THREE ESSAYS ON MARKET INSTITUTIONS

by

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A Dissertation
Submitted to the
Graduate Faculty
of
George Mason University
In Partial fulfillment of
The Requirements for the Degree
of
Doctor of Philosophy
Economics

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Three Essays on Market Institutions

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy at George Mason University

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Acknowledgments

I would like to thank my advisors and coauthors, Cesar Martinelli and Jason Shachat, for their generosity with their time, knowledge and wisdom. I am also indebted to my other committee members, Daniel Houser and Kevin McCabe, for their guidance and supports, and another coauthor, Jianxin Wang, for his inputs in the second chapter. This dissertation also benefits from helpful discussions with other members of the Finance and Economics Experimental Laboratory and the Interdisciplinary Center for Economic Science, and the audiences of several seminars and conferences.

The studies in this dissertation are financed by the National Nature Science Foundation of China [Grant 71131008], the Fujian Overseas High Talent Organization, and the Interdisciplinary Center for Economic Science at George Mason University.

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Abstract

THREE ESSAYS ON MARKET INSTITUTIONS

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George Mason University, 2020

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This dissertation focuses on the equilibrium and efficiency of market institutions, a major determinant of market outcomes. The three chapters of the dissertation study market institutions in the presence of classic challenges: incomplete contracts, few traders, and incomplete information.

Chapter 1 compares two mechanisms, posted-offer and posted-bid, in a procurement setting with incomplete contracts. Reciprocity has been identified in recent literature as a behavioral trait that mitigates moral hazard problems in the presence of incomplete contracts, along with repeated interactions and reputation concerns. This study builds a model of reciprocity based on inequality aversion, and takes it to the lab. In the laboratory experiment, the posted-offer mechanism induces higher level of inequality aversion on sellers, resulting in higher efficiency than the posted-bid mechanism.

Chapter 2 studies minimal conditions for competitive behavior with few agents, adapting price-quantity strategic market games to an indivisible good environment, and taking it to the lab. In the proposed mechanism, all Nash equilibrium outcomes with active trading are competitive if and only if there are at least two buyers and two sellers willing to trade at every competitive price. In the laboratory experiment, this condition is enough to induce competitive results. Moreover, the performance of a sealed-bid auction following the rules of the strategic market game approaches that of its dynamic counterpart, the double auction, over time.

Chapter 3 surveys the theoretical and experimental literature on the k-double auction. A k-double auction is a multi-unit sealed-bid call market in which the price is determined giving weight $k \in [0,1]$ to the upper bound of market-clearing prices and $(1-k)$ to the lower bound. When agents' values of a commodity are private information, this institution features convergence to efficient outcomes in equilibria as the size of the market grows, supporting the use of Walrasian model as an asymptote of market outcomes in the absence of complete information. This chapter includes a history of the development of the theory, a summary of methods and results, the use in experimental economics, and the relation to other mechanisms.

Chapter 1: Suggested versus Offered Gifts: How Alternative Market Institutions Mitigate Moral Hazard

1.1 Introduction

Almost all procurement contracts fail to specify standards for all possible supplier performance dimensions, making moral hazard a ubiquitous presence in supply chain relationships. To explain the widespread use of such incompletely specified contracts and accompanying lack of supplier malfeasance, standard theory relies upon models of repeated strategic interactions and reputational equilibrium. We show that the market institution, via its effects on the buyer-supplier relationship, can be an alternative force that supports this phenomenon.

We consider a setting in which the buyer wishes to acquire a single object from a set of possible suppliers. The buyer's valuation of the object depends on its quality, such as promptness of delivering the product, the amount of labor put into production, etc. The buyer can form a contract with one of the suppliers, and the price of the object is specified in the contract. After signing a contract, the supplier determines the quality level of the object during production. In our setting, a higher quality object is more valuable for the buyer, but more costly to produce. Hence the supplier, without any contractual restriction or reputation concern, has an incentive to supply an object of the lowest quality.

Besides the use of incentive contract, repeated game, and reputation system, behavioral economists have identified gift exchange and reciprocity as a remedy for moral hazard problem and an explanation for the existence of efficiency wage (Falk et al., 1999; Fehr et al., 1993, 1998). In this paper, we show that the extent of gift-exchange is subject to the trading institution used. We compare results of two commonly used auctions in procurement: the posted-bid mechanism for labor market as in Fehr et al. (1993, 1998), and the posted-offer mechanism for commodity market as in (Fugger et al., 2019). In our controlled laboratory experiments, we found posted-offer mechanism leads to higher extent of gift-exchange, shown as higher price and quality, and higher efficiency. We explain it as a result of the higher level of other-regarding in the supplier's preference induced by the posted-offer mechanism.

In controlled laboratory experiments with designs that rule out the possibility of dynamic relationships and reputation building, suppliers choose higher quality when the contracted price is higher in both labor market settings (Fehr et al. (1993, 1998), hereafter FKR) and commodity market settings (Fugger et al., 2019). These studies found it Pareto-improving to allow the buyer to offer a price higher than minimum (FKR), or to allow the buyer to accept a price other than the lowest offer from sellers (Fugger et al., 2019). The results in these studies are explained by the other-regarding preference the contracted supplier has: when the supplier is awarded a contract that has a price higher than the minimum, she acquires sentiment for the buyer's payoff, and thus chooses a quality level higher than minimum requirement (Akerlof, 1982).

As Fehr and Schmidt (1999) pointed out, "the economic environment determines the preference type that is decisive for the prevailing behavior in equilibrium" (p. 819). A supplier's willingness to reciprocate may also depend on her perceived kindness of the buyer (Cox et al., 2007; Rabin, 1993). The two institutions we investigate here are symmetric: one side of the market propose a price, the other side chooses whether to trade or not. The buyer is not required to accept/propose the minimum price in either mechanism. However, in the posted-offer mechanism, suppliers propose prices to the buyer, hence the size of the gift is demanded; in the posted-bid mechanism, the buyer extends the gift to the suppliers by offering a price to them. We show that if whether a gift is demanded or offered has no effect on the supplier's sentiment for the buyer, the two mechanisms shall generate the same price and quality in the Nash equilibrium. In the experiment, we observe higher price and quality in markets under the posted-offer mechanism, together with higher payoffs for both the buyer and the supplier. In other words, when the supplier demands a gift, the size of the gift is larger, so is the size of reciprocation. This implies that the two mechanisms induce different preferences for the supplier.

We also test the robustness of our results in thicker markets. Studies of procurement auction for commodities focus on the monopsonist scenario, where one buyer faces multiple suppliers. In labor market experiments such as in FKR, there are more than one buyers in the market, which may drive the outcome more competitive. We keep the buyer-seller ratio the same in our experiment, and compare the performance of the mechanisms in monopsonist markets and the competitive markets with six buyers. When the posted-offer mechanism is used, a thicker market does not have an effect on average price, quality or payoffs. When the posted-bid mechanism is used, a thicker market has no effect on average price and the supplier's payoff, but reduces the average quality and the buyer's payoff.

In the procurement auction literature, the posted-offer mechanism we use here is a buyer-determined auction. It is investigated in the monopsony setting. When the object is not homogeneous an quality matters in the procurement, the buyer-determined auction outperforms the first price auction, regardless of the existence of the reputation system (Brosig-Koch and Heinrich, 2014; Fugger et al., 2019).

A closely related study is Charness et al. (2012). The experiment in Charness et al. (2012) shows that delegating the wage choice to the supplier is Pareto improving when the buyer is matched with a supplier before the wage determination process. Since there is no competition on the suppliers' side, the findings in Charness et al. (2012) can be explained by the different equilibrium outcomes under different mechanisms, without a change in the supplier's preference. Our design rules out this possibility. Having suppliers competing against each other in our settings, the Nash equilibrium outcomes are identical under the two mechanisms as long as the extent of the supplier's altruism does not change. Hence we can reach to the conclusion that the different results from our experiment is due to the change in the supplier's preference.

The rest of the paper is organized as follows. Section 1.2 gives a formal model of the procurement. Section 1.3 describes the experimental design and hypotheses. Section 1.4 presents the results. Section 1.5 concludes.

1.2 Model

1.2.1 Setup

We model a procurement scenario in which a buyer attempts to form a contract with one of multiple potential suppliers to produce a single object. The quality of the object, denoted by q, is determined by the amount of costly effort exerted by the supplier and can't be verified by a third party. Further, the contract terms can't be conditioned on the quality. The contract is characterized by the naming of a supplier and the price of the object, p, prior to production. After the awarding of the contract, the named supplier chooses the quality of the object.

We use the payoff structure of the classic studies of FKR. The quality of the object is between 1 and 10. Suppliers' cost for producing the object, $c(q) \approx aq^2 + bq + d$, where $a = 0.1061$, $b = 0.8333$ and $d = 28.933$, is an increasing function of quality in the range.

A supplier's payoff if sells is

$$\Pi_S = p - c(q).$$

If a supplier does not sell, her payoff equals 0.

The buyer's payoff if she buys is

$$\Pi_B = (126 - p) \times \frac{q}{10}.$$

Also, we have $p \in [30, 126]$ to guarantee non-negative payoffs for buyers involved in trades.

We consider two mechanisms to award the contract: a posted-bid mechanism, and a posted-offer mechanism. The posted-bid mechanism is the one-sided double auction used in FKR (FKR hereafter), where the buyer makes a price offer to all suppliers, and contract with one of the suppliers who are willing to accept the offer. The posted-offer mechanism is called buyer-determined auction (BDA hereafter) in the literature, where each supplier makes a price offer and the buyer picks one of them to trade with.

Table 1.1 illustrates the procedure used in the two mechanisms. Both the buyer and the suppliers take part in deciding the transaction price in both mechanisms: one of the parties proposes a price, the other party approves. The posted-bid mechanism works as follows. In the first stage, the buyer sends a bid to all potential suppliers, who in the second stage choose whether to accept the price. One of the suppliers who accept the price is awarded the contract. In the third stage, the contracted supplier chooses a quality level. The posted-offer mechanism differs in the first two stages: in the first stage, each supplier sends a sealed offer to the buyer, who in the second stage decides which supplier to contract with. The third stage is identical in both mechanisms.

Table 1.1: Procedure

	Posted-bid (FKR)	**Posted-offer (BDA)**
Stage 1: Proposal	The buyer proposes a price to all suppliers.	Each supplier proposes a price to the buyer.
Stage 2: Contract	One of the suppliers who accept the proposal forms a contract with the buyer. The buyer pays the price she proposed to the supplier.	The buyer picks one of the suppliers to form the contract with. The buyer pays the price the contracted supplier asked.
Stage 3: Production	The contracted supplier chooses the quality level of the product.	The contracted supplier chooses the quality level of the product.

1.2.2 Nash equilibrium

Here we look for subgame-perfect Nash equilibrium. We assume all suppliers are identical. Knowing the supplier's best response in the last stage, we can find the buyer's globally preferred price and the minimum price the seller is willing to trade at. It can be shown that the optimal price the buyer can get, denoted by p^*, is the maximum of the two.

The Nash equilibrium of the posted-bid mechanism is where the buyer offers p^* to suppliers, and forms the contract with a random one of them; The suppliers accept the proposal from the buyer, and the contracted one offers the quality that maximizes her utility. The buyer would not be better off at a different price, and suppliers prefer trading than not.

In the Nash equilibrium of the posted-offer mechanism, all suppliers offer p^*, the buyer contracts with a random one of them, and the contracted supplier chooses the quality level that maximizes her utility given p^*. If the suppliers offer different prices, a supplier with an unchosen offer can be better off deviating to the chosen price. If suppliers all offer a price different from p^*, a supplier can make a small deviation towards p^*, and increases the probability of trade to 1.

Without any shift in the preference, p^* and the corresponding optimal quality for suppliers are the same under the two mechanisms. Thus the outcomes of Nash equilibrium are the same under both mechanisms: the price equals p^*, and the quality equals the supplier's optimal quality at p^*. [1]

The rest of this section explores the equilibrium outcomes under alternative assumptions on agents' preference.

Profit-maximizing agents

Suppose all agents maximize their own payoffs. In the last stage of the game, the supplier chooses $q = 1$. Knowing this, the buyer obtains the highest profit at price $p^* = 30$.

The Nash equilibrium outcome is $p^* = 30$ and $q = 1$ under both mechanisms.

Agents with other regarding preference

Suppose the buyer and the supplier involved in the contract care about the payoff for the other party, and the preference can be represented by the utility function from Fehr and Schmidt (1999).

Denote by B and S the buyer and the supplier involved in the transaction. The utility for a buyer who buys is

$$U_B = \Pi_B - \alpha_B \max\{\Pi_S - \Pi_B, 0\} - \beta_B \max\{\Pi_B - \Pi_S, 0\}.$$

[1] This is different from the equilibrium in Charness et al. (2012). In Charness et al. (2012), when the supplier sets the price, she sets it at her preferred price. The supplier's preferred price is higher than the buyer's unless they value equality in payoffs highly.

The utility for a supplier who sells is

$$U_S = \Pi_S - \alpha_S \max\{\Pi_B - \Pi_S, 0\} - \beta_S \max\{\Pi_S - \Pi_B, 0\}.$$

The αs and βs in the functions represent the levels of aversion an agent has towards unequal payoffs. Following Fehr and Schmidt (1999), we assume that $\beta_S, \beta_B \in [0,1)$, $\beta_B \leq \alpha_B$ and $\beta_S \leq \alpha_S$. Different values of αs and βs may be induced by the institutions.

Without loss of generality, we further assume that the utility for a supplier who sells is always higher than the utility if she does not sell, and the utility for a buyer who buys is always higher than if she does not buy. Hence p^* equals to the buyer's globally preferred price. [2]

Supplier's choice of quality

With the other-regarding preference described in this section, the quality the supplier chooses could be higher than 1.

When $\Pi_S < \Pi_B$, the supplier has disutility from obtaining lower payoff than the buyer. Given a price, decreasing the quality increases supplier's utility in two ways: it decreases the disutility from unequal payoffs, and it increases the supplier's payoff. Therefore, if $\Pi_S < \Pi_B$, the supplier would want to lower quality as much as possible. The value of α_S does not affect the supplier's choice of quality as long as it's positive.

When $\Pi_S > \Pi_B$, the supplier has disutility from obtaining higher payoff than the buyer. Given a price, increasing the quality decreases the distutility from unequal payoff, but decreases the supplier's payoff as well. The supplier weighs these two factors, and chooses the quality according to how much she values equality.

Hence, we have the following proposition.

Proposition 1. *The supplier chooses $q = 1$ at all prices if $\beta_S < 0.11$. At each price, the supplier's choice of quality is increasing in β_S.*

[2] Alternatively, we can assume U_0=0. Then the minimum price a seller is willing to trade at depends on α. The equivalence of Nash equilibrium outcomes in the two institutions still holds if preferences are the same across institutions.

The proof of the proposition is in the appendix. Figure 1.1 depicts the optimal quality choices at each price. Different colors indicate different values of β_S. The shade indicates that $\Pi_S < \Pi_B$ in the area. At the boundary of the shaded area, the payoffs are equal for both buyer and seller.

If β_S is smaller than 0.11, as shown by the yellow line, the supplier chooses $q = 1$ at all price levels.

When β_S is larger than 0.11, the supplier may choose a quality higher than 1 at some prices. As shown in figure 1.1, when the price is low, the quality chosen by the supplier equals 1; As the price goes higher, the optimal quality starts to increase, and then decreases after the price reaches a certain level. The the highest quality choice on the equal-payoff curve is the point that the supplier's marginal utility from increasing own payoff equals the marginal utility from decreasing the inequality. The same goes to the points on the decreasing segments of the quality. Even though the supplier values equality, the quality level is decreasing in price in these segments. This feature comes from the structure of the payoff functions. As the price goes higher, the marginal effect of quality on buyer's payoff drops, while its effect on supplier's payoff is constant. Therefore, high quality is not necessarily efficient in our settings. When price goes high enough, by lowering the quality, the increase in the supplier's own payoff exceeds the increase in the disutility from inequality. The higher β_S is, the higher the price and quality are at the turning point. When $\beta_S > 0.43$, the preferred quality level is bounded by 10 in certain price ranges.

Buyer's preferred price

Denote by P_1 the price at which the buyer and the supplier have equal payoffs when $q = 1$. Given the supplier's quality choice, denote by $\hat{P}(\beta_S)$ the highest price at which the buyer and the supplier have equal payoff. From proposition 1, $\hat{P}(\cdot)$ is a non-decreasing function.

We have the following proposition.

Proposition 2. *A buyer's globally preferred price is 30 if* $\beta_B < 0.09$ *and* $\beta_S < 0.11$. *Otherwise the preferred price is* $\hat{P}(\beta_S)$. *A buyer's globally preferred price is increasing in* β_S.

The proof is in the appendix. Intuitively, when the price exceeds $\hat{P}(\beta_S)$, the quality chosen by the supplier is not increasing in p, and the buyer has lower payoff than the supplier. Therefore,

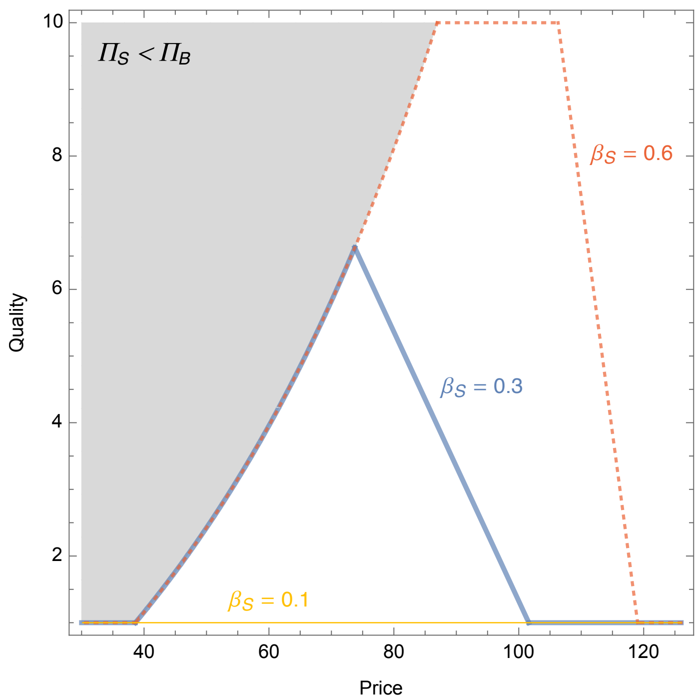


Figure 1.1: Quality choice by the supplier

the buyer prefers $\hat{P}(\beta_S)$ to all prices higher than it. In the price range between p_1 and $\hat{P}(\beta_S)$, the buyer prefers $\hat{P}(\beta_S)$ because it gives the highest payoff, while there is no disutility from inequality in the range. In the range of price between 30 and p_1, the buyer prefers p_1 if he has strong enough preference for equality, i.e. $\beta_B > 0.09$, and prefers 30 if $\beta_B < 0.09$. Hence, if β_S is high so that $\hat{P}(\beta_S)$ generates higher utility than when $p = 30$, or if $\beta_B > 0.09$, the buyer's globally preferred price is $\hat{P}(\beta_S)$.

In figure 1.2, the asterisks indicate the Nash equilibrium outcomes at different levels of β_S. Lines in colors are the best responses by suppliers of different β_Ss, same as the ones in figure 1.1. In a subgame-perfect Nash equilibrium, the buyer chooses a point on the supplier's best response curve that yields the highest utility. The black lines in the background are the buyer's indifference curves.

In panel (a), $\beta_B \leq 0.09$. The buyer's utility increases in the direction of northwest. Depending on the slope of indifference curves, when β_S is low, i.e. $\beta_S \leq 0.11$, the Nash equilibrium price can be between 30 and 39.59. When $\beta_S > 0.11$, as shown in proposition 2, the Nash equilibrium price is $\hat{P}(\beta_S)$, which is non-decreasing in β_S. In panel (b), $\beta_B > 0.09$, the buyer obtains highest utility at the highest kink of the indifference curves. The Nash equilibrium price is $\hat{P}(\beta_S)$.

Overall, at any level of β_B, the Nash equilibrium price and quality are non-decreasing in β_S. When $\beta_S \in [0.11, 0.43]$, the Nash equilibrium outcome is strictly increasing in β_S regardless of β_B.

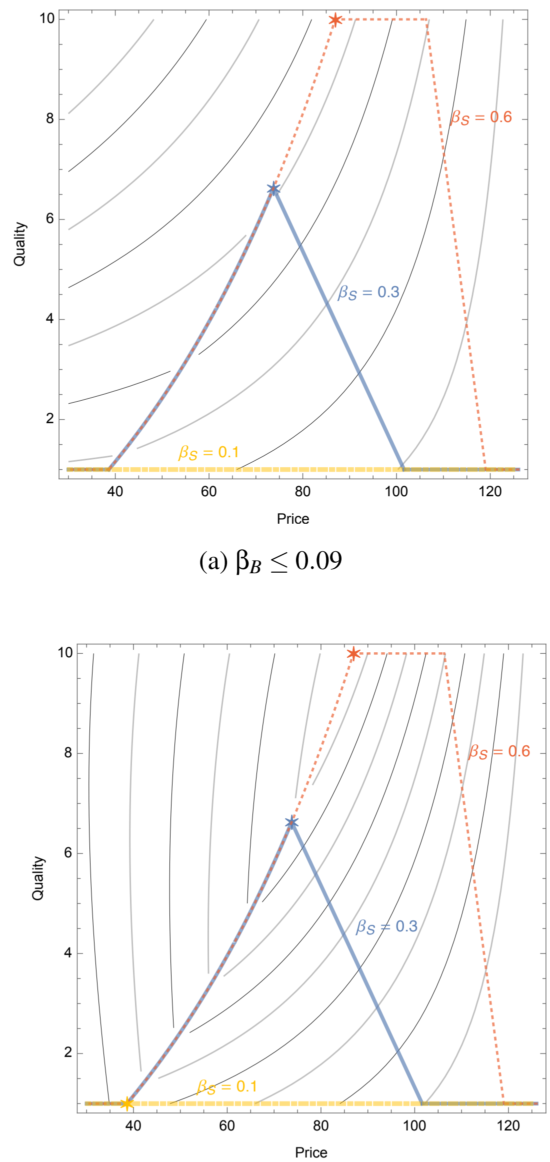
If the two institutions induce different values of β_S, the resulted Nash equilibrium price and quality may be different. As discussed in Charness et al. (2012), when a supplier offers the price, the supplier may feel more responsible for the outcome and care more about the inequality, thus offer a higher quality. If this is true, the price and quality could be higher under the posted offer mechanism.

1.3 Experimental design and hypotheses

The experiment compares gift-exchange behavior under posted-offer and posted-bid mechanisms. We study both a monopsonist environment (1 buyer versus 2 suppliers, 1:2 hereafter) and a thick market environment (6 buyers versus 12 suppliers, 6:12 hereafter). The two environments have the same buyer-supplier ratio. In the standard theory with homogeneous good, a thicker market reduces the market power possessed by the monopsonist, and generates higher efficiency. Here we test whether the proposition holds in an incomplete contract environment.

In every period, each buyer can form an incomplete contract to buy one unit of product from one of the suppliers. A supplier can produce no more than one unit in a period. The product is made upon request: no cost is incurred if the supplier doesn't make a transaction.

A period begins with an auction to award the contract and determine the transaction price(s). One side of the market offer price(s) and the other side choose whether or which to accept. Once an offer is accepted, an incomplete contract is formed, and the corresponding two participants trade one unit of product at the price they agree on. After a contract is awarded, the supplier who gets



(a) $\beta_B \leq 0.09$

(b) $\beta_B > 0.09$

Figure 1.2: Nash equilibrium with reciprocity

the contract chooses the quality of the product. Supplier's choice on quality is revealed only to the corresponding buyer in that transaction. The quality choices a supplier made is not revealed to any other subject in any other part of the experiment. All levels of quality are feasible options for the supplier. The cost scheme of quality levels is common knowledge for all participants. All transactions and bids are anonymous. Bids are only revealed to participants in the same market after the bidding stage.

Under posted-offer mechanism, suppliers submit sealed offers in the bidding stage. In the 1:2 market, each supplier submits an offer anonymously, the buyer in the same group chooses one of the offers and trade with the corresponding supplier at that price. In the 6:12 market, after each supplier submits an offer, buyers see all the offers, and take turns in a random order to form contracts.

The posted-bid mechanism replicates the settings in Fehr et al. (1993, 1998). When the market is open, each buyer submits a bid, and sellers in the group choose whether to accept it. If multiple sellers accept the same bid, the one who accepts it first gets the contract. An unaccepted bid can be revised by raising the price.

The experiment was conducted at the Finance and Economics Experimental Laboratory in Xiamen University. Subjects were undergraduate or master students in the university, recruited via ORSEE (Greiner et al. (2004)). Four sessions were conducted for each treatment. Each session lasted for no more than 90 minutes.

There were 18 participants in every session. Participant were assigned the role of a buyer or a seller, which remained the same throughout the session. There were 30 periods in each session. In the monopsonist environment, subjects were randomly assigned into groups of 1 buyer and 2 sellers, and were randomly rematched after each period.

The experiment was programmed in z-Tree (Fischbacher (2007)). Subjects were seated at computer terminals separated by partitions upon arrival. Instruction was read out loud to guarantee the public information known to all subjects, and the experiment began after every subject indicated understanding of the instruction and got familiar with the payoffs under different prices.

The exchange rate was private information for subjects. To give similar payoffs to both roles, the exchange rate was 16 experimental dollars=1 CNY for buyers and 8 experimental dollars = 1

CNY for sellers. Average payoff was 63.2 CNY (around 7 dollars) including show-up fee of 10 CNY.

1.3.1 Parameters

For simplicity of choices, we use discrete schemes in the experiment. Prices (p) are integers from 30 to 126. Qualities (q) are integers from 1 to 10.

The cost scheme for suppliers is listed in Table 1.2.

Table 1.2: Sellers' cost scheme

Quality q	1	2	3	4	5	6	7	8	9	10
Cost $c(q)$	30	31	32	34	36	38	40	42	45	48

If a contract is formed, the payoffs for the two parties are,

Buyer's payoff $\Pi_B = (126 - p) \times q/10$,

and seller's payoff $\Pi_S = p - c(q)$.

For buyers and sellers who does not trade, $\Pi_0 = 0$.

1.3.2 Hypotheses

Our first hypothesis relates to the existence of gift-exchange behavior. When sellers have high enough other regarding preference, the equilibrium price and quality are above the minimum level, as was observed in FKR and others studies. We expect to observe the same in our experiment.

Hypothesis 1. *The price and quality are above the minimum levels.*

Whether a gift is suggested or extended may have an effect on how reciprocal the seller is, but there is no clear evidence on how the difference would be under the two institutions. Without having an effect on the seller's preference, the different institutions yield the same equilibrium outcome.

Hypothesis 2. *Price and quality are the same in posted-bid and posted-offer mechanisms.*

The third hypothesis corresponds to the effect of the thickness of the market. It is possible that intensifying competition in the market on both sides improves the efficiency.

Hypothesis 3. *Efficiency is higher in thicker markets.*

1.4 Results

1.4.1 Overview

Figure 1.3 shows the average price and quality in each period in the four treatments. The blue lines are for the posted-offer mechanism, and the red lines are for the posted-bid mechanism. Three inferences can be drawn. Firstly, in favor of hypothesis 1, the price and quality are higher than the minimum levels. The average price and quality in each treatment are listed in table 1.3. Average prices in the four treatments range from 58.35 to 79.09, and are significantly higher than 30 (t-test p-value < 0.001 for each treatment). Average qualities are higher than 1 (t-test p-value < 0.001 for each treatment). Secondly, contrasting hypothesis 2, in both 1:2 and 6:12 markets, price and quality are higher in the posted-offer mechanism (t-test $p-value < 0.001$ for both 1:2 market and 6:12 market). Thirdly, the thickness of market does not have an obvious effect on the price or quality. In fact, under both mechanisms, the two-sided t-tests on the effect of market thickness show no significant effect on price ($p-values > 0.1$), and only under posted-bid mechanism the quality is different when the market thickness changes (two-sided t-test $p-value > 0.1$ for posted-offer treatments, and $p-value = 0.006$ for posted-bid treatments).

Consistent with previous literature, behavior of subjects in our experiment is closer to the predictions made with other-regarding preference. Both price and quality deviate from the profit-maximizing Nash equilibrium of $(p,q) = (30,1)$. Correlations between quality and price are positive and significant through all treatments (Spearman's rank correlations > 0 and p-values < 0.01).

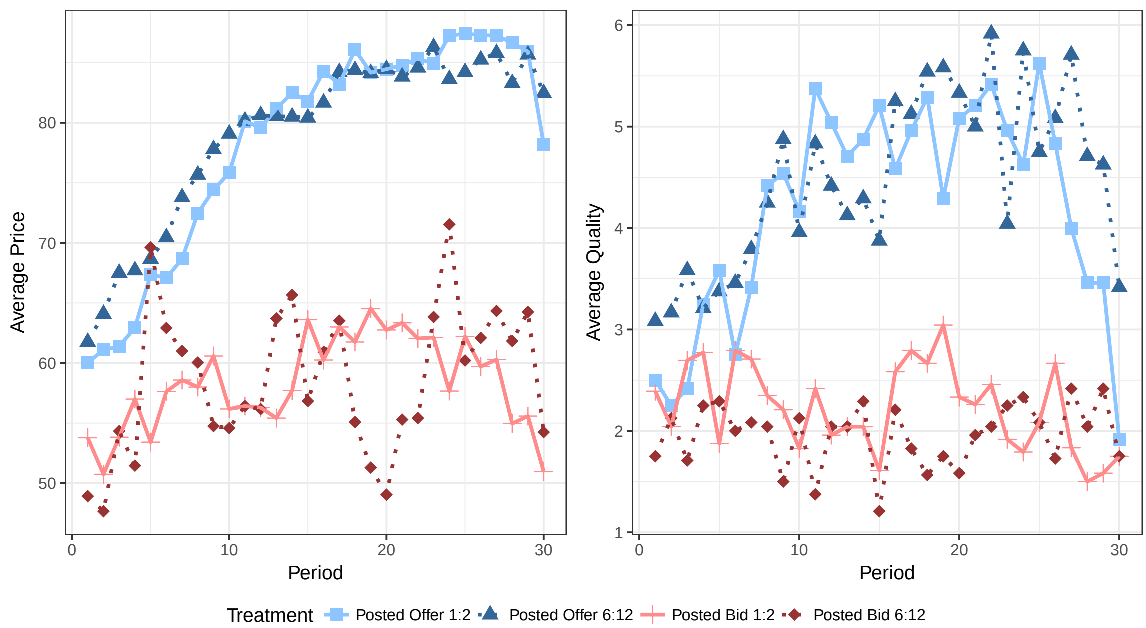


Figure 1.3: Average price and quality over time

Table 1.3: Mean values of variables

Mechanism	Posted Offer		Posted Bid	
Number of Buyer(s): Number of Sellers	1:2	6:12	1:2	6:12
Number of trades [a]	720	720	710	716
Price	78.46	79.09	58.35	58.55
	(13.03)	(13.99)	(20.53)	(21.1)
Quality	4.21	4.47	2.23	1.96
	(3.37)	(3.44)	(1.89)	(1.82)
Spearman's rank correlation (p,q)	0.33	0.37	0.52	0.48

Note: standard deviations in parentheses.

[a] There are 720 potential trades in each treatment, which were all achieved in posted-offer treatments. In posted-bid treatments, several trades were not made because no seller accepted the bid or the buyer didn't bid during the auction. One trade was deleted in posted-bid 6:12 treatment due to a error in the program that allowed price to be zero. Both subjects involved in this trade noticed the problem and informed the experimenter. Due to simultaneous clicks by sellers in posted-bid 6:12, two sellers were accidentally involved in two transactions. The analysis provided in this paper excludes the extra sellers. No distinguishable change happens if they are included.

1.4.2 Effect of institution

Price and quality distribution

Figure 1.4 demonstrates the joint distribution of price and quality in each treatment. The redder the color, the more trades happen at that price-quality combination. Two inferences could be drawn from the graph. Firstly, the range of prices and qualities are similar for all treatments, except that only in the posted-bid treatments do trades happen at the lowest price range. Secondly, in the posted-bid treatments, trades happen most often at the lowest price range with the lowest quality; in the posted-offer treatment, trades happen most often at the price range of 80 – 90, with the quality ranging from 1 – 10 in this range. Overall, the trades in the posted-offer treatments cluster more on the upper-right part of the graphs than the trades in the posted-bid treatments.

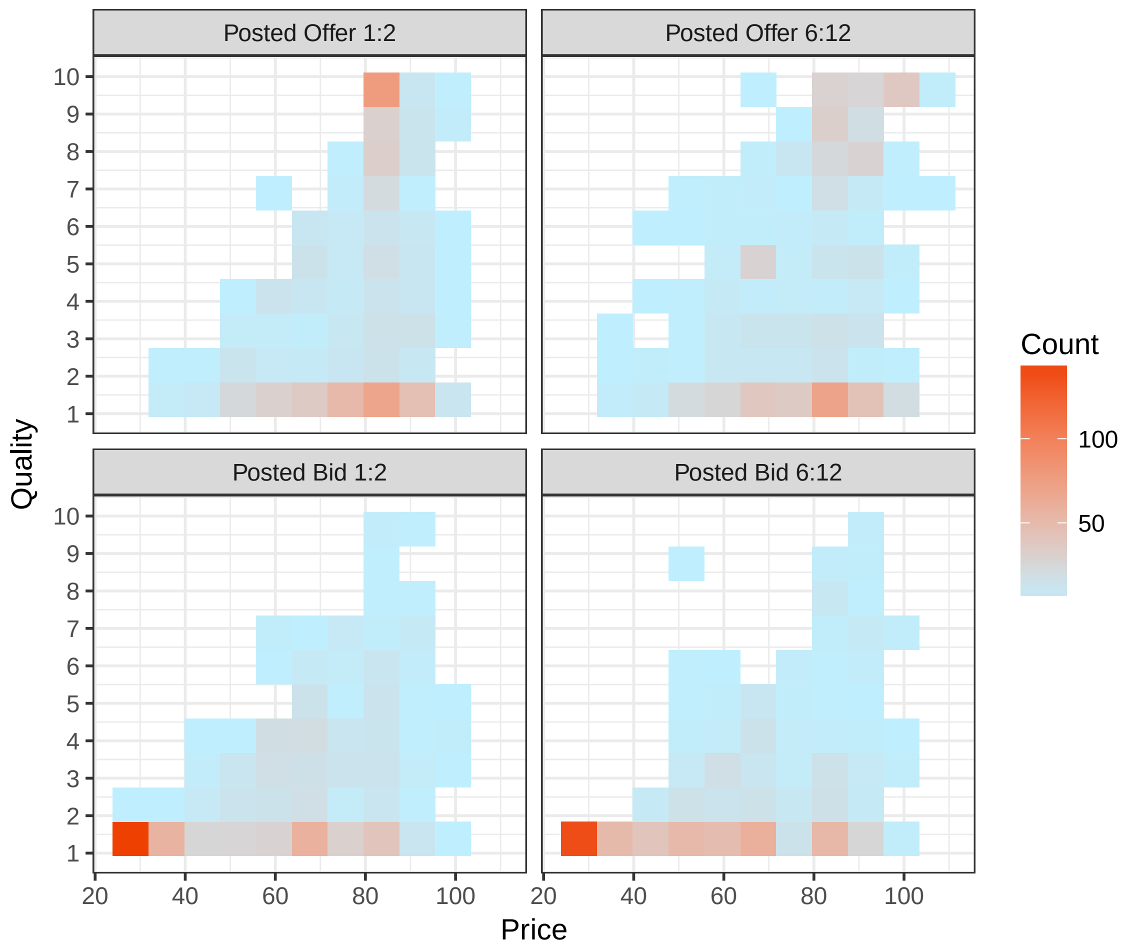


Figure 1.4: Price and quality distribution

Quality

We run several Tobit regressions to test whether the difference in institutions affects the seller's choice of quality. Results are shown in table 1.4.

From regression (1), consistent with previous results, quality is increasing in price. The seller chooses a lower quality when a given price is offered by the buyer, but the market thickness does not have a significant effect. Shown in regression (2), if we look at the effect of mechanism and market thickness on the coefficient of the price, only the mechanism has a significant effect: when the bid is offered by the buyer, increasing the price has a lower effect on quality. However, if we consider both level effect and slope effect of the mechanism together, as in regression (3) and (4), neither the level effect nor the slope effect is significant. This could due to the high correlation between the treatment variable and its interaction term with price. We test whether variables containing mechanism and market thickness have an effect on the quality jointly in the regressions. Results from F tests for regression (1) to (4), in which the effects of the mechanism are included, indicate that there is a treatment effect in each of the regressions. In regression (5), which only contains the effect of market thickness, the F-test cannot reject the null hypothesis that the treatment variables do not have an effect on quality. Therefore, we reach to the conclusion that the mechanism has an effect on the level of reciprocity, although the level effect and slope effect cannot be disentangled.

Table 1.4: Tobit regression: treatment effect on quality

Dependent Variable: Quality	(1)	(2)	(3)	(4)	(5)
Constant	-9.37***	-10.35***	-10.03***	-9.44***	-12.16***
	(1.18)	(1.01)	(2.23)	(1.91)	(1.61)
Price	0.16***	0.17***	0.17***	0.16***	0.19***
	(0.01)	(0.01)	(0.03)	(0.03)	(0.02)
Posted-bid	-1.71***		-1.42	-1.37	
	(0.63)		(1.93)	(1.93)	
Thin market	0.26		1.24		1.05
	(0.63)		(1.70)		(1.75)
Price $\times$ Posted-bid		-0.02**	0.00	0.00	
		(0.01)	(0.03)	(0.03)	
Price $\times$ Thin market		0.00	-0.01		-0.01
		(0.01)	(0.03)		(0.03)
F-statistic for treatment variables	3.70**	3.31**	2.30*	4.05**	0.33
Number of observations	2868	2868	2868	2868	2868
Pseudo R^2	0.10	0.10	0.10	0.10	0.09

Note: Robust standard errors in parentheses. Standard errors clustered by seller. Significance at 1%, 5% and 10% are denoted by ***, **, * respectively.

Payoffs

As depicted in figure 1.5, in the procurement, the average payoffs for both the buyer and the seller are higher in the posted-offer treatments (one-sided t tests p-values < 0.001 for both 1:2 and 6:12 markets).

Table 1.5 contains the average social surplus and payoffs in transactions. The average social surpluses are around 60 in the treatments that sellers offer bids, and around 40 in the treatments that

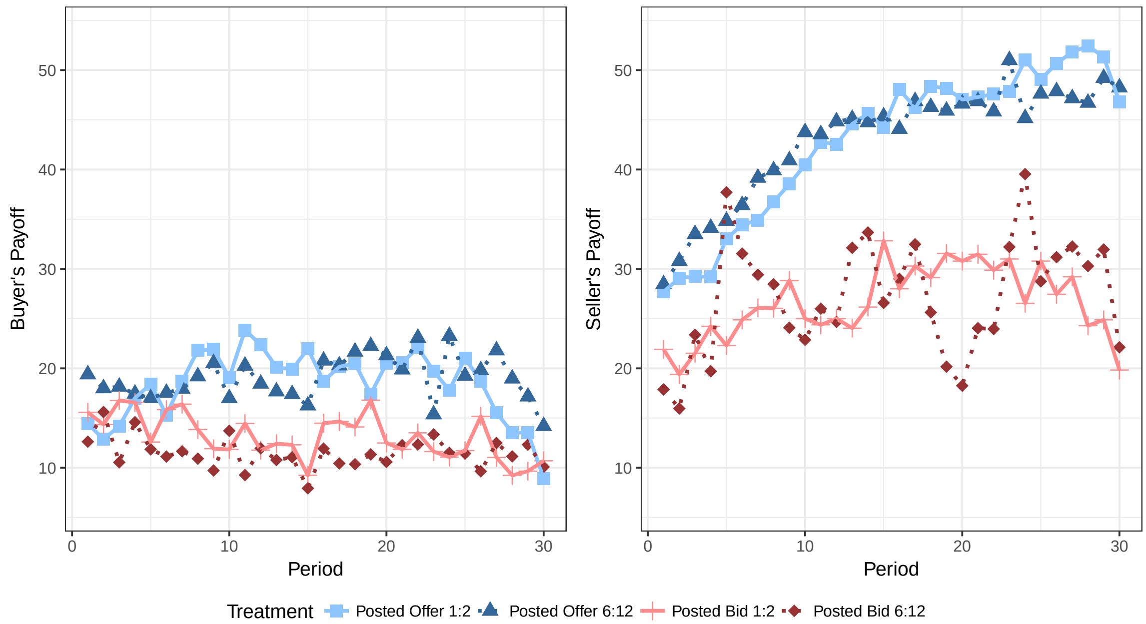


Figure 1.5: Buyer's and seller's payoff over time

buyers offer bids. At the same thickness of the market, using the mechanism in which sellers offer bids generates a significantly higher social surplus (one-sided t test p-value< 0.001 for both 1:2 and 6:12 markets).

Contradicting hypothesis 3, the thickness of market does not have an effect on the social surplus (two-sided t test p-value $= 0.29$ for posted-offer treatments, 0.36 for posted-bid treatments) or seller's payoff (two-sided t test p-value $= 0.84$ for posted-offer treatments, 0.58 for posted-bid treatments). When the mechanism in which sellers offer bids is used, the thickness of the market does not have an effect on the buyer's payoff (two-sided t test p-value $= 0.33$). Only when the buyer submits bid, a thicker market reduces the payoffs for the buyers (one-sided t test p-value < 0.001).

Table 1.5: Mean values of variables

Treatment	Posted Offer		Posted Bid	
	1:2	6:12	1:2	6:12
Social surplus in a period	61.25	62.07	39.72	38.66
	(14.95)	(14.67)	(21.85)	(21.60)
Buyer's payoff in a period	18.36	19.04	13.12	11.50
	(13.32)	(13.38)	(8.51)	(7.94)
Seller's payoff in a period	42.90	43.03	26.60	27.17
	(12.15)	(12.67)	(19.24)	(20.03)

Note: The values are in experimental dollars. Standard deviations are in the parentheses. Only data in transactions are included.

1.4.3 Price and quality dynamics

From figure 1.3 we can see, the average price in the posted-offer treatments increase gradually as participants gain experience in the experiment. Although the prices in the posted-bid treatments have reached the same levels as the posted-offer treatments begin with, they don't exhibit the same increasing trend afterwards. This can be explained by the results in the previous sections: the price-quality relationship is stronger under the posted-offer mechanism, driving the equilibrium price and quality higher than the posted-bid mechanism.

A possible explanation that suppliers collude and offer high prices in posted-offer treatments is not plausible here. A collusion breaks down if the buyer always accepts the lowest offer. On average, the probability a buyer chooses an offer higher than average is 49.72% in the thin market, and 42.64% in the thick market.

In the posted-offer treatments, the buyer learns from previous experience, and chooses whether to accept a offer higher than average accordingly. Table 1.6 contains the results of Logit regressions for factors that may affect whether the buyer chooses the higher bid. Regression (1) checks whether the distribution of the bids has any effect on the choice, and neither the mean or the standard deviations of offers has a significant effect on the choice. We add other factors to regression (2) and (3).

Table 1.6: Logit regression for offer selection

Dependent Variable: Whether chooses a high offer price	(1)	(2)	(3)
Constant	-0.07	-0.11	0.11
	(0.57)	(0.58)	(0.58)
Average of offers	0.00	-0.01	-0.01
	(0.01)	(0.01)	(0.01)
Standard deviation of offers	-0.01	0.00	0.00
	(0.01)	(0.01)	(0.01)
Whether selected a high offer in previous round		1.08^{***}	-0.02
		(0.13)	(0.21)
Quality level in periods round			-0.09^{***}
			(0.03)
Quality level in periods round$\times$ Whether selected a high offer in previous round			0.25^{***}
			(0.04)
Number of Observations	1440	1392	1392

Notes: Logit regression on panel data of buyers' choices over time. Only posted offer treatments are included. Standard errors in parentheses. Significance at 1%, 5% and 10% are denoted by *** , **,* respectively. The Im-Pesaran-Shin for unit root tests for all variables reject the null hypotheses that the panels contain unit roots.

From these regressions, a buyer who accepted a high offer in the previous round is more likely to have the same choice in the current round. Suggested by regression (3), if the buyer selected a high offer in the previous round and the quality was high, it's more likely that the buyer keeps choosing a high offer in the current round, and if the quality was high without choosing the high offer in the previous round, the buyer is more likely to accept a lower offer in the current round.

Since the seller is more altruistic in the posted-offer treatments, the market moves towards the high-price-high-quality equilibrium gradually as the buyer learns from previous experience.

Table 1.7 further illustrates how sellers adjust their offers in the posted offer treatments. Sellers adjust their offers towards the accepted offer in the previous period, although the groups are randomly reformed in every period. This is verified by the significant negative coefficient for ($\text{Offer}_{i,t-1} - \text{Average accepted offer}_{t-1}$) in the regressions: if the offer was higher than the average

Table 1.7: Sellers' offer adjustment in posted offer treatments

Dependent Variable: $\text{Offer}_{i,t}$-$\text{Offer}_{i,t-1}$	(1) Posted Offer 1:2	(2) Posted Offer 6:12
Constant	9.12^{***}	8.65^{***}
	(2.48)	(2.90)
Offer_{t-1}	-0.12^{***}	-0.09^{**}
	(0.03)	(0.04)
$\text{Offer}_{i,t-1}-\text{Average accepted offer}_{t-1}$	-0.66^{***}	-0.58^{***}
	(0.04)	(0.06)
$\text{Offer accepted}_{i,t-1}$	1.11^{**}	0.00
	(0.48)	(0.53)
Number of Observations	1392	1392

Note: GLS regression with random effects. Robust standard errors in parentheses. Standard errors clustered by individual. Significance at 1%, 5% and 10% are denoted by *** , **,* respectively. The Im-Pesaran-Shin for unit root tests for all variables reject the null hypotheses that the panels contain unit roots.

accepted offer, sellers lowers their offer in the next period.

In the posted offer 1:2 treatment, when the offer was not accepted in the previous period, the seller adjusts her offer towards the previous accepted offer; when the offer was accepted, the seller increases the offer in the current period. The adjustment does not depend on whether the offer was accepted or not in the thick market.

Analogously, in posted bid treatments, buyers adjust their bids according to the information and results in the previous period. When the quality from previous period was high, the buyer increases the bid. In the thick market, buyers adjust their bids towards the average bids on the market in the previous period, without knowing the resulted quality of each price. Next section provides a possible explanation for this behavior.

Table 1.8: Buyers' bid adjustment in posted bid treatments

Dependent Variable: $\text{Bid}_{i,t}$-$\text{Bid}_{i,t-1}$		
	(1)	(2)
	Posted Bid 1:2	Posted Bid 6:12
Constant	26.32^{***}	28.67^{***}
	(3.49)	(4.50)
$\text{Bid}_{i,t-1}$	-0.54^{***}	-0.60^{***}
	(0.06)	(0.08)
$\text{Quality}_{i,t-1}$	2.40^{***}	3.37^{***}
	(0.29)	(0.43)
$\text{Bid}_{i,t-1}-\text{Average bid}_{t-1}$		-0.32^{***}
		(0.10)
Number of Observations	1386	695

Note: GLS regression with random effects. Robust standard errors in parentheses. Standard errors clustered by individual. Significance at 1%, 5% and 10% are denoted by *** , **,* respectively. The Im-Pesaran-Shin for unit root tests for all variables reject the null hypotheses that the panels contain unit roots. Regression (2),(3) include only data in the thick markets.

1.4.4 Reference points

Information in the market may affect sellers' perception of the kindness, and hence how much they care about inequality. Regression results in table 1.9 shows that available offers in the market serve as reference points for sellers in posted offer treatments. Price is positively correlated with whether it is higher than average offer. The buyer's payoff increases when the price is higher, as shown in regression (1). Hence, the quality increases with price as shown in table 1.3, and the increase is in a large enough scale to bring the buyer higher payoff. Regression (2) shows that this benefit of a high price comes from the sellers' comparison with the average offer on the market: the payoff is higher if the chosen offer is higher than average. The buyer's forgoing of low prices is reciprocated. Other factors such as market thickness in regression (3) does not have a statistically significant effect.

Shown in table 1.10, the same result holds in the thick posted-bid markets, as the information of other bids are available. This result provides a possible explanation for bid adjustment in table

1.8: although the resulted qualities were not revealed to buyers, they understand that sellers would compare bids on the market, thus adjust their bids towards the average if they are too low.

Table 1.9: OLS regression: Buyer's payoff in posted offer treatments

Dependent Variable: Buyer's payoff	(1)	(2)	(3)
Constant	8.58**	11.16***	11.72***
	(3.66)	(3.80)	(4.18)
Price	0.13**	0.08	0.07
	(0.05)	(0.05)	(0.05)
Whether offer is higher than average		3.24**	3.33**
		(1.36)	(1.31)
Whether buyer:seller=1:2			−0.87
			(2.27)
Adjusted R^2	0.02	0.03	0.03
Number of observations	1.440	1.440	1.440

Note: Data include only posted offer treatments. Robust standard errors in parentheses. Standard errors clustered by seller. Significance at 1%, 5% and 10% are denoted by *** , **,* respectively.

Table 1.10: OLS regression: Buyer's payoff in posted bid 6:12

Dependent Variable: Buyer's payoff		
	(1)	(2)
Constant	8.53***	9.61***
	(0.60)	(0.66)
Price	0.05***	0.01
	(0.01)	(0.02)
Whether offer is higher than average		2.43**
		(0.92)
Adjusted R^2	0.02	0.03
Number of observations	716	716

Note: Data include only posted bid 6:12 treatment. Robust standard errors in parentheses. Standard errors clustered by seller. Significance at 1%, 5% and 10% are denoted by *** , **,* respectively.

1.5 Conclusion

In this paper, we provide a comparison of two commonly used institutions in procurement – the posted-bid mechanism and the posted-offer mechanism – in an incomplete contract environment. While the moral hazard problem is hindered by sellers' reciprocity, the institution plays a role in the level of reciprocity sellers demonstrate. We interpret this difference as a result of the different preferences induced by the institutions. Taken as a gift, the contract is suggested by the seller in the posted-offer mechanism, and extended to the seller in the posted-bid mechanism. If stemmed from the same preference, the two mechanisms yield the same outcomes in Nash equilibrium. In our laboratory experiment, the posted-offer mechanism leads to a higher level of reciprocity, shown as higher levels of price and quality, together with a higher correlation between the two. This is consistent with the effect of a higher level of other-regarding in the preference.

A few possible explanations can be applied. One is from the conjecture in Charness et al. (2012): in posted-offer treatments, when a seller's proposal is accepted, the seller feels more responsible for the outcome than if the proposal is made by the buyer, thus cares more about equality. Another explanation is a reference-point story: sellers use other prices on the market as reference points, and become more other-regarding if the buyer lets go of lower offers to accept theirs in the posted offer treatments. Since buyers choose offers higher than average over 40% of the time in posted-offer treatments, the sellers exhibit higher aversion to inequity. The second explanation is not as plausible because the effect of reference points also exists in the thick markets under posted-bid mechanism, but does not result in higher prices and qualities.

Buyers and sellers in our experiment adjust their choices given the information they observed previously. In line with the first explanation, since sellers has a higher inequality aversion under the posted-offer mechanism, the market reaches the equilibrium of higher price and higher quality after learning. The overall surplus is also higher under the posted-offer mechanism.

An alternative way of modelling the scenario is to incorporate incomplete information and heterogeneity on preferences. When the quality each seller can offer is determined exogenously, the offers can be indicators of the pre-determined qualities (Janssen and Roy, 2010). Our setting is more complicated as the qualities are endogenous as functions of prices, but there may be a separating equilibrium in which offers are signals of sellers' types under the posted offer mechanism, hence the efficiency is enhanced by the reduction of information asymmetry.

We are aware that the results from our experiment can be driven by specific features of the parameters and payoff structures used in this study. Future research using various payoff structures can provide a more thorough understanding of our results.

Chapter 2: Competition with Indivisibility and Few Traders

2.1 Introduction

Ever since the classic contributions of Cournot (1838) and Bertrand (1883), the question of whether a market with a small number of traders can achieve competitive outcomes has been a matter of debate. The literature on strategic market games, pioneered by Dubey and Shubik (1980), revisits this topic in an environment in which buyers and sellers submit price-quantity pairs to a clearing house, which acts as a profit-maximizing middleman, and allocates trades accordingly. In line with Bertrand's argument, Dubey (1982), Simon (1984) and Benassy (1986) prove that having two active sellers and two active buyers in a Nash equilibrium is sufficient to make the outcome competitive. In this paper, we propose a strategic market game applicable to markets with indivisible commodities, we derive conditions for the equivalence between Nash and competitive equilibrium, and we test the equivalence in the lab.

We provide a necessary and sufficient condition for equivalence between Nash equilibrium and competitive equilibrium outcomes with indivisible commodities. Essentially, our condition requires that on each side of the market there are two inframarginal traders, in the sense that they are *willing* to trade at every competitive price.[1] Unlike previous work, our condition relies on the characteristics of the set of competitive equilibria, and place no requirement on Nash equilibria other than the occurrence of trade. Notably, our equivalence result includes contestable markets, in which a single active seller sells in the market at the competitive price.[2]

To test our results in the lab, we conduct market experiments with two buyers and two sellers—the minimal size allowing for the equivalence of Nash equilibrium and competitive equilibrium outcomes, and thus adequate for a stringent test. We consider two market environments: one in

[1] Our exact condition, spelled precisely in the statement of theorem 3, is slightly weaker.

[2] To prove our equivalence result, we first extend results from previous literature to our indivisible commodity setting in theorems 1 and 2. The proof of theorem 3 builds on those results and handles the additional contestable market case.

which the two buyers and the two sellers are inframarginal, so that all Nash equilibrium outcomes are competitive, and one in which the two buyers but only one of the sellers are inframarginal (i.e. there is monopoly power) so that some Nash equilibrium outcomes are non-competitive. In each environment, we consider two market institutions: a sealed-bid auction and a double-oral auction (following the rules of Smith (1962)), which are static/dynamic versions of each other.

In our laboratory experiments, as in other market experiments, traders are informed about their own valuations but not about the valuations of other traders. Thus, in using a strategic market game to explain behavior in the lab, we are following what Friedman and Ostroy (1995) call the "as-if Nash equilibrium complete information approach," the underlying idea of which is that "although traders' information in the experiments is far from complete, it is possible for them to learn to use the relevant 'complete information' strategies" (p. 23). The double-oral auction institution is known to facilitate learning of the relevant information for traders when compared to call markets, with as few as eight traders (see e.g. Smith, 1982; Smith et al., 1982), and hence provides a useful benchmark for assessing the equivalence result.

In the absence of monopoly power, the results from our experiment confirm the double auction institution's convergence to competitive outcomes, though we have fewer traders than previous experiments.[3] Efficiency under the sealed-bid institution remains below efficiency under the double-oral auction, but seemingly converges over time, in line with the results obtained by Smith et al. (1982) and Friedman and Ostroy (1995) for larger numbers of traders. Under both institutions, trading prices lay mostly in the competitive range in the absence of monopoly power, consistent with equilibrium predictions.

When monopoly power exists, higher trading price, lower trading volume and an efficiency loss can be observed under the double-oral auction compared to the environment without monopoly, as expected. Under the sealed-bid institution, trading volume is lower compared to the environment without monopoly, but the efficiency loss is not significant, and prices seem to converge to competitive levels over time. This surprising result may be either a consequence of the inability of the monopolist to gather enough information about the other side of the market to exploit monopoly

[3]See figure 2.7 for a comparison between efficiency in our experiment and others in the literature.

power under the sealed-bid institution, or a consequence of coordination on a low-price outcome, which remains a Nash equilibrium outcome under monopolistic conditions. It is an interesting and open question whether the convergence to competitive outcomes for the sealed-bid institution even in the presence of monopoly power is robust to learning with a longer horizon and to variations in the parameters describing the economy.

In previous experimental work on market games, Duffy et al. (2011) explore a quantity strategic market game with divisible commodities, where traders retain market power. They compare outcomes with two and with ten traders per side, and obtain higher efficiency and more coherence to competitive behavior if there are more traders. Dufwenberg and Gneezy (2000) also obtain—somewhat surprisingly—a beneficial effect of the number of traders in an experiment on Bertrand competition, comparing outcomes with two, three, and four traders per side. We differ from both in that we explore the boundary between competitive and noncompetitive environments.

The rest of the paper is organized as follows. Section 2.2 gives a formal description of the economy. Section 2.3 gives a detailed explanation of the strategic market mechanism. Section 2.4 contains the theorems of coincidence of Nash equilibrium and competitive equilibrium. Section 2.5 presents the experimental design and hypotheses. Section 2.6 describes the results. Section 2.7 concludes. Proofs for the main results are collected in the appendix, and additional proofs, graphs, and experimental instructions and quizzes are collected in the online appendix.

2.2 The economy

We describe a general equilibrium model related to laboratory experiments. Our notation follows Friedman and Ostroy (1995). There are two goods, a divisible 'money' and a traded good that can only be traded in indivisible units. Let $I = B \cup S$ be the set of individuals, classified as either buyers (B) or sellers (S). Each $i \in I$ is defined by a vector $(r_{i1}, \ldots, r_{ik})$, where r_{ij} indicates the reservation value for the j^{th} unit of the traded good. The parameter $k \geq 1$ indicates the maximum number of units of the traded good that an individual can buy or sell. For each $i \in B$, reservation values decrease with the quantity demanded: $r_{i1} \geq \cdots \geq r_{ik} \geq 0$. For each $i \in S$, reservation values increase with the quantity supplied $0 \leq r_{i1} \leq \cdots \leq r_{ik}$.

Each trader's utility is given by

$$u_i(q_i,m_i) = \begin{cases} \delta_i \sum_{j=1}^{|q_i|} r_{ij} + m_i & \text{if } q_i \neq 0 \\ m_i & \text{if } q_i = 0 \end{cases}, \quad \text{with} \quad \delta_i = \begin{cases} 1 & \text{if } i \in B \\ -1 & \text{if } i \in S \end{cases},$$

where $q_i \in Q_i$ is the quantity of the good traded by i and $m_i \in \Re$ are the money holdings of i. We let $Q_i = \{0, 1, \ldots, k\}$ if $i \in B$ and $Q_i = \{0, -1, \ldots, -k\}$ if $i \in S$, so that supply is described as negative demand. We assume that initial endowment of money of each individual is equal to 0; note that individuals are allowed negative money holdings.

Keeping fixed the sets of buyers and sellers and k, an economy $r \in \Re_+^{k|I|}$ is described by a set of vectors of reservation values that are weakly decreasing for each buyer and weakly increasing for each seller, as described above. Given an economy r, an *allocation* (of the indivisible good) is a vector $q = (q_i) \in \times_{i \in I} Q_i$ and an *outcome* is a vector (q,m) where q is an allocation and $m \in \Re^{|I|}$.

Denote by $\xi(r)$ the set of competitive equilibria for an economy r. A *competitive equilibrium* $(p,q) \in \xi(r)$ is a price $p \in \Re_+$ and an allocation q such that

1. (utility maximization) for each i, $u_i(q_i, -pq_i) \geq u_i(q_i', -pq_i')$ for all $q_i' \in Q_i$.

2. (market clearance) $\sum_{i \in I} q_i = 0$.

By utility maximization, if (p,q) is a competitive equilibrium for economy r, then

- for every $i \in B$, either $q_i = 0$ and $r_{i1} \leq p$, or $0 < q_i < k$ and $r_{iq_i} \geq p \geq r_{i,q_i+1}$, or $q_i = k$ and $r_{ik} \geq p$.

- for every $i \in S$, either $q_i = 0$ and $r_{i1} \geq p$, or $-k < q_i < 0$ and $r_{i|q_i|} \leq p \leq r_{i,|q_i|+1}$, or $q_i = -k$ and $r_{ik} \leq p$.

Note that (p,q) induces the outcome $(q,m) = (q, (-pq_i))$.

It is easy to prove that for any economy r, there is a competitive equilibrium. We can order the units that sellers can supply in ascending order according to their reservation values, and the units that buyers can demand in descending order according to their reservation values, to obtain the familiar supply and demand curves. Equilibrium prices and allocations can be obtained by the crossing of the supply and demand curves. As it is well-known for economies with quasi-linear preferences, the set of competitive allocations is the set of solutions to the problem of maximizing social surplus, that is

$$\max_{q \in Q} \sum_{i \in I} \sum_{0 \leq j \leq |q_i|} \delta_i r_{ij},$$

where $Q = \{q : q_i \in Q_i, \sum_i q_i = 0\}$ is the set of feasible allocations.

Trade is positive in every competitive equilibrium if and only if

$$\min_{i \in S} r_{i1} < \max_{i \in B} r_{i1}. \tag{A}$$

As we will see, an important condition for the equivalence between competitive equilibrium outcomes and the outcomes of a strategic game is that there are at least *two* trading individuals on each side of the market.

Related work on price-quantity strategic market games feature divisible commodities under the usual assumptions of continuous, increasing marginal costs for each seller, and continuous, decreasing marginal utility of consumption for each buyer. Note that in economies with divisible units active traders compete "at the margin," in the sense that in a competitive equilibrium the marginal utility of consumption and the marginal cost of production for the last unit are equated to the price for all active traders. Our results illustrate that competition at the margin is unnecessary for the equivalence between competitive and strategic outcomes.

2.3 The strategic market game

Each individual submits a price-quantity offer $(\widetilde{p}_i, \widetilde{q}_i)$ to the clearing-house, where $\widetilde{p}_i \geq 0$ and $\widetilde{q}_i \in Q_i$. Intuitively, each individual offers to trade up to $|\widetilde{q}_i|$ units of the traded good at the price $\widetilde{p}_i$.

Denote the set of feasible offers for individual i by

$$W_i = \{(\widetilde{p}_i, \widetilde{q}_i) : \widetilde{p}_i \geq 0;\ \widetilde{q}_i \in Q_i\}.$$

Given an offer profile $w \in W = \times_{i \in I} W_i$, the set of feasible allocation vectors for the clearing house is

$$Y(w) = \{(y_1, ... y_n)\colon 0 \leq y_i \leq \widetilde{q}_i,\ \text{if } i \in B; \tag{2.1}$$

$$0 \geq y_i \geq \widetilde{q}_i,\ \text{if } i \in S; \tag{2.2}$$

$$\textstyle\sum_i y_i = 0; \tag{2.3}$$

$$y_i \in \mathbb{Z}\}. \tag{2.4}$$

Note that $Y(w)$ is a finite set. Conditions (2.1) and (2.2) guarantee that trade is voluntary, i.e. individuals do not end up trading more than what they offered. Condition (2.3) ensures that the market clears and the clearing house keeps no inventory. Condition (2.4) conveys the assumption that the good is indivisible.

After the clearing house chooses an allocation $y = (y_1, \ldots, y_n) \in Y(w)$, individual i receives y_i units of the traded good and earns an amount of money equal to $-\widetilde{p}_i y_i$. We assume that the clearing house allocates trade to maximize the arbitrage profit, $\sum_{i \in I} y_i \widetilde{p}_i$, as if the clearing house buys units from the sellers and sells them to buyers at the agents' proposed prices. Thus, given an offer profile w, the resulting allocation y must satisfy

$$y \in \Pi(w) = \{y \in Y(w) : \sum_{i \in I} y_i \widetilde{p}_i \geq \sum_{i \in I} y'_i \widetilde{p}_i \text{ for all } y' \in Y(w)\}.$$

Intuitively, as in Dubey (1982), buying offers are ranked in a descending order by price while the quantities offered are accumulated to form the demand curve, and selling offers are ranked in an ascending order by price while the quantities offered are accumulated to form the supply curve.

The clearing house extracts the surplus between the supply and demand, as Figure 2.1 illustrates. That is, the clearing house chooses a competitive equilibrium allocation for a fictitious economy $\tilde{r}$ given by

$$\widetilde{r}_{ij} = \begin{cases} \widetilde{p}_i & \text{if } 1 \leq j \leq |\widetilde{q}_i| \\ 0 & \text{if } |\widetilde{q}_i| < j \leq k \text{ and } i \in B \\ +\infty & \text{if } |\widetilde{q}_i| < j \leq k \text{ and } i \in S \end{cases},$$

and appropriates the social surplus.

In scenario (a) of Figure 2.1, $\Pi(w)$ is a singleton set. To maximize the arbitrage profit, the clearing house would fulfill all demand and supply to the left of the dashed line. The dotted area is the profit for the clearing house, and the profit is positive in this case. In scenario (b), $\Pi(w)$ contains two allocations if units A and B are offered by different sellers, depending on which of the two sellers is allowed to sell the last unit. In scenario (c), the clearing house gets the same profit allocating q_1 or $q_2 > q_1$ units. Similarly, in scenario (d), buying and selling q units gives the same profit for the clearing house as making no trade.

To make trade happen whenever possible, following Simon (1984), we assume that the clearing house chooses an allocation from the set

$$\begin{aligned} F(w) = \{y \in \Pi(w) : \text{there is no } \phi \in \Pi(w) \\ \text{such that } \phi \neq y \text{ and } |\phi_i| \geq |y_i| \text{ for all } i \in I\}. \end{aligned}$$

That is, the clearing house does not choose allocations that are ray-dominated. Then in scenario (c), q_2 units will be bought and sold, and in scenario (d), q units will be traded. We still have two allocations in $F(w)$ in scenario (b) if units A and B are offered by different sellers. We assume that the clearing house chooses randomly according to the distribution μ_w that gives probability $\mu_w(y) > 0$ to each allocation $y \in F(w)$ and probability $\mu_w(y) = 0$ to every other allocation in $Y(w)$

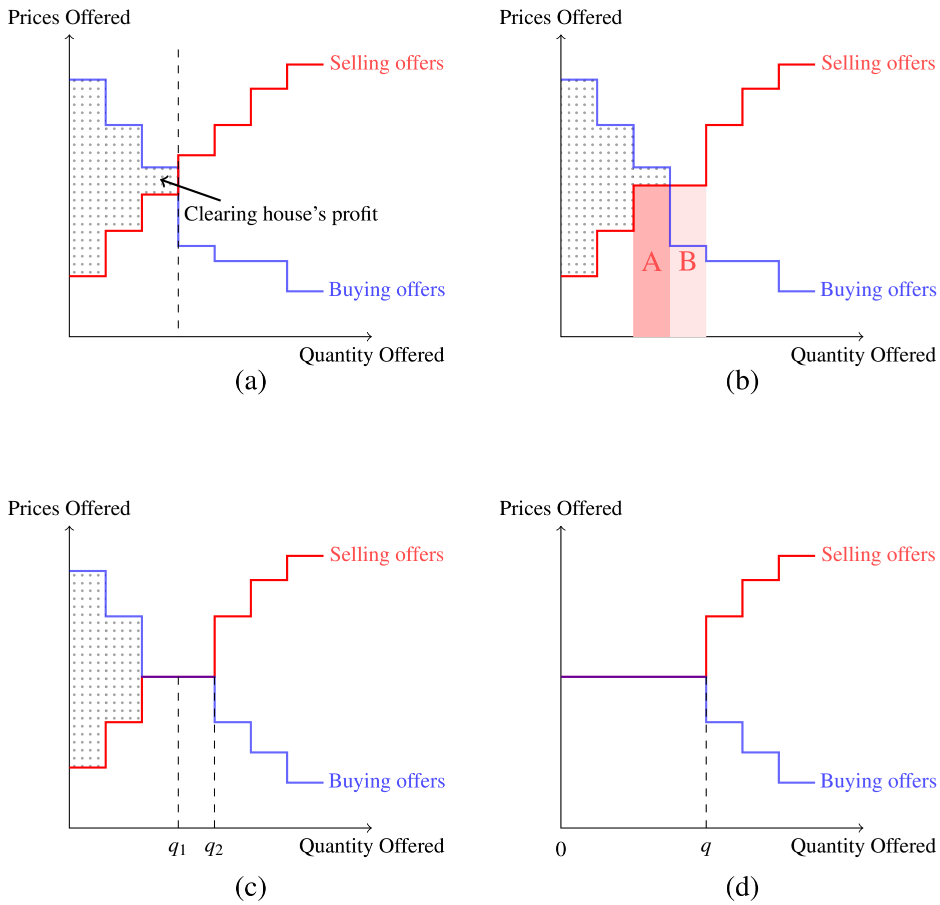


Figure 2.1: Arbitrage profit for the clearing house

such that $\sum_{y \in F(w)} \mu_w(y) = 1$. Propositions 3–8 in the Appendix provide a characterization on $F(w)$.

Given this market mechanism, define an *active trader* given offer profile w as a trader that has positive probability to trade. In other words, agent i is an *active trader* given offer profile w if there exists $y \in F(w)$ such that $y_i \neq 0$. Furthermore, denote by $AS(w)$ the set of active sellers, and $AB(w)$ the set of active buyers given offer profile w.

2.4 Nash equilibrium and competitive outcomes

Note that each offer profile $w \in W$ induces a lottery over outcomes. Each outcome $(y, (-\widetilde{p}_i y_i))$ is realized with probability $\mu_w(y) > 0$ if $y \in F(w)$, and $\mu_w(y) = 0$ if not. Given an offer profile $w \in W$, the expected utility for each trader is,

$$Eu_i(w) = \sum_{y \in F(w)} \mu_w(y) u_i(y_i, -\widetilde{p}_i y_i).$$

A (pure strategy) *Nash equilibrium* for an economy r is an offer profile $w^* \in W$ such that for every $i \in I$,

$$Eu_i(w_i^*, w_{-i}^*) \geq Eu_i(w_i', w_{-i}^*) \text{ for all } w_i' \in W_i.$$

As in other price-quantity strategic market games, every competitive equilibrium outcome can be reached by with probability one by at least one Nash equilibrium offer profile, and all the positive probability outcomes of a Nash equilibrium are competitive as long as in the Nash equilibrium there are at least two active traders on each side of the market.

Theorem 1. *For every competitive equilibrium, there is a Nash equilibrium that induces the same outcome with probability one.*

To prove the theorem, we consider an offer profile such that each agent offers the trading price and quantity she obtains in the competitive equilibrium, and show that such offer profile is a Nash equilibrium and yields exactly the same outcome as in the competitive equilibrium. Agents have no incentive to deviate from the proposed offer profile: since the quantity offered in the profile is utility-maximizing given the competitive price, obtaining a different quantity at the same price does not increase the payoff for the individual; given other agents are offering the same price, increasing offer price as a seller or decreasing offer price as a buyer, regardless of the quantity offered, reduce the chance of trade to 0, and thus cannot be payoff-improving; decreasing offer price as a seller or increasing offer price as a buyer reduces the payoff for sure as the new price is less preferred to the competitive price, even at its corresponding utility-maximizing quantity.

As long as condition (A) is satisfied (which is, of course, the case of interest), there are Nash equilibria that induce noncompetitive allocations. For instance, any offer profile such that $\widetilde{q}_i = 0$ for all i, or such that $\min_{i \in S} \widetilde{p}_i > \max_{h \in B} r_{h1}$ and $\max_{i \in B} \widetilde{p}_i < \min_{h \in S} r_{h1}$, is a Nash equilibrium. Those Nash equilibria result in no trade. We restrict our attention on Nash equilibria such that trade happens with positive probability, so that $AS(w)$ and $AB(w)$ are nonempty. We have

Theorem 2. *In every Nash equilibrium with at least two active traders on each side, every positive probability outcome is competitive.*

To prove theorem 2, we first show that in any given Nash equilibrium, all active traders offer the same price. Then we show that in every allocation induced by a Nash profile, the quantity that an active trader is allocated is utility-maximizing given the Nash price. The intuition is that if an active buyer/seller does not get the utility-maximizing quantity at the Nash price, the buyer/seller can always obtain a more preferable quantity by offering a slightly higher/lower price.

Note that there is a gap between the statement of theorem 2 and the no-trade examples preceding the statement of the theorem. Theorem 2 leaves open the possibility that there are Nash equilibria with active trading but with noncompetitive outcomes and in which there is only one active trader in at least one of the two sides of the market. In the proof of the theorem, we rely on two or more active sellers in order to show that there is no Nash equilibrium in which one seller produces less than the competitive allocation requires. Intuitively, these situations would correspond to the single active seller behaving as a monopolist and charging a price above the competitive level. Similarly, there could be situations in which there is a single active buyer behaving as a monopsonist and charging a price below the competitive level. Finally, there could be situations in which there is a single active buyer and a single active seller, and competitive outcomes are not reached even if the price is competitive because of a coordination failure: both the buyer and the seller offer suboptimal quantities.

In what follows, we provide necessary and sufficient conditions for all the outcomes of *every*

Nash equilibrium with trade to be competitive. Define the *buyers' marginal value*, v_b, as the maximum of the lowest reservation value for buyers' units traded in competitive equilibria, that is,

$$v_b = \max_{(p,q)\in\xi(r)} \min_{q_i>0} r_{i,q_i}.$$

Similarly, define the *sellers' marginal value*, v_s, as the minimum of the highest reservation value for sellers' units traded in competitive equilibria, that is,

$$v_s = \min_{(p,q)\in\xi(r)} \max_{q_i<0} r_{i,|q_i|}.$$

In economies such that (A) is satisfied, v_b and v_s are well-defined, since in every competitive equilibrium at least some $i' \in S$ with the minimum cost (i.e. $r_{i'1} = \min_{i\in S} r_{i1}$) must have $q_{i'} < 0$, and at least some $i'' \in B$ with the maximum reservation value (i.e. $r_{i''1} = \max_{i\in B} r_{i1}$) must have $q_{i''} > 0$. As shown in the Appendix, v_b and v_s are equal, respectively, to the lowest reservation value of buyers' traded unit(s) and the highest reservation value of sellers' traded unit(s) in any competitive equilibrium with the smallest number of transactions. Moreover, if (A) is satisfied, we must have $v_b > v_s$, because if there is a competitive equilibrium such that both the marginal buyer and the marginal seller are indifferent (i.e. $\min_{q_i>0} r_{i,q_i} = p = \max_{q_i<0} r_{i,|q_i|}$), there is another competitive equilibrium in which one fewer unit is traded.

Denote by $\overline{p}$ and $\underline{p}$ the highest and lowest competitive price respectively. It is easy to check that

$$v_s \leq \underline{p} \leq \overline{p} \leq v_b.$$

The first and third inequalities above follow from the fact that for every equilibrium $(p,q) \in \xi(r)$ we must have $\max_{q_i<0} r_{i,|q_i|} \leq p \leq \min_{q_i>0} r_{i,q_i}$.

We say that $i \in B$ is an *inframarginal buyer* if $r_{i1} \geq v_b$. Similarly, we say that $i \in S$ is an *inframarginal seller* if $r_{i1} \leq v_s$. Intuitively, an inframarginal trader is someone who is willing to trade at every competitive equilibrium price. Note that in economies satisfying (A), there is at least one inframarginal trader on each side of the market, since every seller with the minimum cost and every buyer with the maximum reservation value is inframarginal.

We say that $i \in B$ is a *weakly inframarginal buyer* if $r_{i1} > v_s$ and $r_{i1} \geq \underline{p}$. Similarly, we say that $i \in S$ is a *weakly inframarginal seller* if $r_{i1} < v_b$ and $r_{i1} \leq \overline{p}$. Intuitively, a weakly inframarginal trader is someone who would generate positive social surplus if matched in pairwise trade with an inframarginal trader on the other side of the market. Using $v_b > v_s$ and $v_s \leq \underline{p} \leq \overline{p} \leq v_b$, it is easy to check that, in economies satisfying (A), all inframarginal traders are also weakly inframarginal (justifying our nomenclature).

If an economy has competitive equilibria in which only one unit is traded, then all outcomes of every Nash equilibrium profile with trade are efficient.[4] From here on, we focus on economies such that all competitive equilibria involve trading two or more units, which is a more demanding condition than (A).

We have

Theorem 3. *In economies such that all competitive equilibria involve trading two or more units, every positive probability outcome from every Nash equilibrium with active trade is competitive if and only if there are at least two inframarginal traders on one side of the market, and at least two weakly inframarginal traders on the other side.*

Intuitively, rivalry between two traders on the same side of the market who can exploit mutually advantageous trades with at least two traders on the other side of the market both eliminates monopoly and monopsony power and precludes coordination failures. In the coordination failure example proposed above, we have $v_s = 1$, $v_b = 3$, and all traders are weakly inframarginal but only one seller and one buyer are inframarginal.

The condition $r_{i1} \geq \underline{p}$ for $i \in B$ and $r_{i1} \leq \overline{p}$ for $i \in S$ to be a weakly inframarginal trader ensures that the trader has a value "close enough" to the competitive range, so that the trader weakly prefers to trade in the competitive equilibrium. Without this condition, there may be noncompetitive Nash

[4]The reason is that if one unit is traded in a given outcome induced by an equilibrium profile, both the active buyer and the active seller must be offering the same price. If any buyer has a reservation price higher than the Nash price and is not trading, the buyer can offer a price that is slightly higher and grab the trade, so that in equilibrium the buyer who trades must be the one with the highest reservation price. A similar argument applies on the supply side. (However, the trading price may not be competitive in the Nash equilibrium.)

equilibrium outcomes. Consider the economy $S = \{1,2\}$, $B = \{3,4\}$, $k = 3$, and $r_{11} = r_{21} = 1$, $r_{12} = r_{13} = r_{22} = r_{23} = 4$, $r_{31} = r_{32} = r_{33} = 3$, $r_{41} = r_{42} = r_{43} = 1$. Seller 1, seller 2, and buyer 3 are inframarginal traders, while buyer 4 satisfies one of the conditions to be weakly inframarginal but not $r_{i1} \geq \underline{p}$. Here buyer 4 strictly prefers not to trade in the competitive equilibrium, and the range for Nash equilibrium prices is $[2,3]$, including prices that are not competitive.

It is worth noticing that theorem 3 includes the contestable market scenario (Baumol et al., 1982), in which there is only one active seller but all outcomes from Nash equilibria are competitive. An example is the economy $S = \{1,2\}$, $B = \{3,4\}$, $k = 2$, and $r_{11} = r_{12} = r_{21} = r_{22} = 2$, $r_{31} = 4$, $r_{41} = 3$, $r_{32} = r_{42} = 1$. The competitive equilibrium price is 2 in this economy, and two units are traded in every competitive equilibrium. We have $v_b = 3$ and $v_s = 2$ for this economy, so all traders are inframarginal and the condition in theorem 3 holds. One of the Nash equilibria in this economy is $w = ((2,-2),(2,0),(2,1),(2,1))$, in which seller 1 is the only active seller, but the outcome is competitive. The presence of seller 2, a non-active seller in the Nash equilibrium, brings enough competition to the market to make the outcome competitive.

2.5 Experimental design and hypotheses

2.5.1 Experimental design

We test the predictive ability of our market game model in laboratory experiments. We consider two markets with indivisible commodities. Each market has two buyers $B = \{B1, B2\}$ and two sellers $S = \{S1, S2\}$, and each trader can either buy or sell two units. We assign the first and third highest demand reservation values to one buyer, and the second and fourth to the other buyer. By assigning the units to sellers in different ways, we create a market that satisfies the condition in theorem 3, and a market that does not. This design is similar to one implemented by Davis and Holt (1994).

In our competitive market, the two supply units that can be traded in competitive equilibrium are assigned each to each one of the two sellers. Thus, as shown in the left part of figure 2.2, there are two inframarginal traders on each side of the market. By theorem 3, Nash equilibrium outcomes with trading of the strategic market mechanism coincide with competitive equilibrium outcomes.

That is, both units with lowers costs should be traded, and the price should be in the competitive price range, \$15–\$19. Correspondingly, efficiency (as percentage of the maximum possible surplus) should be 100%.

In our monopoly market, instead, the two low cost units are assigned to the same seller, as shown in the right part of figure 2.2. The set of Nash equilibrium outcomes with trade includes the set of competitive equilibria just described, as well as monopolistic market equilibria in which only the unit with the lowest cost is traded, and the price is between \$19 and \$30. Efficiency of monopolistic equilibria is $(32-2)/(32-2+19-15)$, that is approximately 88%.

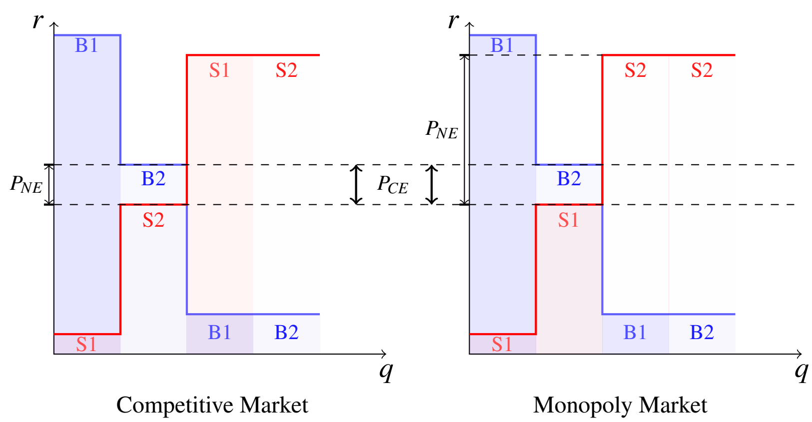


Figure 2.2: Competitive and monopolistic markets

In the experiment, we compare the performance of our strategic market mechanism (Clearing House, or CH hereafter) with the continuous time double auction (Double Auction, or DA hereafter), in the two markets.

In the clearing house institution, each trader submits a price-quantity pair to the clearing house. The clearing house then decides trade by the rules described in section 2.3, and reports the trader's own transaction price and quantity, together with the price and quantity traded in the market. We let $\mu_w(y) = 1/|F(w)|$ for all $y \in F(w)$ in the experiment; that is, the clearing house assigns equal probability to all arbitrage-maximizing allocations. When making decisions, traders are given their own values, but not other traders' values or offers.

In the double auction institution, the traders buy/sell the good unit by unit. Each trader can submit limit offers for one unit, and each limit offer has to reduce the bid-ask spread to be valid. Valid offers are listed on the screen as public information for all traders in the market, with bids ranked from high to low and asks ranked from low to high. A transaction happens automatically if a valid bid is no lower than a valid ask. In each transaction, a bid will always be matched with the highest-ranked ask, and an ask will be traded with the highest-ranked valid bid. The trading price will be the price in the pair that was submitted later. After a trader has the first unit traded, he/she can submit limit offers for the second unit. All valid offers and transactions are shown in real time to all traders in the market.

The experiment was conducted in the Interdisciplinary Center for Economic Science (ICES) lab in George Mason University. In total, 240 subjects participated in the 18 sessions, and each session lasted for no more than 100 minutes.[5] Each subject participated in only one treatment, playing the same role (B1, B2, S1 or S2) in the same market for 20 rounds. The final payoffs ranged from \$5 to \$36. The average payoff is \$11.25 including a \$5 show-up fee.

The experiment was computerized, and programmed in oTree (Chen et al., 2016). At the beginning of the session, the participants were seated at partitioned computer stations and allowed 10 minutes to read the instructions on their own. Then an experimenter read the instructions out loud to all participants. Afterward, a quiz was handed out, and the experiment began after each participant gave correct answers to all the questions in the quiz.[6] Then the role a participant had in the experiment was revealed to him/her, and the participants were given a practice round before the formal rounds began. There were 20 formal rounds, one of which was randomly chosen for payment. After the 20 formal rounds, each participant was informed of the round chosen for payment and his/her own payoff. The payment was made privately.

[5] For the CH competitive treatment, there were three sessions with 16 and one session with 20 participants; for the CH monopoly treatment, there were four sessions with 16 participants; for the DA competitive treatment, there was one session with 8 and three sessions with 16 participants; and for the DA monopoly treatment, there were five sessions with 8 and one session with 12 participants.

[6] Instructions and quizzes for each treatment are provided in the online supplementary material.

2.5.2 Hypotheses

Our first set of empirical hypotheses correspond to treatment effects. Because the set of equilibria under monopolistic conditions includes inefficient outcomes with prices above competitive levels, we expect treatments with competitive markets to exhibit lower prices and higher efficiency, together with higher trading volume, higher surplus for buyers and lower surplus for sellers. And given the advantage for learning of the double auction institution over the clearing-house, we expect treatments under the double auction institution to exhibit higher efficiency.

(H1) Under the double auction institution, prices and sellers' total surplus are lower, and efficiency, trading volume and buyers' total surplus are higher, in competitive markets than in monopolistic markets.

(H2) Under the clearing-house institution, prices and sellers' total surplus are lower, and efficiency, trading volume and buyers' total surplus are higher, in competitive markets than in monopolistic markets.

(H3) In competitive markets, efficiency under the double auction institution is higher than under the clearing-house institution.

(H4) In monopolistic markets, efficiency under the double auction institution is higher than under the clearing-house institution.

Our second set of empirical hypotheses correspond to the convergence to competitive prices in the long run if the market has a competitive structure under both institutions.

(H5) Under the double auction institution, prices converge to the competitive range in competitive markets.

(H6) Under the clearing house institution, prices converge to the competitive range in competitive markets.

Our next set of empirical hypotheses correspond to predictive success of equilibrium notions. Because the set of equilibria under monopolistic conditions is a strict superset of the set of equilibria under competitive conditions, we expect competitive predictions to perform better in the latter case. And given the advantage for learning of the double-oral auction over the clearing-house, we expect Nash predictions to perform better in in the former case.

(H7) Competitive predictions perform better in competitive markets than in monopolistic market under the double-oral auction.

(H8) Competitive predictions perform better in competitive markets than in monopolistic markets under the clearing-house auction.

(H9) Competitive predictions perform better under the double-oral auction than under the clearing-house in competitive markets.

(H10) Nash predictions perform better under the double-oral auction than under the clearing-house in monopolistic markets.

2.6 Results

2.6.1 Treatment effects

Overview

Table 2.1 presents treatment effects using the last ten rounds. Efficiency is defined as the percentage of the maximum social surplus realized. Trading volume is defined as the number of units traded divided by two (the number of inframarginal units), in percentage. Buyers' and sellers' surplus are defined as percentage of the maximum possible social surplus. In agreement with H1, under the double auction institution, trading prices are lower, and efficiency, trading volume, and buyers' are significantly higher in competitive markets than in monopolistic markets. Average sellers' surplus is higher in monopolistic markets but the difference is not significant. In agreement with H2, under the clearing-house institution, trading volume and buyer's surplus are significantly higher in competitive markets than in monopolistic markets. Differences in trading prices, efficiency, and sellers' surplus are not significant. In agreement with H3, in competitive markets, efficiency, trading volume, buyers' surplus, and sellers' surplus are higher under the double auction than under the clearing house

Table 2.1: Treatment effects

			Trading price	Efficiency (%) [a]	Trading volume (%) [b]	Buyers' surplus (%) [c]	Sellers' surplus (%) [c]
Mean values	CH Competitive	(17 markets)	15.73	69.45	58.53	36.32	30.81
			(2.70)	(32.39)	(28.80)	(21.33)	(17.62)
	CH Monopoly	(16 markets)	15.78	71.95	52.19	31.36	32.51
			(4.60)	(30.17)	(25.30)	(21.02)	(18.66)
	DA Competitive	(14 markets)	16.89	88.55	80.36	44.37	44.18
			(2.28)	(22.77)	(26.62)	(15.88)	(15.08)
	DA Monopoly	(13 markets)	18.18	75.07	57.31	33.06	42.00
			(5.41)	(33.57)	(31.81)	(21.27)	(24.05)
Kruskal-Wallis test			$<.001$	$<.001$	$<.001$	$<.001$	$<.001$
Mann-Whitney U test [d]	Competitive vs Monopoly	CH	.951[-]	.420[+]	.015[+]	.017[+]	.476[-]
		DA	$<.001$[-]	$<.001$[+]	$<.001$[+]	$<.001$[+]	.290[-]
	Clearing House vs Double Auction	Competitive	$<.001$[-]	$<.001$[-]	$<.001$[-]	.008[-]	$<.001$[-]
		Monopoly	$<.001$[-]	.011[-]	.051[-]	.722[-]	$<.001$[-]

Note: Data includes only the last 10 rounds. Standard deviations in parentheses. p-values are reported for statistical tests.

[a] Percentage of maximum social surplus realized. Efficiency = sellers' surplus + buyers' surplus + clearing house's profit.

[b] Number of unit traded in the market over two – the number of units traded in competitive equilibrium.

[c] As a percentage of maximum social surplus.

[d] $[+]$ and $[-]$ signs indicate the tested hypothesis being the difference is positive or negative.

institution. In agreement with H4, in monopolistic markets, efficiency, trading volume, and sellers' surplus are higher under the double auction than under the clearing house institution. Average buyers' surplus is higher under the double auction but the difference is not significant. Summing up, there is significant evidence in favor of H1 and H3, and some evidence in favor of H2 and H4.

Prices

Figure 2.3 shows average trading prices in each round in the four treatments. Two inferences can be drawn from figure 2.3. First, average prices adjust over time and stay in the competitive price range in the second half of the experiment in all treatments (average trading prices range from \$15.73 to \$18.18). The learning process takes longer under the clearing-house institution: the average price starts low, and reaches the competitive range over time. The upward sloping trend is not as strong under the double auction institution: the average trading price starts within the competitive range. Second, compared to competitive markets, monopolistic markets bring forth a higher average trading price under the double auction institution, but not so clearly under the clearing-house institution.

Efficiency

Figure 2.4 plots the average efficiency in each round in the four treatments. Efficiency is defined as the percentage of the maximum social surplus realized. Similar to what is shown in figure 2.3, learning takes longer under the clearing-house institution; hence, average efficiency under the clearing-house institution presents a stronger upward trend over time. Under the clearing-house institution, the average efficiencies start at levels lower than under the double auction institution, and remain statistically lower in the second half of the experiment. Nevertheless, we can observe from figure 2.3 that the upward trend of the efficiencies in clearing-house treatments persist over time, and at the end of the experiment, the efficiency levels from the two institutions are close.

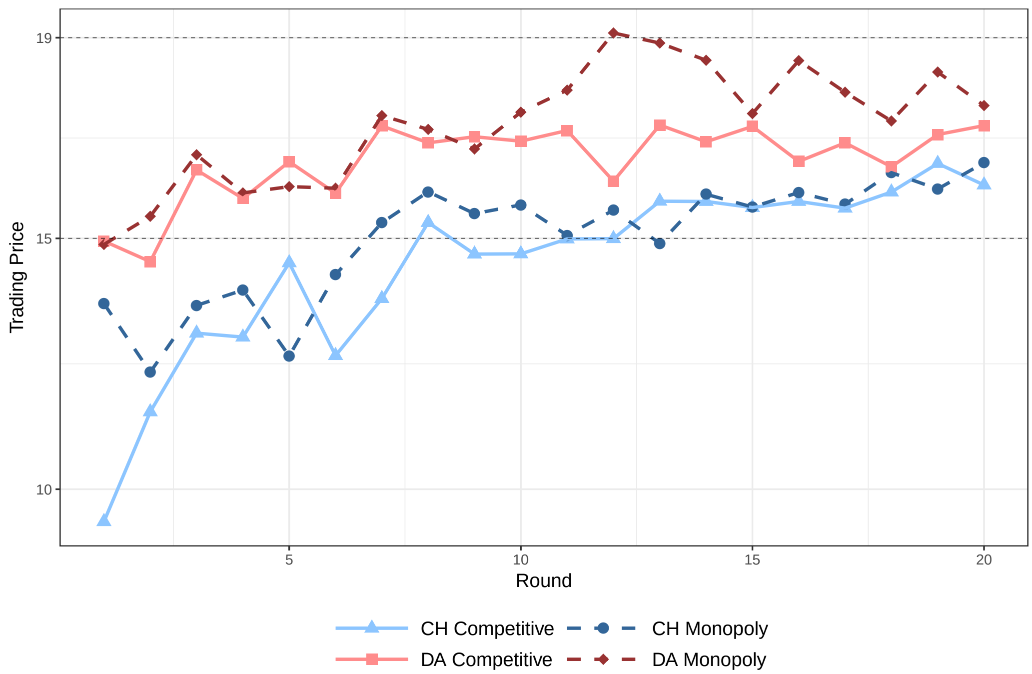


Figure 2.3: Average trading price

Trading volume

In our setting, supramarginal trade occurs if a seller sells a unit with a cost of 30, or if a buyer buys a unit with a valuation of 4. In our experiment, in the last ten rounds, supramarginal trade occurred in 3 out of 199 trades in the CH Competitive treatment and in 4 out of 149 trades in the DA Monopoly treatment, and did not occur in other treatments. Thus, trading volume reflects inframarginal trading. Figure 2.5 and table 2.1 illustrate that in the second half of the experiment, under both institutions, there are fewer trades in the monopolistic markets than in the competitive markets. Under the double auction, lower trading in monopolistic markets explains the advantage of competitive markets in terms of social efficiency and corroborates our hypotheses H1 and H2. As figure 2.5 and table 2.1 show, the clearing-house institution results in less trade than the double

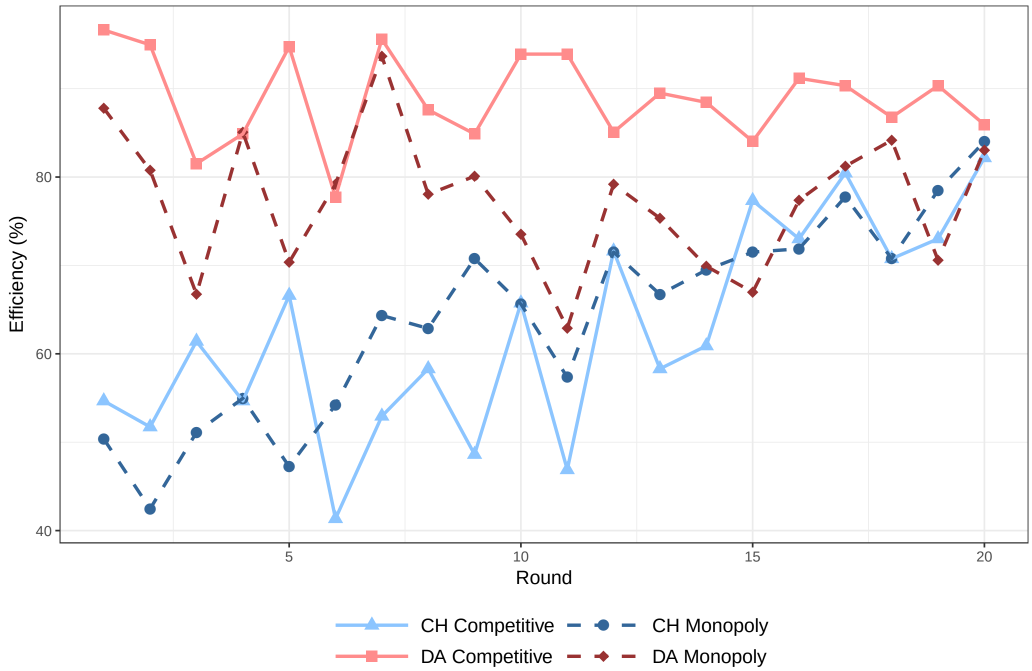


Figure 2.4: Average efficiency

auction institution, corroborating our hypotheses H3 and H4.

2.6.2 Convergence to competitive prices

Following Noussair et al. (1995), we estimate

$$p_{it} = \alpha_1 D_1 \frac{1}{t} + \ldots + \alpha_i D_i \frac{1}{t} + \ldots + \beta \frac{t-1}{t} + \varepsilon_{it}$$

for each treatment, where p_{it} is the average price in market i at round t, D_i is an indicator for a specific market, which equals 1 if the market is i and 0 otherwise, β is the asymptote for the average price in the treatment, and ε_{it} is an error term. In using this statistical model, we assume

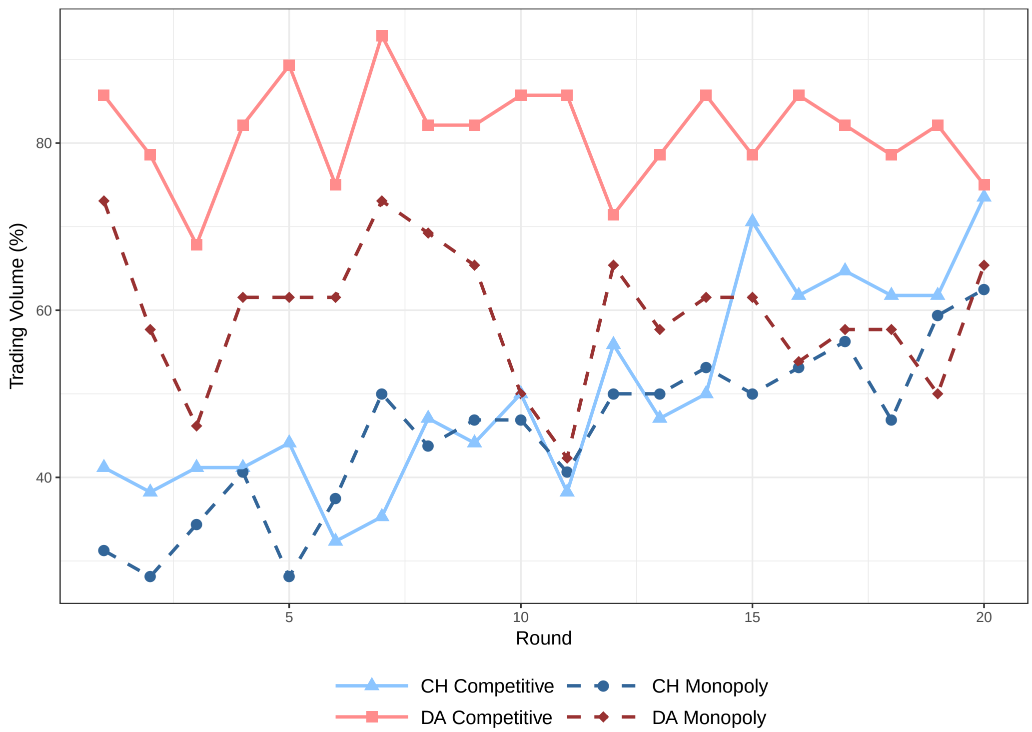


Figure 2.5: Average trading volume

that although each market has its own pattern of convergence, there is a common asymptote by treatment.

Table 2.2 lists the estimated β for each treatment. For competitive markets, the 95% confidence interval for long run prices is contained in the competitive price range under both the double auction and the clearing-house institution, providing corroborating support for H5 and H6. For monopolistic markets, the 95% confidence interval for long run prices is contained in the competitive price range for the clearing-house institution but not for the double auction. In fact, the confidence intervals are nested under the clearing-house institution but are disjoint under the double auction.

Figure 2.6 shows the distribution of trading prices in the last 10 rounds in different treatments. The DA Monopoly treatment has a heavy right tail outside of the competitive price range but within

Table 2.2: Average price asymptote

Treatment	$\hat{p}^*$	95% Confidence interval
CH Competitive	16.12	$[15.68, 16.57]$
CH Monopoly	16.02	$[15.90, 16.14]$
DA Competitive	16.98	$[16.65, 17.32]$
DA Monopoly	19.20	$[18.51, 19.90]$

Feasible generalized LS estimation with AR(1) correction.

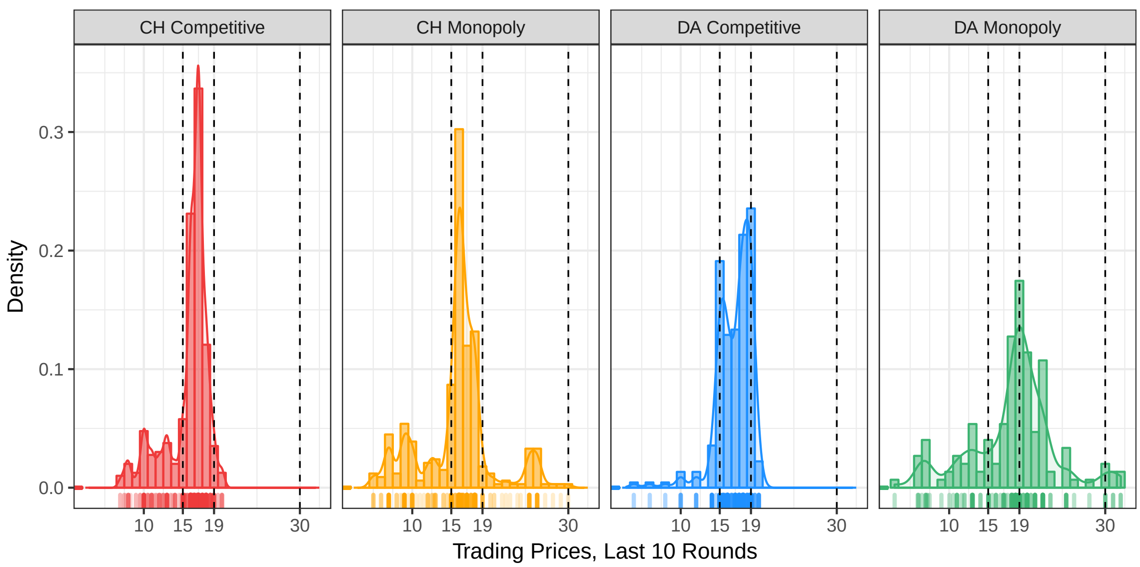


Figure 2.6: Distribution of trading prices

the Nash equilibrium range for that environment. In the CH Monopoly treatment, instead, most of the trading price within the Nash equilibrium range is also in the competitive price range. In both competitive treatments, trading prices cluster in the competitive price range.

Table 2.3: Predictive success index

	Predicted price range	
	Competitive ($15–$19)	Competitive + Monopolistic ($15–$30)
CH Competitive	**63.81%**	28.89%
DA Competitive	**75.11%**	42.44%
CH Monopoly	52.54%	**26.35%**
DA Monopoly	26.27%	**26.51%**

Except for the DA Competitive treatment, all treatments have a heavy left tail. The heavy left tail may be due to slow learning, due to (i) lack of within-round feedback under the clearing-house institution, and (ii) less experimentation about possible prices when there is only one rather than two inframarginal sellers. Prices below the competitive equilibrium level were also observed by Smith and Williams (1990) in two monopolistic markets, perhaps for a similar reason.

2.6.3 Predictive success

To explore whether Nash equilibrium is a good predictor for the experimental results, we use Selten (1991) predictive success index. Selten's index is defined as the difference between the 'hit rate' (the percentage of data that is coherent with the prediction of the model) and the 'area' (the percentage of all possible outcomes that is coherent with the prediction of the model). Nash equilibrium predicts the range of competitive prices ($15–$19) for the competitive environment, and the range including both competitive prices and monopolistic prices ($15-$30) for the monopolistic environment. Given that participants cannot submit a price that may cause a loss, the possible price range in our experiment is $2–$32. Thus, the area equals 13.33% for the competitive range, and 50% for the combination of competitive and monopolistic price ranges.

Tables 2.3 summarizes the predictive success index of the two price ranges. The indices for Nash equilibrium are in bold. In agreement with our hypotheses H7 and H8, the competitive price range is a better prediction for competitive markets than in monopolistic markets, under both institutions and regardless of the index. In agreement with hypothesis H9, the competitive price range is a better prediction for competitive markets under the double auction than under the clearing-house institution. Opposite to hypothesis H10, Nash predictions perform similarly in monopolistic markets under both institutions. In fact, under the clearing-house institution, the competitive price range predicts better than the Nash range for monopolistic markets. Overall, predictive success indices indicate that learning to play equilibrium happens more easily in competitive markets, especially under the dynamic institution.

2.7 Conclusions

In this paper, we aim to fill a gap in the theoretical and experimental literature about markets with few participants and indivisible commodities. First, we provide a necessary and sufficient condition for the equivalence of Nash equilibria of price-quantity strategic market games and competitive equilibrium outcomes. Second, we conduct market experiments in a competitive environment and in a monopolistic environment. We consider two market institutions, a sealed-bid auction (call market), following closely the rules of the market game, and a double-oral auction, which has been known to be successful in inducing competitive outcomes and prices in the lab.

Our lab experiments involve the minimum number of traders using the double auction that we know of. Figure 2.7 compares the efficiency level in our double auction markets with a few double auction markets in previous studies (Friedman and Ostroy, 1995; Kachelmeier and Shehata, 1992; Kimbrough and Smyth, 2018; Smith, 1982; Smith and Williams, 1990; Smith et al., 1982). Double auction markets conducted in previous studies are mostly used for testing the robustness of the mechanism, so disturbances may have been introduced during the session, and different settings have been used in these studies. Efficiency in thicker markets is higher than in our four-trader market, although the difference is not large when markets are competitive.

Under the call market institution, efficiency is below that under the double auction in our experimental competitive markets. We interpret the advantage of the double auction as a result of better

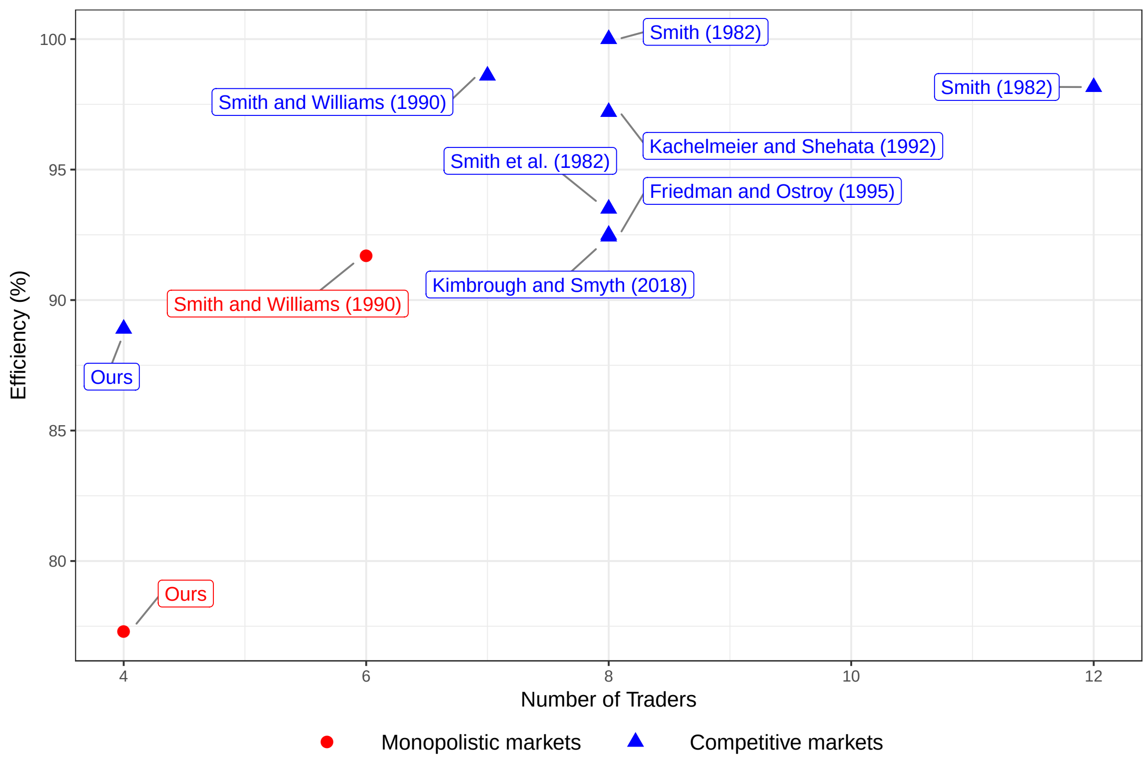


Figure 2.7: Efficiency and number of traders in the double-oral auction literature. All the points are average efficiency using all rounds.

opportunities for learning. Nevertheless, the efficiency of the call market increases over time and gets closer to the double auction institution as traders in the market gradually learn. The approximation to competitive equilibrium outcomes is obtained without traders' knowledge of others' values under both institutions. Our results provide supportive evidence for the Hayek hypothesis (Hayek, 1945; Smith, 1982) in a limit setting with few traders: using appropriate institutions, markets can work with very limited information. Under the call market institution, transaction prices are the only information revealed to each trader other than their own value. This information appears to be sufficient for achieving equilibrium outcomes, although it may take a few trials.

In our experimental monopolistic markets, buyers' surplus and trading volume remains below that in our experimental competitive markets under both the double auction and the call market. The loss of total surplus in monopolistic markets is significant under the double auction although not under the call market. Tantalizingly, under the call market, prices are not in average higher in monopolistic than in competitive markets. Whether these observations about long-run prices can be generalized is left as an open question. Generally, Nash predictions from our strategic model do much worse in monopolistic markets. Learning enough to behave as if possessing complete information is seemingly much harder in monopolistic than in competitive markets.

Chapter 3: A Survey on k-double Auctions

3.1 Introduction

The k-double auction is a trading mechanism applicable to markets of any size. In the simplest setting, each trader has one unit of demand or supply over an indivisible commodity, the value of which is private information. In a k-double auction, traders submit limit orders to the clearing house privately, who then determines a single trading price that clears the market, using a convex combination of the marginal bid(s) and/or ask(s), and executes the orders. Studies on k-double auction began with the one buyer and one seller setting, in which k is the weight of the buyer's bid in the trading price. Later studies on multilateral scenario inherited this notion, and used k to denote the weight the upper bound of market-clearing prices has in the trading price.

A k-double auction is a sealed-bid uniform price call market—orders are executed at the designated time according to a price that maximizes reported social surplus at that moment, and no feedback of the order flow is provided before the market is called. While the continuous double auction is prevalently used in stock exchanges where a large number of traders are presented, the k-double auction is widely used to organize thin markets. Examples include trading quotas for agricultural goods, and wholesale electricity markets. When there are few traders in the market, executing orders frequently results in very few orders at each call. The k-double auction induces a thicker market than the continuous double auction by accumulating orders over a period of time before executing them.

The multiplicity of equilibria from a k-double auction with $k \in (0,1)$ mirrors the indeterminacy of outcomes in multilateral bargaining games. In an incomplete information setting, as the market size increases, equilibrium strategies in a k-double auction converge to truth-telling, verifying Edgeworth's insight that increasing the number of traders resolves the indeterminacy of bargaining (Rustichini et al., 1994).

In an incomplete information setting, equilibrium outcomes in the k-double auction can range from zero efficiency to full efficiency. As a result of convergence to truth-telling strategy, the mechanism features diminishing expected inefficiency as the market grows. The k-double auction provides a model for price formation, and addresses how the competitive equilibrium can be reached without the price-taking assumption. The asymptotic result of this non-tatonnement process provides a support to the use of Walrasian model as an asymptote of market outcomes in the absence of complete information (Wilson, 1985).

In terms of the speed of convergence to efficient outcome as the market grows, the k-double auction has the same order as the optimal mechanism in Gresik and Satterthwaite (1989). Evaluated in their least favorable trading environments for each possible size of the market, among plausible mechanisms, the k-double auction is shown to force the worst-case inefficiency to zero at the fastest possible rate (Satterthwaite and Williams, 2002).

The rest of the paper is organized as follows. Section 3.2 provides a brief history of the k-double auction. Section 3.3 and 3.4 describe the classic environment and a few methods to find Bayesian Nash equilibria. Section 3.5 discuss the efficiency of the mechanism under the settings in section 3.3. Section 3.6 contains applications of k-double auction in experimental economics. Section 3.7 lists alternative settings in the literature and related mechanisms. Section 3.8 concludes.

3.2 A brief history of the k-double auction

Harsanyi and Selten (1972) first studied the two person bargaining problem under incomplete information, applying the concept of Bayesian Nash equilibrium to a bargaining situation with fixed threats. Following a suggestion by Howard Raiffa, Chatterjee and Samuelson (1983) first proposed the k-double auction as a mechanism for bilateral bargaining under incomplete information. Chatterjee and Samuelson (1983) characterized a class of equilibria, and identified a set of equilibria for uniformly distributed independent redemption values, in which strategies are linear functions of redemption values. When the values are independently uniformly distributed between 0 and 1, the $k = 0.5$ double auction has a linear equilibrium described by Chatterjee and Samuelson (1983) that obtains the highest expected gains from trade among all mechanisms that satisfy individual rationality (Myerson and Satterthwaite, 1983).

Satterthwaite and Williams (1989a) further studied bilateral k-double auction. They focused on differentiable equilibrium strategies, which include but are not limited to the linear case. In particular, they used a set of differential equations to show both existence of equilibrium and a numerical way to construct it. Leininger et al. (1989) demonstrated the existence of two types of equilibria with $k = 0.5$ in the bilateral setting with uniformly distributed independent values: the ones in which strategies are differentiable functions of values, and ones in which strategies are step functions. As pointed out in Leininger et al. (1989), “the equilibria in the uniform distribution case range from second best to worthless, so that equilibrium theory provides no basis for recommending the sealed-bid mechanism in practice.” Results from laboratory experiment in Radner and Schotter (1989) alleviate this concern by showing that in such settings, subjects tend to apply linear strategies.

The mechanism was later on adapted to markets with multiple buyers and multiple sellers. Wilson (1985), in particular, proved that the k-double auction is interim incentive efficient when the market is sufficiently large. For redemption values drawn from the unit interval, Rustichini et al. (1994) characterized equilibria with positive trading opportunity and non-dominated strategies for market with equal numbers of buyers and sellers. They provided a characterization of equilibrium strategies using first order conditions, as well as a convergence rate to truth-telling and efficient outcome. Satterthwaite and Williams (2002) show that k-double auction is worst case asymptotic optimal among all interim individually rational and ex ante budget balanced mechanisms.

Kadan (2007) provided sufficient conditions for the existence of an increasing equilibrium in the bilateral setting when private values are affiliated, and Fudenberg et al. (2007) proved the existence of a pure strategy, symmetric, increasing equilibrium with correlated, conditionally independent private values in large markets. Proofs for existence of equilibrium in more general settings can be found in Cripps and Swinkels (2006); Jackson and Swinkels (2005); Reny and Perry (2006).

The mechanism is called buyer’s bid double auction (BBDA) when $k = 1$, since the price is determined by the buyer in the bilateral case. In the multilateral setting, the price may be determined by an ask as well. With truth-telling being the dominant strategy, this mechanism eliminates the strategic behavior on the seller side in the equilibrium, hence simplifies the analysis. Satterthwaite and Williams (1989b) studies the rate of convergence to efficient outcome as the market grows. Williams (1991) extended it to the market of unequal number of buyers and sellers, and proved the existence of equilibria. Zacharias and Williams (2001) further showed that when the number of buyers is arbitrarily larger than the number of sellers, the order of expected inefficiency of BBDA is consistent with the results from Rustichini et al. (1994). Satterthwaite et al. (2018) and Satterthwaite et al. (2019) extended the analysis to interdependent redemption values settings.

Satterthwaite and Williams (2018) surveys the theory on k-double auction until 1991, and discusses possible contributions experimental economists can make in this area. A handout for literature on k-double auction can be found at Williams (2016).

3.3 Basics

There are two goods in the economy, the divisible money an indivisible commodity. There are $(m+n)$ traders, each assigned a redemption value for one unit an indivisible commodity. m of the traders are buyers (B), and the rest are sellers (S). Each trader $i \in S$ is endowed with one unit of the commodity. Traders $i \in B$ are not endowed with the commodity. Denote by $G(v)$ and $F(c)$ the cumulative distribution function for a buyer's redemption value v_i, which is called a "value", and a seller's redemption value c_i, which is called a "cost". For now we focus on the scenario in which redemption values are independent. Extensions to interdependent values and multiple units per trader are discussed in section 3.7. Redemption values are private information. A buyer's payoff is zero if the buyer does not make a purchase, and is $v_i - p$ if the buyer buys a unit at the market price p. Analogously, a seller who does not sell has zero payoff, and the payoff is $p - c_i$ if the seller sells one unit at the market price p.

A clearing house solicits a bid from each buyer and an ask from each seller, and determines a unit price p for the market after receiving bids and asks. A bid indicates the highest price a buyer is willing to pay for the unit; an ask indicates the lowest price a seller is willing to sell the unit at. Rank the bids and asks from low to high, and denote by $e_{(j)}$ the jth lowest order price. In a k-double auction where $k \in [0,1]$, the clearing house sets the uniform price at $p = (1-k)e_{(m)} + ke_{(m+1)}$.

Buyers who bid higher than p and sellers who ask lower than p trade with probability one. If tied bids or asks at the price p cannot be fulfilled, the clearing house uses a lottery to determine the allocation of transaction on the long side.

The interval $[e_{(m)}, e_{(m+1)}]$ is the range of market clearing price given the indicated supply and demand. Thus for any k, $p = (1-k)e_{(m)} + ke_{(m+1)}$ is a market clearing price. To see this, suppose there are t asks in $\{e_{(1)}, \ldots, e_{(m)}\}$, then in $\{e_{(m+1)}, \ldots, e_{(m+n)}\}$, there are exactly t bids, all of which are no lower than the asks in the first set. These t asks and t bids are the tradable ones. In the rest of the orders, the bids are no higher than the asks.

Alternatively, the allocation rule of k-double can be written as follows. Each trader i submits a price e_i. Denote by $x = (x_1, \ldots, x_{m+n})$ the allocation of the commodity. We have $x_i \in \{0,1\}$ for $i \in B$, and $x_i \in \{0,-1\}$ for $i \in S$, in which $x_i = 0$ if the trader doesn't trade. Given the order profile $w = (e_1, \ldots, e_{m+n})$, the feasible set of allocation for the clearing house is

$$
\begin{aligned}
X(w) = \arg \max_{(x_1,\ldots,x_{m+n})} \sum_{i \in B \cup S} x_i e_i, \quad \text{such that} & \sum_{i \in B \cup S} x_i = 0, \\
& x_i \in \{0,-1\}, \forall i \in S, \\
& \text{and } x_i \in \{0,1\}, \forall i \in B.
\end{aligned}
$$

The clearing house chooses allocation $x \in X(w)$ such that $\sum_{i \in B} x_i \geq \sum_{i \in B} x'_i, \forall x'_i \in X(w)$; that is, the allocation that maximizes trade among feasible allocations.

3.4 Bayesian Nash equilibria

The k-double auction has multiple Bayesian Nash equilibria, some of which involve no trade. Studies focus on the non-trivial ones—the Bayesian Nash equilibria in which trade happens with positive probability. Existing literature provide various methods to find Bayesian Nash equilibria with trade.

The analysis is different for the bilateral and multilateral case, so we discuss them separately here. In this section, we consider symmetric equilibria where each buyer i bids $B(v_i)$ and seller j asks $S(c_j)$. Cripps and Swinkels (2006) relaxes the symmetry assumption. For the rest of this section, for simplicity, we restrict our attention to redemption values drawn from the unit interval.

3.4.1 Bilateral case

Chatterjee and Samuelson (1983), Leininger et al. (1989) and Satterthwaite and Williams (1989a) provide a few methods to look for Bayesian Nash equilibria in the bilateral case ($m = n = 1$) when $k \in [0,1]$. Both Chatterjee and Samuelson (1983) and Satterthwaite and Williams (1989a) focus on differentiable equilibrium strategies. Leininger et al. (1989) show the existence of equilibrium strategies that are step functions.

When $k = 1$, the price equals the buyer's bid. The seller's dominant strategy is to ask his redemption value, and the buyer has a unique best response. For $k = 0$, the buyer has a dominant strategy to bid his redemption value, and the seller has a unique best response.

Chatterjee and Samuelson (1983) shows that in nontrivial equilibria, bids and asks are increasing in the redemption values. For the class of equilibria with strictly increasing differentiable strategies that are bounded above and below, the ask given cost c, denoted by $S(c)$, and the bid given value v, denoted by $B(v)$, satisfy the following conditions.

$$B^{-1}(S(c)) = S(c) + kS'(c)\frac{F(c)}{f(c)} \tag{3.1}$$

$$S^{-1}(B(v)) = B(v) - (1-k)B'(v)\frac{1-G(v)}{g(v)} \tag{3.2}$$

Equations 3.1, 3.2 are first order conditions from maximizing the expected utilities of the buyer's and the seller's.

Denote by $\pi_B(v,\lambda)$ the expected payoff if a buyer with value v bids λ, and by $\pi_S(c,\lambda)$ the expected payoff for a seller who has a cost c and bids λ.

The buyer's best response

$$B(v) = \arg\max_{\lambda} \pi_B(v,\lambda) = \arg\max_{\lambda} \int_0^{S^{-1}(\lambda)} [v - k\lambda - (1-k)S(c)] f(c) dc,$$

where $S(c)$ is the seller's best response, i.e.,

$$S(c) = \arg\max_{\lambda} \pi_S(c,\lambda) = \arg\max_{\lambda} \int_{B^{-1}(\lambda)}^{1} [kB(v) + (1-k)\lambda - c] g(v) dv.$$

Satterthwaite and Williams (1989a) shows that for admissible distributions (F,G), equations 3.1,3.2 are also sufficient conditions for regular equilibria. A equilibrium is regular if the strategies are continuous and strictly increasing, undominated, differentiable in the tradable range and truth-telling outside of it. Further more, let $\dot{c} \equiv \frac{dS^{-1}(\lambda)}{d\lambda} = \frac{1}{S'(c)}$ and $\dot{v} \equiv \frac{dB^{-1}(\lambda)}{d\lambda} = \frac{1}{B'(v)}$, the first order conditions $\frac{\partial \pi_B(v,\lambda)}{\partial \lambda} = 0$ and $\frac{\partial \pi_S(c,\lambda)}{\partial \lambda} = 0$ lead to

$$\dot{c} = \frac{k}{v-\lambda} \frac{F(c)}{f(c)} \tag{3.3}$$

$$\dot{v} = -\frac{1-k}{c-\lambda} \frac{1-G(v)}{g(v)} \tag{3.4}$$

Equations 3.3, 3.4 and $\dot{\lambda} \equiv \frac{d\lambda}{d\lambda} = 1$ together create a vector field in a part of (c,v,λ) space. Take an interior point within the tetrahedron that $0 \leq c \leq \lambda \leq v \leq 1$, and trace out the path in both directions following the vector field, the collection of points defines an equilibrium. Different equilibria can be found this way by starting at different interior points.

As shown by Satterthwaite and Williams (1989a), the regular equilibrium exists for admissible pairs of redemption value distributions. It is unique for $k \in \{0,1\}$. For $k \in (0,1)$, the regular equilibria in the k-double auction form a two-parameter family.

An example of differentiable equilibria is the linear equilibrium provided in Chatterjee and

Samuelson (1983) for uniformly distributed redemption values, which is ex ante incentive efficient when $k = 0.5$ (Myerson and Satterthwaite, 1983).

Linhart et al. (1989) summarizes a few other bilateral bargaining models.

3.4.2 Multilateral case

Rustichini et al. (1994) characterizes Bayesian Nash equilibria for $c, v \in [0,1]$ with $m \geq 2$ buyers and $n \geq 2$ sellers. For any equilibria in which strategies are undominated and induce trade with positive probability, there exist thresholds of cost and value, denoted by $\overline{c}$ and $\underline{v}$ respectively, such that: a seller has positive probability to trade if and only if the cost is lower than $\overline{c}$, a buyer has positive probability to trade if and only if the value is higher than $\underline{v}$; $S(\cdot)$ and $B(\cdot)$ are increasing for costs lower than $\overline{c}$ and values higher than $\underline{v}$; $\overline{c}$ and $\underline{v}$ are also the upper bound of buyers' bids and lower bound of sellers' asks respectively; seller's ask and buyer's bid converge to truth-telling when the cost goes to $\overline{c}$ and the value goes to $\underline{v}$ respectively. These propositions rule out possibilities of ties in the tradable range, hence simplifies the analysis.

Consider differentiable and strictly increasing strategies for $k \in [0,1]$. We have

$$\pi_B(v,\lambda) = \int_0^{\lambda} \int_x^1 [v - (1-k)x - k\min\{\lambda, y\}] e(x,y)dydx,$$

where $x = \zeta_{(m)}$, $y = \zeta_{(m+1)}$ are the mth and $(m+1)$th lowest orders from other $(m+n-1)$ traders, and $e(x,y)$ is the probability density function for the joint distribution of x and y.

The first order condition is

$$\begin{aligned}
0 &= \frac{\partial \pi_b(v,\lambda)}{\partial \lambda} \\
&= (v-\lambda)\int_{\lambda}^1 e(\lambda,y)dy - k\int_0^{\lambda}\int_{\lambda}^1 e(x,y)dydx \\
&= (v-\lambda)f_x(\lambda) - kPr(x < \lambda < y).
\end{aligned}$$

Here $f_x(\cdot)$ is the pdf of x. Let $c = S^{-1}(\lambda)$. From Rustichini et al. (1994),

$$f_x(\lambda) = nK_{n,m}(\lambda)\frac{f(c)}{S'(c)} + (m-1)L_{n,m}\frac{g(v)}{B'(v)},$$

where

$$K_{n,m}(\lambda) = \sum_{\substack{i+j=m-1\\0\le i\le m-1\\0\le j\le n-1}} \binom{m-1}{i}\binom{n-1}{j} G(v)^i F(c)^j (1-G(v))^{m-1-i}(1-F(c))^{n-1-j},$$

$$L_{n,m}(\lambda) = \sum_{\substack{i+j=m-1\\0\le i\le m-2\\0\le j\le n}} \binom{m-2}{i}\binom{n}{j} G(v)^i F(c)^j (1-G(v))^{m-2-i}(1-F(c))^{n-j}.$$

Let $M_{n,m}(\lambda) \equiv Pr(x < \lambda < y)$, then

$$M_{n,m}(\lambda) = \sum_{\substack{i+j=m\\0\le i\le m-1\\0\le j\le n}} \binom{m-1}{i}\binom{n}{j} G(v)^i F(c)^j (1-G(v))^{m-1-i}(1-F(c))^{n-j}.$$

Let $\dot{c} \equiv \frac{dS^{-1}(\lambda)}{d\lambda} = \frac{1}{S'(c)}$ and $\dot{v} \equiv \frac{dB^{-1}(\lambda)}{d\lambda} = \frac{1}{B'(v)}$, the first order condition can be written as

$$0 = (v-\lambda)\left[nK_{n,m}(\lambda)f(c)\dot{c} + (m-1)L_{n,m}(\lambda)g(v)\dot{v}\right] - kM_{n,m}(\lambda). \tag{3.5}$$

The intuition is as follows. Suppose a buyer with a value $v \in [\underline{v}, 1]$ bids λ. If he increases the bid by $\Delta\lambda$, the payoff can be affected in two ways: (a) If λ is too low to trade, but $\Delta\lambda$ is tradable, then the payoff increases by $v - \lambda - \Delta\lambda$. The probability of this event is $f_x(\lambda)$. $f_x(\lambda)$ consists of two parts: the probability $\lambda + \Delta\lambda$ surpasses a bid, and the probability it surpasses an ask. (b) The buyer trades at λ, and increasing the bid increases the price the buyer pays. This happens only if $\lambda = s_{(m+1)}$ and $\lambda + \Delta \le s_{(m+2)}$. In this case, the price is increased by $k\Delta\lambda$. The probability is $Pr(x < \lambda < y)$. If $< S, B >$ is an equilibrium, we have $\frac{\partial \pi_b(v,\lambda)}{\partial \lambda} = (v-\lambda)f_x(\lambda) - kPr(x < \lambda < y) = 0$.

Analogously, the first order condition for the seller is

$$
\begin{aligned}
0 &= \frac{\partial \pi_S(c,\lambda)}{\partial \lambda} \\
&= -(\lambda - c)\left[(n-1)J_{n,m}(\lambda)f(c)\dot{c} + mK_{n,m}(\lambda)g(v)\dot{v}\right] + (1-k)N_{n,m}(\lambda), \qquad (3.6)
\end{aligned}
$$

where

$$
J_{n,m}(\lambda) = \sum_{\substack{i+j=m-1 \\ 0\le i\le m \\ 0\le j\le n-2}} \binom{m}{i}\binom{n-2}{j} G(v)^i F(c)^j (1-G(v))^{m-i}(1-F(c))^{n-2-j},
$$

$$
N_{n,m}(\lambda) = \sum_{\substack{i+j=m-1 \\ 0\le i\le m \\ 0\le j\le n-1}} \binom{m}{i}\binom{n-1}{j} G(v)^i F(c)^j (1-G(v))^{m-i}(1-F(c))^{n-1-j}.
$$

Equations 3.5, 3.6 and $\dot{\lambda} \equiv \frac{d\lambda}{d\lambda} = 1$ defines a system of ordinary differentiable equations, which can be used numerically to obtain Bayesian Nash equilibria. The procedure is the same as $m = n = 1$.

For $k = 1$, it can be verified from the equations that the dominant strategy for a seller is $\lambda = c$, and the equilibrium is unique (Rustichini et al., 1994; Satterthwaite and Williams, 2018). The case $k = 0$ is analogous.

3.5 Efficiency

A market is ex post efficient if given the realized redemption values, the maximal social surplus is achieved. In a Bayesian Nash equilibrium, a trader who holds private information of his own redemption value "may have an incentive to misrepresent his preferences in order to influence the market price in his favour." (Williams, 1991) Consequently, in a Bayesian Nash equilibrium some trades inefficiently fail to occur. As the size of the market grows, the influence a trader has on the market price becomes smaller. Hence "he has little incentive to misrepresent and the market is therefore almost fully efficient" (Williams, 1991).

As shown by Myerson and Satterthwaite (1983), in a bilateral trade under incomplete information setting, the ex post efficiency cannot be achieved by any mechanism satisfying incentive compatibility, individual rationality, and an ex post budget constraint. This result is extended to multilateral setting by Williams (1999). Using the definition of efficiency from Holmström and Myerson (1983), in the bilateral case, the k-double auction is ex ante incentive inefficient with generic distribution when $k \in (0,1)$ (Satterthwaite and Williams, 1989a), and is ex ante incentive efficient for some class of distribution when $k \in \{0,1\}$ (Myerson, 1981; Williams, 1987).

Wilson (1985) shows that in a class of environment, k-double auction is interim incentive efficient (Holmström and Myerson, 1983) when the market is large enough, in the sense that there is no other trading rule surely preferred by each agent.

Gresik and Satterthwaite (1989) first studies how market size affect the inefficiency caused by strategic behavior. The maximal expected inefficiency from an optimal mechanism is $O((\ln \tau/\tau^2))$, where τ is an index for number of traders. Satterthwaite and Williams (1989b) shows the relative inefficiencies in BBDA is of the same order as the optimal mechanism in Gresik and Satterthwaite (1989): in Bayesian Nash equilibria for a BBDA with m buyers and m sellers, the maximal amount a trader misrepresent his value is $O(\frac{1}{m})$, regardless of distributions of the redemption values. For BBDA markets with unequal numbers of buyers and sellers, Williams (1991) shows that the buyers' misrepresentation is $O(1/\tau)$. Zacharias and Williams (2001) further relaxes it to the market with arbitrarily more buyers than sellers, i.e. $m \geq \beta n$ for a constant $\beta > 1$ the misrepresentation of buyer's value is $O(1/m)$, and the resulting expected inefficiency is $O(n/m^2)$. Note that this rate is consistent with the one from Rustichini et al. (1994), which is for general k-double auction and requires relatively balanced market such that $1/\beta \leq n/m \leq \beta$.

Rustichini et al. (1994) establishes that in any equilibrium, the maximal amount by which a trader misreport is $O(1/m)$, where m is the number of buyers, and the expected inefficiency is $O(1/m^2)$. Moreover, for a sample of redemption values, the difference between the price determined by k-double auction and a competitive price is $O(q(n,m))$, which is equivalent to $O(1/n)$ and $O(1/m)$ if n/m is bounded above and away from zero. This is coincidentally of the same order as the maximal distance between a core allocation and its nearest Walrasian allocation when a regular

Arrow-Debrew economy is replicated m times, as well as the difference between a competitive price and a Nash equilibrium price in a Cournot competition with a homogeneous good.

Satterthwaite and Williams (2002) use a maxmin approach to evaluate mechanisms by their rates of convergence to efficiency, in a setting of m buyers and m sellers. Mechanisms that are interim individual rational and ex ante budget balanced for each m and each environment are compared. The k-double auction is worst-case asymptotic optimal such that its worst-case error over a set of environments converges to zero at the fastest possible rate among all plausible mechanisms.

3.6 Experiments

Experimental studies can shed light on the equilibrium selection in the k double auction. In the bilateral setting, subjects, sellers in particular, tend to adopt the linear equilibrium strategy. Non-linear equilibrium strategies, such as the step-function strategy provided in Leininger et al. (1989), are observed with experienced subjects. Although the ex post efficiency ranges from zero to 100% in the prediction of Bayesian Nash equilibrium for the bilateral case, when tested in laboratory experiment, on average 85% to 92% of the available surplus is obtained Radner and Schotter (1989).

Bayesian Nash equilibria predict convergence to truth-telling strategies as the market grows. Kagel (2004) test the performance of the BBDA mechanism and continuous double auction in markets of two buyers and two sellers, and markets of eight buyers and eight sellers. Redemption values are drawn from publicly known distribution each round. Both thicker market and the dynamic institution induce higher efficiency in the markets. The performance of the markets are closer to the prediction of Bayesian Nash equilibrium than the zero-intelligent traders. Unlike markets with stationary demand and supply, convergence to the competitive equilibrium price across rounds is not observed.

Kagel and Vogt (2018) tests the performance of BBDA and a modified BBDA (MBBDA) in markets with $m = n = 2$ and $m = n = 8$. A dual-market technique is employed, in which a subject bids in both $m = 2$ and $m = 8$ markets with the same redemption value, and a random one of the two markets pays off. In BBDA, the average achieved efficiency is lower than the prediction of the Bayesian Nash equilibrium, but higher than the prediction from zero-intelligent traders. Majority of the seller deviate from the dominant strategy, and for $m = 2$, buyers on average over bid. In MBBDA, the market price is set to the lowest tradable bid, which may not be market-clearing. While sellers in a MBBDA could benefit from asking less than the costs, subjects tend to ask more than their costs in the experiment.

Cason and Friedman (1997) studies the predictive power of Bayesian Nash equilibrium for k-double auction, with $k = 0, 0.5, 1$ in markets of four buyers and four sellers. The authors take into account the experience level of subjects, and use programmed traders to reduce the complication of opponents' strategy. Subjects' behavior is closer to the prediction of Bayesian Nash equilibrium than competitive equilibrium, or zero-intelligence traders. Overall, experienced subjects tend to be more truth-revealing, and the Bayesian Nash equilibrium has the best prediction when the subject plays against robots that adopt linear Bayesian Nash equilibrium strategies.

Eliciting belief about others' strategies is helpful for understanding the bidding behavior. (Neri, 2015) provides a method to elicit probabilistic beliefs of continuous choices, and tests it in a 0.5-double auction with four, six and eight traders. Besides submitting bids and asks, subjects also indicate the probability they think another buyer's bid falls into each of the price intervals, and the probability they think another seller's ask falls into each of the price intervals. A parametric distribution is then fitted over the data to approximate each subject's belief of others' orders. This elicited subjective beliefs help explain observed bidding choices better than Bayesian Nash equilibrium beliefs and empirical beliefs in the experiment.

3.7 Extensions

3.7.1 Interdependent values

Jackson and Swinkels (2005) and Cripps and Swinkels (2006) allows for multiple units per trader and interdependent values. Jackson and Swinkels (2005) provides the first general existence result for k-double auctions. They prove the existence of equilibrium in distributional strategies in a large class of private value auctions, and the existence of nontrivial equilibrium k-double auction. Cripps and Swinkels (2006) relaxes the independence assumption to z-independence, and further proves that all nontrivial equilibria of auctions are asymptotically efficient.

Reny and Perry (2006) establishes the existence of a pure and nondecreasing equilibrium for sufficiently large markets with sufficiently fine discrete set of prices. The setting in Reny and Perry (2006) is general: redemption values can be interdependent, and information can be affiliated. The unique fully revealing rational expectations equilibrium of a market with continuum of traders and continuous price can be approximated by an equilibrium from the discrete case.

Kadan (2007) studies the bilateral case with dependent private values. The sufficient conditions is given for existence of regular equilibrium. The FGM copula, a class of distributions that a single parameter determines the dependency without affecting marginal distributions, is used as an example to show how dependency of distribution affects bids in BBDA, and the existence of regular equilibria for $k \in (0,1)$. Equilibria of the independent value setting can be obtained as the limit of a sequence of equilibria with affiliated values.

For large markets with correlated, conditionally independent private values, Fudenberg et al. (2007) proves the existence of pure strategy equilibria that are symmetric and increasing. The maximal misrepresentation in equilibria is $O(1/N)$, where $N = n+m$.

Satterthwaite et al. (2018) and Satterthwaite et al. (2019) applies BBDA to correlated and interdependent values setting. Satterthwaite et al. (2018) shows that in finite market, the strategic behavior generated by private information has only marginal effect on allocational efficiency and information aggregation. With a less general environment than Reny and Perry (2006), Satterthwaite et al. (2018) uses a numerical result to show the existence of equilibrium and the rate of convergence to efficiency. Satterthwaite et al. (2019) identifies the asymptotic limits of distributions in the first order conditions for optimal bidding/asking.

3.7.2 Alternative decision rules

The following studies explore the situations when traders are not expected payoff maximizers.

Agastya (2004) applies the mechanism to a complete information setting, allowing for unneutral risk preferences. Stochastical stability, as a criterion for equilibrium selection, is applied to look for price distributions that are likely to prevail in the longrun as errors goes away. Under this criterion, the price is decreasing in k, and the equilibria approximate an asymmetric Nash bargaining solution.

Rasch et al. (2012) investigates the bilateral trade scenario of Myerson and Satterthwaite (1983) with inequity averse traders. A incentive compatible direct mechanism acquiring ex post efficiency exists when traders exhibiting strong enough inequity aversion a la Fehr and Schmidt (1999). Truth-telling, hence full efficiency, can be implemented by a k-double auction with $k = 0.5$ in this case.

Shafer (2015, 2016) study the mechanism under Knightian uncertainty (Knight, 1921), assuming traders are regret-minimizing. Unlike expected utility maximizers, minimax regret traders do not react to the increase of market size. The strategy of these traders does not approximate price-taking in large markets, nor does the strategy of traders minimizing their expected maximum regret. Convergence to truth-telling is possible with Γ-minimax regret (Stoye, 2011) traders, who minimize their maximum expected regret, under certain prior Γ. The third chapter in Shafer (2015) offers preliminary results under a common value setting.

Ahmad (2020) presents a linear equilibrium in the bilateral case when both the buyer and the seller have reference-dependent preferences, i.e. the difference between the realized terms of trade and their reference points is of concern, and their values are independently drawn from a uniform distribution over $[0,1]$. The ex post efficiency of this equilibrium decreases in the level of agents' reference dependency as their reference points differ.

3.7.3 Related mechanisms

The k-double auction is also a strategic market game with one trading post, in the sense that each trader submits an sealed order to the centralized market place to determine the terms of trade. Different from the strategic market game from Shapley and Shubik (1977), orders are not always fulfilled in a k-double auction, and each trader has to specify a price for only one unit. Despite the completeness of information, with finite number of traders, the Nash equilibrium of the strategic market game a la Shapley and Shubik (1977) is generically inefficient under certain conditions (Dubey, 1980). This inefficiency can be overcome by having a continuum of traders (Dubey et al., 1980).

A main feature of BBDA is its simplification of analysis compared to general k-double auction. McAfee (1992) provides another sealed-bid double auction in which truth-revealing is a dominant strategy for both buyers and sellers. The mechanism forgoes the trade of the least socially profitable unit, and charges buyers and sellers prices of the lowest tradable bid and the highest tradable ask. A clearing house obtains the arbitrage profit. Counting only the profit of traders, the expected inefficiency is $O(1/m)$.

For interdependent values and multiple units setting, Kojima and Yamashita (2017) proposes a asymptotically efficient double auction that satisfies ex post incentive compatibility, individual rationality, feasibility, nonwastefulness, and no budget deficit. In the mechanism, a market is divided into submarkets. The price in each submarket is determined by the orders from other submarkets.

EPA designed an institution for trading allowances to emit sulfur dioxide, which resembles k-double auction but uses a discriminatory pricing rule. Bids and asks are ranked and trades are allocated as k-double auction does; the buyer and the seller of the same ranking trade at the bid the buyer submits. Cason and Plott (1996) compares the performance of this institution with a k-double auction where $k = 0.5$ in a laboratory experiment. Subjects have constant redemption values that are private informations in some sessions, while in other sessions the values are independently drawn from a uniform distribution over $[0, 250]$ in earlier rounds and $[0, 300]$ in later rounds. They observe higher efficiency and less misrepresentation of value under the uniform pricing rule, together with more rapid discovery of equilibrium price. Cramton and Stoft (2007) provides a discussion on why it is a good choice to adopt the uniform-priced k-double auction instead of other mechanisms with a discriminatory pricing rule in the spot market for whole sale electricity.

Transactions in electric market happen through power networks, thus can be limited to the capacity of the transmission line, and subject to fees line owners charge. Singer (2002) investigates outcomes from a revised k-double auction in this scenario. When the transmission capacity is binding, the revised mechanism gives priority to higher bids and lower asks as the standard k-double auction does, but charge buyers the marginal bid and pay sellers the marginal ask. The difference of the two prices is extracted by the line owner. The mechanism results in higher misrepresentation of values than the standard k-double auction in equilibria, but the asymptotic price-taking behavior persists.

When each trader is allowed to bid/ask for multiple units, the k-double auction is equivalent to the uniform price auction in experimental economics literature. The rest of this section includes some studies of the sealed-bid uniform price auction. Typically, k is set to 0.5 in them.

Quotas have been imposed on selling dairy and farming laying hens in Canada, as part of the country's supply management for a few agricultural products. The k-double auction is used to trade dairy quota in Québec since the market is relatively thin. Doyon et al. (2008) uses laboratory experiments to check whether two modifications, taxing unsold asks and excluding the highest bid and the lowest ask, can help pushing the quota price down. Neither of these interventions has a significant effect on driving price low, nor if they are implemented together. Both of them reduce efficiency in the market. Doyon et al. (2010) proposes a truncated k-double auction (T-kDA) for the laying hen permit market to restrain market power of oligopolies, and tests its performance in a laboratory experiment. In this experiment, $k = 0.5$, and subjects have private information of their own values without knowledge of the value distribution. The truncated k-double auction(T-kDA) results in lower prices with a moderate loss of efficiency, compared to the standard k double auction. The T-kDA allocates trades on the seller side in the same way as the standard k-double auction; the trading price is also a weighted sum of the highest tradable ask and the lowest tradable bid. Differently, T-kDA fulfills bids from low to high, starting from the first one that exceeds or equals to the highest tradable ask, which may not be allocated in a standard k-double auction. This priority on lower bid reduces buyers' incentive to submit high prices. In both Doyon et al. (2008) and Doyon et al. (2010), a trader may submit a unit price and a quantity if he wishes to trade multiple units.

The k-double auction is a static version of the continuous double auction (Smith, 1962). Attempts at game theoretic models for the continuous double auction include Friedman (1984), Wilson (1987), and Friedman (1991). In the absence of complete information, the results from a continuous double auction in laboratory experiments can still be well-approximated by a complete information model due to adequate information revelation (Friedman and Ostroy, 1995). Comparisons have been made in laboratory experiments under the environment that each trader has demand/supply over multiple units. When traders have private signals on their own redemption values, the continuous double auction achieves higher efficiency than the k-double auction ($k = 0.5$) on average, and the k-double auction that allows each agent to submit different prices for the units performs better than the one in which each trader submits one price for all his units (Smith et al., 1982). The continuous double auction also yields higher efficiency when the value of the commodity is uncertain, i.e. the commodity is an asset that pays dividends depending on the state (Friedman, 1993). An interesting result from the asset market setting is that providing order flow information enhances efficiency in the continuous double auction, but has an opposite effect on the k-double auction.

The k-double auction and the continuous double auction differ in both the allowance of information flow and the number of times a market is called. A mechanism that falls in between these two is the single call market with feedbacks, which is prevalently used for setting the opening prices in stock exchanges. The mechanism resembles the tâtonnement procedure: feedbacks on the clearing price and supply-demand imbalance are provided, and traders can adjust their orders till the market calls. Variations of this mechanism are compared in Friedman (1993), McCabe et al. (1992), Bronfman et al. (1996) and McCabe et al. (2018). Another mechanism that links the k-double auction and the continuous double auction is a multiple-call market, where trades happen in multiple stages, each consists of a single call market. Comparisons of this mechanism with the the k-double auction and the continuous double auction in the laboratory experiment can be found in Van Boening (1991) and Cason and Friedman (1999). Frequent uniform price call markets has been considered a good alternative for financial exchanges to mitigate high-frequency trading arms race caused by the continuous double auction (Aldrich and Vargas, 2018; Budish et al., 2014, 2015). Plott and Pogorelskiy (2017) uses two sequential call markets to understand the price formation process.

3.8 Conclusion

The k-double auction is a plausible mechanism for thin markets due to it simplicity to implement and fast convergence to efficient outcome. When adopting the mechanism to real markets, beware that small revisions can lead to drastic change in the results. An example is the MBBDA treatment in Kagel and Vogt (2018), which is also discussed by Satterthwaite and Williams (2018).

Given the complication of Bayesian Nash equilibrium in the k-double auction, experimental economics can help understanding whether it is a reasonable approximation for traders' behavior in the market. Suggested by Satterthwaite and Williams (2018), we can learn from experiments about how long it takes for subjects to learn to play an equilibrium strategy, and how to make learning process easier.

Appendix A: Appendix

A.1 Proofs for Chapter 1

Proof of Proposition 1

Proof. Given price p, the seller's optimization problem is

$$\max_q \Pi_S(p,q) - \alpha_S \max\{\Pi_B(p,q) - \Pi_S(p,q), 0\} - \beta_S \max\{\Pi_S(p,q) - \Pi_B(p,q), 0\}$$

For each β_S, denote by $\hat{P}$ the highest price at which the seller's optimal quality equalizes buyer's and seller's profits. We can show that $\hat{P} \in [P_1, P_2]$, where $P_1 = 38.61$, $P_2 = 86.94$, and $\hat{P}$ is a function of β_S.

The blue lines in figure A.1 are examples of the quality choices at different β_Ss.

1. If $\Pi_S \leq \Pi_B$, we have

$$U_S(p,q) = \Pi_S(p,q) - \alpha_S(\Pi_B(p,q) - \Pi_S(p,q)).$$

   In this range, $\frac{\partial U_S}{\partial q} < 0$, the seller obtains the highest utility at the lowest quality level that satisfies $\Pi_S \leq \Pi_B$ and $q \geq 1$.

   When the price is lower than P_1, the optimal quality level for a seller is 1 regardless of the value of β_S.

2. If $\Pi_S \geq \Pi_B$, we have

$$U_S(p,q) = \Pi_S(p,q) - \beta_S(\Pi_S(p,q) - \Pi_B(p,q)).$$

   In this range, $\frac{\partial U_S}{\partial q} = -(1-\beta_S)(2aq+b) + \beta_S(126-p)/10$.

If $\beta_S \leq 0.11$, $\frac{\partial U_S}{\partial q} < 0$ when $\Pi_S \geq \Pi_B$. The optimal quality is 1 for all $p \in [30, 126]$.

As shown in panel (c) and (d) of figure A.1, if $\beta_S > 0.11$, there can be several segments for the optimal quality.

When price is smaller than $\hat{P}$, $\frac{\partial U_S}{\partial q} > 0$ for (p, q) if $\Pi_S \geq \Pi_B$, the optimal quality is the highest quality in the range of $\Pi_S \geq \Pi_B$, which is the quality that makes $\Pi_S = \Pi_B$.

Denote by P_3 the price at which $\frac{\partial U_S}{\partial q}|_{q=1} = 0$, thus $P_3 = 126 - 10.455 \times \frac{1-\beta_S}{\beta_S}$. When $p \in [\hat{P}, P_3]$, we have $\frac{\partial U_S}{\partial q} = 0$ and $\Pi_S \geq \Pi_B$ for the optimal quality, as depicted by the downward sloping segment in panel (c) and (d) in figure A.1.

When price is higher than P_3, we have $\frac{\partial U_S}{\partial q} < 0$ for $q \in [1, 10]$, the optimal quality equals 1.

Furthermore, if $\beta_S > 0.43$, as in panel (d) of figure A.1, the constraint of $q \leq 10$ is binding for some quality choices. When price is between P_2 and $P_4 = 126 - 29.553 \times \frac{1-\beta_S}{\beta_S}$, we have $\frac{\partial U_S}{\partial q} > 0$ for $q \in [1, 10]$, the optimal quality in this range is 10.

Therefore, denote the quality choice by $q(p)$, we have

- $\beta_S \in [0, 0.11]$

$$q(p) = 1 \text{ for } p \in [30, 126]$$

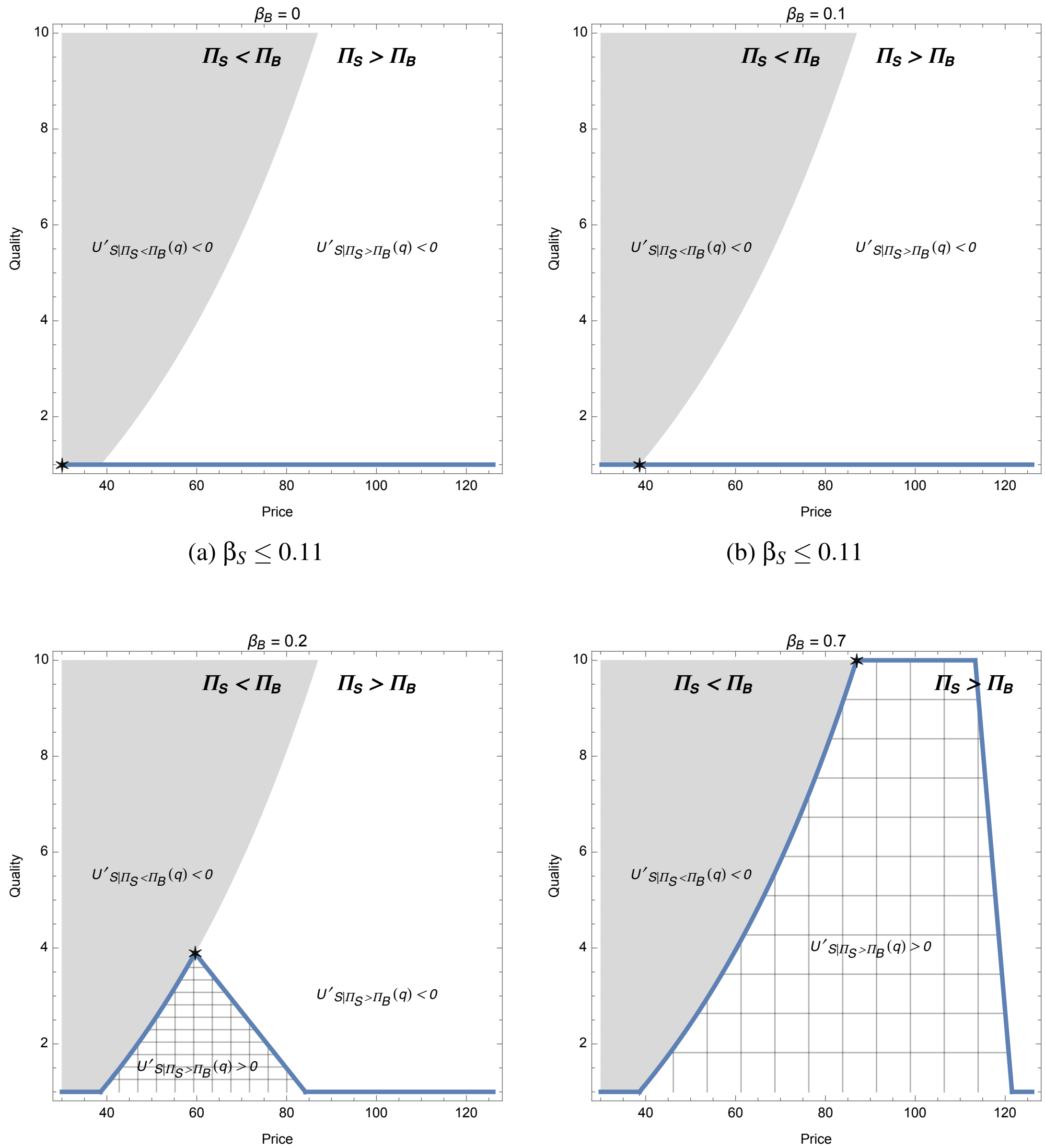
- $\beta_S \in (0.11, 0.43]$

$$q(p) = \begin{cases} 1 & p \in [30, P_1] \cup [P_3, 126] \\ q^e(p) & p \in [P_1, \hat{P}] \\ q^0(p) & p \in [\hat{P}, P_3] \\ 1 & p \in [P_3, 126] \end{cases}$$

- $\beta_S \in (0.43, 1]$

$$q(p) = \begin{cases} 1 & p \in [30, P_1] \cup [P_3, 126] \\ q^e(p) & p \in [P_1, P_2] \\ 10 & p \in [P_2, P_4] \\ q^0(p) & p \in [P_4, P_3] \\ 1 & p \in [P_3, 126] \end{cases}$$

Here $q^e(p)$ is the quality level such that $\Pi_S = \Pi_B$ given price p, which is increasing in p when $p \in [30, 126]$, and is not a function of β_S. Also, $q^0(p) = 0.4713 \times \frac{\beta_S}{1-\beta_S}(126 - p) - 3.9270$ is the quality level such that $\frac{\partial U_S}{\partial q}|_{\Pi_S \geq \Pi_B} = 0$, which is decreasing in p and increasing in β_S.



(a) $\beta_S \leq 0.11$

(b) $\beta_S \leq 0.11$

(c) $0.11 < \beta_S \leq 0.43$

(d) $0.43 < \beta_S < 1$

Figure A.1: Optimal quality and prices

Note that $\hat{P}$, P_3 and P_4 are non-decreasing functions of β_S.

The following shows that $q(p;\beta_S)$ is increasing in β_S. Take $\beta_1, \beta_2 \in (0,1]$, and suppose $\beta_1 \leq \beta_2$.

a) If $\beta_1 \leq 0.11$, we have $1 = q(p;\beta_1) \leq q(p;\beta_2)$ for all prices.

b) If $\beta_1, \beta_2 \in (0.11, 0.43]$,

   i) For $p \in [30, P_1]$, the quality is 1 for any β_S, $q(p;\beta_1) = q(p;\beta_2)$.

   ii) For $p \in [P_1, \hat{P}(\beta_2)]$, we have $q(p;\beta_2) = q^e(p)$, and $q(p;\beta_1) = \min\{q^e(p;\beta_1), q^0(p;\beta_1)\}$. Hence, $q(p;\beta_1) \leq q^e(p;\beta_1) \leq q^e(p;\beta_2) = q(p;\beta_2)$.

   iii) For $p \in [\hat{P}(\beta_2), P_3(\beta_2)]$, we have $q(p;\beta_2) = q^0(p;\beta_2)$, and $q(p;\beta_1) = \max\{q^0(p;\beta_1), 1\}$. Therefore, $q(p;\beta_1) \leq q^0(p;\beta_1) \leq q(p;\beta_2)$.

   iv) For $p \in [P_3(\beta_2), 126]$, we have $q(p;\beta_1) = q(p;\beta_2) = 1$.

   Therefore, $q(p;\beta_1) \leq q(p;\beta_2)$ if $\beta_1, \beta_2 \in (0.11, 0.43]$.

c) If $\beta_1 \in (0.11, 0.43]$, and $\beta_2 \in [0.43, 1]$,

   i) For $p \in [30, P_1]$, the quality is 1 for any β_S, $q(p;\beta_1) = q(p;\beta_2)$.

   ii) For $p \in [P_1, P_2]$, same as when $\beta_1, \beta_2 \in (0.11, 0.43]$, $q(p;\beta_1) \leq q^e(p;\beta_2) = q(p;\beta_2)$.

   iii) For $p \in [P_2, P_4(\beta_2)]$, $q(p;\beta_2) = 10 \geq q(p;\beta_1)$.

   iv) For $p \in [P_4(\beta_2), P_3(\beta_2)]$, same as when $\beta_1, \beta_2 \in (0.11, 0.43]$, we have $q(p;\beta_1) \leq q^0(p;\beta_1) \leq q^0(p;\beta_2) = q(p;\beta_2)$.

   v) For $p \in [P_3(\beta_2), 126]$, $q(p;\beta_1) = q(p;\beta_2) = 1$.

   Therefore, $q(p;\beta_1) \leq q(p;\beta_2)$ in this price range.

d) If $\beta_1 > 0.46$, we have $q(p;\beta_1) = q(p;\beta_2)$ for $p \in [30, P_4(\beta_1)]$. For $p \in [P_4(\beta_1), P_4(\beta_2)]$, $q(p;\beta_2) = 10 \geq q(p;\beta_1)$. For $p \in [P_4(\beta_2), P_3(\beta_2)]$, we have $q(p;\beta_2) = q^0(p;\beta_2) > \max\{q^0(p;\beta_1), 1\} = q(p;\beta_1)$. For $p \in [P_3(\beta_2), 126]$, we have $q(p;\beta_1) = q(p;\beta_2) = 1$.

   Therefore, $q(p;\beta_1) \leq q(p;\beta_2)$ in this price range.

Hence, we have $q(p;\beta_1) \leq q(p;\beta_2)$, $q(p;\beta_S)$ is increasing in β_S. $\square$

Proof of Proposition 2

Proof. Given the seller's quality choice in the last stage, we can find the price that maximizes the buyer's utility.

- If $p \leq P_1$, we have $\Pi_S \leq \Pi_B$ and $q(p) = 1$. In this range, the buyer's utility

$$U_B(p, q(p)) = (1 - \beta_B)(126 - p) \times 0.1 + \beta_B(p - c(1)).$$

  Therefore a buyer with $\beta_B < 0.09$ prefers $p = 30$ in this range, and a buyer with $\beta > 0.09$ prefers $p = P_1$ in this range.

- If $p \in [P_1, P^*]$, where $P^* = \min\{\hat{P}(\beta_S), P_2\}$, the seller chooses $q^e(p)$ that equalizes Π_S and Π_B. In this range, $U_B = \Pi_B = \Pi_S$. Since $q(p)$ is a bijective function in this range, we can write p as a function of q, such that $p(q) = q^{-1}(q)$. From $\Pi_B = \Pi_S$ we have,

$$p'(q) = -\frac{-c'(q) - (126 - p)/10}{1 + q/10}.$$

  Therefore,

$$\begin{aligned} \frac{dU_B}{dq} =& p'(q) - c'(q) \\ =& \frac{1}{1 + q/10} \cdot [c'(q) + (126 - p)/10 - c'(q)(1 + q/10)] \\ =& \frac{1}{10 + q} \cdot [-c'(q) \cdot q + (126 - p)]. \end{aligned}$$

  We have $\frac{dU_B}{dq} > 0$ when $p \in [P_1, P_2]$ and $q \in [1, 10]$. In this range, the buyer prefers the highest price, $\min\{\hat{P}(\beta_S), P_2\}$, which leads to the highest quality. Note that $\hat{P}(\beta_S) \leq P_2$ when $\beta_S \leq 0.43$, and $\hat{P}(\beta_S) > P_2$ otherwise. Hence the buyer's preferred price is increasing in β_S.

- If $p \in [P^*, 126]$, we have $\Pi_S > \Pi_B$, thus $U_B = \Pi_B - \alpha(\Pi_S - \Pi_B)$. In this range, $q(p)$ is a decreasing function of p, thus $\frac{dU_B}{dp} < 0$, and the buyer prefers P^*.

Therefore, the buyer prefers $P^* = \min\{\hat{P}(\beta_S), P_2\}$ to any other price in the range of $[P_1, 126]$, and if the buyer has $\beta \geq 0.09$, P^* is the buyer's globally preferred price.

If the buyer has $\beta_B < 0.09$, and the seller has $\beta_S < 0.11$, we have $U_B(q(30), 30) > U_B(q(\hat{P}(\beta_S)), \hat{P}(\beta_S))$, the buyer's globally preferred price is 30. If $\beta_S \geq 0.11$, then $U_B(q(30), 30) < U_B(q(P^*), P^*)$, the buyer prefers P^*.

The preferred prices are marked by asterisks in figure A.1.

In summary, the globally preferred price is 30 if $\beta_B < 0.09$ and $\beta_S < 0.11$, and P^* otherwise. Since P^* is increasing in β_S, and $P^* > 30$, the buyer's globally preferred price is increasing in β_S.

□

A.2 Proofs for Chapter 2

Characteristics of $F(w)$

The following propositions characterize the set of allocations chosen by the clearing house with positive probability after the offer profile w, and are used in later proofs.

Proposition 3. *For any $y \in F(w)$, if $y_b > 0$ and $y_s < 0$ for a buyer b and a seller s, then $\widetilde{p}_b \geq \widetilde{p}_s$.*

Proof. Suppose $\widetilde{p}_b < \widetilde{p}_s$. Consider an alternative allocation y' such that $y'_i = y_i$ if $i \neq b, s$ and $y'_b = y_b - 1$, $y'_s = y_s + 1$. Since $y \in F(w) \subseteq Y(w)$, we have $y' \in Y(w)$, and

$$\sum_{i \in I} y'_i \widetilde{p}_i = \sum_{i \in I} y_i \widetilde{p}_i + (\widetilde{p}_s - \widetilde{p}_b) > \sum_{i \in I} y_i \widetilde{p}_i.$$

Then $y \notin F(w)$, contradicting the assumption.

□

Proposition 4. *Given an offer profile w and a buyer b and a seller s such that $\widetilde{p}_b \geq \widetilde{p}_s$, there cannot be an allocation $y \in F(w)$ such that $y_b < \widetilde{q}_b$ and $y_s > \widetilde{q}_s$.*

Proof. For a given offer profile w such that there is a buyer b and a seller s that $\widetilde{p}_b \geq \widetilde{p}_s$, suppose there is an allocation vector $y \in Y(w)$ such that $y_b < \widetilde{q}_b$ and $y_s > \widetilde{q}_s$. We can show that $y \notin F(w)$. Take an alternative allocation vector y', let $y'_i = y_i$ if $i \neq b, s$, and $y'_b = y_b + 1$, $y'_s = y_s - 1$. We have $y' \in Y(w)$. The arbitrage profit for the clearinghouse by allocating y' is

$$\sum_{i \in I} y'_i \widetilde{p}_i = \sum_{i \in I} y_i \widetilde{p}_i + (\widetilde{p}_b - \widetilde{p}_s) \geq \sum_{i \in I} y_i \widetilde{p}_i,$$

where the last term is the clearing house's profit if it allocates y. Therefore, if $\widetilde{p}_b > \widetilde{p}_s$, then $\sum_{i \in I} y_i \widetilde{p}_i < \sum_{i \in I} y'_i \widetilde{p}_i$ and $y \notin \Pi(w)$; and if $\widetilde{p}_b = \widetilde{p}_s$, then $\sum_{i \in I} y_i \widetilde{p}_i = \sum_{i \in I} y'_i \widetilde{p}_i$ but y is ray dominated by y'. Either way we have $y \notin F(w)$.

□

Proposition 5. *If, for a given offer profile w, seller a and seller b offer $\widetilde{p}_a < \widetilde{p}_b$, and seller b is an active trader, then for all $y \in F(w)$ we have $y_a = \widetilde{q}_a$. Symmetrically, if in a given offer profile w, buyer a and buyer b offer $\widetilde{p}_a > \widetilde{p}_b$, and buyer b is an active trader, then for all $y \in F(w)$ we have $y_a = \widetilde{q}_a$.*

Proof. We will show the proof for the sellers' case, since the buyers' case is symmetric. By definition, if seller b is an active trader, there exists an allocation $y^* \in F(w)$ such that $y^*_b < 0$. First we show that $y^*_a = \widetilde{q}_a$.

Suppose $y^*_a > \widetilde{q}_a$. Take an alternative allocation vector y' given by $y'_a = y^*_a - 1$, $y'_b = y^*_b + 1$, and $y'_i = y^*_i$ for $i \neq a, b$. It is easy to see that $y' \in Y(w)$. The profit for the clearing house by allocating y' equals

$$\sum_{i \in I} y'_i \widetilde{p}_i = \sum_{i \in I} y^*_i \widetilde{p}_i + (\widetilde{p}_b - \widetilde{p}_a) > \sum_{i \in I} y^*_i \widetilde{p}_i.$$

The last term is the profit of the clearing house if y^* is allocated. Hence $y^* \notin \Pi(w)$, so $y^* \notin F(w)$.

Therefore, if $y_b^* < 0$ and $y^* \in F(w)$, we must have $y_a^* = \widetilde{q}_a$. By the same argument, we must have $y_a = \widetilde{q}_a$ for every allocation $y \in F(w)$ such that $y_b < 0$.

Now suppose there is an allocation $\hat{y} \in F(w)$ such that $\hat{y}_b = 0$ and $\hat{y}_a > \widetilde{q}_a$. According to the result in the first part of the proof, for any seller h that offers $\widetilde{p}_h > \widetilde{p}_a$, $\hat{y}_h = 0$, otherwise $\hat{y} \notin F(w)$. Hence

$$\sum_{\{h \in S: \widetilde{p}_h > \widetilde{p}_a\}} \hat{y}_h \;=\; 0 \;>\; y_b^* \;\geq \sum_{\{h \in S: \widetilde{p}_h > \widetilde{p}_a\}} y_h^*.$$

According to the first part of the proof, $y_i^* = \widetilde{q}_i$ for $i \in S$ if $\widetilde{p}_i < \widetilde{p}_b$. Since $\widetilde{p}_b > \widetilde{p}_a$, we have

$$\sum_{\{h \in S: \widetilde{p}_h \leq \widetilde{p}_a\}} \hat{y}_h \;\geq \sum_{\{h \in S: \widetilde{p}_h \leq \widetilde{p}_a\}} \widetilde{q}_h \;= \sum_{\{h \in S: \widetilde{p}_h \leq \widetilde{p}_a\}} y_h^*.$$

Therefore,

$$\sum_{i \in B} y_i^* = -\sum_{i \in S} y_i^* > -\sum_{i \in S} \hat{y}_i = \sum_{i \in B} \hat{y}_i.$$

Since $\sum_{i \in B} \hat{y}_i < \sum_{i \in B} y_i^*$, there exists at least one buyer, say buyer e, such that $0 \leq \hat{y}_e < y_e^* \leq \widetilde{q}_e$. Since $y_e^* > 0$ and $y_a^* < 0$, from proposition 1 we have $\widetilde{p}_e \geq \widetilde{p}_a$. Therefore $\widetilde{p}_e \geq p_a$, $\hat{y}_a > \widetilde{q}_a$, and $\hat{y}_e < \widetilde{q}_e$, violating proposition 2.

□

Proposition 6. *Given an offer profile w, if buyer $b \in AB(w)$ and seller $s \in AS(w)$, then $\widetilde{p}_b \geq \widetilde{p}_s$.*

Proof. Since $b \in AB(w)$ and $s \in AS(w)$, there must be some $y, y' \in F(w)$ such that $y_b > 0$ and $y'_s < 0$. If $y = y'$, the desired result follows from Proposition 1. Suppose $y \neq y'$. Since $y, y' \in \Pi(w)$, we have

$$\sum_{i \in AB(w)} y_i \widetilde{p}_i + \sum_{i \in AS(w)} y_i \widetilde{p}_i = \sum_{i \in AB(w)} y'_i \widetilde{p}_i + \sum_{i \in AS(w)} y'_i \widetilde{p}_i.$$

Suppose $\widetilde{p}_b < \widetilde{p}_s$. From Proposition 1, we have $y_s = 0$ and $y'_b = 0$. From proposition 3, then, there is no active seller submitting a price higher than p_s, and no active buyer submitting a price lower than p_b. Denote by $\overline{AB}$ the set of active buyers that offer p_b, and $\underline{AS}$ the set of active sellers that offer p_s. From proposition 3, for $i \in AB(w) \backslash \overline{AB}$ and $i \in AS(w) \backslash \underline{AS}$,

$$y'_i = y_i = \widetilde{q}_i.$$

Thus,

$$\sum_{i \in \overline{AB}} y_i p_b + \sum_{i \in \underline{AS}} y_i p_s = \sum_{i \in \overline{AB}} y'_i p_b + \sum_{i \in \underline{AS}} y'_i p_s,$$

which is equivalent to

$$p_b \cdot \left(\sum_{i \in \overline{AB}} y_i - \sum_{i \in \overline{AB}} y'_i \right) = p_s \cdot \left(\sum_{i \in \underline{AS}} y'_i - \sum_{i \in \underline{AS}} y_i \right).$$

Given $\widetilde{p}_b < \widetilde{p}_s$, the equation above implies either

$$\sum_{i \in \overline{AB}} y_i - \sum_{i \in \overline{AB}} y'_i = \sum_{i \in \underline{AS}} y'_i - \sum_{i \in \underline{AS}} y_i = 0$$

or

$$\sum_{i \in \overline{AB}} y_i - \sum_{i \in \overline{AB}} y'_i > \sum_{i \in \underline{AS}} y'_i - \sum_{i \in \underline{AS}} y_i.$$

Since $y_b > 0$, in the first case there must be some buyer c such that $y'_c > 0$ and $\widetilde{p}_c = \widetilde{p}_b < \widetilde{p}_s$. But since $y'_s < 0$, proposition 1 implies $\widetilde{p}_c \geq \widetilde{p}_s$, a contradiction.

In the second case we have

$$\sum_{i \in \overline{AB}} y_i + \sum_{i \in \underline{AS}} y_i > \sum_{i \in \underline{AS}} y'_i + \sum_{i \in \overline{AB}} y'_i,$$

which implies

$$\sum_{i \in I} y_i > \sum_{i \in I} y'_i.$$

But since $y, y' \in F(w) \subseteq Y(w)$, we have $\sum_{i\in I} y_i = \sum_{i\in I} y'_i = 0$, a contradiction.

$\square$

Proposition 7. *If $y \in F(w)$, then either $y_i = \widetilde{q}_i$ for all $i \in AS(w)$ or $y_i = \widetilde{q}_i$ for all $i \in AB(w)$. Furthermore, if there is $y^* \in F(w)$ such that $y^*_i = \widetilde{q}_i$ for all $i \in AS(w)$, then $y_i = \widetilde{q}_i$ for all $y \in F(w)$ for all $i \in AS(w)$. Symmetrically, if there is $y^* \in F(w)$ such that $y^*_i = \widetilde{q}_i$ for all $i \in AB(w)$, then $y_i = \widetilde{q}_i$ for all $y \in F(w)$ for all $i \in AB(w)$.*

Proof. For the first part, from proposition 4, if buyer b and seller s are active, we have $p_b \geq p_s$. Thus, from proposition 2, there is no $y \in F(w)$ such that $y_s > \widetilde{q}_s$ and $y_b < \widetilde{q}_b$. Therefore, for every $y \in F(w)$, either $y_i = \widetilde{q}_i$ for all $i \in AS(w)$, or $y_i = \widetilde{q}_i$ for all $i \in AB(w)$.

For the second part, we show the proof for the active sellers' case, since the buyers' case is symmetric. Suppose there is $y^* \in F(w)$ such that $y^*_i = \widetilde{q}_i$ for all $i \in AS(w)$, and $y' \in F(w)$ such that $y'_a > \widetilde{q}_a$ for some $a \in AS(w)$. Then

$$\sum_{i\in B} y^*_i = -\sum_{i\in S} y^*_i > -\sum_{i\in S} y'_i = \sum_{i\in B} y'_i.$$

Therefore there must be an active buyer, say b, such that $y'_b < y^*_b \leq \widetilde{q}_b$. From proposition 4, we have $\widetilde{p}_b \geq \widetilde{p}_a$. But, from proposition 2, $y'_a > \widetilde{q}_q$ and $y'_b < \widetilde{q}_b$ imply $\widetilde{p}_b < \widetilde{p}_a$, a contradiction. $\square$

Proposition 8. *If seller a offers $(p, \widetilde{q}_a)$ in offer profile w and $a \in AS(w)$, then seller b who offers $(p, \widetilde{q}_b)$ where $\widetilde{q}_b < 0$ is also an active seller. Symmetrically, if buyer a offers $(p, \widetilde{q}_a)$ in strategy profile w and $a \in AB(w)$, then buyer b who offers $(p, \widetilde{q}_b)$ where $\widetilde{q}_b > 0$ is also an active buyer.*

Proof. We show the proof for the sellers' case, since the buyers' case is symmetric. Suppose $y_b = 0$ for all $F(w)$. Provided seller a is an active seller, there exists $y \in F(w)$ such that $y_a > 0$. Consider an allocation vector y' that $y'_i = y_i$ for $i \neq a, b$ and $y'_a = y_a - 1$, $y'_b = 1$. It's easy to see that $y' \in Y(w)$. The profit for the clearing house if y' is allocated is equal to

$$\sum_{i\in I} y'_i \widetilde{p}_i = \sum_{i\in I} y_i \widetilde{p}_i + p - p = \sum_{i\in I} y_i \widetilde{p}_i,$$

so that $y' \in \Pi(w)$. Thus, either $y' \in F(w)$ or there is $y'' \in F(w)$ such that $y'' \neq y'$ and $|y''_i| \geq |y'_i|$ for all i, so that in either case b is an active trader, contradicting the assumption. Therefore, as long as seller a is an active seller, so is seller b.

$\square$

Proof of Theorem 1

Suppose (p,q) is a competitive equilibrium. First we claim that the offer profile $w = ((p,q_i))$ induces the same outcome with probability one. To see this, since $\widetilde{p}_i = p$ for all i, the arbitrage profit for the clearing house is 0 for each $y \in Y(w)$, so that $\Pi(w) = Y(w)$. Clearly $q \in Y(w)$ since by definition of a competitive equilibrium $q_i \in Q_i$ and $\sum_i q_i = 0$. Moreover, by definition of $Y(w)$, for every $y \in Y(w)$ we have $|y_i| \leq |q_i|$. Hence q ray-dominates any other allocation in $\Pi(w)$ and is the unique element of $F(w)$. Thus, w induces the outcome $(q,(-pq_i))$ with probability one. This is precisely the outcome induced by the competitive equilibrium.

Next, we show that no individual i has an incentive to deviate from the offer profile $w = ((p,q_i))$. We consider deviations for buyers, since the proof for sellers is symmetric. We classify possible individual deviations for $i \in B$ from w into three categories, and show that none of them is profitable.

(i) Consider $w'_i = (p,q'_i)$ such that $Q_i \ni q'_i \neq q_i$. In any outcome with positive probability after that deviation, the utility for i is $u_i(y,-py)$ for some $y \in Q_i$. Since $q_i \in \arg\max_{q \in Q_i} u_i(q,-pq)$, we have that the expected utility after the deviation cannot be larger.

(ii) Consider $w'_i = (p'_i,q'_i)$ such that $q'_i \in Q_i$ and $p'_i < p$. Since every seller $s \in S$ is asking $\widetilde{p}_s = p$, by proposition 3 we must have that in any outcome with positive probability after that deviation $y_i = 0$. But then the expected utility after the deviation is 0, and since $q_i \in \arg\max_{q \in Q_i} u_i(q,-pq)$, we have $u_i(q_i,-pq_i) \geq u_i(0,-p \times 0) = 0$.

(iii) Consider $w'_i = (p'_i,q'_i)$ such that $q'_i \in Q_i$ and $p'_i > p$. Denote by w' the new offer profile. For any $y' \in F(w')$, buyer i gets a payoff of $u_i(y'_i,-p'_iy'_i)$. Note that $u_i(y'_i,-p'_iy'_i) < u_i(y'_i,-py'_i) \leq u_i(q_i,-pq_i)$, where the first inequality follows from $p'_i > p$ and the fact that $u_i(q,-pq)$ is decreasing in p, and the second from $q_i \in \arg\max_{q \in Q_i} u_i(q, -pq)$. It follows that $Eu_i(w') < Eu_i(w)$.

Proof of Theorem 2

The proof comes in several steps. Note that the 'thick market' condition (at least two active traders in the Nash equilibrium) is invoked only in the last step.

Lemma 1. *In each Nash equilibrium, all active sellers offer the same price, and all active buyers offer the same price.*

Proof. We prove the result for the sellers; the proof for the buyers is analogous. Take any offer profile w^* such that two active sellers offer different prices $\overline{p}$ and $\underline{p}$, with $\overline{p} > \underline{p}$. We can claim that the seller, say trader l, who offers $(\underline{p}, \widetilde{q}_l)$ would be better off submitting $(p', \widetilde{q}_l)$ such that $\underline{p} < p' < \overline{p}$.

To see this, from proposition 5, since there is another active seller offering the price $\overline{p}$, seller l sells $|\widetilde{q}_l|$ units when she offers $w_l^* = (\underline{p}, \widetilde{q}_l)$. We can show that seller l sells $|\widetilde{q}_l|$ units as well when she offers $w' = (p', \widetilde{q}_l)$. Suppose there is $y' \in F(w')$ such that $y_l' > \widetilde{q}_l$. As in the last step of the proof of proposition 5, for any $y \in F(w^*)$ we have

$$\sum_{i \in B} y_i = -\sum_{i \in S} y_i > -\sum_{i \in S} y_i' = \sum_{i \in B} y_i'.$$

Therefore there must be an active buyer at w^*, say b, such that $y_b' < y_b \leq \widetilde{q}_b$. From proposition 6, we have $\widetilde{p}_b \geq \overline{p}$. But, from proposition 4, at profile w' we have $y_l' > \widetilde{q}_l$ and $y_b' < \widetilde{q}_b$ implying $\widetilde{p}_b < p' < \overline{p}$, a contradiction.

Thus, by offering $(p', \widetilde{q}_l)$, seller l gets

$$u_l(\widetilde{q}_l, -p'\widetilde{q}_l) = -p'\widetilde{q}_l - \sum_{j=1}^{|\widetilde{q}_l|} r_{lj} > -\underline{p}\widetilde{q}_l - \sum_{j=1}^{|\widetilde{q}_l|} r_{lj},$$

where the last term is the payoff seller l gets by offering $(\underline{p}, \widetilde{q}_l)$. Hence the seller gets better off by offering $(p', \widetilde{q}_l)$, so that w^* cannot be a Nash equilibrium. $\square$

Lemma 2. *In each Nash equilibrium, all active traders offer the same price.*

Proof. Consider an offer profile w^* such that there is trade and such that all active buyers offer the same price, say p_b, and all active sellers offer the same price, say p_s. From lemma 1, we know that only such profiles, or some profiles such that there is no trade, can be Nash equilibria. By proposition 6, we have $p_s \leq p_b$. We will show that if $p_s < p_b$, at least one active trader has an incentive to deviate, so that w^* cannot be a Nash equilibrium.

If $p_s < p_b$, following proposition 7, we have that either $y_i = \widetilde{q}_i$ for all $y \in F(w^*)$ for all $i \in AS(w^*)$, or $y_i = \widetilde{q}_i$ for all $y \in F(w^*)$ for all $i \in AB(w^*)$. Suppose $y_i = \widetilde{q}_i$ for all $y \in F(w^*)$ for all $i \in AS(w^*)$ (the argument for the other case is analogous). Note that if there are inactive sellers in w^*, for any such seller h we have $\widetilde{p}_h > p_s$ or $\widetilde{q}_h = 0$. If $\widetilde{p}_h < p_s$, then following proposition 5 we have $y_h = \widetilde{q}_h$ for all $y \in F(w^*)$, so the seller can be inactive only if $\widetilde{q}_h = 0$. If $\widetilde{p}_h = p_s$, according to proposition 8 there must be some $y \in F(w^*)$ that $y_h < 0$ unless $\widetilde{q}_h = 0$. Denote by $p_{\neg s}$ the lowest price offered with a non-zero quantity by inactive sellers, if there is any, and note that in that case $p_{\neg s} > p_s$.

We claim that if $p_s < p_b$, an active seller, say a, would have an incentive to deviate from $w_a^* = (p_s, \widetilde{q}_a)$ to $w_a' = (p', \widetilde{q}_a)$, where $p' \in (p_s, \min\{p_b, p_{\neg s}\})$ if there are non-zero quantity inactive sellers, and $p' \in (p_s, p_b)$ otherwise.

To prove the claim, we argue first that for any $y' \in F(w')$ we have $y_a' = \widetilde{q}_a$. Suppose there is a $y' \in F(w')$ such that $y_a' > \widetilde{q}_a$. Then, from proposition 5, inactive sellers at w^* remain so at w' since $p_{\neg s} > p'$. Therefore $\sum_{i \in S} y_i' > \sum_{i \in S} y_i$ for every $y \in F(w^*)$. Thus, for any $y \in F(w^*)$,

$$\sum_{i \in B} y_i' = -\sum_{i \in S} y_i' < -\sum_{i \in S} y_i = \sum_{i \in B} y_i.$$

Therefore there must be an active buyer at w^*, say h, who offers p_b and gets $y_h' < y_h \leq \widetilde{q}_h$. But, from proposition 4, at profile w' we have $y_a' > \widetilde{q}_a$ and $y_h' < \widetilde{q}_h$ implying $p_b < p'$, a contradiction.

From the previous argument, by offering w' instead of w^*, seller a is allocated $\widetilde{q}_a$, and gets a utility of

$$u_a(\widetilde{q}_a, -p'\widetilde{q}_a) = -p'\widetilde{q}_a - \sum_{j=1}^{|\widetilde{q}_a|} r_{aj} > -p_s\widetilde{q}_a - \sum_{j=1}^{|\widetilde{q}_a|} r_{aj} = u_a(\widetilde{q}_a, -p_s\widetilde{q}_a).$$

Hence seller a gets better off by offering w', so that w^* cannot be a Nash equilibrium. $\square$

Lemma 3. *In each Nash equilibrium, every trader is indifferent between all outcomes that occur with positive probability.*

Proof. Consider an offer profile w^* such that all active traders, if there is any, offer the same price, say p. From lemma 2, we know that only such profiles can be Nash equilibria if trades happen with positive probability. Take trader a, a seller, for example. If seller a is inactive, then her utility is 0 for any positive probability outcome. Suppose a is active, and moreover there are $y, y'' \in F(w^*)$ that $u_a(y_a, -py_a) > u_i(y''_a, -py''_a)$. We can show that in this case, w cannot be a Nash equilibrium.

Since $F(w^*)$ is finite, there is some $y^* \in F(w^*)$ such that $u_a(y^*_a, -py^*_a) \geq u_a(y_a, -py_a)$ for all $y \in F(w^*)$ and moreover $u_a(y^*_a, -py^*_a) > u_a(y''_a, -py''_a)$. Since y'' has positive probability, $u_a(y^*_a, -py^*_a) > Eu_a(w)$. By continuity, there is some $p' < p$ such that $u_a(y^*_a, -py^*_a) > u_a(y^*_a, -p'y^*_a) > Eu_a(w)$.

We claim that if seller a offers $w'_a = (y^*_a, p')$, then $y'_a = y^*_a$ for every $y' \in F(w')$, so that the seller obtains $u_a(y^*_a, -p'y^*_a)$ which is a profitable deviation from w^* by the inequality above. The claim implies that w^* cannot be a Nash equilibrium. To verify the claim, suppose first that there is another seller h that is active at w'; since seller h offers the price $p > p'$, the claim follows from proposition 5. Suppose that no other seller is active at w', then if $y'_a > y^*_a$ we get for any $y' \in F(w')$,

$$\sum_{i \in B} y'_i = -\sum_{i \in S} y'_i < -\sum_{i \in S} y^*_i = \sum_{i \in B} y^*_i.$$

Then there must be some buyer, say b, such that $y'_b < y^*_b \leq \widetilde{q}_b$. Since there is also a seller, seller a, such that $y'_a > y^*_a$ and moreover this seller offers a price p' below the price offered by the buyer, we get a contradiction with proposition 4. $\square$

Lemma 4. *In every positive probability outcome of a Nash equilibrium with active trading, given the price offered by all active traders, all non-active traders are allocated utility-maximizing quantities.*

Proof. In the proof of this and the following lemmas, let w^* be a Nash equilibrium with active trading, and (invoking lemma 2) let p^* be the price offered by all active traders. We focus on sellers; the proof for the buyers is analogous.

As shown in the second paragraph of lemma 2, non-active sellers offer $\widetilde{p}_i > p^*$ or $\widetilde{q}_i = 0$. Therefore, they get $y_i = 0$ for all $y \in F(w^*)$, and thus obtain $Eu_i(w^*) = 0$. We claim that for inactive sellers, $y_i = 0$ is utility-maximizing given price p^*. Equivalently, we claim that $r_{i1} \geq p^*$.

To see this, suppose trader i is an inactive seller and $r_{i1} < p^*$. Consider a deviation for i to $w'_i = (p^*, -1)$. By proposition 8, if seller i is inactive under the offer profile w', so is every seller in $AS(w^*)$ under the offer profile w', and by proposition 5 so is every seller. But this would violate proposition 4, since there are trades in each side of the market active under w^* and thus offering p^* should induce positive probability to trade. Hence, there exists $y' \in F(w')$ such that $y'_i = -1$. Since $u_i(y'_i, -p^* y'_i) = p^* - r_{i1} > 0$, by deviating to offer $(p^*, -1)$, agent i would have $Eu_i(w') > 0$, so that w^* would not be a Nash equilibrium.

□

Lemma 5. *In every positive probability outcome of a Nash equilibrium with active trading, given the price offered by all active traders, all active traders are allocated quantities that are either utility-maximizing or involve less in absolute value than the utility-maximizing trade.*

Proof. For a given active seller, say s, let $\underline{\delta}_s$ and $\overline{\delta}_s$ be the minimal and the maximal element, respectively, of the set $\arg\max_{q_s \in Q_s} u_s(q_s, -p^* q_s)$, so that $-k \leq \underline{\delta}_s \leq \overline{\delta}_s \leq 0$. From the utility maximization problem, it follows that every $x \in Q_s$ such that $\underline{\delta}_s \leq x \leq \overline{\delta}_s$ is also a utility maximizer.

We claim that for every $y \in F(w^*)$ we have $y_s \geq \underline{\delta}_s$ so that either the seller is allocated an optimal trade or a smaller (in absolute value) than optimal trade. For suppose there is $y \in F(w^*)$ such that $y_s < \underline{\delta}_s$ so that $u_s(y_s, -p^* y) < u_s(\underline{\delta}_s, -p^* \underline{\delta}_s)$. If $\underline{\delta}_s = 0$ or $p^* = 0$, it follows that $u_s(y_s, -p^* y) < 0$, and by lemma 3, $Eu_s(w^*) < 0$. But then trader s can deviate to $(p^*, 0)$ and guarantee an expected utility of zero, so that w^* cannot be a Nash equilibrium. Suppose instead that $\underline{\delta}_s < 0$ and $p^* > 0$. By continuity, there is some $p' < p^*$ such that

$$u_s(y_s, -p^* y) < u_s(\underline{\delta}_s, -p'\underline{\delta}_s) < u_s(\underline{\delta}_s, -p^*\underline{\delta}_s).$$

Now consider a deviation by s to $w'_s = (p', \underline{\delta}_s)$. We show that such deviation guarantees $y'_s = \underline{\delta}_s$ for all $y' \in F(w')$, so that by Lemma 3, $Eu_s(w') = u_s(\underline{\delta}_s, -p'\underline{\delta}_s) > Eu_s(w^*)$. To see this, suppose there is some $y' \in F(w')$ such that $y'_s > \underline{\delta}_s$. Since $p' < p^*$, and all other sellers offer a price equal or larger than p^* or a quantity equal to zero, it follows from proposition 5 that for all other $i \in S$ we have $y'_i = 0$. Therefore

$$\sum_{i \in B} y_i = -\sum_{i \in S} y_i \geq -y_s > -\underline{\delta}_s = -\sum_{i \in S} y'_i = \sum_{i \in B} y'_i.$$

But then there must be a buyer, say a, such that $y'_a < y_a \leq \widetilde{q}_a$ offering price $p^* > p'$, contradicting proposition 4.

□

Lemma 6. *In every positive probability outcome of a Nash equilibrium with active trading, given the price offered by all active traders, if there are two or more active traders on the same side of the market, then all traders on this side of the market are allocated utility-maximizing quantities.*

Proof. We claim that if there are at least two active sellers, then every $y \in F(w^*)$ satisfies $\underline{\delta}_s \leq y_s \leq \overline{\delta}_s$ and is therefore a utility maximizer.

In lemma 5 we have shown in every positive probability allocation y, active sellers are allocated quantities that are either utility-maximizing given the price or involve less trade ($\underline{\delta}_s \leq y_s \leq 0$) so we need only focus on active sellers.

Now suppose there are two active sellers, say s and h. If $y_s < \underline{\delta}_s$ for any $y \in F(w^*)$, we have that w^* cannot be a Nash equilibrium by the previous step. If $\underline{\delta}_s \leq y_s \leq \overline{\delta}_s$, the claim follows from lemma 3. In the last part of this proof, we show that if there is a $y \in F(w^*)$ such that $y_s > \overline{\delta}_s$, w^*

cannot be a Nash equilibrium.

Since $|y_s| < |\overline{\delta}_s|$, from the utility maximization problem we must have $r_{|y_s|+1} < p^*$. Hence $u_s(y_s - 1, -p^*(y_s - 1)) - u_s(y_s, -p^* y_s) = p^* - r_{|y_s|+1} > 0$. By continuity, there is some $p' < p^*$ such that

$$u_s(y_s, -p^* y) < u_s(y_s - 1, -p'(y_s - 1)) < u_s(y_s - 1, -p^*(y_s - 1))).$$

Also, for any $y, y'' \in F(w^*)$, we have $y_s = y''_s$. Suppose there exists $y, y'' \in F(w^*)$ such that $y_s < y''_s$, then

$$u_s(y''_s, -p^* y''_s) - u_s(y_s, -p^* y_s) = -p^*(y''_s - y_s) + \sum_{j=|y''_s|+1}^{|y_s|} r_{sj} < 0,$$

contradicting lemma 3.

Now consider a deviation by s to $w'_s = (p', y_s - 1)$. We show that such deviation guarantees $y'_s = y_s - 1$ for all $y' \in F(w')$, so that by Lemma 3, $Eu_s(w') = u_s(y_s - 1, -p'(y_s - 1)) > Eu_s(w^*)$. To see this, suppose there is some $y' \in F(w')$ such that $y'_s > y_s - 1$. Since $p' < p^*$, and all other sellers offer a price equal or larger than p^* or a quantity equal to zero, it follows from proposition 5 that for all other $i \in S$ we have $y'_i = 0$. Therefore, take any $y'' \in F(w^*)$ such that $y''_h < 0$,

$$\sum_{i \in B} y''_i = -\sum_{i \in S} y''_i \geq -y_s - y''_h \geq -y_s + 1 > -y'_s = -\sum_{i \in S} y'_i = \sum_{i \in B} y'_i$$

But then there must be a buyer, say a, such that $y'_a < y''_a \leq \widetilde{q}_a$ offering price $p^* > p'$, contradicting proposition 4. $\square$

Since the market clearing condition in the equilibrium definition is satisfied by any allocation induced by any offer profile, theorem 2 follows from lemma 6.

Properties of v_b and v_s

Lemma 7. *In every competitive equilibrium $(p,q) \in \xi(r)$ that contains the smallest number of transactions, the lowest reservation value of buyers' traded unit(s) is equal to v_b, and the highest reservation value of sellers' traded unit(s) is equal to v_s.*

Proof. We show the proof for v_b; the proof for v_s is analogous. By definition of v_b, there is a competitive equilibrium $(\hat{p},\hat{q})$ such that every unit bought has a buyer's valuation greater than or equal to v_b. Suppose there is a competitive equilibrium $(\tilde{p},\tilde{q})$ such that a buyer, say $i \in B$, buys a unit with valuation strictly below v_b. Then it must be the case that $\tilde{p} < v_b$. But then we have that $\tilde{q}_i > \hat{q}_i$ and for every $j \in B \setminus \{i\}$, $\tilde{q}_j \geq \hat{q}_j$, so that strictly more units are traded in $(\tilde{p},\tilde{q})$ than in $(\hat{p},\hat{q})$. $\square$

Proof of Theorem 3

First we prove the condition in the statement of the theorem is sufficient. Suppose w^* is a Nash equilibrium with active trading, and suppose there are at least two inframarginal sellers and at least two weakly inframarginal buyers. (The other case is analogous.) From lemma 1 and lemma 2, all active traders offer the same price, say p^*. Denote $\underline{\delta}_i$ and $\overline{\delta}_i$ the minimal and maximal element, respectively, of the set $\arg\max_{q_i \in Q_i} u_i(q_i, -p^* q_i)$. From lemma 5, for any $y \in F(w^*)$, we have $\underline{\delta}_i \leq y_i \leq 0$ for every active seller i, and $0 \leq y_i \leq \overline{\delta}_i$ for every active buyer i, and moreover from lemma 4, non-active traders acquire utility-maximizing quantities given p^*. That is, no one trades in excess of their utility-maximizing quantity.

Consider first the case $p^* > v_s$. We claim that every inframarginal seller must be active. For suppose an inframarginal seller i is not active; then the seller is making a payoff equal to zero in every allocation $y \in F(w^*)$. But by deviating unilaterally to $w_i'(p,1)$ for any $v_s < p < p^*$, the seller can guarantee herself a positive payoff $u_i(-1,p) = -r_{i1} + p > -v_s + p^* > 0$ in every allocation with positive probability given the new offer profile. Hence, two or more sellers are active in w^*. If two or more buyers are active in w^*, then applying theorem 2, p^* is a competitive price and all the outcomes from the Nash equilibrium are competitive.

If only one buyer is active in w^*, say buyer a, we must have that at least one weakly inframarginal buyer, say buyer c, who is not active in w^*. Since c is not active in w^*, we must have $p^* \geq r_{c1}$; otherwise c has a profitable deviation. Therefore $p^* \geq r_{c1} \geq \underline{p}$. If $p^* > \overline{p}$, then for every $y \in F(w^*)$,

$$\sum_{i \in B} y_i \leq \sum_{i \in B} \overline{\delta}_i < -\sum_{i \in S} \underline{\delta}_i \leq -\sum_{i \in S} y_i,$$

violating the allocation rule of the clearing house. The first and the last inequality comes from lemma 6 which implies that for all the active sellers $y_i \in [\underline{\delta}_i, \overline{\delta}_i]$ since there are at least two of them, and from lemma 5 which implies that for any active buyer $y_i \leq \overline{\delta}_i$. The strict inequality in the middle is a result of the price being higher than any competitive price. Hence $\underline{p} \leq p^* \leq \overline{p}$ so that p^* is a competitive price.

Now suppose that there is an allocation $y \in F(w^*)$ such that $y_a < \underline{\delta}_a$. Since p^* is competitive, in any competitive equilibrium allocation (q_i), we have $-\sum_{i \in S} q_i \geq \underline{\delta}_a$. Thus in every competitive equilibrium at price p^*, there exists at least one seller s that has $q_s < y_s$. Since $y_s, q_s \in [\underline{\delta}_s, \overline{\delta}_s]$, we have $r_{s,|q_s|} = p^*$. Hence for any competitive equilibrium at p^*, there is at least a traded unit with reservation value p^* for a seller. By definition of v_s this implies $p^* \leq v_s$, a contradiction to the assumption. Therefore for the only active buyer a, $y_a \in [\underline{\delta}_a, \overline{\delta}_a]$ for every $y \in F(w^*)$. Hence, all traders obtain utility-maximizing quantities given p^*, and every outcome $y \in F(w^*)$ is competitive.

Consider the remaining case $p^* \leq v_s$. Since $p^* < r_{1i}$ for every weakly inframarginal buyer, it follows that there are at least two active buyers in Nash equilibrium and moreover every buyer chooses utility-maximizing quantities given p^*. As in the previous proof, if there are two or more active sellers, then, from theorem 2, all outcomes in $F(w^*)$ are competitive. Similarly, if there is a unique active seller a and $y_a \in [\underline{\delta}_a, \overline{\delta}_a]$ for every $y \in F(w^*)$, then all traders obtain utility-maximizing quantities given p^*, and every outcome $y \in F(w^*)$ is competitive. The remaining case is that there is a unique active seller a and $\overline{\delta}_a < y_a < 0$, so that $\sum_{i \in B} y_i = -y_s < -\overline{\delta}_s$.

Suppose $p^* = v_s = \underline{p}$. Since p^* is a competitive price, in every competitive equilibrium allocation (q_i), we have $\sum_{i \in B} q_i \geq -\overline{\delta}_s$; i.e. aggregate demand should be able to meet an individual seller's supply. Thus in every competitive equilibrium at p^*, there exists at least one buyer b that has $q_b > y_b$. Since $y_b, q_b \in [\underline{\delta}_b, \overline{\delta}_b]$, we have $r_{b,q_b} = p^*$. Hence in every competitive equilibrium at p^*, there is at least one traded unit with reservation value p^* for a buyer. By definition of v_b, this implies $p^* \geq v_b$. Using $v_b > v_s$ we get a contradiction to the assumption $p^* = v_s$.

Finally, suppose $p^* = v_s < \underline{p}$ or $p^* < v_s$. In either case, $p^* < \underline{p}$, and

$$-\sum_{i \in S} y_i \leq -\sum_{i \in S} \overline{\delta}_i < \sum_{i \in B} \underline{\delta}_i \leq \sum_{i \in B} y_i,$$

violating the allocation rule of the clearing house. The first and the last inequality comes from lemma 5 which implies that for any active seller $y_i \geq \underline{\delta}_i$ and from lemma 6 which implies that for all the active buyers $y_i \in [\underline{\delta}_i, \overline{\delta}_i]$ since there are at least two of them. The strict inequality in the middle is a result of the price being lower than any competitive price.

This finishes the proof of sufficiency. We now prove that the condition is necessary. Since at least two units are traded in every competitive equilibrium, there is at least one inframarginal trader on each side of the market. Possible violations of the condition in the theorem are that, among the remainder of traders, either (a) there is no additional weakly inframarginal trader on one side of the market, or (b) there is no additional inframarginal trader in either side.

Consider case (a), and suppose without loss of generality that trader 1 is the unique weakly inframarginal seller, so that every seller $i \in S \setminus \{1\}$ is such that either $r_{i1} \geq v_b$ or $r_{i1} > \overline{p}$; recall that each of these conditions imply $r_{i1} > v_s$. Take a competitive equilibrium that has the smallest number of units traded, and denote the allocation by $\hat{q} = (\hat{q}_i)$. From lemma 7, $\hat{q}_i = 0$ for every seller $i \in S \setminus \{1\}$. From lemma 7 as well, a unit of value v_b is bought by at least one buyer, say without loss of generality buyer 2, and moreover for every buyer j such that $q_j > 0$ we must have $r_{j,\hat{q}_j} \geq v_b$. Recall that the highest equilibrium price $\overline{p}$ satisfies $\overline{p} \leq v_b$, and moreover $(\overline{p}, \hat{q})$ is a competitive equilibrium.[1] Suppose first that $\overline{p} = v_b$. Consider the offer profile w such that $w_1 = (v_b, \hat{q}_1 + 1)$ (seller 1 sells one fewer unit than in the competitive equilibrium), $w_2 = (v_b, \hat{q}_2 - 1)$ (buyer 2 buys one fewer unit), and $w_i = (v_b, \hat{q}_i)$ for every $i \neq 1,2$. It is easy to check that no trader has a profitable deviation; buyer 2 in particular is indifferent between buying one more unit or not.

Now suppose that $\overline{p} < v_b$. Define

$$\tilde{p} = \begin{cases} \min\left\{\min_{i \in S \setminus \{1\}} r_{i1}, v_b\right\} & \text{if } S \setminus \{1\} \neq \emptyset \\ v_b & \text{if } S \setminus \{1\} = \emptyset \end{cases},$$

and consider the offer profile $\tilde{w}$ such that $\tilde{w}_i = (\tilde{p}, \hat{q}_i)$ for all $i \in S \cup B$. It is easy to check that no trader has a profitable deviation. But the induced outcome is not competitive since $\tilde{p} > \overline{p}$.

Consider case (b), and suppose without loss of generality that trader 1 is the unique inframarginal seller and that trader 2 is the unique inframarginal buyer, so that for every seller $i \in S \setminus \{1\}$ and buyer $j \in B \setminus \{2\}$, $r_{i1} > v_s$ and $r_{j1} < v_b$. Take a competitive equilibrium $(\hat{p}, \hat{q})$ that has the smallest number of units traded. Since $v_s \leq \hat{p} \leq v_b$, traders 1 and 2 are the only traders who are trading in $\hat{q}$. Consider the offer profile $w_1 = (\hat{p}, -1)$, $w_2 = (\hat{p}, 1)$, and $w_k = (\hat{p}, 0)$ for every $k \in S \cup B \setminus \{1,2\}$. No trader has a profitable deviation, but this offer profile induces an allocation which is not competitive under the assumption that at least two units are traded in competitive equilibrium.

A.3 Instructions and Quizes for Chapter 2

A.3.1 Instructions for CH treatments

Instructions

Welcome to today's experiment! You have earned \$5 for showing up on time. The following instructions will explain how you can make decisions and earn more money, so please read them carefully. During the experiment, please keep your cell phone turned off, and refrain

[1] In quasilinear economies, the set of competitive equilibria is the product of the set of competitive allocations and the set of competitive equilibrium prices.

from talking to other participants. If at some point you have a question, please raise your hand, and we will address it with you privately.

In the experiment, you will be grouped anonymously with three other participants, whose identities will not be revealed. Two of the participants in your group will be buyers, and the other two will be sellers. Your group and your role will remain the same throughout the experiment. Your role will be revealed to you at the beginning of the experiment.

There will be 20 formal rounds. In each round, each of the two buyers has the opportunity to buy up to 2 units of the good from the two sellers in the same group, and each of the two sellers has the opportunity to sell up to 2 units of the good to the two buyers in the same group.

Obtaining each unit of the good generates a value for the buyer, and selling each unit of the good incurs a cost to the seller. The values to a buyer and the costs to a seller may vary by unit. Values may vary between buyers and costs may vary between sellers.

Your own values (if you are a buyer) or costs (if you are a seller) will be revealed to you at the beginning of the experiment. Your values/costs remain constant throughout the experiment. The values/costs of other participants will NOT be revealed to you.

Payoffs

The values and costs are in US Dollars. A buyer's payoff in one round equals the value she obtains from the unit(s) she buys minus the total price she pays for her purchase. A seller's payoff in one round equals the revenue she gets from the sale minus the cost incurred for the unit(s) she sells.

Buyer's payoff = value obtained from purchase – payment for purchase

Seller's payoff = revenue from sale – cost incurred for sale

For example, suppose Buyer A generates a value of \$4 from buying the first unit, and \$3 from buying the second. If Buyer A obtains 2 units at the unit price of \$2, then

$$\text{Buyer A's payoff} = \underbrace{(\$4+\$3)}_{\text{Values}} - \underbrace{(\$2+\$2)}_{\text{Payment}} = \$3$$

Suppose Seller A sells 1 unit at the price of \$5.6, and her cost is \$1 for selling the first unit and \$3 for selling the second. Then

$$\text{Seller A's payoff} = \underbrace{\$5.6}_{\text{Revenue}} - \underbrace{\$1}_{\text{Cost(s)}} = \$4.6$$

Since Seller A does not sell the second unit, only the cost of the first unit incurs.

If a participant does not trade in a round, her payoff from that round is \$0.

The payoffs from different rounds do not accrue. At the end of the experiment, one of the 20 formal rounds will be randomly chosen. Your total earnings in this experiment will be your payoff from the chosen round, plus the \$5 show-up bonus.

How to trade

Each group trades in its own market. In each round, the market opens for 2 minutes, during which each participant can submit an offer. In a buying offer, a buyer submits a unit price, together with how many units (1 or 2) she would like to buy for that price. In a selling offer, a seller submits a unit price, and how many units (1 or 2) she would like to sell for that price. The offer you submit will NOT be shown to any other participant.

Please note that you can submit only ONE offer in each round, and you cannot revise your offer once you submit it.

After two minutes, or once every participant has submitted a unit price and quantity, transactions will be determined under the rules below, as demonstrated in the following example.

Example

Suppose the submitted offers are as follows.

Buyer A: buying offer for 1 unit, at the unit price of \$3

Buyer B: buying offer for 2 units, at the unit price of \$1

Seller A: selling offer for 1 unit, at the unit price of \$4

Seller B: selling offer for 1 unit, at the unit price of \$2.

Please note that this example is only for demonstration of the procedure, the submitted offers will NOT be shown to any participant in the experiment.

- **Sort Orders** Firstly, buying offers and selling offers will be sorted separately. If an offer contains two units (eg. Buyer B's offer), it will be split into TWO IDENTICAL offers, each containing one unit. Buying offers for each unit will be queued in descending order, and selling offers for each unit will be queued in ascending order, as the following table shows.

Buying offers for one unit (high to low)	Selling offers for one unit (low to high)
\$3 (from Buyer A)	\$2 (from Seller B)
\$1 (from Buyer B)	\$4 (from Seller A)
\$1 (from Buyer B)	

  In case of tied buying offers or tied selling offers, the order of them will be randomly determined.

- **Trade Units** After the orders are sorted, each buying offer and selling offer at the same position in the queues will be compared. As long as the buying price is no lower than the selling price, the corresponding buyer and seller make a trade.

  The first buying offer in the queue (\$3 from Buyer A) and the first selling offer (\$2 from Seller B) make a trade since 3>2. The second buying offer and selling offer cannot trade since the buying price (\$1 from Buyer B) is lower than the selling price (\$4 from Seller A). The third buying offer cannot be fulfilled since there is not a selling offer corresponding to it. By this procedure, the buying offer with higher price is more likely to be fulfilled, and so is the selling offer with lower price.

- **Prices** When a trade happens, the buyer will pay the price she offered and get one unit of the good, and the seller will receive the price she asked for and sell one unit of the good. In this example, one unit of the good is traded. Buyer A pays \$3 for the unit she bought, as she offered to. Seller B gets \$2 for the unit she sells, as she asked for.

In each round, a participant who does not submit any offer will not make any trade. To prevent losing money, a buyer/seller cannot submit an offer that could cause a loss for her.

Summary of Each Round

The market for each group opens at the beginning of each round. After each participant in your group submits an offer or when the market closes, you will be informed of how many units you trade, and your payoff in the current round. Also, the price(s) for each traded unit in your market will be revealed anonymously to all participants in your group. You will NOT be informed of the buying/selling offers that do not result in trade.

This is the end of the instructions. We now proceed to a quiz to ensure everyone understands the instructions. The experiment will begin after everyone gives a correct answer to each question. Before the formal rounds begin, there will be a practice round, which does not count towards payment.
Again, if you have any question at any point of the experiment, please raise your hand and an experimenter will assist you.

A.3.2 Quiz for CH treatments

Quiz

1. True or False. Circle your answers.

Your role (buyer or seller) will remain the same in all of the rounds.	T	F
Your group does not change throughout the experiment.	T	F
In each round, you can revise your offer after you submit it.	T	F
Your costs or values will not change between rounds.	T	F
Your offer in each round will not be shown to other participants.	T	F

2. Suppose the offers submitted are as follows.

Buyer A: buying offer for 2 units, at the unit price of $3

Buyer B: buying offer for 1 unit, at the unit price of $5

Seller A: selling offer for 2 units, at the unit price of $1

Seller B: selling offer for 1 unit, at the unit price of $2.

(a) Use the procedure demonstrated in the instructions, fill out the buying and selling offers in the table.

Buying offers for one unit (high to low)	Selling offers for one unit (low to high)
\$5 (from Buyer B)	\$1 (from Seller A)
\$_____ (from Buyer _____)	\$1 (from Seller A)
\$3 (from Buyer A)	\$_____ (from Seller _____)

(b) How many units does Buyer A buy? ______ unit(s)

(c) How much does Buyer A pay for the unit(s) she buys <u>in total</u> ? \$______

(d) Suppose the first unit Buyer A obtains will generate a value of \$5 to her, and the second unit she obtains will generate \$4. What is Buyer A's payoff here?

$$\text{Buyer A's payoff} = \underbrace{\$\underline{\hspace{2cm}}}_{\text{Value(s)}} - \underbrace{\$\underline{\hspace{2cm}}}_{\text{Payment}} = \$\underline{\hspace{2cm}}$$

(e) Suppose the first unit Seller B sells will cost her \$0.5, and the second unit she sells will cost \$2.5. What is Seller B's payoff here?

$$\text{Seller B's payoff} = \underbrace{\$\underline{\hspace{2cm}}}_{\text{Revenue}} - \underbrace{\$\underline{\hspace{2cm}}}_{\text{Cost(s)}} = \$\underline{\hspace{2cm}}$$

A.3.3 Instructions for DA treatments

Instructions

Welcome to today's experiment! You have earned \$5 for showing up on time. The following instructions will explain how you can make decisions and earn more money, so please read them carefully. During the experiment, please keep your cell phone turned off, and refrain from talking to other participants. If at some point you have a question, please raise your hand, and we will address it with you privately.

In the experiment, you will be grouped anonymously with three other participants, whose identities will not be revealed. Two of the participants in your group will be buyers, and the other two will be sellers. Your group and your role will remain the same throughout the experiment. Your role will be revealed to you at the beginning of the experiment.

There will be 20 formal rounds. In each round, each of the two buyers has the opportunity to buy up to 2 units of the good from the two sellers in the same group, and each of the two sellers has the opportunity to sell up to 2 units of the good to the two buyers in the same group.

Obtaining each unit of the good generates a value for the buyer, and selling each unit of the good incurs a cost to the seller. The values to a buyer and the costs to a seller may vary by unit. Values may vary between buyers and costs may vary between sellers.

Your own values (if you are a buyer) or costs (if you are a seller) will be revealed to you at the beginning of the experiment. Your values/costs remain constant throughout the experiment. The values/costs of other participants will NOT be revealed to you.

Payoffs

The values and costs are in US Dollars. A buyer's payoff in one round equals the value she obtains from the unit(s) she buys minus the total price she pays for her purchase. A seller's payoff in one round equals the revenue she gets from the sale minus the cost incurred for the unit(s) she sells.

Buyer's payoff = value obtained from purchase – payment for purchase

Seller's payoff = revenue from sale – cost incurred for sale

For example, suppose Buyer A generates a value of \$4 from buying the first unit, and \$3 from buying the second. If Buyer A obtains the first unit at the price of \$2 and the second unit at the price of \$1, then

$$\text{Buyer A's payoff} = \underbrace{(\$4+\$3)}_{\text{Values}} - \underbrace{(\$2+\$1)}_{\text{Payment}} = \$4$$

Suppose Seller A sells 1 unit at the price of \$5.6, and her cost is \$1 for selling the first unit and \$3 for selling the second. Then

$$\text{Seller A's payoff} = \underbrace{\$5.6}_{\text{Revenue}} - \underbrace{\$1}_{\text{Cost(s)}} = \$4.6$$

Since Seller A does not sell the second unit, only the cost of the first unit incurs.

If a participant does not trade in a round, her payoff from that round is \$0.

The payoffs from different rounds do not accrue. At the end of the experiment, one of the 20 formal rounds will be randomly chosen. Your total earnings in this experiment will be your payoff from the chosen round, plus the \$5 show-up bonus.

How to trade

Each group trades in its own market. In each round, the market opens for a maximum of two minutes, during which each participant can submit offers. In a buying offer, a buyer submits a price she is willing to buy a unit at. In a selling offer, a seller submits a price she is willing to sell a unit at. For each participant, only after her first unit is traded can she trade her second unit.

The timer on the screen counts down the time remaining for the current round. The timer starts from two minutes at the beginning of each round, then jumps to 20 seconds once a participant attempts to submit an offer, and restarts from 20 seconds every time a participant attempts to submit an offer. The round finishes if two minutes elapse, or if no new attempt occurs within 20 seconds of the last attempt, whichever occurs first.

The attached pages are screen shots of the interface for a seller and a buyer in the same market. Screen shot 1 is for the seller. Screen shot 2 is for the buyer.

From left to right in the upper part of the interface are the ***Submit Your Offer*** section, where you can enter the price for each of your offers; the section for ***general information***, where you can see the number of rounds, your role, time remaining in the current round, and your real-time payoff in the current round; ***Your Values/Costs*** section, where you can see the values or costs for your units and whether they are traded or not.

On the lower part of the interface, from left to right are the ***Selling Offers*** section, which lists the selling offers from low to high; the ***Buying Offers*** section, which lists the buying offers from high to low; the ***Transactions*** section, which displays all transactions in your market in the current round. Your own offers and transactions will be highlighted on the lists.

- **How to Sell**

- **Offer to Sell**

  You can offer to sell one unit by submitting a price in the ***Submit Your Offer*** section. When you make an offer, the price has to be lower than the lowest selling offer at the time, which is the top one on the ***Selling Offers*** list. If you make a new offer, it will replace your previous offer.

  As shown in the screen shots, the lowest selling offer is $3, so if any of the sellers wants to make a new offer, she has to offer a price lower than $3.

  To prevent losing money, you cannot submit an offer that could cause a loss for you.

- **Accept A Buying Offer**

  You can sell one unit by submitting a price equal to the highest buying offer, which is the top one on the ***Buying Offers*** list. By doing so, you sell the unit to the buyer and incur the cost, the buyer pays you the price you submitted. (If you submit a price lower than the highest buying offer, you sell the unit at the price you submit.) In the example from the screen shots, the highest buying offer is $2, if a seller submits an offer of $2, she sells the unit to the buyer, and the buyer pays her $2.

- **Transactions**

  There are two ways you sell one unit. Your selling offer is accepted by a buyer, or you accept a buying offer. When you sell one unit, your offer for that unit will be removed from the list, the transaction will be recorded, and your payoff will be updated. Then you may offer to sell your second unit or accept another buying offer on the list. The rules are the same as for the first unit.

* **How to Buy**

  - **Offer to Buy**

    You can offer to buy one unit by submitting a price in the ***Submit Your Offer*** section. When you make an offer, the price has to be higher than the highest buying offer at the time, which is the top one on the ***Buying Offers*** list. If you make a new offer, it will replace your previous offer.

As shown in the screen shots, the highest buying offer is \$2, so if any of the buyers wants to make a new offer, she has to offer a price higher than \$2.

To prevent losing money, you cannot submit an offer that could cause a loss for you.

– **Accept A Selling Offer**

You can buy one unit by submitting a price equal to the lowest selling offer, which is the top one on the ***Selling Offers*** list. By doing so, you buy the unit from the seller and obtain the value, and pay the seller the price you submitted. (If you submit a price higher than the lowest selling offer, you buy the unit at the price you submit.) In the example from the screen shots, the lowest selling offer is \$3, if a buyer submits an offer of \$3, she buys the unit from the seller, and pays the seller \$3.

– **Transactions**

There are two ways you buy one unit. Your buying offer is accepted by a seller, or you accept a selling offer. When you buy one unit, your offer for that unit will be removed from the list, the transaction will be recorded, and your payoff will be updated. Then you may offer to buy your second unit or accept another selling offer on the list. The rules are the same as for the first unit.

Summary of Each Round

The market for each group opens at the beginning of each round. A seller can make selling offers, or accept buying offers, by submitting prices on the interface. A buyer can make buying offers, or accept selling offers, by submitting prices on the interface. When an offer is accepted, a transaction happens. Offers, transactions and your payoff in the current round will be displayed on your screen.

This is the end of the instructions. We now proceed to a quiz to ensure everyone understands the instructions. The experiment will begin after everyone gives a correct answer to each question. Before the formal rounds begin, there will be one practice round, which does not

count towards payment.

Again, if you have any question at any point of the experiment, please raise your hand and an experimenter will assist you.

A.3.4 Quiz for DA treatments

Quiz

1. True or False. Circle your answers.

Your role (buyer or seller) will remain the same in all of the rounds.	T	F
Your group does not change throughout the experiment.	T	F
Your costs or values will not change between rounds.	T	F
You can submit offers for both of your units at the same time.	T	F

2. Suppose you are a **buyer**, and the lists of offers are as follows. Your offer is highlighted.

Selling Offers	Buying Offers
\$4	\$3
\$5	\$1

(a) Which of the following prices can you submit as a new offer? Circle your answer.

A. 2 B. 0.5 C. 3.7 D. 1.5

(b) Which of the following prices can you submit to accept the selling offer of \$4? Circle you answer.

A. 4 B. 2.5 C. 1.2 D. 3

(c) If you accept the lowest selling offer on the list, and your values for the first and second unit are \$7 and \$6 respectively, what is your payoff?

$$\text{Your payoff} = \underbrace{\$\_\_\_\_\_\_\_\_}_{\text{Value(s)}} - \underbrace{\$\_\_\_\_\_\_\_\_}_{\text{Payment}} = \$\_\_\_\_\_\_\_\_$$

3. Suppose you are a **seller**, and the lists of offers are as follows. Your offer is highlighted.

Selling Offers	Buying Offers
\$4	\$3
\$5	\$1

(a) Which of the following prices can you submit as a new offer? Circle your answer.

A. 6 B. 2.1 C. 4 D. 5

(b) Which of the following prices can you submit to accept the buying offer of \$3? Circle you answer.

A. 3.5 B. 4.1 C. 5 D. 3

(c) Suppose the first and second unit you sell will cost \$0.1 and \$0.4 respectively, and you accept both buying offers on the list. What is your payoff?

$$\text{Your payoff} = \underbrace{\$\_\_\_\_\_\_\_\_\_\_}_{\text{Revenue}} - \underbrace{\$\_\_\_\_\_\_\_\_\_\_}_{\text{Cost(s)}} = \$\_\_\_\_\_\_\_\_\_\_$$

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