A DESIGN RESEARCH PROJECT: INVESTIGATING AN INTEGRATED MATHEMATICS INSTRUCTIONAL SYSTEM (IMIST) FOCUSED ON INQUIRY LEARNING AND PROFICIENCIES IN SYMBOLIC LITERACY, CONCEPTUAL LITERACY, AND PROBLEM-SOLVING FOR HIGH SCHOOL MATHEMATICS

by

Laura C. McConnaughey
A Dissertation
Submitted to the
Graduate Faculty
of
George Mason University
in Partial Fulfillment of
The Requirements for the Degree
of
Doctor of Philosophy
Education

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Date: ________________________________ Fall Semester 2019
George Mason University
Fairfax, VA

A Design Research Project: Investigating an Integrated Mathematics Instructional System (IMIST) Focused on Inquiry Learning and Proficiencies in Symbolic Literacy, Conceptual Literacy, and Problem-Solving for High School Mathematics

A Dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy at George Mason University

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Dedication

I dedicate this project to my family. I could never have made or completed the journey without your love and support.

First, to my cheerleaders – my sister, Daphne Gill, and my stepfather, Donald Brian Ward. Daph, you were with me everyday. Saying “thank you” does not express the depth of my gratitude and love. Dad, I know how much you wanted to be here for the finish. Thank you for always being proud of me. I know you are here in spirit.

Next, to my grandmothers – Helen Cornell Powers Pottenger Hops and Esther Louise Campbell McConnaughey (ABD). Potchie, you always supported me in my academic endeavors. Grandmother McC, you told be to make sure to finish. I hear you saying, “Get her done!” You both inspired my efforts and my finish.

To my children – Ian, Scott, Jamie, and Leslie. Thank you for giving up time with your mom so that I could lay the groundwork for the rest-of-my-life. Thank you for letting me help my “other kids.” Always know that you are my “real kids.” I have loved the support, feedback, correction, and concern. It has been a joy to see you grow and become such awesome people. I love you all so much and look forward to seeing what the next thirty years will bring!

To Kevin – Thank you for giving up time with your wife. When you ask me what I am doing, the automatic response will no longer be “grading papers.” What a change! Love you, sweetheart.

Acknowledgements

I would like to acknowledge the primary chair of my dissertation committee, Dr. Priscilla Norton. Thank you for taking an orphaned math teacher under your wing and for mapping out my journey. I am a better researcher, scholar, and writer thanks to your tutelage. I am deeply grateful. I also would like to thank Dr. Dawn Hathaway and Dr. Toya Frank for supporting me through this process. It was a privilege to study with you and to learn from you both. Thank you for the effort you have made on my behalf.

I would like to acknowledge and thank my dear colleagues and mentors who have worked with me at schools and as advisors during this dissertation phase of my journey: Pat Butler (Norwell), Mary Ann Donohue (TJ, Editor-in-Chief), Anna Frome, Jerry Berry, and Pat Gabriel (TJ), Susan Steiner (Kilmer), Dr. Christi Bergin and Dr. David Bergin (Mizzou/University of Missouri). All of you have made invaluable contributions to the educator that I have become. Thank you for your guiding influences.

I appreciate and acknowledge the administration and mathematics department staff at BDJO. Thank you for providing me with the opportunity to work with your students. The research and lessons learned from my small class provided valuable insights for this project.

Finally, I would like to acknowledge the hundreds of students who have helped me gain experience and perspectives on mathematics teaching and learning over the years. A special shout out goes to all my case study students, especially Howard, Hailey, Bobcat, and Kaya. Thank you for your effort, your time, and most of all, your wonderful insights and feedback.

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Abstract

A DESIGN RESEARCH PROJECT: INVESTIGATING AN INTEGRATED MATHEMATICS INSTRUCTIONAL SYSTEM (IMIST) FOCUSED ON INQUIRY LEARNING AND PROFICIENCIES IN SYMBOLIC LITERACY, CONCEPTUAL LITERACY, AND PROBLEM-SOLVING FOR HIGH SCHOOL MATHEMATICS

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George Mason University, 2019

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The combination of stakeholders' concerns about mathematics education in the United States – students' lack of proficiencies, problem-solving skills, and preparation for 21st century careers in stem fields – together with the conflict between educators using different and often fragmented curricula and instructional methods (traditional versus reform-based) suggested a need to develop an integrated framework to help teachers design curricula that used the best research-based practices to improve students' achievement, engagement, and proficiencies in mathematics.

This study was the second phase or enactment phase of a design research project that examined prototypical units for Algebra 2 and geometry. This study investigated the IMIST (Integration of Mathematical Inquiry, Symbolic literacy, and Technology) instructional system developed using principles from design thinking, a design pattern

approach, and activity theory as a possible solution to help educators design curricula to meet the learning needs of students and to support students' development of strong mathematical foundations in symbolic literacy, conceptual literacy, and problem-solving applications. The research questions for this study examined the impact of two IMIST units on students' overall achievement and mastery of symbolic, conceptual, and problem-solving literacies; on students' attitudes, engagement, and confidence in mathematics; and reports of their experiences using the IMIST unit learning activities.The IMIST system framework used unit learning objectives to identify authentic, contextual problem-solving applications with supporting symbolic and conceptual learning activities to build and scaffold learning needed for deep understanding of mathematical language and applications.

Thirteen students, ages 10 to 15, participated in four separate case studies: two individual, one paired, and one small class. The treatment for each case study was a packet of learning activities designed and written by the researcher to support the learning objectives of a unit on quadratic functions for Algebra 2 students and a unit on right triangles and right triangle trigonometry for the geometry student. The instructor provided online or in-class lessons and discussions that introduced student-centered activities with summaries, reviews, and practice.

Formal data collection instruments included demographic surveys, Math Attitudes and Perceptions Surveys, assessment data, pre-intervention interviews, and post-intervention interviews and surveys. The researcher kept a journal with observations and students’ comments as well as annotated class notes written during online and face-to-face classes to capture students’ questions and comments during discussions.

A cross-case data analysis was developed to examine similarities and trends relevant to the study’s research questions. Data analysis of the individual and small class case studies examined students’ achievement scores overall and in the core literacies linking their comments from interview and survey data to support and explain their learning outcomes. Similarities and trends in students’ reports provided insights into the impact of the IMIST unit on attitudes and confidence as well as evaluations of learning experiences. The data analysis of the case studies and cross-case analysis identified themes describing how students learned using the IMIST unit activities.

Chapter One

There is continuing concern about the state of mathematics education in the United States especially in light of students' low performance on international and national assessments of mathematics skills and literacy (Cheung & Slavin, 2013; Schmidt, 2012; NSB, 2016; Thompson, 2009). Stakeholders have recognized that proficiency in mathematics is a gatekeeper for careers in STEM fields as well as careers in economics, business, and finance. Students who lack mathematical skills and literacy may limit their access to 21st century careers in these fields (Herzig, 2005; Miller & Kimmel, 2012). In response, the professional organization of mathematics teachers, the National Council of Teachers of Mathematics (NCTM), published a series of standards, values, and strategies aimed at improving mathematics education focusing on student-centered learning, the promotion of coherent learning progressions, and real-word contextual understandings. (NCTM, 1989; NCTM, 2000; NCTM, 2014). Using these guidelines, there have been modest gains at the elementary and middle school levels, but student achievement in mathematics at the secondary level shows a lack of significant improvement (NCTM, 2014; NSB, 2016; Schmidt, 2012).

Educational stakeholders, policy makers, district leaders, teachers, and parents are divided as to the best curricular approaches for improvement and reform (Klein, 2003). Traditional educators focus on teaching mathematical skills, algorithms, procedures, and assessments (Chandler, Fortune, Lovell, & Scherrer, 2016; Klein, 2003; Schoenfeld, 2004). Educators who endorse the NCTM standards, referenced as standards-based or reform-based, argue for an emphasis on student-centered learning, inquiry, and contextual understandings (Ellis & Berry, 2005; Klein, 2003; NCTM, 2014; Schoenfeld, 2004). Traditional educators argue against adoption and implementation of NCTM's standards and practices as well as corresponding national curricula standards such as the Common Core State Standards for Mathematics (CCSSM). They state lack of professional development, instructional materials, and appropriate assessments (Chandler, et al., 2016; Editorial, IRA, 2009; Robbins, 2013; Toscano, 2013). Yet, neither traditional curricula nor reform-based curricula have demonstrated consistent improvement in student learning, engagement, and achievement particularly at the high school level (NSB, 2016; Schmidt, 2012).

Current curricula, policies, or resources have not met states' and districts' desired goals for student proficiency and improvement (Cheung & Slavin, 2013; NSB, 2016; Schmidt, 2012). In addition, researchers in mathematics education suggest mathematics curricula in the United States lack focus, rigor, and coherence as compared to mathematics curricula taught in top performing nations in the world. Furthermore, they state students in the United States are not taught in a conceptual manner consistent with research on the ways in which students learn mathematics (Ellis & Berry, 2005; Hirsch, 2004; NCTM, 2014; Schmidt, 2012; Schoenfeld, 2014). Research suggests that educators and curriculum developers need to support students' development of mathematical fluencies and literacies in symbolic competency, conceptual understanding, procedures, and problem-solving using appropriate research-based learning activities. (Jacobs, 2011; Milgram, 2007; Schoenfeld, 2014). To meet the learning needs of students, there is a need to design a curriculum framework or system that supports students' development of both strong symbolic and conceptual literacies as well as strong problem-solving skills integrating the strengths of both traditional and reform-based learning activities.

Background

Proficiencies and achievement of U.S. students in mathematics at all levels continues to be a major concern for educators and policy makers (Cheung & Slavin, 2013; Schmidt, 2012; NSB, 2016; Thompson, 2009). Results from international standardized tests such as the Program for International Student Assessment (PISA, Organisation for Economic Co-operation and Development, 2012b), show students from the United States performed below average in mathematics, ranking 26th of thirty-four participating countries. The percentage of students proficient in mathematics skills and procedures on the 2013 administration of the National Assessment of Education Progress (NAEP) declined from 42% of fourth graders to only 26% of 12th graders who scored at or above proficiency (NSB, 2016).

Although the comparative statistics of low proficiency have fallen below the goal of 50%, the NAEP trends in elementary mathematics (4th grade) and middle school mathematics (8th grade) have increased over the period of 2000 to 2013 (NSB, 2016). However, the scores for 12th graders have remained flat. A disproportionate number of students who attend college require remediation in mathematics (Schmidt, 2012). Stakeholders, policymakers, educators, and parents are concerned that U.S. students lack the basic quantitative skills required to complete in the international, global economy of the 21st century and are not prepared for domestic careers in STEM, business, and economics (Herzig, 2005; Miller & Kimmel, 2012; Schmidt, 2012).

Solutions are not broadly agreed upon and represent at least two distinct perspectives (NRC, 2001). Schoenfeld (2004) delineated two camps in mathematics education both historically and as curricula – the traditional view and the reform-based or standards-based view. Traditional education, founded in behaviorist and positivist philosophies, focuses on social efficiency, rigor, and the perpetuation of privilege (Ellis & Berry, 2004; Klein, 2003; Schoenfeld, 2004). Traditional mathematical curricular content typically emphasizes teacher-centered, guided instruction focused on skill acquisition, facts, algorithms, procedures, and assessments (Wu, 1999; Kirschner, Sweller, & Clark, 2006; Klein, 2003; Schoenfeld, 2004). Research has supported the merits of the traditional mathematics perspective which emphasizes arithmetic symbolic skills and procedures (Klein, 2003; Wu, 1999) but has not shown that these curricula are effective in promoting problem-solving and understanding of mathematics in context (Post et al., 2008; Schoenfeld, 2004).

Reformed-based mathematics education, grounded in constructivist theories of learning, focuses on the goal of making mathematics learning accessible to all students (Herzig, 2005; NCTM, 1989, 2000, 2014; Schoenfeld, 2004). Reform-based mathematics curricular content emphasizes student-centered, inquiry learning focused on mathematical thinking, context, meaning, and problem-solving (Hmelo-Silver, Duncan, & Chinn, 2006; NCTM, 2014; Schoenfeld, 2004). The emergence of new or different pedagogical practices for teaching and learning have resulted from development of reform-based curricula. Many activities focus on discovery or inquiry learning and include problem-based learning, investigations, collaborative learning, and mathematical discourse (Hmelo-Silver et al., 2006; NCTM, 1989; NCTM, 2000; NCTM, 2014). These practices actively use technology such as calculators, computer applications, and videos to flip traditional in-class activities to homework or outside-of-class activities making time for inquiry-based learning in class (Bergmann & Sams, 2012; Educause, 2012; Fulton, 2012; Goodwin & Miller, 2013; Rose, 2009; Strayer, 2012; Sweet, 2014; Tucker, 2012).

Research on reform-based curricula has found increased student engagement and improvement in students' attitudes about mathematics, but findings have been mixed related to the development or maintenance of arithmetic skills and symbolic procedures (Klein, 2003; Post et al., 2008). Mathematicians and mathematics educators have expressed concerns about students' abilities to move from manipulatives and visualization of simple arithmetic relationships to symbolic abstractions and processes vital for the study of high school mathematics (Wu, 1999). The development of basic skills with symbolic literacy and fluency has been identified as a consistent weakness in reformed-based curricula (Hirsch, 2004; Klein, 2003; Wu, 1999).

The consequence of the division between two ideological and pedagogical perspectives is a highly fragmented and variable curriculum across the United States (Schmidt, 2012). Mathematics and curriculum researchers have called for better curricular focus, specificity, rigor, and coherence in mathematics curriculum (Hirsch, 2004; Polly, 2016; Schmidt, 2012). Schoenfeld (2004, 2014) and others (e. g., Hirsch, 2004; NRC, 2001; Wu, 1999) have called for a middle ground between traditional and reform-based mathematics educational practices, explaining that students need to have mathematical skills and need to understand mathematics in context. “If students are to become proficient in mathematics, teaching must create learning opportunities both constrained and open” (NRC, 2001, p.xiv). Traditional activities and inquiry-based activities can scaffold both students’ conceptual understanding and the development of problem-solving strategies (Hmelo-Silver, Duncan, & Shin, 2006). “The mark of powerful learning is the ability to solve problems in new contexts or to solve problems that differ from the ones one has been trained to solve” (Schoenfeld, 2004, p. 262). To create powerful classrooms, students need learning opportunities that develop both skills and contextual understanding. They need elements of both traditional and standards-based mathematics curricula (NRC, 2001; Schoenfeld, 2004; Schoenfeld, 2014).

To bridge the gap and to integrate the best practices of traditional curricula, (skills and symbolic literacy and fluency) with reformed-based curricula (student-centered learning, inquiry, applications, and context), educators need guidelines, a design system, or framework to provide specific, rigorous, and coherent curricula that engages and motivates students to meet their needs and improve their learning in mathematics particularly at the high school level. The process of standards adoption, whether those of NCTM or the CCSSM, often lack professional development or instructional materials, and mathematics educators are challenged to select the best curricular content, the most effective instructional practices, the best activities, and the most appropriate learning technologies to support mathematics learning. The researcher for this design research project conceptualized and developed the Integration of Mathematical Inquiry, Symbolic literacy, and Technology (IMIST) instructional system using a design pattern approach to create and provide an instructional system that integrates the most effective practices to meet teaching objectives and learning needs in mathematics instruction.

A design pattern approach is an analytical framework with four elements: specifying a pattern name, describing the problem, creating the core of a solution, and detailing pattern consequences (Hathaway & Norton, 2013). The IMIST system was developed using this analytical framework and is proposed as a system of instruction that allows secondary mathematics educators to identify appropriate content activities to support student learning, both symbolic and conceptual. It engages educators in the identification of content activities matched with the affordances of traditional and newer technologies. Educators can determine which types of activities are best suited for in-class activities and which are best-suited for homework or out-of-class activities. The IMIST instructional system offers a possible framework to create an integrated systemic approach to designing curricular units in secondary mathematics.

The Research Problem

One approach to devise a solution to address U. S. students' low performance and lack of improvement in mathematics is to begin with students themselves. Research has shown that student adaptability, self-beliefs, confidence, and engagement are positively correlated with achievement in mathematics (Barkatsas, Kasimatis, & Gialamas, 2009; Collie & Martin, 2017; Skaalvik, Federici, & Klassen, 2015; Valentine, DuBois, & Cooper, 2004) An in-depth examination of students' attitudes, beliefs, confidence, and achievement using different learning activities that combine inquiry learning, symbolic literacy, conceptual literacy, and problem-solving may help educators choose the best learning activities to engage and motivate students to study mathematics thereby improving their mathematical literacy and proficiencies to meet the goals of major educational stakeholders. These learning activities need to integrate effective activities from reform-based practices focused on student-centered inquiry learning and problem-solving with effective traditional practices that build strong foundations and fluency in mathematical skills, literacy, algorithms, and procedures. This study was designed to investigate students' learning using units designed with the IMIST instructional design system as compared to their prior experiences with mathematics instruction. Specifically, the problem of this study was to study the impact of units designed using the IMIST system on students' attitudes, beliefs, confidence, and achievement in mathematics.

Specific research questions include:

1. What is the impact of a mathematics unit designed using the IMIST system on students' understanding of mathematics and it core literacies?
2. What is the impact of a mathematics unit designed using the IMIST system on students' attitudes, confidence, and engagement with mathematics?
3. What do students report about their mathematical learning experiences when learning is structured by a unit designed using the IMIST system?

Conceptual Framework

Figure 1 presents a map of the conceptual framework for this study. This concept map models the relationships between the research participants, the intervention, and the variables (Maxwell, 2013).

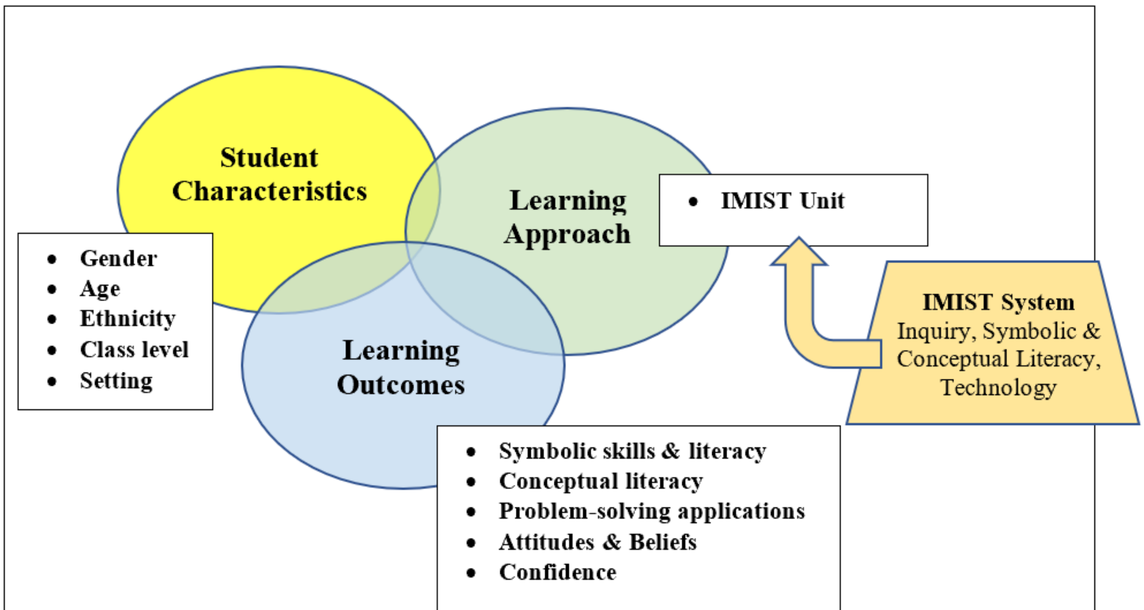


Figure 1. This concept map illustrates the components of the research study of the IMIST system and its impact on students' attitudes and learning outcomes.

Figure 1. Conceptual Framework - Research Study of the Impact of Integrated Instructional Learning Activities on Students' Learning Outcomes

Student characteristics. The first component of the conceptual framework for this study centers on student characteristics and educational background. Although teaching and learning in the mathematics classroom is situated within broader educational, social, and political contexts where issues of students' race, class, gender, ethnicity, language, may come into play (Di Martino & Zan, 2015; Diversity in Mathematics Education Center for Learning and Teaching (DMECLT), 2007), this study investigated a small number of individual students whose primary differences were their educational settings and backgrounds. Researchers and mathematics educators not only have to be concerned about teaching mathematics but also need to consider how teaching activities and approaches affect students with diverse educational experiences and backgrounds (DMECLT, 2007; NCTM, 2014). Herzig (2005) explained that it is a "unique challenge for us to build a context for mathematics education that is truly accessible and inviting to a broad range of students" (p. 253). This small study examined four students who were independent learners and had been homeschooled for most of their education as well as nine students enrolled in Algebra 2 at a mid-sized private high school who came from a variety of educational experiences in traditional middle schools. Due to the small number of students involved in the study, the demographic and background information collected is informative and supports awareness of each students' educational story.

Learning approach. This study investigated students' experiences and the impact of units designed with the IMIST instructional system on students' attitudes, beliefs, confidence, and achievement. The IMIST units were designed using the design pattern analytical framework which identified the recurring problem, developed the core of the solution, and described the consequences of the process. With specific learning objectives in mind, the recurring problem was to identify the appropriate content activities to support student learning, symbolic and conceptual, when designing curricular units for secondary mathematics. The core of the solution was to match content activities to the affordances of traditional or newer technologies, to determine what types of activities were best suited for in-class activities, and to identify which activities were best suited for homework or out-of-class activities. Using the IMIST instructional system, curricular units were created choosing learning activities that met two learning goals: to build a strong foundation of skills with symbolic and conceptual literacy and to promote inquiry learning and problem-solving using authentic applications.

Learning outcomes. To evaluate the mathematical learning outcomes which included students' attitudes, beliefs, and confidence in mathematics as well as students' achievement of skills and symbolic literacy, conceptual literacy, and problem-solving applications, data were collected using surveys, interviews, and assessments.

Math attitudes, beliefs, and confidence. Data collected for this study were qualitative and came from questions posed in semi-structured interviews, written surveys as well as a more comprehensive Likert survey. The construct of mathematical attitudes is complex, and as a result, there has been a shift from data collected by quantitative survey-type instruments to qualitative data collected from open-response questions, interviews, and student narratives (Code, Merchant, Maiejewski, Thomas, & Lo, 2016; Di Martino & Zan, 2015).

Students' mathematical attitudes affect course choices and willingness to take more advance math courses; gender and culture play a role in students' mathematical attitudes independent of cognitive ability; and mathematical attitudes are closely linked to students' emotions, perceived achievement, and vision or perceived value of mathematics

(Di Martino & Zan, 2015; Enderson & Ritz, 2016; Schöber, Schutte, Koller, McElvany, & Gebauer, 2018; Skaalvik, Federici, & Klassen, 2015). Code et al. (2016) explained that the goal of mathematics education is to help students develop "productive dispositions" (p. 917) towards mathematics in an effort to encourage students to become enculturated in the mathematical community of practice and to move forward to higher levels of mathematics education.

Student achievement. To investigate students' mathematics proficiencies in core literacies, students were given formative and summative assessments during the implementation of the IMIST unit. Their scores were collected consistent with data collected by most quantitative standardized testing methods. Questions or items on these assessments were initially coded into the categories of learning outcomes or proficiencies: symbolic literacy, conceptual literacy, or problem-solving applications.

Students' symbolic literacy or fluency represented their level of mastery of basic arithmetic skills and procedures as well as their understanding and use of correct mathematical vocabulary and symbolic representations (NRC, 2001; Schoenfeld, 2007b; van Jaarsveld, 2016; Wu, 1999). Symbolic literacy is the critical foundation needed for students to develop mathematical conceptual understanding (NRC, 2001; Schoenfeld, 2007b; van Jaarsveld, 2016; Wu, 1999; Yackel & Cobb, 1996). Students' conceptual literacy or understanding is a measure of their ability to use and choose the correct mathematical procedures to solve symbolic exercises and their ability to recognize and make connections between multiple representations of mathematical relationships (Berger, 2017; NCTM, 2014; Schoenfeld, 2007b). Conceptual literacy and procedural fluency are the critical understandings which help students apply mathematical reasoning and thinking to solve contextual, authentic, or real-world problems (Berger, 2019; NCTM, 2014). For the purposes of this study, conceptual literacy and procedural fluency were combined and referenced simply as conceptual literacy. NCTM (2014) considered students' ability to use mathematics to problem-solve as a significant goal of mathematics education and explained that mathematical activities and tasks should develop students' ability to reason and problem-solve. Schoenfeld (2007b) explained that problem-solving requires students to extend known results, find new results, and to apply known mathematical results in new contexts.

To measure students' mastery for each level of learning outcome, students' subscores were collected for each literacy: symbolic literacy, conceptual literacy, and problem-solving. Grading rubrics awarded points for symbolic literacy questions, conceptual literacy questions, and problem-solving applications and delineated points awarded for questions that combined two or more literacies. Scores reflected students' ability to communicate mathematically and to show their reasoning. These subscores provided a deeper picture of students' understanding of these learning outcomes (NCTM, 2014; Schoenfeld, 2007a).

Significance of Study

As early as 1999 and 2004 (e.g. Wu, 1999 and Schoenfeld, 2004), mathematicians' and mathematics educators' recommendations aimed at improving the low mathematics achievement of United States students on international and national assessments. They believed that students in elementary through secondary levels needed both symbolic literacy (skills) and contextual understanding as well as authentic, real-world problem-solving in mathematics (NRC, 2001). Since teacher and student resources such as textbooks and other curricular materials that integrate the two are not commonly used (Remillard et. al., 2007), the primary purpose and development of the IMIST instructional system was to provide an integrated mathematics approach and to investigate whether this focused approach would improve student attitudes about learning mathematics and their mathematics achievement as has been shown in other studies (Barkatsas, Kasimatis, & Gialamas, 2009; Collie & Martin, 2017; Skaalvik, Federici, & Klassen, 2015; Valentine, DuBois, & Cooper, 2004). The success of the IMIST instructional system might provide an alternative approach to the design of mathematical instruction. The consequences might be an improvement in students' scores on secondary mathematics achievement tests, as well as improvement in students' disposition and choices to continue onto higher level of mathematics courses.

The IMIST instructional system might offer a potential solution to additional concerns about effective instructional design frameworks, effective use of instructional time, and a system to guide appropriate use of technology. Positive results using the IMIST system could potentially provide a framework to help teachers choose and integrate learning activities that address students' need for conceptual understanding and problem-solving with a strong foundation in skills and symbolic literacy. It might play a role in assisting teachers to conceptualize and understand what activities address these different learning goals and how to structure their instructional sequences effectively. The analysis of the IMIST system might help teachers choose and structure student learning activities that promote student engagement, motivation, and learning as well as making better use of instructional time inside and outside the classroom.

Scope of Study

The thirteen student participants in this study came from two groups. Four were independent learners from homeschooled backgrounds, and nine were freshman enrolled at a small private high school from the Mid-Atlantic region of the United States. The home-schooled students participated in pre- and post-intervention interviews. Three of these students participated online and one participated face-to-face. The nine freshmen were part of a small class. They participated in a classroom setting and filled out a post-intervention survey. All students responded to the Math Attitudes and Perceptions Survey (MAPS, Code et al., 2016) and took a series of two formative and one summative assessment over the course of the IMIST unit implementation.

To answer the research questions, this study used a primarily qualitative methodology with elements of action research and quantitative research. Because of the small sample size, the quantitative data was analyzed using descriptive statistics that were informative, but not statistically significant. The coded assessment data provided insights to answer the first research question about the impact or effects of the instructional intervention (IMIST) on student learning by examining the trends in the percentage achievement by subcore for each of the literacies or variables studied.

The second research question examined students' attitudes, beliefs, and confidence about mathematics. The third research question examined students' learning experiences with the IMIST unit activities. Students provided information about their background, attitudes, and perceptions on demographic questionnaires and MAPS (Code et al., 2016). The independent learners also participated in pre- and post-interviews. The students in the small class filled out a post-intervention survey about their experiences with IMIST instructional activities.

Definition of Terms

Conceptual Literacy – Conceptual literacy develops from a deep understanding of arithmetic skills and symbolic literacy (Wu, 1999). “Skills are the requisite vehicles to convey conceptual understanding” (Wu, 1999, p. 1). Conceptual literacy is the ability to move from concrete examples into symbolic abstractions and models, and to apply general conceptual principles into new contexts and applications (Schoenfeld, 2004). Conceptual understanding is demonstrated by “the meaningful and flexible use of procedures to solve problems” (NCTM, 2014, p. 7). In this paper, conceptual literacy includes the construct of procedural fluency as described by NCTM and other authors. Applications of conceptual literacy and understanding may be advanced use of symbolic mathematical language, procedures and manipulations, which may or may not use real-world or authentic problems.

Design Pattern Approach – A design pattern approach is an analytical strategy used to provide a potential solution to a problem that occurs over and over in a field of practice. In the context of education, a design pattern is “an analytical strategy to guide thoughtful consideration of design problem” which may include designing curriculum and/or instructional activities (Hathaway & Norton, 2013, p. 5 and p. 8). A design pattern approach researches and describes the problem, names the pattern, provides the core of a solution, and considers the consequences of implementation of the design pattern.

IMIST Instructional System – Integration of Mathematical Inquiry, Symbolic literacy, and Technology system. This is the name of the design pattern or instructional system used in this study. It provides an analytical framework to consider the four parts that contribute to research-based, effective practices for mathematics teaching and learning addressing inquiry learning, skills and symbolic literacy, conceptual literacy, and problem-solving knowledge and applications. The IMIST instructional system considers the affordances of learning technologies to meet learning objectives and to leverage students' interests and abilities to use technology for learning.

Inquiry learning – A student-centered learning approach based on constructivist theory that includes investigations, experiential learning, problem-based learning, mathematical modeling, and problem-solving strategies. This paper uses inquiry learning (IL) as the general term which includes the learning activities listed above. In inquiry learning, students learn content, reasoning skills, and strategies organized around relevant authentic or real-world questions (Hmelo-Silver, Duncan, & Chin, 2009).

Learning Activities – "Learning activities are the strategies and practices that engage learners in 'doing' or knowing'" (Hathaway & Norton, 2013, p. 8). These may include traditional paper-and-pencil activities, worksheet and textbook exercises, as well as technology-enhanced activities, explorations, and investigations using calculators, computer applications, videos, and video micro lectures.

Mathematics Confidence – Barkatsas, Kasimatis, and Gialamas (2009) defined mathematics confidence as "students' perception of their ability to attain good results and their assurance that they can handle difficulties in mathematics" (p. 564). This is reflected in the literature as the relation between self-beliefs and achievement using different constructs or terms such as self-concept, self-esteem, and self-efficacy (Valentine, DuBois, & Cooper, 2004). Bandura (1977) described the construct of perceived self-efficacy as "beliefs in one's capabilities to organize and execute the course of action required to manage prospective situations" (p. 2). In research with students that use interviews and surveys, self-efficacy has been described in lay terms as confidence (Barkatas, Kasimatis, & Gialamas, 2009; Code et al., 2016; Skaalvik, Federici, & Klassen, 2015).

Problem-solving Applications – In this study, problem-solving applications or problem-solving will be the term used to describe applications that reference real-world, contextual, and/or authentic applications of mathematical thinking and modeling. As such, mathematical problem-solving is considered the highest order of mathematical thinking skills at the secondary level (Avetisyan & Hayrapetayan, 2017). Problem-solving applications integrate students' knowledge of skills and symbolic literacy with conceptual understandings. Students demonstrate both inquiry strategies, critical thinking, and problem-solving abilities by using different strategies, approaches, and techniques. "The mark of powerful learning is the ability to solve problems in new contexts or to solve problems that differ from the ones one has been trained to solve" (Schoenfeld, 2004, p. 262).

Symbolic Literacy (Skills) – At elementary levels, the term skills is used to describe knowledge of basic arithmetic facts, skills, and arithmetic algorithms which provide a foundation for abstractions of Algebra and secondary mathematics courses (Wu, 1999). At higher levels, the definition of skills expands to include symbolic literacy. Symbolic literacy and skills include understanding vocabulary and representations of data associated with conceptual ideas associated with mental arithmetic and mathematical patterns. In addition, symbolic literacy includes the ability to decode and encode real-world or authentic situations into mathematical language (Eisner, 1994; Wu, 1999). Symbolic literacy moves beyond simple arithmetic skills and algorithms and moves into work with equations, larger symbolic patterns, and arrays. Symbolic literacy or fluency represents a knowledge and ability to work with symbolic equations and patterns in abstract ways and with multiple representations (numerically, graphically, and analytically).

Reform-based mathematics education – The reform-based view of mathematics education is founded in constructivist philosophies. Research literature in mathematics education refers to reform-based mathematics education as "standards-based" which comes from the reference to the NCTM standards of 1989 and 2000. Given the pervasive use of the word "standards" in reference to state standards, national standards, Common Core State Standards (CCSS), this paper refers to the reform efforts started in the late 20th century as reform-based mathematics education. Reformed-based learning activities are student-centered and include inquiry-based activities, group-based projects, hands-on experiences, co-operative or collaborative learning, the use of computer technologies, and the use of calculators (Thompson, 2009). Reform-based mathematics education promotes activities that help students make connections, communicate mathematically, solve real-world problems, derive meaning, and understand context (NCTM, 1989; NCTM, 2000; NCTM 2014; Thompson, 2009).

Traditional mathematics education – The traditional view of mathematics education is based in behaviorist and positivist philosophies (Ellis & Berry, 2004; Klein, 2003; Schoenfeld, 2004). Traditional learning activities are teacher-centered with lecture or guided instruction. Learning activities focus on skill acquisition, facts, algorithms, and procedures using textbooks, worksheets, and word-problems. Proponents of traditional mathematics education value accuracy, rigor, and achievement (Wu, 1999; Kirschner et al., 2006; Klein, 2003; Schoenfeld, 2004).

Chapter Two

The conceptual framework for the research problem in this study is rooted in mathematics education in the United States – its history, curricula, and students' experiences. Examination of the IMIST system as the center of this research is rooted in design thinking, design patterns, and design research. The following literature review presents literature related to the state of mathematics education in the United States and indicators used to highlight issues in achievement, attitudes, and confidence in student learning, proficiencies, and mastery of mathematics. In addition, this review presents literature related to the process of using a design approach to solve a problem with a discussion of design principles, design thinking, and a design pattern approach (Hathaway & Norton, 2013) to solve a problem. Following guidelines established by a design pattern approach, an analysis of the design problem is explored through a discussion of mathematics curricula and the most effective practices and activities proposed by both traditional and reformed-based curricula as well as research on students' attitudes about and confidence in doing mathematics. The final section presents the core of the solution which is based on activity theory – activities that support students' learning in mathematics, and the proposed solution, the IMIST instructional system. The pattern consequences and pattern summary are presented followed by the design research methods used to investigate the IMIST instructional system.

State of Mathematics Education in the United States

Indicators and measures. Scores on high stakes international and national mathematics assessments have been the primary measures used to compare, analyze, and predict the strength of the educational systems within the United States and throughout the world (Bodovski, Byun, Chykina, & Chung, 2017; Hanushek, Woessman, Jamison, & Jamison, 2008; OECD, 2016a). The assessment results demonstrate that students in the United States are underachieving in mathematics as compared to those in other nations, that there is poor mathematics achievement across U.S. students, and that mathematical dispositions and attitudes are poor.

Local, state, and national stakeholders in the United States who include parents, teachers, administrators, and policy makers are concerned about the low performance of U.S. students on these assessments (Cheung & Slavin, 2013; Schmidt, 2012; NSB, 2016; Thompson, 2009) and the ability of the United States to maintain its place as a leader in the global economy (Hanushek et al., 2008; Herzig, 2005; Milgram, 2007; NSB, 2016; Schoenfeld, 2007a).

The goal of national policy makers (NCES, NSB, NAEP) has been to increase the percentage of students who pass at or above proficiency to 50% (NSB, 2016). However, the data show that students are not close to attaining that goal. The percentage of students scoring at or above proficiency in mathematics skills and procedures on the 2015 administration of the National Assessment of Education Progress (NAEP) declined to 40% of fourth graders, to 33% of eighth graders and to only 25% of 12th graders. This low math proficiency among graduating seniors means that a disproportionate number of students who attend college require remediation in mathematics (Schmidt, 2012). Furthermore, the lack of significant improvement over the last forty-two years, suggests that mathematics education in the United States has not improved enough to make-up the difference and to catch up to the mathematics education in the highest performing countries.

Mathematical dispositions, attitudes, and confidence. The Trends in International Math and Science Study (TIMSS) survey data from all students taking the 2015 assessment reported links between students' achievement, attitudes towards mathematics, and engagement in mathematics (Mullis, Martin, Foy, & Hooper, 2016). The analysis showed that students with positive attitudes towards mathematics tend to have higher achievement. However, the percentages of students in the United States with positive attitudes towards mathematics as well as their personal confidence level about their abilities and understandings in mathematics showed substantial decline by 8th grade. Also, fewer students reported being engaged in mathematics lessons in 8th grade (Mullis et al., 2016). Mathematics educators and policymakers should be concerned about why students are disengaging from mathematics between 4th and 8th grades. Educators need to consider whether these trends continue into secondary and high schools. Furthermore, these trends illustrate the need to identify what can be done to improve students' attitudes towards mathematics and to reengage students in mathematics throughout the United States.

Additionally, the PISA (2015) results indicated that U.S. students were satisfied with their schools and teachers but were not highly motivated to learn mathematics. Only

50% of the students responded that they were interested in learning mathematics which was below the OECD average of 53%. Again, this result has negative implications for the number of students in the U.S. prepared to take on the rigor of more mathematics and science courses in high school and higher education (Hanushek et al., 2008; National Mathematics Advisory Panel (NMP), 2008; Schmidt, 2012).

Given the results on the TIMSS, PISA, and NAEP, stakeholders, policymakers, educators, and parents continue to be concerned that students in the United States lack the basic quantitative skills required to complete in the international, global economy of the 21st century (Hanushek et al., 2008; Milgram, 2007; NSB, 2016; Schoenfeld, 2007a, Schmidt, 2012). This points to a vital need to examine, evaluate, and reform the teaching and learning of mathematics in the United States.

Using a Design Approach to Solve the Problem

How can mathematics educators “identify and create optimal conditions for the kind of learning and development especially important for effectively functioning in the 21st century?” (Dai, 2012, p. ix). Specifically, how can mathematics educators provide curricular content and materials to help students develop deep understandings of core mathematical concepts, theories, and principles so they can develop causal reasoning, critical thinking, and creative problem-solving abilities (Dai, 2012)? Mathematics educational researchers have proposed a need for mathematics curricula and materials designed to build strong foundations and mathematical proficiencies in symbolic literacy, conceptual literacy, and problem-solving applications (Jacobs, 2011; Milgram, 2007; Ramaley, 2007; Schoenfeld, 2007a).

Standardized testing whether at the international, national, or local level may highlight issues and discrepancies in mathematics education in the United States, but these assessments do not address the complex causes and students' need for deep understandings and mastery of mathematical proficiencies to make progress in assessment outcomes. Given the disappointing standardized test scores and concerns with U.S. student attitudes about mathematics, there is a need to devise solutions that offer the potential for successful academic achievement and positive dispositions towards mathematics and its role in 21st century education. Mathematics educators, teachers, and leaders need to address the questions: What is an appropriate process that will enable educators to improve mathematics education in the United States, and how might success be achieved?

Over the last century, stakeholders – researchers, teachers, administrators, policy makers, parents, and students – interested in the development and improvement of mathematics curricula have proposed solutions to improve teaching and learning in mathematics (Klein, 2003; Schoenfeld, 2004; Schmidt, 2012). The shared goal of these stakeholders is to build curricula, activities, and materials to improve students' understanding in mathematics symbolically, conceptually, and for authentic problem-solving to close the achievement gap between students in the United States and the students in high-achieving countries (Cheung & Slavin, 2013; Schmidt, 2012; NSB, 2016; Thompson, 2009). However, proposed solutions have been elusive and under debate (Hirsch, 1996; Klein, 2003; Schoenfeld, 2004; Schmidt, 2012). In an effort to devise a solution to the mathematics education dilemma, researchers have turned to new approaches to solve issues in teaching and learning using lessons and techniques from design, design thinking, and design research (Crouch & Pearce, 2012; Dai, 2012; Liedtka, King & Bennett, 2013).

Educational systems consisting of a hierarchy of administrators, educators, teachers, students, and parents in combination with curriculum, content, standards, and assessments are dynamic, iterative, and repetitive complex systems (Bannan-Ritland, 2003; Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; Lesh & Kelly, 2000; McKenney & Reeves, 2012; Plomp & Nieveen, 2013). In addition, these complex systems include educational practices and curricula that are socially, culturally, and politically situated (Brown, Collins, & Duguid, 1989; Popkewitz, 2004; Frank, 2016). Within the complexities of the educational system, design thinking, design patterns, systems design strategies, and design research have successfully produced solutions to complex problems using collaboration, iterative studies, and holistic approaches in many fields including education (Crouch & Pearce, 2012; Dai, 2012; Liedtka, King & Bennett, 2013; McKenney & Reeves, 2012; Plomp & Nieveen, 2013). Rather than focusing on content and assessment data, educators are able to focus on design issues such as creating optimal learning environments focused on the individual needs of all learners and leveraging personal interests and resources to promote higher-ordered thinking and proficiency (Dai, 2012).

The science of learning design is a domain of inquiry and a field of practice relevant to all disciplines and all levels of educational practice (Dalziel, 2015). Learning design is “the creative and deliberate act of devising new practices, plans of activity, resources and tools aimed at achieving particular educational aims in a given context" (Mor & Craft, 2012, p. 86). Underlying the creative process of devising new practices and strategies is a set of fundamental design principles that promote thinking, analysis, integration, and the creation of new ideas or solutions (Liedtka, King & Bennett, 2013).

Design Principles and Design Thinking

An explicit assumption of design thinking is that design solutions are transformative or designed to make significant changes in practices, products, and outcomes (Crouch & Pearce, 2012; Dai, 2012: McKenney & Reeves, 2012). Design is a cognitive mode, and design knowledge underlies systematic habits of thinking related to a specific discipline or domain (Hathaway & Norton, 2018). Design thinking is driven by asking questions to determine the context and conditions, the needs of the stakeholder and users, and to look at the materials and processes (Crouch & Pearce, 2012; Dai, 2012). Design thinking can be structured into stages that encourage analysis and understanding of problems and their solutions. This can be done on a smaller scale or micro-cycle which may or may not be within a larger research study (McKenney & Reeves, 2012). Designers use questions to define the four stages of systematic design thinking. These questions require the researcher or designer to think about, ponder, and generate ideas. The first stage is to answer the question, "What is?" (Liedtka et al., 2013). In the context of the complexity of educational systems and curricular design, the designer must seek to understand the problem within its educational context to determine the teaching needs of teachers and the learning needs of students (Crouch & Pearce, 2012; Liedtka et al., 2013). To answer this question, designers need to identify specific learning objectives, the content and topics, effective practices, and the learning needs of students from prior research, a review of the literature, educational experiences, interviews, surveys, and observations (Bannan-Ritland, 2003; Hathaway & Norton, 2018; McKenney & Reeves, 2012)

The second stage of design-thinking is to determine “What if?” In this stage, the designer proposes new hypotheses, approaches, or ways to solve a problem (Liedtka et al., 2013). At this stage, the designer examines research data, identifies patterns, gains insights, and translates this understanding into prototypes, interventions, systems or a learning design (Liedtka et al., 2013). The question posed in the second stage naturally combines with the question in the third stage, “What wows?” To construct a workable prototype or system, it must appeal to or engage the users in educational settings. In this context, the answers to the question “What wows?” has been linked to strategies and methods that leverage student engagement and student motivation (Dai, 2012; Dalhstrom & Bichsel, 2015; Tapscott, 2009).

The final stage of the design thinking process occurs after the learning design has been implemented. It is the reflective stage which determines “What works?” This becomes a reflective and evaluative part of the design process. To determine if the learning design is successful or effective, teachers and researchers evaluate student outcomes through feedback and measures of student attitudes, perceptions, learning, and achievement. Designers use this feedback to “inform their design thinking and to modify their designs” (Hathaway & Norton, 2018). By using this feedback, not only can teaching and learning strategies improve, but the most effective new strategies are documented, generalized, and shared with the larger educational community (Crouch & Pearce, 2012; Dalziel, 2015; Liedtka et al., 2013; Maina, Craft, & Mor, 2015).

The Design Pattern Approach

The general design principles from research literature have been organized into guidelines for educational practitioners that recommend a design pattern approach to curricular content design. The design pattern approach enables researchers, practitioners, and educators to focus on the development of content, activities, and materials that meet content learning goals for students. This approach supports practitioners' abilities to implement a design thinking approach to develop a solution to a recurring problem in their educational context (Hathaway & Norton, 2013).

A design pattern is a description of a problem that occurs repeatedly in a field of practice. The pattern presents solutions to the problem in a generalized and practical way that can also be used repeatedly in different applications, with different contexts, and with different outcomes. The design pattern approach has been used in the fields of architectural design, computer science, website design, and online course design (Hathaway & Norton, 2013, 2018). The design pattern approach is "an analytical strategy to guide thoughtful consideration of design problems" (Hathaway & Norton, 2013, p. 6). The four essential components of a design pattern are a pattern name, a description of the problem, the core of the solution, and the pattern's consequences and context (Hathaway & Norton, 2013, 2018).

The first component of a design pattern is to name the pattern. This name should describe the design problem, its solutions, and its consequences. The name should reference important elements of the pattern and may serve as a “shorthand reference to the solution” (Hathaway & Norton, 2018, p. 30). The name represents an abstraction or generalization of the design pattern and provides an easy reference that promotes designers’ communication, documentation, and discussions of the solution(s) (Hathaway & Norton, 2018).

The second component of the design pattern approach is to craft a description of the design problem that explains the problem and its context. To help describe the pattern, the designer may review the literature, identify problems with similar content and contexts, and seek to identify the recurring nature and the common attributes of the design problem (Hathaway & Norton, 2018). After this analysis, the designer states “the design problem as a broad, generalizable abstraction” (Hathaway & Norton, 2018, p. 31).

The third component of a design pattern approach is to state the core of the solution. Like the study and analysis for stating the problem, stating the core of the solution may involve a review of literature pertinent to the characteristics and features of the design problem, interviews, needs analyses, and observations (Bannan-Ritland, 2003; Hathaway & Norton, 2018). The designer may use the solution to organize learning activities consistent with the design problem, to specify elements or features of the solution and their relationship to each other, and to state the design solution in the form of an instruction (Hathaway & Norton, 2018).

The final component of the design pattern approach is a description of the pattern consequences. Again, to understand the broader context of the problem and context in which the design pattern is situated, the designer may review research and relevant literature. The designer needs to identify issues and considerations associated with implementing the design solution and to determine what should precede implementation of the design pattern and what should follow it (Hathaway & Norton, 2018).

Understanding and Stating the Design Problem

Finding solutions to the issues and problems in mathematics education in the United States, student achievement in mathematics, and the concerns of multiple stakeholders is a complex problem with roots in two hundred plus years of debate. The arguments center on mathematics as a discipline, and how mathematics is defined and taught as a curricula and content area (Ellis & Berry, 2005; Popkewitz, 1987; Schoenfeld, 2004; Stein, Remillard, & Smith, 2007). “The choice of curriculum involves philosophical, political, and ethical questions” (Popkewitz, 1987, p. 16). Choices that govern curricular content and implementation – what is taught and who is taught – are often controlled by various stakeholders and groups with power at particular times in history. School content is shaped and directed by social issues in the larger structures of power – school administrations, school boards, colleges, and universities (Popkewitz, 1987). Mathematics curriculum is an important social construct (Stein & Kaufman, 2010). “In mathematics, curriculum has traditionally been viewed as the key policy lever for improving instruction and learning on a large scale” (Stein & Kaufman, 2010, p. 664).

To design a solution to guide curricular and instructional strategies to improve mathematics education in the United States at all levels – the district, the school, or the classroom, the focus of this project – it is important to understand the two dominant pedagogical approaches to mathematics curricula, teaching, and learning that have emerged over the second half of the 20th century – a traditional view and a reform-based view. Research has shown that both approaches have merit, but both also have limitations and constraints (NRC, 2001). In addition, research in educational psychology has contributed to the understanding of students' disposition, attitudes, and confidence, and their impact on student learning and achievement.

Traditional mathematics education and curricula. As mathematics emerged as an important content area during the public-school movement of late 19th and 20th centuries, traditional mathematics curricula had foundations in the governing educational theories and philosophies of the early 1900s (Ellis & Berry, 2005; Popkewitz, 1987; Schoenfeld, 2004). Traditional mathematics instruction, pedagogy, and curricula were based on behaviorist and positivist philosophies focused on social efficiency, rigor, and the perpetuation of privilege (Ellis & Berry, 2004; Klein, 2003; Schoenfeld, 2004). This philosophical and functional view of education gave rise to two types of math instruction with a humanistic and social efficiency focus. The first taught children how to think and reason. This type of mathematics content and instruction would prepare students with the mental discipline and character appropriate for university studies and leadership in society or business. The second focused on non-college bound students, teaching everyday arithmetic and mathematics appropriate for budgets and shop clerks (Popkewitz, 1987).

Traditional mathematics educators viewed mathematics as a hierarchy of mental habits and topics which should be taught in a specific sequence and with practice (Ellis & Berry, 2005). Grades 1 to 8 focused on arithmetic; grade 9 on algebra; grade 10 on geometry; and grades 11 and 12 focused on precalculus (Schoenfeld, 2004). The progressive or utilitarian view proposed by Dewey and others, within the social efficiency framework, promoted tracking and ability grouping so that learning was connected to students' experiences and interests. The progressives also believed that children should develop naturally and that their perceived interests should be the motivation for work rather than the learning of critical math concepts (Ellis & Berry, 2005).

Proponents of traditional mathematics instruction believed that the best way for students to learn mathematics was to do activities focused on drill and practice. The curricula were skill-based and lacked focus on reasoning about concepts, problem-solving, and real-world experience or applications (Ellis & Berry, 2005). Traditional mathematical instructional approaches typically emphasized teacher-centered, guided instruction focused on skill acquisition, facts, algorithms, procedures, and assessments (Kirschner et al., 2006; Klein, 2003; Schoenfeld, 2004; Wu 1999). This traditional view held that the role of school was to provide authoritative knowledge. Answers were either right or wrong, and it was "the responsibility of the teacher to say what is right and to make sure students learn it" (Schoenfeld, 2004, p. 271).

The basic lesson format for a traditional program has three parts: 1) homework check and review, 2) direct teaching which includes definitions, review of formulas, or instruction on a fundamental mathematical idea or concept, and 3) student practice solving examples, textbook problems, and/or solving some textbook word problems as time permits (Jacobs, 2011). This format is often referred to in the literature as a "teacher-centered" approach. (Jacobs, 2011; Stein, Remillard, & Smith, 2007).

Research and competencies from traditional curricula. Mathematics educational research during the 1970s and 1980s primarily focused on student achievement and skill competency which supported the traditional view of mathematics curricula and sequence. The study of traditional curricula and its relationship to student learning was not a topic of deep scholarly research since most of the curricula focused on arithmetic computation especially in elementary education (Stein, Remillard, & Smith, 2007).

Research showed that traditional curricula's focus on practice, rehearsal, or repetition of facts had a positive effect as it helped students gain automaticity in number sense and operations giving them a strong foundation in math facts (Jacobs, 2011; Wu, 1999). This strong foundation is a necessary prerequisite for conceptual literacy and applications (Wu, 1999). Research also showed that students using traditional curricula showed slightly improved test scores on some standardized assessments especially for underserved students. However, traditional curricula's focus on memorization did not prepare students for the cognitive demand, thinking, and understanding of higher-level coursework (Ellis & Berry, 2005).

Although the strength of traditional mathematics curricula has been its emphasis on student learning of arithmetic symbolic skills and procedures (Klein, 2003; Wu, 1999), research has not shown that this curricular focus is effective in promoting problem-solving and understanding of mathematics in context (Post et al., 2008;

Schoenfeld, 2004). Classroom instruction that focused exclusively on content or base knowledge of facts, algorithms, and formulas deprived students of problem-solving knowledge and strategies (Schoenfeld, 2004).

Research in the 1990s into the 2000s found that in traditional programs, teachers were still the center of authority, teaching rote skills and procedural knowledge. Students worked individually on problem sets to internalize knowledge (Ellis & Berry, 2005). However, research showed that students did not retain what they learned after a few weeks (Battista, 1999). Traditional curricula also stratified and tracked students by their abilities. This became a barrier both educationally and socially for segments of the population (Boaler & Staples, 2008; Ellis & Berry, 2005).

Critics of traditional mathematics programs often referenced the rote practices of mimicry, explanation, lists, and practice in which students memorize facts and procedures without sense-making or contextualization (Boaler & Staples, 2008; Jacobs, 2011; Milgram, 2007). Furthermore, the traditional approach to mathematics was shown to be ineffective in promoting students' growth in mathematical thinking (Battista, 1999). The lack of student engagement and the decline in student interest in mathematics has been tied to the rote memorization and decontextualized nature of traditional mathematics programs (Schoenfeld, 2004). Reform-based mathematics education and curricula arose in response to these concerns.

Reform-based mathematics education and curricula. The beginning of the reform movement in mathematics can be attributed to the emerging science of psychology and the psychology of learning (Popkewitz, 1987). The reform-based curricula movement had its epistemological and philosophical roots in the constructivist framework of cognitive development by Piaget and his followers. Piaget's theories about the development of cognition in children influenced research, curriculum planning, preschool programs, and many areas of psychology and education finding fruition in the many student-centered activities in reform-based curricula (Pulaski, 1971).

The first wave of mathematics curricular reform came in response to the economic and scientific concerns of the 1950s as well as the attrition in the number of high school students taking math. The National Science Foundation's (NSF) built a rationale for the development and support of "New Math" curricula in the 1960s (Schoenfeld, 2004; Ellis & Berry, 2005). The School Mathematics Study Group (SMSG), attempted to modernize traditional curricula including topics such as set theory, modular arithmetic, and symbolic logic (Ellis & Berry, 2005; Stanic, 1987; Schoenfeld, 2004). The curriculum, strongly influenced by the developmentalist or constructivist philosophies of Piaget introduced manipulatives and discovery learning (Ellis & Berry, 2005).

However, the adoption of the New Math curriculum and its implementation failed mostly due to the lack of teacher education and support for these pedagogical innovations (Ellis & Berry, 2005). Stakeholders were disenfranchised. Teachers did not feel prepared or comfortable teaching the new curriculum. Parents did not feel competent and could not help their children with homework. Stakeholders did not see the value in the new content or approaches (Schoenfeld, 2004). Critics of SMSG felt that the curricula were too abstract and not related to real-world applications (Ellis & Berry, 2005).

The second wave of mathematics curricular reform came from NCTM's The Standards (1989) that proposed a set of vision statements or a set of criteria for curriculum development very unlike the traditional scope and sequence of content proposed by traditional mathematics curricula (Ellis & Berry, 2005). NCTM defined a standard to be "a statement that can be used to judge the quality of a mathematic curriculum or methods of evaluation. Thus, standards are statements about what is valued" (NCTM, 1989, p. 2).

The early goals of the standards-based reform movement or reform-based mathematics curricula were to educate all students to become mathematically literate workers, life-long learners with opportunities for all, and to become an informed electorate (Schoenfeld, 2004). Reform-based curricula, being grounded in constructivist philosophy, assumed that learning is an active process. NCTM's five goals for students were: 1) to learn to value mathematics; 2) to become comfortable in their ability to do mathematics; 3) to become mathematical problem-solvers; 4) to learn to communicate mathematically; and 5) to learn to reason mathematically (NCTM 1989, 2000; Schoenfeld, 2004). The reform movement stressed the importance of students' development of deep, interconnected understandings of math concepts, procedures and principles versus memorization of formulas and applications of procedures (Stein, Grover, & Henningsen, 1996).

The most recent articulation of reform efforts in mathematics education are embodied in the Common Core State Standards for Mathematics (CCSSM) which began development in 2009. Underlying the development of the CCSSM was the new construct of hypothetical learning trajectories (HLT) introduced in by Simon in 1995. HLTs were built on the developmental approach to teaching and learning in mathematics and were constructs developed in response to reform-based curriculum development. HLTs are guiding current research in mathematics education and the development of innovative curricula by teachers and practitioners (Clements & Sarama, 2004).

In 2014, NCTM published Principles to Actions in an effort to articulate the conditions, structures, and policies that must be in place to ensure mathematics is accessible to all students. NCTM emphasized the need for professional development and support for mathematics teachers. For students, NCTM defines five strands that constitute mathematical proficiency: 1) conceptual understanding; 2) procedural fluency; 3) strategic competence; 4) adaptive reasoning; and 5) productive disposition. Their framework serves as a guide for teachers or framework of elements that describe effective teaching and learning practices and beliefs with examples of tasks, representations, discourse, and questioning. NCTM encourages the development of procedural fluency from conceptual understandings. They acknowledge productive struggle as part of the learning process and emphasize the importance of student reflection and thinking. These recommendations currently serve as a framework for the development curricula, textbooks, and teaching resources as well as the next wave of research in mathematics education.

In response to standard-based or reform-based curricula, instructional practices and textbooks which have evolved for reform-based curricula have been significantly different from traditional instruction of an orderly, sequential presentation of content, formulas, and practice problems. Instructional materials often featured colorful illustrations and pictures with assignments including stories with fun names, and topics focused on real-world applications or math history (Schoenfeld, 2004). Reform-based curricula usually have an articulated curriculum map tied to state or national standards like the CCSSM (Jacobs, 2011). Reform-based classroom instructional practices provide hands-on or experiential learning activities with manipulatives, inquiry learning activities focused on mathematical thinking with context and meaning. They use questioning rather than lecture, provide challenging, interesting and complex problems built on students' prior knowledge, collaborative inquiry learning opportunities, and promote time for student reflections and communication. The literature references this type of classroom as student-centered rather than teacher-centered (Hmelo-Silver, Duncan, & Chinn, 2006; Jacobs, 2011; NCTM, 2014; Schoenfeld, 2004; Thompson, 2009). These inquiry-based practices have actively used technology such as calculators, computer applications, videos, and the Internet (Bergmann & Sams, 2012; Educause, 2012; Fulton, 2012; Goodwin & Miller, 2013; Rose, 2009; Strayer, 2012; Sweet, 2014; Tucker, 2012).

In addition to the new format of content and curricula, reform-based curricula required new teaching practices and teacher development. Teachers were encouraged to move from teacher-centered practices of lecture and guided instruction to student-focused practices of experiments, investigations, and collaborative group work. Teachers had to balance projects, group and individual assignments, with discussions and explanations between teacher and students and between students themselves (NCTM, 1989; Schoenfeld, 2004).

Research and competencies from reform-based curricula. Reform-based practices that significantly improved student achievement and attitudes in mathematics included hands-on activities using manipulatives, self-assessment, cooperative project-based group activities, and activities which used computer and calculator technologies (Thompson, 2009). Reform-based curricula designed and implemented using culturally relevant content with high cognitive demand gave access to traditionally underserved populations (Boaler & Staples, 2008; Ellis & Berry, 2005, Holmes & Hwang, 2016). Students using reform-based curricula believed that mathematics was more useful and saw more connections between mathematical ideas (Holmes & Hwang, 2016; Star & Hoffman, 2005). Students learned to communicate meaningfully and effectively about mathematical thinking and outperformed students using traditional curricula on measures of applications and understanding. Students learning mathematics with reform-based curricula had stronger interest in mathematics and more motivation to do mathematics. Reform-based learning tasks valued students' abilities to make sense of math through meaningful learning opportunities (Boaler & Staples, 2008; Ellis & Berry, 2005; Holmes & Hwang, 2016).

Although research on reform-based curricula has found increased student engagement and improvement in students' attitudes about mathematics, it has been mixed on the development or maintenance of arithmetic skills and symbolic procedures (Holmes & Hwang, 2016; Klein, 2003; Post et al., 2008). Mathematicians and mathematics educators have expressed concerns about students' abilities to move from manipulatives and visualization of simple arithmetic relationships to symbolic abstractions and processes vital for the study of high school mathematics (Wu, 1999). The development of basic skills with symbolic literacy and fluency has been a consistent weakness in reformed-based curricula (Hirsch, 2004; Klein, 2003; Wu, 1999). A strong foundation in basics skills and symbolic literacy is necessary to move onto courses with higher cognitive demand and facilitates problem-solving (Wu, 1999).

Although promising results in problem-solving and mathematical attitudes have resulted, the increase in standardized testing focused on skill competencies and basic literacies have interfered with full implementation of reform-based mathematics education in classrooms across the United States. Additional reasons for this lack of implementation or acceptance focus on the beliefs and values of educational stakeholders, the need for teachers' professional development, and a lack of planning and instructional time, (Blair, 2014; Fontichiaro, 2014; Schoenfeld, 2004).

Reform-based curricula require teachers to have deeper conceptual knowledge of mathematics and to be more flexible in their understandings to support students in these new learning environments (Ellis & Berry, 2005; Ball, Thames, & Phelps, 2008). Teachers who learned mathematics from proceduralist-formalist or traditional instructional practices needed to learn how to approach reform-based curricula. Again, the resistance of some stakeholders and teachers has provided a barrier to implementation of reform-based curricula (Nie et al., 2013; Schoenfeld, 2004). In addition, teachers had to learn how to direct exploration and mathematical sense-making (Schoenfeld, 2004; Ellis & Berry, 2005) and many felt ill-prepared and needed to learn how to structure learning environments with student-centered activities that allow for mathematical discourse and making connections between mathematical ideas (Ellis & Berry, 2005).

However, the biggest constraint to the implementation of reform-based inquiry learning is time for both teachers and students. Teachers need time for professional development, lesson planning, and instructional time (Blair, 2014; Fontichiaro, 2014). Even teachers who have had professional development in reform-based instructional strategies are two and a half times more likely to adopt traditional instructional activities rather than reform-based activities because of the lack of time (Thompson, 2009). Given limited time for classroom activities, students often do not have time to explore, may not have the skills to do inquiry activities independently for homework, and students may not be able to cope with the open nature of inquiry activities, experiments, and investigations (Blair, 2014).

Research in attitudes and confidence in mathematics. In concert with the development of reform-based standards and a student-centered focus on learning mathematics has been the emergence of research in educational psychology which recognizes the positive relationship between students' attitudes, confidence, academic self-efficacy, interest, effort and subsequent achievement in mathematics (Di Martino & Zan, 2015; Schöber et al., 2018; Skaalvik, Federici, & Klassen, 2015). Attitudes in mathematics are described as a pattern of beliefs and emotions associated with students' experiences learning and studying mathematics (Di Martino & Zan, 2015). A shared goal of NRC (2001), NCTM (2014), and Code et al. (2016) is that mathematics educators need to help students develop productive dispositions or positive attitudes towards math. Their research shows that helping students develop a cohesive view of math as an interconnected web of knowledge is positively correlated with course grade which highlights importance of students' view of a subject in relation to the academic performance. Di Martino and Zan (2015) emphasize the importance of researching and understanding the complex construct of attitude which includes students' intentional actions in a complex context involving their beliefs and emotions and acting as a bridge between them.

Students' self-efficacy and academic self-efficacy have also been found to be important contributors to students' academic achievement in mathematics (Schöber et al., 2018; Skaalvik, Federici, & Klassen, 2015). Self-efficacy refers to an individual's belief or confidence in his or her ability to exert control over his or her own motivation, behavior, and social environment (Bandura, 1977). Zimmerman (1995) extended this construct to academic self-efficacy which are "personal judgments of one's capabilities to organize and execute courses of action to attain designated types of educational performances" (p. 203). Academic self-efficacy or academic confidence influences and has a mediating role in students' level of effort, persistence, and choice of activities. Because confidence in a domain or academic self-efficacy has a positive influence on the effort students make, it is an important precursor of engagement in that domain and has a positive effect on achievement. The moderating variables between confidence, self-efficacy, and achievement in mathematics include students': 1) perceived difficulty of mathematics; 2) homework behavior; 3) intrinsic motivation or interest in mathematics; and 4) extrinsic motivation such as the perception of a higher utility value of mathematics if they are interested in a career that uses mathematics (Schöber et al., 2018). Understanding students' attitudes towards and confidence in mathematics plays a vital part in their learning (Di Martino & Zan, 2015).

A Synthesis of Traditional and Reform-based Approaches

To achieve improvement in mathematics learning, goals for math instruction needed to be much broader than the traditional curricular focus on proficiency at skills and content mastery. Reform-based instructional practices have improved students' abilities to learn to think mathematically and problem-solve, but they also need to gain content knowledge (NRC, 2001; Schoenfeld, 2004). Mathematics educational stakeholders share the goal for all students to gain complete mathematical proficiency. The core proficiencies listed by NCTM (2014) were initially proposed in NRC's Adding It Up: Helping Children Learn Mathematics (2001) and have been articulated differently by other researchers as three literacies: symbolic literacy, conceptual literacy, and problem-solving applications (Jacobs, 2011; Milgram, 2007). NCTM's (2014) five strands of proficiency list conceptual understanding, procedural fluency with strategic competence and adaptive reasoning. The first two describe conceptual literacy and the second two describe problem-solving. The fifth strand references students' attitudes and the importance of productive disposition which is a proficiency separate from content knowledge (Code et al., 2016, NCTM, 2014; Schoenfeld, 2007a). Researchers not only emphasize that students need to learn and master these proficiencies or literacies, but teachers need to identify and implement instructional strategies, either traditional or reform-based, that are most effective at promoting each. To do this, teachers need a deeper understanding of each of the core proficiencies. (Jacobs, 2011; NRC, 2001; NCTM, 2014; Milgram, 2007; Schoenfeld, 2007b).

Schoenfeld (2007a) describes mathematical proficiency in four parts: knowledge base, strategies, metacognition, and students' beliefs and dispositions. The knowledge base, often referred to as skills, includes students' ability to use arithmetic facts, definitions, and symbols (Jacobs, 2011; Ramaley, 2007; Schoenfeld, 2007a; Wu, 1999). When describing the knowledge base, foundational skills, or symbolic literacy needed for deeper understandings in mathematics, researchers use words like rigor, flexibility, precision, fluency, symbols, literacy, and definitions (Hirsch, 1996; Milgram, 2007; Ramaley, 2007; Schoenfeld, 2007a, 2007b; Wu, 1999). In addition, content specifics such as knowing operations, relations, and what symbols mean are essential to being symbolically literate. (Kilpatrick, Swafford, & Findell, 2001; Ramaley, 2007).

Traditional curricular instructional strategies providing practice, vocabulary, and skill development have proven to build and maintain students' basic symbolic literacies (Jacobs, 2011; Milgram, 2007; Wu, 1999). Students' precision and fluency with basic knowledge and skills are the foundations for conceptual understanding and literacy (Wu, 1999). "Students must learn precision because if they do not, they will fail to develop mathematical competency" (Milgram, 2007, p. 56). Often procedural skills or symbolic literacies have been referenced in research literature or in mathematical standards as procedural literacy, but this gets confused with doing mathematics and proficiencies in conceptual literacy (Schoenfeld, 2007a).

The next level of proficiency, conceptual understanding or literacy, is strategic competence or the ability to formulate, represent, and solve mathematical problems (Ramaley, 2007; Schoenfeld, 2007a; Wu, 1999). One component of conceptual understanding is moving from concrete or intuitive examples to the abstract, or "from primitive skills to sophisticated ones, such is the normal progression in mathematics" (Wu, 199, p. 2). Moving from symbolic literacy to conceptual literacy moves students from knowing the definitions and symbols to using them in equations and applications (Milgram, 2007). Conceptual understanding or conceptual literacy in mathematics is the beginning of students' abilities to do math – when students master the mathematical processes and algorithms which become the tools for further inquiry and problem solving (Schoenfeld, 2007b; Wu, 1999). Both traditional instructional strategies of examples and practice and reform-based instructional strategies using manipulatives, inquiry, and experiments have proven to be effective in developing students' conceptual literacies (Jacobs, 2011; Milgram, 2007; Ramaley, 2007; Schoenfeld, 2007b).

The third proficiency consistently referenced in the literature is problem-solving. Most mathematicians will explain that all mathematics is problem-solving (Milgram, 2007; Schoenfeld, 2007b; Wu, 1999). To problem-solve, mathematicians extend known results, find new results, or apply known results in new contexts (Schoenfeld, 2007b). The goal for mathematics educators is to help students become good problem solvers who are flexible and resourceful (Milgram, 2007; Schoenfeld, 2007a). Reform-based instructional strategies developed in alignment with NCTM's (2000) process standards which include problem-solving, reasoning, making mathematical connections, and communicating mathematics orally and in writing have been shown to be effective in building students' problem-solving proficiencies (Jacobs, 2011; Schoenfeld, 2007a; Thompson, 2009).

It is important to understand that there is a difference between routine problem-solving and proficiency in problem solving. Traditional curricula have word problems which are solved in formulaic, algorithmic ways or ways that are rigid, list-making approaches (Milgram, 2007). Students must have the opportunity to solve problems "where the answer is not immediate and requires a novel idea from the student" (Milgram, 2007, p. 47). There are verbal and non-verbal parts of problem solving and students need total fluency with basic concepts to become successful problem solvers in mathematics (Milgram, 2007).

Another dimension to problem-solving proficiency means developing an understanding of one's own thinking or metacognition (Schoenfeld, 2007a). "Effective problem solvers behave differently – they have learned to become more efficient at monitoring and self-regulation" (Schoenfeld, 2007a, p. 67).

The challenge for mathematics educators is to find or create sensible and well-posed problems. These problems can be authentic or real-world applications which are mathematically and developmentally appropriate for students at different levels (Milgram 2007; Schoenfeld, 2007a). In their essay on situated cognition, Brown, Collins, and Duguid (1989), the precursors of inquiry-based learning (IL) or problem-based learning (PBL), introduced the concept of authentic problems or contextual problems which engage student in real-world problem-solving (Hmelo-Silver, Duncan, & Chinn, 2006;

Jacobs, 2011; NCTM, 2014; Schoenfeld, 2004; Schoenfeld & Kilpatrick, 2013; Thompson, 2009) Building students' problem-solving abilities and proficiencies takes practice and experience (Jacobs, 2011; Milgram, 2007).

The NRC (2001) and Schoenfeld (2007a) both referenced the final proficiency as a "productive disposition." The NRC (2001) stated that students need a "habitual inclination to see math as sensible useful and worthwhile" (p. 5). This final proficiency is related to students' attitudes about mathematics based on their personal and classroom experiences. To change or improve students' attitudes about mathematics, the nature of the mathematical activities used to develop symbolic literacy, conceptual literacy, and authentic problems-solving proficiencies must change. Research suggests that inquiry learning, investigations, problem-based learning and collaboration can foster productive dispositions (Starr & Hoffman, 2005). To help students develop productive dispositions students need interesting mathematical experiences, real-world or relevant activities, and engaging pedagogical approaches to content (Jacobs, 2011; Milgram, 2007).

In summary, extreme positions were and are the problem in mathematics education. "An exclusive focus on basics leaves students without the understandings that enable them to use mathematics effectively. A focus on 'process' without attention to skills deprives students of the tools they need for fluid, competent performance" (Schoenfeld, 2004, p. 280). The consequence of the division between two ideological and pedagogical perspectives is a highly fragmented and variable curriculum across the United States (Jacobs, 2011; Polly, 2016; Schmidt, 2012). Mathematics and curriculum researchers have called for better curricular focus, specificity, rigor, and coherence in mathematics curriculum (Hirsch, 2004; Schmidt, 2012). There needs to be a synthesis of traditional and reform-based mathematics educational practices to address students' needs: students need to have mathematical skills and need to understand mathematics in context (Hirsch, 2004; Jacobs, 2011; Schoenfeld, 2004, 2014; Wu, 1999). Both traditional and reform-based practices have strengths and effective practices that can build students' core mathematical proficiencies.

Milgram (2007) recommended that the entire mathematical educational system in the United States be rebuilt on a foundation of symbolic literacy, conceptual literacy, and authentic problem-solving applications. Ramaley (2007) suggested that "perhaps a more integrative model can move us toward shared understanding about what math must be taught, how and to what end" (p. 19). The National Mathematic Advisory panel (2008) recommended that a balance should be struck between basic mathematical processes and problems-solving suggesting that both skill development and problem-solving can be taught together. To improve the mathematical achievement and dispositions for students in the United States a solution may be the design of an integrated curricular approach that uses effective activities and exercises from both traditional and reform-based practices targeted at each level of proficiency – symbolic, conceptual, and authentic problem-solving –to make a better use of instructional time both inside and outside the classroom.

Given two perspectives – each offering part of the curriculum – and the call to merge the two perspectives around the three core literacies, it is possible to name the design pattern and state the design problem.

NAME: Integration of Mathematical Inquiry, Symbolic literacy, and Technology (IMIST) System

Design PROBLEM: How can educators integrate mathematics learning opportunities for students that promote mastery of symbolic and conceptual literacies, foster problem-solving abilities, and situate mathematics in authentic learning contexts?

The Core of a Solution

To help students gain mathematical knowledge and mastery of the key proficiencies, the core of the solution focuses on choosing and designing appropriate learning activities. To learn is to gain knowledge. One branch of learning theory suggests that knowledge is constructed and that knowledge construction is an active process in which learning and doing cannot be separated. (Dai, 2012; Dougherty, 2012; Hmelo-Silver et al., 2007). Understanding activity theory may help guide practitioners to choose learning activities which support inquiry learning and the development of each of the key mathematical proficiencies: symbolic literacy, conceptual literacy, authentic problem-solving strategies and skills.

Activity theory. Activity theory is about "who is doing what, why and how" (Hasan & Kazlauskas, 2014, p. 9). It has is its foundations in the work of Vygotsky (1978), Leont'ev (1981), and Engestrom (1987). "Activity theory rests on the assumption that all intentional human actions are goal-directed and tool-mediated" (Venkat & Adler, 2008, p. 129). Activity theory investigates "the relationship between the subject (the human doer) and the object (the thing being done)" by looking at the focus or purpose of the activity, the outcomes, and the motivation behind the activity (Hasan & Kazlauskas, 2004, p. 9). Activity theory also investigates the mediating effects of different artifacts or tools which help the doer accomplish the activity.

Artifacts and tools may be classified into three categories. Primary artifacts or tools are physical tools such as instruments, machines, computers, or calculators. Secondary tools include language, symbols, ideas, and models. Tertiary tools are communities, contexts, or environments. (Hasan & Kazlauskas, 2014; Nardi, 1996)).

In the context of education, activities are the strategies and practices that engage learners in doing and knowing. The purpose of activities is to create opportunities for learners to master learning goals (Hathaway & Norton, 2013). "Carefully structured activities provide students with new ways of looking at what occurs around them every day and develop an appreciation of how to think, talk, and act like" professional practitioners in a discipline (Dai, 2012, p. 10). When activities are set in the context of an assignment, those "assignments hold the potential to make learning and teaching more focused and relevant because in the crafting process teachers must be deliberate and highly aware of the context, content, and charge involve in an assignment" (Dougherty, 2012, p. 7).

Activity theory provides a language and a set of frameworks to conduct research and/or to construct activities. In complex situations, such as educational environments, there may be many inter-related activities forming a system of activities. To construct or design a system of educational activities, practitioners must first identify the important activities of the system considering the activity's doers, purpose, motivation, and learning goals. Second, practitioners must identify the actions and mediating tools (primary, secondary, and tertiary) required to perform the activity. Finally, there is a tension or dynamic with and between the activities that can be guided or mediated by the practitioner to connect the purposes and learning goals between activities (Hasan & Kazlauskas, 2014). Using this framework, researchers and practitioners implement learning activities by identifying underlying process and mechanisms, how information gets encoded and retrieved for use, how the users' or students' new responses are strengthened and habitualized, and the situation or learning environment (Dai, 2012).

In addition, activities have operational aspects – the way the action is carried out either consciously or habitually. Initially, the activity or action may have an explicit goal to which the user has to consciously pay attention in order to carry out the activity. With practice, the action becomes an automatic response that is operational or habitualized. In the context of education, initial activities engage learners in conceptual and procedural understanding. With practice, these concepts and procedures become skills for later use and retrieval to use for activities with higher cognitive demand. In activity theory, the components of activities are not fixed but change dynamically as conditions change. Goals, activities, and operations change as conditions and context change (Nardi, 1996).

There is also a motivational component in activity theory. Activities should be shaped or designed to support learner responsiveness and motivation (Dai, 2012; Nardi, 1996). Students' desire for knowledge may be driven by choosing activities that are socially important or personally meaningful. Learning and doing cannot be separated. Human motivations are deeply involved in any socially organized, goal-directed learning activities. It is important that students feel a need to know. (Dai, 2012). Recent studies on active learning with instructional practices structured using student-centered activities show that these types of learning activities promote increases in student performance in science, engineering, and mathematics (Freeman et al., 2014; Terada, 2019).

Activities to support student learning of core mathematical proficiencies. . Using an activity theory system approach, the literature that frames the design problem supports the use of inquiry learning and reform-based activities as an overarching framework to motivate and engage students in learning and doing mathematics. Inquiry learning engages students in thinking and reflection which is in stark contrast to learning as regurgitation and passive absorption of prescribed knowledge (Dai, 2012). The IMIST instructional system addresses the core of the solution using an activity theory system approach focusing on each component of the design pattern and using design questions to identify content, learning goals, effective activities, appropriate tools, and authentic applications. Successful or effective pedagogical approaches and activities help students: (a) understand more clearly, (b) remember more accurately, (c) perform in assessments more competently, and (d) transfer and apply knowledge and skills in new contexts (Holmes & Hwang, 2016).

The introductory component to inquiry learning, problem-based learning, and/or mathematical modeling, requires a “well-posed problem” (Milgram, 2007, p. 49). Well-posed problems may be situated in the real-world, culturally relevant, or interdisciplinary in nature. These problems may have many different solutions depending on how students define variables, frame their thinking, and communication solutions. Teachers and practitioners should choose problems that support learning objectives, justify the content that needs to be learned, and engage student learners (Jacobs, 2011; Milgram, 2007; Schoenfeld, 2004).

Inquiry learning structured around a well-posed problem requires scaffolding activities, guiding student inquiry, and is compatible with the cognitive structures that help students learn (Hmelo-Silver et al., 2007; Schmidt, Loyens, van Gog, & Pass, 2007). Supporting activities should be chosen to scaffold student learning and understanding of the core proficiencies and mathematical disposition – symbolic literacy, conceptual literacy, and problem-solving applications in authentic contexts (Hmelo-Silver et al., 2007; Schmidt, Loyens, van Gog, & Paas, 2007; Schoenfeld, 2004; Wu, 1999).

Inquiry Learning (IL) activities, together with Problem-Based Learning (PBL) and mathematical modeling activities, are centered around relevant, authentic questions to develop mathematical thinking and problem-solving strategies. IL activities motivate students to learn content, strategies, self-directed learning skills, and reasoning skills. Activities are often collaborative and focus on students' sense-making, explanations, and communication (Hmelo-Silver et al., 2007; Holmes & Hwang, 2016; Jacobs, 2011; Schoenfeld, 2004). IL environments "present learners with opportunities to engage in complex tasks that would otherwise be beyond their current abilities. Scaffolding makes the learning more tractable for students by changing complex and difficult tasks in ways that make these tasks accessible, manageable, and within students' zone of proximal development" (Hmelo-Silver et al., 2007, p. 100). Inquiry learning increases students' deeper understanding of mathematical concepts, helps them retain knowledge, and helps them apply knowledge in real-world applications (Holmes & Hwang, 2016).

Inquiry learning has been shown to engage students and to improve attitudes in mathematics (Blair, 2014; Boaler & Staples, 2008; Holmes & Hwang, 2016). It answers two important questions:

1. Why do we have to learn this?
2. Where is the mathematics?

IL activities provide students with the value and relevance behind mathematical topics and content as well as justifying their study and learning of skills and concepts necessary to solve problems (Blair, 2014; Hmelo-Silver et al., 2007; Holmes & Hwang, 2016). Inquiry learning and problem-solving are closely linked. Using inquiry learning to introduce an authentic real-world problem or application at the beginning of a unit design sets the stage and the rationale for the development of students' symbolic literacy and conceptual literacy for the specific content topic. Introduction and scaffolding of IL activities prepares and helps students to extend and apply this knowledge to solve problems flexibly, effectively, and successfully (Jacobs, 2011; Milgram, 2007; Schoenfeld, 2007a). Two components of the IMIST instructional system use inquiry learning activities aimed at problem-solving. The first component focuses on situated or authentic problem-solving activities used as introductory and/or culminating activities for the unit. The second type of problem-solving activities are scaffolding activities that help develop fluency, conceptual connections, and strategies that can be extended to solve bigger problems in situated or authentic contexts.

Situated/authentic inquiry activities. The initial component of the IMIST instructional system is to present situated or authentic inquiry activities to give students opportunities to practice and think like mathematicians (Schoenfeld, 1989). Research has shown that learning is socially and culturally constructed, and inquiry learning leverages this type of learning by using authentic situations, scenarios, and cognitive apprenticeships enculturating students in the activities of mathematicians (Brown et al., 1989; Dai, 2012; Schoenfeld, 1992). These activities are designed to promote mathematical thinking and problem-solving in natural ways of trial and error, guess and check, and using prior knowledge. Inquiry learning activities are often complex, nonroutine problems that require students to seek solutions, explore patterns, and formulate conjectures. Inquiry learning activities help students to develop mathematical habits and dispositions (Schoenfeld, 1992).

These activities may be interdisciplinary, modeling relationships in science, chemistry, or physics, or they can be based in real-world, culturally significant contexts using examples from sports, entertainment, or videogames. Answers to inquiry learning activities often vary and are dependent on the way students identify key variables, concepts, and generate their models and solutions. The authentic stories that situate mathematical activity culturally and socially provide more engagement and motivation for students to do mathematics and to develop mathematical habits of mind (Brown et al., 1989; Schoenfeld, 1992).

Symbolic literacy activities. A scaffolding component of the IMIST instructional system to support IL focuses on choosing activities that develop students' symbolic literacies. Pre-requisite knowledge and basic mathematical skills are the foundation of symbolic literacies. Practice activities which develop precision and fluency of these skills support rigor and are required to develop deeper conceptual understanding and problem-solving proficiency (Hirsch, 1996; Wu, 1999). In addition, students must be enculturated in the vocabulary and language of mathematics by definition, symbols, graphical representations, and context (Brown et al., 1989; Milgram, 2007; van Jaarsveld, 2016; Wu, 1999).

Students' mastery of symbolic literacy is important to IL as students with superficial knowledge of basic skills and concepts have trouble seeing patterns, building conceptual understanding, and problem-solving applications (Hmelo-Silver et al., 2007; Holmes & Hwang, 2016). Symbolic literacy activities encourage practice and rehearsal, repetition, and application of math facts, basics skills, and pre-requisite knowledge that prepare students to do tasks with higher cognitive demand such as applications in modeling, solving equations, and inquiry learning activities (Adams, 2010; Jacobs, 2011; Milgram, 2007). Tools that mediate symbolic literacy activities include traditional worksheet skill drills and practice as well as flash cards, computer software activities and mobile apps which gamify practice and repetition. The purpose or learning goal for these activities is to help students develop operational, automatic, or habitualized responses and knowledge of these basic skills (Hasan & Kazlauskas, 2014; Nardi, 1996).

A second focus of symbolic literacy is to help students know and use mathematical language correctly. Mathematical language and definitions need to be exact or precise to help develop critical thinking and rigor (Adams, 2010; Milgram, 2007; van Jaarsveld, 2016). Correct use of mathematical definitions and language prepares students to be able to communicate mathematical thinking – their own and others', justify their solutions and methodology, and analyze and evaluate problems in context. These skills are critical to help students move onto higher levels of conceptual understanding and problem-solving (Adams, 2010; Milgram, 2007).

Effective symbolic literacy activities may use language literacy strategies such as reading, writing, and vocabulary development to help students with fluency of basic skills and concepts (Adams, 2010). Mathematics educators need to help students learn to read and summarize mathematical texts. Strategies such as pre-reading, anticipation guides, and verbalization prepare and help students engage with new material (Adams, Pegg, & Case, 2015; Roepke & Gallagher, 2015). Writing or "writing to learn" activities increase understanding, achievement and problem-solving skills in mathematics. Effective vocabulary instruction supports students' sense-making, their ability to make connections to prior or pre-requisite knowledge, and their ability to recall knowledge for later use (Adams, 2010).

Conceptual literacy activities. Another scaffolding design component of the IMIST instructional system focuses on the development of students' conceptual literacy of mathematics procedures and applications. Conceptual literacy is often described as "doing" mathematics or "using" mathematics (Schoenfeld, 2007b; Wu, 1999). Conceptual knowledge can be compared to a set of tools and using them in mathematical contexts helps build deeper understanding of both symbolic and real-world applications (Brown et al., 1989). Instructional activities focused on conceptual literacy build on what students know, their pre-requisite knowledge and skills, and lead to procedural fluency (Jacobs, 2011). Students need to use their knowledge base to be able formulate, represent and solve mathematical problems using equations, graphs, diagrams, and different representations of content. They need to develop flexibility and abilities to extend knowledge in new ways to develop new tools and strategies for doing mathematics (Milgram, 2007; Schoenfeld, 2007a). Understanding procedural algorithms and why they work is an important part of conceptual literacy (Wu, 1999).

Included in conceptual literacy activities would be those focused on NCTM's (2000) process standards such as reasoning, proof, communication, connections, and representations. Reasoning and proof activities provide opportunities for students to learn and do mathematics that includes sense-making, justification, and explanation of why something works or is true. Communication activities in mathematics require students to understand and share their thinking and methodologies in writing, using diagrams, or other representations. Communication at this level reveals students' deep conceptual understanding of why mathematics works. Activities that build conceptual understanding may come from traditional sources such as worksheets and textbooks which provide different examples and types for practice and pattern recognition. Students should be asked to reflect, summarize, and articulate their understandings from these activities. Other activities may relate to the guiding inquiry learning activities and questions asking students to analyze data and to build symbolic models, preparatory for problem-solving activities (Jacobs, 2011).

Scaffolding problem-solving applications. Proficiency in problem-solving is not the ability to solve routine problems represented by typical textbook word problems. It is the ability to solve a problem "where the answer is not immediate and requires a novel idea from the student” (Milgram, 2007, p. 47). In general terms, problem-solving approaches include: 1) understanding the problem, 2) breaking it down into smaller parts and replacing each part by a precise mathematical question; 3) devising a method or plan to solve each of the mathematical questions; 4) carrying out the methods or plans; 5) evaluating, refining, and revising the plans; and 6) making sense of the results in context (Milgram, 2007; Polya, 1945).

Scaffolding activities that help students build problem-solving skills begin with simple contextual problems which help students use strategies to decode written language and encode the information into mathematical symbols, variables, and models. These problem-solving applications can also be contextual and real-world. More complex or open-ended activities can build upon these simpler applications. Traditional textbooks may provide sources for introductory problem-solving activities. However, practitioners may look to reform-based activities, problem-based activities, and at situated, real-word, culturally relevant applications to engage student learners (Boaler & Staples, 2008; Holmes & Hwang, 2016; Jacobs, 2011; Milgram, 2007).

As students wrestle with large nonroutine inquiry learning activities, these smaller more “routine” problem-solving activities can scaffold knowledge and strategies that students may be able to use and apply to larger, non-routine problems (Schoenfeld, 1992). They can illustrate mathematical techniques and methods using smaller, more focused contexts. These scaffolding problem-solving activities help students make connections between conceptual understandings, abstractions, and larger, complex problem-solving activities. Specifically, they help students define variables and develop different representations to find answers and solutions (Hmelo-Silver et al., 2007; Milgram, 2007, Wu, 1999).

To choose and identify tools to mediate problem-solving and problem-solving applications, teacher practitioners need to consider the affordances of different types of tools and how they meet learning objectives. Also, leveraging and using technology tools has been shown to motivate and engage students in problem-solving activities (Dai, 2012; Tapscott, 2009). As examples, teachers may use computer applications and software, spreadsheets, calculator applications and graphing utilities, data analysis and statistical software, as well as Internet resources for data bases and real-world problems.

Activities that support students' mastery of symbolic literacy and conceptual literacy provide students with the knowledge, skills, and strategies to become problem-solvers and to apply mathematics in authentic, real-world contexts (Milgram, 2007; Schoenfeld, 2007a).

IMIST instructional system – design pattern components. To answer the design problem, "How can educators integrate mathematics learning opportunities for students that promote mastery of symbolic and conceptual literacies and foster problem-solving abilities and situate mathematics in authentic learning contexts?," the IMIST instructional system uses inquiry learning strategies and activities integrated with scaffolded learning activities to build new knowledge from previously studied concepts. The design pattern can address the recurring problem of designing content instruction to develop students' conceptual understandings and ability to use this new knowledge to successfully solve problems (Jacobs, 2011).

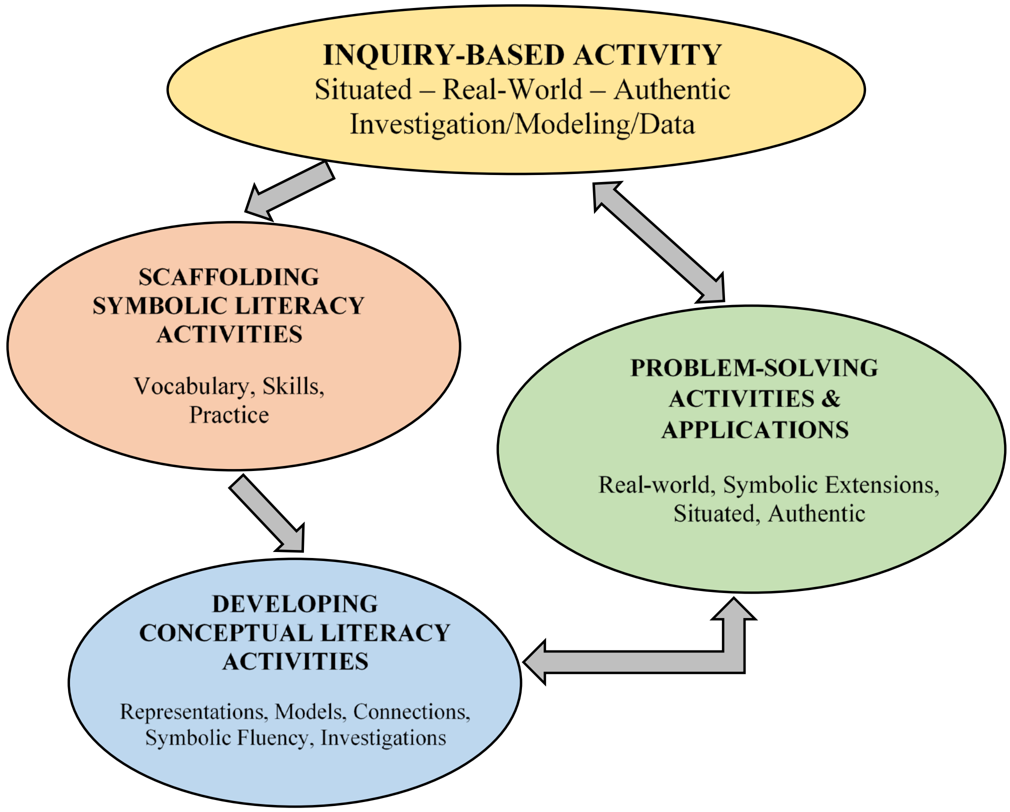


Figure 2. This concept map illustrates the IMIST instructional design pattern components and their interrelationships. It includes examples of the types of learning activities that support inquiry learning, scaffolding symbolic literacy, scaffolding conceptual literacy, and scaffolding problem-solving.

Figure 2. IMIST Instructional System Design Pattern

The IMIST instructional system design pattern has four main components: an introductory and/or culminating inquiry-based, situated learning activity, supported by scaffolding symbolic literacy activities, scaffolding conceptual literacy activities, and scaffolding problem-solving activities.

As illustrated in the figure above, there is an interrelationship between introductory inquiry learning activities and problem-solving applications. A problem which may have been too difficult to solve at the beginning of a unit may be revisited after symbolic and conceptual literacies and skills have been learned and/or mastered. Likewise, conceptual literacy investigations could be linked to problem-solving applications and vice versa to provide more context behind the development of conceptual understanding and procedural fluency. It is important to understand that not all problem-solving applications may have real-world applications. For example, geometric proofs or symbolic proofs are situated in the context of mathematical language and practice and often involve highly conceptual and abstract thinking important to preparation for mathematics in higher education.

When using the IMIST instructional system, practitioners or teachers need to ask specific design questions addressing each component of the design pattern – inquiry learning, scaffolding symbolic competence, developing conceptual procedures, and using knowledge to solve and communicate solutions to real-world, authentic problems. These questions may guide the teacher practitioner to make content and activity decisions and choices to help design instruction and strategies for a given topic or unit.

Two design questions need to be answered by practitioners using IL. What authentic, real-world, or situated activities support content learning objectives and associated mathematical knowledge? What scaffolded activities support students' construction of that knowledge and problem-solving proficiencies?

To choose effective practices and activities to help students build and master symbolic literacies, teacher practitioners need to answer two questions. What vocabulary and pre-requisite knowledge and skills do students need to master the new content and learning goals? What strategies and activities support students' construction of their knowledge and fluency with basic skills, symbols, and mathematical language?

To design effective practices and strategies to help students build conceptual literacy, practitioners need to answer two questions. What procedures, applications, tools, and strategies do students need to understand, use, and apply the content to meet learning goals? What activities support students' development of conceptual literacy and applications?

To choose problems and activities to help students build problem-solving strategies and communication, teacher practitioners need to answer two questions. What strategies do students need to decode and encode written information, to identify variables, and to mathematically model authentic problems? What activities and technologies support students' abilities to engage in authentic problem-solving, sense-making, and communication?

Pattern Consequences

The pattern consequences are issues and tasks that need to be addressed before implementing a design pattern within the context of the instructional or school setting. The pattern consequences help the practitioner or teacher identify and evaluate the needs, requirements, and considerations that support a successful implementation of the pattern (Hathaway & Norton, 2013, 2018). To implement the IMIST instructional system to design curricular units in a secondary or high school mathematics classroom, teacher practitioners need to consider: (a) the curricular standards of learning mandated by the school, the district, or the state; (b) their beliefs about pedagogical practices of traditional versus inquiry learning pedagogical approaches; (c) the social and behavioral norms of their classroom learning environment; (d) instructional strategies and plans for activities inside and outside the classroom; (e) assessments practices; and (f) resources and technology.

One pattern consequence is the need for teacher practitioners to be well-versed in the content standards of their school, district, or state. A deep understanding of the content knowledge, skill requirements, and authentic, situated problem-solving applications will help them identify the activities that support student mastery and help students extend content knowledge into new contexts and applications.

Another pattern consequence is for teacher practitioners to reflect on their own personal beliefs, their strengths and weaknesses regarding instructional strategies and pedagogy. Teachers from traditional instructional backgrounds may need professional development or support to implement inquiry learning strategies into their classrooms. They need to be comfortable with the inquiry learning process of student-centered investigations and experiments that may produce a variety of methods and answers.

The learning environment both inside and outside the classroom is another pattern consequence. Teacher practitioners need to consider not only the instructional strategies used inside and outside the classroom, but the social and behavioral norms that need to be discussed and implemented to promote student learning in both places. Teacher practitioners need to consider the constraints of instructional learning time and balance time required to do activities that promote symbolic and conceptual literacies with activities that build problem-solving strategies. They need to leverage the types of activities best suited to outside of class or homework to promote inquiry learning inside the classroom. This may require that they redesign their homework practices. As a result, they need to consider how to keep students accountable for their individual learning when doing homework activities as well as hold them accountable for investigatory collaborative group activities done in class.

A second part of this pattern consequence is to develop social and behavioral norms and expectations for both independent and collaborative work such as sharing responsibilities, breaking down tasks into different parts, and taking turns. Teacher practitioners may need to use scaffolding activities to build social skills, communication, and responsibility between students.

Another important pattern consequence for teacher practitioners to consider is the type of assessments they need to provide learning support in the classroom. They may need pre-assessments to determine what students know or their pre-requisite knowledge, and if they need remediation preparatory to learning new content. As teaching and learning progresses, teachers need to make informal and/or formative assessments to help guide and support student learning and to address needs for remediation and review. Finally, teacher practitioners must design summative assessments that will evaluate whether students have mastered the core literacies – symbolic, conceptual, and problem-solving.

Finally, teacher practitioners need to think about the technology tools that will support learning activities and may engage students in the learning process. They may need to consider what types of tools students themselves have available, what tools the school has available, and what type of tools may need to be acquired. These tools span the range from traditional tools of colored pencils, markers, rulers, compasses, and graph paper to technology tools such as hand-held calculators, computers, and cell phones. Access to the Internet may also be a key consideration both at home and at school for the types of activities teachers can assign for homework and in-class activities.

Pattern Summary

The IMIST instructional system provides a framework for teacher practitioners to address the recurring problem of designing instructional units of mathematical content. Teacher practitioners need to design curricular content that addresses the key concerns of mathematics educational stake holders by integrating effective practices from reform-based inquiry learning activities that promote problem-solving abilities and student engagement with effective practices that build strong foundations in symbolic and conceptual literacies. The four components include an introductory and/or culminating well-posed problem activity situated in real-world mathematical context. The second component is a scaffolding component designed around symbolic literacy activities to support mathematical skills and vocabulary development of the new content. The next scaffolding component is designed around conceptual literacy activities which help students make connections between symbolic literacy, procedural skills, and different representations using equations, models, graphs, and tables to provide strategies for problem-solving. The final component is designed using scaffolding problem-solving activities, both symbolic and authentic, which support strategies and mathematical thinking that can be used to extend knowledge into new or more complex non-routine problems and activities.

Design Research

The principles of design, activity theory, and the design pattern approach used to conceptualize the IMIST instructional system provide a potential solution to the complex recurring problem of designing relevant, meaningful, and effective curricular units to support student learning in mathematics. This study of the IMIST instructional system is one part or phase of a larger design research project.

Design research as a genre of scientific inquiry is a process that seeks to develop theoretical insights and practical solutions simultaneously using real-world settings and contexts (McKenney & Reeves, 2012). Design research seeks to advance design, research, and practice concurrently. Design research "identifies real world problems, suggests actions to improve the status quo, and involves teachers in the research process" (Hathaway & Norton, 2018, p. 5). Design research provides possibilities for creating innovative learning environments, contextualizing theories of learning and teaching, constructing cumulative design knowledge, and promoting the human capacity for innovation (Hathaway & Norton, 2018).

Design research is a complex and multi-faceted research process which is: "adaptive, collaborative, contextual, flexible, goal-oriented, grounded, integrative, interactive, interventionist, iterative, methodologically inclusive, multilevel, pragmatic, process-focused, theoretical, transformative and utility-oriented" (McKenney & Reeves, 2012, p. 13). The design research process may use an iterative framework of three phases: (a) Analysis and Exploration, (b) Design and Construction, and (c) Evaluation and Reflection (McKenney & Reeves, 2012). Another Integrative Learning Design Framework (ILDF) has four phases: (a) Informed Exploration, (b) Enactment, (c) Evaluation: Local Impact, and (d) Evaluation: Broader Impact (Bannan-Ritland, 2003). For each phase of a design research study, the research methodology chosen should be appropriate to the type of problem, research question, and context under study (Bannan-Ritland, 2003; Crouch & Pearce, 2012; McKenney & Reeves, 2012). Each phase, study, or micro-cycle of design research may use traditional quantitative or qualitative data collection methodologies such as experiments, surveys, observations, interviews, assessments, and document analysis (Bannan-Ritland, 2003; Crouch & Pearce, 2012; Dai, 2012; McKenney & Reeves, 2012).

In the first phase of design research, Analysis and Exploration or Informed Exploration, the research focus is to analyze and define the problem. A problem is a discrepancy between the existing state and the desired state (Hathaway & Norton, 2018). In the context of learning, this could be a set of instructional strategies and curricula that are not producing the desired learning outcomes and achievement. To fully understand the problem, the researcher or designer may collect and analyze data and background information from a variety of sources including a review of the literature, educational experiences, needs analyses, surveys, interviews, observations, and assessment results (Bannan-Ritland, 2003; Hathaway & Norton, 2018; McKenney & Reeves, 2012).

The second phase of design research, Design and Construction or Enactment, focuses on intervention design, prototype design, or system design which proposes a solution to the problem and implements the intervention or solution in context (Bannan-Ritland, 2003). Following a coherent conceptual process, a tentative intervention is developed, designed, and documented. The design process involves rational and purposeful consideration of available knowledge and the interrelationships of that knowledge with techniques and strategies to meet the needs of users and to provide tentative solutions. Core ideas underpinning the design, both theoretical and/or practical, are articulated to design the framework or system. Construction is the process of taking the conceptual framework built by the design concepts and creating a prototype to be implemented in context (McKenney & Reeves, 2012).

After the second phase of initial design and construction of the prototype, the third phase of design research, Reflection and Evaluation or Evaluation: Local Impact evaluates the intervention, prototype, or system using empirical testing, surveys, and feedback from the users to determine how well the intervention or solution addressed the needs of the users, and if and how it addressed the issues of the problem in context. Analysis of this data is used to adapt and improve the intervention or system, and to prepare it to be implemented again in another cycle of research or generalized and implemented into a broader context or theory (Bannan-Ritland, 2003; McKenney & Reeves, 2012).

The final stage articulated in the ILD Framework is Evaluation: Broader Impact. After evaluation, reflection, and improvements are made to the design framework from implementing the prototype and using it in local or smaller contexts, the final stage generalizes the prototype or system and uses it in a broader context. This includes the dissemination of the research via publications or presentations. This final stage focuses on the adoption of the research practices and interventions in the larger community (Bannan-Ritland, 2003).

This study focused on the second phase of design research – the Design and Construction (McKenney & Reeves, 2012) or the Enactment phase (Bannan-Ritland, 2003). The IMIST instructional system was used to design two curricular content units or prototypes to implement with high school or secondary mathematics students. The first unit was an Algebra 2 unit on quadratic functions, and the second was a geometry unit on right triangles and right triangle trigonometry. Appendix A presents the IMIST design document for the quadratics unit, and Appendix B presents the IMIST design document for the geometry unit. Each design document lists learning activities, participants, and literacies. This study investigated the impact of these units on students' learning in the core proficiencies – symbolic literacy, conceptual literacy, and problem-solving – as well as students' mathematical attitudes and confidence in doing mathematics.

Summary

This literature review discussed ongoing concern about students' low mathematics achievement in the United States, differences in educational philosophies governing instructional materials (traditional and reform-based) and research and recommendations that promote the integration of the most effective instructional strategies and activities from both perspectives. This study investigated a potential solution or framework to help educators design instructional materials to promote student engagement, active-learning, and better achievement in mathematics. As curriculum, unit, and lesson design is a recurring task or problem for teachers and educators, the researcher used a design pattern approach grounded in design principles and thinking to develop and name the IMIST instructional system. After understanding the design problem, the core of the solution has its foundations in activity theory and choosing the most effective learning activities to promote students' proficiencies in the core literacies of mathematics – symbolic, conceptual, and problem-solving. This study investigated the impact of a unit designed using the IMIST instructional framework on student achievement, attitudes, and confidence and solicited feedback and evaluation of their learning using the IMIST learning activities.

Chapter Three

This project was part of a larger design research study. It was the second phase or evaluation phase investigating prototypical, instructional units in small or local implementations (Bannan-Ritland, 2003). The study focused on the construction and implementation of units developed using a design pattern approach evaluated by student users (Hathaway & Norton, 2018; McKenney & Reeves, 2012). Unit activities were designed following a series of design research exploratory or informed exploration phases which had never been field-tested (Bannan-Ritland, 2003; McKenny & Reeves, 2012). The purpose was two-fold. First, the research sought to determine how these units affected students' perceptions, attitudes, engagement, and problem-solving strategies in the core mathematical literacies of symbols, concepts, and problem-solving. Second, the research sought to obtain user feedback to evaluate prototypes developed using the integrated IMIST framework which present mathematics instruction and activities combining the best, research-based practices of inquiry learning and core literacy activities (Bannan-Ritland, 2003; McKenney & Reeves, 2012).

The research questions for this study were:

1. What is the impact of a mathematics unit designed using the IMIST system on students' understanding of mathematics and its core literacies?

2. What is the impact of a mathematics unit designed using the IMIST system on students' attitudes, confidence, and engagement with mathematics?
3. What do students report about their mathematical learning experiences when their learning is structured by a unit designed using the IMIST system?

Research Design

As this study was the second phase or evaluation of a design research project, it combined two different research methodologies: case study and action research. To provide rigor to the iterative case study process, each of the four case studies used both qualitative and quantitative elements for data collection instruments and analysis (Glanz, 2016; Yin, 2014).

Case study. Given the total of four case studies, this project would best be described as a collective or multiple case study (Glesne, 2011; Maxwell, 2013). Each case study included an element of action research given that the researcher served in the role of teacher/instructor presenting the unit materials and activities. Qualitative data were collected via semi-structured interviews, surveys, and observations. Quantitative data were collected via graded assessments. Due to the small sample size, the quantitative results were informative and descriptive rather than statistically significant. For the purposes of analysis, this study used primarily qualitative methodology.

Yin (2014) defined case study as an empirical methodology of inquiry that "investigates a contemporary phenomenon...within its real-world context" and "relies on multiple sources of evidence, with data needing to converge in a triangulating fashion" (p. 16-17). Willis (2014) explained that case study research "provides a nuanced, empirically rich, holistic account of specific phenomena" (p. 14). Case study methodology was appropriate for this research because students' mathematical learning is a contemporary issue under the constant scrutiny of stakeholders. To better understand and improve students' mathematical learning, stakeholders can gain valuable insights by studying students' perspectives and achievement directly. Furthermore, this study sought students' feedback as they used integrated activities in real-world, authentic contexts. Multiple data sources came from surveys, semi-structured interviews, researcher's observations and notes, and assessment data which provided triangulation, convergence, and validity for themes and findings of the study (Maxwell, 2013; Yin, 2014). These data sources provided insights to address the research questions.

Students who participated in this study provided the individual cases, although the cases themselves varied by delivery method and content. The first case study examined a pair of students using the IMIST unit materials to study an Algebra 2 quadratics unit delivered online. The next two were individual case studies. One student studied the Algebra 2 quadratics unit online, and the second student studied a geometry unit presented face-to-face. A final case study collected data from students in a small Algebra 2 class taught by the researcher/instructor in a face-to-face school setting. Data for each student participant was collected individually. By comparing and analyzing the group of individual cases, this research gained the added dimension of being a collective or multiple case study (Glesne, 2011; Maxwell, 2013). Students used unit materials and activities designed with the IMIST system framework to investigate how integration of inquiry activities as well as more traditional literacy activities supported their learning, engagement, and confidence in mathematics. Using the individual case studies as a collective provided the opportunity to compare students' experiences and perceptions for similarities, themes, and patterns across the individual case studies (Glesne, 2011; Maxwell, 2013; Saldaña, 2016).

Action research. The element of action research came from the researcher's role as teacher/instructor presenting and summarizing the materials, content, and activities. The goal of action research is to improve practice through acting, observing, and reflecting (Glesne, 2011). Practical action research is often used by teachers to address issues that focus on procedures, activities, and to solve classroom problems. Practical action research studies often involve teachers as researchers, studies of student learning, and implementation of new learning plans or activities (Creswell, 2012; Glanz, 2016). The researcher's direct involvement as teacher/instructor introducing and summarizing unit activities as well as evaluating student understanding using new learning activities added the element of action research to this study. The qualitative data collected in researcher's notes and memos provided insights for the second and third research questions. These data focused on students' attitudes, perceptions, and confidence documented by observations of students' reactions, comments, beliefs, and perspectives about activities during real-time and direct communication with students.

Quantitative data. Quantitative data for each case study were derived from grading and analysis of unit assessments, formative and summative. Each question was assigned a point-value based on question type (e. g., short answer, multiple choice, free response), content, and steps or procedures required to provide an answer. Students' overall scores, points correct as a percentage of total points, provided students with feedback on their mastery and achievement. To study students' achievement in the core literacies, the researcher used descriptive codes to categorize each assessment question using the researcher-generated constructs or core literacies – symbolic (s), conceptual (c), and problem-solving (p). The researcher used magnitude coding to break down points for assessment questions by literacy and found the cumulative sum for each literacy to determine students' level of mastery and understanding of each (Saldaña, 2016). "Magnitude Coding is a method that applies numbers or other symbols to data and even to codes themselves that represent values on a scale" (Saldaña, 2016, p. 72). Due to the small sample size, information derived from magnitude coding of cumulative points in each core literacy could not be analyzed for statistical significance. Descriptive statistics using means and percentage achievement on assessments provided insights and evidence of students' learning and understanding in each core literacy.

Qualitative data. Although elements of action research (qualitative) and quantitative research (assessments) were employed, the primary research methodology for this study was qualitative. The researcher used qualitative data from five sources: (a) questionnaires to collect students' demographic information; (b) surveys to assess students' attitudes and perceptions about mathematics; (c) transcripts of pre-intervention, semi-structured interviews [case studies of individual students only]; (d) researcher notes from teacher-led, online and in-class unit lessons presentations, activities, and discussions; and (e) transcripts of post-intervention, semi-structured interviews [case studies of individuals students] or written post-intervention surveys [students in small class case study].

After identification of participants for the individual case studies and completion of the assent/consent forms, each student completed a Math Attitudes and Perceptions Survey (MAPS) (See Appendix C) adapted for high school students from the survey developed by Code, Merchant, Maciejewski, Thomas, and Lo (2016). Students in the individual case studies returned survey answers to the researcher before the IMIST unit intervention. Due to time constraints, student participants in the small class case study filled out the MAPS at the end of the study and returned them to the researcher. A questionnaire was included with MAPS which collected demographic information including gender, age, ethnicity, and grade level. For the participants in the individual case studies, students were asked to include the number of years they had been studying mathematics as independent learners or home-schooled students. This data helped establish learner profiles, background, and baseline for comparison for each student and his or her relative interest, experience, and confidence in mathematics. Although the survey was a Likert-type survey used to quantify student responses, the sample size was small, and data analysis used magnitude coding to understand each student's perceptions about him or herself as a learner of mathematics on a scale or continuum (Saldaña, 2016).

Using individual case study answers to MAPS as a guide, the researcher conducted semi-structured interviews with each student in-person or online (See Appendix D). Interview data provided additional insights into students' attitudes, perceptions, and experiences learning mathematics. Each interview was recorded (audio) and transcribed by the researcher. Each student chose a pseudonym for use in the transcript and the study to protect his or her identity. The interview questions established the length of time that the student had been an independent learner, and how that learning was structured. Students were asked to clarify and to explain their answers on MAPS, particularly their perceptions of themselves as mathematics students. They were asked about curricular resources and learning activities they used to study and practice mathematics. They were asked about familiarity and confidence in their mathematical literacy and knowledge of symbols, concepts, and problem-solving. Students were asked about the role of technology in their learning as well as the role of practice and assessments. Students were asked about how they got help, got questions answered, made corrections, and handled mistakes or misconceptions about mathematics.

Following initial MAPS and interview data collection, each student completed a mathematics unit designed using the IMIST system framework. Implementation was different in delivery and the amount of content covered for the student participants in the individual case studies as compared to student participants in the small class case study. Algebra 2 students studied the IMIST quadratics unit, and one student studied an IMIST unit on right triangles and trigonometry. Student participants in the individual case studies attended virtually or in-person researcher/teacher led introductions and summaries of each of the learning activities. Each student was provided with a unit outline of activities and a set of handouts with guided notes and activities. For online students, these sessions occurred twice per week for one to one and one-half hours during the four-week unit. The geometry student met face-to-face with the researcher for one to one and one-half hours per week. Students in the individual case studies were asked to supplement with practice activities from their textbook or other curricular resources. As independent learners, students were responsible for their own learning and practice with support from parents or others. As these varied for each student, this was a practical solution for the purposes of this study.

During online lessons, the researcher annotated class notes slides creating a log of student responses given during the lesson. She also kept a research journal summarizing and reflecting on the activities and responses given by students during the lesson. Both the annotated notes and the researcher's journal provided data for qualitative analysis and evaluation of the instructional activities and presentations.

For the implementation part of the study, student participants in the small class case study met face-to-face with the researcher/instructor four to five times per week for 45-minutes in a classroom setting. Instructor's presentation of introductory instructions for activities, summarization and discussion of students' reflections and "lessons-learned" as well as teacher-led lessons were similar in content and format to the case studies for students in the individual case studies since they were guided by the same class notes and activities provided in the IMIST unit packet. However, students in the small class setting were given specific assignments in the school's textbook to reinforce learning and practice in-line with the school's and math department's expectations. Students were given time in each class to review answers and to ask questions about homework. The instructor also held students accountable for the textbook work by doing a daily homework check. The small class case study was constrained by time and by the content covered by the school's syllabus. Students in the Algebra 2 small class case study only used the first seven sections of the IMIST unit as compared to the nine sections taught in the Algebra 2 individual case studies.

At the completion of the IMIST unit, individual case study participants participated in post-intervention interviews. For participants in the small class case study, students completed the MAPS and written post-intervention surveys (See Appendix E). Post-intervention interviews and post-implementation MAPS provided data about students' perceptions, experiences, attitudes, and confidence using content units designed with the IMIST system. The interview and survey questions addressed the research questions in detail focusing on student understanding and learning of mathematics and its core literacies, students' attitudes and engagement with the IMIST unit activities, and students' confidence in doing mathematics. Students were asked whether they liked or did not like the IMIST unit activities, and how they were the same or different from learning activities they had used in the past. They were asked to evaluate the unit structure, delivery, and if it helped them learn. Specifically, they were asked which activities helped them learn vocabulary, symbols, and symbolic literacies; which activities helped them learn and understand multiple representations, equations, and conceptual literacies; which activities helped them learn problem-solving strategies. In addition, students were asked to evaluate the use of technology, guided notes, worksheets, instructions, and activities. Finally, they were asked whether they felt they had learned more or less with the IMIST unit activities as opposed to curricula they had used before. Interviews for participants in the individual case studies were recorded and transcribed using students' pseudonyms. Students in the small class case study filled out and returned the written surveys to the researcher.

Participants and Setting

Students in this study came from two groups: students who were independent learners or homeschooled in mathematics and students who were members of a traditional Algebra 2 class taught as a medium-sized, private high-school in the Mid-Atlantic area. They were chosen based on their qualifications and willingness to participate. It was a purposeful selection (Maxwell, 2013).

To recruit independent/homeschooled students, the researcher solicited assistance via email and direct communication from individuals including educational professors, home-school liaisons, and parents to identify and provide contact information for parents of students willing to participate in the study. Specifically, the researcher sought to identify students who were prepared to study geometry or Algebra 2 for the 2018-19 school year. Most students studying mathematics courses at this level have had prior experience with high school curricula and with independent learning and/or online learning environments. Parents and students who participated in the study had identified particular curricula for the school year for either geometry or Algebra 2. The sample included two boys and two girls who typified independent learners who had experience in homeschooled settings. This included two students who learned mathematics independently via textbook, and two-students involved with a facilitator-lead group using online curricula or a combination of these learning formats.

Parents and participating students received a “Study Information Sheet” presenting a summary of the study’s goals, purposes, research procedures (See Appendix F). They also received consent and assent forms as part of the Institutional Review Board (IRB) process (Appendices G, H, and I). The researcher mailed two hard copies of the research summary and the forms with a self-addressed stamped envelope to students and families participating online. After receiving signed consent and assent forms, the researcher emailed MAPS, unit materials, outline, and preliminary interview questions. The researcher was in email communication with the participants and their parents providing her email address and cell phone information. During the recruitment processes, the researcher spoke individually with all the parents and students to answer all questions that they had about the study via Google Hangouts or in person. The researcher kept a log and made notes of these conversations as part of her research journal to identify issues and concerns relevant to the study. For the geometry student, all forms, materials, and interviews were completed face-to-face.

The setting for the online Algebra 2 classes was at the discretion of both the researcher and the participants. The lessons and activities included both synchronous and asynchronous elements. Communication, interviews, lessons, and document exchange was completed via email or Google Hangouts. The researcher/instructor and students worked in whatever environment gave them the best access to the Internet. This was most often at home or at a library. The setting for the IMIST unit instruction for the geometry student was in-person at the researcher’s home. Instructions, introductions, summaries, discussions, and assignments were given weekly and completed by students at home.

In addition, to maintain confidentiality and to protect the students' identity, students chose pseudonyms to provide anonymity in reporting their information, responses, and experiences for this study. Only the primary researchers involved in this study had access to a list of students and their pseudonyms that has been kept in a secure location by the researchers and will be destroyed after a period of five years.

Students who participated in the small class case study attended a private high school. The researcher initiated communication about the study with teachers and administrators at the school during the spring and summer of 2018 hoping to implement the study. When one of the Algebra 2 teachers took a leave-of-absence, the teachers and administrators asked the researcher to serve as a long-term substitute. Because the students were studying the two units prior to the quadratics unit as well as the quadratics unit, the researcher was given permission to use the IMIST instructional materials that correlated to the school's Algebra 2 unit on quadratics during her tenure as a substitute teacher and to do a small case study for one of the three classes she was teaching.

Similar to the student participants in the individual case studies, parents and students in the small class case study were provided with the Study Information Sheet with consent and assent forms preparatory for the implementation of the IMIST unit (Appendices F, G, and H). There were eighteen students in the class. Nine chose to participate – six girls and three boys. The research was conducted in a chemistry classroom setting with some technology issues.

Data collected during the study included class observations, homework completion/effort grades, and assessment data. At the end of the intervention, nine student participants completed the Math Attitudes and Perceptions Survey as well as a written survey which used some of the questions asked in the post-intervention interviews given in the individual case studies (Appendices C and E). To protect the identities of participating students, the researcher assigned pseudonyms to the students when referencing class comments, observations, and in the data analysis. The pseudonym assignment was random not purposeful. Only the primary researchers involved in this study had access to a list of students and their reference names. These were kept in a secure location and will be destroyed after a period of five years. Table 1 lists student participants by study pseudonyms, type of case study, learning environment, and IMIST delivery structure.

Table 1

Summary of Student Participants, Learning Environments and Delivery Structure for IMIST Intervention

Student	Case Study	Course	Learning Environment	IMIST Delivery
Hailey	Pair/Individual	Algebra 2	Virtual Classroom	Online/Face-to-face
Bobcat	Pair/Individual	Algebra 2	Virtual Classroom	Online/Face-to-face
Kaya	Individual	Algebra 2	Virtual Classroom	Online/Face-to-face
Howard	Individual	Geometry	One-on-one	Face-to-face
Abigail	Small Class	Algebra 2	Classroom	Face-to-face
Brenda	Small Class	Algebra 2	Classroom	Face-to-face
Chad	Small Class	Algebra 2	Classroom	Face-to-face
Don	Small Class	Algebra 2	Classroom	Face-to-face
Erin	Small Class	Algebra 2	Classroom	Face-to-face
Frank	Small Class	Algebra 2	Classroom	Face-to-face
Gloria	Small Class	Algebra 2	Classroom	Face-to-face
Helen	Small Class	Algebra 2	Classroom	Face-to-face
Jackie	Small Class	Algebra 2	Classroom	Face-to-face

Data Collection Instruments

This study used five data collection instruments to answer the research questions. A demographic survey collected data on student characteristics such as gender, age, class level, grade, and ethnicity. Four additional instruments collected data designed to answer the research questions on achievement, attitudes, confidence, and experiences using the IMIST unit activities. These instruments included the IMIST assessments, MAPS, and the guiding questions used for the pre- and post-interviews and/or post-intervention surveys. The following will discuss the purpose, design, composition, and validity of the instruments.

IMIST achievement assessments. There were three assessments for each IMIST unit to assess students' mastery of the core literacies – symbolic, conceptual, and problem-solving. These assessments were developed by the researcher from assessments previously constructed and administered by Algebra 2 and geometry colleagues at different secondary schools. Two formative assessments examined students' mastery of the learning objectives, standards, and core literacies in the first three and second three sections of the IMIST unit respectively. The unit assessment assessed learning objectives, standards, and core literacies for the entire unit. The learning objectives and standards were compiled from state standards, textbook standards, and the CCSSM for quadratics and right-triangle geometry (Virginia Department of Education, Mathematics Standards of Learning for the Commonwealth of Virginia, 2018; CCSSM, 2019; Textbook standards: Larson, Boswell, et al., 2007, 2012; Larson Boswell, & Stiff, 2003; Prentice-Hall, 2008).

Student participants in the individual case studies completed the assessments on their own time, outside of instructional time. These assessments measured learning at an honors level. Student participants in the small class case study completed the assessments during class time. The assessments given to these students were shorter, assessed standard level proficiencies, and did not assess the last two sections of the Algebra 2 unit.

To assess mastery of learning objectives and literacies, the assessment questions were designed using similar formats and structures found in the activities and exercises used in the IMIST unit. They reflected the content, context, and types of exercises used for learning and practice. The formative assessments had short answer questions requiring students to show work and procedures when appropriate and to write their answers. Questions that assessed problem-solving literacy were free-response or essay-type questions. Students had to identify the variables, encode the context into variables and symbolic language, work through the mathematics, and provide a contextual answer written as a sentence. The only multiple-choice assessment questions appeared as three questions on the Algebra 2 summative assessment. The rest of the questions on the unit assessments were short answer or free-response types. Although there was no formal validation process, the questions and question types had been vetted over years of practice and assessment in both Algebra 2 and geometry classes by the researcher and her colleagues.

Each question on each assessment was allocated points and coded by literacy (s: symbolic, c: conceptual, p: problem-solving). Many questions assessed combinations of literacies and the total points for the question where divided to reflect the appropriate points for each literacy. Many students received partial credit as a result. For example, they might choose a correct or appropriate method for solving a particular question and receive credit for conceptual literacy but lose points in symbolic literacy when they made an arithmetic or algebra error. Appendix J presents the unit test for student participants in the small class case study showing coding by literacy and points. Further discussion of the coding and point allocation is in the data collection section.

MAPS. The Math Attitudes and Perceptions Survey used for this study was adapted from a survey designed and validated by Code et al. (2016) that was administered to freshmen and sophomores in university level mathematics courses. They stated, “One goal of an undergraduate education in mathematics is to help students develop a productive disposition towards mathematics. A way of conceiving … this is as helping mathematical novices transition to more expert-like perceptions of mathematics” (p. 917). This goal statement is consistent with the standard of “productive disposition” introduced by the NRC (2001) and supported by NCTM (2014). The purpose of the MAPS survey was to assess changes in university students’ attitudes and perceptions at the beginning and end of a mathematics course. As the intervention for this study was only one unit of IMIST instruction, MAPS was administered to the students only once to assess students’ attitudes, beliefs, and confidence about their experience in mathematics education at the middle and high school levels.

Code et al. (2016) developed MAPS from different survey instruments and studies including the study done by Petocz et al. (2007), the Conceptions of Mathematics Survey (CMQ; Crawford, Gordon, Nicholas, & Prosser, 1998), and the Colorado Learning Attitudes about Science Survey adapted for the domain of mathematics (CLASS; Adams, Perkins, Dubson, Finkelstein, & Wieman, 2006). Code et al. solicited input from experts – instructors, teachers, and educators of mathematics – to explore and confirm categories that identified and classified categories of mathematical beliefs. The categories included confidence in mathematics, persistence in problem-solving, growth mindset, interest in mathematics, relationships between mathematics and the real-world, and sense-making. MAPS was validated using student interviews and by administering the survey to students (N = 3411). They found a Cronbach's alpha value of 0.87 or 95% confidence interval for the whole instrument indicating good reliability. Alpha values for individual categories were slightly lower due to a small number of questions in each category.

This study used 32 questions taken directly from MAPS as written by Code et al. (2016). The only difference was to reduce the number of categories from seven to five to correlate more precisely with the research questions for this study. Survey questions on growth mindset, interest, sense-making, and answers where combined into two categories titled "attitudes" and "learning." Questions in the "attitudes" category focused on general beliefs about mathematics, and questions in the "learning" category focused on students' personal beliefs about learning mathematics. Questions about real-world, persistence (work ethic), and confidence were categorized in the same way as in the Code et al. survey (Appendix C).

Pre-intervention question guide. Pre-intervention questions (Appendix D) were designed to collect data on demographics, academic background, mathematical learning experiences and curricula as well as students' attitudes towards learning mathematics from students in the individual case studies. Students in this group came from a variety of homeschooled or independent learner backgrounds. This information helped guide the researcher to understand the students' level or academic readiness for the IMIST unit activities, discussions, and practice exercises.

The first two questions asked students about demographics and to classify themselves as mathematics students by level, interest, and work ethic. These questions followed up on the students' answers to MAPS. The next set of questions asked students about the curricula they had used to study mathematics and the format for that instruction such as virtual, online, or face-to-face. The questions asked about parent involvement and help-seeking strategies.

Next, questions asked students about their understanding of the core literacies, symbolic, conceptual, and problem-solving, and if they could link learning activities or strategies that helped them with mastery of these literacies. The researcher took this opportunity to define and explain the meaning and what type of learning was reflected in each of the core literacies. In addition, students were asked about their confidence in learning mathematics for each of the core literacies.

Questions asked students about the roles of assessments, homework, practice, and technology in their mathematics learning. They were asked to describe their favorite and least favorite thing about mathematics, and whether they ever had an *aha* moment in mathematics when they made a significant leap in understanding. For all questions, students were asked to explain, give examples, and provide insights into their thinking.

Post-intervention question guide. The post-intervention question guide asked questions of student participants in the individual case studies and were used to create written surveys completed by the student participants in the small class case study

(Appendix E). The questions were designed to investigate students' attitudes and experiences using the IMIST learning activities. Most of the questions asked students for a positive or negative response (yes/no, like/did not like, or more/less) and to provide an explanation, example, or rationale for their responses.

The first series of questions assessed students' attitudes and asked students whether they liked/did not like learning with the IMIST activities and whether they liked/did not like the unit structure. The next series of questions asked students whether the IMIST activities improved their understanding of the core literacies, symbolic, conceptual, and problem-solving. Students were asked to include specific activities or examples. Students were asked whether they learned more or less with the IMIST activities than they learned in their previous experiences studying mathematics. Students were asked about the unit delivery and whether the class instruction was helpful and whether the practice worksheets/homework helped them learn. Consistent with design research goals, students were asked to evaluate and make recommendations about how the unit could be improved. They were asked to comment about structure, organization, clarity, delivery, and/or types of activities. Finally, and as a follow up to the pre-intervention survey, students were asked to describe their favorite and least favorite IMIST learning activity(ies) and whether they had an aha moment during the IMIST unit when they made a deeper connection or had deeper understanding of a specific concept taught during the unit.

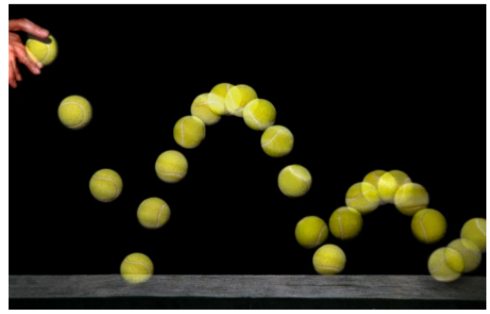
Treatment

The researcher designed two units using the IMIST instructional design system described in Chapter Two. The Algebra 2 unit on quadratic functions was given in its entirety to the students in the individual case studies. The algebra students in the small class case study only covered the first seven sections of the IMIST unit per the school's syllabus and course content requirements. The researcher designed an additional geometry unit for the single geometry case study on right triangles and trigonometry. All IMIST unit activities were designed for honors level instruction. To accommodate the students in the standard level Algebra 2 small class case study, the researcher made adjustments to practice exercises given for homework assignments and to the depth and breadth of the content measured on the assessments. The unit activities reflected the IMIST system design framework by identifying authentic problem-solving applications and choosing the symbolic and conceptual learning activities to review, support, and scaffold mathematical skills and content needed for the unit's learning objectives. The researcher chose additional authentic scaffolding problem-solving applications to support symbolic and conceptual literacies as well as to build students' problem-solving strategies (Hmelo-Silver et al., 2006).

The product of the IMIST system design process or the treatment for each case study was a packet of learning activities designed and written by the researcher to support the learning objectives for Algebra 2 and geometry units. Each unit was subdivided into sections with specific learning objectives/content to scaffold learning.

Using the IMIST framework, each unit began with an authentic, introductory investigation and a project to situate the unit topic in a real-world, non-school context. For example, both the quadratics and the right triangle units began with a discussion about what students remembered about the topic followed by an Internet investigation of applications and professions that used quadratics or right triangles as part of their professional practice. The second part of the introductory section for the quadratics unit solicited students' help with a game-design question asking students how they could model or mathify the path of a bouncing ball on a screen. The activity provided them with a time-lapse photograph of a bouncing tennis ball for analysis. Figure 3 illustrates the questions and discussion used as a summary of the introductory activity for the quadratics unit and as an introduction into solving quadratic equations. The introductory right triangle project asked the students to take pictures of triangles at work in the world around them, and then to analyze the pictures mathematically by providing measures, lengths, and angles. These introductory investigations and projects built a framework to scaffold the learning objectives for subsequent sections in each unit.

Unit Project: Equations & Solutions of Quadratic Functions



- What does it mean to "solve" a quadratic equation?
- Think about the Wintendo project. If you had to pick three points that are the most important to the path, which would you pick?
- Describe how you would measure those points. What variables did you describe?
- How would you assign numbers to those points? (Think scales!)
- What *variable and value* would you assign to the ground?
- Can you guess what the solution to your model might be?

<u>Complete the Unit Project: Wintendo Games</u>
Based on what you learned above, go back and complete the unit project. Find an equation model for the path of the ball bounce!

Figure 3. This learning activity asks students to discuss and answer questions that were asked in the introductory lesson to the quadratics unit. After exploring quadratic equations in vertex form, this activity may be used to introduce intercept form for quadratic equations.

Figure 3. Quadratics Section 3 IMIST Learning Activity - Discussion of Unit Project, Developing Equation Models, and Solving Quadratic Equations

The quadratics unit for the standard-level Algebra 2 class identified seven scaffolding learning objectives which became topics for each section. Student participants in the individual case studies were given two additional content sections, and exercises corresponded to the depth and breadth of an honors-level course. The honors geometry IMIST unit identified eight scaffolding learning objectives or sections.

Five of the seven sections for the small class case study treatment on quadratics included student-centered, inquiry investigations followed up by a summary discussion of Lessons Learned. Seven of the nine sections of the treatment for the individual case studies had student-centered investigation activities. An example of a student-centered learning activity used in the first section for all case studies was a technology-enhanced learning activity called “Connect the Dots” in which students used tables, equations, graphs, and calculators to explore quadratic graphs. Follow-up questions encouraged students to think and make connections between numerical parameters in the equations and transformations and critical features of parabolas such as the vertex, axis of symmetry, and orientation (See Figure 4). A second student-centered learning activity used experimental data and regression to derive the gravitational constant for acceleration. This constant is used in physics and in authentic applications of dropping objects and projectile motion. During summary discussions, the instructor and students discussed what they learned and reviewed vocabulary and symbolic literacy skills to build conceptual understanding of multiple representations and applications.

CONNECT THE DOTS!!!

Algebra 2 Unit – Quadratic Functions Name: ______________________________

Investigation 5-1: Graphing Quadratics & Transformations

Technology Enhanced Activity Class/Period: ________ Date: ___________

Technology: Graphing Calculator or Web-based Graphing Utility (Desmos: https://www.desmos.com/calculator)

Investigation 5-1: Graphing Quadratics & Transformations

Directions: Complete the table(s) using and graphing *anchor points* for x-values: {-3, -2, -1, 0, 1, 2, 3} for all graphs listed below. Then use a graphing calculator or graphing utility to check your answers.

1. **Parent Function:** $y = x^2$
   a. Complete the table.
   b. Use these anchor points to graph the parent function for quadratics on the coordinate grid.

➢ Describe the graph. Include its shape, orientation, and critical point(s).

x	$y = x^2$
−3	
−2	
−1	
0	
1	
2	
3	

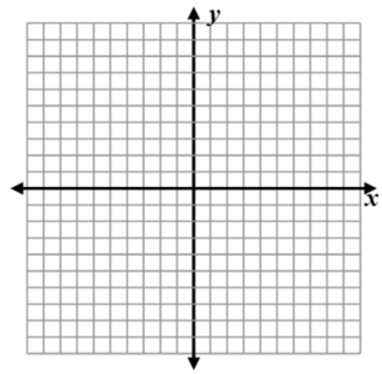


Figure 4. Student technology enhanced learning activity introducing the exploration of the quadratic parent graph and transformations.

Figure 4. Quadratics Section 1 IMIST Technology Enhanced Learning Activity - Representations of Quadratic Functions Using Tables, Graphs, and Equations

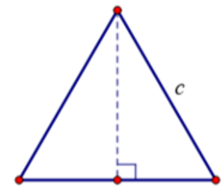
In the treatment for quadratics, four of the sections focused on the development of conceptual literacies focused on solving quadratic equations using different methods. Situating the concept of solving using the time-lapse photo of a bouncing ball from the introductory project, the discussion focused on what points are interesting in the picture such as the top of the bounce or vertex and the points where the ball bounces on the ground. Students were able to contextualize the concepts of the ground being a horizontal axis and height being the vertical axis. By applying a scale to the photo, students explored the concepts of zeroes and factors (See Figure 3). They were also able to connect the different forms of quadratics equations and parameters to model and describe the parabolas mathematically. To support students' understanding of solving, learning activities explored and reviewed symbolic literacy skills such as factoring, taking the square root, completing the square, and using the discriminant and quadratic formula. Students did additional practice using worksheets or textbook exercises. One of the additional sections for the individual case studies, investigated quadratic inequalities and systems of inequalities. These activities extended students' knowledge of graphing and solutions to represent a range of solutions or a region in the coordinate plane.

For the small class case study, four of the seven sections had authentic application activities to promote problem-solving. These applications were core quadratics learning objectives that modeled dropping objects and projectile motion. The individual case study students had these same activities and one additional section devoted to authentic applications in other non-school, real-world applications. These examples included economic modeling, optimization, and architectural applications. These additional applications followed up on discoveries students made during the introductory discussion. A complete list of learning activities used in the IMIST quadratic function unit is in Appendix A. Each activity is classified by literacy.

Every section in the geometry unit included student-centered investigations and/or "think about" reflection activities. Students were expected to think about geometric relationships and make conjectures by writing their ideas in English, writing the relationship in symbols, and following up with drawings, diagrams, or applications. These activities included reflections about the Pythagorean Theorem and different types of geometric proofs, investigating similar triangles (See Figure 5) and the ratios of side lengths preparatory to trigonometry, and explorations of the Laws of Sines and Cosines using diagrams and coordinate proof. Similar to the Algebra 2 sections, these activities were followed by summary discussions or Lessons Learned during which the instructor reviewed vocabulary and reviewed pre-requisite symbolic literacy skills such as simplifying and working with irrational numbers or radicals. The instructor also provided symbolic applications and authentic real-world applications as example exercises. Conceptual learning activities were also included in all sections helping students to apply the theorems using triangle diagrams preparatory for real-world applications.

Investigation 9-4b: Special Right Triangles

Construct an equilateral triangle then fold it in half. Label the side length "*c*". Look at the new triangles formed.



➢ What do you notice about the angle measures?

➢ What do you notice about the side lengths?
- The length of the hypotenuse?
- The length of the short leg?
- Use the Pythagorean Theorem to find the length of the long leg.

Lessons Learned – Investigation 9-4b: Special Triangles

➢ Write a conjecture about what you observed in the investigation.

- What type of right triangle is created?
- Describe the angle measures.
- Draw a diagram.
- Describe the relationship between the side lengths.

***Conjecture*:** ____

____

____

➢ Check your work!
To "Say it in Symbols" complete ***Example 2!***

Figure 5. Geometry IMIST student investigation into 30°-60°-90° special triangles – their side and angle measures

Figure 5. Geometry Section 5 IMIST Learning Activity – Student Investigation of Special Triangles

To build problem-solving strategies, all eight sections of the IMIST geometry unit provided conceptual and authentic applications for student practice and problem-solving. Authentic applications considered right triangles in construction and architecture, finding areas and volumes for gardening applications, and navigational applications to find the distances between ships at sea. A complete list of learning activities used in the IMIST right triangle unit is in Appendix B. Each activity is classified by literacy.

Data Collection

Table 2 presents the timeline for the implementation, participant recruitment, unit development, data collection, and data analysis for this research study. The preliminary work and development of the instructional units' materials and assessments occurred during the spring and summer of 2018. The researcher submitted the research proposal for defense and followed up with the Institutional Review Board (IRB) to gain necessary permissions and guidelines for the study. Concurrently, the researcher began the development of the IMIST units and recruited parents and their students to participate in the study. The study was conducted during the 2018 – 2019 academic year. Assent and consent forms were given to students and their parents prior to the implementation of instruction unit. This was followed by the administration of the demographic questionnaire and MAPS for the student participants in the individual case studies. All forms and surveys were collected by the researcher via email or hard copy. For the individual case studies, pre-intervention interviews were conducted and recorded prior to the implementation of the units. The researcher transcribed the interview data and then began a cyclical coding process with pre-coding, highlighting, and matrices using descriptive and In Vivo codes (Saldaña, 2016).

The first case study of two Algebra 2 students began online sessions, activities, and assessments in October 2018. The researcher made notes, memos, and collected and graded all student assessments during this first implementation. Graded assessments were returned to students via scan/email to provide achievement feedback. During December, the researcher conducted the post-intervention interviews on the IMIST unit followed by transcription and initial coding analysis.

The small class case study began prior to the post-intervention interviews for the first case study. The researcher/instructor began work with the class during the last week in October. The approximately four and a half week IMIST unit case study began in mid-November and finished in mid-December.

In January 2019, the researcher implemented the second individual, online case study of the IMIST quadratics unit following the same protocol of providing the Study Information Sheet, assent form, consent form, and unit materials as she had done for the first individual case study. As the student was taking an online course for Algebra 2 with a workbook that did not specifically correlate to the IMIST unit, the instructor also mailed her an Algebra 2 textbook (McDougal-Littell, 2001) to be used as a reference and for practice activities. Data collected included a pre-intervention interview, researcher's notes and memos, annotated class notes, and three assessments. The final post-intervention interview was conducted in March followed by transcription and data analysis.

The fourth case study, the individual case study of a geometry student, began in mid-March and concluded in mid-May. The researcher followed the same IRB research protocols and collected the same data gathered in the previous individual case studies. The two primary differences were that a different IMIST unit for geometry and right triangles was implemented and the researcher met weekly with the student participant face-to-face.

Table 2

Timeline for Research on the IMIST Unit Design System

Task	**Time**
Proposal Defense	April 2018 – Revised August 2018
IRB Submission	June 2018 – Revised August 2018
Design Algebra 2 Unit: Quadratics	Spring/Summer 2018
Recruitment of Parents and Students	August - December 2018
Assent/Consent Forms	September - December 2018
Demographics & Math Attitudes and Perceptions Survey (MAPS)	Individual Case Study 1: October 2018 Small Class Case Study: November 2018 Individual Case Study 2: December 2018 Individual Case Study 3: November 2018
Pre-Intervention Interview	Individual Case Study 1: October 2018 Individual Case Study 2: January 2019 Individual Case Study 3: March 2019
Intervention: Algebra 2 Instruction & Assessment – Initial Data Analysis	Individual Case Study 1: October - November 2018 Small Class Case Study: October - November 2018 Individual Case Study 2: January 2018 – March 2019
Design Geometry Unit: Right Triangles and Trigonometry	Fall/Winter 2018-2019
Intervention: Geometry Instruction, Assessments, and Initial Data Analysis	Individual Case Study 3: March – May 2019
Post Intervention Interviews, Transcription, and Analysis	Individual Case Study 1: December 2018 Individual Case Study 2: March 2019 Individual Case Study 3: May 2019
Final Data Analysis	May – August 2019

Data Analysis

The primary data sources for this study were demographic questionnaires (Appendix C), Math Attitudes and Perceptions Surveys (MAPS, Appendix C), formative and summative assessments, transcripts of pre- and post-intervention interviews (individual case studies), written response post-intervention surveys (small class case studies), annotated notes, and researcher's journal memos written during the implementation of the IMIST units (Appendices D and E). Data collected provided insights into the students' confidence as mathematics learners and their attitudes and perceptions about mathematics. Analysis of the researcher's notes and memos, the assessment data, and the post-intervention interviews and surveys (Appendix E) provided insights to answer the research questions and to compare and understand students' learning, attitudes, and perceptions – how they were the same or different for each of the students. The annotated notes and researcher's journal memos provided additional insights into how students interacted with the unit material and activities as well as their reactions and evaluation of the activities. Data collected from the demographic questionnaires was presented for each case study and compiled in a table for cross-case analysis.

Students' initial responses to MAPS were compiled in two tables for the two types of case studies – individual and small class. There were 32 questions on the Likert-type survey with responses scaled from 1 (Strongly Disagree) to 5 (Strongly Agree). The researcher used descriptive codes of positive (+) and negative (-) to classify each question to reflect a positive/mature or negative/less mature attitude towards mathematics. Student responses to negative questions were re-coded on the scale of one to five so that low cumulative scores would reflect less positive or negative attitudes or perceptions about mathematics and high cumulative scores would reflect positive attitudes about the students' beliefs about themselves and mathematics in general. The cumulative scores could range from 31 (low/negative attitudes and perceptions) to 155 (high/positive attitudes and perceptions.) The case studies report cumulative scores for student participants.

The cross-case analysis combined MAPS data to compare students' responses for all case studies. The researcher classified the questions into two primary categories: general attitudes and personal beliefs. Eight questions focused on general attitudes about mathematics which made statements about math as an innate ability and how people understand mathematics. The range of scores could be 8 to 40. There were 23 questions about personal beliefs which were further divided into sixteen questions about personal learning, four questions about work ethic and/or perseverance, and three questions about value and usefulness of mathematical knowledge and the real-world. The final category examined five questions from the personal beliefs that addressed students' confidence in both work and learning. High scores in each category represented positive attitudes or mature beliefs about mathematics, and low scores represented the opposite. Table 3 displays the categories for the MAPS questions, the number of questions in each category, and the numerical range for the scores possible in each category.

Table 3

MAPS Questions Classified by Category and Range

	Total	Attitudes	Learning	Work Ethic Perseverance	Real-world	Confidence
# Questions	31	8	16	4	3	5
Range	31-155	8-40	16-80	4-20	3-15	5-25
High	110-131	39	58-71	15-16	14	24
Medium	90-105	28-32	48-50	12-13	9-11	15-20
Low	82-88	25-26	33-47	10-11	6-8	6-14

Transcripts of pre-intervention interviews for the four individual case study student participants provided data to classify and understand similarities and differences between students as part of their background learner profiles. The researcher transcribed and read each of the interviews for initial coding analysis (Maxwell, 2013; Saldaña, 2016). The coding of the pre- and post-interview transcripts went through three cycles of coding. For pre-coding, descriptive codes were used to classify and categorize responses to identify and link keywords, patterns, and themes (Glesne, 2011; Maxwell, 2013; Saldaña, 2016). Each transcript was pre-coded using color highlighting, circling, and underlining. The preliminary descriptive codes used the researcher-generated constructs of the core literacies – symbolic, conceptual, and problem-solving. The pre-codes highlighted and identified learning activities that helped students learn and understand in each of the literacies. Additional pre-codes included colors for educational background, work ethic, and attitudes and beliefs. Descriptive subcodes for attitudes and beliefs were ability/confidence, work ethic, and assessments. Table 4 summarizes the pre-coding color-coding used on the interview transcripts.

Table 4

Pre-coding Color-scheme for Descriptive Codes on Pre-intervention Interview Transcripts

Descriptive Code	**Subcode**	**Color**
Background		Light Blue
Literacy		
	Symbolic	Orange
	Conceptual	Pink
	Problem-solving	Yellow
Technology		Green
Attitudes and Beliefs		
	Ability/Confidence	Dark Blue
	Work Ethic	Brown
	Assessments	Gray

The third part of the study was the delivery and implementation of the IMIST unit. As part of the design pattern approach, the researcher/instructor created a design document selecting learning activities to support inquiry, symbolic literacy, conceptual literacy, and problem-solving (Hathaway & Norton, 2018). She prepared class notes for annotation with accompanying student study guides and worksheets. Data collected from these annotated notes and the researcher's notes and memos were made after each online session. Data included student responses and the researcher's reflections on students' comments about understanding and improvements that could be made in the resources and examples. These responses served as part of the evaluation of the IMIST unit materials. The researcher also recorded students' attitudes and perceptions as math students during the discussion as well as commentary they made during the lessons. Using this data, the researcher was able to do a preliminary assessment of students' learning and attitudes using the IMIST activities. Again, the researcher's notes and memos were subject to coding analysis for student learning, attitudes, and confidence in doing mathematics using the descriptive codes described above.

In addition to the qualitative data from the annotated notes and researcher's journal, students were given two formative assessments (quizzes) and one summative assessment (unit test) for the IMIST unit. To study students' achievement in the core literacies, the researcher used abbreviations of the descriptive codes to categorize assessment questions using the researcher-generated constructs for the core literacies – symbolic (s), conceptual (c), and problem-solving (p). The researcher used magnitude coding to allocate the points for each question to determine students' level of mastery and understanding in each of the literacies (Saldaña, 2016). Since many of the questions used combinations of the core literacies, points awarded for the question were allocated to reflect the combination of literacies assessed. For example, a question might combine conceptual and symbolic literacy. Conceptual literacy points would be awarded for setting up the appropriate process or procedure for solving the question, and symbolic literacy points would be awarded for the algebraic and arithmetic evaluation of the solution. This often gave students partial credit on a particular question. This a common pedagogical or grading practice on short answer and free-response questions given in mathematics because students might get the conceptual literacy part correct and then make an algebra or arithmetic mistake or vice versa. Students' subscores for each of the literacies or literacy combinations were compiled for analysis and reported as percentages of total points for each of the individual and small class case studies. The researcher provided feedback to students using descriptive statistics reporting their percentage achievement overall for every assessment.

The final data collected were the post-intervention interview transcripts from the student participants in the individual case studies and the written post-intervention surveys from the student participants in the small class case studies. The researcher went through cycles of coding, initially to identify descriptive codes for the activities and students' perspectives for each literacy and then to look for In Vivo codes which used the students' own words to describe their attitudes and beliefs about their learning with the IMIST unit (Saldaña, 2016). The researcher transcribed each interview and then pre-coded the transcripts by highlighting, circling, and underlining for an initial line-by-line analysis of the data (Maxwell, 2013; Saldaña, 2016). For the student responses to the post-interview surveys, the researcher entered all data/responses into a large matrix so student responses to each question were side-by-side. Again, the researcher pre-coded this data for descriptive codes. This initial analysis was completed to identify and link themes, and patterns, and to identify In Vivo codes or key words. Table 5 lists the color scheme and marking for the pre-coding of the transcripts and response matrix as well as some of the In Vivo codes that emerged from the data.

Table 5

Pre-coding Color-scheme for Descriptive Codes on Post-intervention Interview Transcripts, Student Response Matrix, and In Vivo Codes

Descriptive Code	**Subcode**	**Color**
Evaluation		Light Blue
	Unit Structure	
	Investigations	
	Lessons	
	Worksheets	
Literacy		
	Symbolic	Orange
	Conceptual	Pink
	Problem-solving	Yellow
Technology		Green
Attitudes and Beliefs		
	Ability/Confidence	Dark Blue
	Work Ethic	Brown
	Assessments	Gray
In Vivo		
	Explain/Understand	Boxed
	Packets	Circled/Pen
	Work/Pacing	Underlined

After the initial coding and In Vivo coding, the researcher used magnitude coding to quantify student responses looking for patterns and categories in attitudes, perceptions, and understanding of mathematics. The researcher particularly looked for codes that expressed students' attitudes, (positive, negative, or mixed), views on work or practice (positive, negative, mixed), views on the values of the activities (inquiry, symbolic, conceptual, or problem-solving), and indications of understanding or learning as compared to previous learning in math class (more, less, or mixed). The final coding matrix grouped student data by code to reveal common themes and patterns. This was done for the individual case studies individually and for the small class case study as a collective.

The data collected for all the case studies was combined for a cross-case analysis. The data collected for the students' demographics, MAPS, and assessment scores were combined. The goal was to create a relational framework of categories for the data to review the data as a collective case study (Glesne, 2011).

Cross-Case Analysis.

Cross-case analysis "facilitates the comparison of commonalities and difference in the events, activities, and processes that are the units of analysis in case studies" (Khan & VanWynsberghe, 2008, p. 1). Each case study represented rich, holistic experiences for the student participants. As cases shared themes of achievement in the core literacies and students' attitudes, perceptions, and confidence in mathematics, comparing the data can construct meaningful links, insights, and inferences (Khan & VanWynsberghe, 2008).

The cross-case analysis for this project compiled data from all case studies to provide insights to answer the three research questions. To understand how the IMIST unit impacted student learning, assessment data were compiled for all thirteen students in tables to examine proficiencies in each of the core literacies and proficiencies overall. The tables included the percentage scores as well as descriptions of the types of questions used to assess the literacies. The data analysis linked students' comments and explanations that came from pre- and post-intervention interviews from the students in the individual case studies and from the post-interview survey given to the students in the small class case study in each of the core literacies. These student comments gave further insights into understanding students' achievement and mastery of the content.

To investigate the second research question, data collected during the case studies to assess students' attitudes, perceptions, confidence, and engagement included class observations, homework effort grades, and MAPS (Code et al., 2016). These data were compiled for all thirteen students in a table for comparison. Again, data collected from the students in the individual cases studies during the pre- and post-intervention interviews and data from the small class students' responses on the post-intervention survey were linked to support and explain observations, results, and trends.

To assess engagement and work ethic, the researcher compiled data from class observations and homework effort grades. The researcher/instructor kept notes on class participation for both online and face-to-face classes in her researcher journal and annotated notes. Initially, she used plus signs (positive/active) and minus signs (negative/passive) or a combination to indicate the level of engagement and class participation for each student. She also kept a record of individual student responses and observations. She used up and down arrows to indicate improving or declining attitudes or engagement during implementation of the IMIST unit. She used magnitude coding or numerical values to quantify and interpret the level of engagement on a scale of 1 to 5 to quantify the level of engagement for comparison between students. A score of "1" indicated negative attitude and/or low engagement. A score of three indicated a moderate level of class engagement or activity. For example, a score of "3" was given to a student who did not raise his or her hand to answer questions but would answer when called upon. A score of "5" indicated a high level of engagement and a willingness to volunteer answers and participate in class discussion.

To assess students' attitudes, beliefs, and confidence, the researcher made a detailed analysis of the students' responses to MAPS using a meta-matrix (Glesne, 2011; Khan & VanWynsberghe, 2008; Maxwell, 2013). Rather than only reporting total scores, the researcher classified the questions into two primary categories: general attitudes and personal beliefs. Eight questions focused on general attitudes about mathematics which made statements about math as an innate ability and how people understand mathematics. The range of scores could be 8 to 40. There were 23 questions about personal beliefs which were further divided into sixteen questions about personal learning, four questions about work ethic and/or perseverance, and three questions about value and usefulness of mathematical knowledge and the real-world. The final category examined five questions from the personal beliefs that addressed students' confidence in both work and learning. High scores in each category represented positive attitudes or mature beliefs about mathematics, and low scores represented the opposite. Students were placed in a table based on their score for each category and classified into the appropriate range – high, medium, or low.

The final part of the cross-case analysis, synthesized students' responses given during the post-intervention interviews and written on the post-intervention surveys in a meta-matrix (Glesne, 2011; Khan & VanWynsberghe, 2008; Maxwell, 2013). The data were examined in three sets dividing students into groups who answered "yes," those that answered "no," and those who were mixed in their responses. Students' explanations for answers were included to support the tabular data in each group. The first set examined student responses to what they liked or did not like about the IMIST unit structure, lessons, and materials. The second set examined what students, liked or did not like learning with the IMIST activities, whether they learned more or less using the IMIST activities, and what activities supported their learning of symbolic literacy, conceptual literacy, or problem-solving. The third set examined students' comments about learning experiences in which they gained deeper understanding of the content.

This part of the cross-case analysis also examined In Vivo codes or students' responses that emerged during the interviews and surveys (Saldaña, 2016). These responses considered the differences in learning environments; the differences in learning objectives, standards, and the level at which the content was taught; and the types of technology students had used before and after the IMIST unit.

Limitations

Limitations for this study resulted from the case study methodology and from researcher subjectivity or bias as part of the action research (Glanz, 2016; Glesne, 2011; Maxwell, 2013). The case studies were limited by access to participants, investigating only one instructional unit, and content and time constraints. Some of these limitations were beyond the control of the researcher/instructor while she had direct influence on others.

The case studies were limited to a small number of students who volunteered to participate. These students represented independent, homeschooled students and students in a classroom environment. This limited the demographics and diversity of students as compared to larger and/or different student populations. As a result, the external validity or the generalizability of the research and potential impact on student populations is limited.

The case studies investigated the impact of only one unit of mathematics instruction over a four-week period rather than a full, year-long mathematics course. The treatment limitation of only one unit of mathematics instruction offered in the middle of a semester affected student attitudes and effort. Most year-long high school mathematics courses offer between ten and twelve different units. The implementation of the IMIST unit represented a significant change in instructional patterns, expectations, and learning activities. Students' willingness or unwillingness to change and work differently influenced their attitudes, achievement, and effort. Students unwilling to complete the investigations for homework did not make the same learning gains as those who completed the homework. Students' accountability for work outside of the class (online or face-to-face) became an important factor influencing their learning. Also, since the implementation was only one unit, there was no comparative data. Due to these limitations, the researcher was unable to assess changes in achievement or attitude before or after implementation and was only able to assess achievement and student experiences during the IMIST learning activities. Again, the implementation of only one instructional unit limited the external validity and generalizability of the study as compared to a study implemented over a full-year course.

Time was a constraint or limitation in all case studies but felt most profoundly in the small class case study. Coupled with time constraint was the depth, breadth, and level of content that could be covered by in-class activities and outside-of-class activities assigned for homework. The limitation of time and content affected the ability of the instructor to fully implement all of the IMIST unit activities as originally planned. The IMIST packet provided an in depth, honors level of learning activities and practice for both Algebra 2 and geometry. However, student participants in the individual case studies had additional course work other than math which limited their time to complete all activities and additional exercises assigned for practice. They were able to take assessments independently out-of-class which meant more time for class instruction. Homework completion was an issue, and instructional time was spent on homework review rather than on discussion.

For the small class case study, in-class instructional time was restricted by the 45-minute class periods. Class time had to include homework review, discussions, learning activities, and assessments. The trade-off was to reduce the depth and breadth of content covered. This limited full implementation of IMIST investigations both in and outside of class. Again, student willingness to do assigned homework impacted the effectiveness of student-centered learning and investigations. The limitation of time affected full implementation of the IMIST activities for the unit and may have affected student learning gains.

Researcher subjectivity or bias. Researcher subjectivity or bias was a limitation to this study. The researcher served as designer, instructor, assessor, and analyst. The researcher developed the IMIST conceptual framework and the treatment or instructional packets based on her experience both as a teacher/practitioner in middle and high school mathematics classrooms as well as her extensive study and literature review of most effective practices in mathematics education. She had instructional experiences using both reformed-based instructional practices and traditional practices and had taught students at all ability levels from learning disabled to advanced or gifted learners.

The instructor's teaching experience and research informed her choices of activities. All the activities, worksheets, and assessments were written by the instructor and were adopted from activities, exercises, and assessments given in prior Algebra 2 and geometry classes. There was no peer review or discussion of learning activities and assessment questions as effective instruments to teach and/or assess the core literacies. Such a peer review with discussion and feedback might have improved the validity of the interventions used in this study. Other teachers/practitioners may have chosen other learning activities they perceived as better fitting for their students based on prerequisite skills, interests, and cultural relevancy.

The element of action research with the researcher serving as the instructor also introduced limitations for generalizability and external validity of this study. Students' attitudes towards the instructor and her methods of teaching affected their willingness to participate in learning activities. In addition, the instructional practices were different than their prior experiences, and students experienced a learning curve especially during investigations when they were challenged to think differently, to think mathematically, and when answers could vary. The instructor's personal biases were reflected in her observations especially when challenged by negative attitudes and/or a lack of work effort.

The researcher designed the data collection instruments and interview question guidelines. There was no peer review or discussion of assessment questions or their point-allocation or coding and classification into the core literacy. Again, peer feedback and discussion could have improved the validity of the data collection instruments and the subsequent analysis of the data in this study.

In summary, as there was only one researcher/instructor who designed, implemented, and taught using the IMIST instructional framework, her findings and experiences are limited and may not be representative of other instructors who try to implement the IMIST instructional framework.

Chapter Four

The data collected for this study came from a series of four case studies – a pair of students, two individuals, and one small class. Students for the paired case study and the two individual case studies were homeschooled students. Three of the students – Hailey, Bobcat, and Kaya – were studying Algebra 2, and the fourth, Howard, was studying geometry. All homeschooled Algebra 2 students met with the researcher/instructor online twice per week for introductions to activities, summaries and discussions of "lessons-learned," additional lessons with instruction, and explanations of homework activities. The paired case study (Hailey and Bobcat) met from October through November 2018. The third Algebra 2 student, Kaya, met one-on-one online with the instructor once or twice per week from January to mid-March. The fourth student, Howard, worked with the instructor face-to-face from November to April. Howard did the IMIST unit on Right Triangles beginning in March.

The fourth case study was an implementation of the IMIST unit on quadratics in a classroom setting of freshmen taking Algebra 2 at a private high school in the Mid-Atlantic region of the United States. Nine of the eighteen students agreed to participate in data collection for the study. Table 6 summarizes the case studies, the IMIST unit, timeframes for the intervention, the method of unit delivery, the types of data collected, and the dates collected. Class observations and assessments were collected over the timeframe for the intervention.

Table 6

IMIST Case Studies Implementation and Data Collection

Student(s)	IMIST Unit	Time	Method	Data collected	Date
Hailey & Bobcat	Quadratics	October – November 2018	Online	MAPS survey Pre-Interview Class observations Assessments Post-Interview	10/1/2018 12/21/2018
Kaya	Quadratics	January – March 2019	Online	MAPS survey Pre-Interview Class observations Assessments Post-Interview	1/8/2019 3/15/2019
Howard	Right Triangles	March 2019 – May 2019	Face-to-face	MAPS survey Pre-Interview Class observations Assessments Post-Interview	3/4/2019 5/27/2019
Class	Quadratics	October – November 2018	Face-to-face	MAPS survey Class observations Assessments Post-Survey	12/11-13/2018 12/11-13/2018

Data for the paired and individual case studies are organized into five sections. The first section provides background for each case study. It includes demographics, schooling experiences, curricula, approaches to studying and learning mathematics, and a summary of the Math Attitudes and Perceptions Survey (MAPS) supported by student responses about their attitudes and beliefs. The second section presents the thematic analysis of the pre-intervention interviews focused on student understanding of IMIST proficiencies (symbolic, conceptual, and problem-solving), student use of technology, and other themes that emerged during the interviews. The third section is a summary of assessment data from formative assessments or quizzes given during the units and the summative assessment given at the end of the intervention. The final section summarizes evaluation of the IMIST unit and perceptions of learning and understanding using the IMIST activities. It also includes student recommendations for improvement in the materials, delivery, and structures for the unit.

Hailey and Bobcat

Background. Hailey and Bobcat were both thirteen-year-old, homeschooled students. Both were white and the eldest in their families. They were recruited for the study through a recruitment email sent to a small network of homeschoolers known to the researcher. Hailey considered herself an eighth grader and had been homeschooled for the previous eight years or all of her elementary and middle school education. Bobcat also placed himself in the 8th grade and had been homeschooled for six years. When asked about the type of math student they considered themselves to be (an honors student or a standard student), Hailey needed clarification about levels and pacing. She did not know how she compared to non-homeschooling students. She explained, “I never really thought about it before.” Bobcat considered himself an honors student. Given their placement in Algebra 2 as eight-graders, two to three years above math courses taught to most 8th grade students, both students would be considered accelerated students. Initially and without knowing the depth and breadth of their individual curriculum and mastery, it was hard to determine if they were studying at an honors level or just a standard level. Hailey shared that her favorite subject was English and writing but that she thought she was a good math student. Bobcat considered himself a good math student "because I get most of the answers right." Both Hailey and Bobcat explained their parents made the decision to homeschool. Neither offered additional information or insights as to why the decision was made.

When asked about curriculum and how they studied math, each student had a different approach. Hailey explained that her mother had studied math in college and had done research on the math curriculum she chose for her children. For the most part, Hailey studied math independently from a textbook chosen by her mother. She read each section or lesson in the book and worked through all the odd-numbered exercises. This meant she usually did 30 or so homework exercises for practice. She explained she did problems from all levels in each section, some from the A-, B-, and C-leveled problems. Hailey would do a self-check with the answers in the back of the book. Her father reviewed her work daily and answered any questions she might have. Hailey had studied both Algebra 1 and Geometry in the years prior to this study using the Prentice-Hall series of textbooks, and she was currently using the Prentice-Hall Algebra 2 book. She explained she was one-third of the way through the Algebra 2 book and had been working on square roots, cube roots, and "stuff." She explained she was working on quadratics concurrently with the start of the IMIST study and was in the middle of

Chapter 7 in her textbook. When asked how she studied math she explained, "I practice it a lot." She had also had the opportunity during the summer to help another student with Algebra 1. She liked teaching other people because it helped her "touch up" on the topics and liked "helping someone else learn it."

Bobcat used a technology-integrated program called Math-U-See which provided video lessons and accompanying textbook. He did not read the book but would do the practice exercises and worksheets following each lesson. He would check his answers in an answer book that was provided. He explained that he chose his own practice and limited the time he spent on math. "If it's really hard, and it takes me a long time, I do less. But if it's kind of simple, I do work a certain amount." Bobcat had completed pre-algebra, Algebra 1, geometry, and Algebra 2 prior to the IMIST intervention. He had had a tutor for five weeks the previous year to help him learn the Algebra 2 content.

Neither Hailey nor Bobcat used regular assessments as part of their learning. Hailey used the assessments in her textbooks and did a self-check/review with her dad. In her current Algebra 2 book, there was a cumulative assessment every four chapters, but she had only taken one. She did not "do a ton of reviewing." She did not take final exams or standardized tests. She also had never done a project in mathematics. Bobcat did not take any assessments during the math course he was studying but took a standardized achievement test at the end of each course based on the learning standards of the State of California. He also had never done a project in mathematics.

MAPS – students' attitudes and beliefs. As part of the pre-intervention and background assessment, each student completed the Math Attitudes and Assessment

Survey (MAPS, Code et al., 2016), a Likert-type survey adapted by the researcher for high school students. Survey data helped the researcher frame questions for the pre-intervention interviews, provided deeper insights into the attitudes and beliefs of each student, and established a baseline for comparison to attitudes and beliefs of the students after the IMIST intervention. Total MAPS scores could range from 31 to 155 with a high score representing a positive and mature attitude about mathematics both generally and personally and a lower score representing a more traditional (negative attitude) and less-mature attitude about the study of mathematics.

Hailey's and Bobcat's total scores on MAPS reflected very different beliefs and attitudes about mathematics and learning mathematics. Hailey had a total score of 131 out of 155 indicating a strong positive attitude about mathematics in general and a strong belief in her personal ability to study and learn math. Bobcat's score of 86 out of 155 revealed a much more traditional and negative view of mathematics and suggested a lack of confidence and perseverance in his ability to study math.

In the pre-intervention interview, Hailey explained that she enjoyed doing math problems. In fact, Hailey's favorite thing about math was problem-solving or doing "word problems" that made her "do a lot of like logical thinking and stuff." She really liked getting the right answer. By contrast, when Bobcat was asked about activities that made math fun, he replied simply, "It's not fun." However, he believed that he was good at math "because I get most of the answers right." Both students found it discouraging when they got the wrong answer to a math problem.

To probe more deeply into "getting a wrong answer" and as a follow-up to MAPS, students were asked how they dealt with productive struggle (Warshauer, 2015) when they did not understand a concept, and how they got help or support. Hailey explained she really did not struggle when learning math, but when she struggled or had trouble understanding concepts or topics, she would ask her dad or go to Khan Academy to find videos with similar problems with worked-out solutions. Hailey was in the habit of reviewing and correcting her work so that she would understand her mistakes and that made her "feel better." Bobcat explained that if he was stuck on a math problem, he was likely to give up. He did say that he would "look at the answer book and try … to figure it out." He also said he would ask his mom and would use Khan Academy videos for extra help.

Each student had a different approach to the time they were willing to spend on math. Hailey was willing to do extra practice using the even-numbered problems in the textbook for additional practice if she needed it. Bobcat found time a barrier. If his math was taking a long time, he would "do less."

Hailey's responses to MAPS illustrated a more mature attitude about mathematics in that she understood the value of mathematics to understand "how the world works" and that "math can be helpful to me in my everyday life." By contrast, Bobcat did not see the value in understanding formulas. "I think as long as they work, it doesn't really matter where they came from."

Hailey and Bobcat had used traditional curricula for the topics, scope, and sequence of Algebra 2. Hailey's study related directly to the lessons presented in the textbook. She studied by reading, working through examples, and practice. Bobcat used videos to present the lesson and practiced using worksheets provided by the program. Neither had experience with reform-based activities, open-ended investigations, or projects in mathematics. Hailey had a much more positive attitude and was confident about her ability to do mathematics based on her experiences with textbook learning. Although Bobcat felt he was good at math, he was more easily discouraged and was not as willing to put in extra time or effort to gain deeper understandings of mathematics concepts and formulas.

IMIST pre-intervention interviews thematic analysis. The pre-intervention interview questions asked students to reflect on different types of proficiencies in mathematics and the learning activities they used to gain understanding of each as well as technology use in mathematics. During the interview, the researcher needed to clarify math proficiencies and the meanings of symbolic literacy, conceptual literacy, and problem-solving. Hailey had discussed the questions with her mother prior to the interview and had a better understanding of these questions than Bobcat. She was much more articulate during the interview than Bobcat.

Symbolic literacy. Both students needed help to understand the meaning of symbolic literacy. The researcher explained and gave examples of vocabulary and the types of skills they had learned in the past. Since Hailey was roughly one-third to halfway through the Algebra 2 course and Bobcat had finished it, she asked them to suggest what types of skills they needed in Algebra 2. Hailey explained that she had trouble understanding the concept of functions and had trouble connecting "the names" or vocabulary with what she studied. Hailey explained that “a skill is when it’s something that you worked really hard for.” She suggested that “knowing when to factor” is an Algebra 2 skill. Also, she could not really describe activities that helped her learn math other than reading the book, doing practices exercises, reviewing her answers, and seeking help from her dad or Khan Academy. Bobcat explained that a skill in math was “being good at it” and knowing formulas. Similar to Hailey, he could not list specific activities that helped him learn vocabulary but explained that he did the exercises in the workbook. Both recognized they needed to know math vocabulary to search by topic when they use videos on Khan Academy.

Conceptual literacy. It was challenging for both students to explain their understanding of conceptual literacy. Bobcat could not articulate the difference between a concept and a skill even after a number of different examples were given. Hailey had a generalized understanding of concept. She explained, “A concept is like an idea about something.” “Concepts are like the ideas kind of like when it’s being introduced … when it’s something new … It’s where you start.” She explained she thought the idea of imaginary numbers was a concept introduced in Algebra 2.

Problem-solving. Both students had a better understanding of what it meant to “problem-solve.” Both linked problem-solving to traditional, textbook word problems. The theme that emerged from the discussion was that both understood math better when there was a story or context behind the numbers. Hailey explained that she found “math makes a lot more sense when it’s in perspective and stuff.” When asked what she meant by perspective, she explained, “… if it’s just an equation that doesn’t make as much sense as if it's like this is the equation for how many Ohms that's like in this resistor or something like … that makes more sense to me." Bobcat explained that doing word problems was better than just doing skills and practice because "just numbers are often confusing." Bobcat explained he did not really have a set of strategies to solve word problems. Hailey described problem-solving strategies as "more like when you have like all the resources you need, but you don't know which ones you need. So, you kind of have to figure out and … kind of like, you, know, everything, but you need to know which ones are important."

Technology. Both students had used videos and calculators. Bobcat's use of video was integral to his study of math through his Math-U-See curriculum. He was dependent on the videos to explain and illustrate what he needed to learn through working through examples, and they prepared him to do the practice exercises. Bobcat liked video learning "because I … can see like what they're teaching." Both students used Khan Academy videos when they were struggling with math concepts and needed extra explanation and help. Hailey used online videos at Khan Academy but only when she was "desperate" and found them to be "really helpful." She also found that many of the Khan Academy videos had too much information, and it was hard to find what she was looking for.

Both students had used calculators for four-function operations such as adding, subtracting, multiplying, or dividing large numbers. Bobcat had also used his calculator for trigonometric ratios to solve right triangles in geometry. Neither had used calculators as learning tools for graphing functions or data analysis. Hailey explained she had seen her dad use his calculator to do both. He had tried to show her, but she did not remember how to do it herself.

Assessments. As part of data collection, Hailey and Bobcat took three assessments. The first two were formative assessments or quizzes. Quiz 1 covered content in the Sections 1 to 3, and Quiz 2 covered content in Sections 4 to 7. The summative or unit assessment covered content taught in all nine sections of the unit. Similar to the learning activities, questions on the assessments often covered more than one literacy. There were questions that focused specifically on symbolic literacy or the understanding of vocabulary and skills. Most questions combined conceptual literacy with symbolic literacy. For these questions, the conceptual literacy part required the student to set-up the model or equation properly/conceptually while the symbolic literacy part usually involved algebraic or arithmetic operations to complete or answer the question. There were also questions focused strictly on conceptual literacy and focused strictly on problem-solving.

Quiz 1 had thirteen questions for a total of 70 points. Quiz 2 had twenty questions for a total of 62 points. The comprehensive unit assessment had 31 questions for a total of 142 points. Table 7 shows the number of questions, the total point value represented on the assessment for each of the literacies or combinations.

Table 7

IMIST Assessments – Questions and Number of Points by Literacy(ies)

	Quiz 1 Sections 1 to 3		Quiz 2 Sections 4 to 7		Unit	
Question Literacy	# of Questions	Points	# of Questions	Points	# of Questions	Points
Symbolic	4	9	6	15		
Symbolic and Conceptual	8	56	9	27	24	105
Conceptual			3	12	5	19
Problem-solving	1	5	2	8	2	18
Total	13	70	20	62	31	142

The following discussion presents the data from all three assessments for each type of literacy. It shows the levels of mastery and trends for each expressed as points earned and percentages of total possible points for each literacy.

Symbolic literacy. Questions that measured symbolic literacy in all assessments either addressed vocabulary or skills and/or content taught in Algebra 1 as concepts. Quiz 1 had four questions for 9 points that assessed symbolic literacy with students identifying zeros from factored expressions. Quiz 2 had six symbolic literacy questions for 15 points assessing radical number operations. Both sets of questions addressed skills developed from Algebra 1. No questions on the unit assessment focused strictly on symbolic literacy.

With the exception of Bobcat's score on Quiz 1, students showed mastery (percentage score above 80%) of the symbolic literacy skills. Bobcat's score on Quiz 1 showed a lack of understanding of factored form. On the two questions he missed, he left out the greatest common factor (the leading coefficient) so that the factored form was no long equivalent to the original expression. Hailey's score on Quiz 2 reflected new learning of radical number operations. She had learned simplification and operations incorrectly in previous years. The feedback given to both students cleared up misconceptions from Algebra 1. Table 8 shows the points and percentage performance of each student.

Table 8

IMIST Assessment Data for Symbolic Literacy

Symbolic Literacy	**Quiz 1** 4 Questions 4 points	**Quiz 2** 6 Questions 15 points
	Percent	Percent
Hailey	100%	86.7%
Bobcat	66.7%	90%

Conceptual and symbolic literacies. Questions which assessed comprehension of a combination of conceptual and symbolic literacies represented the highest percentage or weighting on each assessment. Quiz 1 had eight questions for 56 points or 80% of the total points. Quiz 2 had nine questions for 27 points or 44% of the total points. The unit assessment had twenty-four questions for 105 points or 74% of the total points. The conceptual literacy part of these questions assessed new content understanding, procedural understanding, and/or the student's ability to set-up or apply an appropriate model or equation. The symbolic literacy part was the algebraic or arithmetic mechanics required to provide the answer or solution.

Bobcat showed mastery of conceptual and symbolic literacy on both of the formative quizzes scoring 83.4% and 92.6%. Bobcat had taken Algebra 2 before, and these quizzes reflected standard level learning objectives. However, on the unit assessment, his score dropped to a 69.7% reflecting some misunderstanding of new concepts such as linear and quadratic inequalities and applications of completing the square. On Quiz 1, Hailey struggled with the questions on graphing and describing graphs and transformations. On Quiz 2, she demonstrated strong mastery and understanding of equations, symbols, and procedures. On the unit assessment, Hailey showed consistent improvement in her ability to graph, set-up models, and follow procedures. She lost points on the vocabulary section consistent with her commentary about her weaknesses linking words and symbols. Table 9 summarizes the students' performance on the questions both by points scored and percentage correct.

Table 9

IMIST Assessment Data for Conceptual and Symbolic Literacies

Conceptual and Symbolic Literacies	**Quiz 1** 8 questions 56 points	**Quiz 2** 9 questions 27 points	**Unit** 24 questions 105 points
	Percent	Percent	Percent
Hailey	67.9%	100%	77.1%
Bobcat	83.4%	92.6%	69.7%*

* Omitted a six-point graph due to a printing error at home. Percentage score adjusted for omission.

Conceptual literacy. Questions that assessed conceptual literacy looked at new Algebra 2 concepts that were applied using new conceptual procedures introduced in the unit. Quiz 1 did not have any questions that addressed conceptual literacies and procedures. Both Quiz 2 and the unit assessment had questions using the domain of Complex numbers. Students were expected to extend their knowledge of applications and procedures over the domain of Real numbers to include applications and procedures over the domain of Complex numbers. Also, they needed to understand the meanings of solutions algebraically and graphically. Procedures included completing the square, naming and describing transformations, and finding a quadratic equation model from a graph or given points.

Hailey's performance on Quiz 2 and the unit assessment again showed gains in conceptual literacy moving from 58.3% to 78.9% respectively. Her scores illustrated good progress in learning and understanding as she studied, practiced, and worked with the domain of Complex numbers. She lost points on Quiz 2 because she had not mastered the procedure of completing the square to find solutions or to rewrite a standard form equation in vertex form. On the unit test, she demonstrated improved knowledge of completing the square, finding an equation from a graph, and explaining transformations and graphs. The points she lost were simple mistakes such as missing a sign or not correctly naming transformations although her descriptions improved.

Bobcat showed a decline in his understanding of new concepts and conceptual literacy, scoring 75% on Quiz 2 and 55.2% on the unit assessment. Bobcat did well on Quiz 2 as he had learned to complete the square in his previous class. However, on the unit assessment, he was still unable to demonstrate the proper names and descriptions of graphing transformations and was unable to find an equation of a quadratic function given the vertex and a point. Table 10 summarizes students' performance on the questions both by points scored and percentage correct.

Table 10

IMIST Assessment Data for Conceptual Literacy

Conceptual Literacy	**Quiz 1**	**Quiz 2** 3 questions 12 points	**Unit** 5 questions 19 points
		Percent	Percent
Hailey		58.3%	78.9%
Bobcat		75%	55.2%

Problem-solving. All three assessments had problem-solving applications appropriate for the content covered. Quiz 1 included an area maximization application similar to some done in Algebra 1. Unlike Algebra 1 applications which often explicitly provide all numbers and walk students through the steps, the question was situational and opened-ended only giving one linear measure. Students had to extend their knowledge of perimeter and area on their own to find a solution. Quiz 2 had two contextual application problems using the dropping object model derived in the investigation for Section 4. Again, the students were given one or two measures and had to use the numbers appropriately to provide answers in context. The unit assessment had two applications. The first was a story with an underlying projectile motion model. The second was a data analysis problem using experimental data looking at fluid dynamics.

For all application questions, students were expected to encode the data by defining variables and parameters, set-up an appropriate equation model, solve the equation model, then provide a written answer responding in context to the situation given. Both problems on the unit assessment asked the students to assess the model in context to determine if the model made sense or not. The data analysis regression model determined by the calculator did not make sense in the context of the problem.

Hailey showed progress in her problem-solving abilities. She had not attempted to do the perimeter and area problem on Quiz 1. During the discussion and feedback, she realized she needed to use perimeter and area, but this had always been explicitly stated in problems she had done before. On Quiz 2, she mastered quadratic modeling for dropping objects showing an ability to use the contextual parameters in the dropping object model and interpret her results. She did not do the throwing object modeling application on the unit assessment but completed the data analysis required for the fluid dynamic problem deriving a model to illustrate the data. She missed two points for not labeling her graph and not interpreting the validity of the model.

Similar to trends in conceptual literacy, Bobcat showed a decline in his problem-solving strategies and understanding as the applications shifted from more traditional Algebra 1 applications to more complex open-ended applications. He scored well on the first area and perimeter application, showing a blend of drawings and symbolic reasoning. He lost a point for not explicitly defining the variables or providing the answer in context. Bobcat also did well on the dropping object modeling on Quiz 2. On the first question, he completely answered the question in context, showing all the appropriate work as his justification. On the second question, he only provided an answer in context with no work or justification of how he arrived at the answer, resulting in a loss of two points. On the unit assessment, Bobcat did not attempt the projectile motion application. He started the data analysis application but only received three points for his initial scatterplot with the pattern indicating an appropriate regression model. He did not attempt to run a regression of the data, provide a model, or interpret his results. Table 11 summarizes students' performance on the questions both by points scored and percentage correct.

Table 11

IMIST Assessment Data for Problem-solving.

Problem-solving	**Quiz 1** 1 question 5 points	**Quiz 2** 2 questions 8 points	**Unit** 2 questions 18 points
	Percent	Percent	Percent (#1/#2)
Hailey	0	100%	0/80%*
Bobcat	80%	75%	0/30%*

* Both students chose not to do question #1, the projectile motion application, and only worked on the data analysis application. The percentages reflect the score for work completed.

IMIST post-intervention thematic analysis. The post-intervention interviews asked students to evaluate and share their thoughts about the unit structure. They also shared their perceptions and attitudes about their learning, how the activities supported learning and understanding of the core literacies, and their recommendations for improvement in structures, activities, and unit organization. The interviews were analyzed and coded in a matrix for major themes, similarities, and differences.

Unit structure. Both students initially found working through the activities and materials to be confusing. Bobcat explained the structure was different from what he had used before because “there weren’t like activities like things.” He said, “It was kind of confusing because there were a bunch of things that were like investigations, like class notes, and all that stuff.” Hailey shared that it was “Kinda like confusing … like the homework … I got kinda confused about which parts we’re doing ’cause some of the labeling was a little confusing.” The instructor responded to these concerns with a more detailed assignment list and attempted to differentiate between types of learning activities for each section. As explained previously, some of the confusion was due to block scheduling and assigning two or more investigations to be done preparatory for subsequent lessons while assigning a practice worksheet(s) to follow-up previous or current lessons. As the unit progressed, students figured out the organization and were better able to follow along.

Both students characterized the investigations as “confusing.” The investigations were open-ended, and students did not know what to do. Bobcat explained, “like they kind of like told you what to do, but not like how exactly [how to] work it.” Both students were accustomed to following worked-out examples, having procedures explained, and doing examples that practiced the same procedures. Both struggled with the problem-solving part of the investigations and mathematical thinking: defining the problem, defining the quantities or variables, and encoding/decoding information into mathematical symbols and models.

Hailey wanted more examples than were provided in the class notes and worksheets. “It would have been nice if like the problems are kinda like steps up … ‘cause the problems are more like what I would find in the challenge section of my Algebra book.” She wanted “some easier ones … to get you used to like using the different formulas and stuff, and then, like the hard ones that you had.” Hailey supplemented her study by reading her textbook and working through examples to make “sure that I had covered all the bases.” She also used her notes and worksheets to help her review. “There were like parts that I would like go back to … there were a couple of tables that like reminded me like the different equations like the names cause I kinda got some of those confused with like the … quadratic formula and … all of the different forms that it can be in."

Bobcat felt the worksheets and going over or reviewing the worksheets was helpful. He felt that between the class notes and practice worksheets, there was enough practice. Hailey thought the worksheet instructions were easy to follow, and she liked the solution sheets because she could check the "ones I got wrong."

Both students preferred the online lesson time or real-time with the instructor as compared to the curriculums they had used before. They appreciated the explanations and being able to ask questions. Hailey thought the online lessons were "nice because I was able to go over what I had gotten wrong, and you were able to explain it better." She continued, "I do better with like when someone is like telling me the instructions rather than just reading the book. So, it was easier that way because you were kinda like verbally telling us before we went and did the homework." She explained it "was kinda nice to have someone explain all of that 'cause normally my dad doesn't explain it. He just goes over my homework." Bobcat thought the online lessons were "helpful" and that he could not have done the unit without the lessons. The delivery was similar to what he had done before with his video instruction, but he liked the online lessons better "'cause I could ask questions." In fact, being able to ask questions was Bobcat's favorite thing about the quadratics unit.

Attitudes, perceptions, and confidence. Hailey and Bobcat were clear about their attitudes and perceptions of learning with the IMIST unit discussing workload, the time it took to do the work and investigations and comparing IMIST to how they learned before. Both students agreed they learned more with the IMIST units.

Bobcat and Hailey explained they thought that the unit took too much time. Hailey explained that the work “dominated like a lot of my schoolwork this fall.” Bobcat thought there was too much information as he spent three hours per day four days per week in class and working on the investigations and practice exercises. When asked if he did any outside practice, he explained, “I didn’t have time for practice.” He also did not go back to self-correct his work with the worked-out solution sheets. He would have liked the unit better “I think if it wasn’t as long.”

Since this was Hailey’s first time through the unit material, she experienced a certain amount of frustration. She explained that she had not done any graphing of parabolas as part of her Algebra 1 curriculum. “I think [this] is part of the frustration I have is because I wasn’t really … I was pretty new to it.” She explained that the IMIST unit was “definitely harder because … it was kinda geared up for a follow-up … but I hadn’t learned that yet.” “It was a lot different from the normal learning that I do, ’cause I’m typically doing it in a textbook.” She also was not used to taking notes from the book and had no experience taking notes during class. She struggled to complete a lot of the problems. She explained that she like learning with the textbook better “in a lot of ways, mostly because I kinda have like a rhythm of what I know what to do.” But she also explained, “I feel like I learned more. I did not learn less” with the IMIST unit activities. “It was fine … it wasn’t like I hated it, but it wasn’t like I really loved it. It varied you know.”

Because Bobcat had studied quadratics the previous year, he felt he had learned less with the IMIST unit activities because he had learned it before. He had learned about equations, solving equations, and already knew about graphing. However, he learned more about solving word problems as he had not done "a lot of that before." Bobcat explained, "I think I understood it a little better, but it was still like it was hard … Well, there was a lot more work, and it [was] more like complicated."

Symbolic literacy. Hailey and Bobcat acknowledged there was "a lot" of vocabulary and definitions to learn. Hailey found the class notes and worksheets helped her learn vocabulary, and she would use them to go back and review to remind her of the names and different forms of equations. This is something she had struggled with in previous classes. Bobcat did not feel the IMIST activities helped him understand symbols and vocabulary better, because he "already knew some of them," but he did learn more about the different forms of the equations for quadratic functions.

Conceptual literacy. Both Hailey and Bobcat felt the IMIST unit activities helped them understand the relationships between graphs, tables, and equations better. Bobcat explained, "It seemed to make it easier," and there was enough practice. He explained he already knew how to graph but felt he learned more about different procedures for solving quadratic equations. The conceptual content was new for Hailey. She explained, "I hadn't been introduced at all to it before." As mentioned previously, she struggled with graphing through the first part of the unit. She felt she had a lot to learn and was frustrated sometimes and had to work harder. Her hard work seemed to pay off as she explained, "There was definitely problems where I was like, 'Yeah, I've got the hang of this.'" However, when asked if she felt she gained a deeper understanding of quadratics using the IMIST materials, she explained, "I probably would have done just fine by myself."

Problem-solving. Hailey and Bobcat felt that of all the literacies, their problem-solving abilities improved the most with the IMIST activities. Both explained the primary reason for this was the lack of problem-solving in their previous curricula. Hailey explained that her book "didn't help or prepare" her for problem-solving and that she tended to use Khan Academy. Bobcat had not done a lot of problem-solving before. However, both explained that they thought problem-solving "was hard." When asked to explain what she meant by "hard," Hailey explained that because many of the applications were open-ended, "It was kinda hard for me to know exactly what you wanted, and it was kinda a little confusing." She thought they were harder because of "the lack of example problems, because I typically work with example problems, like, I read through all of them in my Algebra book and that really helps explain it to me. And so, to have the lack of [examples] … that was kind of hard for me." She particularly liked the authentic application problems such as the "rocket" problems. They helped her "learn the vocabulary, symbols – like formulas." Bobcat liked the application problems and "to see where it like was used." He was not as confident about his mathematical abilities because "I don't think I got most of them right." Although he was given solution sheets to evaluate his work, his response indicates that he did not go back to correct his work or to critically think about his mistakes and errors. When asked about his learning, Bobcat explained, "I think I understood it a little better, but it was still like it was hard … Well, there was a lot more work, and it [was] more like complicated."

Technology. Bobcat and Hailey felt the use of computer technology was fine for communication and online lessons. Hailey explained, "I felt like we were able to go over the homework enough that I … I thought it was fine." They disagreed about the value of the TELA activity which had them explore careers and applications of quadratics. Hailey said, "I kinda didn't like that because that was my online time was pretty limited." Bobcat thought that the exploration was valuable as it explained the reason why studying quadratics was important.

Neither Hailey nor Bobcat had used graphing calculators other than for four function calculations or trigonometric values prior to the IMIST unit. Bobcat felt the "calculator stuff" was confusing because he "hadn't done it before, and he felt that it was hard to do the activities because he did not have calculator skills. "I think like if your calculator doesn't like work very … or like you don't know how to use it … it doesn't help much." Hailey explained, "It was kinda hard for me to like step into like graphing and ask the calculators," but overall, she thought that the use of calculators was fine.

IMIST recommendations. Hailey and Bobcat had specific recommendations about how to improve the IMIST unit. First, they wanted better clarification between the types of activities – investigations, class notes, and homework worksheets – and what was assigned for independent work or homework and what was assigned for instructor/classwork. Hailey suggested that she would like more scaffolding exercises – starting with easier examples and moving to harder examples – and more workspace on the worksheets. She said she would have liked to have a copy of the annotated notes made during the online classes for reference as sometimes she did not have notes or did not know how to do the follow-up activities for homework. She also suggested that a list of reference videos would be helpful as resources. Bobcat addressed the issue of calculators. He said there should be more tutorials on calculators or do “not use them as much.” When asked if they would like to do another IMIST unit on exponentials and logarithms, both said no. They both felt the unit needed to be shorter, took too much time, and required too much work.

Kaya

Background. The student in second individual case study for the Algebra 2 unit on quadratics was Kaya, a fifteen-year old, white, high school girl who had been homeschooled since second grade for the last nine to ten years. She considered herself a tenth grader “because I take classes at that level through online curriculums, and I’m up on those.” She had taken pre-algebra and Algebra 1 in preparation for Algebra 2. She had not taken geometry so she would be considered at grade-level placement in a traditional course sequence. She was the youngest of three children, and her parents made the decision to homeschool their children because all three were diagnosed with dyslexia and dyscalculia. Kaya considered herself okay at math stating, “I get pretty good grades … and I wouldn’t say I’m great at it.” However, Kaya explained that she struggles with math due to reading and memorization. She finds studying math “hard,” using the word “hard” eight times during her pre-intervention interview when describing her learning experiences in math.

Kaya used online curriculum through most of her homeschooling for high school. She took most of her subject classes with Columbia Virtual Academy but only did their math curriculum for one year because she did not like it. She explained that most of her math experience has been through Teaching Textbooks. “They still teach … the nice, one-way to do it … so, you don’t follow those other different kind of confusing ways.” Both math programs were entirely self-study and self-paced. Teaching Textbooks has CDs and a textbook with daily lessons on CD or online. Lessons present five examples using a guided instruction teaching model. Examples can be worked out using the textbook or online lesson, and she likes using the video lessons. After completing a lesson, Kaya works through twenty-four graded practice questions online. She likes the structure, clear expectations, and traditional curriculum. She likes using the textbook to review or “refresh” her knowledge. When asked about getting help when she is struggling, she explained that she re-watches the videos, re-works the practice examples with guided “hints,” and re-reads the textbook. Sometimes she uses search engines to help understand vocabulary or to find answers to questions, but she typically does not get help from outside sources. She has no one to answer her questions, and there are no additional practice exercises beyond what is provided in each lesson.

All Kaya’s online work was assessed. “I am always being graded on every lesson … The questions are always being graded.” Kaya has quizzes on current and previous material. She struggles with the expectation of knowing “all the formulas,” and has a “hard time” with assessments when she has to recall past work, lessons, and processes. The program has no summative assessments although “The quizzes are just a compilation of … what could be in previous lessons. So, you don't know what they are giving you. There's not a study guide." Kayla explained that she has never done a project in mathematics.

MAPS – Attitudes and beliefs. Kaya was not confident about her ability to do math. Her MAPS score was low (73 out of 155). Most of the low scores on her survey were from questions about confidence. Her verbal responses during the pre-intervention interview supported her MAPS responses. "I would say in math, I never feel confident in anything … I don't feel very confident in math as a whole." "I wouldn't say I love it, and I wouldn't say I'm great at it." "Like even if I know how to do something, I'll always be like, 'Oh, I don't know if I'll get the correct answer.'" Kaya was concerned about how "hard it was" to remember formulas and procedures as she progressed through the lessons. "You don't know what like formulas are going to be there. What you'll need to know. There's like no way of telling what they're going to have for you." She does not enjoy solving math problems, but she does "kind of like when I get it right," and her favorite thing is "When I get a lesson done."

Both on the MAPS and in her pre-interview responses, Kaya showed a mature attitude about mathematical concepts, work ethic, and perseverance. She strongly agreed with the statements that "Nearly everyone is capable of understanding math if they work at it," and "Understanding math means being able to recall something you've read or been shown." She had good work habits and good self-discipline. "I do lessons everyday … it's one of my courses that I do very consistently."

IMIST pre-intervention thematic analysis. Kaya was well-prepared and thoughtful about her answers to the pre-intervention interview questions. She was confused and had some difficulty understanding definitions and vocabulary when describing math activities. Specifically, she used the word “formula” both when describing vocabulary or symbolic literacy activities and when describing processes and activities used to develop conceptual literacy. She was also somewhat general in her responses with answers that did not specifically tie together activities with the core proficiencies. Most of the learning activities she described could be classified as traditional, guided-instructional practices and exercises.

Symbolic literacy. Kaya explained she had trouble with memorization of vocabulary and formulas. “I would definitely say that I don’t know all the vocabulary.” Sometimes, she would write down “like formulas I know I’ll forget. So, I can make sure I know like what formula I need.” “If you don’t know the formula and have everything memorized in my mind, it’s like I … can’t get that done.” True/false learning activities help her memorize vocabulary. “One of the things that they have you do in my curriculum is there’s a true or false … So, like when they give you like a new word … So, it kind of helps you with the vocabulary a bit.” She also uses a hardcover math dictionary and will search online to get help understanding vocabulary. Her skill practice was limited to the five example questions and twenty-four practice questions that are part of each lesson. When asked about Algebra 2 skills such as factoring, she explained she learned that as part of her Algebra 1 curriculum.

Conceptual literacy. Kaya explained that seeing and working through example problems in each lesson helped her gain conceptual literacy. “When I can see the problem, and I can feel kind of comfortable with, ‘Okay, I know how to do this.’ Like … I understand what I need to do to get the answer.” To Kaya, being able to get the correct answer means understanding. She explained sometimes she has trouble because “I’m getting to the point now where also sometimes there’s no correct answer … and that really … bothers me, ’cause I’m like, I just did a whole problem and there’s no answer.”

Kaya also wants to understand why “the steps” work. She wants “to make sure I was doing it correctly, ’cause I was getting the problem right, but I didn’t know why.” She likes the “fill-in-the-blanks” activities that come with the online lessons that can be done with the video lessons. “They’re teaching you while having you do the problem a bit step by step.” She says, “It really helps when someone like works out the problem with me, because then I can see how they do it, and I can see like they kind of analyze it … Like it helps me kinda see more of how to do it.” When asked specifically about conceptual activities like graphing and equations, she explained that she had done that in Algebra 1 but, “Farther along the program, we’ll get into graphs and stuff. So, I mean, it’s going to get there, but so far, this year, I haven’t gotten there yet.”

Problem-solving. Kaya clearly explained that she did not like word problems. Her least favorite thing about math is “messy problems and word equations.” Kaya does not feel confident in her ability to problem-solve and work with applications. “I would say I really ... don’t know how to really do it. Like ... I’m never really good at them. ... It’s really hard to like draw out. ... Of course, the simpler ones you know at the very beginning, those are really easy. Like when you get into ... you have to put it into this ... it's like ... I'm just ... I'm lost ... I don't know."

Technology. Kaya said she was comfortable with online technology and doing her course work through online platforms. Her program allowed her to use calculators, but she was limited to four functions and square roots. She used calculators to speed up computation and to complete problems. She did not have a graphing calculator and had never used a calculator as an exploratory tool. This became an issue in the study as she was unable to get a graphing calculator so some of those activities had to be eliminated.

Assessments. Kaya explained in her pre-intervention interview that she was used to taking assessments because "I am always being graded every lesson … The questions are always being graded." However, the format of these online assessments was multiple choice or fill-in-the-blank and often asked questions on content learned in previous lessons. She was accustomed to doing her work on a whiteboard not having to show or organize her work using a pencil and paper. As she took the first IMIST unit assessments, she explained that she "did not understand the question." After discussion with the principal researcher, Kaya was offered the accommodations of having the directions for assessment questions read and explained to her and having extended time. However, even after reading and explaining the directions, Kaya did not always go back to finish the questions that she omitted. She explained that she "forgot." She was encouraged to take pictures of her whiteboard work with her phone so that the instructor could understand her thinking and award partial credit. However, she never took advantage of that accommodation. As a result, her lack of completion of the assessments led to gaps in the data collected and made it hard to assess trends.

Symbolic literacy. Symbolic literacy questions that appeared on Quizzes 1 and 2 reviewed Algebra 1 skills (i.e., finding the zeros from linear factors using the zero-product property and irrational number operations). Kaya had not learned these skills to a level of mastery in her prior classes. Her quiz scores (0 out of 4 and 9.5 out of 15) together with the warm-up activities, highlighted Kaya's need for remediation and re-teaching in preparation for applications in Algebra 2. After reviewing Quiz 1, Kaya explained she did not understand the instructions, specifically the vocabulary "zero-product property." On Quiz 2, she was able to do more the complex radical operations of multiplication and division but had trouble simplifying radicals (i.e. factoring out perfect squares). This highlighted her weaknesses in number sense and arithmetic. Table 12 summarizes her quiz scores.

Table 12

IMIST Assessment Data for Symbolic Literacy

Symbolic Literacy	**Quiz 1** 4 Questions 4 points	**Quiz 2** 6 Questions 15 points
	Percent	Percent
Kaya	0%	63.3%

Conceptual and symbolic literacies. On Quiz 1, Kaya did well on the conceptual literacy skills of analyzing graphs and writing equations from graphs, scoring 14 out of

20 points. She also did well on factoring quadratic expressions, scoring 15 of 19 points. Her errors were symbolic literacy errors – not writing coordinate pairs with parentheses, not writing the equation form (x =) of the line of symmetry, and omitting the exponent when writing the equation in vertex form. Her mistakes on the factoring portion came from not factoring completely and not recognizing perfect squares. She did not show any work on the test making it hard to assess her thinking. She did her work on a whiteboard but erased after each question.

Kaya's score on Quiz 2 (6 out of 27 or 22.2%) revealed a significant lack of understanding of vocabulary and a lack of practice and preparation. Kaya explained she "did not know how to start," "did not understand the words," and did not know "what the question was asking." She was again offered accommodations. The questions she missed focused on different methods or procedures for solving quadratics and solving quadratics over the complex numbers. She defaulted to the use of the quadratic formula but made calculation errors. She had not learned how to complete the square and did not learn basic complex/imaginary number operations.

On the unit assessment, Kaya showed considerable improvement in both her mastery of symbolic and conceptual literacies, scoring 55.5 out of 105 points (adjusted for omission of a graph due to a printing error.) She missed conceptual vocabulary questions, scoring 8 of 14. However, she had a perfect score on the multiple-choice questions (12 out of 12). She showed no work on the assessment for both of these sections but was familiar with the question format and worked on her whiteboard. She improved on her complex number operations scoring 14 out of 18. Although she could do operations with complex numbers, she missed the conceptual understanding of complex numbers – graphing, classification, and absolute value – scoring only 2 of 15 points. She continued to have some trouble graphing parabolas but was able to locate most critical features and anchor points (vertex, axis of symmetry, etc.). Her mistakes on graphing and equations were similar to her symbolic literacy mistakes on Quiz 1 – notation and forgetting the exponent in vertex form. Table 13 summarizes her assessment scores for conceptual and symbolic literacies.

Table 13

IMIST Assessment Data for Conceptual and Symbolic Literacies

Conceptual and Symbolic Literacies	**Quiz 1** 8 questions 56 points	**Quiz 2** 9 questions 27 points	**Unit** 23 questions 105 points*
	Percent	Percent	Percent
Kaya	73.2%	22.2%	52.9%*

* Omitted a six-point graph due to a printing error at home. Percentage score adjusted.

Conceptual literacy. Kaya's low scores on conceptual literacy questions assessing her learning about new concepts and procedures indicated she did not learn or engage with the material. She did not attempt two of the questions on Quiz 2 (solving by completing the square) and made an attempt at the third but not enough to score any points. On the unit assessment, she only attempted two of the three questions that assessed conceptual literacy. She did not complete the two questions that followed-up the methodology questions (completing the square) she missed on Quiz 2. However, on the questions that she did attempt, she scored well. She showed improvement in her ability to name and describe transformations from a graph scoring 3.5 out of 4 points. She also received full credit for finding a quadratic equation model given two points – the vertex and one other. This tested her learning following the review lesson on this topic. She asked for help (accommodations) to understand how to set up a system to solve the product and sum question. Although the directions were explained, she did not go back to finish the question. Table 14 summarizes assessment data for conceptual literacy.

Table 14

IMIST Assessment Data for Conceptual Literacy

Conceptual Literacy	**Quiz 1**	**Quiz 2** 3 questions 12 points	**Unit** 5 questions 19 points
		Percent	Percent
Kaya		0%	39.5%
Score on questions completed		0%	93.8%

Problem-solving. Kaya showed good effort and made an attempt on all contextual problem-solving questions on all assessments. On Quiz 1, she demonstrated problem-solving strategies by drawing a picture and labeling the variables of width and length. She did not complete the problem because she did not recall the perimeter and area formulas for a rectangle. Quiz 2 included two questions using the dropping object model and provided the equation. Kaya attempted to do both problems but switched the parameter and variable of initial height and falling height as a function of time. Although she did the procedural steps correctly on the second application question, by missing the set-up to the solution, she only scored 2 of 8 points. On the unit assessment, Kaya was only given the 10-point application questions on projectile motion that did not require a graphing calculator or data analysis. Her work was messy and disorganized, but she used the model and the quadratic formula correctly. She included units in her answers and responded in context. Table 15 presents assessment data for problem-solving questions.

Table 15

IMIST Assessment Data for Problem-solving

Problem-solving	**Quiz 1** 1 question 5 points	**Quiz 2** 2 questions 8 points	**Unit** 1 question 10 points*
	Percent	Percent	Percent
Kaya	40%	25%	70%*

*Data analysis problem omitted for lack of a graphing calculator. Total points and percentage score adjusted.

Although Kaya did not always complete assessments or omitted questions she did not feel confident about, work on her assessments clearly differentiated her level of mastery for the core proficiencies. The assessments themselves became learning tools as Kaya and the instructor went over directions she did not understand and questions missed. Kaya showed improvement in symbolic literacy gaining a better understanding of vocabulary and meaning over the course of the three assessments but still struggled with arithmetic skills. She made progress in conceptual literacy, mastering concepts connecting graphs, equations, and transformations, but still showed weakness in procedural applications and methods of solving equations. Although she had indicated she did not like “doing word problems,” she used different problem-solving strategies and showed good work and effort when doing the application problems on all three assessments.

IMIST post-intervention thematic analysis. Kaya’s post intervention interview was conducted in March almost three weeks after the completion of instruction. She took extra time to complete the unit assessment as part of an accommodation for her dyslexia. Kaya and the instructor discussed her performance on the unit assessment prior to the interview where she shared her perceptions and experiences with the IMIST unit. Kaya had used a number of different formats for her homeschool mathematics curricula working online independently, working online with an instructor, using video lessons, and online text lessons. During her post-intervention interview, she gave many examples comparing and contrasting her learning experiences with past curricula and her experiences with the IMIST unit activities.

Unit structure. Kaya liked the organization of the unit overall but “felt like it was a bit hard to follow at times.” Given the block scheduling of online lessons meeting once or twice per week, she was assigned activities from one or two sections making it difficult for her to follow or complete. She explained that at the beginning of a section or topic sometimes “it didn’t make sense, or it was kind of like fast.” If she did not understand, she would stop work and wait until the next online lesson for clarification. As a result, Kaya did not really differentiate between student investigation activities and the teacher-led summaries, discussions, and lessons. Kaya felt that the online lessons were very helpful “because I think it’s easier for someone to tell you about it and show you, than just to … get … answers back.” Her favorite thing about using the IMIST unit activities was “getting to work face-to-face with someone.”

Kaya described the unit as being “compacted.” “I feel like it was kinda like, ‘Let me show you all of … Algebra 2 in like two months!’” But she liked having “more time to work with stuff.” She liked the examples and taking notes in the packet. She explained the packet was a good resource. She thought the practice worksheets were “helpful, because they kinda work with what you know … but if you’re confused on what you’re doing, they’re not helpful.” Kaya explained she would have liked examples that tied the sections together or reviewed work from previous lessons. This was in the unit design, but she just did not make the connection between sections and examples. She also thought that the answer sheets with solutions and annotated notes were helpful. “Just knowing the answer doesn’t have you know what the problem is.” She explained going over the assessments helped her learn. “I like to be able to understand … why I got something wrong” or “like why it works.”

Attitudes, perceptions, and confidence. When asked whether she liked learning using the IMIST unit activities, Kaya responded, “I did and I didn’t … I did because there was an actual person I could tell,” and there was more time to work on “stuff.” She did not like the IMIST unit structure for feedback on her work as she compared it to her current online program. With her online program she explained, “when you get a problem wrong, it … shows you how to do it.” With IMIST, she had to wait to get the solution sheet, annotated notes, or to ask a question in the online class. This sometimes made her feel "lost." Overall, she thought that learning with the IMIST unit "was really beneficial."

Kaya felt she learned more content in the IMIST unit than she had learned in other curricula, and she would be willing to continue studying math using the IMIST units although it was "a lot of work." She "tried to do it every day, and every day that I would do math, it would … could take from one to two hours." In her post-intervention interview, the concept of "work" or "working" emerged as a significant theme. Kaya referenced "work" over twenty-five times, and she never described the work as being "hard" which was a theme in her pre-intervention interview. She explained, "if you are going to try to do ... stuff like that or do any kind of standardized testing or want to get in college, like sure, if you just want to get out of high school, then no you probably won't need to like spend all this time on stuff, because you obviously don't really care about what you're getting into." Kaya has learned to work hard in her studies "just because for me I know things can take me forever to learn. So, like I'm the kinda person who like, 'Yeah, you've got to put in the work for it.'"

Kaya's goal was understanding. "I'll take more time doing something than like maybe everybody else, but I mean, if that's what it takes to get the same amount or even more so of understanding than they do, well like, I want to understand it. I'm going to put this work into it, you know." She continued, "I'm just the kinda person who just I know things are going to take work. Like you don't get anything out of not working." When asked about the IMIST workload as compared to her other studies, she replied, "I think it was a lot, but I mean, I would say it's not any more than my other online classes." "You have to pace this yourself. You know how much time you have. You have to know how you can work through this."

At times Kaya felt that she could have used more practice and repetition in the IMIST unit. When asked if she used the textbook as a resource for additional practice exercises, she explained, "I did some additional problems, but if I did additional problems, they were ones that I was supposed to be learning then." By contrast, her current curriculum "goes over the same stuff over and over and over again," and "if you are doing something over and over again you keep renewing it and getting it." She would have liked more review exercises to spiral and "refresh" her learning. "I'm like this repetition person where I need the problem like, 'Okay, here's this.' I use this formula again and again and again and again, and then I get it. Like that's kind of how I work."

The interview revealed that Kaya still did not have a great deal of confidence in her ability to do math. Although she clearly enjoyed the online units, working with the instructor, taking notes, and working through the exercises, Kaya explained, "It's really hard for me to remember things" and "I am really bad with word problems."

Symbolic literacy. Kaya explained the lessons, notes packet, and repetition helped her learn the vocabulary. "Every time I had a note, I would take this sticky note, and I would put it on the book." Her notes also helped her review and study. She explained she liked to have "reference points. So, for me those notes were kinda like my reference points where I was like, 'Oh, yeah, we did this which was for this,' and that really helped me." Articulating the vocabulary in her own words also helped her understand meaning. She often made connections with vocabulary and descriptions that she created independently such as describing vertical reflection as the "graph goes into an arch." The warm-up reviews, particularly for Algebra 1 skills, also helped her remember and practice her skills. Sometimes she had trouble organizing her notes and references. "I'll be like, 'Where's the note on this? Or where's the note on that?'"

Conceptual literacy. Kaya explained she had a number of *aha* moments gaining conceptual understanding using the IMIST graphing activities for quadratics and parabolas. "Just graphing. I hadn't done any graphing before. So, I mean I went into like 'I don't know anything about that really,' … because I was only barely getting into it with my other program. So, I ... really went into it like, 'I don't know this really at all.'" She found graphing using a table of values confusing, but as she practiced graphing with transformations she explained, "You've just got to find what the vertex is. I'm like, 'Oh, my gosh that's so easy.'" She was able to have a "picture" in her head.

She felt the IMIST unit activities made a better connection between vocabulary and concepts than programs she had used before. "Before like it'll give you vocabulary … and stuff … It'll give you different stuff, and it'll teach you to solve it," but the approach is "more like 'we're going to solve it, so we're solving it.'" In her previous curricula, they never explained "why" they were solving an equation or even what "solving" meant. "I feel like I understood the reason more. Maybe not necessarily understanding how to do it more, 'cause I felt like some of the stuff I already kinda knew how to do. But just understanding like the reasons, and how it fits in – in a sense to map as a whole – was kinda neat."

The theme that emerged in her post-intervention interview from the discussion of the IMIST learning activities was "wanting to know why." Kaya felt that the IMIST unit activities did a better job of explaining the "reason" or rationale for learning things as well as the "how to" or procedures. She mentioned wanting to understand or "know why" ten times in her responses to the questions. For example, she wanted to "understand why I got something wrong." She explained, "the hard thing is like just knowing the answer doesn't have you know what the problem is." She explained her previous program had introduced "the quadratic formula before any graphing. Zero graphing… So, I'm just like, cool it's just another formula. Like I had no idea, 'Why am I using this?' 'What's it for?' So, now … I can kinda think when it does give me those problems … I can be like, 'Oh, this is for like the … parabola.'" She could see it in her head and understand the meaning of zeros, *x*-intercepts, and/or solutions.

Kaya continued to use the word "formula" to discuss vocabulary as well as procedural knowledge. She mentioned still confusing the "formulas" such as equations for intercept and vertex forms of the equations. Then she described the procedure of finding the equation from the graph of the parabola as a formula. In her words, having the "formula" or procedure written out as an example "really helped me." She also felt she needed more practice. "I think though for me, it definitely works better if something comes up over and over and over again, and I just at least do two problems on it … again and again." It was not clear how much independent practice she was doing either from the reference textbook or on the practice worksheets. She had "trouble remembering."

Problem-solving. When asked whether the IMIST unit activities helped her with problem-solving, Kaya replied, “I am really bad at word problems.” She explained that “I’m the kind of person who likes to have the equation, but not just the equation, I like to know how like to put it into use, because once I do that, and once I keep practicing that, I’ll get it.” However, in real-life applications, she explained, “I probably wouldn’t use math for it. I wouldn’t be like, ‘Oh, I’m just gonna do this equation,’ because for me, my brain just doesn’t work with converting a real-life problem into math.” When asked if the story or context helped her understand the problem, she explained, “It’s not the story. See, I would’ve gotten yours wrong, or I wouldn’t even of answered, because I would have had no idea if I hadn’t known the formula that I need to use.”

Kaya’s performance on problem-solving applications on all three assessments provided evidence that continued to show her beliefs and attitudes were in conflict with her actual ability to problem-solve. She did solve the problems and could apply the “formulas” or procedures that she remembered. On the first quiz, she did not remember perimeter and area formulas so she could not provide an answer for the question but attempted to solve the question by drawing and defining the variables. She demonstrated she did learn the standard quadratic applications of dropping and throwing objects using IMIST unit activities in the second quiz and unit assessment. She explained her concern, “It’s just with word problems, if I don’t know like in a sense what they’re asking me to do, and how they’re asking me to use the problem, then like I’ll have no idea.”

Kaya did not have access to a graphing calculator so the IMIST unit activities using the calculator for explorations and data analysis were omitted. She explained she did use the calculator for multiplication of "such big numbers." She thought the online delivery of the IMIST unit activities, lessons, and discussions was good. She explained the online lessons were helpful because, "It's like one thing to like see the materials and to see how you're understanding it. That's another… to have like a teacher who's like, 'Oh, well it's this way,' and like help you with the problems." She liked the use of the white board. "Writing on the board, I would say is really good … and that really helps me see it." As mentioned previously, she "liked getting to work face-to-face." "I think it's easier for someone to like tell you about it and show you."

Recommendations. Kaya had a number of recommendations to help improve the structure of the IMIST unit, its exercises and review materials, based on her understanding of her own learning style. She would have liked to see more spiral learning and review between sections or lessons. "I would've liked the [new] work to have old work with it." She explained, "When you get something new, do those, and then have some ones from the last lesson … just kinda make it so then there's a little bit of everything…You'd like switch it up." She says just "doing the same thing … sometimes I feel like in my case that makes me brain dead … you can be so focused on this you forget something else." She indicated the directions on the activities and worksheets were usually clear and easy to understand but encouraged that the directions to be written as "bluntly as possible."

Kaya stated she would have liked to have real-time feedback for solutions and answers to the practice exercises so that "I can immediately look at how I got it wrong or what I did wrong, and it'll like show me how to do it right which I think is good, because right then and there you know. While it's kinda hard with meeting up, you know, and having to like wait for that, and then you're kinda already not be in a sense lost it, but you know it's not right then fresh in your mind."

Summary. Kaya was positive and enthusiastic about learning math during the online lessons. She was honest when she struggled or was unable to complete the activities, often saying, "I don't get it" or "I couldn't read it." She self-advocated and asked for help when she needed it. Specifically, she asked for detailed notes on how to set-up a substitution table and how to find the leading coefficient from the graph of a parabola. Also, she seemed to understand in the moment, specifically during the online class discussion, but needed repetition, reinforcement, and verbal practice. It was evident that she really appreciated learning new things. When she "got it," she explained she felt she had "accomplished something."

Howard

Background. Howard, a white, ten-year-old, fifth grader, came to the instructor in response to an online posting for geometry tutoring. Unlike the participants in the other individual case studies who worked with the researcher/instructor online, Howard worked with her face-to-face before, during, and after the IMIST intervention. Howard had been homeschooled for six years, as he described it, "All my life." As a fifth grader, he was accelerated in mathematics studying geometry two to four years earlier than traditional placement. Howard considered himself "advanced in math" but "just regular for other courses." Until this year, Howard's mother had been his math teacher. Because of his acceleration in mathematics, Howard's mother had been looking for a way to enrich his study of mathematics and had enrolled him in an Art of Problem-Solving (AoPS) class. In this program, Howard was one of two fifth graders and explained, “It was just a school for really advanced kids, and … I was one of the youngest kids in the class and … I just don’t think I was ready for that kind of thing yet.” Howard and his mother agreed that he was not ready for AoPS, and he started with the researcher/instructor in December after withdrawing from the AoPS program in an effort to provide more depth, breadth, and challenge to his study of geometry.

For his regular curriculum, Howard’s mother chose the Saxon textbook series through pre-algebra and Algebra 1 and the Jacobs *Geometry* book for the current year. The Saxon course content is noted for its strong traditional instructional focus on skills, symbolic literacy, and conceptual literacy. The geometry text, Jacobs *Geometry*, required a deeper level of conceptual understanding and reasoning. Howard’s mother was concerned about her ability to answer Howard’s questions, to develop his reasoning skills, and develop his ability to write proofs. She felt she needed additional support and instruction to meet Howard’s needs. Howard and his mother were interested in the researcher/instructor’s IMIST research and readily volunteered to participate as one of the case studies. The researcher used the IMIST system framework to design a unit on right triangles and trigonometry specifically for this case study.

Howard’s mathematics learning and content had been driven by the textbook he used. He did independent study, reading, and practice exercises. Howard explained, “I just read the lesson, and then sometimes I pick my own exercises, but usually my mom picks exercises from that lesson to do for homework.” When asked about the amount of homework or number of practice exercises, Howard explained, “About twenty … Sometimes ten if they’re hard, like they’re hard ones.” “We tried to do [math] every … day, and we usually do it every day.” Other than his AoPS program which was outside of his core curricular studies, Howard did no extra practice or worksheets and had never done a project in mathematics. He did not separate the concept of homework and practice exercises in learning math. He just considered it the “lesson” for the day. When asked about the role of assessments, he explained that he did not take any regular assessments. At the end of each academic year, he took the end-of-the-year series of Iowa Test of Basic Skills (ITBS) for all subjects including math, but he did not study or review for them.

MAPS – attitudes and beliefs. Howard was confident about his ability to do math. When asked how he felt about math, he exclaimed, “LOVE math!” He was given the Math Attitudes and Perceptions Survey (MAPS) before beginning his work with the researcher/instructor. His MAPS score was 126 of 155 indicating a high level of belief in his ability to do math and a good work ethic. His responses indicated he believes he can figure out how to do math problems; he does not give up; and he does not get stuck on math concepts very often. Furthermore, he wants to understand processes and to know why formulas work.

Howard explained he did not feel that he needed to take notes on reading, “I just read the lessons … pretty short. There’s like three pages or something, and I make sure I understand it, and I’m not just reading it just to get through it. I make sure I understand what I’m reading.” When asked how he knows he understood what he was reading, he explained, “When I’m not confused about what they’re saying … ’cause sometimes I read it and like, ‘What does this mean?’ And I … go back and read it again. Make sure I understand it, and then I move on.” When asked what he did when he did not understand something or needed help, he explained, “If I didn’t understand something, I would go to my mom, and then if she didn’t understand it, we would look it up.” Howard explained they usually used Internet resources to find definitions and explanations. They would also look at video micro lectures on Khan Academy for help with procedures.

In describing his studies in mathematics, he used the strong descriptor “definitely” when he talked about learning from mistakes, doing corrections, and when having *aha* moments. When trying to understand his mistakes, he would “just look at the problem … I try to find my mistake, and then if I can’t find any mistakes, I ask my mom if I can go and look at the answer key … and then I go through the steps in the answer key and compare to mine, and then once I come upon a mistake in mine, I realized that I’ve made a mistake there.” Howard described an *aha* moment, “One time there was a proof that I was doing. It’s something to do with … the diagonals in an isosceles trapezoid are congruent. And I just thought about it for a while. I couldn’t think of it, and then it was like, ‘Oh, I get how to do that’ … It was … I love that kind of moment.” Howard’s responses demonstrated he is a deep thinker, willing to work at understanding, and feels great satisfaction when he figures out a solution or answer to a question. When asked about his favorite thing in mathematics, Howard replied, “Everything.” When asked about his least favorite thing, he replied, “That’s a really hard question.”

His MAPS survey indicated that he did not necessarily understand how other people learned or struggled with math. He could not speak to others' experiences which was age appropriate and a function of his more insolated homeschooling experience. Before beginning work with the researcher/instructor, he strongly agreed with the statement, "There is usually only one correct approach to solving a math problem." At the time of the pre-intervention interview, the researcher/instructor had been working with Howard for two and a half months and asked him if he still strongly agreed with the statement. Howard replied, "No … Well, in geometry for example with proofs … I don't know about usually, but sometimes there's another way to do the proof ... There was one proof I did and … where the answer key went about it in a different way. But, I did it my own way." When asked whether he had confidence that his proof or argument was correct, he replied, "Yes. Pretty sure."

IMIST pre-intervention thematic analysis. Given that Howard was only ten, his answers to the pre-intervention interview were often short, one-word responses. The researcher/instructor asked detailed follow-up questions to help him explain his thinking and to provide examples to support his responses and beliefs. Howard did not understand the differences between the core literacies, and he and the researcher/instructor discussed the meaning of symbolic and conceptual literacies in detail with examples. He also did not really differentiate types of learning activities. His study of mathematics had been traditional, textbook work based on reading, independent study, and the book's exercises. The researcher/instructor acknowledged there was a question as to whether to classify the process of proof as a conceptual learning activity or an authentic problem-solving activity. Proof is authentic mathematical problem-solving activity for higher level mathematics. For the purposes of this study, the researcher/instructor classified proof at this introductory conceptual level as a reasoning process and a writing activity and defined problem-solving activities as those focused on non-school or real-world applications.

Symbolic literacy. The researcher explained that symbolic literacy in mathematics describes a person's knowledge of vocabulary and skills. Howard explained there were not specific activities that helped him learn vocabulary and skills. He learned vocabulary by reading the textbook. Howard and the researcher/instructor also discussed the "un-definitions" of geometry, and how concepts like point, line, and plane become vocabulary both with symbols and pictures.

Conceptual literacy. It was much easier for Howard to describe conceptual learning activities and how he gained understanding of geometric concepts. He learned from textbook examples. "They pretty much like showed an example. Explained what they did. Showed several more examples." He explained that learning to prove something was an activity and process that helped him build understanding. He explained writing proofs and comparing them to examples given in the answer key to determine if his argument was valid or if he "got it right" really helped him learn. He stated he has to think "about it for a while" to understand how to write a proof.

Problem-solving. When discussing problem-solving, Howard referenced his AoPS class that did many competition math problems. He struggled with the AoPS class explaining "That was really hard." He did not feel ready to answer the types of questions they were asking or sometimes how to even start them. For his current coursework, he explained his process for problem-solving, “The first thing we do, I think about it and then … If I need to read it again, I do, and sometimes I read it many times.” In geometry, “I just draw a sketch.” He did not have much experience with authentic or non-school applications in either his Algebra 1 or geometry course, explaining that he did not do them “very often.” He could not think of any real-world applications he had done. When asked if he wanted more experience with authentic applications or non-school applications, he replied, “I don’t really care.” The researcher asked him, “Then why study math?” At that point, he realized there might be more to learn.

Technology. Howard explained he did use computers and calculators but “very rarely.” He and his mother used the Internet and Google to find resources to explain problems and concepts they did not understand. They have used videos on Khan Academy. “We usually watch it if … we don’t understand something.” “We just watch it. We don’t actually do … like the whole … all the lessons.” Howard thinks Khan Academy is “pretty good.” Howard has “rarely” used calculators and does not really use them for computations. He did not remember doing data analysis, curve-fitting, or regression as part of his Algebra 1 curriculum.

Assessments. Similar to the IMIST unit for quadratics, Howard completed three assessments – two formative assessments or quizzes and a final assessment. The first quiz focused on symbolic and conceptual literacies building the foundational knowledge of vocabulary and procedures preparatory to using them to solve right triangles for problem-solving. Quiz 2 introduced questions that required all literacies – problem-solving to draw and apply, conceptual to set up the proportional or functional relationships, and symbolic using arithmetic to solve, round, and include units and/or include a justification using a definition, postulate or theorem. Neither quiz included any authentic, problem-solving applications. The unit assessment included questions that used both conceptual and problem-solving skills to decode relationships in a diagram with multiple triangles. The processes needed to “solve the diagram” could vary and use a number of different theorems and/or techniques. The unit test also included four authentic problem-solving questions considering the dimensions of a box, the height of kite, the height of the Washington Monument, and an angle of depression from the top of the monument. Table 16 summarizes the quizzes, questions, points, and percentages measuring the core literacies.

Table 16

IMIST Assessments for Geometry Unit on Right Triangles and Trigonometry – Questions and Points by Literacy(ies)

Literacy	**Quiz 1 Sections 1 to 3**		**Quiz 2 Sections 4 to 6**		**Unit Sections 1 to 8**	
	# of Questions	Points	# of Questions	Points	# of Questions	Points
Symbolic	2	5				
Symbolic and Conceptual	13	43	10	49	9	78
Conceptual Problem-solving					2	13
Symbolic Conceptual Problem-solving			4	18	1	16
Problem-solving					4	15
Total	15	48	14	67	16	122

Howard's overall achievement on all three assessments was excellent and is summarized in Table 17.

Table 17

IMIST Points and Percentage Scores on Howard's Assessments

	Quiz 1	**Quiz 2**	**Unit**
	Percentage	Percentage	Percentage
Howard	92%	93%	87%

Symbolic literacy. The first quiz was the only assessment that had questions focused strictly on symbolic literacy. Howard was asked to justify his work by completely stating the theorems he used with hypothesis/conditions and conclusions. Howard provided abbreviated answers and did not state the theorem for the first and forgot the symbolic representation for the second. Table 18 presents assessment data for symbolic literacy.

Table 18

IMIST Assessment Data for Symbolic Literacy

Symbolic Literacy	**Quiz 1** 2 Questions 5 points
	Percent
Howard	60%

Conceptual and symbolic literacies. Howard's scores for questions on conceptual and symbolic literacies were strong on all three assessments – 95%, 90%, and 88% respectively. On the first quiz, he lost two points for an arithmetic error that prevented him from getting the correct answer. He made a similar mistake on the second quiz, missing four points due to computational errors in multi-step processes. He lost the fifth point due to a calculator error in the trigonometric section. He had the correct calculator-ready expression but did not enter it correctly. Howard's mistakes were simple computational or symbolic literacy errors. Overall, his conceptual understanding and mastery was evident.

On the unit assessment, Howard made a significant conceptual error when using the converse of the Pythagorean Theorem to classify triangles by angles. He forgot to determine if the theorem applied by adding side lengths to determine if the side lengths formed a triangle. Two of the sets of side lengths did not form a triangle, so he missed 6 points. On a trigonometry question on the unit assessment, similar to questions on Quiz 2, he lost four points for computational, rounding, and unit errors. These were symbolic literacy errors not conceptual. Table 19 presents assessment data for conceptual and symbolic literacies.

Table 19

IMIST Assessment Data for Conceptual and Symbolic Literacies

Conceptual and Symbolic Literacies	**Quiz 1** 13 questions 43 points	**Quiz 2** 10 questions 49 points	**Unit** 9 questions 78 points
	Percent	Percent	Percent
Howard	95%	90%	87%

Conceptual and problem-solving literacies. Quiz 1 and 2 did not have questions that targeted conceptual and problem-solving literacies. Two questions on the unit assessment assessed conceptual literacy and problem-solving. The first question was a diagram composed of many different triangles. There were a variety of choices/procedures to find the different side lengths. Howard misread one of the triangles and used an incorrect justification for his answer losing two points. He lost the third point for a calculator error in the final problem of the assessment. Table 20 summarizes assessment data for conceptual and problem-solving literacies.

Table 20

IMIST Assessment Data for Conceptual and Problem-solving Literacies

Conceptual and Problem-solving	**Unit** 2 questions 13 points
	Percent
Howard	77%

Symbolic, conceptual, and problem-solving literacies. Howard did well on the questions which assessed all three proficiencies. His only mistake was a computational error on Quiz 2. Table 21 summarizes assessment data for symbolic, conceptual, and problem-solving literacies.

Table 21

IMIST Assessment Data for Symbolic, Conceptual, and Problem-solving Literacies

Symbolic Conceptual Problem-solving	**Quiz 2** 4 questions 18 points		**Unit** 1 question 16 points	
	Points	Percent	Points	Percent
Howard	17	94%	16	100%

Problem-solving. The unit assessment had four authentic problem-solving applications. Howard missed two points for mislabeling an angle that resulted in an incorrect answer. This was a transposition error. The second point missed was a conceptual/reading error. Howard misread the distance from the "base" of the monument and calculated the line-of-sight distance instead. Both of these errors were not significant. Howard's work on the assessments showed clear interpretation and understanding of the situations and stories showing good problem-solving strategies using labeled diagrams and equations. Table 22 summarizes assessment data for problem-solving.

Table 22

IMIST Assessment Data for Problem-solving

Problem-solving	**Unit** 4 questions 15 points
	Percent
Howard	80%

IMIST post-intervention thematic analysis. Howard's post-intervention interview was delayed due to the timing of spring break and the completion of his unit assessment. Similar to the pre-intervention interview, Howard's answers were often short, one- or two-word responses, and the researcher had to rephrase and clarify the questions to elicit longer answers and explanations. She also asked him to compare and contrast his learning activities before the IMIST unit with learning he did using the

IMIST unit activities. Because of the lapse in time and starting the next unit, Howard often stated that he "could not remember" or could not give specific examples.

Unit structure. Howard explained what he thought about the IMIST unit activities and packet. "I like how it's structured … better than I was doing math before." Howard explained the exercises used in the IMIST unit were similar to those in his textbook, but the real difference were the application problems and the number of proofs. He liked the worksheets and examples, and since he was used to reading, he found the instructions easy to follow. He explained, "Before I would just read the lesson and then … just do the homework … I would do it with my mom … like read the lesson with my mom, and she would assign homework." In the IMIST unit, "There's a lot of different stuff in the packets like investigations." When asked to explain what he meant by "stuff," Howard explained, "Like different kinds of problems, and there were more … there were more proofs than in the book, I think, and I like the investigation … and using the Internet and stuff, 'cause that wasn't in the … book. It never told us to do that."

Howard liked the face-to-face lessons "'cause this is like how I've always done it. Just one-on-one." He likes one-on-one and self-study because "you go at your own pace and like the teacher asks questions." He also likes being able to ask questions. "I mean you can still ask questions in a class, it's just" not as easy. Howard explained that he liked the questions, reflections, and "Think About" questions in the packet.

Attitudes, perceptions, and confidence. Howard loves math. He brought his interest and gentle enthusiasm to each class. He was always prepared having completed the homework exercises assigned and had framed thoughtful questions about exercises that he could not complete. Howard was driven to deeply understand concepts. He was deliberate in his answers and has complete confidence that if he works to understand and thinks about things, he will learn what he needs.

Howard “liked using IMIST.” He found the workload, “I think just about right,” and “I think just about the right amount of practice,” and he did not think he needed more practice. He liked the challenge exercises. “They were hard, but I like … I like hard.” He explained he did not struggle with activities, and he did not mind spending extra time or doing extra practice. He felt the “real-world” applications in the IMIST unit helped him learn more because “I think so ’cause like before I was doing math, and I didn’t really know why I was doing it, and then like, this showed me the … real-world applications and stuff.” When asked about his confidence about writing proofs, he explained “I don’t know how to quantify” confidence. He “feels better” about doing proofs after the IMIST unit. He stated he felt better about being able to start and complete proofs, what he and the instructor called “attack.” He was much more comfortable about using multiple ways to come to an answer especially when proving something. He explained that doing the work in the packets, the practice worksheets, and assessments, “I think it gives me confidence.” When talking about assessments, he did not think about them as learning tools, and he “did not know” if they helped him learn. He also could not really determine if he learned more or less with the IMIST unit.

Symbolic literacy. Howard and the researcher discussed the IMIST unit activities that helped build his understandings of symbolic literacies and vocabulary during the unit. Howard explained the vocabulary seemed to be “just part of the reading.” When asked what activities helped, he explained, “Just practice. Like do the exercises, and like it would say justify your equation or whatever with theorem, and like I just got familiar with uses.” He found the instructions to justify his equations, work, or an answer with a definition, postulate, or theorem was more helpful than just writing a number answer. He was not really aware that he was learning vocabulary when completing these activities. When asked to contrast his study of vocabulary with previous math units, he stated his lessons usually started with a vocabulary list. The IMIST unit had more writing and vocabulary embedded in the activities with context and drawings. Howard explained, “I like that.” However, he does like a list “as long as I learn it.”

Conceptual literacy. When asked about activities related to conceptual learning, Howard found “Just practice” and doing the exercises helped him learn. He explained that the directions for conjecture-making activities directions to “Say it in English. Say it in symbols. Draw a picture and apply it,” really helped him understand. He had not done those steps before. Analyzing the drawings/diagrams also helped him understand how to set up proofs or the “steps.” He liked the questions in the packet. They were different than in the textbook, and he thought they were “helpful” to him when thinking about procedures and writing proofs. Howard thought that doing “more writing” with the IMIST unit activities was helpful.

Problem-solving. Howard had not done many “real-world” applications before. Actually, his book presented standard word-problems but without context. Before the IMIST unit, Howard stated he “didn’t really know why I was doing it, and then like, this showed me the … real-world applications and stuff.” He found the unit projects useful.

He liked taking pictures, and he worked on a self-directed project using right triangles in design. The applications activities encouraged him to “maybe it’s just like seeing math in more things.” Also, he had not drawn many pictures as a problem-solving strategy. He thought drawing pictures helped, and “Yeah. It’s fun.” Drawing pictures made math fun. He explained the IMIST unit activities made him think about math “differently.”

Technology. Howard’s experience with technology for geometry, the computer and calculators, was quite different than the other case studies. Unlike other students, Howard participated in face-to-face lessons, so he did not experience online classes. The only directed Internet or computer use was for the first section investigation. Howard explained, “I never used the Internet … like before.” He had used the Internet as a resource for symbolic literacy, vocabulary, and procedures but not to investigate real-world applications or careers in mathematics. He liked understanding “why” he was learning geometry.

When asked about how much he used calculators, Howard explained, “Pretty much none.” However, he recognized that it was important and necessary to use calculators for trigonometry because otherwise you just “like set-up the equation and say, ‘Oh, this is the answer,’ ’cause you can’t simplify it anymore without a calculator. A calculator expression makes no sense in the real world.” Using the calculator to find a value made sense of the answer. Howard liked using the calculator particularly for work with the real-world applications. He felt confident in his ability to use the calculator and believed he will be able to remember how to use it in the future.

Recommendations. Howard did not have many recommendations on how to improve the IMIST geometry unit packet on right triangles. He felt the workload was good and that he did not need more practice. He suggested that all illustrations and diagrams need to be clear. He would use units designed with IMIST activities in the future. He liked the IMIST unit "better than a textbook" primarily because of the variety of activities and the different ways to learn "stuff."

Freshmen Algebra 2 Class

Background. This case study implemented selected materials from the IMIST quadratics unit in a small class of freshmen Algebra 2 students. Chronologically, the data for this case study was collected after the study of Hailey and Bobcat and before the case studies of Kaya and Howard. Because this case study consisted of a small class or group of students instead of individuals, the intervention was different in its implementation, structure, and data collection.

The researcher was asked to substitute for an Algebra 2 teacher at a medium-sized private high school in the Mid-Atlantic area. The administration at the high school had been informed about the IMIST research project in the spring of the previous year. When the researcher approached them about using the IMIST unit materials during her time as a substitute teacher, the administration and current teacher gave permission to use the materials in three classes but wanted her to collect data only from the standard-paced third period class. There were eighteen students in the class, and nine agreed to participate in the study – six girls and three boys. Students are referenced by pseudonyms to protect their identities. The regular teacher, for whom the researcher was a substitute, will be referred to in the discussion as Ms. K. Demographics for the participants are summarized in Table 23.

Table 23

Demographics for Participants in the IMIST Small Class Case Study

Student	Gender	Age	Grade	Ethnicity
Abigail	F	15	9	White/Asian
Brenda	F	14	9	White
Chad	M	14	9	White
Don	M	14	9	Hispanic
Erin	F	14	9	White
Frank	M	15	9	White
Gloria	F	14	9	Asian
Helen	F	14	9	White/Black/Biracial
Jackie	F	15	9	White

All students enrolled in the class were freshmen and came to the high school from many different elementary and middle schools. They had self-selected their math placement based on their prior courses. All had taken a course titled Algebra 1 at their middle school. The school's Algebra 2 class had a standard-paced or standard-level curriculum as opposed to an honors curriculum so only the first seven sections of the IMIST quadratics packet were used.

Data collected during the study included class observations, homework completion/effort grades, and assessment data. At the end of the intervention, nine participants completed the Math Attitudes and Perceptions Survey (MAPS) as well as a written survey composed of questions asked in the individual case studies post-intervention interviews.

Attitudes and perceptions. Students' attitudes and perceptions about math were observed and measured four ways. The researcher/instructor wrote comments in her class observation journal. She checked homework daily for completion and effort, and students completed MAPS. Due to timing and permissions, the instructor was unable to give MAPS at the beginning of the study, so it was given at the end of the unit instead. Because MAPS was only given once, the data can only be interpreted as a measure of each student's overall attitudes and perceptions about math at a point in time with no correlation to the IMIST intervention. The fourth measure was the written survey students completed at the end of the unit with questions similar to the individual case studies' post-intervention interviews.

Class observations. The following discussion highlights some general class attitudes and highlights specific comments and actions by individuals observed by the instructor over the course of the unit. The instructor was met with some hostility at the beginning of her teaching assignment. This was due to differences between her instructional practices and those of Ms. K. The instructor was more structured, had higher expectations about class participation and homework, and held the students accountable for their work. Students were used to a traditional instructional pedagogy of guided instruction, explained examples, and homework practice. They were not accustomed to investigations, learning activities, and productive struggle (NCTM, 2014; Warshauer, 2015) associated with student-centered active learning.

Over the course of the IMIST unit, the instructor was able to encourage some students to change their attitudes towards class work evidenced by improved work ethic. Brenda, in particular, was initially vocal about the amount of work, not understanding, and generally had a negative attitude. However, she had an excellent work ethic, and although she mumbled and was disgruntled about doing the work, she saw an improvement in her understanding and learning. Her attitude generally improved. At the end of the unit, she became one of the students volunteering to answer questions and participating in the class discussion. The instructor observed that Brenda saw her hard work pay off in understanding and achievement.

Erin was an example of a student who did not have good work ethic. Her work on the Section 1 TELA investigation was incomplete, and she spent a fair amount of time “not getting it” until she finally made the connection between the parent graph and the effects of transformations described earlier. She made the comment, “Math has always been easy,” and “This is hard.” Erin came around and had an *aha* moment during the Section 6 activities. Her attitude also became more positive and accepting. However, she also was representative of a number of her peers who did not like the “amount of work” associated with the student-centered activities and the change in the instructional practices.

Frank was bright and had a positive attitude in math class. His comments during the initial part of the unit showed intuitive understanding and were insightful. It was clear that he liked math and made connections easily. Unfortunately, he got sick during the unit and fell behind in his work, and although he was given time to make up his homework, he never caught up in his understanding of skills and procedures.

Chad was unhappy to be in the standard level Algebra 2 class. He wanted to be moved to the honors class. Ms. K. and his counselors did not support his move based on his background and prior achievement. He did all the homework demonstrating good work habits but, unfortunately, was not an enthusiastic participant in class. He was not quick to make conceptual connections. It took him longer to learn some of the concepts and his work often showed mistakes. He did do the TELA investigation for Section 4 but more to get the extra credit than for the love of math. He seemed to be more motivated and concerned about grades than learning.

Helen struggled throughout the unit primarily due to a lack of preparation from Algebra 1. She tried to do her best but clearly was discouraged and lacked confidence. At the end of the unit, she opted to drop back to Algebra 1. This was a difficult decision for her, but the instructor and Ms. K. both believed that continuing in Algebra 2 would have destroyed her confidence to be able to study and learn mathematics.

Homework. The instructor assessed homework daily for effort and completion. Her purpose was to encourage students to learn through practice and doing as well as to hold them accountable. Knowing who was doing the homework versus who was not doing the homework was also a clue as to which students had good work habits and attitudes and those who did not. In general, the students who did not consistently keep up with the homework practice for whatever reason struggled with their learning.

Trends in the homework grades reflect students' willingness to work and indirectly their attitudes. A second important homework activity was self-correction and homework review at the beginning of each class. Students who completed their homework and self-corrected their mistakes benefitted from working on the exercises the night before, correcting their thinking, and learning from their mistakes. This opportunity for self-correction and reflection was lost to those students who did not do the homework.

In general, the weakest performance on homework was in weeks 1 and 2 when students were assigned the student-centered investigations and activities with follow-up practice from the textbook. As students realized the value of doing homework, homework effort and completion improved in weeks 3 to 5. Students with high completion and high effort grades of 95% or above (Abigail, Brenda, Chad, Helen, and Jackie) tended to participate in class discussion, worked well with their partners, and generally had positive attitudes in class. They were actively engaged at the beginning of each class, correcting their work, learning from their mistakes, and asking the instructor for clarification.

There were a couple of noticeable trends which supported the instructor's class observations about attitudes. Helen who had struggled with Algebra 1 skills made a good effort on homework but only completed 3 of 6 points during the last week. She had given up at the end of the unit and was discouraged.

Erin did only 43.8% of the homework assigned during weeks 1 and 2. This supported her lack of understanding, her disrupting comments during class, and her general lack of understanding. She made a significant turnaround in weeks 3 through 5, completing 100% of the homework in the second half of the unit. This supported the instructor's class observations about a change in attitude.

Don's work ethic declined over the 4 and a half weeks. He started with 100% in weeks 1 and 2, then scored 77.8% and 66.7% in weeks 3-4 and week 5 respectively. His homework grades do not explain the drop in his effort or attitude. In class, he was pleasant and answered questions when called upon but did not proactively engage in class activities or discussions. Table 24 summarizes the homework performance of the participating students.

Table 24

Class Homework Effort and Completion Grades for the IMIST Quadratics Unit

	Homework Weeks 1-2		TELA Section 1	Homework Weeks 3-4		Homework Week 5		Homework Total	
Student	Points	%	3	Points	%	Points	%	Points	%
Total Points	16			18		6		40	
Abigail	15	93.8	2	18	100.0	6	100.0	39	97.5
Brenda	14	87.8	3	18	100.0	6	100.0	38	95.0
Chad	16	100.0	3	18	100.0	6	100.0	40	100.0
Don	16	100.0	3	14	77.8	4	66.7	34	85.0
Erin	7	43.8	2	18	100.0	6	100.0	31	77.5
Frank	13*	81.3	2	17	94.4	3	50.0	33	82.5
Gloria	11	68.8	2	17	94.4	6	100.0	34	85.0
Helen	14	87.8	2	17	94.4	5	83.3	36	90.0
Jackie	16	100.0	3	18	100.0	4	75.0	38	95.0

*Frank was absent for three days during the grading period and did not make up all of the assigned homework.

MAPS. MAPS data collected at the end of the unit were the third data source providing insights about mathematics attitudes. There were 32 questions on MAPS, and the scores could range from a low of 31 to 155 which correlates to a low to high (more positive) attitude or perception about mathematics looking at both general beliefs and beliefs about self.

The scores of the eight students who completed the survey ranged from 82 to 110. Helen scored 82 which was reflective of her lack of confidence and discouragement about her abilities to do math. Don and Gloria with high scores of 110 and 104 respectively showed the most positive attitudes and perceptions about mathematics. There were three students clustered around 100 (Abigail, Erin, and Frank), and two students clustered around 90 (Brenda and Jackie). Table 25 shows the total scores for each of the student participants.

Table 25

Small Class MAPS Scores after the IMIST Quadratics Unit

Student	**MAPS Score**
Abigail	99
Brenda	91
Chad	Incomplete*
Don	110
Erin	96
Frank	100
Gloria	104
Helen	82
Jackie	88

* Chad only completed the first page of the survey, 12 questions. However, his subscore was the lowest of the group. He scored a 33 as compared to the range for the other students of 36 to 43.

Since MAPS was given after the IMIST unit, the researcher found three questions relevant to the research questions concerning attitudes about math and the real-world applications. None of the students “strongly agreed” with statement 12, “I enjoy solving math problems.” Only Don responded with “Agree.” Five students were “undecided” about how they felt about solving math problems (Abigail, Chad, Erin, Frank, and

Gloria), and three disagreed with the statement (Brenda, Helen, and Jackie). These responses indicated that the students may not have been intrinsically motivated to do math and had ambivalent attitudes. Students were also “undecided” or “disagreed” with statement 14, “Learning math changes my ideas about how the world works.” This suggested they had not made the connection between in-school math activities and real-world applications. However, four students (Brenda, Don, Frank, and Jackie) saw value in studying math because they agreed with the statement, “Reasoning skills used to understand math can be helpful to me in my everyday life.”

Don’s MAPS responses were interesting. He had the highest score on the survey indicating a positive and more mature attitude and perception about mathematics. He was the only student who “liked to solve math problems,” and he saw value in learning mathematical reasoning. However, his homework effort declined over the course of the unit. His MAPS score seemed inconsistent with his attitude and performance in class. It may have been his specific reaction to the IMIST unit structure and materials as compared to a more traditional approach to learning math or due to having studied the material before and being bored. The questions and student responses are presented in Table 26. An answer of 1 corresponded to “Strongly Disagree” and an answer of 5 corresponded to “Strongly Agree.”

Table 26

MAPS Questions about Personal Beliefs and Mathematics

Question	Abigail	Brenda	Chad	Don	Erin	Frank	Gloria	Helen	Jackie
12. I enjoy solving math problems.	3	2	3	4	3	3	3	2	1
13. Learning math changes my ideas about how the world works.	3	3		2	3	3	3	2	3
15. Reasoning skills used to understand math can be helpful to me in my everyday life.	3	4		4	2	4	3	3	4

Assessments. Students completed three graded assessments in the small class case study. The first two were formative assessments or quizzes followed by the unit assessment. Due to scheduling and class time, Quiz 1 and the unit assessment were given in two parts over two consecutive days. The assessments were also shorter than the assessments used in the individual case studies because of time constraints. Quiz 1 covered content in the Sections 1 to 3, and Quiz 2 covered content in Section 4. The unit assessment covered content taught in Sections 1 through 7 of the unit. Quiz 1 had fourteen questions for a total of 56 points. However, questions 1 and 2 had multiple parts as they assessed students' abilities to interpret graphs and equations and vice versa. Quiz 2 had twelve questions for a total of 43 points. Part 1 of the unit assessment had thirteen questions worth 42 points, and Part 2 had seventeen questions worth 77 points. Table 27 and Table 28 show the IMIST assessment questions, total points and percentages coded by literacies.

Table 27

IMIST Assessments – Quiz 1 and 2 – Questions and Points by Literacy(ies)

Question Literacy	**Quiz 1 Sections 1 to 3**		**Quiz 2 Sections 4 to 7**	
	# of Questions	Points	# of Questions	Points
Symbolic	4	9	4	12
Symbolic and Conceptual	10	47	7	27
Problem-solving			1	4
Total	14	56	12	43

Table 28

IMIST Unit Assessment – Questions and Points by Literacy(ies)

Question Literacy	**Unit Assessment Part 1**		**Unit Assessment Part 2**	
	# of Questions	Points	# of Questions	Points
Symbolic	1	9		
Symbolic and Conceptual	12	33	7	42
Conceptual			9	24
Problem-solving			1	11
Total	13	42	17	77

The following discussion presents the data from all assessments for each type of literacy. It shows the levels of mastery and trends for each expressed as points earned and percentages of total possible points for each literacy.

Symbolic literacy. On Quiz 1, the four symbolic literacy questions were a review of an Algebra 1 skill using Algebra 2 vocabulary. The directions asked students to "Use the Zero-Product property to solve each equation." Seven of the nine students had perfect scores, and one only missed one point which was due to a transposition error. Helen answered only four questions correctly. The rest were left blank. The algebraic work was to solve a one-step solution, but students had to know how to set-up the solution. Most students did not show work and did the math mentally which is appropriate in Algebra 2.

On Quiz 2, the first three symbolic literacy questions were irrational number operations: simplifying, multiplying and dividing. Abigail, Brenda, Chad, Don, and Erin scored 6 out of 6, and Frank, Gloria, and Jackie scored 5 out of 6 making small arithmetic errors. Helen only simplified the first number and left the next two questions blank. She seemed to lack the pre-requisite knowledge to do the operations.

The fourth question on Quiz 2 was a simple vocabulary question worth 6 points. Directions asked students to "Name and write the three forms of quadratic function equations we have studied in this unit." This question required students to write their answers and proved to be challenging to some although the answers should have been memorized by this time. No student received a perfect score. Gloria scored 5.5 out of 6. Four scored 5 out of 6 (Brenda, Chad, Don, and Erin), and two students scored 4.5 out of 6 (Frank and Jackie). The most common mistake was not writing an equation, writing an expression or leaving out the leading coefficient. Abigail and Helen left the question blank, scoring zero points. This was evidence of the students' lack of connection between the desired learning objective of recognizing and working with the three forms of quadratic equation and knowing them by name. Since the questions were blank it was unclear whether the students ran out of time or simply did not know the answers.

The symbolic literacy question on Part 1 of the unit assessment was a combination of vocabulary, identifying the type of complex number, and the second assessed graphing skills on the complex plane similar to graphing real numbers on the coordinate plane. The differences were the axis labels, scales, and interpreting the form of the complex number as coordinates on the Complex plane. The learning outcome/expectation was that these questions should be done using mental math. Unlike the assessment question for students in the individual case studies, the question did not ask them to find the absolute value of the complex number or its distance from the origin. Five of nine students had perfect scores (Abigail, Brenda, Gloria, Frank, and Jackie), and two students only missed one point (Don and Erin). The most common error was to omit the scales on the axes or to mislabel them. Chad scored 6.5 out of 9 points misclassifying two of the numbers and transposing the coordinates of a point on the graph. Helen missed three points: one point for not labeling the axes and two points for misclassifying the numbers. Table 29 summarizes students' scores on the symbolic literacy questions on the two quizzes and Part 1 of the unit assessment. The second part of the unit assessment did not have any questions that only assessed symbolic literacy.

Table 29

IMIST Small Class Assessment Data for Symbolic Literacy

Symbolic Literacy	**Quiz 1 4 Questions 9 Points**	**Quiz 2 4 Questions 12 Points**	**Unit Part 1 1 Question 9 Points**
Student	Percent	Percent	Percent
Abigail	100%	50%	100%
Brenda	100%	92%	100%
Chad	100%	92%	72%
Don	100%	92%	88%
Erin	100%	92%	88%
Frank	89%	79%	100%
Gloria	100%	88%	100%
Helen	44%	17%	67%
Jackie	100%	79%	100%

Symbolic and conceptual literacies. The highest percentage of questions and points on each assessment evaluated student mastery of the combination of symbolic and conceptual literacies. The scores for these literacies were the most variable.

Quiz 1 was the formative assessment on graphing, transformations, critical features, and equations. Scores for Quiz 1 for the top eight students ranged from 57% to 85%. Quiz 1 showed the second greatest variability in student scores with a mean for the top eight students of 72% with Standard Deviation of 12.1%. Helen's score was 23% overall three questions.

Quiz 1 had three questions that assessed symbolic and conceptual literacies, and each had multiple parts. Question 1 asked students to identify critical features from a graph of a parabola and to write the three forms of the equation for the graph. Students performed poorly on this question scoring between 1 to 7 points out of a possible 11 – a range of 9.1% to 63%. The second question also had multiple parts and was procedural as well as conceptual. Students were asked to interpret an equation in vertex form, to give the critical features and alternate forms of the equation, describe transformations, and then to graph the equation. Students did better on this question scoring between 5 and 14 out of a possible 15 points – a range of 33.3% to 93.3%. Questions 3 through 10 were factoring questions. The first three were simple quadratic expressions with leading coefficient "1." The other five questions had expressions with different leading coefficients and asked students to "factor completely." Students performed the best on this factoring section. Three students (Brenda, Don, and Gloria) had perfect scores. Other scores ranged from 14.5 to 19 out of 21 points – a range of 76.3% to 90.5%. Helen scored only 5 out of 21 or 23.8%.

Student scores for Quiz 2 showed significant improvement and less variability as compared to scores on Quiz 1. The questions were procedural and focused on solving quadratic equations by completing the square and finding square roots. Questions 4 to 7 assessed solving by taking square roots. Five students had perfect scores (Brenda, Chad, Erin, Gloria, and Jackie). The range of scores for the other students was 9 to 12 points out of 13. For questions 8 and 9, students used "completing the square" to solve the equation. Students had more difficulty with this procedure. Three students had perfect scores of 8 out of 8 (Abigail, Brenda, and Gloria). Other student scores ranged from 3 to 7. Question 10 was the most difficult conceptually and proved to be the most challenging. The directions instructed students to use “completing the square” to transform a standard form equation into vertex form. No student was able to do this correctly. Scores ranged from 1 to 5 points out of 6. Overall for Quiz 2, the mean score for the top eight students was 63.9% with a standard deviation of 9.1% points. The percent average was 82%. Again, Helen scored 13 out of 27 points or 48%.

Student scores for Part 1of the unit assessment showed improvement for six of the nine students as compared to the formative Quizzes 1 and 2 for mastery of symbolic and conceptual literacies. The first six questions combined factoring and identifying the axis of symmetry from a standard form equation. Two students had perfect scores of 18 out of 18 (Brenda and Gloria) and two students only missed one point (Chad and Don). Other students’ scores ranged from 12 to 16 points with Helen at 7.5. Questions 7 to 12 assessed complex number operations. Three students had perfect scores of 15 out of 15 (Don, Erin, and Gloria). Two students only missed one half of a point scoring 14.5 out of 15 (Brenda and Chad), and three students missed 1 to 1½ points scoring 13 or 13.5 out of 15 (Abigail, Frank, and Jackie). Only Helen had trouble scoring 4.5 out of 15. The mean of the top eight students was 91% with a standard deviation of 8.2%. Helen scored 36%.

Student scores for Part 2 of the unit assessment were lower than the scores on Part 1 and showed the greatest variability. There were two reasons for this. The first was the questions assessed a higher level of conceptual understanding, and the second reason was time. The assessment was 77 points and the highest score was 57.5 out of 77 or 75%. The scores on this portion of the test averaged 15 to 20 percentage points lower than on Part 1 primarily because students ran out of time and had to choose what problems they would complete. Final scores were adjusted to reflect this issue. Questions 9 and 10 were multiple choice and asked students to identify the vertex and axis of symmetry from vertex form equations. Six of the nine students answered both questions correct. Two students missed one and Frank missed both (perhaps due to his absence from class discussion). Questions 11 and 12 reassessed students' ability to solve by "completing the square," however the second question was more difficult because the students had to work with fractions. Two students (Brenda and Gloria) got both questions correct. The other students scored 0 to 3 points out of 4 due to arithmetic errors. Questions 13 and 14 reassessed students' ability to graph, solve, transform, and identity critical features. The directions asked students to use the quadratic formula in Question 13. Time played a factor in these two questions because graphing takes time. Two students (Abigail and Helen) left question 14 blank, and two students (Chad and Helen) did not do the graph for question 13. Omitting those students, the scores on question 13 ranged from 5.5 to 12 out of 13 points and scores on question 14 ranged from 6 to 16 out of 16 points. Both sets of scores showed improvement from similar questions on Quiz 1.

Question 16, the final question on the unit assessment that combined symbolic and conceptual literacies, asked students to find a quadratic equation given the vertex and a point on the curve for 4 points. Two students omitted the question (Abigail and Helen). There was only one perfect score (Erin), and the scores for the other students ranged from

2 to 3.5 points. The top eight students scored between 29% and 83%. The mean was 77% with the highest standard deviation of 18.1%. Helen scored 23%.

The means and standard deviations for the percent correct are summarized in Table 30. Helen's scores continued to be lower than other students' (with the exception of Part 2 of the unit assessment). Her scores were omitted from the calculation of the mean and standard deviation.

Table 30

IMIST Small Class Assessment Data for Symbolic and Conceptual Literacies

Symbolic and Conceptual	**Quiz 1 10 Questions 47 Points**	**Quiz 2 7 Questions 33 Points**	**Unit Part 1 12 Questions 36 Points**	**Unit Part 2 7 Questions 42 Points**
Student	Percent	Percent	Percent	Percent
Abigail	61%	91%	89%	43%
Brenda	85%	93%	98%	82%
Chad	68%	81%	95%	74%
Don	81%	69%	97%	83%
Erin	57%	67%	82%	81%
Frank	74%	74%	83%	29%
Gloria	89%	96%	100%	71%
Helen	23%	48%	36%	23%
Jackie	61%	85%	79%	68%
Mean*	72%	82%	91%	71%
SD	±12.1%	±9.1%	±8.2%	±18.1%

* The mean score/percent was calculated for the top eight students. Helen's scores were omitted as they were significantly lower than those of the other students.

Conceptual literacy. Part 2 of the unit assessment was the only assessment with questions that measured conceptual literacy independently. However, part of Question 2 (2f) on Quiz 1and Question 15 (15b) on Part 2 of the unit assessment assessed students' ability to "name and describe" the transformations applied to the parent function to get to a new function and its graph. This ability to describe transformations was classified as a conceptual literacy proficiency. Since this was identified as an important learning objective for the IMIST unit and for NCTM standards for Algebra 2, the student scores for these parts were reviewed. The balance of the conceptual literacy questions on the unit assessment are reported together. Percentage scores of seven of the nine students stayed the same or improved showing improvement in their conceptual understanding of naming and describing transformations of quadratic equations and graphs (Brenda, Chad, Don, Erin, Frank, Gloria, and Helen). Abigail did not answer the question, and Jackie's answer was incomplete.

Seven of the nine conceptual literacy questions on the unit assessment were short answer questions similar to a true/false format, but students needed to identify or answer the question in one or two words. These questions assessed student knowledge of concepts and their ability to describe concepts. Question 8 was a three-point multiple choice question. Seven of the nine students answered correctly (Abigail, Glenda, Chad, Don, Erin, Helen, and Jackie). Part (b) of question 15 asked students to find a quadratic equation from a graph. Three students did not answer the question (Abigail, Helen, and Gloria). Of the six students who answered, three scored 3 out of 3 (Don, Erin, and Frank), two scored 2 out of 3 (Brenda and Chad), and Jackie scored one point. The most omissions or non-answered questions occurred on this page as it was the “graphing” page. Graphing and writing take the most time for students to complete, and it appeared that many of the students saved this page for last. If they ran out of time, they did not complete this page. Omitting Helen who left most of the questions blank, students’ scores ranged from a low of 7 to 14 out of 14 points or 50% to 100%. The questions, points, and student percentage scores are shown in Table 31.

Table 31

IMIST Small Class Assessment Data for Conceptual Literacy

Conceptual Literacy	**Quiz 1 Question 2f 4 Points**	**Unit Part 2 Question 15b 4 Points**	**Unit Part 2 9 Questions 20 Points**
Student	Percent	Percent	Percent
Abigail	50%	0%*	65%*
Brenda	63%	88%	65%
Chad	88%	88%	73%
Don	63%	88%	28%
Erin	50%	75%	70%
Frank	50%	50%	75%
Gloria	75%	75%	70%
Helen	50%	75%	10%
Jackie	63%	38%	69%

* Abigail omitted question 15.

Problem-solving. As mentioned previously, the school had only two learning objectives for problem-solving, modeling dropping objects and projectile motion using quadratic equations. Quiz 2 had one question that was modeled by the dropping object equation worth 4 points, and the unit assessment had one projectile motion problem with four parts worth 11 points. Both questions were framed as stories, and students were expected to answer with a complete sentence written in context. The equation for the dropping object model was included on the quiz, but the equation for the projectile motion model was not provided on the unit assessment.

Students did an excellent job on the application problem on Quiz 2. Six students received full credit or 4 out of 4 (Abigail, Brenda, Chad, Don, Erin, and Gloria), Frank scored 3.5 out of 4, and Helen and Jackie scored 3 out of 4 or 75%. All three students who missed points did not interpret the answer correctly and/or did not write the answer as a sentence.

The application question on the unit assessment was the last question. Since most students did not have enough time to finish the assessment, scores on the application question were low due to incomplete answers. However, all students who attempted the problem received credit for writing the correct equation model (2 points). Those receiving 3 points also provided the time that the clam reached its maximum height by finding the value of the time at the axis of symmetry. Brenda was the most successful on this question. She wrote the incorrect equation model but used her equation correctly to determine when and where the clam reached its maximum height and also made an estimate for when the clam would hit the water. Table 32 presents assessment data for problem-solving literacy.

Table 32

IMIST Small Class Assessment Data for Problem-solving Literacy

Problem-solving Literacy	**Quiz 2 1 Question 4 Points**	**Unit Part 2 1 Question 11 Points**
Student	Percent	Percent
Abigail	100%	27%
Brenda	100%	59%
Chad	100%	18%
Don	100%	32%
Erin	100%	18%
Frank	88%	27%
Gloria	100%	18%
Helen	75%	18%
Jackie	75%	0% *

* Jackie omitted the problem.

IMIST post-intervention survey thematic analysis. At the end of unit, the students were given a post-intervention survey. The survey had sixteen questions (Appendix E). The initial questions asked students whether they liked or disliked using the IMIST unit activities, if they were the same or different than the learning activities they had used before, and if they liked or disliked the unit structure. The second set of questions asked the students about their learning using the IMIST activities and how the activities supported their learning for symbolic, conceptual, and problem-solving literacies. The final questions asked students to list their favorite and least favorite activities and how they could have improved their study of math mathematics.

Students wrote their responses to the questions. One limitation that became apparent was that all of the IMIST activities were included in one packet. Although the page titles differentiated Class Notes (CN), Investigations, and Work Sheets (WS), students tended to make inclusive or generalized statements about the “packets” or activities, and it was not easy to discern what specific activities they referenced in their comments.

Unit structure. Student reactions to the unit structure were mixed: four students liked the unit structure (Abigail, Brenda, Chad, and Frank); four students did not (Don, Erin, Helen, and Jackie); and one felt mixed (Gloria). Abigail liked the unit structure because “each topic was built off of the one covered before,” and they “made me more organized, and they were easy to find.” Brenda explained, “It helped me learn because we did many of the same problems, which helped to reinforce my understanding of the topic.” Chad explained that “Most things made sense because it was a lot of thinking work.” His comment about “thinking work” described one of the learning objectives of the IMIST unit, to help the students think more deeply and to understand mathematics conceptually and as applications.

Students who did not like the unit structure stressed their confusion, organization, and explanations. Don explained, “I was always lost or confused.” Erin said there was “too much paper and assignments.” Gloria explained, “Some problems didn’t make sense because of the wording, and I didn’t know the vocabulary.” Helen explained that “not everything made sense because it wasn’t fully explained.” The first three students had issues with work ethic, completion of the homework, and taking notes in class. Helen had trouble throughout the unit due to a lack of course preparation in her Algebra 1 class as documented in the class observations.

Students' evaluation of the in-class lessons was also mixed but more favorable: five students liked the in-class lessons (Abigail, Brenda, Chad, Frank, and Gloria); three did not (Don, Helen, and Jackie); and one was mixed (Erin). The contrast between those who liked the lessons and those who did not focused on explanations. Students who liked the lessons stated there was enough explanation, and those who did not like the lessons did not think there was enough explanation. Because most lessons required some pre-reading or preparatory activity, those who came to class prepared were more inclined to understand and to participate in class discussion than those students who did not prepare for class. Abigail explained that the lessons "explained how things worked," and Brenda explained that she learned to "fix my mistakes and further understand the topic." Chad thought the lessons showed "me how to solve/simplify a problem very thoroughly." Gloria brought up the issue of pacing. "The lessons were a little too fast for me, so it was hard to keep up, but when I started doing the problems, I was fine." Seven of the students wrote that they felt "rushed," that there was "too much information," or that the "pace was a little fast" or "too fast for me" (Abigail, Brenda, Chad, Erin, Frank, Gloria, and Jackie). Students included that they would have liked more opportunities to do/redo homework, primarily the investigations. Redoing homework or late homework was offered by the instructor for this unit. It was not the policy of their teacher Ms. K. Students would have liked to have more time in class to ask questions. Don wanted the instructor to “teach more depth” which may have meant more explanations, a sentiment shared by Gloria, Helen, and Jackie.

When asked about the amount of practice, the investigations, the examples, the textbook homework, and the worksheets, student responses varied: four students thought there were enough practice activities (Brenda, Chad, Don, and Gloria); four students were mixed (Abigail, Erin, Frank, and Jackie); and Helen did not think there was enough practice. This question did not differentiate between the types of activities, so the answers were generalized to “practice activities” and “worksheets.” Those students thought there was “enough practice” using the textbook exercises and the worksheets, and the other half said there was “too much.” Gloria explained the practice helped improve her understanding because of repetition and “showed us examples, and that “there was a lot of problems to practice and some questions made you think harder to get the answer.” Chad liked the worksheets because they showed “the many scenarios for a certain problem.” Frank referenced the investigations in Sections 1 and 2. He said they “helped with transforming equations from form to form.” The learning objectives for those activities were to help students make connections between graphing, transformations, tables, and equations and the previous comments indicated that these students learned through these different activities.

Don, who was negative about almost everything in the unit, found the worksheets “helpful,” but they “took up too much space and were hard to keep track of.” Some found the number of worksheets “overwhelming” or “complicated” (Erin and Frank). Helen thought the worksheets were “helpful” but “not really. I think I can do better if I do more problems and the worksheets didn't really let me do that. The homework didn't really either. I don't know." This comment shows that Helen was at a loss about how to learn during the unit. Her confusion likely came from a lack of pre-requisite knowledge so the type of practice either in the packet or in the textbook was beyond her ability. Jackie was mixed about the amount of practice and the worksheets. In reference to whether the worksheets and practice exercises were helpful or not, she explained, "Some were, some weren't. Some helped me understand the material better, while others were either too much work or busy work or did not help me understand the material and made me more confused." Jackie did not differentiation between which were helpful or not and which were confusing or not.

The instructor acknowledges that some of the negative comments about the "packets" and "too much material" were partly due to her lack of access to technology and a course management system. Everything had to be provided to students as hard copy, and although the class notes and activities were grouped, stapled and hole-punched, some students had trouble keeping the packet materials organized in their notebooks.

When asked about whether the instructions in the activities were easy to follow, student responses varied: four said yes, they were "straight forward" (Abigail, Chad, Don, and Helen); Jackie said no; and four were mixed (Brenda, Erin, Frank, and Gloria). Erin explained that the instructions "did not give much information about processes to where and how they got that answer." Perhaps Erin was not taking notes during class on "the processes" or reviewing the homework solutions sheets which provided detailed solutions on "how they got that answer." Frank thought that "a little more instruction on the worksheets could help/also background." Frank had missed some classes and, as a result, missed some of the explanation and discussion of processes. It was unclear if Gloria or Helen were taking notes or reading the book since both explained that they did not "know the meaning of the vocab words" or had "trouble understanding some vocab" in some of the instructions.

Attitudes and perceptions. Students were asked specifically whether they liked "learning using the IMIST activities". Student responses varied but were more negative: three students liked learning using IMIST (Brenda, Gloria, and Jackie); four students did not (Abigail, Chad, Don, and Erin); and two students felt mixed (Frank and Helen). Brenda liked learning using the IMIST activities, because it was "easier to stay organized and do better on the quizzes." Even though Jackie liked the IMIST activities, she said, "Sometimes it can be too challenging or difficult to understand." Jackie was one of the two students who completed the investigation for Section 4. She was willing to take on challenging work but, due to time or lack of explanation from the instructor, she did not always understand or was "confused." Gloria thought that the IMIST unit "was a little hard to understand at first, but once I got it, it was easier."

Frank had mixed feelings about the IMIST unit. He found the amount of work in the packets "too much," but the investigations "help me to understand certain topics." In the post-intervention surveys, students made nine references to "too much work," "too much homework", and/or "too much in the packets." However, Chad made an insightful comment, "Math is just a subject that requires work and isn't supposed to be very easy." Chad was the second student who did the extra credit project, the investigation in Section

4. His homework effort and willingness to do extra work demonstrated good work ethic and discipline although he had mixed feeling about his learning.

Don also felt mixed. He responded he did not like the IMIST unit but that “the teacher talked a little bit more about it,” meaning that the instructor did provide more discussion or explanation than he had experienced before. Abigail explained, “I am left not completely understanding a couple of topics.” Brenda found “a lot of it as extra and some things seemed unnecessary.” Don cited the “overuse of worksheets and packets.” Erin found the IMIST unit “confusing, too fast, and relatively disorganized.”

When students were asked whether they felt they had learned more or less using the IMIST unit activities, four said they learned more (Brenda, Chad, Frank, and Gloria); three said they learned less (Don, Erin, and Jackie); and two felt mixed (Abigail and Helen). Brenda felt she had learned more. “I could not only do problems, but I could understand them.” Chad said, “I learned a lot more because the course was much more rigorous and required more work.” Gloria explained, “I learned a bit more because they included word problems that apply to everyday life, which I can use in the future.”

Students who felt they learned less again cited confusion and not understanding. Don felt he learned less “because I always got confused, and then we moved to a more complicated subject,” and “All of this is new stuff I’ve never learned before.” Erin explained, “I was often confused and never really got the hang of any units. I understood the gist, but not all.” She also stated, “I spent a lot of time teaching myself how to do things.” Here, she may be referencing the investigations and student-centered activities – a new instructional practice for most students. Jackie felt that “I did not have a good enough understanding in the beginning and that accumulated over time." Abigail felt mixed. "I feel like we went over many topics, but I didn't understand some of them." Helen stated, "I felt like I learned nothing during this unit."

Jackie was the most articulate about her lack of confidence, and her sentiments were reflective of some of the other students who felt confused and did not feel like they mastered the topics of graphing and solving quadratics. Jackie explained, "I feel like I don't completely understand parabolas and quadratic equations, and I find myself struggling on most test(s)." "I am not confident in graphing parabolas at all, I felt like I was working through problems without understanding anything and that reflected in my test." She wrote about feeling too rushed. "I felt the lessons were too rushed, and I never understood in depth what I was doing."

In the survey data, students referenced pacing eleven times: seven mentioned the pacing was too fast, and four recommended a slower pace. Since the pacing had been set by the school and the Algebra 2 teaching team, the instructor did not have much flexibility to accommodate pacing changes.

When asked what they could do to improve their learning and understanding of mathematics, five students said they could improve their homework practice (Abigail, Brenda, Frank, Gloria, and Helen). Three students said they could have read the textbook to help them learn vocabulary (Abigail, Erin, and Gloria). Brenda thought she should ask more questions in class. Four students said they should study more and spend more time on math (Chad, Don, Frank, and Gloria). Helen said she needed to get a tutor. These responses provided insights into why some of the students felt negatively about mathematics. They may not have been taking notes, reviewing them, completing homework and correcting their mistakes, reading the textbook, or preparing for class.

Symbolic literacy. The post-intervention survey asked students whether the IMIST unit activities helped them with their symbolic literacy – their understanding of vocabulary, symbols, and skills. Six students thought the activities helped them (Abigail, Brenda, Erin, Frank, Gloria, and Jackie), and three did not (Chad, Don, and Helen). Abigail thought the TELA investigation for Section 1 "taught the vocabulary for transforming the parent functions on a graph." Brenda explained, "I learned more about the formulas because now I understand what each part of the formula means and is used for." Erin shared that "some of the terminology was different," and "I thought the best one was the imaginary number one. It was the most clear." Frank explained, "The IMIST activities definitely helped me understand the symbols and vocabulary more than the textbook." Gloria said that the IMIST activities "improved understanding because it kept repeating it and showed us examples." Jackie explained, "I learned more vocabulary than I used to."

Abigail and Brenda also shared that some of the activities reviewed skills they learned in Algebra 1, particularly factoring. The IMIST activities and practice exercises helped them master these skills. When asked what their favorite thing about the quadratics unit was, three students replied "factoring" (Brenda, Gloria, and Jackie). Brenda commented, "I really enjoyed factoring," and Gloria shared, "factoring, because it was easy, and I understood it." In Algebra 1, factoring is introduced as a concept. As a learning objective in Algebra 2, factoring moves from a conceptual literacy to a symbolic literacy as a skill or mental math activity. This change represents mastery of this mathematical proficiency. These students expressed their confidence in their level of mastery. However, in contrast, Helen said that one of her least favorite things included factoring. “I find factoring and graphing difficult, and I never really understood it.”

Conceptual literacy. When asked whether the IMIST activities helped them learn conceptual literacies – the connections between tables, graphs, and equations, transformations of a parent graph, and the procedures for solving quadratic equations – five students thought the activities helped (Abigail, Brenda, Chad, Frank and Gloria), and four did not (Don, Erin, Helen, and Jackie). Abigail explained “it showed me how math is used and shown in real-world situations.” Brenda shared, “I learned more about quadratics because I now know how to solve a problem much quicker.” Chad provided an example equation and an explanation of what he had learned. “I know well how to find vertex, axis of symmetry, and describing transformations.” Frank and Gloria explained what they had learned about the relationships between graphs and tables. “The activities greatly enhanced my knowledge of graphing & backed up my knowledge of tables,” and “It helped me realize that you can use tables to find points on a graph if you don’t know how to find the points directly.” Gloria also referenced “thinking.” “There was a lot of problems to practice and some questions made you think harder to get the answer.”

Don, Erin, Helen, and Jackie responded “no” to the conceptual literacy questions without much explanation. They may not have fully understood the question/vocabulary or it could have been that they still felt confused and did not feel they had mastered the content. Erin wrote that she had spent time teaching herself and felt confused. Jackie, as described before, felt confused throughout most of the unit.

Some students referenced conceptual literacies as their favorite thing about the quadratics unit. Some of them also explained they had a significant understanding or an *aha* moment during the IMIST conceptual learning activities. Abigail's favorite thing was "understanding the formulas," and she had an *aha* moment "When I learned how the discriminant found how many and what kinds of roots where were for the equations." Chad's favorite thing was "describing the transformations of a quadratic expression." Frank liked the "focus on parabolas & every way to solve them," and he had an *aha* moment for "complex numbers, I really had a click." Gloria liked the process of "completing the square" to solve quadratic equations. She also had an *aha* moment "When we were talking about different translations and dilations, I realized where you could find the different translations and dilations in vertex form." Jackie's *aha* moment happened during the activities for "factoring or imaginary numbers. I had a moment where I finally understood it." The comments made by these five students support their feelings, understanding, and mastery of these conceptual literacies.

By contrast, four of the students expressed that one of their least favorite things was graphing quadratics (Brenda, Erin, Gloria, and Jackie). Gloria explained, "My least favorite thing was graphing because it took too long and was tedious." Graphing using tables of points is introduced in Algebra 1, and the learning objective in Algebra 2 is to learn a faster method – graphing by transformations of the parent function. This takes practice, and although there was significant practice in both the IMIST packet and some in the textbook, these students still needed more time and effort to become proficient.

Problem-solving. Five students felt the IMIST activities helped them with problem-solving (Abigail, Brenda, Chad, Frank, and Gloria) and three did not (Chad, Erin, and Helen). Students referenced “solving equations” as problem-solving rather than specifically looking at application problems such as the dropping object or projectile motion problems. Abigail explained, “When there was a problem or formula I didn’t recognize, I would try to figure it out through process of elimination.” Brenda shared, “I learned how to better read equations, and how knowing what an equation means can help you solve problems.” Brenda was probably referencing the discussion about the effects of the parameters such as gravity and initial velocity in the applications. Chad said, “I can now solve quadratics fairly well and am much better at the word problems than I used to be.” Frank and Gloria specifically described “problem-solving” as a conceptual literacy in this research. Frank explained that “the IMIST activities helped with transforming equations from form to form” (conceptual/algebraic process), and Jackie shared, “I now know multiple ways to solve an equation” (conceptual/algebraic process). Two students who struggled with problem-solving provided short explanations. Erin explained, “I didn’t ever really understand the problem-solving stuff,” and Frank struggled with the “velocity & time problems.”

Technology. Unlike the case studies with online class delivery of the IMIST quadratics unit, the small class was held face-to-face. SMART Board technology was used in class to project class notes, assignment sheets, and the homework solution sheets.

The annotated notes, assignment sheets, and solution sheets, however, could not be shared electronically with students outside the classroom which was different than for the participants in the other case studies.

The primary technology tools used in class were graphing calculators. Eight of the students liked using calculators, and Erin did not. Students explained that calculators were “helpful,” made things “easier” and “faster,” and they were able to “check work.” Erin who did not find calculators helpful explained, “We barely used technology. Rarely used calculators. Also, when we did the instruction was confusing.” Frank felt similarly when he said, “I wish we had in-class lessons with calculators.”

Recommendations. Students’ recommendations to “improve the structure and delivery of the learning activities” focused on time, pacing, and explanations not on the activities themselves. Students made eleven references to pacing. Seven comments stated the pace was “too fast,” and four recommended “slowing down.” The comments about explanations and understanding are tied into the issue of class time. Students had a daily opportunity to come into the math office for extra help. Students who made these comments did not avail themselves of that opportunity.

Summary. Trends and behaviors emerged from the data for the small class case study. There were students who were consistently positive in attitude, constructive in their comments, and had good work ethic and discipline (Abigail, Brenda, Chad, Frank, and Gloria). There were students who were consistently negative in their attitudes, comments, and less willing to work (Don, Erin, and Helen). Jackie was mixed about her feelings and learning. These characteristics were not strictly related to assessment grades.

Rather, these characteristics related more closely to personal beliefs, effort, and work ethic. Table 33 presents the unit assessment grades scaled by adding 10 points to compensate for the lack of time and the students' homework effort grades for comparison.

Table 33

Small Class Scaled Unit Assessment Grades and Homework Effort Grades

	IMIST Responses	**Unit Assessment Scaled**	**Homework Effort**
Student		Percent	Percent
Abigail	+	70.9	97.5
Brenda	+	93.2	95.0
Chad	+	84.4	100.0
Don	–	89.8	85.0
Erin	–	84.0	80.0
Frank	+	67.6	90.0
Gloria	+	86.5	87.5
Helen	–	39.0	82.5
Jackie	±	87.2	95.0

As a measure of core learning objectives and proficiencies, all students with the exception of Helen demonstrated mastery of the symbolic and conceptual literacy questions in the first part of the unit assessment with scores above 80%. Six of the students passed the final assessment with a scaled score of 84% or better (Brenda, Chad, Don, Erin, Gloria, and Jackie). Three of those students were consistently positive about learning with the IMIST activities (Brenda, Chad, and Gloria). Two students were consistently negative in their comments (Don and Erin), and they also had two of the lowest scores on homework effort. Jackie scored well on the unit assessment, made excellent effort on homework, but her feelings were mixed about learning with IMIST. Her success on the unit assessment was inconsistent or surprising given her comments on her survey about learning, confusion, and not doing well on tests. Although they did not score highly on the unit assessment, both Abigail and Frank were generally positive and constructive about their learning with the IMIST activities. As expected, Helen failed the unit assessment, had low homework effort due to discouragement at the end of the unit, and had a negative attitude towards her learning experiences.

Students' negative comments about IMIST activities were tied more closely to work and effort focusing mainly on the amount of work, pacing, and explanations and not about the investigations and learning activities. Students shared they had learned symbolic literacy, conceptual literacies, and problem-solving using the IMIST activities but "struggled" and needed more time and explanation. Most of these students demonstrated good effort on homework and good participation in class. Frank's lower scores on the assessment and in homework were due to absence. However, he still liked the IMIST activities with his active participation in class demonstrating his ability to make connections and explain his learning. The most negatively vocal students in class with similar written responses were Don and Erin. Even though they did not like the workload or activities, Don and Erin scored 89.8% and 84.0% respectively on the final assessment. This demonstrated they may have had a good foundation in their Algebra 1 classes and did enough of the work during the unit to learn new material. They just did not like doing it.

Chapter Five

Cross-Case Analysis

The previous case studies told the stories of thirteen students – two as a pair, two individually, and nine as a small class. This section examines data collectively for all students framed by each research question. The first part compares students' understanding, learning, and achievement in each of the core literacies and overall. The next section examines MAPS data and students' responses in interviews and on written surveys to examine students' attitudes, confidence, and engagement with mathematics. The next section summarizes students' comments and evaluation of their learning experiences with the IMIST unit. The final section examines emergent themes and considerations that arose during the implementation of the IMIST unit not directly related to the research questions.

Learning and achievement.

Summative assessment. The summative assessment or unit assessment was comprehensive and covered learning objectives and standards for the quadratics unit in Algebra 2 and the right triangle/trigonometry unit in geometry. Questions for the assessments were adapted from assessments given prior to this study. These assessments had equivalent content and had been vetted by the researcher and colleagues in previous Algebra 2 and geometry classes. Hailey, Bobcat, and Kaya studied two additional

Algebra 2 sections beyond the students in the small class. Therefore, their summative assessment included questions for the additional sections and had a greater total point score of 142 points as compared to 112 points on the assessment given to students in the small class. Howard's geometry assessment was worth a total of 118 points. The instructor recommended that Hailey, Bobcat, Kaya, and Howard set aside one and one-half hours to take the assessment, but they could take as much time as they needed. Time was not a constraint. However, students in the small class were constrained by time and completed the assessment in two parts over two class periods or roughly one hour and fifteen minutes. For all students, percentage scores for total points were low if compared to a one hundred-point, letter-grading scale. However, given a classroom grading scenario adjusting for time, completion, and grading scales, Hailey, Bobcat, and Kaya's scores would have been scaled by adding 20 percentage points. Scores for the students in the small class were scaled by adding 10 percentage points which correlated more closely to percentage achievement and letter grades. Unscaled scores are used for the purposes of discussion and achievement in the core literacies.

For students in the Algebra 2 paired case studies, Hailey did an outstanding job on her unit assessment, omitting one problem but attempting to answer all questions. Her scaled score (93.2%) set the bar for students who understood the material and showed a high level of mastery. Similarly, Brenda's score (93.2%) in the small class indicated the highest level of mastery on that assessment. Seven students fell in the range of high achievers (above 84%) – Hailey, Brenda, Chad, Don, Erin, Gloria, and Howard. This group mastered the material and would likely have passed at the advanced level on a state standards multiple-choice assessment. Five students passed at a standard level of achievement (67 to 78%), preparing them to move on to the next unit – Bobcat, Kaya, Abigail, Frank, and Jackie. These students attempted most of the assessment but either omitted questions or left sections blank. It is likely that this group of students would have passed proficient on a state standard multiple-choice assessment. Helen was the only student who did not demonstrate overall mastery of the unit content. Table 34 presents students' unscaled and scaled scores for the summative assessment.

Table 34

Summative Assessment Data for IMIST Units as Percentages of Total Points – Unscaled and Scaled

	Total Score Unscaled	**Total Score Scaled**
Student	**Percent**	**Percent**
Hailey	73.2	93.2
Bobcat	58.1	78.1
Kaya	54.3	74.5
Small Class		
Abigail	60.9	70.9
Brenda	83.2	93.2
Chad	74.4	84.4
Don	79.8	89.8
Erin	74.0	84.0
Frank	57.6	67.6
Gloria	76.5	86.5
Helen	29.0	39.0
Jackie	67.2	87.2
Geometry		
Howard	87.3	

Symbolic Literacy. Questions assessing mastery of symbolic literacies examined students' understanding of arithmetic skills, vocabulary, and Algebra 1 skills for solving equations using transformations. Examining unscaled scores on formative assessments (Quiz 1 and Quiz 2), six students showed strong mastery with scores ranging from 80 to

90% (Hailey, Brenda, Chad, Don, and Gloria). Brenda explained she had "already learned factoring" and "really enjoyed factoring." For Brenda, learning the vocabulary for imaginary numbers was new. Although Chad and Don showed good mastery of symbolic literacy, Chad did not think that the IMIST activities helped "improve my understanding of vocabulary, symbols, etc.," and neither did Don.

Five students showed mixed levels of mastery on symbolic literacy items with scores in the 70% range or having one high and one low score. These students included Bobcat, Abigail, Erin, Frank and Jackie. Frank explained, "The IMIST activities definitely helped me understand the symbols and vocabulary more than the textbook." Gloria said the IMIST unit "improved some understanding because it kept repeating, and it showed us examples." Jackie wrote, "I learned more vocabulary than I used to." Both Kaya and Helen showed a lack of proficiency on both quizzes scoring 30% to 40%.

Trends for the three assessments show some differences and patterns. Four students had relatively consistent scores over the three assessments. These included Brenda (88 to 91), Don (80 to 84), Erin (72 to 76), and Howard (83 to 88). Brenda, Don, and Howard demonstrated high levels of symbolic literacy on all three assessments, and Erin demonstrated moderate proficiency. There were seven students who did significantly better on the symbolic literacy questions in the formative assessments than on the unit assessment – Hailey (92 dropped to 80), Bobcat (90 dropped to 69), Abigail (70 dropped to 59), Chad (87 dropped to 78), Frank (86 dropped to 62), Gloria (93 dropped to 79), and Jackie (82 dropped to 74). Although Kaya's scores showed low mastery of symbolic literacy (46 to 54), she alone showed improvement over the three assessments. Helen's scores showed a downward trend from 41 to 27%.

Examining the data more carefully, drops in both Hailey's and Bobcat's scores came from points lost due to algebraic/arithmetic errors, not labeling/scaling their axes on their graphs, and in Bobcat's case, omitting a graph. Omitting the graph was significant since there were four graphs on the assessment. Most of these mistakes were careless errors rather than errors in understanding and probably come from lack of prior grading feedback since neither had much experience with formal assessments, rubrics, and expectations. Similarly, students in the small class whose scores dropped on the unit assessment missed symbolic literacy points in graphing (labeling/scaling axes) or missed points by omitting questions due to lack of time. Abigail, Frank, Helen, and Gloria missed between 5 and 12 points by omitting questions 14, 15, and 16 in part or in total. Jackie omitted parts of 15 and 16. Chad lost points for not labeling/scaling axes on graphs. The total points possible for symbolic literacy on the small class unit assessment were 43 points. Omitting 5 to 12 points represented a 12 to 28% drop in score for these students which was most likely not representative of their understanding. Their symbolic literacy scores may have been higher if they been given more time to complete the assessment.

Two students showed a lack of mastery of symbolic literacy, Kaya and Helen. Researcher's notes for class observations indicate early in the study that Kaya had trouble understanding vocabulary. Kaya explained, "It's really hard for me to remember things." The instructor made accommodations for her and spent extra time explaining vocabulary and directions. Kaya made some improvement in her symbolic literacy scores as a result. She acknowledged that the lessons and repetition helped her learn vocabulary. Helen's problem with symbolic literacy was a lack of arithmetic and factoring skills. Her lack of prerequisite knowledge contributed to her decline in performance throughout the unit. Table 35 presents assessment data (reported as percentages) for symbolic literacy over the three assessments.

Table 35

Assessment Data for Symbolic Literacy on Formative and Summative Assessments

	Symbolic Literacy (Percent)		
Student	**Quiz 1**	**Quiz 2**	**Unit**
Hailey	90.4	92.3	79.6
Bobcat	76.9	90.4	68.9
Kaya	46.2	48.1	54.4
Small Class			
Abigail	79.3	70.0	59.3
Brenda	91.4	88.0	90.7
Chad	86.2	86.0	77.9
Don	87.9	80.0	83.7
Erin	72.4	76.0	76.7
Frank	86.2	74.0	61.6
Gloria	93.1	90.0	79.1
Helen	41.4	32.0	26.7
Jackie	77.6	82.0	74.4
Geometry			
Howard	83.3	87.5	86.8

In summary, due to issues of timing and completion, the unit assessment may not have been representative of all Algebra 2 students' level of mastery of symbolic literacy. The scores on the formative assessments or quizzes show that all but two students reached proficiency in symbolic literacy.

Conceptual literacy. Questions assessing mastery of conceptual literacies examined students' understanding of graphing by transformations, understanding quadratic functions in the forms of graphs, tables, and equations as well as choosing and using methods to solve quadratic equations with procedural fluency. Conceptual literacy questions assessed content that built and extended knowledge gained in Algebra 1 or was entirely new. Almost all Algebra 2 students demonstrated an improvement in conceptual literacy from Quiz 1 to Quiz 2. Only Kaya, Don, and Howard showed a drop in performance. Howard's change in score was not significant since he dropped from 100% to 96%, a high level of mastery on both.

For Algebra 2, Quiz 1 assessed more advanced factoring skills which extended skills learned in Algebra 1. With the exception of Helen, students did well on those questions. The second part of the conceptual content on the quiz focused on graphing quadratics by transformations, identifying critical features, and expressing a quadratic function equation in all three forms. This was new content for most of the students. The learning objective included naming and describing the transformations applied to the parent function to graph the new function. All three students in the Algebra 2 individual case studies improved their scores when reassessed on the unit assessment, and the scores of seven of nine students in the small class stayed the same or improved on the unit assessment. Only two students (Jackie and Abigail who omitted the problem) did not improve their scores. Frank explained, “The [IMIST] activities greatly enhanced my knowledge of graphing and backed up my knowledge of tables.” Chad also thought the IMIST activities helped him learn because “I now know well of how to find vertex, axis of symmetry, and describing transformations.” Earlier, Chad had commented that the activities did not help him learn vocabulary and symbols. In this response, he demonstrated a knowledge of vocabulary when he wrote the vertex form of a quadratic function in symbols from looking at a graph. His responses were slightly contradictory and showed proficiency in both symbolic and conceptual literacy. Gloria explained that IMIST “helped me realize that you can use tables to find points on a graph if you don’t know how to find the points directly.” Frank made a significant link between the characteristics of parabolic graphs and the different forms of quadratic equations when he was able to interpret the standard form of an equation to find the axis of symmetry and the vertical dilation effect of the leading coefficient. Bobcat shared that the activities helped him understand the relationship between graphs, tables and equations because IMIST activities “seemed to make it easier,” and there was enough practice.

The conceptual literacy questions on Quiz 2 focused on solving equations using square roots and by completing the square. Again, with the exception of Kaya and Helen, students did well on the conceptual literacy questions. Five students were high achievers with scores above 84% (Hailey, Bobcat, Abigail, Brenda, and Jackie). Brenda wrote, “I learned more about quadratics because I now know how to solve a problem much quicker." Both her work on the assessments and her response illustrated her procedural fluency – being able to identify a solution method and work through it with accuracy. Hailey explained, "There was definitely problems where I was like, 'Yeah, I've got the hang of this.'" Bobcat said he learned about equation types and solving. Four students demonstrated moderate proficiency (Chad, Don, Erin, and Frank) with scores ranging from 68 to 82%. Frank wrote, "The IMIST activities helped with transforming equations from form to form." Jackie shared, "I now know multiple ways to solve an equation."

Scores on the Algebra 2 unit assessment for conceptual literacy were deceptive because of omissions or incomplete work. However, ten of the students demonstrated improvement and/or mastery of conceptual literacy content with evidence of making connections between multiple representations of quadratic functions and solving equations using different methods. Six students scored between 75 and 85% (Hailey, Brenda, Chad, Don, Erin, and Gloria), and four students scored between 60 and 74% (Bobcat, Abigail, Frank, and Jackie). Although, Kaya's scores improved from Quiz 2 to the unit assessment (10.7 to 48.3%), she still fell short of mastery as did Helen.

Although Howard's conceptual literacy scores declined over the three assessments, (100% to 96.4% to 85.2%), he still demonstrated mastery of the content. The questions required him to use correct theorems and principles to solve for different side lengths and angles of triangles and to justify his reasoning. On the unit assessment, he made an error in application of the Pythagorean converse used to classify triangles as right, acute, or obtuse when he did not check to see if the side lengths given formed a triangle by applying the Triangle Inequality Theorem. This cost him six points, and on review of the assessment he recognized that he “fell into the trap!” and did not check to make sure he was working with triangles. Howard also explained the he liked it when the activities “would say justify your equation or whatever with a theorem, and like, I just got familiar with uses.” He liked the “Think About” or reflection activities and thought that the questions were “helpful” to build his understanding. Table 36 summarizes students’ percentage scores for conceptual literacy over the three assessments.

Table 36

Assessment Data for Conceptual Literacy on Formative and Summative Assessments

	Conceptual Literacy (Percent)		
Student	**Quiz 1**	**Quiz 2**	**Unit**
Hailey	56.0	83.9	75.9
Bobcat	78.6	85.7	60.7
Kaya	69.0	10.7	48.3
Small Class			
Abigail	53.7	92.9	67.7
Brenda	83.3	100	82.3
Chad	59.3	82.1	81.5
Don	79.6	67.9	85.4
Erin	55.6	71.4	81.5
Frank	66.7	78.6	60.0
Gloria	88.9	100	84.6

Problem-solving. Although many problem-solving applications with students in Algebra 2 were done either in class or for homework, there were few problem-solving applications given on the assessments mostly due to assessment length and time constraints. Therefore, scores were not necessarily good indicators of problem-solving ability. Students' comments were better indicators.

The percentage scores on the application problems were low because of the low number of points awarded to each question (between 4 and 10 points). The paired and individual case study students had problem-solving applications on all assessments – one on Quiz 1, two on Quiz 2, and two on the unit assessment. Students in the small class had no problem-solving applications on Quiz 1 and only one question on both Quiz 2 and the unit assessment. Howard had problem-solving applications for geometry on Quiz 2 and on his unit assessment.

Because of differences in the assessments between those given to students in the paired and individual cases studies and the small class case study, students in the paired and individual case studies are the primary focus of discussion. Overall, the student with the most impressive improvement in problem-solving was Hailey from the individual case studies. She did not understand how to start work on the application question on the first quiz, scored 100% on the two questions on quiz two, and had the highest problem-solving score on the unit assessment given to students in the Algebra 2 individual case studies.

On the unit assessment, Hailey and Bobcat were given the choice to do one of two application questions. The first question was a story about projectile motion, and the second required data analysis and regression using data from a real-world context. Both students had worked through Section 4's TELA activity with the instructor but had trouble using, applying, and interpreting their results on the assessment. Hailey scored

80% – good proficiency, and Bobcat scored 30% – low proficiency. It is noteworthy that both chose to do the data analysis/calculator regression problem since neither had prior experience using the calculator as a statistical tool prior to the IMIST unit. Bobcat liked the application problems “to see where it was like used.” He explained, “I think I understood it a little better, but it was still like it was hard … Well, there was a lot more work, and it was more like complicated.” He also thought that his problem-solving strategies improved. Hailey found that the context of the application problems “like the rocket ones, helped me learn the vocabulary, symbols, like formulas.” Hailey’s response illustrates the valuable connection between real-world context and the meaning of symbols used in equation models for applications.

Kaya did not have a graphing calculator and had not learned how to do regression, so she was assessed on the projectile motion application and scored 7 of 8 points or 87.5%. Kaya showed good improvement from Quiz 1 to the unit assessment. Also, an examination of her written work showed good problem-solving strategies. Once, she understood the meaning of the parameters in the projectile-motion model, she knew how to interpret the problem. However, during her post-intervention interview, she stated, “I am really bad with word problems,” and “It’s just with word problems, if I don’t know like in a sense what they’re asking me to do and how they’re asking me to use the problems … like, I’ll have no idea.” Although the assessment data shows that she consistently used a variety of problem-solving strategies, her post-intervention interview responses reflected her continued lack of confidence in her problem-solving abilities.

Quiz 2 provided the equation for the dropping object model. Students had to use the contextual information in the stories appropriately to answer the questions. Hailey and Bobcat did well on these problems. Bobcat lost points on both because he did not interpret the answer in context. Six of the students in the small class received full credit for the problem on Quiz 2. The three students who lost points (Frank, Helen, and Jackie) made the same error as Bobcat and did not interpret the answer correctly. All of the Algebra 2 students showed problem-solving proficiency for this application.

On the unit assessment, students in the small class case study were presented with a scenario that required a projectile motion model similar to Kaya's. Unlike Quiz 2, the symbolic equation for projectile motion was not provided on the assessment. Students needed to use information from the story to write the model and then use the equation to answer questions. As described in the small class case study, all students except for Jackie, who omitted the problem, were able to develop an equation that modeled the projectile motion giving them a base score of 18.2%. This demonstrated a basic or proficient knowledge of projectile motion. Most did not complete the problem due to a lack of time but received partial credit based on the work they did interpreting questions on the assessment. Brenda did the best interpretation of the question but did not set-up the correct equation model, so she lost points for that portion of the problem.

Students' comments provided evidence of value, learning, and an understanding of problem-solving. Abigail wrote that the IMIST unit "showed me how math is used and shown in real-world situations." Gloria felt she "learned a bit more because they included word problems that apply to everyday life, which I can use in the future." She also shared that “some questions made you think harder to get the answer.” Chad wrote that he is “much better at the word problems than I used to be.” Brenda wrote, “I learned how to better read equations, and how knowing what an equation means can help you solve problems.” The most notable quote came from Frank who explained during class, “I never thought … never realized that the math we did in school … could actually be used … I mean like in the real-world.” Not all students felt they had improved in problem-solving. Erin explained, “I didn’t ever really understand the problem-solving stuff.”

Howard did an exceptional job on the problem-solving applications that used right-triangle trigonometry. His only error was an error in reading and interpretation. When he was asked if the IMIST unit activities improved his problem-solving, he replied, “I think so ’cause like before I was doing math, and I didn’t really know why I was doing it, and then like, this showed me the … real-world applications and stuff.” “Maybe it’s just like seeing math in more things.”

Summary. In summary, four students showed high levels of mastery on all three assessments consistently scoring above 80% (Brenda, Chad, Gloria, and Howard). Two students showed improvement over the three assessments. Hailey showed significant improvement starting at 64.4% with no problem-solving skills and finishing with 93.2% on the unit assessment. Erin also showed an improving trend starting with 64.3% and finishing with 84.0%.

Five students had mixed results on the assessments but demonstrated moderate proficiency over all assessments (Bobcat, Kaya, Abigail, Don, and Jackie). Two students did not demonstrate consistent mastery of the content (Frank and Helen). Although Frank did moderately well on the first two quizzes, his unit assessment scaled score was low at 67.6%. As noted in the case study, this was due to absence and his inability to catch up/make-up work independently. Helen consistently performed poorly on all assessments. This unit served as a diagnostic for her and highlighted skills that needed remediation or that she did not learn in Algebra 1.

Attitudes, perceptions, confidence, and engagement. To investigate the second research question, data collected during the case studies to assess students' attitudes, perceptions, confidence, and engagement included class observations, homework effort grades, and the Math Attitudes and Perceptions Survey (MAPS, Code et al., 2016). In addition, data were collected from students in the paired and individual cases studies during the pre- and post-intervention interviews as well as written responses from student participants in the small class on the post-intervention survey. To examine students' engagement and effort, magnitude codes from data collected in the researcher's observational notes and homework effort scores were analyzed. The researcher used MAPS to examine students' attitudes and perceptions about mathematics in general and personal beliefs by placing students into matrix categories with ranges such as high, medium, and low.

Class engagement and work ethic. The researcher/instructor kept notes on class participation for both online and face-to-face classes in her researcher journal and annotated notes. Initially, she used plus signs (positive/active) and minus signs (negative/passive) or a combination to indicate the level of engagement and class participation for each student. She also kept a record of individual student responses and observations. She used up and down arrows to indicate improving or declining attitudes or engagement during implementation of the IMIST unit. She used magnitude coding to interpret the level of engagement on a scale of 1 to 5 to quantify the level of engagement for comparison between students. A score of “1” indicated negative attitude and/or low engagement. A score of three indicated a moderate level of class engagement or activity. For example, a score of “3” was given to a student who did not raise his or her hand to answer questions but would answer when called upon. A score of “5” indicated a high level of engagement and a willingness to volunteer answers and participate in class discussion. The scores for the students in the individual case studies is a little deceptive because the instructor was constantly asking questions one-on-one, and the students were on the spot to answer.

Five students were highly engaged in class discussion (Hailey, Kaya, Howard, Frank, and Gloria). Hailey and Kaya were always quick to answer questions or to ask questions when they did not understand. Both took charge of their own learning and understanding. Howard did an excellent job in discussing his work and understanding. Frank readily volunteered answers and insights. As mentioned in the small class case study, two students in the small class improved in both attitude and engagement over the intervention – Erin and Brenda. Erin was highly engaged in class discussion. Most of her early comments were complaints, negative, and whining. However, by the end of the unit, her contributions were more constructive as she gained understanding. She also helped her study partner by explaining what she had learned or understood. Brenda was very vocal at the beginning of the unit about her concerns with the amount of work and not understanding, but she diligently did all the assigned work and investigations. Her work and good assessments scores led to a significant change in her attitude in class. Further evidence of her change in attitude came during the discussion for Section 3 on solving quadratic equations. Both Brenda and Gloria were leaders in the discussion, volunteering answers and explanations. Five students were moderately engaged in class discussion and would answer when called upon (Bobcat, Abigail, Chad, Don, and Jackie). Helen showed declining participation and engagement in class.

As discussed in the small class case study, the instructor did a daily homework check and assigned students an effort grade for completion. The same markings or codes that were used for class participation were used to indicate homework effort. This was particularly useful for the online students in the individual case study who were expected to do their own homework and to self-check against answer sheets and notes. Again, a scale of 1 to 5 was used as magnitude coding to quantify the level of effort. For the students in the small class, their percentage effort grade was divided by twenty and rounded to the nearest whole number plus or minus 0.5. The trends in the homework grades reflect students' willingness to work and indirectly their attitudes.

Five students (Howard, Abigail, Brenda, Chad and Jackie) showed excellent homework effort. Hailey and Erin showed improvement in their homework effort especially when they realized they needed to do the homework to understand the concepts, particularly graphing and graphing by transformations. Hailey explained that she "always used a solutions sheet, but I'm not cheating or anything … I'd only check my problems." Kaya, Frank, and Helen all struggled with homework and completion.

Kaya had trouble reading and understanding the homework. She felt it was a consequence of her dyslexia. She could not always remember what had been done in class and had not taken good enough notes. It helped her to have the annotated notes emailed to her as a review resource to help with homework, and she always checked her answers using the solution sheets. Frank struggled with homework due to his absence. He was unable to make up all the work. Helen always made an attempt at the homework, but it was often incomplete. Bobcat made very little effort to do homework and practice. He had taken the course before. This proved to be a problem when he was introduced to new material as part of the honors level content in the IMIST unit activities. He was not used to doing homework and did not do enough practice to master new concepts.

Overall, four students were highly engaged in class and had excellent homework effort using the IMIST unit activities (combined scores of 9 or 10). These included Hailey, Howard, Brenda, and Gloria. Seven students showed positive engagement and homework effort (combined scores of 7 or 8). These were Kaya, Abigail, Chad, Don, Erin, Frank, and Jackie. Helen continued to struggle, and, although she asked questions, the instructor was unable to provide enough explanation to meet her individual needs and to remediate her lack of content knowledge. She would have benefitted from seeking extra help offered outside of class. Bobcat had taken the class before and had the lowest score for effort and engagement. Table 37 summarizes observations of class participation or engagement and students' homework effort.

Table 37

Data Collected for Students' Engagement and Effort from Class Observations and Unit Achievement

	Class Observations					
	Class Participation		Homework Effort		TOTAL	Unit Assessment
Range		1-5		1-5	2-10	Percentage
Student						
Hailey	+	5	±, ↑	4	9	93.2
Bobcat	±	3	–	1	4	78.1
Kaya	+	5	±	3	8	74.5
Howard	+	5	+	5	10	87.3
Abigail	±	3	+	5	8	70.9
Brenda	±, ↑	4	+	5	9	93.2
Chad	±	3	+	5	8	84.4
Don	±	3	±	4	7	89.8
Erin	±, ↑	4	±	4	8	84.0
Frank	+	5	±	3.5	8	67.6
Gloria	+	5	+	4	9	86.5
Helen	±, ↓	2	±,↓	4.5	6.5	39.0
Jackie	±	3	+	5	8	87.2

Math attitudes and perceptions survey. The next data source for students' attitudes, beliefs, work ethic/perseverance, and confidence came from their responses to MAPS. The time frame for the IMIST implementations was approximately four weeks for all cases, so MAPS served as a single measurement or "snapshot" of students' attitudes and perceptions. Given the short implementation time and the small number of students, MAPS cannot be used to correlate changes in attitudes to experiences with the IMIST units. Student responses on MAPS gave evidence to support their attitudes about mathematics and served to validate class observations and students' comments. Students in individual case studies were given MAPS before implementation of the IMIST unit, and students in the small class case study were given MAPS after. Categorized scores reported on MAPS placed students into ranges of high, medium, and low for their general beliefs, beliefs about themselves as math students, their work ethic and confidence. Most students' classifications matched with their overall achievement scores with the exception of work ethic and confidence. Some students' responses placed them higher in the work ethic category than what was observed in class, and other students placed themselves lower in the confidence category than they demonstrated on assessments.

Hailey was the only student whose scores placed her in the highest range for all attitudes and beliefs categories. Her scores were well-above the next student in attitudes and personal beliefs about real-world/math and confidence. Her results were consistent with her assessment grades and the researcher's class observations of her high level of participation and engagement.

Three students (Howard, Don, and Gloria) placed medium to high in all categories. Howard and Gloria's performance on assessments and classroom observations were consistent with their high levels of participation and engagement. Abigail's responses placed her in the medium range for all categories. This was consistent with her assessment performance, her work ethic, and thoughtful comments on the post-intervention survey.

Four students (Frank, Brenda, Erin, and Bobcat) had mixed responses placing them in all response levels, or medium to low depending on the category. Frank's scores placed him in the highest level for Work Ethic/Perseverance which was inconsistent with his homework effort score. Bobcat was in the low range for all categories but Work Ethic/Perseverance in which he placed in the medium range. The researcher observed his lack of effort for the last three sections of the unit which corresponded to his decline in assessment scores and was inconsistent with his personal beliefs about himself as a math student.

Three students (Kaya, Helen, and Jackie) had scores that placed them in the lowest range for all categories. These scores were consistent with achievement grades for Kaya and Helen. However, for Jackie, the scores did not correspond to her assessment scores nor the level of effort observed by the researcher. Jackie was one of two students who did one of the two the extra credit assignments and was very detailed and thoughtful in her homework as well as her written responses to the post-intervention survey.

The category that did not seem consistent with the researcher's class observations and homework effort scores was "Work Ethic/Perseverance." Don's and Frank's scores placed them in the high range although their homework effort scores were low. Don and Frank perceived themselves to be hard workers, but the evidence was not observed or recorded. Because Don's achievement was high, it is possible that he did study or did work outside of class that was unmeasurable. Overall, with the exception of work ethic, the MAPS data seemed consistent with assessment data and achievement which supported the researcher's observations about students' attitudes, participation, and engagement in class.

Hailey outscored her peers in confidence scoring 24 out of 25 points. Six students scored in the medium range for confidence (in descending order: Howard, Don, Gloria, Frank, Abigail, and Erin). Howard scored 20 points which meant that he agreed (4) with all five statements which put him in the medium to high range in confidence. Five students scored in the low range for confidence (in descending order: Bobcat, Jackie, Kaya, Brenda, and Helen).

Hailey explained that she has a high confidence in her ability to figure math problems out on her own, and that she knows she can get help from her dad or from online videos. In her pre-intervention interview she explained, "I really liked [Geometry]. I thought it was really fun." She has a good work ethic, and "I practice a lot." Howard's favorite thing about math is "Everything!" He likes it all. Howard also knows that if he does not understand a concept, he can ask his mom or go to an online resource. Both Hailey and Howard take the initiative to understand mathematical concepts completely before they move on. They are also in control of their own learning and pacing as independent homeschooled students.

Common In Vivo code responses for students who fell in the medium to low confidence group were feeling "lost", "confused," and "not understanding." They also wanted/needed more time for "explanation" and felt that the pacing of the class was "too fast." A significant observation about this group, with the exception of Kaya and Bobcat, was that they had the opportunity to come in for extra help, or to exhibit “help seeking behavior” (Skaalvik, Federici, & Klassen, 2015). The instructor was available for 45 minutes every day for student assistance outside of class. Their lack of confidence could be attributed to not fully understanding the content when facing an assessment. Jackie articulated this well when she wrote, “I still feel like I don’t completely understand parabolas and quadratic equations, and I find myself struggling on most test(s).” Helen wrote, “Everything that was ‘explained’ was not fully explained.” Erin wrote, “I was often confused and never really got the hang of any units. I understood the gist, but not all.” When the students were asked what they could do to improve their learning in mathematics, the assumption being that improving learning would improve confidence, they responded that they could do more homework practice, read the textbook, ask questions, study more, and spend more time on mathematics. Although many of these students wrote that the IMIST unit activities improved their understanding of symbols, concepts, and problem-solving, the short unit intervention of four weeks was only one unit of instruction and did not impact students’ confidence in their ability to do math.

Bobcat’s low score in confidence was consistent with his medium to low attitude about math. Bobcat said, “Math is not fun.” Although, he feels that he “is pretty good at it … because I get most of the answers right.” Bobcat has come up against concepts or procedures that he cannot do and will ask his mother for help or look for online videos. But he just quits if it “takes too long.” Bobcat’s lack of effort to master learning and to seek out answers to concepts he does not understand may affect his confidence in learning mathematics. Even though Kaya uses additional support such as online videos or her math dictionary, her lack of confidence is directly related to her disability, and she was very candid about her lack of confidence because she has “trouble remembering” and is constantly working hard to memorize, use sticky notes, and to understand. She also mentioned that she needs constant review and “refreshers” to keep understanding. The researcher believed that Kaya was a better math student than she thinks she was because of her work ethic and diligence. She just finds math “something I think that’s really hard.”

Students’ were placed into categories and ranges (high, medium, low) based on their responses to the 32 questions on the MAPS (See Appendix C for all questions and classification categories.). MAPS data is presented in Table 38. The column at the left describes the range for student responses (high, medium, low), and the ranges in the row represent lower and upper values of students’ scores within each range. The lower bound of medium range was the number of questions in the category multiplied by three which corresponded to answers that were undecided or above (agree/strongly agree). Students are listed by category and range into which their score fell. The last column presents students’ scores for confidence. Confidence and self-efficacy were the least observable in class but were expressed to some extent in student interviews and surveys. Each of the five questions on the MAPS specifically addressed confidence in student learning, understanding, or work effort. Table 38 summarizes students’ scores for each set of categorical questions on the MAPS and places them in that category by range (high, medium, and low).

Table 38

MAPS Data – Students Placed by Category and Range.

	Total	Attitudes	Learning	Work Ethic Perseverance	Real-world	Confidence
# Questions	31	8	16	4	3	5
Range	31-155	8-40	16-80	4-20	3-15	5-25
High	110-131	39	58-71	15-16	14	24
	Howard Hailey Don	Hailey	Howard Hailey Don	Hailey Gloria Don Frank	Hailey	Hailey
Medium	90-105	28-32	48-50	12-13	9-11	15-20
	Gloria Frank Abigail Erin Brenda	Howard Brenda Gloria Erin Abigail Don	Frank Abigail Gloria	Abigail Bobcat Howard Erin	Brenda Jackie Howard Frank Abigail Don Gloria	Howard Don Gloria Frank Abigail Erin
Low	82-88	25-26	33-47	10-11	6-8	6-14
	Jackie Bobcat Kaya Helen	Jackie Frank Helen Kaya Bobcat	Erin Bobcat Jackie Brenda Helen Kaya	Jackie Kaya Brenda Hellen	Kaya Helen Erin Bobcat	Bobcat Jackie Kaya Brenda Helen

* Chad did not complete the second page of the MAPS, so his results are not included in the table.

Students' experiences and evaluation of IMIST. Post-intervention interviews and surveys collected students' reports on their experiences using the IMIST unit activities. As a phase of a larger design research project, student users were asked to review, evaluate, and give feedback about the IMIST unit activities and their learning experiences.

Unit structure, lessons, and materials. All students received a packet of IMIST unit activities that included investigations, lessons-learned summaries, class notes, and practice worksheets. Over the three Algebra 2 implementations, the packet of materials was edited and changed based on feedback from students in the previous implementation. Hailey and Bobcat used the first draft of the IMIST unit materials, and both found it a little bit "confusing" to follow. Part of this was due to meeting twice per week rather than daily. Bobcat found it hard to distinguish between the investigations, "class notes, and all that stuff." Hailey was confused "about like … the homework … like I got kinda confused about which parts we're doing 'cause some of the labeling was a little confusing." The instructor acknowledged Hailey's and Bobcat's confusion and was careful to create an assignment sheet that documented class activities and homework activities as the unit progressed. As a result of this feedback, the instructor made a conscious effort to clearly communicate what the assignments and activities were in subsequent implementations.

Kaya and Howard both liked the unit organization. Kaya liked "that portion of it being organized, but in reference to the packet she said, "I felt like it was a bit hard to follow at times." She suggested that the pages should be numbered sequentially at the bottom rather than by section. Howard commented, "I liked how it's structured. Well, better … better than I was doing math before." Abigail liked the unit structure because "each topic was built off of the one covered before," and they "made me more organized, and they were easy to find." Brenda explained, "It helped me learn because we did many of the same problems, which helped to reinforce my understanding of the topic." Chad explained that "Most things made sense because it was a lot of thinking work."

Students who did not like the unit structure described their experiences using words like "confused," "too much paper", or a lack of "explanations." Confusion, organization, and explanation were common In Vivo codes that emerged in the post-intervention interviews and surveys. Don explained, "I was always lost or confused." Erin said there was "too much paper and assignments." Gloria explained, "Some problems didn't make sense because of the wording, and I didn't know the vocabulary." Helen explained that "not everything made sense because it wasn't fully explained."

Students gave more favorable feedback about the IMIST unit's online or classroom lessons with the instructor. Nine students liked the lessons. One student was mixed in her feeling about the lessons, and three students did not like the lessons. Online students all shared that they liked being able to ask questions and get explanations from an instructor in real-time. In their prior curricula, they were doing independent study or watching video lessons. They explained that having a face-to-face instructor provided them with a better learning experience. Bobcat explained that he liked online lessons "'cause I could ask questions." Hailey said, "I think that it was nice because I was able to go over what I had gotten wrong, and you were able to explain it better." Kaya shared, "They were very helpful," and "I liked getting to work face-to-face with someone." Although, Howard did not work online he liked working "one-on-one." "You go at your own pace and like the teacher asks questions." Howard liked discussion and answering questions.

Similar to the comments from online students, students in the small class who liked the lessons appreciated explanations and being able to ask questions. Abigail explained that the lessons "explained how things worked," and Brenda explained that she learned to "fix my mistakes and further understand the topic." Chad thought the lessons showed "me how to solve/simplify a problem very thoroughly." Gloria brought up the issue of pacing. "The lessons were a little too fast for me, so it was hard to keep up, but when I started doing the problems, I was fine." Criticisms of the lessons focused on pacing or the amount of information. Students wrote that they felt "rushed," that there was "too much information," or that the "pace was a little fast" or "too fast for me." Don wanted the instructor to "teach more depth" which may have meant more explanations, a sentiment shared by Gloria, Helen, and Jackie.

Eight students thought that the worksheets and practice assignments were helpful, and five gave a mixed review. No students thought that the worksheets and practice were not helpful to their learning. Hailey thought the worksheets were good for review. She said, "There were like parts that I would like go back to." Both Kaya and Howard though there was the right amount of practice and review. When asked about the level of the exercises, Howard explained, "They were hard, but I like … I like hard." Students in the small class who thought that the worksheets were helpful agreed that there was enough practice and review. Gloria explained that the practice helped improve her understanding because of repetition, and it "showed us examples, and "there was a lot of problems to practice and some questions made you think harder to get the answer." Chad liked the worksheets because they showed "the many scenarios for a certain problem." Frank specifically referenced the investigations in Sections 1 and 2. He said they “helped with transforming equations from form to form.” Don, who was negative about almost everything in the unit, found the worksheets “helpful” but they “took up too much space and were hard to keep track of.”

Students who were mixed in their reviews found the number of worksheets “overwhelming” or “complicated.” Students who were mixed in their responses also felt “confused” or “needed more explanation.” Table 39 shows six students who liked the unit structure, three who felt mixed about the unit structure, and four who did not like the unit structure and summarizes students’ responses to questions about the IMIST unit structure, lessons, and practice materials.

Table 39

Summary of Student Responses to Learning with IMIST – Unit Structure, Activities, Lessons

Question	**Yes**	**Mixed**	**No**
3. Did you like the unit structure? Did it help you learn?	Kaya Howard Abigail Brenda Chad Frank	Hailey Bobcat Gloria	Don Erin Helen Jackie
8. Were the online/classroom lessons helpful?	Hailey Bobcat Kaya Howard Abigail Brenda Chad Frank Gloria	Erin	Don Helen Jackie
9. Were the worksheets/practice helpful?	Hailey Bobcat Kaya Howard Brenda Chad Don Gloria	Abigail Erin Frank Helen Jackie	

Learning, activities, and literacies. Students provided responses to questions concerning their learning with IMIST unit activities, whether they liked learning with IMIST or not, and whether they felt that they learned more or less during the IMIST unit as compared to their learning experiences with previous curricula. Students also shared how they felt the IMIST unit impacted their learning in the core literacies. Students were asked to describe their favorite experience learning with the IMIST unit and whether they experienced any deeper understandings, connections, or aha moments using IMIST unit activities.

When asked whether they liked using the IMIST unit learning activities, the students' responses were the most varied. Three students liked learning using the IMIST unit; five were mixed; and four did not. Brenda liked learning using the IMIST unit activities, because it was "easier to stay organized and do better on the quizzes." Gloria wrote that the IMIST unit "was a little hard to understand at first, but once I got it, it was easier."

Bobcat's feelings were mixed as he explained, "They were kind of confusing." Bobcat had not used a graphing calculator before, so both TELA investigations were difficult for him because he had not learned "calculator skills." Hailey explained, "I had some frustration with it." "It was a lot different from the normal learning that I do" and "I kinda struggled to complete a lot of the problems." "I was like frustrated … which I think … is because … I was pretty new to it." Kaya explained that the activities were "kind of like fast." Frank was also mixed in his feelings about the IMIST unit. He found the amount of work in the packets "too much," but the investigations "help me to understand certain topics." Don also felt mixed. He responded that he did not like the IMIST unit but that "the teacher talked a little bit more about it," meaning that the instructor did provide more discussion or explanation than he had experienced before. Criticisms included Abigail's explanation, "I am left not completely understanding a couple of topics." Brenda found "a lot of it as extra and some things seemed unnecessary." Don cited the “overuse of worksheets and packets.” Erin found the IMIST unit “confusing, too fast, and relatively disorganized.”

When asked if they learned more or less with the IMIST unit activities, six thought they learned more; three said that they learned about the same; and four said that they learned less. Both Don and Bobcat shared they learned less because they had had the unit before. Hailey explained that she learned more and that the IMIST unit was “definitely harder” than how she studied math before and took more time. Kaya learned more explaining, “I feel like it was so compacted. Like I feel like it was kinda like, ‘Let me show you all of … Algebra 2 in like two months!’” Brenda felt she had learned more. “I could not only do problems, but I could understand them.” Chad said, “I learned a lot more because the course was much more rigorous and required more work.” Gloria explained, “I learned a bit more because they included word problems that apply to everyday life, which I can use in the future.”

When asked if he learned more or less with the IMIST unit, Howard was honest, “I don’t know ’cause like I can’t really tell.” Other students who felt they learned less again cited confusion and not understanding. Don felt he learned less “because I always got confused, and then we moved to a more complicated subject,” and “All of this is new stuff I’ve never learned before.” Erin explained, “I was often confused and never really got the hang of any units. I understood the gist, but not all.” She also stated, “I spent a lot of time teaching myself how to do things.” Jackie felt that “I did not have a good enough understanding in the beginning and that accumulated over time.” Abigail felt mixed. “I feel like we went over many topics, but I didn't understand some of them." Helen stated, "I felt like I learned nothing during this unit."

When looking at student responses about learning the core literacies using the IMIST unit activities, with the exception of Kaya and problem-solving, the students were either positive or negative. For symbolic literacy and conceptual literacy, nine students felt that the IMIST unit activities improved their learning and understanding and four did not. For problem-solving, eight students felt the IMIST unit activities improved their problem-solving strategies. Kaya was mixed, and three did not feel they learned problem-solving strategies.

For symbolic literacy, Hailey felt the real-world problems helped her understand the vocabulary. For her, the contexts helped the numbers and equations make sense. Brenda agreed when she wrote, "I learned more about the formulas because now I understand what each part of the formula means and is used for." Kaya explained she learned a lot more vocabulary with the IMIST unit activities than her other program. Frank also explained he learned more about "symbols and vocabulary than the textbook." Howard and Gloria felt the practice, examples, and repetition helped them learn. Don and Bobcat both had studied quadratics before and felt they already knew "some definitions from before."

For conceptual literacy, Hailey, Bobcat, Kaya, Chad, Frank and Gloria thought they learned more about graphing than they had learned before. Bobcat shared that graphing by transformations "seemed to make it easier" and that there was enough practice. Hailey explained about graphing, “I hadn’t been introduced at all to it before.” “I think I learned it pretty well.” Kaya shared, “I hadn’t done any graphing before.”

Kaya also shared that when it came to solving quadratic equations, she felt “like I understood the reason more … Just understanding like the reasons, and how it fits in … In a sense to map as a whole was kinda neat.” Brenda shared, “I learned more about quadratics because I now know how to solve a problem much quicker.”

For problem-solving, Bobcat, Chad, and Gloria felt they did better on word problems after the IMIST unit. Bobcat liked “to see where it was like was used … I think I understood it a little better.” Chad also wrote that he is “much better at the word problems than I used to be.” Gloria wrote, “It made me realize that are other ways to solve problems.” Howard felt better about doing proofs and thought problem-solving with geometry was fun because drawing pictures, “It’s fun.”

Kaya was mixed in her feelings about problem-solving and the IMIST unit. Her responses were all about confidence. She was “afraid” that if she does not know the “formula” she cannot do the word problem. She feels that part of this is because of her reading and comprehension. Table 40 summarizes students’ responses to learning with the IMIST unit learning activities.

Table 40

Summary of Student Responses to Learning with IMIST – Learning, Activities, and Literacies

Question	Yes	Mixed	No
1. Did you like learning using the IMIST activities?	Brenda Gloria Jackie	Hailey Bobcat Kaya Frank Helen	Abigail Chad Don Erin
12. Do you feel like you learned more or less with the IMIST unit activities than you have before?	Hailey Kaya Brenda Chad Frank Gloria	Howard Abigail Helen	Bobcat Don Erin Jackie
4. Did the IMIST activities improve your understanding of vocabulary, symbols, and symbolic literacies?	Hailey Kaya Howard Abigail Brenda Erin Frank Gloria Jackie		Bobcat Chad Don Helen
5. Did the IMIST activities improve your understanding of graphing, tables, equations, and your conceptual literacy?	Hailey Bobcat Kaya Howard Abigail Brenda Chad Frank Gloria		Don Erin Helen Jackie
6. Did the IMIST activities improve your problem-solving strategies?	Hailey Bobcat Howard Abigail Brenda Chad Frank Gloria Jackie	Kaya	Don Erin Helen

Deeper understandings and connections. The IMIST unit activities were designed to support specific learning objectives and goals. They were designed to help students gain deeper understandings and meanings of mathematical concepts. Students articulated such learning moments on additional questions in the post-intervention interviews and surveys. When asked what their favorite part was during the IMIST unit and if they experienced an *aha* moment when they gained a significant insight in math using the IMIST unit activities, Frank remarked after the first investigation he realized that the math being taught in school actually had real-world or outside-of-school applications. Bobcat and Kaya explained that their favorite thing about the IMIST unit was talking, asking questions, and getting to work directly with the instructor during the online lessons. The discussions and lessons-learned summaries helped them understand concepts better. Brenda, Gloria, and Jackie described that they gained confidence in their ability to factor quadratics, a symbolic literacy. Factoring was introduced in Algebra 1, and they really felt they mastered the process doing the IMIST unit.

Other students described moments when they made deeper conceptual literacy connections. Chad's favorite thing was learning to describe and apply transformations to graphs. Gloria had an *aha* moment about transformations when "I realized where you could find the different translations and dilations in vertex form." Frank and Gloria gained insights into procedures and conceptual literacy as they liked the focus on parabolas and every way to solve them including completing the square and factoring. Brenda mentioned having an *aha* moment in conceptual literacy when she understood how the discriminant "found how many and what kinds of roots there were for the equation." Both Frank and Jackie had *aha* moments with complex and imaginary numbers. Frank wrote, "I really had a click," and Jackie wrote, "I had a moment where I finally understood it." Abigail's favorite thing was related to problem-solving. She liked "understanding the formulas" and equations.

Emergent themes and considerations. To evaluate iterations of a prototype phase of a design research project, feedback provides useful insights into considerations and improvements that can be made in future implementations. Over the course of all four implementations, many issues and considerations became evident. Students' made comments, recommendations, and criticisms of the IMIST unit including pacing, workload, keeping track of assignments, and the amount of content or information covered in the packets. These issues are products of the educational environment: the level of the course and associated learning standards; the workload associated with reform-based, student-centered activities; and technology or calculator use.

Educational environment. There were different types of educational environments for the four implementations of IMIST: (a) the online lesson environment with two students with one and one-half hour virtual classes twice per week; (b) the online lesson environment with one student and one and one-half hour virtual classes twice per week; (c) the one-on-one instructor/student, face-to-face lessons that met for one to one and one half hours weekly; and (d) a small classroom setting in a school which met daily for 45-minute periods.

On the issue of pacing, the online and face-to-face environments for individual case studies were very flexible. The instructor was able to adjust the pacing of the course to fit the needs of the students. Hailey, Bobcat, and Howard did not make any comments about pacing. Kaya made one comment that a couple of the sections went "a little fast." In contrast, students in the small class referenced the pacing of the course seven times as "too fast" or "needed to be slower." The pacing and content for the course was determined by the mathematics department and the Algebra 2 teaching team, and the instructor had to accommodate the timeline she was given. Although, students in the small class did not complete two of the quadratics sections that were completed by the students in the individual case studies, students still made comments about pacing. Assessments also limited the amount of instructional time in the small class case study. Students in the individual case studies took all the assessments on their own, outside of class; whereas, time had to be allotted in class for students in the small class case study to take assessments.

Related to pacing were the students' comments about wanting more time for "explanation." Again, these concerns came from students in the small class. The instructor was constrained by the department schedule and 45-minute periods. The department's pacing schedule was set at the beginning of the year. The schedule was a departmental expectation and had been used in past years. Another consideration about pacing and time for explanation relates to the work the students were doing to prepare for class and to keep up with assignments. This was a class of freshmen, and they were experiencing their first high school math class which had higher expectations and a faster pace than what they had done in middle school. By contrast, because time was flexible in the individual case studies, the instructor could spend as much time on explanation as needed. Some online sessions were one and one-half hours which allotted enough time for explanations, review, discussions, and new topics. Students felt they had enough explanation.

Another issue to consider about the educational environment is identify what is being taught – course level and corresponding learning standards. The ideal is to meet students "where they are" in their learning of mathematics – to assess readiness, review prerequisite knowledge, and to teach content standards at the appropriate level. Learning objectives and content for the IMIST units covered the Algebra 2 learning standards for quadratics and the geometry learning standards for right triangles and trigonometry at the depth and breadth of an honors level course. Students in the small class case study did not cover the last two sections of the IMIST unit – one on applications and the second on inequalities and systems – and were only assessed on applications of motion – dropping objects and projectile motion. Units taught to the Algebra 2 student participants in the individual case studies included additional sections and content and were taught and assessed at an honors level with more applications. Both Hailey and Bobcat used the word "hard" to describe some of the topics they had to learn. However, both classified themselves as accelerated, honors students so the content would have been appropriate for that level. Their prior curricula were not at the honors level, so they found the unit content and assessments challenging. In fairness to Kaya, she was given the same level of honors content but probably should have been taught at a standard level of challenge to accommodate her learning disability. The last two sections were very challenging for her. Her attitude, willingness to work, and to do the best she could were admirable. Howard, as the youngest student, was well prepared to handle a challenging, honors level course as indicated by his enthusiastic comments and his high assessment scores. When asked if the students would like to use IMIST units to study math in the future, both Hailey and Bobcat said “no,” but both Kaya and Howard said, “yes.” Hailey and Bobcat felt the unit activities required too much work. In contrast, Kaya expected to “work hard” because of her disability, and she felt she learned more and understood more with the IMIST unit. Howard’s willingness to do more IMIST units was no surprise. He liked “hard.”

The content and standards for students in the small class case study were set by the school and were driven by the standard level content in the school’s textbook. The content, exercises and examples did not include many fractions or decimals and reinforced arithmetic skills learned in prior classes. The IMIST unit activities were designed to complement these standards and to provide student-centered learning activities and investigations to promote better learning. The researcher believes students’ issues with content, learning, and explanations were a function of their preparation, prerequisite knowledge, and their learning experiences in middle school. Students commented they gained a mastery of factoring and graphing which extended Algebra 1 content knowledge. The more serious concern was Helen who was placed in the class having taken Algebra 1 the previous year, but the content and level taught in her middle school class did not prepare her for Algebra 2 in high school.

Workload and reform-based, student-centered activities. All students, with the exception of Kaya and Howard, made comments about the workload and the packets associated with the IMIST unit activities. Students were accustomed to traditional mathematics instruction at a standard level. They were not used to engaging in mathematical activities and projects. They were used to lectures and guided instruction. The IMIST unit activities required students to work differently and to spend time "thinking harder." This represented a change from what they had done before. As a result, students made eleven references to "too much information" or "long packets" in their comments. Students did not connect doing the work with learning. Their comments suggested they did not value discovery learning and working mathematically even though nine of the thirteen students made comments that stated that the IMIST learning activities improved their learning of the three core literacies.

Technology and support. Another issue that emerged during the implementation of the IMIST units was one of technology – Internet use and calculators. All four of the students in the independent case studies had experience using the Internet to research topics and to look for video and online support of their learning. Hailey even recommended that the instructor make her annotated notes available and provide a list of support videos to accompany the IMIST unit. The students in the small class were not accustomed to using the internet as a resource and struggled with the first investigation. If the instructor had modeled an Internet search, more students may have been willing to engage with the activity.

Calculator experience was another issue. Students in the individual case studies did not have experience using a graphing calculator as an exploratory tool or for data analysis. Even though Hailey's textbook had calculator activities, she never felt she needed to do them. The instructor spent time during two online classes demonstrating how to graph functions with the calculator and how to run regressions. Both of these activities are usually presented as part of Algebra 1 standards but were introductory to Hailey and Bobcat. Their lack of experience made TELA activities "frustrating" and "confusing." Kaya did not have a graphing calculator, so she did the transformation activity using tables and did not do the data analysis activity. In Howard's case, the instructor introduced trigonometry and expected to teach calculator skills as part of the curriculum. Using the calculator was much easier for Howard in geometry since no prior experience was necessary.

In the small class case study, students came from a variety of middle school backgrounds, and although eight of nine students "liked using calculators" and thought they were "helpful" and made things "easier," some students would have liked "lessons with calculators" and "needed instruction." Ms. K. (their regular math instructor) had used some calculator activities in the beginning of the year, but some students felt that they needed more support and explanation.

Chapter Six

The combination of stakeholders' concerns about mathematics education in the United States – students' lack of proficiencies, problem-solving skills, and preparation for 21st century careers in stem fields (Cheung & Slavin, 2013; Herzig, 2005; Miller & Kimmel, 2012; Schmidt, 2012) – together with the conflict between educators using different and often fragmented curricula and instructional methods – traditional versus reform-based (Klein, 2003; Chandler et al., 2016; Polly, 2016; Schoenfeld, 2004) – suggested a need to develop an integrated framework to design curricula that used the best research-based practices to improve students' achievement, engagement, and proficiencies in mathematics (Jacobs, 2011; Milgram, 2007; Schoenfeld, 2014).

This study investigated the IMIST (Integration of Mathematical Inquiry, Symbolic literacy, and Technology) instructional system developed using principles from design thinking, a design pattern approach (Hathaway & Norton, 2018), and activity theory as a possible solution to help educators design curricula to meet the learning needs of students and to support students' development of strong mathematical foundations in symbolic literacy, conceptual literacy, and problem-solving applications. The goal of the IMIST unit learning activities was to help students identify and build mathematical problem-solving skills and strategies to use in real-world, contextual applications.

The IMIST system framework used unit learning objectives to identify authentic, contextual problem-solving applications with supporting symbolic and conceptual learning activities to build and scaffold learning needed for deep understanding of mathematical language and applications. This study was the second phase or enactment phase (Bannan-Ritland, 2003) of a design research project that examined prototypical IMIST units for Algebra 2 and geometry. The research questions for this study examined the impact of two IMIST units on students' overall achievement and mastery of symbolic, conceptual, and problem-solving literacies; on students' attitudes, engagement, and confidence in mathematics; and reports of their experiences using the IMIST unit learning activities.

Thirteen students, ages 10 to 15, participated in four separate case studies: two individual, one paired, and one small class. The treatment for each case study was a packet of learning activities designed and written by the researcher to support the learning objectives of a unit on quadratic functions for Algebra 2 students and a unit on right triangles and right triangle trigonometry for the geometry student. The instructor provided online or in-class lessons and discussions that introduced student-centered activities with summaries, reviews, and practice.

Formal data collection instruments included demographic surveys, Math Attitudes and Perceptions Surveys (MAPS; Code et al., 2016), assessment data, pre-intervention interviews, and post-intervention interviews and surveys (Appendices C, D, E, and J). The researcher kept a journal with observations and students' comments as well as annotated class notes written during online and face-to-face classes to capture students' questions and comments during discussions.

Data analysis of the individual and small class case studies examined students' achievement scores overall and in the core literacies linking their comments from interview and survey data to support and explain their learning experiences with the IMIST unit. A cross-case analysis was developed to examine similarities and trends relevant to the study's research questions. Similarities and trends in the assessment data and students' reports provided insights into students' learning and the impact of the IMIST unit on their achievement, attitudes, and experiences. As a result of the data analysis of the case studies and cross-case analysis, it was possible to identify a number of conclusions about how students learned using the IMIST unit.

Summary of Findings

Achievement. The following findings for the first research question, what is the impact of a mathematics unit designed using the IMIST framework on students' understanding of mathematics and its core literacies, indicate overall positive achievement results for students whose mathematics learning was structured using the IMIST framework. Eleven students achieved a level of proficiency or above. Two students did not perform well on unit assessments. One student did not have the prerequisite knowledge from Algebra 1 to succeed in Algebra 2 and needed remediation or to repeat Algebra 1. The second student had learning disabilities and was taught and assessed at an honors level which lowered her achievement scores.

Overall. Using the IMIST unit activities, students mastered the learning standards and objectives at a proficient level or higher.

Symbolic literacy. Using the IMIST unit activities, students mastered vocabulary, skills, and procedural standards at a proficient level or higher.

Most students reported that the IMIST unit learning activities improved their learning and retention of symbolic literacy skills and vocabulary more than traditional curricula.

Conceptual literacy. Using the IMIST unit activities, students mastered conceptual standards of graphing, tables, and solving equations at a proficient level or higher.

Most students reported that the IMIST learning activities improved their learning and understanding of conceptual literacy, procedures, and fluency more than traditional curricula.

Problem-solving. Most students reported more confidence in their ability to solve problems using context and real-world applications.

Students reported that understanding vocabulary and context helped them with problem-solving strategies more than traditional methods used before.

Attitudes, perceptions, and confidence. Students' attitudes about mathematics in general and their personal beliefs about mathematics were consistent with their level of achievement.

Students' level of confidence in their ability to do math was not measurably impacted by only one implementation of the IMIST unit.

The instructor’s observed levels of students’ classroom engagement or participation was not a predictor of student learning and mastery.

Students’ level of achievement and learning was influenced by individual characteristics such as willingness to work, effort made to complete activities and homework assignments, and preparation, retention, and mastery of prerequisite knowledge and skills.

The learning environment structures such as online versus classroom, content standards, learning objectives, class level, and time constraints impacted the delivery of the IMIST units and influenced students’ attitudes about the IMIST unit.

For students in the individual and paired case studies, the online or face-to-face learning environment with an instructor was a positive learning experience.

Responses to the IMIST unit. Most students reported learning more content and information with the IMIST unit activities than they had previously using traditional curricula.

Students reported having to work harder or differently with the IMIST unit activities than they had previously using traditional curricula. Some liked the work and activities, and others did not.

Some students reported being confused or needing more explanation using the IMIST unit activities.

For students in the small class case study, the learning environment caused concerns about the pacing, amount of material and content, and a need for more explanation using the IMIST unit activities.

Discussion

The purpose of this research study was to evaluate mathematics instructional units designed using successful research-based learning activities from traditional mathematics instruction with successful research-based learning activities from reform-based mathematics instruction. Mathematics educational leaders (Jacobs, 2011; Milgram, 2007; Schoenfeld, 2014) have recommended and called for integrated instructional approaches, but teachers and educators have struggled to find instructional materials that integrate the research-based, most effective practices of both. The IMIST instructional framework used research-based, authentic, student-centered inquiry learning activities. The framework supported these activities mathematically by providing learning activities that built understanding of vocabulary, context, and meaning with conceptual procedural skills and methods that students needed to solve real-world problems. Examination of the data sought to evaluate the impact of the IMIST units on students' learning, attitudes, and experiences to illustrate similarities, differences, and improvements as compared to using only traditional or reform-based instructional practices.

Achievement. Students' achievement overall met or exceeded state and national learning standards for quadratic functions (Algebra 2) and right triangles and trigonometry (geometry) with the exception of two students with learning issues (Virginia Department of Education, Mathematics Standards of Learning for the Commonwealth of Virginia, 2018; CCSSM, 2019; Textbook standards: Larson, Boswell, et al., 2007, 2012; Larson Boswell, & Stiff, 2003; Prentice-Hall, 2008). This suggests the IMIST instructional units provided sufficient learning activities and opportunities for most students to gain understanding and mastery of the content standards and learning objectives similar to many traditional curricula. This level of mastery and understanding prepared Algebra 2 students to progress and extend their knowledge to subsequent units such as polynomial and rational functions which require symbolic and conceptual knowledge gained in the quadratics unit.

Examining achievement for two students with learning issues, the first student's issue was a lack of readiness that could not be addressed by changes to the IMIST unit, but the second student's learning disabilities might have been addressed by making accommodations earlier in the unit. For the first student, readiness for algebra is an important factor in placement in high school algebra courses (Ketterlin-Geller, Gifford, & Perry, 2015; Sahal & Ozdemir, 2019). The solution to the first student's issues was independent of instruction using the IMIST unit as she required significant remediation for Algebra 1 skills. The second student did better with the IMIST unit after certain accommodations were made – reading through the instructions for worksheets and assessments and providing annotated notes with worked-out examples. In addition, she may have benefitted from a slower pace or more frequent meetings with the instructor.

Following up on overall achievement, students mastered symbolic literacy of vocabulary and skills and conceptual literacy of procedures and methods at a level of proficiency similar to content taught in traditional curricula. However, students reported the focus of the IMIST unit activities on reading vocabulary and context helped them understand equations and problem-solving applications more than traditional instructional methods they had used before. This finding is significant since educators or districts have moved away from encouraging students to read mathematics textbooks, have reduced use, or removed mathematics textbooks as a resource (Berger, 2019; Darling, 2013; Taylor, 2013). Students' reports support findings by Darling (2013) and Adams (2010) that reading and understanding vocabulary is a critical strategy to help students with constructing meaning and problem-solving. In addition, students reported that improved understanding of skills such as factoring developed their conceptual understanding of graphing and solving equations which support findings by Wu (1999).

Attitude, perception, engagement, and confidence. Findings from this study indicate students' learning experiences with the IMIST unit activities, their achievement, and understanding of core literacies were influenced by their personal attitudes and beliefs, their adaptability or willingness to change (Collie & Martin, 2017), and their work ethic.

Findings from students' reports of improvement in attitude, perception, engagement, and confidence in problem-solving abilities were a consequence of or related to students' reports of improved understanding of symbolic literacies and vocabulary. Nine of thirteen students reported that the IMIST unit learning activities improved their problem-solving skills compared to traditional word problems done before. This finding separates the impact of the IMIST unit activities on problem-solving strategies from those of a traditional curriculum. Of additional significance is that five of the nine students were in the small class case study which had limited exposure to problem-solving applications as compared to the four students in the individual and paired case studies. The small class case study did not work through the eighth section of the Algebra 2 IMIST unit that focused on modeling and problem-solving. Two students in the paired case study reported significant improvement in understanding of problem-solving as a result of the Section 8 modeling activities and discussed this at length in their post-intervention interviews. The complete IMIST Algebra 2 unit was designed for all students, not just accelerated or honors students, and students in the small class case study may have benefitted from completing the unit resulting in similar positive changes in attitudes and confidence about problem-solving. For geometry, the student discussed the relevance and insights he gained through problem-solving and real-world applications. In summary, students' comments about connections made between what they were doing in mathematics class and authentic real-world applications indicated an increase their engagement and interest in learning which supported findings by many educators who support inquiry learning (Blair, 2014; Hmelo-Silver et al., 2006; Kirschner et al., 2006).

These findings indicate that using the IMIST system framework to design instructional units – integrating traditional symbolic and conceptual literacy learning activities with authentic problem-solving applications – improves students' attitudes and perceptions about problem-solving. However, students' attitudes about mathematics in general as reported on MAPS were consistent with their level of achievement in the course. Research supports this finding and illustrated that the implementation of only one unit of IMIST learning activities was not enough to change students' general beliefs or self-beliefs about mathematics (Valentine et al., 2004).

The observational data also indicated that students' engagement in class – answering questions, participating in class discussion – did not correlate to their achievement, learning, or mastery of content standards and learning objectives (Collie & Martin, 2017). However, students' characteristics such as work ethic, perseverance, and willingness to complete IMIST activities and practice exercises did impact and positively influenced understanding, engagement, and achievement. This finding was consistent with studies done by Collie and Martin (2017) who used the construct of "adaptability" or the capacity to adjust thoughts, behaviors, and emotions to manage change or new demands as a predictor of engagement. Students' willingness to do the class activities and complete homework was evidence of their adaptability and a better measure of engagement. Studies that link time spent on homework and increased achievement or test scores suggest that students' adaptability and willingness to work on the IMIST unit activities may have improved their understanding of core literacies in mathematics (Cheema & Sheridan, 2015; Cooper, 2008; Maltese, Tai, & Fan, 2012). Together these findings indicate that work ethic and willingness to complete the IMIST activities and homework exercises were more important than engagement in class discussion to help students gain mastery of the core literacies. The implication of these findings for teachers/instructors is to be up-front with students about required work and effort. They need to set-up norms and expectations for student work, discourse, and discussion, hold students accountable, and be actively engaged in questioning and guiding student discussions.

Another factor that influenced students' attitudes and engagement was the implementation of the IMIST unit in the middle of a sequence of instruction with a change in course delivery and differences in expectations, activities, and the type and amount of work. The level of cognitive demand of the activities and the student-centered investigations represented a different type of learning activity than traditional learning activities students had experienced before. Most students struggled with open-ended, student-centered activities and the amount of time they had to spend thinking mathematically. Many felt they needed more time or more explanation. As a result, whether a student liked or did not like learning with the IMIST unit activities was a consequence of students' personal beliefs about mathematics and their adaptability to change. For example, two students who were high achievers did not like the amount of work and effort or perhaps the change in the type of work required by the IMIST unit activities. However, the student with learning disabilities expected to work hard and would have liked to continue learning math with IMIST structured units. The implications of these findings for teachers/instructors is to scaffold and explain how students need to approach active learning and to help them develop and implement new learning practices and strategies. It would be best to do this at the beginning of a course or at the beginning of instruction using student-centered active learning and investigations.

Confidence also influenced students' perceived ability to do and learn using the IMIST unit activities. Students who reported low confidence in their ability to do mathematics on MAPS, also reported low confidence, and confusion about their learning experiences. Research by Kisantas, Cheema, and Ware (2011) state that educators need to focus on the self-efficacy and confidence of all students to improve their understanding and achievement in mathematics. Unfortunately, to measure the influence of the IMIST unit learning activities on students' confidence was beyond the scope of this study. Again, the implementation of only one unit of IMIST over a short period of time was insufficient for the researcher/instructor to make a measurable impact on the self-efficacy or confidence of the students.

Learning environment. The study findings indicate that learning environment affected students' attitudes, engagement, and willingness or ability to work with the IMIST unit activities. There were two general learning environments in this study: (a) a flexible, online or in-person, face-to-face instruction with the researcher, or (b) face-to-face instruction in a traditional classroom setting with time constraints. In all case studies, except for the geometry case study, and consistent with research done on student-centered learning activities, having time to complete the activities and/or assigning them for in-class or outside of class activities was challenging for both the instructor and students (Bergmann & Sams, 2012; Educause, 2012; Fulton, 2012; Goodwin & Miller, 2013; Rose, 2009; Strayer, 2012; Sweet, 2014; Tucker, 2012). The geometry student had outstanding independent work habits and did most of his work as student-centered, independent study. Students' attitudes in the individual Algebra 2 case studies were positive about the pace of the course because of its flexibility, but two students were concerned about the level of difficulty, challenge, and workload. As a result, neither wanted to continue learning with IMIST units. Students in the small class case study, where instructional time was limited, were very concerned about pacing and the amount of work. Reflecting these concerns, students' comments tended to be negative or criticisms about the IMIST unit. Students felt that they did not have enough time to ask questions or get the explanations they needed although they had daily access to the instructor outside of class time during a special help period but did not take advantage of that opportunity.

On the positive side, online students felt that the virtual class environment or working face-to-face with the instructor gave them plenty of time to ask questions and get explanations. All three online students mentioned that working with a teacher and being able to ask questions was their favorite thing about the IMIST unit.

Another strategy that kept students motivated and engaged with the work was holding students' accountable for completing assigned work. This was effective in the small class case study as the instructor did daily homework checks. However, checking students' homework was difficult to do in the online environment. Students were expected to self-check with answer sheets, but it was evident from the questions and discussions that the students did not always follow through with assigned work. Students' reports suggest that holding students accountable for doing their mathematics work may have helped both learning and achievement.

In addition, online students had access to the annotated notes completed during the class sessions, whereas the students in the small class only had access to answer sheets during class and a textbook for reference. It was not clear if this was an advantage to the online students, but it would have been a supportive resource had it been provided to the students in the small class, especially those who were absent from class. Students reported that the answer sheets and access to annotated notes was helpful.

Recommendations for Practice

The goal of this design research project was to create and evaluate a framework to guide good mathematical teaching practices, to identify appropriate learning activities, and to help educators in their lesson planning and development. Chapter Two discussed the IMIST design pattern consequences that presented concerns and issues that teacher/practitioners must consider when designing a unit using the IMIST system. The following discussion summarizes concerns and issues in the context of this research study.

The IMIST instructional system integrated NCTM (2014) reform-based standards for teaching and learning mathematics with a practical focus to help teachers and educators choose mathematical learning activities to strengthen students' symbolic literacy, conceptual literacy, and problem-solving strategies (Jacobs, 2011; Milgram, 2007; Schoenfeld, 2014). Since teaching and learning is a fundamental recurring process in educational settings, using a design pattern approach to implement and use the IMIST system framework became a reflective and iterative process (Hathaway & Norton, 2018). To implement practices associated with the IMIST system framework, teachers and educators need to consider their teaching environment, norms and classroom expectations, and time available for instruction, activity, and practice inside and outside of the classroom. In addition, teachers and educators need to be familiar with content learning standards and objectives so they can choose learning activities to support learning in the core literacies as well as to engage students in authentic, contextual problem-solving activities relevant to them.

A major consideration or change required to put the IMIST system framework into practice is to focus on student-centered learning as compared to more traditional lecture or guided instruction. Teachers need to be aware that students who have been taught using traditional methods of math instruction such as lecture and using worksheets are going to find the work of learning mathematics in an active learning environment different than what they have done before. Students will have to work and think differently. Teachers need to communicate and model these changes clearly by establishing expectations for collaborative work, communication, discussion and by demonstrating problem-solving strategies. Teachers need to scaffold students' understanding of how to work with authentic contextual problems in real-world settings compared to traditional, textbook word problems. NCTM (2014) described the move from dominant traditional instructional beliefs and practices to reform-based practices as an obstacle that needs to be overcome. Teaching with a student-centered focus requires new ways of thinking and professional development. Specifically, teachers need to establish classroom expectations and norms for new ways to work in and outside of math class and to hold students accountable. This study examined only one unit of instruction dropped in the middle of an instructional sequence which made it challenging to set up norms, expectations, work habits, and accountability. This was a limitation of the study. A recommendation for practice, which has the best effects on achievement, is to establish the expectations and norms for new or different learning practices early or at the beginning of a mathematics course (Hattie, 2009; Haystead & Marzano, 2009).

As part of this focus on student-centered learning, teachers and educators need to consider the time required for inquiry and student-centered learning. Decisions need to be made about what activities can or should be done in class as paired or collaborative group work, and what can be done outside of class as homework activities (Bergmann & Sams, 2012; Fulton, 2012; Goodwin & Miller, 2013; Rose, 2009; Strayer, 2012; Sweet, 2014; Tucker, 2012). Consistent with beliefs that mathematics learning is an active process and that learning is done by doing math (NCTM, 2014), students need to be encouraged and held accountable for doing assigned work. As evident in this study, students' work effort as measured by the completion of learning activities improved their assessment scores and, by their own reports, improved their learning of the core literacies.

As modeled in this study, the IMIST instructional units were written to be used by students at any level of their mathematics instruction – honors, standard, or special education – adjustments were made to pacing, practice exercises, and level of challenge, and number of assessments. Students in the small class case study did all the same investigations and learning activities as those in the Algebra 2 individual cases studies except for the last two sections. Positive responses from students in individual cases studies suggest that students in the small class case studies might have benefitted and gained more understanding of problem-solving strategies had they been able to complete the IMIST unit.

In order to implement the IMIST system framework successfully, teachers and educators need to have a deep understanding of the content, standards, and learning objectives to identify and choose learning activities which meet the goal of building strong foundations in the core literacies (Ball, Thames, & Phelps, 2008). Although teachers may choose to work independently, teacher collaborative teams could work on the units together to share insights, ideas, and activities.

To begin, teachers need to anticipate or assess pre-requisite knowledge or skills and to be prepared to choose activities to support, review, scaffold, and remediate student learning. Teachers need to consider what technology they would like to use as part of instruction. As evident in this study, some students did not know how to use graphing calculators to investigate graphs or do data analysis and regression. Teachers and educators may need to allot instructional time to prepare, model, and teach students technology skills. Teachers need to acknowledge that it is challenging to teach students skills like drawing, constructing, and/or graphing, and some of these skills take time and practice. Teachers need to integrate and model thinking and problem-solving skills and strategies into their practice (Tishman, Perkins, & Jay, 1995). Teachers also need to practice reflection, preparation, and evaluation as part of their practice to improve, adapt, and make changes to their activities and to make the content and authentic applications culturally relevant to their student populations (Boaler & Staples, 2008; Holmes & Hwang, 2016; Nasir, Hand, & Taylor, 2008).

Finally, when teachers implement a unit designed using the IMIST system framework, they need to evaluate what worked and what did not work and to make changes to their lessons and activities. The flexibility of the IMIST system framework allows teachers to decide what activities best suit and address the interests and learning needs of their students.

Recommendations for Research

This study was an enactment and evaluation phase of a larger design research project. It examined the local impact of one instructional unit in four case studies (Bannan-Ritland, 2003). Generalizability of the study findings is limited as they reflect experiences of thirteen students with one unit of instruction. Additional research is recommended to increase the size and scope of implementation of IMIST units (broader impact); to do comparative studies of IMIST units with other instructional methods on learning outcomes such as achievement, attitudes, and confidence; and for more teachers and educators to test the framework for designing instruction in their own classrooms. Additional research could be done to identify and evaluate the most effective learning activities to support the components of the IMIST instructional system for the core literacies – symbolic, conceptual, and problem-solving.

The next phase of a design research project would be implementation with broader impact to evaluate learning and student experiences (Bannan-Ritland, 2003). This could take the form of a series of IMIST units designed to cover multiple learning objectives for a full-year course. It could also take the form of IMIST units implemented over a larger student population or school.

Research using comparative studies examining students' learning outcomes would be valuable to assess whether learning with integrated IMIST units is more effective than other instructional methods such as Problem-Based Learning or traditional instruction. These studies could compare students' achievement in core literacies and the impact of the units on students' attitudes and perceptions.

Teacher/practitioners could do focused research on the individual components of the IMIST instructional system framework. Teacher/practitioners could research and investigate the most effective learning activities for each of the core literacies – symbolic, conceptual, and problem-solving – as determined by content learning standards and objectives for the courses that they teach.

As the goal of the development of the IMIST instructional system was to provide teachers with a framework to design learning for their classrooms and to choose effective learning activities for mathematics instruction of the core literacies, the fulfillment of that goal and to evaluate the framework lies in the hands of teacher themselves. Teachers could be encouraged to conduct action research using the IMIST instructional framework to see if it works for them.

Appendix A

IMIST Design Pattern Implemented as an Algebra 2 Unit for Quadratic Functions

Unit Topics, Learning Activities, and Literacy Classifications

Name: Integration of Mathematical Inquiry, Symbolic Literacy, and Technology (IMIST) Framework			
The Problem: How can educators integrate mathematics learning opportunities for students that promote mastery of symbolic and conceptual literacies, foster problem-solving abilities, and situate mathematics in authentic learning contexts?			
The Core of a Solution: To choose and create content that builds students' mathematical knowledge and mastery of the key proficiencies using inquiry learning activities with scaffolding activities in symbolic literacy, conceptual literacy, and authentic problem-solving applications.			
Inquiry Learning Activities Authentic, real-world studies that provide students with rationales for learning and applying content topics. Build mathematical thinking and problem-solving strategies in heuristic ways.	**Symbolic Literacy Activities** Provide students with the vocabulary, symbolic language, and foundational skills for more advanced conceptual understandings.	**Conceptual Literacy Activities** Move students to tasks with higher cognitive demands, processes, multiple representations, and symbolic applications.	**Problem-solving Applications** Authentic and/or real-world applications of content topics that build problem-solving strategies to help students define and model more open-ended problems.

Section Number Topic	**Learning Activity**	**Participants**	**Symbolic Literacy**	**Conceptual Literacy**	**Problem-Solving**
0 Introduction	Investigation 5-0 Internet Exploration - Careers	Intro: Teacher Activity: Students			X
	Project 5-0 Unit Project – Wintendo Games	Intro: Teacher Activity: Students			X
1 Graphing & Transformations	Investigation 5-1 Technology Enhanced Learning Activity/Graphs, Tables & Equations	Intro: Teacher Activity: Students		X	
	Lessons Learned: Inv 5-1	Discussion: Teacher & Students	X	X	

2 Vertex & Standard Forms	Investigation 5-2 Symmetry & Equations	Intro: Teacher Activity: Students	X	X	
	Lessons Learned: Inv 5-2	Discussion: Teacher & Students	X	X	
	Worksheet 5-2 Practice: Graphing with Vertex & Standard Forms	Practice: Students	X	X	
1 – 2 Review	Worksheet 5-1 to 5-2 Warm-up Review 5-3	Practice: Students	X #1, 2, 5, 6	X #3	X #4
3 Intercept Form, Factoring & Solving	Packet 5-3 Factoring Review	Practice: Students	X	X	
	5-0: Unit Project – Solution of a quadratic function	Discussion: Teacher & Students	X	X	X
	Lesson 5-3a Vocabulary, Intercept Form & Factoring	Discussion: Teacher & Students	X	X	
	Investigation 5-3a Factors, Solutions & *x*-intercepts	Intro: Teacher Activity: Students		X	
	Lessons Learned: 5-3a	Discussion: Teacher & Students	X	X	
	Lesson 5-3b Factoring & Solving Non-standard Form Equations	Discussion: Teacher & Students	X	X	
	Investigation 5-3b Writing Equations Given Roots & Leading Coefficient "*a*"	Intro: Teacher Activity: Students	X	X	
	Lessons Learned: Inv 5-3b	Discussion: Teacher & Students	X	X	
	Worksheet 5-3 #1 Practice: Graphing, Transformations, & Equations	Practice: Students	X	X	
	Worksheet 5-3 #2 Applications & Modeling *(Added Case Study #3)*	Practice: Students		X	X
4 Square Roots & Solving Quadratic Equations	Warm-up Review Worksheet 5-4 Square Roots & Radicals	Practice: Students	X	X	
	Lessons Learned: Review Worksheet 5-4	Discussion: Teacher & Students	X	X	

	Lesson 5-4 Solving Quadratics by Finding Square Roots	Discussion: Teacher & Students	X	X	
	Investigation 5-4 (TELA) Falling Objects	Intro: Teacher Activity: Students		X	X
	Lessons Learned: Inv 5-4 Applications, Falling Object Model	Discussion: Teacher & Students		X	X
5 Completing the Square & Solving Quadratic Equations	Warm-up Review 5-5 Perfect Square Trinomials	Practice: Students	X	X	
	Lesson 5-5 Completing the Square: The Story & Process	Discussion: Teacher & Students	X	X	
	Challenge 5-5 Deriving the Quadratic Formula	Activity: Students		X	X
	Lessons Learned: Challenge 5-5	Discussion: Teacher & Students	X	X	
6 Complex Numbers & Operations	Investigation 5-6 The Nature of Roots	Intro: Teacher Activity: Students		X	
	Lessons Learned: Inv 5-6	Discussion: Teacher & Students	X	X	
	Lesson 5-6 Complex Numbers – Vocabulary & Operations	Discussion: Teacher & Students	X	X	
	Worksheet 5-6 Complex Number Operations	Practice: Students	X	X	
7 The Quadratic Formula & Discriminant	Lesson 5-7 The Quadratic Formula & Solving Quadratics	Discussion: Teacher & Students	X	X	
	Investigation 5-7a Solutions, Graphs & the Determinant	Intro: Teacher Activity Students	X	X	
	Lessons Learned: Inv 5-7a The Discriminant & the Nature of roots	Discussion: Teacher & Students	X	X	
	Applications: Vertical Motion Models Rocketry & NASA	Intro: Teacher Activity: Students	X	X	X
8 Modeling with Quadratic Functions	Lesson 5-8 Applications	Intro: Teacher Activity: Students	X	X	X
	Worksheet 5-8 #1 Economics: Yearbook Publishing	Activity: Students		X	X

	Worksheet 5-8 #2 Motion, cooling, architecture	Activity: Students		X	X
	Worksheet 5-8 #3 College Costs	Activity: Students		X	X
9 Quadratics, Inequalities & Systems	Warm-up & Review 5-9 Linear Inequalities & Systems	Practice: Students	X	X	
	Investigation 5-9a Graphing & Solving Quadratic Inequalities	Intro: Teacher Activity: Students	X	X	
	Lessons Learned 5-9a Graphing Linear & Quadratic Inequalities in Two Variables	Discussion: Teacher & Students	X	X	
	Investigation 5-9b Solving Systems	Intro: Teacher Activity: Students	X	X	
	Lessons Learned: Investigation 5-9b	Discussion: Teacher & Students	X	X	
	Worksheet 5-9 Practice: Solving Linear Inequalities & Systems	Practice: Students	X	X	
Unit Review	Worksheet Unit Review Practice	Practice: Students	X	X	X

Appendix B

IMIST Design Pattern Implemented as a Geometry Unit for Right Triangle & Right Triangle Trigonometry

Unit Topics, Learning Activities, and Literacy Classifications

Name: Integration of Mathematical Inquiry, Symbolic Literacy, and Technology (IMIST) Framework			
The Problem: How can educators integrate mathematics learning opportunities for students that promote mastery of symbolic and conceptual literacies, foster problem-solving abilities, and situate mathematics in authentic learning contexts?			
The Core of a Solution: To choose and create content that builds students' mathematical knowledge and mastery of the key proficiencies using inquiry learning activities with scaffolding activities in symbolic literacy, conceptual literacy, and authentic problem-solving applications.			
Inquiry Learning Activities Authentic, real-world studies that provide students with rationales for learning and applying content topics. Build mathematical thinking and problem-solving strategies in heuristic ways.	**Symbolic Literacy Activities** Provide students with the vocabulary, symbolic language, and foundational skills for more advanced conceptual understandings.	**Conceptual Literacy Activities** Movs students to tasks with higher cognitive demands, processes, multiple representations, and symbolic applications.	**Problem-solving Applications** Authentic and/or real-world applications of content topics that build problem-solving strategies to help students define and model more open-ended problems.

Section Number **Topic**	**Learning Activity**	**Participants**	**Symbolic Literacy**	**Conceptual Literacy**	**Problem-Solving**
0 Introduction	Investigation 9-0 Internet Exploration - Careers	Intro: Teacher Activity: Students	X		X
	Project 9-0	Intro: Teacher Activity: Students			X

	Unit Project – Right Triangles at Work: Photo Collage				
1 Similar Right Triangles	Investigation 9-1a Altitudes & Similarity of Right Triangles	Intro: Teacher Activity: Students		X	
	Lessons Learned: Investigation 9-1a Applications	Discussion: Teacher & Students	X	X	
	Investigation 9-1b Altitudes/Sides & Similarity of Right Triangles	Intro: Teacher Activity: Students		X	X
	Lessons Learned: Investigation 9-1b Altitude as mean/Legs as means Theorems	Discussion: Teacher & Students	X	X	
	Applications 9-1: Using Right Triangle Similarity Theorems	Activity: Students	X	X	X
	Worksheet 9-1: Right Triangle Similarity & Means	Practice: Students	X	X	X
2 The Pythagorean Theorem	Think About/Reflection 9-2 The Pythagorean Theorem	Intro: Teacher Activity: Students	X	X	
	Lessons Learned: Think About 9-2	Discussion: Teacher & Students	X	X	
	Proof Activity: 9-2 Three Pythagorean Proofs	Activity: Students	X	X	X
	Lessons Learned: Proof Activity 9-2 Pythagorean Triples	Discussion Student Presentations	X	X	X
	Applications 9-2: Pythagorean Theorem	Activity: Students	X	X	X
	Worksheet 9-2: Applications of Pythagorean Theorem	Practice: Students	X	X	X
	Extension Worksheet 9-2: Investigating Pythagorean Triples	Activity: Students	X	X	X
3 The Converse of the Pythagorean Theorem	Think About 9-3: The Converse of the Pythagorean Theorem	Activity: Students	X	X	
	Investigation 9-3: Using the Converse of the Pythagorean Theorem	Activity: Students	X	X	X
	Lessons Learned: Investigation 9-3 Triangle Inequality Theorem	Discussion: Teacher & Students	X	X	
	Applications 9-3: Classifying Triangles – Corollary to the Pythagorean Converse	Intro: Teacher Activity: Students	X	X	X
1 – 3 Review	Worksheet 9-1 to 9-3	Practice: Students	X #1-5; 6-9; 20	X #1-5; 10-16	X #17-19

4 Special Right Triangles	Investigation 9-4a: Special Right Triangles (45°-45°-90°)	Intro: Teacher Activity: Students	X	X	
	Lessons Learned: Investigation 9-a Applications	Discussion: Teacher & Students	X	X	
	Investigation 9-4b: Special Right Triangles (30°-60°-90°)	Intro: Teacher Activity: Students	X	X	
	Lessons Learned: Investigation 9-a Applications	Discussion: Teacher & Students	X	X	
	Worksheet 9-4: Special Right Triangles Challenge	Activity: Students	X	X	X
5 Introduction to Trigonometric Ratios	Investigation 9-5: Triangle Ratios	Intro: Teacher Activity: Students		X	X
	Lessons Learned 9-5: Trigonometric Ratios	Discussion: Teacher & Students	X	X	
	Lesson 9-5 & TELA Activity: Finding Trigonometric Ratios	Discussion: Teacher & Students	X	X	
	Think About/Reflection 9-5: Using Trigonometric Ratios & Angles	Discussion: Teacher & Students		X	
	Applications 9-5: Using Trigonometric Ratios to find Lengths	Activity: Students	X	X	X
	Worksheet 9-5: Trigonometry Challenge	Activity: Students	X	X	X
6 Solving Right Triangles	Think About/Refection 9-6: Solving Right Triangles	Intro: Teacher Activity: Students		X	
	Lesson 9-6: Inverse Trigonometric Functions (TELA)	Discussion: Teacher & Students	X	X	
	Investigation 9-6 (TELA): Finding Angle Measures	Intro: Teacher Activity: Students	X	X	X
	Lessons Learned 9-6: Finding Angle measures	Discussion: Teacher & Students	X	X	
	Applications 9-6: Solving & Using Right Triangles Surveyors & Air Traffic Control	Intro: Teacher Activity: Students	X	X	X
	Worksheet 9-6: Solving Right Triangles & Applications	Practice: Students	X	X	X
4 – 5 Review	Worksheet 9-4 to 9-6: Special Triangles & Right Triangle Trigonometry	Practice: Students	X #2-6; 10-13	X #2-6	X #7-9; 14,15
7 The Law of Cosines	Think About/Reflection: Solving Non-right Triangles	Discussion: Teacher & Students		X	

	Investigation 9-7: The Law of Cosines Coordinate Proof	Intro: Teacher Activity Students	X	X	
	Lessons Learned Investigation 9-7: The Law of Cosines & Applications	Discussion: Teacher & Students	X	X	
	Applications: The Law of Cosines Heron's Formula	Activity: Students	X	X	X
8 The Law of Sines	Think About/Reflection: Congruent Triangles Postulates & Theorems	Discussion: Teacher & Students		X	
	Investigation 9-8a: The Law of Sines Coordinate Proof	Intro: Teacher Activity: Students		X	X
	Lessons Learned Investigation 9-8a: The Law of Sines & Applications	Discussion: Teacher & Students	X	X	X
	Investigations 9-8b: Areas of Triangles	Activity: Students		X	X
	Lessons Learned 9-8b: Areas of Triangles & Applications	Discussion: Teacher & Students	X	X	X
	Think About/Reflection: The Ambiguous Case (SSA)	Discussion: Teacher & Students		X	
	Lesson 9-8: Solving the Ambiguous Case	Discussion: Teacher & Students	X	X	
	Applications 9-8: Solving Triangles	Activity: Students	X	X	X
	Worksheet 9-7 to 9-8: Law of Sines, Cosines & the Ambiguous Case	Practice: Students	X	X	X
Unit Review	Worksheet Unit Review Practice	Practice: Students	X	X	X

Appendix C

IMIST: Demographics & Math Attitudes and Perceptions Survey (MAPS)

Name: ____ Age: ______________

Circle one for each category below:

Gender: M F Grade Level: F S Jr Sr

Ethnicity: White Asian Black Hispanic Other: ______________________

Number of years as an independent learner/homeschool student: ________________

MAPS Survey (Adapted from Code et al., 2016)

This is a survey of your attitudes and perceptions about math; these statements all have the response choices: Strongly Disagree, Disagree, Neutral, Agree, Strongly Agree. Please chose the response that matches your opinion, not what you think your teacher might say or what to hear.

Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
1	2	3	4	5

Code*		Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
B,WC	1. After I study a topic in math and feel that I understand it, I have difficulty solving problems on the same topic.	1	2	3	4	5
A	2. There is usually only one correct approach to solving a math problem.	1	2	3	4	5
B	3. I'm satisfied if I can do the exercises for a math topic, even if I don't understand how everything works.	1	2	3	4	5
B	4. I do not expect formulas to help my understanding of mathematical ideas, they are just for doing calculations.	1	2	3	4	5
B	5. Math ability is something about a person that cannot be changed very much.	1	2	3	4	5
A	6. Nearly everyone is capable of understanding math if they work at it.	1	2	3	4	5

A	7. Understanding math means being able to recall something you've read or been shown.	1	2	3	4	5
B,W	8. If I am stuck on a math problem for more than ten minutes, I give up or get help from someone else.	1	2	3	4	5
A	9. I expect the answers to math problems to be numbers.	1	2	3	4	5
B C	10. If I don't remember a particular formula needed to solve a problem on a math test, there's nothing much I can do to come up with it.	1	2	3	4	5
B	11. In math, it is important for me to make sense out of formulas and procedures before I use them.	1	2	3	4	5
B	12. I enjoy solving math problems.	1	2	3	4	5
RW	13. Learning math changes my ideas about how the world works.	1	2	3	4	5
B	14. I often have difficulty organizing my thoughts during a math test.	1	2	3	4	5
RW	15. Reasoning skills used to understand math can be helpful to me in my everyday life.	1	2	3	4	5
B	16. To learn math, the best approach for me is to memorize solutions to sample problems.	1	2	3	4	5
B C	17. No matter how much I prepare, I am still not confident when taking math tests.	1	2	3	4	5
A	18. It is a waste of time to understand where math formulas come from.	1	2	3	4	5
	19. Please select Agree (not Strongly Agree) for this question.	1	2	3	4	5
B,W C	20. I can usually figure out a way to solve math problems	1	2	3	4	5
RW	21. School mathematics has little to do with what I experience in the real world.	1	2	3	4	5

A	22. Being good at math requires natural (i.e. innate, inborn) intelligence in math.	1	2	3	4	5
B,W	23. When I am solving a math problem, if I can see a formula that applies then I don't worry about the underlying concepts.	1	2	3	4	5
B,W C	24. If I get stuck on a math problem, there is no chance that I will figure it out on my own.	1	2	3	4	5
B	25. When learning something new in math, I relate it to what I already know rather than just memorizing it the way it is presented.	1	2	3	4	5
B	26. I avoid solving math problems when possible.	1	2	3	4	5
B	27. I think it is unfair to expect me to solve a math problem that is not similar to any example given in class or the textbook, even if the topic has been covered in the course.	1	2	3	4	5
B	28. All I need to solve a math problem is to have the necessary formulas.	1	2	3	4	5
B,W	29. I get upset easily when I am stuck on a math problem.	1	2	3	4	5
B	30. Showing intermediate steps for a math problem is not important as long as I can find the correct answer.	1	2	3	4	5
A	31. For each person, there are math concepts that they would never be able to understand, even if they tried.	1	2	3	4	5
B	32. I only learn math when it is required.	1	2	3	4	5

*The codes did not appear on the student surveys. The codes were used as part of the data analysis and classified the questions into: A – general attitudes about mathematics; B – personal beliefs about mathematics; B,W – personal beliefs about work ethic and perseverance; RW – personal beliefs about mathematics and the real-world; C – personal beliefs about work, math, and confidence.

Appendix D

Pre-intervention Interview Questions Guide

1) Demographics: Gender, Age, Grade level, Level, courses?

2) How long have you been an independent learner or homeschool student? Why? What circumstances?

3) What type of math student are you? Do you like math? Do you struggle?
    a. Review student's MAPS survey responses

4) How have you studied mathematics in the past – what structures, classes, organizations?

5) How have you and/or your parents chosen a mathematics curriculum?

6) What resources do you use to **study** math?
    a. Textbook?
    b. Study group – collaboration?
    c. Online lecture – videos?
    d. Teacher resources?

7) What resources do you use to **practice** math?
    a. Independent study & reading?
    b. Textbook exercises?
    c. Worksheets?
    d. Study group – collaboration?
    e. Online lecture – videos?
    f. Teacher? Group leader?

8) What learning activities do you use to study math?
    a. What activities are successful or helpful to you? Unsuccessful/unhelpful?
    b. What types of activities engage you – make you excited about learning math?

c. What types of activities help you learn and become proficient in math – prepare you an assessment, project, or test?

9) What types and how do learning activities support your development of proficiencies in mathematics?
    a. How do you learn symbolic literacies? Vocabulary? Symbols? Practice?
        i. How confident are you about your learning and understanding?
    b. How do you learn conceptual literacies?
        i. Representations – graphs, tables, equations? Symbolic problem-solving?
        ii. How confident are you about your learning and understanding?
    c. How do you learn problem solving?
        i. What types of problems? (applications of formulas, data analysis, statistics, real-world, authentic?)
        ii. How confident are you about your learning and understanding?

10) What is the role of technology in your learning of mathematics?
    a. Calculators
    b. Search engines
    c. Online lessons – format?
    d. Video micro lectures?

11) What is the role of practice and assessment in your study of mathematics?
    a. Homework
    b. Formative assessments? Quizzes?
    c. Projects?
    d. Summative assessment? Unit tests? Standardized tests?

12) How do you get help when you do not understand?

13) Do you make corrections to your homework and assessments?

14) What is your favorite thing about math?

15) What is your least favorite thing about math?

16) Have you ever had an *aha* moment in mathematics – a time when you gained a significant insight in math? Describe it.

Appendix E

Post IMIST Intervention Questions Guide

1) Did you like learning using the IMIST activities? Why or Why not?

2) What was the same? What was different?

3) Did you like the unit structure? Did it make sense/help you learn? How, why or why not?

4) Did the IMIST activities improve your understanding of vocabulary, symbols, and symbolic literacies?
    a. Which ones? How?

5) Did the IMIST activities improve your understanding of graphing, tables, equations and your conceptual literacy?
    a. Which ones? How?

6) Did the IMIST activities improve your problem-solving strategies?
    a. Which ones? How?

7) Did you like the use of technology in the investigations?

8) Were the online lessons helpful? How, why or why not?
    a. Was the amount of information or discussion too much, too little, or just right?

9) Were the worksheets helpful? How, why or why not?

10) Were the instructions clear and easy to follow? Which could be improved? How?

11) What could be done to improve the structure or delivery of the learning activities?

12) Do you feel like you learned more or less with the IMIST unit activities than you have before? Why or why not?

13) What is your favorite thing about the IMIST unit?

14) What is your least favorite thing about the IMIST unit?

15) Did you have an “Ah-ha” moment using the IMIST activities – a time when you gained a significant insight in math? Describe it.

Appendix F

Study Information Sheet – Dissertation Research Study

IMIST: Case Studies of the Perceptions, Attitudes, and Experiences of Students Using an Integrated Mathematics Instructional System Focused on Inquiry Learning and Proficiencies in Symbolic Literacy, Conceptual Literacy and Problem-solving for High School Mathematics

Dr. Priscilla Norton/Laura McConnaughey * George Mason University * 2018-19

Personal Background: Laura McConnaughey taught mathematics at secondary levels over the last 35 years. Until last spring, she taught mathematics at Thomas Jefferson High School for Science and Technology where she synthesized and developed curricular materials to teach Geometry, Algebra 2, Precalculus, and Calculus. Prior to that, she taught at Joyce Kilmer Middle School (Vienna, Virginia) and Norwell Middle School (Massachusetts). She is currently a doctoral candidate at George Mason University in Learning Technologies Design Research, Integration of Learning Technologies in Schools, and Mathematics Educational Leadership working on her dissertation research.

Research Interests: As a mathematics educator, she has been deeply involved in developing curriculum designed to help students apply math in real-world contexts. Research has shown that inquiry learning activities based on real-world problem-solving engages students in the study of mathematics and provides them with rationales, purposes, and motivation to study math. Inquiry learning activities support the need for students to develop key mathematical proficiencies in symbolic literacy, conceptual literacy, and problem-solving. Her hypothesis is that an integrated mathematics curriculum using inquiry learning activities and activities aimed at building strong foundations in symbolic literacy, conceptual literacy, and problem-solving promotes better learning and stronger mathematical abilities in our students.

Description of Study: This collective case study will investigate student learning in secondary mathematics, specifically Algebra 2, using instructional units designed with an integrated system framework (IMIST) focused on inquiry learning activities and activities that develop symbolic, conceptual, and problem-solving proficiences. It will study students understanding, perceptions, attitudes, and confidence in learning and doing mathematics.

The study has six parts: 1) a survey to assess students' attitudes and perceptions about mathematics and to collect demographic information; 2) a pre-intervention, semi-structured interview; 3) participation in the first of two teacher-led, online unit lessons presentations, activities, and assessments; 4) a post-intervention, semi-structured interview; 5) participation in the second teacher-led, online unit lesson presentations, learning activities, and assessments; and 6) a final, post-intervention, semi-structured interview.

Participants procedures and activities

- **Study Information Sheet:** Mrs. McConnaughey will provide interested parents and students with this study information sheet.
- **Technology:** For this study, students must have a reliable internet connection with a device that has good audio and visual capability to work with Google Hangouts. They must also have a graphing calculator (Suggestion: TI-83 or TI-84 family) or access to graphing emulator software (Suggestion: DESMOS (free: desmos.com)).
- **Textbook and/or Curricular Resource:** Mrs. McC will provide a packet of unit activities and worksheets. Students will need to have an additional curricular resource or textbook to support their learning and homework assignments. Students will be asked to do reading and select practice exercises to solidify their learning from this resource.
- **Consent and Assent Forms:** Parents with students willing to participate in this study will be sent consent and assent forms via email.
- **Demographics & MAPS Survey:** After parents and students return the signed consent and assent forms (email or regular mail), Mrs. McC will send them the Demographic and Math Attitudes & Perceptions Survey (MAPS) to start building a learner profile for each student.
- **Pre-Intervention Interview:** After the completed Demographic & MAPS survey is returned (email or regular mail), Mrs. McC will schedule a pre-intervention interview with each student which will be recorded and transcribed. Students will choose a pseudonym to protect their identity which will be used on all reports of research. She will email the interview questions to the parents and students prior to the interview.
- **All interviews may be conducted in person or via Google Hangouts.**
- **Online Lessons:** Mrs. McC will send all participating students a unit outline with a schedule, lessons, and assignments with accompanying unit activities, notes and worksheets. Students will need to use a textbook or other curricular resource to supplement and practice the topics and concepts as they need during the unit. Students will email/mail a copy of the completed outline with practice (Homework!) exercises to Mrs. McC. During October (or per student's curriculum), Mrs. McC will conduct 60- to 90-minute online lessons for the student participants twice per week. These lessons, will introduce, discuss, and summarize the learning activities for the first unit on quadratics. **Students must participate in these online lessons as part of the research study.**
- **Assessments:** Mrs. McC will mail sets of unit assessments (two quizzes and one unit test) to the parents of participating students. These assessments need to be taken in a secure setting – no books, notes, or other assistance. Each assessment may have a non-calculator and calculator portion. Mrs. McC would like these assessments scanned and emailed to her for timely grading and feedback. She will email the graded assessments back to each student for achievement and feedback. No letter grades will be assigned.
- **Post-Intervention Interview:** After the completion of the first unit, Mrs. McC will schedule an interview with each participating student. The interview will be recorded and

transcribed. She will email the interview questions to the parents and students prior to the interview.

- **REPEAT!! Unit 2:** Mrs. McC will conduct the second unit in the same manner as the first. Students will receive the unit materials, online lessons, and assessments. After the completion of the unit, she will conduct the second-post intervention survey.

Questions and Clarifications: This research is being conducted by Laura McConnaughey, a doctoral candidate at George Mason university under the supervision of Dr. Priscilla Norton at the College of Education and Human Development at GMU. Dr. Norton may be reached at (703) 993-2015 for questions or to report a research-related problem. Laura McConnaughey may be reached at (703) 389-9943 or email: lmcconna@masonlive.gmu.edu. You may contact the George Mason University Institutional Review Board (IRB) Office at (703) 993-4121 if you have questions or comments regarding your rights as a participant in the research. This research is under review according to George Mason University procedures governing your participation in this research.

Appendix G

INFORMED CONSENT FORM: Parent/Guardian

IMIST: ***Case Studies of the Perceptions, Attitudes, and Experiences of Students Using an Integrated Mathematics Instructional System Focused on Inquiry Learning and Proficiencies in Symbolic Literacy, Conceptual Literacy, and Problem-solving for High School Mathematics***

Dr. Priscilla Norton/Laura McConnaughey * George Mason University * 2018-19

INFORMED CONSENT FORM: Parent/Guardian

RESEARCH PROCEDURES

This research is being conducted to investigate changes in students' understanding, perceptions, attitudes, and confidence using an integrated approach to mathematics instruction focused on inquiry learning activities and activities designed to build strong foundations in symbolic literacy, conceptual literacy, and problem-solving. This study is designed to gather student feedback and evaluation of learning activities, materials, and experiences.

Your student will be taught two units by the researcher/teacher of this study. The topics for the two units will be chosen from quadratics, exponentials & logarithms, and/or sequences & series. Lessons and materials have been designed using the IMIST framework of inquiry activities, technology enhanced learning activities, and activities to build strong foundations in symbolic, conceptual, and problem-solving literacies. The IMIST instructional materials meet or exceed learning objectives of both state and national standards and exemplary textbook content. Students will participate in online classes offered twice per week for 60 to 90-minutes via Google Hangouts for each of the units. The times chosen will be at the convenience of the participants and the researcher/teacher.

At the beginning of the 2018-19 school year, we will need your signature on this consent form. If you consent to your student's participation, your student will fill out an information sheet which will provide information about gender, age, class level, and ethnicity. In addition, your student will fill out a Math Attitudes and Perceptions Survey (MAPS; Code et al., 2016) which will provide background and a student profile of your student's beliefs about mathematics. All participating students will choose a pseudonym which will be used to keep all information confidential and referenced on all data reported.

Your student will participate in three interviews: A pre-intervention interview to follow up on the MAPS survey and two post-intervention interviews which will occur after each instructional unit. These interviews will ask students to share their learning experiences using the IMIST units. Each interview will be 15 to 20-minutes and conducted in person or via Google Hangouts at a time convenient for the student and the researcher. The interviews will be audio-recorded and transcribed. Again, your student will not be identified by their name in the recording. The total face-to-face time required for this study, instruction, and interviews, is approximately 15 to 21 hours.

The researcher will also provide formative and summative assessments (quizzes and unit test) for each instructional unit. These assessments will be graded numerically to provide feedback on your student's understanding and achievement. No letter grades will be assigned.

RISKS

There are no potential risks regarding the content, assessments, interviews, or the identity of your student.

BENEFITS

There are no benefits to your student other than to further research in understanding effective practices in mathematics instruction and students' attitudes, perceptions, and confidence in learning mathematics.

CONFIDENTIALITY

The data in this study will be confidential. Your student's name will not be included on collected data. A pseudonym will be placed on the interview and assessment data. Only the researchers will know the link between your student's identity and the linked pseudonym, and only the researchers will have access to the audio-recordings and transcriptions. The data from this study (the audio-recordings, transcripts, and assessment data) will be stored electronically on the principle investigator's password-protected university computer. The electronic data files will be destroyed after a five-year period. Identifiers may be removed from the data and the de-identified data could be used for future research without additional consent from participants Those who participate via Google Hangouts may review Google's website for information about their privacy statement: https://www.google.com/intl/en/privacy.html.

PARTICIPATION

Your student's participation is voluntary, and you may withdraw him/her from the study at any time and for any reason. If you decide not to have your student participate or if you withdraw him/her the study, there is no penalty or loss of benefits to which he/she is otherwise entitled. There are no costs to you or any other party.

CONTACT

This research is being conducted Laura McConnaughey, a doctoral candidate at George Mason University under the supervision of Dr. Priscilla Norton at the College of Education and Human Development at GMU. Dr. Norton may be reached at 703-993-2015 for questions or to report a research-related problem. Laura McConnaughey may be reached at 703-389-9943 or email: lmcconna@masonlive.gmu.edu. You may contact the George Mason University Institutional Review Board (IRB) Office at 703-993-4121 if you have questions or comments regarding your student's rights as a participant in the research. This research has been reviewed according to George Mason University procedures governing your student's participation in this research.

CONSENT

I have read this form, all of my questions have been answered by the research staff, and I agree to allow my child to participate in this study.

______________________________ Student Name (Printed)

______________________________ Parent Signature Date

______________________________ Parent Name (Printed)

Appendix H

ASSENT FORM: Students

IMIST: ***Case Studies of the Perceptions, Attitudes, and Experiences of Students Using an Integrated Mathematics Instructional System Focused on Inquiry Learning and Proficiencies in Symbolic Literacy, Conceptual Literacy, and Problem-solving for High School Mathematics***

ASSENT FORM: Students

My name is Laura McConnaughey, and I am a doctoral student at George Mason University in Learning Technologies Design Research and Mathematics Educational Leadership.

I want to talk to you about a research study I am doing. In our study, we want to learn more about how to help students learn and study math in better ways. Your parents have already agreed that you may take part in the study, so feel free to talk with them about it before you decide whether you want to join the study.

What will happen to me in the study?

We would like you to participate to help us understand how Algebra 2 students, like yourself, study and learn math. If you would like to participate in the study, you will be asked to participate in online classes and the activities provided by the researcher/teacher of this study – me! There would be 4 to 6 online classes offered twice per week for 60 to 90-minutes via Google Hangouts for each of the units. The classes will be offered at a convenient time for you and me. I am requesting your permission to use data I collect through interviews, classes, surveys, and assessments to understand your learning, perceptions, attitudes, and confidence about learning and doing mathematics. I also would like to have information providing your age, class level, gender, and ethnicity. You will choose a pseudonym for this study which will be used to keep all information confidential.

You will be asked to fill out a Math Attitudes and Perceptions Survey (MAPS; Code et al., 2016) and to participate in three interviews: A pre-intervention interview to follow up on the MAPS survey and two post-intervention interviews which will occur after each instructional unit. You will be asked to share your learning experiences using the IMIST units. I will give you a copy of the questions in advance of the interview. Each interview will be 15 to 20-minutes and conducted in person or via Google Hangouts at a time convenient for you and me. The interviews will be audio-recorded and transcribed. Again, you will not be identified by name in the recording or transcription to protect your privacy. The total face-to-face time for this study, instructional time and interviews, is approximately 15 to 21 hours.

I will also provide formative and summative assessments (quizzes and unit test) for each instructional unit. These assessments will be graded numerically to provide you with feedback on your understanding and achievement. No letter grades will be assigned.

What are the risks?
There are no potential risks regarding the content, assessments, interviews, or the use of your identity.

What are the benefits?
There are no benefits to you other than to further research in understanding effective practices in mathematics instruction and students' attitudes, perceptions, and confidence in learning mathematics.

Will anyone know that I am in the study?
The data in this study will be confidential. Your name will not be included on collected data; a pseudonym you choose will be placed on all interview and assessment data. Only the researchers will know the link between your identity and the linked study pseudonym, and only the researchers will have access to the data. The data from this study (the audio-recordings, transcripts, and assessment data) will be stored electronically on the principle investigator's password-protected university computer. Electronic data files will be destroyed after a five-year period. Identifiers may be removed from the data and the de-identified data could be used for future research without additional consent from you.

What if I do not want to participate or decide later to withdraw?
Being in this study is voluntary. You don't have to participate in this study, and you can stop being in the study at any time.

Will I receive anything for being in the study?
You will not receive any compensation for being in this study.

Who can I talk to about this study?
If you have questions about the study or have any problems, you can talk to you parents, or call Dr. Priscilla Norton, the principal investigator of this study (703) 933-2015. If you have questions about the study but want to talk to someone else who is not a part of the study, you can call the Institutional Review Board (IRB) Office at George Mason University at (703) 993-4121.

Your signature below means that you have read the above information about the study, have had a chance to ask questions to help you understand what you will do in this study, and you are willing to be in the study. Your signature also means that you have been told that you can change your mind later if you want to.

____________________________ ____
Student Name (Printed) Student Signature Date

Parent Name (Printed)

Appendix I

IRB Approval

Reply all | Delete Junk | •••

IRBNet Board Action

BD Bess Dieffenbach <no-reply@irbnet.org>

Today, 12:19 PM

Priscilla Norton; lmcconna@masonlive.gmu.edu

Reply all |

Inbox

Please note that George Mason University IRB has taken the following action on IRBNet:

Project Title: [1318697-1] IMIST: Case Studies of the Perceptions, Attitudes, and Experiences of Students Using an Integrated Mathematics Instructional System focused on Inquiry Learning and Proficiencies in Symbolic Literacy, Conceptual Literacy, and Problem-solving for High School Mathematics
Principal Investigator: Priscilla Norton, EDD

Submission Type: New Project
Date Submitted: September 7, 2018

Action: APPROVED
Effective Date: September 24, 2018
Review Type: Expedited Review

Should you have any questions you may contact Bess Dieffenbach at edieffen@gmu.edu.

Thank you,
The IRBNet Support Team

www.irbnet.org

Appendix J

Algebra 2 – IMIST Unit Assessment

Algebra 2 – IMIST Unit Assessment Name: ____________________________

Quadratics – Part 1A

Non-calculator Section Class/Period: ________ Date: ________

Solve each of the following by factoring.

- **Write the solutions on the line at the left.**
- **Then write the equation of the axis of symmetry of the parabola.** (3 points each:1s/2c)

1. $x^2 - 4x - 32 = 0$	2. $x^2 - 17x - 42 = 0$	1. ________________ ________________ 2. ________________ ________________
3. $x^2 - 16x + 64 = 0$	4. $4x^2 - 25 = 0$	3. ________________ ________________ 4. ________________ ________________
5. $x^2 - 5x = 0$	6. $2x^2 - 7x - 15 = 0$	5. ________________ ________________ 6. ________________ ________________

Simplify each of the following. Write in your answer *standard form*. (2 to 3 points each: 1s/1c or 1s/2c)

7. $\sqrt{-48}$	8. $(6 + i) - (3 + 2i)$	7. ______________ 8. ______________
9. $(2 + i) - (3 + 4i) + 3i$	10. $(-2 - 3i)(1 + 2i)$	9. ______________ 10. ______________
11. $(2 + 5i)^2$	12. $\dfrac{2+3i}{4+i}$	11. ______________ 12. ______________

13. For each of the following complex numbers A – D: (9 pts)

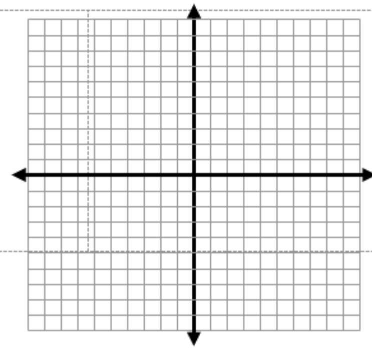
- **Graph as a point in the complex plane.** (5s)
- **Classify the number as *real*, *imaginary*, or *pure imaginary*, or *neither*.** (4s - vocabulary)

A. -5

B. $3 + 6i$

C. $-5 - 2i$

D. $-6i$



A. ________________

B. ________________

C. ________________

D. ________________

Algebra 2 – IMIST Unit Assessment
Quadratics – Part 2A
Calculator Section

Name: ___________________________

Class/Period: ________ Date: ________

Short Answer. Write the answer to each question on the space at the right. (2 points each: 2c)

1. Given $y = ax^2 + bx + c$, "c" is the ______.	________________
2. Given $y = ax^2 + bx + c$ with $0 < a < 1$, as a increases to 1, the graph gets______.	________________
3. Given $y = ax^2 + bx + c$, the x-coordinate of the vertex is______.	________________
4. If i is the imaginary unit, then i^{81} = ______.	________________
5. If i is the imaginary unit, the multiplicative inverse of i is:	________________
6. Given $y = ax^2 + bx + c$ with $a < 0$, the parabola opens ______.	________________
7. If the discriminant of a quadratic equation is zero, it has ____ ______ root(s).	________________

Multiple Choice. Write the letter of best answer to each question on the space at the left. (3 pts each)

_____(3c)	8. Give the value of the discriminant and describe the nature of the roots of the equation $4x^2 - 8x + 4 = 0$ A. 0; 1 real root B. –128; 2 complex roots C. 128; 2 real, irrational roots D. 128; 2 real rational roots E. 128; 2 complex roots
___(1s/2c)	9. What is the vertex for the equation defined by $y = -2(x-3)^2 + 5$? A. (-3, 5) B. (-3, -5) C. (3, 5) D. (-6,5) E. (-6, 0)
___(1s/2c)	10. What is the equation of the axis of symmetry of the parabola defined by $y = \frac{1}{3}(x-6)^2 + 1$? A. $x = 2$ B. $x = -2$ C. $x = -18$ D. $x = 6$ E. $x = -6$

Solve each of the following for x using the given method. Leave answers in simplest radical form.

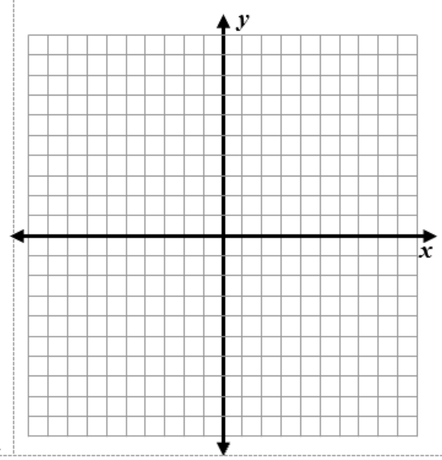
SHOW ALL WORK! (2 pts each: 1s/1c)

11. Solve by finding square roots: (3pts) $\frac{1}{5}(x-4)^2 = 7$	12. Solve by completing the square: (4pts) $2x^2 + x - 9 = 0$	11. ____________ 12. ____________

13. Given the domain of <u>Complex Numbers</u>, solve the following by using the quadratic formula. (12 pts)

List answers in standard form. SHOW WORK!

a. $y = 2x^2 - 3x + 4$



b. Write the equation in vertex form.

c. Graph the parabola. (2s/2c)
Label critical points and features.

a. __________(2s/2c)

b. __________(2s/2c)

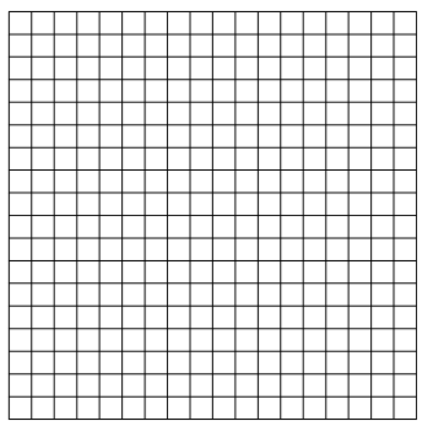
14. Given $f(x) = -7 + x^2 - 6x$, complete each of the following. SHOW ALL WORK! (16 pts)

a. Rewrite the equation in standard form.

b. Rewrite the equation in intercept form.

c. List the roots of the equation.

d. Rewrite the equation in vertex form.



e. Find the coordinates of the vertex.

f. Graph the function (3s/3c)
Label all critical points and features on the graph.

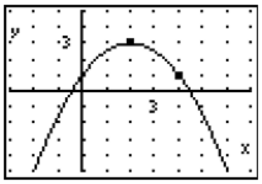
a. ____________(1s)

b. ____________(2s)

c. ____________(2s)

d. __________(1s/2c)

e. __________(1s/1c)

15. Write an equation of the function for the graph below. (7 pts)  **Name and describe the transformations and their effects on the graph as applied to $y = x^2$.** (4c)	15. ______________(3c)

____

____

____

____

16. Write a quadratic equation in ***vertex form*** of the function with vertex (−4, 6) passing through the point (−3, 52) __(4 pts: 2s/2c)	16. ________________

17. Mrs. McMath has traveled north to Cape Cod on her way to visit friends in Duxbury, MA. She is standing in the middle of the Bourne Bridge 148 feet above the water. If she throws a quahog clam (pronounced: co-hog) upward with an initial velocity of 24 feet per second, find the following information.

ALL WORK AND CALCULATOR-READY EQUATIONS __(11 pts: problem-solving)

a. Write the equation that models the situation (Think throwing object!)	a. ___________________
b. How long did it take the quahog to reach its maximum height?	b. ___________________
a. What is the maximum height reached by the quahog?	c. ___________________
c. When will the quahog hit the water? SHOW CALCULATOR READY EQUATIONS!	d. ___________________

References

Adams, A. (2010). Rehearsal or reorganization: Two patterns of literacy strategy use in secondary mathematics classes. The Montana Mathematics Enthusiast, 7(2 & 3), 370-390.

Adams, A. E., Pegg, J., & Case, M. (2015). Anticipation guides: Reading for understanding mathematics. Mathematics Teacher, 108(7), 498-504. doi:10.5951/mathtacher.108.7.0498

Adams, W. K., Perkins, K. K., Podolefsky, N. S., Dubson, M., Finkelstein, N. D., & Wieman, C. E. (2006). New Instrument for measuring student beliefs about physics and learning physics: The Colorado learning attitudes about science survey. Physical Review Special Topics - Physics Education Research, 2, 010101, 1-14. doi:10.1103/PhysRevSTPER.2.010101

Akers, R. (2017). A journey to increase student engagement. Technology and Engineering Teacher, 76(5), 28-32.

Avetisyan, N., & Hayrapetyan, L. (2017). Mathlet as a new approach for improving critical and creative thinking skills in mathematics. International Journal of Education Research, 12(1), 34-44.

Ball, D., Thames, M. & Phelps, G. (2008). Content knowledge for teaching: What makes it special? Journal of Teacher Education (59)5, 389-407. doi:10.1177/0022487108324554.

Bandura, A. (1977). Self-efficacy: The exercise of control. New York, NY: Freeman.

Bannan-Ritland, B. (2003). The role of design in research: The integrative learning design framework. Educational Researcher, V32(21). doi:10.3102/0013189x0320002021. Retrieved from http://edr.sagepub.com/content/32/1/21

Barell, J. (2007). Problem-based learning: An inquiry approach. Thousand Oaks, CA: Corwin Press.

Barkatsas, A., Kasimatis, K., & Gialamas, V. (2009). Learning secondary mathematics with technology: Exploring the complex interrelationships between students' attitudes, engagement, gender and achievement. Computers and Education, 52, 562-570. doi: 10.1016/j.compedu.2008.11.001

Battista, M. T. (1999). The mathematical miseducation of America's youth: Ignoring research and scientific study in education. Phi Delta Kappan, 80(6), 425-433.

Berger, M. (2019). Different reading styles for mathematics text. Educational Studies in Mathematics, 100(2), 139-159. doi:10.1007/s10649-018-9871-y

Bergmann, J., & Sams, A. (2012). Flip your classroom: Reach every student in every class every day. Eugene, OR: International Society for Technology in Education. doi:10.1177/073989131401100120

Blair, A. (2014). Inquiry maths: An idea whose time has come. Mathematics Teaching, 240, 32-35.

Boaler, J., & Staples, M. (2008). Creating mathematical futures through an equitable teaching approach: The case of Railside School. Teachers College Record, 110(3), 608-645.

Bodovski, K., Byun, S., Chykina V., & Chung H. J. (2017). Searching for the golden model of education: Cross-national analysis of math achievement. Compare: A Journal of Comparative and International Education, 47(5), 722-741. doi:10.1080/03057925.2016.1274881

Bower, M. (2008). Affordance analysis: Matching learning tasks with learning technologies. Educational Media International, 45(1), 3-15. doi:10.1080/09523980701847115

Brown, J. S., Collins, A., & Duguid, P. (1989). Situated cognition and the culture of learning. Educational Researcher, 18(1), 32-42. doi:10.3102/0013189X018001032

Chandler, K., Fortune, N., Lovell, J. N., & Scherrer, J. (2016). What should common core assessments measure? Kappan, 97(5), 60-63. doi:10.1177/0031721716629660

Cheema, J. R., & Sheridan, K. (2015). Time spent on homework, mathematics anxiety and mathematics achievement: Evidence from a US sample. Issues in Educational Research, 25(3), 246-259. Retrieved from http://www.iier.org.qu/iier25/cheema.pdf

Cheung, A. C. K., & Slavin, R. E. (2013). The effectiveness of educational technology applications for enhancing mathematics achievement in K-12 classrooms: A meta-analysis. Educational Research Review, 9, 88-113. doi:10.1016/j.edurev.2013.01.001

Clements & Sarama (2004). Learning trajectories in mathematics education. Mathematical Thinking and Learning, 6(2), 81-89. doi:10.1207/s15327833mtl0602_1

Clopton, P., Bishop, W., & Klein, D. (2001). Standards-based mathematics assessments: Appearances may be deceiving. In S. Stotsky (Ed)., What's at stake in K-12 standards wars: A primer for educational policy makers (pp. 59-77). New York, NY: Peter Lang Publishing. doi:10.1007/s12129-000-1056-2

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. Educational Researcher, 32(1), 9-13. doi:10.3102/0013189X0320010009

Code, W., Merchant, S., Maciejewski, W., Thomas, M., & Lo, J. (2016). The Mathematics Attitudes and Perceptions Survey: An instrument to assess expert-like views and dispositions among undergraduate mathematics students. International Journal of Mathematical Education in Science and Technology, 47(6), 917-937. doi: 10.1080/0020739X.2015.1133854

Collie, R. J., & Martin, A. J. (2017). Students' adaptability in mathematics: Examining self-reports and teachers' reports and links with engagement and achievement outcomes. Contemporary Educational Psychology, 49, 355-366. doi:10.1016/j.cedpsych.2017.04.001

Common Core State Standards Mathematics Initiative. (2010). Common Core State Standards for Mathematics (CCSSM). Washington, DC: National Governors Association Center for Best Practices, Council of Chief State School Officers. Retrieved from http://www.corestandards.org/Math/. doi:10.2139/ssrn.3331689

Cooper, H. (2008). Homework: What the research says. Research Brief National Council of Teachers of Mathematics. Retrieved from https://www.nctm.org/Research-and-Advocacy/Research-Brief-and-Clips/Homework_-What-Research-Says/

Crawford, K., Gordon, S., Nicholas, J., & Prosser, M. (1998). University mathematics students' conceptions of mathematics. Studies in Higher Education, 23(1), 87-94. doi: 1080/03075079812331380512

Creswell, J. W. (2012). Educational research: Planning, conducting, and evaluating quantitative and qualitative research (4th ed.). Boston, MA: Pearson.

Crouch, C., & Pearce, J. (2012). Doing research in design. London, UK: Bloomsbury.

Dai, D. Y. (Ed.). (2012). Design research on learning and thinking in educational settings: Enhancing intellectual growth and functioning. New York, NY: Routledge. doi:10.4324/9780203849576

Dahlstrom, E., & Bichsel, J. (2014). ECAR study of undergraduate student and information technology. Louisville, CO: EDUCAUSE Center for Analysis and Research (ECAR). Retrieved from: htttp://www.educause.edu/ecar

Dalziel, J. (2015). ASLD 1. Reflections on the art and science of learning design and the Larnaca Declaration. In Maina, M., et al (Eds.). The art & science of learning design, pp. 3-14. Rotterdam: Sense Publishers.

Darling, N. (2013). Reading and writing strategies for mathematics. AURCO Journal, 19, 178-188.

Davis, J. D., & Shih, J. C. (2007). Secondary options and post-secondary expectations: Standards-based mathematics programs and student achievement on college mathematics placement exams. School Science and Mathematics, 107(8), 336-346. doi: 10.1111/j.1949-8594.2007.tb17798.x

Di Martino, P., & Zan, R. (2015). The construct of attitude in mathematics education. In B. Pepin & B. Roesken-Winter (Eds.), From beliefs to dynamic affect systems in mathematics education (pp. 51-72). Cham, Switzerland: Springer International Publishing. doi:10.1007/978-3-319-06808-4_3

Dimitrov, D. M. (2010). Quantitative research in education: Intermediate and advanced methods. New York, NY: Whittier Publications, Inc.

Diversity in Mathematics Education Center for Learning and Teaching (DMECLT) (University of California, Los Angeles; University of Wisconsin, Madison; University of California Berkeley). (2007). Culture, race, power, and mathematics education. In F. K. Lester (Ed.), Second handbook of research on mathematics teaching and learning (pp. 405-434). Charlotte, NC: Information Age Publishing. doi:10.1093/ww/9780199540884.013.266578

Dougherty, E. (2012). Assignments matter: Making the connections that help students meet standards. Alexandria, VA: ASCD.

Editorial: IRA: Elevate our expectations for America’s students: National standards should be high, credible, research-based, teacher oriented [Editorial]. (2009-2010). Reading Today, 27(3), 6.

Educause. (2012). 7 things you should know about microlectures. Retrieved from https://net.educause.edu/ir/library/pdf/ELI7090.pdf

Eisner, E. (1994) Cognition and curriculum: Reconsidered. New York, NY: Teachers College Press.

Ellis, M. W., & Berry, R. Q. (2005). The paradigm shift in mathematics education: Explanations and implications of reforming conceptions of teaching and learning. The Mathematics Educator, 15(1), 7-17.

Enderson, M. C., & Ritz, J. (2016). STEM in general education: Does mathematics competence influence course selection. Journal of Technology Studies. 42(1), 30-40. doi:10.21061/jots.v42i1.a.3

Engestrom, Y. (1987). Learning by expanding: An activity-theoretical approach to developmental research. Helsinki, Finland: Oreinta-Konsultit. Retrieved from http://lchc.ucsd.edu/MCA/Pater/Engestrom/exapanding/toc.htm. doi:10.1017/cbo0781139814744.001

Fan, L., Zhu, Y., & Zhenzhen, M. (2013). Textbook research in mathematics education: Development status and direction. ZDM Mathematics Education, 45, 633-646. doi:10.1007/s11858-013-0538-x

Fontichiaro, K. (2014). Finding time for inquiry learning. School Library Monthly, 31(1), 50-51.

Frank, T. (2016, November 9). EDCI 856: Mathematics education curriculum design and evaluation [Class notes]. College of Education and Human Development, George Mason University, Fairfax, Virginia, United States of America.

Freeman, S., Eddy, S. L., McDonough, M., Smith, M. K., Okoroafor, N., Jordt, H., & Wenderoth, M. P. (2013). Active learning increases student performance in science, engineering, and mathematics. Proceedings of the National Academy of Sciences, USA, 111(23), 8410-8415. doi:10.1073/pnas.1319030111

Fulton, K. P. (2012). 10 reasons to flip. Phi Delta Kappan, 94(2), 20-24. doi:10.1177/003172171209400205

Glanz, J. (2016). Action research by practitioners: A case study of a high school's attempt to create transformational change. Journal of Practitioner Research, 1(1), 1-23. doi:10.5038/2379-9951.1.1.1027

Glesne, C. (2011). Becoming qualitative researchers (4th Ed.). Boston, MA: Pearson.

Goodwin, B., & Miller K. (2013). Evidence on flipped classrooms is still coming in. Educational Leadership, 70(6), 78-80.

Green, J. C. (2007). Mixed methods in social inquiry. San Francisco, CA: John Wiley & Sons, Inc.

Greenberg, J., & Walsh, K. (2008). No common denominator: The preparation of elementary teachers in mathematics by America's education schools. Retrieved from National Council on Teacher Quality website: http://www.nctq.org

Hannula, M., DiMartino, P., Pantziara, M., Zhang, Q., Morselli, F., … Goldin, G. (2016). Attitudes, beliefs, motivation, and identity in mathematics education. In G. A. Golden (Ed.), Attitudes, beliefs, motivation and identity in mathematics education (pp. 1-35). ICME-13 Topical Surveys. Cham, Switzerland: Springer. doi:10.1007/978-3-319-32811-9_1

Hanushek, E. A., Woessmann, L., Jamison, E. A., & Jamison, D. T. (2008). Education and economic growth. Education Next; Stanford, 8(2), 1-12.

Hasan, H., & Kazlauskas, A. (2014). Activity theory: Who is doing what, why and how. In H. Hasan (Ed.), Begin practical with theory: A window into business research (pp. 9-14). Wollongong, Australia: THEORI.

Hathaway, D., & Norton, P. (2013). Designing an online course content structure using a design patterns approach. Educational Technology, 53(2), 3-15.

Hathaway, D., & Norton, P. (2018). Understanding problems of practice: A case study in design research. Cham, Switzerland: Springer. doi:10.1007/978-3-319-77559-3

Hattie, J. A. C. (2009). Visible learning: A synthesis of over 800 meta-analyses relating to achievement. New York, NY: Routledge. doi:10.4324/9780203887332

Haystead, M. W., & Marzano, R. J. (2009) Meta-analytic synthesis of studies conducted at Marzano Research Laboratory on instructional strategies. Englewood, CO: Marzano Research Laboratory.

Herzig, A. H. (2005). Goals for achieving diversity in mathematics classrooms. The Mathematics Teacher, 99(4), 253-259.

Hirsch, E. D., Jr. (2004, May). Mental fight: A prescription for K-12. In What our universities and ours schools owe each other. Symposium conducted at the meeting of the National Association of Scholars (NAS) in New York City.

Hmelo-Silver, C. E., Duncan, R. G., & Chinn, C. A. (2006). Scaffolding and achievement in problem-based and inquiry learning: A response to Kirschner, Sweller, and Clark. Educational Psychologist, 42(2), 99-107. doi:10.1080/00461520701263368

Holmes, V. L., & Hwang, Y. (2016). Exploring the effects of project-based learning in secondary mathematics education. The Journal of Educational Research, 109(5), 449-463. doi: 10.1080/00220671.2014.979911

Jacobs, J. (2011). A winning formula for mathematics instruction. Alexandria, VA: Educational Research Institute.

Ketterlin-Geller, L. R., Gifford, D. B., & Perry, L. (2015). Measuring middle school students' algebra readiness: Examining validity evidence for three experimental measures. Assessment for Effective Intervention, 41(1), 28-40. doi:10.1177/1534508415586545

Khan, S., & VanWynsberghe, R. (2008). Cultivating the under-mined: Cross-case analysis as a knowledge mobilization. Forum Qualitative Sozialforschung/Forum: Qualitative Social Research, 9(1), Art 24. Retrieved from http://www.qualitative-research.net/index.php/fqs/article/view/334/729

Kilpatrick, J., Swafford, J., & Findell, B. (Eds) (2001). Adding it up: Helping children learn mathematics. Washington, DC: National Academy Press.

Kirschner, P. A., Sweller, J., & Clark, R. E. (2006). Why minimal guidance during instruction does not work: An analysis of the failure of constructivist, discovery, problem-based, experiential, and inquiry-based teaching. Educational Psychologist, 41, 75–86. doi:10.1207/s15326985ep4102_1

Kisantas, A., Cheema, J. & Ware, H. W. (2011). Mathematics achievement: The role of homework and self-efficacy beliefs. Journal of Advanced Academics, 22(2), 310-339. doi:10.117/1932202X1102200206

Klein, D. (2003). A brief history of American K-12 mathematics education in the 20th century. In J. Royer (Ed.), Mathematical Cognition (pp. 175-259). Greenwich, CT: Information Age Publishing. Retrieved from www.csun.edu/~vcmth00m?AHistory.html

Kvale, S., & Brinkmann, S. (2009). Interviews: Learning the craft of qualitative research interviewing. Los Angeles, CA: Sage.

Larson, R., Boswell, L., et al. (2007). Algebra 2. Austin, TX: Holt McDougal Publishing.

Larson, R., Boswell, L., et al. (2012). Algebra 2. Austin, TX: Holt McDougal Publishing.

Larson, R., Boswell., L., & Stiff, L., (2003). Geometry. Austin, TX: Holt McDougal Publishing.

Leont'ev, A. N. (1981). Problems of the development of the mind. Moscow. USSR: Progress.

Lesh, A., & Kelly, A. (2000). Multitiered teaching experiments. In A. Kelly & R. Lesh (Eds.), Research design in Mmathematics and science dducation (pp. 197-230). Mahwah, NJ: Lawrence Erlbaum Associates.

Li, Q., & Ma, X. (2010). A meta-analysis of the effects of computer technology on school students' mathematics learning. Educational Psychology Review, 22(3), 215-243. doi:10.1007/s10648-010-9125-8

Liedtka, J., King, A., & Bennett, K. (2013). Solving problems with design thinking: 10 stories of what works. New York, NY: Columbia Business School Publishing.

Maina, M., Craft, B., & Mor, V. (Eds.) (2015). The art & science of learning design. Rotterdam, The Netherlands: Sense Publishers. doi:10.1007/978-94-6300-103-8

Maltese, A. V., Tai, R. H., & Fan, X. (2012). When is homework worth the time? Evaluating the association between homework and achievement in high school science and math. The High School Journal, 96(1), 52-70. doi:10.1352/hsj.2012.0015

Maxwell, J. A. (2013). Qualitative research design: An interactive approach. Los Angeles, CA: SAGE.

McKenney, S. & Reeves, T. C. (2012). Conducting educational design research. New York, NY: Routledge.

Milgram, R. J. (2007). What is mathematical proficiency? In A. Schoenfeld (Ed.), Assessing mathematical proficiency (pp. 31-58). Cambridge UK: Cambridge University Press. doi:10.1017/cbo9780511755378.007

Miller, J. D., & Kimmel, L. G. (2012). Pathways to a STEMM profession. Peabody Journal of Education, 87(1), 26-45. doi:10.1080/0161956X.2012.642274

Mor, Y., & Craft, B. (2012). Learning design: Reflections upon the current landscape. Research in Learning Technology, 20. doi:10.3402/rlt.v20i0.19196

Mullis, I. V. S., Martin, M. O., Foy, P., & Hooper, M. (2016). TIMSS 2015 international results in mathematics. Chestnut Hill, MA: TIMSS & PIRLS International Study

Center. Retrieved from http://timssandpirls.bc.edu/timss2015/international-results/wp-content/uploads/filebase/full%20pdfs/T15-International-Results-in-Mathematics-Grade-8.pdf

Nardi, B. A. (1996). Studying in context: A comparison of activity theory, situated action models, and distributed cognition. In B. A. Nardi (Ed.), Context and consciousness: activity theory and human-computer interaction (pp. 69-100). Cambridge, MA: MIT Press. doi:107551/mitpress/2137.001.0001

Nasir, N. S., Hand, V., & Taylor, E. V. (2008). Culture and mathematics in school: Boundaries between "cultural" and "domain" knowledge in the mathematics classroom and beyond. Review of Research in Education, 32, 187-240. doi:10.3102/009173X07308962

National Assessment for Educational Progress (NAEP). (2012). Trends in academic progress. Washington, DC: Institute of Educational Sciences (IES). Retrieved from https://nces.ed.gov/nationsreportcard/subject/publications/main2012/pdf/2013456.pdf

National Assessment for Educational Progress (NAEP). (2015). 2015 assessment report cards. Washington, DC: National Center for Educational Statistics (NCES). Retrieved from https://www.nationsreportcard.gov/reading_math_2015/files/2015_Results_Appendix_Math.pdf

National Center for Education Statistics (NCES). (2003). The condition of education 2003 (NCES 2003-067). Washington, DC: U.S. Department of Education, National Center for Education Statistics. Retrieved from https://nces.ed.gov/puts2003/2003067.pdf.

National Center for Educational Statistics (NCES). (2017a). NAEP: History and background. Retrieved from https://nces.ed.gov/nationsreportcard/about/naephistory.aspx

National Center for Educational Statistics (NCES). (2017b). Trends in International Mathematics and Science Studies (TIMSS). Retrieved from https://nces.ed.gov/timss/

National Council of Teachers of Mathematics. (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics. Reston, VA: Author.

National Council of Teachers of Mathematics. (2014). Principles to actions: Ensuring mathematical success for all. Reston, VA: Author.

National Mathematics Advisory Panel (NMP). (2008). Foundations for success: The National Mathematics Advisory Panel final report. Washington, DC: U.S. Department of Education.

National Research Council (NRC). (2001). Adding it up: Helping children learn mathematics. Washington, DC: National Academies Press. Retrieved from http://nap.edu/9822. doi:10.17226/9822

National Research Council (NRC). (2013). Monitoring progress toward successful K-12 STEM education: A nation advancing? Washington, DC: The National Academies Press.

National Science Board (NSB). (2016). Elementary and secondary mathematics and science education. In Science and Engineering Indicators 2016 (NSB-2016-1). Arlington, VA: National Science Foundation. Retrieved from https://www.nsf.gov/statistics/2016/nsb20161/#/report/chapter-1/student-learning-in-mathematics-and-science

Nie, B., Freedman, T., Hwang, S., Wang, N., Moyer, J. C., & Cai, J. (2013). An investigation of teachers' intentions and reflections about using standards-based and traditional textbooks in the classroom. ZDM Mathematics Education, 45, 699-711. doi:10.1007/s11858-0130493-7

Norris, C., & Soloway, E. (2016). Educational technology is at a fork: Instruction vs construction. The Journal. Retrieved from https://thejournal.com/articles/2016/05/16/educational-technology-is-at-a-fork.aspx

Organisation for Economic Co-operation and Development (OECD). (2012a). PISA 2012 results in focus. Retrieved from https://www.oecd.org/pisa/keyfindings/pisa-2012-results-overview.pdf

Organisation for Economic Co-operation and Development (OECD). (2012b). Country note: Programme for International Student Assessment (PISA) results from PISA 2012. Retrieved from https://www.oecd.org/unitedstates/PISA-2012-results-US.pdf

Organisation for Economic Co-operation and Development (OECD). (2016a). PISA 2015 results in focus. Retrieved from https://www.oecd.org/pisa/pisa-2015-results-in-focus.pdf

Organisation for Economic Co-operation and Development (OECD). (2016b). Country note: Key findings from PISA 2015 for the United States. Retrieved from https://www.oecd.org/pisa/PISA-2015-United-States.pdf

Petocz, P., Reid, A., Wood, L., Smith, G., Mather, G., Harding, A., … Perrett, G. (2007). Undergraduate students' conception of mathematics: An international study. International Journal of Science and Mathematics Education. 5(3), 439-459. doi:10.1007/s10763-006-9059-2

Plomp, T., & Nieveen, N. (2013). Educational design research part A: An introduction. Netherlands: Netherlands Institute for Curriculum Development (SLO). Retrieved from https://www.researchgate.net/profile/Brenda_Bannan/publication/263733328_The_Integrative_Learning_Design_Framework_114_-_133_An_Illustrated_Example_from_the_Domain_of_Instructional_Technology/links/0046353bc22945ecfd000000/The-Integrative-Learning-Design-Framework-114-133-An-Illustrated-Example-from-the-Domain-of-Instructional-Technology.pdf#page=12

Polly, D. (2016). Evaluating mathematics curricular resources. Teaching Children Mathematics, 22(6), 325-326. doi:10.5951/teacchilmath.22.6.0325

Polya, G. (1945). How to solve it: A new aspect of mathematical method. Princeton, NJ: Princeton University Press.

Popkewitz, T. (Ed.). (1987). The formation of school subjects: The struggle for creating an American institution. New York, NY: Falmer Press. doi:10.2307/368860

Popkewitz, T. (2004). The alchemy of the mathematics curriculum: Inscriptions and the fabrication of the child. American Educational Research Journal, 41(1), 3-34. doi:10.3102/00028312041001003

Post, T. R., Harwell, M. R., Davis, J. D., Meaeda, Y., Cutler, A, Anderson, E. A., … Norman, K. W. (2008). Standards-based mathematics curricula and middle-grades students' performance on standardized achievement tests. Journal for Research in Mathematics Education, 39(2), 184-212.

Prentice Hall. (2008). Algebra 2. Upper Saddle River, NJ: Prentice Hall Publishing.

Programme for International Student Assessment (PISA). (2017). About. Retrieved from http://www.oecd.org/pisa/aboutpisa/

Pulaski, M. A. S. (1997). Understanding Piaget: An introduction to children's cognitive development. New York, NY: Harper & Row Publishers.

Ramaley, J. A. (2007). Aims of mathematics education. In A. H. Schoenfeld (Ed.), Assessing mathematical proficiency (pp. 17-21). Cambridge, UK: Cambridge University Press. doi:10.1017/CBO9780511755378.004

Robbins, J. (2013). Uncommonly bad. Academic Questions, 26(1), 9. doi:10.1007/s12129-013-9336-9

Roepke, T. L., & Gallagher, D. K. (2015). Using literacy strategies to teach precalculus and calculus: Instructional strategies used by language arts teachers are adapted for the mathematics classroom. Mathematics Teacher, 108(9), 672-678. doi:10.5951/mathteacher.108.9.0672

Rose, K. K. (2009). Student perceptions of the use of instructional videos in online and face-to-face classes. Journal of Online Learning and Teaching, 5(3). Retrieved from http://jolt.merlot.org/vol5no3/rose_0909.htm

Sahal, M., & Ozdemir, A. S. (2019). Examination of activities and problems oriented toward algebra readiness indicators proposed by pre-service mathematics teachers. Acta Didactica Napocensia, 12(1), 165-176. doi:10.24193/and.12.1.12

Saldaña, J. (2016). The coding manual for qualitative researchers. Los Angeles, CA: SAGE Publications.

Schmidt, H. G., Loyens, S. M. M., van Gog, T., & Paas, F. (2007). Problem-based learning is compatible with human cognitive architecture: Commentary on Kirschner, Sweller, and Clark (2006). Educational Psychologist, 42(2), 91-97. doi:10.1080/00461520701263350

Schmidt, W. H. (2012). At the precipice: The story of mathematics education in the United States. Peabody Journal of Education, 87(1), 133-156. doi:10.1080/0161956X.2012.642280

Schöber, C., Schutte, K, Koller, O., McElvany, N., & Gebauer, M. M. (2018). Reciprocal effects between self-efficacy and achievement in mathematics. Learning and Individual Differences, 63, 1-11. doi:10.1016/j.lindif.2018.01.008

Schoenfeld, A. H. (1989). Explorations of students' mathematical beliefs and behavior. Journal for Research in Mathematics Education, 20(4), 338-355. doi:10.2307/749440

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. New York, NY: Macmillan.

Schoenfeld, A. H. (2004). The math wars. Educational Policy, 18(1), 253-286. doi:10.1177/0895904803260042

Schoenfeld, A. H. (2006). What is mathematical proficiency and how can it be assessed? In A. Schoenfeld (Ed.), Assessing mathematical proficiency (pp. 59-73). Cambridge, UK: Cambridge University Press. doi:10.1017/CBO9780511755378.008

Schoenfeld, A. H. (Ed.). (2007a). Assessing mathematical proficiency. Cambridge, UK: Cambridge University Press. doi:10.1017/CBO9780511755378

Schoenfeld, A. H. (2007b). Issues and tensions in the assessment of mathematical proficiency. In A. H. Schoenfeld (Ed.), Assessing mathematical proficiency (pp. 3-16). Cambridge, UK: Cambridge University Press. doi:10.1017/CBO9780511755378.003

Schoenfeld, A. H. (2014). What makes for powerful classrooms, and how can we support teachers creating them? A story of research and practice, productively intertwined. Educational Researcher, 43(8), 404-412. doi:10.3102/0013189X14554450

Schoenfeld, A. H., & Kilpatrick, J. (2013). A US perspective on the implementation of inquiry-based learning in mathematics. ZDM, 45(6), 901-909. doi:10.1007/s11858-013-0531-5

Skaalvik, E. M., Federici, R. A., & Klassen, R. M. (2015). Mathematics achievement and self-efficacy: Relations with motivation for mathematics, International Journal of Educational Research, 72, 129-136. doi:10.1016/j.ijer.2015.06.008

Stanic, G. M. A. (1987). Mathematics education in the United States at the beginning of the twentieth century. In T S. Popewitz (Ed.), The formation of school subjects: The struggle for creating an American institution (pp. 147-189). New York, NY: Falmer.

Star, J. R., & Hoffman, A. J. (2005). Assessing the impact of standards-based curricula: Investigating students' epistemological conceptions of mathematics. The Mathematics Educator, 15(2), 25-34.

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. American Educational Research Journal, 33(2), 455-488. doi:10.3102/00028312033002455

Stein, M. K., & Kaufman, J. H. (2010). Selecting and supporting the use of mathematics curricula at scale. American Educational Research Journal, 47(3), 663-693. doi:10.3102/0002831209361210

Stein, M. K., Remillard, J., & Smith. M. S. (2007). How curriculum influences student learning. In F. K. Lester (Ed.), Second handbook of research on mathematics teaching and learning (pp. 319-370). Charlotte, NC: Information Age Publishing.

Stouraitis, K., Potari, D., & Skott, J. (2017). Contradictions, dialectical oppositions and shifts in teaching mathematics. Educational Studies in Mathematics, 95, 203-217. doi:10.1007/s10649-017-9749-4

Strayer, J. (2012). How learning in an inverted classroom influences cooperation, innovation, and task orientation. Learning Environments Research. 15(2), 171-193. doi:10.1007/s10984-012-9108-4

Sweet, D. (2014). Microlectures in a flipped classroom: Application, creation, and resources. Mid-Western Educational Researcher, 26(1), 52-59.

Tapscott, D. (2009). Grown up digital. New York, NY: McGraw Hill.

Taylor, M. W. (2013). Replacing the “teacher-proof” curriculum with the “curriculum-proof” teacher: Toward more effective interactions with mathematics textbooks. Journal of Curriculum Studies, 45(3), 295-321. doi:10.1080/00220272.2012.710253

Terada, Y. (2019). Students think lectures are best, but research suggests they’re wrong. Edutopia, http://www.edutopia.org/. Retrieved from https://www.edutopia.org/article/students-think-lectures-are-best-research-suggests-theyre-wrong

Thompson, C. J. (2009). Preparation, practice, and performance: An empirical examination of the impact of standards-based instruction on secondary students’ math and science achievement. Research in Education, 81(1), 53-62. doi:10.7227/RIE.81.5

Tishman, S., Perkins, D. N., & Jay, E. (1995). The thinking classroom: Learning and teaching in a culture of thinking. Boston, MA: Allyn and Bacon.

Toscano, M. (2013). The common core: Far from home. Academic Questions, 26, 411-428. doi:10.1007/s12129-013-9389-9

Tran, D. (2016). Statistical association: Alignment of current U. S. high school textbooks with Common Core State Standards for Mathematics. School Science and Mathematics, 116(5), 286-296. doi:10.1111/ssm.12179

Tucker, B. (2012). The flipped classroom. Education Next, 12(1), 82-83.

Valentine, J. C., DuBois, D. L., & Cooper, H. (2004). The relation between self-beliefs and academic achievement: A meta-analytic review. Educational Psychologist, 39(2), 111-133. doi:10.1207/s15326985ep3902_3.

van Jaarsveld, P. (2016). Making a case for exact language as an aspect of rigor in initial teacher education mathematics programmes. Perspectives in Education, 34(1), 150-166. doi:10.18820/2519593X/pie.v34i1.11

Venkat, H., & Adler, J. (2008). Expanding the foci of activity theory: Accessing the broader contexts and experiences of mathematics education reform. Educational Review, 60(2), 127-140. doi:10.1080/00131910801933914

Virginia Department of Education (VA DOE). (2016). Standards of learning and SOL testing: Mathematics 2016. Retrieved from http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/2016/index.shtml

Vygotsky, L. S. (1978). Mind and society: The development of higher mental processes. Cambridge, MA: Harvard University Press.

Warshauer, H. K. (2015). Strategies to support productive struggle. Mathematics Teaching in the Middle School, 20(7), 390-393. doi:10.5951/mathteacmiddscho.20.7.0390

Willis, B. (2014). The advantages and disadvantages of single case study analysis. Retrieved from https://www.e-ir.info/2014/07/05/the-advantages-and-limitations-of-single-case-study-analysis/

Wu, H. (1999). Basic skills versus conceptual understanding. American Educator, 23(3), 14-19.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. Journal for Research in Mathematics Education, 27, 458-477. doi:10.2307/749877

Yin, R. K. (2014). Case study research: Design and methods. Thousand Oaks, CA: SAGE Publications.

Zimmerman, B. J. (1995). Self-efficacy and educational development. In A. Bandura (Ed.), Self-efficacy in changing societies (pp. 202-231). Cambridge, UK: Cambridge University Press. doi:10.1017/CB09780511527692.009

Biography

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