# QUANTIFYING RELEVANCE: THE DEVELOPMENT AND VALIDATION OF AN INITIAL SCALE TO MEASURE SECONDARY STUDENTS' PERCEPTIONS OF THE RELEVANCE OF MATHEMATICS CONTENT

by

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of
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Quantifying Relevance: The Development and Validation of an initial Scale to Measure Secondary Students' Perceptions of the Relevance of Mathematics Content

A Dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy at George Mason University

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#### **Dedication**

Gary, there aren't enough words to express my gratitude for you and for your unwavering support in this journey. However, I will try. Thank you for being a constant source of strength and being so supportive from the beginning when I got accepted into this program and until the very end. Thank you for providing me with unconditional love, for believing in me when I questioned myself, for drying my tears when I became overwhelmed, and for assuring me that my sacrifices would be worthwhile. You were there through all the tears, milestones, setbacks, and accomplishments constantly cheering me on. Without you none of this would have been possible. Thank you for helping me keep a positive perspective in life while you sacrificed for me to follow my dreams. Thank you for being my anchor and partner through everything life has thrown our way. We truly have been through it all during this program. Starting with the highs, we started this program planning our wedding, lived in 2 states, became parents to two princesses, traveled, ventured into real estate investing, and advanced in our careers. With the highs we have also experienced extreme lows. We supported one another while trying to mend our broken hearts from the loss of your mother, kept the faith while our first born was fighting for her life after being born at 25weeks, dealt with a pandemic, and you have been my anchor as I suffered a head on collision from a drunk driver leaving my ankle and shin broken and having to relearn to walk again. Out of everything we have been through we have always chose each other and you have been my light. You have taught me through your actions what unconditional love truly means. It's because I know that you will catch me if I fall that I am able to take risks. My heart belongs to you. WE did it!

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**Abstract** 

QUANTIFYING RELEVANCE: THE DEVELOPMENT AND VALIDATION OF AN INITIAL SCALE TO MEASURE SECONDARY STUDENTS' PERCEPTIONS OF THE RELEVANCE OF MATHEMATICS CONTENT

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Learning mathematics content is often portrayed as being irrelevant and not useful in secondary students' lives. The literature suggests that making mathematics content relevant to students' lives, their community, and society plays a vital role in their academic motivation, attitudes toward mathematics, engagement, academic achievement, and future course selection and careers. Furthermore, there has been a call to shift the direction of research on students of color and mathematics, given that research has highlighted the nature of their underachievement and failure in the subject over their successes and motivations for learning mathematics. Thus, this study is committed to understanding what it means to learn relevant mathematics while being Black and Hispanic. By learning through research what Black and Hispanic secondary students perceive as mathematically relevant to their lives, we can then tailor curriculum to strengthen positive student perceptions of mathematics. We can learn how Black and

Hispanic students can best attain and maintain excellence when learning mathematics. This dissertation details the development of a scale that measures secondary students' perceptions of their mathematics content as being relevant to their lives. A scale of this nature is needed given that mathematics relevance research is typically explored through qualitative methodologies, and the voices of secondary students of color who will be first-generation college students have largely been ignored in empirical research on achievement motivation. Thus, this study weaves together expectancy-value theory and teaching for social justice pedagogy to create a scale that will provide feedback on how understudied student populations—academically successful Black and Hispanic students—perceive the relevance of mathematics content. I conducted a literature review of expectancy value theory, social justice pedagogy, and existing measurement. I also interviewed students about their perceptions of mathematical relevance. I validated the instrument via expert validation and then engaged in item development and exploratory factor analysis. This study suggests how students respond to items measuring their perceptions of mathematics content as being relevant to their lives is directly related to the lived experiences of those students. This study will help increase our understanding of high-ability students from economically vulnerable families and their perceptions of mathematics content to help identify factors associated with their academic success.

**Keywords:** relevance; mathematics relevance; academic motivation; expectancy value theory; social justice pedagogy; scale development; measurement

# **Chapter One: Importance**

## **Introduction and Statement of the Problem**

The National Research Council (NRC, 2001) described the conditions and structures that they propose as necessary for all students to learn and be proficient in mathematics. They identified five interwoven strands of mathematical proficiency (Figure 1), the necessary components for any student to learn mathematics successfully.

#### Intertwined Strands of Proficiency

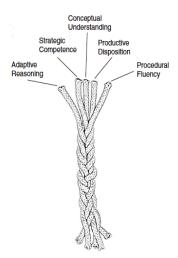


Figure 1
Strands of Mathematical Proficiency

Note. From Adding It Up: Helping Children Learn Mathematics, by National Research Council, 2001, National Academy Press (https://doi.org/10.17226/9822).

These strands encompass conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Conceptual understanding is defined as understanding mathematical concepts, understanding why they are important and useful, and being able to connect the procedures with concepts and different representations. Procedural fluency is defined as having the skillset to carry out procedures accurately and efficiently. Strategic competence means understanding how to formulate, represent, and solve mathematical problems. Adaptive reasoning is having the ability to think logically about mathematical relationships. The NRC broadened how mathematics education defined successful learning of mathematics by including an affective dimension: productive disposition. Productive disposition is defined as "the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics" (NRC, 2001, p. 131).

The NRC incorporated productive disposition into the definition of mathematical proficiency given the argument that students' attitudes toward mathematics can influence mathematical thinking (NRC, 2001), performance (Means et al., 1997; Wooley et al., 2013), motivation (Beyers, 2001; Hubert, 2014; Means et al., 1997), engagement (Gutstein, 2003; Wooley et al., 2013), and students' future opportunities and decisions (NRC, 2001; National Council of Teachers of Mathematics [NCTM], 1989, 2018). Students need to be able to do more than understand, compute, solve, and reason in mathematics. To be mathematically proficient, students need to also believe that

mathematics can be used outside of the classroom.

Productive disposition is a fairly new research topic within mathematics education in the context of teaching and learning. Philipp and Siegfried (2015) reviewed research that has been done on productive disposition and identified three themes: (a) pursuing an area of interest that is connected to productive disposition; (b) using productive disposition as a lens through which to view their area of interest; and (c) directly studying productive disposition. The majority of the research has centered on productive disposition for teaching, specifically focusing on the relationship between teachers' knowledge, pedagogical beliefs, and practice. However, limited research focuses on the utility aspect of productive disposition: the tendency to perceive mathematics as useful and worthwhile.

Despite NRC's (2001) emphasis on productive disposition as an important component of becoming mathematically proficient, the majority of qualitative research suggests secondary students perceive school mathematics as meaningless (Boaler, 2000; Brown et al., 2008; Matthews, 2018; Murray, 2011; Onion, 2004; Sealey & Noyes, 2010). For example, Murray (2011) interviewed secondary students about their perspectives on why there is declining participation in secondary school mathematics and their potential solutions to the problem. Students reported that they viewed mathematics as irrelevant, not practical, and not useful in real life. Similarly, findings from student focus groups suggest that students perceive that mathematics is only useful for classwork and exams (Onion, 2004), and that the nature of their mathematics content is meaningless (Boaler, 2000). Teachers often hear their students ask, "Why do I have to learn math?" or

"When will I ever use this?" Specifically, when Black and Latino students ask these same questions, they may not only question when they will use math in the real world, but they may be questioning how mathematics can inform who they are and who they can become, and want to understand how mathematics can help them navigate their immediate environment (Matthews, 2018). These findings suggest the need to make mathematics education relevant to students' lives.

The need for students to view mathematics as useful and worthwhile is not a new topic of interest, but has spanned several decades. In the late 1960s, mathematician Freudenthal (1968) argued that students do not apply their mathematical classroom experiences in other subjects or in their daily lives. He urged for mathematics to be connected to reality so that students have the opportunity to make sense of the real world through mathematics. More than five decades later, researchers and journalists are still highlighting that students are lacking the connection between school mathematics and why it is useful and worthwhile outside of the classroom. NCTM convened a group of experts to highlight the grand challenges facing mathematics education that research should address (Stephan et al., 2015). Experts mentioned changing the public's perception about the role of mathematics in society, and stressed the importance of people viewing mathematics as something that human beings normally do and that has relevance. More recently, NCTM (2018) addressed ways to promote change in high school mathematics learning and instruction, and has initiated the critical conversation around expanding the purposes of mathematics education. They highlight that the purpose of learning mathematics goes beyond students' preparation for postsecondary

education or career readiness, and is for students to use mathematics to understand and critique the world, to have an appreciation of the usefulness of mathematics.

A closely researched term, *relevance*, has received a considerable amount of attention in the literature. Within the field of mathematics education, relevance is conceptualized and operationalized in multiple ways; thus, there is no generally accepted definition or model of relevance.

# **Defining Mathematics Relevance**

I operationalized *mathematics relevance* using several sources of literature.

Mathematics relevance is defined as the extent to which students perceive mathematics content to possess at least one of the following characteristics (a) practical utility in students' everyday lives (Eccles [Parsons] et al., 1983; Sealey & Noyes, 2010); (b) exchange value in preparing students' for future careers (Eccles [Parsons] et al., 1983; Wooley et al., 2013); and (c) illuminating social inequities that exist within students' communities and society (Dover, 2009; Gutstein, 2003, 2007, 2013).

# Relevance Emerging from Science Education

When it comes to the topic of relevance, a number of studies originated within the field of science education (Newton, 1988; Roberts & Henke, 1997; Siegel & Ranney, 2003; Stuckey et al., 2013). Dating back to the 1980s, Newton (1988) defined relevance in science education as a pedagogical device for enhancing interest, motivation, learning, and retention. This work originated from the critique of researchers who believed that students were not seeing the utility of science content and how it could be applied outside of school. Newton urged for a consistent and unambiguous definition of relevance in

science education to better align the goals of policy makers, educators, and researchers to make science more meaningful for students. Building off this earlier work, Stuckey et al. (2013) made the first attempt at connecting different dimensions of relevance into a single model. The result of their analysis was that relevance consisted of three different dimensions: (a) individual, for understanding scientific phenomena and learners' everyday life; (b) societal, for becoming effective future citizens in society; and (c) vocational, preparing students for potential careers in science and engineering (Stuckey et al., 2013). These three dimensions of relevance encompass present and future components as well as intrinsic and extrinsic components.

This study models Stuckey et al. (2013) by providing a critical analysis of the term *relevance* within mathematics education. This study provides an overview of how the term is used and operationalized in mathematics education, discusses theories of relevance, and makes the first attempt at connecting the different dimensions into one single model in order to clarify the ambiguity of the term in the field of mathematics education. From this, I created a scale that measures secondary students' perceptions of their mathematics content as being relevant to their lives.

### Relevance Within Mathematics Education

In mathematics lessons, educators present content that they believe is important, is worth knowing, and makes learning mathematics richer. Lessons are typically grounded in state and district standards. However, research has shown that despite this alignment to standards, students rarely find mathematics relevant to their everyday lives, communities, and future decisions (Boaler, 2000; Brown et al., 2008; Matthews, 2018;

Murray, 2011; Onion, 2004; Sealey & Noyes, 2010). If students were to develop an awareness of the relevance of mathematics, it could influence their motivation, achievement, and attitudes toward the subject (Boaler, 2000; Gaspard et al., 2015; Gutstein, 2003; Ladson-Billings, 1994; Musto, 2008; Onion, 2004; Sealey & Noyes, 2010; Wooley et al., 2013).

Within the literature, the four components of mathematics relevance are discussed in great detail. Researchers have defined mathematics relevance as the significance of mathematics in students' lives, specifically in their current or future lives (Boaler, 2000; Gaspard et al., 2015; Hulleman et al., 2010; Murray, 2011; Onion, 2004; Sealey & Noyes, 2010). This definition is very broad and describes the meaning making of the subject and applying learned mathematics content in everyday life and in the future. Researchers have also discussed mathematics relevance in terms of the exchange value of mathematics (Musto, 2008; Onion, 2004; Sealey & Noyes, 2010; Wooley et al., 2013). The exchange value is in terms of students perceiving that learning mathematics content would help them access STEM fields and financial security. Sealey and Noyes's (2010) findings reveal that students perceive learning mathematics content as providing skills that translate into other subjects (e.g., process relevance). Specifically, learning mathematics helps students with logical thinking, which is a skill that can be used in all subjects. Process relevance was geared to helping students think more logically and become problem solvers in any area of their lives.

The last dimension of relevance is helping students understand the significance of mathematics in recognizing, analyzing, and critiquing inequities within society.

Culturally relevant pedagogy (CRP) is a theory developed by Ladson-Billings (1992, 1994, 1995a, 1995b, 2014) and emerged from the studying of successful educators of Black students. These educators supported students in becoming academically successful by utilizing their students' cultures as a vehicle for learning. Ladson-Billings (1995) defines culturally relevant pedagogy as supporting students who can (1) achieve academically; (2) maintain their cultural integrity; (3) extend learning beyond the classroom and teach students to recognize, analyze, and critique social inequities that exist within their communities and society as a whole. CRP has shifted the view of teaching and learning to ways that encourage students are welcome to bring their whole selves into the classroom and be academically successful. By affirming students cultures and developing skills of social political consciousness, educators can better meet the needs of our most vulnerable students. Specifically, focusing on extending the learning beyond the classroom and building students critical intellectualism, social justice pedagogy has evolved.

SJP opens up opportunities for students to use mathematics as a tool to analyze the world and issues like racism, sexism, gentrification, and resource inequities. SJP, also known as teaching for social justice, changes the sense of how mathematics is taught: in addition to mathematics-related objectives, mathematics is connected to students' everyday lives and they learn that mathematics is not just for math class, but has a lot more meaning. Furthermore, SJP is intended to enhance equity within the classroom across social identity groups (e.g., race, class, gender, sexual orientation, ability), to foster critical thinking, and to promote social action (Carlisle et al., 2006). Teaching for

social justice within K-12 classrooms must follow the six-principle framework (see Figure 2).

# Principles of Teaching for Social Justice in K-12 Classrooms

- Assume all students are participants in knowledge construction, have high expectations for students and themselves, and foster learning communities;
- Acknowledge, value, and build upon students' existing knowledge, interests, cultural and linguistic resources;
- 3) Teach specific academic skills and bridge gaps in students' learning;
- Work in reciprocal partnership with students' families and communities;
- 5) Critique and employ multiple forms of assessment; and
- Explicitly teach about activism, power, and inequity in schools and society

Figure 2

Principles of Teaching for Social Justice in K-12 Classrooms

Dover (2009) discusses how this framework is an adaptation of Cochran-Smith's (1999, 2004) six principles of socially-just teacher education. It integrates multiple aspects of equity and justice-oriented reforms, which research shows has a positive effect on students' academic, motivational, and attitudinal outcomes. The SJP framework is

grounded in principles of social justice education (e.g., Adams et al., 1997, 2007; Carlisle et al., 2006; Lalas, 2007; Poplin & Rivera, 2005; Shakman et al., 2007), culturally responsive education (e.g., Gay, 2000; González et al., 2005; Irvine & Armento, 2001; Ladson-Billings, 1995; Murrell, 2000, 2001; Villegas & Lucas, 2002), multicultural education (e.g., Banks, 1995; Grant & Sleeter, 2007; Nieto, 1999, 2002; Sleeter & Grant, 1999; Suzuki, 1984), critical pedagogy (e.g., Frankenstein, 1990; Freire, 1970/2002), and democratic education (e.g., Dewey, 1916/2007; Parker, 2003; Westheimer & Kahne, 2004; Woodruff, 2005). The six principles require (a) teachers to have high expectations of their students and themselves (Carlisle et al., 2006; Cochran-Smith, 1999, 2004; Poplin & Rivera, 2005); (b) teachers to value and build upon students' existing knowledge and interests; (c) teachers to focus on developing students' academic skills (Darling-Hammond et al., 2002; Poplin & Rivera, 2005; Shakman et al., 2007); (d) teachers to foster learning communities (Cochran-Smith, 1999, 2004; Villegas & Lucas, 2002); (e) teachers to employ multiple forms of assessment; and (f) teachers to explicitly teach about the problems within the U.S. educational systems and society and to engage students in developing critical habits of mind, understanding multiple perspectives, and learning to be advocates for change.

A growing body of literature addresses what teaching for social justice looks like in mathematics classrooms and its impact on students' academic, behavioral/motivational, and attitudinal outcomes (e.g., Gutstein, 2003, 2007, 2013). Gutstein (2003, 2007, 2013) studied how he used SJP as a way to make mathematics relevant to students' lives. His goals for SJP included helping students (a) develop social

and political consciousness, (b) develop a sense of agency, and (c) develop positive social and cultural identities. The mathematics-specific goals included developing students' ability to (a) use mathematics to understand their sociopolitical context; (b) confidently engage in complex mathematical tasks; (c) communicate their ideas and results effectively; (d) value mathematics and engage actively in learning it; and (e) have a positive disposition toward mathematics. Within Gutstein's research, students were given opportunities to discuss racism, discrimination, power, and injustices within their communities and society as a whole. An example project was understanding neighborhood displacement, where 12th-grade students used mathematics concepts from precalculus, algebra, discrete mathematics, and statistics along with interpreting data, graphs, pictures, maps, and text to better understand the causes and roots of displacement in their neighborhoods (Gutstein, 2013). Resulting from Gutstein's research, when mathematics is taught in a way that connects students' lives to the content and demonstrates how mathematics can be used as a tool to critique students' communities and society, students begin to value and develop positive dispositions toward mathematics (e.g., view mathematics as a relevant tool, connected to their lives and experiences).

Gutstein (2003, 2007) identified academic and attitudinal outcomes from teaching social justice in mathematics classrooms. He found increases in (a) students' confidence in their mathematical abilities, (b) students' ability to apply mathematical reasoning and problem-solving skills to real-world contexts, (c) students' ability to pass eighth-grade mathematics courses and standardized tests, and (d) students' sociopolitical

consciousness. Furthermore, students changed their sense that mathematics was useful outside of math class. In fact, students began to believe that mathematics could be used to solve problems in the world, that mathematics had more meaning and could help explain injustices (Gutstein, 2003).

In summary, there are four dimensions suggested for defining *mathematics* relevance:

- Relevance for helping students understand the significance of mathematics in their own lives;
- 2. Relevance for preparing students for potential careers in STEM;
- 3. Relevance for students becoming logical thinkers across all subjects;
- 4. Relevance for empowering students to merge culture and learning, and to critique societal inequalities.

Building from prior research, I define *mathematics relevance* as the extent to which students perceive mathematics content to possess at least one of the following characteristics: (a) practical utility in students' everyday lives (Eccles [Parsons] et al., 1983; Sealey & Noyes, 2010), (b) exchange value in preparing students for future careers (Eccles [Parsons] et al., 1983; Wooley et al., 2013), or (c) illuminating social inequities that exist within students' communities and society (Dover, 2009; Gutstein, 2003, 2007, 2013). My definition includes all four dimensions of mathematics relevance found in the literature; process relevance, helping students think more logically and become problem solvers in any area of their lives, is included in the definition of practical utility. Students' everyday lives include their experiences within school as well as outside of school, where

students define relevance as being able to transfer skills across all subjects.

## **Student Perceptions of Relevance**

The research on students' perceptions of mathematics has been typically qualitative and centered on focus groups of secondary students (e.g., Boaler, 2000; Brown et al., 2008; Onion, 2004; Murray, 2011; Sealey & Noyes, 2010). The research questions that drove these studies have focused on how students perceive their mathematics classrooms and how their perceptions might influence future course enrollment, career aspirations, and knowledge production and use (Boaler, 2000; Brown et al., 2008; Murray, 2011; Onion, 2004; Sealey & Noyes, 2010). Research suggests that secondary students view mathematics as irrelevant, not useful, and not practical in reallife (Boaler, 2000; Brown et al., 2008; Murray, 2011; Onion, 2004). Matthews (2018) studied over 400 Black and Latino secondary students' perceptions of learning mathematics, and his findings revealed that students acknowledged mathematics was important and had practical uses like counting and money purposes and to attain STEM careers; however, they struggled to find its meaning in their everyday lives. Furthermore, students in another study expressed how they were unaware of the importance of mathematics content (Brown et al., 2008; Murray, 2011) and how they found basic mathematics useful in their daily life, unlike their high school mathematics content (Onion, 2004). Even students who received high grades in mathematics ruled out taking additional mathematics courses due to their perceptions that the subject was not useful for their future degree or in life (Brown et al., 2008). Somehow, messages about the importance and usefulness of mathematics are not being translated or believed by

secondary students, impacting their future course decisions and careers.

Within the expectancy-value theory (EVT) literature, researchers have examined the relationship between relevance and utility. EVT is situated in achievement motivation and provides a lens to understand how students' perceptions of mathematics content can influence their achievement, choices, and persistence in the subject. To be optimally motivated and engaged in a task, students need to both feel confident that they can achieve success (e.g., expectancy) and believe that what they are learning is worthwhile (e.g., subjective task value; Eccles [Parsons] et al., 1983). Among the four aspects of subjective task values, research suggests relevance directly affects students' sense of utility value (Hulleman et al., 2010).

Teasing apart relevance and utility value, research has shown that students' dispositions toward the subject influence their motivation, achievement, and attitude toward mathematics (Boaler, 2000; Gaspard et al., 2015; Musto, 2008; Onion, 2004; Sealey & Noyes, 2010; Wooley et al., 2013). Research has also shown that utility value has been associated with course enrollment decisions (Durik et al., 2006; Updegraff et al., 1996), leisure time activity choices (Durik et al., 2006), interest in specific school subjects (Harackiewicz et al., 2008; Hulleman et al., 2008), and performance (Bong, 2001; Cole et al., 2008; Durik et al., 2006; Hulleman et al., 2008; Mac Iver et al., 1991; Simons et al., 2004). For example, fifth- through 12<sup>th</sup>-grade students' values of mathematics predicted their course enrollment intentions (Eccles [Parsons], 1984; Wigfield & Eccles, 1989). Additionally, eighth- through 10<sup>th</sup> grade students' values of mathematics strongly predicted their actual decisions for course enrollment later in their

high school careers (Eccles, 1984), and utility value specifically has been found to predict college students' intentions to enter graduate school (Battle & Wigfield, 2003).

Updegraff et al. (1996) tested the expectancy-value model for predicting high-school mathematics course enrollment and found perceived task utility yielded the strongest predictor with number of high school math courses taken, even after current achievement levels were controlled. Given how students' values impact their future intentions and decisions for course enrollment and future careers, it is important to know what influences how students value tasks. Thus, an examination of the role of mathematics relevance and task value is warranted.

When relevance and utility value have been studied together, research has shown that relevance interventions in the classroom influence students' value beliefs, interest, and performance (Gaspard et al., 2015; Hulleman et al., 2010). When students are provided with various examples of the relevance of mathematics in everyday life and future careers, it fosters positive value beliefs (e.g., higher utility value; Gaspard et al., 2015; Hulleman et al., 2010; Hulleman & Harackiewicz, 2009), engagement (Hidi & Harackiewicz, 2000; Hidi & Renninger, 2006), interest (Hulleman et al., 2010; Hulleman & Harackiewicz, 2009), and performance (Eccles & Wigfield, 1995; Hulleman et al., 2010). Thus, encouraging students to discover the relevance of what they are learning increases their perceived utility value.

A common finding across the literature on student perceptions of mathematics is that students believed *how* mathematics was taught influenced their attitudes (Boaler, 2000; Murray, 2011; Sealey & Noyes, 2010). In other words, in order to have positive

learning experiences in mathematics, meaning making was central. Classroom context impacted students' perceptions of mathematics content being relevant (Boaler, 2000; Hubert, 2014; Ladson-Billings, 1995, 2014; Sealey & Noyes, 2010; Tate, 1995). Students expressed having no opportunities in class for meaning making or to connect how the mathematics learned in the classroom could be applied to their everyday lives. Students expressed that classroom practices heavily enforced rote learning and memorization with lack of connections, which in turn impacted their perceptions. In turn, students' attitudes toward mathematics impacted their future mathematics course enrollment (Boaler, 2000; Murray, 2011). Drawing from EVT of achievement motivation, the values that students place on tasks directly influence their choices, effort, and persistence (Eccles, 2005; Eccles [Parsons] et al., 1983; Simpkins et al., 2006; Wigfield & Eccles, 2000; Wigfield et al., 2009). Additionally, drawing from Gutstein's (2003, 2007, 2013) work, how students learn mathematics content influences their dispositions toward mathematics as well as their academic achievement.

Across the highlighted studies, students had opportunities to provide possible solutions to help change their perceptions of the subject. They suggested having educators demonstrate the importance of mathematics by connecting what they were learning in the classroom to their everyday lives and the lives of others. The literature supports that lack of mathematics relevance is a factor in students not continuing with mathematics and choosing alternative careers (Brown et al., 2008; Murray, 2011). It appears that students need to experience a curriculum that targets conveying the relevance of mathematics content within the classroom.

#### **Need for New Measurement**

With an increased emphasis on promoting the relevance of mathematics content comes a need to assess how students perceive these efforts. Earlier studies of relevance primarily focused on assessing students' perceptions of mathematics relevance—in other words, ways to make mathematics curriculum relevant to students' lives—and how these interventions influence students' beliefs (e.g., utility value), mathematical thinking, mathematics achievement, and interest. Extensive reviews of the literature yielded no instruments that measure secondary students' perceptions of the relevance of mathematics content, where *relevance* is defined as the extent to which students perceive mathematics content to possess at least one of the following characteristics: (a) practical utility in students' everyday lives (Eccles [Parsons] et al., 1983; Sealey & Noyes, 2010); (b) exchange value in preparing students for future careers (Eccles [Parsons] et al., 1983; Wooley et al., 2013); and (c) illuminating social inequities that exist within students' communities and society (Dover, 2009; Gutstein, 2003, 2007, 2013). The interventions research did not define mathematics relevance to include illuminating social inequities that exist within students' communities and society (Gaspard et al., 2015; Hulleman, 2010; Hulleman & Harackiewicz, 2009).

It is important to depict the similarities and differences between mathematics relevance and utility value for the purposes of this study. Utility value, situated in EVT, can refer to short- and long-term goals in a variety of life domains, including school, daily life, and social life. On the other hand, I am situating mathematics relevance in both EVT *and* SJP, unlike utility value. Thus, mathematics relevance can refer to students'

short- and long-term goals as well as expanding the idea that mathematics can be used as a tool to critique societal inequities. Developing a scale that includes this added part of the definition is imperative.

Earlier studies of secondary students' perceptions of their mathematics learning have been primarily qualitative (Boaler, 2000; Brown et al., 2008; Murray, 2011; Onion, 2004; Sealey & Noyes, 2010). Data collection has been primarily drawn from student interviews and focus groups where the research has been situated outside of the United States (e.g., Australia, England; Boaler, 2000; Brown et al., 2008; Onion, 2004; Murray, 2011; Sealey & Noyes, 2010). It is important to run a quantitative analysis with larger sample sizes in order to generalize the common perceptions among secondary students within the United States. Furthermore, along with studies selecting high-achieving schools to collect their data, the vast majority of research has been conducted with White students who are in the middle to upper class. Far less is known about African American and Hispanic students' perceptions of the relevance of mathematics content, particularly those who will be first-generation college students upon graduation. Research suggests students' dispositions toward mathematics directly influence their future intentions and decisions (Eccles, 1984, 2005; Eccles [Parsons], 1984; Eccles [Parsons] et al., 1983; Simpkins et al., 2006; Wigfield & Eccles, 1990; Wigfield et al., 2005). Thus, it is imperative to increase our understanding of the psychological and social factors that influence the success of high-ability students from economically vulnerable families who will become first-generation college students. It is important to learn about their perceptions of their mathematics content to help identify factors associated with their

academic success. Lastly, prior research has analyzed how relevant mathematics tasks can influence students' value beliefs (Gaspard et al., 2015; Hulleman et al., 2010; Hulleman et al., 2017), attitudes toward mathematics (Musto, 2008); engagement (Means et al., 1997; Wooley et al., 2013), interest (Hulleman et al., 2010, 2017), mathematical achievement (Keller, 1987; Means et al., 1997; Wooley et al., 2013), and sociopolitical consciousness (Bonner & Adams, 2012; Dover, 2009; Gutstein, 2003, 2007, 2013; Hubert, 2014; Tate, 1994). However, there is no instrument that measures mathematics relevance directly, only how exposing students to relevant mathematics tasks can influence and predict these outcomes. As in the case of students' perceptions of the relevance of mathematics content, clarifying the concept of mathematics relevance and its definition is crucial in exhausting the factors that can influence students' value beliefs, attitudes toward mathematics, engagement, interest, mathematics thinking, mathematical achievement, and sociopolitical consciousness.

# **Purpose of the Study**

The purpose of this study is to develop an instrument that measures secondary students' perceptions of mathematics content as relevant to their lives. *Mathematics relevance* is defined as the extent to which students perceive mathematics content to possess at least one of the following characteristics: (a) practical utility in students' everyday lives (Eccles [Parsons] et al., 1983; Sealey & Noyes, 2010), (b) exchange value in preparing students for future careers (Eccles [Parsons] et al., 1983; Wooley et al., 2013); and (c) illuminating social inequities that exist within students' communities and society (Dover, 2009; Gutstein, 2003, 2007, 2013). Useful theoretical and pedagogical

frameworks for understanding the role of mathematics relevance in student academic motivation, achievement, and future decisions are the EVT framework (Eccles [Parsons] et al., 1983) and SJP (Dover, 2009; Gutstein, 2003, 2007, 2013). Results from this scale are intended to inform curriculum writers and educators about ways to increase Black and Hispanic secondary student motivation and engagement while learning mathematics. Black and Hispanic students were chosen as the focus of this study in response to a call to shift the direction of research on Black students and mathematics, given that the nature of their underachievement and failure in the subject has been highlighted in research over their successes and motivations for learning mathematics (Martin, 2012). Thus, this study is committed to understanding what it means to learn mathematics while Black and Hispanic.

- a. The goals of this study are to,
  - Articulate the need for the Mathematics Relevance Scale (MRS);
  - Describe the creation of the instrument; and
  - Assess the instrument's construct validity by way of
    - a. Content validity
    - b. Substantive validity
    - c. Structural validity
    - d. Generalizability validity
    - e. External validity

### **Significance and Contributions**

In the field of mathematics education, educators and researchers share a collective interest in making mathematics relevant to students' lives. However, researchers have defined the term *mathematics relevance* in different ways, which has led to a lack of clarity in the field. After a thorough review of the literature, this study is the first attempt at conceptualizing and operationalizing the term *mathematics relevance*.

Furthermore, there are significant gaps in the literature. Primarily, data have been collected in Australia and England, which raises an issue with generalizability within the United States. Moreover, the majority of the research has been qualitative, with data primarily collected from focus groups. Extensive reviews of the literature yielded no instruments that measured secondary students' perceptions of the relevance of mathematics content; thus, this study will fill that gap.

Additionally, the voices of students of color who will be the first to attend college in their families have largely been ignored in empirical research on achievement motivation. Research suggests that when students place high value on tasks, it directly influences their intentions and decisions to enroll in additional mathematics courses (Eccles, 1984, 2005; Eccles [Parsons], 1984; Eccles [Parsons] et al., 1983; Simpkins et al., 2006; Wigfield & Eccles, 1990; Wigfield et al., 2005). Furthermore, taking advanced mathematics courses in high school is associated with a higher rate of enrollment in a 4-year institution (Choy, 2001). Given that more than 3.4 million students who achieve in the top quartile academically are low income (Wyner et al., 2008), this study will help increase our understanding of high-ability students from economically vulnerable

families and their perceptions of their mathematics content in turn to help identify factors associated with their academic success.

Thus, I integrated mathematics education, educational psychology, and SJP to create a scale that in turn will provide feedback on how this understudied student population perceives the relevance of mathematics content. The results from the scale will be especially critical given that students' beliefs about mathematics are important determinants regarding their future involvement and performance in mathematics classes and careers. Understanding students' perceptions of the relevance of their mathematics content provides a means to more effectively cater professional development for their teachers. With the creation of the MRS, researchers can attempt to draw a causal link between secondary students' perceptions of mathematics relevance, academic motivation, and achievement. Further, if this relationship can be established, then researchers can attempt to design professional development experiences for both pre-service and inservice teachers to increase students' mathematics relevance in an attempt to increase motivation, engagement, and achievement in mathematics.

### **Potential Limitations**

Despite my efforts to be thorough in the development of the MRS, there are some noteworthy limitations to my research. First, there were some limitations in the student interviews. I asked students to reflect on how they defined mathematics relevance and to discuss a specific lesson or project in their mathematics class that they believed had relevance to their lives. Second, I chose the voices of students of color who would be first-generation college students at the completion of their college preparatory program

and graduation of high school. The students who have been admitted into this program are unique in their perceptions of mathematics relevance and motivational attributes. The development of the scale, its factors, and its relationships to outcomes could be different with other samples of students. An important and necessary component area of future research for scale development is verifying the factor structure of the items with other samples (e.g., confirmatory factor analysis) and investigating the invariance of the MRS with students of color who will be first-generation college students but are not part of a college preparatory program.

### **Definition of Key Terms**

Mathematics Relevance: The extent to which students perceive mathematics content to possess at least one of the following characteristics: (a) practical utility in students' everyday lives (Eccles [Parsons] et al., 1983; Sealey & Noyes, 2010); (b) exchange value in preparing students for future careers (Eccles [Parsons] et al., 1983; Wooley et al., 2013); or (c) illuminating social inequities that exist within students' communities and society (Dover, 2009; Gutstein, 2003, 2007, 2013).

Expectancy-Value Theory: Students' academic motivation depends on both expectation and value. Students are motivated to engage in a task that they expect to be able to perform successfully and that they value (Eccles [Parsons] et al., 1983; Wigfield & Eccles, 1992).

Social Justice Pedagogy: "The conscious and reflexive blend of content and process intended to enhance equity across multiple social identity groups (e.g., race,

class, gender, sexual orientation, ability), foster critical perspectives, and promote social action" (Carlisle et al., 2006, p. 57).

Subjective Task Values: The perceived importance of a task because (a) it is useful or relevant for other tasks or aspects of an individual's life (utility value); (b) it is enjoyable and fun to engage in (intrinsic value); (c) doing well on the activity influences the individual's self-concept, self-worth, and identity (attainment value); and (d) there are perceived negative aspects of engaging in the activity (cost value; Eccles [Parsons] et al., 1983).

# **Chapter Two: Literature Review**

## **Search of Existing Literature**

Ancestry and descendent searches were conducted as well as previous literature reviews and meta-analyses to locate relevance in mathematics education studies. My criteria for inclusion were studies conducted over the last five decades (1965–2021) and involving secondary and first-generation students. The ancestry search was conducted by reviewing reference sections in selected publications to determine if additional studies should be included. The following electronic databases were searched: The Social Sciences' 134 Citation Index, PsychInfo, APA PsycNet, Education Full Text, Academic Search Complete, Education Research Complete, Science Direct, and ERIC. Each database was searched using the following keywords: mathematics, mathematics education, productive disposition, relevance, usefulness, meaningfulness, teaching for social justice, social justice pedagogy, expectancy-value theory, and utility value. In addition, a search was conducted of journals in the field of mathematics education and educational psychology that commonly publish studies about student dispositions toward mathematics (e.g., Journal for Research in Mathematics Education, Journal of Mathematical Behaviour, Developmental Psychology, American Education Research Journal, Equity & Excellence in Education, Journal of Educational Psychology, and Journal of Research on Adolescence). Only research published in peer-reviewed journals was considered. The screening process of selecting articles entailed reviewing article abstracts, descriptions of participants, relationships among constructs, and measures to

determine if studies met inclusion criteria. Studies that met inclusion criteria were then thoroughly reviewed. Studies were excluded if (a) the study was not peer-reviewed (e.g., dissertation, master's thesis) or (b) the study focused on relevance in literacy and social studies.

#### **Theories of Mathematics Relevance**

### Expectancy-Value Theory

EVT, a theory situated in achievement motivation, provides a lens to understand how students' perceptions of mathematics content can influence their achievement, choices, and persistence in the subject. Motivation is a key factor for student success in mathematics. The domain of motivation is concerned with answering questions such as why some students engage deeply, enjoy learning, and perform better than others (Kaplan et al., 2012). It is the study of why individuals behave as they do (Graham & Weiner, 2012). A review of the expectancy-value literature will give greater detail on the relationship between mathematics relevance and the utility of mathematics, and how they play a seminal role in motivating students in the subject.

Students' achievement motivation depends on both expectation and value (Eccles [Parsons] et al., 1983). Students are motivated to engage in a task that they *expect* to be able to perform successfully and that they *value*. *Value* is defined as the perceived importance of the task because (a) it is useful or relevant for other tasks or aspects of an individual's life (utility value); (b) it is enjoyable and fun to engage in (intrinsic value); (c) doing well on the activity influences the individual's self-concept, self-worth, and identity (attainment value); and (d) there are perceived negative aspects of engaging in

the activity (cost value), such as effort, fear of failure, or performance anxiety (Eccles [Parsons] et al., 1983). To be optimally motivated and engaged in a task, students need to both feel confident that they can achieve success (e.g., expectancy) and believe that what they are learning is worthwhile (e.g., subjective task value). The majority of the work that Eccles and her colleagues have done has focused on students' decisions whether to continue studying mathematics and the factors that influenced those decisions (Wigfield & Eccles, 1992). Eccles and her colleagues have carried out three longitudinal studies investigating how elementary through high school students' task values change over time and how those values relate to performance and choice. As students transitioned from elementary school through 12<sup>th</sup> grade, the values they placed on mathematics declined, which in turn impacted their performance, persistence, enrollment, and career choices (Eccles et al., 1989, 1993; Eccles & Wigfield, 1995; Eccles [Parsons] et al., 1983; Fredericks & Eccles, 2002; Jacobs et al., 2002; Meece et al., 1990; Watt, 2004; Wigfield et al., 1991, 1997; Wigfield & Eccles, 2000).

What Does Task Utility Value Predict? The values that students place on tasks directly influence their choices, effort, and persistence (Eccles, 2005; Eccles [Parsons] et al., 1983; Simpkins et al., 2006; Wigfield & Eccles, 2000; Wigfield et al., 2005). More specifically, placing high value on tasks directly influences their intentions and decisions to enroll in additional mathematics courses (Eccles, 1984, 2005; Eccles [Parsons], 1984; Eccles [Parsons] et al., 1983; Simpkins et al., 2006; Wigfield & Eccles, 1990; Wigfield et al., 2005). For example, fifth- through 12<sup>th</sup>-grade students' values of mathematics predicted their course enrollment intentions (Eccles [Parsons], 1984; Wigfield & Eccles,

1989). Additionally, eighth- through 10<sup>th</sup>-grade students' values of mathematics strongly predicted their actual decisions for course enrollment later in their high school careers (Eccles, 1984), and utility value specifically has been found to predict college students' intentions to enter graduate school (Battle & Wigfield, 2003). Updegraff et al. (1996) tested the expectancy-value model for predicting high-school mathematics course enrollment and found perceived task utility yielded the strongest predictor with number of high school math courses taken, even after current achievement levels were controlled. Given how students' values impact their future intentions and decisions for course enrollment and future careers, it is important to know what influences how students value tasks. Thus, an examination of the relationship between mathematics relevance and task value is warranted.

### **Mathematics Relevance Interventions**

The approaches to studying how to make mathematics curricula relevant can be addressed in three distinct ways: (a) students reading and evaluating texts about the relevance of mathematics, (b) incorporating occupational examples within lesson content regarding how mathematics is used outside of the classroom, and (c) fusing culture within learning mathematics. These approaches have begun to answer ways to influence students' value beliefs, sociopolitical consciousness, mathematical thinking, mathematics achievement, and academic motivation. Studies of students evaluating texts have been conducted (Gaspard et al., 2015; Hulleman et al., 2010; Hulleman & Harackiewicz, 2009; Hulleman et al., 2017; Musto, 2008), along with intervention studies incorporating occupational examples and awakening sociopolitical consciousness have been carried out

(Bonner & Adams, 2012; Gutstein, 2003, 2006, 2007, 2013; Hubert, 2014; Means et al., 1997; Tate, 1995; Wooley et al., 2013).

Prior research (Gaspard et al., 2015; Hulleman et al., 2010, 2017; Hulleman & Harackiewicz, 2009; Musto, 2008) examined whether secondary students' value beliefs in mathematics could be promoted through reading quotations and writing text about the relevance of mathematics. The quotes described how young adults applied classroom mathematics in their careers (e.g., thermofluids, engineering, meteorologist, organic pizza owner) in an effort to help secondary students see the relevance of the mathematics they experienced in school. Students' attitudes toward mathematics improved, along with their awareness of the relevance of mathematics needed in careers and everyday life (Musto, 2008). Similarly, Hulleman et al. (2010) examined whether undergraduate students' value beliefs in mathematics could be manipulated through writing about the four-step method for solving two-digit multiplication problems. Students were instructed to describe how this technique had relevance to their lives and the lives of other college students. Findings revealed this intervention had the strongest effects on students with low performance expectations; these students experienced positive effects on their interest and performance when they perceived value in the mathematics course material. These results built upon Hulleman and Harackiewicz's (2009) intervention with ninth-graders that encouraged students to make connections between course material and their lives. The findings were similar: students who experienced making connections between learned content and their lives showed increased interest and performance, particularly those who were most at risk for being disengaged in school. Hulleman et al. (2017) found that the

frequency with which undergraduate students connected material to their lives was a unique and significant predictor of increased interest and students' perceived utility value in the course. Overall, findings revealed that relevance interventions had positive effects on (a) student learning (Hulleman & Durik, 2008; Hulleman & Harackiewicz, 2009; Musto, 2008), (b) student values of mathematics (Hulleman et al., 2010; Musto, 2008), and (c) student interest in the subject (Hulleman & Durik, 2008; Hulleman & Harackiewicz, 2009).

In addition to mathematics relevance interventions, positive effects have emerged from infusing occupational examples within mathematics lessons (e.g., why probabilities are important to doctors). They influenced students' motivation and achievement (Means et al., 1997; Wooley et al., 2013), mathematical thinking (Bonner & Adams, 2012; Tate, 1995), and value beliefs and perceptions of mathematics (Hubert, 2014). In the 1980s, Keller (1987) was one of the first researchers to discuss relevance as a teaching strategy in a model called ARCS, which stands for attention, relevance, confidence, and satisfaction, four conditions that have to be met for people to become and remain motivated. The ARCS model is a method for improving the motivational appeal of instructional materials. Particularly looking at relevance, instructional materials were deemed relevant if they touched on how lessons can be connected to present and future career opportunities for students. The purpose of this model was to improve middle school instruction in order to motivate students to learn. Means et al. (1997) implemented the ARCS model with undergraduate students and found that relevance as an instructional strategy promoted both higher student motivation to learn and academic achievement (N

= 100; p < .001). Similar findings were reported by Wooley et al. (2013), who investigated introducing middle school students to how mathematics relates to future career opportunities and its impact on motivation and achievement. Example lessons included why it is important for caterers to know how to perform operations with fractions, why a meteorologist needs to understand integers, graphs, and spreadsheets, and how mathematics relates to art and architecture. Wooley et al.'s data verified that when relevant occupational examples were incorporated in lessons, students' mathematics performance increased (N = 3,295; p < .001). Their research demonstrates that relevant mathematics content affords students an opportunity to see school mathematics as useful outside the classroom. The work of Means et al. and Wooley et al. informs researchers and educators that infusing lessons with occupational relevance in middle school and undergraduate mathematics courses can improve students' math performance and make them more motivated to study the materials perceived as relevant.

# **Social Justice Pedagogy**

SJP affords students the opportunity to use mathematics as a tool to analyze society and issues like racism and resource inequities. SJP changes how mathematics is taught and connects it to students' everyday lives so that it has a deeper meaning. In fact, SJP is intended to enhance equity within the classroom across social identity groups (e.g., race, class, gender, sexual orientation, ability), to foster critical thinking, and to promote social action (Carlisle et al., 2006). SJP is grounded in principles of social justice education (e.g., Adams et al., 1997, 2007; Carlisle et al., 2006; Lalas, 2007; Poplin & Rivera, 2005; Shakman et al., 2007), culturally responsive education (e.g., Gay, 2000;

González et al., 2005; Irvine & Armento, 2001; Ladson-Billings, 1995; Murrell, 2000, 2001; Villegas & Lucas, 2002), multicultural education (e.g., Banks, 1995; Grant & Sleeter, 2007; Nieto, 1999, 2002; Sleeter & Grant, 1999; Suzuki, 1984), critical pedagogy (e.g., Frankenstein, 1990; Freire, 1970/2002), and democratic education (e.g., Dewey, 1916/2007; Parker, 2003; Westheimer & Kahne, 2004; Woodruff, 2005).

Often in research there is an overlap of culturally responsive teaching (CRP) and SJP as theoretical frameworks that support students in becoming critically conscious. It is important to note CRP supports students' academic success, cultural competence, and development of critical consciousness (Ladson-Billings, 1994, 1995). CRP leans toward incorporating the lived experiences and cultures of students while they learn mathematics, and thus issues of societal inequity are not always the focus. However, SJP differs from CRP in that SJP explicitly deals with addressing larger societal inequalities and the marginalization of specific groups of people (Leonard et al., 2010). For the purposes of this study, SJP will be a focus as a theoretical framework.

Teaching for social justice requires (a) teachers to have high expectations of their students and themselves (Carlisle et al., 2006; Cochran-Smith, 1999, 2004; Poplin & Rivera, 2005); (b) teachers to value and build upon students' existing knowledge and interests; (c) teachers to focus on developing students' academic skills (Darling-Hammond et al., 2002; Poplin & Rivera, 2005; Shakman et al., 2007); (d) teachers to foster learning communities (Cochran-Smith, 1999, 2004; Villegas & Lucas, 2002); (e) teachers to employ multiple forms of assessment; and (f) teachers to explicitly teach about the problems within the U.S. educational systems and society and to engage

students in developing critical habits of mind, understanding multiple perspectives, and learning to be an advocate for change. This framework will be included in the MRS, given that how students learn mathematics content influences their dispositions toward mathematics (e.g., mathematics relevance) as well as their academic achievement (e.g., Gutstein, 2003, 2007, 2013). When mathematics is taught in a way that connects students' lives to the content and demonstrates how mathematics can be used as a tool to critique students' communities and society, students begin to value and develop positive dispositions toward mathematics.

## Merging EVT and SJP

For the purposes of this study, mathematics relevance research is grounded in EVT and SJP. EVT, situated in achievement motivation, explains that a student chooses to take on a challenging task—such as persisting in a high-school mathematics course or choosing to become a mathematics major—if the student (a) values the task and (b) expects that he or she can succeed at the task. The values that students place on tasks directly influence their intentions and decisions to enroll in additional mathematics courses, future career decisions, effort, persistence, and achievement (Eccles et al., 1989, 1993; Eccles & Wigfield, 1995; Eccles [Parsons] et al., 1983; Fredericks & Eccles, 2002; Jacobs et al., 2002; Meece et al., 1990; Watt, 2004; Wigfield et al., 1991, 1997; Wigfield & Eccles, 2000). Moreover, *how* students learn mathematics can influence their task values (Gaspard et al., 2015; Hulleman et al., 2010, 2017; Hulleman & Harackiewicz, 2009; Musto, 2008). Adopting an integrated lens of EVT and SJP provides a space to understand and interpret the perceptions of secondary students' mathematics content as

being relevant to their lives. This means the context of how students learn mathematics in the classroom plays a role in their perceptions of their mathematics content. How students learn mathematics and their attitudes toward the subject are both critically important in predicting student engagement, persistence, academic motivation, and future course choices.

When relevance and utility value have been studied together, research has shown that relevance interventions in the classroom influence students' value beliefs, interest, and performance (Gaspard et al., 2015; Hulleman et al., 2010). When students are provided with various examples of the relevance of mathematics in everyday life and future careers, it fosters positive value beliefs (e.g., higher utility value; Gaspard et al., 2015; Hulleman et al., 2010; Hulleman & Harackiewicz, 2009), engagement (Hidi & Harackiewicz, 2000; Hidi & Renninger, 2006), interest (Hulleman et al., 2010; Hulleman & Harackiewicz, 2009), and performance (Eccles & Harold, 1991; Eccles & Wigfield, 1995; Hulleman et al., 2010). Thus, by encouraging students to discover the relevance of what they were learning increases their perceived utility value.

### **Need for MRS**

With an increased emphasis on promoting the relevance of mathematics in students' everyday lives comes a need to assess its development. Earlier studies have primarily focused on assessing students' perceptions of mathematics relevance, assessing ways to make mathematics curriculum relevant to students' lives, and researching how these interventions influence students' values, mathematical thinking, mathematics achievement, and interest. To date, my extensive search of the literature yielded no

instrument that measured secondary students' perceptions of the relevance of mathematics content. Earlier studies primarily focused on either students' perceptions of the relevance of mathematics through qualitative methodology or assessing the impact of curriculum designed to enhance the relevance of mathematics. These studies did not grasp the overall notion of mathematics relevance. To date, no model of mathematics relevance encompasses the multifaceted nature of the construct.

There exist different approaches to defining and measuring the effects of implementing the relevance of mathematics content within the curriculum. Findings suggest positive dispositions toward mathematics are crucial in promoting students' success in mathematics (Means et al., 1997; NRC, 2001; NCTM, 1989; Wooley et al., 2013). Thus, there is a need for further clarification of how to operationalize the term *mathematics relevance* and how to measure it.

The current scales found in the literature are designed to measure students' attitudes toward the subject of mathematics, and may have a few items that touch on the relevance of mathematics. Some scales phrase the mathematics relevance items in terms of the exchange value of mathematics for future careers (Guzey et al., 2014; Hendy et al., 2014; Huang & Lin, 2015; Ingels et al., 2011; Luttrell et al., 2010; Yanez-Marquina & Villardon, 2016), whereas other items are phrased for the value of using mathematics in everyday life (Huang & Lin, 2015; Luttrell et al., 2010; Mullis et al., 2016; Yanez-Marquina & Villardon, 2016), and valuing mathematics for illuminating societal inequalities (Wilkins, 2010; Yanez-Marquina & Villardon, 2016). While results from these scales might provide an understanding of how students view the subject,

researchers and educators cannot use the current measures to assess students' perceptions of the mathematics content they are learning as (a) having practical utility in students' everyday lives, (b) having exchange value in preparing students' for their future careers, or (c) illuminating social inequities that exist within students' communities and society.

The goal of Wilkins (2010, 2016) was to build and examine a measurement model of quantitative literacy. Quantitative literacy was defined as the interrelationship among a person's everyday understanding of mathematics, his or her beliefs about mathematics, and his or her disposition toward mathematics (e.g., an understanding of its nature and societal value and utility; Wilkins, 2010). Quantitative literacy, situated in the expectancy-value framework (Eccles & Wigfield, 1995; Eccles [Parsons] et al., 1983; Pintrich & Schunk, 2002), consisted of three factors: Mathematical Beliefs, Mathematical Cognition, and Mathematical Disposition. The Mathematical Disposition factor had three subscales: Self, Society, and Motivate. The subscale Society (e.g., social utility) measured the social utility of mathematics. For example, students were asked to rate their agreeableness to "Most applications of mathematics have practical use on the job," "Mathematics is useful in solving everyday problems," and "It is important to know mathematics to get a good job." Wilkins used exploratory factor analyses to refine the items and confirmatory factor analyses to investigate the structure of quantitative literacy. The Cronbach's alpha reliability of the items was between 0.72 and 0.86. The seven items for social utility focused on mathematics content being useful to attain a future career and to solve everyday problems, with no specifics to the community or society.

The TIMSS 2015 Students Value Mathematics Scale (Mullis et al., 2016)

included nine items. Five are focused on future careers/occupations and two are focused on everyday life (one is focused on using mathematics for other school subjects). The Cronbach's alpha reliability of the items was 0.89. Sample items include "I think learning mathematics will help me in my daily life," "I need mathematics to learn other school subjects," and "It is important to learn about mathematics to get ahead in the world." Items focused on students' perceptions of the need or importance of mathematics instead of whether they perceive what they are learning in mathematics class as having relevance to their lives or their community/society.

The High School Longitudinal Study (Ingels et al., 2011) was a national longitudinal study of over 23,000 ninth-graders from 944 U.S. schools. The study included surveys of students, their parents, math and science teachers, school administrators, and school counselors. The focus was on how students made decisions about postsecondary options and what factors influenced them following through with STEM majors and careers. Within the mathematics experiences section of the survey, three items assess students' perceptions of their mathematics content as being (a) useful for everyday life, (b) useful for college, and (c) useful for a future career.

Guzey et al. (2014) developed an instrument to assess elementary school students' attitudes toward science, technology, engineering, and mathematics (STEM). The 28-item survey consisted of a four-factor structure: (a) Personal and Social Implications of STEM, (b) Learning of Science and Engineering and the Relationship to STEM, (c) Learning of Mathematics and the Relationship to STEM, and (d) Learning and Use of Technology, with Cronbach's alpha for the entire survey of 0.91. Example items include

"It is important to know mathematics in order to get a good job," "Science, technology, engineering, and mathematics makes our lives better," "Science, technology, engineering, and mathematics are very important in life," and "Learning mathematics helps me learn science, mathematics, or engineering."

Huang and Lin (2015) developed an instrument to measure student attitudes toward calculus, with four factors: Self-Confidence, Value, Enjoyment, and Motivation. Value was defined as measuring students' beliefs about the usefulness, relevance, and worth of calculus in their life now and in the future. Sample items include "Learning calculus can improve my ability in problem solving" and "Applications of calculus are useful in everyday life." The Cronbach's alpha reliability of the value scale was 0.84.

Hendy et al. (2014) created an instrument to measure college students' value of mathematics. The 10-item scale consisted of two factors: Class Devaluation and No Future Value. Sample items include "Getting a bad grade in math will not seriously affect my future financial well-being," "Being good at math will help me in my future professional life," and "Getting a bad grade in math will not seriously affect the completion of my college degree." Cronbach's alpha reliability of the scale was 0.71.

Luttrell et al. (2010) developed a self-report instrument to measure the perceived value of mathematical literacy for general education students. The scale consisted of four factors: Interest, General Utility, Need for High Achievement, and Personal Cost. The General Utility subscale consisted of the following example items: "I see no point in being able to do math," "Having a solid background in mathematics is worthless," "I have little to gain by learning how to do math," and "After I graduate, an understanding

of math will be useless to me." Cronbach's alpha reliability of the scale was 0.89.

Yanez-Marquina and Villardon-Gallego (2016) created a scale to measure attitudes toward mathematics at the secondary level. The scale measured the perceived usefulness of mathematics, and sample items include "Math is very useful," "Everybody needs to learn math," "Math is necessary for life," "Math is important for society development," and "Learning math is important for my future job." Cronbach's alpha reliability of the scale was 0.80.

In summary, the existing scales (see Table 1) measure students' attitudes toward mathematics, value of mathematics, and quantitative literacy. There are no scales that specifically measure students' perceived relevance of mathematics. Thus, the present work becomes more than just a study using Black and Hispanic participants: rather, it presents an opportunity to see what mathematics relevance means to these adolescents and how educators can in turn motivate and engage them while they learn mathematics.

As the mathematics education community advocates for students to see sense in mathematics and perceive it both as useful and worthwhile, measuring students' perceptions of their mathematics content would be valuable (NCTM, 1989; NRC, 2001). This scale would be valuable for researchers, school leaders, and teachers to answer the following questions: Are students seeing sense in learning mathematics content? Are students seeing their mathematics content as relevant outside of the classroom, and how? Developing the MRS may help educators and researchers answer these questions and understand if secondary students are deeply understanding the role of mathematics in their everyday lives, communities, and society and how students see math as meaningful.

Table 1

Items, Factors, Reliabilities, and Limitations of Scales

Instrument	Items/scale	Factors	Reliability	Variance explained	Limitations
Task Utility Value Scale (Eccles, 1997)	6 items; 2-point Likert scale  (e.g., Some things that you learn in school help you do things better outside of class, that is, they are useful. For example, learning about plants might help you grow a garden. In general, how useful is what you learn in math?)	1 factor: utility value	Reliabilities for the utility value scale ranged from .74 to .90 across domains and times of measurement		It has been used and validated with primarily white, middle-class students in Grades 5-12. Incorporates items dealing with students liking mathematics and how important they think math is.
Task Utility Value Scale (Conley, 2012)	4 items; 5-point Likert scale  (e.g., Math will be useful for me later in life)	1 factor: utility value	Cronbach's alpha for the entire survey of 0.80		All items deal with math being useful for students later in life (e.g., college, job). No items that target students' current everyday lives or math being useful in society and their communities.
Fennema-Sherman Mathematics Attitudes Scales (1976) – Usefulness of Mathematics Scale	12 items on math usefulness; 5- point Likert scale  (e.g., Mathematics is of no relevance to my life)				All items deal with math being useful for students' future education, vocation, and other activities. No items that target students' current everyday lives or math being useful in society

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Instrument	Items/scale	Factors	Reliability	Variance explained	Limitations
Musto, G. (2008). Showing you're working: A project using former pupils' experiences to engage current mathematics students. Teaching Mathematics and its Applications, 27(4), 210–217.	5 items that measured perception of confidence in mathematics as well as usefulness of mathematics; 5-point Likert scale  (e.g., I can see the relevance of mathematics to everyday life)	N/A	N/A	N/A	Items deal with enjoying mathematics, interest in mathematics, and importance of mathematics. Only one item on the relevance of mathematics. No items that target students' current everyday lives or math being useful in society and their communities.
Quantitative Literacy - Mathematical Disposition Questionnaire (Wilkins, 2010)	18 items; 6-point Likert scale  (e.g., Mathematics is useful in solving everyday problems)	Interest in Mathematics (Interest) Self-Perception of Ability (Self- Concept) Social Utility (Utility)	Interest: .93 Self-Concept: .91 Utility: .87	66.39%	Social Utility factor incorporates items that targets mathematics content being useful in everyday life or for a job. Does not include utility aspect in society and students' communities (e.g., SJP)  The Grade 9-12 data were collected in the 1980s and 1994. The undergraduate data were collected from psychology courses. Majority female and Caucasian students.
Students Value Mathematics Scale (TIMSS, 2015)	9 items; 4-point Likert scale  (e.g., It is important to learn about mathematics to get ahead in the world)		Cronbach's alpha for the entire survey of 0.89 (United States)	54% (United States)	Five items are focused on future careers/occupations, and two items are focused on everyday life (one is focused on using mathematics for other school subjects). Does not include utility aspect in society and

Instrument	Items/scale	Factors	Reliability	Variance explained	Limitations
				1	students' communities (e.g., SJP)
					Designed for Grade 8 students.
2009 High School Longitudinal Survey by U.S. National Center for Education Statistics (Ingels et al., 2011)	9 items; 3-point Likert scale  1 item on math usefulness (overall 9 mathematics experience questions)  What students learn in this course[Strongly Agree; Agree; Disagree] is useful for everyday life. will be useful for college. will be useful for a future career.				Three items assess students' perceptions of their mathematics content as being (a) useful for everyday life, (b) useful for college, and (c) useful for a future career. Does not include utility aspect in society and students' communities (e.g., SJP).
Attitudes Towards STEM (Guzey et al., 2014)	28 items;  (e.g., Learning mathematics helps me learn science, mathematics, or engineering)	4 factors: Personal and Social Implications of STEM, (b) Learning of Science and Engineering and the Relationship to STEM, (c) Learning of Mathematics and the Relationship to STEM, and (d) Learning	Cronbach's alpha for the entire survey of 0.91		Designed to be used with elementary students. Focus on mathematics being relevant for other STEM subjects and not for students' lives outside of the classroom.

Instrument	Items/scale	Factors	Reliability	Variance explained	Limitations
		and Use of Technology		•	
Attitudes Towards Calculus (Huang & Lin, 2015)	(e.g., Applications of calculus are useful in everyday life)	4 factors: Self- Confidence, Value, Enjoyment, and Motivation	Cronbach's alpha for the entire survey of 0.84		Designed to measure_student attitudes toward calculus. Value is defined as measuring students' beliefs about the usefulness, relevance, and worth of calculus in their life now and in the future.
Value of Mathematics (Hendy et al., 2014)	<ul><li>10 items;</li><li>(e.g., Getting a bad grade in math will not seriously affect my future financial well-being)</li></ul>	2 factors: Class Devaluation and No Future Value	Cronbach's alpha for the entire survey of 0.71		Designed to measure college students' value of mathematics.
Perceived Value of Mathematical Literacy (Luttrell et al., 2010)	(e.g., Having a solid background in mathematics is worthless; I have little to gain by learning how to do math)	4 factors: Interest, General Utility, Need for High Achievement, and Personal Cost	Cronbach's alpha for the entire survey of 0.89		Designed to measure the perceived value of mathematical literacy for general education students.
Attitudes Towards Mathematics (Yanez-Marquina & Villardon, 2016)	(e.g., Math is necessary for life; Math is important for society development)		Cronbach's alpha for the entire survey of 0.80		Created a scale to measure attitudes toward mathematics at the secondary level.

## **Chapter Three: Methodology**

The purposes of this study were to develop and to provide validation evidence for an instrument that measures Black and Hispanic secondary students' perceptions of mathematics relevance. Scores on this instrument are intended to inform researchers, curriculum writers, education leadership, and teachers about factors that influence motivation and engagement and strengthen positive perceptions of mathematics while learning the subject. Mathematics relevance is defined as the extent to which students perceive mathematics content to possess at least one of the following characteristics: (a) practical utility in students' everyday lives (Eccles [Parsons] et al., 1983; Sealey & Noyes, 2010); (b) exchange value in preparing students for future careers (Eccles [Parsons] et al., 1983; Wooley et al., 2013); or (c) illuminating social inequities that exist within students' communities and society (Dover, 2009; Gutstein, 2003, 2007, 2013).

The goals of this study were to:

- 1. Articulate the need for the MRS;
- 2. Describe the creation of the instrument; and
- 3. Assess the instrument's construct validity by way of
  - a. Content validity
  - b. Substantive validity
  - c. Structural validity

- d. Generalizability validity
- e. External validity

Gelbach and Brinkworth's (2011) six-step model for scale development guided the research design. In their model, they delineated the use of (a) literature reviews; (b) interviews with the target population; (c) a synthesis of the information from the literature review and interviews; (d) item development; (e) expert interviews; and (f) cognitive interviews. The six-step approach frontloads validity, which, in turn, is likely to reduce measurement error and enhance the validity of scales (Gehlbach & Brinkworth, 2011). Below I describe in detail each step of the scale development process.

### **Scale Development Process**

The scale development process was broken up into two phases. Phase 1, item generation, involved the writing of items in response to the literature review, student interviews, and synthesis of the two. Phase 2 incorporated item revision and reduction, by way of feedback from expert interviews, cognitive interviews, and exploratory factor analysis (EFA). Ghelbach and Brinworth's (2011) six-step model is demonstrated below in Figure 3. The cyclical nature of the diagram implies that this process welcomes repetition of steps, driven by the literature review. Below, I describe how each phase of this model supported the development process of the MRS.

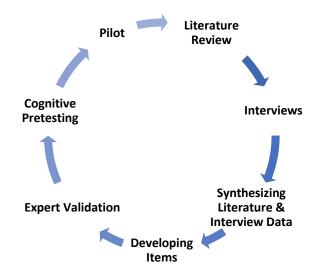


Figure 3

Scale Development Process Based on the Work of Gehlbach and Brinkworth (2011)

### Phase 1: Item Generation

The item generation phase was grounded in a thorough literature review on the construct mathematics relevance, EVT, and SJP, as well as results from student interviews, a synthesis of the literature review and student interviews, and preliminary item development.

Literature Review. The literature review covered three primary areas: (a) EVT for motivating students to engage in tasks that they value (Eccles [Parsons] et al., 1983); (b) SJP for promoting achievement and positive attitudes toward mathematics by making mathematics content connected to real-life issues (Gutstein, 2003, 2007, 2013); and (c) the relevance literature, which considers how the construct has been operationalized within mathematics education.

Student Interviews. The objective of semi-structured student interviews was to determine whether the target population thought about the construct in the same way as the literature defined it, and whether students used the same terminology when describing the construct (Gehlbach & Brinkworth, 2011). These one-on-one interviews began with two students from each level of the 8-12 grade band, which was a total of 10 participants. Sample questions during the interviews included "What do you think of when I say 'Teachers should make mathematics lessons relevant to students' lives'?" and "What does 'mathematics relevance' mean to you? How would you define 'mathematics relevance'?"

Synthesis of Literature Review and Student Interviews. The synthesis of the literature and students' interviews focused on the similarities and discrepancies in the construct observed between the literature and the student interviews. The goal of this step was to provide a thorough definition of the construct that aligned with the students' perspectives as well as the literature regarding relevance (Gehlbach & Brinkworth, 2011). Upon analysis and synthesis of the literature review and student interviews, I noted communalities and discrepancies in order to make sure my hypothesis of three factors was an accurate representation of mathematics relevance. Within the literature, the four components of mathematics relevance are

 Relevance for helping students understand the significance of mathematics in their own lives (Boaler, 2000; Gaspard et al., 2015; Hulleman et al., 2010; Murray, 2011; Onion, 2004; Sealey & Noyes, 2010);

- Relevance for preparing students for potential careers in STEM (Musto, 2008;
   Onion, 2004; Sealey & Noyes, 2010; Wooley et al., 2013);
- Relevance for students becoming logical thinkers across all subjects (Sealey & Noyes, 2010);
- Relevance for empowering students to merge culture and learning, and to critique societal inequalities (Adams et al., 1997, 2007; Carlisle et al., 2006; Gutstein, 2003, 2007, 2013; Ladson-Billings, 1995; Lalas, 2007; Poplin & Rivera, 2005; Shakman et al., 2007).

Writing of Items. After synthesizing the literature and student interviews, I wrote preliminary items that represented the literature while using terminology that was meaningful to the target population (DeVellis, 2003; Gehlbach & Brinkworth, 2011). As I wrote items, clarity, unambiguity, and non-biased language were kept in mind.

Additionally, I wrote items that encompassed the mathematics relevance, EVT, and SJP literature. Based on Crocker and Algina's (1986) criteria for item construction, the MRS items do not exceed 20 words, include language that would be easily understood by the participants, and avoid the use of negatives (e.g., none, never, not) and universal statements (e.g., always, all). Furthermore, response anchors were labeled verbally instead of using the combination of verbal and numeric labels, given participants may have conflicts with their implicit meaning (Tourangeau et al., 2000). In other words, the presence of verbal anchors (e.g., strongly disagree, disagree, somewhat disagree, somewhat agree, agree, strongly agree), lets the participants avoid being tasked with inferring what numerical scores mean. Six response anchors (Krosnick, 1999; Weng,

2004) were used to encourage participants to be non-neutral, and there were no reverse-scored items, given they diminish scale reliability and lead to inconsistent participant responses (Cacioppo & Berntson, 1994; Swain et al., 2008).

### Phase 2: Item Revision and Reduction

The item revision and reduction phase was influenced by expert interviews and cognitive interviews, which were integral components of Phase 2. Additionally, EFA supported the reduction of the number of items.

Expert Interviews. A preliminary list of items was reviewed by experts in the fields of mathematics education, EVT, and SJP, who provided feedback on each item. The experts provided feedback on item clarity and language complexity, and judged how well the set of items represented the construct mathematics relevance (Gehlbach & Brinkworth, 2011). Participants were considered experts if they had research experiences in mathematics education, EVT, or SJP. Experts provided feedback on how comprehensible each item was for the anticipated respondent population; how central each item was to the construct; and any aspects of the construct that were not represented or inadequately represented by the scale.

Cognitive Interviews. The purpose of cognitive interviews was to learn how the target population interpreted items. Through two rounds of interviews, a sample of the target population went through each item and provided feedback on any ambiguity of meanings and overly challenging vocabulary (Gehlbach & Brinkworth, 2011). Types of questions the target population were asked included: "As you read this question, what are your thoughts?", "Do the response options make sense?", "Why would you respond in

that way?" and "Are there any other words and/or phrasing that could convey the question more successfully?" The focus of these cognitive interviews was to determine whether students in the target population understood the items in the same way as intended when authored; less attention was paid to their actual responses to the items.

Next, I will describe the participants of this study and the data collection procedures.

#### Methods

## Participants and Setting

The vast majority of mathematics relevance research has been conducted with middle- and upper-class White students. Far less is known about African American and Hispanic students' perceptions of the relevance of mathematics content, particularly those who will be the first to attend college in their families. It is important to learn about their perceptions of their mathematics content to help identify factors associated with their academic success, particularly as their lived experiences may lend them to different perceptions than those in dominant upper- and middle-class communities. Thus, the need for research on this population is ever important. For this reason, I was specific about the selection criteria for the study.

The participants for both Phases 1 and 2 of this study were students in Grades 8-12 who primarily identified themselves as Hispanic or African American and participated in an out-of-school college preparatory program. This program enrolls more than 600 students from seven local public schools and provides access to educational resources for students who will be first-generation college students upon their high school graduation. The participants in this study were future first-generation college students: their parents

did not receive 4-year college degrees (Choy, 2001; Nunez & Cuccaro-Alamin, 1998). Students in this program receive year-round academic enrichment and college preparation, and attend a free 3-week summer program housed on a college campus where they are exposed to coursework they will encounter in the upcoming year. The program designs these opportunities based on the empirical findings that first-generation students tend to experience more academic challenge than their peers and tend to have a significantly higher dropout rate (Snibbe & Markus, 2005). Furthermore, students' college enrollment rates differ based on parent education level. For example, in 1999, 82% of students whose parents held a bachelor's degree or higher enrolled in college. For students whose parents had completed high school but not college, 54% enrolled, and for students whose parents had less than a high school diploma, 36% enrolled (Wirt & Livingston, 2001). Given this difference in enrollment rates along with the 43% increase in the Hispanic population between 2000 and 2010, and a growing number of firstgeneration Hispanic college students (U.S. Census Bureau, 2010), outreach programs have been designed to raise the level of student preparation and readiness to increase college enrollment (Swail & Perna, 2000).

Of the 232 participants in the current study, 84 (36.2%) identified as male and 145 (62.5%) as female, with ages spanning 14 to 18 years old. The majority of the students identified as Hispanic (73.3%) or African American (26.7%) and would be the first in their families to attend college. All students under 18 asked their parents to sign and complete consent forms that were approved by the Institutional Review Board. The consent form informed parents of the study and asked permission to have their child

participate. For all participants, participation was voluntary. Students returned signed consent forms to the associate director of the program. The students who had provided consent to participate in the study were then recruited. Following a brief explanation of the assent form and the purpose of the research, participants were asked to participate in either the student interviews, cognitive interviews, or completing the scale.

## **Qualitative Data Collection Procedures**

The data collection process incorporated data from student interviews, cognitive interviews, expert interviews, student demographics, and student responses to items from the MRS, utility value scales (Conley, 2012; Wigfield et al., 1997), and cost value scales (Conley, 2012; Kosovich et al., 2015). The data collection procedures below are divided into qualitative data (e.g., interviews), which aided in the development of the scale, and quantitative data, which aided in the item reduction process.

## Student Interviews

The goal of the student interviews was to gain feedback from the students' perspective on how they defined relevance in relation to learning mathematics in the classroom. The interviews also included students discussing specific lessons or projects in their math classes that they believed were relevant to their lives.

### **Expert Interviews**

The goal of the expert interviews was for experts to provide feedback on item clarity, language complexity, and how well the set of items represented the construct mathematics relevance, situated in EVT and SJP. There were two rounds of expert interviews and participants were considered experts if they had research experience in

mathematics education, educational psychology, and/or SJP. The two mathematics education experts were an assistant professor and professor whose research interests included equity and access to higher-level mathematics for underrepresented populations, as well as teaching for social justice and critical literacies in an urban, multicultural context. The two educational psychology experts were assistant and associate professors whose research interests included learner motivational beliefs. The transformative teaching expert was an associate professor whose research interests included SJP.

# Cognitive Interviews

Student cognitive interviews provided feedback on item clarity and comprehension before students actually completed the MRS. Unlike the student interviews where students discussed their own definitions of relevance, for the cognitive interviews, students simply went through each item on the scale and provided their recommendations to improve the quality of the scale. Cognitive interviews were one-on-one and included students in Grades 8-12. Round one of these interviews included five students, one from each grade band, and discussions were based around understanding how these students read and responded to the items on this scale. After the initial round of interviews, items were edited based on patterns in responses. The goal of the second round of interviews with five additional students, one from each grade band, was for these students to provide feedback on the revised items. These interviews aided in Phase I and Phase 2 of the scale development process. The remaining data collection was the administration of the MRS.

### **Quantitative Data Collection Procedures**

### Demographic Survey

Students participating in completing the MRS started by completing the background questionnaire form, which included questions regarding their age, gender, ethnicity, mathematics course enrollment patterns, mathematics achievement, and future career aspirations.

#### MRS

The MRS was developed to measure students' perceptions of their mathematics content as being relevant to their lives. Three subscales addressed areas of (a) practical utility in students' everyday lives, (b) exchange value in preparing students for future careers, and (c) illuminating social inequities that exist within students' communities and society, with the following Likert-type responses: *never true for me, rarely true for me, sometimes untrue for me, sometimes true for me, usually true for me, always true for me.* 

## Utility Value Scales

Campbell and Fiske (1959) described the approach to examine the adequacy of a measure of a construct by identifying different measures of similar or distinctly different constructs. To test the validity of the measure, correlations of each pair of measures were computed. Correlations between measures of the same or similar constructs should be high (e.g., convergent validity). Correlations between measures that are distinctly different should be substantially lower than the convergent validity coefficients (e.g., discriminant validity). Given the MRS is situated in EVT, I chose two utility value scales to test for convergent validity. The first is the utility value scale by Wigfield et al. (1997),

which has been extensively used and tested in achievement motivation research. This scale consists of six items (e.g., "Some things that you learn in school help you do things better outside of class, that is, they are useful. For example, learning about plants might help you grow a garden. In general, how useful is what you learn in math"). It has been used and validated with primarily White, middle-class students in Grades 5-12 (Eccles & Wigfield, 1995; Eccles [Parsons] et al., 1983; Wigfield et al., 1997). The factor structure of EVT was explored through both EFA and confirmatory factor analysis (CFA). EFA was used as a data reduction technique where items were eliminated from the measure in order to define the construct more precisely. CFA was used to test predictions given the literature, concerning the factor structure of utility task value. Results indicated that the construct task value separates into three distinct factors: interest, perceived importance, and perceived utility (df = 11; chi-square = 16.78; GFI = .99; Tucker-Lewis Coefficient = .99; Eccles & Wigfield, 1995). Thus, for the purposes of this study, the perceived utility value scale can be used separate from interest and perceived importance to test for convergent validity against the MRS. Wigfield et al. (1997) used the utility value scale to assess changes in elementary school children's competence beliefs over a span of 3 years in the domains of math, reading, instrumental music, and sports. Resulting from EFA, internal consistency reliabilities for the utility value scale ranged from .74 to .90 across domains and times of measurement.

The second utility value scale, chosen to test for convergent validity, is by Conley (2012). This scale was used to measure students' perceived usefulness of mathematics. It consists of four items (e.g., "Math will be useful for me later in life"), and Cronbach's

alpha for this scale is .80. This measure adapted items from the original scale by Eccles (Parsons) et al. (1983); however, this measure has been used with participants similar to those in this study. Conley's participants reflected ethnic and economic diversity in that they were primarily Latino or Hispanic (69%), and more than half (56%) were eligible for free or reduced lunch. This contrasts with the original scale, where participants were primarily European American (95%) and from the middle class (Eccles [Parsons] et al., 1983).

### Cost Value Scales

When testing for discriminant validity, correlations between measures that are distinctly different should be substantially lower than the convergent validity coefficients (Campbell & Fiske, 1959). Within achievement motivation research, cost value and utility value tend to have low correlations (Flake et al., 2015). Cost is defined as what is invested, required, or given up to engage in a task and thus impacts a student's overall value for a given activity or task (Flake et al., 2015). Students may perceive a task to require too much effort to be successful at the task, prevent them from participating in a different activity, or even induce anxiety in response to potential failure at the task.

Therefore, cost value and the MRS, situated in utility value, often have low correlations.

Two cost value scales were chosen to test for discriminant validity. The first is the cost value scale by Conley (2012), which measures the opportunity costs of doing well in mathematics. The scale consists of two items (e.g., "I have to give up a lot to do well in math.") where Cronbach's alpha is .70. Like Conley's utility value scale, it adapted items from the original scale by Eccles (Parsons) et al. (1983), but has been used with

participants who were majority Latino or Hispanic (69%) and eligible for free or reduced lunch (56%). Conley explored what natural patterns of achievement goal, subjective task value, and competence beliefs exist within the domain of mathematics using cluster analysis. Results showed that average-high cost students had low overall subjective task value and that math achievement was lower in high-cost groups than in low-cost groups.

The second cost value scale, chosen to test for discriminant validity, is by Kosovich et al. (2015). The scale was used to measure the opportunity costs of doing well in mathematics. The scale consists of four items (e.g., "I have to give up too much to do well in my math class"), and the reliability of this scale is coefficient omega,  $\omega_{cost} = .86$ (Yang & Green, 2011). This measure also adapted items from the original scale by Eccles (Parsons) et al. (1983), but has been used with participants from a diverse student body where 59% were eligible for free and reduced lunch, 45% had limited English proficiency, and 50% were Hispanic and African American (Kosovich et al., 2015). Additionally, this scale was developed given the non-practical previous use of lengthy EVT scales in classrooms to assess student motivation. Thus, Kosovich et al. developed a 10-item EVT scale that was shown to be valid and easy to implement in the classroom given the two major barriers to psychological measurement, time limitations and delivery constraints (Bryk et al., 2013). Confirmatory factor analyses support the three-factor EVC model ( $\chi^2(32) = 30.46$ , p = .54; RMSEA < .01; CFI > .99; SRMR = .02), which suggests that expectancy, value, and cost are separate factors in mathematics. Kosovich et al. tested for discriminant evidence, and for math achievement, the correlation pattern with cost in the Fall was  $r_{m,c} = -.17$  and in the Winter was  $r_{m,c} = -.17$ . Additionally, utility

value and cost value were negatively correlated as well, r = -.57. These correlations follow prior research where cost and value have been typically negatively and lowly correlated with each other (e.g., Durik et al., 2006; Flake et al., 2015).

### **Data Analysis**

Data analysis consisted of qualitative and quantitative analysis, determined by the various data sets collected for this study. Qualitative data were collected from student interviews, expert interviews, and cognitive interviews. The quantitative data consisted of the students' responses to five scales: the MRS, two utility value scales (Conley, 2012; Wigfield et al., 1997), and two cost value scales (Conley, 2012; Kosovich et al., 2015).

### Qualitative Data Analysis

Student Interviews. Memos were written immediately after the student interviews to facilitate analytical thinking and to note details that might not necessarily be included in the transcripts (Maxwell, 2012). The memos included notes on what was surprising or intriguing to the researcher at the completion of interviews. Dedoose, a qualitative data analysis software package, was used to generate coding categories and to link transcriptions and memos. After reading all memos and transcriptions, coding was done in three stages: open, axial, and selective (Maxwell, 2012). Several preexisting codes from the relevance literature guided the coding process:

A. Meaningfulness (e.g., meaning development)—students define relevance as understanding how specific mathematics knowledge is connected to their lives and the lives of others;

- B. Usefulness/Practicality—students define relevance as seeing the wider significance of mathematics in their own everyday and future lives;
- C. Value—students define relevance as understanding the value of doing well in mathematics for their future selves (e.g., exchange value);
- D. Application—students define relevance as applying the knowledge learned in their mathematics classroom to real-world scenarios;
- E. Daily life;
- F. Future life/future plans.

The codes above represent the initial coding scheme; however, I knew it was possible that student responses would not be captured by these preexisting codes. Thus, I was open to discover new codes that emerged from the data (Creswell et al., 2007). As I reviewed the interview transcripts, I considered: (a) How are students talking about, characterizing, and understanding relevance? (b) What terminology do students use in describing relevance? (c) What assumptions are these students making about relevance? From these codes, major themes in the data were generated and participants' responses were grouped into major themes.

**Expert Interviews.** In order to pilot the MRS for expert review, an assistant professor and doctoral student (refer to Table 3) from the departments of Mathematics Education and Educational Psychology at a large public university were recruited for round one of expert interviews. The purpose of interviewing these two participants as preliminary experts was to aid in revising and reducing the number of items before

sending the measure to a second round of experts in the fields of mathematics education, educational psychology, and transformative teaching to provide further content validity.

Cognitive Interviews. After the cognitive interview templates were completed, students' responses were added (see Table 10). Round one interviews were with five students, and discussions were based around understanding how students read and respond to the items on this scale. All students' responses were recorded and common themes for modifications were highlighted. Modifications were written in the revised items column and then the scale was updated. The goal of the second round of interviews with five additional students was for new students to provide feedback on the revised items. All students' responses were recorded and common themes for modifications were highlighted.

### Quantitative Data Analysis

Data Cleaning. Due to the assumption of univariate and multivariate normality not being applicable for EFA and, thus, not necessary to test (Pituch & Stevens, 2016), data were not screened for univariate and multivariate outliers. Normality was assessed to determine if the distribution of the data was skewed. I did this by creating histograms with a normal curve. I also used multiple imputation to correct for missing data. Multiple imputation is a statistical technique designed to handle missing data by replacing them with two or more imputed values (Rubin, 1987). Lastly, I ran descriptive statistics for the data collected and had SPSS calculate the mean and standard deviation for the scales used. The descriptive statistics were then used to describe characteristics of the sample.

**Exploratory Factor Analysis.** The quantitative data from the online survey were entered and analyzed using SPSS software. EFA was used as a data reduction technique to create factors that explained the most variance possible in the items (Dimitrov, 2012; Urdan, 2010). I followed the six steps of EFA (Figure 5).

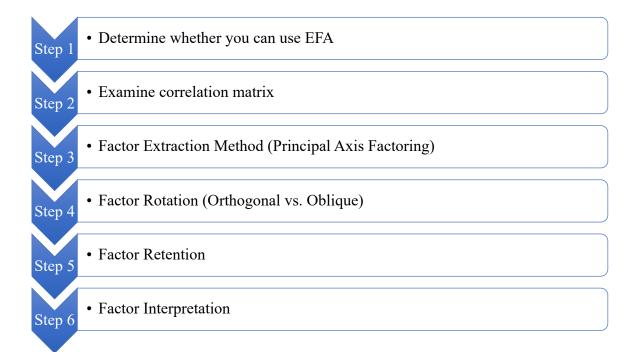


Figure 4

Exploratory Factor Analysis Steps

**Determining to Use EFA.** To determine whether using EFA is appropriate, sample size is key. In the literature there exist varying rules of thumb for sample sizes. They include a minimum of 100 to 250 (Cattell, 1978; Gorsuch, 1983), at least 500 (Comrey & Lee, 1992), or a ratio of at least 10:1 plus 50 for sample size to number of

variables (Meyers, Gamst, & Guarino, 2017). For the purposes of this study, I followed having a minimum of 100 to 250. I computed both Bartlett's test of sphericity and the Kaiser-Meyer-Olkin (KMO) measure of sampling adequacy. It was important to use Bartlett's test of sphericity (Bartlett, 1950) when determining if EFA was necessary because it tests if the correlation matrix is significantly different from an identity matrix. A correlation matrix is only equal to an identity matrix if all correlations between the variables are zero. Thus, if the chi-square value yields a rejection of the null hypothesis (p < .05), then the correlation matrix is significant. The KMO (Kaiser, 1974) determines the degree of common variance among variables (e.g., the items share a common factor). KMO values range from zero to one and I wanted values closer to one (e.g., KMO  $\geq$  .60), meaning it is likely that the observed variables share a common factor (Comrey & Lee, 1992).

**Examine Correlation Matrix.** I examined the correlation matrix to get an idea of whether items had a factor in common. If items were not correlated at all (e.g., < .30), then a factor analysis would not be appropriate.

Factor Extraction Method. Due to assessing latent factors, I chose the principal axis factoring (PAF) extraction method. PAF is commonly reported in social and behavioral science research (Warner, 2013) due to its use when factors are correlated with one another and analyzing the shared variance among the factors (Dimitrov, 2012). More specifically, PAF provides separate estimates of common and unique variance, thus considering the presence of error (Reise et al., 2000).

Factor Rotation. The next step in the EFA process is factor rotation in order to create factors from the set of items. Since all items on the MRS were situated in mathematics relevance literature, EVT, and SJP, I expected theoretically for the factors to correlate. Thus, I used the oblique rotation method, Direct Oblimin, where factors are allowed to correlate and EFA does not try to maximize the distinction between the factors. This rotation method would help with interpretability by maximizing larger factor loadings closer to 1 and minimizing smaller factor loadings closer to 0. I selected for SPSS not to print any factor loadings less than .32, since this indicates the item is not a strong indicator of that factor (Comrey & Lee, 1992). I only considered an orthogonal rotation method (e.g., Varimax) if factors were not correlated.

Factor Retention. To determine the number of factors to retain, I compared the hypothesized three-factor solution to results from the number of eigenvalues greater than 1 and scree plot. First, I referred to the total variance explained output and retained the factors that had eigenvalues greater than 1 (Kaiser, 1960). Factors that explained less than 10% of the total variance in the full set of items were not considered, since they are deemed too weak. I also examined the scree plot to see how many factors should be retained (Cattell, 1966). The scree plot plots the eigenvalues against their factor numbers, and there was autonomy in determining where they level off (e.g., where they form an elbow). The proper number of factors to retain was determined by the number of eigenvalues plotted above the elbow (the space between the steep slope and the leveling off). I also examined the factor loadings (e.g., measure of shared variance) for each item where the loadings should be > .32 (McCoach et al., 2013). For example, if there exists a

loading of .55, then there exists 30% (. 55<sup>2</sup>) shared variance between the item and the factor, which is ideal. Items that cross-loaded onto two or more factors were examined to determine why cross-loading occurred (e.g., item wording needs to be revised), as these items were not accurately discriminating between factors. Per recommendations from McCoach et al. (2013), the final solution was chosen based on factors having at least three items with strong primary loadings (greater than .40), which suggests that the solution is not over-extracted.

The first factor created explained the most variance in all of the items; the second factor created explained the second most variance; and the third factor created explained the third most variance. EFA stops extracting factors when an additional factor would not explain very much additional variance in the items.

Interpret Factors. The last stage of the EFA process was to interpret the retained factors. The literature review guided this interpretation, given the scale was written to have three factors in mind.

Validity. Within the scale development process, testing for validity is an important and key component (Dimitrov, 2012; Messick, 1995). This study is guided by Messick's (1989) definition of validity: "Validity is an integrated evaluative judgment of the degree to which empirical evidence and theoretical rationales support the adequacy and appropriateness of inferences and actions based on test scores or other modes of assessment" (p. 13). In summary, validity tests for whether an instrument measures what it purports to measure (Dimitrov, 2012). When testing for validity of a scale, Cronbach (1951) reminds us that the interpretation of test scores is what is being validated, not the

test itself. This study follows Messick's (1989, 1995) unified construct-based model of validity, which integrates content-related and criterion-related validity into construct validity. The five aspects of construct validity are content, substantive, structural, generalizability, and external (refer to Figure 6).

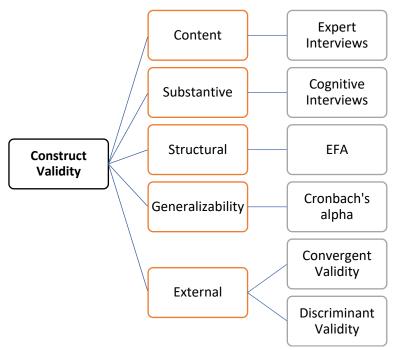


Figure 5

Construct Validity Based on Messick (1995)

All aspects of validity are incorporated throughout the design of this study, which includes both the qualitative and quantitative data collection and analysis. The construct-based validity model by Cronbach and Meehl (1955) will not be used for this study given the researchers defined validity as three separate types: content, criterion, and construct. The problem with defining validity as three separate types is that it misleads survey

developers to believe that content, criterion, and construct validity are comparable and that collecting evidence from any of them is sufficient to label a survey as valid (Dimitrov, 2012). This is avoided in Messick's (1989, 1995) six-step validity model, which integrates content and criterion validity into a unified model of construct validity and thus will be the driver in this study.

Content Aspect. Testing for content validity is necessary to show evidence that the MRS represents the sample of characteristics necessary for a secondary student to perceive mathematics content as relevant. I began investigating content validity (e.g., face validity evidence) by way of expert interviews where experts provided judgments of whether the items were central to the relevance construct of interest. Additionally, they provided face validity evidence regarding readability, suitability, and fairness of items.

Substantive Aspect. Testing for substantive validity is key to show evidence of response consistencies through cognitive modeling of the participants' response processes (Loevinger, 1957). During the cognitive interviews, students provided feedback on their understanding of items, item phrasing, vocabulary, and clarity. Evidence was collected by way of "think aloud" protocols where examinees explained the reasoning they used in responding to each item and how they were interpreting the items. These interviews gave me firsthand understanding of how the target population thought about and responded to items on the MRS, which in turn supported my revision of the items.

**Structural Aspect.** Structural validity evidence is shown through correlational and measurement consistency between the construct and the items; this is done by using EFA. EFA was used, given there is not enough theoretical and empirical research on the

construct mathematics relevance to determine how many factors underlie the items (Dimitrov, 2012).

Generalizability Aspect. Generalizability validity was examined by the extent to which the interpretation of scores from the scale was accurate, consistent, and replicable (e.g., reliability, Dimitrov, 2012). For instance, when the MRS is administered, test users will need to know that the results can be replicated if the same participants are tested repeatedly under the same circumstances (Crocker & Algina, 1986). Internal consistency reliability was tested using Cronbach's alpha (Cronbach, 1951). Cronbach's alpha tests how well the items as a group hold together. Conceptually, the idea is that all survey items that are supposed to measure the construct mathematics relevance should be answered similarly by participants. However, if a participant gives varying answers to items that are supposed to be measuring relevance, it is difficult to argue that these items offer a reliable measure of the construct relevance (Urdan, 2010). Cronbach's alpha is sensitive to sample size, and thus the more items there are, the higher the Cronbach's alpha will be (Urdan, 2010). I wanted a Cronbach's alpha > .7 (Nunnally, 1978). As a reminder, when looking at the "item-total statistics" and the column "Cronbach's alpha if item deleted," it is important to note that only one item can be deleted at a time and then it needs to be rerun.

External Aspect. As described by Messick (1995), the external aspect of validity includes convergent and discriminant evidence. The convergent validity evidence indicates similarity between measures of the same trait, whereas the discriminant evidence indicates a distinctness from measures of other traits. Convergent validity was

examined by demonstrating the high intercorrelations of the items from the MRS and the already validated utility value scales (Conley, 2012; Wigfield et al., 1997). This provided evidence that the items on both were most likely related to the same construct, mathematics relevance. Discriminant validity was examined by demonstrating the low intercorrelations of the items from the MRS and the already validated cost value scales (Conley, 2012; Kosovich et al., 2015). This provided evidence of whether the newly developed instrument, MRS, was measuring something unintended. Both convergent and discriminant validity can be assessed using Pearson's r to examine the relationship between factors on the newly established instrument and already validated instruments.

In this chapter, I described the procedures for the creation of the MRS which follows Gelbach and Brinkworth's (2011) six-step model. I also detailed how this instrument's construct validity will be assessed following Messick's (1989, 1995) unified construct-based model. In Chapter 4, I describe the results of the qualitative and quantitative analyses as outlined above.

# **Chapter Four: Results**

The purposes of this study were to develop and validate an instrument to measure secondary students' perceptions of mathematics relevance, particularly for students of color. This chapter describes the results of the qualitative and quantitative analyses discussed in Chapter 3. I describe in detail how the MRS evolved from 45 items to 35 items intended to be administered to secondary students (please refer to Figure 7 for item development process).

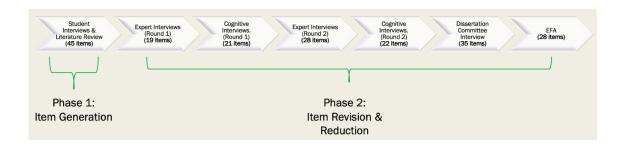


Figure 6

Item Development Process

This chapter begins with the results of the initial round of expert interviews.

Following, the two rounds of cognitive interview data are reported and discussed, and the findings from EFA are described. Ultimately, the validity results are reported.

## **Initial Round of Expert Interviews**

In order to pilot the MRS for expert review, an assistant professor and doctoral student (refer to Table 3) from the departments of Mathematics Education and Educational Psychology at a large public university were recruited for round one of expert interviews. The purpose of interviewing these two participants as preliminary experts was to aid in revising and reducing the number of items before sending the measure to a second round of experts in the fields of mathematics education, educational psychology, and transformative teaching to provide further content validity.

Table 2

Original Scale (45 Items) for Expert Interviews Part I

Dimension A: practical utility in students' everyday lives (EVT)

Math is important in my daily life.

I often use the math I learned in elementary school.

Knowing math helps me in other subjects.

I already know more math than I need in my daily life.

Math helps me in my daily life.

I use math outside of school.

Studying math helps me with problem solving in other subjects.

The things I do in math have nothing to do with my daily life.

I apply my math skills outside of school.

I use math to make decisions outside of school.

I often use the math I am learning in high school.

Knowing math helps me outside of school.

I don't need math in my daily life

I often use the math I learned in middle school.

# Dimension B: exchange value in preparing students for future careers (EVT)

Math may play an important role in my career.

My career will involve math.

The jobs I am interested in don't use math.

I need to know math to earn a living.

My math skills may help me in my professional life.

My math knowledge may help me in my professional life.

I will use math problem-solving skills in my career.

I need math knowledge to be successful in my career.

Math will not be useful in my career.

I am not interested in jobs that use math.

Knowing math gives me more career choices.

Learning math will help me to get the job I want.

Math will be useful no matter what career I choose.

Math will prepare me to succeed in my chosen career.

Knowing math will make me more competitive in the job market.

Dimension C: illuminating social inequities that exist within students' communities and society (SJP)

Math can help me to better understand the world we live in. I can use my math knowledge to analyze problems in society.

I can use my math skills to help others.

I can use my math knowledge to create a more just society.

Math can help me to find solutions for societal problems.

I don't see how the math I know can improve society.

I can use my math skills to explore societal issues.

Math helps me to evaluate the information I receive about the world.

I can use my math skills to improve the community I live in.

I can use my math knowledge to justify my views about the world.

My math knowledge makes me a more valuable member of society.

Studying math prepares me to investigate the world's problems.

Math equips me to advocate for my community.

Math helps me to question potential inequities in society.

Math helps me to justify my reasoning when making decisions.

 Table 3

 Experts for Round I of Expert Interviews

Date	Last	First	Department	Specialty
	name	name		
Fall	Frank,	Toya	Mathematics	Equity
2016	PhD		Education (GMU)	

Fall	Klee	Holly	Educational	Student
2016		•	Psychology (GMU)	beliefs,
			, ,	motivation

*Note.* Holly Klee has now received her PhD.

During the preliminary expert interviews, experts were each given a handout (refer to Figure 4) listing each item (45 items total). The experts were to provide feedback on each item, which entailed (a) rating how understandable each item was with space to provide revision notes, (b) rating how central each item was to the construct of interest with space to provide suggestions, and (c) indicating any aspects or characteristics of the construct that were inadequately represented by the MRS. In order to ensure the experts were clear on the construct being measured, the definition of mathematics relevance was provided.

Thank you again for agreeing to participate in the review of the items on the **Mathematics Relevance Scale (M.R.S.)** that I am developing. Below I define the construct and provide a list of questions about each of the items on the survey. Please begin by familiarizing yourself with this background information and the construct definition, and then please review the specific instructions for completing the face and content validation.

- I. Construct definition: Relevance is defined as the extent to which students perceive mathematics content to possess at least one of the following characteristics (a) practical utility in students' everyday lives and the lives of others, (b) exchange value in preparing students' for future careers, and (c) illuminating social inequities that exist within students' communities and society (Eccles (Parsons) et al., 1983; Gutstein, 2006; Sealey & Noyes, 2010; Wooley, Rose, Orthner, Akos, & Jones-Sanpei, 2013).
- II. **Purpose of Instrument:** The purpose of this scale is to measure secondary students' perceptions of relevance of mathematics content.

A. In this section I would like to know how comprehensible each item is for the anticipated respondent population. Please rate how understandable each of the following items are by using the scales below. If you have ideas for how to clarify the meaning of an item please note your thoughts beneath each item.

Dimension A: Practical utility in students' everyday lives and the lives of others.

1. I use math to help me make decisions outside of school.

Not at all understandable	Slightly understandable	Somewhat understandable	Extremely understandable			
Suggestions:						
Knowing math helps me in other school subjects.						
Not at all understandable	Slightly understandable	Somewhat understandable	Extremely understandable			

Figure 7

Expert Interview Handout

Upon receiving the reviews from experts, all qualitative responses were recorded in Google Docs. Next, the qualitative responses were examined and final item revisions and reductions occurred to prepare the measure for further expert review (refer to Table 4). From this initial round of expert reviews, the total number of items on the MRS was reduced from 45 to 19 items (refer to Table 4). The decision to remove items was based on both experts agreeing that an item was redundant or unclear. Furthermore, based upon suggestions by the experts, items that included the phrase "math skills" were changed to

"math" for fear of the phrase math knowledge vs. math skills being confusing to students. Items which included the phrase "societal issues" were changed to "problems in my community" or "problems in society" based on the experts articulating that there will be a range of students' interpretations of "societal issues." These revisions were made to help with the clarity and meaning of items. All the items remaining on the MRS met or exceeded expectations for both clarity and fitting the construct. Table 4 provides several details of comments that the experts provided.

Table 4

Expert Interview Comments

Items	Reviewer qu	Reviewer qualitative notes		
	Toya Frank, PhD	Holly Klee, PhD		
Math skills are useful in my daily life.		"Can students decipher the difference between knowledge and skills?"	Math is useful in my daily life.	
I need math knowledge to be successful in my career.	"Need a description BEFORE the word mathdoing math, knowing math, etc."	"Can students decipher the difference between knowledge and skills?"	Knowing math will help me to be successful in my career.	

Items	Reviewer au	alitative notes	Revised items
	Toya Frank, PhD	Holly Klee, PhD	
I can use my math skills to explore societal issues.	"How will students interpret 'societal issues'?"	"I was more comfortable when I read questions about helping my community."	I can use the math I am learning to analyze problems in society.
			I can use the math I am learning to analyze problems in my community.
Math helps me to think about solutions for societal problems.	"How will students interpret 'societal issues'?"	"I was more comfortable when I read questions about helping my community."	Math helps me to think about solutions for problems in society.  Math helps me to think about solutions for problems in my community.

# **Second Round of Expert Interviews**

Three experts, two from SJP and one from learner motivational beliefs, reviewed the 21-item MRS that resulted from the cognitive interviews with the five students as described in Chapter 3.

Table 5
Scale for Expert Interviews Part II (21 Items)

Dimension A: practical utility in stu	idents' every	day lives (EV	T)			
I use math to help me make decisions outside of school.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Knowing math helps me in other school subjects.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Knowing math helps me think more logically.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Math is useful in my daily life.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
I find the content of my math courses to be personally useful outside of school.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Dimension B: exchange value in pro	eparing stude	ents for future	e careers (EVT)			
Learning math will help me to get the job I want.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Math will play an important role in my future career.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me

Knowing math will help me to be successful in my career.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Knowing math will make me more competitive on the job market.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Studying math gives me more career choices.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Dimension C: illuminating social ir	nequities that	exist within	students' communi	ties and society	(SJP)	
I can use the math I am learning to explore problems within my community.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Math helps me to think about solutions for problems within my community.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
I can use the math I am learning to analyze problems in my community.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Math can help me to better understand the world we live in.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Math helps me to explore fairness in society.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Knowing math helps me understand neighborhood displacement.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me

Knowing math helps me understand deportation.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Knowing math helps me to judge other people's points of view in politics.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Knowing math makes it easier to understand income inequality.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Knowing math makes it easier to understand gentrification	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Math has to do with community issues, such as foreclosures.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me

Table 6

Experts for Round II of Expert Interviews

Date	Last name	First name	Department	Specialty
Mon July 9, 2018	View, PhD	Jenice	Transformative Teaching (GMU)	SJP
Fri July 13, 2018	Gutstein, PhD	Eric	Mathematics Education (UIC)	SJP
Mon July 16, 2018	Miller, PhD	Angela	Educational Psychology (GMU)	Learner motivational beliefs

Three experts—two who study SJP, both within and outside of mathematics, and one who studies learner motivational beliefs—reviewed the 21-item MRS. The purpose of interviewing these three experts was to provide further content validity by revising and reducing the number of items before sending the measure to participants. Angela Miller, PhD, an assistant professor at the time of the interview, was chosen as an expert in the field of educational psychology given her research focuses on student motivation in the classroom context and the impact of classroom characteristics on student motivation.

Jenice View, PhD, an associate professor at the time of the study, was chosen as an expert due to her research in teaching for social justice. Eric (Rico) Gutstein, PhD, a noted Professor in the field of mathematics education, was chosen as an expert given his highly respected research about social justice pedagogy and content in mathematics education. Dr. Gutstein's research challenges the field to consider teaching mathematics

through incorporating students' experiences with the goal of using mathematics as a tool for social justice and for analyzing and making sense of, and working to improve society.

During the second round of expert interviews, each expert was given a handout listing each item by dimension. The experts were to provide feedback on each item, which entailed (a) discussing how understandable each item was, (b) discussing how central each item was to the construct of interest, and (c) noting any aspects or characteristics of the construct that were inadequately represented by the MRS. Experts were encouraged to give suggestions to make items clearer and also to suggest any deletions or additions of items. In order to analyze the responses from the expert interviews, all responses were inputted into Microsoft Word. They were examined and final item revisions and reductions occurred to prepare the measure for participants. From this second round of expert interviews, the total number of items on the MRS was increased from 21 to 28 items. The details of revisions are discussed below.

# Types of Revisions

The three expert reviewers revised the 21 items on the MRS. First, items were edited to reflect better word choice and to be more concise. Next, items were edited to include definitions of terminology to avoid multiple interpretations from participants.

Lastly, items were added to exhaustively reflect SJP and EVT, in which the construct mathematics relevance is situated.

Word Choice and Conciseness. Experts read through each item on the MRS and made suggestions to revise items to be clearer and more concise. The following is one example of how word use was changed: "Knowing math helps me to judge other people's

points of view in politics" to "Knowing math helps me to evaluate other people's points of view in politics." One of the SJP experts described how the use of evaluate instead of judge casts mathematics in an objective stance instead of as a weapon, and the wording of the item was changed to reflect this concern (personal communication, Jenice View, PhD, July 10, 2018). The learner motivational beliefs expert suggested the word *logically* was very abstract within the item "Knowing math helps me think more logically," which could lead to multiple student interpretations. Thus, it was suggested to add the item "Knowing math helps me to justify my reasoning." By replacing logically with the phrase justify my reasoning, I would enhance the logic question twice and try to minimize multiple interpretations (personal communication, Angela Miller, PhD, July 16, 2018). Furthermore, it was recommended to delete the following item due to wordiness: "I find the content of my math courses to be personally useful outside of school." The learner motivational beliefs expert noted this item was repetitive of two other items, "I use math to help me make decisions outside of school" and "Math is useful in my daily life," and suggested deleting it (personal communication, Angela Miller, PhD, July 16, 2018).

**Definitions.** Experts read through each item on the MRS and made suggestions on how understandable each item was and how to clarify the meaning of certain items. One of the SJP experts raised concerns about the student population having varying interpretations of the terms *neighborhood displacement*, *gentrification*, *foreclosure*, and *deportation*. To prevent varying interpretations for items that included these terms, it was advised to provide concise definitions in smaller font after the item. For example, the item "Math has to do with community issues, such as foreclosures" was changed to

"Math has to do with community issues, such as foreclosures. (Foreclosure: banks taking property when a person fails to keep up their mortgage payments)" (personal communication, Jenice View, PhD, July 10, 2018). The second SJP expert agreed that adding definitions would prevent varying interpretations for items. Additionally, the definition of neighborhood displacement was edited following the concern that the original definition, "the buying and re-selling of houses and stores in lower income communities by upper- or middle-income people, raising property values," did not provide a distinction between the causes of displacement and the actual physical displacement. Thus, the definition was edited to reflect this distinction: "The forced movement of people from their homes caused by the buying and re-selling of houses and stores in lower income communities by upper- or middle-income people, resulting in raising property values" (personal communication, Eric Gutstein, PhD, July 13, 2018). These descriptors alongside these items were added to avoid varying interpretations having an effect on how students responded to these items.

Addition of Items. The MRS reviewed by these experts had 21 items. Additional items were added to the MRS to include issues of educational, racial, gender identity, sexual orientation, language, and learning ability inequalities. In response to a suggestion from the SJP expert, items were added to exhaust the construct of SJP that were originally not represented by this scale. Example items that were added to the MRS include "Knowing math makes it easier to understand educational inequality" and "Knowing math makes it easier to understand racial inequality" (personal communication, Jenice View, PhD, July 10, 2018). Lastly, it was suggested to add the

item "Circle 'Always true for me' if you are reading this!" as a way to spot check if participants were actually reading items on the MRS when completing the scale (personal communication, Angela Miller, PhD, July 16, 2018). This way, scales where participants simply circled answers without reading the items could be easily targeted and discarded before data analysis was run.

# Summary of Expert Interviews

At the end of the expert interviews, the MRS contained 28 items: 5 for the Practical Utility factor, 5 for the Exchange Value factor, and 18 for the Social Inequities factor.

**Table 7**Topics That Arose from the Second Round of Expert Interviews

Items		Reviewer qualitative notes		Revised items
	Jenice View, PhD	Eric Gutstein, PhD	Angela Miller, PhD	
I can use the math I am learning to analyze problems in my community.	n/a	"'Analyze problems'—'explore problems'—'think about solutions'—these seem too similar.  Are you expecting that "analyze"	n/a	Math helps me to think about solutions for issues within my community.
		and "explore" are sufficiently different that you might get different answers? Sure, we can explore w/out analyzingbut you can't think		I can use the math I am learning to analyze issues in my community.
		about solutions w/out doing both exploration and analysis, at some levelI'd try to collapse these three into two"		COMBINED items: Learning math helps me to analyze and think about solutions to issues within my community.
Knowing math helps me to judge other people's points of view in politics.	"Use the word' evaluate' instead of 'judge'. The word 'evaluate' casts mathematics as objective instead of as a weapon."	n/a	n/a	Knowing math helps me to evaluate other people's points of view in politics.
Knowing math helps me understand neighborhood displacement.	"Will students know what the terms neighborhood displacement means? Gentrification? Foreclosure? Suggestion is to add simple definitions in smaller font so that	(Neighborhood displacement: the buying and re-selling of houses and stores in lower income communities by upper- or middle-income people, raising property values)  "This definition says nothing about displacement—that is, the connection to what's listed and the actual physical displacement is not	n/a	Knowing math helps me understand neighborhood displacement. (Neighborhood displacement: the buying and re-selling of houses and stores in lower income communities by upper- or

Items		Revised items		
	Jenice View, PhD Eric Gutstein, PhD Angela Miller,		Angela Miller, PhD	
	you know students are interpreting these terms how you want	explicit. The causes of displacement are not the same as displacement itself"		middle-income people, raising property values).
	terms now you want them to."	RSCII		UPDATED DEFINITION: Neighborhood displacement: The forced movement of people from their homes caused by the buying and re-selling of houses and stores in lower income communities by upper- or middle-income people, resulting in raising property values
Knowing math makes it easier to understand educational inequality.	n/a	"You might want to add some definitions hereyouth may not recognize what you mean by this."	n/a	Knowing math makes it easier to understand educational inequality. (Educational inequality: The unequal distribution of academic resources, including school funding, qualified and experienced teachers, books, and technologies. This leads to major differences in the educational success of these individuals.)
Knowing math helps me understand deportation.	"Will students know what the terms neighborhood displacement means? Gentrification? Foreclosure?	n/a	n/a	Knowing math helps me understand deportation. (Deportation: people being pushed out of the country).

Items		Revised items		
	Jenice View, PhD	Eric Gutstein, PhD	Angela Miller, PhD	
	Suggestion is to add simple definitions in smaller font so that you know students are interpreting these terms how you want them to."			
Knowing math makes it easier to understand gentrification.	"Will students know what the terms neighborhood displacement means? Gentrification? Foreclosure? Suggestion is to add simple definitions in smaller font so that you know students are interpreting these terms how you want them to."	n/a	n/a	Knowing math makes it easier to understand gentrification. (Gentrification: the buying and re-selling of houses and stores in lower income communities by upper- or middle-income people, raising property values).
Math has to do with community issues, such as foreclosures.	"Will students know what the terms neighborhood displacement means? Gentrification? Foreclosure? Suggestion is to add simple definitions in smaller font so that you know students are interpreting these terms how you want them to."	n/a	n/a	Math has to do with community issues, such as foreclosures. (Foreclosure: banks taking property when a person fails to keep up their mortgage payments).

Knowing math makes it easier to understand language inequality.	n/a	"I think you need to define what you mean here. For ostensibly monolingual speakers (sidestepping the discussion about Ebonics being its own viable language), the meaning of "language inequality" may not be clear. The "English Only" ideological frames people are enculturated into make it hard to recognize language inequality. And, if you mean the "politics of correctness," (that is, that Ebonics speakers are "wrong" and must be "corrected," which to me is definitely an aspect of language inequality/discrimination and hegemony), that's a complicated matterso how you define this for young people is key. I don't think it's a commonly understood idea."	n/a	DELETE
Knowing math makes it easier to understand learning abilities inequalities.	n/a	"And I think some of the same concerns from #17 apply herewhat are "learning abilities inequalities?" I don't even have a clear ideaare we talking about tracking in school? That whites and Asians are perceived (by whom?) to be "better" in mathematics than Black and Brown people?"	n/a	DELETE
n/a	"Include an item on educational inequities (achievement gap)."	n/a	n/a	ADD: Knowing math makes it easier to understand educational inequality.

Reviewer qualitative notes Eric Gutstein, PhD

Revised items

Angela Miller, PhD

Items

Jenice View, PhD

	Items		Revised items		
		Jenice View, PhD	Eric Gutstein, PhD	Angela Miller, PhD	
	n/a	"Include items geared towards inequities found in the literature."	n/a	n/a	ADD: Knowing math makes it easier to understand race, gender identity, sexual orientation, language, learning abilities inequalities?
	n/a	"Include an item that is straight forward in terms of students' perceptions that they can use math to make changes in their community."	n/a	n/a	ADD: I can use math to make changes in my community.
89	Knowing math helps me think more logically.	n/a	n/a	"The term 'logically' is very abstract whereas justify reasoning is more concrete for students. Thus, add one additional item to enhance the logic question twice by using the phrase justify reasoning."	ADD: Knowing math helps me to justify my reasoning.
	I find the content of my math courses to be personally useful outside of school.	n/a	n/a	"This item is too wordy and basically is repeating the following two items 'I use math	DELETE

Items		Revised items		
	Jenice View, PhD	Eric Gutstein, PhD	Angela Miller, PhD	-
			to help me make decisions outside of school' and 'Math is useful in my daily life'so don't need it."	
n/a	n/a	n/a	"A great way to check if participants are actually reading your items make sure to add an item that says bubble or circle if you are reading."	ADD: Circle 'Always true for me' if you are reading this!

## **Cognitive Interviews**

# Initial Round of Cognitive Interviews

The purpose of the initial cognitive interviews was to gain feedback on the MRS items in terms of clarity and comprehension before participants actually completed the MRS. Unlike the student interviews where students discussed their own definitions of mathematics relevance, for these cognitive interviews, students went through each item and provided feedback on the phrasing and clarity, their interpretation of the items, and recommendations to improve the quality of the scale. Cognitive interviews were conducted with five students to determine how they responded and viewed the items on the MRS.

After the recordings of the cognitive interviews were analyzed, the participants' responses were organized into two categories to represent the emerging themes of the interviews: (a) ease of understanding of items and (b) wording clarity of items. Items were edited after considering the common suggestions made across the participants.

 Table 8

 Cognitive Interview Round I Participants Self-Reported Demographic Data

Student	Age	Grade	Gender	Ethnicity	Math course	Career interest
					currently	
					enrolled in	
Student A	15	10	Male	Hispanic/Latino	Algebra	Health Care
					II/Trig	
Student B	15	10	Female	Hispanic/Latino	Geometry	Psychology,
						American Sign
						Language
Student C	18	12	Female	Asian	N/A	Nursing

Student	Age	Grade	Gender	Ethnicity	Math course	Career interest
					currently	
					enrolled in	
Student D	17	11	Female	Asian	Algebra	Communication
					II/Trig	Arts &
						Animation
Student E	14	9	Female	Hispanic/Latino	Honors	Business &
				1	Geometry	Marketing

**Table 9**Revised Scale for Cognitive Interviews Part I (19 Items)

Dimension A: practical utility in students' everyday lives (EVT)

I use math to help me make decisions outside of school.	Strongly Disagree	Disagree	Somewhat Disagree	Somewhat Agree	Agree	Strongly Agree
Knowing math helps me in other school subjects.	Strongly Disagree	Disagree	Somewhat Disagree	Somewhat Agree	Agree	Strongly Agree
Knowing math helps me think more logically.	Strongly Disagree	Disagree	Somewhat Disagree	Somewhat Agree	Agree	Strongly Agree
Knowing math helps me become a problem solver in my community.	Strongly Disagree	Disagree	Somewhat Disagree	Somewhat Agree	Agree	Strongly Agree
I use math outside of school.	Strongly Disagree	Disagree	Somewhat Disagree	Somewhat Agree	Agree	Strongly Agree
Math is useful in my daily life.	Strongly Disagree	Disagree	Somewhat Disagree	Somewhat Agree	Agree	Strongly Agree
I find the content of my math courses to be personally useful outside of school.	Strongly Disagree	Disagree	Somewhat Disagree	Somewhat Agree	Agree	Strongly Agree

Dimension B: exchange value in preparing students for future careers (EVT)

Learning math will help me to get the job I want.	Strongly Disagree	Disagree	Somewhat Disagree	Somewhat Agree	Agree	Strongly Agree
Math will play an important role in my future career.	Strongly Disagree	Disagree	Somewhat Disagree	Somewhat Agree	Agree	Strongly Agree
Knowing math will help me to be successful in my career.	Strongly Disagree	Disagree	Somewhat Disagree	Somewhat Agree	Agree	Strongly Agree
Knowing math will make me more competitive on the job market.	Strongly Disagree	Disagree	Somewhat Disagree	Somewhat Agree	Agree	Strongly Agree
Studying math gives me more career choices.	Strongly Disagree	Disagree	Somewhat Disagree	Somewhat Agree	Agree	Strongly Agree
My career will involve doing math.	Strongly Disagree	Disagree	Somewhat Disagree	Somewhat Agree	Agree	Strongly Agree

Dimension C: illuminating social inequities that exist within students' communities and society (SJP)

I can use the math I am learning to explore problems within my community.	Strongly Disagree	Disagree	Somewhat Disagree	Somewhat Agree	Agree	Strongly Agree
Math helps me to think about solutions for societal problems.	Strongly Disagree	Disagree	Somewhat Disagree	Somewhat Agree	Agree	Strongly Agree
I can use the math I am learning to analyze problems in society.	Strongly Disagree	Disagree	Somewhat Disagree	Somewhat Agree	Agree	Strongly Agree
Math can help me to better understand the world we live in.	Strongly Disagree	Disagree	Somewhat Disagree	Somewhat Agree	Agree	Strongly Agree
Math helps me to explore fairness in society.	Strongly Disagree	Disagree	Somewhat Disagree	Somewhat Agree	Agree	Strongly Agree
Math helps me to explore fairness in community.	Strongly Disagree	Disagree	Somewhat Disagree	Somewhat Agree	Agree	Strongly Agree

Ease of Understanding of Items. Students were asked to read each item and put the question in their own words. In addition, students were asked if the words used in the items were clear. These item prompts were imperative to make sure the 19 items on the MRS were being interpreted in the way the researcher intended. For example, for the item "I use math to help me make decisions outside of school," Student A stated, "I think of math for money since I work as a cashier," whereas Student B stated, "I think of math for taxes, financial, in order to make decisions, I think of probability of doing something and what are the chances." Here Students A and B were able to express that this item was easy to understand by quickly giving examples of how mathematics could be used outside of school. Moreover, their understanding of the item matched the researcher's intent, implying no edits were needed for this item.

Table 10

Data Analysis Matrix for Round I of Cognitive Interviews: Example MRS Items Participants Found "Easy" to Answer

Items			Student qualitative	notes		Revised items
	A	В	С	D	Е	-
I use math to help me make decisions outside of school.	"To make decisions, I think about the probability of doing something and what are the chances."	"Math for taxes."	"Math for money since I work as a cashier sometimes."	"I'm a complete nerd and play a lot of DND and so I need to know the probability of rolling [DND: Dungeons and Dragons]"	"Financial."	No Change
Knowing math helps me in other school subjects.	"Sciences I guesshelps in chemistry because of basic math skills, word problems, algebra involved."	N/A	N/A	"Some of it helps surprisingly in English classI have to read not only literature but scientific journals because there is a lot of statistics in them. I have to be able to interpret graphs in there."	"Helps in chemistry as well."	No Change
Knowing math will make me more competitive on the job market.	"You need math in most jobs I guesslike measurements."	"More skills you have the higher successful you are."	N/A	"If you know math or science then it helps you."	N/A	No Change
Knowing math helps me think more logically.	N/A	"Try to think in multiple perspectives."	"Think reasonably, like my geometry teachers says."	"Think inductively/deductively."	N/A	No Change

Wording Clarity of Items. The second category for revisions included items that students described as being "unclear" or "confusing." To help students clarify items being unclear, the following probes were used: "Are the words in the item clear?", "Is the phrasing of the item clear?", "Are there any other words and/or phrasing that could convey the question more successfully?" Overall, two items fell into this category of being confusing for students.

One example of confusion stemmed from the item "Math helps me to think about solutions for societal problems." Students felt more comfortable answering questions regarding their community and less comfortable answering questions about society. To them, community meant their neighborhood, school, or even their group of friends. When asked what types of societal problems they could think of, students mentioned renewable energy, changing power sources, and how these help people save money over time. The items from Dimension C were then revisited to determine if the items were best written to reflect the SJP framework, given students' difficulty in the swapping of society and community. Out of the six items within Dimension C, four of the items were worded in terms of society. The items "Math can help me to better understand the world we live in" and "Math helps me to explore fairness in society" were kept. All items within Dimension C with the word *society* were not changed to *community* given that the SJP research framework is situated to reflect students' communities and society as a whole. However, the remaining items were edited to reflect students' communities. For example, the item "Math helps me think about solutions for societal problems" was changed to "Math helps me think about solutions for problems within my community." The item "I

can use the math I am learning to analyze problems in society" was changed to "I can use the math I am learning to analyze problems in my community." Lastly, one item was added to Dimension C: "I can use math to make changes in my community."

The final change to items on the MRS was based on students' confusion over the word *problems* in the item "I can use the math I am learning to explore problems within my community." Students asked, "What do you mean by the word 'problems'?" and "Do you mean the problems that anyone is having?" Thus, the item was revised in response to students' suggestions and the word *problems* was changed to *issues* in order to make the item clearer.

# Second Round of Cognitive Interviews

After the first round of cognitive interviews, items on the MRS were revised and the scale included 28 items. In order to determine if these changes were sufficient to clarify the meaning of the revised items, a second round of cognitive interviews with students occurred. Five new students were interviewed for this round. The procedure followed the same format as the preliminary cognitive interviews. The purpose was to still gain feedback on the items in terms of clarity and comprehension before participants actually completed the MRS. Again, students went through each item and provided feedback on the phrasing and clarity, their interpretation of the items, and recommendations to improve the quality of the scale. The only change in the second round of interviews was getting feedback on response anchors from students.

After the recordings of the second round of cognitive interviews were analyzed, the participants' responses were organized into four categories to represent the emerging

themes of the interviews: (a) ease of understanding of items, (b) wording clarity of items, (c) repetition, and (d) response anchors. Items were edited after considering the common suggestions made across the participants.

 Table 11

 Cognitive Interview Round II Participants Self-Reported Demographic Data

Student	Age	Grade	Gender	Ethnicity	Math course currently enrolled in	Career interest
Student A	16	11	Female	Hispanic/Latino	Algebra II	TBD
Student B	15	11	Female	Hispanic/Latino	Algebra II/Trig	Nurse
Student C	14	9	Male	Hispanic/Latino	Geometry	Lawyer
Student D	16	12	Female	African- American	IB Math Higher-Level II	Physician
Student E	14	9	Female	Multi- ethnic/Multi- race	Honors Geometry	Medical Field

Ease of Understanding of Items. Participants demonstrated ease of understanding when reading through items by showing the ability to rephrase items in their own words and describe how they would answer items in detail. For example, when reading the item "Knowing math makes it easier to understand educational inequality," students responded with examples of educational inequality by discussing how student/teacher ratios vary by school and how students receive a better education from

schools that have a smaller student/teacher ratio. After reading the item "Knowing math makes it easier to understand racial inequality," students responded that by knowing statistics they can understand racial inequality through prison rates and tobacco selling in low-income communities. Furthermore, after reading the item "Knowing math will make me more competitive on the job market," students responded that by knowing math they will be better candidates and have more opportunities than their peers. After reading the item "Math can help me to better understand the world we live in," students responded by giving examples of how math helps to better understand science, nature, carbon dioxide in the air, and the ozone layer, and how math helps them think logically about facts they learn.

Wording Clarity of Items. The second category for revisions included items that students described as being "unclear "or "confusing." To help students clarify items being unclear, the following probes were used: "Are the words in the item clear?", "Is the phrasing of the item clear?", "Are there any other words and/or phrasing that could convey the question more successfully?" Students found the item "I find the content of my math courses to be personally useful outside of school" to be too wordy and confusing compared to the item "Math is useful in my daily life." Thus, I decided to delete "I find the content of my math courses to be personally useful outside of school," given it was redundant. Furthermore, students had a hard time understanding the term gentrification compared to neighborhood displacement, even with definitions added at the end of the items. They responded, "The phrase 'neighborhood displacement' is clearer and to the point without having to read the definition," compared to when I used

the term *gentrification* in the item "Knowing math makes it easier to understand gentrification." Thus, the gentrification item was deleted and "Knowing math helps me understand neighborhood displacement" was retained.

When students came across the item "I can use math to make changes in my community," they immediately started giving examples of what changes they thought of. They mentioned cosmetic changes to their communities, for instance, needing to improve their community by planting trees, fixing potholes, and paving roads. Once I gave examples of what I meant, referencing lessons from SJP research about issues such as too many liquor stores in a neighborhood, students then said, "Oh, like building more libraries and schools." Thus, to make student and researcher interpretations match for this item, the item was changed to "I can use math to make changes to improve my community."

Repetition. Participants expressed that a few items were repetitive and not needed on the MRS, such as the items "Knowing math makes it easier to understand gender identity inequality" and "Knowing math makes it easier to understand sexual orientation inequality." Students suggested these two items be combined into one item that reflected the LGBT community. Thus, these two items were combined into "Knowing math makes it easier to understand inequities in the LGBTQ community. (LGBTQ: Lesbian, Gay, Bisexual, Transgender, Queer or Questioning)."

Next, students suggested that "Math helps me to think about solutions for issues within my community" and "I can use math to make changes in my community" were repetitive and suggested getting rid of one of them. As the researcher, I intended these to

examine two different things: thinking about solutions and taking action to bring about change. However, the students did not see a difference in these statements. They expressed being more comfortable thinking about solutions in mathematics. For the purposes of this study, I was more interested in determining whether students perceived that mathematics could be used to find solutions to issues in their communities without creating pressure around being able to actually bring about those changes. Thus, I decided to delete the item "I can use math to make changes in my community."

Response Anchors. After further research into response anchors and their effects on participants' responses to items, I decided to assess my participants' preferences between two options for response anchors. Fowler (2009) and Krosnick (1999) advise to avoid using agree-disagree response anchors, given asking respondents to rate their level of agreement with items is a cognitively demanding task that ends up increasing respondent error and reduces their effort in completing the scale. Thus, I asked students in the second round of cognitive interviews to discuss their preference between agree-disagree statements and never true for me-always true for me statements. Students expressed preferring the never true for me-always true for me statements because they made them think about themselves in every item, while the agree-disagree response anchors made them think about how others would respond to the items on the MRS Thus, the response anchors on the MRS were changed after hearing students' preferences.

Regarding the *never true for me-always true for me* statements, particularly the *untrue*, I asked an expert reviewer, Angela Miller, about her thoughts on this response anchor. She showed no signs of hesitation about me using these anchors and noted she

had seen them used before. As I was using scales published by other researchers, I was hesitant to change their response anchors, as the reliabilities and validities were reported as is. Thus, I did not feel confident to change anything when reusing the same scales in my dissertation.

## Summary

Overall, following these two rounds of cognitive interviews, there were no changes in Dimension B, items written in terms of exchange value. Students perceived these items to be written clearly, and there were no differences in the interpretability of items between the students and the researcher. The majority of the students' confusion and thus item revisions came from Dimension C, items written and situated in SJP. At the beginning of the initial cognitive interviews, the MRS had 28 items. Four items were deleted due to repetition or confusion, two items were combined to reflect the question regarding the LGBTQ community, and two items were edited to make them clearer. Thus, the final number of items on the MRS for the participants to complete was 22.

The MRS started with 45 items and ultimately had 22 items after two rounds of student cognitive interviews and two rounds of expert interviews. The items then underwent two more rounds of edits under the guidance of my dissertation committee. The first round of edits transformed the MRS from 22 items to 29 items. Feedback received included the need for an 8-10 item minimum per subscale. Furthermore, on the teaching for justice subscale, there was only one item geared toward the LGBTQ community. Thus, to incorporate this feedback, items were added to all dimensions to account for the minimum of eight items per subscale. One more item was also added to

reflect the LGBTQ community: "Learning math helps me to analyze inequalities in the LGBTQ community. (LGBTQ: Lesbian, Gay, Bisexual, Transgender, Queer or Questioning)." The second and last round of edits in response to my dissertation committee transformed the MRS from 29 items to 35 items. The committee advised me to revise rather than delete the items originally deleted due to confusion in student cognitive interviews. Thus, five original items that were deleted due to feedback were now reworded, e.g., "Knowing math makes it easier to understand inequality of access to the English language education in schools. (English language education: The teaching and learning conditions of English Language Learners (ELL) in schools.)."

#### Administration of the MRS

The MRS provided in Appendix D reflects the 35 items administered to participants. The MRS consists of three subscales that address areas of (a) practical utility in students' everyday lives, (b) exchange value in preparing students for future careers, and (c) illuminating social inequities that exist within students' communities and society, with the following Likert-type responses: never true for me, rarely true for me, sometimes untrue for me, sometimes true for me, usually true for me, always true for me.

In addition to the MRS, participants completed four established scales to test for convergent (Conley, 2012; Wigfield et al., 1997) and discriminant validity (Conley, 2012; Kosovich et al., 2015). Prior to scales being completed, parental consent was obtained by sending consent forms home in the first week of the summer academy with the intent that students would return them by the end of the week. All 550 secondary students enrolled in the college preparatory program were eligible to participate in this

study. Mentors were in charge of collecting consent forms and providing students with a QR code that they could scan or a survey link to enter on their computers or smartphones. All scales were completed online using the Qualtrics survey platform, and the average time to complete the survey was 20 minutes. Out of 550 students in the program, 341 completed the surveys (62% return rate).

# **Exploratory Factor Analyses**

EFA, the process outlined in detail in Chapter 3, is a common technique used in scale development that uses variance to determine how items are functioning together within an instrument (Comrey, 1988; DeVellis, 2003). Prior to carrying out EFA, the data from the 341 participants were cleaned to ensure the data were acceptable for use. The data were then analyzed using the following six-step process outlined in Chapter 3.

- 1. The data were cleaned to get rid of outliers, data entry mistakes, and to prepare data to ensure assumptions of EFA were met.
- 2. Bartlett's test of sphericity and KMO test statistics were examined to ensure that it was appropriate to use EFA.
- 3. Correlations among items were analyzed to determine the extraction (i.e., Principal Components Analysis [PCA] or PAF) and rotation methods of EFA.
- 4. Examination of eigenvalues and the scree plot to decide the number of factors to extract.
- 5. Factor loadings were used in determining deletion and retention of items.
- 6. The factor structure was analyzed to interpret the factors.

#### Data Cleaning

Prior to carrying out EFA, the data from the 341 participants were cleaned to ensure the data was acceptable for use with EFA. First, data from participants who disagreed with participating in the study were deleted in order to honor the assent process. Additionally, data from students who answered questions MRS\_13 & MRS\_21 ("Select 'Always true for me' if you are reading this") incorrectly were deleted to ensure participants were fully present while answering the items. The assumption of univariate and multivariate normality was not applicable for EFA and thus it was not necessary to test (Pituch & Stevens, 2016).

Missing data were analyzed and results showed that 6.28% of the values in the data set were missing. In order to run EFA without compromising inferences, multiple imputation, a strategy used to handle data sets with missing values, was used. Instead of filling in a single value for each missing value, Rubin's (1987) multiple imputation procedure handles missing data by replacing each missing value with a set of plausible values (e.g., five data sets). Next, this procedure requires averaging the values of the estimates across the five missing value samples in order to obtain a single point estimate. After the data cleaning process, the final data set consisted of 232 participants.

#### Descriptive Statistics

The participants were students in Grades 8-12 who had been selected to participate in an out-of-school college preparatory program. This program enrolls over 500 students from seven local public schools, and provides access to educational resources and mentoring for students who will be first-generation college students upon

their high school graduation. These students receive year-round academic enrichment, college preparation, and attend a free 3-week summer program housed on a college campus where they are exposed to coursework they will encounter in the upcoming year with hopes of increasing their college readiness. Of the 232 participants who completed the MRS, 84 (36.2%) were male and 145 (62.5%) were female, with ages spanning 14 to 18 years. The majority of the participants identified themselves as Hispanic (73.3%) or African American (26.7%). The majority of the participants identified themselves as in Grade 9 (39.2%), 10 (28.9%), or 11 (22%). Furthermore, 85% reported they typically earned As and Bs in their math courses, and 100% of the participants intended to go to college upon graduating high school.

Mean composites were created for all established scales by combining the items that represented that scale to create a score, or data point, for that scale. Thus, each item on a particular scale was totaled and then divided by the total number of items on that scale: e.g., Cost Value Scale: (item 1) + (item 2) + (item 3) + (item 4) = /4. Mean composites were created for the MRS and the four established subscales.

**Table 12**Descriptive Statistics for Participants

	Final	sample
	(N =	232)
	f	%
Gender		
Male	84	36.2
Female	145	62.5
Race/Ethnicity		
African American	62	26.7
Hispanic	170	73.3
Grade		
8	2	.9
9	91	39.2
10	67	28.9
11	51	22.0
12	20	8.6
	M	SD
MRS	3.69	0.87
Task Utility Value Scale (Wigfield et al., 1997)	5.04	1.17
Task Utility Value Scale (Conley, 2012)	3.94	0.87
Cost Value Scale (Conley, 2012)	2.37	1.00
Cost Value Scale (Kosovich et al., 2015)	3.04	1.04

## **MRS**

To ensure the data were appropriate for statistical analyses, means, standard deviations, skewness, and kurtosis were calculated for the 35-item MRS. The items on the MRS were measured on a six-point scale (*never true for me–always true for me*) with item means between 2.24 and 4.99, and standard deviations ranged from 1.08 to 1.67. Additionally, skewness, a measure of the asymmetry of the distribution, and kurtosis, a measure of whether there is a very narrow distribution with most of the responses in the

center, analyses revealed some extreme values for a subset of items on the MRS. A guideline for a skewed distribution is greater than +1 or lower than -1. For kurtosis, greater than +1 means the distribution is too peaked and less than -1 indicates a distribution that is too flat. The following MRS items had highly negatively skewed distributions (skewness ranged less than -1 or greater than 1): MRS 23: I need to know math to earn a living (-1.22); MRS 25: Studying math gives me more career choices (-1.32); MRS 27: Knowing math will help me to be successful in my career (-1.09); MRS 29: Math will be useful no matter what career I choose (-1.01). These items shed light on this study's participants in terms of more students seeing the relevance that mathematics has for career choices. Alternatively, the following MRS items had moderately positively skewed distributions (skewness between -1 and -0.5 or between 0.5 and 1): MRS 24: Learning math helps me to analyze inequalities in the LGBTQ community (.973); MRS 36: Knowing math makes it easier to understand inequalities in the LGBTQ community (.970). These items illustrate that more participants found it difficult to see the connection between mathematics and LGBTQ inequalities.

**Table 13**Skewness, Kurtosis, Mean, and Standard Deviation for the Original 35-Item MRS

Item	Mean	SD	Skewness	Kurtosis
MRS 1: I use math to help me make decisions outside of	3.51	1.28	33 .16	96 .31
school.				
MRS 2: Math helps me to explore fairness in society.	2.78	1.33	.19  .16	-1.19 .31
MRS_3: I will use math problem-solving skills in my	4.36	1.37	83 .16	.05   .31
career.				
MRS_4: Knowing math helps me understand neighborhood	3.30	1.51	01 .16	1.04 .31
displacement. (Neighborhood displacement: The forced				

Item	Mean	SD	Skewness	Kurtosis
movement of people from their homes caused by the			224000	
buying and re-selling of houses and stores in lower				
income communities by upper- or middle-income people,				
resulting in raising property values).				
MRS_5: Math provides me the tools to make informed	3.67	1.33	46 .16	61 .31
decisions outside of school.			1 -	1 -
MRS 6: I can use the math I learned in elementary school	3.74	1.50	30 .16	92 .31
to explore problems within my community.				·
MRS 7: I can use the math I learned in middle school to	3.64	1.34	52 .16	60 .31
explore problems within my community.			•	·
MRS_8: I can use the math I learned in high school to	3.35	1.56	10 .16	-1.21 .31
explore problems within my community.				
MRS_9: Knowing math makes it easier to understand	2.86	1.49	.36 .16	-1.04 .31
inequality of access to English language education in				
schools. (English language education: The teaching and				
learning conditions of English Language Learners (ELL)				
in schools.)				
MRS_10: Learning math will help me to get the job I want.	4.53	1.35	89 .16	.09 .31
MRS_11: Knowing math makes it easier to understand	4.05	1.31	53 .16	25 .31
income inequality.	2.02	1.20	26116	051.01
MRS_12: I apply my math skills outside of school.	3.83	1.39	36 .16	85 .31
MRS_14: Knowing math helps me understand	3.09	1.56	.13 .16	-1.19 .31
gentrification. (Gentrification: The process of renovating				
and improving a house or district. Thus, forcing the				
movement of people from their homes in lower income				
communities.) MRS 15: Math can help me to better understand the world	3.90	1.31	42 .16	32 .31
we live in.	3.90	1.51	42 .10	32 .31
MRS_16: Math is useful in my daily life.	4.12	1.45	48 .16	44 .31
MRS_17: Math has to do with community issues, such as	4.05	1.53	48 .16	72 .31
foreclosures. (Foreclosure: banks taking property when a	1105	1.00		.,2 .51
person fails to keep up their mortgage payments).				
MRS 18: Math will play an important role in my future	4.53	1.37	90 .16	.18 .31
career.	-	-	1	1
MRS_19: Knowing math helps me to justify my reasoning.	3.73	1.30	47 .16	45 .31
MRS_20: Learning math helps me to analyze and think	3.43	1.40	18 .16	86 .31
about solutions to issues within my community.				
MRS_22: Knowing math helps me to evaluate other	2.85	1.40	.18 .16	92 .31
people's points of view in politics.			•	·
MRS_23: I need to know math to earn a living.	4.80	1.18	-1.22 .16	1.68 .31
MRS_24: Learning math helps me to analyze inequalities in	2.27	1.50	.97 .16	16 .31
the LGBTQ community. (LGBTQ: Lesbian, Gay,				
Bisexual, Transgender, Queer or Questioning).				
MRS_25: Studying math gives me more career choices.	4.99	1.16	-1.31 .16	1.83 .31
MRS_26: Knowing math helps me in other school subjects.	4.55	1.08	74 .16	.82 .31
MRS_27: Knowing math will help me to be successful in	4.72	1.32	-1.09 .16	.77 .31
my career.	4.56	1 2 4	06116	221.21
MRS_28: Knowing math will make me more competitive	4.56	1.34	86 .16	.22 .31
on the job market.	4.70	1 20	1.011.17	70  21
MRS_29: Math will be useful no matter what career I	4.78	1.38	-1.01 .16	.78 .31
choose.				

Item	Mean	SD	Skewness	Kurtosis
MRS_30: Knowing math makes it easier to understand the inequalities within tracking students in schools.  (Tracking: Assigning students to different classes based on their perceived ability in that subject.)	3.43	1.45	18 .16	85 .31
MRS_31: Knowing math makes it easier to understand racism.	2.49	1.53	.63 .16	73 .31
MRS_32: Knowing math helps me think more logically.	4.49	1.28	90 .16	.72 .31
MRS_33: Knowing math makes it easier to understand educational inequality. (Educational inequality: The unequal distribution of academic resources, including school funding, qualified and experienced teachers, books, and technologies. This leads to major differences in the educational success of these individuals.)	3.41	1.56	16 .16	-1.06 .31
MRS_34: Knowing math helps me understand deportation. (Deportation: people being pushed out of the country).	2.70	1.66	.57 .16	92 .31
MRS_35: I can use math to make changes in my community.	3.59	1.45	17 .16	79 .31
MRS_36: Knowing math makes it easier to understand inequalities in the LGBTQ community. (LGBTQ: Lesbian, Gay, Bisexual, Transgender, Queer or Questioning).	2.24	1.51	97 .16	12 .31
MRS_37: Studying math helps me with problem solving in other subjects.	4.40	1.14	69 .16	.70 .31

*Note.* SD = standard deviation. For skewness and kurtosis: Statistic | Std. Error.

# Determining to Use EFA

EFA is a statistical method used to identify the underlying relationships between measured variables. To determine whether using EFA is appropriate, the sample size is key. In the literature, there exist varying rules of thumb for sample sizes. Data were collected from 341 participants, and after data cleaning, 232 participants remained for the EFA; this sample size was over the minimum of 100 to 250 outlined by Cattell (1978) and Gorsuch (1983). Both Bartlett's test of sphericity and the KMO measure of sampling adequacy were calculated to determine if an EFA was acceptable to run (Bartlett, 1950; Kaiser, 1974). The KMO = .930 determines the degree of common variance among

variables, and it met the requirement of wanting values closer to one (preferable KMO > .6), which means it is likely that the observed items on the MRS share a common factor (Comrey & Lee, 1992; Kaiser, 1974). Correlations among items ranged between r = .073 and r = .815 (p < .01), signifying low to strong positive relationships between items on the MRS. Given the wide range in correlations, it was important to apply Bartlett's sphericity test, which tests the null hypothesis that the items in correlation matrix are uncorrelated (Pituch & Stevens, 2015). The Bartlett's test of sphericity was significant,  $\chi^2$  (232) = 5800.056, p < .001, meaning there is a redundancy between items that can be summarized with factors. Thus, I was able to run EFA on the 35 MRS items.

#### Factor Extraction

Due to assessing latent factors, I chose the PAF extraction method. PAF is commonly reported in social and behavioral science research (Warner, 2013) due to its use when factors are correlated with one another and analyzing the shared variance among the factors (Dimitrov, 2012). More specifically, PAF provides separate estimates of common and unique variance, thus considering the presence of error (Reise et al., 2000).

#### Factor Rotation

The next step in the EFA process was factor rotation in order to create factors from the set of MRS items. Since all items on the MRS are situated in mathematics relevance literature, EVT, and SJP, I expected theoretically for the factors to correlate. Thus, I used the oblique rotation method, Direct Oblimin, where factors are allowed to correlate and EFA does not try to maximize the distinction between the factors. This

rotation method helped with interpretability by maximizing larger factor loadings closer to 1 and minimizing smaller factor loadings closer to 0. Factor loadings less than .32 indicated the item was not a strong indicator of that factor (Comrey & Lee, 1992).

#### Factor Retention

Once extraction and rotation methods were determined, eigenvalues and the scree plot determined the number of factors to extract. Seven rounds of the factor analysis were performed until a final factor structure was found. All rounds used PAF as the extraction method and Direct Oblimin as the factor rotation method.

Round 1, an exploratory round of the analysis, consisted of allowing the factor extraction to be based on eigenvalues greater than 1. The KMO = .930 and the Bartlett's test of sphericity were significant,  $\chi^2(595) = 5800.056$ , p < .001. Initial eigenvalues indicated that the first three factors explained 39.13%, 9.67%, and 5.27% of the variance respectively. The fourth and fifth factors had eigenvalues just over one, and each explained 2.89% and 2.01% of the variance respectively. The three-factor solution, which explained 58.99% of the variance, was not preferred despite the hypothesized three-factor solution because of (a) the "leveling off" of eigenvalues on the scree plot after who factors; and (c) the insufficient number of primary loadings and difficulty of interpreting the third factor and subsequent factors.

Round 2 consisted of allowing the factor extraction to be based on fixing the number of factors to 2. The KMO = .930 and the Bartlett's test of sphericity were still significant,  $\chi^2(595) = 5800.056$ , p < .001, and did not change from Round 1 of the analysis. Initial eigenvalues indicated that the first two factors explained 38.86% and

9.29% of the variance respectively. The following items were deleted because they failed to meet the minimum criterion of having a primary factor loading of .4 or above in addition to close cross-loading: MRS\_12: I apply my math skills outside of school (.419, -.441); MRS\_16: Math is useful in my daily life (.391, -.386); MRS\_5: Math provides me the tools to make informed decisions outside of school (.362, -.423); MRS\_1: I use math to help me make decisions outside of school (.410, -.319); and MRS\_15: Math can help me to better understand the world we live in (.441, -.519).

Round 3 of the analysis consisted of rerunning the EFA after deleting items MRS\_12, MRS\_16, MRS\_5, MRS\_1, and MRS\_15. The KMO = .918 and the Bartlett's test of sphericity were significant,  $\chi^2(435) = 4733.73$ , p < .001. Initial eigenvalues indicated that the first two factors explained 37.40% and 10.79% of the variance respectively and the two-factor solution explained 48.20% of the variance. One item (MRS\_32: Knowing math helps me think more logically) was deleted because it was the only item that cross-loaded on both factors, .353 and .454 respectively, without a strong primary loading on any factor.

Round 4 of the analysis consisted of rerunning the EFA after deleting item MRS\_32. The KMO = .917 and the Bartlett's test of sphericity were significant,  $\chi^2$  (406) = 4530.655, p < .001. Initial eigenvalues indicated that the first two factors explained 37.13% and 11.10% of the variance respectively and the two-factor solution explained 48.24% of the variance. One item (MRS\_19: Knowing math helps me to justify my reasoning) was deleted because it was the only item that cross-loaded on both factors, .507 and .345 respectively, without a strong primary loading on any factor.

Round 5, the final round of the analysis, consisted of rerunning the EFA after deleting item MRS\_19. The KMO = .914 and the Bartlett's test of sphericity were significant,  $\chi^2(378) = 4321.87$ , p < .001. Initial eigenvalues indicated that the first two factors explained 36.55% and 11.50% of the variance respectively and the two-factor solution explained 48.06% of the variance. All items in this analysis had primary loadings over .35. None of the items had any cross-loadings and all of the factor loadings ranged between .352 and .905, well above the recommended loading of at least .30 (DeVellis, 2003). Eighteen items loaded onto Factor 1 with factor loadings of .435 to .770, ten items loaded onto Factor 2 with factor loadings of .384 to .905. The factor loading matrix for this final solution is presented in Table 14.

Internal consistency for each of the scales was examined using Cronbach's alpha. The alphas were high: .931 for practical utility in students' everyday lived experiences (18 items); .914 for exchange value in preparing students for future courses and careers (ten items). No substantial increases in alpha for any of the scales could have been achieved by eliminating more items.

**Table 14**Factor Loadings for Final 28-Item MRS

Item	Comm.	nm. Factor loadii	
		Oblique f	actors
		1	2
MRS_30 Knowing math makes it easier to understand the	.522	.676	.094
inequalities within tracking students in schools. (Tracking:			
Assigning students to different classes based on their perceived			
ability in that subject.)			

Item	Comm.	Factor lo	adings
		Oblique	
		1	2
MRS_7 I can use the math I learned in middle school to explore	.466	.597	.160
problems within my community.			
MRS_33 Knowing math makes it easier to understand educational	.591	.770	003
inequality. (Educational inequality: The unequal distribution of			
academic resources, including school funding, qualified and			
experienced teachers, books, and technologies. This leads to			
major differences in the educational success of these individuals.)			
MRS_8 I can use the math I learned in high school to explore	.349	.467	.210
problems within my community.			
MRS_2 Math helps me to explore fairness in society.	.330	.440	.223
MRS_20 Learning math helps me to analyze and think about	.626	.732	.117
solutions to issues within my community.			
MRS_4 Knowing math helps me understand neighborhood	.393	.626	.001
displacement. (Neighborhood displacement: The forced			
movement of people from their homes caused by the buying and			
re-selling of houses and stores in lower income communities by			
upper- or middle-income people, resulting in raising property			
values)			
MRS_11 Knowing math makes it easier to understand income	.457	.653	.048
inequality.			
MRS_6 I can use the math I learned in elementary school to explore	.240	.435	.103
problems within my community.			
MRS_35 I can use math to make changes in my community.	.513		
MRS_9 Knowing math makes it easier to understand inequality of	.408	.624	.033
access to English language education in schools. (English			
language education: The teaching and learning conditions of			
English Language Learners (ELL) in schools.)			
MRS_37 Studying math helps me with problem solving in other	.322	.630	.162
subjects.	2.1-	220	
MRS_26 Knowing math helps me in other school subjects.	.247	.229	.352
MRS_22 Knowing math helps me to evaluate other people's points	.500	.704	.007
of view in politics.	500	<b>5</b> 04	0.47
MRS_14 Knowing math helps me understand gentrification.	.523	.701	.047
(Gentrification: The process of renovating and improving a house			
or district. Thus, forcing the movement of people from their			
homes in lower income communities.)	.277	150	126
MRS_17 Math has to do with community issues, such as foreclosures. (Foreclosure: banks taking property when a person	.211	.458	.126
fails to keep up their mortgage payments.)			
MRS_18 Math will play an important role in my future career.	.754	093	.905
MRS 27 Knowing math will help me to be successful in my career.	.800	019	.903
MRS_10 Learning math will help me to get the job I want.	.671	019	.857
MRS 23 I need to know math to earn a living.	.616	.042	.766
MRS 25 Studying math gives me more career choices.	.534	.042	.717
MRS_3 I will use math problem-solving skills in my career.	.585	035	.780
MRS_28 Knowing math will make me more competitive on the job	.442	.078	.627
market.	.¬ <b>¬</b> ∠	.076	.027
MRS 29 Math will be useful no matter what career I choose.	.428	.179	.555
17110 _ 7 171011 WIII OF USEIGI IIO MARTEL WHAT CAICEL I CHOUSE.	.720	.1/7	.555

Item	Comm.	Factor loa	adings
		Oblique f	actors
		1	2
MRS_36 Knowing math makes it easier to understand inequalities	.458	.743	212
in the LGBTQ community. (LGBTQ: Lesbian, Gay, Bisexual,			
Transgender, Queer, or Questioning)			
MRS_24 Learning math helps me to analyze inequalities in the	.440	.708	124
LGBTQ community. (LGBTQ: Lesbian, Gay, Bisexual,			
Transgender, Queer, or Questioning)			
MRS_31 Knowing math makes it easier to understand racism.	.450	.719	134
MRS_34 Knowing math helps me understand deportation.	.516	.769	140
(Deportation: people being pushed out of the country.)			

### Factor Interpretation

After completing the factor analysis and determining the final items on the MRS, the factors were interpreted. Mathematics relevance is defined as follows: Factor 1—practical utility in students' everyday lived experiences (18 items) and Factor 2—exchange value in preparing students for future courses and careers (ten items). Each of these factors is described in detail in Chapter 5.

## Validity

Within the scale development process, testing for validity is an important and key component (Dimitrov, 2012; Messick, 1995). Validity tests for whether an instrument measures what it purports to measure (Dimitrov, 2012). When testing for validity of a scale, Cronbach (1951) reminds us that the interpretation of test scores is what is being validated, not the test itself. This study followed Messick's (1989, 1995) unified construct-based model of validity, which integrates content-related and criterion-related validity into construct validity. The five aspects of construct validity are content, substantive, structural, generalizability, and external (refer to Figure 6). Content validity,

substantive validity, and structural validity were discussed in detail in the beginning of this chapter.

### **Generalizability**

Generalizability validity was examined by the extent to which the interpretation of scores from the scale was accurate, consistent, and replicable (e.g., reliability, Dimitrov, 2012). For instance, when the MRS is administered in the future, participants will need to know that the results can be replicated if the same participants are tested repeatedly under the same circumstances (Crocker & Algina, 1986). Thus, internal consistency reliability was tested using Cronbach's alpha (Cronbach, 1951). Cronbach's alpha tests how well the items as a group hold together. Conceptually, the idea is that all of the survey items that are supposed to measure the construct mathematics relevance should be answered similarly by participants. However, if a participant gives varying answers to items that are supposed to be measuring mathematics relevance, it is difficult to argue that these items offer a reliable measure of the construct (Urdan, 2010).

Cronbach's alpha is sensitive to sample size, and thus the more items there are, the higher the Cronbach's alpha will be (Urdan, 2010). A rule of thumb is that Cronbach's alpha should be greater than .7 (Nunnally, 1978).

Internal consistency for each of the scales was examined. For Factor 1 of the MRS the reliability was  $\alpha = .931$ , Factor 2 had a reliability of .914, which are both significantly higher than the minimum. Reliabilities were calculated for the established scales as well and are listed in Table 15.

**Table 15**Generalizability Aspect

Variable	Items	Alpha
MRS	MRS_1,,MRS_37 (OMITTED MRS_1, MRS_5; MRS_12, MRS_15, MRS_16,	$\alpha = .939$
	(OMITTED MRS_1, MRS_5, MRS_12, MRS_13, MRS_10, MRS_19, MRS_32)	
MRS_Factor1	MRS_2, MRS_4, MRS_,6 MRS_7, MRS_8, MRS_9, MRS_11, MRS_14, MRS_17,MRS_20, MRS_22, MRS_24, MRS_30, MRS_31, MRS_33, MRS_34, MRS_35, MRS_36	$\alpha = .931$
MRS_Factor2	MRS_3, MRS_10, MRS_18, MRS_23, MRS_25,MRS_26, MRS_27; MRS_28; MRS_29; MRS_37	$\alpha = .914$
CVS_Conley	CVS_Conley1, CVS_Conley2	$\alpha = .870$
CVS_Kosovich	CVS_Kosovich1,CVS_Kosovich2, CVS_Kosovich3, CVS_Kosovich4	$\alpha = .838$
UVS_Conley	UVS_Conley1, UVS_Conley2, UVS_Conley3, UVS_Conley4	$\alpha = .914$
UVS_Wigfield	UVS_Wigfield1,UVS_Wigfield2, UVS_Wigfield3,UVS_Wigfield4, UVS_Wigfield5, UVS_Wigfield6	$\alpha = .880$

# External Validity

As described by Messick (1995), the external aspect of validity includes convergent and discriminant evidence. The convergent validity evidence indicates similarity between measures of the same trait, whereas the discriminant evidence indicates a distinctness from measures of other traits. Convergent validity was examined by demonstrating the high intercorrelations of the items from the MRS and the already validated utility value scales (Conley, 2012; Wigfield et al., 1997). This would provide evidence that the items on both scales were most likely related to the same construct,

mathematics relevance. Discriminant validity was examined by demonstrating the low intercorrelations of the items from the MRS and the already validated cost value scales (Conley, 2012; Kosovich et al., 2015). This provided evidence of whether the newly developed instrument, MRS, was measuring something unintended. Both convergent and discriminant validity were assessed using Pearson's r to examine the relationship between factors on the newly established instrument and already validated instruments.

Mean composites were created for all established subscales. Composites were created by taking the average from multiple indicators into a single score that represented the underlying construct (DeVellis, 2013). To create mean composites, I summed the item scores for each item on a particular subscale and then divided that by the total number of items on the scale: e.g., Cost Value Scale: (CVS\_Kosovich1) + (CVS\_Kosovich2) + (CVS\_Kosovich3) + (CVS\_Kosovich4) = /4. The purpose of creating these composites was to compare scores on subscales instead of individual items (i.e., each subscale had one score averaged from all item scores comprising that subscale). Three mean composites were created for the MRS subscales, two for the cost value subscales, and two for the utility value subscales.

Convergent Validity. Two mean composites were created for the two utility value subscales (Conley, 2012; Wigfield et al., 1997) and two mean composites were created for the two MRS factors. MRS Factor 1 (situated in EVT and SJP) and the Conley (2012) utility value subscale were moderately positively correlated, r = .42, p < .01. MRS Factor 2 (situated in EVT) and the Conley utility value subscale were highly positively correlated, r = .86, p < .01.

Similarly, the Wigfield et al. (1997) utility value subscale and the MRS were analyzed. MRS Factor 1 (situated in EVT and SJP) and the Wigfield et al. utility value subscale were moderately positively correlated, r = .480, p < .01. MRS Factor 2 (situated in EVT) and the Wigfield et al. utility value subscale were moderately positively correlated, r = .680, p < .01.

**Discriminant Validity.** Two mean composites were created for the two cost value subscales (Conley, 2012; Kosovich et al., 2015) and two mean composites were created for the two MRS factors. MRS Factor 1 (situated in EVT and SJP) and the Conley (2012) cost value subscale were x correlated, r = -.035, p < .01. MRS Factor 2 (situated in EVT) and the Conley cost value subscale were weakly negatively correlated, r = -.054, p < .01.

Similarly, the Kosovich et al. (2015) cost value subscale and the MRS were analyzed. MRS Factor 1 (situated in EVT and SJP) and the Kosovich et al. cost value subscale were weakly negatively correlated, r = .011, p < .01. MRS Factor 2 (situated in EVT) and the Kosovich et al. cost value subscale were weakly negatively correlated, r = .115, p < .01.

## **Summary**

This chapter described how experts in the fields of mathematics education, EVT, and SJP examined the items on the MRS for clarity and relevance. After multiple revisions were made to the MRS items, cognitive interviewing with a sample of secondary students ensured that the MRS was a sound instrument before engaging in exploratory analyses. After two rounds of expert interviews and two rounds of cognitive

interviews, the initial MRS was revised from 45 items to the final 35-item instrument and was ready for exploratory analyses.

EFA consisted of the six-step procedure outlined above and in Chapter 3. Data were collected from 550 Black and Hispanic secondary students and used to conduct exploratory analyses. After descriptive statistics and EFA, 28 of the original 35 items remained. The 28 items loaded onto two factors—practical utility in students' everyday lived experiences (18 items) and exchange value in preparing students for future courses and careers (ten items), and explained 48.06% of the variance equally across the two factors (Factor 1: 36.55%, Factor 2: 11.51%). Cronbach's alpha was used to establish reliability (i.e., generalizability validity) and all values were greater than .83.

External validity was also evaluated by way of testing convergent and discriminant validity using four already validated instruments (Conley, 2012; Kosovich et al., 2015; Wigfield et al., 1997). Convergent validity was established for the MRS subscales through significant weak to highly positive correlations with the utility value subscales (Conley, 2012; Wigfield et al., 1997). Further, discriminant validity was established for the MRS subscales through weakly negative correlations with the cost value subscales (Conley, 2012; Kosovich et al., 2015). In the following chapters, I will discuss key findings, address validity, explain limitations to this study, and offer future directions and implications of this research for various stakeholders.

# **Chapter Five: Discussion**

The purpose of this study was to develop and validate an instrument for measuring secondary students' perceptions of mathematics content as relevant to their lives. Useful theoretical and pedagogical frameworks for understanding the role of mathematics relevance in student academic motivation, achievement, and future decisions are the expectancy-value framework (Eccles [Parsons] et al., 1983) and SJP (Dover, 2009; Gutstein, 2003, 2007, 2013). Chapter 4 discussed the results of conducting an EFA which resulted in a 28-item MRS. The purpose of this chapter is to address the validity of the MRS, present the limitations, and offer interpretations of the findings in Chapter 4.

# Validity

In order to reduce measurement error and enhance the validity of the MRS, Gelbach and Brinkworth's (2011) six-step model for scale development guided the research design. This model delineates the use of (a) literature reviews; (b) interviews with the target population; (c) a synthesis of the literature review and interviews; (d) item development; (e) expert interviews; and (f) cognitive interviews. This study also followed Messick's (1989, 1995) unified construct-based model of validity, which integrates content-related and criterion-related validity into construct validity. Following, the construct validity is assessed by describing content, substantive, structural, generalizability, and external validity.

### **Content Validity**

Testing for content validity was necessary to show evidence that the MRS represented the sample of characteristics necessary for a secondary student to perceive mathematics content as relevant. With a preliminary list of items, experts in the fields of mathematics education, EVT, and SJP provided judgments on item clarity, language complexity, how well the set of items represented the construct mathematics relevance, and any aspects of the construct that were not represented or inadequately represented by the scale (Gehlbach & Brinkworth, 2011). Experts read through each item on the MRS and made suggestions to revise items to be clearer and more concise. For example, the item "Knowing math helps me to judge other people's points of view in politics" was changed to "Knowing math helps me to evaluate other people's points of view in politics" after one of the SJP experts described how the use of evaluate instead of judge casts mathematics in an objective stance instead of as a weapon (personal communication, Jenice View, PhD, July 10, 2018). One of the SJP experts raised concerns about the student population having varying interpretations of the terms neighborhood displacement, gentrification, foreclosure, and deportation. To prevent varying interpretations for items that included these terms, it was advised to provide concise definitions in smaller font after the item. For example, the item "Math has to do with community issues, such as foreclosures" was changed to "Math has to do with community issues, such as foreclosures. (Foreclosure: banks taking property when a person fails to keep up their mortgage payments)" (personal communication, Jenice View, PhD, July 10, 2018).

Adapting Gehlbach and Brinkworth's (2011) six-step approach to scale development (see Chapter 3 for details) allowed me to successfully frontload validity for the MRS. By using the literature and expert interviews, I was able to construct new theoretically meaningful items and further revise existing items to ensure they were correctly measuring the intended construct.

# Substantive Validity

Testing for substantive validity is key in showing evidence of response consistencies through cognitive modeling of the participants' response processes (Loevinger, 1957). Conducting cognitive interviews enabled me to see firsthand how my target population responded to items on the MRS and allowed me to refine items during the development stage. A sample of students from the target population provided feedback on their understanding of items, item phrasing, vocabulary, and clarity by way of "think aloud" protocols where students explained their interpretation when responding to each item. Participants' responses were organized into two categories to represent the emerging themes of the interviews: (a) ease of understanding of items and (b) wording clarity of items. Overall, two items were categorized as confusing for students. For the item "Math helps me to think about solutions for societal problems," students felt more comfortable answering questions regarding their community compared to society, as they interpreted community to mean their neighborhood, school, or even their group of friends. When asked what types of societal problems they could think of, students mentioned renewable energy, changing power sources, and how these help people save money over time. In response to their feedback, items from Dimension C were then

revisited to determine if the items were best written to reflect the SJP framework, given students' difficulty in the swapping of the terms *society* and *community*. Furthermore, students were asked to discuss their preferences between options for response anchors. Students expressed preferring the *never true for me–always true for me* statements instead of the *agree-disagree* statements, indicating that the *never true for me–always true for me–always true for me* statements encouraged them to think about themselves when answering items, while the *agree-disagree* response anchors encouraged them to think about how others would respond to the items on the MRS. Thus, the response anchors on the MRS reflected the students' preferences.

Through the use of cognitive interviews where students provided feedback on their understanding of items, item phrasing, vocabulary, and clarity, I was able to further revise existing items and response anchors on the MRS. These cognitive interviews supported me in revising items so that the MRS became a good measure of the mathematics relevance construct.

# Structural Validity

Structural validity of the MRS was assessed through EFA, a common technique used in scale development that uses variance to determine how items are functioning together within an instrument (Comrey, 1988; DeVellis, 2003). EFA was used as a data reduction technique to create factors that explain the most variance possible in the items (Dimitrov, 2012; Urdan, 2010). The following MRS items had highly negatively skewed distributions (skewness ranged less than -1 or greater than 1): MRS\_23: I need to know math to earn a living (-1.23); MRS\_25: Studying math gives me more career choices (-

1.32); MRS\_27: Knowing math will help me to be successful in my career (-1.09); MRS\_29: Math will be useful no matter what career I choose (-1.02). These negatively skewed items shed light on this study's participants in that the majority of this sample interpreted the relevance that mathematics has for career choices compared to practical utility and illuminating social issues. This finding mirrors the research (Boaler, 2000; Brown et al., 2008; Matthews, 2018; Murray, 2011; Onion, 2004) in showing that the majority of the sample acknowledged learning mathematics served as a bridge to attaining a future career, but struggled to find relevance for their everyday lives.

Overall, the hypothesized factors of the MRS before the EFA was run included three subscales: (a) practical utility in students' everyday lives, (b) exchange value in preparing students for future careers, and (c) illuminating social inequities that exist within students' communities and society (see Table 16 for items).

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Table 16

MRS Items Before EFA (35 Items)

Factor 1: practical utility in students' everyday lives	Factor 2: exchange value in preparing students for future	Factor 3: illuminating social inequities that exist within students' communities and society
I use math to help me make decisions outside of school.	Learning math will help me to get the job I want.	Learning math helps me to analyze and think about solutions to issues within my community.
Knowing math helps me to justify my reasoning.	Math will play an important role in my future career.	Math can help me to better understand the world we live in.
Studying math helps me with problem solving in other subjects.	I need to know math to earn a living.	Math helps me to explore fairness in society.
Knowing math helps me think more logically.	Knowing math will help me to be successful in my career.	Learning math helps me to analyze inequalities in the LGBTQ community. (LGBTQ: Lesbian, Gay, Bisexual, Transgender, Queer or Questioning)
Math is useful in my daily life.	Math will be useful no matter what career I choose.	Knowing math helps me understand neighborhood displacement. (Neighborhood displacement: The forced movement of people from their homes caused by the buying and re-selling of houses and stores in lower income communities by upper- or middle-income people, resulting in raising property values)
I apply my math skills outside of school.	Knowing math will make me more competitive on the job market.	Knowing math helps me understand deportation. (Deportation: people being pushed out of the country).
Knowing math helps me in other school subjects.	Studying math gives me more career choices.	Knowing math helps me to evaluate other people's points of view in politics.
Math provides me the tools to make informed decisions outside of school.	I will use math problem-solving skills in my career.	Knowing math makes it easier to understand income inequality.
		Math has to do with community issues, such as foreclosures. (Foreclosure: banks taking property when a person fails to keep up their mortgage payments).

Factor 1: practical utility in	Factor 2: exchange value in	Factor 3: illuminating social inequities that exist within students'
students' everyday lives	preparing students for future	communities and society
	careers	
		Knowing math makes it easier to understand educational inequality
		(Educational inequality: The unequal distribution of academic
		resources, including school funding, qualified and experienced
		teachers, books, and technologies. This leads to major difference
		in the educational success of these individuals).
		Knowing math makes it easier to understand racism.
		Knowing math makes it easier to understand inequalities in the
		LGBTQ community. (LGBTQ: Lesbian, Gay, Bisexual,
		Transgender, Queer or Questioning)
		Knowing math makes it easier to understand inequality of access t
		the English language education in schools. (English language
		education: The teaching and learning conditions of English
		Language Learners (ELL) in schools.)
		Knowing math makes it easier to understand the inequalities within
		tracking students in schools. (Tracking: Assigning students to
		different classes based on their perceived ability in that subject.
		I can use the math I have learned in elementary school to explore
		problems within my community.  I can use the math I have learned in middle school to explore
		problems within my community.
		I can use the math I have learned in high school to explore problem
		within my community.
		I can use math to make changes in my community.
		Knowing math helps me understand gentrification. (Gentrification
		The process of renovating and improving a house or district so
		that it conforms to middle-class taste. Thus, forcing the moveme
		of people from their homes in lower income communities.)

*Note.* One item omitted that checked for participants' reading of items.

Table 17

MRS Items After EFA (28 Items)

Factor 1: practical utility in students' everyday lived experiences	Factor 2: exchange value in preparing students for future courses and careers
Math helps me to explore fairness in society.	Math will play an important role in my future career.
Learning math helps me to analyze and think about solutions to issues within my community.	Knowing math will help me to be successful in my career.
Knowing math helps me understand neighborhood displacement. (Neighborhood displacement: The forced movement of people from their homes caused by the buying and re-selling of houses and stores in lower income communities by upper- or middle-income people, resulting in raising property values)	Learning math will help me to get the job I want.
Knowing math makes it easier to understand income inequality.	I need to know math to earn a living.
Knowing math makes it easier to understand inequality of access to English language education in schools. (English language education: The teaching and learning conditions of English Language Learners (ELL) in schools.)	Studying math gives me more career choices.
Knowing math makes it easier to understand educational inequality. (Educational inequality: The unequal distribution of academic resources, including school funding, qualified and experienced teachers, books, and technologies. This leads to major differences in the educational success of these individuals.)	I will use math problem-solving skills in my career.
Knowing math makes it easier to understand the inequalities within tracking students in schools. (Tracking: Assigning students to different classes based on their perceived ability in that subject.)	Knowing math will make me more competitive on the job market.
Knowing math makes it easier to understand racism.  Knowing math helps me to evaluate other people's points of view in politics.	Math will be useful no matter what career I choose. Knowing math helps me in other school subjects.
Knowing math helps me understand gentrification. (Gentrification:  The process of renovating and improving a house or district.  Thus, forcing the movement of people from their homes in lower income communities.)	Studying math helps me with problem solving in other subjects.

٠	Factor 1: practical utility in students' everyday lived experiences	Factor 2: exchange value in preparing students for future courses and careers
	Knowing math helps me understand deportation. (Deportation:	
	people being pushed out of the country.)	
	I can use math to make changes in my community.	
	Math has to do with community issues, such as foreclosures.	
	(Foreclosure: banks taking property when a person fails to keep	
	up their mortgage payments.)	
	I can use the math I have learned in middle school to explore	
	problems within my community.	
	I can use the math I have learned in high school to explore problems	
	within my community.	
	I can use the math I have learned in elementary school to explore	
	problems within my community.	
	Knowing math makes it easier to understand inequalities in the	
	LGBTQ community. (LGBTQ: Lesbian, Gay, Bisexual,	
	Transgender, Queer, or Questioning).	
	Learning math helps me to analyze inequalities in the LGBTQ	
	community. (LGBTQ: Lesbian, Gay, Bisexual, Transgender,	
	Oueer or Questioning)	

Queer or Questioning)
Note. One item omitted that checked for participants' reading of items.

After the factor analysis was complete, the rotated solution determined two interpretable factors: Factor 1—practical utility in students' everyday lived experiences (18 items); and Factor 2—exchange value in preparing students for future courses and careers (10 items). The two-factor solution explained 48.07% of the total variance. The final items on the MRS are listed in Table 17.

The greatest change in factors after the EFA was within Factors 1 and 3. The definition of Factor 1 changed from practical utility in students' everyday lives to practical utility in students' everyday lived experiences. All items from Factor 3 (illuminating social inequities that exist within students' communities and society) shifted to Factor 1 because this sample of students associated everyday life with an awareness of the inequities and unjust conditions that exist in their communities. For example, these items originally hypothesized for Factor 3 were now associated with Factor 1: "Knowing math helps me understand neighborhood displacement," "Knowing math makes it easier to understand inequality of access to English language education in schools," and "Math has to do with community issues, such as foreclosures." Details of the interpretation of the two factors are below.

Practical Utility in Students' Everyday Lived Experiences. The first factor,
Practical Utility in Students' Everyday Lived Experiences (PULE), reflects the belief that
learning mathematics has practical relevance and use for a student's day-to-day life
outside the classroom. Items that loaded on this factor reflect the belief that learning
mathematics helps students make informed decisions outside of school and justify their
reasoning, meaning making, and providing an argument or rationale rooted in evidence

(NRC, 2001). Additionally this factor aligns with the belief that learning mathematics helps students understand the significance of mathematics in recognizing, analyzing, and critiquing social inequities within society (e.g., Gutstein, 2003, 2007, 2013). The PULE factor aligns with research showing that when Black and Latino students respond to items reflecting their beliefs on the practicality of learning mathematics, they may be questioning how mathematics can inform who they are and can become, and want to understand how mathematics can help them navigate their immediate environment, their community (Matthews, 2018). These items migrated from Factor 3. These items that loaded on the PULE factor reflect the belief that learning mathematics helps a student explore and think critically about fairness in society, analyze and think about solutions for inequalities within a student's community, and understand neighborhood displacement, income inequality, gentrification, foreclosures, and resource inequities like the inequality of access to English Language education in schools. Thus, how students answer the items in the PULE factor will directly relate to the lived experiences of those students. Of the 232 participants who completed the MRS, majority of the students identified themselves as Hispanic (73.3%) or African American (26.7%) and will become the first in their families to attend college. Thus, given the demographics of this sample, how students answered the items on the PULE factor was directly related to the lived experiences of them. Cronbach's alpha for the PULE factor was .931.

Exchange Value in Preparing Students for Future Courses and Careers. The second factor, Exchange Value in Preparing Students for Future Courses and Careers (EVFCC), reflects a student's belief that the learning of mathematics has exchange value

in terms of providing skills that translate in the future for other courses, STEM fields, and providing students with financial security. These elements of the EVFCC factor align with prior qualitative research on students' beliefs about mathematics relevance (Musto, 2008; Onion, 2004; Sealey & Noyes, 2010; Wooley et al., 2013) and utility value research situated in EVT (e.g., Eccles [Parsons] et al., 1983) where the practical use of math is aligned to its exchange value. Items that loaded on the EVFCC factor reflect the belief that learning mathematics will broaden students' career options, make them more competitive in the job market in the future, and provide them with financial security. Some of the items that loaded on the EVFCC factor align with the belief that learning mathematics provides skills that translate into any area of a student's day-to-day life and varying coursework where these skills include problem solving (Sealey & Noyes, 2010). Cronbach's alpha for the EVFCC factor was .914.

### Generalizability

Generalizability validity was examined by the extent to which the interpretation of scores from the scale was accurate, consistent, and replicable (e.g., reliability, Dimitrov, 2012). Cronbach's alpha was used to test for generalizability and how well the items on the MRS as a group held together. Conceptually, the idea is that all survey items that are supposed to measure the construct mathematics relevance should be answered similarly by participants. A rule of thumb is that Cronbach's alpha should be greater than .7 (Nunnally, 1978). Internal consistency for each of the scales was examined; for Factor 1 of the MRS the reliability was  $\alpha = .931$ , and Factor 2 had a reliability of  $\alpha = .914$ , which are both significantly higher than the minimum. Thus, we can infer the items on

the MRS do indeed tap into an underlying construct of mathematics relevance among respondents.

The MRS scale evidenced good reliability where the intercorrelations among the subscales were significant, indicating the subscales (practical utility in students' everyday lived experiences; exchange value in preparing students for future courses and careers) measured two related yet distinct constructs.

### External Validity

As described by Messick (1995), the external aspect of validity includes convergent and discriminant evidence. The convergent validity evidence indicates similarity between measures of the same trait, whereas the discriminant evidence indicates a distinctness from measures of other traits.

Convergent Validity. Convergent validity was used to ensure that the newly established instrument, the MRS, measured a construct similar to that in an already validated instrument (Crocker & Algina, 1986). This was examined by carrying out correlational analyses with the MRS and the already validated utility value scales (Conley, 2012; Wigfield et al., 1997). Correlation is measured by r, the correlation coefficient, on a scale of -1 to 1, where r = -1 is a perfect negative correlation, r = 1 is a perfect positive correlation, and r = 0 is no correlation at all. For convergent validity, values closer to r = 1 would show similarity between the MRS and the utility value scales (Conley, 2012; Wigfield et al., 1997).

MRS Factor 1 (practical utility in students' everyday lived experiences) and the Conley (2012) utility value subscale were moderately positively correlated, r = .422, p <

.01. MRS Factor 2 (exchange value in preparing students for future courses and careers) and the Conley utility value subscale were highly positively correlated, r = .861, p < .01. Similarly, MRS Factor 1 (practical utility in students' everyday lived experiences) and the Wigfield et al. (1997) utility value subscale were moderately positively correlated, r = .480, p < .01. MRS Factor 2 (exchange value in preparing students for future courses and careers) and the Wigfield et al. utility value subscale were moderately positively correlated, r = .680, p < .01.

The moderate to high correlations between Factors 1 and 2 of the MRS and the utility value scales (Conley, 2012; Wigfield et al., 1997) demonstrate that the MRS and these utility value scales are measuring similar constructs, but not too similarly.

**Discriminant Validity.** Discriminant validity was used to ensure that the newly established MRS measured a different construct than an already validated instrument (Crocker & Algina, 1986). This was examined by carrying out correlational analyses with the MRS and the already validated cost value scales (Conley, 2012; Kosovich et al., 2015). Correlation is measured by r, the correlation coefficient, on a scale of -1 to 1, where r = -1 is a perfect negative correlation, r = 1 is perfect positive correlation, and r = 0 is no correlation at all. For discriminant validity, values closer to r = 0 show the measures are not related and measure different constructs.

MRS Factor 1 (practical utility in students' everyday lived experiences) and the Conley (2012) cost value subscale were weakly negatively correlated, r = -.035, p < .01. MRS Factor 2 (exchange value in preparing students for future courses and careers) and the Conley cost value subscale were weakly negatively correlated, r = -.054, p < .01.

Similarly, MRS Factor 1 (practical utility in students' everyday lived experiences) and the Kosovich et al. (2015) cost value subscale were weakly correlated, r = .011, p < .01. MRS Factor 2 (exchange value in preparing students for future courses and careers) and the Kosovich et al. cost value subscale were weakly negatively correlated, r = -.115, p < .01. The weak correlations between the MRS and the cost value scales demonstrate that the MRS and the cost value scales are measuring different constructs.

These findings contribute to the research on mathematics relevance because it offers another construct that merges EVT and SJP. The MRS can provide opportunities for future research to attempt to establish a more thorough construct of mathematics relevance.

#### Limitations

Although I conducted this study using rigorous qualitative and quantitative methods within Gehlbach and Brinkworth's (2011) six-step approach, limitations still arose during the development and validation of the MRS. A limitation of this study worth noting is the generalizability aspect of this study. Far less is known about students' perceptions of the relevance of mathematics content with high-ability African American and Hispanic populations, particularly those who will be the first to attend college in their families. It is important to learn about their perceptions of their mathematics content in turn to help identify factors associated with their academic success, particularly as their lived experiences may lend them to different perceptions than those in dominant upper-and middle-class communities. Thus, in describing the results of this study, they are specifically geared to learning about how high-ability Black and Hispanic students

respond to items reflecting their perceptions of mathematics relevance. Despite these limitations, the validation of the MRS remain noteworthy for illustrating how students of color see math as meaningful.

### Administering the MRS

This instrument is designed for secondary students in Grades 8-12 who identify themselves as Black or Hispanic and who will become first-generation college students upon their high school graduation. The development of this instrument is aligned with goals of increasing our understanding of high-ability students from economically vulnerable families and their perceptions of their mathematics content. Additionally, the development of this instrument will help education leaders tailor curriculum to strengthen positive student perceptions of mathematics.

The MRS consists of two subscales, the Practical Utility in Students' Everyday Lived Experiences (PULE) and the Exchange Value in Preparing Students for Future Courses and Careers (EVFCC). The subscales are typically administered together; however, each subscale can also be administered separately. For example, a school district, school, or individual math classroom may be only interested in the effectiveness of the math curriculum as impacting students' perceptions of mathematics being used as a tool in recognizing, analyzing, and thinking critically of inequities in society. In turn they might use the PULE subscale and track students' perceptions of their mathematics content over time and in turn not use the EVFCC subscale. The MRS can be completed via computer (e.g., Qualtrics) or via hard copy, was designed to be given in class, and takes approximately 15-20 minutes to administer (refer to Appendix E).

### Scoring the MRS

Upon completing the MRS, students rate themselves on a 6-point Likert scale from *never true for me* to *always true for me*. The MRS is constructed by taking the mean of the items that make up the scale. For example, the PULE subscale has 18 items. An individual's PULE score would be computed by summing the 18 items and taking the average.

### Uses of the MRS

Previous research (e.g., Hulleman et al., 2010) has shown that how students learn mathematics in the classroom plays a role in their perceptions of the subject. How students learn mathematics and their attitudes toward the subject are both critically important in predicting student engagement, persistence, academic motivation, and future course selection. The MRS is intended to provide teachers, math education leaders, stakeholders, and researchers a tool to learn about how and what math is meaningful to Black and Hispanic students. By using the MRS, results will provide useful information to determine which factors of mathematics relevance (practical utility, exchange value) students need more exposure to when it comes to their mathematics curriculum. In turn, school districts can build out and/or offer curriculum suggestions to deepen students' understanding of the real-world applications of math, the exchange value of math for students' futures, and the potential of math for investigating and understanding the world they live in. The MRS could also be used to determine the effectiveness of these curricular experiences as researchers track changes over time in students' perceptions of the subject. Understanding students' perceptions of the relevance of their mathematics

content provides a means to more effectively cater professional development for their teachers. This scale should be used in combination with other assessment data, such as academic achievement data.

In Chapter 6, I will discuss implications of the findings from this research and offer future directions for different stakeholders.

### **Chapter Six: Implications**

There has been a call to shift the direction of research on students of color, specifically Black and Hispanic students and mathematics, given that conversations regarding students of color are often unfairly centered on failure and do not account for their successes and motivations for learning mathematics. This study was committed to understanding how Black and Hispanic secondary students perceive mathematics as relevant to their lives through the creation and validation of the MRS. The goal was to offer researchers a tool to learn about how and what math is meaningful to Black and Hispanic students. By learning this, it can be a window into tailoring curriculum. This study included creating and developing the MRS. After interviews with students and experts across fields related to the study and validation procedures described in this study, the final scale results in 28 items with two factors: PULE and EVFCC. In this chapter, I discuss the implications for this work and how it could potentially support various stakeholders within and beyond mathematics education.

### **Implications**

### Implications for Research

With the creation of the MRS, future research should consider to draw a causal link between secondary students' perceptions of mathematics relevance, academic motivation, and achievement. Further, if this relationship can be established, then researchers can design targeted professional development experiences for both preservice and in-service teachers to increase students' mathematics relevance in an attempt

to increase motivation, engagement, and achievement in mathematics. This study and future research presents an opportunity for researchers to understand how mathematics can inform high ability Black and Hispanic students about who they are, who they can become, and how mathematics can help them in recognizing, analyzing, and think critically about inequities in their communities.

### Implications for Policy

Previous research (e.g., Hulleman et al., 2010) has shown that how students learn mathematics in the classroom plays a role in their perceptions of the subject. How students learn mathematics and their attitudes toward the subject are both critically important in predicting student engagement, persistence, academic motivation, and future course selection. The creation of the MRS offers decision makers the ability to ensure our students of color have access to learning, resources, and math instruction that deepens their understanding of the real-world applications of math, the exchange value for math for their future careers, and the potential of math to investigate, critique, understand, and find solutions for the inequities that exist in their communities. Decision makers are often faced with budgetary decisions that impact the financial means, resources, and support needed to provide meaningful mathematics teaching and learning. Thus, the creation of the MRS offers decision makers an opportunity to assess districts' current curriculum and its impact over time on students' perceptions of mathematics relevance.

#### Implications for Practice

Given that more than 3.4 million students who achieve in the top quartile academically are low-income students (Wyner et al., 2008), this study and the MRS will

help increase our understanding of high-ability students from economically vulnerable families and their perceptions of mathematics content to help identify factors associated with their academic success. As previously noted, relatively little is known about perceptions of the relevance of mathematics content among first-generation college students within African American and Hispanic populations. Research suggests students' dispositions toward mathematics directly influence their future intentions and decisions (Eccles, 1984, 2005; Eccles [Parsons] et al., 1983, 1984; Simpkins et al., 2006; Wigfield & Eccles, 1990; Wigfield et al., 2005). If educators were to draw on what students tell us is relevant to their lived experiences, it could influence students' motivation, achievement, and attitudes toward the subject (Boaler, 2000; Gaspard et al., 2015; Gutstein, 2003; Ladson-Billings, 1994; Musto, 2008; Onion, 2004; Sealey & Noyes, 2010; Wooley et al., 2013). As mathematics education researchers, our goal is to strengthen the student experience and support our educators and curriculum writers in providing students an opportunity to find relevance within the mathematics that they are learning.

Implications for Professional Development and Instruction. The creation of the MRS offers teachers, math education leaders, stakeholders, and researchers a means to develop professional development targeted around secondary students' of color perceptions of mathematics content being relevant based upon the two factors: (a) practical utility in students' everyday lived experiences and (b) exchange value in preparing students for future courses and careers. Previous research (e.g., Hulleman et al., 2010) has shown that how students learn mathematics in the classroom plays a role in

their perceptions of the subject. How students learn mathematics and their attitudes toward the subject are both critically important in predicting student engagement, persistence, academic motivation, and future course selection. Further research should be conducted using the MRS as a tool to determine which factors of relevance (practical utility, exchange value) students need more exposure to when it comes to their mathematics curriculum. Researchers and stakeholders can build out curriculum suggestions to deepen students' understanding of the real-world applications of math, the exchange value of math for students' futures, and the potential of math for investigating and understanding the world they live in. The MRS could also be used to determine the effectiveness of these curricular experiences as researchers track changes over time in students' perceptions of the subject. Understanding students' perceptions of the relevance of their mathematics content provides a means to more effectively cater professional development for their teachers.

Implications for Teacher Education. This study was a first attempt at merging EVT and SJP to create a scale that measures students' perceptions of the relevance of mathematics content. With the MRS, teachers and leaders can gain a deeper understanding of their students' perceptions of the subject and target where students need to gain a better understanding of the relevance of mathematics. Having students understand the relevance of mathematics also means them recognizing, understanding, and critiquing current social inequities. This in turn presumes that teachers themselves need to be able to recognize social inequities and their causes (Ladson-Billings, 1995). Thus, the MRS could support the creators of professional development for teachers to

learn how and where to incorporate real-world applications and social justice tasks within the curriculum. Designing professional development experiences for teachers might help them increase students' perceptions of mathematics relevance in an attempt to increase motivation, engagement, and achievement while learning mathematics. Preservice secondary teacher education programs can also incorporate the three ways of making mathematics relevant for students (practical utility, exchange value) as they prepare the next generation of teachers.

#### **Recommendations for Future Research**

This study focused on the initial development and validation of the MRS and will lead to future research to confirm these findings so that the MRS may be used across multiple samples. Thus, further evidence in support of the reliability and validity of the MRS is needed. First, CFA should be conducted using the two-factor structure of mathematics relevance in order to confirm the validity of the factor structure and factors' relationships with the set of items (Dimitrov, 2012), using a new sample of secondary students. After CFA is conducted and discriminant/convergent validity are further established, the MRS can be utilized to assess secondary students' perceptions of mathematical relevance. Additionally, a new sample could be surveyed with the MRS to determine if there are group differences in how students perceive mathematics relevance. Furthermore, further analysis can be conducted to explore the factor structure with a new sample of secondary math students who identify themselves as African American and Hispanic, however are not identified as high achieving. This would be a first step in exploring ways to change the trajectory of low achieving secondary students of color and

motivating these students to understand the relevance of mathematics and become interested in the STEM fields.

### **Summary**

The MRS is the first instrument designed to measure mathematics relevance and is situated in the frameworks of EVT and SJP. The passion behind developing this instrument was influenced by the need to increase our understanding of the psychological and social factors that influence the success of high-ability students from economically vulnerable families who will become first-generation college students. It is important to learn about their perceptions of their mathematics content to help identify factors associated with their academic success.

Gelbach and Brinkworth's (2011) six-step model for scale development was followed, and the process included a literature review, student interviews, the synthesis of this information, item writing, expert interviews, and cognitive interviews. Engaging in these qualitative analyses before conducting EFA created the opportunity to frontload validity and in turn likely reduce measurement error and enhance the validity of the scale development process. Professional development experiences and curriculum writing can be targeted for both pre-service and in-service teachers to develop math learning experiences where secondary students make connections between the math content they are learning and how mathematics can help them think critically about their communities and society, as well as being a stepping stone to endless career opportunities.

### Appendix A

### IRB Approval Letter



# Office of Research Development, Integrity, and Assurance

Research Hall, 4400 University Drive, MS 6D5, Fairfax, Virginia 22030 Phone: 703-993-5445: Fax: 703-993-9590

DATE: April 1, 2019

TO: Toya Frank, PhD

FROM: George Mason University IRB

Project Title: [1148686-3] Quantifying Relevance: The Development and Validation of a

Scale to Measure Secondary Students' Perceptions of the Relevance of

**Mathematics Content** 

SUBMISSION TYPE: Amendment/Modification

ACTION: APPROVED
APPROVAL DATE: April 1, 2019
EXPIRATION DATE: November 27, 2019
REVIEW TYPE: Expedited Review

REVIEW TYPE: Expedited review category #7

Thank you for your submission of Amendment/Modification materials for this project. The George Mason University IRB has APPROVED your submission. This submission has received Expedited Review based on applicable federal regulations.

Please remember that all research must be conducted as described in the submitted materials.

Please remember that informed consent is a process beginning with a description of the project and insurance of participant understanding followed by a signed consent form unless the IRB has waived the requirement for a signature on the consent form or has waived the requirement for a consent process. Informed consent must continue throughout the project via a dialogue between the researcher and research participant. Federal regulations require that each participant receives a copy of the consent document.

Please note that any revision to previously approved materials must be approved by the IRB prior to initiation. Please use the appropriate revision forms for this procedure.

All UNANTICIPATED PROBLEMS involving risks to subjects or others and SERIOUS and UNEXPECTED adverse events must be reported promptly to the IRB office. Please use the appropriate reporting forms for this procedure. All FDA and sponsor reporting requirements should also be followed (if applicable).

All NON-COMPLIANCE issues or COMPLAINTS regarding this project must be reported promptly to the IRB.

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# Office of Research Development, Integrity, and Assurance

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### Appendix B

### Recruitment Letter for Students

Dear students,

I am a doctoral student at George Mason University in the Mathematics Education Department and I am conducting research on students' perceptions of mathematics content as being relevant to their lives. I developed a scale that will measure how useful students perceive their math content is to their lives. The goal of this dissertation study is to gain feedback on this scale. This study will aide in understanding how relevance can serve a role in motivating students in seeing the usefulness of math in hopes of promoting greater achievement. You have been selected since your parent/guardian has given consent for you to participate in this study.

(Students are selected to participate in ONE of the three stages of this study. Refer to the section below for the recruitment script which is based on the particular stage chosen for the student).

**Stage 1:** If you agree to help with this study, you will be asked to provide feedback on how you define relevance in your own words, when it comes to learning mathematics in the classroom. This interview will be one-on-one and this interview will last no longer than 30minutes, which will also include you completing the demographic survey.

Before we begin, however, I'd like to share the assent form for this study. Feel free to ask any questions or express any concerns. Also know that you are free to quit this study at any time. Any questions/concerns?

<u>Stage 2:</u> If you agree to help with this study, you will be asked to discuss items on the instrument that you find confusing or difficult to answer, including the instructions and response set categories. This interview will be one-on-one and this interview will last no longer than 30minutes, which will also include you completing the demographic survey.

Before we begin, however, I'd like to share the assent form for this study. Feel free to ask any questions or express any concerns. Also know that you are free to quit this study at any time. Any questions/concerns?

<u>Stage 3:</u> If you agree to help with this study, you will be asked to complete the survey with a section on demographics. Completing the survey will last no longer than 30minutes.

Before we begin, however, I'd like to share the assent form for this study. Feel free to ask any questions or express any concerns. Also know that you are free to quit this study at any time. Any questions/concerns?

# **Appendix C**

# Original Version of MRS

### Original Scale (45 items)

Dimension A: practical utility in students' everyday lives (EVT)

Math is important in my daily life.
I often use the math I learned in elementary school.
Knowing math helps me in other subjects.
I already know more math than I need in my daily life.
Math helps me in my daily life.
I use math outside of school.
Studying math helps me with problem solving in other subjects.
The things I do in math have nothing to do with my daily life.
I apply my math skills outside of school.
I use math to make decisions outside of school.
I often use the math I am learning in high school.
Knowing math helps me outside of school.
I don't need math in my daily life
I often use the math I learned in middle school.

Dimension B: exchange value in preparing students for future careers (EVT)

Math may play an important role in my career.
My career will involve math.

The jobs I am interested in don't use math.

I need to know math to earn a living.

My math skills may help me in my professional life.

My math knowledge may help me in my professional life.

I will use math problem-solving skills in my career.

I need math knowledge to be successful in my career.

Math will not be useful in my career.

I am not interested in jobs that use math.

Knowing math gives me more career choices.

Learning math will help me to get the job I want.

Math will be useful no matter what career I choose.

Math will prepare me to succeed in my chosen career.

Knowing math will make me more competitive in the job market.

Dimension C: illuminating social inequities that exist within students' communities and society (SJP)

Math can help me to better understand the world we live in.
I can use my math knowledge to analyze problems in society.

I can use my math skills to help others.

I can use my math knowledge to create a more just society.

Math can help me to find solutions for societal problems.

I don't see how the math I know can improve society.

I can use my math skills to explore societal issues.

Math helps me to evaluate the information I receive about the world.

I can use my math skills to improve the community I live in.

I can use my math knowledge to justify my views about the world.

My math knowledge makes me a more valuable member of society.

Studying math prepares me to investigate the world's problems.

Math equips me to advocate for my community.

Math helps me to question potential inequities in society.

Math helps me to justify my reasoning when making decisions.

# Appendix D

### MRS Administered to Participants

# Mathematics Relevance Scale Please indicate the degree to which you agree with the following statements (CIRCLE)

Dimension A: practical utility in students' everyday lives (EVT)

I use math to help me make decisions outside of school.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Knowing math helps me to justify my reasoning.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Studying math helps me with problem solving in other subjects.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Knowing math helps me think more logically.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Math is useful in my daily life.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
I apply my math skills outside of school.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me

Knowing math helps me in other school subjects.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Math provides me the tools to make informed decisions outside of school.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me

# Dimension B: exchange value in preparing students for future careers (EVT)

Learning math will help me to get the job I want.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Math will play an important role in my future career.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
I need to know math to earn a living.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Knowing math will help me to be successful in my career.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Math will be useful no matter what career I choose.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me

Knowing math will make me more competitive on the job market.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Studying math gives me more career choices.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
I will use math problem- solving skills in my career.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me

Dimension C: illuminating social inequities that exist within students' communities and society (SJP)

Learning math helps me to analyze and think about solutions to issues within my community.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Math can help me to better understand the world we live in.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Math helps me to explore fairness in society.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Learning math helps me to analyze inequalities in the LGBTQ community. (LGBTQ: Lesbian, Gay, Bisexual, Transgender, Queer or Questioning)	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me

Knowing math helps me understand neighborhood displacement. (Neighborhood displacement: The forced movement of people from their homes caused by the buying and re-selling of houses and stores in lower income communities by upper- or middle-income people, resulting in raising property values)	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Knowing math helps me understand deportation. (Deportation: people being pushed out of the country).	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Knowing math helps me to evaluate other people's points of view in politics.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Knowing math makes it easier to understand income inequality.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Math has to do with community issues, such as foreclosures. (Foreclosure: banks taking property when a person fails to keep up their mortgage payments).	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Knowing math makes it easier to understand educational inequality. (Educational inequality: The unequal distribution of academic resources, including school funding, qualified and experienced teachers, books, and technologies. This leads to	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me

major differences in the educational success of these individuals.)						
Circle 'Always true for me' if you are reading this.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Knowing math makes it easier to understand racism.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Knowing math makes it easier to understand inequalities in the LGBTQ community. (LGBTQ: Lesbian, Gay, Bisexual, Transgender, Queer or Questioning)	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Knowing math makes it easier to understand inequality of access to the English language education in schools.  (English language education: The teaching and learning conditions of English Language Learners (ELL) in schools.)	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Knowing math makes it easier to understand the inequalities within tracking students in schools.  (Tracking: Assigning students to different classes based on their perceived ability in that subject.)	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
I can use the math I have learned in elementary and middle school to explore problems within my community.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me

I can use math to make changes in my community.	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me
Knowing math helps me understand gentrification. (Gentrification: The process of renovating and improving a house or district so that it conforms to middle-class taste. Thus, forcing the movement of people from their homes in lower income communities.)	Never true for me	Rarely true for me	Sometimes <u>un</u> true for me	Sometimes true for me	Usually true for me	Always true for me

# Appendix E

### MRS After EFA (28 items)

MRS After EFA (28 items)	
Factor 1: practical utility in students' everyday	Factor 2: exchange value in preparing students for
lived experiences	future courses and careers
Math helps me to explore fairness in society.	Math will play an important role in my future
Main helps me to explore fairness in society.	career.
I coming moth holes me to analyze and think shout	
Learning math helps me to analyze and think about solutions to issues within my community.	Knowing math will help me to be successful in my career.
Knowing math helps me understand neighborhood	Learning math will help me to get the job I want.
displacement. (Neighborhood displacement: The	Learning math will help me to get the job I want.
forced movement of people from their homes	
caused by the buying and re-selling of houses	
and stores in lower income communities by	
upper- or middle-income people, resulting in	
raising property values)	
Knowing math makes it easier to understand	I need to know math to earn a living.
income inequality.	<u> </u>
Knowing math makes it easier to understand	Studying math gives me more career choices.
inequality of access to English language	
education in schools. (English language	
education: The teaching and learning conditions	
of English Language Learners (ELL) in schools.)	
Knowing math makes it easier to understand	I will use math problem-solving skills in my
educational inequality. (Educational inequality:	career.
The unequal distribution of academic resources,	
including school funding, qualified and	
experienced teachers, books, and technologies.	
This leads to major differences in the educational success of these individuals.)	
Knowing math makes it easier to understand the	Knowing math will make me more competitive
inequalities within tracking students in schools.	on the job market.
(Tracking: Assigning students to different	on the job market.
classes based on their perceived ability in that	
subject.)	
Knowing math makes it easier to understand	Math will be useful no matter what career I
racism.	choose.
Knowing math helps me to evaluate other people's	Knowing math helps me in other school subjects.
points of view in politics.	
Knowing math helps me understand gentrification.	Studying math helps me with problem solving in
(Gentrification: The process of renovating and	other subjects.
improving a house or district. Thus, forcing the	

Factor 1: practical utility in students' everyday lived experiences

Factor 2: exchange value in preparing students for future courses and careers

movement of people from their homes in lower income communities.)

Knowing math helps me understand deportation. (Deportation: people being pushed out of the country.)

I can use math to make changes in my community. Math has to do with community issues, such as foreclosures. (Foreclosure: banks taking property when a person fails to keep up their mortgage payments.)

I can use the math I have learned in middle school to explore problems within my community.

I can use the math I have learned in high school to explore problems within my community.

I can use the math I have learned in elementary school to explore problems within my community.

Knowing math makes it easier to understand inequalities in the LGBTQ community. (LGBTQ: Lesbian, Gay, Bisexual, Transgender, Queer, or Questioning).

Learning math helps me to analyze inequalities in the LGBTQ community. (LGBTQ: Lesbian, Gay, Bisexual, Transgender, Queer or Questioning)

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### **Biography**

Monique Williams graduated from an all girls' college preparatory school, Garrison Forest School, Owings Mills, MD, in 2003. She went on to receive her Bachelors of Arts in Mathematics from University of Maryland Baltimore County in 2009. She was then employed with her first job as a high school mathematics teacher at Friends School of Baltimore and went on to accept her first college teaching job at Catonsville Community College. She then went on to receive her Masters of Science in Mathematics Education at Towson University in 2012 before being accepted into George Mason's PhD in Education program with a Dean's Scholarship. Monique was a Statistics tutor for doctoral students at GMU as well as a graduate teaching assistant for Mathematics Education and Research Methods, and an adjunct professor for both. She now is a Manager of Secondary Mathematics for District of Columbia Public Schools Central Office, where she supports approximately 50 schools, 18,000 students, and 3,000 teachers in the district. She develops curriculum, creates professional development workshops, creates district-wide assessments to monitor student progress towards grade-level outcomes, as well as promotes instructional best practices through co-teaching, and coplanning models. She also guides school and classroom level data-driven discussions to inform best practices, co-plans with teachers and math coaches, leads classroom observations and debriefs, as well as provides targeted feedback on how to be an effective math coach and assistant principal in transforming student math achievement. She often participates in instructional walkthroughs to collect evidence of classroom practices and lead discussions with Principals and Superintendents to identify high leverage next steps for instructional development. Outside of working with DC Public Schools she is an Adjunct Professor at GMU and she also is an independent contractor and works with an education company to build secondary math teachers and school leaders knowledge of high-quality instructional materials, college and career-ready standards, and equitable teaching practices through professional development across multiple states.