# THE AP CALCULUS EXAM READING EXPERIENCE: IMPLICATIONS FOR TEACHER CLASSROOM PRACTICE AND STUDENT COMPREHENSION

by

Mimi Corcoran
A Dissertation
Submitted to the
Graduate Faculty
of
George Mason University
in Partial Fulfillment of
The Requirements for the Degree
Doctor of Philosophy
Education

Committee:

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Date: ________________________________ Summer Semester 2017
George Mason University
Fairfax, VA

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## Dedication

This is dedicated to my loving, patient and immensely encouraging and insightful father, James E. Corcoran.

Dad, you taught me the important lessons in life. You guided me in faith. You nurtured in me a love of logic, science and mathematics, coupled with humanity. You modeled honesty, ethical behavior, a high moral code and humility every day. You are the best person I have ever known. I hope you are looking down on me from heaven with pride. I will always love you deeply and I miss you immeasurably.

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## List of Abbreviations

Advanced Placement ........ AP
Advanced Placement Incentive Program ........ APIP
American Educational Research Association ........ AERA
American Society for Engineering Education ........ ASEE
Assistant Chief Reader ........ ACR
Association for Supervision and Curriculum Development ........ ASCD
Center for Outreach in Mathematics Professional Learning and Educational Technology ........ COMPLETE
Center for the Study of Teaching and Policy ........ CTP
Chief Reader ........ CR
Cognitively Guided Instruction ........ CGI
College Admission with Advanced Standing ........ CAAS
College Board ........ CB
Common Content Knowledge ........ CCK
Consortium for Policy Research in Education ........ CPRE
Educational Testing Service ........ ETS
Exam Reader ........ RD
Exam Leader ........ EL
Fund for the Advancement of Education ........ FAE
George Mason University ........ GMU
Grade Point Average ........ GPA
Horizon Content Knowledge ........ HCK
Impact on Student Learning ........ ISL
Individually Prescribed Instruction ........ IPI
International Baccalaureate ........ IB
International Congress on Mathematical Education ........ ICME
Knowledge of Contents and Curriculum ........ KCC
Knowledge of Contents and Students ........ KCS
Knowledge of Contents and Teaching ........ KCT
Looking at Student Work ........ LASW
Left Rectangular Area Method ........ LRAM
Mathematical Knowledge for Teaching ........ MKT
National Assessment of Educational Progress ........ NEAP
National Association for Research in Science ........ NARST
National Association of Secondary School Principals ........ NASSP
National Center for Educational Accountability ........ NCEA

| | |
|---|---|
| National Council of Teachers of Mathematics | NCTM |
| National Council of Supervisors of Mathematics | NCSM |
| Pedagogical Content Knowledge | PCK |
| Professional Development | PD |
| Question Leader | QL |
| Right Rectangular Area Method | RRAM |
| Specialized Content Knowledge | SCK |
| Science, Technology, Engineering and Mathematics | STEM |
| Specialized Content Knowledge | SPK |
| Table Leader | TL |

**Abstract**

THE AP CALCULUS EXAM READING EXPERIENCE: IMPLICATIONS FOR TEACHER CLASSROOM PRACTICE AND STUDENT COMPREHENSION

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George Mason University, 2017

Dissertation Director: Dr. Jennifer Suh

This dissertation explores the views and experiences of high school calculus teachers and college mathematics professors on the professional development which occurs at the annual national AP Calculus exam reading. This professional development experience comes in several forms: the exam briefing sessions, the actual reading of the exams, the collegial interactions, and the optional formal and informal sessions, which include professional development, which are offered on most evenings during the reading week. Second, this study examines the impact which this professional development has on high school teachers' and professors' classroom practice, as reported by participants. Third, the study addresses changes which results in their students' achievement in Calculus, as perceived by the educators. Fourth, the goal of the study is to develop a working level grounded theory of how the varied aspects of the exam reading influence teacher knowledge, classroom practice, and teachers' perceptions and observations of the influence of their reading experiences on their students' comprehension of calculus. Data

on was collected through an online survey in which both high school teachers and college professors who serve as AP Calculus exam readers voluntarily participated and for which their identities were anonymous. A second data collection was executed through 17 telephone interviews with volunteers from the participant pool. Data were analyzed through descriptive statistics, qualitative commentary, and $\chi^2$ analysis when possible.

## Chapter One

### Introduction

As college admission becomes increasingly competitive, high school students augment their résumés with sports, arts, other extracurricular activities, and community service. However, the holy grail of ambitious college-bound high school students is a transcript with a litany of Advanced Placement (AP) courses. The main advantages of successful completion of AP courses are the prospect of college credit and a record of high achievement through a rigorous curriculum which impresses college admissions officers (Judson & Hobson, 2015). Steinberg (2009) cites a study from the Thomas B. Fordham Institute which found that 90% of over 1,000 surveyed advanced placement teachers attributed increased student interest in AP classes and exams to improving the content of their college applications. Schools also like to bask in the pride of having a website with a competitive number of AP course offerings and numerous student AP success stories. And, given the fierce competition for admission to the nation's top colleges, it is reasonable that students would feel pressure to enroll in AP courses; the AP program experiences strong growth as a result (Bressoud, 2004).

These courses are college freshman level classes, taught as year-long high school courses and usually taken by high school juniors and seniors. There are currently 36 AP courses available. See Appendix B.

These courses are designed to be much more rigorous than regular high school courses. And, to acquire permission to use the AP© identifier on their courses, secondary schools must submit all their AP course syllabi to the College Board for audits. These audits are an attempt to maintain quality control to ensure that AP teachers are teaching college-level content. However, as with any behemoth system, these syllabi submitted for audit are merely a representation of what is intended to be taught; not necessarily what is actually taught. As de Vise (2008) points out, once a school district had a course syllabus approved, it could be shared with everyone in that district and all the audits would be approved. This could be of concern if teachers merely copy the work of a colleague but do not follow the syllabus. Such abuse, if there is any, would be difficult for the College Board to weed out; however, the College Board has indicated that they will monitor courses through several methods, for example, course descriptions on school websites (de Vise, 2008).

Because of this extra course rigor and status, many secondary schools reward AP students with a one-point grade boost, which helps bolster their grade point averages (GPA). For example, on a 4-point scale, a grade of B would be counted as 3.0, B+ as 3.3, A- as 3.7, etc. With a one-point grade boost, a grade of B would be counted as 4.0, B+ as 4.3, A- as 4.7, etc. Thus, it is not uncommon for students with multiple AP courses on their transcripts to have GPAs greater than 4.0.

At the end of the school year, during the first two weeks of May, nationwide AP exams are administered throughout the United States, Canada, and at American schools overseas. These exams are strictly controlled. A particular subject's exam is administered on the same day at the same time, with adjustments for time zones, throughout the United States and no electronics, except graphing calculators, if appropriate, are permitted. This is intended to maintain the academic integrity of the exams by preventing responses from being shared among students. Time adjustments are made to administer the exams in a reasonable time slots for students in Alaska, Hawaii, Pacific territories and American schools overseas. Alternate forms of the exam are available for students who are unable to participate on the exam date due to, for example, illness or participation in championship sporting events and for students who need to complete several alternate exams.

Each year, in June, college professors and high school AP teachers gather in selected cities in the United States and spend one week grading the AP examinations. During Week I, usually the first full week in June, about half of the AP exam readings are accomplished. In 2016, there were four cities, Cleveland, OH, Louisville, KY, Salt Lake City, UT, and Kansas City, MO, which served as sites for these AP readings. AP Calculus has held its annual reading at the Kansas City Convention Hall in Kansas City, MO since 2008. The schedule for AP Calculus for the past several years has been for readers to arrive in Kansas City in mid-week, begin reading duties the next morning, and depart the following week, eight days after arrival. Readers are on-site working from 8 a.m. until 5:00 p.m., each day, including Saturday and Sunday. Other disciplines may arrange their schedules differently; for example, AP U.S. History readers travel on Saturday, read from Sunday through the next Saturday, and travel home on Sunday.

In 2016, there were over 900 AP Calculus readers who shared the huge Kansas City convention center with readers of other disciplines. Each discipline has their own areas for briefings, the actual reading work and professional development. Dining facilities are shared by all disciplines. The schedule for the June 2016 readings is in Appendix C.

In addition to the actual grading of exams, the reading consists of formal training in executing grading rubrics, collegial discussions, socializing and optional sessions, including professional development, which are offered in the evenings. Readers frequently discuss exams with their reading partners throughout the day; insights into calculus students' thinking result because one reader may understand what the student is doing and the other does not. For example, the researcher read an exam in which the student used the acronym LRAM, which she did not recognize. Her reading partner informed her that it means Left Rectangular Area Method (and RRAM means Right Rectangular Area Method). As a college professor, the reading partner said those terms were commonly used in her courses; yet, the researcher had never seen those terms in all the calculus books she owns. Another example is a student who used the shell method to find the volume of a solid of revolution. The AP Calculus curriculum requires students to learn disk and washer methods. The shell method is a mathematically sound approach to a solid of revolution problem; however, it is not part of the AP Calculus curriculum. Although some teachers may opt to teach it, other teachers may not be familiar with it. Because the question did not specify a method for finding the volume, a correct, Calculus-based shell method presentation was awarded full credit. And, another example is that reading partners also help each other decipher unclear or almost-illegible handwriting.

This study explores the professional development, which Calculus educators experience at these varied activities at AP Calculus exam reading and how they report that these experiences influence teachers' classroom practice and their students' learning in their Calculus courses, as perceived by the educators.

**Origins of AP Calculus**

After the end of World War II, the Fund for the Advancement of Education (FAE), established by the Ford Foundation, sponsored a study which led to the creation of the Advanced Placement (AP) program. The conclusions of this study were that (a) secondary schools and colleges should work together to avoid repetition in course work in both institutions, (b) motivated students should have opportunities to study at their peak capabilities, and (c) these students could enter college with advanced standing based on achievement exams (College Board, 2004; Nugent & Karnes 2002). In a 1951 FAE study, eleven colleges formed the School and College Study of Admission with Advanced Standing (CAAS) and launched a study to determine how curricula could be revised to never-before-seen levels of challenge and accommodation for strong secondary school students; and, "to encourage able students in strong secondary schools to pursue a liberal arts education at a pace appropriate to their abilities and their teachers' interests and skills" (Cornog, 1957, p. 49). The mathematics test was beta-tested in 1954; participants included 120 secondary school students and freshmen from twelve colleges (Bressoud, 2010).

That fall, the College Entrance Examination Board, now known as the College Board, assumed management of the program. The Educational Testing Service (ETS) was chosen as the administrators of the examinations, charged with creating the first CAAS exams to enable students to take college-level work before graduating from high school (College Board, 2003; Handwerk, Tognatta, Coley, & Gitomer, 2008). In 1955, the first CAAS exams, including the mathematics exam, were given in ten different subjects; the next year, 1956, the name was changed to Advanced Placement (Bressoud, 2010; Potter & Morgan, 2000).

Although shortening the time spent in college was a possible outcome of these credits for AP work, this was not the intention of the CAAS study. The intention always was to "provide enriching learning experiences, to ensure that our best students are given challenging material that develops the quality of their understanding, rather than the quantity of what they have learned (Bressoud, 2010)." In these early exams, Calculus was not a big part of the content of the mathematics exam; instead the exam focused on many mathematical topic areas. However, by the end of the decade, “AP Calculus offered what was unmistakably a calculus exam. As the program grew, the emphasis shifted from assessing problem-solving ability to testing knowledge of calculus” (Bressoud, 2015).

In the late 1960's, the AP Calculus course was modified to include two courses. AP Calculus AB covers differential and integral calculus, roughly equivalent to Calculus I at the college level. AP Calculus BC, which is roughly equivalent to Calculus I and II at the college level, covers all the concepts of AP Calculus AB plus the additional topics of: improper integrals, infinite series, including Taylor and Maclaurin polynomials, polar functions, parametric functions, and vector functions. The topics common to both course require similar depths of knowledge; the BC course is intended as an extension of the AB course, not an enhancement (Dossey, Halvorsen, & McCrone, 2008).

For the 2016-2017 academic year, the new concept outline for the AP calculus course was published; it includes four “Big Ideas,” limits, differentiation, integration and the Fundamental Theorem of Calculus, and, for the BC course only, series (College Board, 2016).

**The AP Calculus Exams**

In 1955, the first mathematics exam was taken by 285 students (Broussard, 2010). In the ensuing years, the numbers have steadily increased, passing the 100,000 (AB and BC exams combined) mark in 1993, the 200,000 mark in 2003, and the 300,000 mark in 2009. In June 2016, 435,000 students sat for the exam. See Figure 1. The AP Calculus exam consists of four main parts. Time allowance is 3 hours and 15 minutes, plus a 15-minute break. See Table 1. Part I of the exam includes two multiple choice question sections; one section permits calculator use and one does not. Part II consists of six free response questions, each of which contain three or four parts; Part II also has one section which permits calculator use and the other does not. The exam allows a specific amount of time for each section; including a 15-minute break between Parts I and II. The multiple-choice sections are completed by filling in answers on a “bubble” sheet; these are graded electronically. The number of multiple choice questions in each section fluctuates from year to year. For 2017, there will be 30 questions in the non-calculator

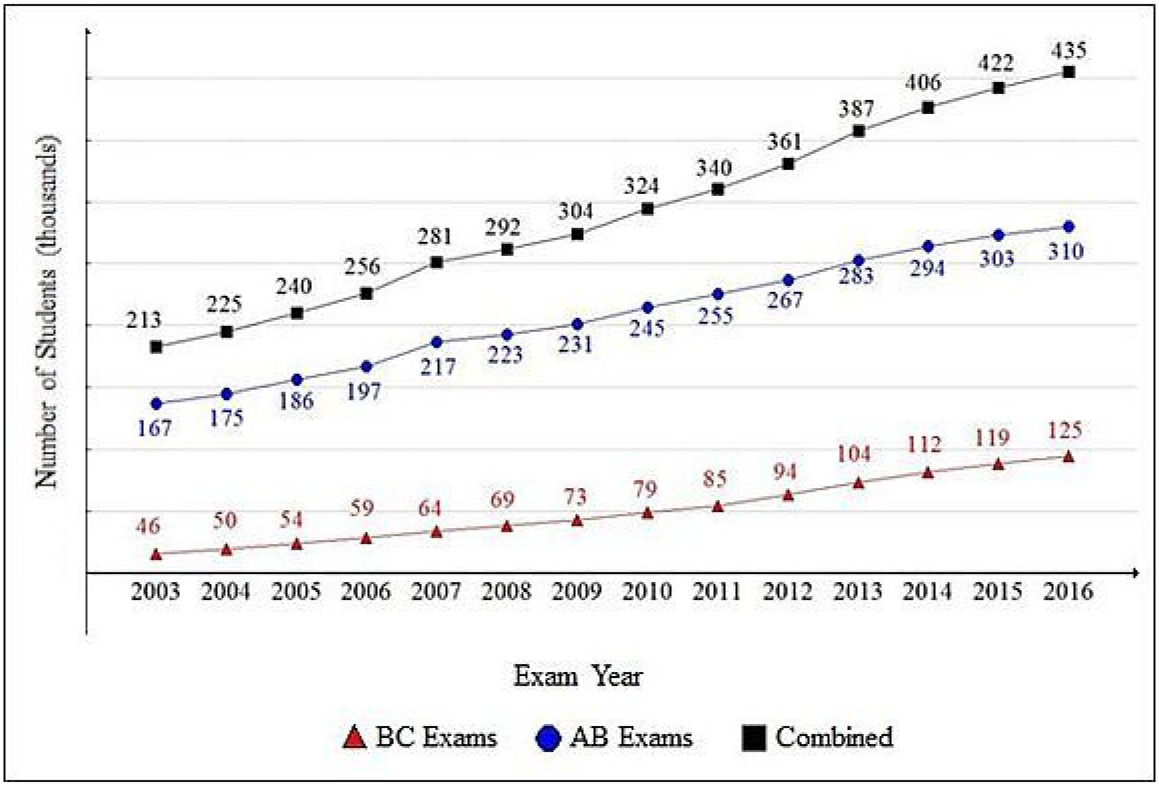

*Figure 1.* Participation in AP Calculus Exams, 2003-2016.

Table 1

*AP Calculus Exam Sections and Time Limitations, 2016*

| Exam Part | Quantity and Type of Questions | Time Allowance | Calculator Permitted |
|---|---|---|---|
| Part I A | 25 Multiple Choice Questions | 55 minutes | NO |
| Part I B | 15 Multiple Choice Questions | 50 minutes | YES |
| Break | --- | 15 minutes | --- |
| Part II A | 2 Free Response (multi-part) Questions | 30 minutes | YES |
| Part II B | 4 Free Response (multi-part) Questions | 60 minutes | NO |

portion. The free response questions require an analysis of written work completed by the students and, therefore, must be graded by humans.

The two versions, AB and BC, of the exam have three free response questions in common and three which are not in common. These questions are referred to by their version, AB, BC or both, and the question number. For example, the first question is designated as AB/BC-1 because the AB exam and the BC exam both have the same first question. AB-2, however, is different than BC-2. Brief summaries of the topics of the exam questions for 2013, 2014, 2015 and 2016 are shown in Appendix D.

Even with the enormous growth in registration for AP classes, the core of the AP Calculus program is the development of conceptual understanding of interconnected mathematical ideas, in concert with procedural skill. In the AP Calculus course and exam description, the College Board (2016) states:

> AP Calculus AB and AP Calculus BC focus on students' understanding of calculus concepts and provide experience with methods and applications. Although computational competence is an important outcome, the main emphasis is on a multi-representational approach to calculus, with concepts, results, and problems being expressed graphically, numerically, analytically, and verbally. The connections among these representations are important.

The reading of the AP Calculus exams is focused on the facets of interconnected conceptual understandings, not just procedural skill. Verbal justification for computations are usually required to earn credit; therefore, a student who is limited to only procedural knowledge will probably not do well on the exam. For example, one free-response question from the 2015 exam provided the equation for a first derivative of a continuous function $f(x)$ on the open interval $(a, b)$ and required the student to determine if/where $f(x)$ had a relative maximum on the open interval $(a, b)$. The first step is to determine at what values of *c*, with $a < c < b$, the first derivative of $f(x)$ is equal to zero, does not exist, or is undefined, making *c* a critical number of $f(x)$ on $(a, b)$. Students were expected to use the first derivative test to show that the first derivative of $f(x)$ was positive on the interval (*a, c*) and negative on the interval $(c, b)$. Using a sign chart, example at Figure 2, is insufficient. Although the sign chart correctly indicates the sign, or value equal to zero, of the first derivative, $f'(x)$, without a narrative explanation, the student has not sufficiently explained why a relative maximum must occur at $x = c$. A sign chart is not required; but, a sign chart alone is insufficient justification. Merely showing that for some selected value of *x* less than *c*, $f(x)$ has a positive slope, and, that for some selected value of *x* greater than *c*, $f(x)$ has a negative slope, was insufficient. Such an argument does not convey the requisite conceptual

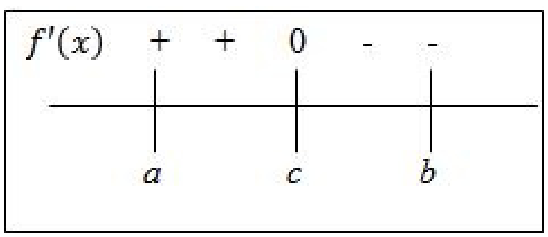


*Figure 2.* Sign Chart Example.

understanding of a relative extremum on an interval. A student could also elect to use the second derivative test; the requirements for this option also focus on conceptual understanding. Students needed to not only state that *c* is a critical number of $f(x)$ on $(a, b)$ and that the second derivative of $f$ was negative at $x = c$, which could simply be guessed, but also to provide the specific value of the second derivative at this particular value of *x*; show complete and correct computations; and, explain why a negative second derivative, indicative of downward concavity, would denote a relative maximum. Students who confuse *increasing* and *positive* or *decreasing* and *negative* when discussing functions and the slopes of their tangent lines also showed a lack of conceptual fluency and were awarded no points.

**Reading the AP Calculus Exam**

Rubrics for the exam grading are developed well ahead of the reading. Before the readers arrive at the reading, the table leaders and question leaders meet to review over 10,000 exams to look for patterns, common mistakes, and any other noteworthy observations in the exams. They reconcile their findings with the rubrics and decide how each anomaly will be treated. The rubric for each AP Calculus question is compacted into one page. It would not be possible to include the voluminous scenarios on one page; so, readers supplement this rubric with pages of notes from the briefings. Although the rubric has been formulated before readers arrive at the reading, readers are encouraged to give feedback on the grading of any question in which they participated. The forms for each of the nine questions (3 AB only, three BC only and three AB/BC) are printed on different colors of paper to avoid confusion. These forms and the collection boxes are located on the far side of the reading area and are easily accessible to all readers.

Before grading any exams, readers are briefed on the rubric and each section is dissected in minute detail and numerous exemplars are discussed. After an approximately 2-hour briefing, readers return to their reading tables and grade two exemplars. Table leaders ensure all readers have graded correctly. Each reader then retrieves a folder of 25 exams and grades them. The table leaders review, or back read, each folder for grading consistency. Table leaders discuss any grading discrepancies with each of the 16 readers at the table. Once this process has been completed several times and the table leaders are confident that the readers are correctly applying the rubric, the back-reading stops. As questions crop up, readers go to the table leaders for direction. The flow of readers to the table leaders is an ongoing process all day long. Over the course of a week, readers usually read three different questions. If one question is taking its readers a longer than expected time to complete, other tables may be briefed on a fourth question to grade.

To ensure fairness and consistency in reading the exams, all readers must grade the exams according to the rubric. Teachers and professors have their own ideas on how something should be graded; but, the rubrics are not up for debate. While the AP reading is a valuable grading experience, readers do have differences of opinion on the way student responses are graded. However, the grading standards are reasonably clear, with some subtleties which can at times be problematic, and all readers grade to the same standard, whether they agree with it or not. For example, question AB/BC-1 on the 2013 exam read as follows:

> On a certain workday, the rate, in tons per hours, at which unprocessed gravel arrives at a gravel processing plant is modeled by a cosine function $G(t) = 90 + 45cos(t^2/18)$, where *t* is measured in hours on the closed interval [0,8]. At the beginning of the workday, when $t = 0$, the plant has 500 tons of unprocessed gravel. During the hours of operation, the plant processes gravel at a constant rate of 100 tones per hour.
>
> (a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

Each of the six free response questions (FRQ) is valued at nine points. Part (a) of this question was worth two points. The first point was earned for supplying the correct answer to three decimal places, $G'(5)$= 24.588 or 24.587. The second point was earned for correctly explaining that $G'(5)$ is the rate at which the rate of gravel arrival is changing at $t$ = 5 in tons per hour squared. In other words, $G'(5)$ is the rate of change of the rate of change at time = 5 hours. Most students did not earn the second point. However, students who gave a correct explanation, including the units, in tons per hour squared, but did not reiterate that this was occurring at $t$ = 5, lost the second point. Some readers thought that the inclusion of a correct explanation coupled with the inclusion of the correct units was convincing evidence that the student should get the second point. They believed that the omission of reiterating $t = 5$ was trivial because writing $G'(5)$ specifies that $t = 5$. Rudd (1985) discusses a problem from the 1982 exam, for which he was a reader, and notes that even if a justification of an answer is not required, a student may be docked points if a correct answer is accompanied by an incorrect justification.

**College Credit for AP Calculus Exams**

Currently, many colleges and universities offer credit for introductory level courses based on the AP exam score. The American Council on Education and the College Board have developed a matrix for universities to use in developing a policy to provide advanced placement or award credit for AP coursework (College Board 2017a). See Table 2. Colleges and universities have their own concerns about granting credit, among them, loss of tuition and uncertainty about a student's actual course content knowledge. Lucas and Spivey (2011) suggest a transition course to bridge any such gaps.

By exposing students to college math experiences, such summer programs can help to increase students' academic preparedness and attract and retain STEM majors; and, effective academic emphasis can strongly impact student retention (Raines, 2012).

Table 2

*AP Calculus Exam Score Interpretations*

| AP Exam Score | Interpretation |
|---|---|
| 5 | Extremely Well Qualified |
| 4 | Well Qualified |
| 3 | Qualified |
| 2 | Minimally Qualified |
| 1 | Not Qualified |

Of course, it is the purview of each university or college to establish their own policies on credit awards. In 2013, Dartmouth announced that it would no longer grant credit for AP courses. The college stated that it wanted its students to benefit from maximum exposure to the institution's faculty; they reported no loss in application volume (Adams, 2014). Some universities will grant credit only for scores of 5; others will also grant credit for scores of 4 or 3. It is the policy at some institutions to allow AP credit to serve as a prerequisite for a more advanced course, even though the institution will not grant credit for the AP course (Laurent, 2009). Kelleher (2004a) discusses a teacher whose AP chemistry students scored 1s and 2s on the AP Chemistry exam; these same students passed the chemistry placement test in university, allowing them to register for their chemistry class without taking a remedial course. Industrious students can earn more than a year's worth of credits and bypass freshman year, or even more.

Some top universities, where high-achieving students are apt to be enrolled, are tightening the reins on awarding credit for AP; however, public institutions with their smaller proportion of high-achieving students with a surplus of AP courses, are generally more accepting (Adams, 2014). However, correlation between class grade for the AP Calculus course and the AP exam grade is not always so clear. Earning an A in the AP course does not necessarily mean the student will earn a 5 on the exam; in fact, some students who earn a grade of A in their Calculus course do poorly on the exam.

There is some disquiet that, as a wider net for AP students is cast and the number of students who enroll in AP courses continues to grow, the content will be watered down to ensure some less-capable students will pass (Koebler, 2012; Manzo, 2004). There are some school administrators who push for making participation in AP courses mandatory for all students (Mathews, 2003). However, Mattimore (2009), while agreeing that there are valid concerns about the expansion of AP, points to the high failure rates on the exams, leading to apprehension about the college preparedness of students who complete these courses. College Board has made available "AP Potential" software which identifies students, based on PSAT scores, who may be capable of succeeding in AP courses. Klopfenstein (2004a) warns that such measures must be used judiciously, ensuring that no student is excluded from an AP class based on only one criterion. Nitta, Holley and Wrobel (2010) found that consolidation of schools in rural areas meant that AP classes were available to students who otherwise would have had no access to them due to insufficient number of AP teachers. Shaw (2014) recommends that, to keep capable but lower achieving students in AP classes, that extra weekly meetings in small groups be held; the sessions could be used for tutoring, using Khan Academy resources, or locally developed materials. Pape and Barnes (2010) advocate for augmenting classroom instruction with asynchronous online instruction during study halls or free periods. For students who are struggling in AP classes, these additional resources may motivate students to rise to the challenge instead of dropping out, because they know they have a strong support system in place. The expansion of AP has allowed students, who previously would not have been thought capable of AP rigor, to show that they are rising to the challenges of high expectations and mastering demanding course material (Deneen, 2005).

Robinson (2003) claimed that over 90% of U.S. colleges and universities offer credit, placement in advanced courses or both to students with good scores on the AP exams. There is currently no comprehensive list of which institutions grant credit and their criteria for granting credit. However, using the 2016 U.S. News and World Report list of top universities in the United States, the researcher compiled information on the AP credit posture of each institution from information obtained from the College Board website (College Board, 2017b). The top 101 universities are included because there was a three-way tie for 99th place. Universities which require a grade of 5 for either the AB or BC exam are mostly at the top of the list. See Appendix E. Data are summarized in Table 3.

For some time, advanced placement classes were exclusively for select high achievers; after all, the beginnings of AP are rooted in that perspective. Clearly, with the number of AP exams taken each year growing steadily, that viewpoint has dissipated. As

Table 3

*Summary of U.S. News and World Report 2016 List of Top 100 Colleges and Universities Offering Course Credit and/or Placement for AP courses*

| Credit Offered | AB | BC |
|---|---|---|
| No Credit/No Placement | 7 | 4 |
| Credit for score of 3 or higher | 29 | 50 |
| Credit for score of 4 or higher | 53 | 41 |
| Credit for score of 5 | 11 | 5 |
| Other | 1 | 1 |
| Total | 101 | 101 |

more students are expected to enter colleges, our high schools need to offer challenging classes to prepare those students for college (Kelleher, 2004a). However, getting some capable students to register for AP classes can be problematic, because they do not want to do the work or they are afraid of failing. Schools want to have students in those AP classes. However, when commenting on the school board adopting some of the most rigorous graduation requirements in California, Maxwell (2006) noted that if the goal for all students was to increase both academic standards as well as expectations, then the students would rise to the challenge and more of them would be going to college.

There have been suggestions of incentivizing students by paying cash for excellent AP exam scores. Bennett and Arnold (2008) argue both for and against: monetary incentives serve as motivators, yet, they serve to degrade value of knowledge. Paneque (2008) discusses the Advanced Placement Incentive Program (APIP), which advocates cash reward for both students and teachers. A more mainstream incentive recognizes that AP courses are more rigorous and demanding than traditional high school classes; so, schools use a variety of methods to account for these differences in computing GPA and class rank (Sadler & Tai, 2007).

Jeong (2009) reviewed a nationally representative AP exam data set, taken from the Education Longitudinal Study of 2002 and found that "the AP exam fee (approximately $90 per exam) exemption, the most prevalent incentive, leads to an increase in the likelihood of AP course enrollees taking the exam-in particular, the disadvantaged. In contrast, little evidence was found that performance-based incentives, to which several states link AP test results, are helpful for improving AP exam participation and performance (p. 1)." Manzo (2004) points out that the College Board has been repackaging the AP classes as being welcoming to any student who is ready to do the work, notwithstanding their academic status.

**Positionality of the Researcher**

The researcher has served as an AP Calculus exam reader every year since 2008. She has found the experiences at the readings to be beneficial for her teaching practice and wondered if this was true of most readers. After her first reading experience, she believed that she had grown in four separate areas: (1) better understanding of the exam scoring rubrics which could be passed on to students; (2) development of a better sense of verbal precision, e.g., refer to $f(x)$ as $f(x)$ not as "it" or "the function;" refer to $f'(x)$ as $f'(x)$, not "the derivative," or "the function's derivative;" (3) expression of clear and precise justification for conclusions; and, (4) the importance of using units in context. Meticulous attention to the use of mathematical notation and vocabulary in calculus problems is essential to the students' development of mathematical literacy (Mahavier &

Mahavier, 2008). In Calculus classes, the terms “limit” and “continuity” have been problematic because of the connotations from their non-mathematical meanings or their less rigorous use in lower level classes (Edwards & Ward, 2004). The researcher has stressed all of these in her AP Calculus classes ever since. She believes that her own practice has improved because of her participation in the readings. Her knowledge of how students think, and how they mathematically express themselves when they do not understand, have increased; and, her appreciation for varying approaches to teaching concepts of calculus has broadened. Her own understanding of calculus has also escalated; and, her relationships with her AP Calculus students has enjoyed heightened credibility. While improvement is to be expected with years of experience, her belief is that a significant percentage of the credit for the positive amendments to her own practice as well as changes in her students' understanding and achievement can be attributed to the professional development, in its varied forms, which she experiences at the AP Calculus exam readings.

From her experiences, the educators who work as AP readers are very dedicated, and would, presumaby, want to help with research in mathematics education. No one attends a reading for fun. It is mentally grueling, tiresome work for seven straight days, while sitting in uncomfortable chairs in an environment which is frequently chilly. The modest stipend has never been named as a driving factor, or even an important one, for serving as a reader, to her knowledge. This is not to imply that anyone would do this without reasonable compensation. In conversations with her, readers have mentioned the stipend as a nice way to make some money in the summer; but, the much more prevalent interest is advancing the understanding of mathematics.

At the readings, participants frequently express that they believe they are much better teachers because they had attended the AP reading. Throughout the reading, readers are heard saying that they are learning so much and that they will be changing the way they teach. This begs the questions if that really happens and what the results are. The researcher's own students have asked if they had an advantage because their teacher is an exam reader. The researcher's answer is always, "of course!" The researcher believes that her reading experiences have positively influenced her own mathematical knowledge, her classroom strategies and her students' understanding of Calculus. This knowledge may be useful to other teachers. Additionally, her student's average AP exam scores have increased in the past several years. The goal of this study is to determine if there is a causal connection in general.

**Statement of the Problem**

This study is concerned with high school teachers and college professors who read the national AP Calculus exam each June. The reading is touted by the College Board as excellent professional development for several reasons: (a) professional connections with hundreds of mathematics educators from across the country, (b) learning more about mathematics and teaching strategies, (c) examining voluminous amounts of authentic student work and discussing it with colleagues, (d) sharing ideas and materials with other educators, (e) improving classroom practices, and (f) gaining insights into student thinking. However, no study has been conducted to determine if the professional development at the reading actually does lead to teachers' improved classroom practices and their student achievement gains. This study is designed to determine if and how the reading experiences impact teacher knowledge, teacher classroom practices and student achievement. Because the study deals only with mathematics educators who read the annual AP Calculus exam, it is not generalizable to the entire population of Calculus educators.

**Statement of the Purpose**

Numerous research reports point to success in AP courses as predictors of: academic success at the university level (Ackerman, Kaufer, & Calderwood, 2013; Adelman, 1999; Burney, 2010; Dougherty, Mellor, & Jian, 2006; Hargrove, Godin, & Dodd, 2008; McKillip &Rawls, 2013); the higher expectation of graduation (Adams, 2014; Adelman, 1999, College Board, 2014)); higher College GPA (Hargrove et al., 2008); more proclivity to enter a STEM field (Robinson, 2003; Robinson, Fadali, Ochs, & Willis, 2002; Sadler, Sonnert, Hazari, & Tai, 2014); and, issues such as minority status and gender (Klopfenstein, 2004a; Klopfenstein, 2004b; Shapiro & Williams, 2012; Whitehead, 2006). However, for a high school student who does not earn at least a 3 on the AP Calculus exam, there is no correlation with success or failure in Calculus I in college (Bressoud, 2015). Sadler and Sonnert (as cited in Bressoud, 2017) found that students who do not earn top grades in Algebra, Geometry and Precalculus are unlikely to earn higher than B- in a college Calculus class. In a joint statement, NCTM and MAA state, "students who enroll in a calculus course in secondary school should have demonstrated mastery of algebra, geometry, trigonometry, and coordinate geometry"

(NCTM & MAA, 2012, p. 1). However, this is not always the case. Bressoud (2017) points out that roughly 19% of students who complete calculus in high school take Calculus II in their college freshman year; approximately 50% take business Calculus, a lower level math course or no math course at all. Klopfenstein and Thomas (2009) agree that taking AP courses is positively correlated with college success; however, they also dismiss the idea of causality and warn against less-capable students overestimating the college readiness afforded by AP classes. That is, merely taking AP classes is not an assurance of college success; students who do well in multiple AP classes tend to be students who challenge themselves and are motivated to do the requisite work. These are characteristics which are likely to lead to college success. While AP classes are no longer restricted to the privileged elite, AP classes are for the prepared (Grier, 2002). And, AP Calculus is not a college preparatory class; it is a college level class (Klopfenstein & Thomas, 2009).

However, Bressoud (2015) warns that success in high school calculus is not a one-way ticket to success in college; students must be able to think critically and construct a "personal, coherent, and functional mental structure for the many concepts of calculus" (p. 179).

Because of her involvement as a research archivist and instructor for teacher professional development summer at several of the George Mason University Center for Outreach in Mathematics Professional Learning and Educational Technology (COMPLETE) professional development institutes the past several summers, the researcher has become increasingly fascinated and interested in teacher professional development and how it impacts their practices. She wants to know what impact being an AP Calculus reader has on each teacher's practice and their students' understanding. The research examined if and how participation in the AP Calculus exam reading: (a) influences teachers' beliefs about their own practice; (b) influences teachers' own understanding of calculus; (c) leads teachers to change their classroom practices; and (d) propagates an impact on student learning (ISL), as perceived by the teachers. This study sought to explore in what ways these changes are occurring or not occurring, how these changes are being implemented, and how students are affected, as perceived by their teachers.

**Primary Research Questions**

The research questions are designed to determine if there are connections between the readers' professional development experiences at the AP Calculus exam reading and their teaching mathematical thinking, and/or their perceptions of their teaching practice and/or their perceptions of their students' earning

Research Question 1: How do AP Calculus teachers and college Calculus professors perceive the professional development experiences at the AP Calculus exam reading?

Research Question 2: How does participation in an AP Calculus national exam reading affect teachers' and professors' classroom practice, as perceived by the educators?

Research Question 3: How do exam readers report that their participation in an AP Calculus national exam reading influences their students' success, as perceived by the educators?

**Significance of the Study**

The AP Calculus exam questions from previous years are readily available at College Board's AP website. The grading rubrics, as well as sample exam responses which received low, medium and high scores, are also published online. Teachers who have never participated in the AP reading can access these materials and effectively use them in their classes. Students can also access these materials for a limited number of recent years. However, there is much more occurring at an AP Calculus exam reading than grading exams. The one-page rubric for each question gives a general snapshot of how the nine points for each question are assigned. The many and varied subtleties of the grading, however, are not included. Each reader takes voluminous notes at each briefing, shares those notes with their reading partners, and uses them to engage in conversations at the reading table. During the question briefings, the mathematical concepts and philosophy for the grading of the question is explored. This is only available to the reading participants. Additionally, exam readers are invited to what are described as professional development sessions in the evenings. Educational Testing Service (ETS) (2017) describes these sessions: “formal and informal voluntary activities are offered, such as subject-specific professional presentations, receptions and local outings. Some subjects host evening workshops providing opportunities to network with colleagues and exchange best practices.” For AP Calculus, these sessions usually include one meeting with the College Board representatives about AP Calculus topics and any changes on the horizon. Other presentations may include strategies for explaining a specific Calculus concept or a demonstration of classroom activities for all levels of secondary mathematics classes. Usually, there are one or two sessions which are mostly for entertainment, but could have some utility in the classroom, such as "magic tricks" with mathematics or mathematic Jeopardy games. The sessions are usually well attended (at least 100 people) and attendees are encouraged to ask questions and be involved.

A teacher who does not experience the reading is not exposed to the many and varied opportunities discussed both during the work day, after work and at evening sessions. The population of AP Calculus exam readers, is, therefore, different from the general population of AP Calculus high school teachers and college professors in an important way. And, this difference begs exploration. There is no published work on this topic. If participation in the exam reading does positively affect classroom practices and student success, then the mathematics education world needs to know this. It is intended that this study will lead to development of a working grounded theory on the influence of the breadth of AP Calculus exam reading experiences on teacher knowledge, classroom practice, and teacher observations of ISL, as perceived by the educators.

**Research Design**

A mixed methods approach was employed for this study. Quantitative analysis includes descriptive statistics of responses to Likert scale questions as well as Chi-Square or Fisher's Exact, tests for association or independence. Qualitative analysis entailed the teacher commentary from the online survey as well as the interview responses. The data collection tool had two phases. The first phase was a four-part survey which participants completed online. This survey was designed by the author and piloted during the AP Calculus reading in June 2014. Therefore, there are no formal tests of validity or reliability. However, Maxwell (2013) suggests the creation of a validity matrix to organize possible threats to validity, delineate how validity may be threatened, and define strategies address validity threats. For example, a teacher may be unwilling to give accurate information if that information is unflattering; teachers may feel that they are being judged. To mitigate any such fears, participants were guaranteed anonymity, guarding any connection of their identity with their responses to the online survey. See Validity Matrix, Appendix F.

In structuring the survey, Fink (2008) and Miles, Huberman, and Saldaña (2014) were consulted. Questions were designed to be clear, unbiased, and single purpose. Participants were asked about their opinions and perceptions. For example, the question on classroom practice asked, “if your experiences as a reader have influenced your teaching, in what ways has this happened?” It was not assumed that any influence had occurred, nor was it assumed that any influence was necessarily positive. Participants were also asked to elucidate on the ways, if there were any, in which their teaching has changed. They were not asked to justify specific pre-packaged changes such as, emphasizing specific topics, giving assessments more or less frequently, or, using different resources for the classroom.

The survey included four parts: (a) informed consent, (b) Likert scale questions, (c) open-ended questions, and (d) demographic information about the educators and their institutions. The high school teachers were asked to provide historical, quantitative data about their students' AP exam scores. This data is archived by school counselors. The second phase of the study was a series of interviews via telephone. The baseline questions for these live sessions were developed before the online survey was launched. However, based on the results of the initial survey, it was expected that some issues may arise which would require clarification, elucidation or more discussion. The population is a diverse group in age, location, type of institution, number of years teaching, and number of students, etc.; the participants reflect that diversity, as do the volunteers for the interviews.

**Assumptions, Limitations and Scope**

All AP Calculus readers are mathematics educators who applied to participate in a reading. Presumably everyone who is invited to the reading wants to be there. The researcher believes anyone who actually agrees to spend seven full days grading exams does so for a combination of four reasons; these are: (1) to learn more about the exam to share with their students, (2) to improve thier own classroom practices, and (3) personal reasons such as getting away for a week and earning a modest stipend ($1600). The relative importance of these reasons, will, of course, vary from person to person. As opined by many readers with whom the researcher has spoken, the stipend is secondary to the many opportunities to both network with such a vast and diverse pool of many enthusiastic educators as well as the exposure to a wide-ranging assortment of student responses to the exam questions and the insight into student thinking which is gained.

People who serve as readers have waited several years after being approved and added to the pool of potential readers before they actually received an invitation to serve as a reader. As of 2016, there are approximately 900 readers needed for the AP Calculus exam. Approximately 75% of the readers in any year are experienced readers returning for another session. In order to be considered for a reading position, an educator must have been teaching AP Calculus for at least three years. College professors must have taught calculus at least once in the past three years. At the reading, each reader's speed, compared to others at the same reading table, and accuracy, as measured by how often the table leader needs to correct the reader's grades, are measured. These statistics, coupled with the table leaders' recommendations and the number of new and experienced readers needed each year, determine if the reader receives an invitation to return the next year. The pool of readers is largely comprised of experienced people who have performed well enough to be repeatedly invited back, supplemented by a much smaller number of eager, new readers. It is possible that these educators are already impacting their students' learning at a higher level than their non-AP reading colleagues. Some readers even give up a week of their own time to participate in the reading. However, this study attempts to compare these educators against themselves, not against teachers who do not read the AP exam.

The researcher assumed that those who choose to answer the survey gave accurate and truthful information. Anonymity was guaranteed and she had neither any way of ascertaining any particular participant's identity nor any way to link survey responses to any particular person. Even though some requested information may be unflattering, for example, a low exam pass rate, she believed that these educators did not falsify the information which they submitted.

The number of respondents to the survey was an initial concern. However, given that she expected to have approximately 1200 email addresses, even a response rate of only 5% would have given her an acceptable sample size of 60. However, she expected a much higher number of participants. The final total of survey participants was 155. Recruitment email is at Appendix G.

Obtaining the quantitative data on exam scores from all participants was not expected. Some teachers may be uncomfortable sharing information on their students' scores, even though no identifying information is included. The school counselors, who hold the data, may be unwilling to share it with the teachers. Some educators may not teach at the AP level every year; and, some may have limited AP reading experience. However, even if exact information was not available, these teachers could have discussed their observations of any changes in their students' scores over a period of time.

**Summary**

Effective professional development for mathematics teachers includes mathematical as well as pedagogical content, analysis of student work and working in collaboration with colleagues. Hoy (2009) found what she described as "critical links among student achievement, professional development, and instructional practices as perceived by educators" (p. 85). These are discussed in detail in Chapter 2.

## Chapter Two

### Conceptual Framework and Literature Review

In researching professional development, specifically for mathematics teachers, the bulk of research was found for elementary and middle school. However, the principles delineated in this research was pertinent to high school level classes. Several overarching premises were prevalent in the literature. The research commenced with studying the varying components of the teacher's mathematical knowledge and the components of effective professional development. The idea of collaboration with other education professionals arose as a dominant theme. From these ideas of "communities of practice," the specific notion of examining student work to gain insight into student thinking emerged. The ideas of collegial interaction, sharing observations, and reaching conclusions as a cooperative venture, not a critique of the teacher, was poignant. Relating this research specifically to the highest level of high school mathematics, however, reveals of paucity of relevant literature. Existing studies tend to be parochial in scope and focused mainly on teaching practices, not on teachers' professional learning.

Another germane offshoot of the idea of looking at student work as professional development is the recognition of the power of both writing narratives as a method of mathematical problem solving and the importance of analyzing incorrect work for content and student thinking.

There is limited literature of student achievement in AP Calculus. These examine a range of thoughts from the reasons why students enroll in AP Calculus in the first place, to the appropriateness of using highly sophisticated graphing calculators and to the likelihood of these students completing college and pursuing STEM careers.

Finally, for the AP Calculus exam itself, there is a scarcity of literature. There are some anecdotal accounts of various readers' experiences. And, there are some encouraging pieces which proclaim that even if a student fails the AP exam, he still has had a valuable classroom experience. There is, however, no research which specifically addresses the professional development which occurs at the AP exam reading and its ramifications in teacher knowledge, classroom practice, and student learning.

It is reasonable to presume that a teacher cannot teach what the teacher does not know. This refers to the transmission of knowledge. So, if a teacher has no knowledge of quadratic equations, for example, then the teacher will not be able to transfer knowledge about quadratic equations, the quadratic formula, the meaning of the discriminant, or solutions either by completing the square method or by factoring method to the students. However, teaching is more than just the one-way sharing of information from teacher to students. Teachers are deeply guided by their own experiences, their views of their students, and their appraisal of the official curriculum; teachers tend to adapt the official curriculum to fit their own active practice and decide what is important for their students to understand (McDonald, 2002) and, therefore, what knowledge to transmit.

However, mathematical content knowledge is only one part of teacher professional development. There are connections between teacher knowledge and teacher ability to transmit knowledge; and, therefore, an understanding of the resultant ISL is part of a teacher's enterprise. Professional development also envelops collaboration among teachers, examination of student work for understanding of student thinking, improving teaching strategies, and incorporating new technologies and materials. These elements are all indissolubly intertwined.

The study focuses the professional development, in its varied forms, which occurs at the AP Calculus exam reading and its connections, if any are found, to the educators' critical thinking, the educators' classroom practices, and the educators' perceptions of changes in their students' comprehension of Calculus concepts and procedures. Therefore, the literature review addresses the essential elements of professional development for secondary and post-secondary mathematics teachers. The literature review begins with the theoretical framework for mathematics teacher knowledge. Teacher professional development is then discussed, followed by a discussion on learning communities and their importance in teacher professional development. A brief discussion of the paucity of research in STEM professional development is followed by the ideas of looking at student work and using problem writing in mathematics classes are examined; from incorrect solutions, teachers can gain insight into student understanding. Lastly, student achievement in AP courses and research on the AP Calculus exam in particular are presented.

## Teacher Knowledge and Theoretical Framework

Subject matter expertise is an expectation of teachers. However, exactly what mathematical knowledge really qualifies as expertise is not definitely decided. Hill, Sleep, Lewis and Ball (2007) review and assess the numerous methods which are used to quantify the mathematical knowledge of mathematics teachers in the U.S. Merely measuring mathematical knowledge through teacher testing ignores much of what teaching involves. Shulman (1986) coined the phrase "pedagogical content knowledge (PCK)" as a tier of teacher knowledge which meshes both content and pedagogy. What exactly should be included in that knowledge has been the subject of discussion for some years (Ball, Thames & Phelps, 2008; Hill, Rowan & Ball, 2005; Ma, 1999; McCrory, Floden, Ferrini-Mundy, & Senk, 2012; Mewborn, 2003; Sleep & Eskelson, 2012.). Ball and Bass (2003) referred to these skills as "mathematical knowledge for teaching (MKT)" and described them as including knowledge, competencies and skills. While Shulman (1986) laments “the absence of focus on subject matter among various research paradigms for studying teaching” (p. 6); not all skills needed for teaching mathematics evolve from mathematical content. Schifter (2001) states these necessary skills arise from:

> attending to the mathematics in what one's students are saying and doing, assessing the mathematical validity of their ideas, listening for the sense in children's mathematical thinking even when something is amiss, and identifying the conceptual issues on which they are working (p. 131).

The interwoven relationships among teachers' knowledge, beliefs and attitudes are still being studied; researchers continue to examine how these relationships relate to teachers' classroom practice (Philipp, 2007). Content knowledge is one piece of a teacher's enterprise. The teacher's knowledge of students and learning as well as the teacher's knowledge of pedagogy are also vital elements of the classroom practice. In his study of AP Calculus teachers, Utter (1997) found great variation in the teachers’ “pedagogical content beliefs about mathematics, curriculum and instruction in AP Calculus” (p. 86); these beliefs were strongly correlated to their classroom practices. Shulman (1986) describes essential types of teacher knowledge which he broadly summarized under the categories "Subject Matter Content Knowledge," "Pedagogical Content Knowledge,” and “Curricular Knowledge." Gardner (1989) describes “knowledge-for-practice” which he bases on the idea that there is a distinctive knowledge base in teaching which affords teachers a bank of knowledge not possessed by others. Cochran-Smith and Lytle (1999) describe three concepts for teacher learning: knowledge-for-practice, knowledge-in-practice, and knowledge-of-practice. The National Council of Supervisors of Mathematics (NCSM, 2017) delineates the necessary elements of mathematical knowledge as mathematics content knowledge, pedagogical content knowledge, and mathematics curriculum knowledge.

Ball, Thames and Phelps (2008) had refined these into their “Domains of Mathematical Knowledge for Teaching” (p. 403) model, shown in Figure 3. Subject matter knowledge is comprised of three components; and, pedagogical content knowledge also consists of three components. This model will serve as the theoretical framework of this study.

This model is especially appropriate for this study because of the interweaving of multiple perspectives on teacher knowledge. AP Calculus readers are accomplished educators whose experiential perspectives of their professional development at the AP Calculus exam reading can shed light on the outcomes of the professional development;

that is, how their various types of knowledge changed or not, and, why they believe these changes have or have not occurred. The experiences at the reading also affect their

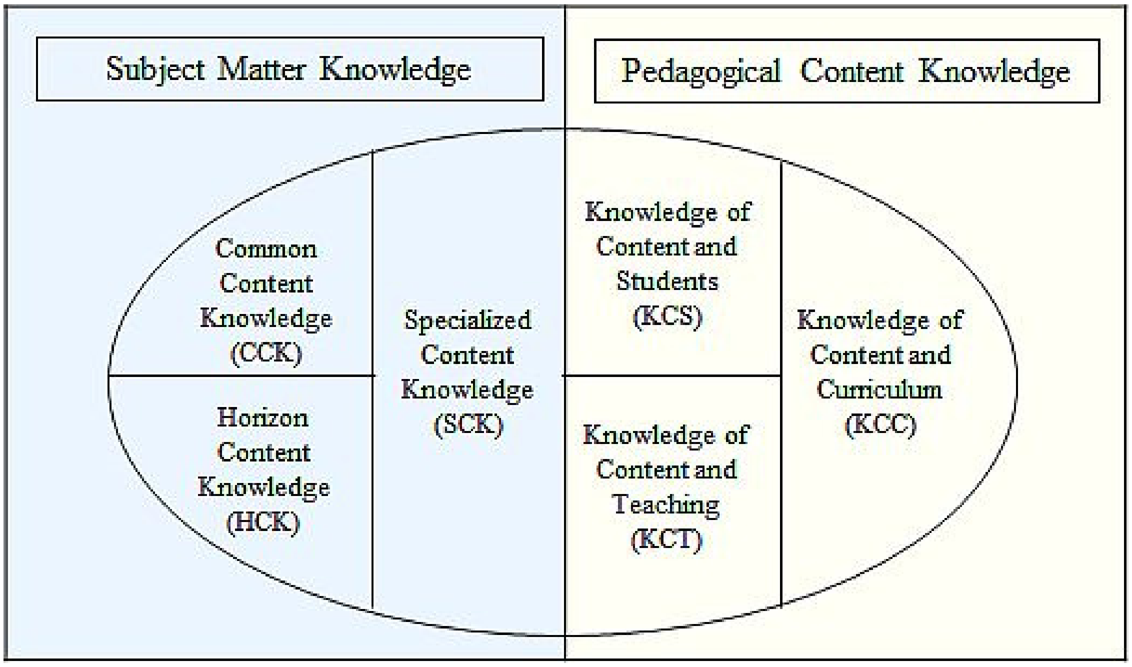

*Figure 3.* Domains of Mathematical Knowledge for Teaching Model.

curriculum knowledge; how will this be incorporated into their practice and how will it influence their knowledge of students? Ultimately, the study seeks to determine if participation in the AP Calculus reading does make a teacher a *"better teacher,"* as perceived by the teachers themselves, and, possibly as evidenced by their students' exam performance. Teachers may describe themselves as becoming better teachers because of increased knowledge or course content, ability to understand exam grading rubrics, strategies for focusing on conceptual development, or higher AP exam pass rates. The aim of this study is to determine if teachers really do see themselves as improved, and how they assess any changes in their practice and ISL.

**Definition of Mathematical Knowledge Terms**

Ball, Thames and Phelps (2008) define the six elements in their Mathematical Knowledge for Teaching Model as follows:

- Common Content Knowledge (CCK): mathematical knowledge we would expect a well-educated adult to know
- Horizon Content Knowledge (HCK): awareness of how mathematical topics are related over the span of mathematics included in the curriculum
- Specialized Content Knowledge (SCK): mathematical knowledge beyond that expected of any well-educated adult but not yet requiring knowledge of students or knowledge of teaching
- Knowledge of Content and Students (KCS): pedagogical content knowledge that combines knowing about students and knowing about mathematics
- Knowledge of Content and Teaching (KCT): knowledge that combines knowing about teaching and knowing about mathematics
- Knowledge of Content and Curriculum (KCC): (not explicitly defined, meaning is inferred) knowledge of what mathematical content is appropriate for teaching at what grade levels and knowledge of appropriate materials for student learning.

**Teacher Professional Development**

The College Board claims that serving as an AP exam reader leads to becoming a better teacher, through increased content and curriculum knowledge, collegial interactions, and examining thousands of student work exemplars (College Board, 2017c). Bressoud (2004), former president of the Mathematical Association of America (MAA), stated that all college mathematics departments “should encourage at least one individual to attend the annual AP reading (the grading of the free response questions), to work with local AP Calculus teachers, or to help prepare and support those who will teach calculus in high school” (p. 7). This study seeks to determine if the promise of better teaching is realized. And, if so, what evidence supports it. Analysis of changes in teachers' knowledge in the categories of the model was used to analyze the impact of the readers’ professional growth.

Effective professional development is conducted in a collegial and professional environment and provides teachers with opportunities to work together sharing their perspectives and seeking solutions to common problems (Guskey, 1995). When people share perspectives, they build meaning. People learn best when they construct meanings rather than having those meanings given to them. Such an approach results in better teaching of mathematics than non-participatory, non-communal, more traditional methods (Bruner, 1996). Similarly, Lamon (2007) contends that our traditional teaching for computational ability has left us pedagogically bankrupt for an age which values meaning and connections. Instead of teaching our students to use algorithms robotically to arrive at correct but meaningless solutions, we need to focus on Bruner's "meaning making." Mathematical concepts and procedures need to be conjoined, a package deal, and they must make sense; else, the "correct" answer has no value to the student beyond a check mark on an assessment. Professional development for mathematics teachers, therefore, must provide a platform for developing the requisite knowledge, skills and outlook to become effective teachers (Sowder, 2007).

Kennedy (1999a) asserts that the extent to which learning occurs seems to be proportional to the extent to which thinking occurs; thinking and learning ought to go hand in hand. Students need to exercise their brain cells and think. Specifically, students need to understand logical relationships between mathematical terms and need to be able to see structural connections behind the algorithms (Skovsmose, 2005). The same argument could be made for teachers.

In summer institutes conducted by the GMU COMPLETE Center, a seemingly simple, yet multifaceted problem always garners considerable discussion. See Figure 4. Known as the pipeline problem, this task can be solved using algebra, trigonometry, geometry or calculus. Teachers tend to use an approach with which they are most familiar and comfortable. However, when they see their colleagues solving the same problem in a completely different way, the connections begin to emerge and teachers have new skills to take to their classrooms. Not dissimilarly, an AP reader may not understand what a student has written; however, the reading partner may know exactly what the student is doing. A different approach, an unfamiliar term, or an unrecognized symbol may well be familiar to a reading partner.

Deepening teachers' subject matter knowledge and pedagogical content knowledge is the goal of professional development. Effective professional development provides teachers with new knowledge, such as the use and integration of new technology

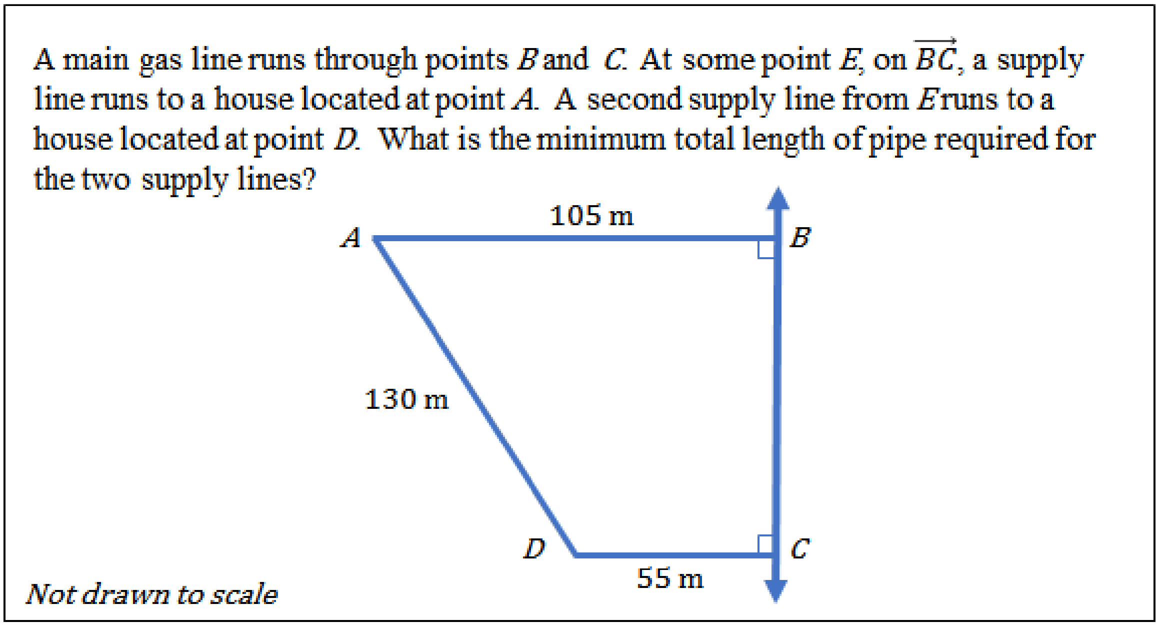


*Figure 4.* The Pipeline Problem.

into their classrooms (Ganser, 2000). Teachers need to ruminate about thinking and learning as joint issues. In mathematics education, professional development must be designed to help teachers entwine their knowledge of learning with their knowledge of teaching (Driscoll & Moyer, 2001). Herrington and Yezierski (2014) claim that high quality professional development needs to be designed for collective participation of the teachers and active learning which leads the teachers to learn about student learning. With AP mathematics and science curricula focusing more on conceptual understanding over rote memorization, Loucks-Horsley, Love, Stiles, Mundry and Hewson (2003) warn against simply changing the PD delivery method without modifying the duration, content and focus; they find that "activities that are content focused, but do not increase teachers' knowledge and skills, have a negative association with changes in teacher practice" (p. 934).

Effective professional development includes the following components: content focus, active learning, coherence with other PD, duration and collective participation (Desimone, 2011; Hochberg & Desimone, 2010). Since the early 1990's, teacher professional development has evolved into a social, participatory, interactive venture (Cochran-Smith & Lytle, 1999). Teacher professional development must, therefore, be centered on teachers' pedagogical skills as well as deepening their content knowledge. And, it cannot be rushed. Meaningful professional development requires the allotment of sufficient time as well as other resources (Guskey, 2003). Just as we expect with students, teacher learning is a slow and uncertain process if it is going to be meaningful (Schifter & Fosnot, 1993). And, we cannot expect an equal amount of change in every teacher; some will change more than others; some will change differently than others (Fennema, Franke, Carpenter & Carey, 1993; Franke, Carpenter, Levi, & Fennema, 2001; Knapp & Peterson, 1995).

There is resistance to professional development in some arenas because of the time and finances involved. Desirable features of professional development require substantial lead time, planning, and finances which may not be available (Birman, Desimone, Porter, & Garet (2000). Holloway (220) states that there are still school districts which view professional development as a singular experience.

Professional development varies in effectiveness in student learning gains; but, professional development which is "research-based, thoughtfully conceived and delivered, and focused on the right things can, indeed, impact learning" (Guskey, 2005, p. 38). Bobrowsky, Marx and Fishman (2001) found that an important factor in evaluating professional development effectiveness is gleaned from studies in which the teachers volunteered to participate; they contend that these volunteers "tend to be innovators, actively seeking new ideas that challenge their present thoughts on teaching and learning. They do not see change as a flaw in themselves and are risk-takers" (p. 2).

**Learning Communities**

The logistics of school schedules and physical school structure are usually conducive to teacher isolation. Therefore, it is not surprising that professional development, "especially when done in community with others, where the learning is richer and deeper, has not been the norm in the education community" (Hord, 2007, p. 39). However, it is crucial that educators recognize the connection between the student learning in their classrooms and their own learning with their colleagues (Franke & Kazemi, 2001a; Lambert, 2003). Hord (2015) states that "the professional learning community is the most powerful structure and strategy for enhancing educators' effectiveness and increasing students' successful learning" (p. 38).

Learning constructively requires an environment in which learners work collegially and is situated in authentic activities and contests (Vgotsky, 1978). The most effective professional development endeavors are planned to ensure teacher engagement on social, intellectual and emotional levels. Considering both effectual student learning and teacher professional development as constructivist activities is warranted; the optimum professional development for teachers focuses on continual individual, collegial and organizational progress and improvement (Corcoran, 1995). Similarly, stressing the meanings of the words professional, learning and community, Hord (2009) states, "the professional learning community encourages constructivism by providing the setting and the working relationships demanded of constructivist learning" (p. 41). The context most supportive of the learning of professionals is the professional learning community (Hord & Hirsh, 2008).

Connecting the opportunities and experiences of all its members, a learning community provides faculty with opportunities to envision practices which will enrich student learning in fundamental areas such as critical thinking and knowledge making (Condon, Iverson, Manduca, Rutz, & Willett; 2016; Little, 2002). Ciesemier (2003) argues that teachers who examine student work in a learning community setting will reap new knowledge about both teaching and learning.

Several studies point out the difficulties teachers face in working collaboratively to analyze and discuss student work, such as teachers becoming defensive, and teachers not being able to schedule time with colleagues (Deuel, Nelson, Slavit, & Kennedy, 2009; Hord, 2007). Even when educators are able to collaborate, the outcomes depend on the approaches which are used. Charalambos and Silver (2008) draw sharp contrasts between the teachers who want to prove that their students have learned and those who use the data in the student work to reflect on their own teaching practice. In the research of Deuel et al. (2008), groups which used the “proving approach” tended to make assumptions about their students’ abilities, such as overestimating the understanding of high-achieving students. The teachers who used the “improving approach” analyzed a student’s work so they could grasp how and what the student was thinking. In that view, teachers believe that “teaching can always be improved in relation to students’ conceptual understandings” (Nelson, LeBard, & Waters, 2010, pp. 39-40).

“Peers learning and working collaboratively create an ideal environment for a constructivist learning approach that benefits both teachers and students. The elements that foster professional learning communities include time, space, use of data, and careful consideration of community membership” (Hord, 2009, 43).

**Science, Technology, Engineering and Mathematics (STEM)**

In STEM areas, there is little educational research on teaching or student learning which bridge higher education communities (Henderson, Beach, & Finkelstein 2011). However, these studies largely center on correlations between teaching practices and ungeneralizable student outcomes; they do not, however, pinpoint the professional learning activities which championed the changes in teaching practice.

Robinson (2003) found that, although many in his study were undecided, the majority of high school students who take AP Calculus and AP science courses in high school had expectations of pursuing STEM-related career tracks in college. Student completion of AP exams is positively correlated to both college grades and graduation rates (Tyson, 2011). More than half of the students who matriculate as full-time students to a four-year college have completed a calculus course in high school (Bressoud, 2017). Ackerman et al. (2013) cites a Georgia Tech study which found that the best predictors of student persistence in STEM majors were college credit for their AP Calculus course and successful completion of at least three AP exams in STEM subjects. Sadler, Sonnert, Hazari, and Tai (2014) found that taking calculus or an advanced science class alone did not correlate to a stronger propensity toward STEM careers, but taking “calculus and a second year of chemistry, or one or two years of physics all predict large increases in STEM career interest” (p. 1).

**Using Student Work as Teacher Professional Development**

NCTM (2000) states, "when assessment is an integral part of mathematics instruction, it contributes significantly to students’ mathematics learning. Assessment should inform and guide teachers as they make instructional decisions." Analyzing student work not only for correct responses, but also for critical thinking and logical thought processes are crucial for teachers to understand their students' levels of understanding. The correct answer is insufficient to assess student understanding or cognitive abilities. Even Erlwanger's (1973) "Benny" managed to produce a correct answer occasionally, despite his following nonsensical "procedures" and possessing minimal understanding of elementary mathematical processes.

Condon et al. (2016) assert that the education community needs to investigate for causal connections between teacher professional development, their resultant changes in classroom practices, and specific student learning gains. With a well-defined purpose and consistent use, “student’s work can be a catalyst for effective professional development for teachers" (Driscoll & Moyer, (2001, p. 287). With the goal of guiding student thinking, teachers need to understand the connections between the students' ideas and the key ideas in the discipline (Schifter & Fosnot, 1993). In their research, Kazemi and Franke (2004) found that analysis of student work allowed teachers to learn details of their students' thinking and strategies and to "recognize that students had powerful mathematical ideas" (p. 223).

In the past two decades, there has been a growing focus on the professional development phenomenon of "looking at student work," whereby a teacher does not merely check for right and wrong answers, but analyzes the work to assess the student's level of understanding. Ball and Cohen (1999) assert that "simply looking at students' work would not ensure that improved ways of looking at and interpreting such work will ensue" (p. 16). Detailed and probing analysis is needed. Baron (2008) states that such "careful examination of student work, guided by an inquiry question, is the most powerful catalyst for changing and improving teacher practice" (p. 68). Learning activities and contexts are a fundamental part of how people learn (Greeno, Collins, & Resnick, 1996), implying that their own classrooms are formidable scenarios for their own learning (Ball & Cohen, 1999; Putnam & Borko, 2000). And, the feedback teachers get from the performance of their students are powerful reinforcers for teaching behaviors (Tschannen-Moran, Hoy, & Hoy, 1998). However, this does not mean that all meaningful teacher learning occurs in the classroom. Looking at student work in collaboration with peers in a professional development setting gives teachers opportunities to explore each other's classroom practices and student learning (Ball & Cohen, 1999; Carpenter, Fennema, Peterson, Chiang & Loef, 1989; Little, Gearhart, Curry, & Kafka, 2003). The range of benefits of such collaborative work extends from deeper understanding of a teacher's classroom practice to the fine-tuning of high quality instruction (Flowers, Mertens, & Mulhall, 2005). All too often, teachers are not asked to provide input for the professional development sessions which they will attend; however, when teachers collaboratively analyze student work, there is an investment in the process because the teachers were involved in the creation of the experiences (Cameron, Loesing, Rorvig & Chval, 2009). Kazemi and Franke (2003) contend:

> Discussions of student work allow teachers to raise their own questions about practice and to deliberate about what it is that they want and need students to learn. Such professional inquiry can allow teachers to form generalizations and conclusions from the particular instances of students' reasoning that would guide future interactions in their classroom (p. 6).

A teacher's belief in his/her own abilities will influence the amount of effort the teacher is willing to expend and how the teacher deals with stress, failures, and obstacles (Bandura, 1997). And, collaboration with teacher peers has been correlated with higher efficacy (Chester & Beaudin, 1996; Rosenholtz, 1989). However, Tschannen-Moran et al. (1998) warn that a negative may also result if communal inefficacy leads to rejection of innovative methods.

Considering mathematics teaching, Crockett (2002) asserts that analysis of student thinking is a powerful vehicle for guiding teachers to analyze their own teaching and the learning of mathematics. Not only looking at the students' work, but also listening while students navigate through meaningful mathematical tasks, contribute to a teacher's professional development (Krebs, 2005). Langer and Colton (2005) claim that collaboratively analyzing student's progress over time, using a structured inquiry approach, is vital for school improvement. In student work analysis, "teachers might discover from students' responses that the questions they ask confuse students, and, therefore, do not fully elicit what students really know" (Nelson et al., 2010, p. 39).

There are several related but distinct reasons for looking at student work: understanding students' thinking, evaluating achievement, and improving instruction (Herbel-Eisenmann & Phillips, 2005; Zydney & Holovach, 2004). The purpose will drive the goals and approach to the activities. Charalambos and Silver (2008) suggest that teachers adopt the "improving stance" as they look at student work collaboratively; this approach can expose significant complexities in both teaching and learning which may otherwise remain unnoticed when the focus stays on achievement scores. "When looking at students' work, an 'inquiry stance' is key; in other words, it is important to look for what it is one can learn, rather than to try to see what one already thinks one knows" (Baron, 2008, p. 66).

Using "improving" techniques to look at student work, teachers view the activities as a way to help their students improve their learning, not as a castigation or evaluation of the teachers (Williams, 1999a). In such teacher groups, the improving stance led to analysis of students' work to understand student thinking; with this insight, teachers could more effectively plan future instruction (Ojose, 2008). Collaboratively analyzing student work affords teachers numerous opportunities to focus their attention on both learning issues as well as issues of pedagogy (Driscoll & Moyer, 2001).

Purposefully looking at student work changes the focus from what is occurring in the classroom to the work which students are producing, and this in turn leads to discussion of teaching practice (Baron, 2008). Certainly, not all students in a class are at the same level of understanding. It would be beneficial for teachers to be cognizant of the levels at which their students are functioning, which would assist in adjusting the cognitive levels in their classes (Ojose, 2008).

In discussing the meager 1% passing rate of a question on a national Assessment of educational Progress (NEAP) exam, taken by high school seniors, D'Ambrosio, Kastberg and dos Santos (2010) conclude that each exemplars of student work, rife with incorrect and illogical statements, provided unique insights into what the students are thinking.

Goodwyn and Hein (2016) write about a professor, who, as a graduate student, was asked how two different papers, one was formulaic and "structurally sound," and the other a scrawled, crumpled mess on torn paper which told a captivating story, albeit with misspelled words, would be assessed; the 'productive dialogue' shaped the English professor's classroom practice. They claim, "that's the promise of teachers looking at student work together. Focusing teacher dialogue on real artifacts can surface tacit differences in our expectations and teaching practices" (p. 79).

In Boston, a school rejuvenation plan led off with examination of student work, leading teachers to tie together the problem which they saw in the writing assignment with an examination of how they taught writing (Williams, 1999b). One elementary school principal said, "This told me that looking at student work was the starting point.

You can't develop a professional development plan or anything else until you know what you need and why, and you don't know that until you look at student work” (Williams, 1999b, p. 1).

Williams (1999a) discusses a high school mathematics teacher examining the work of her students for indications of what is and what is not working in her classroom; The teacher says, “Now, I look at the kinds of mistakes my students make. I figure out where they went wrong, and I ask myself, `What do I need to do differently so they get it?' “(p. 1).

In a first-grade classroom, a teacher asked her class how they can find the length of a boat, without giving them any instruction on measurement. The children’s curiosity drove the discussions and led to discovery; through the constructivist approach, the teacher gained insight into her students’ thinking (Schifter, 1996).

Williams and Weissmann (1999) discuss a certification process which requires teachers to submit a selection of student work examples accompanied by the teacher’s comprehensive analysis. They quote one teacher as saying, “Before, I didn't look at student work as carefully as I do now. Looking at student work with others taught me you have to be self-critical; I now have a better feel for how my kids learn. I ask myself all the time, did they get what I wanted them to get out of this lesson?" (p. 10).

Preferring the phrase “studying student work” over “looking at student work,” McDonald (2002) eschews the ideas of teachers becoming student of their own students and of teaching being predominantly limited to the transmission of knowledge. He states that teaching is “far more complicated than the transmission view of teaching suggests;”

he continues that teachers' practices are deeply influenced by their own experiences both as teachers and as students. Analyzing student work with algebraic proofs which require explanations reap insights into student thinking and understanding, which could serve as the basis for adapting instruction to address student reasoning (D'Ambrosio et al., 2010).

Looking at student work underscores the bond between teacher learning and changes in classroom practice; it has the power to transform a school from a culture of teacher isolation to one of teacher collaboration (Baron, 2008). Judging student work can be a potent form of professional development (Ciesemier, 2003; Krebs, 2005; Wolk, 2007). This process has been formalized into protocols such as the Harvard's Project Zero, the Coalition of Essential Schools, the Academy for Educational Development, and Descriptive Review at the Prospect Archive and Center in Vermont (Little et al., 2003). And, one web site, www.lasw.org, is devoted to collaboratively assessing student work. All of these projects had at their core a goal of connecting a comprehension of student understanding to the student work. There are several parallels to these protocols, such as the design of the lesson study process; however, the focus is on the student work, not on the teacher and the lesson. While in lesson study, observing teachers are not permitted to intervene during the lesson, in the student work analysis models, the teacher whose artifacts are being discussed is excluded from vocalizing any information during portions of the discussions. Research on learning communities suggest that collaborative analysis of student work reaps benefits for both students and teachers including new knowledge and deeper understanding (Hord, 1997). "In analyzing students' work together, teachers have numerous opportunities to focus on learning issues and how those concerns link with pedagogy” (Driscoll & Moyer, 2001, p. 282).

**Using Mathematical Problem Writing**

Presumably, all teachers, at some point, have believed, based on performance on assessments, that their students thoroughly understood a concept, only to discover that their understanding is, in reality, lacking. Drake and Barlow (2007) conducted a multi-year study with teachers who faced such assessment issues; problem writing by the students was one of the strategies which they employed. Writing activity “engages students in creating mathematical word problems (…) designed to match the mathematics being studied” (Drake & Barlow, 2007, p. 272). Writing about mathematics is an effective vehicle to help students think about what they are doing and how they can use math in problem solving (Mayer & Hillman, 1996).

Reading assignments in a variety of mathematical content areas with emphases on notation, vocabulary, and concepts, are appropriate for calculus classes as well as mathematics classes at other levels (Stickles & Stickles, 2008).

Advocating for incorporation of problem writing into elementary school classrooms, Barlow and Cates (2006) point to the resulting positive influences on teachers’ ability to assess their students’ mathematical understanding and problem-solving skills. "Writing in mathematics can also help students consolidate their thinking because it requires them to reflect on their work and clarify their thoughts about the ideas" (NCTM. 2000, p. 61). As mathematics students write about their mathematical problems, they reveal their thinking, and not only their understandings, but their misunderstandings, in rich ways that traditional assessments cannot discern (Drake & Barlow, 2007). Student reflection and communication of ideas are inextricably intertwined processes and are brought to light by the emphasis on writing (Pugalee, 2001). To provide fruitful student work products for analysis, the writing assignments must be created with an eye on what will provide insight into students' mathematical understandings (D'Ambrosio et al., 2010; Drake & Barlow, 2007; Whiten, 2004). Students who write about their approaches to solving a problem may employ an agonizingly circuitous route to the solution when a much more straightforward approach is available. Others may make unsupported assumptions or use diagrams which reveal their grasp of concepts as well as their misconstructions and confusion. All of this is valuable information for the teacher. Written response provides a metacognitive level of detail about the students thinking and actions (Pugalee, 2001).

The approach which teachers take in analyzing student work will impact both their analyses and their outcomes. Charalambos and Silver (2008) discuss the stark differences between a "proving" approach and an "improving" "approach; the former focuses on proving student learning gains, that the students "got it." The latter spotlights some to improving their practice through reflecting on data and using it to understand student thinking. "Switching from a proving to improving approach will yield more worthwhile discussions around student work – discussions that enrich our teaching as well as our students' understanding" (Charalambos & Silver, 2008). Williams (1999a) quotes a high school mathematics teacher who indicated that she was much more focused on getting her students away from memorization and more centered on thinking and understanding; one method she employed to accomplish this goal was to have students write about an object in motion for a lesson on velocity.

While most of traditional teacher professional development centers on teaching strategies, looking at student work strengthens the connections between learning and changes in instruction and curriculum (Baron, 2008). Teachers need to balance teaching meaningful skills with teaching so that students pass standardized tests. In studying three AP social studies teachers, Goss (2004) found that accomplishing both is possible, by providing teacher training in varying teaching methods (pedagogical knowledge) and incorporating weekly written assignments for the students which the teachers would analyze for student content knowledge.

“The goal of teaching mathematics for conceptual understanding entails an instructional practice that treats mathematics as a realm of ideas to be explored rather than exclusively a set of facts, procedures, and definitions to be memorized. In this way, we provide students with the capability to reason mathematically and to solve the new kinds of problems they will inevitably face in the future” (NCTM, 2000).

Professional development programs for teachers should focus on creation and selection of mathematically meaningful activities which increase cognitive demand and improve student participation (Copur-Gencturk, 2015). Referring to his more than ten (at the time) years as an AP Calculus exam reader, Kennedy (1999b,) contends that even seemingly simple problems have "remarkable subtleties buried just beneath the surface” (p. 346), and that these opportunities to learn from student work and learn more about calculus are important professional development benefits. Similarly, Cannon and Wise

(2004), former chair of the AP Biology development committee and former chief reader for AP Biology, respectively, claim that serving as an AP Biology exam reader will lead participants to be better biologists. They describe the reading as “a wonderful opportunity for professional development” (p. 807) and cite the exchange of ideas between high school Biology teachers and college professors as well as the chance to learn about test and rubric development.

Schools have begun putting teachers together to both look at student work and to assess classroom performance; these are both activities which teachers generally do on their own (Little et al., 2003). In researching technology-enhanced learning materials, Matuk, Linn and Eylon (2015) concluded that "customizations based in evidence from student work lead to improved learning outcomes; they define customization as "making small adjustments to tailor given materials to particular scenarios and settings" (p. 229).

In interviewing several students after they had completed a geometric problem, Jung (2015) found that even though a student had computed a correct set of sides for a right triangle with hypotenuse = 5, the student was not sure why the Pythagorean Theorem led to the correct answer and she was unable to explain how she would solve the problem if the angles were different than she assumed. The student had merely "plugged in" the values for *a*, *b* and *c* into $a^2 + b^2 = c^2$ and assumed integer values for *a* and *b*.

NCTM's listing of "Professional Development Guides" includes numerous works on assessing student work. For example, Drake and Barlow (2007) suggest that teachers share a new teaching idea they have gained from discussing student work. Driscoll and

Moyer (2001) recommend small groups of teachers analyze student work, discuss what they have learned about students' algebraic thinking, and deliberate on how their analyses of student work will affect their classroom practices. D'Ambrosio et al. (2010) propose instructional sequencing for algebraic proofs and ask teachers if analysis of student work provides any new revelations about student thinking. Krebs and Keiser (2005) describe an activity for creating a rule for the total number of toothpicks needed to make a figure; after teachers have explained their rules and the reasons why these rules function as desired, the teachers then compare their own strategies and explanations of reasoning with the strategies and reasoning shown in student work. From this examination of student work, teachers can gain insights into student thinking.

When teachers engage in mathematical tasks and assessments, they develop proficiency in evaluating the work of their own students; this professional development leads teachers to engage in beneficial practices, such as, heightened recognition of their students' cognitive abilities; strengthening their own mathematical understanding; and, collegial discussions about assessment (Krebs, 2005).

Formalized professional development sessions, off-campus courses and workshops as well as professional days on-campus, can be of enormous help to both new and experienced teachers. The big drawback is the time required and time is a precious commodity for teachers and schools. For the classroom teacher, assessments are usually completed alone, or possibly discussed with students or parents; but, these generally occur in isolation from colleagues (Little et al., 2003). However, the absence of opportunities to collaboratively assess with colleagues does not leave a teacher without valuable resources. When discussing struggles with teaching special right triangles for the first time, Jung (2015) states, "I learned that I need to start by analyzing my students' mathematical reasoning before jumping into teaching a topic" (p. 33).

A quality vs. quantity model for assessing student work proposed by McNeill, Bellamy and Burrows (1999) is claimed to have had "a profound impact on the quality of student work and performance" (p. 486). At the conclusion of a longitudinal study, Copur-Gencturk (2015) concluded that the CKT which teachers gained over the three years was a significant factor in the quality of the classroom practices.

There is currently substantial research on how teachers learn while working together to create lesson plans, authentic activities to use during their lessons, and in looking at student work. However, the clear majority of this research is focused on elementary and middle school teachers. Research on collaborative work done by secondary school teachers is not so voluminous. This research concerns not only secondary school teachers but also includes college professors. The researcher hopes to make a meaningful contribution to the literature with this study of collaborative work of both categories of educators.

**The Power of the Wrong Answer**

"One of the single most powerful ways to make an impact on young children's thinking is by accepting incorrect answers or ideas as a natural part of doing mathematics and pursuing them in the same way as correct solutions" (Kline, 2008, p. 146). In discussing a study performed by Dr. Philip Sadler, Kamenetz (2016) describes middle school science teachers trying to identify the answers which their students had chosen on a quiz; the study found that the teachers had low knowledge of common misconceptions of their students. However, the teachers with more robust knowledge of their students' weaknesses led their students to learning significantly more science, according to a retest at the end of the year. Sadler and Sonnert (2016) found that "teachers' subject-matter knowledge should be considered a necessary, but not sufficient, precondition of knowledge of students' misconceptions" (p. 30).

Teachers need to be able to address their student's misconceptions and steer their students toward discovering the correct concepts, without devaluing the students' original thinking. "It's very expensive in terms of mental effort to change the ideas that you come up with yourself. It's a big investment to say, 'I'm going to abandon this thing that I came up with that makes sense to me and believe what the book or the teacher says instead' " (Sadler & Sonnert, 2016).

**Student Achievement in AP Courses**

Steinberg (2009) reports that about 2/3 of the teachers in a survey of 1,000 believed that students enrolled in AP courses for pragmatic reasons and not for academic achievement, and about 75% of the teachers believed that school administrators were increasing AP course offerings to step up the school's status and reputation.

The idea of scaffolding, introduced by Vygotsky (1978), is the altering of the amount of support a teacher gives a student, dependent on the student's learning and cognitive development. This is a dynamic process in which the teacher continually assesses and modifies the level of intervention. Goss (2004) states that as schools are casting wider nets for students in AP class, we can still teach meaningful content while still "teaching to the test," and have high student success. Strategies for accomplishing these goals in AP social studies courses include frequent written assignments, using former AP exams for practice, and, vocabulary development. Wanke (1975) was concerned that the expansive breadth of the AP Biology course content resulted in students who had a superficial glimpse of everything of which a biology major would have, instead of developing true mastery of some aspect of biology. The AP Biology teacher at the researcher's school has lamented the growing number of topics she must cover and the reality that she must choose which topics to emphasize and which to minimize, much as described by Wanke over 40 years ago.

Recently, however, similar to AP Calculus, the AP Biology Curriculum Framework has been undergoing changes which "reflect a larger effort to transform biology education as a whole; emphasis on breadth of knowledge to an approach based on core content knowledge, understanding of science as a process, and inquiry-based learning" (French, 2012, p. 300).

In referring to the AP Chemistry move to focus on "conceptual understanding of chemistry content over more rote memorization of facts and algorithmic problem solving" (Herrington and Yezierski, 2014, p. 1369) assert that the needed inquiry-based instructional changes will require a change in teacher professional development. This PD must engage teachers in "activities to help them develop a deep and personal understanding of these theories" (p. 1379). Ma (1999) advocated for the same "deep understanding" of mathematics which she found in Chinese teachers, but lacking in mathematics teachers in the U.S. at the time. Kennedy (2014) writes about addressing the "big ideas" of chemistry in his AP Chemistry classes; the topics of chemistry are not supposed to be taught as separate entities in a chapter by chapter, sequential manner, but rather as interconnected concepts which merge to form a cohesive totality of knowledge and skills. Loucks-Horsley et al. (2003) state that limiting the focus of professional development to only the student learning outcome misses the critical ideas of changes in teachers' knowledge, changes in classroom practice, and incorporation of new strategies.

In 2014, the AP Calculus Curriculum Framework from College Board was remodeled around the four "big ideas" of Calculus, i.e., limits, differentiation, integration, and, for BC classes, series. The intent was well-defined focus on the "big ideas" so that students could find connections between the central concepts of the course (Dossey, McCrone, & and Halverson, 2016).

Although the AP Calculus course is unambiguously written and the exams are well aligned to the course content, critics point to the breadth of the course at the expense of depth (D. Klein, 2007), specifically, concerning the time spent on the Fundamental Theorem of Calculus and on the Mean Value Theorem for integrals.

Progression to AP Calculus in the senior (or earlier) high school year is predicated upon accelerated placement. Enrollment in AP Calculus is preceded by Honors Precalculus as a junior and Algebra II (or Advanced Algebra II) as a sophomore. Algebra I and Geometry can be completed in any order as a seventh or eighth grader or a high school freshman. Those who enroll in Algebra I, Geometry, and Algebra II for their first three years of high school, do not have the requisite Precalculus skills to qualify for AP Calculus. Ma (2010) describes accelerated mathematics students as those who complete

Algebra I in either 7th or 8th grade. With Algebra I, and possibly another math course, completed before high school, the student can complete both Algebra II, Geometry and Precalculus in high school and still have high school time remaining to complete AP mathematics courses. Ma (2010) found that even gifted students who were not accelerated were unlikely to take calculus in high school.

Focusing on international awareness, Kelly (2004) states that as the AP program expands, classroom teachers are sharing their instructional materials on the internet; teacher educators, including those who provide teacher education will need to adapt their own teaching. Nichol, Szymczyk and Hutchinson (2014) developed a model to support the "shift to more conceptual and inquiry-based teaching and learning in AP chemistry" (p. 1318), the AP Chemistry Curriculum Framework focuses on concept development and understanding over the more traditional rote memorization and calculations.

Thirty years ago, Dickey (1986) concluded that students who complete AP Calculus BC achieve at a level comparable to that of their counterparts completing a first-year college calculus course and suggests that these students be awarded college credit for a full year of calculus. However, he did not include AP Calculus AB students in his study.

Dick, Dion and Wright (2003) laud the incorporation of graphing calculators into the curriculum as complementary to the multi-representational nature of AP Calculus; that is, calculators not only eliminate the need for tedious pencil and paper calculations, they also make topics such as Euler's method, slope fields and zoom-in/out for functional analysis much more accessible. These are significant tools in helping students visualize concepts through a graphical approach. For example, graphing the infinitely oscillating behavior of the graph of $f(x) = \frac{sin(x)}{x}$, the student can zoom in to more and more narrow intervals and see that the oscillation never stops. Dick, et. al. (2003) also conclude that both low-tech and high-tech tools serve to help teachers focus on "improving students' understanding of and ability to use good mathematics" (p. 596). The best teaching does not showcase a heroic protagonist because the best teachers are generally not entertainers yammering away at the front of the room, but are more often found, almost invisible, coaching from the back; truly great teaching is less about the performing teacher and more about the performing student (Gillard, 2012), which is the intent of teaching.

**Research on the AP Calculus Exam**

Colleges and universities have viewed performance in AP classes and scores on the AP exams to be predictors of post-secondary academic success (Ackerman et al., 2013; Kelleher, 2004b; A. Klein, 2007). However, Viadero (2006b) points to studies which cast doubt on the AP program being a miracle cure for the United States lagging behind other nations in mathematics and science. On the other hand, Viadero (2006a) also cites a study which supports the idea that actually passing the AP exam is a more important factor in on-time graduations that simply taking AP courses.

Barbour and Mulcahy (2006) studied the differences in both student achievement and retention in AP courses between students in Newfoundland who used a web-based course delivery method and those who experienced AP classes in a traditional classroom setting. In both urban and rural settings, a higher percentage of classroom students earned a 3 or above on the AP exam. The importance of teacher presence would seem to be established.

Students who earn low scores on the AP exam should not be discouraged. Answering some of the questions correctly indicates that the student did learn something. In arguing the value of hard work, John View, a director of financial aid for the State University of New York, states, “I would rather see a solid student who took AP Calculus and perhaps scored low on the AP test, than one who chose an extra year of keyboarding or a foreign language in which the student had ample classroom time. This is especially true for schools where the competition is intense or for schools that have tough academic programs centered on math and science" (Fox & View, 2006, p. 4).

Several studies have shown that higher college GPA, higher college graduation rates and more credits earned are among the benefits gained from passing AP exams when compared to simply completing AP courses without sitting for the exam (Hargrove et al., 2008).

**Research on Professional Development for AP Readers**

Teacher professional development offers the possibility of positive impact on teacher classroom practice and, by extension, on affecting student achievement (Ganser, 2000; Payne & Wolfson, 2000; Wenglinsky, 2002).

Hoy (2009, 85) found what she described as "critical links among student achievement, professional development, and instructional practices as perceived by educators." Franke and Kazemi (2001b) contend that "focusing on students' mathematical thinking remains a powerful mechanism for bringing pedagogy, mathematics, and student understanding together" (p. 108). Marzano (2003) views professional development for teachers as a key element of collegiality and professionalism.

Franke and Kazemi (2001b) describe professional development, where teachers engage in learning collaboratively, as a part of the teacher's practice. And, Marshall and Sorto (2012) found significant effects of the teacher's SCK and CCK on overall student achievement gains.

Blank, de las Alas, and Smith (2008) described the goal of education research to establish a direct relationship between teacher learning and student learning as an important challenge for the mathematics education community. While a few published commentaries about AP exam readers' experiences are available, there is no research which specifically addresses the professional development which occurs at the AP Calculus exam reading.

**Working Grounded Theory**

While much research is based on using accurate facts to test or validate existing theories, in 1967, Glaser and Strauss presented the idea of discovering a theory from the data. They dubbed this process as "grounded theory," because the theory was grounded in the data. They "reiterated that the ambition of grounded theory is not verification of a preconceived theory, or capacious description; rather it is unambiguously defined by its exclusive endeavor to discover an underlying theory arising from the systematic analysis of data" (Kenny & Fourie, 2014, p. 2). Glaser and Strauss (1967) also stipulate that grounded theory researchers must approach their studies without any preconceived notions or intention to prove or disprove. The quest needs to be focused on revealing “and ultimately conceptualizing the principal concern of participants” (Kenny & Fourie, 2014, p. 2).

Moore (2009) contends that grounded theory does not possess clearly elucidated epistemological assumptions and that this has led to misinterpretation and misuse of method. Urquhart and Fernández (2013) point out several prevalent misconceptions of grounded theory. However, they identify as harmful the incorrect belief that the researcher needs to be a blank slate. They opine that delving into research without reviewing pertinent literature is harmful; they hold that it is folly for the grounded theorist is “to forget what they know in order to learn what they need” (p. 226). Simply because the researcher must separate herself from existing theories, Urquhart and Fernández (2013) contend that there is no implication that “grounded theory researchers must ignore the existing literature and become a tabula rasa” (p. 226).

Charmaz (2006) contends that the guideline for collection and analysis of qualitative data in grounded theory are both systematic as well as flexible. And, she claims that “grounded theory methods have the additional advantage of containing explicit guidelines that show us how we may proceed” (p. 3). These guiding principles were formulated by Glaser and Strauss (1967) as: (a) concurrent data collection and data analysis; (b) developing data-based, not theory based, codes; (c) develop concepts by repeatedly comparing data, categories and concepts; (d) simultaneous theory development throughout the collection and analysis processes; (e) memo writing; (f) use sampling for construction of theory, not concern for generalizability to the entire population; and, (g) conduct data analysis before delving into the literature review. They advised grounded theorists to postpone their literature review, and thus avoid the influence of extant ideas, and to cultivate original theories (Charmaz, 2006).

Kenny and Fourie (2014) assert that the researcher reaches a theory at the end of the research and that this theory encompasses the main issues of the study. It is intended that this study will lead to the discovery of a working theory evolving from analysis of data on the influence of the breadth of AP Calculus exam reading experiences on teacher knowledge, classroom practice and teacher observations of impacts on their students' understanding.

Charmaz (2006,) asserts that grounded theory "is distinguished from others (methods) since it involves the researcher in data analysis while (simultaneously) collecting data. We use this data analysis to inform and shape further data collection" (pp. 187-188).

**Summary**

We know that student achievement is linked to teacher knowledge and teacher professional development. We see the positive correlations between AP exam reading participation and ISL. AP Calculus readers talk about the massive benefits to their practice and their students' achievement because of their AP Calculus exam reading experiences. Yet, there are currently no studies of the impact of the professional development at the exam reading for the educators who grade those AP exams. This study is intended to bridge that gap.

# Chapter Three

## Introduction

The purpose of this study is to examine teachers' beliefs about how their participation in the AP Calculus exam reading affects or influences their classroom practices, and, how their participation has affected or influenced their students' understanding, as perceived by the educators.

The researcher's own classroom practice as a secondary Mathematics/Calculus teacher has evolved because of her experiences as an AP Calculus exam reader. She has also noticed an improving trend in her AP Calculus students' understanding of the course content. And, based on both her own experiences as well as those her colleagues, from a student's perspective as well as that of the parents and the school administrators, a teacher who grades the AP exams has high credibility on AP exam matters. The researcher has analyzed her students' AP Calculus exam scores and have found that there is no clear upward trend in average scores since she started reading the AP Calculus exam. However, if she considers only the students who had the qualifications and motivation to be admitted to the class, an upward trend in the percentage of students scoring at the 4 and 5 levels is apparent. Given that her AP Calculus classes are generally small, averaging 12-18 students for AP Calculus AB and 6-8 students for AP Calculus BC, this improvement could be an anomaly of small sample sizes or due to increased teacher experience. Clearly more data is needed before any reasonable inferences can be drawn.

This study explores the professional development which Calculus educators experience at the varied activities at AP Calculus readings and how they report that these experiences affect teachers' classroom practices and their students' learning in their Calculus courses. The logistics of the reading organization, the research questions, the pilot study, and, the methodology will be discussed in the following sections.

**The Exam Reading**

In this section, the logistics of participation in the AP Calculus reading, including reader selection, on-site training, and the actual reading activities are explored.

**AP Calculus Exam Hierarchy**

The hierarchy of the reading starts with the Chief Reader and the Assistant Chief Reader, who oversee the entire reading. There are two exam leaders, one for the AB exam and one for the BC exam. There are also three more exam leaders, one for each of the alternate exams. And, each of the free response questions (FRQ) in each exam has a Question Leader. At the AP Calculus reading, there are approximately fifty reading tables, each with two table leaders and 16 readers. The table leaders ensure that the readers are following the grading standards, answer questions, and resolve discrepancies. Readers are assigned to one seat at a specific table for the entire reading. The person on either the left or the right of a reader is the reader's reading partner, with whom the reader can consult, if needed.

In earlier years, the AP Calculus readings were held at college campuses and the readings took place in classrooms. With the growth of AP Calculus, a larger venue was required. The reading is now held on the second floor of a large convention center; the area is a huge, warehouse-looking space which stretches for several city blocks. The areas for reading are cordoned off with poles and blue curtains similar to the one from an industrial advertisement shown in Figure 5. Typically, four tables are cordoned off together. This grouping is called a pod. The individual table areas, however, are still referred to as rooms. The layout of a reading room is shown in Appendix H.

*Figure 5.* Typical Curtain Partitions for Reading Pods.

During the week before the reading, the Chief Reader, exam leaders and others at the top of the reading structure arrive in Kansas City and grade several thousand exams to ascertain any common problems with the answers, to determine any repetitive student errors and to fine tune the grading rubrics. The table leaders arrive at the reading site two days before the readers to gain experience with the grading standards.

**AP Calculus Readers**

The College Board encourages experienced AP Calculus high school teachers and college mathematics professors to apply to become AP Calculus readers. After being accepted, a teacher may wait on the list of approved readers and not actually be invited to a reading for several years. Readers are chosen based on experience and the needs of the College Board. Experienced readers are invited to return the next year based on their reading speed and accuracy and on the number of readers needed. Accuracy is measured by adherence to the grading rubric and all its idiosyncrasies; this is evaluated by the table leaders. Speed is evaluated by comparing how many exams a reader grades compared to others at the same reading table.

The Educational Testing Service (ETS), who administers the exams for the College Board, attempts to invite approximately equal numbers of high school teachers and college professors, and approximately equal numbers of men and women. They also attempt to have as many states as possible represented; teachers from American schools overseas are also included.

The first invitations are issued via email in January; invitees have three weeks to respond. After the College Board assesses the number of acceptances, they issue another set of invitations in March. A third set of invitations is issued in May to ensure that a sufficient number of readers will be present for the reading. After accepting the invitation via email, a reader is directed to the reader registration site where information on travel, lodging, transportation and other logistics can be found. Using links from this site, a reader can make airline reservations, request a specific roommate, read announcements from the Chief Reader, and access a list of the current year participants. The ETS pays all travel expenses, including air fare, meals and transportation costs on the two travel days. Meal service, hotel accommodations, and frequent bus service to and from the hotels and the reading site are provided on-site for the duration of the reading.

The College Board© touts the benefits of reading the AP exams as: exchanging ideas with faculty, teachers and AP development committee members; establishing friendships in a worldwide network of faculty members; and, becoming familiar with AP scoring standards, which provide valuable knowledge for scoring your own students' work (College Board, 2017a).

The AP readings are currently held in four different U.S. cities and occur over approximately a two-week period, with the exception of Salt Lake City, UT, which hosts for one week. AP U.S. History has the highest number of readers, 1700 in year 2015, an increase of 400 over the previous year. AP Calculus comes in sixth at 884 readers, behind English Language, English Literature, World History and Spanish Language. Except for AP U.S. History which reads for eight days, all readings last for seven days, including full work days on Saturday and Sunday. A complete schedule of dates and locations for the 2016 readings is at Appendix C.

In 2016, there were over 900 AP Calculus readers who share the huge Kansas City convention center with readers of other disciplines. Each discipline has their own areas for briefings, the actual reading work and professional development. Dining facilities are shared by all disciplines.

One the first day of the reading, all exam graders, referred to as readers, report to a large open space on the top floor of the Kansas City Convention Center which has approximately 800 chairs and two large screens on a large stage. Readers are welcomed and the mechanics of the reading are explained. Readers are given their table assignments. The Chief Reader discusses the reading, gives some statistical information and introduces key personnel. Then, about half of the readers are directed to another large room and briefed on question AB/BC-1. The remaining 450 readers are briefed on question AB/BC-3. At the beginning of the question briefing session, the readers receive a grading rubric, a packet of about 15-20 samples of student work, and paper for notes. While the assistants display the information on the large screens on the stage, the question leader discusses the rubric, and all items which readers need to consider. The question leader shares thoughts on the philosophy of the question, including the conceptual understanding and computational skills the student was expected to demonstrate. Criteria for awarding partial credit consumes the lion's share of the briefing time. Rubrics are not up for discussion or debate. Question leaders do not attempt to persuade or reach agreement on the rubrics; they instruct the readers how to use the rubric. There is little doubt that readers would grade the questions differently if left to their own judgements. Among the readers, there is always some disagreement with grading rubrics, some trivial, some more substantial. These briefings, which include numerous examples, help to ensure uniformity of grading. Then, the briefer reviews several exemplars from the packet, pointing out nuances in the grading standard. After a thorough review, the briefer directs the readers to a specific student sample and instructs everyone to grade it on their own. After a minute or two, the briefer announces the correct score and then discusses each item in detail and justifies the score. This is repeated several more times. These briefing sessions usually approximately 2 hours. Readers then report to their reading tables, meet their table leaders and fellow readers. They then read two more student samples and the table leaders confirm the correct score. If anyone disagrees, one of the table leaders reviews the scoring with the reader. Readers are now ready to start their work. A breakdown of reader activities during the first day of the reading are shown in Table 4. On subsequent days, readers will be briefed on a second question and the cycle begins again. Readers typically read three different questions at the reading; on occasion, if one question is requiring more time than expected to grade, some tables may be briefed on a fourth question.

Collegial discussions at the AP reading suggest that there is a positive correlation between teacher participation as an exam reader and student achievement. Jennifer Pfannerstill, former Assistant Chief reader for AP Biology, says on a College Board video that "the best professional development is in the dining halls. Oh, to eat at night wherever you might be. Because just sitting across from another AP teacher, from a college professor, finding out what they think is important in the curriculum, finding out how they do a lab, finding out maybe what their tests are like, sharing email addresses, so throughout the year you can exchange ideas, that's really important" (College Board, 2017a).

Educators in varied disciplines claim that they have become better teachers because of their AP reading experiences. Just how they are measuring "better" and to exactly what experiences they are referring are somewhat elusive except for very general descriptions. Of course, these will vary somewhat for each individual educator.

Table 4

*Typical First Reading Day Schedule*

| Time | Activity | Lead |
|---|---|---|
| 8:00 - 9:00 | Welcome Remarks<br>Explanation of Overall Logistics<br>Distribution of Table Assignment | Chief Reader<br>Assistant Chief Reader |
| 9:00-9:15 | Break<br>Readers are split into two groups; one moves to another location and one remains | All |
| 9:15-10:30 | Briefing on Question AB/BC-1 (Group 1) or Question AB/BC-3 (Group 2) | Question Leader and Assistants |
| 10:30-10:40 | Readers arrive at assigned tables<br>Introductions of readers and table leaders | All |
| 10:40-10:50 | Readers grade the two assigned questions<br>Table leaders check that everyone graded correctly | Readers and Table Leaders |
| 10:50-12:30 | Reading | Readers and Table Leaders |
| 12:30-1:30 | Lunch | All |
| 1:30-3:30 | Reading | Readers and Table Leaders |
| 3:30-3:45 | Break | All |
| 3:45-5:30 | Reading | Readers and Table Leaders |

## Research Questions

The major research questions address what these educators mean by "better" and to investigate how they assess that. The major research questions are:

Research Question 1: How do AP Calculus teachers and college Calculus professors perceive the professional development experiences at the AP Calculus exam reading?

Research Question 2: How does participation in an AP Calculus national exam reading affect teachers' and professors' classroom practice, as perceived by the educators?

Research Question 3: How do exam readers report that their participation in an AP Calculus national exam reading influences their students' success, as perceived by the educators?

## Pilot Study Design and Execution

Before finalizing the survey questions, a pilot study was planned which occurred at the AP Calculus Exam reading in June 2014. During the AP Calculus reading, the opportunity to interact with other AP Calculus readers and get their input for a few baseline questions was availed. The intentions were to gather initial data to assess the educators' openness to discuss the research topics; and, expose any area or idea which may have not been included.

The AP readings are accomplished in four different cities in two time periods. The AP Calculus reading has been held in Kansas City, MO, every year since 2008. During the week of the 2014 AP Calculus reading, several other disciplines, e.g., Statistics, Biology, and Psychology also held their readings in Kansas City. However, AP Calculus is the largest group by far. All readers wear name tags and each discipline has their own color. This way, it is easy to identify the discipline of any reader. While each discipline has their separate areas for their work and evening activities, all readers share the same dining room. The large dining hall houses approximately 150 tables, each of which seat 8-10 people. Readers may sit at any table at any meal. There is no assigned seating.

When designing her pilot study, the researcher included the open-ended questions which she intended to use in the survey. She did not ask demographic questions or Likert scale questions because she was not trying to do a mini version of the survey. She wanted to know if she was asking the best questions in order to determine if they would lead to other relevant questions. She tried to achieve some degree of randomness in the people she asked to participate, never asking anyone with whom she had had a previous conversation. She was able to interview educators at different levels (reader, table leader, question leader, assistant chief reader) in the AP Calculus reading community and she conducted 12 interviews over the seven days. She obtained written informed consent for audio taping of all interviews.

**Pilot Study Results**

The first question for each person was why she/he had applied to become an AP Calculus reader. Every person replied that they had a colleague who recommended they apply. The colleagues had all had positive experiences and were encouraging about the reading experience. The researcher found this surprising as she had sought out the reading application on her own. No one at her school was an AP reader, in any subject area. She had no colleagues with AP reading experience.

The second question asked if the readers had changed anything about their course content and/or their teaching approach because of their experiences at the AP Calculus exam reading. The common responses were an increased emphasis on interpretation in context and using correct units. The researcher received the most varied responses to her question about what was the most beneficial aspect of becoming and AP Calculus exam reader. Most teachers expressed more than one reason for assessing the reading experiences as beneficial. Several cited the friendships which had started at a reading; others focused on the variety of the professional development available. Almost all the interviewees said that they were more meticulous and clear in their classroom lectures because they had seen how seemingly small mistakes revealed lack of understanding. For example, a student who states that a rate is decreasing by negative 10 centimeters per second, is stating that the rate is increasing. As a result, several readers said that they have stressed this point with their classes. A similar confusion with positive and negative versus increasing and decreasing is evident on the exams. Again, from table conversations, the researcher gleaned that many readers stress these concepts and their differences because of their AP reading experiences.

Based on the results of her pilot study, the researcher sensed that she had designed the best questions; but, she was not asking enough questions to answer her research questions. Her interview questions have increased from 10 basic questions to 19 probing questions. For example, instead of asking if a teacher's or professor's students had benefitted from the teacher's participation in the reading, the refined questions concerned the teacher's impression of how the reading experience had affected student learning, teacher classroom practice, a teacher's perceptions of her/his own competence, and what measures are used to assess student success. Additionally, questions were added to ask about balancing conceptual and procedural knowledge with students; teacher gains in understanding of content knowledge, curriculum, reframing of concepts and/or dispelling misconceptions; and, any obstacles which educators encounter in changing classroom practice.

**Method and Instruments**

The focal points of this research are both identifying the commonalities in the experiences which have led AP Calculus exam readers to view themselves as changed, or not changed, as well as quantification or description of any such changes. To these ends, this study was designed to employ a mixed methods approach. First, a pilot study was conducted to ensure the right questions were being asked and to determine if other questions were needed. Second, the data collection through the online survey opened to a large population. As an incentive, a random drawing for gift cards was offered. And, third, after the survey, 17 interviews were conducted to follow-up on and gather elucidating information gleaned from the survey. A second random drawing for gift cards was also held for the interviewees. For example, interviewees were able to provide a wealth of information on their social interactions at the reading as well as specific issues with grading the exams, and to inject emotion into their responses with tone of voice. These could not be extracted from the written survey. Surveys collected data on experiences, attitudes and opinions, as well as demographic data. Quantitative data included Likert scale questions about the value of the professional development at the reading, attendance at evening PD sessions, and, desire to return to the reading in future years, to name a few, and demographic information. Collection of usable, quantitative AP exam score information was problematic; however, ample qualitative assessments of changes in students' conceptual understandings occurring after the teacher's participation in the AP Calculus reading were offered. AP exams are completed only by high school students. Therefore, college professors were asked to quantify any changes in their students' performance by test grades and/or course grades. However, most gave qualitative summaries. Institutional Review Board approval is at Appendix A.

**Online Survey**

The survey included four parts: informed consent, Likert scale questions, open-ended questions and demographic information about the educators and their institutions. Two versions of the survey were produced, one for high school teachers and one for college professors. The two versions were essentially equal; however, the wording of some questions was adjusted. For example, both high school teachers and college professors were asked about the type of institution at which they taught; however, the answer choices differed and were applicable to their respective institution types. A summary of educator age, type of institution, and location for high school teachers and college professors are shown in Table 5 and Table 6, respectively.

However, one question was present only in the high school version. The high school teachers were asked to supply student scoring information and to identify any trends in the students' score since the teacher became an AP Calculus reader. College professors could not provide scoring information; but, they were asked if they noticed any differences in their students who had taken AP Calculus in high school and those who had not.

Table 5

*Type of Institution and Educator Age vs. Institution Location for High School Teachers*

| | Location | | | |
|---|---|---|---|---|
| Type of Institution | Inner City | Urban | Suburban | Rural |
| Public | | | | |
| Age 31-45 | 1 | 4 | 10 | 1 |
| Age 46-60 | | 2 | 15 | 6 |
| Age 60+ | | | 4 | |
| Private/Independent | | | | |
| Age 31-45 | | 2 | 4 | |
| Age 46-60 | | 2 | 1 | 1 |
| Age 60+ | | 1 | 1 | 1 |
| Private/Religious | | | | |
| Age 31-45 | | | 1 | |
| Age 46-60 | | 2 | 1 | 1 |
| Age 60+ | | 1 | 1 | |
| Magnet/Charter | | | | |
| Age 31-45 | | | 2 | |
| Age 46-60 | | | | |
| Age 60+ | | | | |
| Total | 1 | 14 | 40 | 10 |

Initial emails invitations were sent to 902 potential participants. The email briefly explained the intended research and requested that they participate. At the end of the

Table 6

*Type of Institution and Educator Age vs. Institution Location for College Professors*

| | Location | | | |
|---|---|---|---|---|
| Type of Institution | Inner City | Urban | Suburban | Rural |
| Public 4-year College | | | | |
| Age 31-45 | | 2 | 2 | 4 |
| Age 46-60 | 1 | 7 | 2 | 2 |
| Age 60+ | | 8 | | 4 |
| Private 4-year College | | | | |
| Age 31-45 | | 7 | 6 | 2 |
| Age 46-60 | | 3 | 4 | 4 |
| Age 60+ | | 3 | | 1 |
| Religious or Women's College | | | | |
| Age 31-45 | | 1 | 1 | 1 |
| Age 46-60 | | 1 | 2 | 1 |
| Age 60+ | | 2 | | |
| Community College | | | | |
| Age 31-45 | | | | |
| Age 46-60 | | 2 | 7 | 3 |
| Age 60+ | | 1 | 1 | 1 |
| Total | 1 | 37 | 25 | 23 |

email, there are two links to the research survey, one for high school teachers and one for college professors. When accessing the link online, a participant was taken to the survey.

Part I of the survey is the informed consent. Participants may give their consent online. The participants are told that without their consent, their input cannot be used. Part II of the survey consists of eleven Likert-scale, agree/disagree, questions. Part III included several open response questions. And, Part IV was on demographic data. See

Appendix I through Appendix L for the high school teacher survey; see Appendix M through Appendix P for the college professor survey.

About half of the high school teachers are in the 46-60 age bracket; about 62% of high school teachers are located in suburban areas. For the college professors, roughly 45% are in the 46-60 age group and highest proportion, 43%, are located in urban areas.

It was expected that some issues would arise from the survey which would need clarification and elaboration. The purpose of the interviews was to address these issues. The researcher wanted to ensure that she cast a wide net to ensure that she captured a gamut of opinions and experiences. The survey questions were designed to obtain educators' opinions on the areas of professional development, classroom practice and student learning and they align directly with the research questions. The number of questions was kept reasonably short to avoid survey fatigue and to ensure that most participants would complete the survey. Demographic data was collected to determine if age, physical location, years in teaching, years of exam reading, etc. are associated with particular viewpoints and/or outcomes.

**Interview Plan**

Interviews were planned to occur after the survey closed and had been initially analyzed. It was anticipated that the interviews would garner more information than could be collected from the surveys. The plan was to interview at least five high school teachers and at least five college professors.

Because there is no precedent for this research, the survey and the interview protocols were developed by the researcher and were piloted during the AP Calculus reading in June 2014. Therefore, there are no formal tests of validity or reliability. However, the author did create a validity matrix as suggested by Maxwell's (2013) text on Qualitative Research and consulted Fink (2008) on the conduct of surveys. See Validity Matrix in Appendix F.

At the end of the online survey, participants were asked if they would we willing to be contacted for a 30-45-minute interview. A total of 72 participants agreed to this. Using a random number generator, 20 individuals were selected and contacted. However, because not all volunteers responded, two more iterations of randomly choosing interviewees were conducted. Several people procrastinated in responding affirmatively for several weeks. By then, the researcher had already randomly chosen and contacted other potential interviewees. However, all who responded were included in the interviews. A total of eleven high school teachers and six college professors participated in the interviews. All seventeen interviewees were included in the second random drawing. Interview questions are at Appendix Q.

**Composition of the Study**

In the following paragraphs, the population of interest and sample selection, participant response, characteristics of the responding sample, and the interviews are discussed.

**Population and Sample Selection**

In recent years, more than 900+ educators serve as AP Calculus readers each year. These educators are from all 50 states and territories, as well as from American schools overseas. As the number of students who complete the exam increases, the number of readers increases proportionately. While many readers return for several years, each year, new readers are needed to replace those who do not return or have retired. The College Board publishes a list of all readers on the AP Calculus reader website. The reader lists contain the readers' name, position and institution. The city and state of the institution is not listed. Using the lists from the 2013, 2014 and 2015 readings, the researcher found that several hundred-people appeared on two or all three lists. In building the master list, these duplicates were removed. The names of people in upper management positions were also removed; only readers and table leaders were included. The aim of the survey was to obtain the perceptions of the people who read exams all day, not the people who organize and direct the reading. The researcher also removed anyone with whom she had any know association; this included the researcher’s former table partners, table leaders, and roommates for her years at the readings. Lastly, she removed the names of anyone with whom she had even a remote association, for example, someone with whom she had once had a conversation and whose name she recognized. The final master list included 1316 names.

To obtain email contact information for the potential participants, internet searches of names and institutions were conducted. Some potential participants are employed at institutions which do not offer contact information. These people could not be contacted. Other potential participants are employed by institutions which do not provide direct email addresses but require an email request with sender contact information. These people were not contacted because the invitation could not fit in the limited space allowed for messages. Otherwise, all readers from 2013-2015 readings for whom email contact information could be located were contacted via email invitation to participate. The email invitation is at Appendix G. In total, 902 potential participants were contacted via email and asked to complete the online survey. The survey was open for participation from mid-July to the end of September 2015 to ensure that those who do not access their email during the summer would still have the opportunity to participate. For data collection, a survey is the only practical method of reaching this number of people who are geographically dispersed.

The study began with an online survey in four parts, including informed consent. All responses were anonymous. Demographic information was collected; no information was sought which could jeopardize anonymity. At the end of the survey, each participant was asked to click on the Submit button. After doing so, the participant was electronically sent to another survey in which they were asked two questions. Because they are in another survey, their responses cannot be linked back to their responses to the main survey. The first question asks them if they would like to be entered into a random drawing for one of three Amazon.com gift cards. See Appendix R. If they answer "yes," they are asked to supply their name and email address so that they can be notified if they win. The second question asks if they would be willing to participate in a one-on-one interview via telephone in a few months. See Appendix S. If they answer affirmatively, then, again, they are asked to supply their name and email address so that they can be contacted for the interview. They are assured that their choice to participate in the interviews has no bearing on their chances of winning a gift card in the first drawing. At this point, those who volunteered to participate in the interviews were given another opportunity to enter a second random drawing for one of another three Amazon.com gift cards.

Winners of the random drawing for the online survey were selected by random draw and were notified in December 2015. Winners of the random drawing for the interviews were selected by random draw and were notified in March 2016.

## Participant Response

A total of 102 college professors completed the survey. However, 14 did not check the "consent to participate" box; their responses were not used. A total of 88 useable responses from college professors were received. A total of 83 high school teachers completed the survey. However, 2 did not complete the survey and checked the "do not consent to participate" box. And, 14 did not check the "consent to participate" to give consent even though they completed the survey; their responses were not used. A total of 67 useable responses from high school teachers were received. The total number of participants included in the survey is 155.

## Characteristics of the Sample

Over half of the high school teachers work at a public school; 85% of the teachers are in the age group 31-60. There was only one participant who works at an inner-city school; this person was included in the Urban category. There was only one person in the 21-30 age category; this person is included in the 31-45 age group. One high school teacher did not report age; one did not report location; and, one did not report type of high school.

Several educators teach classes at both a high school and a college. In the case of educators who teach at both levels, they were listed as high school teachers if they taught an AP Calculus class, and, therefore, could comment on any changes in their students' AP exam scores. A few people indicated that they taught at a school which is public and charter; they were classified as teaching at a charter school.

Two college professors did not report age. One college professor chose the location option of rural but noted that the actual course delivery is online. Two professors wrote that their location was a small town; they were coded as rural. The college professors did not have anyone in the 21-30 age group. No professors chose Men's College, Ethnically Based College, or Ivy League. Seven of the professors are retired. Seven professors chose the Religiously Based College option, one chose both religiously based and Women's College; and, one chose both Private College and Women's College. All nine were grouped in the Religiously-based or Women's category. All of the professors, except one who is an adjunct at a public university, reported that they have full-time employment. Seven professors also have part-time employment at other institutions; they were coded for the type of institution which is their full-time employment. Two did not report their work as full-time or part-time. Those on sabbatical were coded as full-time. 72% of the college professors teach at four-year institutions; 30% of the college professors are aged 31-45, 46% are aged 46-60, and 24% are aged over 60.

A summary of all survey participants by number of years of teaching experience and years of AP Calculus exam reading experience is at Table 7. Approximately 75% of the readers in any year are experienced readers returning for another session.

Table 7

*Summary of High School Teachers and College professors by Years of Teaching Experience and Years Reading AP Calculus Exams*

| | Years of AP Calculus Exam Reading | | | | |
|---|---|---|---|---|---|
| Years of Teaching | 1-5 | 6-10 | 11-15 | 16-20 | 21+ |
| 1-10 | | | | | |
| High School Teacher | 9 | | | | |
| College Professor | 4 | | | | |
| 11-20 | | | | | |
| High School Teacher | 13 | 2 | | | |
| College Professor | 9 | 18 | 4 | | |
| 21-30 | | | | | |
| High School Teacher | 18 | 8 | 1 | | |
| College Professor | 5 | 8 | 6 | 2 | 2 |
| 31-40 | | | | | |
| High School Teacher | 6 | 4 | 2 | 1 | |
| College Professor | 5 | 5 | 6 | 1 | 1 |
| 41-50 | | | | | |
| High School Teacher | 2 | | | 1 | |
| College Professor | 4 | 2 | 1 | 1 | 2 |
| Total | 75 | 47 | 20 | 6 | 5 |

Two people omitted data. About half of the respondents have between 1 and 5 years of reading experience; and, a little less than one-third have between 6 and 10 years of reading experience. Almost two-thirds of the participants have between 11 and 30 years of teaching experience; about 20% have 31-40 years teaching experience. And, 13 people have over 40 years of teaching experience. So, while less than 10% of the sample has fewer than 10 years of teaching experience, almost 80% of the sample has fewer than ten years reading experience.

## Interviews

Seventy-two people provided their email addresses to participate in post-survey interviews. In the first round of interviews, a random number generator was used to choose 20 of these volunteers; they were contacted and asked to indicate an agreeable time for their interview. Nine people responded. A second round of 20 interview requests, again chosen by a random number generator, garnered 5 more interviews. Eleven-of the interviewees were high school teachers. Therefore, for the third round of interview requests, a random number generator was used to choose college professors,

only. Three more interviews were arranged. In total, 17 interviews were accomplished 11 high school teachers and 6 college professors. The interviews were conducted over a three-month period from November 2015 through February 2016. Interviews ranged in time from approximately 25 minutes to 55 minutes. The average time was about 35 minutes. Transcriptions of all interviews were completed in August 2016.

## Data Collection

The specific questions of each type, Likert scale, open ended and demographic are discussed below.

**Data Collection for Research Question 1**

Research Question 1: How do AP Calculus teachers and college Calculus professors perceive the professional development experiences at the AP Calculus exam reading?

The first research question focuses on the perceptions of AP Calculus teachers and college Calculus professors of the various forms of professional development at the AP Calculus reading. The professional development takes on several forms: the briefings, the actual reading of exams, the collegial interactions, and professional development sessions in the evenings. Relevant data was collected by asking participants Likert scale questions concerning the professional development. And, in open-ended questions, they were asked to comment on their reasons for becoming a reader, the relationship between their reading experiences and their perceived professional growth and their classroom practices. The specific questions which were asked in support of research question 1 are shown at Table 8.

Table 8

*Survey and Interview Questions Designed to Answer Research Question 1: How do AP Calculus teachers and college Calculus professors perceive the professional development experiences at the AP Calculus exam reading?*

| Type of Questions | Questions |
|---|---|
| Likert Scale | Participating in the AP Calculus exam reading was worthwhile professional development for me. |
| | I find the interaction with other AP Calculus readers to be professionally beneficial. |
| | Professional development sessions are offered in the evenings at the reading. I found these sessions informative and useful. |
| | The way I think about teaching Calculus has been influenced by my experiences as an AP Calculus exam reader. This means your ideas about what should be taught and how it should be taught. Ideas, not your actions. |
| | I encourage my colleagues to become AP readers.<br>If scheduling allows, I intend to attend future AP Calculus readings |
| Open-Ended Survey Questions | Why did you decide to become an AP Calculus reader? What benefits/challenges did you experience? |
| | What are your opinions on the professional development at the reading? Please comment on the briefings, the exam reading, the collegial interactions, and the formal professional development sessions? |
| | Please share whether you find the opportunity to interact with other AP Calculus teachers and college professors useful, and in what ways? |
| | Do you have any other comments on your AP Calculus reading experiences and their relationship to your own professional development? |

| | |
|---|---|
| Interview Questions | Why did you decide to become an AP Calculus reader?<br><br>What did you expect to gain from your reading experience?<br><br>Was the reading experience what you expected it to be? How or how not?<br><br>What, if any, experiences at the reading stand out for you?<br><br>Have your experiences at the AP reading influenced your competence as an AP Calculus teacher? If so, how? What evidence do you use to support your thoughts?<br><br>What are your impressions of the AP Calculus reading as professional development for you?<br><br>What have you gained in your own understanding of content knowledge, curriculum, reframing of concepts and/or dispelling misconceptions? |

**Data Collection for Research Question 2**

Research Question 2: How does participation in an AP Calculus national exam reading affect teachers' and professors' classroom practice, as perceived by the educators?

The second research question centers on application of any professional development for the AP Calculus reading in the actual classroom. The question seeks to find out if educators report actually changing their classroom practice, what practices they may be changing and why the educators implemented these changes. Again, relevant data was collected by asking participants Likert scale questions concerning any impact on their classroom practices. And, in open-ended questions, they were asked to comment on their reasons for changing their practice as a result of their AP reading experiences, the relationship between their reading experiences and their perceived professional growth and their classroom practices. Quantitative data, including years of teaching, years as a reader, and level of impact on classroom practice were collected. Qualitative data on the changes in classroom practice (i.e. sequence, problem types, strategies) which these teachers and professors with AP Calculus exam reading experience implemented because of their reading experiences? The specific questions which were asked in support of research question 2 are shown at Table 9.

**Data Collection for Research Question 3**

Research Question 3: How do exam readers report that their participation in an AP Calculus national exam reading influences their students' success, as perceived by the educators?

The third research question addresses the impact on student learning. This question examines the participants' perceptions of how involvement in the AP Calculus exam reading has impacted their students' understanding of calculus as a result of their professional learning. Originally, this question was designed to analyze any linkage between teachers' participation in AP reading and the students' understanding as measured by the AP Calculus exam score. However, collecting the exam score data was problematic. Ultimately, quantitative analysis was not possible due to lack of data; however, plentiful qualitative commentary was available. Again, relevant data were collected by asking participants Likert scale questions concerning any impact on their students' understanding. And, in open-ended questions, they were asked to comment on

Table 9

*Survey and Interview Questions Designed to Answer Research Question 2: How does participation in an AP Calculus national exam reading affect teachers' and professors' classroom practice, as perceived by the educators?*

| Type of Questions | Questions |
|---|---|
| Likert Scale | **I** have changed my classroom practice because of my experiences as an AP Calculus exam reader. This means any changes you have actually implemented. Actions, not ideas. |
| | **I** use former AP Calculus Exam questions and rubrics to assess my students' Calculus understanding. |
| Open-Ended Survey Questions | In what ways, if any, have your experiences as a reader influenced your thinking (ideas, not actions) about teaching Calculus? |
| | In what ways, if any, have your experiences as a reader influenced your actual teaching (actions, not ideas) of Calculus? Why did you make these changes? |
| | If you have decided to change your classroom practice based on your AP reading experiences, what obstacles have you encountered in introducing those changes? How did you deal with them? |
| | Do you have any other comments on your AP Calculus reading experiences and their relationship to your classroom practice? |
| Interview Questions | Have your experiences at the AP reading influenced your classroom practice? If so, why and how? |
| | Have your views of how to teach your students changed since you became an exam reader? |

their reasons for any changes in their students' understanding as a result of their AP reading experiences. Quantitative data, including years of teaching, years of teaching calculus, years as a reader, and class size, as well as qualitative data on differences in student learning were collected. The specific questions which were asked in support of research question 3 are shown at Table 10.

Table 10

*Survey and Interview Questions Designed to Answer Research Question 3: How do exam readers report that their participation in an AP Calculus national exam reading influences their students' success, as perceived by the educators?*

| Type of Questions | Questions |
|---|---|
| Likert Scale | I believe that my students' conceptual understanding of Calculus has increased since I became an AP Calculus exam reader<br><br>I believe that my students are better prepared for the AP Calculus exam since I became an AP Calculus exam reader.<br><br>I believe that my students' AP exam scores have increased after I became an AP Calculus exam reader. |
| Open-Ended Survey Questions | Given the changes in your classroom practice, if any, which you described in the previous question, please describe the results you have observed or experienced.<br><br>Do you believe that your experiences as an AP Calculus exam reader have influenced your students' understanding and success in your Calculus classes? Why or why not?<br>Please provide a summary of your students' scores on the AP Calculus exam. |

| | |
|---|---|
| Interview Questions<br>High School Teachers and College Professors | How have your AP reading experiences affected your students' learning? Are other factors at play with your AP experiences?<br><br>Do you believe that your students' understanding of Calculus has changed since you became an exam reader?<br><br>How do you balance conceptual and procedural knowledge with your students? |
| High School Teachers | Have your students' AP exam scores increased since you became a reader?<br><br>In addition to the AP exam scores, what measures do you use to assess your students' success in your class? |
| College Professors | Do you notice any differences in students who take AP Calculus in high school and those who do not? |

**Data Analysis**

The research questions were designed to determine if there are connections between the dynamics of the professional learning experiences of the AP Calculus exam reading and how the educators perceive these experiences impact on their own classroom practices and their students' learning of Calculus. To address each research question, statistical analyses were conducted on numerous sets of data. Relevant commentary from the survey and from the interviews were incorporated as explanatory evidence. The matrices of planned statistical analyses for research questions 1, 2 and 3 are at Table 11, Table 12, and Table 13, respectively.

Table 11

*Statistical Analysis Matrix for Research Question 1: How do AP Calculus teachers and college Calculus professors perceive the professional development experiences at the AP Calculus exam reading?*

| Statistical Analyses/Comparisons | Methods |
|---|---|
| Likert Scale Survey Questions | Descriptive Statistics |
| Regarding my teaching of Calculus, the AP reading, was worthwhile professional development for me | Table 24, Table 25, Table 26, and Table 27 |
| Professional development sessions are offered in the evenings at the reading. I found these sessions informative and useful. | Table 28, Table 29, Table 30, Table 31 |
| Do you attend the evening PD sessions? How many sessions do you attend? | Table 32 and Table 33 |
| I find the interaction with other college mathematics professors and AP Calculus teachers to be professionally beneficial. | Table 34, Table 35 |
| I encourage my colleagues to become readers? | Table 36, Table 37, Table 38 |
| I intend to attend the AP Calculus reading next year? | Table 39, Table 40, Table 41, Table 42 |
| Changes in Educator's Thinking | Descriptive Statistics and Chi-Square Analysis |
| Changes in Educator's Thinking vs. Value of the PD | Table 43 and Chi-Square Analysis |
| Changes in Educator's Thinking vs. Age Group | Table 44 and Chi-Square Analysis |
| Changes in Educator's Thinking vs. Educator Level | Table 45 and Chi-Square Analysis |
| Changes in Educator's Thinking vs. Changes in Classroom Practice | Table 46 and Chi-Square Analysis |

| | |
|---|---|
| Changes in Educator's Thinking vs. Student Understanding | Table 47 and Chi-Square Analysis |
| Changes in Educator's Thinking vs. Years Reading Exams | Table 48 |
| Changes in Educator's Thinking vs. Years Teaching | Table 49 |
| Open-Ended Survey Questions and Interviews | Narrative Summaries |
| If you find the opportunity to interact with other AP Calculus teachers and college professors useful, please explain why. | |
| Do you have any other comments on your AP Calculus reading experiences and their relationship to your own professional development? | |

Table 12

*Statistical Analysis Matrix for Research Question 2: How does participation in an AP Calculus national exam reading affect teachers' and professors' classroom practice, as perceived by the educators?*

| Statistical Analyses/Comparisons | Methods |
|---|---|
| Likert Scale Survey Questions | Descriptive Statistics |
| The way I teach Calculus has been influenced by my experiences as an AP Calculus reader. | Narratives |
| Educator's Classroom Practice | Descriptive Statistics and Chi-Square Analysis |
| Changes in Educator's Classroom Practice vs. Educator Level | Table 50 and Chi-Square Analysis |
| Changes in Educator's Classroom Practice vs. Age Group | Table 51 and Chi-Square Analysis |

| | |
|---|---|
| Changes in Educator's Classroom Practice vs. Years Teaching | Table 52 and Chi-Square Analysis |
| Changes in Educator's Classroom Practice vs. Years Reading Exams | Table 53 and Chi-Square Analysis |
| Changes in Educator's Classroom Practice vs. Student Understanding | Table 54 and Chi-Square Analysis |
| Open-Ended Survey Questions and Interviews | Narrative Summaries |
| If your experiences as a reader have influenced your teaching, in what ways has this happened? | |
| Do you have any other comments on your AP Calculus reading experiences and their relationship to your classroom practice? | |

Table 13

*Statistical Analysis Matrix for Research Question 3: How do exam readers report that their participation in an AP Calculus national exam reading influences their students' success, as perceived by the educators?*

| Statistical Analyses/Comparisons | Methods |
|---|---|
| Likert Scale Survey Questions | Descriptive Statistics and Chi-Square Analysis |
| My students' understanding of AP Calculus has improved since I have become an AP Calculus Exam reader. | Table 55<br>Table 56<br>Table 57<br>Table 58 and Chi-Square Analysis |

| | |
|---|---|
| Changes in Student Understanding | Descriptive Statistics and Chi-Square Analysis |
| Changes in Student Understanding vs. Use of Former AP FRQ | Table 59 and Chi-Square Analysis |
| Use of AP FRQ vs. Age Group | Table 60 and Chi-Square Analysis |
| Use of AP FRQ vs. Educator Level | Table 61 and Chi-Square Analysis |
| Use of AP FRQ vs. Valuable PD | Table 62 and Chi-Square Analysis |
| Use of AP FRQ vs. Years of Teaching | Table 63 and Chi-Square Analysis |
| Use of AP FRQ vs. Years of Reading | Table 64 and Chi-Square Analysis |
| Open-Ended Survey Questions and Interviews | Narrative Summaries |
| If your experiences as a reader have influenced your teaching, in what ways has this happened? | |
| If you have decided to change your classroom practice based on your AP reading experiences, what If you have decided to change your classroom practice based on your AP reading experiences, why did you decide to do so and what results have you seen? | |

## Data Coding

In order to conduct the $\chi^2$ analyses for association or independence among the variables in the data collections, coding of the categorical data was necessary. The coding schemes used for Likert scale questions and demographic questions are discussed below.

### *Likert Scale Questions*

The Likert scale data from the online survey was transferred to an Excel spreadsheet. Responses were translated into scale values as shown in Table 14. As expected, some people wrote in the commentary box. These were saved in a separate file and analyzed to determine if they brought any added value. Demographics were analyzed to determine the overall opinions of the professional development at the reading from the responding sample. Results from the high school teachers were also compared to the results from the college professors.

Table 14

*Coding of Likert Scale Questions*

| Response | Scale Value |
|---|---|
| Strongly Disagree | 1 |
| Disagree | 2 |
| Neutral | 3 |
| Agree | 4 |
| Strongly Agree | 5 |
| No response | blank |

***Demographic Questions***

The Demographic data from the online survey were transferred to an Excel spreadsheet. Responses were coded to allow descriptive statistics to be generated. For example, the coding for the type of institution at which the educator is employed is shown at Table 15. For high school teachers, Charter School and Magnet School were separate choices. However, given the small number of people who chose either of these,

Table 15

*Institution Choices for High School Teachers and College Professors*

| Code | High School Choices | University/College Choices |
|---|---|---|
| | Public High School | Private Four-year College or University |
| 2 | Private/Independent High School | Public Four-year College or University |
| 3 | Private/Religious High School | Religious Based or Women's College or University |
| 4 | Magnet or Charter School | Community College |

they were combined. Similarly, for college professors, there were several choices which no one chose; and, only two professors chose Women's College, so they were combined with the religious based college, given that they both have special interests.

Both teachers and professors had a wide range of actual years of teaching and years of experience as a reader. Most readers had less than 20 years of reading experiences; but, the years of teaching experience had a broader range. Coding for these two variables is shown in Table 16. Coding for age group, school location and attendance at evening professional development sessions at the reading are shown at Table 17.

## Quantitative Analysis

The Likert scale data were compared to determine if there were any connections between teacher age, years teaching, years reading the exam, type of institution, and location and opinions on the professional development at the reading, influence on their thinking, and impact on their observations of their students' understanding. Collection of exam scores for high school teachers proved problematic; at least twenty high school teachers provided comprehensive data about their students' exam scores over periods of several years. However, the data formats were inconsistent. Some teachers provided a mean score and standard deviation but did not include class size. Some teachers merely estimated that their class averages were "about 3" or "about 4." Also, there was a broad range of years covered by their data; so, consistency in comparing before and after could not be achieved. However, qualitative analysis was conducted on the teacher's observations and perceptions of any changes in their students' understanding of calculus.

Table 16

*Coding for Years of Reading AP Calculus Exams and Years of Teaching Experience*

| Scale Value | Years of AP Exam Reading | Years of Teaching Experience |
|---|---|---|
| 1 | 1-5 | 1-10 |
| 2 | 5-10 | 11-20 |
| 3 | 11-15l | 21-30 |
| 4 | 16-20 | 31-40 |
| 5 | 22+ | 41+ |
| blank | No response | No response |

Table 17

*Coding for Demographic Data on Age, Location and Evening Sessions*

| Code | Age Group | Location | Attend PD Sessions |
|---|---|---|---|
| 1 | 21-30[a] | Inner City[b] | 1 |
| 2 | 31-45 | Urban | 2 |
| 3 | 46-60 | Suburban | 3 |
| 4 | 61+ | Rural[c] | All |
| 5 | | Remote[d] | |
| blank | None | None | None |

*Note.* [a]Only one person in the age category 21-30; this person was included with the 31-45 age group. [b]Only one person in the Inner-City location category; this person was included with the Urban location group. [c]Only one college professor delivers class instruction online; this person was included in the Rural group. category; this person was included with the Rural category. [d]Only one college professor is in the Remote Location category; this person was included with the Rural category.

## Qualitative Analysis

The purpose of this study is to develop a theory of what connections, if any, exist between experiences at AP Calculus exam readings, educator professional development, classroom practice and student learning. The construction of this working level grounded theory is obtained through qualitative data analysis, augmented with meaningful quantitative analyses. To structure the analysis of the voluminous data which was collected, data from the research questions were sorted according to the research questions with which they are associated. For each of these data sets, analysis rendered several recurring themes throughout the commentary. As suggested by Saldaña (2009), the first cycle coding method utilized in vivo coding of both the interviews and the narrative responses to the survey questions. Specific attributes, accompanied by relevant commentary from participants, were incorporated. These were then distilled down into axial codes, again as suggested by Saldaña (2009), for the second cycle coding. The dominant axial codes provide the themes for the qualitative analysis in Chapter 4.

For Research Question 1, the data was refined into five major themes: self-improvement; collegial interaction; the value of professional development; thought-provoking, beneficial work; and, logistics concerns. See Table 18 for open-coding to axial coding for Research Question 1.

Table 18

*Open-Coding and Axial Coding for Research Question 1*

| Open Code | Properties | Participants' Comments | Axial Codes |
|---|---|---|---|
| Colleague recommendations; making contacts; increase professional knowledge; deepen content knowledge; know how the exam is graded; explore student thinking; learn from others | Seeking growth; increase knowledge | I wanted to improve my AP scores and thought reading would help that goal; the rich professional alliances created by the reading; insight into what topics cause struggle for students across the country; I have learned to understand the meaning of calculus concepts to a much higher degree than what I learned as a student; I have a much better understanding of the ways that other people approach grading before encountering the rubric style | Evolving as a more knowledgeable and capable educator |
| Collegiality; formal and informal interactions; mutual academic interests | Team Work; energizing; insightful | Love the briefings, they are helpful to think about the philosophy of grading; working with a table partner is very helpful in really understanding how to score papers; I find the briefings to be a bit boring and | Beneficial Collaboration |

| | | | |
|---|---|---|---|
| | | drawn out. I think there are some ways they could be more efficient in terms of the grading; this is the only place in the world where I tell a joke and almost everyone gets it and laughs... and comes back with another joke. | |
| Openness to change; eagerness to learn from others and share; chance to discuss pedagogy; using rubrics; learning the nuances of the grading | Differing forms and worth of professional development; applicability of PD; | It's been by far the best source of professional development I've had as a calculus teacher; I view teaching calculus differently after my involvement with the AP Reading; It is not my only professional development but it has certainly enhanced my Calculus courses; It's affected my teaching, test writing, and perspectives of learning that has reached all of my classes, not just AP; the briefings can be confusing, depending on the question leader; I think more precisely. I want to know the details and how you explain something mathematically. | Value and usability of PD |
| Conceptual thinking; reassessing student thinking; evaluating student capabilities; challenging student to best performance; accurate notation and verbiage; justification of solutions | Accuracy and completeness of student work | I have become a strong believer in conceptual understanding over performance; I understand much better why my college freshmen find university level math course such a shock; I am more aware of the things I thought my students understood that they do not in fact understand; I realized I should expect more from my students. That I was not challenging them enough; I realized I should expect more from my students. That I was not challenging them enough; By not accepting sloppy answers; I have seen how poor written | Thought provoking lessons; precise verbal communication |

| | | | |
|---|---|---|---|
| | | communication can affect a student's score; I include topics that I have not included before, and, I also ask tougher questions nowadays; I've learned not to be that predictable in my exams. | |
| Shortfalls of the reading; decreasing benefits over time; tiring schedule with increased demands; dates are not compatible with school schedule | No stipend increase; heavy workload; evening sessions too tiring | They need to do real professional development in the evenings for instructors to attend; I believe that I have probably gotten all the professional development out of the experience that I will get; They were good experiences, just exhausting; Some of the readers are there just for the paycheck. Some are there because it is a "free" trip to another city. Some have actually told me that they come to the reading so that they can miss a week of work without being penalized; pay has not changed in years but the hours have increased. | Scheduling and pay logistics |

For Research Question 2, the data was condensed into three major themes: mathematical fluency and communication, incorporating exam strategies, and scope of practice. See Table 19 for open-coding to axial coding for Research Question 2.

For Research Question 3, the data was fine-tuned into four major themes: problem solving skills; scope of understanding; optimal use of instruction time; and, AP exam preparation and performance. See Table 20 for open-coding to axial coding for Research Question 3.

Table 19

*Open-Coding and Axial Coding for Research Question 2*

| Open Code | Properties | Participant Comments | Axial Code |
|---|---|---|---|
| Focusing on concepts; making connections among concepts; justification; accuracy of notation and verbiage; vocal explanations | Conceptual understanding; mathematical connections | I now think about Calculus in new ways, my teaching has incorporated many more hands-on activities involving genuine mathematical inquiry; I spend more time having my students focus on connections between concepts; most significant changes I made since I began reading is the way I manage homework; greater attention to specific topics like the Fundamental Theorem of Calculus, differential equations, and modeling with integrals, not only because of their likelihood of being on the exam, but because of their importance in understanding calculus; I made sure I ask my students to show me different ways to solve a problem; It has changed in the way that I ask questions on my exams; I am a better teacher, so I do a better job teaching. | Mathematical fluency and communication |
| Changing assessment strategies; using knowledge of the exam; student self-evaluation; comprehensive assessments; thinking differently about assessments; | Assessment content and format; rubrics, grading focus | I have students learn how to evaluate their own work as a way to reinforce the material; I changed the way I grade free response questions, by using the rubric; I add more FR questions throughout the year rather than waiting until the end of the year; tests now included questions from many different parts of Calculus, not | Incorporation of exam strategies |

| | | | |
|---|---|---|---|
| more incorporation of AP materials | | just what we have been working on most recently; I am a much more strict grader; I grade with more leniency (for example, students can leave answers unsimplified); I communicate many mathematical principles that I would otherwise neglect or underserve; more reliable assessment results, and assessments that are easier to grade; the rubrics I provide help them to understand their mistakes so that they do better. | |
| Teacher content knowledge; sharing new ideas; learning from each other; teacher confidence; teacher growth; peer connections and mutual support; experience leading to change | Expanding teacher learning; incorporating self-prescribed changes | I'm more knowledgeable about the topic for sure, but I think more importantly, I see much deeper connections between topics within Calculus; I'll just repeat that teachers need a deep understanding of the material they teach. The questions on AP calculus test deep learning; I have learned the concepts in more depth, learned more about the format of the exam itself, and giving me confidence to work to improve student understanding; I believe that the AP reading has improved teaching in all areas, not just Calculus. I have learned and been able to apply assessment standards with inter-rater reliability; I enjoy hearing about how my colleagues deal with various challenges in the classroom; the reading and peer interaction/discussions | Breadth and depth of teaching practice |

| | | | |
|---|---|---|---|
| | | with colleagues while reading have been immensely helpful as we share about teaching practices - differentiation and ways to present/explain challenging concepts | |

Table 20

*Sam Open-Coding and Axial Coding for Research Question 3*

| Open Code | Properties | Participant Comments | Axial Code |
|---|---|---|---|
| Student content knowledge; ability to analyze and solve problems; real-world applicability; algebra skills | Precalculus skill; procedural and conceptual understanding | My students understand Calculus much better today than they did prior to the time I became a reader; my students in all classes, not just calculus, show better understanding in that they are able to make connections; students are clearer about their responses to standard AP questions; the students as more understanding of Calculus concepts from their ability to pointed questions; I have been able to separate a student's perform algebraic computations; because of the changing expectations, students were making it to calculus class who did not have the needed algebra skills to be successful in calculus; rather than the AP professional development helping me enable more students to pass the exam, it's probably enable me to help those students, who might've already passed, gain a better, deeper understanding of many concepts. | Problem solving skills |

| | | | |
|---|---|---|---|
| Calculus themes; making connections among concepts; meaningful explanations, justifications, and vocabulary | Meaning across multiple ideas; proficient verbal and written expression of composite models | My students are giving better explanations and justifications; the themes of calculus emerge to the class, not just the question; their answers on justifications and interpretation questions are more concise/focused; the students spend more time in class talking about the concepts; making students fully explain their answers increases their ability to translate mathematics to the real world and also see mathematics in everyday situations; I have witnessed a greater understanding of the questions that I am asking. They know how to look for key ideas and the big picture of calculus; they write justifications better; understanding both verbally and in their written work. | Comprehensive, compound understandings |
| Time management; altering which topics to stress and which to condense; teachers more efficient at grading; assessments more meaningful; depth of content | Voluminous course content; efficiency in grading and class time | I can be more efficient in writing and grading exams and therefore I can spend more time thinking about instruction; I believe that the semester is too short and jammed packed; I have improved my grading times (without sacrificing accuracy) for exams; I now spend a week covering the Fundamental Theorem. Before, I'd spend about 20 minutes; I am much more efficient with time management; I don't think my experiences as a reader have influenced my students. The biggest impact has been on my efficiency. | Optimal use of instruction time |

| | | | |
|---|---|---|---|
| AP Exam scores; higher scores; better preparation; increased confidence; thinking about concepts, not algorithms | Familiarity with former AP questions; focus on big themes | My students' scores on the FRQ's have increased; my students' scores continue to improve; more students scoring higher; students feel better prepared for the free response portion of the AP exams; their answers on justifications and interpretation questions are more concise/focused; | AP exam preparation and performance |

**Summary**

The response to the surveys was more voluminous than expected. This bounty of data provided a breadth of experiences and viewpoints of 155 mathematics educators. Although quantitative data for changes in student exam scores could not be reliably analyzed, qualitative commentary from teachers was plentiful. In vivo coding was distilled down to axial codes, which set the framework for analyzing linkages among data categories. Despite some participants not answering all of the questions, there were still an ample number of responses which could be examined for connections and for which a working theory could be developed.

# Chapter Four

## Introduction

The purpose of this study is to develop a working theory of the connections and associations which may exist between the professional development which educators experience at AP Calculus exam readings, their personal growth, their classroom practice, and their students' learning, as perceived by the educators. Professional development is present at the exam readings in a variety of forms: the briefings, the actual exam readings, the collegial interaction and the evening sessions, some of which were professional development. The construction of this working grounded theory was sought through qualitative data analyses, augmented with meaningful quantitative analyses.

A mixed methods approach was used. The data collection was accomplished in two phases. The first phase is a four-part survey which was completed online. This survey was created by the author and was piloted during the AP Calculus Exam reading in June 2014. Therefore, there are no formal tests of validity or reliability. However, a validity matrix was created as recommended by Maxwell (2013); Fink (2008) was consulted on the conduct of surveys. Validity matrix is at Appendix F.

The survey includes four parts: informed consent, Likert scale questions, open-ended questions and demographic information about the educators and their institutions. The high school teachers were asked to provide historical, quantitative data about their students' AP exam scores. The second phase was a series of interviews via telephone. A total of 67 high school teachers and 88 college professors completed the online survey and gave consent. Over 70 participants volunteered to participate in a telephone interview. Seventeen people were interviewed.

**Data Collection**

A total of 185 educators replied to the survey. However, 30 people failed to indicate their consent by checking the consent box on the consent form. The people who completed the survey did so anonymously through a link sent to them in my invitation email. There was no way to discern the identities of the people who had not given consent explicitly. Therefore, they could not be contacted to obtain consent; and, their responses were not viewed or used in any way for this study. A total of 67 high school teachers and 88 college professors completed the online survey and gave consent. See Table 21, Table 22, and Table 23 for age group distribution, location, and type of institution, respectively. Not every person answered every question; therefore, the total number of responses varies from question to question.

Interviews were conducted after the survey closed and had been initially analyzed. The purpose of the interviews was to garner more information than could be collected from the surveys. It was anticipated that several issues would emerge which required more information and/or clarification. For example, teachers may want to convey more information about their own personal views of education or their views of the AP reading than would be practical in the survey. The plan was to interview at least five high school

Table 21

*Educator Age Group*

| | Age Groups | | |
|---|---|---|---|
| Educator Level | 31-45 | 46-60 | 60+ |
| High School Teacher | 25 | 33 | 9 |
| College Professor | 27 | 39 | 21 |
| Total | 52 | 72 | 30 |

Table 22

*Location*

| | Location | | |
|---|---|---|---|
| Educator Level | Urban | Suburban | Rural |
| High School Teacher | 15 | 40 | 11 |
| College Professor | 38 | 25 | 23 |
| Total | 53 | 65 | 34 |

Table 23

*Institution Type*

| | Type of Institution | | | |
|---|---|---|---|---|
| Educator Level | Public | Private/ Independent | Private/ Religious | Magnet/ Charter/Other |
| High School Teacher | 44 | 13 | 7 | 2 |
| College Professor | 34 | 36 | 1 | 15 |
| Total | 78 | 49 | 8 | 17 |

teachers and at least five college professors. A total of 11 high school teachers and 6 college professors were interviewed. The educators shared their viewpoints on the exam reading experiences, including the importance of collegial interaction, the value of learning new strategies to use with their students and their overall opinion of the varied professional development opportunities at the reading.

**Perceptions of Professional Development**

The AP Central website touts the exam reading experience: “AP Readers often describe the AP Reading as one of the best professional development experiences they have ever had, (College Board, 2017b). The professional development is present at the exam readings in varying forms. During the briefings for each question, the question leader discusses the purpose of the question, that is, the mathematical reasoning and conceptual knowledge as well as computational skills which are being assessed. Grading standards are discussed, including a myriad of possible errors which students may commit and how they will be graded. During the actual exam reading, the educators regularly consult their reading partners and table leaders to ensure accuracy and consistency. The collegial interaction is present throughout the week, in the dining areas, the hospitality suite, and the walks back and forth to the convention center. And, there are optional evening sessions, some formal professional development, some more informal, scheduled for most evenings. These include a meeting with the AP Calculus question development committee, a question and answer meeting the senior reading personnel, a guest lecture on a mathematical topic, and an entertainment session such as Mathematics Jeopardy games.

**Value of Professional Development**

Survey responses bolster the claim of valuable professional development. 34% of participants agreed and 62% strongly agreed with assessing the reading experience as valuable professional development. Only 6 people were either neutral of disagreed. See Table 24 and Table 25 for comparison by educator level and by age group. While most readers value the professional development at the reading, high school teachers tend to strongly agree. College professors were about evenly distributed in the Agree and Strongly Agree categories. Of the six people who responded negatively or neutrally: four were in the young age group; two were in the middle age group; and, none were in the 60+ age group.

Table 24

*The Reading is Valuable Professional Development vs. Educator Level*

| | The Reading is Valuable Professional Development | | | | |
|---|---|---|---|---|---|
| Educator Level | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| High School Teacher | 1 | 0 | 1 | 9 | 55 |
| College Professor | 1 | 3 | 0 | 43 | 40 |
| Total | 2 | 3 | 1 | 52 | 95 |

Years of teaching experience and years of exam reading experience appear to have no relation to the positive opinion of the reading experience. 145 of 151 respondents in all teaching experience levels, and 145 of 151 respondents in all exam reading

Table 25

*The Reading is Valuable Professional Development vs. Age Group*

| | The Reading is Valuable Professional Development | | | | |
|---|---|---|---|---|---|
| Age | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| 31-45 | 0 | 3 | 1 | 19 | 29 |
| 46-60 | 2 | 0 | 0 | 23 | 45 |
| 60+ | 0 | 0 | 0 | 9 | 21 |
| Total | 2 | 3 | 1 | 51 | 95 |

experience levels, either agreed or strongly agreed that the reading is valuable professional development. See Table 26 and Table 27.

Table 26

*The Reading is Valuable Professional Development vs. Years of Teaching*

| | The Reading is Valuable Professional Development | | | | |
|---|---|---|---|---|---|
| Years of Teaching | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| 1-10 | 0 | 1 | 0 | 3 | 8 |
| 11-20 | 0 | 2 | 0 | 18 | 26 |
| 21-30 | 2 | 0 | 1 | 16 | 30 |
| 31-40 | 0 | 0 | 0 | 10 | 21 |
| 41-50 | 0 | 0 | 0 | 5 | 8 |
| Total | 2 | 3 | 1 | 52 | 93 |

**Formal and Informal Evening Sessions**

Although the majority viewed the evening professional development sessions positively, they were less enthusiastic than the responses to the value of the reading as a

Table 27

*The Reading is Valuable Professional Development vs. Years of Exam Reading*

| | The Reading is Valuable Professional Development | | | | |
|---|---|---|---|---|---|
| Years of Exam Reading | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| 1-5 | 1 | 1 | 1 | 22 | 48 |
| 6-10 | 0 | 1 | 0 | 19 | 27 |
| 11-15 | 0 | 1 | 0 | 6 | 13 |
| 16-20 | 1 | 0 | 0 | 2 | 3 |
| 21+ | 0 | 0 | 0 | 2 | 3 |
| Total | 2 | 3 | 1 | 51 | 94 |

Table 28

*Evening Professional Development Sessions vs. Educator Level*

| | Evening PD Sessions are Valuable | | | | |
|---|---|---|---|---|---|
| Educator Level | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| High School Teacher | 1 | 4 | 24 | 31 | 4 |
| College Professor | 2 | 7 | 0 | 60 | 0 |
| Total | 3 | 11 | 24 | 91 | 4 |

whole. 20 fewer people answered this question; but, 14 (10.5%) disagreed or strongly disagreed; and 24 (18%) were neutral. 91 (68.5%) agreed but only 4 (3%) strongly agreed. And, there are no distinctions between educator levels or among age groups. See Table 28 and Table 29.

Neither years of teaching experience nor years of exam reading experience appear to be associated with an educator's opinion of the value of the evening professional

Table 29

*Evening Professional Development Sessions vs. Age Group*

| | Evening PD Sessions are Valuable | | | | |
|---|---|---|---|---|---|
| Age Group | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| 31-45 | 1 | 5 | 11 | 25 | 2 |
| 46-60 | 1 | 6 | 9 | 44 | 0 |
| 60+ | 1 | 0 | 4 | 21 | 2 |
| Total | 3 | 11 | 24 | 91 | 4 |

development sessions. 71% viewed the evening session favorably; 18% were neutral. See Table 30 and Table 31.

37% of the respondents reported attending none of the evening sessions. 48% attend between 1 and 3 of the sessions. And, about 15% of the participants attend all the evening sessions. However, the distribution of the high school teachers is relatively even across the number of sessions attended, ranging from a low of 11 to a high of 15; the college professor distribution is skewed right. 45% of college professors do not attend any evening sessions; only 27% of high school teachers do not attend any evening sessions. The proportion of readers who do not attend any of the evening sessions decreases as age increases. See Table 32 and Table 33.

Table 30

*Evening Professional Development Sessions are Valuable vs. Years of Teaching*

| | Evening PD Sessions are Valuable | | | | |
|---|---|---|---|---|---|
| Years of Teaching | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| 1-10 | 1 | 1 | 3 | 7 | 1 |
| 11-20 | 0 | 5 | 6 | 24 | 0 |
| 21-30 | 1 | 2 | 10 | 29 | 2 |
| 31-40 | 0 | 3 | 4 | 21 | 1 |
| 41-50 | 1 | 0 | 1 | 9 | 0 |
| Total | 3 | 11 | 24 | 90 | 4 |

Table 31

*Evening Professional Development Sessions are Valuable vs. Years of Exam Reading*

| | Evening PD Sessions are Valuable | | | | |
|---|---|---|---|---|---|
| Years of Exam Reading Experience | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| 1-5 | 1 | 6 | 18 | 39 | 2 |
| 6-10 | 0 | 5 | 4 | 29 | 2 |
| 11-15 | 1 | 0 | 1 | 14 | 0 |
| 16-20 | 1 | 0 | 1 | 4 | 0 |
| 21+ | 0 | 0 | 0 | 4 | 0 |
| Total | 3 | 11 | 24 | 90 | 4 |

**Collegial Interaction**

The value of collegial interaction was seen as positive by all but three respondents. High school teachers were more apt to strongly agree (SA=80%) than agree (A=17%); college professors were almost equally likely to strongly agree (SA=50%) and agree (A=49%). See Table 34. Age group comparisons show that the three participants

Table 32

*Number of Evening PD Sessions Attended vs. Educator Level*

| | Number of Evening PD Sessions Attended | | | | |
|---|---|---|---|---|---|
| Educator Level | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| High School Teacher | 15 | 11 | 14 | 15 | 55 |
| College Professor | 33 | 22 | 15 | 4 | 74 |
| Total | 48 | 33 | 29 | 19 | 129 |

Table 33

*Number of Evening PD Sessions Attended vs. Age Group*

| | Number of Evening PD Sessions Attended | | | |
|---|---|---|---|---|
| Age Group | None | 1-2 | 3 | All |
| 31-45 | 21 | 12 | 7 | 5 |
| 46-60 | 22 | 13 | 17 | 11 |
| 60+ | 4 | 8 | 5 | 3 |
| Total | 47 | 33 | 29 | 19 |

Table 34

*Value of Collegial Interactions vs. Educator Level*

| | Collegial Interaction is Valuable | | | | |
|---|---|---|---|---|---|
| Educator Level | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| High School Teacher | 1 | 0 | 1 | 11 | 53 |
| College Professor | 1 | 0 | 0 | 44 | 43 |
| Total | 2 | 0 | 1 | 55 | 96 |

who did not view the collegial interaction favorably are all in the middle age group. Every age group was more likely to strongly agree than agree: young age group, A=38%, SA=62%; middle age group, A=38%, SA=58%; and, older age group, A=27%, SA=73%. See Table 35.

Table 35

*Value of Collegial Interaction vs. Age Group*

| | Collegial Interaction is Valuable | | | |
|---|---|---|---|---|
| Age Group | None | 1-2 | 3 | All |
| 31-45 | 0 | 0 | 0 | 20 |
| 46-60 | 2 | 0 | 1 | 27 |
| 60+ | 0 | 0 | 0 | 8 |
| Total | 2 | 0 | 1 | 55 |

Collegial interaction received an almost unanimously positive rating. Some commentary was non-specific, such as, "the collegial interactions in and out of the reading day are invaluable," or, "the collegial interactions are the most useful for me." However, most people cited a specific reason or example. These specifics fell into four overlapping categories: social; learning and exchanging ideas; insights into student thinking; and, interaction with educators at different levels.

A high school teacher who was interviewed said, "our table developed good camaraderie, we all have a deep sincere interest in the topic; I felt like a closeness to everybody." In praising the social aspects of the reading, survey participants stated:

> The collegial interactions are priceless and in some ways more valuable than the actual formal professional development.

> Not only have these interactions helped me better understand the topic to better support my students in learning it, it has also given me a huge set of professional colleagues to call on for ideas and help when I need it.
> Other than being able to interact with others of strong intellect and interest in mathematics, I find it quite enlightening to talk about how to teach certain topics, what concepts are more important than others, and specific lessons and activities I can use in my own class. I also make/keep personal relationships with other teachers, and sometimes contact them with questions or comments throughout the year.
>
> Learning from each other and discussing teaching strategies, classroom practices,

assessments and materials was another prevalent them. Participant commentary includes:

> There is no other more authentic activity that an AP Calculus teacher can attend to gain specific, detailed information on how to prepare students for the exam. I say all the time that the reading is by far the best professional development experience there is. You get to interact with hundreds of other Calculus teachers at every point in time.
>
> Collegial interactions are also great professional development. A group of us started a Google drive group to share calculus materials.
>
> Friendships are formed at the AP Reading and discussion about classroom pedagogy, both formally and informally, happens just by the nature of the reading. It is the most important reason why I continue to grade AP Calculus even though I do not teach in high school anymore.
>
> I love meeting with the other HS and College teachers at the reading. There are many websites, tools, strategies, and resources that I've brought back to my own teaching. Two specific examples: I learned about Desmos, an online graphing tool which will be instrumental in some of my teaching this coming year. I learned more about flipping my classroom (videos) and have implemented some strategies I learned from seasoned "flippers."
>
> The best professional development is interaction with other readers. I learn things that they have tried, and how effective they were, and I'm able to bounce ideas off of other people in a larger context than my own small college department.
> Hearing about what other teachers consider important is useful. I have been teaching a long time; and, so, I have pretty strong opinions; but, it is always good to hear what others think. I have especially been stimulated by the slope field and differential equations questions and scoring rubrics.

> I get outside my own teaching bubble and learn what others are doing around the country, where my classes are stronger, where we can improve. It is also a place to learn the political dynamics around the country and reflect on how education everywhere is dealing with over-testing and government interference. Makes me feel like I'm not alone and strengthens my fight against it. Many of us stay connected beyond the reading as well so this dialogue continues.

In summary, the colleague-to-colleague dialog in which all readers participate is highly valued. Participants specifically point to new teaching strategies which they learn, sharing materials and lesson plans, developing friendships and communicating throughout the school year, and interesting discussions on pertinent mathematical topics such as notational fluency, as the powerful social aspects of the reading experience.

Educators also pointed to the insights they gain from looking at the volume of student work in the same problems and being able to discuss the work with other educators. Representative commentary:

> There were a couple of responses that I saw on test I was reading through that I thought wow I would never have viewed that type of a question from that perspective before. And, there are inside of our classrooms, kids who think differently than what I do. And, the more different views that I can see then I can address inside the classroom.

> I also learn from the students. Often, they will present solutions that are much better than what I originally did to work the problem. I think I have learned a lot about calculus as a result of looking at thousands of papers and working with other calculus teachers. Back at home at work, I don't have anyone to discuss calculus with, so it is nice to have that at the Reading.

> When we see mistakes that are common, we discuss what we do in the classroom to prepare students not to make those mistakes. We discuss test strategies, classroom resources, etc.

> There have been some topics that I was surprised to find are not a mandatory part of the AP curriculum, such as L'Hopital's rule for AB calculus or all of the different forms of partial fraction decomposition for BC calculus. This helps me to understand, not only some of the quirks in the ways that students approach the

> problems on the exam, but also some of the things that I see them doing in my classes.

In summary, educators benefit from seeing the gamut of valid approaches students use in problem solutions. These mirror the differing though processes of their own students. Often, teachers will add these approaches to their arsenal of examples to use in their classrooms. Some of the student approaches are viewed by educators as an improvement over their own and they adopt the new approach for their classroom. The volume of student work also gives educators an expansive sampling of student errors, which leads to insight into methods to help student avoid such errors.

Although the reading is largely viewed as a positive experience, it does have its flaws. These, fortunately, are correctable. And, it is clear from extensive commentary, that readers feel sufficiently strongly about their complaints to include them in their otherwise mostly positive observations and experiences. Participants frequently voiced concern, anger, and frustration at opposing grading philosophies both from one year to the next as well as from one question to another in the same exam year. A high school teacher with over 20 years of reading experience said,

> I'd like to see more consistency from question to question and from year to year. I mean, if it's important in this question, then it should be important in another question. And, then a few years ago, the differential was so important. And, this past year it wasn't. The differential is important and should be there; it's mathematically correct. This just doesn't seem right to me.

Other interviewees said:

> I get frustrated when, on one question, the kid has to point out something, like a negative sign, twice, but on another question, even an answer which does not make a whole lot of sense can get a point or two.

> A Riemann approximation question required a number of rectangles; but, it wasn't good enough if the student found the area of each rectangle and added them up and got the right answer. They had to show how they got each area, length times width, I mean, these were not numbers you would pull out of the air. It's obvious how the kid got each area, but they got docked points for not showing some basic algebra. But, then, in the same question, you might get a point for something which does not require any mathematical thinking. Sometimes, it doesn't make sense.

Both high school teachers and college professors lauded the benefits of interacting with educators at both the high school and college levels. Comments include:

> As a college professor, I learned important ideas from the high school teachers. I gained greater respect for the high school teachers and had new insights regarding the backgrounds of some of the students who had been in my college calculus classes.

> I liked talking to high school teachers about how they teach. It is interesting, and it gave me insight into how the students were trained to think in certain situations. Of course, these are really good teachers. I was fairly confident in the students they produced.

> The informal interactions during the actual scoring of the exams are very valuable. I have learned much over the years from the HS teachers about how to teach calculus.

> It a helpful to communicate with other high school teachers as we use the opportunities to share ideas and personal experiences in the classroom. Interacting with college professors helps to get perspective on the future math courses for my students.

In summary, both high school teachers and college professors found that their interactions with each other were beneficial for understanding both where the student has been as well as where the student is going. They also report benefitting from each other's teaching strategies.

Indicators of how beneficial readers assess the reading experience could also be found in both the rate with which they return to the reading year after year as well as their recommendations for their colleagues to participate in the reading. For the 140 out of 150 respondents, 85 agreed and 55 strongly agreed that they encourage their colleagues to apply to become AP readers. The 10 remaining were 5 high school teachers who were neutral and 5 college professors who disagreed. Similar results were obtained when sorting by age groups. Of the 10 who disagree or were neutral, 3 were in the 31-45 age group, 3 were in the 46-60 age group, and 3 were in the 60+ age group. Similarly, those few who disagreed or were neutral were similarly distributed among the categories of years of teaching. However, all of the ten people who would not encourage a colleague be become a reader have 10 or fewer years of reading experience themselves. 8 of the 10 have 5 or fewer years of reading experience. There is no statistical association between encouraging colleagues to become readers and level of educator, age group, or years of exam reading experience. See Table 36, Table 37, and Table 38.

Table 36

*Encourage Colleagues to Attend vs. Educator Level*

| | Encourage Colleagues to Attend Readings | | | | |
|---|---|---|---|---|---|
| Educator Level | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| High School Teacher | 1 | 0 | 5 | 25 | 31 |
| College Professor | 0 | 4 | 0 | 60 | 24 |
| Total | 1 | 4 | 5 | 85 | 55 |

A large majority of the readers reported that they intend to return to future readings. 92% of high school teachers intend to serve as readers again (A=16%,

Table 37

*Encourage Colleagues to Attend vs. Age Group*

| | Encourage Colleagues to Attend Readings | | | | |
|---|---|---|---|---|---|
| Age Group | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| 31-45 | 0 | 3 | 1 | 25 | 21 |
| 46-60 | 1 | 0 | 2 | 49 | 17 |
| 60+ | 0 | 1 | 2 | 10 | 17 |
| Total | 1 | 4 | 5 | 84 | 55 |

Table 38

*Encourage Colleagues to Attend vs. Years of Exam Reading Experience*

| | Encourage Colleagues to Attend | | | | |
|---|---|---|---|---|---|
| Years of Exam Reading Experience | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| 1-5 | 1 | 3 | 4 | 39 | 26 |
| 6-10 | 0 | 1 | 1 | 25 | 19 |
| 11-15 | 0 | 0 | 0 | 14 | 6 |
| 16-20 | 0 | 0 | 0 | 4 | 1 |
| 21+ | 0 | 0 | 0 | 2 | 3 |
| Total | 1 | 4 | 5 | 84 | 55 |

SA=76%); and, 93% of college professors intend to serve as readers again, (A=40%, SA=53%). When asked if they would be returning to the reading the following year, only 11 of 148 people replied either negatively or neutrally. Both high school teachers and college professors were more likely to respond as Strongly Agree than Agree. However, 16% of high school teachers and 40% of college professors responded with Agree. A clear majority in each age group responded as Strongly Agree to return the reading the future. Across all years of teaching experience, educators reported that they plan to return. And, across all years of exam reading experience, only eleven educators responded neutrally or negatively. Of these 11, 7 have 5 or fewer years of reading experience. See Table 39, Table 40, Table 41, and Table 42.

Table 39

*Intentions for Attending Future Readings vs. Educator Level*

| | Will Attend Future Readings | | | | |
|---|---|---|---|---|---|
| Educator Level | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| High School Teacher | 2 | 1 | 2 | 10 | 47 |
| College Professor | 0 | 5 | 1 | 35 | 46 |
| Total | 2 | 6 | 3 | 45 | 93 |

Table 40

*Intentions for Attending Future Readings vs. Age Group*

| | Will Attend Future Readings | | | | |
|---|---|---|---|---|---|
| Age Group | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| 31-45 | 0 | 4 | 1 | 14 | 31 |
| 46-60 | 0 | 1 | 1 | 23 | 44 |
| 60+ | 2 | 1 | 1 | 7 | 18 |
| Total | 2 | 6 | 3 | 44 | 93 |

Table 41

*Intentions for Attending Future Readings vs. Years of Teaching Experience*

| | Will Attend Future Readings | | | | |
|---|---|---|---|---|---|
| Years of Teaching | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| 1-10 | 0 | 1 | 0 | 2 | 8 |
| 11-20 | 0 | 4 | 1 | 14 | 25 |
| 21-30 | 0 | 0 | 1 | 17 | 32 |
| 31-40 | 1 | 1 | 0 | 6 | 22 |
| 41-50 | 1 | 0 | 1 | 6 | 5 |
| Total | 2 | 6 | 3 | 45 | 92 |

Table 42

*Intentions for Attending Future Readings vs. Years of Exam Reading Experience*

| | Will Attend Future Readings | | | | |
|---|---|---|---|---|---|
| Years of Exam Reading Experience | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| 1-5 | 1 | 3 | 3 | 20 | 44 |
| 6-10 | 1 | 1 | 0 | 16 | 28 |
| 11-15 | 0 | 2 | 0 | 6 | 12 |
| 16-20 | 0 | 0 | 0 | 2 | 4 |
| 21+ | 0 | 0 | 0 | 0 | 5 |
| Total | 2 | 6 | 3 | 44 | 93 |

A large majority of readers indicated their intention to return to a reading in the future. They cited several reasons for wanting to return to experience the reading again. Four themes were recurrent: the professionally enjoyable experience of living and working with hundreds of like-minded mathematicians for a week; learning more about mathematics education; opportunities to form lasting friendships, and, expectations of helping their students better understand Calculus. Representative commentary on reasons why an educator keeps returning to the AP reading include:

> I really enjoy again being … somebody told me this once, they said it feels like summer camp … and it does. It's summer camp and I get to go back and see friends and I really enjoy that. I love being together with math teachers for a week and get to *talk shop*. I still learn things but I'm not learning at the same pace I was originally because it was new. But, I always *come back with something new*, of course, last year was a big year for me after talking to the gentleman about the standards-based grading. So, I always feel like I learn something and I appreciate that. Of course, I do appreciate having the money that we get paid for it. That’s not the only reason I do it.
>
> I keep going back because I *learn* something every time. It's like I tell my student teachers, I guess if I didn't get anything out of it then I wouldn't go back. And I really, here's another thing I tell the kids, I don’t do it for the money, I'm not saying I'd do it without the money, but that is not the driving force. I'm not going to get $1600 where Kansas takes my money and they never give it back to me. And if I would get like how much I made per hour, of my time, I could stay home and tutor and make a lot more money. So, definitely not for that. But, I learn things and I enjoy … it's not all work, I enjoy your … my friends I've made, and there's a whole group of us we kind of just hooked up the first year and we can't wait until we see each other, we look forward to it, and bear through graduations and a parent's death and so it's more than just going to a job. You now have a *group of colleagues that you respect academically* but also have a relationship with personally.
>
> Just the overall experience of being around people *with similar interests*, people with the similar desire to see students succeed, and just the knowledge that you get in terms of a takeaway for what you can do to make the classroom better.
>
> I enjoy the experience. I gain from the experience and I feel my students benefit from my experiences there.

In summary, the key reason for returning is learning. Educators point to the mathematical knowledge they gain, but also the teaching strategies, discussions about the many layers of mathematics education, the success stories from other teachers, and the belief that they return to their classrooms better equipped to help their Calculus students succeed.

## Changes in Educators' Thinking

The participants were asked to ponder any changes in their thinking which have occurred as a result of their exam reading experiences. They then evaluated the effects on their thinking about Calculus and their actions in their classrooms, which were causally linked to their AP reading experiences. The thinking question specified thoughts versus actions; the classroom practice question specified actions versus thoughts. They also evaluated the resultant effects on their students' understanding of calculus.

Comparing the educators' assessments of changes in their own thinking to their opinions on the value of the professional development to, 127 out of 149 respondents chose to either agree or strongly agree to both statements. See Table 43.

Table 43

*Changes in Educators' Calculus Thinking vs. Value of PD*

| | Changes in Educators' Calculus Thinking | | | | |
|---|---|---|---|---|---|
| The Reading is Valuable PD | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| Strongly Disagree | 1 | 0 | 0 | 2 | 0 |
| Disagree | 0 | 0 | 0 | 3 | 0 |
| Neutral | 0 | 1 | 0 | 0 | 0 |
| Agree | 3 | 5 | 1 | 36 | 4 |
| Strongly Agree | 0 | 4 | 2 | 40 | 47 |
| Total | 4 | 10 | 3 | 81 | 51 |

An approximately equal percentage of high school teachers (91%) and college professors (87%) reported that their thinking about Calculus had changed because of their AP reading experiences. However, the high school teachers who agreed were more apt to report strong agreement (SA=50%) than agreement (A=41%); the college professors more often agreed (A=65%) than strongly agreed (SA=22%). See Table 44.

Table 44

*Changes in Educators' Calculus Thinking vs. Educator Level*

| | Changes in Educators' Calculus Thinking | | | | |
|---|---|---|---|---|---|
| Educator Level | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| High School Teacher | 1 | 2 | 3 | 26 | 32 |
| College Professor | 3 | 8 | 0 | 55 | 19 |
| Total | 4 | 10 | 3 | 81 | 51 |

When all five categories of responses Strongly Disagree (SD), Disagree (D), Neutral (N), Agree (A) and Strongly Agree (SA) were utilized, the expected counts in more than 20% of the cells were less than 5, making a $\chi^2$ test unusable. To eliminate this problem, the categories of SD and D were combined; and, the categories of A and SA were also combined. The low expected cell count problem persisted because of the neutral category, which contains only three entries. The neutral responses were ignored, resulting in no cells with an expected value less than 5. The two-side test results, $\chi^2(1) = 2.64, p = .104$, are not indicative of an association between level of educator and their beliefs about their Calculus thinking undergoing any changes due to the reading experience.

Comparing the participants' thinking about Calculus to their ages (combining the two older age groups to achieve appropriate expected cell counts), the two-sided Fisher's Exact test yields $p = .551$, again showing no association. 52% of the older educators agreed and 40% strongly agreed. 58% of the middle-aged educators agreed; and, 33% strongly agreed. For the younger educators, 59% agreed and 23% strongly agreed. 18% of the younger educators either disagreed or were neutral; for the older group, 8% disagreed or were neutral. The large majority (88.5%) of all respondents either agreed or strongly agreed. See Table 45.

Table 45

*Changes in Educators' Calculus Thinking vs. Educator Age Group*

| | Changes in Educators' Calculus Thinking | | | | |
|---|---|---|---|---|---|
| Age Group | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| 31-45 | 1 | 5 | 3 | 30 | 12 |
| 46-60 | 2 | 4 | 0 | 40 | 23 |
| 60+ | 1 | 1 | 0 | 10 | 16 |
| Total | 4 | 10 | 3 | 80 | 51 |

Comparing educators' beliefs about their own thinking about Calculus with their beliefs about positive changes in their classroom practices, positive responses to changes in educator's calculus thinking occur with positive changes in their classroom practices in

76% of the cases. Negative responses to changes in educator's calculus thinking occur with negative responses to changes in student understanding in 6.8% of the cases. 17 cases (11.6%) are negative to positive and positive to negative responses. Neutral responses to either question comprise 5.5% of the cases. See Table 46. Because of the low number of negative responses, $\chi^2$ analysis could not be performed. However, the table illustrates that there is a strong association between agreement with changes in one's Calculus thinking and agreement with changes in classroom practice.

Table 46

*Changes in Educators' Calculus Thinking vs. Changes in Classroom Practice*

| | Changes in Educators' Calculus Thinking | | | | |
|---|---|---|---|---|---|
| Changes in Classroom Practice | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| Strongly Disagree | 4 | 0 | 0 | 2 | 0 |
| Disagree | 0 | 6 | 1 | 11 | 0 |
| Neutral | 0 | 0 | 1 | 3 | 2 |
| Agree | 0 | 4 | 1 | 53 | 18 |
| Strongly Agree | 0 | 0 | 0 | 9 | 31 |
| Total | 4 | 10 | 3 | 78 | 51 |

Comparing educators' beliefs about their own thinking about Calculus with their beliefs about positive changes in their students' understanding of Calculus, positive responses to changes in educator's calculus thinking are associated with positive changes in student understanding in 74.5% of the cases. Negative responses to changes in educator's calculus thinking are associated with negative responses to changes in student understanding in 7.8% of the cases. 16 cases (11.3%) are negative to positive and positive to negative responses. Neutral responses to either question comprise 6.4% of the cases. See Table 47. Because of the low number of negative responses, $\chi^2$ analysis could not be performed. However, the table illustrates that there is a strong association between agreement with changes in one's Calculus thinking and agreement with changes in student understanding.

Table 47

*Changes in Educators' Calculus Thinking vs. Changes in Student Understanding*

| | Changes in Educators' Calculus Thinking | | | | |
|---|---|---|---|---|---|
| Changes in Student Understanding | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| Strongly Disagree | 4 | 1 | 1 | 2 | 0 |
| Disagree | 0 | 6 | 0 | 11 | 0 |
| Neutral | 0 | 0 | 1 | 5 | 1 |
| Agree | 0 | 3 | 1 | 46 | 23 |
| Strongly Agree | 0 | 0 | 0 | 9 | 27 |
| Total | 4 | 10 | 3 | 73 | 51 |

Illustrative commentary, from both the interviews and the survey responses, includes:

> I believe that my students understand Calculus much better today than they did prior to the time I became a reader. I now spend a week covering the Fundamental Theorem. Before, I'd spend about 20 minutes.
> My students are giving better explanations and justifications. I believe students have gained a better conceptual understanding of calculus, more than just the mechanical aspects of it. I have also seen students become more confident as they approach the exam.

> Students ask more pointed questions. They want to know how to correctly justify a point and to look at not only the big picture of what is happening, but the reason why and how that can be accurately explained.
>
> I bring problems back with me and we do them in class and the students seem to have a better grasp of the subject matter after delving into these free response questions.

When considering both years of teaching experience and years of AP exam reading experience, there were relatively few negative responses concerning changes in educators' thinking about calculus as a result of the AP reading. See Table 48 and Table 49. B

Table 48

*Changes in Educators' Calculus Thinking vs. Years of Exam Reading*

| | Changes in Educator's Calculus Thinking | | | | |
|---|---|---|---|---|---|
| Years of Exam Reading Experience | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| 1-5 | 3 | 5 | 3 | 39 | 21 |
| 6-10 | 1 | 4 | 0 | 25 | 16 |
| 11-15 | 0 | 1 | 0 | 12 | 7 |
| 16-20 | 0 | 0 | 0 | 2 | 4 |
| 21+ | 0 | 0 | 0 | 2 | 3 |
| Total | 4 | 10 | 3 | 80 | 51 |

**Changes in Educators' Classroom Practice**

The AP exam reading is touted as valuable professional development. Educators who participate in these readings talk about the exchange of ideas with colleagues, the new strategies which they learn and the excitement they feel in anticipation of

Table 49

*Changes in Educators' Calculus Thinking vs. Years of Teaching*

| | Changes in Educator's Calculus Thinking | | | | |
|---|---|---|---|---|---|
| Years of Teaching | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| 1-10 | 0 | 0 | 0 | 8 | 5 |
| 11-20 | 2 | 6 | 1 | 26 | 9 |
| 21-30 | 1 | 2 | 2 | 24 | 20 |
| 31-40 | 0 | 2 | 0 | 16 | 12 |
| 41-50 | 1 | 0 | 0 | 7 | 4 |
| Total | 4 | 10 | 3 | 81 | 50 |

incorporating new approaches in their classrooms. However, flexibility in the curriculum, or lack of, district requirements, school policies, and student ability and motivation all play a part in how much change an educator can implement.

Participants were asked to explain any changes which they had implemented in their classroom as a result of their exam reading experiences. They then evaluated the effects on these changes on their students' understanding of Calculus.

A higher percentage of the high school teachers (84%) than college professor (73%) reported that their classroom practices had changed because of their AP Calculus exam reading experiences. High school teachers were equally likely to respond in each of the two positive categories (A=42%; SA=42%); college professors were more likely to agree than strongly agree (A=58%; SA=15%). Six high school teachers (9% of high school teachers) and no college professors responded neutrally. Responding negatively were 6% of the high school teachers. However, 27% of college professors either disagreed or strongly disagreed. With the neutral responses removed, the two-sided significance test, $\chi^2(3) = 20.47, p < .001$, shows there is an association; however, 2 cells (25%) had expected counts of less than 5. The two disagree categories were combined and the two agree categories were combined. The Fisher's Exact test, $p = .002$, indicates that there is an association between educators' perceptions of changes in their classroom practices and educator level. High school teachers are more likely to implement changes to their classroom practices. See Table 50.

Table 50

*Classroom Practice Affected vs. Educator Level*

| | Classroom Practice affected by AP Exam Reading Experiences | | | | |
|---|---|---|---|---|---|
| Educator Level | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| High School Teacher | 1 | 3 | 6 | 27 | 27 |
| College Professor | 6 | 17 | 0 | 50 | 13 |
| Total | 7 | 20 | 6 | 77 | 40 |

Commentary from high school teachers generally included placing emphasis on conceptualization over procedural manipulations, changes in their lesson delivery, and fostering deeper mathematical understanding. Commentary from high school teachers includes:

> I spend more time having my students focus on connections between concepts.
>
> One of the most significant changes I made since I began reading is the way I manage homework. I teach in a school that values weekly assignments over daily assignments and, after talking with colleagues at the reading, I changed my approach to daily assignments. Particularly for my weaker students, I saw notable improvements in their performance and scores.

> I have given greater attention to specific topics like the Fundamental Theorem of Calculus, differential equations, and modeling with integrals, not only because of their likelihood of being on the exam, but because of their importance in understanding calculus.
>
> After hearing the presentation from the College Board at the AP Reading and studying the material I picked up from the presentation, I am familiar with the changes in the AP Calculus courses for the 2016-17 school year. For these reasons, I decided to make major changes to my AB Calculus curriculum.
>
> I am much more careful about teaching my students proper presentation (decimal places, linkage errors, saying too much, saying too little). These changes were based on my experience as a reader, when I was unable to give a point to students who clearly knew what they were doing, but didn't present their solutions to the standard specified in the rubric.
>
> I do a better job explaining the use of the 1st and 2nd derivative test as they relate to find/justifying relative extrema and points of inflection. I also do a better job modeling how to solve a differential equation, emphasizing what the graders are looking for in assigning points. I have also changed the way I grade free response questions, by using the rubric and what I have learned as a grader so that student will do a better job especially on justifications.

A high school teacher who serves as a table leader at the readings expressed frustration with adapting her classroom practice to the expectations of the exam, only to have those expectations frequently change. She said:

> I came home from the reading one year and I said to my students, “I don’t want to see an equal sign anywhere, you can use arrows all over the place, but I don’t want you to write an equals sign, because if it's not equal, you are going to lose the point.” Then, one year, it was the year of the parentheses; kids lost 5 points one year because of setting up volume of disks and washers and messing up parentheses. It is so heartbreaking. And, then last year was the year of the *dt*; we've never cared about a *dt*, now all of a sudden, we care about them. So, this year, in my Calculus II class, we’re doing those polars and things now, and I'm saying that we've got to put the *dt;* we've got to put the $d\theta$; we've got to put *d* whatever the heck it is, and we've got to put the initial condition first, because that way, if you write 3 + the integral and forget the *dt*, you'll be fine. But, if you do the integral plus the three, you're going to be penalized.

Commentary from college professors focused on changes other than classroom delivery, as professors generally did not make what they considered to be major changes. However, they did mention the value of a grading rubric frequently, and precise notation and use of symbols. Commentary from college professors includes:

> I realized the value of having a rubric for grading problems. Mine are not as detailed as the AP Reading ones, but they do streamline the process and make sure I look for the same things on each problem.
>
> Being a reader hasn't influenced my actual teaching very much. Sharing rubrics with students before and after the test helps them to understand their mistakes.
>
> I changed my grading habits on exams after being a Reader in that I made better rules for partial credit eligibility on problems. And I think I became a more consistent grader due to my AP Reading experience. But I didn't really change that way I taught calculus in the classroom.
>
> I am more particular about the language (mathematical and English) that I use and that I expect students to use.

However, a few professors reported making changes to their classroom approaches. One professor wrote, "less lecture, more student collaboration, more integration of topics. My teaching actions have essentially closely reflected what was suggested in the AP Calculus Teachers Guide as deserving of more and less attention." Another professor stated that he now places more emphasis on teaching over algorithms. And, another professor stated that his experiences as a reader had not changed "how I teach as much as what I teach," referring to emphasizing interconnections among topics.

Implementing changes to classroom practice appears to be independent of age group. When comparing the changes in participants' classroom practice to their ages (using all three age groups) and eliminating the six neutral responses, the two-sided significance test, $\chi^2(2) = 1.36, p = .507$, shows no association. The 31-45 age group most often agreed (A=44%); they strongly agreed (SA=28%) in equal numbers as those who were neutral or negative. The 46-60 age group also most often agreed (A=60%); they strongly agreed (SA=21%) slightly more frequently than they were neutral or negative (19%). The over-60 age group' agree (A=45%) is closer to their strongly agree total (SA=38%) than the other two age groups; 17% of the over-60 group was neutral or negative. See Table 51.

Table 51

*Classroom Practice Affected vs. Age Group*

| | Classroom Practice affected by AP Exam Reading Experiences | | | | |
|---|---|---|---|---|---|
| Age Group | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| 31-45 | 2 | 9 | 3 | 22 | 14 |
| 46-60 | 3 | 7 | 3 | 42 | 15 |
| 60+ | 2 | 3 | 0 | 13 | 11 |
| Total | 7 | 19 | 6 | 77 | 40 |

Years of teaching experience also appears to be independent from an educator's decision to implement changes to classroom practice. Again, ignoring neutral responses and combining categories, the two-sided significance test, $\chi^2(4) = 9.02, p = 0.061$, does not allow inference of any dependence between the two. See Table 52. Years of reading experience also appear to be independent of educators' decisions to implement classroom changes. The hypothesis test could not be accomplished because of expected cell count deficiencies, even with the combined categories. However, when combining

Table 52

*Classroom Practice Affected vs. Years of Teaching*

| | Classroom Practice affected by AP Exam Reading Experiences | | | | |
|---|---|---|---|---|---|
| Years of Teaching | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| 1-10 | 1 | 1 | 2 | 6 | 3 |
| 11-20 | 3 | 9 | 0 | 20 | 12 |
| 21-30 | 1 | 5 | 3 | 26 | 14 |
| 31-40 | 0 | 2 | 1 | 19 | 8 |
| 41-50 | 2 | 3 | 0 | 5 | 3 |
| Total | 7 | 20 | 6 | 76 | 40 |

three highest reading experience groups, an association with educators' decisions to implement classroom changes was revealed, $\chi^2(2) = 6.35, p = 0.042$.

Those with fewer than 5 years of reading experience have a higher percentage (27%) reporting no changes to their classroom practices than those with 6-10 years of reading experience (15%). For those with over 10 years of reading experience, 2 people (6%) reported no classroom changes. Teachers who have more reading experience are more likely to believe that changes in their classroom practices have been affected by their reading experiences. See Table 53.

Changes in classroom practices would, presumably, be aimed at improving ISL and/or student performance. 104 of 144 (72%) agreed or strongly agreed with both statements. Again, ignoring neutral responses and combining categories, the two-sided significance test, $\chi^2(1) = 73.17, p < .001$, indicates that there is an association between the two. One cell (25%) has an expected count of 4.8. Those who disagreed with changes occurring in their classroom practice tended to also disagree with changes in

Table 53

*Classroom Practice Affected vs. Years of Exam Reading*

| Years of Exam Reading Experience | Classroom Practice affected by AP Exam Reading Experiences Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
|---|---|---|---|---|---|
| 1-5 | 4 | 4 | 0 | 0 | 0 |
| 6-10 | 0 | 12 | 0 | 7 | 0 |
| 11-15 | 0 | 1 | 4 | 2 | 0 |
| 16-20 | 1 | 3 | 0 | 56 | 14 |
| 21+ | 0 | 0 | 2 | 10 | 24 |
| Total | 5 | 20 | 6 | 75 | 38 |

their students understanding; conversely, those who reported that they had implemented changes in their classroom practices tended to also report changes in their students' understanding of calculus. See Table 54.

Relevant commentary from positive responders includes:

> I have found the students to have a far deeper grasp of Calculus than before. They know any topic may appear on the next test; so, they don't compartmentalize their understanding.

> I think my students are more comfortable with AP type questions. They are also more prepared about what is on the exam and how it is set up as well as strategies. Trying to teach young people with concepts using visuals and auditory leaning is important. The information I've received from the reading has helped.

> I have heard from colleagues that students coming out of my classes seem to be more precise in their responses than they used to be. Being better able to prepare students to move on to other courses is a great thing.

> We often talk about things like linage errors, units and rounding throughout the year. I make sure my students write their answer as a sentence all year so that when they get to questions that ask them to explain or justify, they are quite comfortable with that.

From those who responded either neutrally or negatively, some simply stated that they did not know or did not have any way to measure. Some cited wide variability in student skill sets, motivation and experience in their classes from year to year. Others outright stated that the reading experience had no influence. Representative commentary includes:

> I am not sure of any specific change in my teaching or success of my students has resulted.
>
> The influence is not solely from the reader experience
>
> Possibly. I have had other professional growth experiences in the same period as the AP exams, so it is difficult to identify how much the AP experiences have influenced the changes I have seen.
>
> I don't feel that my experiences have had an effect on my students. Having taught calculus for 25 years prior to becoming a reader, I feel that my strategies and techniques are well developed.

Table 54

*Classroom Practice vs. Changes in Student Understanding*

| | Classroom Practice affected by AP Exam Reading Experiences | | | | |
|---|---|---|---|---|---|
| Changes in Student Understanding | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| Strongly Disagree | 4 | 4 | 0 | 0 | 0 |
| Disagree | 0 | 12 | 0 | 7 | 0 |
| Neutral | 0 | 1 | 4 | 2 | 0 |
| Agree | 1 | 3 | 0 | 56 | 14 |
| Strongly Agree | 0 | 0 | 2 | 10 | 24 |
| Total | 5 | 20 | 6 | 75 | 38 |

## Changes in Student Understanding

High school teachers who serve as AP exam readers have insights with the exam grading which could be advantageous to their students. For example, readers would know that functions need to be identified specifically as $f(x)$ or $g(x)$ and that the first derivative of $f(x)$ needs to be identified by $f'(x)$. Using the words "it," "the function," or "the derivative" lacks specificity and will cost a student valuable points on the free response questions. Also, knowing that numerical answers do not need to be simplified can save students considerable time. For example, if the student leaves an answer as $1 + 2 + 3$ and the correct answer is 6, then the student is marked as correct. The answer may not include any variables; $sin(x)$ is not numeric, but, $sin\left(\frac{\pi}{3}\right)$ is numeric and is an acceptable answer. Answers may also contain radicals in the denominator.

## AP Exam Scores

Actual changes which teachers had observed in their classes were sought. And, for quantitative analysis, high school teachers were asked to provide quantitative data on their students' AP Calculus exam scores for the past several years. Uniformity of response was problematic. Some teachers reported their average score but did not include the number of students. Others submitted their best guess; others estimated their mean scores and standard deviations, some with and some without class sizes. There were a number of teachers who reported both AB and BC scores; some teachers distinguished between the two; others did not. The number of years reported, and the actual years themselves, varied a great deal. Some teachers do not teach AP every year; other teachers have taught AP for only a short time and do not yet have a track record.

However, most high school teachers reported that they did not have access to their scores from previous years and either offered their best estimates or declined to submit data because they had no confidence in its accuracy. Quantitative analysis to determine if exam scores do increase after a teacher has served as a reader, therefore, was not possible. Teachers did, however, comment abundantly on how they saw their students' understanding responding, or not responding, to changes in the teacher's classroom approaches.

**Student Knowledge**

It must be borne in mind that natural variation in class size, student motivation, and student ability, occurs in classes from year to year. One year, a class may be significantly smaller or larger than the previous year. Previous experience, motivation and abilities of the students will vary also. These factors will also factor into exam scores. Teachers explained that the changes they had made to their approaches in the classroom which were based on their reading experiences could have a substantial impact on capable and motivated students; but, they lamented that some students simply lack interest and motivation. Representative commentary from high school teachers include:

> I'd like to think that my kids have improved their understandings of the material and their results, but I can't say definitively. Perhaps the one thing I can say with certainty is that my kids and their parents seem to have confidence in me. That perhaps is due in part to my ability to speak to what readers look for.
>
> We often talk about things like linkage errors, units and rounding throughout the year. I make sure my students write their answer as a sentence all year so that

when they get to questions that ask them to explain of justify, they are quite comfortable with that.

I think my Reading experiences have helped to understand the ideas more clearly myself, so I can do a better job explaining things to the students.

I believe that my students understand calculus better because I grade AP Calculus. I bring the problems back with me and we do them in class and the students seem to have a better grasp of the subject matter after delving into these free response problems.

Before I went to the reading, my scores were below average. I was happy if a student passed with a 3 or higher with very few 5's. This year, I was pleased to see many 5's and 4's, which confirmed my belief that changing my teaching practices was working. I even see the students retaining the information for longer amounts of time. My students from last year remember most of the conceptual and procedural topics.

I think my students are providing better, clearer, more concise mathematics. Now the kids think of the word "it" as a swear word in the class. Students are clearer about their responses to standard AP questions. They write justifications better.

Students do better when they know little details can help make a difference. Most explanation need three parts and one is often forgotten, so teaching them to think about all three each time they explain something in context is really helpful. Also, having them look over many problems that have a context so they can see how the Calculus is really the same benefits them a lot. I can see improvement on the tests they take in class.

My students are able to communicate their understanding both verbally and in their written work. Their conversations center on the understanding of the concepts. It is always fun to listen in on their conversations as they speak "math." They enter the exam feeling very prepared. Furthermore, they go off to college feeling like they can take the next step in their math careers.

College professors were asked if they had changed their classroom practice because of their AP reading experiences. Substantial variation in their responses was noted. Some stated that their reading experience had no effect on their practice or their students. Several stated that they did not know, had no way to measure, or did not teach college freshmen. Others discussed how they used the free response questions in their classes to emphasize concepts over calculations. Sample responses are:

> I have always emphasized the need for my students to understand the calculus concepts as well as to do the symbolic manipulations. My AP experience has encouraged me to do more of this and to have the students spend more time in class talking about the concepts.
>
> Making students fully explain their answers increases their ability to translate mathematics to the real world and also see mathematics in everyday situations. I try to incorporate more real-world examples that are applicable to my students' chosen field of study or interest. I hope my students have developed a stronger appreciation of Calculus and its usefulness in the real world.
>
> My students have a better grasp of how to attack something when they have no idea where to begin. That is what the employers want in the real world. Being able to think critically is a skill we sometimes forget when teaching math. We focus too often on teaching skills and not thinking.
>
> I think that my AP Calculus reading experiences have increased my awareness of the backgrounds of some of the students in my classes. However, I am not sure of any specific change in my teaching or success of my students that has resulted. Probably not.
>
> As I interacted with teachers at the Reading, I tried to learn their perspective on teaching of calculus. But I don't think that what I learned influenced my students understanding of calculus.
>
> Not really -- I don't believe I have transferred my AP Calculus reading experiences to the classroom.
>
> No, I don't feel that I benefit from the reading at all in this regard.

When asked about how their experiences as a reader had influenced their own thinking about Calculus, numerous educators discussed multiple representations: numerical, graphical, symbolic, tabular and narrative. Commentary includes:

> I think about ways to get the students using multiple representations and making connections about several concepts at once.

> I have learned and incorporated multiple representations of functions. I have built new examples for understanding the relationships between derivatives and functions. I have learned that communication is as important as accuracy.
>
> It became clear to me that the study of calculus is accessible to more students that I might have though before attending the readings. It led me to emphasize much more, conceptual understanding of calculus in part via multiple representations and multiple teaching approaches.

The importance of precise and accurate justifications to accompany mathematical conclusions was also discussed frequently by the participants. Commentary from high school teachers include:

> The most noticeable influences have been in communicating justifications and expressing meaning. I have used calculus materials for many years which really emphasizes the Rule of 4 (numerical, graphical, symbolic, and verbal) representations; but, the AP Calculus questions provide additional support for emphasizing clear and thorough communication.
>
> The reading experience confirmed the importance of well-written justifications and interpretations of results, as well as the need to prioritize tasks that are application centered and bring multiple concepts together for problem solving.
>
> My idea about teaching Calculus has changed from just getting across the basic concepts, to making sure the students are well-prepared in being able to communicate the math verbally.
>
> I teach my students how to write their justifications in very particular ways. We spend real time on how to phrase your writing in mathematics.
>
> I have my students explain their work to the class. They are more active learners. My students seem to be more confident about the AP Calculus exam because of these exercises and experiences. Their answers on justifications and interpretation questions are more concise/focused.

College professors also commented on the performance of college freshman who had taken AP Calculus in high school and those who had not. No clear trend was noted. A few professors noted that they could see a difference between the two groups; others stated that they taught more advanced Calculus and would not have freshmen in their classes. Most said that they would not have any way of knowing which of their freshman students had AP Calculus experience.

Most college professors indicated that they had not changed their classroom practices due to their AP reading experiences. However, incorporating the free response questions into their classroom activities was a popular theme.

Although the majority of high school teachers (83%) and college professors (72%) reported positive changes in student understanding, high school teachers were more likely to report strong agreement (SA=44%; A=39%) than college professors (SA=10%; A=62%). 28% of the college professors disagreed with any changes in student understanding and none responded neutrally; high school teachers responded negatively less frequently (6%) and did responded neutrally (11%). See Table 55. For the two-sided $\chi^2$ test for independence, the two negative categories were combined, the two positive categories were combined and the seven neutral responses were ignored. The results, $\chi^2(1) = 9.50, p = .002$, indicate that an association exists between the level of the educator and their opinion of positive changes in student understanding. High school teachers and college professors are both more likely to observe positive changes in student understanding; however, high school teachers are more likely to hold this belief strongly. College professors are more likely than high school teachers not to observe changes in student understanding.

No association was found between educator age group and observation of positive changes in student understanding. See Table 56. The two negative categories were again

Table 55

*Changes in Student Understanding vs. Educator Level*

| | Changes in Student Understanding | | | | |
|---|---|---|---|---|---|
| Educator Level | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| High School Teacher | 2 | 2 | 7 | 25 | 28 |
| College Professor | 6 | 17 | 0 | 51 | 8 |
| Total | 8 | 19 | 7 | 76 | 36 |

Table 56

*Student Understanding vs. Age Group*

| | Changes in Student Understanding | | | | |
|---|---|---|---|---|---|
| Age Group | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| 31-45 | 5 | 8 | 3 | 19 | 12 |
| 46-60 | 2 | 8 | 3 | 38 | 17 |
| 60+ | 1 | 2 | 1 | 19 | 7 |
| Total | 8 | 18 | 7 | 76 | 36 |

combined to achieve acceptable cell counts for the statistical analysis; the two positive categories were also combined. $\chi^2$ analysis was executed with the seven neutral responses removed, $\chi^2(2) = 5.17, p = .075$; no evidence to reject independence was obtained.

No association was found between educator years of teaching and observation of positive changes in student understanding. See Table 57. The two negative categories were again combined to achieve acceptable cell counts for the statistical analysis; the two

Table 57

*Student Understanding vs. Years of Teaching*

| | Changes in Student Understanding | | | | |
|---|---|---|---|---|---|
| Years of Teaching | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| 1-10 | 1 | 0 | 2 | 5 | 3 |
| 11-20 | 4 | 10 | 0 | 17 | 11 |
| 21-30 | 2 | 4 | 2 | 30 | 11 |
| 31-40 | 0 | 4 | 2 | 15 | 9 |
| 41-50 | 1 | 1 | 1 | 8 | 2 |
| Total | 8 | 19 | 7 | 75 | 36 |

No association was found between educator years of reading AP exams and observation of positive changes in student understanding; Fisher's Exact test yields $p =$ .598. See Table 58.

Table 58

*Student Understanding Affected vs. Years of Exam Reading*

| | Changes in Student Understanding | | | | |
|---|---|---|---|---|---|
| Years of Exam Reading Experience | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| 1-5 | 5 | 9 | 7 | 31 | 44 |
| 6-10 | 3 | 6 | 0 | 27 | 28 |
| 11-15 | 0 | 4 | 0 | 11 | 12 |
| 16-20 | 0 | 0 | 0 | 3 | 4 |
| 21+ | 0 | 0 | 0 | 4 | 5 |
| Total | 8 | 19 | 7 | 76 | 93 |

positive categories were also combined. $\chi^2$ analysis was executed with the seven neutral responses removed, $\chi^2(4) = 7.41, p = .116$; no evidence to reject independence was obtained.

**Classroom Use of Former AP Free Response Questions**

An association was found between educators' use of former free response questions (FRQ) and their observation of positive changes in student understanding. See Table 59. For both variables, the two negative categories were again combined to achieve acceptable cell counts for the statistical analysis; the two positive categories were also combined. $\chi^2$ analysis was executed with the neutral responses removed, $\chi^2(1) = 23.40, p < .001$. More than two-thirds of the educators report using AP Calculus Free Response Questions (FRQ) from previous years in their classrooms. Eighty-six percent of those educators also report agreeing or strongly agreeing with seeing changes in their students' understanding of Calculus. However, 44 of the educators do not use the FRQs. None of them strongly agree with seeing changes in their students' understanding; twenty-four agreed and twenty were neutral or negative.

No association was found when comparing the use of FRQs, neutral response ignored, to age group, $\chi^2(6) = 7.70, p = .261$. See Table 60. Sixty-five percent of the younger age group indicated use of the FRQs; for the middle age groups, 64%; and, for the older group, 83%. However, when separated by educator level, an association is revealed ($\chi^2(3) = 93.58, p < .001$). Ninety-five percent (A: 11%, SA: 84%) of high school teachers report that they use the FRQs in their classroom. Forty-seven percent (A: 36%, SA: 11%) of college professors report using the FRQs. Fifty-three percent of

Table 59

*Use Former AP FRQ in My Classroom vs. Changes in Student Understanding*

| | Use Former AP FRQ in My Classroom | | | | |
|---|---|---|---|---|---|
| Changes in Student Understanding | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| Strongly Disagree | 7 | 0 | 0 | 0 | 1 |
| Disagree | 4 | 8 | 0 | 6 | 1 |
| Neutral | 1 | 0 | 0 | 1 | 5 |
| Agree | 7 | 17 | 0 | 26 | 25 |
| Strongly Agree | 0 | 0 | 1 | 5 | 29 |
| Total | 19 | 25 | 1 | 38 | 61 |

Table 60

*Use Former AP FRQ in My Classroom vs. Age Group*

| | Use Former AP FRQ in My Classroom | | | | |
|---|---|---|---|---|---|
| Age Group | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| 31-45 | 11 | 7 | 0 | 12 | 21 |
| 46-60 | 8 | 16 | 1 | 17 | 28 |
| 60+ | 2 | 3 | 0 | 11 | 13 |
| Total | 21 | 26 | 1 | 40 | 62 |

college professors do not use the FRQs in their classrooms. Categories were not combined for this $\chi^2$analysis. See Table 61.

Very few people indicated negative or neutral value for the professional development at the reading. Comparing only positive responses to the valuable PD question and including all five levels of response for the FRQ questions, an association is indicated, $\chi^2(4) = 30.47, p < .001$. Those who strongly agree that the reading offers valuable PD tend to also strongly agree about using the FRQs in their classrooms. Only one person was neutral. Sixty-seven percent of participants agreed with both question; twenty-nine percent agreed or strongly agreed on valuable PD but disagreed or strongly disagreed with using the FRQs in their classrooms. See Table 62. No associations were found between the use of former FRQs in the classroom and either years of teaching or years of reading AP exams. See Table 63 and Table 64.

Table 61

*Use Former AP FRQ in My Classroom vs. Educator Level*

| | Use Former AP FRQ in My Classroom | | | | |
|---|---|---|---|---|---|
| Educator Level | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| High School Teacher | 2 | 0 | 1 | 7 | 55 |
| College Professor | 20 | 26 | 0 | 33 | 7 |
| Total | 22 | 26 | 1 | 40 | 62 |

Table 62

*Use Former AP FRQ in My Classroom vs. Valuable PD*

| | Use Former AP FRQ in My Classroom | | | | |
|---|---|---|---|---|---|
| Valuable PD | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| Strongly Disagree | 1 | 1 | 0 | 0 | 0 |
| Disagree | 1 | 1 | 0 | 0 | 0 |
| Neutral | 0 | 0 | 0 | 1 | 0 |
| Agree | 12 | 14 | 0 | 19 | 7 |
| Strongly Agree | 7 | 10 | 1 | 19 | 55 |
| Total | 21 | 26 | 1 | 39 | 62 |

Table 63

*Use Former AP FRQ in My Classroom vs. Years of Teaching*

| | Use Former AP FRQ in My Classroom | | | | |
|---|---|---|---|---|---|
| Years of Teaching | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| 1-10 | 3 | 1 | 0 | 1 | 8 |
| 11-20 | 9 | 8 | 0 | 14 | 14 |
| 21-30 | 3 | 10 | 1 | 13 | 23 |
| 31-40 | 4 | 5 | 0 | 8 | 13 |
| 41-50 | 3 | 2 | 0 | 3 | 4 |
| Total | 22 | 26 | 1 | 39 | 62 |

Table 64

*Use Former AP FRQ in My Classroom vs. Years of Exam Reading*

| | Use Former AP FRQ in My Classroom | | | | |
|---|---|---|---|---|---|
| Years of Exam Reading Experience | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
| 1-5 | 11 | 6 | 1 | 15 | 41 |
| 6-10 | 8 | 9 | 0 | 18 | 11 |
| 11-15 | 2 | 7 | 0 | 5 | 5 |
| 16-20 | 1 | 1 | 0 | 1 | 3 |
| 21+ | 0 | 2 | 0 | 1 | 2 |
| Total | 22 | 25 | 1 | 40 | 62 |

**Summary**

Most participants have positive views on most aspects of the reading. However, there are differences in the aggregate between high school teachers' opinions and college professors' opinions. Years of teaching and years of reading the exams also appear to be influential characteristics in the formation of some opinions.

To achieve a cell count of at least 5 for every cell for $\chi^2$ analysis, the negative categories of Strongly Disagree and Disagree were frequently combined. In consonance, the positive categories of Strongly Agree and Agree were also combined. The neutral category was also ignored when necessary to achieve appropriate cell counts. Attempts were made to obtain the largest degrees of freedom possible, by either combining categories, ignoring a small count of neutral responses, or both. When the analysis was condensed to a 2 x 2 matrix, the Fisher's Exact Test was used.

# Chapter Five

## Introduction

This study investigated possible associations between participation in the reading of AP Calculus exams and the changes reported by participants in their educator professional development, classroom practice and student learning. This chapter discusses the results of the study, which was constructed around the following three research questions:

1. How do AP Calculus teachers and college Calculus professors perceive the professional development experiences at the AP Calculus exam reading?
2. How does participation in an AP Calculus national exam reading affect teachers' and professors' classroom practice, as perceived by the educators?
3. How do exam readers report that their participation in an AP Calculus national exam reading influences their students' success, as perceived by the educators?

A representative sample of 155 AP Calculus exam readers provided data through an online survey; seventeen of these people were also interviewed via telephone. Collected data were both qualitative and quantitative.

**Synopsis of Likert Scale Data Analysis and Significance Tests for Association or Independence**

Chi-Square analysis was attempted on a number of Likert scale data. However, more than 20% of the expected cell counts were less than 5 for a majority of the cases. For these, the strongly disagree (SD) and disagree (D) categories were combined and the agree (A) and strongly agree (SA) categories were combined. The neutral counts were usually low and these were ignored. When more than 20% of the expected cell counts were less than 5, even with category combinations, the Fisher's Exact test was used.

**Professional Development**

The large majority, 96%, of participants view the professional development at the AP Calculus reading as valuable. Level of educator (high school teacher or college professor), educator age group, years of teaching and years of reading experience were not significant.

Although the majority viewed the evening professional development sessions positively, they were less enthusiastic than the responses to the value of the reading as a whole. 71.5% value the evening sessions but there was little strong agreement. 24 high school teachers and no professors reported neutral opinions. No significance was found in age groups, years of teaching experience or years of exam reading experience. More than 1/3 of the participants do not attend any of the evening sessions. Almost half, 48%, attend between 1 and 3 of the sessions. And, about 15% of the participants attend all the evening sessions. However, the distribution of the high school teachers is relatively uniformly distributed across the number of sessions attended; the college professor distribution is skewed right. 45% of college professors do not attend any evening sessions; only 27% of high school teachers do not attend any evening sessions.

### Collegial Interaction

Collegial interaction received an almost unanimously positive rating. These specifics fell into four overlapping categories: social; learning and exchanging ideas; insights into student thinking; and, interaction with educators at different levels. Because almost all responses were positive, no significance tests were executed. Some commentary addressed complaints readers had concerning areas needing improvement. These included improved consistency in grading from one question to another and from year to year, giving credit for mathematically sound answers which do not fit a preferred format, and not giving credit for answers which were mathematically imprecise or too generalized.

### Intend to Return and Encourage Colleagues to Attend

140 out of 150 respondents stated that they encourage their colleagues to apply to become AP readers. The 10 remaining were 5 high school teachers who were neutral and 5 college professors who disagreed. Similar results were obtained when sorting by age groups, of years of teaching. The 10 who did not agree all had 10 or fewer years of exam reading experience; 8 of the 10 have 5 or fewer years of reading experience. When discussing their own future plans, 92% of high school teachers and 93% of college professors reported that they intend to serve as readers again. Readers cited several reasons for wanting to return to experience the reading again. Four themes were

recurrent: the professionally enjoyable experience of living and working with hundreds of like-minded mathematicians for a week; learning more about mathematics education; opportunities to form lasting friendships, and, the expectations of helping their students better understand Calculus.

**Changes in Educators' Thinking**

Educators were asked to assess changes in their own thinking; their opinions were compared with their educator level, age group, opinions on the value of the professional development; their assessments of changes in classroom practices, and their perceptions of changes in their students' understanding of Calculus. Roughly equal proportions of high school teachers (91%) and college professors (87%) reported that their thinking about Calculus had changed because of their AP reading experiences. However, the high school teachers who agreed were more apt to report strong agreement.

With categories combined and neutral responses removed, the two-side test results, $\chi^2(1) = 2.64, p = .104$, are not indicative of an association between level of educator and their beliefs about their Calculus thinking undergoing any changes due to the reading experience. Similarly, a comparison of participants' thinking about Calculus to their ages (combining the two older age groups to achieve appropriate expected cell counts), the two-sided Fisher's Exact test, $p = .551$, again shows no association.

Comparing educators' beliefs about their own thinking about Calculus with their beliefs about positive changes in their classroom practices, positive responses to changes in educator's calculus thinking are associated with positive changes in their classroom practices in 76% of the cases. Because of the low number of negative responses, $\chi^2$

analysis could not be performed. Comparing educators' beliefs about their own thinking about Calculus with their beliefs about positive changes in their students' understanding of Calculus, positive responses to changes in educator's calculus thinking are associated with positive changes in student understanding in 74.5% of the cases. Because of the low number of negative responses, $\chi^2$ analysis could not be performed.

**Changes in Educators' Classroom Practice**

A higher percentage of the high school teachers (84%) than college professor (73%) reported that their classroom practices had changed because of their AP Calculus exam reading experiences. High school teachers also described changes in strategies, assessments, pacing, concepts to be stressed, and materials. College professors who described changes generally limited changes to incorporation of AP Calculus Exam Free Response Questions (FRQ) from previous years into their lectures and being more mindful in creating rubrics and assessing their students' work.

Implementing changes to classroom practice appears to be independent of age group. The two-sided significance test, $\chi^2(2) = 1.35, p = .507$, shows no association. However, educator level is not independent, Fisher's Exact test, $p = .002$, from changes in classroom practices being put into employed. High school teachers are more likely to implement changes to their classroom practices.

Years of teaching experience also appears to be independent from an educator's decision to implement changes to classroom practice. The two-sided significance test produced, $\chi^2(4) = 9.02, p = .061$, again shows no association.

Years of reading experience, however, appears to be associated with educators' decisions to implement classroom changes, $\chi^2(2) = 6.35, p = 0.042$. The three highest reading experience groups were combined. Those with fewer than 5 years of reading experience have a higher percentage (27%) reporting no changes to their classroom practices than those with 6-10 years of reading experience (15%). For those with over 10 years of reading experience, 2 people (6%) reported no classroom changes. Teachers who have more reading experience are more likely to believe that changes in their classroom practices have been affected by their reading experiences.

Changes in classroom practices are associated with increased ISL and/or student performance, as perceived by teachers. Again, ignoring neutral responses and combining categories, the two-sided significance test, $\chi^2(1) = 73.17, p < .001$, indicates that there is an association between the two. One cell (25%) has an expected count of 4.8. Those who disagreed with changes occurring in their classroom practice tended to also disagree with changes in their students understanding; conversely, those who reported that they had implemented changes in their classroom practices tended to also report changes in their students' understanding of calculus.

**Changes in Student Understanding**

For the two-sided $\chi^2$ test for independence, the two negative categories were combined, the two positive categories were combined and the seven neutral responses were ignored. The results, $\chi^2(1) = 9.50, p = .002$, indicate that an association exists between the level of the educator and their opinion of positive changes in student understanding. High school teachers and college professors are both more likely to observe positive changes in student understanding; however, high school teachers are more likely to hold this belief strongly.

No association was found between observation of positive changes in student understanding and educator age group, $\chi^2(2) = 5.174, p = .075$; no evidence to reject independence was obtained.

No associations were found between either observation of positive changes in student understanding and educator years of teaching, $\chi^2(4) = 7.41, p = .116$, or, observation of positive changes in student understanding educator years of reading AP exams; the two-sided Fisher's Exact test yields $p = .598$.

An association was found between educators' use of former free response questions (FRQ) and their observation of positive changes in student understanding, $\chi^2(1) = 23.40, p < .001$. Those who incorporate the FRQs in their classroom also report improved ISL

No association was found when comparing the use of FRQs, the one neutral response ignored, to age group, $\chi^2(6) = 7.69, p = .261$.

However, when separated by educator level, again, with the one neutral response ignored, an association is revealed, $\chi^2(3) = 93.58, p < .001$. Ninety-five percent of high school teachers report that they use the FRQs in their classroom. Forty-seven percent of college professors report using the FRQs. Fifty-three percent of college professors do not use the FRQs in their classrooms. Categories were not combined for this $\chi^2$analysis.

Very few people indicated negative or neutral value for the professional development at the reading. Comparing only positive responses to the valuable PD question and including all five levels of response for the FRQ questions, an association is indicated $\chi^2(4) = 30.471, p < .001$. Those who strongly agree that the reading offers valuable PD tend to also strongly agree about using the FRQs in their classrooms. No associations were found between the use of former FRQs in the classroom and either years of teaching or years of reading AP exams.

**Research Question 1: Conclusions**

The first research question asked how high school teachers and college professors perceive the professional development at the AP Calculus reading. This study appeared to have validated the claim by the College Board that the exam reading experience is often referred to as “one of the best professional development experiences” an educator can have. The large majority of participants in the AP Calculus reading view the professional development at the reading favorably or very favorably. While positive opinions are widespread on some aspects of this professional development, e.g., collegial interaction, there is considerable disparity in other aspects, e.g., opinions of the formal professional development sessions in the evening.

**Collegial Interactions**

The factor which received the most praise and the most consistently positive commentary was the collegial interactions which occur at the reading, both at the reading site during work hours as well as away from work. Participants reported high value in the colleague-to-colleague interactions in which they participated and filtered these into four overlapping, categories: social; learning and exchanging ideas; insights into student thinking; and, interaction with educators at different levels.

**Social Interaction**

People acclaim the long-lasting friendships which they have formed, the opportunities to share teaching strategies, and. the ability to reach out to a colleague during the school year. A college professor wrote that “for me the personal interactions with others is the best part. I am always meeting new teachers who are smart and like-minded. Although we have a wide diversity in culture and home institutions, all of that gets put aside as we discuss how to best educate students in calculus. The conversations range from very specific issues on a single type of problem, to general philosophies such as graphing and communication. I like to learn the differences in college teaching versus high school teaching. And, so, I like to meet high school teachers and learn from them the issues and challenges as well as the successes they face.” A high school teacher who was interviewed said, “our table developed good camaraderie, we all have a deep sincere interest in the topic; I felt like a closeness to everybody.”

**Learning and Exchanging Ideas**

Participants described experiencing deeper mathematical understanding and feeling more able to teach their students well. The interchange of ideas, teaching strategies, stories of failures and successes in the classroom, are all highly valued. One participant reported that “the best professional development is interaction with other readers. I learn things that they have tried, and how effective they were, and I'm able to bounce ideas off of other people in a larger context than my own small college department. Hearing about what other teachers consider important is useful. I have been teaching a long time; and, so, I have pretty strong opinions; but, it is always good to hear what others think. I have especially been stimulated by the slope field and differential equations questions and scoring rubrics." Another stated, "there is no other more authentic activity that an AP Calculus teacher can attend to gain specific, detailed information on how to prepare students for the exam. I say all the time that the reading is by far the best professional development experience there is. You get to interact with hundreds of other Calculus teachers at every point in time." Another high school teacher commented, "I get outside my own teaching bubble and learn what others are doing around the country, where my classes are stronger, where we can improve. It is also a place to learn the political dynamics around the country and reflect on how education everywhere is dealing with over-testing and government interference. Makes me feel like I'm not alone and strengthens my fight against it. Many of us stay connected beyond the reading as well so this dialogue continues."

**Insights into Student Thinking**

Analyzing abundant student work in concert with colleagues allowed readers to see both the differing approaches which some students make in their solutions as well as errors which student commit. These expand the teachers' cache of instructional tools and helps them to reach more students. One high school teacher wrote, "I also learn from the students. Often, they will present solutions that are much better than what I originally did to work the problem. I think I have learned a lot about calculus as a result of looking at thousands of papers and working with other calculus teachers. Back at home at work, I don't have anyone to discuss calculus with, so it is nice to have that at the reading.” A college professor commented, “There have been some topics that I was surprised to find are not a mandatory part of the AP curriculum, such as L'Hopital's rule for AB calculus or all of the different forms of partial fraction decomposition for BC calculus. This helps me to understand, not only some of the quirks in the ways that students approach the problems on the exam, but also some of the things that I see them doing in my classes.”

**Interaction with Educators at Different Levels**

Past president of NCTM, Gail Burrill stated that there are significant differences between teaching mathematics in high school and in college; and, she advocated for dialogue between teachers and professors to make high school teachers aware of the culture and confines of the college system as well as informing professors of incoming students’ experiences and expectations for mathematics classes” (as cited in Bressoud, 2017).

At the exam reading, high school teachers and college professors both acknowledged the value of interacting with mathematics educators, calculus educators in particular, at differing levels. High school teachers are concerned about properly preparing their students for college calculus classes. Professors find insight into their freshman students’ thinking and what is and is not taught in high school. One professor wrote, “as a college professor, I learned important ideas from the high school teachers. I gained greater respect for the high school teachers and had new insights regarding the backgrounds of some of the students who had been in my college calculus classes.” And, a high school teacher wrote, "it is helpful to communicate with other high school teachers as we use the opportunities to share ideas and personal experiences in the classroom. Interacting with college professors helps to get perspective on the future math courses for my students."

### Formal and Informal Evening Sessions

The optional evening sessions have a range of content. One session may be a question and answer session with members of the curriculum development committee; another could be a presentation on instructional strategies, or a lecture from a well-known author. Still others may be purely intended as entertainment. Even so, approximately 50% of the survey participants do not attend any of these sessions. The most prevalent reasons for non-attendance at these optional meetings are fatigue after a long day of reading, other interests such as socializing or exercising, and/or lack of interest. The participants who attend the evening sessions praised the worth and variety of these sessions, but only if the topics of the session were important or interesting to the individual. As one high school teacher interviewee said, "I think that they are valuable, but, because I don't see a lot of variation from year to year, so I think their value drops off after a couple of years." A college professor noted," they are occasionally really good, depends on who it is. I haven't attended all of them. I've attended some good ones and I've attended some that really were not all that valuable. It's like any talk, you just don't know."

College professors frequently asserted that the evening professional development sessions were both aimed at, as well as most beneficial for, high school teachers. They cited the high school teachers' interest in guiding their students to passing the AP Calculus exam, a concern which is not shared by college professors.

### Research Question 2: Conclusions

The second research question asked how participation in the AP Calculus national exam reading affects educators' classroom practices. Educators who participate in these readings talk about the exchange of ideas with colleagues, the new strategies which they learn and the excitement they feel in anticipation of incorporating new approaches in their classrooms. High school teachers are much more likely to make changes to their classroom practices than are college professors. Specifically, high school teachers focus on three main areas: conceptualization, connections, and idiosyncrasies of the exam, which is referred to as "exam game-play." Teachers aim for their students to not only understand the concepts but also be able to explain their work with appropriate justifications, and to see the links among concepts. High school teachers also place high value on being able to share the idiosyncrasies of exam grading to their students. This can both save the student a lot of unnecessary work, e.g., simplification of numeric answers is not required, as well as inform the students of grading fine points so that they can make simple adjustments to their work to maximize the points awarded.

### Emphasis of Meaning and Concepts

Educators generally reported, in varying degrees, that their AP Calculus reading experiences had led them to concentrate on having students conceptualize ideas, not just run through calculations. Teachers employ several strategies to accomplish this goal, including writing justifications, having students explain to classmates, giving real-world examples, using FRQs in classroom discussions, and using strategies and materials which were shared with them at the reading

**Knowing the Idiosyncrasies of the Exam**

Linkage errors occur when a student, going from one step to another in a calculation, mistakenly uses an equal sign when it is incorrect to do so. For example, a student may be computing $sin\left(\frac{\pi}{3}\right)+2$ and write $sin\left(\frac{\pi}{3}\right)=\frac{1}{2}+2=\frac{5}{2}$. Because this is not true, $sin\left(\frac{\pi}{3}\right)$ does not equal $\frac{1}{2}+2$, the student would be docked points. However, AP Calculus teachers who are aware of this will instruct their students to use arrows instead of an equals sign, to show work in progress. So, $sin\left(\frac{\pi}{3}\right)\rightarrow\frac{1}{2}+2=\frac{5}{2}$ would be marked as correct.

The chronic issue of the use of unspecific terms such as "it," "the function," or "the derivative" are well known to experienced AP Calculus readers. And, they frequently reported continually addressing this issue with their students. One teacher reported that the word "it" was treated as a swear word in her class. Students are told to be specific and refer to $f(x)$ or $f'(x)$, repeatedly, if necessary. For example, an appropriate justification for a relative minimum of a continuous and differentiable function, $f(x)$, at $x=c$ would include an explanation that $f'(x)$ changes from negative to positive as $f(x)$ passes through at $f(c)$ and that $f'(c)=0.$ No "its" or "the function" or "the derivative" are used.

Use of proper vocabulary is also stressed. A student who states that a rate of change is decreasing at negative 10 feet per second is actually stating that the rate is increasing. Teachers report that they instruct their students to state that a rate is *changing* at negative 10 feet per second or decreasing at 10 feet per second. Decreasing and negative are not to be used together to describe a rate of change.

Another common example is the inclusion of the differential *dx* or *dy* in integration. Omitting it or placing it incorrectly can cause a student to be penalized on the exam. Also, students who erroneously interchange these differentials will undoubtedly lead themselves to calculus confusion and be unable to compute a solution.

**Research Question 3: Conclusions**

The third research question asked how participation in an AP Calculus national exam reading by educators affect their students' understanding. College professors noted that, in general, they would have no knowledge of which of their freshmen students had taken AP Calculus. However, many of them valued the knowledge they gained at the reading about the content of the AP Calculus curriculum. One professor summed up the opinions of many by the statement:

> I do think that I have gotten more in touch with where a big part of my student population is coming from. And, I think it's helped me to understand some of the things they do that maybe surprised me initially the first few years when I am teaching. You know, certain details specific to the AP Calculus exam that their teachers have drilled into them as things to do to make sure that you don't lose a point on the AP test, right? So, I can see how some of the AP rubrics and evaluation standards have shaped some of the things that my students are doing and I feel like it's made it easier for me to communicate with them at times about what they are doing and what I am doing.

High school teachers, on the other hand, have an immense stake in the success of their AP Calculus students. They value the AP Calculus reading experiences not only for the benefit of their students but for their own understanding as well. High school teachers described improvements in their own deepening mathematical understanding almost as frequently as they mentioned their students increased understanding. Teachers also described the importance of knowing the exam structure and the "tricks" of the trade; the importance of clear, precise written justifications; and, the ability to connect concepts. These teachers have insights with the exam grading which are advantageous to their students. Again, readers would know that functions need to be identified specifically as $f(x)$ or $g(x)$ and that the first derivative of $f(x)$ needs to be identified by $f'(x)$, and the second derivative of $g(x)$ needs to be identified by $g''(x)$, and so on. Using the words "it," "the function," or "the derivative" lacks specificity and the student will lose points on the free response questions for using such generalities.

## Overall Study Conclusions

Conclusion 1: The overwhelming majority of people who serve as AP Calculus exam readers value the collegial interactions more than any other aspect of the reading experience.

Conclusion 2: The most valued professional development occurs during conversations with fellow readers and table leaders, specifically at the reading tables and during meals.

Conclusion 3: The value of the professional development at the reading is far more important than any monetary compensation. Although no readers indicated that they would be willing to participate in the reading for less compensation, the stipend was viewed as a much less motivating factor than the interactions among colleagues.

Conclusion 4: Those who disagreed with changes occurring in their classroom practice tended to also disagree with changes in their students understanding; conversely, those who reported that they had implemented changes in their classroom practices tended to also report changes in their students' understanding of calculus.

Conclusion 5: Use of former AP Calculus Free Response Questions (FRQs) in the classroom is associated with high school teachers' beliefs that their students' understanding of Calculus has increased.

Conclusion 6: High school teachers value the insights they gain about the idiosyncrasies of the exam grading. Although they are eager to share these insights with their students, they find the most value in the mathematical precision which is, or is not, required. Reducing fractions is not required; radicals can be left in denominators. Students are taught not to waste exam time simplifying answers, nor to risk the possibility of making an error when doing so. Presumably, students in an AP Calculus class would be skilled at these less advanced procedures. However, being clear and precise in justifications, using the "equals" sign correctly, and referring to functions by their specific names, are all desired characteristics of correct mathematical work.

The AP Calculus exam reading is a behemoth grading endeavor. Yet, participants see beyond the goal of grading several hundred thousand exams in seven days. What happens in concert with those folders of exams and bubble grading sheets are the invaluable interactions with other educators, specifically Calculus teachers, which lead to broader and deeper understanding of mathematics. These educators, at the high school level, believe that the level of sharing of ideas, learning from each other, and gaining new perspectives from their colleagues leads to higher levels of understanding and achievement in their students.

The degree of positive collaboration, the scope of mathematical concepts, the longevity of repeated and consistent interaction, and the enduring personal and professional relationships among educators, which are abundant at the reading, are not found in traditional professional development, according to the survey participants. A large number of respondents indicated that they valued the professional development, in its different forms, very highly. One high school teacher stated that he found the professional development at the reading to be “the best professional development of my career;” a college professor stated that the reading was “better than any other professional development” which she had experienced. The reasons for these laudatory comments are varied, but fall into four overlapping categories: social, learning and exchanging ideas, insights into student thinking, and interaction with educators at different levels.

**Development of Working Grounded Theory**

Desimone (2011) presents a model of effective professional development for teachers. She holds that the core features are a focus on content, active learning, coherence, duration, and collective participation. She argues that professional development which contains these key features will lead to more skilled teachers and adjustments to instruction strategies which result in improved learning for the student. See Figure 6.

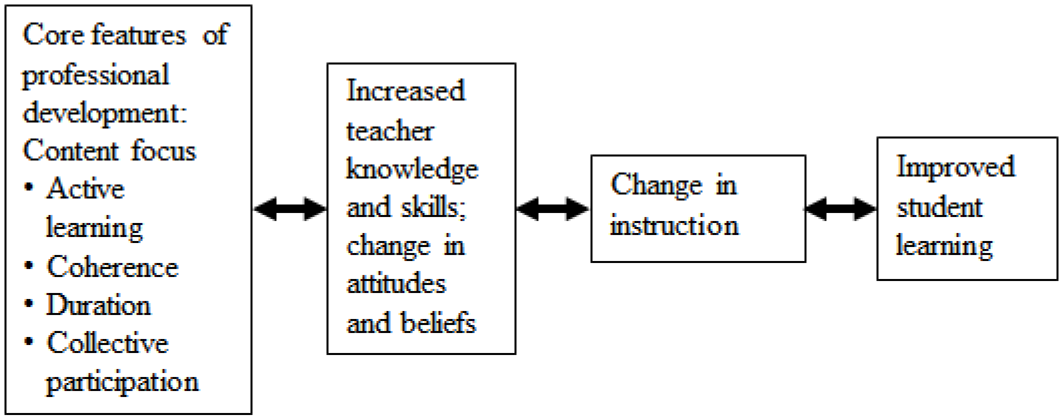

*Figure 6.* Desimone's Model of Effective Professional Development.

All of this model's requirements are present at the AP Calculus reading. And, most of the readers report changes in their own knowledge, both mathematical knowledge and awareness of differing student strategies. From there, high school teachers (readers) widely report making alterations to their classroom practices to increase their students' conceptual understanding, ability to connect ideas, and ability to successfully navigate the AP Calculus exam. College professors, while they do not generally make significant changes to their classroom practice, benefit from interaction with high school teachers, from knowing the content of the exams, and working with established grading rubrics.

The structural requirements for an "authentic professional community," according to Hord (2015), includes (1) set meeting times; (2) supportive relational conditions, mutual respect, collegial conflict resolution; (3) shared values and vision; (4) intentional collective learning; (5) peers supporting peers; and, (6) shared and supportive leadership. Although her descriptors were intended for educators at geographically proximate institutions, and her intent was weekly meetings, the reasoning behind her model can be applied to the AP reading. The possible exception is the shared leadership idea. Although there are limited opportunities for readers to be promoted to table leader, most readers do not receive a promotion. However, at the tables, readers can certainly take on a leadership role in assisting other readers. And, as Hord (2015) concludes, the professional learning community helps educators develop new skills and to "grow in competence and confidence and in developing trust in each other to become true professionals" (p. 39).

Using Desimone's model as the foundation, and incorporating the Ball, Thames and Phelps (2008, 403) model for mathematical knowledge for teaching, a working theory for the phenomena of the AP Calculus reading has been developed. This theory would apply only to those teachers who are willing to commit over a week of their time, be away from their families, stay in hotels with a roommate, work a regimented schedule from 8:00-5:00 each day, including Saturday and Sunday. Clearly, not all teachers would be willing to do so; their reticence, no matter the reason, would most likely affect their attitudes about the experience. The model is predicated upon the teacher desiring to attend the reading and taking action to apply for a reader position. However, for those teachers who would volunteer, if they knew what the reading entailed, the theoretical model for the AP Calculus reading experience is as follows. See Figure 7.

**Horizon Content Knowledge (HCK) Exemplars**

College professor frequently commented on the value of knowing what students are being taught in high school. Even though many college topics are, of course, not

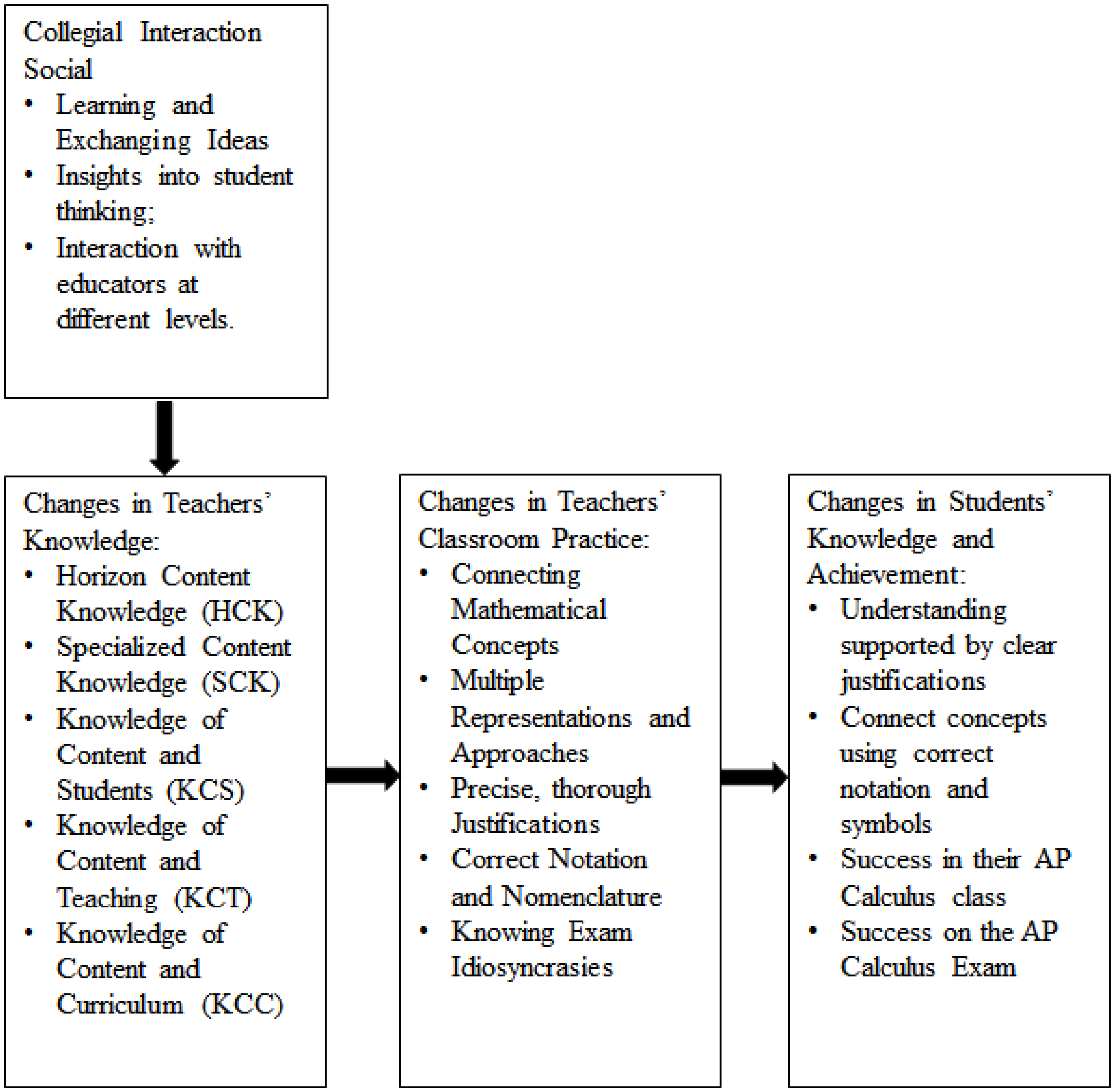

*Figure 7.* Working Grounded Theory Model of Professional Development at AP Calculus Exam Readings.

covered in high school, the groundwork which has been laid gives college professors a platform on which to build their students' knowledge. For example, Fourier series are not included in the AP Calculus curriculum, but BC students are exposed to Geometric Series and Taylor series; so, there is understanding of series with which to work. Partial differential equations are not included the AP Calculus curricula; however, differential equations and slope fields are included. AP Calculus students know how to solve separable differential equations and they can create a slope field for non-separable differential equations. Their future calculus studies can be built on that knowledge and skill set.

Both high school teachers and college professors frequently commented on their increased awareness of the relationships of mathematical concepts in the high school and college curricula. A college Professor stated, "I think that when I am teaching classes I'm honestly more focused on where the students are going to be in the next semester and what they need to be prepared for. I think then that things that I've picked up at the reading gives me those ideas and strategies I'll need." A high school teacher said, "now I feel that I am an established piece in a chain of knowledge from AP central all the way to the minds my students. I'm kind of like, before going to the reading, I was doing okay, but now I am a better instructor for having seen how rigorous certain parts of the FRQs need to be carried out. So, from AP central through the course description to me, with the help of the textbook and a few other materials I use along the way, goes the knowledge of calculus. And, thankfully, I'm in a position to do, leading students who aspire to do great things."

### Specialized Content Knowledge (SCK) Exemplars

Educators were often surprised, despite their advanced degrees and years of teaching experience, that they learned mathematical content or deepened their understanding (SCK) at the reading. One professor mused, "I think through going to the reading I myself have taught the theory deeper and have experimented with different ways to get them to understand the theory, and, therefore, yes, I think I have more students understanding theory better than before. I think I had more students more procedural before and I think I am getting to them the theory/concepts better." A high school teacher focused of his new-found ability to examine different approaches to a solution. He said,

> in terms of professional development, I've never been to anything better than this. I'm spending hours upon hours sifting through massive amounts of answers to questions; and I'm finding out how many different ways students would view a particular question. And, I'm also looking at what is acceptable in terms of answering the question and what is not. And, I think, sometimes to sit in with my classes now, and say, hey, look at this particular way of solving this. Is this accurate? Is this not accurate? And, so we do some great discussions inside the classroom as well. I can't get that from listening to a speaker, I can't get that from sitting down and listening to a webinar. That comes from hands-on experience; so, those hours of going through grading papers and talking to people who were around me, I think that is what makes this PD so unique and makes it so powerful.

**Knowledge of Content and Students (KCS) Exemplars**

At the question briefings, common errors and unusual approaches to solutions are discussed in great detail. These discussions continue throughout the week. The work of a student who has used degrees instead of radians is easy to decipher. However, when a reader encounters student work which he does not understand, consulting the reading partner may clarify the problem. If not, the table leader is consulted. In either event, the reader gains knowledge about how a student is thinking (KCS) and attempting to solve the problem at hand. One educator stated,

> I think because there are so many of the questions with the definite integrals and the Riemann sums, especially the Riemann sums and I teach those differently than I have been doing in the past. So, I feel that my understanding, and there are so many of them, has become much better. And, that makes me teach it better."

Another educator remarked,

> I am endlessly fascinated with the different ways that students can think about mathematics. That's one of the things I love about the AP reading, when you just see the infinite variations on the ways students can make sense of and manage to make nonsense of mathematics situations, I just find it endlessly fascinating. So, even you've read that problem for a whole day, it's kind of laborious to go back, you feel like you've seen every variation possible, I just love to be able to see these different ways. As you are grading that particular problem, you can basically characterize the variety into a number of different kind of pots of common mistakes that people are making and so being able to think through 'why are they making this, what's behind them doing it this way?' I really like that. And, I don't really think the reading takes advantage of that very well. I'm guessing that there could be a lot of people for whom those things would go right on by, I think if they were to focus during the debriefing a little less on the way the problems were scored so you could help students do well on the tests, and a little more on looking at what this tells us about how kids think about this piece of mathematics. I think that could be a lot more educative.

**Knowledge of Content and Teaching (KCT) Exemplars**

Examples of common errors on exam questions include the improper use of the quotient rule when finding the derivative of a function, omitting factors when using the chain rule, incorrect equations when attempting a related rates problem, and elementary algebra errors. Readers commented that, while these mistakes made them sad that students lost points for such avoidable errors, they were focused on ensuring that their classroom practice would include emphasis on these areas so that their students did not make the same mistakes. Readers commented:

> Communicating to my students, trying to communicate to them the importance of the language of mathematics, and the precision with which that language is spoken, and the necessity to keep it precise, those types of things, have been enhanced or changed, impacted by my time at the AP reading.
>
> Because the AP exam is very focused on communicating properly, I'm forcing my students to learn it in such a way that they can communicate, that they actually understand through explanation, through writing, and then in different forms, graphically and numerically, and because I've been taught through the reading that they need to demonstrate in different ways, I feel like they are just learning it better. And, I am not teaching it the one straight way that I know when I first started teaching it that I was teaching it … and through the different forms of their learning how to communicate it, I think they're learning it better.

**Knowledge of Content and Curriculum (KCC) Exemplars**

In an interview, a professor said, "I think the knowledge of (the high school) curriculum is probably the biggest thing that I learned. For me, going from graduate student to full time assistant, now associate professor, as what does the world see as the important things in Calculus? I think what I got out of the AP reading is a sense of knowing what direction Calculus pedagogy is going and what we can do to help here with that." Noting that high school teachers were probably more able and apt to change curricula, another professor said, "I think that for people who may change the curriculum or change the way they teach or have the AP reading affect them more in that sort of way of are probably more on the high school side, because I think they get a bigger sort of bang for the buck out if this. I think it's a bigger professional development opportunity for them. But, I think it's invaluable for the college professor side of things to see what's happening in the high schools. I think it serves a sense of community in that sort of way."

## Implications for Further Research

While high school teachers report increased self-confidence as an AP teacher and their perceptions of improved ability to prepare their students for the AP exam, the question of effect on actual AP exam scores remains unanswered. A quantitative analysis could be used to determine if the distribution of AP Calculus exam scores for students of experienced AP Calculus exam readers is different from the distribution of AP Calculus exam scores for students of teachers who do not read the exams. However, obtaining the AP scores is problematic for a number of legal reasons and privacy concerns. Nonetheless, the observations of dedicated high school AP Calculus teachers who serve as readers must be seen as valuable data. And, they have noted, there are inextricably intertwined connections between their own experiences at the exam readings, the information and procedures they pass on to their students, and their perceptions of changes in their students mathematical understanding. Are the AP Calculus exam scores of students of exam readers different from the exam scores of students whose teachers are not exam readers?

This study has delved into teachers’ perceptions of changes in student understanding. What other methods could be researched to further the study of student knowledge? For example, if teachers who are not AP exam readers use the former AP FRQs in their classrooms, how does that influence ISL? What are the students’ viewpoints? Do teachers who are not exam readers also put emphasis on conceptualization, connections among concepts, written and verbal justifications, and, precise notation and units in context?

Another question which bears investigation is how the formal and informal evening sessions at the AP reading could be designed to entice more attendance or to appeal to a wider audience? Because the repetitiveness and/or lack of personal relevance of the sessions was pointed out in the survey, investigating what topics would be appealing and germane to classroom teaching could be enlightening. College professors comment that the sessions are aimed at high school teachers. How could that be addressed?

Charmaz (2006) states that ethnography is "recording the life of a particular group and thus entails sustained participation and observation in their milieu, community, or social world. It means more than participant observation alone because an ethnographic study covers the round of life occurring within the given milieu (*x*) and often includes supplementary data from documents, diagrams, maps, photographs, and, occasions (*y*), formal interviews and questionnaires" (p. 21). An ethnographic study of AP Calculus exam readers may provide a wealth of information on their pedagogical knowledge and their motivations to increase the positive ISL in their classrooms. It may also delineate characteristics which separate them from the population of Calculus educators who do not read the exams.

Lastly, an exploration of which components of the different elements of mathematical knowledge, both content and pedagogical, are being most impactful in classroom and why would be educative. This could be measured in how the resultant ISL correlates to college preparedness and success in college calculus classes?

**Concluding Remarks**

Because the participants in this generally hold positive viewpoints on most facets of the AP Calculus exam reading, statistically significant results were few. The participants in the study volunteered to complete the survey and the interviewees were all volunteers from the same pool. So, the population, as well as the study sample, is comprised of people who want to be at the reading. In the interviews, every educator was enthusiastic and deeply thoughtful. They frequently asked for clarification to ensure they were understanding my questions clearly. The free offering of their thoughts and experiences was similar to the collegial interactions at the readings which they described.

Although the reading is mentally exhausting work, readers do not view it as burdensome. The benefits of participation are seen as far out-weighing temporary inconveniences.

However, the rich commentary collected in both the surveys and the interviews shows that those who participate in the AP Calculus readings: (1) develop a wider and deeper breadth of mathematical knowledge, (2) learn about student thinking and strategizing from analysis of bountiful exemplars, (3) learn from the student work they analyze, and (4) share ideas and strategies with their students, particularly justifications, precise notation, and conceptual connections, leading to increased ISL of calculus.

Some have argued that all AP Calculus teachers should be required to be AP readers for at least one year. Reasons cited are the insights into exam grading, the growth in specialized content knowledge for the teacher, and the changes in classroom practice which would benefit students. While this study certainly supports those gains, there are a number of reasons why that generalization is not sound. There may be financial and logistic difficulties for districts in having teachers absent for over a week at the end of the school year. But, beyond the plethora of possible personal reasons, it must be recognized that the population of AP Calculus readers is different from the population of all AP Calculus high school teachers and college mathematics professors. All study participants have the common trait of wanting to be an AP reader. Not all teachers have that interest.

However, for those mathematics educators who are interested in the benefits of the reading and are willing to undertake the regimented daily routine, the AP Calculus exam reading experience is an excellent pathway to increasing content knowledge, finding new teaching strategies, making connections with colleagues worldwide, and becoming a teacher who is better equipped to help Calculus students understand and succeed.

# Appendix A

Institutional Review Board Approval

**Office of Research Integrity and Assurance**

Research Hall, 4400 University Drive, MS 6D5, Fairfax, Virginia 22030
Phone: 703-993-5445; Fax: 703-993-9590

| | |
|---|---|
| DATE: | July 15, 2015 |
| TO: | Jennifer Suh, Ph.D. |
| FROM: | George Mason University IRB |
| Project Title: | [547964-1] AP Calculus Exam Readers' Perceptions of the Effects of Their Reading Participation on Their Classroom Practices and Their Students' Calculus Understanding |
| SUBMISSION TYPE: | New Project |
| ACTION: | DETERMINATION OF EXEMPT STATUS |
| DECISION DATE: | July 15, 2015 |
| REVIEW CATEGORY: | Exemption category #2 |

Thank you for your submission of New Project materials for this project. The Office of Research Integrity & Assurance (ORIA) has determined this project is EXEMPT FROM IRB REVIEW according to federal regulations.

Please remember that all research must be conducted as described in the submitted materials.

Please note that any revision to previously approved materials must be submitted to the ORIA prior to initiation. Please use the appropriate revision forms for this procedure.

If you have any questions, please contact Karen Motsinger at 703-993-4208 or kmotsing@gmu.edu. Please include your project title and reference number in all correspondence with this committee.

This letter has been electronically signed in accordance with all applicable regulations, and a copy is retained within George Mason University IRB's records.

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**Appendix B**

Current Advanced Placement Courses

**Arts**

Art History
Music Theory
Studio Art 2-D Design
Studio Art 3-D Design
Studio Art Drawing

**English**

English Language and Composition
English Literature and Composition

**History and Social Sciences**

Comparative Government and Politics
European History
Human Geography
Macroeconomics
Microeconomics
Psychology
United States Government and Politics
United States History
World History

**STEM**

Biology
Calculus AB
Calculus BC
Chemistry
Computer Science Principles
Computer Science A
Environmental Science
Physics 1
Physics 2
Physics C: Electricity and Magnetism
Physics C: Mechanics
Statistics

**World Languages and Cultures**

Chinese Language and Culture
French Language and Culture
German Language and Culture
Italian Language and Culture
Japanese Language and Culture
Latin
Spanish Language and Culture
Spanish Literature and Culture

**Appendix C**

Schedule for the 2016 AP Exam Reading

| Location | Dates | Disciplines |
|---|---|---|
| Cincinnati, OH | | |
| Week 1 | Arrival: Wed, 01 June<br>Reading: 02-08 June<br>Departure: Thurs, 09 June | Environmental Science<br>Human Geography<br>Macroeconomics<br>Microeconomics |
| Week 2 | Arrival: Sat, 11 June<br>Reading: 12-18 June<br>Departure: Sun, 19 June | French Language and Culture<br>German Language and Culture<br>Italian Language and Culture<br>Music Theory<br>Spanish Language and Culture |
| Kansas City, MO | | |
| Week 1 | Arrival: Wed, 01 June<br>Reading: 02-08 June<br>Departure: Thurs, 09 June | Calculus<br>European History<br>Physics |
| Week 2 | Arrival: Fri, 10 June<br>Reading: 11-17 June<br>Departure: Sat, 18 June | Biology<br>Computer Science<br>English Language and Composition<br>Statistics |
| Louisville, KY | | |
| Week 1 | Arrival: Tues, 31 May<br>Reading: 01- 07 June<br>Departure: Wed, 08 June | United States History |

| | | |
|---|---|---|
| Week 2 | Arrival: Thurs, 09 June<br>Reading: 10-16 June<br>Departure: Fri, 17 June | English Literature and Composition<br>Latin<br>Psychology<br>Spanish Literature and Culture |
| Salt Lake City, UT | | |
| Week 1 | Arrival: Thurs, 02 June<br>Reading: 03-09 June<br>Departure: Fri, 10 June | Art History<br>Chemistry<br>Chinese Language and Culture<br>Comparative Government and Politics<br>Japanese Language and Culture<br>Studio Art<br>United States Government and Politics<br>World History |

*Note:* [a]Salt Lake City hosts AP readings for only Week 1.

**Appendix D**

Question Topics on AP Calculus Exams, 2013-2016

2013 Question Topics

| Question Number | Question Topics |
|---|---|
| Question 1 | |
| AB1/BC1 | Rates of change; accumulation; maximum |
| Question 2 | |
| AB-2 | Particle position, direction, acceleration |
| BC-2 | Area of polar region; position vector |
| Question 3 | |
| AB-3/BC-3 | Midpoint sum |
| Question 4 | |
| AB-4/BC-4 | Global/local extrema; concavity; slope of tangent line |
| Question 5 | |
| AB-5 | Area between two curves, volume of solid of revolution; volume of solid with known cross sections |
| BC-5 | Differential equation solution; Euler's Method |
| Question 6 | |
| AB-6 | Differential equation solution; tangent line approximation |
| BC-6 | Taylor polynomial |

2014 Question Topics

| Question Number | Question Topics |
|---|---|
| Question 1 | |
| AB1/BC1 | Rates of change, average value, linear approximation |
| Question 2 | |
| AB-2 | Volume of a solid of rotation; volume of solid with known cross-sections |
| BC-2 | Area of polar region; rate of change of distance |
| Question 3 | |
| AB-3/BC-3 | Graph analysis; compound functions, concavity, tangency |
| Question 4 | |
| AB-4/BC-4 | Intermediate Value Theorem; implicit differentiation; trapezoid sum |
| Question 5 | |
| AB-5 | Fundamental Theorem of Calculus; Mean Value Theorem |
| BC-5 | Volume of solid of rotation, perimeter of curved planar region |
| Question 6 | |
| AB-6 | Slope field; equation of particular solution curve |
| BC-6 | Taylor polynomial; radius of convergence |

2015 Question Topics

| Question Number | Question Topics |
|---|---|
| Question 1 | |
| AB1/BC1 | Rates of change, accumulation, minimum |
| Question 2 | |
| AB-2 | Area between curves; volume of solid with known cross-sections; rate of vertical change |
| BC-2 | Parametric equations; finding coordinates; find time for a particular tangent line slope; find time for particular speed; total distance |
| Question 3 | |
| AB-3/BC-3 | Velocity approximation, Riemann sum, acceleration and average velocity. |
| Question 4 | |
| AB-4/BC-4 | Slope field, second derivative and concavity; particular solution for a differential equation |
| Question 5 | |
| AB-5 | Relative maximum; concavity and decreasing intervals; points of inflection; integral expressions |
| BC-5 | Partial fraction decomposition; relative extrema; tangency; critical points |
| Question 6 | |
| AB-6 | Tangency, vertical tangents, evaluation of second derivative |
| BC-6 | Maclaurin series; radius of convergence |

2016 Question Topics

| Question Number | Question Topics |
|---|---|
| Question 1 | |
| AB1/BC1 | Rates of change, accumulation, Riemann sum |
| Question 2 | |
| AB-2 | AB-2: Particle position, direction, acceleration, distance |
| BC-2 | Parametric equations; particle position, tangent line slope; find time for particular speed; total distance |
| Question 3 | |
| AB-3/BC-3 | Relative extrema, points of inflection, absolute extrema |
| Question 4 | |
| AB-4 | Slope field, tangent line approximation; particular solution for a differential equation |
| BC-4 | Differential equation, Euler's Method |
| Question 5 | |
| AB-5/BC-5 | Average value, volume, related rates |
| Question 6 | |
| AB-6 | Tabular data, tangent line, rate of change of composite function, integration of second derivative |
| BC-6 | Taylor series; radius of convergence, function value approximation |

**Appendix E**

U. S. News and World Report 2016 List of Top 100 Universities in the United States

| Rank | Name | Credit for AP Calculus | Minimum Score AB | Minimum Score BC | Allows placement |
|---|---|---|---|---|---|
| 1 | Princeton University, Princeton, NJ | YES | 5 | 4 | YES |
| 2 | Harvard University, Cambridge, MA | YES | NA | 5 | YES |
| 3 | Yale University, New Haven, CT | YES | 5 | 4 | YES |
| 4[a] | Columbia University, New York, NY | YES | 4 | 4 | YES |
| 4[a] | Stanford University, Stanford, CA | YES | 4 | 3 | YES |
| 4[a] | University of Chicago, Chicago, IL | YES | 5 | 4 | YES |
| 7 | Massachusetts Institute of Technology, Cambridge, MA | YES | NA | 5 | YES |
| 8 | Duke University, Durham, NC | YES | 5 | 3 | YES |
| 9 | University of Pennsylvania, Philadelphia, PA | YES | NA | 5 | NO |
| 10[a] | California Institute of Technology, Pasadena, CA | NO | NA | NA | YES |
| 10[a] | Johns Hopkins University, Baltimore, MD | YES | 5 | 3 | YES |
| 12[a] | Dartmouth College, Hanover, NH | NO | NA | NA | YES |
| 12[a] | Northwestern University, Evanston, IL | YES | 4 | 3 | YES |
| 14 | Brown University, Providence, RI | NO | NA | NA | YES |
| 15[a] | Cornell University, Ithaca, NY | YES | 4 | 4 | YES |
| 15[a] | Vanderbilt University, Nashville, TN | YES | 5 | 4 | YES |
| 15[a] | Washington University in St. Louis, St. Louis, MO | YES | 4 | 4 | YES |
| 18[a] | Rice University, Houston, TX | YES | 4 | 4 | YES |
| 18[a] | University of Notre Dame, Notre Dame, IN | YES | 5 | 5 | YES |
| 20 | University of California at Berkeley, Berkeley, CA | YES | 3 | 3 | YES |
| 21[a] | Emory University, Atlanta, GA | YES | 4 | 4 | YES |
| 21[a] | Georgetown University, Washington, DC | YES | 4 | 4 | YES |
| 23[a] | Carnegie Mellon University, Pittsburgh, PA | YES | 5 | 4 | YES |
| 23[a] | University of California at Los Angeles, Los Angeles, CA | YES | 5 | 4 | YES |

| | | | | | |
|---|---|---|---|---|---|
| 23[a] | University of Southern California, Los Angeles, CA | YES | 3 | 3 | YES |
| 26 | University of Virginia, Charlottesville, VA | YES | 4 | 4 | YES |
| 27[a] | Tufts University, Medford, MA | YES | 4 | 3 | YES |
| 27[a] | Wake Forest University, Winston-Salem, NC | YES | 4 | 3 | YES |
| 29 | University of Michigan at Ann Arbor, Ann Arbor, MI | YES | 4 | 4 | YES |
| 30[a] | Boston College, Chestnut Hill, MA | NO | NA | NA | YES |
| 30[a] | University of N. Carolina at Chapel Hill, Chapel Hill, NC | YES | 3 | 3 | YES |
| 32 | New York University, New York, NY | YES | 4 | 4 | YES |
| 33 | University of Rochester, Rochester, NY | YES | 4 | 3 | YES |
| 34[a] | Brandeis University, Waltham, MA | YES | 4 | 3 | YES |
| 34[a] | College of William and Mary, Williamsburg, VA | YES | 4 | 3 | YES |
| 36 | Georgia Institute of Technology, Atlanta, GA | YES | 4 | 4 | YES |
| 37[a] | Case Western Reserve University, Cleveland, OH | YES | 4 | 3 | YES |
| 37[a] | University of California at SB, Santa Barbara, CA | YES | 3 | 3 | YES |
| 39[a] | University of California at Irvine, Irvine, CA | YES | 3 | 3 | YES |
| 39[a] | University of California at San Diego, La Jolla, CA | YES | 3 | 3 | YES |
| 41[a] | Boston University, Boston, MA | YES | 4 | 4 | YES |
| 41[a] | Rensselaer Polytechnic Institute, Troy, NY | YES | 4 | 3 | YES |
| 41[a] | Tulane University, New Orleans, LA | YES | 4 | 3 | YES |
| 41[a] | University of California at Davis, Davis, CA | YES | 3 | 3 | YES |
| 41[a] | University of Illinois at Urbana-Cham., Champaign, IL | YES | 4 | 4 | YES |
| 41[a] | University of Wisconsin at Madison, Madison, WI | YES | 3 | 3 | YES |
| 47[a] | Lehigh University, Bethlehem, PA | YES | 4 | 4 | YES |
| 47[a] | Northeastern University, Boston, MA | YES | 4 | 4 | YES |
| 47[a] | Penn State University at UP, University Park, PA | YES | 4 | 3 | YES |
| 47[a] | University of Florida, Gainesville, FL | YES | 3 | 3 | YES |
| 51 | University of Miami, Coral Gables, FL | YES | 5 | 4 | YES |
| 52[a] | Ohio State University at Columbus, Columbus, OH | YES | 3 | 3 | YES |
| 52[a] | Pepperdine University, Malibu, CA | YES | 3 | 3 | YES |
| 52[a] | University of Texas at Austin, Austin, TX | YES | 3 | 3 | YES |
| 52[a] | University of Washington, Seattle, WA | YES | 3 | 3 | YES |
| 52[a] | Yeshiva University, New York, NY | YES | 4 | 4 | NO |

| | | | | | |
|---|---|---|---|---|---|
| 57[a] | George Washington University, Washington, DC | YES | 4 | 4 | YES |
| 57[a] | University of Connecticut, Storrs, CT | YES | 4 | 3 | YES |
| 57[a] | University of Maryland at College Park, College Park, MD | YES | 4 | 4 | YES |
| 57[a] | Worcester Polytechnic Institute, Worcester, MA | YES | 4 | 4 | YES |
| 61[a] | Clemson University, Clemson, SC | YES | 3 | 3 | NO |
| 61[a] | Purdue University at West Lafayette, West Lafayette, IN | YES | 3 | 3 | YES |
| 61[a] | Southern Methodist University, Dallas, TX | YES | 4 | 3 | YES |
| 61[a] | Syracuse University, Syracuse, NY | YES | 4 | 4 | YES |
| 61[a] | University of Georgia, Athens, GA | YES | 3 | 3 | NO |
| 66[a] | Brigham Young University at Provo, Provo, UT | YES | 3 | 3 | NO |
| 66[a] | Fordham University, New York, NY | YES | 4 | 4 | YES |
| 66[a] | University of Pittsburgh, Pittsburgh, PA | YES | 4 | 4 | YES |
| 69 | University of Minn. at Twin Cities, Minneapolis, MN | YES | 3 | 1[b] | NO |
| 70[a] | Texas A&M University at C. St., College Station, TX | YES | 4 | 4 | YES |
| 70[a] | Virginia Tech, Blacksburg, VA | YES | 3 | 3 | YES |
| 72[a] | American University, Washington, DC | YES | 4 | 4 | YES |
| 72[a] | Baylor University, Waco, TX | YES | 4 | 3 | YES |
| 72[a] | Rutgers, Piscataway, NJ | YES | 4 | 4 | YES |
| 75[a] | Clark University, Worcester, MA | YES | 4 | 4 | YES |
| 75[a] | Colorado School of Mines, Golden, CO | YES | 4 | 4 | YES |
| 75[a] | Indiana University at Bloomington, Bloomington, IN | YES | 3 | 3 | YES |
| 75[a] | Michigan State University, East Lansing, MI | YES | 4 | 3 | YES |
| 75[a] | Stevens Institute of Technology, Hoboken, NJ | YES | 4 | 4 | YES |
| 75[a] | University of Delaware, Newark, DE | YES | 5 | 5 | YES |
| 75[a] | University of Massachusetts at Amherst, Amherst, MA | YES | 4 | 4 | YES |
| 82[a] | Miami University at Oxford, Oxford, OH | YES | 3 | 3 | NO |
| 82[a] | Texas Christian University, Fort Worth, TX | YES | 3 | 3 | NO |
| 82[a] | University of California at Santa Cruz, Santa Cruz, CA | YES | 3 | 3 | YES |
| 82[a] | University of Iowa, Iowa City, IA | YES | 4 | 3 | YES |
| 86[a] | Marquette University, Milwaukee, WI | YES | 4 | 3 | YES |
| 86[a] | University of Denver, Denver, CO | YES | 4 | 4 | YES |
| 86[a] | University of Tulsa, Tulsa, OK | YES | 3 | 3 | YES |
| 89[a] | SUNY University at Binghamton, Binghamton, NY | YES | 3 | 3 | YES |

| | | | | | |
|---|---|---|---|---|---|
| 89[a] | North Carolina State University at Raleigh, Raleigh, NC | YES | 3 | 2[c] | YES |
| 89[a] | SUNY University at Stony Brook, Stony Brook, NY | YES | 4 | 4 | YES |
| 89[a] | SUNY College of Environmental Science and Forestry, Syracuse, NY | YES | 4 | 4 | NO |
| 89[a] | University of Colorado at Boulder, Boulder, CO | YES | 4 | 3 | YES |
| 89[a] | University of San Diego, San Diego, CA | YES | 3 | 3 | YES |
| 89[a] | University of Vermont, Burlington, VT | YES | 4 | 3 | YES |
| 96[a] | Florida State University, Tallahassee, FL | YES | 3 | 3 | YES |
| 96[a] | Saint Louis University, St. Louis, MO | YES | 4 | 4 | YES |
| 96[a] | University of Alabama, Tuscaloosa, AL | YES | 3 | 3 | YES |
| 99[a] | Drexel University, Philadelphia, PA | YES | 4 | 4 | YES |
| 99[a] | Loyola University Chicago, Chicago, IL | YES | 4 | 4 | YES |
| 99[a] | SUNY University at Buffalo, Buffalo, NY | YES | 3 | 3 | YES |

*Note.* [a]Tie in Ranking. [b]Applicable only if AB subscore is at least equal to 3. Source: http://admissions.tc.umn.edu/academics/ap.html. [c]Requires successful completion of on-campus class to establish eligibility for credit. Source: https://admissions.ncsu.edu/apply/credit-opportunities/

**Appendix F**

Validity Matrix for Online Survey and Telephone Interviews

| Research Component | How Validity is Threatened? | Strategies to Address Threats |
|---|---|---|
| Participation in Survey and Interviews | | |
| Validity Threat: Potential participants may be unclear about who I am, what I am trying to accomplish and what they are being asked to do. | Participants may feel that there is an agenda or that they are being judged. | Participant population is comprised of Calculus educators who served as readers for the AP Calculus exam, ensuring competent and experienced educators are invited. These educators must have taught Calculus for at least 3 years.<br><br>Participation is completely voluntary and anonymous. They will be assured that their responses cannot be linked back to their identities. |
| Validity Threat: Teachers may be defensive if they interpret my inquiry as judgmental. | Teachers may not want to admit to any information which they feel casts them in an unfavorable light. | Teachers will be approached as professionals who have the expertise and knowledge which my study needs. |

| | | |
|---|---|---|
| Validity Threat: Teachers may incorrectly assess their own understanding. | Teachers may know more or less than they think they do. | I will ask teachers to tell me what is important to them and, if appropriate, to give me an example of how they teach a certain lesson or concept. What do they include and do not include, and why? |
| Validity Threat: Reactivity. Teachers may report what they think I want to hear, or what they think paints them as superlative teachers, instead of what they really do in the classroom. | Information is not true | There is only one researcher in this study; so, controlling for differences between researchers is not a concern.<br><br>Eliminating the actual influence of the researcher is not possible (Hammersley and Atkinson, 1995). I present myself as a person who is requesting information from subject matter experts who are both experienced classroom teachers and AP exam readers. They know more than I do. Even more importantly, they know what I want to know. |
| Research Question 1: How do AP Calculus teachers and college Calculus professors perceive the professional development at the AP Calculus reading? | | |
| Validity Threat: Teachers may not fully disclose their views. | Educators may be hesitant to admit that they do not attend the evening PD sessions or that they do not find them useful. | Educators will be asked what they think of the PD at the reading in all of its forms: collegial interaction, the briefings, the actual reading, and the evening PD sessions. They can focus on what they want to talk about. |

| | | |
|---|---|---|
| Research Question 2: How does participation in an AP Calculus national exam reading affect teachers' and professors' classroom practice? | | |
| Validity Threat: Teachers may report that they have made changes which they intend to implement but have not yet done so. They may also be reticent to report changes which they have made which did not, in the teacher's opinion, have the desired impact. | Information is not true or is exaggerated | I present myself as a person who is seeking information. I tell the participants that there are no right or wrong answers, that I am seeking their professional experiences and observations. The educators are the experts; what they have tried and why they tried it is important to me, whether it worked or not. |
| Research Question 3: How does participation in an AP Calculus national exam reading by educators affect their students' success how is that assessed? | | |
| Validity Threats:<br><br>Teachers may not report their students' AP Calculus scores.<br><br>Teachers may not know how to access their students' scores or find someone who can access them.<br><br>Teachers may not want to divulge the scores for many reasons. | Teacher may feel embarrassed because their student scores are lower than they believe is acceptable.<br><br>I may not be able to get the data to answer this question. I consider this question to be secondary to the others and I am not willing to jeopardize the data collection for the first questions. If data for this question cannot be obtained without posing | I may not be able to get the needed data for a quantitative analysis. I will not badger anyone to get it. The actual scores themselves would be nice to have for comparison; but, if I cannot get at least a sample of size 30 with useable data, I will not do a quantitative comparison test. I can do a qualitative analysis of the teachers' perceptions of changes in their students' Calculus understanding and how, |

| | | |
|---|---|---|
| Individual student scores are privacy protected; however, a teacher is free to discuss his/her class mean score. Teachers may be willing to state how their students perform overall; but, this will either be accurate or inflated. If a teacher's mean score is low, he/she may feel demeaned or threatened.<br><br>Some students may have received outside tutoring (which is currently a common practice); this could affect their exam score. | any discomfort, trust issues, or cooperation from the teachers, then it will be abandoned and left for another day. | if at all, that relates to the AP reading experience. |
| Telephone Interviews | | |
| Validity Threat: Researcher Bias: My understanding and teaching of AP Calculus and my experiences as an exam reader. | Interjecting my own opinions and my way of doing something is not the focus of my study, and, doing so would probably stifle the teacher responses. Some may also be insulted or feel pressured to agree with me. | I need to be quiet except when I am asking the questions. No opinions, no gasps of incredulity, no disagreeing, and no interruptions from me. I will certainly answer their questions, ask for clarification if I need it, and offer an "Oh, I see what you mean." I want to remain neutral.<br><br>I do not want the teachers to infer that I know more than they do or thinking that I believe I know more than they do. |

## Appendix G

Recruitment Email for Potential Participants

**From: Mimi Corcoran**
**To: AP Calculus Readers**
**Subject: AP Calculus Exam Readers Needed for Dissertation Research**

Hello,

My Name is Mimi Corcoran and I am completing my Ph.D. in mathematics education at George Mason University in Fairfax, Virginia. I am starting my dissertation research on teachers' perceptions of the professional development they gain from participating in the AP Calculus exam reading and the effects, if any, this professional development has on their classroom practice. I am seeking high school teachers and college professors who have read the AP Calculus exam at least once and who would be willing to complete a survey online during the summer of 2015. This survey will be conducted online via WebSurvey. All responses would, of course, be confidential. No responses would be tied to any particular person. I estimate that completing the survey will take approximately 20-30 minutes.

All participants in the survey will have the opportunity to enter a random drawing for one of three $50 gift cards to Amazon.com. After completing the survey, you will be linked to a separate site at which you can enter your name and email address. This is completely separate from the survey. Your name and email address cannot be linked back to your survey responses.

In late Summer/early Autumn 2015, I will also be conducting a small number of live interviews and/or a focus group via Skype; I will also ask those who respond to the initial survey if they would be willing to participate in a supplemental live interview or focus group.

I will greatly appreciate your willingness to help me with my research. If you have any questions or suggestions, please email me at mcorcor3@masonlive.gmu.edu. If you are willing to participate, then please complete the initial survey by clicking the appropriate link below. Once you start the survey, you can pause it and complete it at a later data. However, I do need the initial surveys to be completed by 31 August 2015.

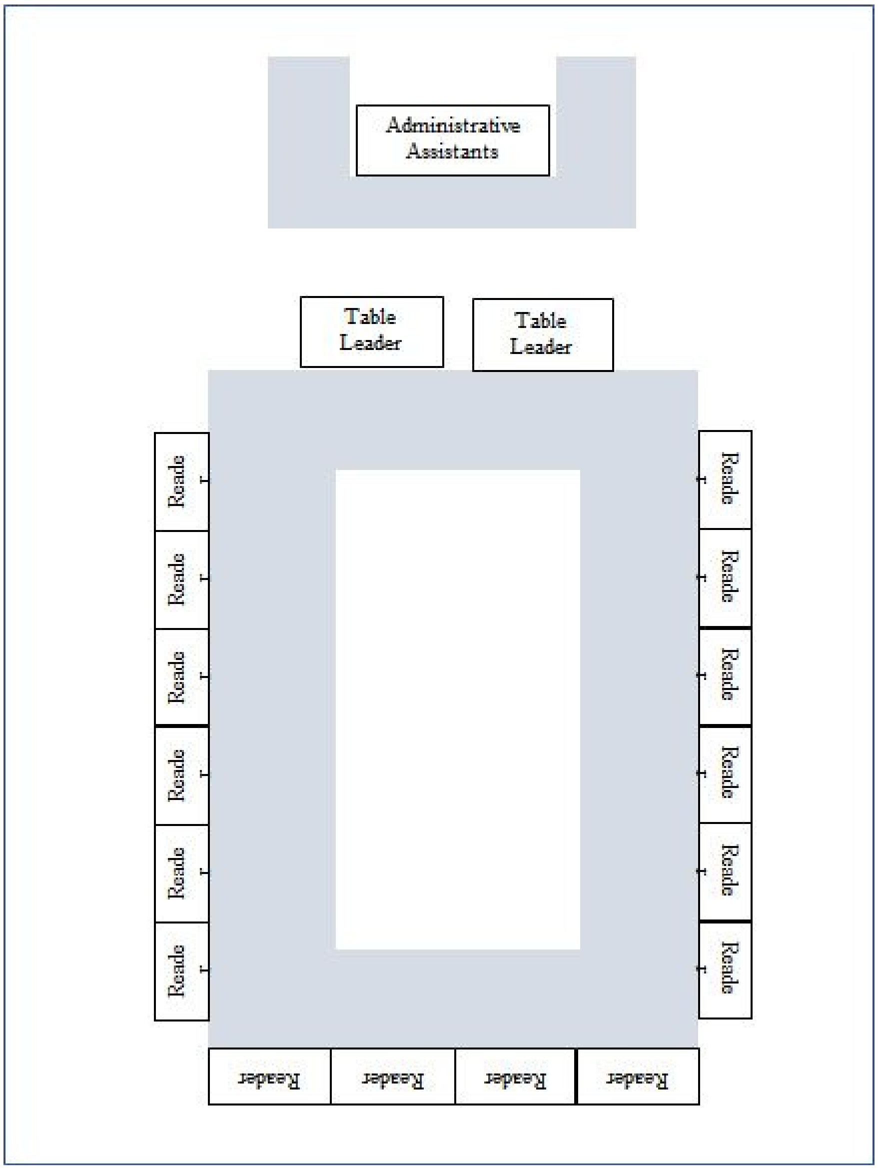
Survey for high school teachers  Survey for college professors

If you are not interested or cannot complete the survey by 31 August 2015, then simply do not respond and I will not contact you again.

Many thanks,
Mimi Corcoran

**Appendix H**

Reading Room Layout



# Appendix I

Part I, Informed Consent for High School Teachers

PROJECT TITLE: The Impact of Teacher Professional Development at AP Calculus Exam Reading on Classroom Practice

RESEARCH PROCEDURE
This research is being conducted to obtain information on potential educational benefits from participation in the annual AP Calculus exam reading. As an experienced AP Calculus exam reader, you will be asked for your opinions on several aspects of your experiences at the AP reading and any impact these experiences may have had on your classroom practice. Your responses are solicited solely for your AP Calculus classes.

The surveys will be made available on WebSurvey. You will be able to access them at your convenience. The survey consists of four parts. The first part is this informed consent in which you indicate your understanding of the project and your willingness to participate in it. The second part consists of Likert-scale questions. The third part asks for your written responses to several open-ended questions. And, the fourth part collects demographic data. In any research report, you will be identified by a pseudonym. Neither your real name nor your institution's name will be used. You will not be asked to provide this information.

At the end of the survey, you will be asked if you wish to participate in follow-up interviews and/or focus groups. Your responses to these questions cannot be linked to your survey responses.

RISKS
There are no foreseeable risks for participating in this research.

BENEFITS
No are no tangible benefits to you other than reflection about your experience as a grader of AP Calculus exams.

COMPENSATION
At the end of the survey, you will be offered the opportunity to link to a separate site and submit your email address to be entered into a random drawing for one of three $50 gift cards to Amazon.com. You will be taken to a separate site to ensure your anonymity. You will also be asked if you wish to participate in follow-up interviews and/or focus groups. Your responses to these questions cannot be linked to your survey responses and your responses have no bearing on your chances of winning one of the gift cards.

CONFIDENTIALITY
The confidentiality of all information collected from you and the other participants is guaranteed. Your name will not be mentioned in the research report or in any published research materials. If you participate in the live interviews, your responses to questions or contributions to interview discussions will be identified only by a pseudonym. You may withdraw from the study at any time without negative consequences. The activities will be conducted in a friendly, non-intimidating, and supportive atmosphere.

PARTICIPATION
Your participation is voluntary. You may withdraw from the study at any time and for any reason. If you decide not to participate or if you withdraw from the study, there is no penalty or loss of benefits to which you are otherwise entitled. There are no costs to you.

CONTACT
This research is being conducted by Dr. Jennifer Suh from the College of Education and Human Development at George Mason University, and by one doctoral student, Mimi Corcoran, who is studying the experiences and perceptions of participants in the annual AP Calculus reading. For questions or to report a research-related problem, Dr. Suh can be reached at (703) 993-9119. You may contact the George Mason University Office of Research Subject Protections at (703) 993-4121 if you have questions or comments regarding your rights as a participant in this research.

This research has been reviewed according to George Mason University procedures governing your participation in this research.

CONSENT:

☐ I have read this form and agree to participate in this study.

☐ I have read this form and do not agree to participate in this study.

Thank you. You have completed Part I: Informed Consent.
If you have indicated your consent to participate, then the data you provide will be used in my research. If you have not indicated your consent to participate, then please exit the survey now. Any data you provide cannot be used and will be discarded.
Thank you for your interest.

## Appendix J

Part II, Likert Scale Questions for High School Teachers

All questions have the following answer choices:

- O Strongly Disagree
- O Disagree
- O Neutral
- O Agree
- O Strongly Agree
- O Commentary, if needed:

1. Participating in the AP Calculus exam reading was worthwhile professional development for me.
2. I find the interaction with other AP Calculus readers to be professionally beneficial.
3. Professional development sessions are offered in the evenings at the reading. I found these sessions informative and useful.
4. The way I think about teaching Calculus has been influenced by my experiences as an AP Calculus exam reader. This means your ideas about what should be taught and how it should be taught. Ideas, not your actions.
5. I have changed my classroom practice because of my experiences as an AP Calculus exam reader. This means any changes you have actually implemented. Actions, not ideas.
6. I use former AP Calculus Exam questions and rubrics to assess my students' Calculus understanding.
7. I believe that my students' conceptual understanding of Calculus has increased since I became an AP Calculus exam reader.
8. I believe that my students are better prepared for the AP Calculus exam since I became an AP Calculus exam reader.

9. I believe that my students' AP exam scores have increased after I became an AP Calculus exam reader.
10. I encourage my colleagues to become AP readers.
11. If scheduling allows, I intend to attend the 2016 AP Calculus reading.

| |
|---|
| Thank you. You have completed Part II: Agree or Disagree |

## Appendix K

Part III, Open-Ended Questions High School Teachers

12. Why did you decide to become an AP Calculus reader? What did you expect? Were your expectations met? Please be as specific as possible.

13. What are your opinions on the professional development at the reading? Please comment on the briefings, the exam reading, the collegial interactions, and the formal professional development sessions. Please be as specific as possible.

14. Please share whether you find the opportunity to interact with other AP Calculus teachers and college professors useful, and in what ways. Please be as specific as possible.

15. In what ways, if any, have your experiences as a reader influenced your thinking (ideas, not actions) about teaching Calculus?

16. In what ways, if any, have your experiences as a reader influenced your actual teaching (actions, not ideas) of Calculus? Why did you make these changes? Please be as specific as possible.

17. Given the changes in your classroom practice, if any, which you described in the previous question, please describe the results you have observed or experienced. Please be as specific as possible.

18. Do you believe that your experiences as an AP Calculus exam reader have influenced your students' understanding and success in your Calculus classes? Why or why not? Please provide as much detail as possible.

19. If you have decided to change your classroom practice based on your AP reading experiences, what obstacles have you encountered in introducing those changes? How did you deal with them? Please be as specific as possible.

**The next two questions are the basis of my quantitative analysis for the fourth research question. I will greatly appreciate your taking the time to give me as much data as possible.**

20. Please provide a summary of your students' scores on the AP Calculus exam for the most recent 3-5 years before you became an exam reader. Please separate AB and BC data.

    Please provide either:

    a. number of students, mean AP score and standard deviation; or,
    b. number of students scoring 5, number of students scoring 4, etc.

    For example,

    a. 2013: 10 students, mean 2.67 standard deviation 1.40
       2012: 12 students, mean 2.55 standard deviation 0.98
       2011: 8 students, ...
       or,
    b. 2013: two scored 5, three scored 4, three scored 3, five scored 2, five scored 1
       2012: two scored 5, two scored 4, six scored 3, four scored 2, seven scored 1
       2011: .......

    *Please indicate if this is actual data or your best estimate. Your school's AP course director, usually the college counselor, should have archival data for at least ten years.*

21. Please provide a summary of your students' scores on the AP Calculus exam for the most recent 3-5 years after you became an exam reader. Please separate AB and BC data.

    Please provide either:

    a. number of students, mean AP score and standard deviation; or,
    b. number of students scoring 5, number of students scoring 4, etc.

    For example,

    a. 2013: 10 students, mean 2.67 standard deviation 1.40
       2012: 12 students, mean 2.55 standard deviation 0.98
       2011: 8 students, ...or,

b. 2013: two scored 5, three scored 4, three scored 3, five scored 2, five scored 1
   2012: two scored 5, two scored 4, six scored 3, four scored 2, seven scored 1
   2011: .......

*Please indicate if this is actual data or your best estimate. Your school's AP course director, usually the college counselor, should have archival data for at least ten years.*

22. Do you have any other comments on your AP Calculus reading experiences and their relationship to your own professional development?

23. Do you have any other comments on your AP Calculus reading experiences and their relationship to your classroom practice?

24. Do you have any other comments on your AP Calculus reading experiences and their relationship to your students' achievement?

| |
|---|
| Thank you. You have completed III: Open Ended Questions |

**Appendix L**

Part IV, Demographic Questions for High School Teachers

25. What is your age group?

- ☐ 21-30
- ☐ 31-45
- ☐ 46-60
- ☐ 61+
- ☐ Commentary, if needed:

26. What is the highest degree which you have earned?

- ☐ B.A.
- ☐ B.S.
- ☐ M.Ed.
- ☐ M.A
- ☐ M.S.
- ☐ D.Ed.
- ☐ Ph.D.
- ☐ Other (please explain):

27. Please indicate the field(s) in which you have earned your degree(s). Please also indicate if you are currently pursuing an advanced degree, type of degree and field of study.

28. How many years of teaching experience do you have?

29. How many years of teaching Calculus do you have?

30. What is your current employment status? (Check all which apply)

- ☐ High School/Secondary School Teacher, Part Time
- ☐ High School/Secondary School Teacher, Full Time
- ☐ Retired High School Teacher
- ☐ Teaching is not my main employment
- ☐ Other (please explain):

31. What levels of Calculus have you taught?

- ☐ Business Calculus
- ☐ Calculus I
- ☐ Calculus II
- ☐ Calculus III
- ☐ Multivariable Calculus
- ☐ Differential Equations
- ☐ Other Calculus courses (please explain):

32. Is teaching your first career or second (or something else)?

- ☐ first career right out of college or graduate school
- ☐ second job. I worked in another field for a few years before coming to teaching
- ☐ second career. I retired from another career field before coming to teaching
- ☐ supplemental work. I teach to supplement my income but teaching is not my main career
- ☐ Other (please explain):

33. At what type of institution(s) do you teach?

- ☐ Public High School
- ☐ Private/Independent High School
- ☐ Private/Religious High School
- ☐ Magnet School
- ☐ Charter School
- ☐ Boarding School - Coed
- ☐ Boarding School - Boys
- ☐ Boarding School - Girls
- ☐ Other (please explain):

34. What is the approximate student population of your high school?

35. How is your teaching delivered?

- ☐ on-site classroom
- ☐ students on-site, teacher live off-site teaching through video/Skype
- ☐ teacher live on-site, students attend synchronously from remote locations
- ☐ online asynchronous with minimal synchronous interaction
- ☐ online asynchronous with no synchronous interaction
- ☐ Other (please explain):

36. How would you describe the physical location of your school?

- ☐ Remote (e.g. Aleutian island)
- ☐ Rural
- ☐ Suburban
- ☐ Urban

- ☐ Inner City
- ☐ Overseas
- ☐ Online, no physical location
- ☐ Other (please explain):

37. How many years have you been a reader? When was your first year as a reader? Have you ever served as a table leader? How many years?

38. Did you attend any of the professional development sessions at the 2015 reading in the evenings?
    - ☐ Mathemagic
    - ☐ The College Board Open Forum
    - ☐ Closing Night "Talent" Show
    - ☐ I attended the 2015 reading; but, I did not attend any of the sessions
    - ☐ I did not attend the 2015 reading
    - ☐ Other (please explain):

39. Did you attend any of the professional development sessions at the readings in the evenings prior to 2015?
    - ○ No, 2015 was my first year at the reading.
    - ○ No, I did not attend any.
    - ○ Yes, I usually attend 1 evening professional development session.
    - ○ Yes, I usually attend 2-3 evening professional development sessions.
    - ○ Yes, I usually attend all of the evening professional development sessions.
    - ○ Other (please explain):

40. What do you consider to be the most important ideas which you bring to your classroom from the AP reading?

- ☐ I have not brought any changes to my classroom
- ☐ clarity in the wording of responses, e.g., refer to a function as *f(x)* and not "it"
- ☐ always including units in all steps of calculations
- ☐ writing clear mathematical sentences
- ☐ making mathematical drawings or graphs
- ☐ focus on responses which make sense in context
- ☐ sharing grading standards for the questions which you graded
- ☐ more practice with former AP Calculus questions
- ☐ judicious use of rounding
- ☐ mathematical reasoning, explaining vs. quoting a theorem
- ☐ Other (please explain):

41. Is there anything else which you wish to share with me about your AP Calculus exam reading experiences?

Thank you. You have completed the survey.
I sincerely thank you for your time and for your participation in my research.
When you click the submit button below, you will be taken to a separate site where you can enter the random drawing for one of the three Amazon.com $50 gift cards. Your name and email address which you use to enter the drawing cannot be linked to responses to this survey. You will also be asked if you are interested in participating in a follow-up interview or focus group. Your response has no bearing on your chances of winning one of the three gift cards.

Submit

## Appendix M

Part I, Informed Consent for College Professors

PROJECT TITLE: The Impact of Teacher Professional Development at AP Calculus Exam Reading on Classroom Practice

RESEARCH PROCEDURE
This research is being conducted to obtain information on potential educational benefits from participation in the annual AP Calculus exam reading. As an experienced AP Calculus exam reader, you will be asked for your opinions on several aspects of your experiences at the AP reading and any impact these experiences may have had on your classroom practice. Your responses are solicited solely for your Calculus classes.

The surveys will be made available on WebSurvey. You will be able to access them at your convenience. The survey consists of four parts. The first part is this informed consent in which you indicate your understanding of the project and your willingness to participate in it. The second part consists of Likert-scale questions. The third part asks for your written responses to several open-ended questions. And, the fourth part collects demographic data. In any research report, you will be identified by a pseudonym. Neither your real name nor your institution's name will be used. You will not be asked to provide this information.

At the end of the survey, you will be asked if you wish to participate in follow-up interviews and/or focus groups. Your responses to these questions cannot be linked to your survey responses.

RISKS
There are no foreseeable risks for participating in this research.

BENEFITS
No are no tangible benefits to you other than reflection about your experience as a grader of AP Calculus exams.

COMPENSATION
At the end of the survey, you will be offered the opportunity to link to a separate site and submit your email address to be entered into a random drawing for one of three $50 gift cards to Amazon.com. You will be taken to a separate site to ensure your anonymity. You will also be asked if you wish to participate in follow-up interviews and/or focus groups. Your responses to these questions cannot be linked to your survey responses and your responses have no bearing on your chances of winning one of the gift cards.

CONFIDENTIALITY
The confidentiality of all information collected from you and the other participants is guaranteed. Your name will not be mentioned in the research report or in any published research materials. If you participate in the live interviews, your responses to questions or contributions to interview discussions will be identified only by a pseudonym. You may withdraw from the study at any time without negative consequences. The activities will be conducted in a friendly, non-intimidating, and supportive atmosphere.

PARTICIPATION
Your participation is voluntary. You may withdraw from the study at any time and for any reason. If you decide not to participate or if you withdraw from the study, there is no penalty or loss of benefits to which you are otherwise entitled. There are no costs to you.

CONTACT
This research is being conducted by Dr. Jennifer Suh from the College of Education and Human Development at George Mason University, and by one doctoral student, Mimi Corcoran, who is studying the experiences and perceptions of participants in the annual AP Calculus reading. For questions or to report a research-related problem, Dr. Suh can be reached at (703) 993-9119. You may contact the George Mason University Office of Research Subject Protections at (703) 993-4121 if you have questions or comments regarding your rights as a participant in this research.

This research has been reviewed according to George Mason University procedures governing your participation in this research.

CONSENT:

☐ I have read this form and agree to participate in this study.

☐ I have read this form and do not agree to participate in this study.

Thank you. You have completed Part I: Informed Consent.
If you have indicated your consent to participate, then the data you provide will be used in my research. If you have not indicated your consent to participate, then please exit the survey now. Any data you provide cannot be used and will be discarded.
Thank you for your interest.

## Appendix N

Part II, Likert Scale Questions for College Professors

All questions have the following answer choices:

- O Strongly Disagree
- O Disagree
- O Neutral
- O Agree
- O Strongly Agree
- O Commentary, if needed:

1. Participating in the AP Calculus exam reading was worthwhile professional development for me.

2. I find the interaction with other AP Calculus readers to be professionally beneficial.

3. Professional development sessions are offered in the evenings at the reading. I found these sessions informative and useful.

4. The way I think about teaching Calculus has been influenced by my experiences as an AP Calculus exam reader. This means your ideas about what should be taught and how it should be taught. Ideas, not your actions.

5. I have changed my classroom practice because of my experiences as an AP Calculus exam reader. This means any changes you have actually implemented. Actions, not ideas.

6. I use former AP Calculus Exam questions and rubrics to assess my students' Calculus understanding.

7. I believe that my students' conceptual understanding of Calculus has increased since I became an AP Calculus exam reader.

8. I believe that my students are better prepared for the mathematics beyond Calculus since I became an AP Calculus exam reader.

9. I believe that my Calculus students' course grades have improved since I became an AP Calculus exam reader.

10. I encourage my colleagues to become AP readers.

11. If scheduling allows, I intend to attend the 2016 AP Calculus reading.

| |
|---|
| Thank you. You have completed Part II: Agree or Disagree |

## Appendix O

Part III, Open-Ended Questions for College Professors

12. Why did you decide to become an AP Calculus reader? What did you expect? Were your expectations met? Please be as specific as possible.

13. What are your opinions on the professional development at the reading? Please comment on the briefings, the exam reading, the collegial interactions, and the formal professional development sessions. Please be as specific as possible.

14. Please share whether you find the opportunity to interact with other AP Calculus teachers and college professors useful, and in what ways. Please be as specific as possible.

15. In what ways, if any, have your experiences as a reader influenced your thinking (ideas, not actions) about teaching Calculus?

16. In what ways, if any, have your experiences as a reader influenced your actual teaching (actions, not ideas) of Calculus? Why did you make these changes? Please be as specific as possible.

17. Given the changes in your classroom practice, if any, which you described in the previous question, please describe the results you have observed or experienced. Please be as specific as possible.

18. Do you believe that your experiences as an AP Calculus exam reader have influenced your students' understanding and success in your Calculus classes? Why or why not? Please provide as much detail as possible.

19. If you have decided to change your classroom practice based on your AP reading experiences, what obstacles have you encountered in introducing those changes? How did you deal with them? Please be as specific as possible.

20. Since you became an AP Calculus exam reader, do you believe that your students' grades in your Calculus classes have been affected? Please provide a synopsis of any before/after changes you have observed?

21. This question is not used for college professors.

22. Do you have any other comments on your AP Calculus reading experiences and their relationship to your own professional development?

23. Do you have any other comments on your AP Calculus reading experiences and their relationship to your classroom practice?

24. Do you have any other comments on your AP Calculus reading experiences and their relationship to your students' achievement?

| |
|---|
| Thank you. You have completed III: Open Ended Questions |

## Appendix P

Part IV, Demographic Questions for College Professors

25. What is your age group?

- ☐ 21-30
- ☐ 31-45
- ☐ 46-60
- ☐ 61+
- ☐ Commentary, if needed:

26. What is the highest degree which you have earned?

- ☐ B.A.
- ☐ B.S.
- ☐ M.Ed.
- ☐ M.A
- ☐ M.S.
- ☐ D.Ed.
- ☐ Ph.D.
- ☐ Other (please explain):

27. Please indicate the field(s) in which you have earned your degree(s). Please also indicate if you are currently pursuing an advanced degree, type of degree and field of study

28. How many years of teaching experience do you have?

29. How many years of teaching Calculus do you have?

30. What is your current employment status? (Check all which apply)

- ☐ Community College Professor, Part Time
- ☐ Community College Professor, Full Time
- ☐ Four-year College or University Professor, Part Time
- ☐ Four-year College or University Professor, Full Time
- ☐ Retired Community College Professor
- ☐ Retired Four-year University Professor
- ☐ Teaching is not my main employment
- ☐ Other (please explain):

31. What levels of Calculus have you taught?

- ☐ Business Calculus
- ☐ Calculus I
- ☐ Calculus II
- ☐ Calculus III
- ☐ Multivariable Calculus
- ☐ Differential Equations
- ☐ Other Calculus courses (please explain):

32. Is teaching your first career or second (or something else)?

- ☐ first career right out of college or graduate school
- ☐ second job. I worked in another field for a few years before coming to teaching
- ☐ second career. I retired from another career field before coming to teaching
- ☐ supplemental work. I teach to supplement my income but teaching is not my main career
- ☐ Other (please explain):

33. At what type of institution(s) do you teach?

- ☐ Community College
- ☐ Public Four-year College or University
- ☐ Private Four-year College or University
- ☐ Men's College
- ☐ Women's College
- ☐ Religious Based College or University
- ☐ Other (please explain):

34. What is the approximate student population of your high school?

35. How is your teaching delivered?

- ☐ on-site classroom
- ☐ students on-site, teacher live off-site teaching through video/Skype
- ☐ teacher live on-site, students attend synchronously from remote locations
- ☐ online asynchronous with minimal synchronous interaction
- ☐ online asynchronous with no synchronous interaction
- ☐ Other (please explain):

36. How would you describe the physical location of your school?

- ☐ Remote (e.g. Aleutian island)
- ☐ Rural
- ☐ Suburban
- ☐ Urban
- ☐ Inner City
- ☐ Overseas
- ☐ Online, no physical location
- ☐ Other (please explain):

37. How many years have you been a reader? When was your first year as a reader? Have you ever served as a table leader? How many years?

38. Did you attend any of the professional development sessions at the 2015 reading in the evenings?

- ☐ Mathemagic
- ☐ The College Board Open Forum
- ☐ Closing Night "Talent" Show
- ☐ I attended the 2015 reading; but, I did not attend any of the sessions
- ☐ I did not attend the 2015 reading
- ☐ Other (please explain):

39. Did you attend any of the professional development sessions at the readings in the evenings prior to 2015?

- ☐ No, 2015 was my first year at the reading.
- ☐ No, I did not attend any.
- ☐ Yes, I usually attend 1 evening professional development session.
- ☐ Yes, I usually attend 2-3 evening professional development sessions.
- ☐ Yes, I usually attend all of the evening professional development sessions.
- ☐ Other (please explain):

40. What do you consider to be the most important ideas which you bring to your classroom from the AP reading?

- ☐ I have not brought any changes to my classroom
- ☐ clarity in the wording of responses, e.g., refer to a function as f(x) and not "it"
- ☐ always including units in all steps of calculations
- ☐ writing clear mathematical sentences
- ☐ making mathematical drawings or graphs
- ☐ focus on responses which make sense in context
- ☐ sharing grading standards for the questions which you graded
- ☐ more practice with former AP Calculus questions
- ☐ judicious use of rounding
- ☐ mathematical reasoning, explaining vs. quoting a theorem
- ☐ Other (please explain):

41. Is there anything else which you wish to share with me about your AP Calculus exam reading experiences?

Thank you. You have completed the survey.
I sincerely thank you for your time and for your participation in my research. When you click the submit button below, you will be taken to a separate site where you can enter the random drawing for one of the three Amazon.com $50 gift cards. Your name and email address which you use to enter the drawing cannot be linked to responses to this survey. You will also be asked if you are interested in participating in a follow-up interview or focus group. Your response has no bearing on your chances of winning one of the three gift cards.

Submit

## Appendix Q

Interview Questions

1. Why did you decide to become an AP Calculus reader?

2. What did you expect to gain from your reading experience?

3. Was the reading experience what you expected it to be? How or how not?

4. What, if any, experiences at the reading stand out for you?

5. Have your experiences at the AP reading influenced your classroom practice? If so, why and how?

6. Have your experiences at the AP reading influenced your competence as an AP Calculus teacher? If so, how? What evidence do you use to support your thoughts?

7. If you could change one aspect of the AP Calculus reading, what would it be? Why would you change it?

8. What are your impressions of the AP Calculus reading as professional development for you?

9. Any difference in students take AP before college and those who do not?

10. How do your students react to your participation in the AP Calculus exam reading?

11. How have your AP reading experiences affected your students' learning? Are other factors at play with your AP experiences?

12. Do you believe that your students' understanding of Calculus has changed since you became an exam reader?

13. In addition to the AP exam scores, what measures do you use to assess your students' success in your class?

14. Have your views of how to teach your students changed since you became an exam reader?

15. How do you balance conceptual and procedural knowledge with your students?

16. What have you gained in your own understanding of content knowledge, curriculum, reframing of concepts and/or dispelling misconceptions?

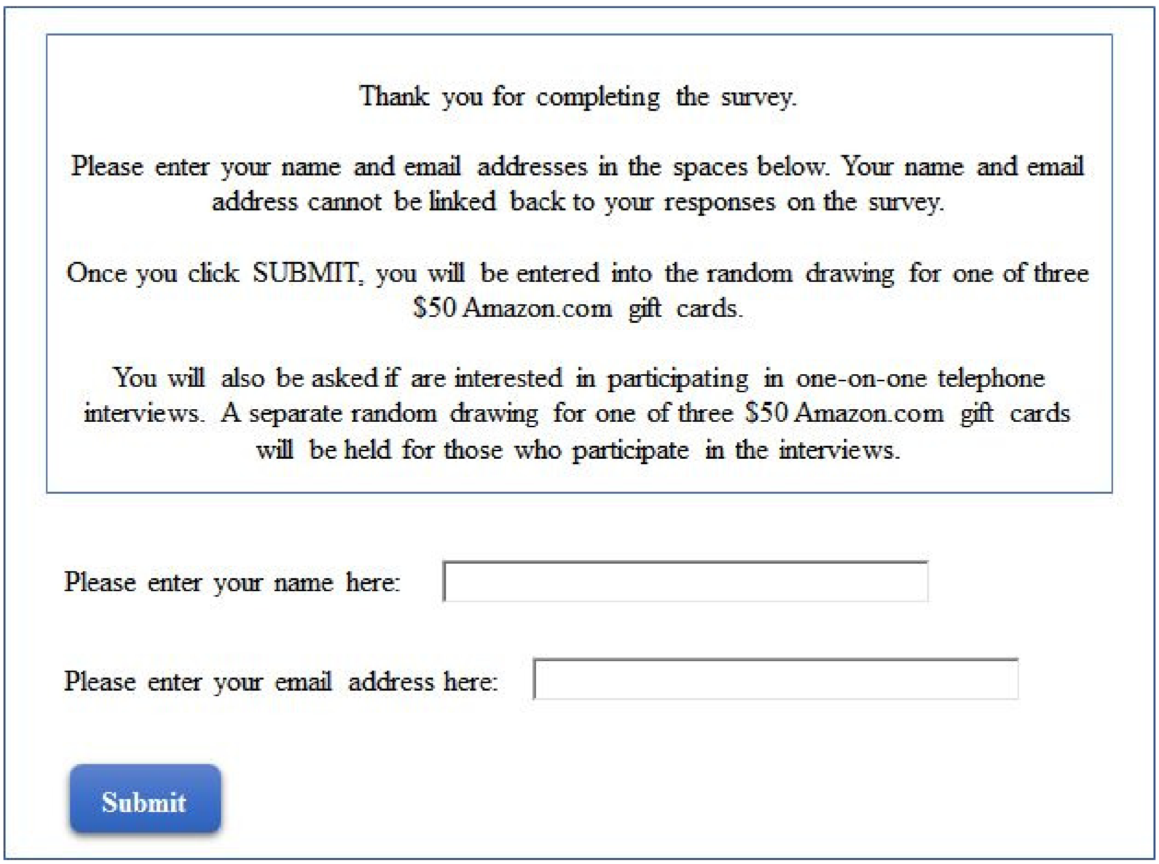
17. The idea of online grading has been discussed for the last several years. What are your opinions of online grading?

18. Why do you keep returning?

19. What did I leave out? What should have asked?

## Appendix R

First Gift Card Drawing



Thank you for completing the survey.

Please enter your name and email addresses in the spaces below. Your name and email address cannot be linked back to your responses on the survey.

Once you click SUBMIT, you will be entered into the random drawing for one of three $50 Amazon.com gift cards.

You will also be asked if are interested in participating in one-on-one telephone interviews. A separate random drawing for one of three $50 Amazon.com gift cards will be held for those who participate in the interviews.

Please enter your name here:

Please enter your email address here:

Submit

## Appendix S

Second Gift Card Drawing

You have been entered into the first random drawing for one of three Amazon.com gift cards.

Please indicate below if you are interested in participating in an interview via Skype or telephone. If you consent and participate, you will be entered into a second random drawing for one of three more Amazon.com gift cards.

You will be contacted via email to schedule a convenient time for the interview.

**Would you be interested in participating in a follow-up interview via telephone or Skype in Summer/Autumn 2015?**
**If you are interested in participating, please be sure that your email address is included below.**

O Yes, I am interested in participating in a one-on-one interview via telephone.
O Yes, I am interested in participating in a one-on-one interview via Skype.
O No, I am not interested in participating.
O Other, please explain:

Please enter your name here:

Please enter your email address here:

Thank you for your feedback. If you have indicated that you are interested in participating in the interviews, you will be contacted by email.

All gift card winners will be contacted via email by 29 February 2016.

Submit

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## Biography

Mimi Corcoran earned her Bachelor of Science in Mathematics from the Pennsylvania State University. She earned her Master of Science in Information Technology from the Naval Postgraduate School. She served as a Naval Officer for twenty-two years, working as an oceanographer, computer specialist and communications specialist. While on active duty, she taught mathematics classes for the University of Maryland and the University of Alaska. After retiring from the Navy, she began her high school teaching career. She has taught mathematics in Virginia independent schools since 2002.