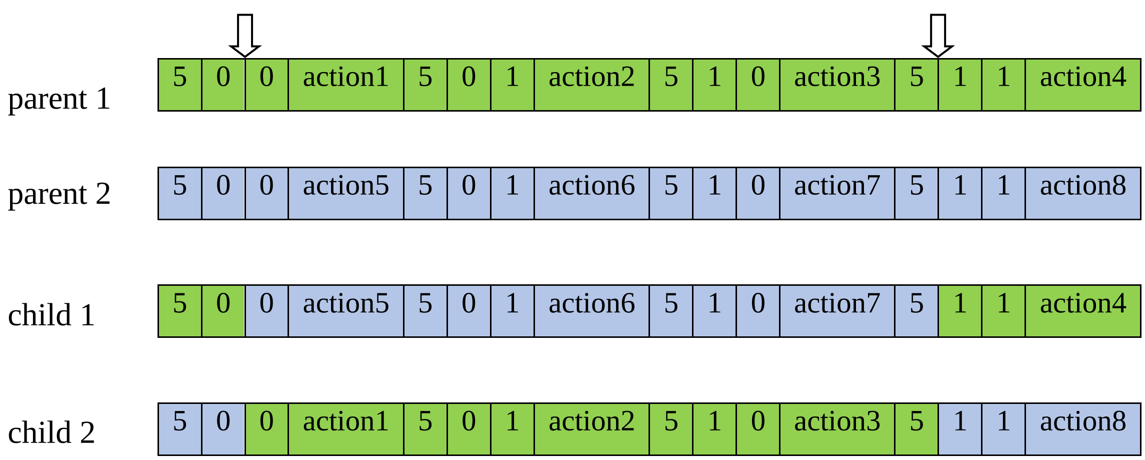


Figure 8: Crossover of rule of market share 5%

4.2.2 Parameters configuration of the agent-based model

Table 2 lists the configurations of the agent-based model for homogeneous consumers. My research builds theoretical models, so the setting of 100 consumers is enough. For homogeneous consumers, the values of reservation price, promotion cost threshold, and the piracy detection cost threshold are $15. The initial price starts from the highest price $25, the initial promotion cost starts from the lowest $5, and the initial piracy detection cost starts from highest $25. The configurations of price and promotion cost meet the general expectations of providers who expect to gain a maximum profit with a minimum cost. Based on the literature review, most scholars believe that piracy is harmful, and a provider should invest to prevent it from happening or reduce its damage. Therefore, in my model, the initial piracy detection cost starts from the highest value which indicates the maximum protection against piracy. The model explores how the piracy detection cost changes and whether it is as useful as most scholars expected. Consumer 5 is defined as the innovation initiator in every network. The maximum change on the current price, promotion cost, and detection cost is 100%. The unit of variation on the price and cost is 5%. For example, a price could be increased at 5%, 10%, 15%, and maximum 100%. The price, promotion cost, and detection cost have upper limit $25 and lower limit $5.

Table 2: Parameters for homogeneous consumers

Parameters	**Value**
Number of consumers	100
Reservation price of a consumer	15
Promotion cost threshold of a consumer	15
Piracy detection cost threshold of a consumer	15
Starting price of a provider	25
Starting promotion cost of a provider	5
Starting piracy detection cost of a provider	25
Position of an innovation initiator	5

Number of an innovation initiator	1
Maximum percent of change	100
Unit of variation on prices and costs	5%
Price range of a provider	[5, 25]
Promotion cost range of a provider	[5, 25]
Detection cost range of a provider	[5, 25]

Table 3 shows the configuration of heterogeneous consumers. The reservation prices, promotion cost threshold, and piracy detection cost threshold are normally distributed with a mean of $15 and standard deviation of 3. It indicates that 95% of these three thresholds is between $9 and $21. Figure 9 to 11 illustrate the distributions of these three thresholds of 100 consumers.

Table 3: Parameters for heterogeneous consumers

Parameters	**Value**
Reservation price of a consumer	N (15,3)
Promotion cost threshold of a consumer	N (15,3)
Piracy detection cost threshold of a consumer	N (15,3)
Starting price of a provider	25
Starting promotion cost of a provider	5
Starting piracy detection cost of a provider	25
Position of an innovation initiator	5
Number of an innovation initiator	1
Maximum percent of change	100
Unit of variation on prices and costs	5%
Price range of a provider	[5, 25]

Promotion cost range of a provider	[5, 25]
Detection cost range of a provider	[5, 25]

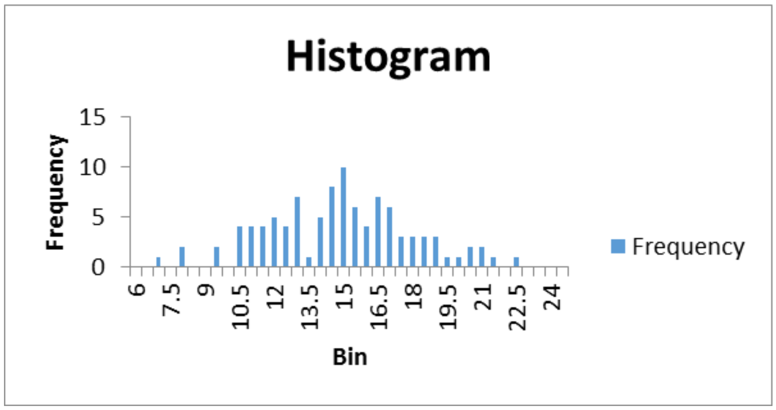


Figure 9: Distribution of reservation prices of 100 consumers

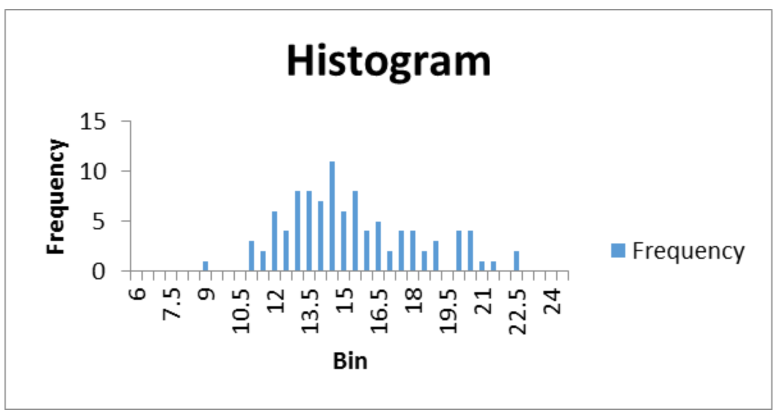


Figure 10: Distribution of promotion cost thresholds of 100 consumers

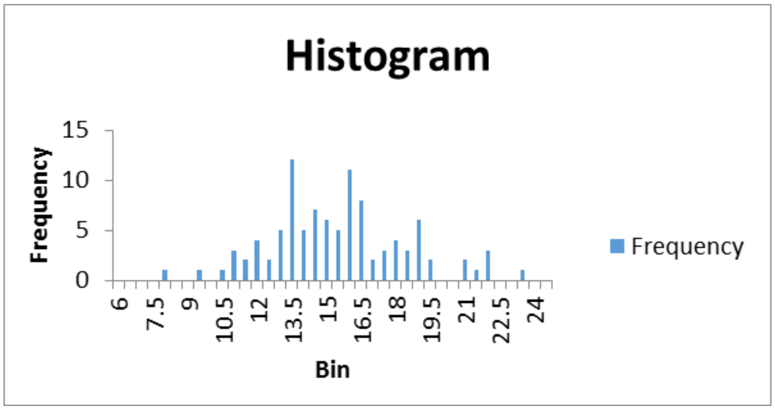


Figure 11: Distribution of piracy detection cost thresholds of 100 consumers

4.2.3 Configurations of network structures

15 small-world networks, three scale-free networks, and three random networks are created. All the small-world networks, scale-free networks, and random networks are illustrated in Appendix A, B, C, and D.

4.2.3.1 Configurations of small-world networks

Networks are created from three random seeds. For each random seed, five rewiring probability, 0.01, 0.03, 0.05, 0.07, and 0.1, are utilized to create five different networks. Tables 4, 5 and 6 list important attributes of each network, including clustering coefficient, average path length, and diameter. As shown in Tables 4, 5, and 6, as the rewiring probability increases, the network becomes less clustered, and the average path length becomes smaller. The reason is simple, as the rewiring probability increases, there are more shortcuts created, therefore, the traveling distance between nodes in the network becomes smaller.

Table 4: Five small-world networks using random seed 1

Rewiring probability	Clustering coefficient	Average path length	diameter
0.01	0.97199	9.392	21
0.03	0.954	8.0908	20
0.05	0.8313	6.3604	16
0.07	0.7426	5.555	12
0.1	0.71	4.7574	9

Table 5: Five small-world networks using random seed 2

Rewiring probability	Clustering coefficient	Average path length	diameter
0.01	0.9513	9.0616	19
0.03	0.9253	7.3824	17
0.05	0.8193	5.2978	11
0.07	0.8219	5.0012	10
0.1	0.8339	4.8502	10

Table 6: Five small-world networks using random seed 3

Rewiring probability	Clustering coefficient	Average path length	diameter
0.01	0.9586	8.854	21
0.03	0.9459	7.5146	19
0.05	0.9106	6.8658	18
0.07	0.75	5.1864	11
0.1	0.756	4.8714	10

4.2.3.2 Configurations of scale-free networks and random networks

The scale-free networks are created for three random seeds respectively. The rewiring probability of random networks is 0.5. Three random networks are created for three random seeds respectively as shown in Table 7.

Table 7: Three random networks using three random seeds

Random seed	Rewiring probability	Clustering coefficient	Average path length	diameter
Seed 1	0.5	0.1972	3.513	7
Seed 2	0.5	0.1365	3.554	7
Seed 3	0.5	0.2595	3.7362	7

CHAPTER 5: MONOPOLY – DYNAMIC AND SELF-LEARNING MARKETING STRATEGY

This chapter illustrates and explains the impacts of the best-so-far strategy on the price, promotion cost, piracy detection cost, and adoption patterns. All monopoly models are used to experiment on three types of networks: a small-world network, a random network, and a scale-free network. In order to have a clear understanding of the impacts, only homogeneous consumers are illustrated and analyzed in this chapter.

5.1 Small-world network

The direct neighborhood of the innovation initiator was defined as L1, and the direct neighborhood of L1 was defined as L2, and so on. Consumer 5 is the innovation initiator in each network. Figure 12 displays the first three levels (Innovation initiator, L1, and L2) of the network of random seed 1 and rewiring probability 0.1. The key agent is located at position 7, right next to the innovation initiator. Among 50 best-so-far strategies, the best of best-so-far marketing strategies is injected into the agent-based model, and its impacts on various marketing factors and adoption patterns are plotted in Figure 13 to 20.

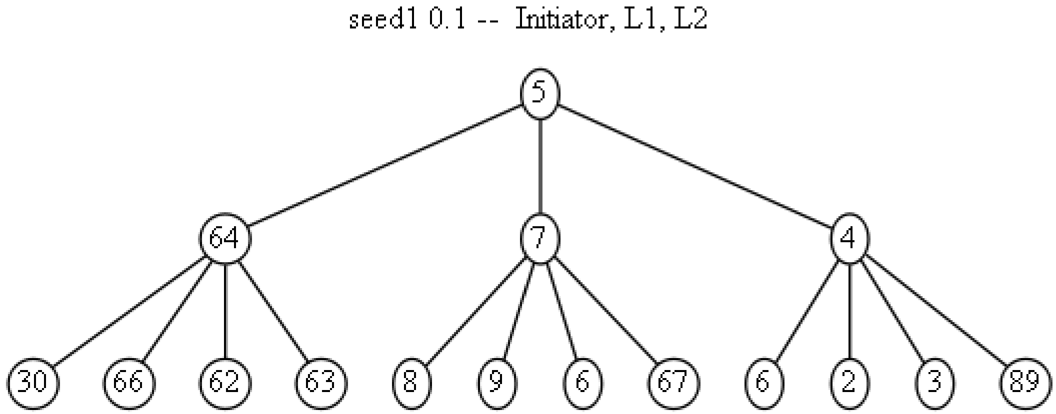


Figure 12: Initiator, L1, and L2 of network seed 1, rewiring probability 0.1

5.1.1 Model 1-1: a key agent is an average consumer

In this section, the key agent is an average consumer who has no influence on the adoption decisions of other consumers.

Figure 13 demonstrates the impacts of the best of best-so-far marketing strategies on the price, promotion cost, and new adoptions. Key agent 7 acquires a gift from the provider at time step 0. The price and promotion cost thresholds of consumers are $15, and they are invisible to the provider, so the self-learning strategy helps the provider adjust his price and promotion cost by moving towards the thresholds of consumers gradually. After reaching the thresholds, the promotion cost and price fluctuate slightly, but they stay close to the thresholds.

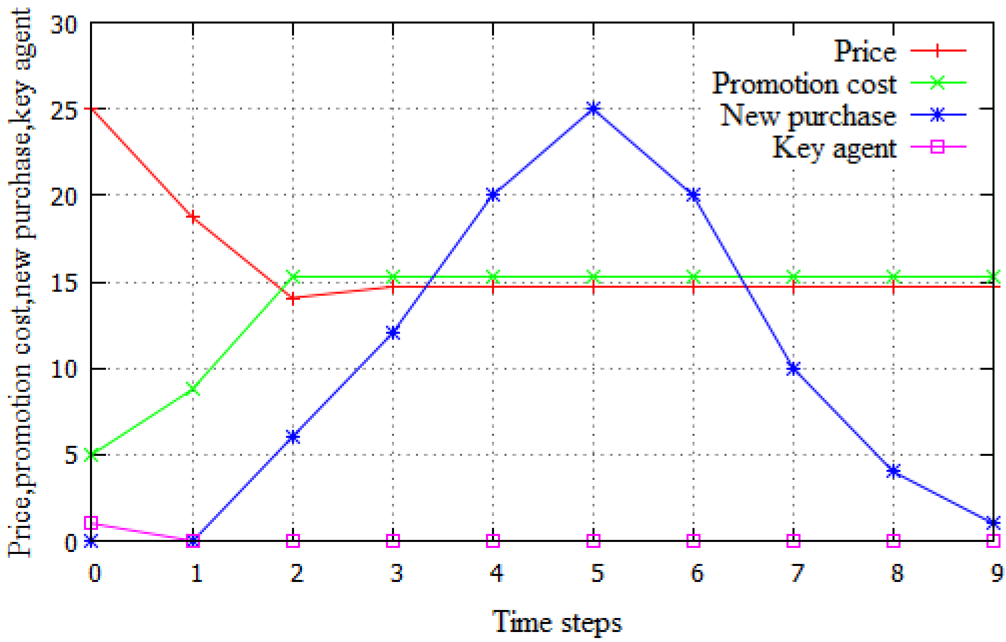


Figure 13: Best of best-so-far strategies on a small-world network (Model 1-1)

5.1.2 Model 1-2: a key agent is a persuasive consumer

In this section, the key agent is a persuader who influences the adoption decisions of other consumers. As shown in Figure 14, the neighbors of the persuader purchase the product before the price and promotion cost reach the thresholds of consumers.

Figures 15 and 16 illustrate the difference on adoption patterns and total profits between Model 1-1 and Model 1-2. In Model 1-2, the neighbors of the persuader adopt the product regardless of the price and the promotion cost. The key agent 7 introduces four adopters at time step 1, and those adopters bought the product at $25 which is high above the reservation price ($15). At time step 1, the promotion cost is $7.50 which is beneath the threshold of a consumer. In Model 1-1, the key agent 7 is not a persuader, thus, when the price and promotion cost are within the acceptable range at time step 2, the adoptions start with six purchases at price $14.06. The promotion cost at time step 2 is $15.31. Four purchase at a high price and a low promotion cost which helps Model 1-2 gain a good start on both profits and diffusion speeds. A good start cannot guarantee final success. An important factor is the network topology. The impacts of the network topology will be explained in detail in chapters 7 and 8. In addition, the adoption curves in Figure 15 demonstrate a typical S-curve as described by Rogers (2003).

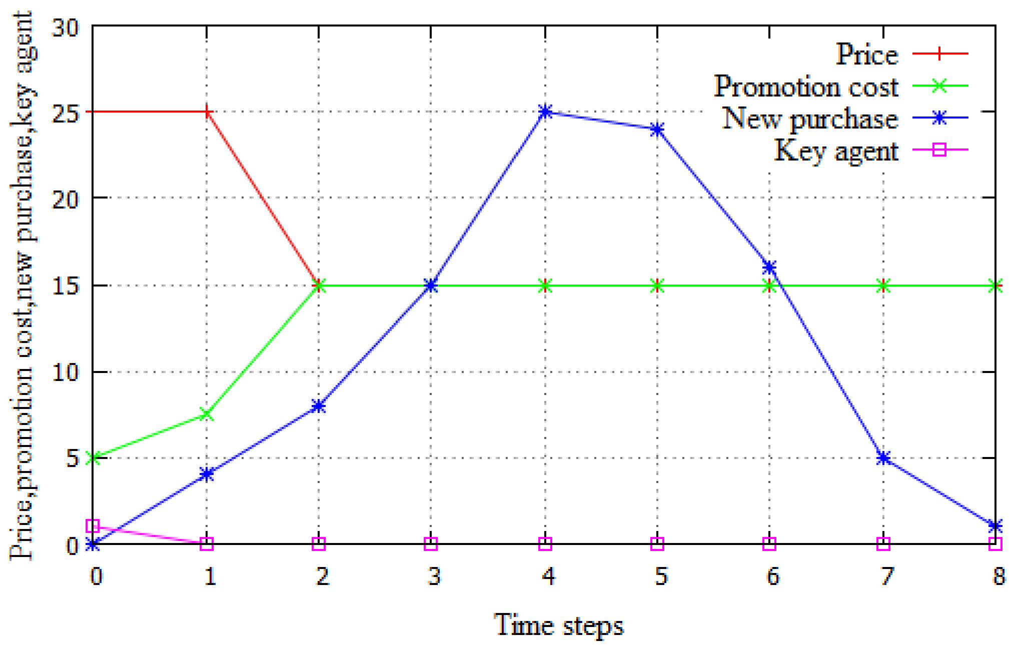


Figure 14: Best of best-so-far strategies on a small-world network (Model 1-2)

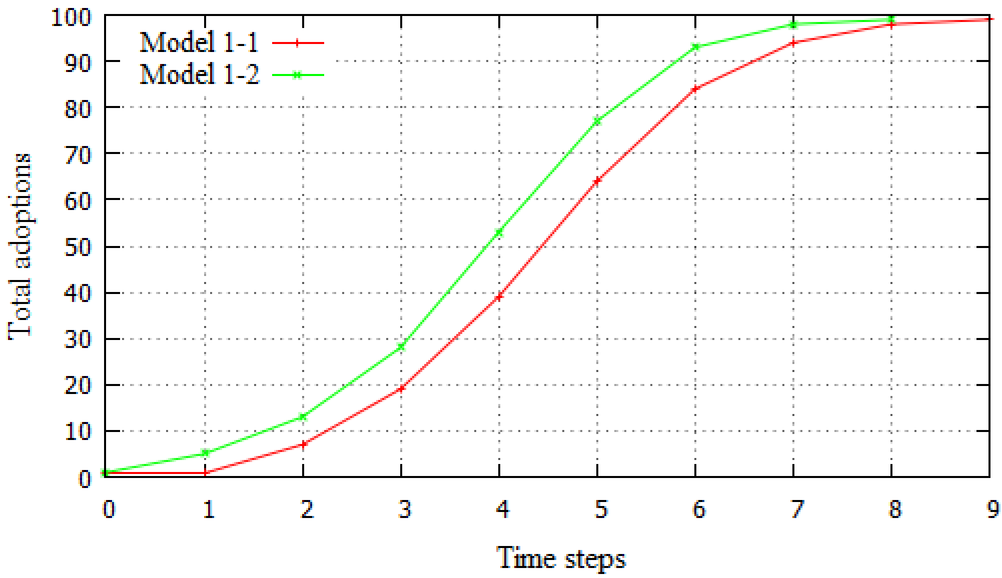


Figure 15: Comparison of total adoptions between Model 1-1 and Model 1-2

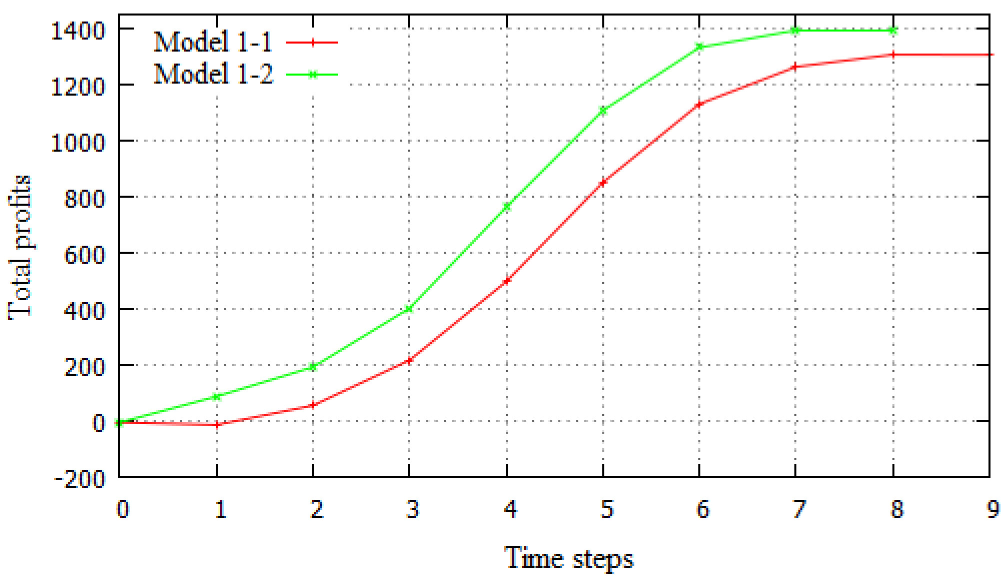


Figure 16: Comparison of total profits between Model 1-1 and Model 1-2

5.1.3 Model 2-1: a key agent is an average pirate

In this section, the key agent 7 is a pirate who is an average consumer. He has no influence on the adoption decisions of other consumers. The major difference between Model 1 and Model 2 is that Model 2 has a piracy detection cost in the marketing strategy, and the pirate gets the free product through piracy. A pirate pirates the product when the piracy detection cost falls beneath the piracy detection cost ($15) of a consumer.

Figure 17 demonstrates the impacts of best of best-so-far marketing strategies on the price, promotion cost, piracy detection cost, the amount of purchase and piracy conducted. Pirate 7 adopted the product at time step 1, after that the piracy detection cost is not used anymore, so all it does is to move downwards. Unlike the key agent in the Model 1 who is given a free product at time step 0, a pirate has to wait until the piracy detection cost fall beneath his detection threshold, then he can pirate the product.

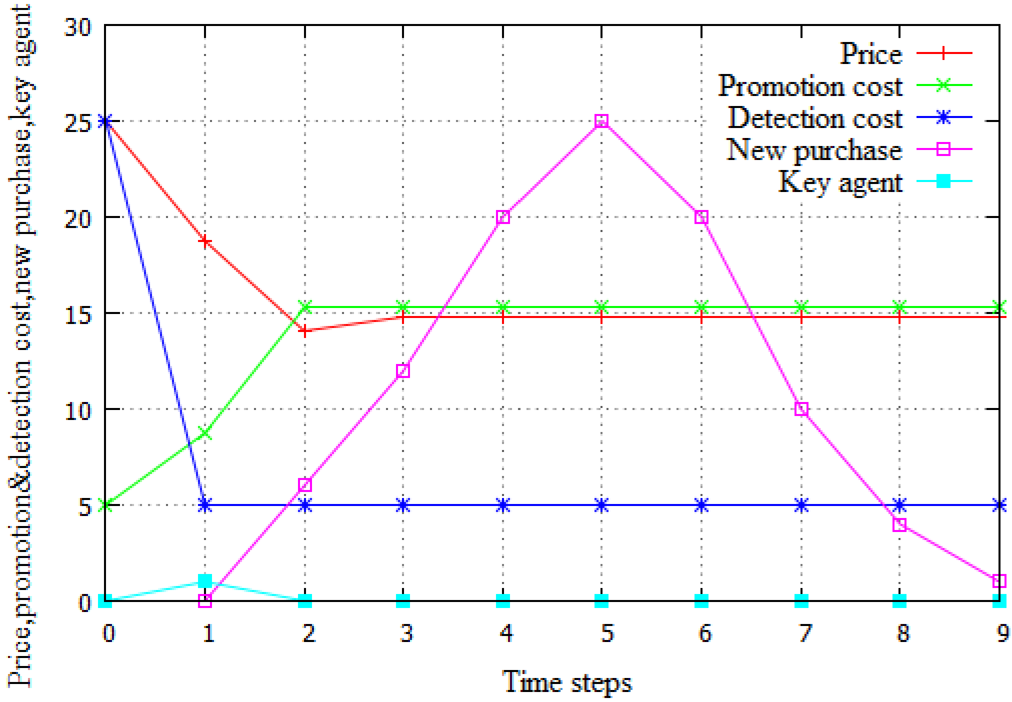


Figure 17: Best of best-so-far strategies on a small-world network (Model 2-1)

5.1.4 Model 2-2: a key agent is a persuasive pirate

Figure 18 demonstrates the impacts of best of best-so-far marketing strategies on the price, promotion cost, piracy detection cost, the amount of purchase and pirates. From Figure 18, I can see that the impacts of the strategy on various marketing factors and adoption patterns are very similar to the impacts illustrated in Figure 14. The key agent is a persuasive pirate. Therefore, the adoptions happen before the price and the promotion cost reach the thresholds of consumers.

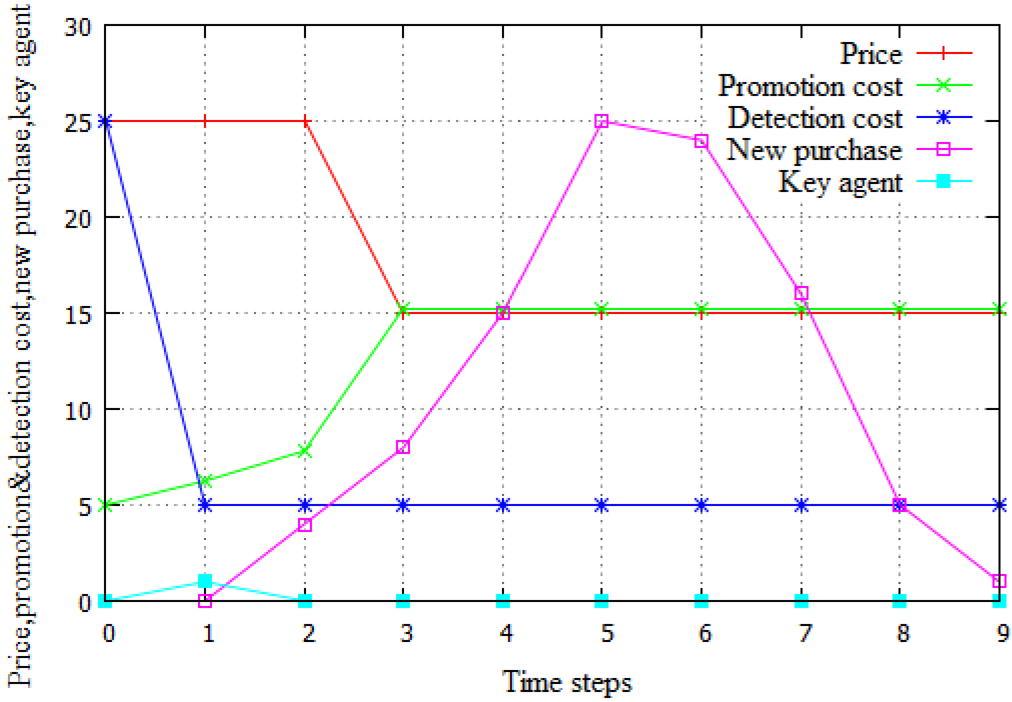


Figure 18: Best of best-so-far strategies on a small-world network (Model 2-2)

Figures 19 and 20 illustrate the difference on adoption patterns and total profits between Model 2-1 and Model 2-2. In Model 2-1, a good start on the profits and diffusion speed does not lead to the final success. The adoption and profits in Model 2-2 catches up after time step 6. The cause of such phenomena is the network topology. The impacts of the network topology will be explained in detail in chapters 7 and 8.

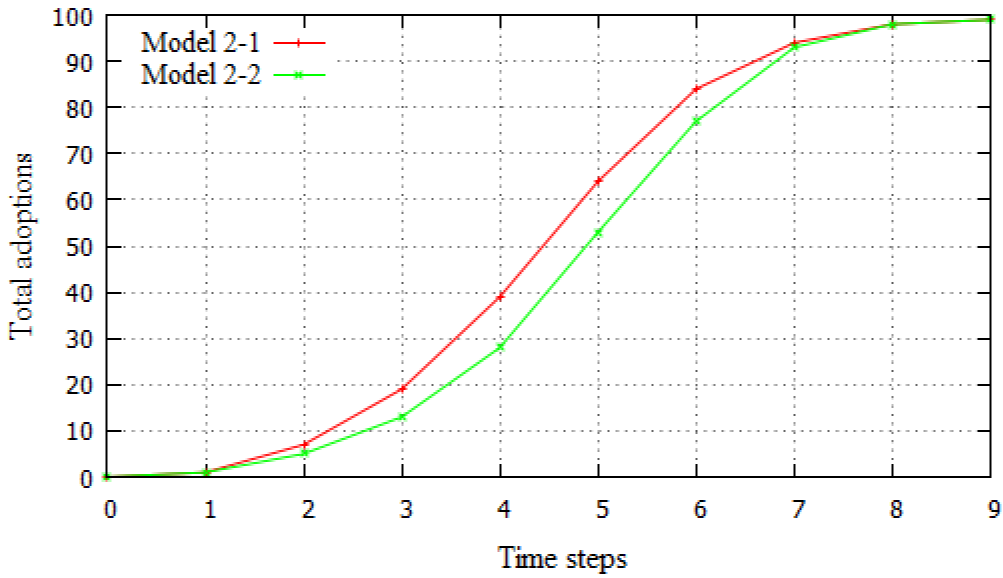


Figure 19: Comparison of total adoptions between Model 2-1 and Model 2-2

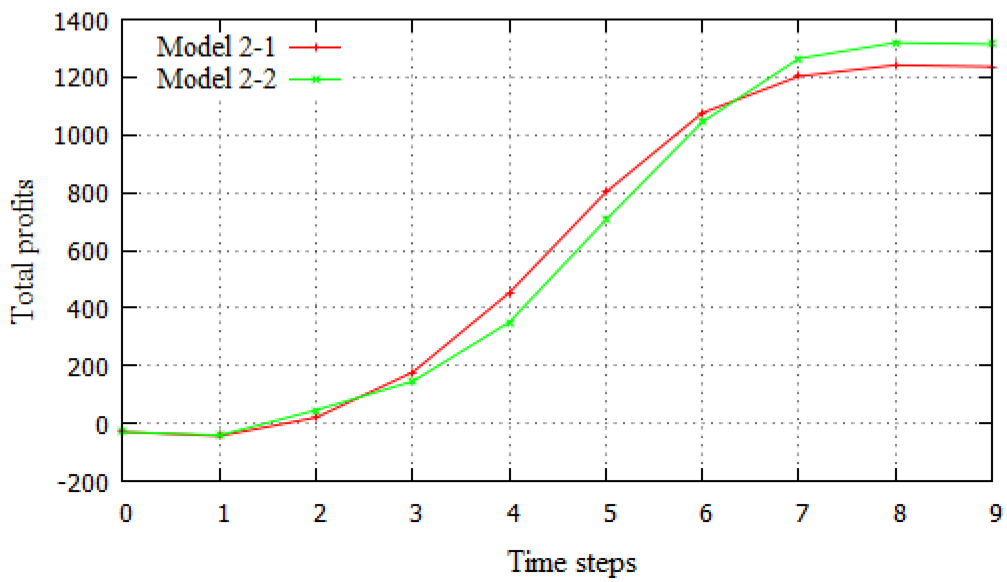


Figure 20: Comparison of total profits between Model 2-1 and Model 2-2

5.2 A scale-free network and a random network

The same experiments were conducted on a scale-free network and a random network. Results show very similar patterns regarding to a strategy's impacts on the marketing factors and adoption patterns.

CHAPTER 6: MONOPOLY – EFFECTS OF VARIATIONS OF NETWORK TOPOLOGY ON THE DIFFUSION SPEEDS AND PROFITS

The diffusion speed and the profit of the best-so-far strategy is recorded for 50 runs on every network. Figure 21 to 32 demonstrate in a small-world network, the impact of network rewiring probabilities on the diffusion speeds and profits of both homogeneous and heterogeneous consumers. The results from Model 2-1, where the pirate is an average consumer, are used to construct the plots from Figure 21 to 32.

6.1 Effects of network rewiring probability on diffusion speeds

Figures 21, 22, and 23 illustrate the diffusion speeds of the 50 best-so-far strategies of homogeneous consumers under non-piracy scenario. The diffusion speeds are plotted for 5 rewiring probabilities of the network of seed 1, seed 2, and seed 3.

Figures 21, 22, and 23 reveal that as the rewiring probability increases, from 0.01 to 0.1, the time steps taken to finish the diffusion process get smaller, in other words, the diffusion gets faster. As shown in Figure 21, the diffusion on the network of rewiring probability 0.01 takes the longest time. For the networks of rewiring probability 0.03 and 0.05, there is not much differences on the time steps taken. The time steps spent in the network of rewiring probability 0.07 are less than 0.03 and 0.05, which indicates that the diffusion on the network of rewiring probability 0.07 is faster. The time steps spent in the network of rewiring probability 0.1 are the least, so the diffusion in the network of rewiring probability 0.1 is the fastest. Five plots reveal that the diffusion is getting faster with the increasing network rewiring probability. The cause of such phenomena is simple, as the rewiring probability increase, there are more shortcuts created (see Appendix A), the network becomes more connected, so the traveling distance between consumers becomes smaller. As a result, it takes shorter time to finish the diffusion process.

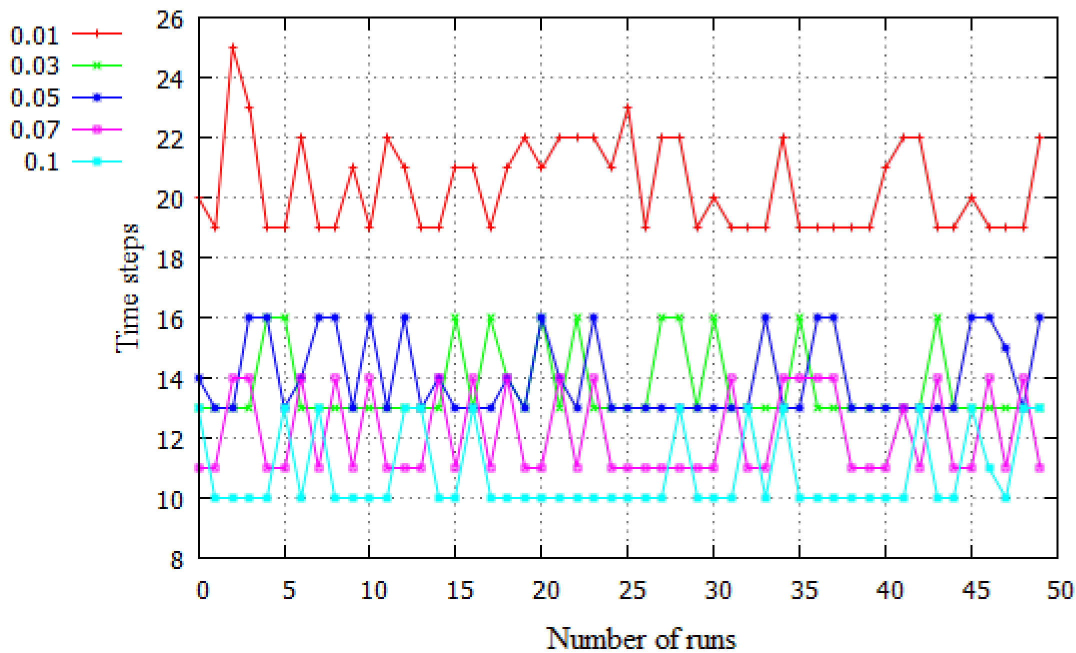


Figure 21: Diffusion speeds of homogeneous consumers on networks of seed 1.

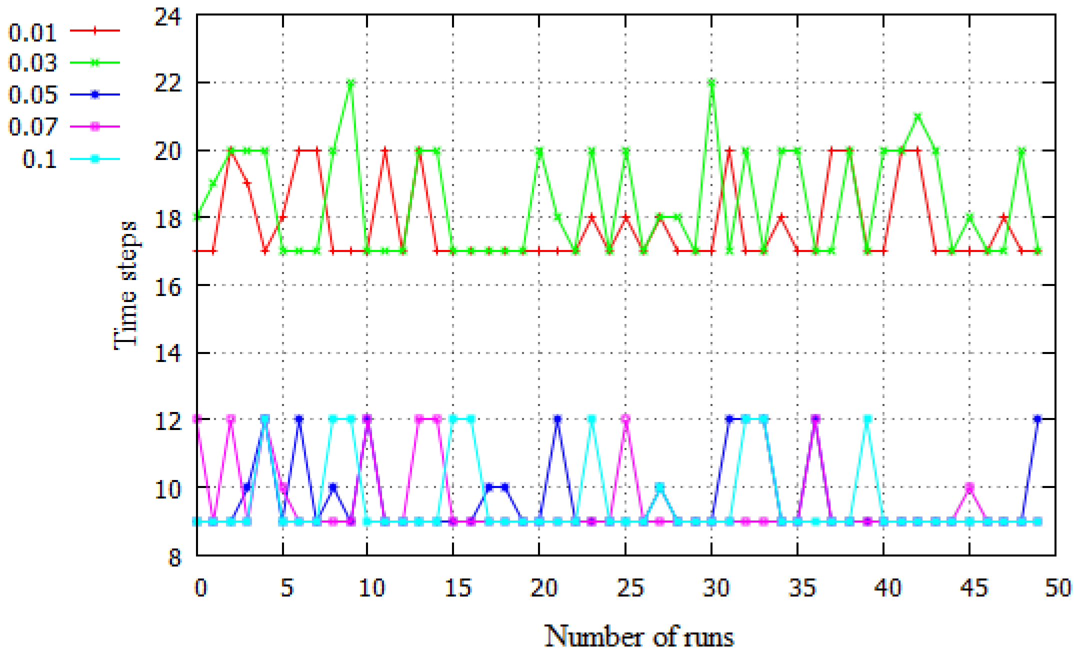


Figure 22: Diffusion speeds of homogeneous consumers on networks of seed 2

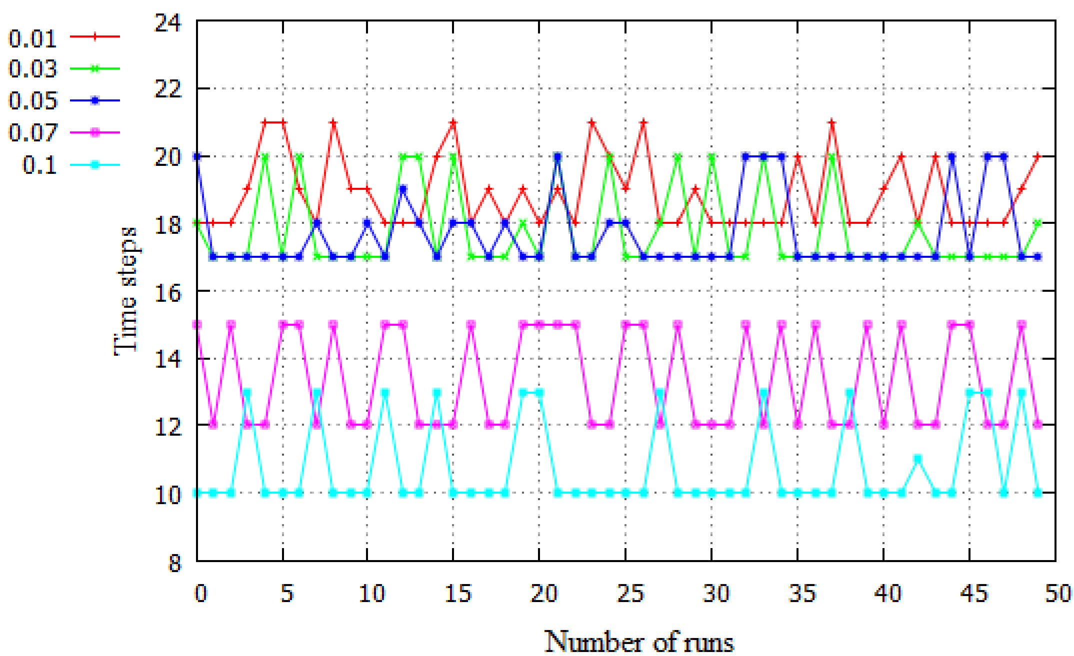


Figure 23: Diffusion speeds of homogeneous consumers on networks of seed 3

Figures 24, 25, and 26 illustrate the diffusion speed of the 50 best-so-far strategies of heterogeneous consumers under non-piracy scenario. The diffusion speeds are plotted for 5 rewiring probabilities of the network of seed 1, seed 2, and seed 3.

Due to the heterogeneity of consumers, the distinction of time steps is not as clear as the homogeneous consumers. As shown in Figure 24, the time steps spent in the network of rewiring probability 0.01 are the longest, so the diffusion on the network 0.01 is the slowest. The plotted time steps of the rest of the networks seem mingled together. However, I can see that the time steps spent in the network of rewiring probability 0.03 are very close to time steps spent 0.05, and the time steps spent in the network of rewiring probability 0.07 are very close to time steps spent 0.1. Time steps spent in the network of rewiring probability 0.03 and 0.05 are both longer than the time steps spent in 0.07 and 0.1, thus the diffusions on the network of 0.03 and 0.05 are slower than the diffusion in the network of 0.07 and 0.1. Therefore, I conclude that for heterogeneous consumers, as the rewiring probability of the network increases, the diffusion gets faster.

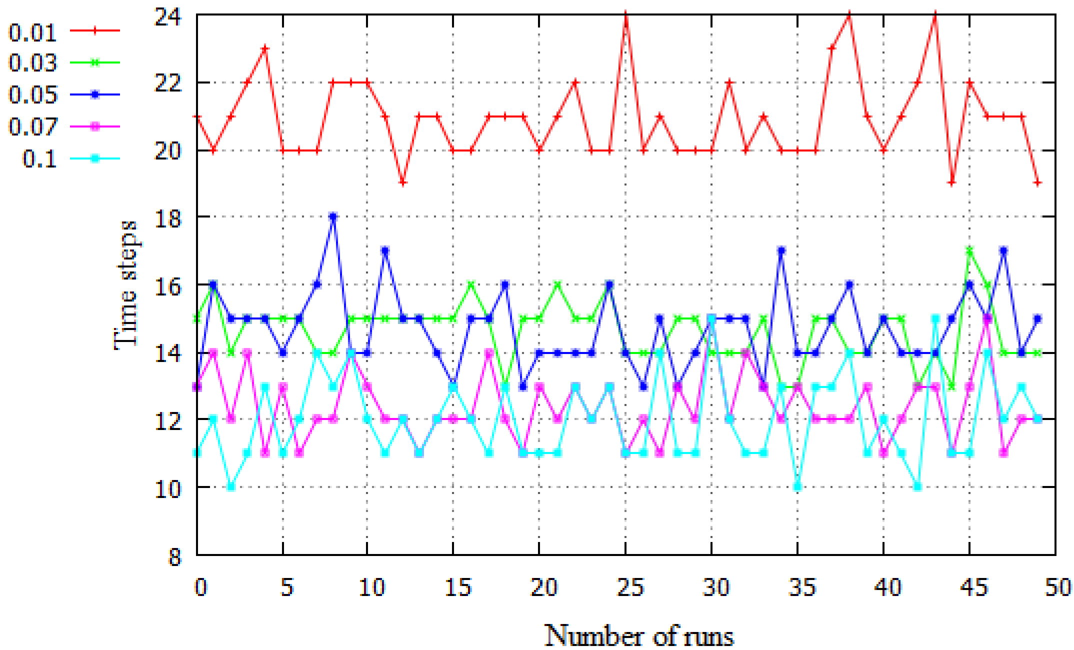


Figure 24: Diffusion speeds of heterogeneous consumers on networks of seed 1.

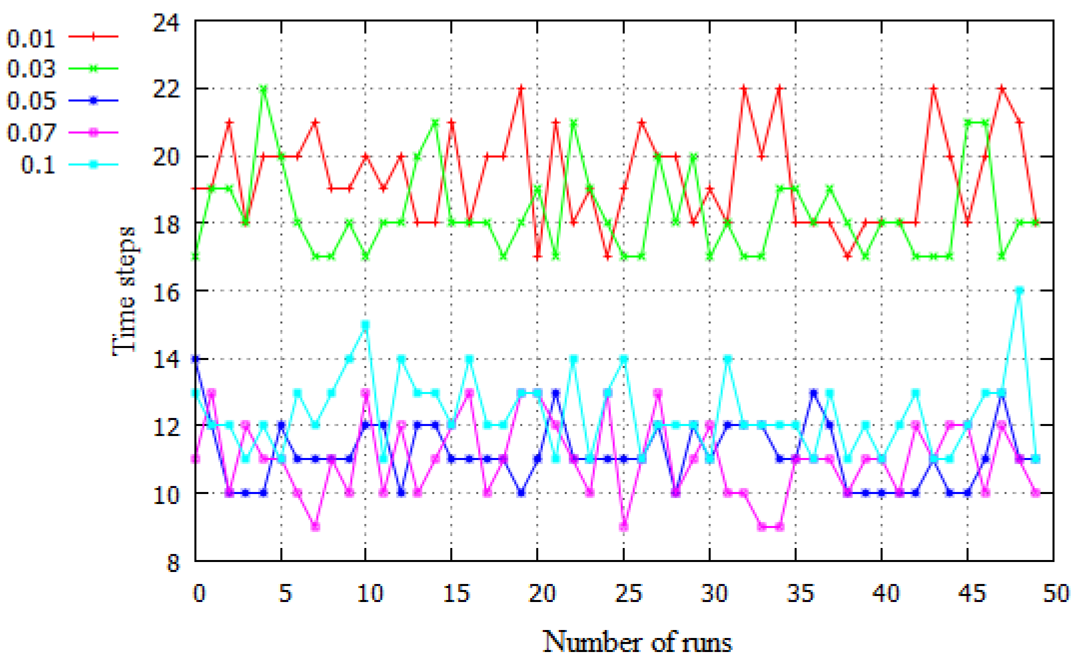


Figure 25: Diffusion speeds of heterogeneous consumers on networks of seed 2

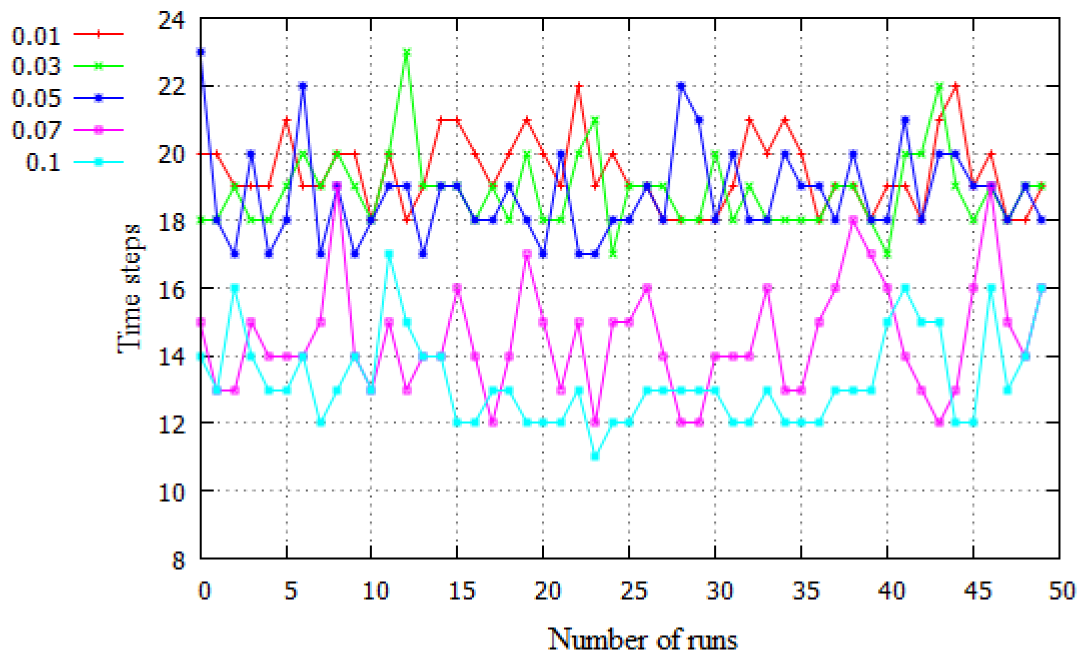


Figure 26: Diffusion speeds of heterogeneous consumers on networks of seed 3.

6.2 Effects of network rewiring probability on profits

Figures 27, 28, and 29 illustrate the profits of the 50 best-so-far strategies of homogeneous consumers under non-piracy scenario. The profits are plotted for 5 rewiring probabilities of the network of seed 1, seed 2, and seed 3.

Figures 27, 28, and 29 reveal that as the rewiring probability increases, from 0.01 to 0.1, the profits increase. As shown in Figure 27, the profits are lowest on the network of rewiring probability 0.01. There is not much difference between profits on the network of rewiring probabilities 0.03 and 0.05. The profits on the network of rewiring probability 0.07 are larger than the profits on network 0.03 and 0.05. The profits on the network of rewiring probability 0.1 are the largest. This indicates that the profits are increasing with the increasing rewiring probability. As the rewiring probability increases, the diffusion gets faster, thus it takes shorter time, and there are less promotion and detection costs put on the market. The total number of consumers is 100, so the total revenue does not change much. Therefore, an almost fixed revenue minus less cost generates a higher profit.

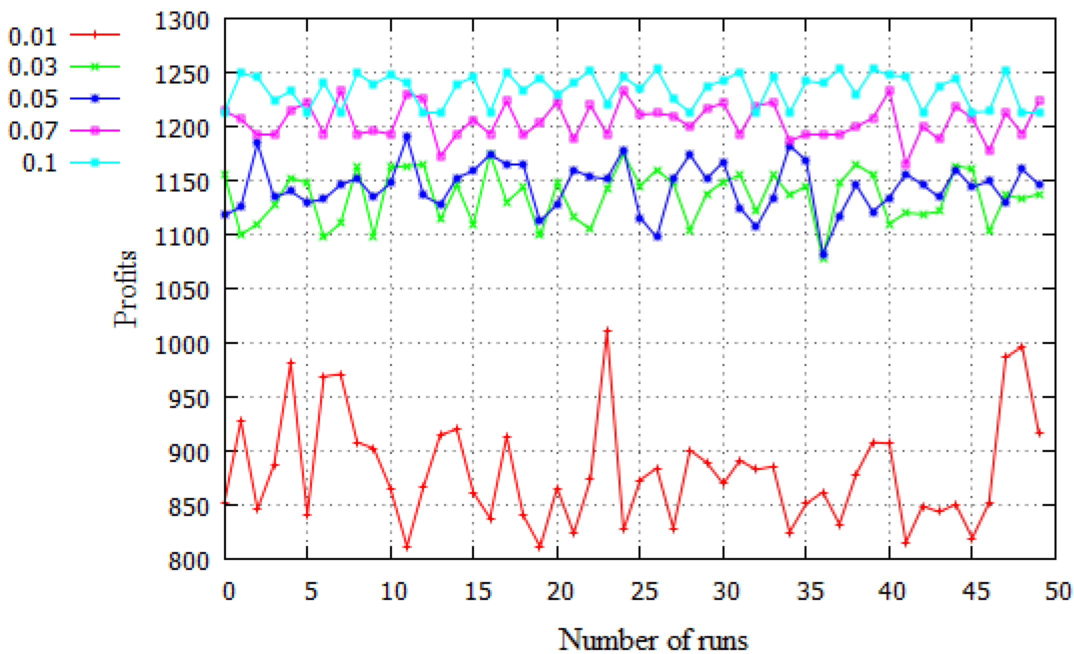


Figure 27: Profits of homogeneous consumers on networks of seed 1.

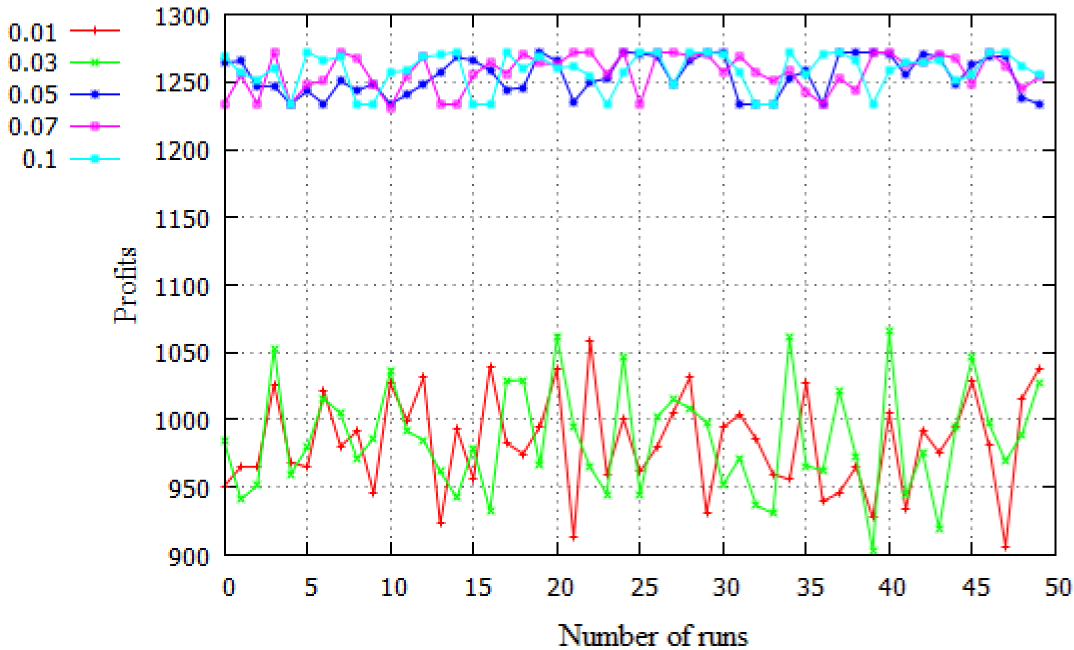


Figure 28: Profits of homogeneous consumers on networks of seed 2.

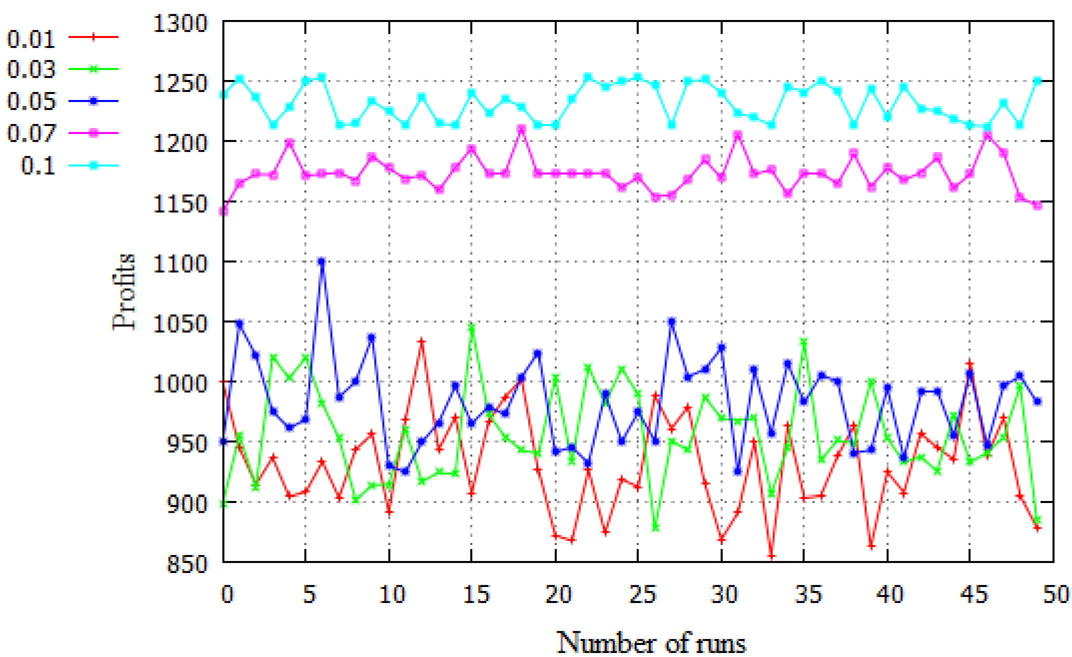


Figure 29: Profits of homogeneous consumers on networks of seed 3.

Figures 30, 31, and 32 illustrate the profits of the 50 best-so-far strategies of heterogeneous consumers under non-piracy scenario. The profits are plotted for 5 rewiring probabilities of the network of seed 1, seed 2, and seed 3.

Figures 30, 31, and 32 reveal that as the rewiring probability increases, from 0.01 to 0.1, the profits increase. As shown in Figure 30, the profits are the lowest on the network of rewiring probability 0.01. The profits on the network of rewiring probability 0.03 are lower than the profits on network 0.05. The profits on the network of rewiring probability 0.07 and 0.1 are close, but the profits on the network of rewiring probability 0.1 are slightly higher than the profits on network 0.07. So I conclude that for heterogamous consumers, the profits are increasing with the increasing network rewiring probability.

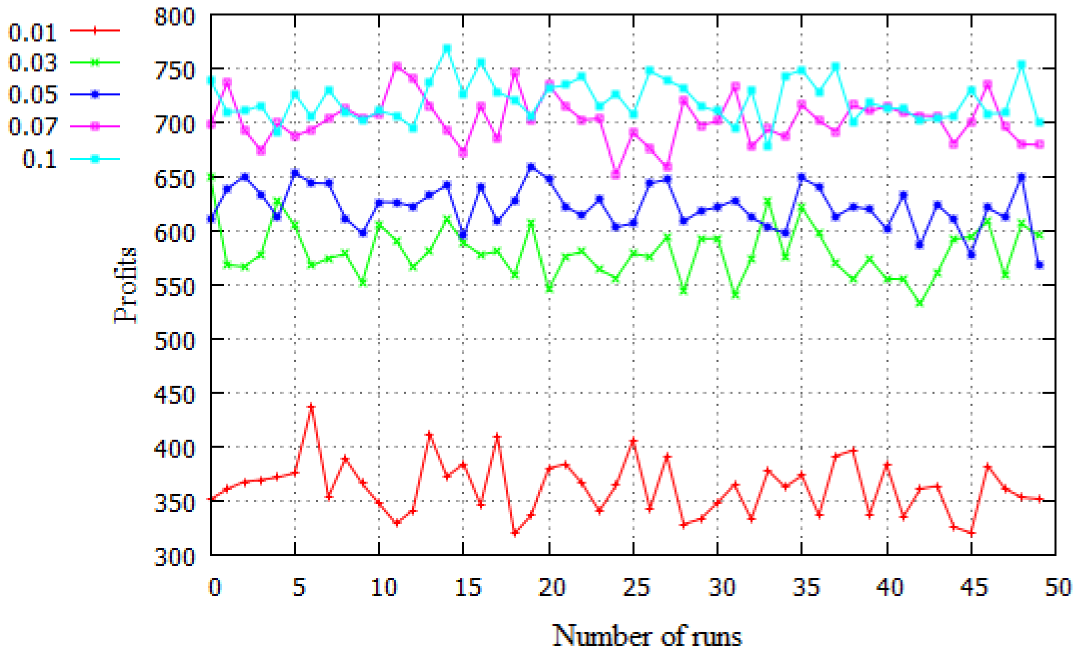


Figure 30: Profits of heterogeneous consumers on networks of seed 1.

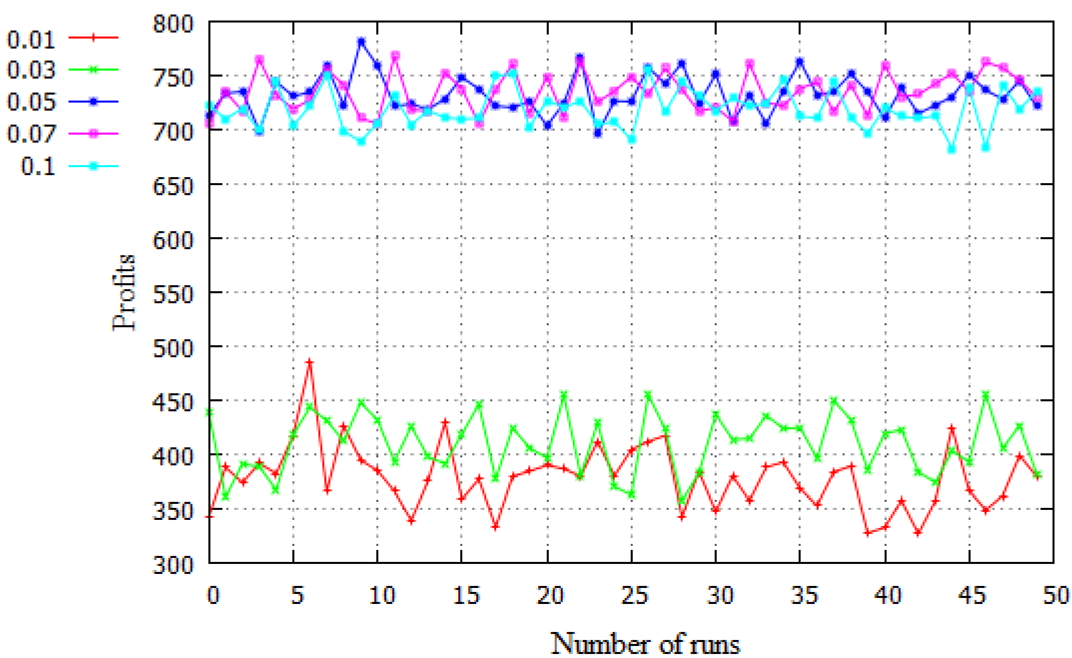


Figure 31: Profits of heterogeneous consumers on networks of seed 2.

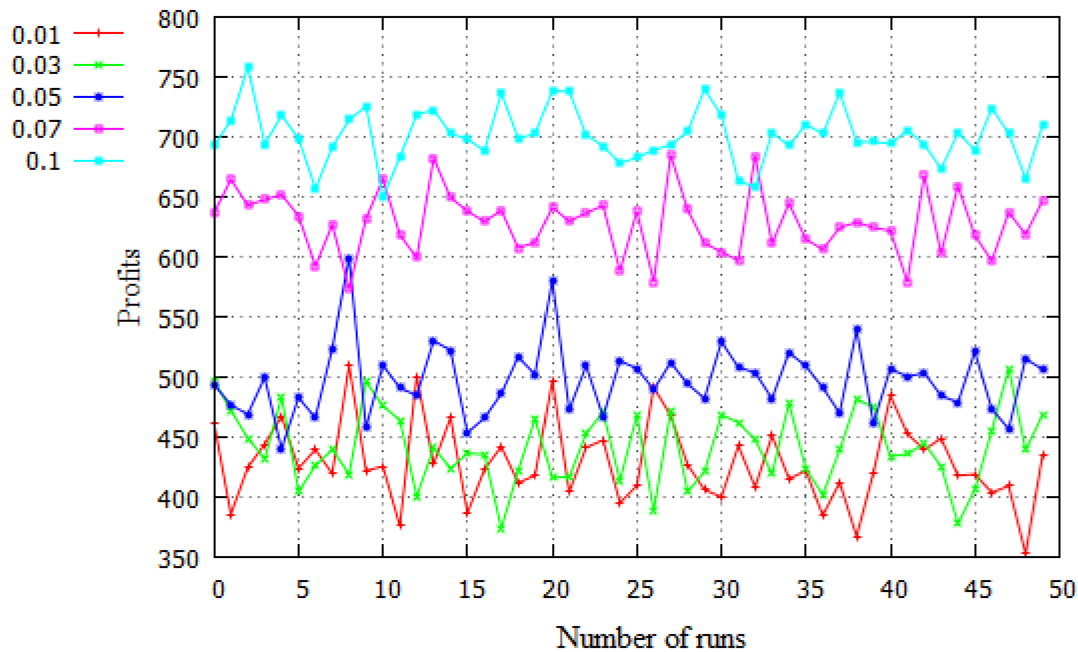


Figure 32: Profits of heterogeneous consumers on networks of seed 3.

6.3 Conclusions concerning the impacts of rewiring probabilities on diffusion speeds and profits

Results show that in a small-world network, more profits are earned and less time is consumed as the rewiring probability increases. The reason is that as rewiring probability increases, the network becomes more connected, and the average path length becomes shorter. As a result, less time is consumed during the diffusion process, and less promotional and detection costs are incurred in the market, and therefore, more profits are earned.

CHAPTER 7: MONOPOLY – IMPACT OF A CONNECTOR

A connecter, defined by Gladwell (2000), is a person who knows a large number of people in the community. Therefore, a connector seems like a good candidate to be a key agent. Unlike a persuader, a maven (Gladwell, 2000), or an opinion leader, besides being highly connected, a connector has no influence on the decision making of other people. We start with a hypothesis.

Hypothesis 1: a connector (consumer who has the most connections) is an ideal candidate to be a key agent.

Model 2-1 is used to experiment with both homogeneous and heterogeneous consumers on three networks: a small-world network, a random network, and a scale-free network. In Model 2-1, the key agent is a pirate. In each network, the consumer has the most directly connected neighbors is selected as connectors, and then those connectors are assigned as a pirate sequentially. The results show that, compared to the base case (NP), assigning a connector as a key agent does not increase profits and speed up diffusion speeds significantly in small-world networks, random networks, and scale-free networks.

7.1 Homogeneous consumers

7.1.1 Small-world network

In a small-world network, a node that has the most neighbors also contains shortcuts. Experiments are conducted on the below network of seed 3 and rewiring probability 0.01. Consumers having the largest number of connections are consumers 11 and 83. In Model 2-1, consumers 11 and 83 are pirates.

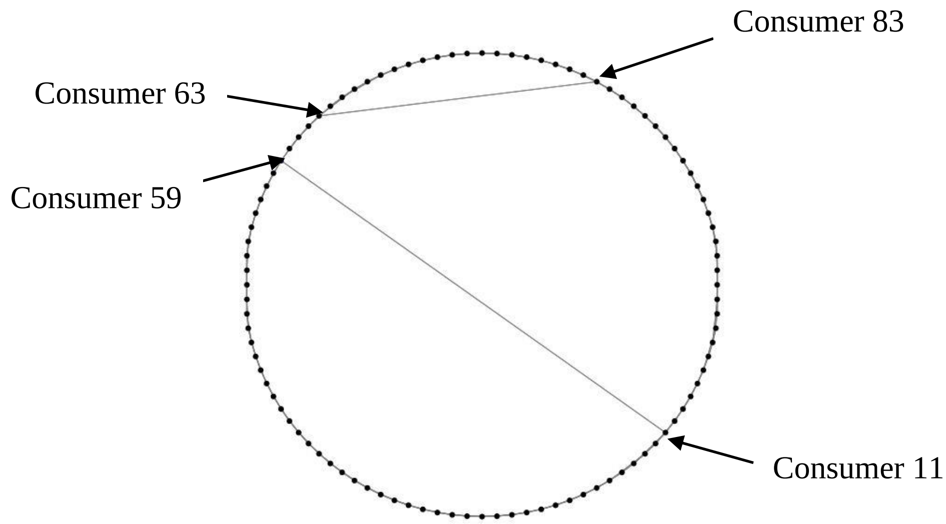


Figure 33: Network of seed 3, rewiring probability 0.01

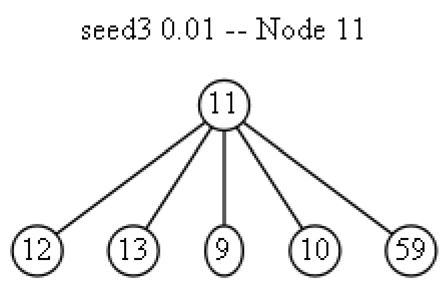


Figure 34: Node 11 and its direct neighbors

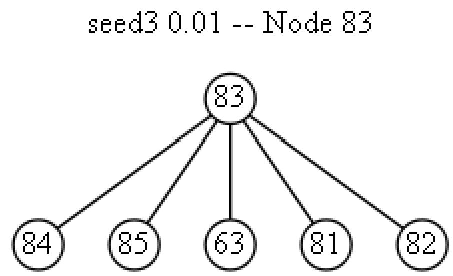


Figure 35: Node 83 and its direct neighbors

7.1.2 Scale-free network

There are three scale-free networks created in chapter 4. As shown in Figure 36, the scale-free network of random seed 1 is used in this experiment. Consumer 2 (red node) who has 17 neighbors is selected as a connector. In Model 2-1, consumer 2 is a pirate.

7.1.3 Random network

There are three random networks created in chapter 4. As shown in Figure 37, the random network of random seed 3 is used in this experiment. Consumers 52 and 4, each has seven neighbors, are selected as connectors. In Model 2-1, consumers 52 and 4 are pirates.

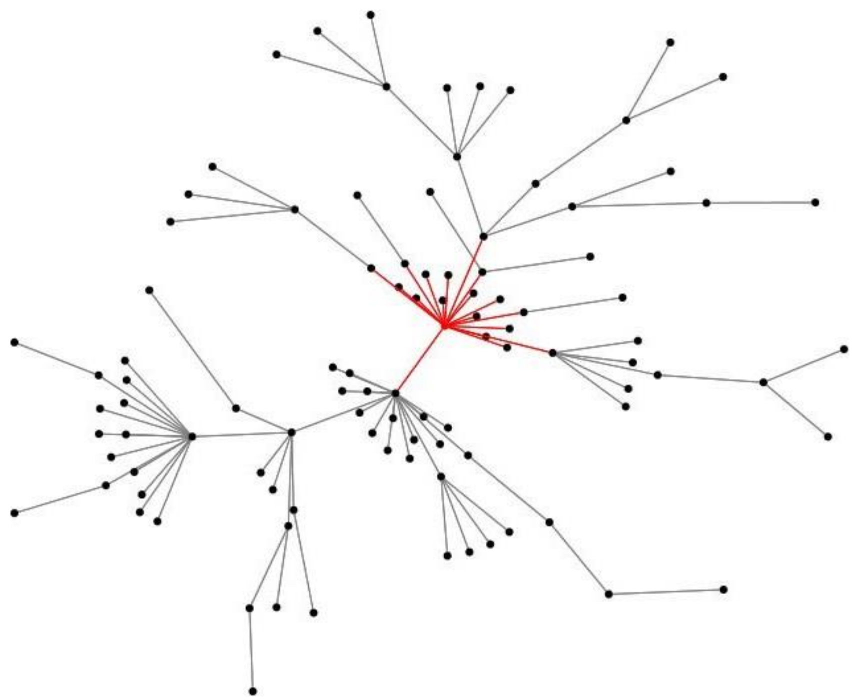


Figure 36: A scale-free network of seed 1

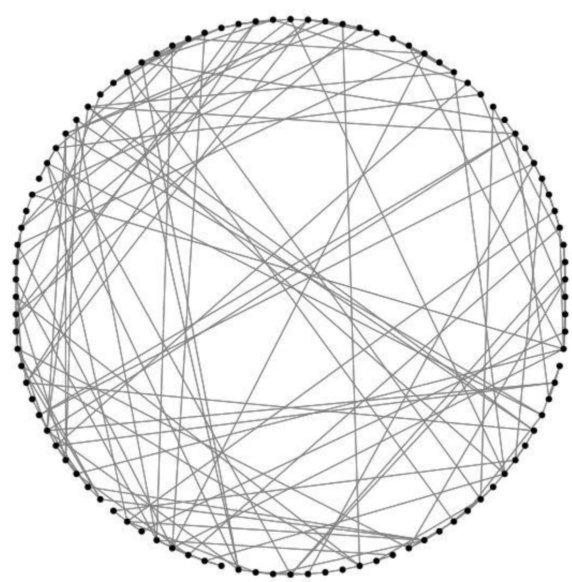


Figure 37: A random network of seed 3

7.1.4 Experimental results and analysis

In each network, results are compared with the base case (NP) at 95% confidence interval using K-S test. In each experiment, the selected connector is assigned as a pirate. P11 indicates the scenario in which the consumer 11 is a pirate. Tables 8 and 9 list the p and mean values of K-S test on the profits and diffusion speed.

Table 8: Comparison of profits for three types of networks

Small-world network			**Scale-free network**			**Random network**		
	Profit			Profit			Profit	
	p	mean		p	mean		p	mean
NP		932.6	NP		1258	NP		1283
P11	0	905.6	P2	0	1247	P52	0	1264
P83	0.009	902.2				P4	0	1266

Table 9: Comparison of diffusion speeds for three types of networks

Small-world network			**Scale-free network**			**Random network**		
	Diffusion speed			Diffusion speed			Diffusion speed	
	p	mean		p	mean			
NP		18.88	NP		8.474	NP		8.38
P11	0.954	18.84	P2	0.413	9.7	P52	0.996	8.56
P83	0.155	19.48				P4	1	8.5

For the small-world network, in the first experiment, consumer 11 is a pirate, referred as scenario P11. The profits are compared with the profits in the non-piracy scenario, referred as NP. The results reveal that the *p_value* is zero, the mean of NP is \$932.6, and the mean of P11 was \$905.6. Therefore, NP is significantly higher than P11 on profits. The diffusion speed is also compared with the diffusion speed in the non-piracy scenario (NP). The results reveal that the *p_value* is 0.954. Therefore, there is no significant difference between NP and P11 on diffusion speed.

In the second experiment, consumer 83 is the pirate, referred as scenario P83. The profits are compared with the profits in the non-piracy scenario (NP). The results reveal that the *p_value* is 0.009, the mean of NP is $932.6, and the mean of P83 is $902.2. Therefore, NP is significantly higher than P83 on profits. The diffusion speed is also compared with the diffusion speed in the non-piracy scenario (NP). The results reveal that the *p_value* is 0.155. Therefore, there is no significant difference between NP and P83 on diffusion speed.

For the scale-free network, consumer 2 is a pirate, referred as scenario P2. The profits are compared with the profits in the non-piracy scenario, referred as NP. The results reveal that the *p_value* is zero, the mean of NP is $1258, and the mean of P2 was $1247. Therefore, NP is significantly higher than P2 on profits. The diffusion speed is also compared with the diffusion speed in the non-piracy scenario (NP). The results reveal that the *p_value* is 0.413. Therefore, there is no significant difference between NP and P2 on diffusion speed.

For the random network, in the first experiment, consumer 52 is a pirate, referred as scenario P52. The profits are compared with the profits in the non-piracy scenario, referred as NP. The results reveal that the *p_value* is zero, the mean of NP is $1283, and the mean of P52 is $1264. Therefore, NP is significantly higher than P52 on profits. The diffusion speed is also compared with the diffusion speed in the non-piracy scenario

(NP). The results reveal that the *p_value* is 0.996. Therefore, there is no significant difference between NP and P_11 on diffusion speed.

In the second experiment, consumer 4 is the pirate, referred as scenario P4. The profits are compared with the profits in the non-piracy scenario (NP). The results reveal that the *p_value* is zero, the mean of NP is $1283, and the mean of P4 is $1266. Therefore, NP is significantly higher than P4 on profits. The diffusion speed is also compared with the diffusion speed in the non-piracy scenario (NP). The results reveal that the *p_value* is 1. Therefore, there is no significant difference between NP and P4 on diffusion speed.

The results of these experiments indicate that a connector is not necessarily a good candidate to be a key agent. Therefore, hypothesis 1 is not true. Why is the connector not necessarily a good candidate for being a key agent? Section 7.1.4.1 provide detailed explanation with one example.

7.1.4.1 Comparison of run 45 among NP, P83, and P11

The best-so-far strategy of each run may have slightly different adjustment of the price, the promotion cost, and the piracy detection cost, therefore, they may pass the consumers' thresholds at different time steps, thus the adoptions may start at different time steps among different runs.

Let's compare the best-so-far strategy of run 45 among NP, P11 and P83, in which all three strategies start adoption at the same time step.

As shown in Table 10, NP, P11, and P83 have the exact same adoption patterns. In the scenario of P83, the pirate 83 pirates the product at time step 8. Also at time step 8, there are eight purchases and the one piracy, so the total adoption amount is nine, which is same as the adoption amount of NP at time step 8. In the scenario of P11, the pirate 11 pirates the product at time step 4. At time step 4, there are three purchases and one piracy for a total adoption amount of four, which is same as the adoption amount of NP at time step 4**.**

Table 10: Comparison of amount of adoptions among NP, P83, and P11

Time step	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
NP	4	4	4	5	7	9	9	12	11	8
P83_buy	4	4	4	5	7	9	**8**	12	11	8
P83_pirate							**1**			
P11_buy	4	4	**3**	5	7	9	9	12	11	8
P11_pirate			**1**							

Time step	*12*	*13*	*14*	*15*	*16*	*17*	*Total*
NP	8	5	4	4	4	1	99
P83_buy	8	5	4	4	4	1	99
P11_buy	8	5	4	4	4	1	99

Each number in Table 11 is rounded to 2 digits after the decimal point. As shown in Table 11, the adoption starts at time step eight. Pirate 11 pirates the product at time step 4, and pirate 83 pirates the product at time step 8. This indicates that other legal buyers have already adopted the products before the pirates 11 and 83 did so. A pirate only makes a difference on diffusion patterns when he is able to influence other consumers while legal buyers did not adopt due to the restrictions of price and promotion costs. After the legal buyers starts the adoption, the provider's price and promotion cost are adjusted to the acceptable levels of legal buyers and detection cost are adjusted to the acceptable level of a pirate. Under such condition, a pirate could not make a difference in the diffusion pattern and the diffusion speed anymore, and the only difference he made is reducing the profit of the provider.

Regarding profits, with P11, the provider lost $17.89 at time step 4 due to piracy, and with P_83 lost $26.35 at time step 8 due to piracy. Therefore, with P11, the provider's total profit was $939.31. With P_83, the provider's total profit was $896.45. With no pirates (NP), the provider's total profit was $1015.2. As a result, with either pirate, the provider had less profit when compared to the scenario with no pirates.

Table 11: Comparison of profits among NP, P83, and P11

Time step	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*
NP	-30	-13.75	35.94	35.94	**35.94**	50	78.13	112.6
P83	-30.0	-33.75	15.93	35.937	35.94	40.31	68.43	96.56
P11	-30	-13.75	35.94	32.11	**18.05**	44.26	72.38	100.5

Time step	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*
NP	**112.6**	156.98	142.11	97.81	90.49	46.19	27.17	27.17
P83	**86.25**	142.5	136.17	92.937	92.94	48.64	29.06	29.06
P11	100.5	138.75	124.69	82.5	92.5	50.32	36.25	36.25

Time step	*16*	*17*	Total
NP	27.17	-17.13	1015.2
P83	27.81	-16.48	898.27
P11	34	-15.94	939.31

7.2 Heterogeneous consumers

The same experiments are conducted on heterogeneous consumers. Tables 12 and 13 list the p and mean values of K-S test on profits and diffusion speed.

Table 12: Comparison of profits for three types of networks

Small-world network			**Scale-free network**			**Random network**		
	Profit			Profit			Profit	
	p	mean		p	mean			
NP		429.1	NP		775.1	NP		819.7
P11	0.017	412.1	P2	0.032	765.1	P52	0	802.2
P83	0.032	412.2				P4	0	802.0

Table 13: Comparison of diffusion speeds for three types of networks

Small-world network			**Scale-free network**			**Random network**		
	Diffusion speed			Diffusion speed			Diffusion speed	
	p	mean		p	mean			
NP		19.42	NP		10.6	NP		10.04
P11	0.358	19.12	P2	0.032	11.12	P52	1	10.02
P83	0.508	19.24				P4	0.996	10

For the small-world network, in the first experiment, consumer 11 is a pirate, referred as scenario P11. The profits are compared with the profits in the non-piracy scenario, referred as NP. The results reveal that the *p_value* is 0.017, the mean of NP is $429.1, and the mean of P11 was $412.1. Therefore, NP is significantly higher than P11 on profits. The diffusion speed is also compared with the diffusion speed in the non-piracy scenario (NP). The results reveal that the *p_value* is 0.358. Therefore, there is no significant difference between NP and P11 on diffusion speed.

In the second experiment, consumer 83 is the pirate, referred as scenario P83. The profits are compared with the profits in the non-piracy scenario (NP). The results reveal that the *p_value* is 0.032, the mean of NP is $429.1, and the mean of P83 is $412.2. Therefore, NP is significantly higher than P83 on profits. The diffusion speed is also compared with the diffusion speed in the non-piracy scenario (NP). The results reveal that the *p_value* is 0.508. Therefore, there is no significant difference between NP and P83 on diffusion speed.

For the scale-free network, consumer 2 is a pirate, referred as scenario P2. The profits are compared with the profits in the non-piracy scenario (NP). The results reveal that the *p_value* is 0.032, the mean of NP is $775.1, and the mean of P2 is $765.1. Therefore, NP is significantly higher than P2 on profits. The diffusion speed is also compared with the diffusion speed in the non-piracy scenario (NP). The results reveal that the *p_value* is 0.032. The mean of NP is 10.6, and the mean of P2 is 11.12. Therefore, NP is significantly faster than P2 on diffusion speed.

For the random network, in the first experiment, consumer 52 is a pirate, referred as scenario P52. The profits are compared with the profits in the non-piracy scenario, referred as NP. The results reveal that the *p_value* is zero, the mean of NP is $819.7, and the mean of P52 is $802.2. Therefore, NP is significantly higher than P52 on profits. The diffusion speed is also compared with the diffusion speed in the non-piracy scenario (NP). The results reveal that the *p_value* is 1. Therefore, there is no significant difference between NP and P52 on diffusion speed.

In the second experiment, consumer 4 is the pirate, referred as scenario P4. The profits are compared with the profits in the non-piracy scenario (NP). The results reveal that the *p_value* is zero, the mean of NP is $819.7, and the mean of P4 is $802. Therefore, NP is significantly higher than P4 on profits. The diffusion speed is also compared with the diffusion speed in the non-piracy scenario (NP). The results reveal that the *p_value* is 0.996. Therefore, there is no significant difference between NP and P4 on diffusion speed.

7.2.1 Comparison of run 10 among NP, P83, and P11

Take run 10 as an example for the analysis of heterogeneous consumers. As shown in Table 14, the P11 and P83 do not have the same adoption patterns as NP due to the heterogeneity of consumers. However, the heterogeneous prices, promotion cost thresholds, and the piracy detection thresholds of consumers are normally distributed with mean $15 and standard deviation 3. It indicates almost 70% of the thresholds are around 15 +/- 3. In other words, between 12 and 18. Figures 9, 10, and 11 illustrate the normal distribution of heterogeneous prices, promotion costs, and piracy detection costs. As shown in Table 14, although the diffusion patterns among NP, P11, and P83 are different, there are no dramatic difference.

For homogeneous consumers, a good strategy adjusts its price, promotion cost, and detection cost gradually gravitates towards the thresholds ($15) of a consumer, after the adoption starts, it keeps the price and promotion cost close to the thresholds and keep the piracy detection cost as low as possible. As long as the strategy follows such rules, the amount of adoptions at each time step is determined by the network topology.

Table 14: Comparison of adoptions among NP, P11, and P83

Time step	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
NP	4	4	4	5	6	7	6	10	11	6
P83_buy	4	4	4	5	6	7	**5**	10	11	6
P83_pirate							**1**			
P11_buy	2	3	**3**	3	4	4	2	16	8	11
P11_pirate			**1**							

Time step	*12*	*13*	*14*	*15*	*16*	*17*	*Total*
NP	7	5	4	4	13	3	99
P83_buy	7	5	4	3	13	4	99
P11_buy	11	8	8	8	4	3	99

In the case of heterogeneous consumers, the consumer adopts the product and the number of consumers who adopt the product at each time step become uncertain due to the heterogeneity of the consumers. A strategy could only propose one price and one promotion cost at each time step. So, what is the optimal price and promotion cost to set at each time step? The strategy makes the decision based on one rule, that is, maximizing its fitness values, which, in this model, are the profits and the diffusion speed. The importance of the self-learning strategy is more obvious in the case of heterogeneous consumers.

7.3 Conclusions concerning a connector

- A connector only makes a difference on diffusion patterns when he is able to influence others while other consumers fail to adopt due to the restrictions of prices and promotion cost.

- Assigning a connector as a key agent does not necessarily increase profits and accelerate diffusion speed. As a result, a connector is not a good candidate for being a key agent.
- Dynamic self-learning marketing strategies alone are not enough to increase profits and accelerate diffusion speed. Network topologies are another important factor which contributes to the performance of profits and diffusion speed.

CHAPTER 8: MONOPOLY – IMPACT OF AN OPINION LEADER

Besides being the most connected consumer, an opinion leader also impacts the adoption decisions of other consumers. Therefore, an opinion leader seems like a good candidate to be a key agent.

The following experiments show whether an opinion leader is an ideal candidate to be a key agent when the social network topologies are considered. We still start with a hypothesis.

Hypothesis 2: the opinion leader (a persuader who has the most connection) is the ideal candidate to be a key agent.

Model 2-2 is used in the experiments with both homogeneous and heterogeneous consumers on three networks: a small-world network, a random network, and a scale-free network. The results show that compare to the base case (NP), an opinion leader does not necessarily increase profits and accelerate diffusion speeds significantly in small-world networks and random networks. However, an opinion leader has a higher probability of generating higher profits and/or accelerating diffusion speeds significantly in a scale-free network.

8.1 Homogeneous consumers

The same experiments are conducted on the same three networks and their corresponding opinion leaders. In this experiment, instead of just being highly connected, those opinion leaders also influence the adoptions decisions of other consumers.

8.1.4 Experimental results and analysis

Experimental results in Tables 15 and 16 show that in the small-world network and the random network, assigning the opinion leader as a pirate does not necessarily increase profits and speed up the diffusion. However, the scale-free network demonstrates a different result. Therefore, hypothesis 2 is only partially true.

Table 15: Comparison of profits for three types of networks

Small-world network			Scale-free network			Random network		
	Profit			Profit			Profit	
	p	mean		p	mean		p	mean
NP		932.6	NP		1258	NP		1283
P11	0.095	912.5	P2	0	1358	P52	0	1266
P83	0.004	904.4				P4	0	1268

Table 16: Comparison of diffusion speeds for three types of networks

Small-world network			Scale-free network			Random network		
	Diffusion speed			Diffusion speed			Diffusion speed	
	p	mean		p	mean			
NP		18.88	NP		9.66	NP		8.38
P11	0.841	19.12	P2	0	10.38	P52	0.358	8.8
P83	0.241	19.34				P4	1	8.42

In the scale-free network, when assigning pirate 2 as an opinion leader, referred as scenario P2, the profits increased dramatically, as shown in Table 15. Why does P2 make such a make difference?

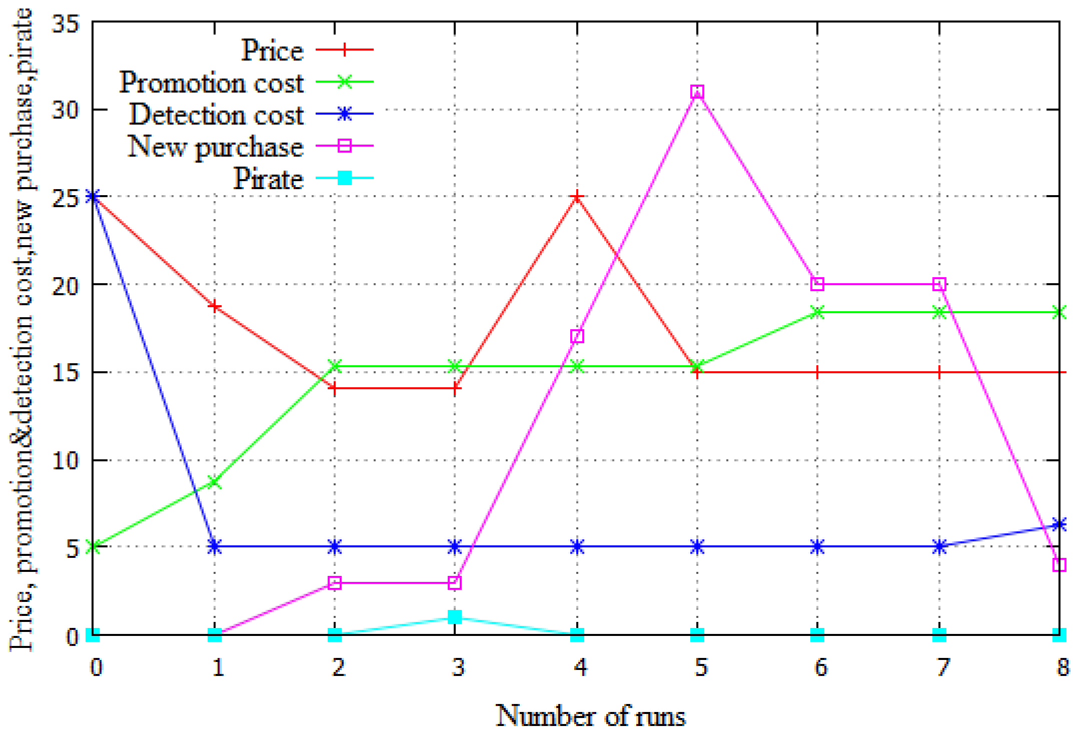


Figure 38: Best of best-so-far strategies of P2 (Model 2-2)

Figure 38 demonstrates the impacts of the best of best-so-far marketing strategies on the price, promotion cost, and new adoptions. I found out that after the adoption starts, instead of staying beneath the price threshold, the price suddenly jumped to the maximum price of $25 at time step 4, accompanied with a large increase of purchases at time step 4. It happened right after pirate 2 pirated the product at the time step 3. Those 17 adoptions are all caused by pirate 2. Because pirate 2 is an opinion leader, all 17 followers purchased the product regardless of the price and the promotion cost. The self-learning marketing strategy evolved by EA is guided by one rule – maximize the fitness values (profits, and then diffusion speeds). As a result, if no consumer cares about the price, the strategy should be to increase the price to the maximum ($25) in order to acquire a higher profit.

The question is why the same phenomena does not happen to a small-world network and a random network. I take the best of best-so-far strategies of P11 in the small-world network as an example for analysis. Pirate 11 has five neighboring consumers: 9, 10, 12, 13, 59. When pirate 11 pirated the product at time step 4, consumer 9 and 10 had already adopted the product, he only has consumer 12, 13, and 59 left to influence. There are another two consumers 97 and 98 who are ready to purchase the product. Now the strategy could either adjust the price close to maximum $25 for consumers 12, 13, and 59, or it could adjust the price within acceptable range and get consumers 12, 13, 59, 97 and 98 to purchase the product. The price at time step 4 is close to $15, and it turned out there is not much difference in profits comparing these two options. As a result, the strategy chose the second option to have all five consumers purchase the product. I do not see a significant increase in the diffusion speed since the marketing strategy has the profit as its highest priority and the diffusion speed is its secondary priority.

If the followers and other regular consumers are both ready for adoptions, the marketing strategy evaluates the profits of two options as above. If the number of followers of the opinion leader is overwhelming, and by adjusting the price to the maximum (and lower the promotion cost), the profit gained compensates the loss due to the discarding of regular consumers, the strategy adjusts its price to the maximum (and lower the promotion cost). Otherwise, if there is no distinctive difference on the number of two groups of adopters, the strategy mostly chose to stay within the range of acceptance thresholds of consumers, which brings more adopters and higher profits.

In summary, since the degree in a scale-free network follows the power-law distribution, the opinion leader in a scale-free network has much higher probability of making a significant contribution to the profits and/or diffusion speeds. The power distribution does not apply to a small-world network and a random network, thus, it is not likely to have the sudden jump of price as illustrated in Figure 38. After the adoption starts, as long as the price and promotion cost stay within the acceptable range of the thresholds of consumers, the opinion leader is no different from a connector. And since the opinion leader gets the free copy either as a gift or through piracy, so all he does is reduce the profit.

8.2 Heterogeneous consumers

The same experiments are conducted on the same three networks for heterogeneous consumers. As shown in Tables 17 and 18, in the small-world network and the random network, assigning the opinion leader as a pirate does not necessarily increase profits and speed up the diffusion. However, pirate 2 in the scale-free network is able to generate higher profits with heterogeneous consumers.

Table 17: Comparison of profits for three types of networks

Small-world network			**Scale-free network**			**Random network**		
	Profit			Profit			Profit	
	p	mean		p	mean			
NP		429.1	NP		775.1	NP		832.6
P11	0.017	412.1	P2	0	961.7	P52	0.678	829.7
P83	0.358	418.8				P4	0.241	838.7

Table 18: Comparison of diffusion speeds for three types of networks

Small-world network			**Scale-free network**			**Random network**		
	Diffusion speed			Diffusion speed			Diffusion speed	
	p	mean		p	mean			
NP		19.42	NP		9.81	NP		10.48
P11	1	19.42	P2	0	11.56	P52	0.954	10.40
P83	1	19.50				P4	0.056	9.74

8.3 Conclusions concerning an opinion leader

- An opinion leader has much higher possibility of generating a high profit in a scale-free network. Therefore, an opinion leader is a good candidate to be a key agent in a scale-free network.
- In a small-world network and a random network, assigning an opinion leader as a key agent does not necessarily increase profits and accelerate diffusion speed. Therefore, an opinion leader is not necessarily a good candidate to be a key agent in a small-world network and a random network.
- Dynamic self-learning marketing strategies alone are not enough to increase profits and accelerate diffusion speed. Network topologies are another important factor which contributes to the performance of profits and diffusion speed.

- Additionally, a self-learning strategy is able to adjust its price and promotion cost by evaluating the potential profits brought by followers of the opinion leaders who pay at a high price and potential profits brought by the average consumers.

CHAPTER 9: MONOPOLY – POSITION OF A KEY AGENT

From experiments in chapters 7 and 8, I conclude that, in order for a key agent to make differences on profits and diffusion speeds, he has to adopt the product before the adoption of other consumers start. How can we make a key agent the first adopter in the network?

The answer lies in the position of the key agent in the network. There is no piracy in Model 1-1 and Model 1-2. As discussed by Barker (2016), the business likes to approach and appoint a customer who has already heard of the product as a key agent. If the key agent is among the direct neighbor of the innovation initiator, he is the first consumer who heard of the product. Now if the provider provides a free copy of the digital information goods to the key agent, he becomes the first adopter. In Model 2-1 and Model 2-2, the key agent is a pirate. If the pirate is among the direct neighbors of the innovation initiator, then he has a higher probability of being in contact with the new product before legal buyers do. Since a pirate is only restricted by the piracy detection cost, he adopts the product much quicker than legal buyers who are restricted by both the price and the promotion cost. It is likely that the moment a pirate adopts the product, a provider is still adjusting his price and promotion cost to meet the reservation prices and promotion cost thresholds of consumers. Does such a design guarantee that the provider achieves higher profits and faster diffusion speed? I start with the third hypothesis. This is tested with homogenous and heterogeneous consumers in small-world networks with a key agent at different distances from the innovation initiator.

Hypothesis 3: positioning the key agent next to the innovation initiator leads to a higher profit and faster diffusion.

A total of 15 small-world networks are created by using three random seeds, and each random seed is coupled with five rewiring probabilities. Three scale-free networks and three random networks are created using the same three random seeds. The direct neighborhood of the innovation initiator is defined as L1, and the director neighborhood of L1 is defined as L2, and so on. Consumer 5 is defined as the innovation initiator in every network. Starting from the first level (L1), second (L2) and third level (L3) neighbors of the innovation initiator, each consumer is made a key agent sequentially, and the experiment is conducted on each scenario. Profits and diffusion speed of each scenario are compared with the profits and diffusion speed for scenario without a key agent (NP).

9.1 Homogeneous consumers in small-world networks

Four models are experimented and results are analyzed. The four models are Model 1-1 in which the key agent is an average consumer, Model 1-2 in which the key agent is a persuasive consumer, Model 2-1 in which the key agent pirate is an average pirate, and Model 2-2 in which the key agent is a persuasive pirate. I start with the Model 2-1.

9.1.1 Model 2-1 (an average pirate)

In Table 19, L1:7 indicates that if consumer 7 in layer 1 is a pirate, he causes higher profits or accelerates diffusion speed significantly. "None" indicates that no pirate could make a difference. The results in Table 19 show that the pirates in L1 are able to increase the profits and accelerate the diffusion speed. In the network of seed 3 and rewiring probability 0.07, besides a pirate in L1, pirate 6 in L2 also accelerate the diffusion process. For the detailed values of D, p, and mean values in below table, please refer to Appendix F.

Table 19: Comparison of profits and diffusion speeds (Homo, Model 2-1)

Rewiring probability	Network of seed 1		Network of seed 2		Network of seed 3	
	Profit	Diffusion speed	Profit	Diffusion speed	Profit	Diffusion speed
0.01	L1: 7	L1: 7	None	L1: 7	L1: 6, 7	L1: 6, 7
0.03	L1: 6	L1: 6, 7	L1: 7	L1: 7	L1; 7	L1: 7
0.05	None	None	L1: 34	L1: 34	L1: 7	L1: 7
0.07	L1: 4, 64	L1: 4, 64	None	None	L1: 7, 96	L1: 7, 96 L2: 6
0.1	L1: 4, 64	L1: 4, 64	None	None	None	None

The results in Table 19 partially confirmed the hypothesis 3, that is, it is possible to increase profits and accelerate diffusion speed by having a pirate among the direct neighbors of the innovation initiator. However, from above results, not every pirate in L1 is able to make a difference. Therefore, hypothesis 3 is only partially true, that is, positioning a pirate among direct neighbors of the innovation initiator has the possibility to increase profits or diffusion speed, but there is no guarantee that every pirate in the direct neighbors makes a difference. This leads to several obvious questions:

1. Why does only certain pirate in L1 make a difference on profits and diffusion speeds?
2. Does positioning a pirate in L1 guarantee a higher profit and faster diffusion?

Above two questions will be answered in the next two sections with detailed examples.

9.1.1.1 Why does only the pirate in the first level (L1) make a difference on profits and diffusion speeds?

Figure 39 displays the innovation initiator, L1, and L2 of the network of seed 1 and rewiring probability 0.01. Consumer 7 is assigned as a pirate. P7 is defined as a scenario in which consumer 7 is the pirate. Similar to P7, NP is defined as a scenario in which there is no pirate. Then the adoption pattern and profit of the best-so-far strategy of P7 is compared with the adoption pattern and profit of the best-so-far strategy of NP. As shown in Tables 20, 21, and 22, P7 is able to achieve both significantly higher profit and faster diffusion speed than NP.

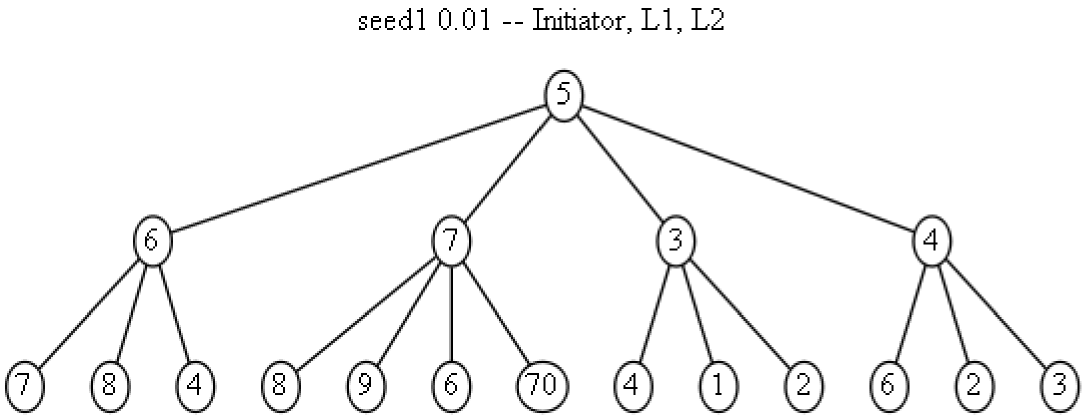


Figure 39: Initiator, L1, and L2 of network of seed 1, rewiring probability 0.01

Why does P7 make a difference? Take run 46 as an example to analyze the difference on diffusion patterns and profits between NP and P7. Table 20 compares the amount of new purchase and piracy between NP and P7. Results show that NP and P7 do not have the same adoption patterns. Also, P7 holds higher amount of adoptions vs. NP except at time step 7 and 9. Table 21 compares the profits earned at each time step for NP and P_7. The drop of adoptions at time step 7 and 9 are reflected in the profits at time step 7 and 9 for P7.

Table 20: Comparison of amount of adoptions between NP and P7

Time step	*1*	*2*	*3*	*4*	*5*	*6*	**7**	8	**9**	*10*
NP	0	4	5	7	10	11	**10**	8	**8**	4
P7_buy	0	6	7	10	11	12	**8**	8	**4**	4
P7_pirate	1									

Time step	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*Total*
NP	4	4	4	4	4	4	4	4	99
P7_buy	4	4	4	4	4	4	4		99

Table 21: Comparison of profits between NP and P7

Time step	*0*	*1*	*2*	*3*	*4*	*5*	*6*	***7***
NP	-30.0	-27.5	10.0	20.56	78.51	122.8	137.6	**122.8**
P7_buy	-30.0	-21.5	51.05	76.69	120.8	137.5	151.0	**92.19**

Time step	*8*	***9***	*10*	*11*	*12*	*13*	*14*	*15*
NP	93.3	**88.3**	29.15	29.15	25.65	25.65	25.65	25.65
P7_buy	92.19	**33.39**	33.39	32.48	31.46	31.46	31.46	28.52

Time step	*16*	*17*	*18*	*Total*
NP	25.65	22.69	26.19	852.02
P7_buy	21.59	12.99		926.75

Table 22 compares the total profits between NP and P7. Although profits of P7 drops at time step 7 and 9 due to less adoptions, it does not affect the total profit of P7 much. The final profit of P7 ($926.75) is still higher than the profit of NP ($852.02).

Table 22: Comparison of total profits between NP and P7

Time step	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*
NP	-30.0	-57.5	-47.5	-26.94	51.58	174.45	312.11	434.99
P7_buy	-30.0	-51.5	-0.45	76.25	197.05	334.55	485.55	577.75

Time step	*8*	*9*	*10*	*11*	*12*	*13*	*14*
NP	528.29	616.59	645.73	674.88	700.53	726.18	751.83
P7_buy	669.95	703.35	736.75	769.23	800.69	832.16	863.63

Time step	16	17	18	Total
NP	803.13	825.83	852.02	852.02
P7_buy	913.75	926.75		926.75

From results in Tables 20, 21, and 22, I find the answer to the first question: why do some pirates in L1 make a difference in profit and diffusion speed? The answer is that a pirate in L1 does not follow the same adoption pattern as NP. Results in Table 20 also show that pirate 7 in L1 adopts the product at time step 1, which is before adoptions of all other legal buyers. From all experiment results on 15 networks, I discovered that any pirate in L1 adopts the product before all other legal buyers do. In addition, besides self-learning marketing strategies, the amount of adoption of pirate 7 at each time step also depends on the topologies of the network.

9.1.1.2 Does positioning a pirate in the first level (L1) guarantee a higher profit and faster diffusion?

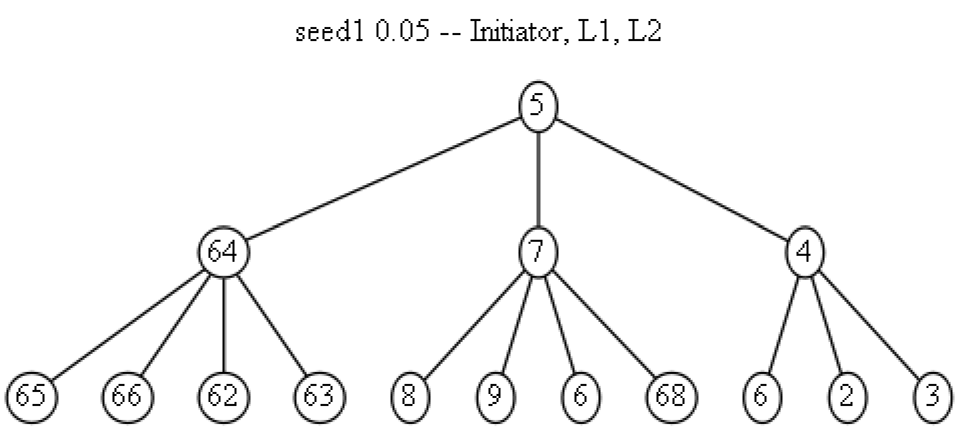


Figure 40: Initiator, L1, and L2 of network of seed 1, rewiring probability 0.05

Figure 40 displays the innovation initiator, L1, and L2 of the network of seed 1 and rewiring probability 0.05. In this network, consumer 7 has many connections and those connections also contain shortcuts. Therefore, consumer 7 seems like a good candidate for being a pirate. Profits are compared between NP and P7. The results show that the p-value is zero. The mean value of NP is $1144, and mean value of P7 is $1123. Therefore, NP is significant higher than P7 on profits. The question is why P7 did not make a difference on profits. The answer still lies in the topology of the network.

Take run 8 as an example to analyze the difference on adoption patterns and profits between NP and P7. As shown in Table 23, pirate 7 pirates the product at time step 1. The adoptions of legal buyers start at time step 5. Although P7 has an early higher adoption rate vs NP, the adoption rate declines after time step 9.

Table 23: Comparison of amount of adoptions between NP and P7

Time step	*1*	*5*	*6*	*7*	*8*	***9***	*10*	*11*	*12*	*13*	*14*	***15***	Total
NP	0	3	10	12	17	**13**	16	10	7	4	4	**3**	99
P7_buy	0	6	11	14	18	**12**	14	8	6	4	4	**1**	99
P7_pirate	1												

Table 24 shows the comparison of profits earned at each time steps between NP and P7. Table 25 shows the comparisons of total profits between NP and P7. As shown in Table 24, from time step 9, NP already catches up to P7 on profits and diffusion. The final step is fatal for the profits of P7. In Table 25, at step 14, the accumulated profit of P7 1136.52, while the accumulated profit of NP is 1127.80. P7 is still winning at step 14. However, the final adoption amount of three in NP turns the tables. The profit of P7 gained at time step 15 is $-6.25 due to the fact that one purchase is not able to cover the cost. However, for NP, the final purchase of three make NP gained the profit of 24.02, and make NP the final winner.

Table 24: Comparison of profits between NP and P7

Time step	*0*	*1*	*2*	*3*	*4*	*5*	*6*
NP	-30.0	-11.25	-12.81	-14.76	-17.2	24.02	127.36
P7	-30.0	-15.0	-12.81	-14.76	-17.20	68.314	142.12

Time step	*7*	*8*	***9***	*10*	*11*	*12*
NP	156.88	230.69	**171.65**	215.93	127.36	83.07
P7	186.41	245.46	**156.63**	186.16	97.58	67.55

Time step	13	14	15	Total
NP	38.79	38.04	24.02	1151.83
P7	38.02	38.02	-6.25	1130.26

Table 25: Comparison of total profits between NP and P7

Time step	*0*	*1*	*2*	*3*	*4*	*5*	*6*
NP	-30.0	-41.25	-54.06	-68.82	-86.03	-62.01	65.35
P7	-30.0	-45.0	-57.81	-72.57	-89.78	-21.47	120.65

Time step	*7*	*8*	*9*	*10*	*11*	*12*
NP	222.24	452.94	624.59	840.53	967.89	1050.97
P7	307.06	552.52	709.16	895.33	992.92	1060.47

Time step	*13*	***14***	***15***	Total
NP	1089.76	**1127.80**	**1151.83**	1151.83
P7	1098.49	**1136.52**	**1130.26**	1130.26

In conclusion, a pirate in L1 does not guarantee a significantly higher profit and diffusion speed. The self-learning marketing strategies and the network topology together determine the adoption pattern and profit with the pirate. This is the answer to the second question.

9.1.2 Model 2-2 (a persuasive pirate)

As shown in Table 26, compared to the results in Model 2-2, with the pirate being a persuader, there are more pirates in L1 that are able to make a difference on profits. The pirate is able to generate higher profits compared to the profits in Model 2-1. The pirate who make the most distinctive improvement in profits is marked as red and bold in the table. For example, for pirate 7 in the network of seed 1, rewiring probability 0.03. The mean of NP is $ 1135.37, and the mean of pirate 7 is $1212.86. For the detailed values of D, p, and mean values in below table, please refer to Appendix G.

Table 26: Comparison of profits and diffusion speeds (Homo, Model 2-2)

Rewiring probability	Network of seed 1		Network of seed 2		Network of seed 3	
	Profit	Diffusion speed	Profit	Diffusion speed	Profit	Diffusion speed
0.01	L1: **7** L3: 11	L1: 7, 4 L2: 1	L1: **7**	L1: 7	L1: 6, **7**	L1: 6, 7
0.03	L1: 6, **7**	L1: 6, 7	L1: **7**	L1: 7	L1: 3,7, **96**	None
0.05	L1: **7,** 64	L1: 7, 64	L1: **34,** 3, 4, 6, 7	None	L1: **7**	L1: 7
0.07	L1: **4,** 64, 7	L1: 4, 64, 7	L1: 34, 3, 4, 6, **7**	None	L1: 3, 4, **7,** 96	L1: 7 L2: 2

0.1	L1: 4, **64**, 7	L1: 4, 64, 7	L1: **34**, 3, 4, 6, 7	None	L1: 3, 7, **96**	None

9.1.3 Model 1-1 (an average consumer)

Besides showing that the key agent in L1 are able to increase the profits and accelerate the diffusion speed, the results in Table 27 also bear great similarities with results of Model 2-1. For the detailed values of D, p, and mean values in below table, please refer to Appendix H.

Table 27: Comparison of profits and diffusion speeds (Homo, Model 1-1)

Rewiring probability	Network of seed 1		Network of seed 2		Network of seed 3	
	Profit	Diffusion speed	Profit	Diffusion speed	Profit	Diffusion speed
0.01	L1: 6, 7	L1: 6, 7	None	L1: 7	L1: 6, 7	L1: 6, 7
0.03	L1: 6, 7	L1: 6, 7	None	L1: 7	L1; 7	L1: 7
0.05	None	L2: 9	None	L1: 34	L1: 7	L1: 7
0.07	L1: 64	L1: 4	None	None	L1: 7	L1: 7, 96
0.1	None	L1: 4, 64	None	None	None	None

9.1.4 Model 1-2 (a persuasive consumer)

As shown in Table 28, there are more key agents in L1 that are able to increase the profits and speed up the diffusion process. The key agent is able to generate higher profits compared to the profits in Model 1-1. The key agent that is able to generate the highest profits is marked as red and bold. Also, the results are very similar to the results of Model 2-2. For the detailed values of D, p, and mean values in below table, please refer to Appendix I.

Table 28: Comparison of profits and diffusion speeds (Homo, Model 1-2)

Rewiring probability	Network of seed 1		Network of seed 2		Network of seed 3	
	Profit	Diffusion speed	Profit	Diffusion speed	Profit	Diffusion speed
0.01	L1: 3, 4, 6, **7**	L1: 3, 4, 6, 7	L1: 3, 4, 6, **7**	L1: 7	L1: 3, 4, 6, **7**	L1: 6, 7
0.03	L1: 3, 4, 6, **7**	L1: 3, 4, 6, 7	L1: 3, 4, 6, **7**	L1: 7	L1; 3, 6, **7**	L1: 7
0.05	L1: 4, **64**, 7	L1: 4, 64, 7 L2: 3, 6 L3: 35, 70	L1: 3, **34**, 4, 6, 7	L1: 3, 34, 4, 6, 7	L1: 4, 6, **7**	L1: 7
0.07	L1: **4**, 64, 7	L1: 4, 64, 7	L1: **3**, 34, 4, 6, 7	L1: 3, 34, 4, 6, 7	L1: 3, 4, **7,** 96	L1: 3, 4, 7, 96
0.1	L1: 4, **64**, 7	L1: 4, 64, 7	L1: 3, **34**, 4, 6, 7	L1: 3, 34, 4, 6, 7	L1: 3, 4, 7, **96**	L1: 3, 7, 96

9.2 Heterogeneous consumers in small-world networks

The same set of experiments is conducted on the 15 small-world networks with heterogeneous consumers. I start with Model 2-1.

9.2.1 Model 2-1 (an average pirate)

Experiments are conducted on the same 15 networks. In Table 29, the results indicate that pirates in L1 either increased profits or diffusion speed, or do not make differences at all. Due to the heterogeneity of the consumers, in the networks of seed 2 (rewiring probability 0.01 and 0.03), pirate 84 in L2 increased profits significantly. Also, due to the heterogeneity of the consumers, there are several occasions, besides pirates in L1, a few pirates in L2 or L3 also increased the profits or diffusion speed significantly. For the detailed values of D, p, and mean values in below table, please refer to Appendix F.

Table 29: Comparison of profits and diffusion speeds (Hetero, Model 2-1)

Rewiring probability	Network of seed 1		Network of seed 2		Network of seed 3	
	Profit	Diffusion speed	Profit	Diffusion speed	Profit	Diffusion speed
0.01	None	L1: 7	L2: 84	None	L1: 7	L1: 7
0.03	L1: 7, 6	L1: 7	L2: 84	None	L1: 7 L2: 9	L1: 7
0.05	L1: 7 L3: 1	L1: 7	L1: 34 L3: 58,	L1: 3 L3: 81, 93	L1: 7 L2: 9	L1: 7
0.07	L1: 7, 64	L1: 64	L1: 34 L2: 84	None	L1: 96	L1: 7
0.1	L1: 7, 64	L1: 4, 7, 64	L1: 34	L1:34, 6, 3, 4	L1: 96	L1: 7

9.2.2 Model 2-2 (a persuasive pirate)

As shown in Table 30, there are more pirates in L1 that are able to increase the profits and speed up the diffusion process. The pirate is able to generate higher profits compared to the profits in Model 2-1. The pirate that is able to generate the highest profits is marked as red and bold. For the detailed values of D, p, and mean values in below table, please refer to Appendix G.

Table 30: Comparison of profits and diffusion speeds (Hetero, Model 2-2)

Rewiring probability	Network of seed 1		Network of seed 2		Network of seed 3	
	Profit	Diffusion speed	Profit	Diffusion speed	Profit	Diffusion speed
0.01	L1: 6, **7**	L1: 7	L1: **7** L2: 84	L1: 7	L1: 6, **7** L2: 9	L1: 7
0.03	L1: 4, 6, **7** L2: 8, 9 L3: 42	L1: 7	L1: **7** L2: 84	None	L1: 6, **7** L2: 9	L1: 6, 7
0.05	L1: 4, 64, **7** L2: 2, 3 L3: 1, 36, 60	L1: 7	L1: **34**, 7 L2: 82, 84 L3: 58, 82, 86, 92	L1: 34	L1: 6, **7** L2: 9	L1: 7
0.07	L1: 64, **7** L2: 2 L3: 36	None	L1: **34,** 6, 7 L2: 84 L3: 82, 83	None	L1: 3, 7, **96** L3: 31	L3: 31
0.1	L1: 4, 64, **7** L2: 3, 63, 8 L3: 31, 32, 40	L1: 4, 7	L1: 3, **34**, 4, 6, 7 L2: 32 L3: 58	L1: 4	L1: 3, 4, 7, **96** L2: 1, 2, 9 L3: 0, 31	L1: 7

9.2.3 Model 1-1 (an average consumer)

Besides showing that the key agents in L1 are able to increase the profits and accelerate the diffusion speed, the results in Table 31 also bear great similarities with results of Model 2-1. For the detailed values of D, p, and mean values in below table, please refer to Appendix H.

Table 31: Comparison of profits and diffusion speeds (Hetero, Model 1-1)

Rewiring probability	Network of seed 1		Network of seed 2		Network of seed 3	
	Profit	Diffusion speed	Profit	Diffusion speed	Profit	Diffusion speed
0.01	L1: 7	L1: 7	L2: 84	L3: 99	L1: 7 L2: 9	L1: 7
0.03	L1: 6, 7 L2: 8	L1: 7	L2: 84	None	L1: 7 L2: 9	L1: 7
0.05	L1: 7	None	L1: 34 L2: 84	None	None	None
0.07	L1: 7	None	L1: 34, 84	L1: 34	L1: 96	L1: 7
0.1	None	L1: 4,7	L1: 34, 6	None	None	L1: 3, 4

9.2.4 Model 1-2 (a persuasive consumer)

As shown in Table 32, there are more key agents in L1 that are able to increase the profits and speed up the diffusion process. The key agent is able to generate more profits compared to the profits in Model 2-2. The key agent that is able to generate the highest profits is marked as red and bold. For the detailed values of D, p, and mean values in below table, please refer to Appendix I.

Table 32: Comparison of profits and diffusion speeds (Hetero, Model 1-2)

Rewiring probabilit	Network of seed 1		Network of seed 2		Network of seed 3	
	Profit	Diffusion speed	Profit	Diffusion speed	Profit	Diffusio n speed
0.01	L1: 3, 4, **7**	L1: 6, 7	L1: **7** L2: 84	None	L1: 3, 6, **7** L2: 9	L1: 6, 7
0.03	L1: 3, 4, 6, **7** L2: 8, 9 L3: 42	L1: 6, 7	L1: **7** L2: 84	None	L1; 3, 6, **7**	L1: 6, 7
0.05	L1: 4, 64, **7** L2: 2, 3 L3: 1, 36	L1: 64	L1: 3, **34**, 4, 6, 7 L2: 82, 84 L3: 22, 37, 82, 86, 93	None	L1: 4, 6, **7**	L1: 7
0.07	L1: 4, **64**, 7 L2: 2 L3: 36	L1: 7, 64	L1: **3**, 34, 4, 6, 7	None	L1: 3, 4, **7,** 96 L3: 31	L1: 3, 4, 7, 96 L3: 31
0.1	L1: 4, 64, **7** L2: 3, 63, 8 L3: 31, 32, 40, 68, 90	L1: 64, 7	L1: 3, **34**, 4, 6, 7 L2: 32, 33, 35	None	L1: 3, 4, 7, **96** L2: 1, 2, 98 L3: 0, 31	L1: 96

9.3. Homogeneous consumers in scale-free networks

The same experiments are conducted on three scale-free networks. From the experiments and analysis in small-world networks, I see great similarities in results between Model 1 and Model 2. So, I only test the Model 2 for the scale-free networks.

Consumer 5 is defined as the innovation initiator in every network. Starting from the first level (L1), second (L2) neighbors of the innovation initiator, and every hub in the network, each consumer is made a pirate sequentially, and the experiment is conducted on each scenario. Profits and diffusion speed of each scenario are compared with the profits and diffusion speed of scenario without a key agent (NP).

9.3.1 Model 2-1 (an average pirate)

As shown in Table 33, only the pirate in L1 is able to make an increase in the profits and accelerate the diffusion speed. This conclusion is no different from the small-world network. For the detailed values of D, p, and mean values in below table, please refer to Appendix J.

Table 33: Comparison of profits and diffusion speeds on scale-free networks (Homo, Model 2-1)

Network of seed 1		Network of seed 2		Network of seed 3	
Profit	Diffusion speed	Profit	Diffusion speed	Profit	Diffusion speed
L1: 1	L1: 1	None	L1: 4	None	L1: 3

9.3.2 Model 2-2 (a persuasive pirate)

As shown in Table 34, a persuasive pirate in L1 is able to generate significant profits, which is not different from other types of networks. However, certain pirate in L2 in all three networks is also able to make an increase in the profits and diffusion speed, especially in the network of seed 1, the pirate generates the highest profits. Besides a pirate in L1 and L2, a pirate on certain hub is also able to generate higher profits. The pirate that is able to generate the highest profits is marked as red and bold. For the detailed values of D, p, and mean values in below table, please refer to Appendix J.

Table 34: Comparison of profits and diffusion speeds on scale-free networks (Homo, Model 2-2)

Network of seed 1		Network of seed 2		Network of seed 3	
Profit	Diffusion speed	Profit	Diffusion speed	Profit	Diffusion speed
L1: 1 L2: 2 Hub: 2, **3**, 12	L1: 1	L1: **4** L2: 2 Hub: 1, 2, **4**	L1: 4	L1: **3**, 14 L2: 2 Hub: 2, **3**	L1: 3, 14

The cause is that when a highly connected hub is assigned as a persuader, he comes an opinion leader. As discussed in chapter 8, the marketing strategy increases the price to the maximum when the number of followers who is ready to adopt the product is overwhelming, therefore, the opinion leader in a scale-free network has the possibility of generating higher profits due to the proper adjustment of price. Table 34 shows that not every hub in the network, when becoming an opinion leader, can generate a higher profit. For example, the hub in the network of seed 2 are 1, 2, 4, 7, 14, and 36. However, only 1, 2, and 4, when becoming an opinion leader, increases the profit.

When encountering both the followers of a persuader and regular consumers during the adoption, a marketing strategy has two options to evaluate. If the number of followers of the opinion leader is overwhelming, and by adjusting the price to the maximum (and lower the promotion cost), the profit gained compensates the loss due to the discarding of regular consumers, the strategy adjusts its price to the maximum (and lower the promotion cost). Otherwise, if there is no distinctive difference on the number of two groups of adopters, the strategy mostly chose to stay within the range of acceptance thresholds of consumers, which brings more adopters and higher profits.

9.4 Homogeneous consumers in random networks

The same experiments are conducted on three random networks created from the same three random seeds. I only test the Model 2 for the random networks.

Consumer 5 is defined as the innovation initiator in every network. Starting from the first level (L1), second (L2) neighbors of the innovation initiator, and every connector (Model 2-1) or opinion leader (Model 2-2) in the network, each consumer is made a pirate sequentially, and the experiment is conducted on each scenario. Profits and diffusion speed of each scenario are compared with the profits and diffusion speed of scenario without a key agent (NP). Results of the random networks are very similar to the results of small-world networks.

9.4.1 Model 2-1 (an average pirate)

As shown in Table 35, only the pirate in L1 is able to make an increase in the profits and accelerate the diffusion speed. This conclusion is no different from the small-world network. For the detailed values of D, p, and mean values in below table, please refer to Appendix K.

Table 35: Comparison of profits and diffusion speeds on random networks (Homo, Model 2-1)

Network of seed 1		Network of seed 2		Network of seed 3	
Profit	Diffusion speed	Profit	Diffusion speed	Profit	Diffusion speed
None	L1: 6	L1: 6	L1: 6	None	L1: 42

9.4.2 Model 2-2 (a persuasive pirate)

As shown in Table 36, only a persuasive pirate in L1 is able to generate significant profits. The persuasive pirate in L2 and other opinion leaders are not able to generate higher profits. The pirate that is able to generate the highest profits is marked as red and bold. For the detailed values of D, p, and mean values in below table, please refer to Appendix K.

Table 36: Comparison of profits and diffusion speeds on random networks (Homo, Model 2-2)

Network of seed 1		Network of seed 2		Network of seed 3	
Profit	Diffusion speed	Profit	Diffusion speed	Profit	Diffusion speed
L1: 3, **6,** 7, 71	L1: 6	None	L1: 6	L1: **42**, 7	None

9.5 Conclusions concerning positions of the key agent

- A key agent in L1 guarantees that he adopts the product before other consumers do.
- The key agent in L1 has the possibility to result in higher profits and faster diffusion speed. However, not every key agent in L1 is able to achieve that. An important factor is the network topology.
- If the key agent in L1 is an average consumer, he indeed has the possibility to make a difference on increasing profits and accelerating diffusion speeds. However, the improvement is not large enough to catch the attention of a provider in realistic marketing environments.
- If the key agent in L1 is a persuader, then besides having the possibility on making a difference in diffusion speeds, he is able to increase the profits significantly.
- An opinion leader has much higher probability of generating a high profit in a scale-free network.
- Dynamic self-learning marketing strategies need to cope with the network topology and the key agent in order to increase profits and accelerate diffusion speed. The role of EA is more evident in the case of heterogeneous consumers and the scale-free network.

In summary, as shown in Table 37, an average key agent has the possibility to increase profits and diffusion speed in L1. However, the difference is not large enough to catch the attention of a provider. A persuader in L1 has the possibility of generating higher profits and diffusion speeds. An opinion leader has the possibility to generate high profits in a scale-free network.

Table 37: Impacts of the position of a key agent, a persuader, and an opinion leader

	Average key agent in L1	Persuader in L1	Opinion leader
Small-world networks	No	Yes	No
Random networks	No	Yes	No
Scale-free networks	No	Yes	Yes

9.6 Timing of a key agent

From the conclusions of position of the key agent, I can also conclude that the adoption of the key agent should take place in the early stage of diffusion before the majority of adoptions start.

CHAPTER 10: COMPETITIONS AMONG DIGITAL CONTENT PROVIDERS

What will happen if there are more than one provider in the market? In reality, there are usually more than one providers that compete for consumers in the same market. Proposing an effective marketing strategy is a challenging task for a provider when there are competitors in the same market. A provider cannot propose strategies without considering the impacts of strategies launched by his competitors. The situation becomes more complicated when digital piracy is involved. Chapter 9 demonstrates that the position of a key agent plays critical roles during the diffusion process of an innovation. Will those claims still be true in a duopoly marketing environment?

In this chapter, there are also four duopoly models. Model 1-1 (an average consumer), Model 1-2 (a persuasive consumer), Model 2-1 (an average pirate), and Model 2-2 (a persuasive pirate). Due to the similarities on results between Model 1 and Model 2, only Model 2-1 and Model 2-2 are experimented on 15 small-world networks and three scale-free networks for both homogeneous and heterogeneous consumers.

10.1 Applying CoEA to a duopoly marketing environment

10.1.1. Design and workflow of digital content providers

When a provider competes with another provider in the same market, he often asks himself such a question: what strategies should I utilize to defeat my opponent?

Inspired by the 2-population parallel competitive coevolution (Luke, 2013), I modified the monopoly model by creating two equal-sized populations of marketing strategies. Each population of strategies represents a provider. Provider 1 is a primary population, and provider 2 is an alternative population. A strategy in each population is selected sequentially and injected into the agent-based model at the same time. Now a consumer is facing two brands of products and has to choose either one of them or give up on both of them based on his own criteria. Consumers' choices are reflected on the fitness values of each strategy.

A winning strategy of provider 1 indicates that it is able to defeat every strategy launched by provider 2. In order to increase the robustness of the strategy and also imitate the real marketing environment, a strategy of provider 1 competes with every strategy of provider 2. For a population of size N, there are N pair of strategies that are injected into the agent-based model sequentially for the evaluation of one single strategy of provider 1. Consumers' choices are reflected on the fitness values (profit and diffusion speed) of each strategy. A single strategy of provider 1 acquires N fitness values through competition with N strategies from provider 2. Hence, the fitness value of a strategy of provider 1 is defined as an average of N fitness values minus standard deviation of N fitness values. The fitness value of each strategy in provider 2 has been acquired through the competition with provider 1.

After every strategy in each population acquired its fitness value, the two populations of strategies evolve independently. The new strategy population of each provider repeats the same operation above until a specified number of generations is reached.

10.1.2. Workflow of a consumer

A consumer faces two brands of a product and has to choose either one of them or none of them based on his own criteria. The product of provider 1 is defined as brand 1, and the product of provider 2 is defined as brand 2. The selection process of a consumer follows the below procedure.

There is still only one innovation initiator and the innovation initiator possesses two brands initially. Although there are two brands in the market, a consumer is only aware of the existence of the brands through interactions with consumers in his direct neighborhood, which is referred as the imperfect information by Pegoretti et al. (2012). By comparing the number of adopters of each brand in his direct neighborhood, a consumer shows preference for the brand with more adopters. In Model 2-1, if the consumer is a legal buyer, and price and promotion cost of his preferred brand pass his thresholds, he buys that brand. In Model 2-2, if the consumer is a legal buyer, and if there is a persuasive pirate in his neighborhood, he buys the product regardless of the price and the promotion cost. Otherwise, the consumer follows the rules of price and promotion cost comparison in Model 2-1. If the consumer is a pirate, and the detection cost of his preferred brand passes his threshold, he pirates that brand.

It is also possible that, for a consumer, the number of adopters of each brand in his neighborhood is equal. For example, the first consumer who gets in touch with the innovation initiator faces such situation because the initiator possesses both brands. How does a consumer choose a brand under such situation? If the consumer is a pirate, he pirates the brand whose piracy detection cost passes his threshold. If the piracy detection costs of both brands pass the threshold, he pirates the brand with the lower piracy detection cost. A lower piracy detection cost indicates less risk for the pirate. If the piracy detection costs of both brands are equal and both pass the threshold, he pirates a brand randomly. In Model 2-1, if a consumer is a legal buyer, and if there is only one brand whose price and promotion cost pass the thresholds, he buys that brand. If the prices and the promotion costs of both brands pass the thresholds, he makes a decision based on several situations. First, the consumer buys the brand with the lower price. Second, if the prices of both brands are equal, the consumer buys the brand with the higher promotion cost. A higher promotion cost indicates a stronger influence on the consumer. Third, if the prices and the promotion costs of both brands are equal, the consumer buys a brand randomly. In Model 2-2, if the consumer is a legal buyer, and if there is a pirate in his neighborhood, he buys the product regardless of the price and the promotion cost. Otherwise, the consumer follows the rules of price and promotion cost comparison in Model 2-1. The details of the workflow of Model 2-1 are in Appendix E.

10.2. Experiments and configurations of conservative strategies on Model 2-1

10.2.1 Configurations of conservative strategies

The configuration of EA in Table 38 is the same as the EA configuration in chapter 4, except the population size is 100 and the generation count is 100.

Table 38: Parameters for EA of duopoly model

Parameter	**Value**
EA Population size (number of strategies)	100
EA generation count	100
Parents selection	Binary tournament
Offspring selection	Truncation
Crossover operator	10-point
Mutation operator	Bit-flip / random value
Mutation probability	0.2F
Simulation runs per evaluation	20

The configuration of homogeneous consumers in Table 39 is the same as the configuration in chapter 4.2.2. The only difference is that the maximum variation on the current price, promotion cost, and detection cost is 20%. The values of reservation price, promotion cost threshold, and the piracy detection cost threshold are $15. The initial price starts from the highest price $25, the initial promotion cost starts the lowest at $5, and the initial piracy detection cost starts from the highest at $25. Consumer 5 is defined as the innovation initiator in every network. The price, promotion cost, and detection cost have upper limit $25 and lower limit $5.

Table 39: Parameters for conservative marketing strategies (Homo)

Parameters	**Value**
Number of consumers	100
Reservation price of a consumer	15
Promotion cost threshold of a consumer	15

Piracy detection cost threshold of a consumer	15
Starting price of a provider	25
Starting promotion cost of a provider	5
Starting piracy detection cost of a provider	25
Position of an innovation initiator	5
Number of an innovation initiator	1
Maximum percent of change	20
Unit of variation on prices and costs	5%
Price range of a provider	[5, 25]
Promotion cost range of a provider	[5, 25]
Detection cost range of a provider	[5, 25]

The configuration of heterogeneous consumers in Table 40 is the same as the configuration in chapter 4.2.2. The only difference is that the maximum variation on the current price, promotion cost, and detection cost is 20%. This is the configuration of heterogeneous consumers. The reservation prices, promotion cost threshold, and piracy detection cost threshold is normally distributed with a mean $15 and standard deviation 3.

Table 40: Parameters for conservative marketing strategies (Hetero)

Parameters	**Value**
Number of consumers	100
Reservation price of a consumer	N (15,3)
Promotion cost threshold of a consumer	N (15,3)
Piracy detection cost threshold of a consumer	N (15,3)
Starting price of a provider	25
Starting promotion cost of a provider	5

Starting piracy detection cost of a provider	25
Position of an innovation initiator	5
Number of an innovation initiator	1
Maximum percent of change	20
Unit of variation on prices and costs	5%
Price range of a provider	[5, 25]
Promotion cost range of a provider	[5, 25]
Detection cost range of a provider	[5, 25]

10.2.2 Experiments and Results

Experiments were conducted on 15 small-world networks. Networks are created using three random seeds, each random seed is coupled with five rewiring probabilities: 0.01, 0.03, 0.05, 0.07, and 0.1. The direct neighborhood of the innovation initiator is defined as L1, and the direct neighborhood of L1 is defined as L2. Consumer 5 is defined as the innovation initiator in every network.

Three scenarios of both homogeneous and heterogeneous consumers are tested on each network: no pirate (NP), consumer 7 in L1 is a pirate (P7), and the consumer 8 in L2 is a pirate (P8). In summary, there were 90 experiments conducted. Each experiment consists of 20 runs. The winning count of each brand was recorded. The competition generates two results: “lock-in” or “winning”. The “lock-in” indicates that the winning provider take over the entire market. The “winning” indicates that the winning provider won over more consumers than his opponent, but did not take over the entire market. There could also be a “tie” situation, but it only happened in two out of 1800 runs.

As shown from Table 41 to 65, the winning count (L + W) of provider 1 is much more than the winning count (L + W) of provider 2. The only exception is that in the network of seed 1 and rewiring probability 0.05, the winning count of provider 1 is one less than the winning count of provider 2. Experiment results prove that the evolution of strategies of provider 1 is successful.

P1 = Provider 1 won

P2 = Provider 2 won

T1 = Provider 1 pirated brand 1

T2 = Provider 2 pirated brand 2

L = Lock-in status

W = Won consumers more than his opponent

A = Won over most market, his opponent did not take any consumers

Tie = Provider 1 and provider 2 won the same amount of consumers

The product of provider 1 = brand 1

The product of provider 2 = brand 2

10.2.2.1 Homogenous consumers

10.2.2.1.1 Experiments on non-piracy

Experiments are conducted on 15 networks. There are no pirates among consumers, and the consumers are homogeneous.

Table 41: Competition results of networks of seed 1 (NP, Homo, conservative)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	12	4		3	1				
0.03	13	6		1					
0.05	9			2	8		1		
0.07	13			1	6				
0.1	8	3		1	8				

Table 42: Competition results of networks of seed 2 (NP, Homo, conservative)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	14	4		2					
0.03	13	3		4					
0.05	12	1		1	6				
0.07	14	4			2				
0.1	14	2		2	2				

Table 43: Competition results of networks of seed 3 (NP, Homo, conservative)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	14	4		2					
0.03	7	11		1	1				
0.05	11	4		4	1				
0.07	11	5		2	2				

0.1	9	1		3	7				

The lock-in and winning scenarios of provider 1 are analyzed below. The analysis of scenario 1 and 2 is also applicable for provider 2.

S1 = adoption amount of provider 1

S2 = adoption amount of provider 2

P1 = price of provider 1

P2 = price of provider 2

A1 = promotion cost of provider 1

A2 = promotion cost of provider 2

D1 = piracy detection cost of provider 1

D2 = piracy detection cost of provider 2

Scenario 1: **provider 1 takes over the entire market**

Take the network seed 1, rewiring probability 0.01, run #3 as an example.

Table 44: Analysis of "lock-in" (NP, Homo)

Time step	S1	P1	A1	D1	S2	P2	A2	D2
0	0	25.0	5.0	25.0	0	25.0	5.0	25.0
1	0	20.0	6.0	20.0	0	22.5	6.0	21.25
2	0	16.0	7.2	16.0	0	20.25	7.2	18.06
3	0	12.8	8.64	12.8	0	18.225	8.64	15.35
4	0	10.24	10.36	10.24	0	16.402	10.36	13.05
5	0	8.192	12.44	8.192	0	14.762	12.44	11.09

6	0	6.553	14.92	6.553	0	13.286	14.92	9.428
7	**4**	**5.2428**	**17.91**	**5.242**	**0**	**11.9**	**17.91**	**8.01**
8	5	6.2914	18.811	5.242	0	10.7	21.49	6.81
9	7	7.5497	19.752	5.242	0	9.68	25.0	5.79
10	10	8.682	17.77	6.29	0	8.7	25.0	5.0
11	11	9.550	16.88	5.34	0	7.8	25.0	5.0
12	10	11.46	20.26	6.41	0	7.0	25.0	5.0
13	8	10.88	22.29	6.41	0	6.3	25.0	5.0
14	8	10.34	25.0	6.09	0	5.7	25.0	5.0
15	4	12.41	23.75	7.31	0	5.1	25.0	5.0
16	4	13.65	23.75	5.85	0	5.0	25.0	5.0
17	4	13.65	25.0	5.85	0	5.0	25.0	5.0
18	4	13.65	22.5	5.55	0	5.0	25.0	5.0
19	4	13.65	20.25	5.28	0	5.0	25.0	5.0
20	4	12.97	17.21	6.07	0	5.0	25.0	5.0
21	4	12.32	17.21	6.37	0	5.0	25.0	5.0
22	4	12.93	17.21	6.69	0	5.0	25.0	5.0
23	4	14.23	16.35	6.69	0	5.0	25.0	5.0
Final profits	452.228				-682.848			

The adoption starts at time step 7. As shown in Figure 41, the consumer 6, 7, 3 and 4 are the direct neighbors (L1) of the innovation initiator. As a result, they are all aware of brand 1 and brand 2, and the number of adopters of brand 1 and brand 2 in their neighborhood is equal. As shown in Table 44, at time step 7, both provider 1 and provider 2 reach the thresholds ($15) of a consumer. However, provider 1 proposes a lower price. Therefore, the consumers in L1 choose the brand 1.

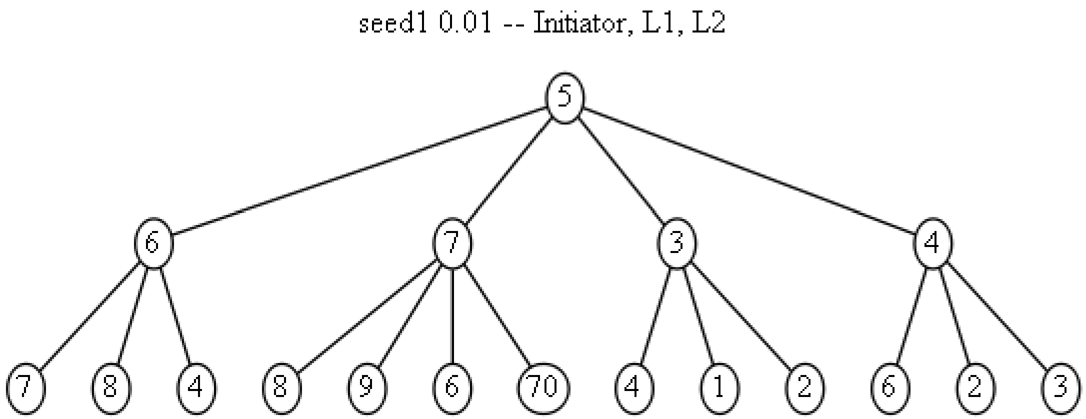


Figure 41: Initiator, L1, and L2 of network of seed 1, rewiring probability 0.01

After the time step 7, no matter how provider 2 lower his price and increase the promotion cost, he is not able to win over any consumers. The reason is simple. A consumer is not aware of all the brands in the market. The only way a consumer gets to know a brand is through his neighboring consumers. Once the entire consumers in L1 adopt the brand 1, the brand 1 becomes the only brand visible to the remaining consumers.

As shown in Table 44, although provider 1 has the lower price initially, after the adoption start at the time step 7, the price continues to increase and approach the reservation price of a consumer. After adoption starts in time step 7, the promotion cost continues to increase a little, then decrease, and gradually move towards the promotion cost threshold of a consumer. For provider 2, he does not realize that he has been blocked by the brand 1. As a result, in order to attract more consumers, he continues to lower the price and increase the promotion cost until it reaches the limits of price and promotion costs.

In addition, the profit of the winning provider is not as high as the profit of monopoly case in chapter 4. The cause is the sacrifice of robustness of strategies. As discussed in chapter 3.4.1, in order to evolve a robust strategy which is able to defeat any strategy launched by provider 2, a strategy of provider 1 competes with every strategy of provider 2, and the fitness of the strategy is defined as the average of all fitness values minus the standard deviation of the fitness values. This fitness calculation removes the best and worst strategies from the population, and keeps the robust strategies. The evolved best-so-far strategies may not be the one with the highest fitness, but it is the most robust one, this is, it is able to defeat any strategy launched by provider 2.

In summary, a provider should not be afraid of lower the price and increase promotion cost initially because whoever wins over all the consumers in L1 will take over the entire market. The price and promotion cost will recover gradually after the adoption starts.

Scenario 2: **provider 1 did not take over the entire market, but provider 1 won over more consumers that provider 2 did.**

Take the network seed 1, rewiring probability 0.03, run #9 as an example. Figure 42 displays the innovation initiator, L1, and L2 of the network of seed 1 and rewiring probability 0.03. Provider 1 did not take over the entire market. Provider 1 won over 65 consumers, and provider 2 won over 34 consumers.

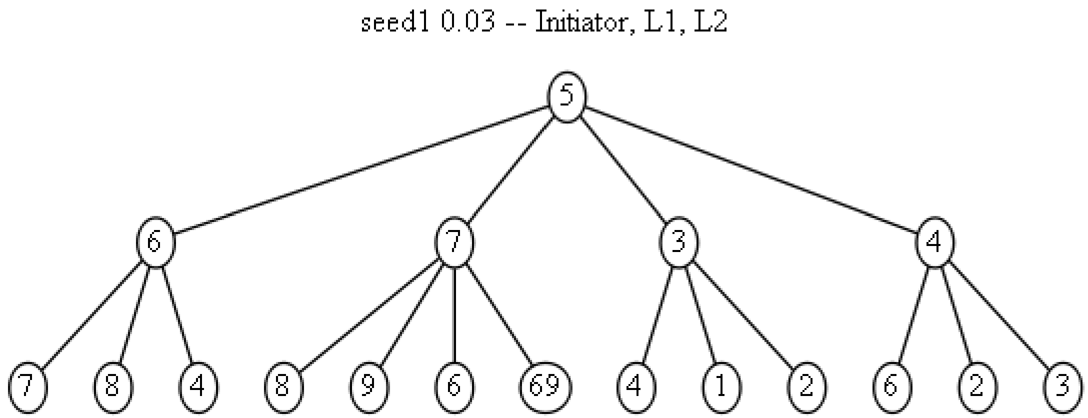


Figure 42: Initiator, L1, and L2 of network of seed 1, rewiring probability 0.03

Table 45: Analysis of "winning" (NP, Homo)

Time step	S1	P1	A1	D1	S2	P2	A2	D2
0	0	25.0	5.0	25.0	0	25.0	5.0	25.0
1	0	22.5	6.0	20.0	0	22.5	6.0	20.0
2	0	20.25	7.2	16.0	0	20.25	7.2	16.0
3	0	18.225	8.64	12.8	0	18.225	8.64	12.8
4	0	16.402	10.368	10.24	0	16.402	10.368	10.24
5	0	14.762	12.441	8.192	0	14.762	12.441	8.192
6	0	13.286	14.929	6.553	0	13.286	14.929	6.553
7	**2**	**11.957**	**17.915**	**5.242**	**2**	**11.957**	**17.915**	**5.242**
8	3	11.957	21.499	5.0	2	9.5659	21.499	6.029
9	5	14.348	21.499	5.0	3	8.1310	21.499	6.933
10	6	11.47	20.42	5.75	5	9.350	20.42	8.32
11	7	13.20	20.42	5.0	6	10.75	21.44	9.15
12	11	10.56	19.40	5.75	4	12.36	24.66	9.15
13	8	12.67	20.37	6.61	4	10.51	23.42	7.32

14	8	10.13	20.37	5.62	4	12.61	23.42	5.85
15	7	10.13	19.35	5.62	1	12.61	21.08	5.0
16	6	12.16	19.35	5.33	2	13.87	22.14	5.0
17	2	12.77	22.25	5.07	1	13.87	19.92	5.0
Final profits	314.73				-98.915			

As shown in Table 45, at time step 7, both providers propose the same initial prices and promotion thresholds and they are both within the acceptable thresholds of a consumer. As shown in Figure 42, consumers 6, 7, 3 and 4 are the direct neighbors (L1) of the innovation initiator. Consumers 6, 7, 3 and 4 are all aware the brand 1 and the brand 2, and the amount of adopters of the brand 1 and the brand 2 is equal. Under such condition, a consumer selects a brand randomly.

Random selection of brands could produce two types of situations. The first situation is that all consumers in L1 chose the same brand randomly, either brand 1 or brand 2, then the situation is same as scenario 1. However, this situation never happened in my experiments. The second situation is that some consumers in L1 chose the brand 1, while other consumers in L1 chose the brand 2, therefore, the remaining consumers are able to see both the brand 1 and the brand 2. Which brand will the remaining consumers choose depends on the network topology and the marketing strategies evolved through EA. As a result, the outcome of the competition is unpredictable.

In Figure 42, the consumers 6, 7, 3 and 4 are the direct neighbors (L1) of the innovation initiator. The adopters of brand 1 and brand 2 are equal in their direct neighborhood, and the both providers propose the same price and promotion cost. Therefore, the consumers 6, 7, 3 and 4 have to choose a brand randomly. The result is that the consumers 3 and 7 chose brand 1, and the consumers 4 and 6 chose brand 2. How such selection results affect the adoption choices of consumers at L2?

Consumers 9, 69, 1, 8, and 2 are consumers in L2. At time step 8, the price and the promotion cost have been adjusted to the acceptable level of a consumer. The direct neighbors of consumer 9 are consumers 10, 11, 7 and 8. The only adopted consumer is 7, and he adopted brand 1. Therefore, consumer 9 adopted brand 1. The direct neighbors of consumer 69 are consumers 70, 7, 67 and 68. The only adopted consumer is 7, and he adopted brand 1. Therefore, consumer 69 adopted brand 1. The direct neighbors of consumer 1 are consumers 2, 3, 0 and 99. The only adopted consumer is 3, and he adopted brand 1. Therefore, consumer 1 adopted brand 1. The direct neighbors of consumer 8 are consumers 9, 10, 6, 7, and 42. The adopted consumers are 6 and 7. Consumer 6 adopted brand 2 and the consumer 7 adopted brand 1. However, as shown in Table 45, at time step 8, provider 2 offers a lower price, therefore, consumer 8 adopted brand 2. The direct neighbors of consumer 2 are consumers 3, 4, 0, and 1. The adopted consumers are 3 and 4. Consumer 3 adopted brand 1 and the consumer 4 adopted brand 2. However, as shown in Table 45, at time step 8, provider 2 offers a lower price, therefore, consumer 2 adopted brand 2.

In summary, when both brands are chosen by the consumers in L1, the choices of brands of the remaining consumers are determined by the network topology and the strategies evolved through EA. As a result, the outcome of the competition, including profits and market shares, is unpredictable.

10.2.2.1.2 Experiments on a pirate in L1

Experiments are conducted on 15 networks. In each network, consumer 7 in L1 is assigned as a pirate. Consumers are homogeneous.

Table 46: Competition results of networks of seed 1 (P7, Homo, conservative)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	11	9						20	
0.03	7	13						20	
0.05	9	6			5			20	
0.07	4	9			6		1	20	
0.1	3	10			7			20	

Table 47: Competition results of networks of seed 2 (P7, Homo, conservative)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	11	9						20	
0.03	9	11						20	
0.05	9	6			5			20	
0.07	7	10			3			20	
0.1	8	9			3			20	

Table 48: Competition results of networks of seed 3 (P7, Homo, conservative)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	5	14			1			20	
0.03	8	11			1			20	
0.05	9	9			2			20	
0.07	11	5			4			20	
0.1	9	10			1			20	

A pirate in L1 is able to see both brands, and he adopts the products before all other legal buyers do. It is possible that the pirate adopts the same brand as other legal buyers in L1 do. However, it is also possible that the pirate adopts a different brand from other legal buyers in L1 do. If two brands are adopted by consumers in L1, the outcome of the completion, as discussed before, is unpredictable.

Take the network seed 1, rewiring probability 0.03, run #16 as an example.

Table 49: Analysis of "winning" (L1, Homo)

Time step	S1	P1	A1	D1	S2	P2	A2	D2
0	0	25.0	5.0	25.0	0	25.0	5.0	25.0
1	0	22.5	6.0	20.0	0	22.5	6.0	20.0
2	0	20.25	7.2	16.0	0	20.25	7.2	16.0
3	**1**	**18.225**	**8.64**	**12.8**	**0**	**18.225**	**8.64**	**12.8**
4	0	16.4025	10.368	10.24	0	16.4025	10.368	10.24
5	0	14.7622	12.441	8.192	0	14.7622	12.441	8.192

6	0	13.2860	14.929	6.553	0	13.2860	14.929	6.553
7	**4**	**11.9574**	**17.915**	**5.242**	**2**	**11.9574**	**17.915**	**5.242**
8	6	12.5552	18.811	5.0	2	10.7616	20.603	5.505
9	9	13.1830	18.811	5.5	2	10.7616	20.603	5.505
10	11	10.546	18.81	5.0	2	11.837	24.72	5.0
11	13	11.073	15.98	5.25	2	13.021	25.0	5.0
12	10	11.073	19.18	5.25	2	13.672	23.75	5.5
13	10	10.520	16.30	5.25	2	14.356	22.56	6.05
14	6	12.098	16.30	5.25	2	14.356	19.17	5.0
15	6	13.912	17.12	5.0	0	16.509	19.17	6.0
16	5	13.217	17.12	5.0	2	14.033	15.34	6.0
Final profits	548.3618				-197.508			

As shown in Table 49, the pirate 7 pirates the product at time step 3. Both providers propose the same piracy detection cost at time step 3, therefore, pirate 7 has to choose a brand randomly, and he chose brand 1. At time step 7, both providers propose the same price and promotion cost and both of them pass thresholds of legal buyers in L1. Under such condition, a legal buyer chooses a brand randomly.

In this case, In L1, for consumers 3 and 4, the only adopted consumer in their neighborhood is the innovation initiator (5). When a consumer faces the situation in which the number of adopters in his neighborhood is equal, and both providers present the same price and promotion cost, a consumer has to choose randomly. In this case, consumers 3 and 4 chose brand 2. For consumer 6, his direct neighbors include consumer 5, 7, 8, and 4, and the number of adopters of brand 1 is two, and number of adopters of brand 2 is one. Therefore, consumer 6 chose brand 1. In summary, in L1, consumers 6 and 7 adopted brand 1, and consumers 3 and 4 adopted brand 2. How do such adoption results in L1 affect the adoption decisions of the consumers in L2?

Consumers in L2 are 9, 69, 1, 8, and 2. The direct neighbors of consumer 8 includes consumers 9, 10, 6, 7, and 42. The only adopted consumer is 7 who adopted brand 1. Therefore, consumer 8 chose brand 1. The direct neighbors of consumer 9 contain consumers 10, 11, 7 and 8. The only adopted consumer is 7 who adopted brand 1. Therefore, consumer 9 chose brand 1. The direct neighbors of consumer 69 contain consumers 7, 67, 68 and 70. The only adopted consumer is 7 who adopted brand 1. Therefore, consumer 69 chose brand 1. The direct neighbors of consumer 2 contain consumers 3, 4, 0 and 1. The adopted consumers are 3 and 4 who adopted brand 2. Therefore, consumer 2 chose brand 2. The direct neighbors of consumer 1 contain consumers 2, 3, 0 and 99. The only adopted consumer is 3 who adopted brand 2. Therefore, consumer 1 chose brand 2.

In summary, a winning provider is able to grab both the pirate and other legal buyers in L1 before his competitor does. When a pirate faces two brands, he chose the one with lower piracy detection cost. When a pirate faces two brands with the same detection cost, he chose a brand randomly. Therefore, the provider who proposes a lower detection cost, a lower price, and a higher promotion cost than his competitor will win the competition.

10.2.2.1.3 Experiments on a pirate in L2

Experiments are conducted on 15 networks. In each network, consumer 8 in L2 is assigned as a pirate. Consumers are homogeneous.

Table 50: Competition results of networks of seed 1 (P8, Homo, conservative)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	14	4		1	1			16	4
0.03	15	1		2	2			15	5
0.05	12				8			12	8
0.07	9	1		3	7			9	11
0.1	10	3		1	6			10	10

Table 51: Competition results of networks of seed 2 (P8, Homo, conservative)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	10	7		3				10	10
0.03	12	7		1				12	8
0.05	9	4		2	5			9	11
0.07	10	7		1	2			11	9
0.1	13	2		1	4			19	1

Table 52: Competition results of networks of seed 3 (P8, Homo, conservative)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	14	4		2				17	3
0.03	8	5		4	3			13	7
0.05	14	5			1			15	5
0.07	9	1		5	5			9	11
0.1	12	2		3	3			12	8

A pirate in L2 adopts the product after all legal buyers in L1 do. As a result, the pirate only able to see the brands adopted by legal buyers in L1. If there is only one brand adopted by the legal buyers in L1, then the pirate in L2 has no other choice but to adopt the same brand. If the legal buyers in L1 adopted two brands, the pirate in L2 faces either one brand or two brands which is determined by the network topology. When the pirate in L2 faces two brands, he picks the brand with the lower detection cost. When the piracy detection cost proposed by two brands are the same detection cost, the pirate makes a random choice. Therefore, the choice of brand of the pirate in L2 is unpredictable. It totally depends on the network topology and strategies evolved through EAs. As shown in Tables 50, 51, and 52, the choices of brand of pirate 8 is random. In summary, a pirate in L2 is not influential on the final competition results.

10.2.2.2 Heterogeneous consumers

For heterogeneous consumers, the situation becomes complicated. For non-piracy situation, there are possibilities for two scenarios. One is that all legal buyers in L1 adopts the same brand, the other scenario is that legal buyers adopts different brands due to the heterogeneity of the reservation price and the promotion cost threshold of a consumer.

Different from the results of the homogeneous experiments, there are more lock-in results in the experiments of heterogeneous consumers. The cause is that due to the heterogeneity of the consumers, it is less likely for EA to evolve strategies which have same prices and promotion costs for both providers, and avoid the situation in which a consumer has to choose a brand randomly.

10.2.2.2.1 Experiments on non-piracy

Experiments are conducted on 15 networks. There are no pirates among consumers, and the consumers are heterogeneous.

Table 53: Competition results of networks of seed 1 (NP, Hetero, conservative)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	18			2					
0.03	20								
0.05	16	3			1				
0.07	13	4		2	1				
0.1	17	3							

Table 54; Competition results of networks of seed 2 (NP, Hetero, conservative)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	20								
0.03	20								
0.05	20								
0.07	20								
0.1	19	1							

Table 55: Competition results of networks of seed 3 (NP, Hetero, conservative)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	19			1					
0.03	19			1					
0.05	18			2					
0.07	1	16			3				
0.1	12	7			1				

The two scenarios: lock-in and winning, of provider 1 are analyzed below. The analysis of scenario 1 and 2 is also applicable for provider 2.

Scenario 1: **provider 1 takes over the entire market**

Take the network seed 1, rewiring probability 0.01, run #1 as an example.

Table 56: Analysis of "lock-in" (NP, Hetero)

Time step	S1	P1	A1	D1	S2	P2	A2	D2
6	2	13.28	14.929	6.553	**0**	13.28	11.565	13.286
7	2	13.95	17.169	6.553	0	11.95	13.300	11.957
8	6	14.64	19.744	6.553	0	10.76	15.295	10.761
9	7	11.71	15.795	7.864	0	9.685	17.589	9.6855
10	9	10.5	16.58	7.47	0	8.71	20.22	8.716
11	7	8.96	17.41	7.47	0	7.84	23.26	7.845
12	3	10.7	14.80	6.35	0	7.06	25.0	7.060
13	2	11.2	14.06	6.98	0	6.35	25.0	6.354
14	1	11.8	13.35	7.68	0	5.71	25.0	5.719
15	2	11.8	16.03	6.53	0	5.14	25.0	5.147
16	7	10.0	17.63	6.20	0	5.0	25.0	5.0
17	6	8.56	17.63	5.0	0	5.0	25.0	5.0
18	12	8.56	19.39	5.0	0	5.0	25.0	5.0
19	10	6.85	21.33	6.0	0	5.0	25.0	5.0
20	4	6.51	21.33	5.7	0	5.0	25.0	5.0
21	7	5.53	24.53	5.7	0	5.0	25.0	5.0
22	4	6.36	24.53	5.13	0	5.0	25.0	5.0
23	4	6.04	25.0	5.64	0	5.0	25.0	5.0
24	4	6.04	25.0	6.77	0	5.0	25.0	5.0
Final profits	285.13				-718.68			

Table 57: Attributes of heterogeneous consumers 3, 4, 6, and 7 in L1

Consumer index	Price	Promotion cost	Piracy detection cost
3	13.5571	15.2709	13.3626
4	17.3407	13.0106	20.7986
6	15.3759	17.7499	17.17835
7	17.4516	12.4977	15.82133

As shown in Table 56, at the time step 6, the proposed price and promotion cost by both providers are within the acceptable thresholds of consumers 4 and 7. Provider 1 and provider 2 proposed the same price, but provider 1 proposed a higher promotion cost, therefore, brand 1 is chosen by the consumer 4 and 7. At the time step 7, only the price and the promotion cost proposed by provider 1 are within the acceptance thresholds of consumers 3 and 6. Therefore, consumers 3 and 6 chose brand 1. After step 6 and 7, all consumers in L1 adopted the brand 1. As we discussed before, provider 1 took over the entire market.

Scenario 2: **provider 1 did not take over the entire market, but provider 1 won over more consumers that provider 2 did.**

Take the network seed 1, rewiring probability 0.05, run #6 as an example. Figure 43 displays the innovation initiator, L1, and L2 of the network of seed 1 and rewiring probability 0.05. Provider 1 did not take over the entire market. Provider 1 won over 87 consumers, and provider 2 won over 12 consumers.

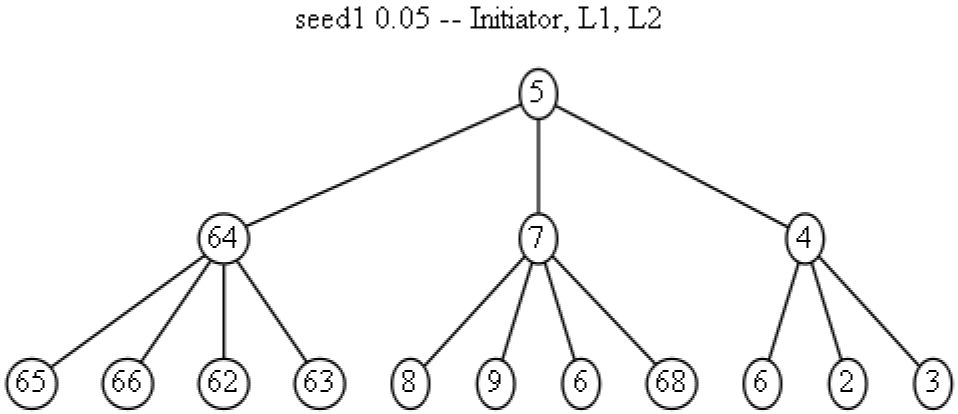


Figure 43: Initiator, L1, and L2 of network of seed 1, rewiring probability 0.05

Table 58: Analysis of "winning" (NP, Hetero)

Time step	S1	P1	A1	D1	S2	P2	A2	D2
6	2	9.43	14.92	9.4287	0	9.43	14.92	25.0
7	4	10.37	17.16	9.4287	1	8.01	17.915	25.0
8	10	10.37	19.74	8.0144	3	8.81	17.915	20.0
9	11	11.92	22.70	8.0144	3	9.69	17.915	16.0
10	8	13.7	24.9	8.014	4	9.69	18.81	16.0
11	5	14.4	23.7	8.415	1	7.75	15.04	16.0
12	3	12.9	21.3	10.09	0	6.98	12.03	16.0
13	7	11.6	19.2	12.11	0	8.37	12.03	16.0
14	8	10.4	20.1	9.694	0	10	12.03	16.0
15	8	10.4	22.1	9.694	0	12	12.03	16.0
16	10	9.44	25.0	11.14	0	14.47	12.03	16.0
17	6	10.8	23.7	10.03	0	17.37	12.03	16.0
18	0	11.9	25.0	8.528	0	20.84	12.03	16.0
19	0	10.1	25.0	8.102	0	25.0	12.03	16.0
20	2	8.63	25.0	7.696	0	25.0	12.03	16.0
21	0	9.49	25.0	8.081	0	25.0	12.03	16.0

22	0	8.07	25.0	7.67	0	25.0	12.03	16.0
23	3	6.86	25.0	7.29	0	25.0	12.03	16.0
Final profits	243.193				-646.55			

Table 59: Attributes of heterogeneous consumers 4, 7, and 64 in L1

Consumer index	Price	Promotion cost	Piracy detection cost
4	17.3407	13.0106	20.7986
7	17.4516	12.4977	15.82133
64	14.1354	15.5770	12.0105

At time step 6, as shown in Table 58, both providers propose the same price and promotion cost, and the price and promotion cost are within the acceptance thresholds of consumers 4 and 7. Therefore, consumers 4 and 7 in L1 select a brand randomly, and both chose the brand 1. At time step 7, both providers are visible to consumer 64, and the prices and the promotion costs proposed by both providers fell into the acceptable range of consumer 64. Consumer 64 selected brand 2 due to its lower price. Other consumers in L2 which adopted the brand 1 at time step 7 are consumers 9, 68, 2 and 3. Not only because the proposed price and promotion cost are within the acceptance thresholds of those consumers, but also the only adopter in their neighborhood adopted brand 1. As a result, consumers 9, 68, 2 and 3 adopted the brand 1. At time step 8, consumer 64 directly influenced the adoption decisions of consumers 63, 65, and 66 because they are the direct neighbors of consumer 66. Without gonging through the adoption decision of every consumer, it is clear to see that if there are two brands adopted in L1, the results of competition are unpredictable.

10.2.2.2.2 Experiments on a pirate in L1

Experiments are conducted on 15 networks. In each network, consumer 7 in L1 is assigned as a pirate. Consumers are heterogeneous.

Table 60: Competition results of networks of seed 1 (P7, Hetero, conservative)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	16	2			2			20	
0.03	17	2			1			20	
0.05	14	5			1			20	
0.07	13	4			3			20	
0.1	14	5			1			20	

Table 61: Competition results of networks of seed 2 (P7, Hetero, conservative)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	20							20	
0.03	20							20	
0.05	20							20	
0.07	20							20	
0.1	19			1				20	

Table 62: Competition results of networks of seed 3 (P7, Hetero, conservative)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	17	3						20	
0.03	17	3						20	
0.05	19	1						20	
0.07	7	6			7			20	
0.1	10	10						20	

10.2.2.2.3 Experiments on a pirate in L2

Experiments are conducted on 15 networks. In each network, consumer 8 in L2 is assigned as a pirate. Consumers are heterogeneous.

Table 63: Competition results of networks of seed 1 (P8, Hetero, conservative)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	17			3				17	3
0.03	17			3				17	3
0.05	12	4		3	1			16	4
0.07	14	5		1				15	5
0.1	15	3		1	1			19	1

Table 64: Competition results of networks of seed 2 (P8, Hetero, conservative)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	19			1				19	1
0.03	20							20	
0.05	20							20	
0.07	20							20	
0.1	19			1				20	

Table 65: Competition results of networks of seed 3 (P8, Hetero, conservative)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	18			2				18	2
0.03	18			2				17	3
0.05	18			2				18	2
0.07	12	7			1			19	1
0.1	13	7						20	0

The situation of a pirate in L1 and a pirate in L2 is the same as the experiments of homogeneous consumers.

10.3 Experiments and configurations of audacious strategies on Model 2-1

The same experiments are conducted using the audacious strategies on Model 2-1 for both homogeneous and heterogeneous consumers. The detailed results are listed in Appendix L.

10.3.1 Configurations of audacious marketing strategies

The configuration of EA is the same as the EA configuration in chapter 4, except the population size is 100 and the generation count is 100.

Table 66: Parameters for EA of duopoly model

Parameter	**Value**
EA Population size (number of strategies)	100
EA generation count	100
Parents selection	Binary tournament
Offspring selection	Truncation
Crossover operator	10-point
Mutation operator	Bit-flip / random value
Mutation probability	0.2F
Simulation runs per evaluation	20

Below configuration is same as the one in chapter 4.2.2. I tried more audacious strategies in which the maximum variation of the price, the promotion cost, and the detection cost is up to 100%. Thus, it increases the searching space of EA and facilitate EA to find better strategies.

Table 67: Parameters for audacious marketing strategies (Homo)

Parameters	**Value**
Number of consumers	100
Reservation price of a consumer	15
Promotion cost threshold of a consumer	15
Piracy detection cost threshold of a consumer	15
Starting price of a provider	25
Starting promotion cost of a provider	5
Starting piracy detection cost of a provider	25
Position of an innovation initiator	5
Number of an innovation initiator	1
Maximum percent of change	100
Unit of variation on prices and costs	5%
Promotion cost range of a provider	[5, 25]
Detection cost range of a provider	[5, 25]

Table 68: Parameters for audacious marketing strategies (Hetero)

Parameters	**Value**
Number of consumers	100
Reservation price of a consumer	N (15,3)
Promotion cost threshold of a consumer	N (15,3)
Piracy detection cost threshold of a consumer	N (15,3)
Starting price of a provider	25
Starting promotion cost of a provider	5
Starting piracy detection cost of a provider	25
Position of an innovation initiator	5

Number of an innovation initiator	1
Maximum percent of change	100
Unit of variation on prices and costs	5%
Price range of a provider	[5, 25]
Promotion cost range of a provider	[5, 25]
Detection cost range of a provider	[5, 25]

10.4 Comparisons between conservative and audacious strategies

Table 95 to 100 in Appendix M list the comparisons on the number of "lock-in" results between conservative and audacious strategies. In general, audacious strategies generate more "lock-in" results than conservative strategies due to the expansion of searching space for EA.

10.5 Experiments of audacious marketing strategies on Model 2-2

In Model 2-2, the pirate is a persuader. If a consumer has a pirate in his direct neighborhood, he buys the product immediately disregarding the price and promotion cost. Does a persuader change the results of the competition? The experiments are conducted on 15 small-world networks. Three scenarios of both homogeneous and heterogeneous consumers are tested on each network: no pirate (NP), consumer 7 in L1 is a pirate (P7), and the consumer 8 in L2 is a pirate (P8).

The results of Model 2-2 are very similar to the results of Model 2-1. As shown from Table 101 to 112 in Appendix N, the winning count (L + W) of provider 1 is much more than the winning count (L + W) of provider 2. As shown in Table 106, there is one “tie” happens in the network of seed3 and rewiring probability 0.1 under scenario P8. Experiment results prove that the evolution of strategies of provider 1 is successful.

The results indicate that the persuader makes no difference on the results. Although the persuader is influential, but it is in L1 and not every consumer in L1 is his follower, therefore, the persuader in L1 does not play an important role in the adoption decisions of other consumers in L1. If the persuader is in L2, his adoption decision is determined by the adoptions decisions of consumers in L1, the network topology, and the marketing strategy.

10.6 Experiments of audacious marketing strategies on scale-free networks

Various experiments have been conducted on the three scale-free networks on both Model 2-1 and Model 2-2. Those experiments include 1) assigning every consumer in L1 and L2 as a pirate sequentially and the experiment is conducted on each scenario, 2) reducing the maximum variation from 100% to 20%, 3) positioning the innovation initiator on the hub, or 4) testing both homogeneous and heterogeneous consumers. All experimental results show that strategies of provider 1 always create lock-in results.

10.7 Conclusions

The adoption choices of consumers in L1 is crucial to the success of the provider. The digital content provider who is able to win over all consumers in L1 will take over the entire market. A winning strategy in Model 2 is to lower the piracy detection cost (if a pirate is in L1) and price, and increase promotion cost faster than its competitor. If a provider is not able to grab all consumers in L1, the competition result is unpredictable.

Besides lowering the price and increasing the promotion cost faster than the competitor, a winning strategy in Model 1 needs to distribute the free copy of the digital information goods to the key agent before the competitor does. If the key agent already received the free copy from the competitor first, no matter how hard the provider tries, there will be two brands presented in L1, and therefore, the outcome of the competition is unpredictable.

A provider should not be afraid of losing profits initially due the lower price and higher promotion cost. As long as he wins over all consumers in L1, after the adoptions start, the price and promotion cost gradually recovers and approaches the thresholds of consumers.

A persuader does not make significant difference to the results of the duopoly models. For scale-free networks, marketing strategies can always guarantee that provider 1 creates a lock-in results.

CHAPTER 11: SENSITIVITY ANALYSIS

The experiments on sensitivity analysis are conducted on Model 2-1 (a pirate is an average consumer) on 15 small-world networks. Chapter 9 demonstrates the similarities of experimental results between Model 1-1 and Model 2-1, and the similarities between Model 1-2 and Model 2-2, so the sensitivity analysis on Model 2-1 is good enough to demonstrate the stability of performance. The various sensitivity analyses and experiments demonstrate that the performance of my models is stable.

11.1 Scalability of the model

The size of consumer used in the model is 100. The processing time increase linearly as I increase the number of consumers from 100, to 500, 1000, 5000, and 10000. Table 69 displays the processing time of one run over different consumer size on the small-world network rewiring probability 0.01. As shown in Figure 44, the processing time increases linearly as the number of consumers increases, thus, the model is scalable.

Table 69: Processing time of different consumer size

Number of consumers	Processing time (sec)
100	48.874
500	253.99
1000	567.004
5000	3134.382
10000	6289.708

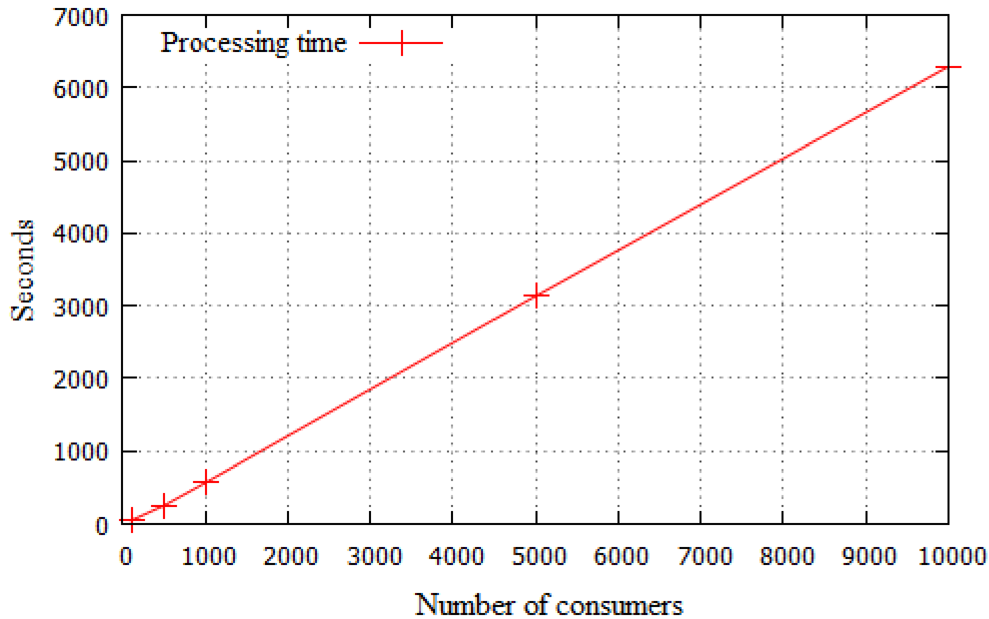


Figure 44: Processing time for different consumer sizes

Figure 45 illustrates that time steps of the diffusion process of 1000 consumers over five rewiring probabilities of network seed 1. Figure 46 illustrates that time steps of the diffusion process of 100 consumers over five rewiring probabilities of network seed 1. In a network with 1000 consumers, the time steps increase a few extra steps compare to the case of 100 consumers.

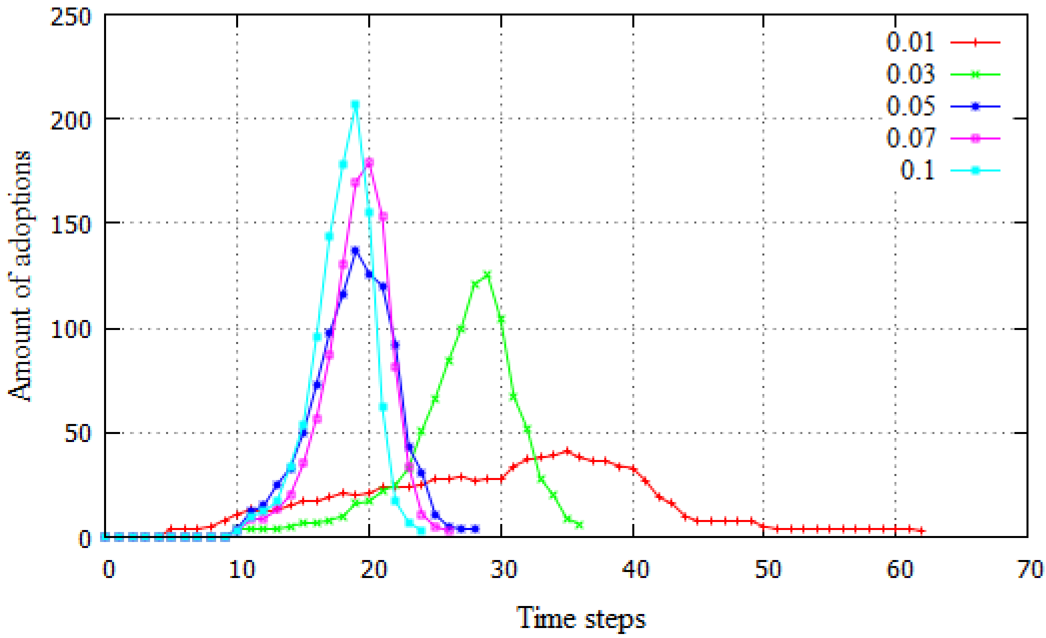


Figure 45: Variations of total diffusion time steps for 1000 consumers

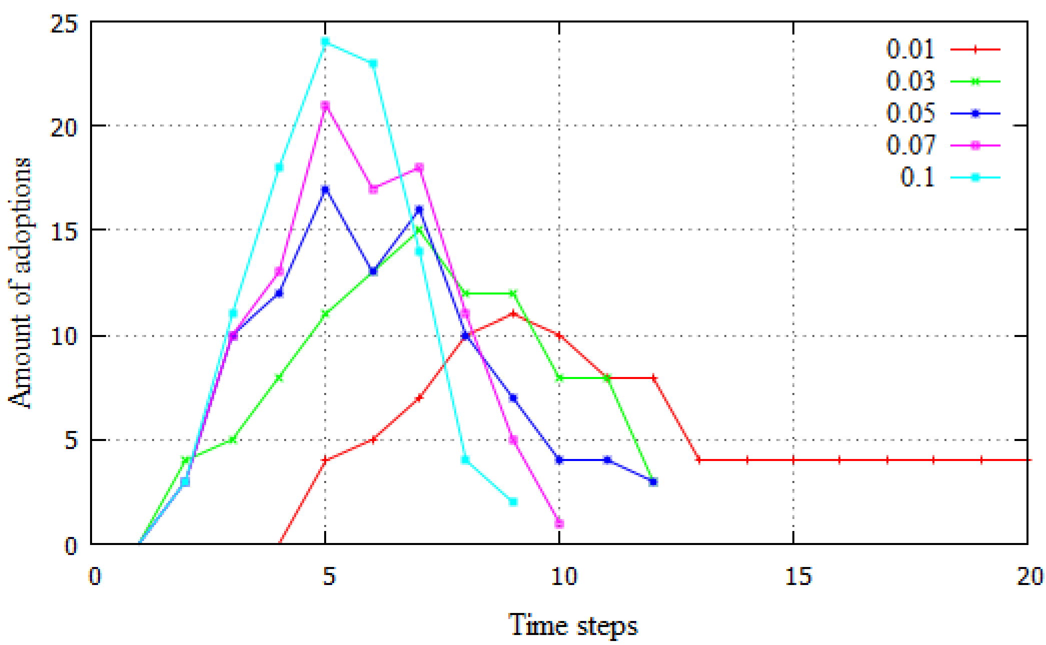


Figure 46: Variations of total time steps for 100 consumers

11.2 Variations of marketing strategies

In the configurations of the marketing strategies, the maximum variation of the price, the promotion cost, and the detection cost is 100%, and unit of variation is 5%.

11.2.1 More conservative variations of prices and costs.

What would be the results of experiments if I reduce the maximum 100% variation of the price, the promotion cost, and the detection cost into a more conservative one, for example, 20%. Tables 70 and 71 show the experiment results for homogeneous and heterogeneous consumers. The results of homogeneous consumers are very similar to results in chapter 9. However, the results of heterogeneous consumers deteriorated slightly compare to experiment results in chapter 9. The cause is that the configuration of the experiments only allows maximum 20% variation on the existing price, promotion cost, and piracy detection cost. From the perspective of EA, those limitations reduce the searching space of marketing strategies which may lead to failures of finding optimal strategies

As shown in Table 70, a pirate in L1 either increase the profits or diffusion speeds, or does not make any difference at all. In the network of seed 3 and rewiring probability 0.05, besides a pirate in L1, a pirate in L2 also increases the diffusion speed. In the network of seed 2 and rewiring probability 0.07, only pirates in L2 could increase the diffusion speed. All results are statistically significant.

Table 70: Comparison of profits and diffusion speeds using conservative strategies (Homo, Model 2-1)

Rewiring probability	Network of seed 1		Network of seed 2		Network of seed 3	
	Profit	Diffusion speed	Profit	Diffusion speed	Profit	Diffusion speed
0.01	L1: 7	L1: 7	L1: 7	L1: 7	L1: 6, 7	L1: 6, 7
0.03	L1: 6, 7	L1: 6, 7	L1: 7	L1: 7	L1; 7	L1: 7
0.05	None	None	L1: 34	L1: 34	L1: 7	L1: 7 L2: 1
0.07	L1: 4, 64	L1: 4, 64	None	L2: 9, 35	L1: 7, 96	L1: 7, 96
0.1	L1: 4, 64	L1: 4, 64	None	None	None	None

Experiments are conducted on the same 15 networks on heterogeneous consumers. Table 71 shows the experimental results of heterogeneous consumers. Compared with the results of homogeneous consumers, there are less pirates in L1 which are able to increase the profits and accelerate diffusion.

Table 71: Comparison of profits and diffusion speeds using conservative strategies (Hetero, Model 2-1)

Rewiring probability	Network of seed 1		Network of seed 2		Network of seed 3	
	Profit	Diffusion speed	Profit	Diffusion speed	Profit	Diffusion speed
0.01	L2: 70	L1: 7 L2: 9 L3: 68, 72	L2: 84	None	L2: 9	None
0.03	L2: 8	None	L2: 84	L2: 2 L3: 10, 11	L2: 8, 9	None

0.05	L3: 1	None	L1: 34 L2: 84	L1: 34	L2: 9	None
0.07	None	None	L1: 34 L2: 84	L1: 34	L1: 96 L2: 1, 8, 9	L1: 96
0.1	L2: 8	None	L1: 34	L1: 34	L1: 96	None

11.2.2 Unit of variation from 5% to 1%

What if I relax the variation unit from 5% to 1%? Tables 72 and 73 contain the results of homogeneous and heterogeneous consumers. The experimental results are still very similar to the results in chapter 9.

Table 72: Comparison of profits and diffusion speeds using variation unit 1% (Homo, Model 2-1)

Rewiring probability	Network of seed 1		Network of seed 2		Network of seed 3	
	Profit	Diffusion speed	Profit	Diffusion speed	Profit	Diffusion speed
0.01	L1: 7	L1: 7	None	L1: 7	L1: 7	L1: 6, 7
0.03	L1: 6	L1: 6, 7	L1: 7	L1: 7	L1: 7	L1: 7
0.05	None	None	L1: 34	L1: 34	L1: 7	L1: 7
0.07	None	L1: 4, 64	None	None	None	L1: 7, 96
0.1	L1: 4, 64	L1: 4, 64	None	None	None	None

Table 73: Comparison of profits and diffusion speeds using variation unit 1% (Hetero, Model 2-1)

Rewiring probability	Network of seed 1		Network of seed 2		Network of seed 3	
	Profit	Diffusion speed	Profit	Diffusion speed	Profit	Diffusion speed
0.01	None	L1: 6, 7	L2: 84	None	L1: 7 L2: 9	L1: 7
0.03	L1: 6, 7	L1: 6, 7	L2: 84	None	None	L1: 7
0.05	None	L1: 7	L1: 34	None	L1: 7	L1: 7
0.07	L1: 7	None	L1: 34 L2: 84	None	L1: 7, 96	L1: 7
0.1	L1: 7	L1: 4	L1: 34	None	None	L1: 7, 4

11.3 Variations of EA parameters

In the EA configuration used the dissertation, the population size is 200 and the generation count is 300.

11.3.1 Population size 400 and generation count 600

In the new experiment, I increase the population size from 200 to 400, and increase the generation count from 300 to 600. Results in Tables 74 and 75 show that the increasing the population size and generation count barely change the relative statistical difference.

Table 74: Comparison of profits and diffusion speeds using population size 400 and generation count 600 (Homo, Model 2-1)

Rewiring probability	Network of seed 1		Network of seed 2		Network of seed 3	
	Profit	Diffusion speed	Profit	Diffusion speed	Profit	Diffusion speed
0.01	L1: 7	L1: 7	None	L1: 7	L1: 6, 7	L1: 6, 7
0.03	L1: 6, 7	L1: 6, 7	L1: 7	L1: 7	L1: 7	L1: 7
0.05	None	None	None	L1: 34	L1: 7	L1: 7
0.07	L1: 4, 64	L1: 4, 64	None	None	L1: 7, 96	L1: 7, 96
0.1	L1: 4, 64	L1: 4, 64	None	None	None	None

Table 75: Comparison of profits and diffusion speeds using population size 400 and generation count 600 (Hetero, Model 2-1)

Rewiring probability	Network of seed 1		Network of seed 2		Network of seed 3	
	Profit	Diffusion speed	Profit	Diffusion speed	Profit	Diffusion speed
0.01	L1: 7	L1: 7	L2: 84	None	L1: 7 L2: 9	L1: 7
0.03	L1: 6, 7	L1: 6, 7	L2: 84	None	L1: 7 L2: 9	L1: 7
0.05	L1: 4, 7 L2: 68 L3: 1	L1: 4, 7	L1: 34 L2: 84 L3: 58	L1: 34, 3	L1: 7 L2: 9	L1: 7
0.07	L1: 7, 64	L1: 7, 64	L1: 34 L2: 84	L1: 34 L2: 8	L1: 96	L1: 7
0.1	L1: 4, 7, 64	L1: 4, 7, 64	L1: 34, 6	None	L1: 3, 4, 7, 96 L3: 0	L1: 7, 4

In summary, through various sensitivity analyses and experiments, I show the performance of my models is stable.

11.4 Models design justifications

Models in the dissertation have been empirically relevant with models from existing research. Watts (2007) utilizes one trend setter to start the diffusion process, which corresponds to one innovation initiator in my models. The piracy models built by Khouja et al. (2007) employs similar parameters for marketing strategies and attributes of consumers. Jeong et al. (2013) uses 1000 consumers in their model. My models use 100 consumers and prove the model is scalable with more consumers.

CHAPTER 12: CONCLUSIONS

12.1 Contributions

In my research, models have been constructed that develop dynamic and self-learning marketing strategies that are able to adapt and dynamically adjust various marketing factors to achieve desired profits and diffusion speed. My models explicitly represent diffusion processes between individual consumers using social networks and agent-based models, and determine effective marketing strategies for digital content providers via learning classifier systems. I have not found any research work which considers all those factors together. I have built these models combining EA and agent-based modeling techniques to overcome the limitations of classical mathematical models.

In both monopoly and duopoly marketing environments, two groups of models are used in experimentation and analyzed. One group has no digital piracy, and the other group contains digital piracy. Each group is further divided into two models based on the level of influence of the key agent: a key agent is either an average individual or a persuader who influences the adoption decisions of other consumers.

12.1.1 Impacts of a key agent in the diffusion of digital information goods

12.1.1.1 Impacts of a connector and an opinion leader

Several conclusions are drawn concerning the connector or the opinion leader as a key agent. First, a key agent only makes a difference on diffusion patterns when he is able to influence others, while other consumers fail to adopt due to the restrictions of prices and promotion cost. Second, assigning a connector as a key agent does not necessarily increase profits and accelerate diffusion speed. Third, assigning an opinion leader as a key agent has the possibility to increase profits or diffusion speeds only in a scale-free network. Last, dynamic self-learning marketing strategies alone are not enough to increase profits and accelerate diffusion speed. Network topologies are another important factor which contributes to the performance of profits and diffusion speed.

12.1.1.2 Position of a key agent

From the experiments on the connector and the opinion leader, it is determined that in order for a key agent to make a difference on profits and diffusion speed, he has to adopt the product before others do. Under what conditions does this happen? The answer lies in the position of the key agent. The results of the model indicate that

- Positioning a key agent among the direct neighbors of the innovation initiator (L1) guarantees that the key agent adopts the product before other consumers do.
- The key agent in L1 has the possibility to result in higher profits and faster diffusion speed. However, not every key agent in L1 is able to achieve that. An important factor is the network topology.

- If the key agent in L1 is an average consumer, he has the possibility to increase profits and accelerate diffusion speeds. However, the improvement is not large enough to catch the attention of a provider in realistic marketing environments.
- If the key agent in L1 is a persuader, then besides having the possibility on making a difference on profits and diffusion speeds, he is able to increase the profits significantly.

The summary of the impacts of a connector, an opinion leader, and the position of a key agent are shown in Table 76. When marketing strategies are involved, an average key agent is not able to contribute significantly to the profits and diffusion speed. An opinion leader has much higher probability of generating a high profit in a scale-free network. A persuader in L1 in any type of networks has the possibility to increase the profits significantly.

Table 76: Impacts of a connector, an opinion leader, and position of a key agent

	Average key agent in L1	Persuader in L1	Opinion leader	Connector
Small-world networks	No	Yes	No	No
Random networks	No	Yes	No	No
Scale-free networks	No	Yes	Yes	No

12.1.1.3 Timing of a key agent

From the results of the positions of a key agent, I conclude that the adoption of a key agent should take place in the early stage of diffusion process before the majority of the adoptions start.

12.1.2 Dynamic marketing strategies

A few scholars have used the Bass model to develop strategies to utilize the positive impacts of digital piracy. However, the Bass model relies on several unrealistic assumptions that diminish the applicability of the strategies. My research develops dynamic and self-learning marketing strategies which are able to continuously adapt and dynamically adjust marketing factors, including prices, promotion costs, and piracy detection costs, based on the current feedback from the market. The self-learning and dynamic adjustments are guided by the goals of the strategies, that is, maximizing profits and accelerating diffusion. Consumers' reservation prices, promotion cost thresholds, and piracy detection thresholds are invisible to the digital content provider. From the demonstration of best-so-far marketing strategies in chapter 5, I see that a successful marketing strategy helps the provider adjust his price, promotion cost, and piracy detection cost by moving closer to the thresholds of consumers gradually. After the price and promotion cost reached the thresholds of consumers, the price and promotion cost fluctuate slightly and stay close to the thresholds of consumers. For the piracy detection cost, after the product has been pirated, the piracy detection cost is of no use further, and it drops quickly to its lowest value.

The definition of fitness (in this case, profits and diffusion speeds) is crucial to the actions of marketing strategy. In order to maximize its fitness value, the dynamic and self-learning marketing strategy is able to adjust the price and promotion cost of the provider at every time step during the diffusion process. In the case of opinion leaders on a scale-free network, the marketing strategy decides to adjust its price and promotion cost by evaluating the potential profits brought by followers of the opinion leaders who pay at a high price and potential profits brought by the average consumers. The case of handling heterogeneous consumers is another example of the wonder of those dynamic and self-learning strategies. When facing the heterogeneities of consumer thresholds, the strategy decides which consumer to approach and how many consumers to adopt at each time step by evaluating the potential profits of every possible action.

12.1.3 Impacts of variations of networks on profits and diffusion speeds

Experiments were performed on 15 small-world networks which are created using three random seeds, each is coupled with five rewiring probabilities: 0.01, 0.03, 0.05, 0.07, and 0.1. Results show that more profits are earned and less time is consumed as the rewiring probability increases. The reason is that as the rewiring probability increases, there are more shortcuts created, therefore, the traveling distance between consumers becomes shorter and the less time consumed during the diffusion process. When there is less time consumed in the diffusion process, less promotion and detection costs are spent in the market, and therefore, more profits are earned.

12.1.4 Relationship between marketing strategies and network topologies

Either the marketing strategy developed by EA or the network topology alone is able to accelerate the diffusion process and increase profits. Dynamic self-learning marketing strategies need to cope with the network topologies and positions of a pirates in order to increase profits and accelerate diffusion speed. The role of EA is more evident in the case of heterogeneous consumers.

12.1.5 Competition between digital content providers

In the duopoly model, a consumer acquires the information about the brands of products through interactions with his neighbors, which is referred to as imperfect information. The outcome of the competition is determined by the brand adopted by the consumers in the direct neighborhood of the innovation initiator (L1). In other words, the adoption decisions of consumers in L1 are crucial to the outcome of the competition. If consumers in L1 adopted only one brand, then the adopted brand will take over the entire market and create a lock-in status. Otherwise, the winning brand of the competition is unpredictable.

The results indicate that the provider who is able to win over all consumers in L1 is the winner of the competition. A winning strategy in Model 2 is to lower the piracy detection cost (if a pirate is in L1) and price, and increase promotion cost faster than its competitor. Besides lowering the price and increasing the promotion cost faster than the competitor, a winning strategy in Model 1 needs to distribute the free copy of the digital information goods to the key agent before the competitor does.

A provider should not be afraid of losing profits initially due the lower price and higher promotion cost. As long as he wins over all consumers in L1, after the adoptions start, the price and promotion cost gradually recovers and approaches the thresholds of consumers.

A persuader does not make significant difference to the results of the duopoly models. For scale-free networks, marketing strategies can always guarantee that provider 1 creates a lock-in results.

12.2. Future work

In my dissertation, each model only includes one key agent. In the future research, I would like to investigate how many key agents each network could tolerate before the profits and diffusion speeds begin to decline. This is an also important question for the digital content provider: How many key agents should a provider hire to maximize the benefits.

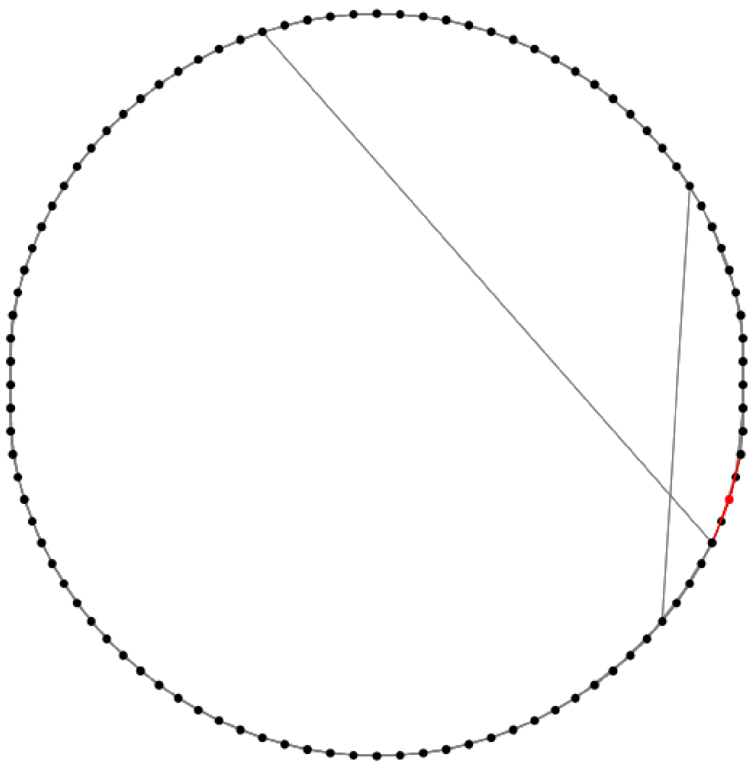
In the duopoly model, a consumer is allowed to acquire only one brand. What if a consumer is allowed to acquire both brands? For example, a consumer may purchase one brand and pirate the other brand. In the future work, I would like to investigate how such behaviors of consumers affect the marketing strategies of the digital content providers.

APPENDICES

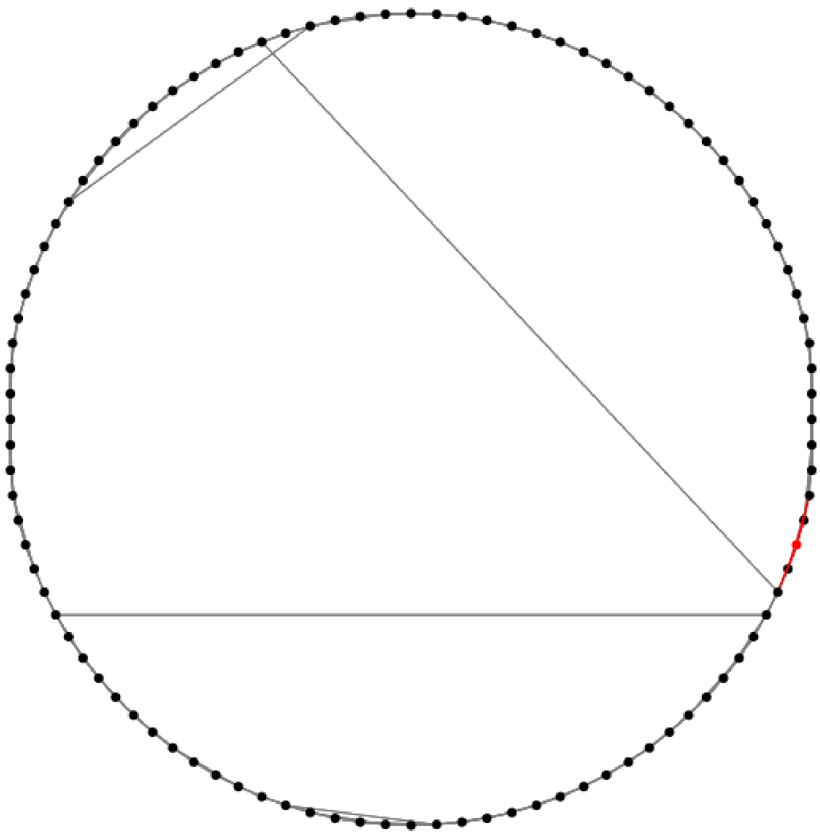
Appendices A, B, C, and D illustrate the network topologies of 15 small-world networks, three scale-free networks, and three random networks. Appendix E demonstrates the flow of consumer adoption decisions in duopoly model in chapter 10. Appendix F provides detailed values of D, p, and mean values for Model 2-1, both for homogeneous and heterogeneous consumers in chapter 9. Appendix G provides detailed values of D, p, and mean values for Model 2-2, both for homogeneous and heterogeneous consumers in chapter 9. Appendix H provides detailed values of D, p, and mean values for Model 1-1, both for homogeneous and heterogeneous consumers in chapter 9. Appendix H provides detailed values of D, p, and mean values for Model 1-2, both for homogeneous and heterogeneous consumers in chapter 9. Appendix J provides detailed values of D, p, and mean values for Model 2-1 and Model 2-2 on scale-free networks, for homogeneous consumers in chapter 9. Appendix K provides detailed values of D, p, and mean values for Model 2-1 and Model 2-2 on random networks, for homogeneous consumers in chapter 9. Appendix L lists the experimental results of audacious marketing strategies in Model 2-1 in chapter 10. Appendix M lists the comparisons of number of “lock-in” between conservative and audacious strategies in chapter 10. Appendix N lists the experimental results of audacious marketing strategies for Model 2-2 in chapter 10.

APPENDIX A: NETWORK TOPOLOGIES OF 15 SMALL-WORLD NETWORKS

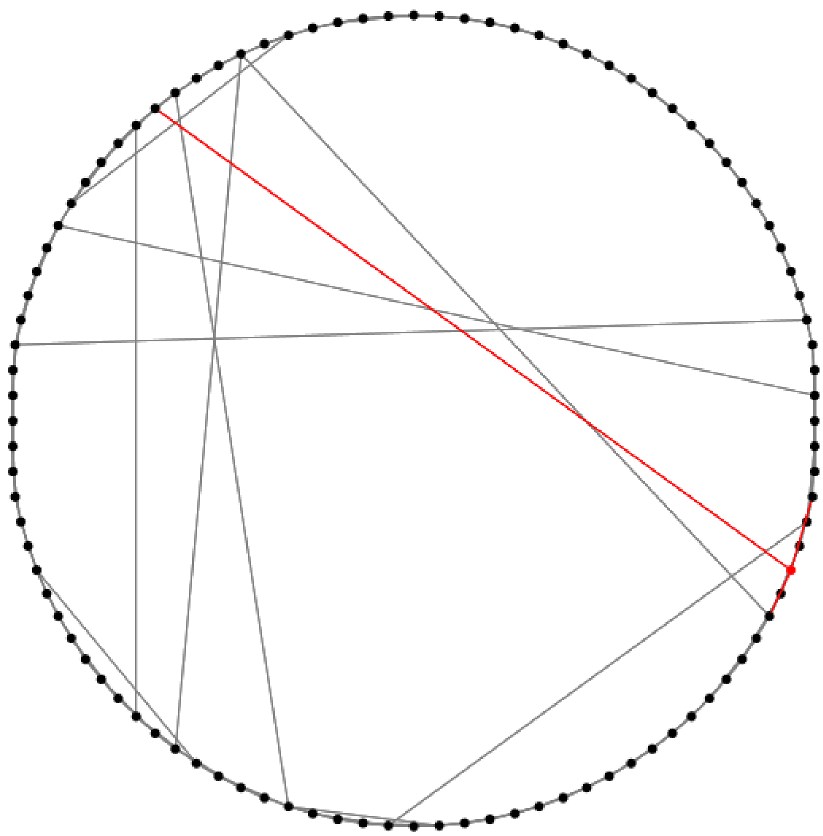
Seed1 0.01



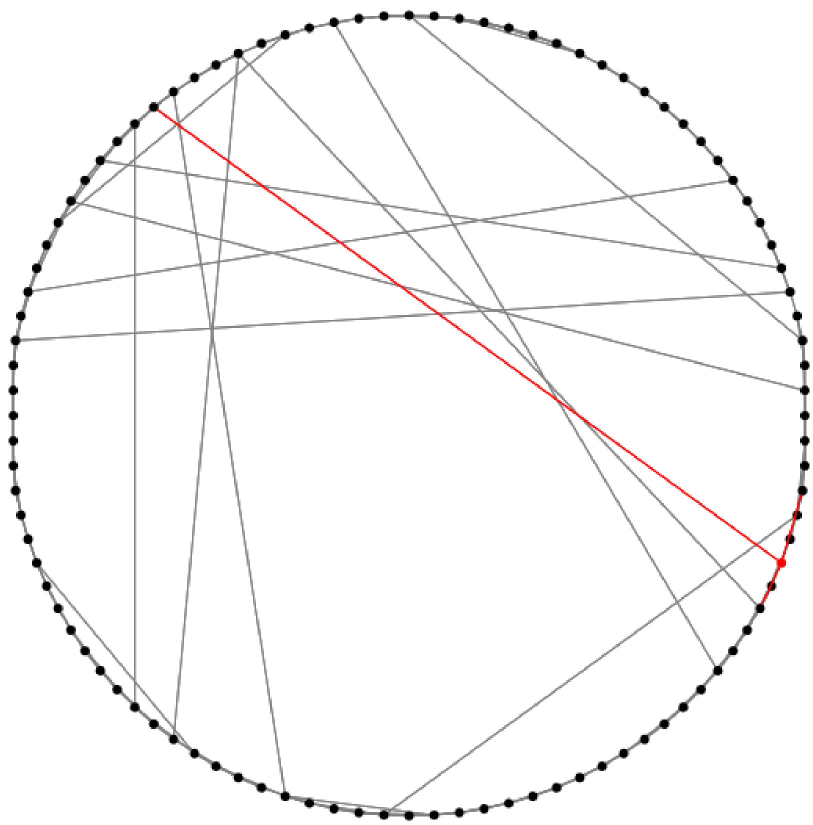
Seed1 0.03



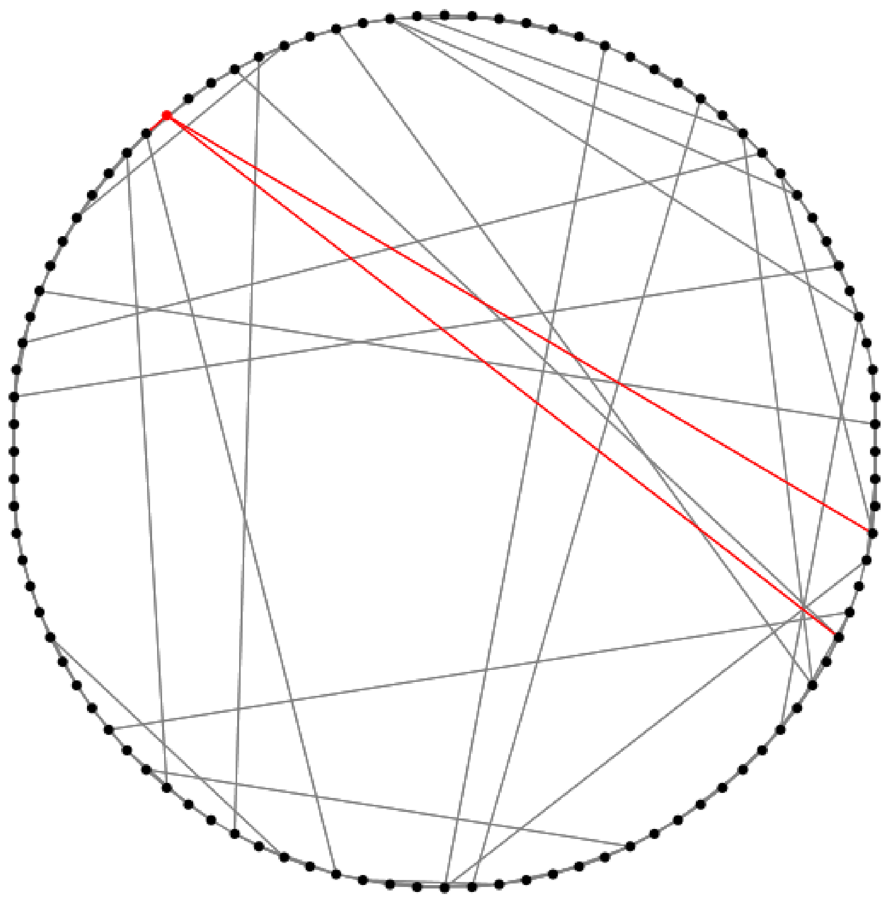
Seed1 0.05



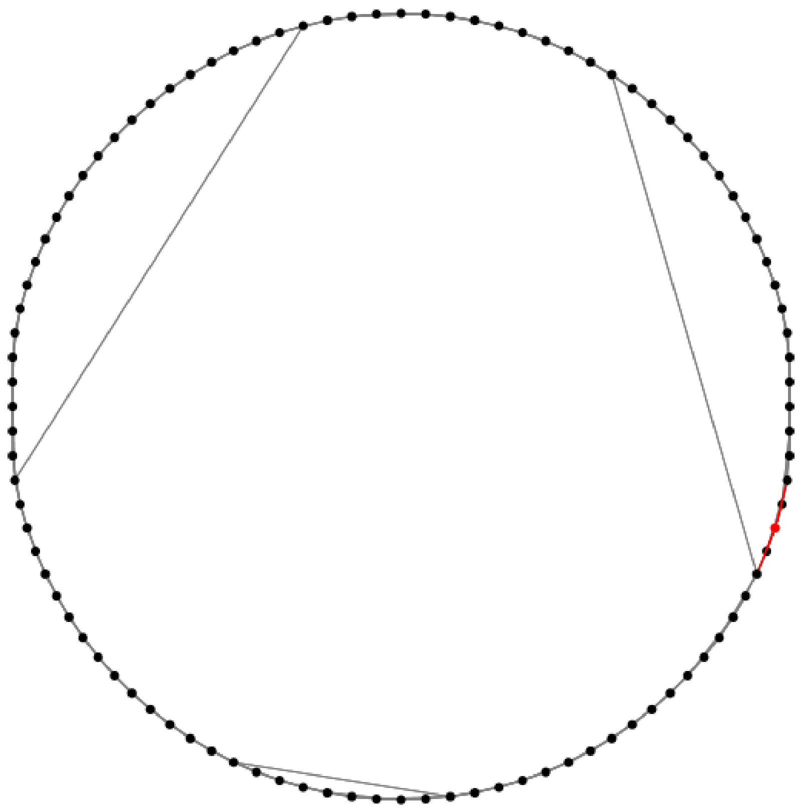
Seed1 0.07



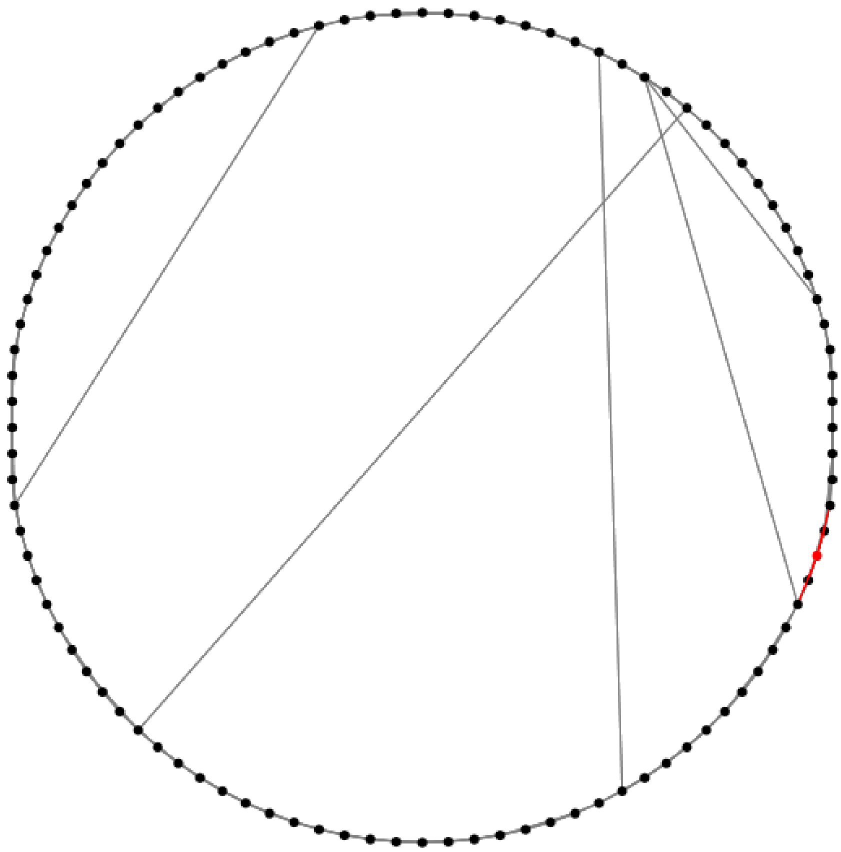
Seed1 0.1



Seed2 0.01



Seed2 0.03



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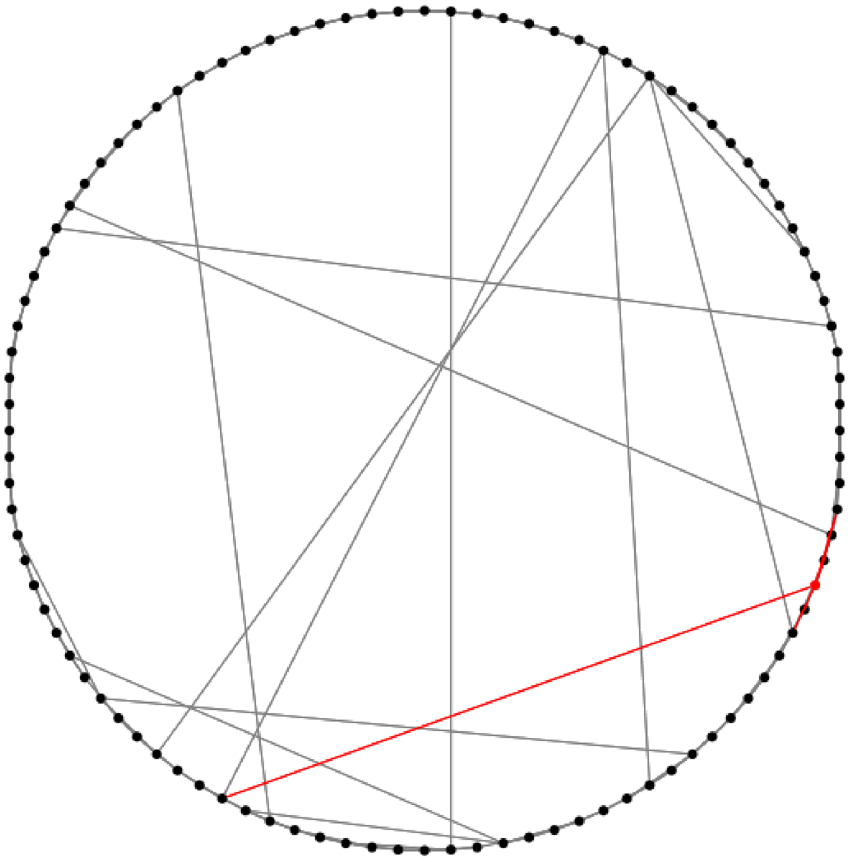

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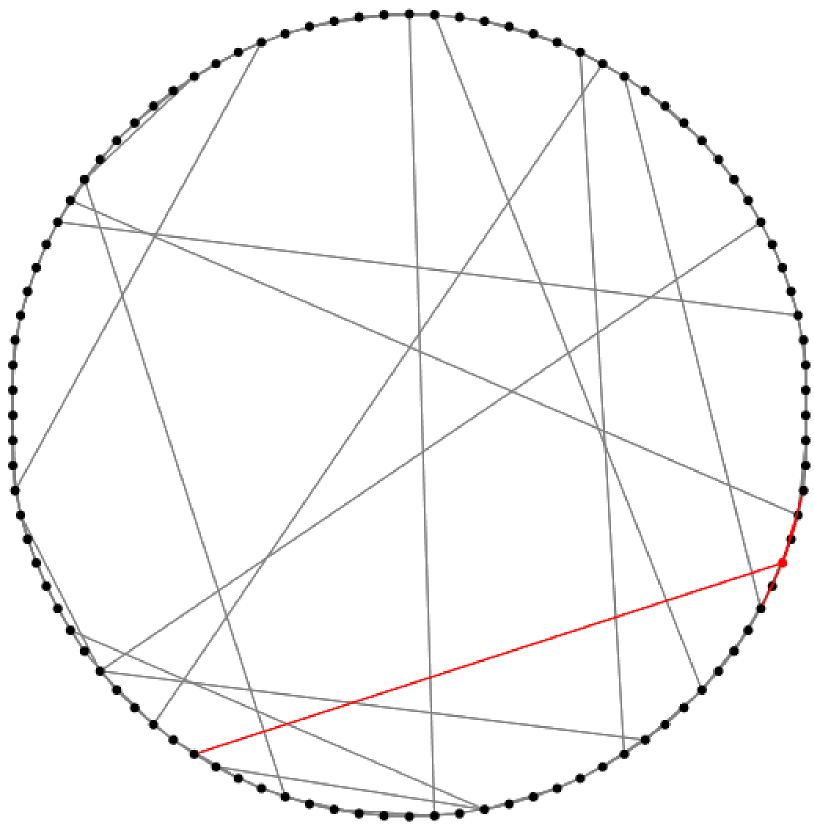
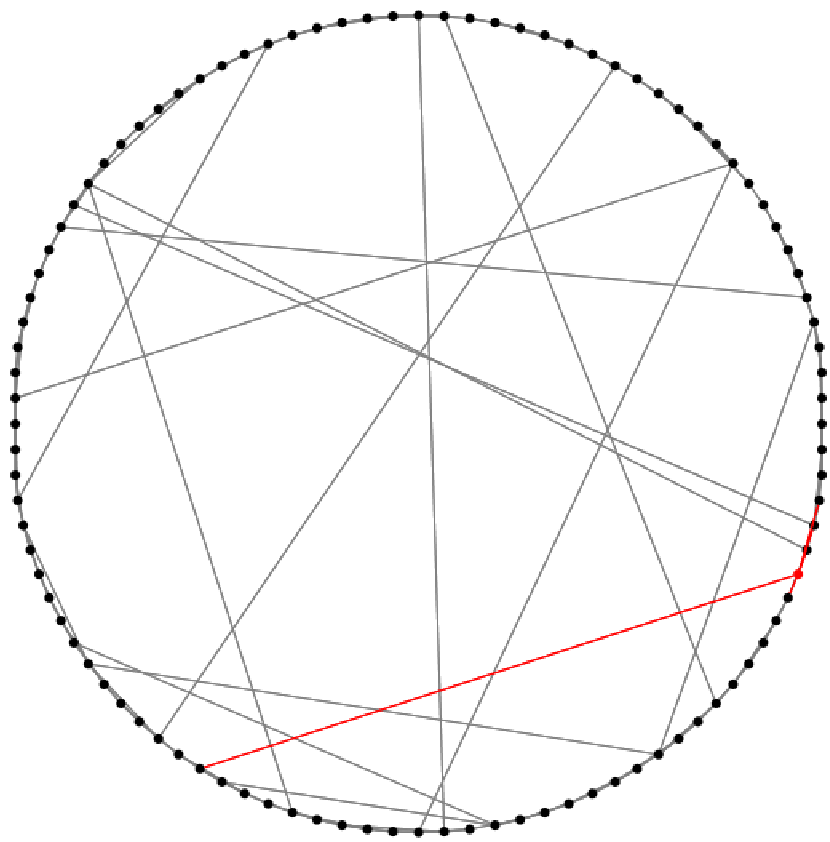
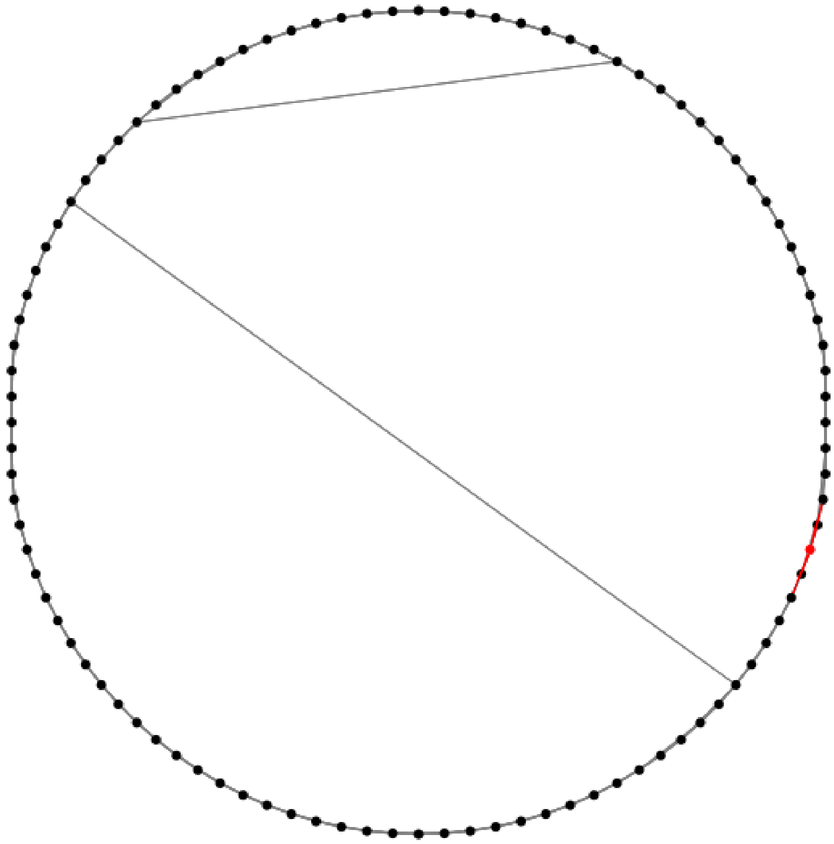


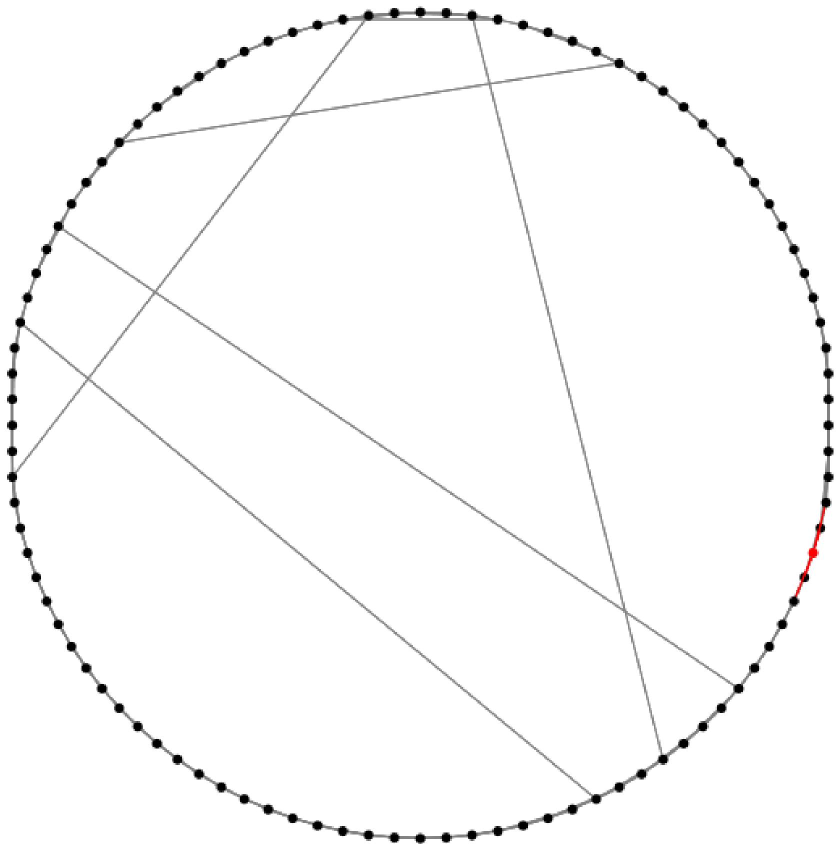
Seed2 0.1



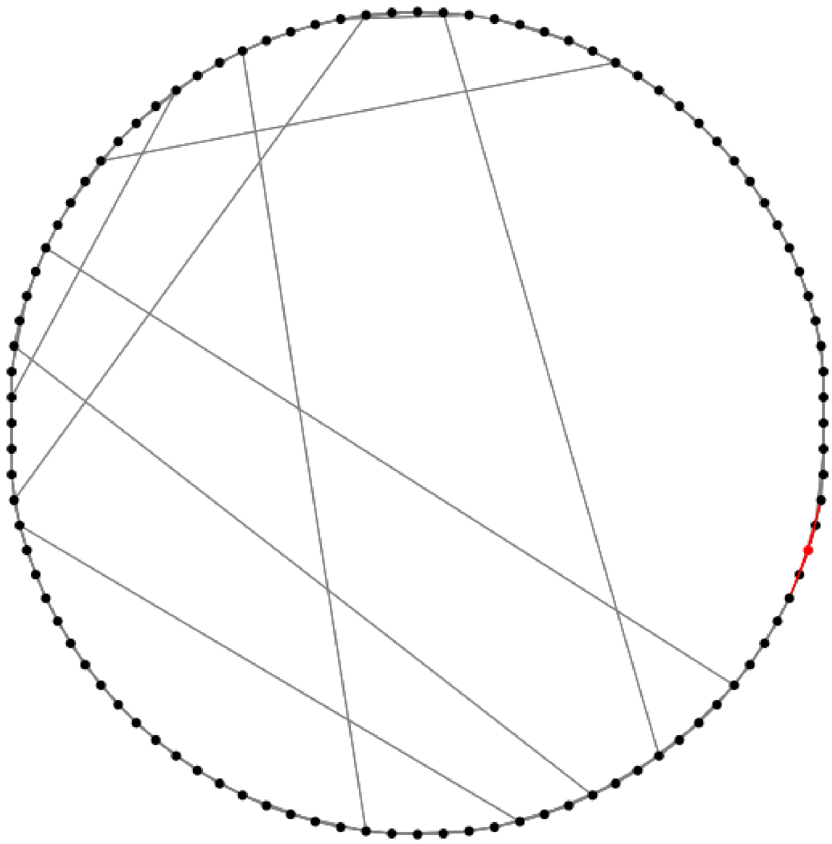
Seed 3, 0.01



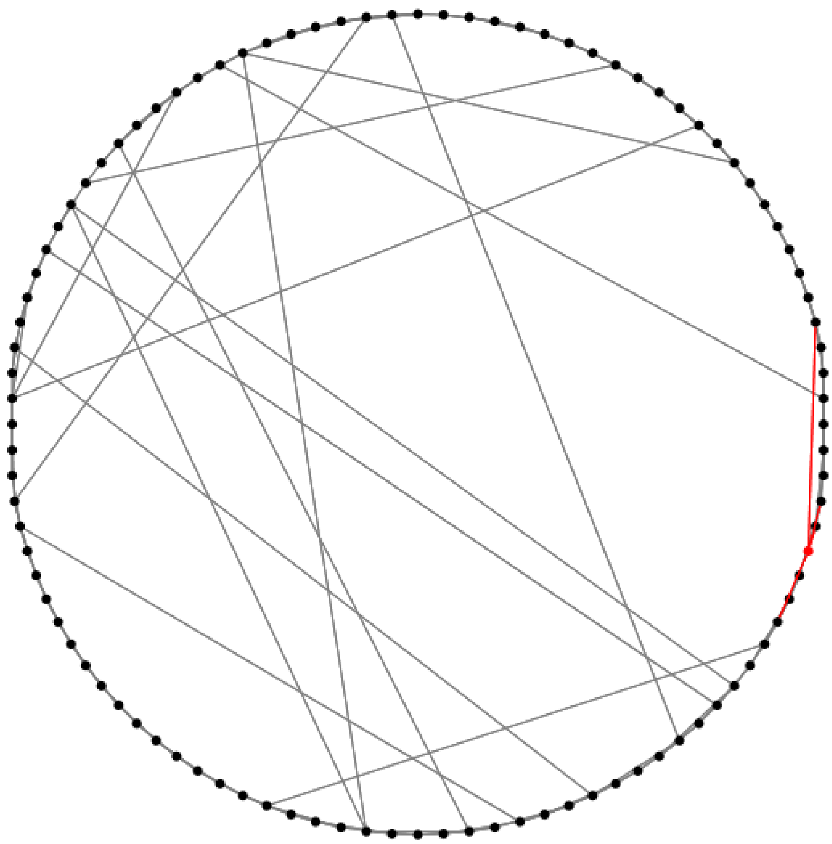
Seed3 0.03



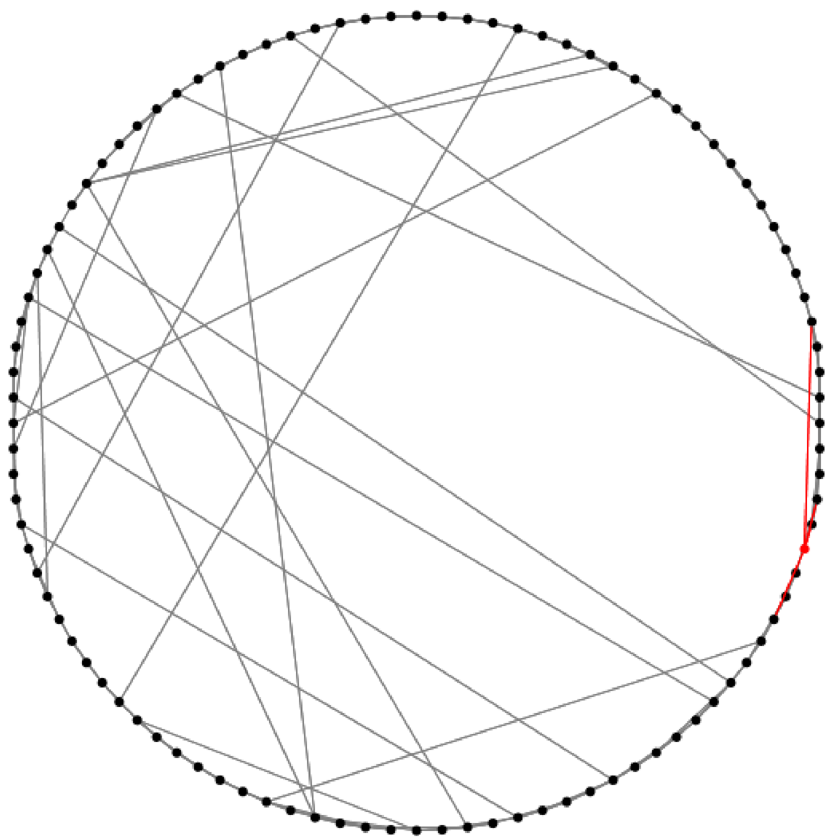
Seed3 0.05



Seed3 0.07

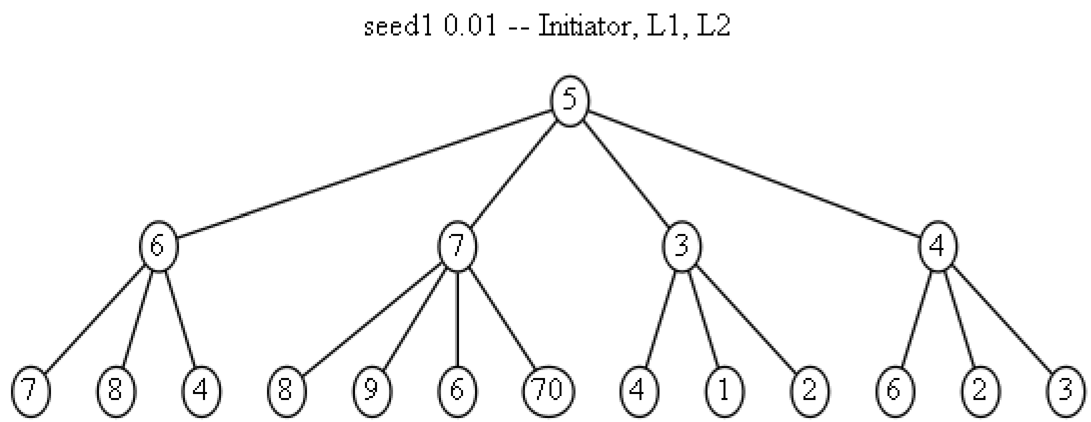


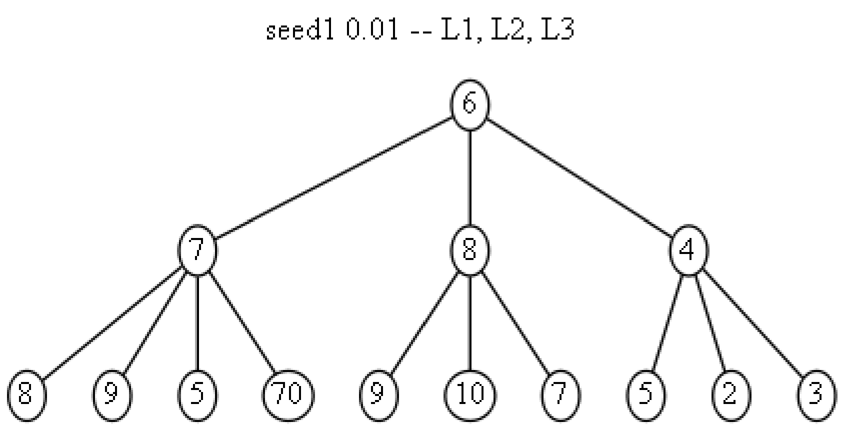
Seed3 0.1



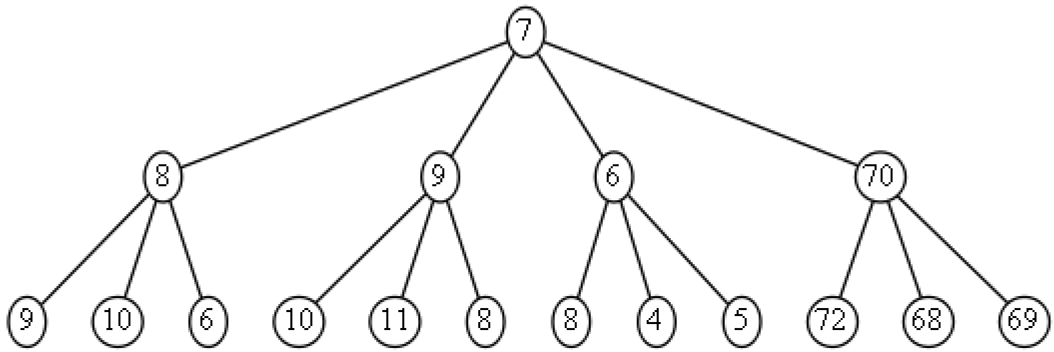
APPENDIX B: INITIATOR, L1, L2, AND L3 OF 15 SMALL-WORLD NETWORKS

Seed 1, rewiring probability 0.01

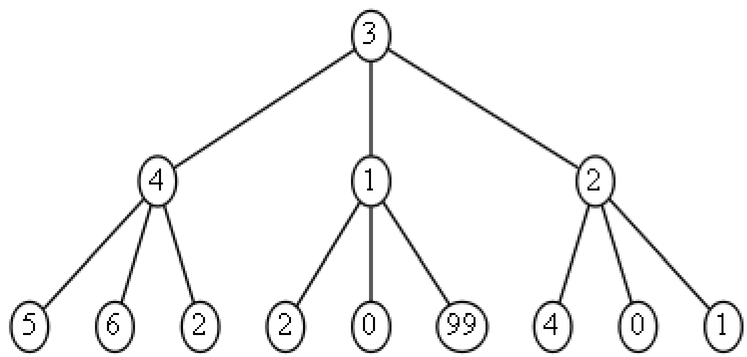




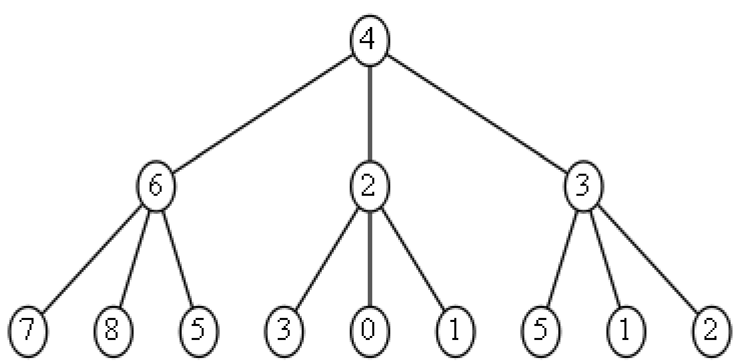
seed1 0.01 -- L1, L2, L3



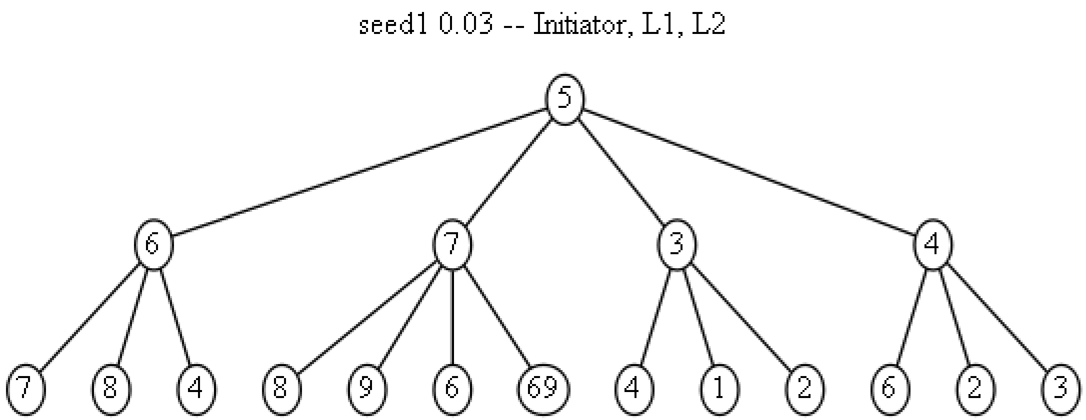
seed1 0.01 -- L1, L2, L3

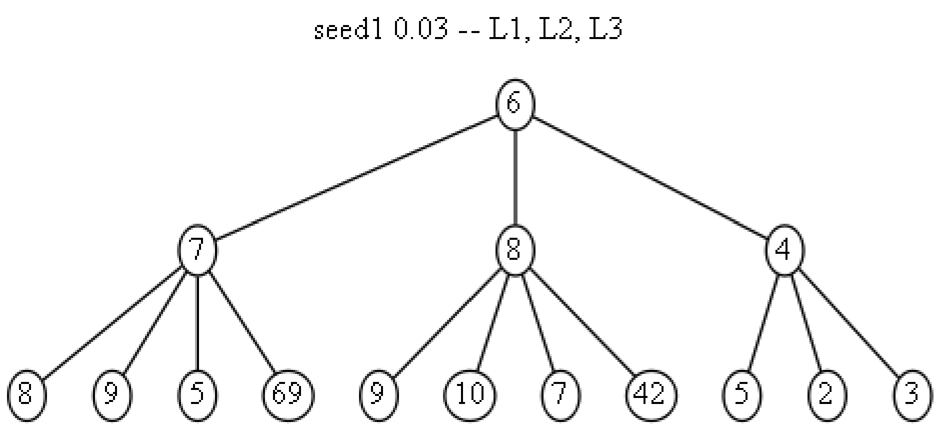


seed1 0.01 -- L1, L2, L3

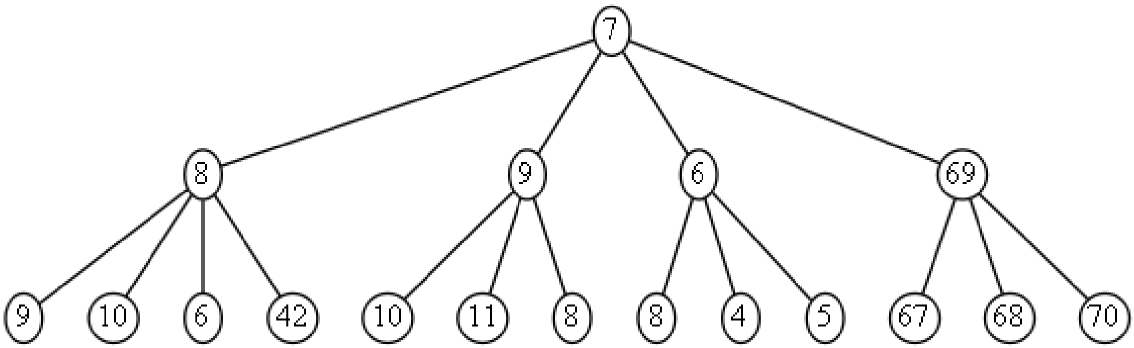


Seed 1, rewiring probability 0.03

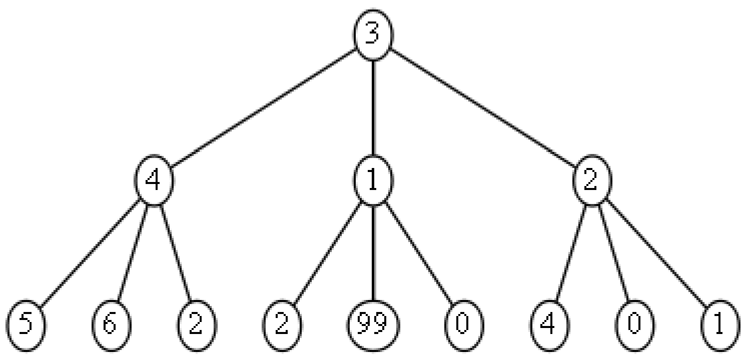




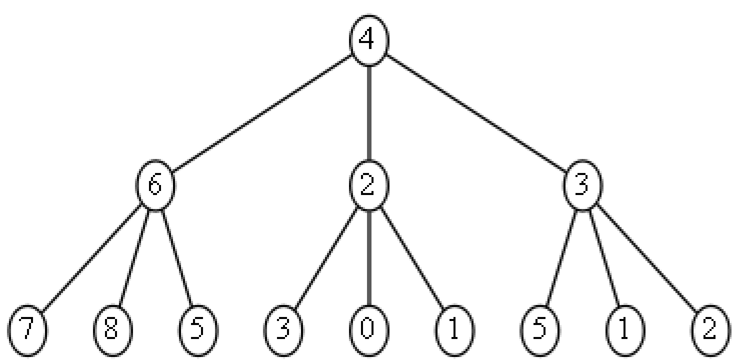
seed1 0.03 -- L1, L2, L3



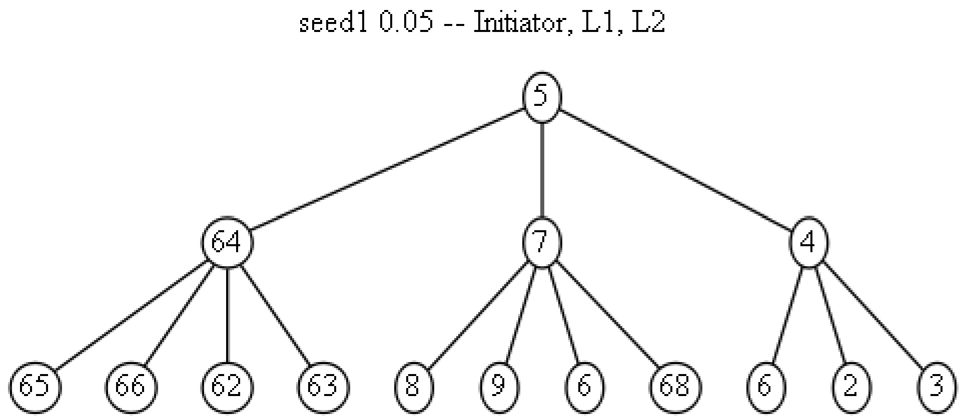
seed1 0.03 -- L1, L2, L3

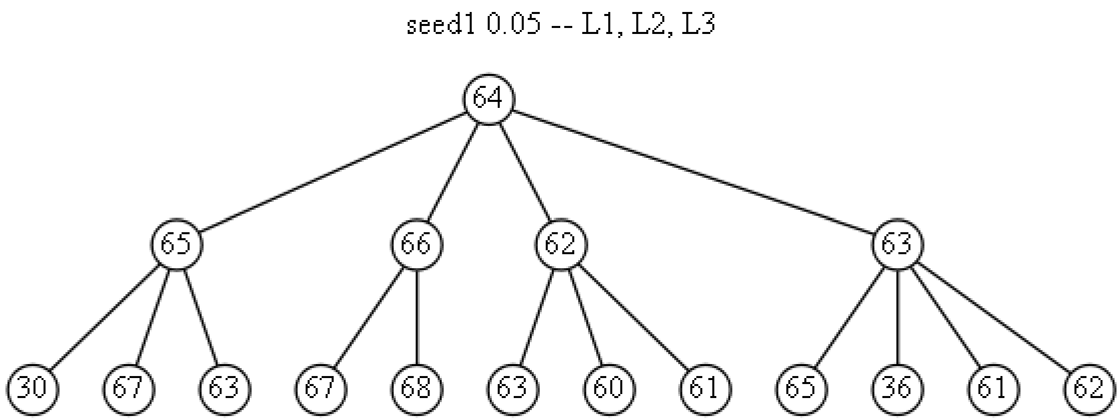


seed1 0.03 -- L1, L2, L3

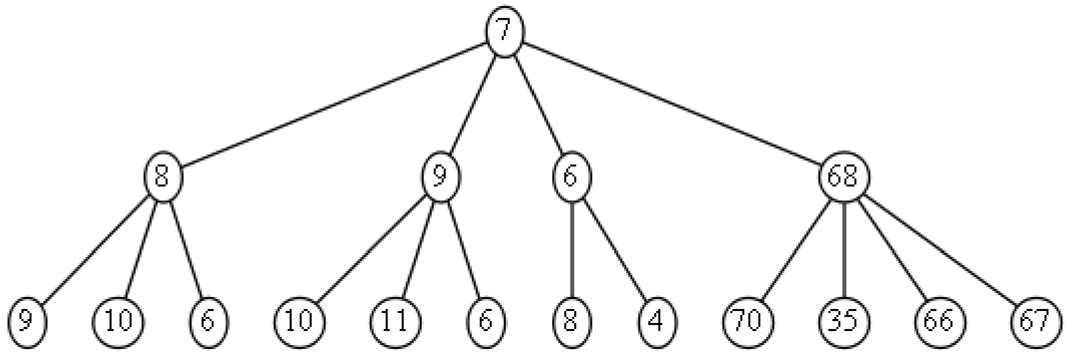


Seed 1, rewiring probability 0.05

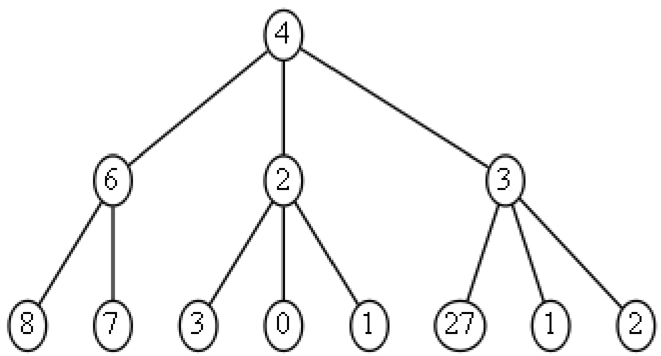




seed1 0.05 -- L1, L2, L3

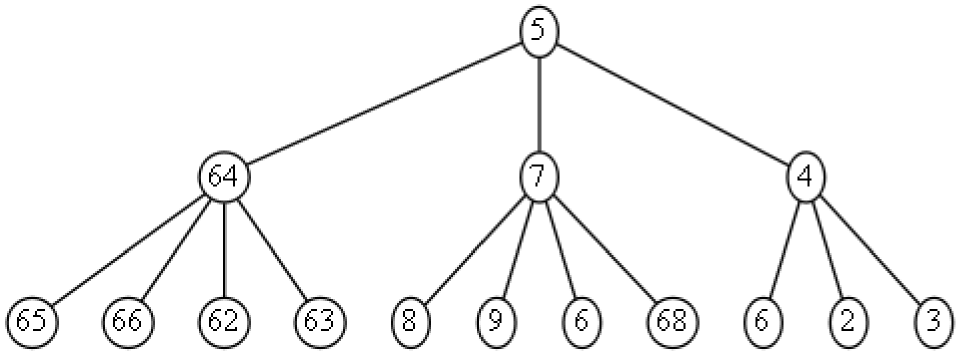


seed1 0.05 -- L1, L2, L3

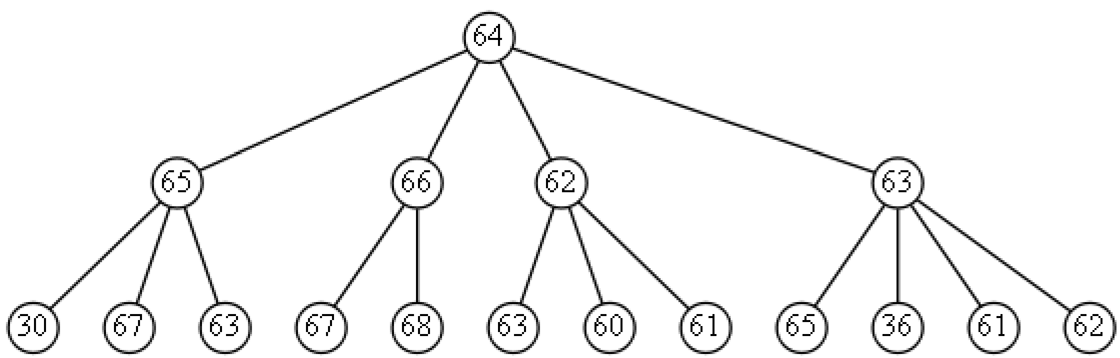


Seed 1, rewiring probability 0.07

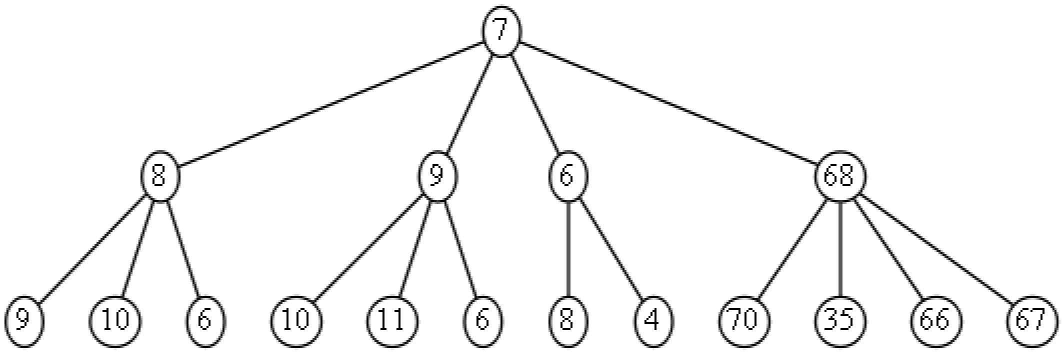
seed1 0.07 -- Initiator, L1, L2



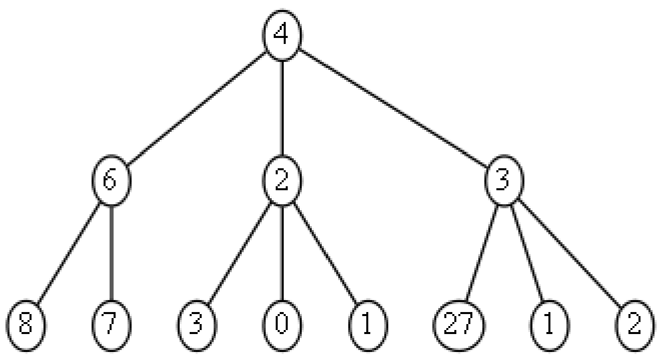
seed1 0.07 -- L1, L2, L3



seed1 0.07 -- L1, L2, L3

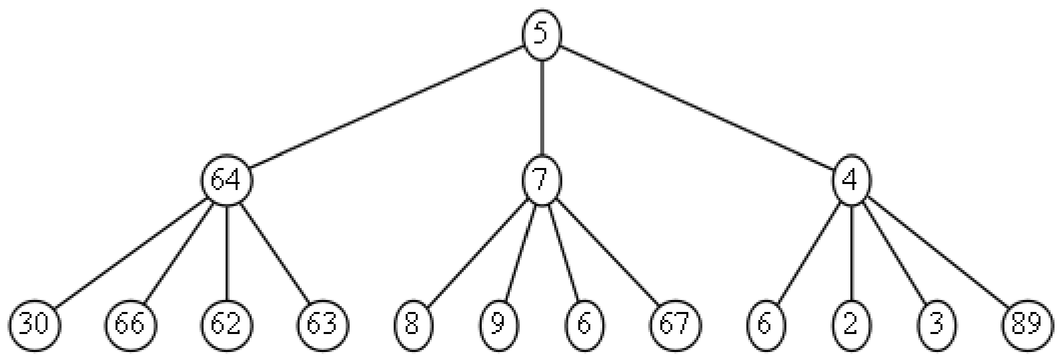


seed1 0.07 -- L1, L2, L3

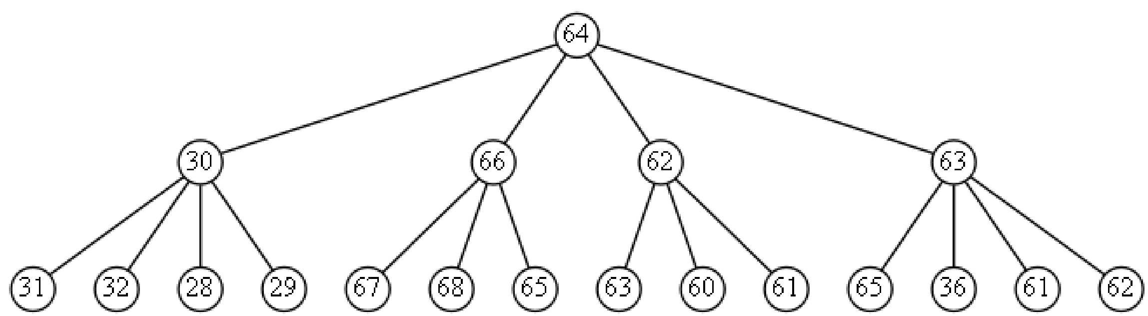


Seed 1, rewiring probability 0.1

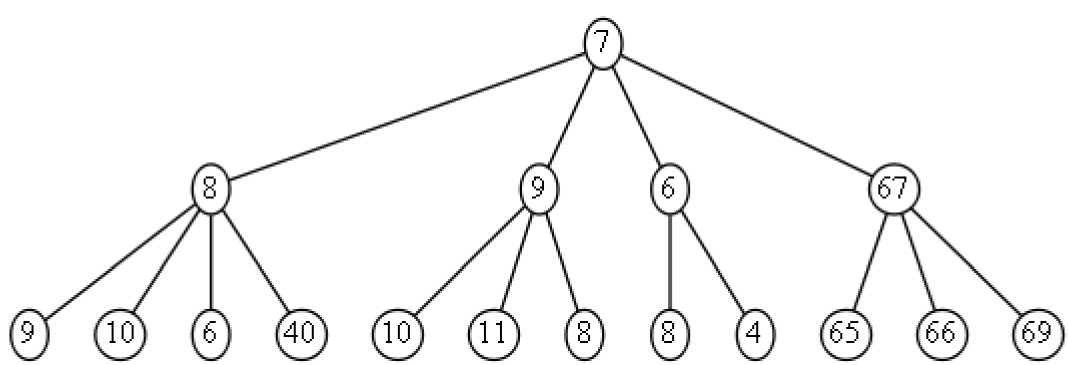
seed1 0.1 -- Initiator, L1, L2



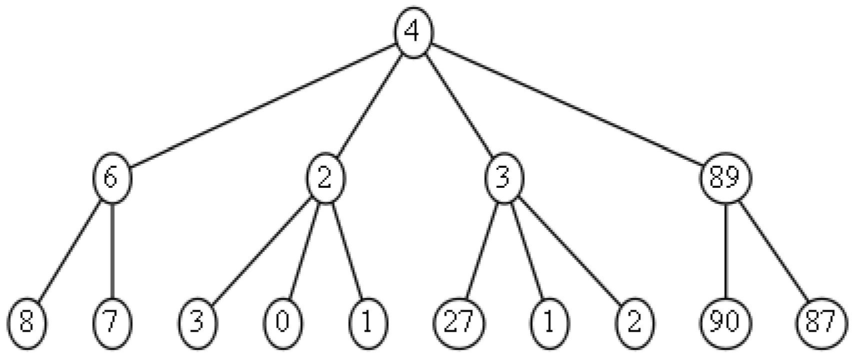
seed1 0.1 -- L1, L2, L3



seed1 0.1 -- L1, L2, L3

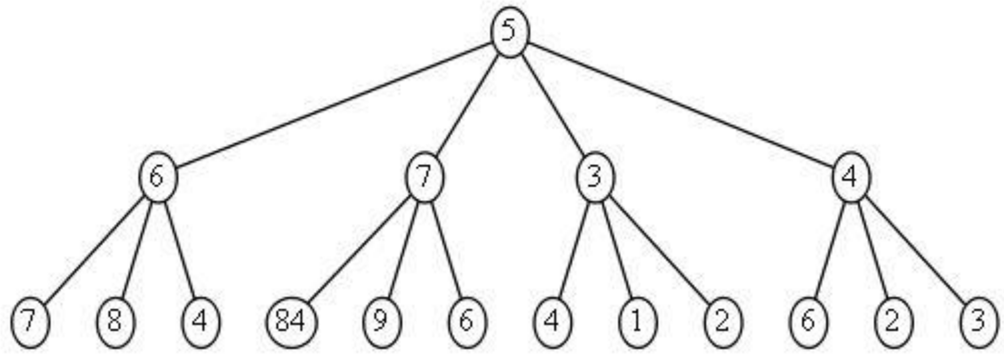


seed1 0.1 -- L1, L2, L3

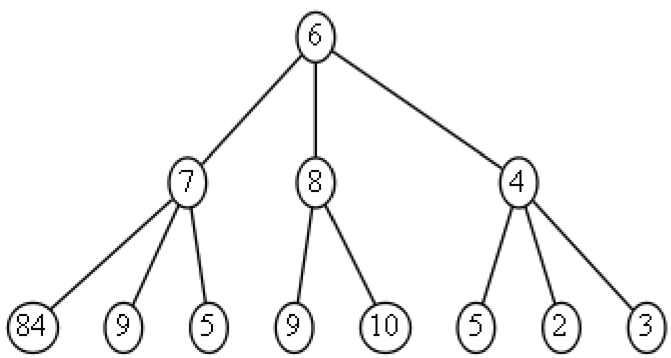


Seed 2, rewiring probaility 0.01

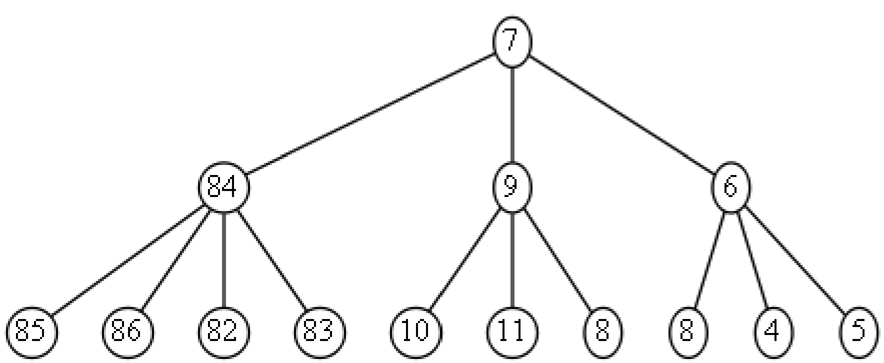
seed2 0.01 -- Initiator, L1, L2

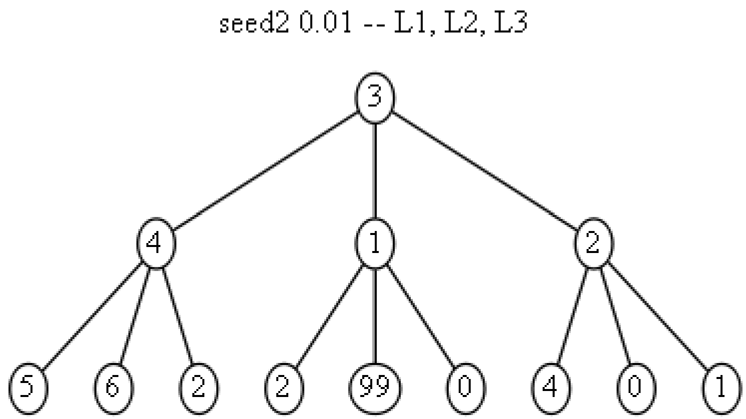


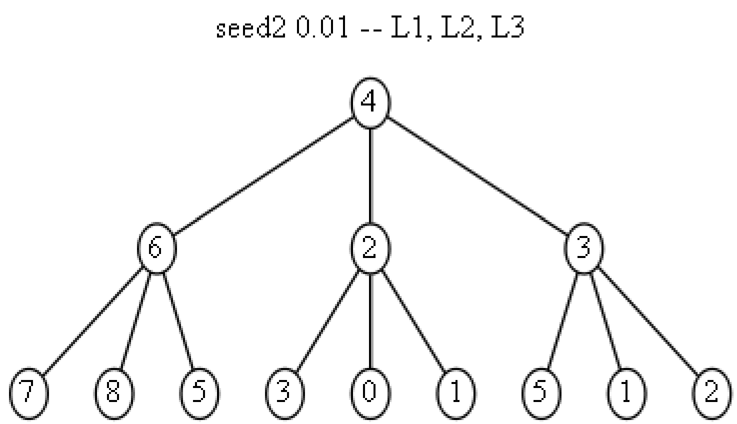
seed2 0.01 -- L1, L2, L3



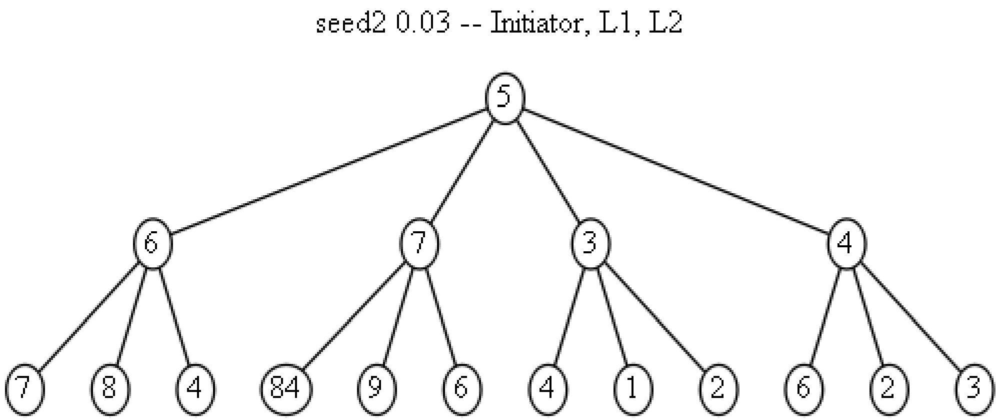
seed2 0.01 -- L1, L2, L3

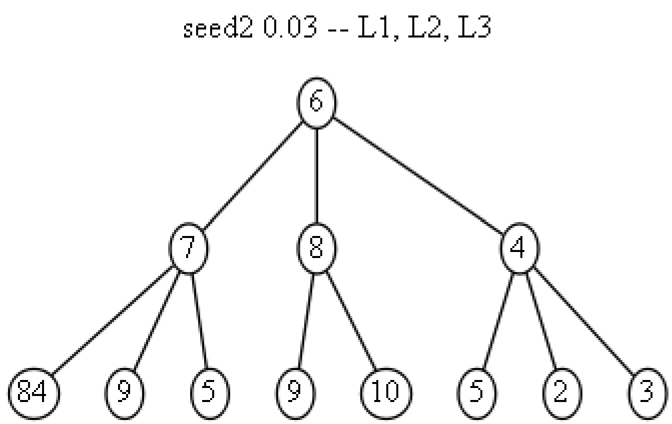



seed2 0.01 -- L1, L2, L3
3
4
1
2
5
6
2
2
99
0
4
0
1

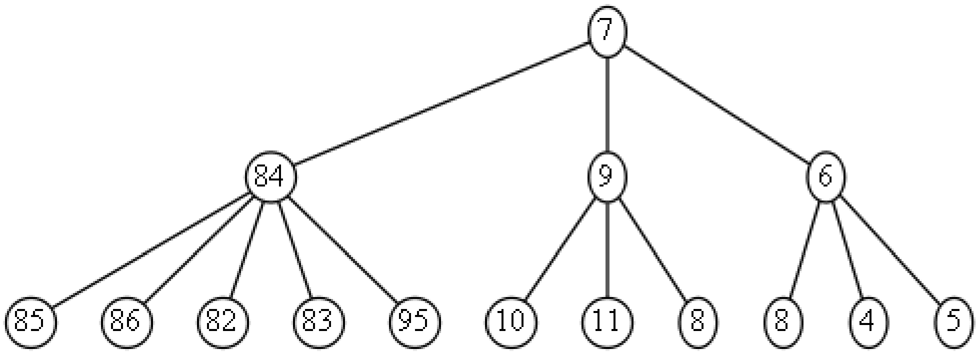

seed2 0.01 -- L1, L2, L3
4
6
2
3
7
8
5
3
0
1
5
1
2

Seed 2, rewiring probaility 0.03

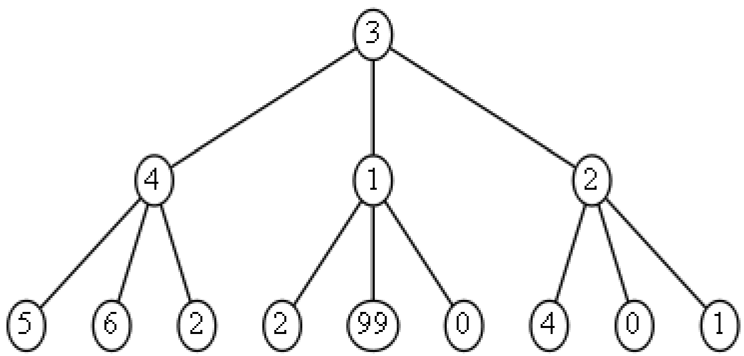




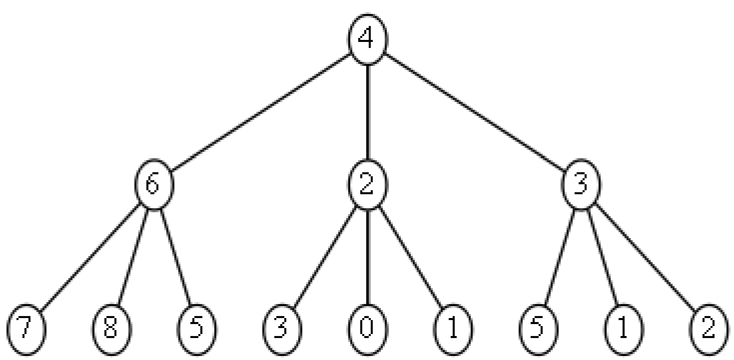
seed2 0.03 -- L1, L2, L3



seed2 0.03 -- L1, L2, L3

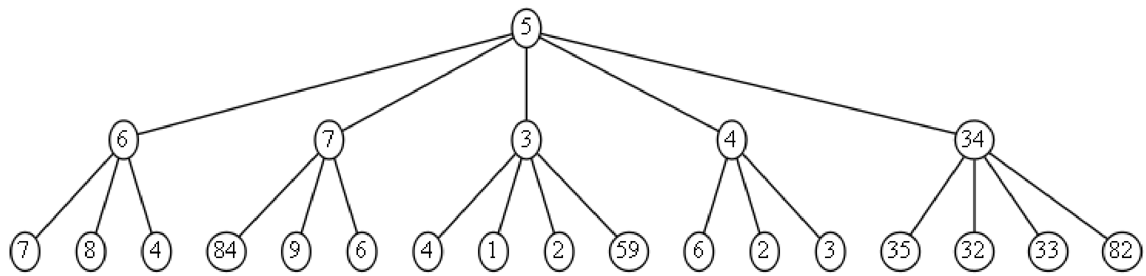


seed2 0.03 -- L1, L2, L3

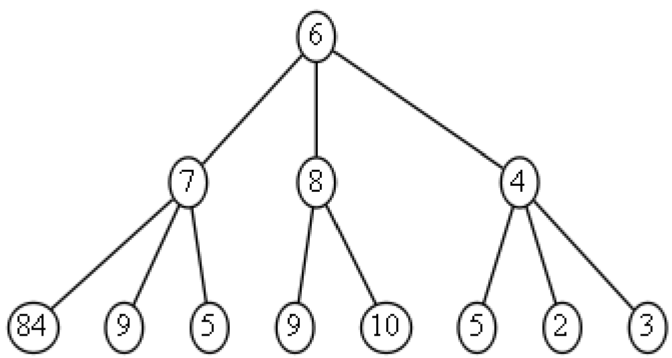


Seed 2, rewiring probaility 0.05

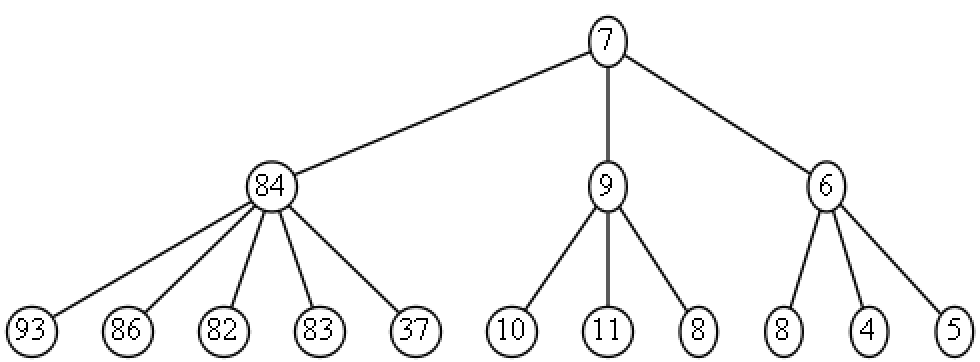
seed2 0.05 -- Initiator, L1, L2



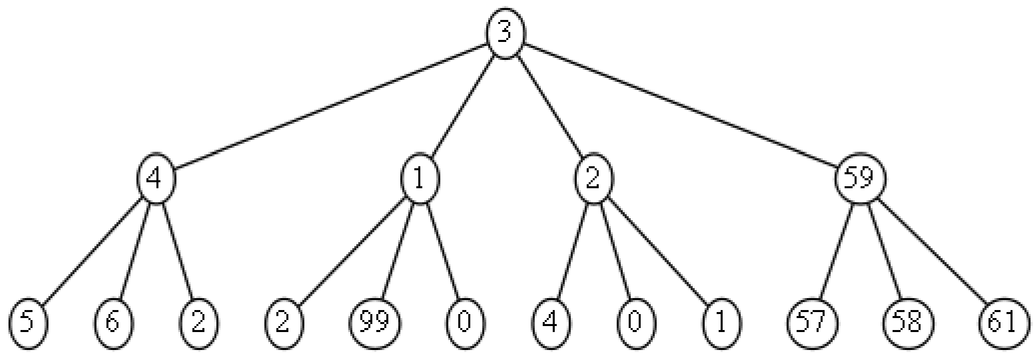
seed2 0.05 -- L1, L2, L3



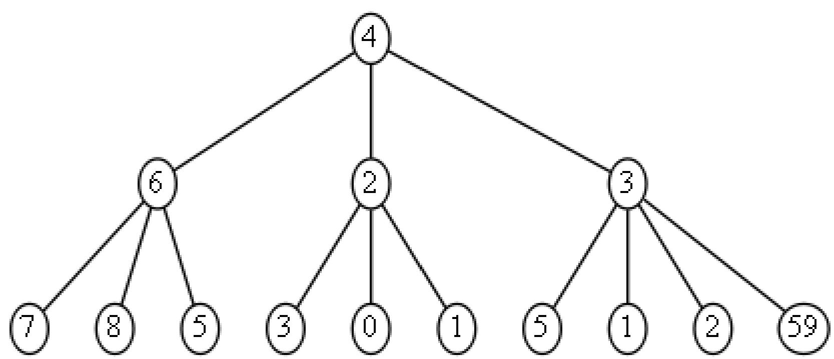
seed2 0.05 -- L1, L2, L3



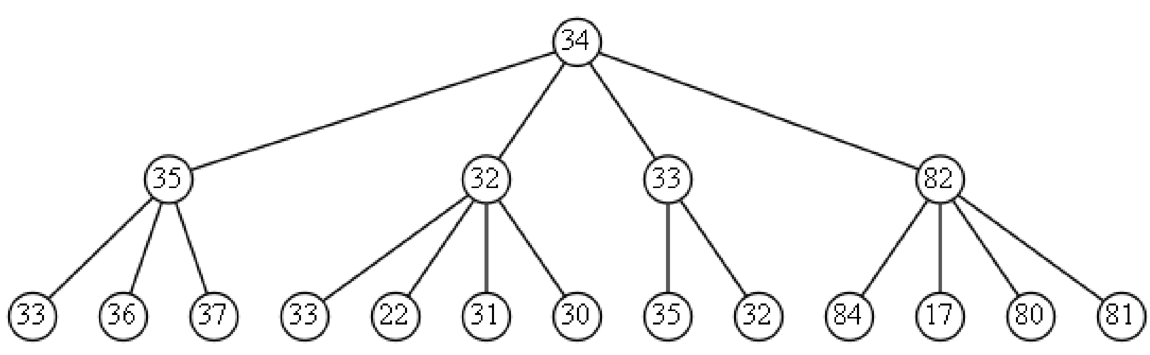
seed2 0.05 -- L1, L2, L3



seed2 0.05 -- L1, L2, L3

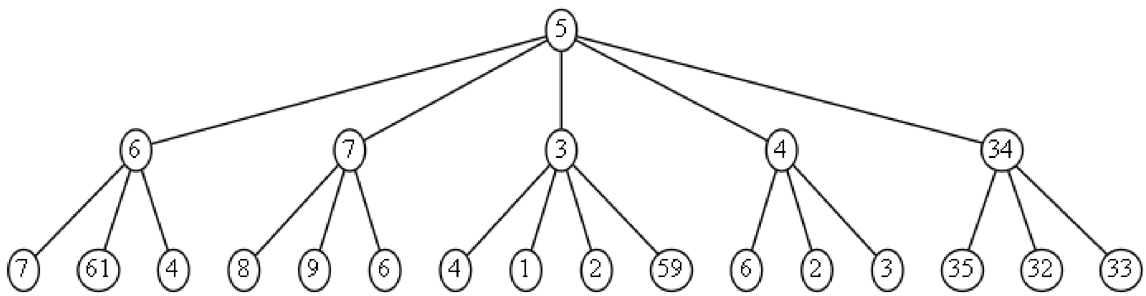


seed2 0.05 -- L1, L2, L3

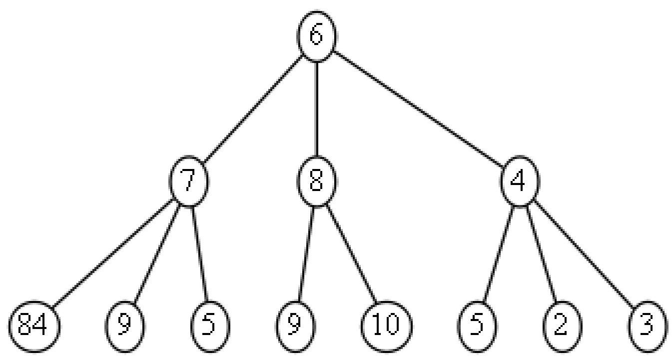


Seed 2, rewiring probaility 0.07

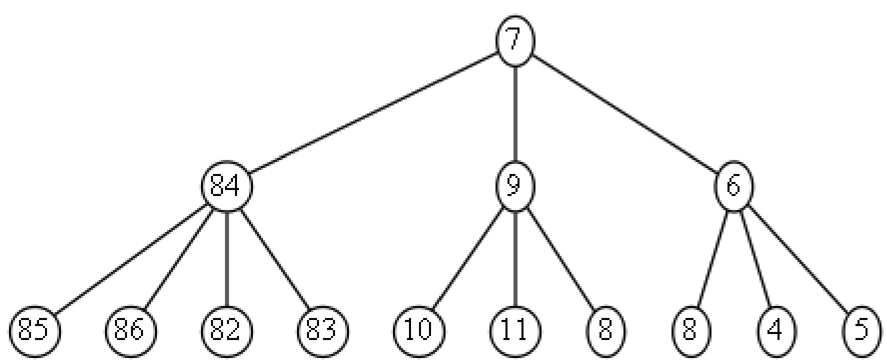
seed2 0.07 -- Initiator, L1, L2



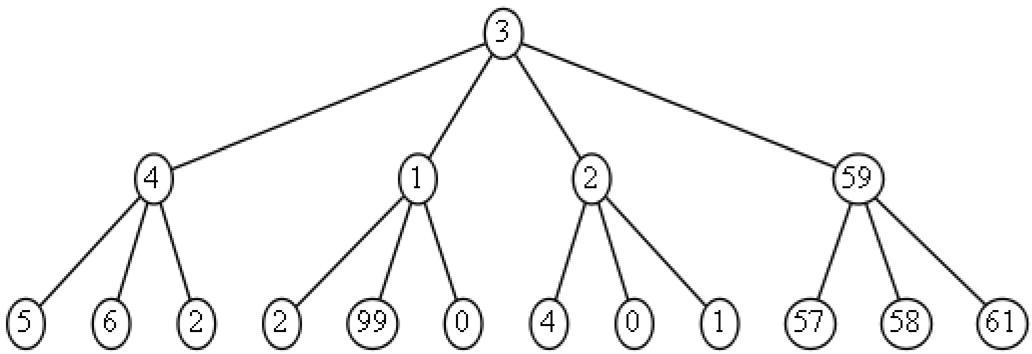
seed2 0.07 -- L1, L2, L3



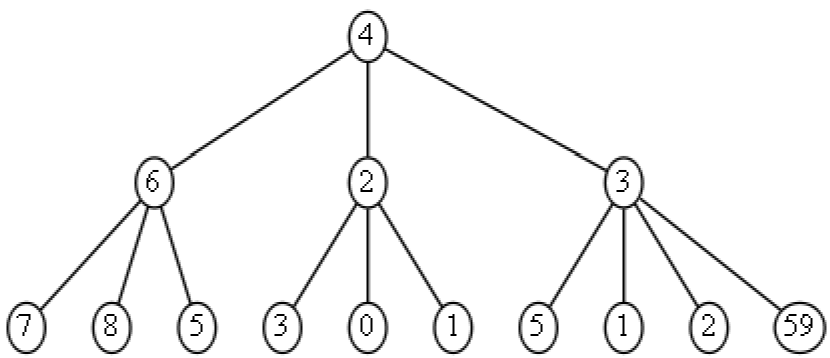
seed2 0.07 -- L1, L2, L3



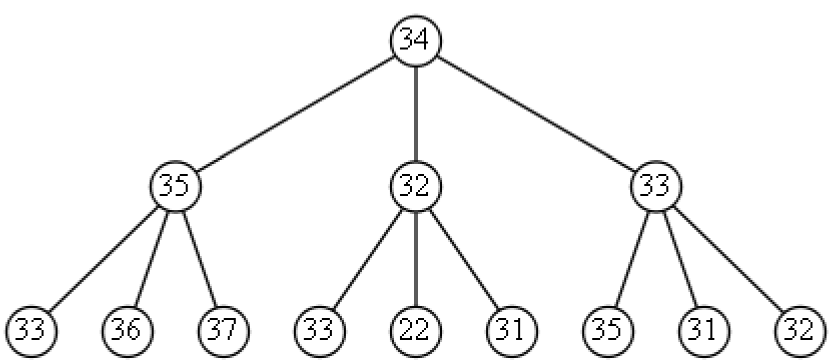
seed2 0.07 -- L1, L2, L3



seed2 0.07 -- L1, L2, L3

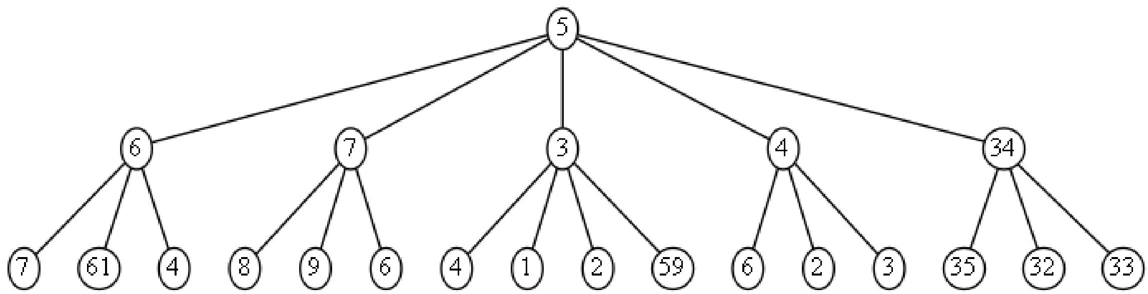


seed2 0.07 -- L1, L2, L3

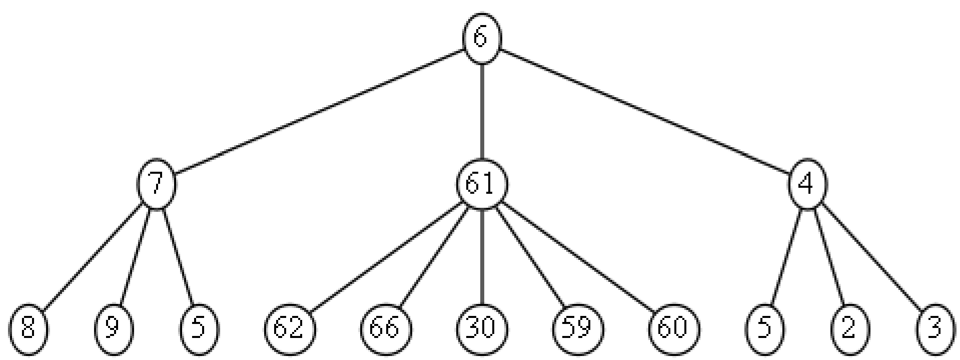


Seed 2, rewiring probaility 0.1

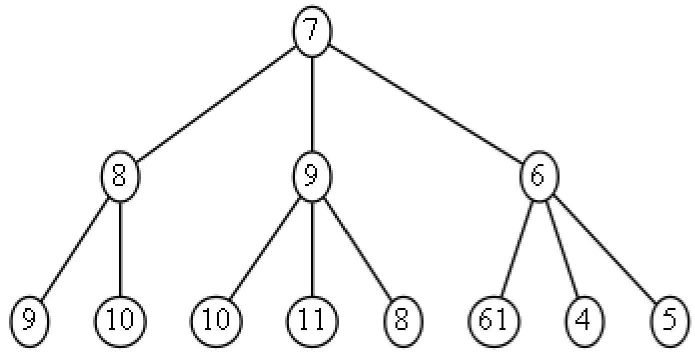
seed2 0.1 -- Initiator, L1, L2



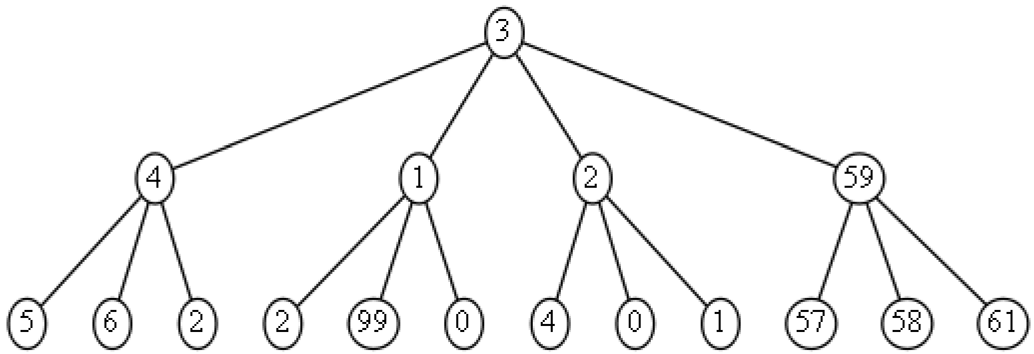
seed2 0.1 -- L1, L2, L3



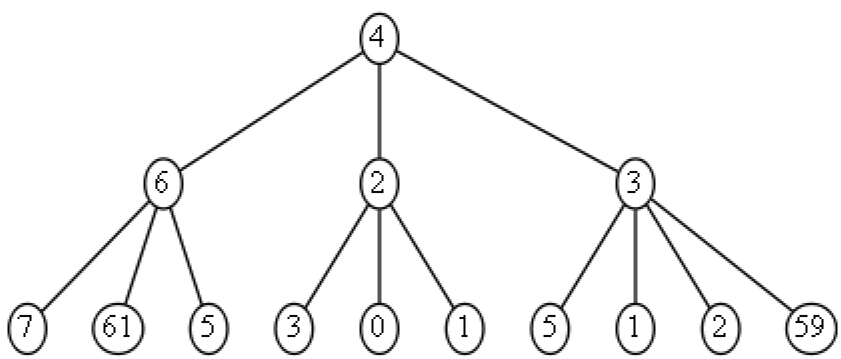
seed2 0.1 -- L1, L2, L3



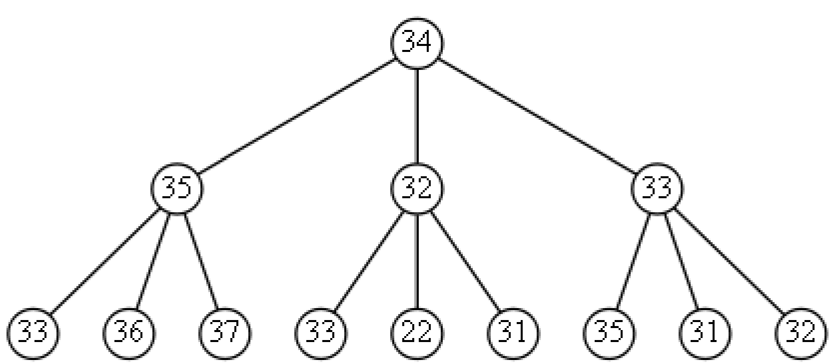
seed2 0.1 -- L1, L2, L3



seed2 0.1 -- L1, L2, L3

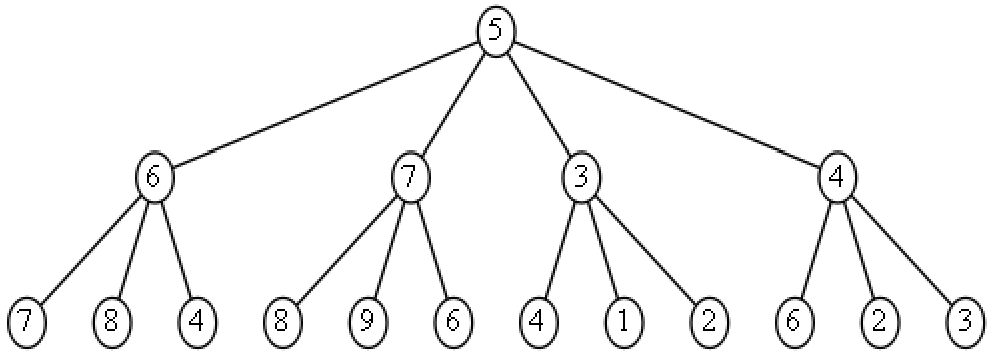


seed2 0.1 -- L1, L2, L3

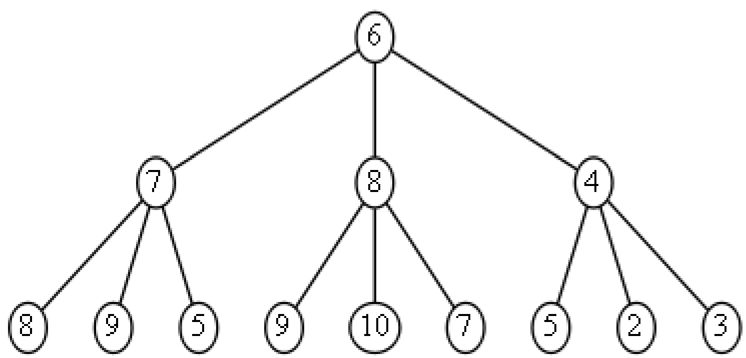


Seed 3, rewiring probaility 0.01

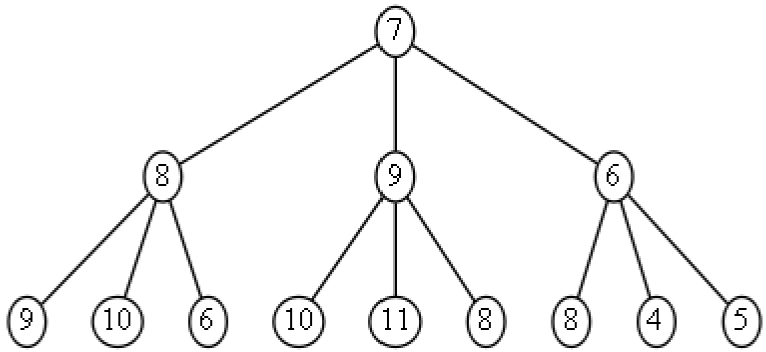
seed3 0.01 -- Initiator, L1, L2



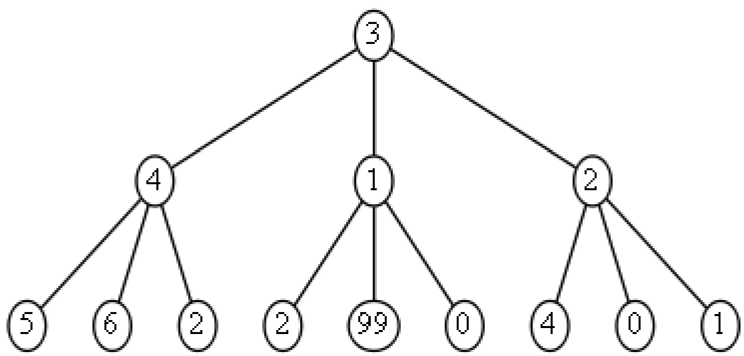
seed3 0.01 -- L1, L2, L3



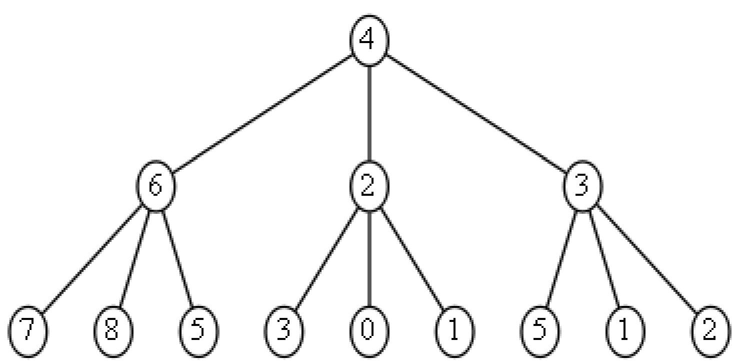
seed3 0.01 -- L1, L2, L3



seed3 0.01 -- L1, L2, L3

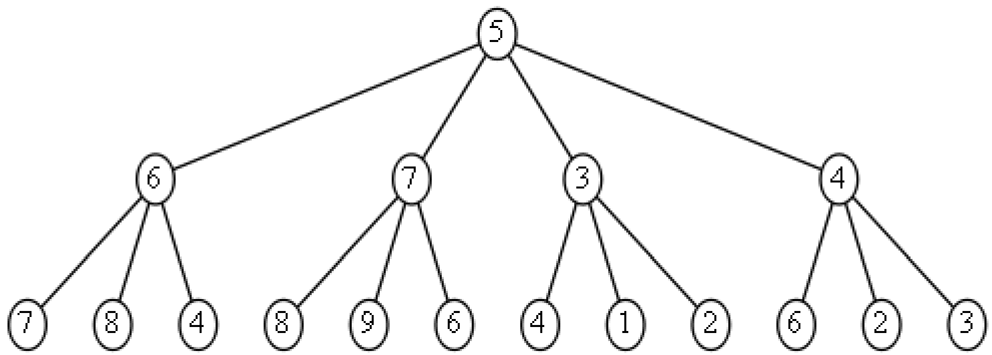


seed3 0.01 -- L1, L2, L3

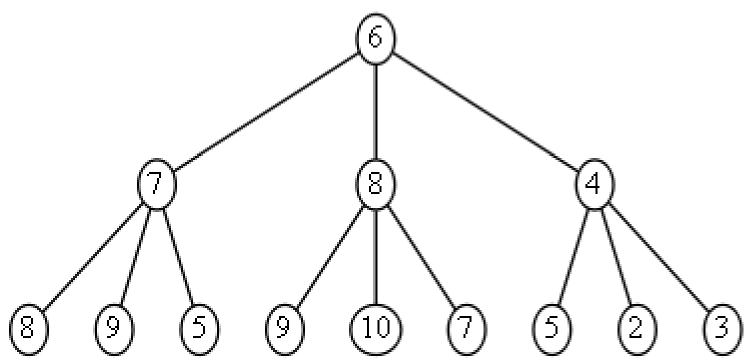


Seed 3, rewiring probaility 0.03

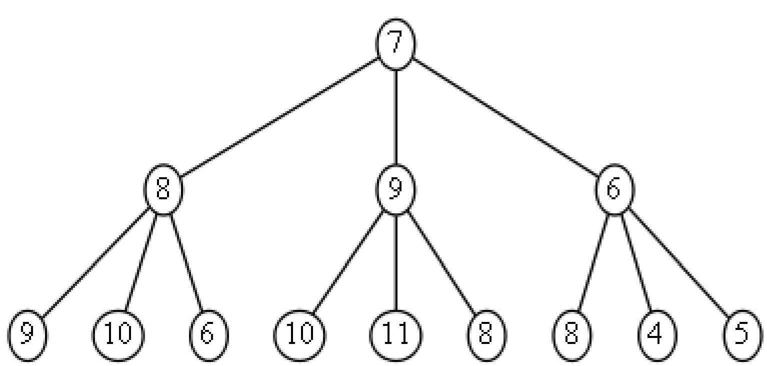
seed3 0.03 -- Initiator, L1, L2



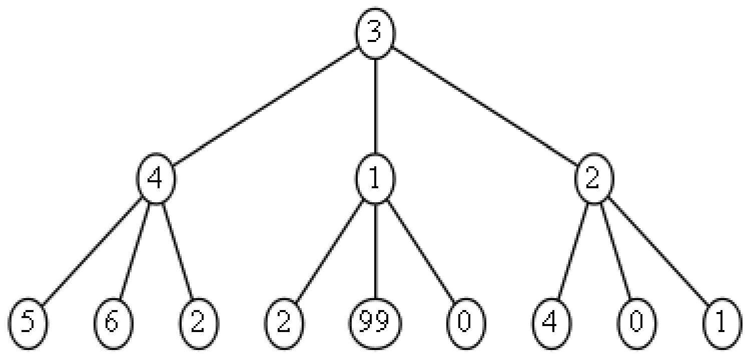
seed3 0.03 -- L1, L2, L3



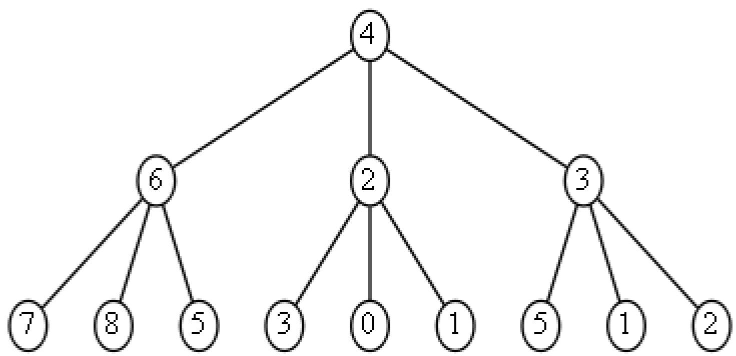
seed3 0.03 -- L1, L2, L3



seed3 0.03 -- L1, L2, L3

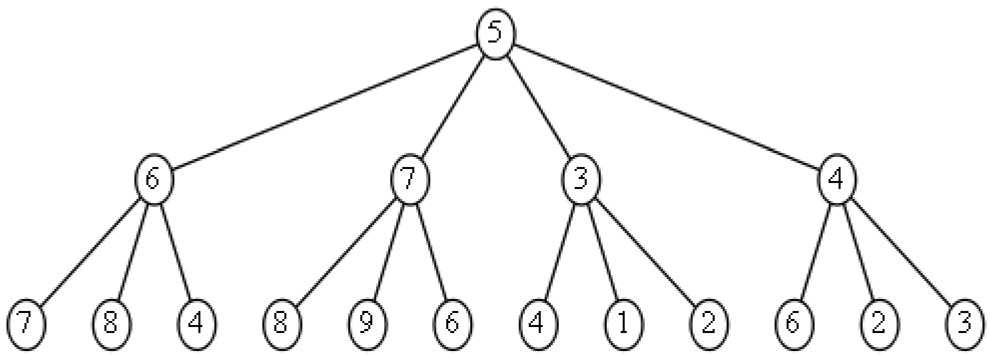


seed3 0.03 -- L1, L2, L3

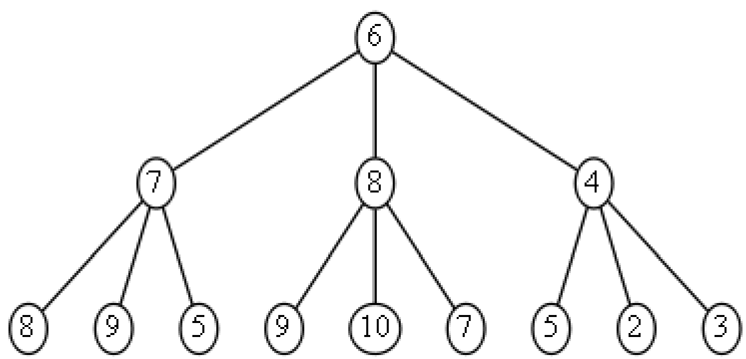


Seed 3, rewiring probaility 0.05

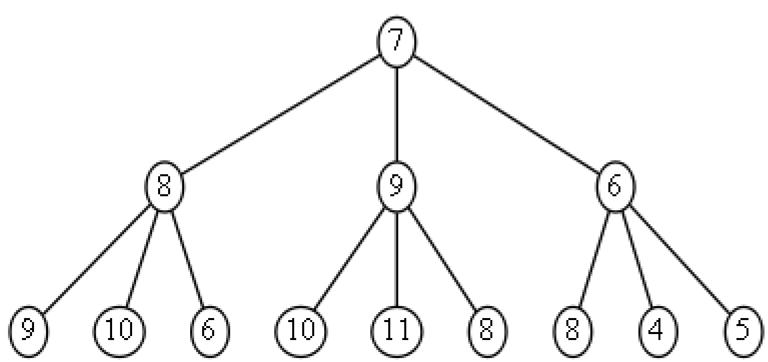
seed3 0.05 -- Initiator, L1, L2



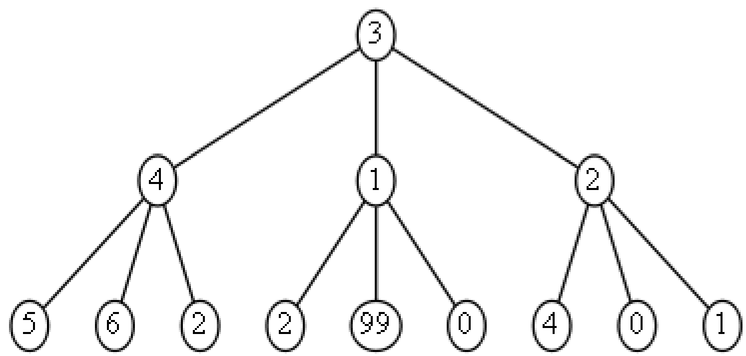
seed3 0.05 -- L1, L2, L3



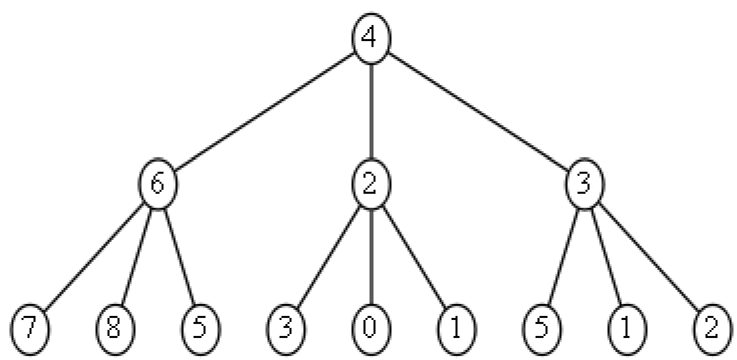
seed3 0.05 -- L1, L2, L3



seed3 0.05 -- L1, L2, L3

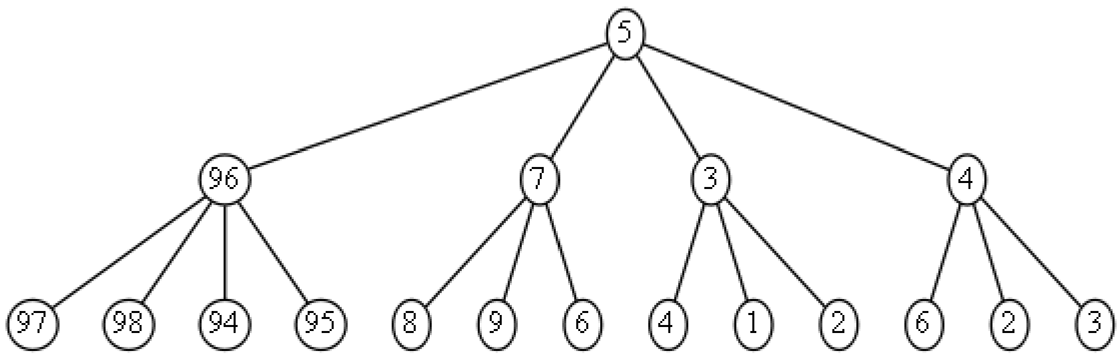


seed3 0.05 -- L1, L2, L3

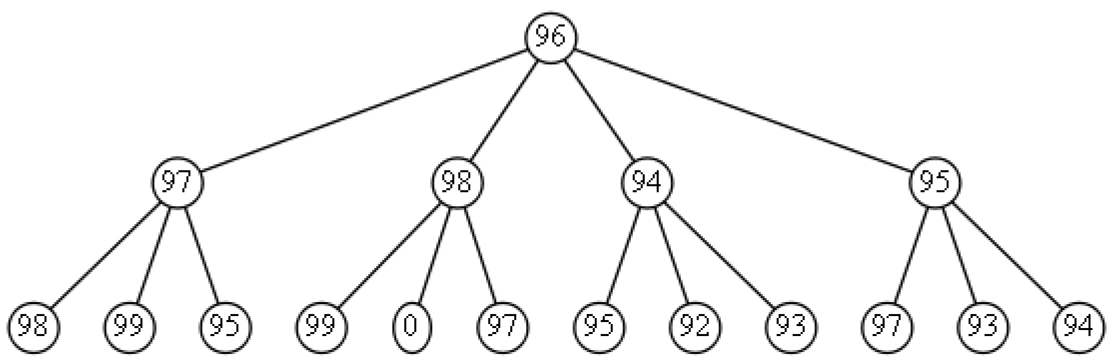


Seed 3, rewiring probaility 0.07

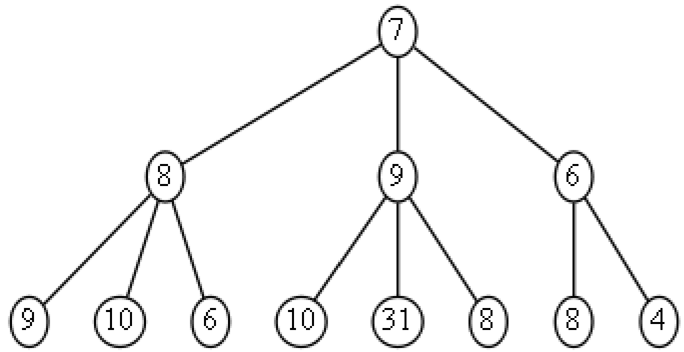
seed3 0.07 -- Initiator, L1, L2



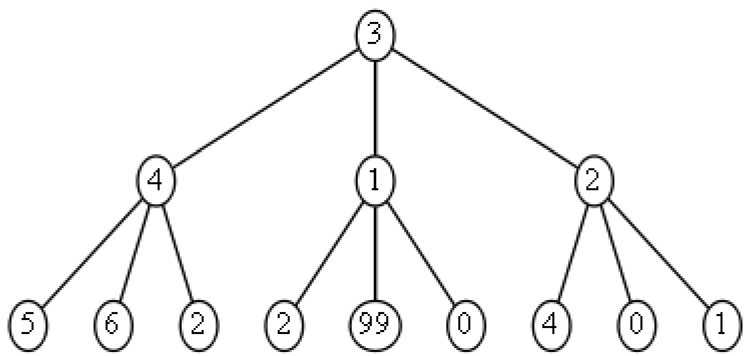
seed3 0.07 -- L1, L2, L3



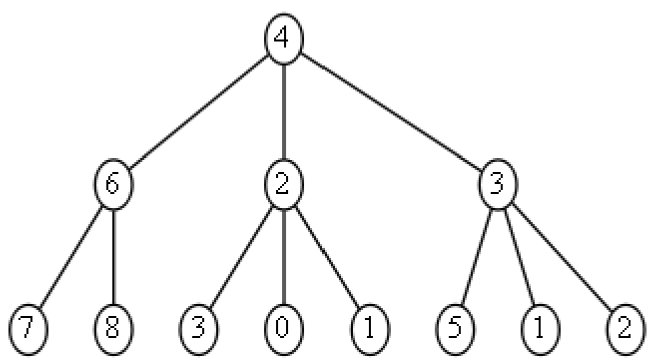
seed3 0.07 -- L1, L2, L3



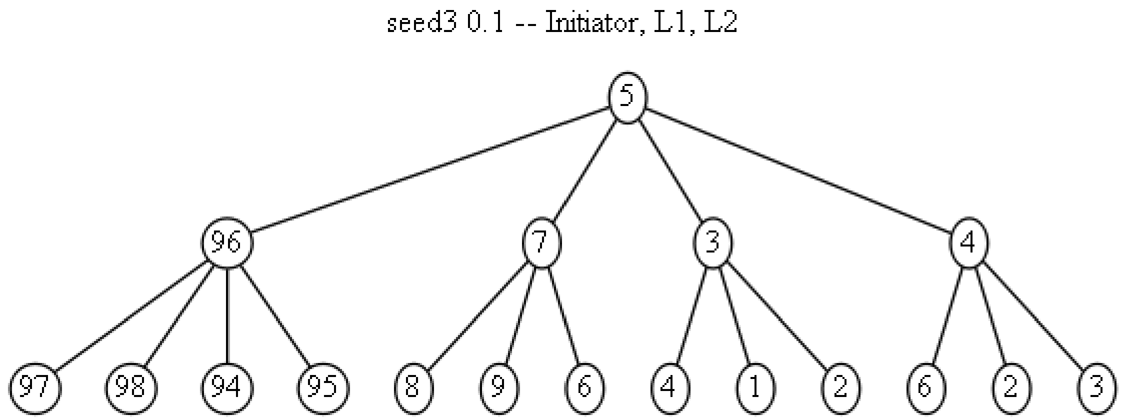
seed3 0.07 -- L1, L2, L3

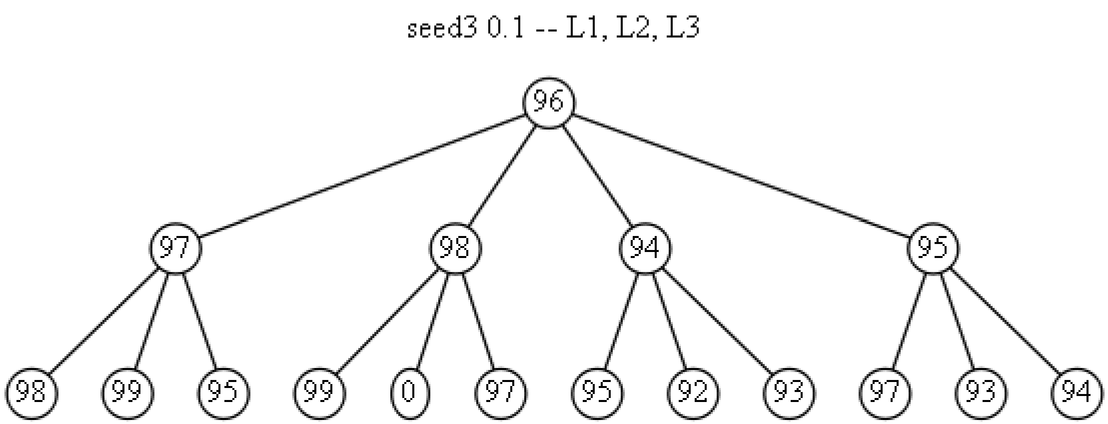


seed3 0.07 -- L1, L2, L3

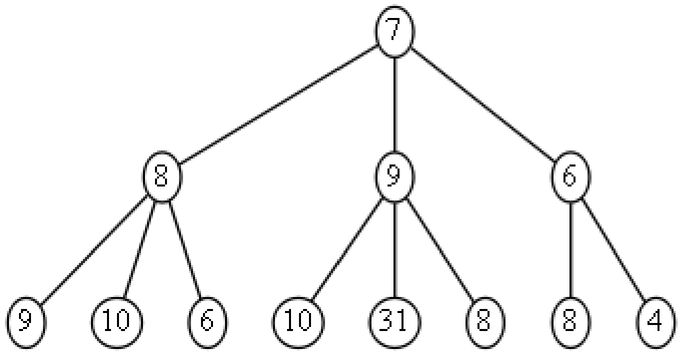


Seed 3, rewiring probaility 0.1

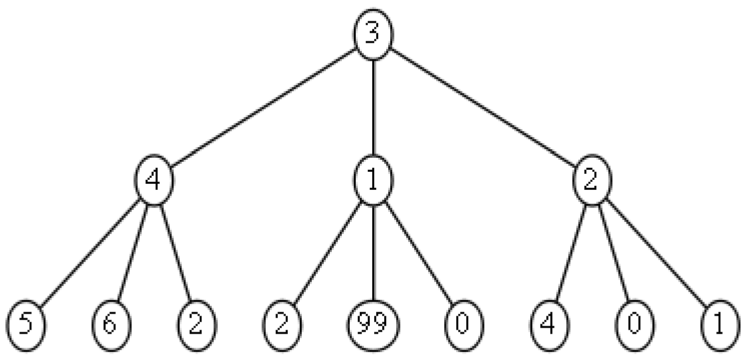




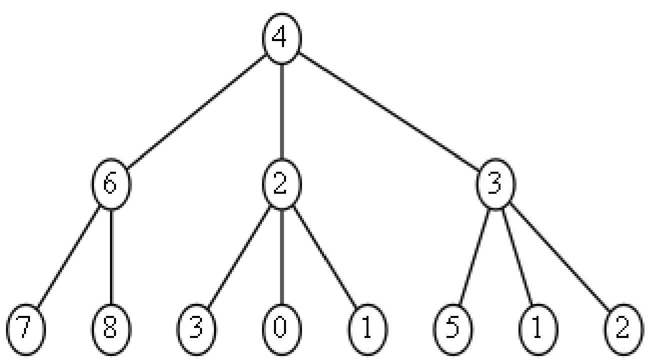
seed3 0.1 -- L1, L2, L3



seed3 0.1 -- L1, L2, L3

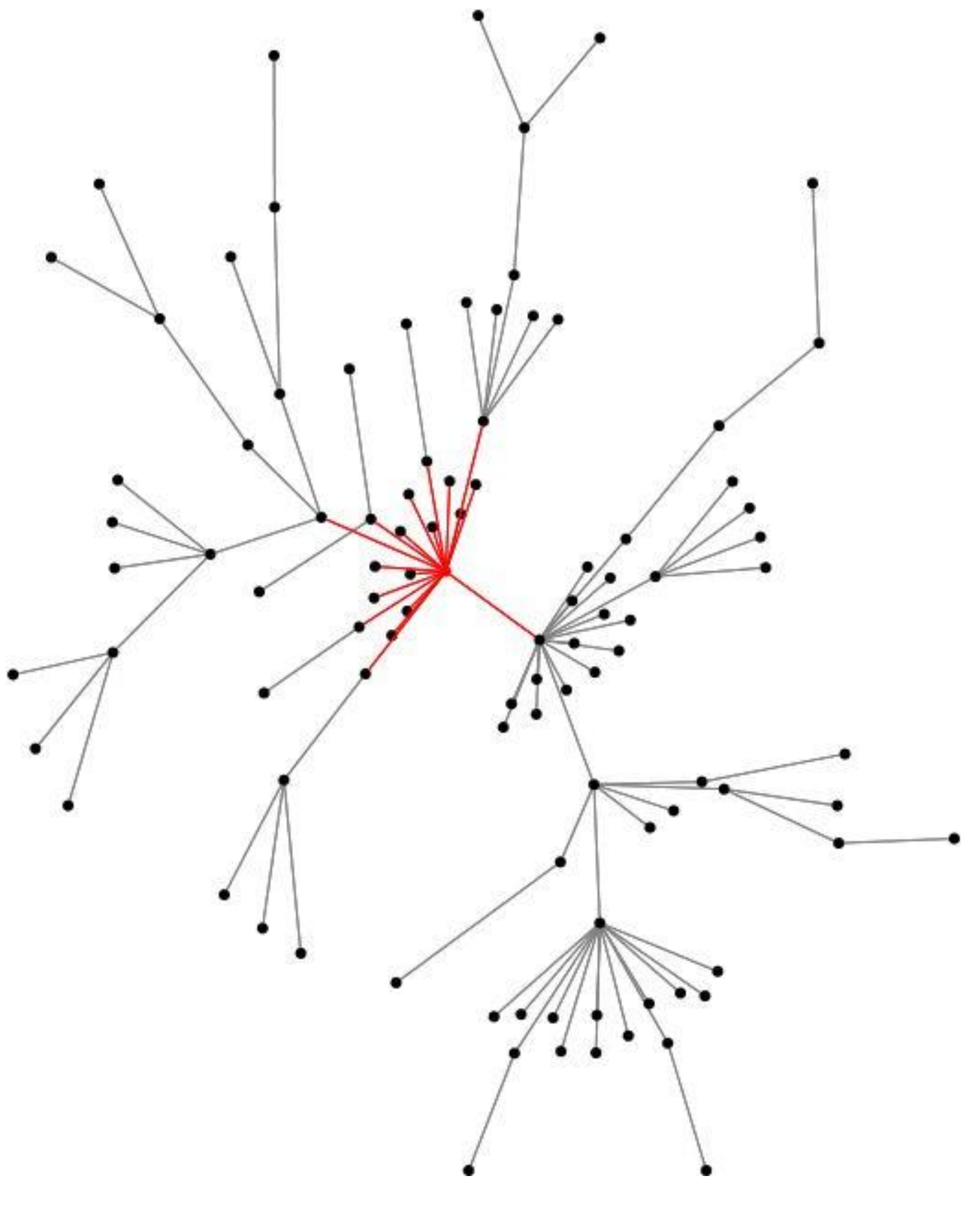


seed3 0.1 -- L1, L2, L3

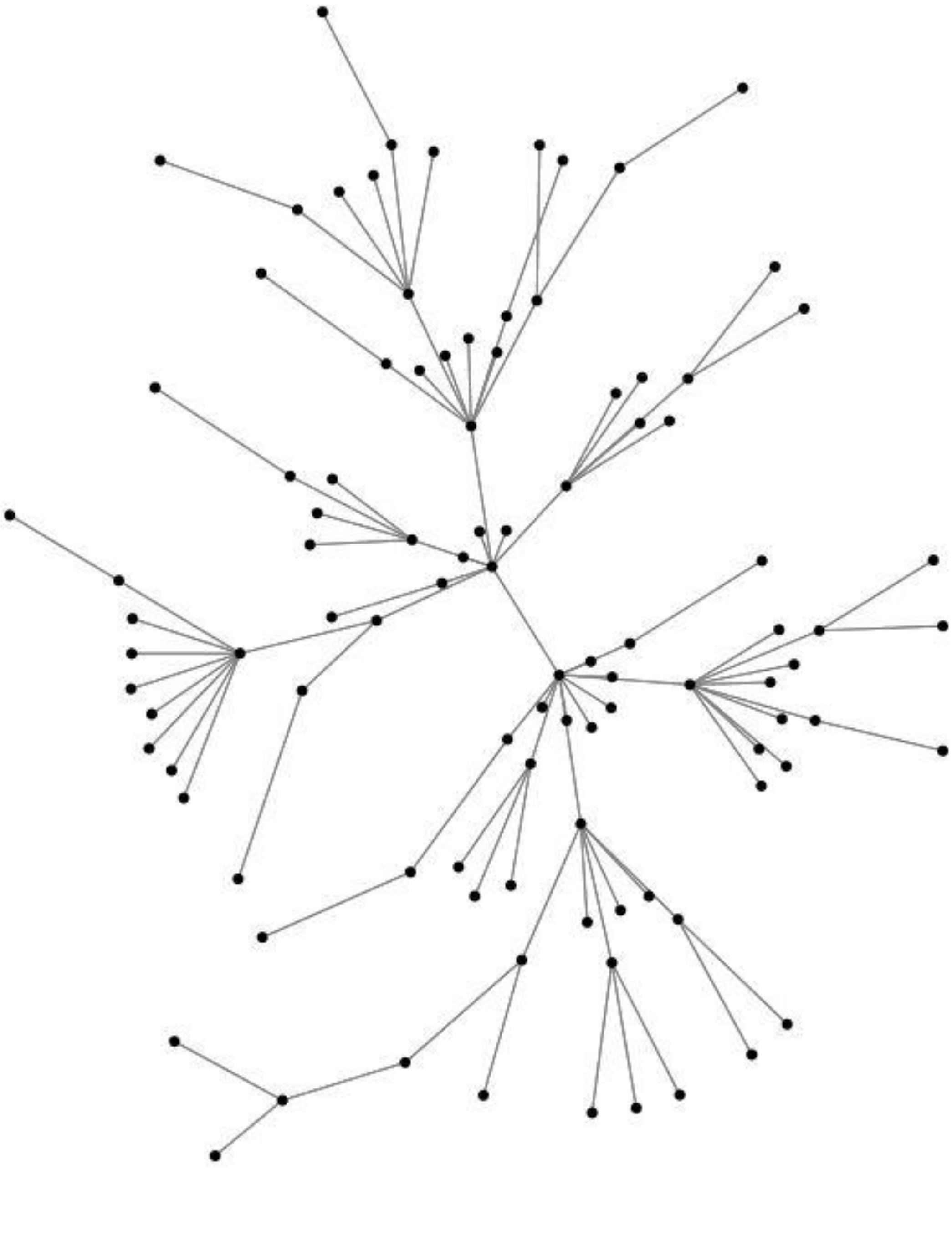


APPENDIX C: NETWORK TOPOLOGIES OF THREE SCALE-FREE NETWORKS

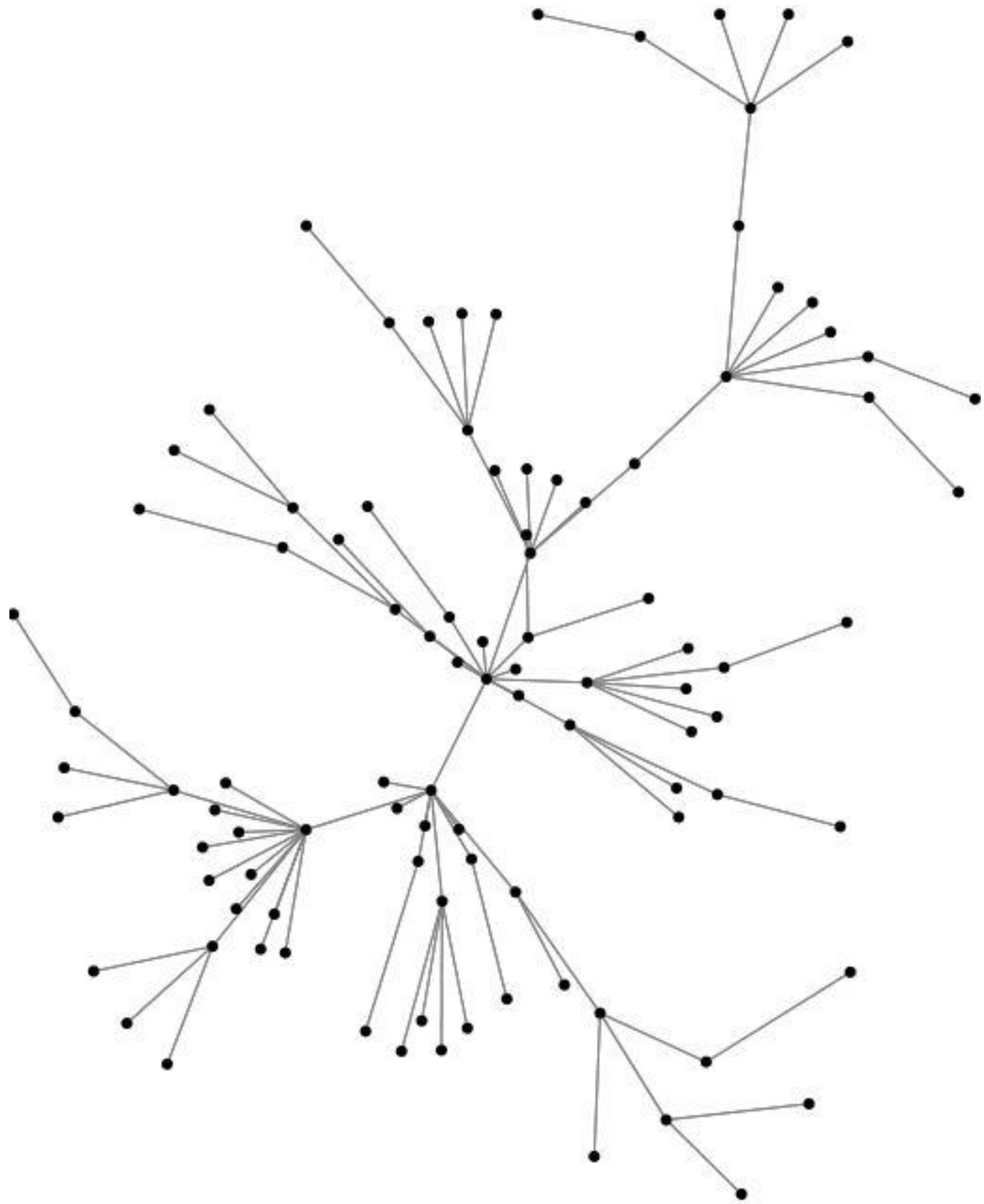
Seed 1



Seed 2

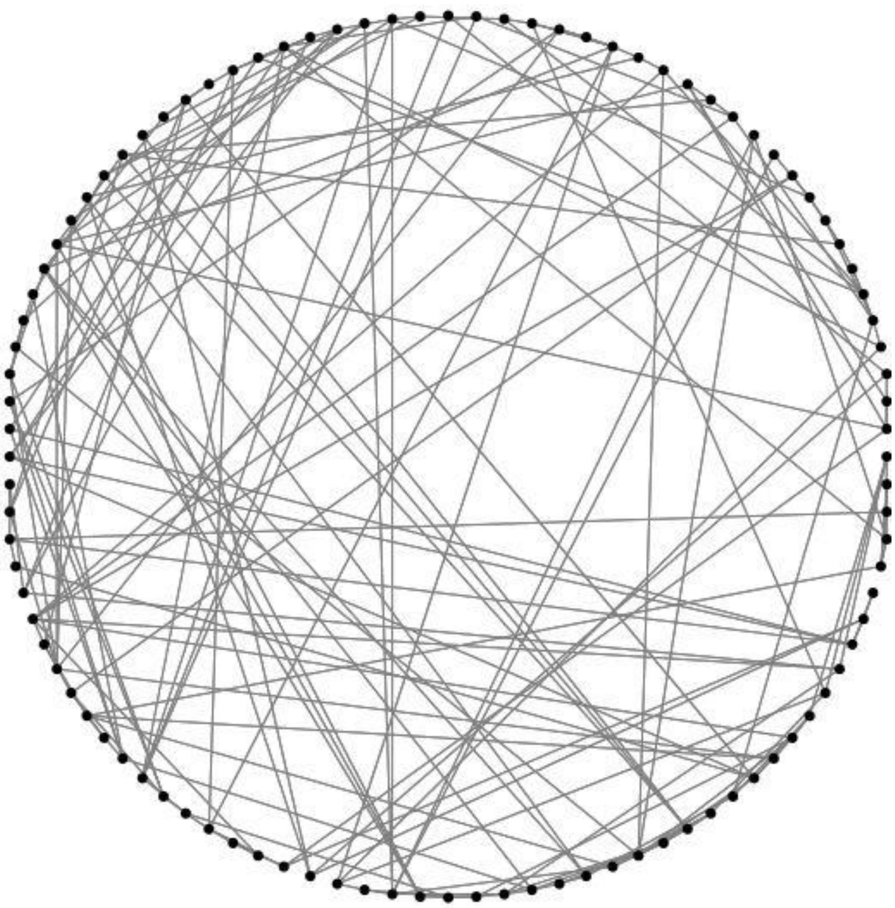


Seed 3

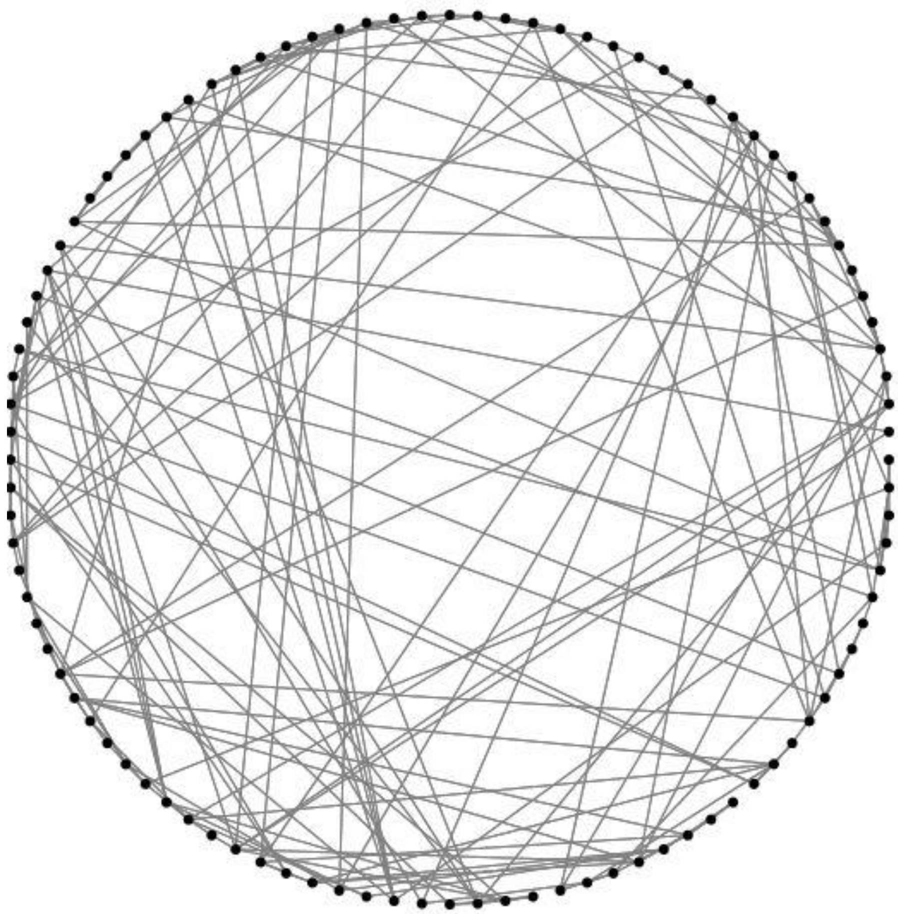


APPENDIX D: NETWORK TOPOLOGIES OF THREE RANDOM NETWORKS

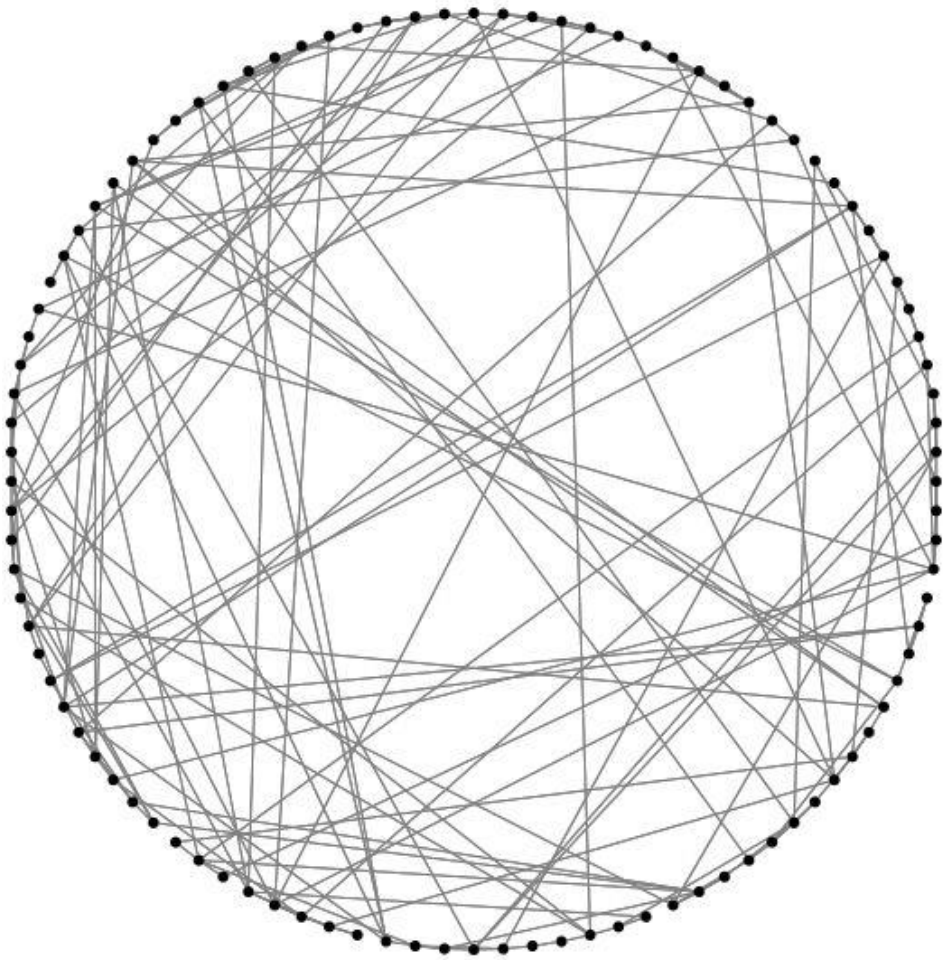
Seed 1



Seed 2

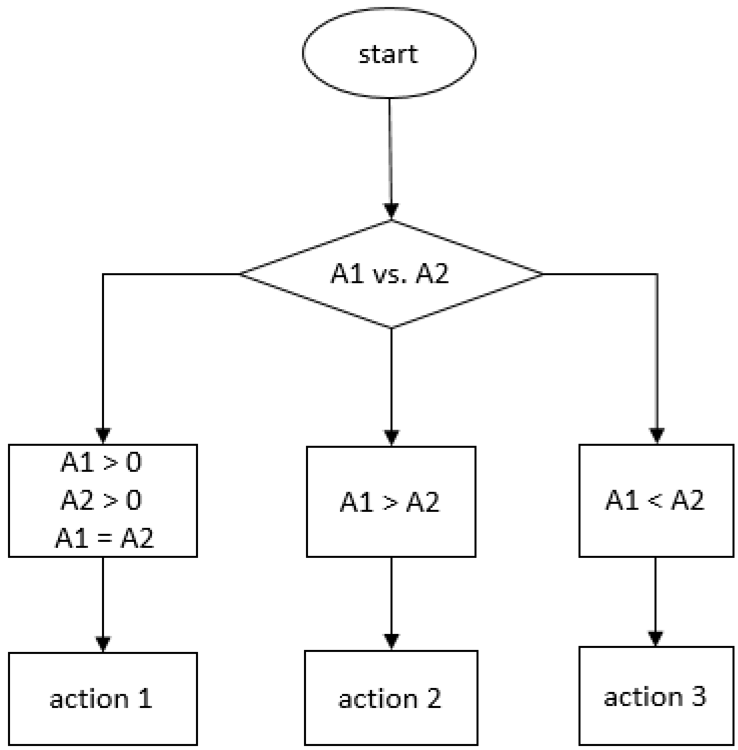


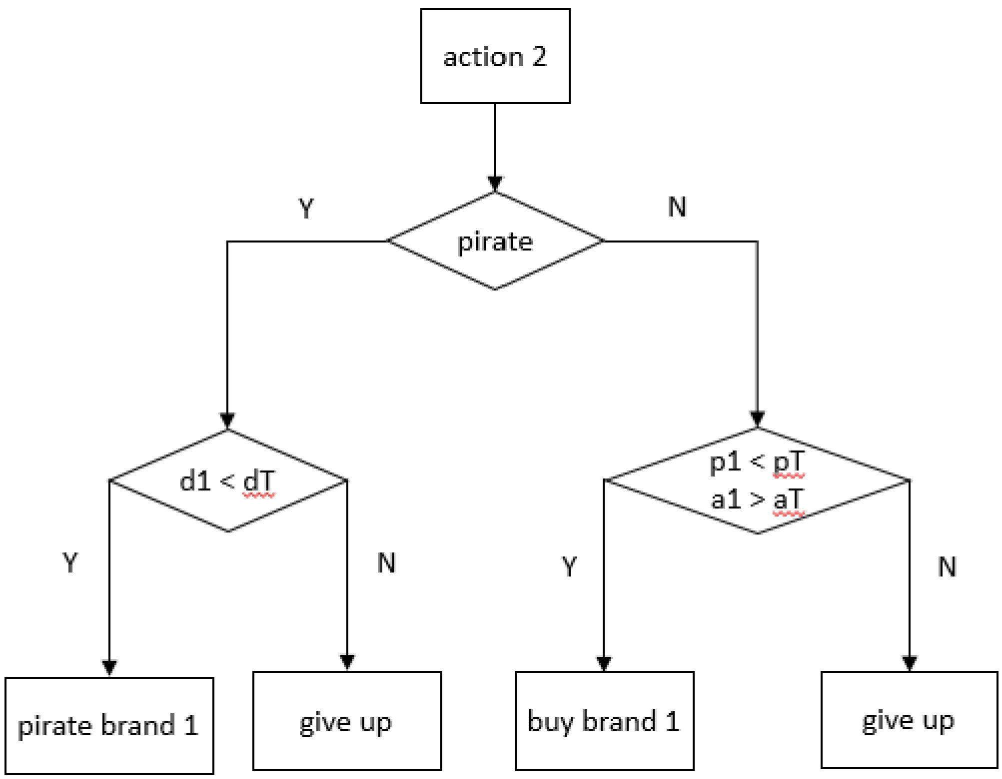
Seed 3

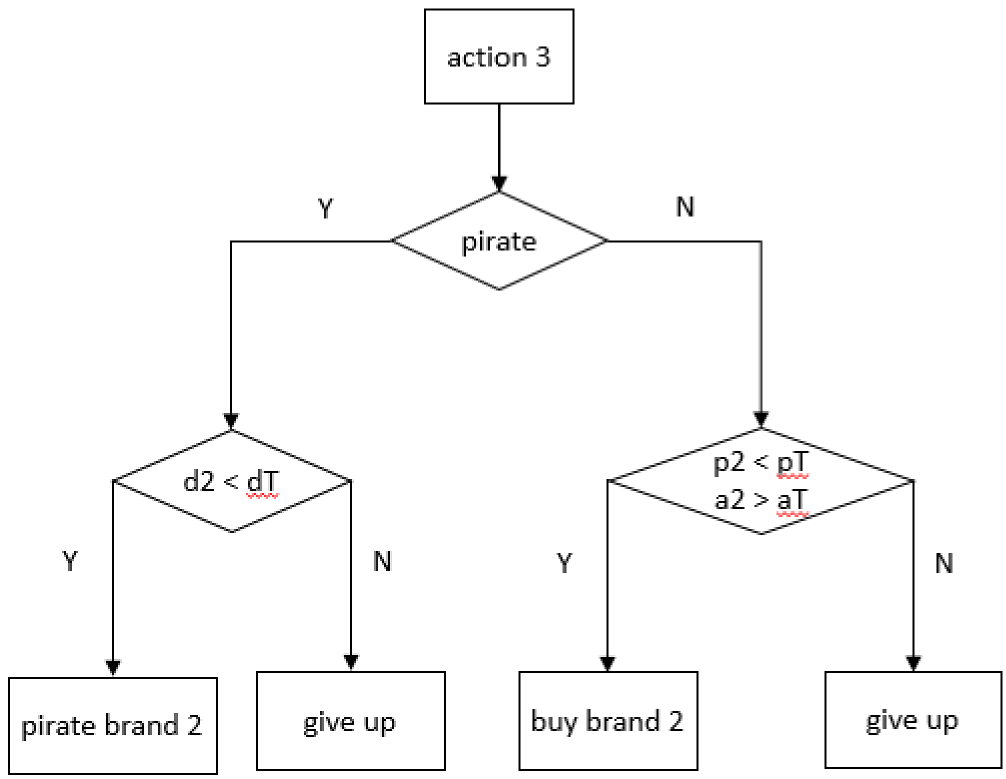


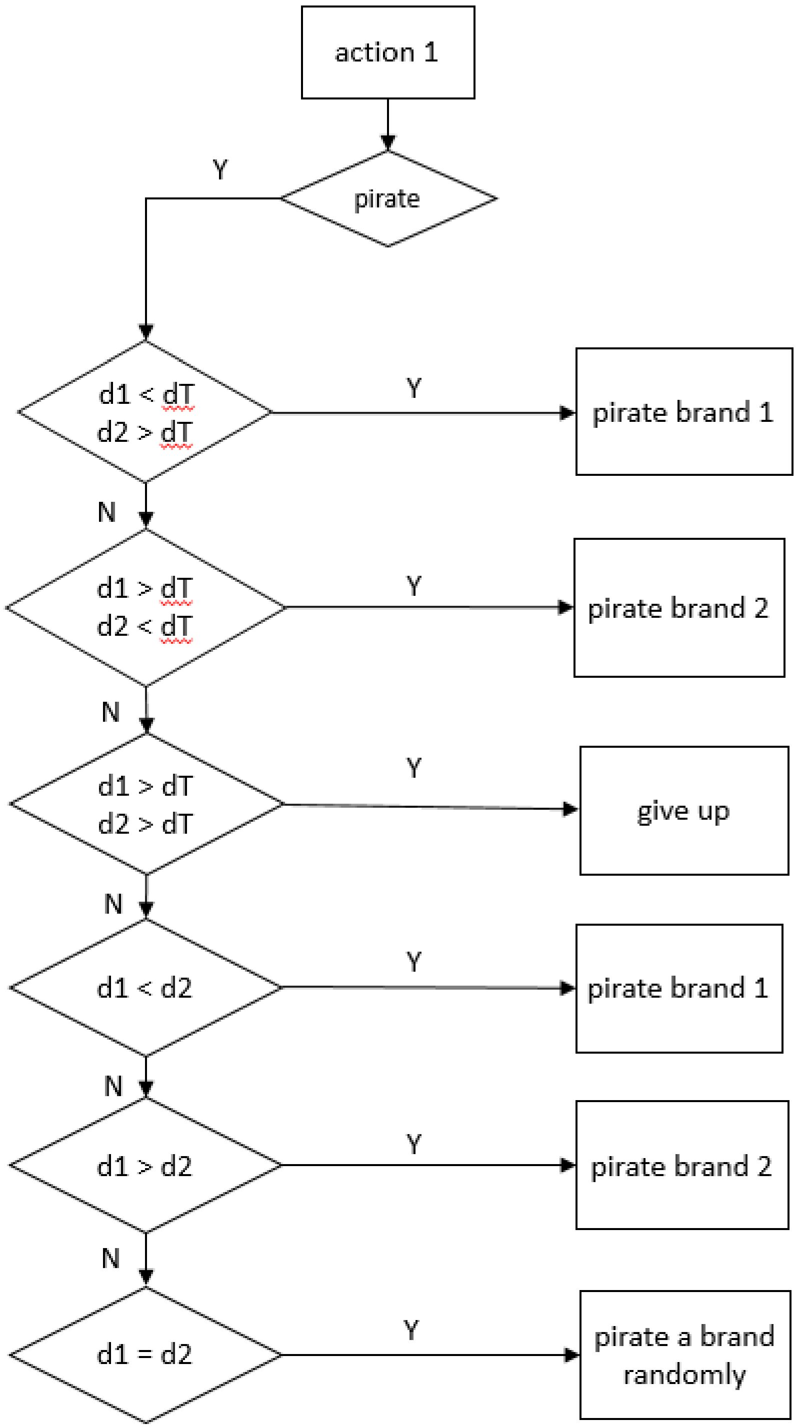
APPENDIX E: FLOW OF CONSUMER ADOPTION DECISIONS IN DUOPOLY MODELS

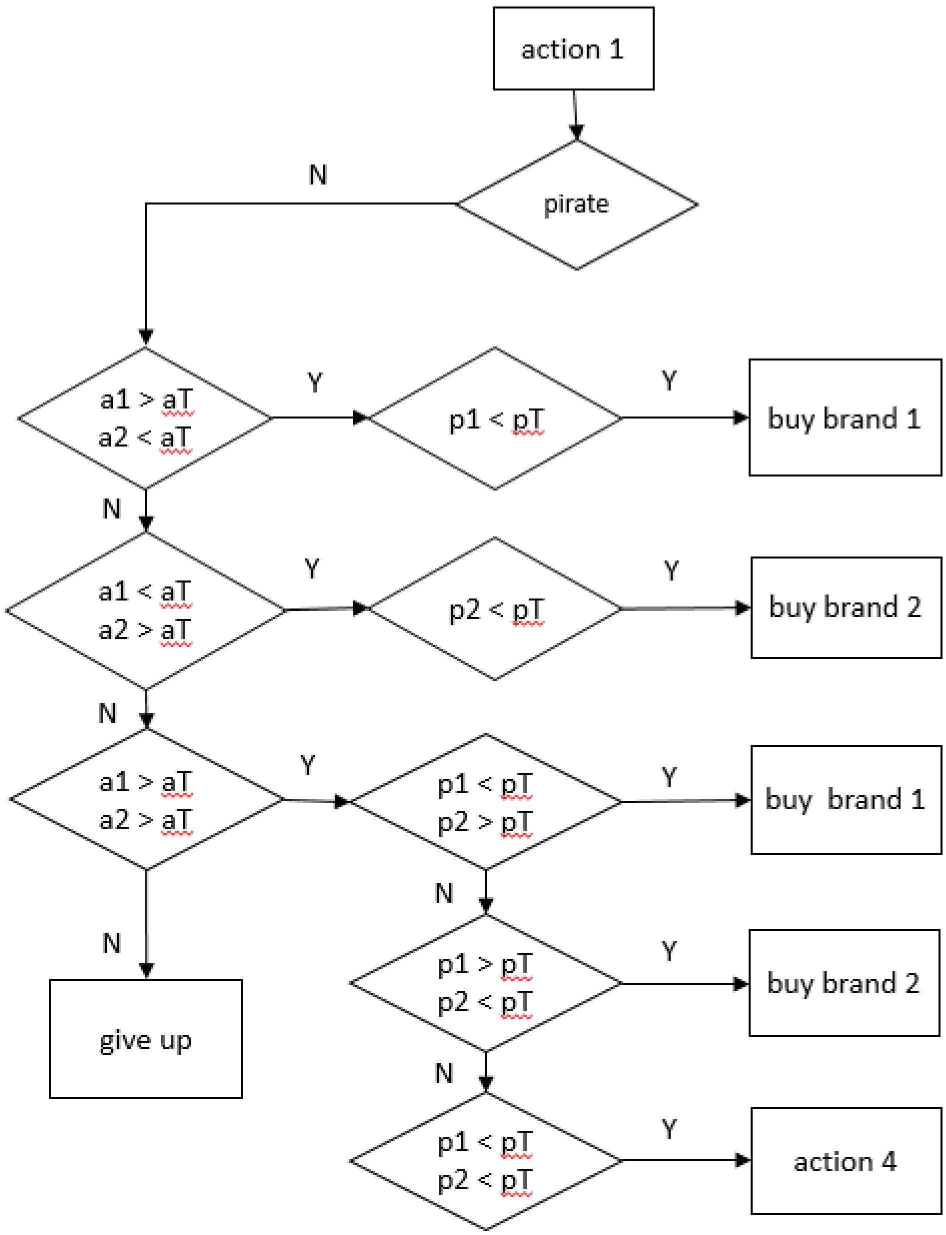
A1: amount of adoption of the brand 1
A2: amount of adoption of the brand 2
pT: reservation price of a consumer
aT: promotion cost threshold of a consumer
dT: piracy detection cost threshold of a consumer
pT: reservation price of a consumer
p1: price of the brand 1,
p2: price of the brand 2
a1: promotion cost of the brand 1,
a2: promotion cost of the brand 2
d1: piracy detection cost of the brand 1,
d2: piracy detection cost the brand 2

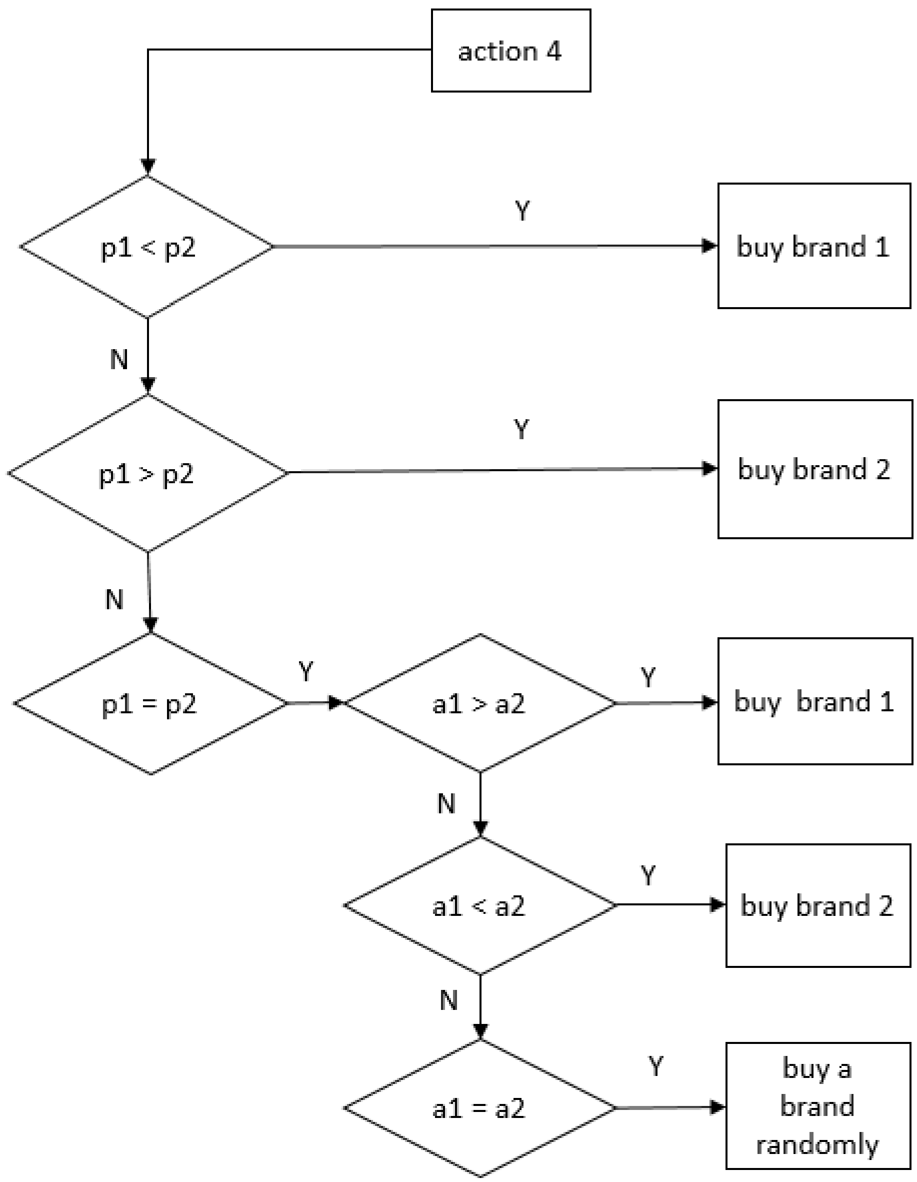

start
A1 vs. A2
A1 > 0
A2 > 0
A1 = A2
A1 > A2
A1 < A2
action 1
action 2
action 3


action 2
Y
pirate
N
d1 < dT
p1 < pT
a1 > aT
Y
N
Y
N
pirate brand 1
give up
buy brand 1
give up


action 3
pirate
Y
N
d2 < dT
p2 < pT
a2 > aT
Y
N
Y
N
pirate brand 2
give up
buy brand 2
give up


action 1
pirate
Y
d1 < dT
d2 > dT
Y
pirate brand 1
N
d1 > dT
d2 < dT
Y
pirate brand 2
N
d1 > dT
d2 > dT
Y
give up
N
d1 < d2
Y
pirate brand 1
N
d1 > d2
Y
pirate brand 2
N
d1 = d2
Y
pirate a brand randomly


action 1
pirate
N
a1 > aT
a2 < aT
Y
p1 < pT
Y
buy brand 1
N
a1 < aT
a2 > aT
Y
p2 < pT
Y
buy brand 2
N
a1 > aT
a2 > aT
Y
p1 < pT
p2 > pT
Y
buy brand 1
N
p1 > pT
p2 < pT
Y
buy brand 2
N
p1 < pT
p2 < pT
Y
action 4
N
give up


action 4
p1 < p2
Y
buy brand 1
N
p1 > p2
Y
buy brand 2
N
p1 = p2
Y
a1 > a2
Y
buy brand 1
N
a1 < a2
Y
buy brand 2
N
a1 = a2
Y
buy a brand randomly

APPENDIX F: VALUES OF D, P, AND MEAN FOR MODEL 2-1

Homogeneous consumers

Seed 1, rewiring probability: 0.01, homogeneous consumers
np: NP_profit_mean: 889.2196, NP_time_mean:26.5
p1_7: profit_mean: 900.5005, time_mean:25.48, profit_p_value: 0.0000, profit_D: 0.46, time_p_value: 0.0000, time_D: 0.76

Seed 1, rewiring probability: 0.03, homogeneous consumers
np: NP_profit_mean: 1032.562, NP_time_mean:20.66
p1_6: profit_mean: 1039.532, time_mean:19.9, profit_p_value: -0.0000, profit_D: 0.84, time_p_value: -0.0000, time_D: 0.86
p1_7: profit_mean: 1040.0747, time_mean:19.94, profit_p_value: -0.0000, profit_D: 0.86, time_p_value: -0.0000, time_D: 0.86

Seed 1, rewiring probability: 0.05, homogeneous consumersgeneous consumers
np: NP_profit_mean: 1032.3519, NP_time_mean:20.82

Seed 1, rewiring probability: 0.07, homogeneous consumers
np: NP_profit_mean: 1076.2585, NP_time_mean:18.96
p1_4: profit_mean: 1082.672, time_mean:17.82, profit_p_value: -0.0000, profit_D: 0.98, time_p_value: -0.0000, time_D: 0.98
p1_64: profit_mean: 1082.3668, time_mean:18.0, profit_p_value: -0.0000, profit_D: 0.98, time_p_value: -0.0000, time_D: 0.98

Seed 1, rewiring probability: 0.1, homogeneous consumers
np: NP_profit_mean: 1097.2795, NP_time_mean:18.0
p1_4: profit_mean: 1103.084, time_mean:16.92, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: -0.0000, time_D: 1.0
p1_64: profit_mean: 1103.0299, time_mean:17.0, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: -0.0000, time_D: 1.0
Seed 2, rewiring probability: 0.01, homogeneous consumers
np: NP_profit_mean: 939.6293, NP_time_mean:24.52
p1_7: profit_mean: 944.00464, time_mean:23.22, profit_p_value: 0.0217, profit_D: 0.3, time_p_value: -0.0000, time_D: 0.8

Seed 2, rewiring probability: 0.03, homogeneous consumers
np: NP_profit_mean: 939.68744, NP_time_mean:24.68
p1_7: profit_mean: 949.1929, time_mean:23.4, profit_p_value: 0.0000, profit_D: 0.58, time_p_value: -0.0000, time_D: 0.88

Seed 2, rewiring probability: 0.05, homogeneous consumers
np: NP_profit_mean: 1118.0927, NP_time_mean:16.88
p1_34: profit_mean: 1123.4565, time_mean:15.96, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: -0.0000, time_D: 0.96

Seed 2, rewiring probability: 0.07, homogeneous consumers
np: NP_profit_mean: 1118.6356, NP_time_mean:16.9
p2_35: time_mean:0.0, time_p_value: -0.0000, time_D: 1.0
p2_9: time_mean:0.0, time_p_value: -0.0000, time_D: 1.0

Seed 2, rewiring probability: 0.1, homogeneous consumers
np: NP_profit_mean: 1118.1786, NP_time_mean:16.9

Seed 3, rewiring probability: 0.01, homogeneous consumers
np: NP_profit_mean: 915.4877, NP_time_mean:24.96
p1_6: profit_mean: 924.7879, time_mean:24.06, profit_p_value: 0.0000, profit_D: 0.5, time_p_value: 0.0000, time_D: 0.58
p1_7: profit_mean: 927.0616, time_mean:24.28, profit_p_value: 0.0000, profit_D: 0.58, time_p_value: 0.0000, time_D: 0.58

Seed 3, rewiring probability: 0.03, homogeneous consumers
np: NP_profit_mean: 936.8276, NP_time_mean:23.82
p1_7: profit_mean: 949.7202, time_mean:23.54, profit_p_value: 0.0000, profit_D: 0.64, time_p_value: 0.0000, time_D: 0.56

Seed 3, rewiring probability: 0.05, homogeneous consumers
np: NP_profit_mean: 941.4377, NP_time_mean:24.26
p1_7: profit_mean: 947.8097, time_mean:23.08, profit_p_value: 0.0058, profit_D: 0.34, time_p_value: 0.0000, time_D: 0.68
p2_1: time_mean:23.6, time_p_value: 0.0006, time_D: 0.28

Seed 3, rewiring probability: 0.07, homogeneous consumers
np: NP_profit_mean: 1054.0262, NP_time_mean:19.84
p1_7: profit_mean: 1062.0511, time_mean:18.64, profit_p_value: -0.0000, profit_D: 0.98, time_p_value: -0.0000, time_D: 0.94
p1_96: profit_mean: 1062.1655, time_mean:18.82, profit_p_value: -0.0000, profit_D: 0.98, time_p_value: -0.0000, time_D: 0.94

Seed 3, rewiring probability: 0.1, homogeneous consumers
np: NP_profit_mean: 1097.1401, NP_time_mean:17.92

Heterogeneous consumers

Seed 1, 0.01, heterogeneous consumers
np: NP_profit_mean: 362.82703, NP_time_mean:20.94
p1_7: time:19.78, p_value: 0.0001, D: 0.4

Seed 1, 0.03, heterogeneous consumers
np: NP_profit_mean: 580.95483, NP_time_mean:14.64
p1_6: profit: 605.84033, p_value: 0.0013, D: 0.38
p1_7: profit_mean: 619.90533, time_mean:13.94, profit_p_value: 0.0000, profit_D: 0.64, time_p_value: 0.0002, time_D: 0.38

Seed 1, 0.05, heterogeneous consumers
np: NP_profit_mean: 622.5683, NP_time_mean:14.72
p1_7: profit_mean: 638.7027, time_mean:13.98, profit_p_value: 0.0217, profit_D: 0.3, time_p_value: 0.0028, time_D: 0.3
p3_1: profit: 648.5214, p_value: 0.0002, D: 0.42

Seed 1, 0.07, heterogeneous consumers
np: NP_profit_mean: 702.33325, NP_time_mean:12.4
p1_64: profit_mean: 708.8176, time_mean:11.72, profit_p_value: 0.0217, profit_D: 0.3, time_p_value: 0.0013, time_D: 0.34
p1_7: profit: 719.5514, p_value: 0.0001, D: 0.44

Seed 1, 0.1, heterogeneous consumers
np: NP_profit_mean: 720.53094, NP_time_mean:12.02
p1_4: time:10.9, p_value: 0.0013, D: 0.34
p1_64: profit_mean: 722.9497, time_mean:10.84, profit_p_value: 0.0392, profit_D: 0.28, time_p_value: 0.0006, time_D: 0.36
p1_7: profit_mean: 738.0246, time_mean:11.06, profit_p_value: 0.0013, profit_D: 0.38, time_p_value: 0.0006, time_D: 0.36

Seed 2, 0.01, heterogeneous consumers
np: NP_profit_mean: 378.99585, NP_time_mean:19.34
p2_84: profit: 458.75247, p_value: -0.0000, D: 0.84

Seed 2, 0.03, heterogeneous consumers
np: NP_profit_mean: 410.09943, NP_time_mean:18.3
p2_84: profit: 497.09714, p_value: -0.0000, D: 0.96

Seed 2, 0.05, heterogeneous consumers
np: NP_profit_mean: 732.7481, NP_time_mean:11.16
p1_3: time:10.48, p_value: 0.0002, D: 0.36
p1_34: profit_mean: 780.8237, time_mean:10.14, profit_p_value: 0.0000, profit_D: 0.78, time_p_value: 0.0000, time_D: 0.42
p3_58: profit: 739.39764, p_value: 0.0392, D: 0.28
p3_81: time_mean:10.56, time_p_value: 0.0115, time_D: 0.28
p3_93: profit_mean: 740.89136, time_mean:10.66, profit_p_value: 0.0115, profit_D: 0.32, time_p_value: 0.0058, time_D: 0.3

Seed 2, 0.07, heterogeneous consumers
np: NP_profit_mean: 734.8642, NP_time_mean:11.02
p1_34: profit: 793.5891, p_value: -0.0000, D: 0.9
p2_84: profit: 760.00604, p_value: 0.0000, D: 0.56

Seed 2, 0.1, heterogeneous consumers
np: NP_profit_mean: 718.935, NP_time_mean:12.36
p1_3: time:11.42, p_value: 0.0000, D: 0.42
p1_34: profit_mean: 769.86597, time_mean:11.7, profit_p_value: -0.0000, profit_D: 0.82, time_p_value: 0.0058, time_D: 0.28
p1_4: time:11.52, p_value: 0.0006, D: 0.32
p1_6: time:11.42, p_value: 0.0000, D: 0.4

Seed 3, 0.01, heterogeneous consumers
np: NP_profit_mean: 429.05655, NP_time_mean:19.42
p1_7: profit_mean: 450.20505, time_mean:17.98, profit_p_value: 0.0028, profit_D: 0.36, time_p_value: 0.0000, time_D: 0.56

Seed 3, 0.03, heterogeneous consumers
p1_7: profit_mean: 466.68542, time_mean:17.84, profit_p_value: 0.0058, profit_D: 0.34, time_p_value: 0.0001, time_D: 0.38
p2_9: profit: 460.00726, p_value: 0.0028, D: 0.36

Seed 3, 0.05, heterogeneous consumers
np: NP_profit_mean: 497.35397, NP_time_mean:18.74
p1_7: profit_mean: 527.9875, time_mean:17.12, profit_p_value: 0.0000, profit_D: 0.48, time_p_value: 0.0000, time_D: 0.54
p2_9: profit: 513.3493, p_value: 0.0392, D: 0.28

Seed 3, 0.07, heterogeneous consumers
np: NP_profit_mean: 628.40796, NP_time_mean:14.5
p1_7: time:13.1, p_value: 0.0001, D: 0.4
p1_96: profit: 647.38947, p_value: 0.0013, D: 0.38

Seed 3, 0.1, heterogeneous consumers
np: NP_profit_mean: 701.24945, NP_time_mean:13.28
p1_7: time:11.84, p_value: 0.0000, D: 0.46
p1_96: profit: 713.7788, p_value: 0.0115, D: 0.32

APPENDIX G: VALUES OF D, P, AND MEAN FOR MODEL 2-2

Homogeneous consumers

Seed 1, rewiring probability: 0.01, homogeneous consumers
np: NP_profit_mean: 880.5747, NP_time_mean:20.38
p1_4: time:19.54, p_value: 0.0013, D: 0.32
p1_7: profit_mean: 986.2556, time_mean:19.68, profit_p_value: -0.0000, profit_D: 0.82, time_p_value: 0.0013, time_D: 0.36
p2_1: time:19.6, p_value: 0.0058, D: 0.3
p3_11: profit_mean: 906.78265, profit_p_value: 0.0115, profit_D: 0.32

Seed 1, rewiring probability: 0.03, homogeneous consumers
np: NP_profit_mean: 1135.3728, NP_time_mean:13.68
p1_6: profit_mean: 1159.1052, time_mean:12.96, profit_p_value: 0.0006, profit_D: 0.4, time_p_value: 0.0000, time_D: 0.54
p1_7: profit_mean: 1212.8641, time_mean:13.1, profit_p_value: -0.0000, profit_D: 0.96, time_p_value: 0.0000, time_D: 0.56

Seed 1, rewiring probability: 0.05, homogeneous consumers
np: NP_profit_mean: 1144.004, NP_time_mean:13.98
p1_64: profit_mean: 1174.1028, profit_p_value: 0.0001, profit_D: 0.44
p1_7: profit_mean: 1184.5767, profit_p_value: 0.0000, profit_D: 0.6

Seed 1, rewiring probability: 0.07, homogeneous consumers
np: NP_profit_mean: 1205.0215, NP_time_mean:12.12
p1_4: profit_mean: 1265.8212, time_mean:10.8, profit_p_value: 0.0000, profit_D: 0.8, time_p_value: 0.0000, time_D: 0.7
p1_64: profit_mean: 1248.6031, profit_p_value: 0.0000, profit_D: 0.62
p1_7: profit_mean: 1245.1549, profit_p_value: 0.0000, profit_D: 0.62

Seed 1, rewiring probability: 0.1, homogeneous consumers
np: NP_profit_mean: 1234.026, NP_time_mean:10.8
p1_4: profit_mean: 1291.6182, profit_p_value: -0.0000, profit_D: 0.9
p1_64: profit_mean: 1315.7097, time_mean:9.36, profit_p_value: -0.0000, profit_D: 0.96, time_p_value: -0.0000, time_D: 0.84
p1_7: profit_mean: 1280.6132, profit_p_value: 0.0000, profit_D: 0.74

Seed 2, rewiring probability: 0.01, homogeneous consumers
np: NP_profit_mean: 983.1693, NP_time_mean:17.76
p1_7: profit_mean: 1047.3875, time_mean:17.56, profit_p_value: 0.0000, profit_D: 0.62, time_p_value: 0.0028, time_D: 0.34

Seed 2, rewiring probability: 0.03, homogeneous consumers
np: NP_profit_mean: 985.78, NP_time_mean:18.44
p1_7: profit_mean: 1063.2761, time_mean:17.88, profit_p_value: 0.0000, profit_D: 0.7, time_p_value: 0.0006, time_D: 0.38

Seed 2, rewiring probability: 0.05, homogeneous consumers
np: NP_profit_mean: 1254.6848, NP_time_mean:9.64
p1_3: profit_mean: 1287.5945, profit_p_value: 0.0000, profit_D: 0.76
p1_34: profit_mean: 1316.8453, profit_p_value: -0.0000, profit_D: 0.96
p1_4: profit_mean: 1264.3098, profit_p_value: 0.0001, profit_D: 0.44
p1_6: profit_mean: 1263.6895, profit_p_value: 0.0013, profit_D: 0.38
p1_7: profit_mean: 1278.912, profit_p_value: 0.0000, profit_D: 0.7

Seed 2, rewiring probability: 0.07, homogeneous consumers
np: NP_profit_mean: 1257.1829, NP_time_mean:9.52
p1_3: profit_mean: 1315.4457, profit_p_value: -0.0000, profit_D: 0.9
p1_34: profit_mean: 1276.725, profit_p_value: 0.0000, profit_D: 0.7
p1_4: profit_mean: 1296.0941, profit_p_value: 0.0000, profit_D: 0.76
p1_6: profit_mean: 1263.5123, profit_p_value: 0.0006, profit_D: 0.4
p1_7: profit_mean: 1304.6091, profit_p_value: -0.0000, profit_D: 0.88

Seed 2, rewiring probability: 0.1, homogeneous consumers
np: NP_profit_mean: 1258.595, NP_time_mean:9.56
p1_3: profit_mean: 1285.4006, profit_p_value: 0.0000, profit_D: 0.72
p1_34: profit_mean: 1301.7854, profit_p_value: -0.0000, profit_D: 0.82
p1_4: profit_mean: 1270.6902, profit_p_value: 0.0000, profit_D: 0.54
p1_6: profit_mean: 1299.9377, profit_p_value: -0.0000, profit_D: 0.82
p1_7: profit_mean: 1266.4772, profit_p_value: 0.0001, profit_D: 0.44

Seed 3, rewiring probability: 0.01, homogeneous consumers
np: NP_profit_mean: 932.55896, NP_time_mean:18.88
p1_6: profit_mean: 983.3513, time_mean:18.52, profit_p_value: 0.0006, profit_D: 0.4, time_p_value: 0.0000, time_D: 0.46
p1_7: profit_mean: 994.5436, time_mean:18.2, profit_p_value: 0.0000, profit_D: 0.56, time_p_value: 0.0000, time_D: 0.52

Seed 3, rewiring probability: 0.03, homogeneous consumers
np: NP_profit_mean: 955.83563, NP_time_mean:17.76

p1_7: profit_mean: 1034.8884, time_mean:17.44, profit_p_value: 0.0000, profit_D: 0.72, time_p_value: 0.0000, time_D: 0.48

Seed 3, rewiring probability: 0.05, homogeneous consumers
np: NP_profit_mean: 982.28467, NP_time_mean:17.68
p1_7: profit_mean: 1027.1254, time_mean:16.92, profit_p_value: 0.0000, profit_D: 0.52, time_p_value: 0.0000, time_D: 0.58

Seed 3, rewiring probability: 0.07, homogeneous consumers
np: NP_profit_mean: 1173.3892, NP_time_mean:13.38
p1_3: profit_mean: 1189.1025, profit_p_value: 0.0000, profit_D: 0.46
p1_4: profit_mean: 1193.5095, profit_p_value: 0.0006, profit_D: 0.4
p1_7: profit_mean: 1224.5675, time_mean:12.34, profit_p_value: 0.0000, profit_D: 0.8, time_p_value: 0.0000, time_D: 0.46
p1_96: profit_mean: 1215.6613, profit_p_value: 0.0000, profit_D: 0.74
p2_2: time_mean:12.66, time_p_value: 0.0217, time_D: 0.28

Seed 3, rewiring probability: 0.1, homogeneous consumers
np: NP_profit_mean: 1231.4739, NP_time_mean:10.74
p1_3: profit_mean: 1262.6567, profit_p_value: 0.0000, profit_D: 0.64
p1_7: profit_mean: 1272.5737, profit_p_value: 0.0000, profit_D: 0.74
p1_96: profit_mean: 1284.3319, profit_p_value: -0.0000, profit_D: 0.86

Heterogeneous consumers

Seed 1, rewiring probability: 0.01, heterogeneous consumers
np: NP_profit_mean: 362.82703, NP_time_mean:20.94
p1_6: profit_mean: 395.45853, profit_p_value: 0.0000, profit_D: 0.46
p1_7: profit_mean: 429.7302, time_mean:19.76, profit_p_value: -0.0000, profit_D: 0.82, time_p_value: 0.0000, time_D: 0.46

Seed 1, rewiring probability: 0.03, heterogeneous consumers
np: NP_profit_mean: 580.95483, NP_time_mean:14.64
p1_4: profit_mean: 597.68835, profit_p_value: 0.0115, profit_D: 0.32
p1_6: profit_mean: 644.7121, profit_p_value: -0.0000, profit_D: 0.82
p1_7: profit_mean: 668.0019, time_mean:13.92, profit_p_value: -0.0000, profit_D: 0.92, time_p_value: 0.0001, time_D: 0.38
p2_8: profit_mean: 603.55334, profit_p_value: 0.0000, profit_D: 0.48
p2_9: profit_mean: 587.85425, profit_p_value: 0.0392, profit_D: 0.28
p3_42: profit_mean: 593.7005, profit_p_value: 0.0115, profit_D: 0.32

Seed 1, rewiring probability: 0.05, heterogeneous consumers

np: NP_profit_mean: 622.5683, NP_time_mean:14.72
p1_4: profit_mean: 649.0277, profit_p_value: 0.0002, profit_D: 0.42
p1_64: profit_mean: 658.0335, profit_p_value: 0.0000, profit_D: 0.58
p1_7: profit_mean: 681.13654, time_mean:13.92, profit_p_value: -0.0000, profit_D: 0.82, time_p_value: 0.0028, time_D: 0.34
p2_2: profit_mean: 659.405, profit_p_value: 0.0000, profit_D: 0.64
p2_3: profit_mean: 661.48663, profit_p_value: 0.0000, profit_D: 0.64
p3_1: profit_mean: 650.5658, profit_p_value: 0.0000, profit_D: 0.48
p3_36: profit_mean: 644.9326, profit_p_value: 0.0058, profit_D: 0.34
p3_60: profit_mean: 638.6272, profit_p_value: 0.0115, profit_D: 0.32

Seed 1, rewiring probability: 0.07, heterogeneous consumers
np: NP_profit_mean: 702.33325, NP_time_mean:12.4
p1_64: profit_mean: 754.10095, profit_p_value: 0.0000, profit_D: 0.78
p1_7: profit_mean: 759.5148, profit_p_value: 0.0000, profit_D: 0.8
p2_2: profit_mean: 714.32697, profit_p_value: 0.0115, profit_D: 0.32
p3_36: profit_mean: 732.20087, profit_p_value: 0.0000, profit_D: 0.64

Seed 1, rewiring probability: 0.1, heterogeneous consumers
np: NP_profit_mean: 720.53094, NP_time_mean:12.02
p1_4: profit_mean: 768.6035, time_mean:10.94, profit_p_value: 0.0000, profit_D: 0.74, time_p_value: 0.0013, time_D: 0.34
p1_64: profit_mean: 772.50494, profit_p_value: -0.0000, profit_D: 0.82
p1_7: profit_mean: 790.87616, time_mean:10.88, profit_p_value: -0.0000, profit_D: 0.88, time_p_value: 0.0006, time_D: 0.36
p2_3: profit_mean: 732.5848, profit_p_value: 0.0006, profit_D: 0.4
p2_63: profit_mean: 740.5096, profit_p_value: 0.0013, profit_D: 0.38
p2_8: profit_mean: 772.52686, profit_p_value: 0.0000, profit_D: 0.78
p3_31: profit_mean: 766.6273, profit_p_value: 0.0000, profit_D: 0.78
p3_32: profit_mean: 795.7756, profit_p_value: -0.0000, profit_D: 0.94
p3_40: profit_mean: 740.4161, profit_p_value: 0.0013, profit_D: 0.38

Seed 2, rewiring probability: 0.01, heterogeneous consumers
np: NP_profit_mean: 378.99585, NP_time_mean:19.34
p1_7: profit_mean: 503.96637, time_mean:18.34, profit_p_value: -0.0000, profit_D: 0.98, time_p_value: 0.0001, time_D: 0.4
p2_84: profit_mean: 462.9093, profit_p_value: -0.0000, profit_D: 0.88

Seed 2, rewiring probability: 0.03, heterogeneous consumers
np: NP_profit_mean: 410.09943, NP_time_mean:18.3
p1_7: profit_mean: 520.8947, profit_p_value: -0.0000, profit_D: 0.98
p2_84: profit_mean: 507.9913, profit_p_value: -0.0000, profit_D: 1.0

Seed 2, rewiring probability: 0.05, heterogeneous consumers

np: NP_profit_mean: 732.7481, NP_time_mean:11.16
p1_34: profit_mean: 833.4168, time_mean:10.42, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: 0.0006, time_D: 0.36
p1_7: profit_mean: 758.82007, profit_p_value: 0.0000, profit_D: 0.58
p2_82: profit_mean: 747.7146, profit_p_value: 0.0001, profit_D: 0.44
p2_84: profit_mean: 775.8825, profit_p_value: 0.0000, profit_D: 0.78
p3_58: profit_mean: 744.3155, profit_p_value: 0.0013, profit_D: 0.38
p3_82: profit_mean: 747.7146, profit_p_value: 0.0001, profit_D: 0.44
p3_86: profit_mean: 742.0859, profit_p_value: 0.0217, profit_D: 0.3
p3_93: profit_mean: 748.81635, profit_p_value: 0.0000, profit_D: 0.48

Seed 2, rewiring probability: 0.07, heterogeneous consumers
np: NP_profit_mean: 734.8642, NP_time_mean:11.02
p1_34: profit_mean: 823.8982, profit_p_value: -0.0000, profit_D: 1.0
p1_6: profit_mean: 750.7394, profit_p_value: 0.0058, profit_D: 0.34
p1_7: profit_mean: 798.008, profit_p_value: -0.0000, profit_D: 0.92
p2_84: profit_mean: 761.70874, profit_p_value: 0.0000, profit_D: 0.58
p3_82: profit_mean: 750.15607, profit_p_value: 0.0028, profit_D: 0.36
p3_83: profit_mean: 752.1878, profit_p_value: 0.0001, profit_D: 0.44

Seed 2, rewiring probability: 0.1, heterogeneous consumers
np: NP_profit_mean: 718.935, NP_time_mean:12.36
p1_3: profit_mean: 751.2152, profit_p_value: 0.0000, profit_D: 0.52
p1_34: profit_mean: 807.69415, time_mean:11.5, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: 0.0006, time_D: 0.32
p1_4: profit_mean: 738.4561, profit_p_value: 0.0006, profit_D: 0.4
p1_6: profit_mean: 748.3971, profit_p_value: 0.0000, profit_D: 0.56
p1_7: profit_mean: 753.07166, profit_p_value: 0.0000, profit_D: 0.6
p2_32: profit_mean: 738.0811, profit_p_value: 0.0013, profit_D: 0.38
p3_32: profit_mean: 738.0811, profit_p_value: 0.0013, profit_D: 0.38
p3_58: profit_mean: 728.37866, profit_p_value: 0.0392, profit_D: 0.28

Seed 3, rewiring probability: 0.01, heterogeneous consumers
np: NP_profit_mean: 429.05655, NP_time_mean:19.42
p1_6: profit_mean: 456.9501, profit_p_value: 0.0002, profit_D: 0.42
p1_7: profit_mean: 486.7267, time_mean:18.26, profit_p_value: 0.0000, profit_D: 0.7, time_p_value: 0.0000, time_D: 0.46
p2_9: profit_mean: 441.77914, profit_p_value: 0.0058, profit_D: 0.34

Seed 3, rewiring probability: 0.03, heterogeneous consumers
np: NP_profit_mean: 441.5622, NP_time_mean:18.88
p1_6: profit_mean: 486.84232, time_mean:18.28, profit_p_value: 0.0000, profit_D: 0.58, time_p_value: 0.0006, time_D: 0.36

p1_7: profit_mean: 499.13858, time_mean:17.92, profit_p_value: 0.0000, profit_D: 0.66, time_p_value: 0.0001, time_D: 0.42
p2_9: profit_mean: 469.25952, profit_p_value: 0.0028, profit_D: 0.36

Seed 3, rewiring probability: 0.05, heterogeneous consumers
np: NP_profit_mean: 497.35397, NP_time_mean:18.74
p1_6: profit_mean: 523.92365, profit_p_value: 0.0006, profit_D: 0.4
p1_7: profit_mean: 551.7851, time_mean:17.72, profit_p_value: 0.0000, profit_D: 0.72, time_p_value: 0.0002, time_D: 0.4
p2_9: profit_mean: 516.7365, profit_p_value: 0.0013, profit_D: 0.38

Seed 3, rewiring probability: 0.07, heterogeneous consumers
np: NP_profit_mean: 628.40796, NP_time_mean:14.5
p1_3: profit_mean: 662.7061, profit_p_value: 0.0000, profit_D: 0.58
p1_7: profit_mean: 668.50354, profit_p_value: 0.0000, profit_D: 0.58
p1_96: profit_mean: 684.03625, profit_p_value: -0.0000, profit_D: 0.82
p3_31: profit_mean: 684.4331, time_mean:13.64, profit_p_value: 0.0000, profit_D: 0.76, time_p_value: 0.0028, time_D: 0.3

Seed 3, rewiring probability: 0.1, heterogeneous consumers
np: NP_profit_mean: 701.24945, NP_time_mean:13.28
p1_3: profit_mean: 725.8911, profit_p_value: 0.0000, profit_D: 0.5
p1_4: profit_mean: 721.0971, profit_p_value: 0.0002, profit_D: 0.42
p1_7: profit_mean: 731.48615, time_mean:12.2, profit_p_value: 0.0000, profit_D: 0.58, time_p_value: 0.0006, time_D: 0.34
p1_96: profit_mean: 754.3144, profit_p_value: 0.0000, profit_D: 0.74
p2_1: profit_mean: 720.86017, profit_p_value: 0.0000, profit_D: 0.46
p2_2: profit_mean: 731.62604, profit_p_value: 0.0000, profit_D: 0.58
p2_9: profit_mean: 710.27576, profit_p_value: 0.0115, profit_D: 0.32
p3_0: profit_mean: 721.6729, profit_p_value: 0.0000, profit_D: 0.48
p3_31: profit_mean: 738.86017, profit_p_value: 0.0000, profit_D: 0.68

APPENDIX H: VALUES OF D, P, AND MEAN FOR MODEL 1-1

Homogeneous consumers

Seed 1, rewiring probability: 0.01, homogeneous consumers
np: NP_profit_mean: 1097.3464, NP_time_mean:20.62
p1_7: profit_mean: 1118.907, time_mean:19.2, profit_p_value: 0.0013, profit_D: 0.38, time_p_value: 0.0000, time_D: 0.54

Seed 1, rewiring probability: 0.03, homogeneous consumers
np: NP_profit_mean: 1252.2063, NP_time_mean:14.84
p1_6: profit_mean: 1259.4691, time_mean:13.5, profit_p_value: 0.0000, profit_D: 0.66, time_p_value: 0.0000, time_D: 0.58
p1_7: profit_mean: 1256.3768, time_mean:13.4, profit_p_value: 0.0000, profit_D: 0.54, time_p_value: 0.0000, time_D: 0.56

Seed 1, rewiring probability: 0.05, homogeneous consumers
np: NP_profit_mean: 1254.5896, NP_time_mean:15.04
p2_9: time_mean:14.44, time_p_value: 0.0217, time_D: 0.28

Seed 1, rewiring probability: 0.07, homogeneous consumers
np: NP_profit_mean: 1293.0057, NP_time_mean:12.32
p1_4: time_mean:11.78, time_p_value: 0.0002, time_D: 0.4
p1_64: profit_mean: 1295.0363, time_mean:11.12, profit_p_value: 0.0001, profit_D: 0.44, time_p_value: 0.0000, time_D: 0.62

Seed 1, rewiring probability: 0.1, homogeneous consumers
np: NP_profit_mean: 1310.5538, NP_time_mean:11.18
p1_4: time_mean:10.74, time_p_value: 0.0000, time_D: 0.44
p1_64: time_mean:10.94, time_p_value: 0.0013, time_D: 0.36

Seed 2, rewiring probability: 0.01, homogeneous consumers
np: NP_profit_mean: 1149.2927, NP_time_mean:18.56
p1_7: time:17.46, p_value: 0.0000, D: 0.48

Seed 2, rewiring probability: 0.03, homogeneous consumers
np: NP_profit_mean: 1157.6523, NP_time_mean:18.54
p1_7: time:18.24, p_value: 0.0006, D: 0.38

Seed 2, rewiring probability: 0.05, homogeneous consumers
np: NP_profit_mean: 1325.9742, NP_time_mean:10.24
p1_34: time_mean:9.78, time_p_value: 0.0006, time_D: 0.38

Seed 2, rewiring probability: 0.07, homogeneous consumers
np: NP_profit_mean: 1325.103, NP_time_mean:10.1

Seed 2, rewiring probability: 0.1, homogeneous consumers
np: NP_profit_mean: 1324.7745, NP_time_mean:10.48

Seed 3, rewiring probability: 0.01, homogeneous consumers
np: NP_profit_mean: 1113.1846, NP_time_mean:19.56
p1_6: profit_mean: 1128.6062, time_mean:18.42, profit_p_value: 0.0392, profit_D: 0.28, time_p_value: 0.0000, time_D: 0.44
p1_7: profit_mean: 1150.0751, time_mean:18.54, profit_p_value: 0.0000, profit_D: 0.6, time_p_value: 0.0001, time_D: 0.42

Seed 3, rewiring probability: 0.03, homogeneous consumers
np: NP_profit_mean: 1140.4078, NP_time_mean:18.58
p1_7: profit_mean: 1171.7251, time_mean:18.22, profit_p_value: 0.0000, profit_D: 0.5, time_p_value: 0.0002, time_D: 0.4

Seed 3, rewiring probability: 0.05, homogeneous consumers
np: NP_profit_mean: 1144.133, NP_time_mean:18.26
p1_7: profit_mean: 1158.9658, time_mean:17.74, profit_p_value: 0.0392, profit_D: 0.28, time_p_value: 0.0000, time_D: 0.44

Seed 3, rewiring probability: 0.07, homogeneous consumers
np: NP_profit_mean: 1273.693, NP_time_mean:13.44
p1_7: profit_mean: 1276.7186, time_mean:12.44, profit_p_value: 0.0002, profit_D: 0.42, time_p_value: 0.0000, time_D: 0.52
p1_96: time_mean:12.72, time_p_value: 0.0000, time_D: 0.44

Seed 3, rewiring probability: 0.1, homogeneous consumers
np: NP_profit_mean: 1312.1019, NP_time_mean:10.54

Heterogeneous consumers

Seed 1, rewiring probability: 0.01, heterogeneous consumers
np: NP_profit_mean: 549.44586, NP_time_mean:21.58
p1_7: profit_mean: 571.3861, time_mean:20.42, profit_p_value: 0.0000, profit_D: 0.5, time_p_value: 0.0115, time_D: 0.28

Seed 1, rewiring probability: 0.03, heterogeneous consumers
np: NP_profit_mean: 713.9768, NP_time_mean:15.4
p1_6: profit_mean: 729.57544, profit_p_value: 0.0028, profit_D: 0.36
p1_7: profit_mean: 732.55505, time_mean:14.72, profit_p_value: 0.0000, profit_D: 0.52, time_p_value: 0.0115, time_D: 0.28
p2_8: profit_mean: 725.40393, profit_p_value: 0.0058, profit_D: 0.34

Seed 1, rewiring probability: 0.05, heterogeneous consumers
np: NP_profit_mean: 764.674, NP_time_mean:14.9
p1_7: profit_mean: 772.77875, profit_p_value: 0.0115, profit_D: 0.32

Seed 1, rewiring probability: 0.07, heterogeneous consumers
np: NP_profit_mean: 819.6116, NP_time_mean:13.26
p1_7: profit_mean: 836.5818, profit_p_value: 0.0013, profit_D: 0.38

Seed 1, rewiring probability: 0.1, heterogeneous consumers
np: NP_profit_mean: 824.4389, NP_time_mean:12.58
p1_4: time_mean:11.78, time_p_value: 0.0115, time_D: 0.3
p1_7: time:11.3, p_value: 0.0000, D: 0.44

Seed 2, rewiring probability: 0.01, heterogeneous consumers
np: NP_profit_mean: 549.87506, NP_time_mean:20.16
p2_84: profit_mean: 630.56134, profit_p_value: -0.0000, profit_D: 0.88
p3_99: time:19.36, p_value: 0.0028, D: 0.3

Seed 2, rewiring probability: 0.03, heterogeneous consumers
np: NP_profit_mean: 578.0256, NP_time_mean:19.24
p2_84: profit_mean: 649.84296, profit_p_value: -0.0000, profit_D: 0.92

Seed 2, rewiring probability: 0.05, heterogeneous consumers
np: NP_profit_mean: 835.50464, NP_time_mean:11.12
p1_34: profit_mean: 879.5493, profit_p_value: -0.0000, profit_D: 0.82
p2_84: profit_mean: 844.2647, profit_p_value: 0.0028, profit_D: 0.36

Seed 2, rewiring probability: 0.07, heterogeneous consumers
np: NP_profit_mean: 843.44305, NP_time_mean:12.28
p1_34: profit_mean: 906.63196, time_mean:11.5, profit_p_value: -0.0000, profit_D: 0.98, time_p_value: 0.0115, time_D: 0.28
p2_84: profit_mean: 865.36523, profit_p_value: 0.0000, profit_D: 0.5

Seed 2, rewiring probability: 0.1, heterogeneous consumers
np: NP_profit_mean: 831.02625, NP_time_mean:12.7
p1_34: profit_mean: 872.2249, profit_p_value: 0.0000, profit_D: 0.78

p1_6: profit_mean: 838.64795, profit_p_value: 0.0058, profit_D: 0.34

Seed 3, rewiring probability: 0.01, heterogeneous consumers
np: NP_profit_mean: 601.7322, NP_time_mean:19.84
p1_7: profit_mean: 620.64264, time_mean:18.44, profit_p_value: 0.0006, profit_D: 0.4, time_p_value: 0.0000, time_D: 0.46
p2_9: profit_mean: 614.497, profit_p_value: 0.0058, profit_D: 0.34

Seed 3, rewiring probability: 0.03, heterogeneous consumers
np: NP_profit_mean: 609.1614, NP_time_mean:19.2
p1_7: profit_mean: 628.2048, time_mean:18.32, profit_p_value: 0.0006, profit_D: 0.4, time_p_value: 0.0058, time_D: 0.32
p2_9: profit_mean: 626.739, profit_p_value: 0.0013, profit_D: 0.38

Seed 3, rewiring probability: 0.05, heterogeneous consumers
np: NP_profit_mean: 671.4601, NP_time_mean:18.62

Seed 3, rewiring probability: 0.07, heterogeneous consumers
np: NP_profit_mean: 758.9921, NP_time_mean:15.58
p1_7: time:14.26, p_value: 0.0058, D: 0.3
p1_96: profit_mean: 774.0757, profit_p_value: 0.0006, profit_D: 0.4

Seed 3, rewiring probability: 0.1, heterogeneous consumers
np: NP_profit_mean: 822.7427, NP_time_mean:13.94
p1_3: time:13.02, p_value: 0.0028, D: 0.32
p1_4: time:12.88, p_value: 0.0002, D: 0.36

APPENDIX I: VALUES OF D, P, AND MEAN FOR MODEL 1-2

Homogeneous consumers

Seed 1, rewiring probability: 0.01, homogeneous consumers
np: NP_profit_mean: 1097.3464, NP_time_mean:20.62
p1_3: profit_mean: 1129.8254, profit_p_value: 0.0028, profit_D: 0.36
p1_4: profit_mean: 1114.3763, profit_p_value: 0.0217, profit_D: 0.3
p1_6: profit_mean: 1141.5582, profit_p_value: 0.0001, profit_D: 0.44
p1_7: profit_mean: 1203.9249, time_mean:18.94, profit_p_value: -0.0000, profit_D: 0.9, time_p_value: 0.0000, time_D: 0.5

Seed 1, rewiring probability: 0.03, homogeneous consumers
np: NP_profit_mean: 1252.2063, NP_time_mean:14.84
p1_3: profit_mean: 1282.4851, time_mean:14.22, profit_p_value: 0.0000, profit_D: 0.58, time_p_value: 0.0000, time_D: 0.44
p1_4: profit_mean: 1316.388, time_mean:12.74, profit_p_value: -0.0000, profit_D: 0.94, time_p_value: 0.0000, time_D: 0.58
p1_6: profit_mean: 1309.8892, time_mean:12.96, profit_p_value: -0.0000, profit_D: 0.92, time_p_value: 0.0000, time_D: 0.5
p1_7: profit_mean: 1351.582, time_mean:11.24, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: -0.0000, time_D: 0.92

Seed 1, rewiring probability: 0.05, homogeneous consumers
np: NP_profit_mean: 1254.5896, NP_time_mean:15.04
p1_4: profit_mean: 1312.6149, time_mean:12.82, profit_p_value: -0.0000, profit_D: 0.9, time_p_value: 0.0000, time_D: 0.62
p1_64: profit_mean: 1321.2275, time_mean:13.1, profit_p_value: -0.0000, profit_D: 0.98, time_p_value: 0.0000, time_D: 0.56
p1_7: profit_mean: 1320.7384, time_mean:13.06, profit_p_value: -0.0000, profit_D: 0.96, time_p_value: 0.0000, time_D: 0.62
p2_3: time_mean:14.14, time_p_value: 0.0006, time_D: 0.38
p2_6: time_mean:14.3, time_p_value: 0.0115, time_D: 0.3
p3_35: time_mean:14.06, time_p_value: 0.0006, time_D: 0.38
p3_70: time_mean:14.38, time_p_value: 0.0115, time_D: 0.3

Seed 1, rewiring probability: 0.07, homogeneous consumers
np: NP_profit_mean: 1293.0057, NP_time_mean:12.32

p1_4: profit_mean: 1378.0624, time_mean:9.02, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: -0.0000, time_D: 1.0
p1_64: profit_mean: 1366.5394, time_mean:10.28, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: -0.0000, time_D: 0.84
p1_7: profit_mean: 1363.2827, time_mean:10.52, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: -0.0000, time_D: 0.8

Seed 1, rewiring probability: 0.1, homogeneous consumers
np: NP_profit_mean: 1310.5538, NP_time_mean:11.18
p1_4: profit_mean: 1388.8413, time_mean:9.14, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: -0.0000, time_D: 0.92
p1_64: profit_mean: 1405.3718, time_mean:8.02, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: -0.0000, time_D: 1.0
p1_7: profit_mean: 1389.0381, time_mean:9.0, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: -0.0000, time_D: 1.0

Seed 2, rewiring probability: 0.01, homogeneous consumers
np: NP_profit_mean: 1149.2927, NP_time_mean:18.56
p1_3: profit_mean: 1168.5122, profit_p_value: 0.0058, profit_D: 0.34
p1_4: profit_mean: 1179.9673, profit_p_value: 0.0013, profit_D: 0.38
p1_6: profit_mean: 1203.6672, profit_p_value: 0.0000, profit_D: 0.64
p1_7: profit_mean: 1241.7128, time_mean:16.32, profit_p_value: -0.0000, profit_D: 0.92, time_p_value: 0.0000, time_D: 0.66

Seed 2, rewiring probability: 0.03, homogeneous consumers
np: NP_profit_mean: 1157.6523, NP_time_mean:18.54
p1_3: profit_mean: 1179.168, profit_p_value: 0.0028, profit_D: 0.36
p1_4: profit_mean: 1174.59, profit_p_value: 0.0217, profit_D: 0.3
p1_6: profit_mean: 1213.7837, profit_p_value: 0.0000, profit_D: 0.78
p1_7: profit_mean: 1248.1305, time_mean:16.68, profit_p_value: -0.0000, profit_D: 0.94, time_p_value: 0.0000, time_D: 0.58

Seed 2, rewiring probability: 0.05, homogeneous consumers
np: NP_profit_mean: 1325.9742, NP_time_mean:10.24
p1_3: profit_mean: 1386.8062, time_mean:9.1, profit_p_value: -0.0000, profit_D: 0.98, time_p_value: 0.0001, time_D: 0.4
p1_34: profit_mean: 1405.3806, time_mean:8.04, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: -0.0000, time_D: 0.96
p1_4: profit_mean: 1377.4133, time_mean:9.1, profit_p_value: -0.0000, profit_D: 0.98, time_p_value: 0.0002, time_D: 0.38
p1_6: profit_mean: 1378.4786, time_mean:9.04, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: 0.0000, time_D: 0.42

p1_7: profit_mean: 1378.8354, time_mean:9.04, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: 0.0000, time_D: 0.44

Seed 2, rewiring probability: 0.07, homogeneous consumers
np: NP_profit_mean: 1325.103, NP_time_mean:10.1
p1_3: profit_mean: 1404.6328, time_mean:8.02, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: -0.0000, time_D: 0.98
p1_34: profit_mean: 1378.1497, time_mean:9.08, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: 0.0006, time_D: 0.36
p1_4: profit_mean: 1395.4578, time_mean:8.02, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: -0.0000, time_D: 0.98
p1_6: profit_mean: 1378.8293, time_mean:9.02, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: 0.0002, time_D: 0.38
p1_7: profit_mean: 1393.8397, time_mean:8.06, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: -0.0000, time_D: 0.96

Seed 2, rewiring probability: 0.1, homogeneous consumers
np: NP_profit_mean: 1324.7745, NP_time_mean:10.48
p1_3: profit_mean: 1388.559, time_mean:9.1, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: 0.0000, time_D: 0.44
p1_34: profit_mean: 1395.0748, time_mean:8.06, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: -0.0000, time_D: 0.98
p1_4: profit_mean: 1373.697, time_mean:9.26, profit_p_value: -0.0000, profit_D: 0.92, time_p_value: 0.0001, time_D: 0.4
p1_6: profit_mean: 1393.9731, time_mean:8.0, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: -0.0000, time_D: 1.0
p1_7: profit_mean: 1376.585, time_mean:9.2, profit_p_value: -0.0000, profit_D: 0.96, time_p_value: 0.0000, time_D: 0.42

Seed 3, rewiring probability: 0.01, homogeneous consumers
np: NP_profit_mean: 1113.1846, NP_time_mean:19.56
p1_3: profit_mean: 1137.11, profit_p_value: 0.0058, profit_D: 0.34
p1_4: profit_mean: 1175.0665, profit_p_value: 0.0000, profit_D: 0.64
p1_6: profit_mean: 1203.9341, time_mean:17.78, profit_p_value: -0.0000, profit_D: 0.88, time_p_value: 0.0000, time_D: 0.5
p1_7: profit_mean: 1209.9409, time_mean:17.72, profit_p_value: -0.0000, profit_D: 0.88, time_p_value: 0.0000, time_D: 0.58

Seed 3, rewiring probability: 0.03, homogeneous consumers
np: NP_profit_mean: 1140.4078, NP_time_mean:18.58
p1_3: profit_mean: 1175.1023, profit_p_value: 0.0002, profit_D: 0.42
p1_6: profit_mean: 1192.0947, profit_p_value: 0.0000, profit_D: 0.54
p1_7: profit_mean: 1238.4559, time_mean:16.68, profit_p_value: -0.0000, profit_D: 0.94, time_p_value: 0.0000, time_D: 0.58

Seed 3, rewiring probability: 0.05, homogeneous consumers
np: NP_profit_mean: 1144.133, NP_time_mean:18.26
p1_4: profit_mean: 1171.0269, profit_p_value: 0.0028, profit_D: 0.36
p1_6: profit_mean: 1205.4633, profit_p_value: 0.0000, profit_D: 0.66
p1_7: profit_mean: 1225.1073, time_mean:16.94, profit_p_value: 0.0000, profit_D: 0.74, time_p_value: 0.0000, time_D: 0.54

Seed 3, rewiring probability: 0.07, homogeneous consumers
np: NP_profit_mean: 1273.693, NP_time_mean:13.44
p1_3: profit_mean: 1340.1329, time_mean:11.24, profit_p_value: -0.0000, profit_D: 0.96, time_p_value: -0.0000, time_D: 0.82
p1_4: profit_mean: 1336.0294, time_mean:11.62, profit_p_value: -0.0000, profit_D: 0.92, time_p_value: 0.0000, time_D: 0.72
p1_7: profit_mean: 1358.9688, time_mean:10.22, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: -0.0000, time_D: 0.96
p1_96: profit_mean: 1348.5944, time_mean:11.44, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: 0.0000, time_D: 0.74

Seed 3, rewiring probability: 0.1, homogeneous consumers
np: NP_profit_mean: 1312.1019, NP_time_mean:10.54
p1_3: profit_mean: 1375.903, time_mean:9.1, profit_p_value: -0.0000, profit_D: 0.98, time_p_value: -0.0000, time_D: 0.96
p1_4: profit_mean: 1349.4836, profit_p_value: 0.0000, profit_D: 0.8
p1_7: profit_mean: 1376.577, time_mean:9.14, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: -0.0000, time_D: 0.9
p1_96: profit_mean: 1385.323, time_mean:9.06, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: -0.0000, time_D: 0.94

Heterogeneous consumers

Seed 1, rewiring probability: 0.01, heterogeneous consumers
np: NP_profit_mean: 549.44586, NP_time_mean:21.58
p1_3: profit_mean: 572.43536, profit_p_value: 0.0000, profit_D: 0.46
p1_6: profit_mean: 593.4387, time_mean:20.52, profit_p_value: 0.0000, profit_D: 0.74, time_p_value: 0.0115, time_D: 0.28
p1_7: profit_mean: 633.29675, time_mean:20.2, profit_p_value: -0.0000, profit_D: 0.88, time_p_value: 0.0002, time_D: 0.36

Seed 1, rewiring probability: 0.03, heterogeneous consumers
np: NP_profit_mean: 713.9768, NP_time_mean:15.4

p1_3: profit_mean: 742.9649, profit_p_value: 0.0000, profit_D: 0.56
p1_4: profit_mean: 750.6502, profit_p_value: 0.0000, profit_D: 0.64
p1_6: profit_mean: 795.2918, time_mean:14.2, profit_p_value: -0.0000, profit_D: 0.96, time_p_value: 0.0006, time_D: 0.38
p1_7: profit_mean: 812.90985, time_mean:13.8, profit_p_value: -0.0000, profit_D: 0.98, time_p_value: 0.0000, time_D: 0.54
p2_8: profit_mean: 743.7562, profit_p_value: 0.0000, profit_D: 0.6
p2_9: profit_mean: 722.1503, profit_p_value: 0.0013, profit_D: 0.38
p3_42: profit_mean: 729.4187, profit_p_value: 0.0013, profit_D: 0.38

Seed 1, rewiring probability: 0.05, heterogeneous consumers
np: NP_profit_mean: 764.674, NP_time_mean:14.9
p1_4: profit_mean: 801.40656, profit_p_value: 0.0000, profit_D: 0.6
p1_64: profit_mean: 812.8878, profit_p_value: 0.0000, profit_D: 0.76
p1_7: profit_mean: 821.9648, time_mean:14.18, profit_p_value: -0.0000, profit_D: 0.86, time_p_value: 0.0006, time_D: 0.34
p2_2: profit_mean: 799.23694, profit_p_value: 0.0000, profit_D: 0.64
p2_3: profit_mean: 797.3582, profit_p_value: 0.0000, profit_D: 0.66
p3_1: profit_mean: 785.49243, profit_p_value: 0.0000, profit_D: 0.48
p3_36: profit_mean: 772.65015, profit_p_value: 0.0217, profit_D: 0.3

Seed 1, rewiring probability: 0.07, heterogeneous consumers
np: NP_profit_mean: 819.6116, NP_time_mean:13.26
p1_4: profit_mean: 837.02905, profit_p_value: 0.0006, profit_D: 0.4
p1_64: profit_mean: 883.364, time_mean:12.18, profit_p_value: -0.0000, profit_D: 0.94, time_p_value: 0.0013, time_D: 0.3
p1_7: profit_mean: 879.7193, time_mean:12.4, profit_p_value: -0.0000, profit_D: 0.94, time_p_value: 0.0013, time_D: 0.3
p2_2: profit_mean: 838.32385, profit_p_value: 0.0000, profit_D: 0.46
p3_36: profit_mean: 846.73047, profit_p_value: 0.0000, profit_D: 0.58

Seed 1, rewiring probability: 0.1, heterogeneous consumers
np: NP_profit_mean: 824.4389, NP_time_mean:12.58
p1_4: profit_mean: 878.2179, profit_p_value: -0.0000, profit_D: 0.9
p1_64: profit_mean: 893.6755, time_mean:10.88, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: 0.0000, time_D: 0.46
p1_7: profit_mean: 914.0722, time_mean:10.98, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: 0.0002, time_D: 0.36
p2_3: profit_mean: 843.48755, profit_p_value: 0.0000, profit_D: 0.56
p2_63: profit_mean: 861.2527, profit_p_value: -0.0000, profit_D: 0.84
p2_8: profit_mean: 883.7762, profit_p_value: -0.0000, profit_D: 1.0
p3_31: profit_mean: 873.4829, profit_p_value: -0.0000, profit_D: 0.94
p3_32: profit_mean: 904.03314, profit_p_value: -0.0000, profit_D: 1.0
p3_40: profit_mean: 849.85443, profit_p_value: 0.0000, profit_D: 0.62

p3_68: profit_mean: 833.51373, profit_p_value: 0.0217, profit_D: 0.3
p3_90: profit_mean: 837.93896, profit_p_value: 0.0028, profit_D: 0.36

Seed 2, rewiring probability: 0.01, heterogeneous consumers
np: NP_profit_mean: 549.87506, NP_time_mean:20.16
p1_7: profit_mean: 687.9888, time_mean:19.02, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: 0.0013, time_D: 0.3
p2_84: profit_mean: 634.5027, profit_p_value: -0.0000, profit_D: 0.9

Seed 2, rewiring probability: 0.03, heterogeneous consumers
np: NP_profit_mean: 578.0256, NP_time_mean:19.24
p1_7: profit_mean: 693.80347, profit_p_value: -0.0000, profit_D: 1.0
p2_84: profit_mean: 675.8313, profit_p_value: -0.0000, profit_D: 0.98

Seed 2, rewiring probability: 0.05, heterogeneous consumers
np: NP_profit_mean: 835.50464, NP_time_mean:11.12
p1_3: profit_mean: 877.4995, profit_p_value: -0.0000, profit_D: 0.82
p1_34: profit_mean: 932.9919, profit_p_value: -0.0000, profit_D: 1.0
p1_4: profit_mean: 860.2722, profit_p_value: 0.0000, profit_D: 0.58
p1_6: profit_mean: 847.25726, profit_p_value: 0.0115, profit_D: 0.32
p1_7: profit_mean: 888.8108, profit_p_value: -0.0000, profit_D: 0.88
p2_82: profit_mean: 848.2215, profit_p_value: 0.0000, profit_D: 0.48
p2_84: profit_mean: 880.7886, profit_p_value: -0.0000, profit_D: 0.82
p3_22: profit_mean: 842.59534, profit_p_value: 0.0392, profit_D: 0.28
p3_37: profit_mean: 848.09216, profit_p_value: 0.0006, profit_D: 0.4
p3_82: profit_mean: 848.2215, profit_p_value: 0.0000, profit_D: 0.48
p3_86: profit_mean: 846.44275, profit_p_value: 0.0028, profit_D: 0.36
p3_93: profit_mean: 858.5041, profit_p_value: 0.0000, profit_D: 0.6

Seed 2, rewiring probability: 0.07, heterogeneous consumers
np: NP_profit_mean: 843.44305, NP_time_mean:12.28
p1_3: profit_mean: 888.34314, profit_p_value: 0.0000, profit_D: 0.8
p1_34: profit_mean: 936.1521, profit_p_value: -0.0000, profit_D: 1.0
p1_4: profit_mean: 864.58514, profit_p_value: 0.0000, profit_D: 0.56
p1_6: profit_mean: 887.4854, profit_p_value: -0.0000, profit_D: 0.84
p1_7: profit_mean: 914.4517, profit_p_value: -0.0000, profit_D: 0.98
p2_84: profit_mean: 869.5513, profit_p_value: 0.0000, profit_D: 0.54
p3_37: profit_mean: 848.3911, profit_p_value: 0.0392, profit_D: 0.28

Seed 2, rewiring probability: 0.1, heterogeneous consumers
np: NP_profit_mean: 831.02625, NP_time_mean:12.7
p1_3: profit_mean: 892.91095, profit_p_value: -0.0000, profit_D: 0.94
p1_34: profit_mean: 912.6612, profit_p_value: -0.0000, profit_D: 0.98
p1_4: profit_mean: 870.324, profit_p_value: 0.0000, profit_D: 0.72

p1_6: profit_mean: 892.9957, profit_p_value: -0.0000, profit_D: 0.92
p1_7: profit_mean: 894.88116, profit_p_value: -0.0000, profit_D: 0.94
p2_32: profit_mean: 855.47424, profit_p_value: 0.0000, profit_D: 0.56
p2_33: profit_mean: 841.9966, profit_p_value: 0.0217, profit_D: 0.3
p2_35: profit_mean: 838.2579, profit_p_value: 0.0392, profit_D: 0.28
p3_32: profit_mean: 855.47424, profit_p_value: 0.0000, profit_D: 0.56

Seed 3, rewiring probability: 0.01, heterogeneous consumers
np: NP_profit_mean: 601.7322, NP_time_mean:19.84
p1_3: profit_mean: 619.20447, profit_p_value: 0.0001, profit_D: 0.44
p1_6: profit_mean: 656.9285, time_mean:18.38, profit_p_value: 0.0000, profit_D: 0.76, time_p_value: 0.0002, time_D: 0.36
p1_7: profit_mean: 672.2553, time_mean:18.66, profit_p_value: -0.0000, profit_D: 0.88, time_p_value: 0.0002, time_D: 0.38
p2_9: profit_mean: 615.9289, profit_p_value: 0.0058, profit_D: 0.34

Seed 3, rewiring probability: 0.03, heterogeneous consumers
np: NP_profit_mean: 609.1614, NP_time_mean:19.2
p1_3: profit_mean: 643.85626, profit_p_value: 0.0000, profit_D: 0.64
p1_4: profit_mean: 633.0864, profit_p_value: 0.0000, profit_D: 0.48
p1_6: profit_mean: 669.6422, time_mean:18.42, profit_p_value: -0.0000, profit_D: 0.9, time_p_value: 0.0006, time_D: 0.38
p1_7: profit_mean: 677.15216, time_mean:17.74, profit_p_value: -0.0000, profit_D: 0.92, time_p_value: 0.0000, time_D: 0.48
p2_9: profit_mean: 624.7474, profit_p_value: 0.0006, profit_D: 0.4
p3_11: profit_mean: 637.38824, profit_p_value: 0.0000, profit_D: 0.52

Seed 3, rewiring probability: 0.05, heterogeneous consumers
np: NP_profit_mean: 671.4601, NP_time_mean:18.62
p1_3: profit_mean: 695.56104, profit_p_value: 0.0013, profit_D: 0.38
p1_6: profit_mean: 721.613, profit_p_value: 0.0000, profit_D: 0.78
p1_7: profit_mean: 735.8488, time_mean:17.52, profit_p_value: -0.0000, profit_D: 0.84, time_p_value: 0.0028, time_D: 0.34

Seed 3, rewiring probability: 0.07, heterogeneous consumers
np: NP_profit_mean: 758.9921, NP_time_mean:15.58
p1_3: profit_mean: 815.21954, time_mean:14.56, profit_p_value: -0.0000, profit_D: 0.82, time_p_value: 0.0058, time_D: 0.3
p1_4: profit_mean: 777.03735, profit_p_value: 0.0058, profit_D: 0.34
p1_7: profit_mean: 811.74713, time_mean:14.72, profit_p_value: -0.0000, profit_D: 0.86, time_p_value: 0.0217, time_D: 0.28
p1_96: profit_mean: 822.3374, profit_p_value: -0.0000, profit_D: 0.94

p3_31: profit_mean: 813.2312, time_mean:14.62, profit_p_value: -0.0000, profit_D: 0.86, time_p_value: 0.0217, time_D: 0.28

Seed 3, rewiring probability: 0.1, heterogeneous consumers
np: NP_profit_mean: 822.7427, NP_time_mean:13.94
p1_3: profit_mean: 864.4043, profit_p_value: 0.0000, profit_D: 0.68
p1_4: profit_mean: 866.04205, profit_p_value: 0.0000, profit_D: 0.74
p1_7: profit_mean: 860.4602, profit_p_value: 0.0000, profit_D: 0.68
p1_96: profit_mean: 878.99585, time_mean:13.12, profit_p_value: 0.0000, profit_D: 0.78, time_p_value: 0.0115, time_D: 0.3
p2_1: profit_mean: 837.87573, profit_p_value: 0.0013, profit_D: 0.38
p2_2: profit_mean: 856.93805, profit_p_value: 0.0000, profit_D: 0.66
p2_98: profit_mean: 831.905, profit_p_value: 0.0028, profit_D: 0.36
p3_0: profit_mean: 835.2665, profit_p_value: 0.0013, profit_D: 0.38
p3_31: profit_mean: 850.02124, profit_p_value: 0.0000, profit_D: 0.62

APPENDIX J: VALUES OF D, P, AND MEAN FOR SCALE-FREE NETWORKS

Model 2-1

Homogeneous consumers

Seed 1:
np: NP_profit_mean: 1257.8394, NP_time_mean:9.66
p1_1: profit_mean: 1263.4899, time_mean:8.64, profit_p_value: 0.0028, profit_D: 0.36, time_p_value: 0.0000, time_D: 0.78

Seed 2:
np: NP_profit_mean: 1261.0607, NP_time_mean:9.3
p1_4: time_mean:8.82, time_p_value: 0.0000, time_D: 0.68

Seed 3:
np: NP_profit_mean: 1211.4052, NP_time_mean:11.98
p1_3: time_mean:11.62, time_p_value: 0.0000, time_D: 0.46

Model 2-2

Homogeneous consumers

Seed 1:
np: NP_profit_mean: 1257.8394, NP_time_mean:9.66
p1_1: profit_mean: 1333.0447, time_mean:8.48, profit_p_value: -0.0000, profit_D: 0.98, time_p_value: -0.0000, time_D: 0.8
p1_12: profit_mean: 1293.7633, profit_p_value: 0.0000, profit_D: 0.8
p1_3: profit_mean: 1373.7738, profit_p_value: -0.0000, profit_D: 1.0
p2_2: profit_mean: 1358.3806, profit_p_value: -0.0000, profit_D: 0.94
p2_5: profit_mean: 1343.572, time_mean:8.16, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: -0.0000, time_D: 0.9

Seed 2:
np: NP_profit_mean: 1261.0607, NP_time_mean:9.3
p1_1: profit_mean: 1302.093, profit_p_value: 0.0000, profit_D: 0.78
p1_4: profit_mean: 1371.6005, time_mean:8.0, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: -0.0000, time_D: 1.0
p2_2: profit_mean: 1300.9304, profit_p_value: 0.0000, profit_D: 0.72

Seed 3:
np: NP_profit_mean: 1211.4052, NP_time_mean:11.98
p1_14: profit_mean: 1221.3672, time_mean:11.86, profit_p_value: 0.0115, profit_D: 0.32, time_p_value: 0.0028, time_D: 0.32
p1_3: profit_mean: 1360.0642, time_mean:10.0, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: -0.0000, time_D: 1.0
p2_2: profit_mean: 1222.8773, profit_p_value: 0.0115, profit_D: 0.32

APPENDIX K: VALUES OF D, P, AND MEAN FOR RANDOM NETWORKS

Model 2-1

Homogeneous consumers

Seed 1:
np: NP_profit_mean: 1301.4949, NP_time_mean:7.76
p1_6: time_mean:7.0, time_p_value: 0.0000, time_D: 0.66

Seed 2:
np: NP_profit_mean: 1284.4796, NP_time_mean:8.4
p1_6: profit_mean: 1285.2921, time_mean:7.68, profit_p_value: 0.0000, profit_D: 0.58, time_p_value: 0.0000, time_D: 0.76

Seed 3:
np: NP_profit_mean: 1283.3918, NP_time_mean:8.38
p1_42: time_mean:8.1, time_p_value: 0.0000, time_D: 0.62

Model 2-2

Homogeneous consumers

Seed 1:
np: NP_profit_mean: 1301.4949, NP_time_mean:7.76
p1_3: profit_mean: 1371.7382, profit_p_value: -0.0000, profit_D: 1.0
p1_6: profit_mean: 1383.4985, time_mean:7.0, profit_p_value: -0.0000, profit_D: 1.0, time_p_value: 0.0058, time_D: 0.28
p1_7: profit_mean: 1346.0798, profit_p_value: -0.0000, profit_D: 0.88
p1_71: profit_mean: 1369.3228, profit_p_value: -0.0000, profit_D: 0.98

Seed 2:
np: NP_profit_mean: 1340.4956, NP_time_mean:9.42
p1_6: time_mean:8.92, time_p_value: 0.0006, time_D: 0.38

Seed 3:

np: NP_profit_mean: 1283.3918, NP_time_mean:8.38
p1_42: profit_mean: 1360.5444, profit_p_value: -0.0000, profit_D: 1.0
p1_7: profit_mean: 1325.9478, profit_p_value: -0.0000, profit_D: 0.9

APPENDIX L: RESULTS OF AUDACIOUS MARKETING STRATEGIES IN MODEL 2-1

L-1. Homogeneous consumers

L-1-1. Experiments on non-piracy

Experiments are conducted on 15 networks. There are no pirates among consumers, and the consumers are homogeneous.

Table 77: Competition results of networks of seed 1 (NP, Homo, Model 2-1)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	16	2		2					
0.03	14	2		3	1				
0.05	18	1			1				
0.07	14	1		3	2				
0.1	15			4	1				

Table 78: Competition results of networks of seed 2 (NP, Homo, Model 2-1)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	13	2		4	1				
0.03	16	2		2					

0.05	15			3	2				
0.07	17			1	2				
0.1	13	2		5					

Table 79: Competition results of networks of seed 3 (NP, Homo, Model 2-1)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	15	5							
0.03	17	2		1					
0.05	17	3							
0.07	12	1		5	2				
0.1	16	1		3					

L-1-2. Experiments on a pirate in L1

Experiments are conducted on 15 networks. In each network, consumer 7 in L1 is assigned as a pirate. Consumers are homogeneous.

Table 80: Competition results of networks of seed 1 (P7, Homo, Model 2-1)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	18	2						20	
0.03	18	1			1			20	
0.05	18				2			20	
0.07	19	1						20	

0.1	18	1			1			20	

Table 81: Competition results of networks of seed 2 (P7, Homo, Model 2-1)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	19	1						20	
0.03	20							20	
0.05	19	1						20	
0.07	18	2						20	
0.1	20							20	

Table 82: Competition results of networks of seed 3 (P7, Homo, Model 2-1)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	18	2						20	
0.03	15	5						20	
0.05	15	3			2			20	
0.07	16	1			3			20	
0.1	15	3			2			20	

L-1-3. Experiments on a pirate in L2

Experiments are conducted on 15 networks. In each network, consumer 8 in L2 is assigned as a pirate. Consumers are homogeneous.

Table 83: Competition results of networks of seed 1 (P8, Homo, Model 2-1)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	17	2		1				19	1
0.03	16	1		2	1			18	2
0.05	19				1			19	1
0.07	12			3	5			12	8
0.1	10	1		2	7			12	8

Table 84: Competition results of networks of seed 2 (P8, Homo, Model 2-1)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	18	1		1				19	1
0.03	20							20	
0.05	15	2		3				15	5
0.07	13	1		4	2			15	5
0.1	11			5	4			15	5

Table 85: Competition results of networks of seed 3 (P8, Homo, Model 2-1)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	18	1		1				19	1
0.03	17	1		1	1			17	3
0.05	16	2		2				17	3

0.07	13			3	4			12	8
0.1	13	1		3	3			13	7

L-2. Heterogeneous consumers

L-2-1 Experiments on non-piracy

Experiments are conducted on 15 networks. There are no pirates among consumers, and the consumers are heterogeneous.

Table 86: Competition results of networks of seed 1 (NP, Hetero, Model 2-1)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	20								
0.03	19	1							
0.05	20								
0.07	19	1							
0.1	18	2							

Table 87: Competition results of networks of seed 2 (NP, Hetero, Model 2-1)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	17	3							
0.03	20								
0.05	20								

0.07	19	1							
0.1	20								

Table 88: Competition results of networks of seed 3 (NP, Hetero, Model 2-1)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	19	1							
0.03	19	1							
0.05	20								
0.07	20								
0.1	20								

L-2-1 Experiments on a pirate in L1

Experiments are conducted on 15 networks. In each network, consumer 7 in L1 is assigned as a pirate. Consumers are heterogeneous.

Table 89: Competition results of networks of seed 1 (P7, Hetero, Model 2-1)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	20							20	
0.03	20							20	
0.05	20							20	
0.07	20							20	
0.1	20							20	

Table 90: Competition results of networks of seed 2 (P7, Hetero, Model 2-1)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	19	1						20	
0.03	18	1			1			20	
0.05	17	1			2			20	
0.07	19	1						20	
0.1	20							20	

Table 91: Competition results of networks of seed 3 (P7, Hetero, Model 2-1)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	19	1						20	
0.03	18	1			1			20	
0.05	18	2					1	20	
0.07	20							20	
0.1	20							20	

L-2-3 Experiments on a pirate in L2

Experiments are conducted on 15 networks. In each network, consumer 8 in L2 is assigned as a pirate. Consumers are heterogeneous.

Table 92: Competition results of networks of seed 1 (P8, Hetero, Model 2-1)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	20							20	
0.03	17			3				17	3
0.05	17			3				17	3
0.07	20							20	
0.1	19			1				19	1

Table 93: Competition results of networks of seed 2 (P8, Hetero, Model 2-1)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	20							20	
0.03	19			1				19	1
0.05	20							20	
0.07	19			1				19	1
0.1	20							20	

Table 94: Competition results of networks of seed 3 (P8, Hetero, Model 2-1)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	18			2				18	2
0.03	18			2				18	2
0.05	20							20	
0.07	20							20	
0.1	20							20	

APPENDIX M: COMPARISONS OF NUMBER OF "LOCK-IN" BETWEEN CONSERVATIVE AND AUDACIOUS STRATEGIES

Seed1_C = competition result of network seed 1 of conservative strategies

Seed1_A = competition result of network seed 1 of audacious strategies

Seed2_C = competition result of network seed 2 of conservative strategies

Seed2_A = competition result of network seed 2 of audacious strategies

Seed3_C = competition result of network seed 3 of conservative strategies

Seed3_A = competition result of network seed 3 of audacious strategies

Table 95: Comparisons of "lock-in" between conservative and audacious strategies (NP, Homo)

Rewiring probability	Seed1_C	Seed1_A	Seed2_C	Seed2_A	Seed3_C	Seed3_A
0.01	12	16	14	13	14	15
0.03	13	14	13	16	7	17
0.05	9	18	12	15	11	17
0.07	13	14	14	17	11	12
0.1	8	15	14	13	9	16

Table 96: Comparisons of "lock-in" between conservative and audacious strategies (P7, Homo)

Rewiring probability	Seed1_C	Seed1_A	Seed2_C	Seed2_A	Seed3_C	Seed3_A
0.01	11	18	11	19	5	18
0.03	7	18	9	20	8	15
0.05	9	18	9	19	9	15
0.07	4	19	7	18	11	16
0.1	3	18	8	20	9	15

Table 97: Comparisons of "lock-in" between conservative and audacious strategies (P8, Homo)

Rewiring probability	Seed1_C	Seed1_A	Seed2_C	Seed2_A	Seed3_C	Seed3_A
0.01	14	18	10	18	14	18
0.03	15	17	13	20	8	17
0.05	12	16	9	15	14	16
0.07	9	13	10	13	9	13
0.1	10	13	13	11	12	13

Table 98: Comparisons of "lock-in" between conservative and audacious strategies (NP, Hetero)

Rewiring probability	Seed1_C	Seed1_A	Seed2_C	Seed2_A	Seed3_C	Seed3_A
0.01	18	20	20	17	19	19
0.03	20	19	20	20	19	19
0.05	16	20	20	20	18	20
0.07	13	19	20	19	1	20
0.1	17	18	19	20	12	20

Table 99: Comparisons of "lock-in" between conservative and audacious strategies (P7, Hetero)

Rewiring probability	Seed1_C	Seed1_A	Seed2_C	Seed2_A	Seed3_C	Seed3_A
0.01	16	20	20	19	17	19
0.03	17	20	20	18	17	18
0.05	14	20	20	17	19	18
0.07	13	20	20	19	7	20
0.1	14	20	19	20	10	20

Table 100: Comparisons of "lock-in" between conservative and audacious strategies (P8, Hetero)

Rewiring probability	Seed1_C	Seed1_A	Seed2_C	Seed2_A	Seed3_C	Seed3_A
0.01	17	20	19	20	18	18
0.03	17	17	20	19	18	18
0.05	12	17	20	20	18	20
0.07	14	20	20	19	12	20
0.1	15	19	19	20	13	20

APPENDIX N: RESULTS OF AUDACIOUS MARKETING STRATEGIES IN MODEL 2-2

N-1 Homogeneous consumers

N-1-1 Experiments on non-piracy

Under NP, the experimental results of Model 2-2 are same as the results of Model 2-1 in the Appendix L-1.

N-1-2 Experiments on a pirate in L1

Experiments are conducted on 15 networks. In each network, consumer 7 in L1 is assigned as a pirate. Consumers are homogeneous.

Table 101: Competition results of networks of seed 1 (P7, Homo, Model 2-2)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	19	1						20	
0.03	19	1						20	
0.05	18	2						20	
0.07	19							20	
0.1	18	2						20	

Table 102: Competition results of networks of seed 2 (P7, Homo, Model 2-2)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	18	2						20	
0.03	18	2						20	
0.05	16	2			2			20	
0.07	19				1			20	
0.1	12	2			6			20	

Table 103: Competition results of networks of seed 3 (P7, Homo, Model 2-2)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	13	7						20	
0.03	17	2			1			20	
0.05	16	4						20	
0.07	19				1			20	
0.1	20							20	

N-1-3 Experiments on a pirate in L2

Experiments are conducted on 15 networks. In each network, consumer 8 in L2 is assigned as a pirate. Consumers are homogeneous.

Table 104: Competition results of networks of seed 1 (P8, Homo, Model 2-2)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	19	1						20	0
0.03	18	1		1				19	1
0.05	14			2	4			14	6
0.07	14	1			5			14	6
0.1	18			1	1			18	2

Table 105: Competition results of networks of seed 2 (P8, Homo, Model 2-2)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	18	2						19	1
0.03	16	1		2	1			16	4
0.05	14			5	1			14	6
0.07	11	3		5	1			11	9
0.1	17	1		2				18	2

Table 106: Competition results of networks of seed 3 (P8, Homo, Model 2-2)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	18	1		1				19	1
0.03	16	3			1			18	2
0.05	18	1			1			19	1
0.07	15	1		2	2			15	5
0.1	12	2		4	1		1	12	8

N-2 Heterogeneous consumers

N-2-1 Experiments on non-piracy

Under NP, the experimental results of Model 2-2 are same as the results of Model 2-1 in the Appendix L-2.

N-2-2 Experiments on a pirate in L1

Experiments are conducted on 15 networks. In each network, consumer 7 in L1 is assigned as a pirate. Consumers are heterogeneous.

Table 107: Competition results of networks of seed 1 (P7, Hetero, Model 2-2)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	15	5						20	
0.03	20							20	
0.05	19			1				20	
0.07	20							20	
0.1	20							20	

Table 108: Competition results of networks of seed 2 (P7, Hetero, Model 2-2)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	16	3			1			20	
0.03	16	4						20	

0.05	19			1				20	
0.07	20							20	
0.1	19				1			20	

Table 109: Competition results of networks of seed 3 (P7, Hetero, Model 2-2)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	15	4			1			20	
0.03	16	3			1			20	
0.05	17	1			2			20	
0.07	18	2						20	
0.1	19				1			20	

N-2-3 Experiments on a pirate in L2

Experiments are conducted on 15 networks. In each network, consumer 8 in L2 is assigned as a pirate. Consumers are heterogeneous.

Table 110: Competition results of networks of seed 1 (P8, Hetero, Model 2-2)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	18			2				18	2
0.03	19			1				19	1
0.05	18			2				18	2
0.07	17			3				17	3

0.1	18			2				18	2

Table 111: Competition results of networks of seed 2 (P8, Hetero, Model 2-2)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	20								
0.03	19			1				19	1
0.05	20							20	
0.07	19			1				19	1
0.1	20							20	

Table 112: Competition results of networks of seed 3 (P8, Hetero, Model 2-2)

Rewiring probability	P1 count			P2 count			Tie count	T1 count	T2 count
	L	W	A	L	W	A			
0.01	18			2				18	2
0.03	18			2				18	2
0.05	17			3				17	3
0.07	20							20	
0.1	20							20	

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