HYPER-ACCELERATION OF ALGEBRA I: NARRATING OPPORTUNITY TO LEARN FROM A SITUATIVE PERSPECTIVE

by

Terrie McLaughlin Galanti
A Dissertation
Submitted to the
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of
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of
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A Dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy at George Mason University

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Dedication

This dissertation is dedicated to the West Valley High School community, without whose support and encouragement I would not have completed this doctoral journey.

To my high school students

You inspire me every day with your willingness to take risks and to trust our work together. You are building your mathematical stories as you ask questions and struggle to make sense out of difficult ideas. You are navigating a complicated world of smartness in mathematics, and your stories need to be heard.

To my fellow teachers

You engage in an endless mission to help each and every student discover their mathematical potential. You are always reflecting and improving your practice with passion and tenacity. You affirm my interest in this work.

To my administrators

You have given me the flexibility to uncover the essence of the phenomenon of hyper-acceleration of Algebra I within and beyond our classroom walls.

Because of your investment in this research project, we can begin to change the narrative about hyper-acceleration of Algebra I and narrow framings of what it means to be good at mathematics.

Acknowledgements

I am deeply thankful for the opportunity to engage in doctoral study of the phenomenon of hyper-acceleration of Algebra I, and it my sincere hope that this dissertation reflects the investment of the George Mason University community in my growth as a scholar. The richness of this journey would not have been possible without the encouragement and affirmation of my professors, my fellow graduate students, and my family.

I could not have asked for stronger mentorship in this pursuit. My advisor, Dr. Toya Jones Frank, challenged me to clearly articulate my shared research interests in acceleration and equity by putting genres of literature "into conversation with each other." Her unwavering expectations of thoughtful justification are reflected in each page of this dissertation. My other committee members, Dr. Courtney Baker and Dr. Meagan Call-Cummings, empowered me to embrace my inner qualitative researcher and to discover the empirical power of narrative inquiry. The leadership and scholarship of Dr. Nancy Holincheck, Dr. Angela Miller, and Dr. Jennifer Suh helped me move this work beyond a personal passion about a singular topic. Each of the aforementioned scholars was a role model who inspired me to construct a broader methodological foundation upon which I can now envision this dissertation as the beginning of a future research agenda.

I also want to express my sincere gratitude to my fellow Mathematics Education Leadership doctoral students. We questioned ourselves at times, we struggled together, and we celebrated our victories. You reminded me each day that the value of this journey is as much within our shared experiences as in our outcomes. My closest colleague, Sara Birkhead, was my patient sounding board, my perpetual critical friend, and my intellectual inspiration from the outset of our doctoral program. We will never stop imagining a more perfect world of mathematics education.

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Advanced Placement	AP
Factors Influencing College Success	FICS
Common Core State Standards for Mathematics	CCSS-M
High School and Beyond	HS&B
International Baccalaureate	IB
International Baccalaureate Higher Level	IB HL
International Baccalaureate Organization	IBO
International Baccalaureate Standard Level	IB SL
Longitudinal Study of American Youth	LSAY
Mathematical Association of America	MAA
National Assessment of Educational Progress	NAEP
National Center for Educational Statistics	NCES
Non-credit Bearing Remedial	NCBR
National Council of Supervisors of Mathematics	NCSM
National Council of Teachers of Mathematics	NCTM
National Education Longitudinal Study of 1988	NELS:88
National Research Council	NRC
Opportunity to Learn	OTL

Abstract

HYPER-ACCELERATION OF ALGEBRA I: NARRATING OPPORTUNITY TO LEARN FROM A SITUATIVE PERSPECTIVE

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George Mason University, 2019

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With increased access to Grade 8 Algebra I in school divisions across the United States, high-achieving students are hyper-accelerating their study of Algebra I to Grade 7. This further acceleration becomes not only a mathematical pursuit but a social expectation as students strive to distinguish themselves from their peers and to improve their competitiveness for admission to elite universities. These students are often deemed successful from a content perspective of opportunity to learn (OTL) which relies on test scores and course taking as metrics of high achievement in secondary mathematics. Yet hyper-acceleration of Algebra I becomes a marker of smartness and status which may diminish opportunities to build foundational mathematics understandings and productive beliefs about mathematics.

This dissertation study synthesizes retrospective narratives of college students' lived experiences with hyper-acceleration of Algebra I to create compelling descriptions of OTL across six years of secondary mathematics. Using a narrative inquiry

methodology from a hermeneutic phenomenological perspective, I layered three narrative analysis techniques to interpret the shared stories of 15 recent graduates from one mid-Atlantic high school. The following research question guided this study: How do recent graduates from one International Baccalaureate (IB) high school within a socioeconomically diverse suburban school narrate their opportunities to learn after hyper-acceleration of Algebra I?

The first article within this manuscript-based dissertation is a research commentary which maps the phenomenon of hyper-acceleration back to broader acceleration of Algebra I to Grade 8. I advocate for research on OTL in this emergent context from a sociocultural perspective. Hyper-acceleration should motivate and empower mathematically promising students. It may instead introduce the potential to reify existing societal power structures and to devalue meaningful mathematics experiences in a potentially misguided and growing "race to calculus".

The second article in this dissertation offers a situative research lens on teaching and learning in hyper-accelerated secondary classrooms. By explaining the what, how, and why of their mathematics experiences, these students help us to better understand how hyper-acceleration may augment or diminish access to meaningful learning. Their adult stories of common middle and high school pathways illuminate a culture and context framed by narrow definitions of what it means to be "good" at mathematics. The students' mathematical identities of competence, bestowed by selection for Grade 7 Algebra I, evolved as their smartness in mathematics was either affirmed or challenged. Interpretation of these storied identities provides an empirical examination of acceleration policies, equitable access to advanced mathematics learning, and persistence on rigorous mathematical pathways.

The third article in this dissertation focuses on the experiences of three hyper-accelerated girls who decelerated their mathematics course-taking trajectories on their pathways toward majoring in STEM fields. Their gendered stories of OTL from a situative perspective were constructed using syllogisms enthymemes as a form of persuasion. These logical arguments about girls and hyper-acceleration revealed a sociocultural interplay of acceleration as an educational advantage and mathematics as a masculine discipline in cultural discourses.

The results of this study can inform the design and implementation of hyper-acceleration policies by parents, teachers, counselors, and administrators. By describing student perspectives on the quality of teaching and learning in these emergent mathematical figured worlds, I offer important empirical evidence of longitudinal beliefs that hyper-accelerated students hold about themselves as productive doers of mathematics. Their narrated identities should spark critical conversations about who has access to this educational advantage and what it means to succeed in secondary mathematics.

Prologue

This dissertation study examines the phenomenon of hyper-acceleration of Algebra I to Grade 7 or earlier from the perspective of college students who narrate their lived mathematics experiences. I have elected to prepare a manuscript-style dissertation for this study, and I would like to offer my personal and professional motivations in this prologue for the pursuit of a non-traditional dissertation in fulfillment of my graduation requirements.

In 2009, I submitted a goal statement for admission to the College of Education and Human Development at George Mason University to begin my secondary mathematics teaching licensure coursework after a career in electrical engineering. Motivated by years of work in school communities as a parent, volunteer, substitute teacher, and mathematics tutor, I asked the following simple yet profound question: "Does the acceleration of algebra to middle school lead to greater difficulty in studying advanced math in later years?"

The mathematics education community has engaged in research on appropriate acceleration of Algebra I for over twenty years. Advocates for broader access to Algebra I in Grade 8 or earlier cite evidence from quantitative correlational studies showing that students who study accelerate Algebra I to middle school are more successful in advanced mathematics courses and more likely to attend college (Ma, 2005a, 2005b;

Rickles, 2013; Smith, 1996, Spielhagen, 2006). As general access to eighth-grade Algebra I has grown, increasing numbers of students are seeking to *hyper-accelerate* Algebra I to Grade 7 or earlier (Simzar, Domina, & Tran, 2016) in order to distinguish themselves in mathematics and increase their competitiveness for admission to elite universities. This hyper-acceleration is a manifestation of sociopolitical privilege and power, and its costs may be far reaching. Early study of formal algebra increases the risk of insufficient attentiveness to foundational conceptual understandings necessary for STEM study (Bressoud, Camp, & Teague, 2012). Hyper-acceleration of Algebra I may also reduce opportunities to develop productive dispositions toward mathematics and detract from STEM interest and persistence.

Nearly ten years after I posed my question about the risks which may accompany increased access to Algebra I in Grade 7 or earlier, I can begin to offer empirical answers through the stories of my 15 doctoral dissertation participants. Yet these answers only serve to further motivate my passion about educational research in the sociocultural and situative experiences of high-achieving mathematics students. As I gain the academic credentialing to pursue this research agenda, I seek to reconcile the compelling need for broader access to rigorous mathematics secondary instruction with emergent structures of hyper-acceleration which can compromise the pursuit of deep understandings of mathematics and perpetuate race and class division. I have grown to understand the empirical power of lived mathematics realities as knowledge which elicits the deeper characterizations of opportunities to learn in secondary mathematics contexts. The power of these realities lies not in the generalizations which we might be tempted to infer from multiple stories but instead in the very individualized ways in which educational systems and structures define success in mathematics. Hyper-acceleration becomes a marker of smartness for young learners which can be affirmed or challenged on their subsequent mathematics journeys. Stories of students on these pathways offer contextual insight which previous large-scale quantitative studies are not able to capture.

In his *Washington Post* education column titled "Why two bright students say I messed up their high school," Jay Mathews (2016a) shared the stories of two graduates of a Maryland high school where broader access to AP mathematics was a policy objective. These accelerated students described reduced access to meaningful calculus learning because "most of the time was spent reviewing precalculus to get students up to speed." One of the students shared that he had placed into remedial mathematics classes at Cal Tech because he did not understand basic calculus enough. While these two students would be deemed successful by standard metrics of high school mathematics achievement, their stories of a diminished quality of calculus learning speak to a potential unintended mathematical consequence of increasing acceleration in our middle schools and high schools. The storied experiences of students should motivate us to further question who is benefitting from this growing race to calculus.

Mathews, the creator of the *America's Most Challenging High School Index*, also published a rebuttal to the students' editorial by a spokesperson from their school division. He cited research showing that greater AP course enrollment "elevates all students and eliminates all gaps" (Mathews, 2016b). Prior research on acceleration of Algebra I typically drives post-positivist policy conversations about increasing access, resources, and outcomes for students who sit on non-accelerated mathematics pathways. This political reality makes it all the more important to revoice the experiences of individual students on the most accelerated pathways to better understand increasing acceleration as an emergent educational phenomenon. I contend that qualitative research using narrative methodologies can describe the what, why, and how of mathematics learning in these classrooms and schools. Their voices should inform policy and practice in the same ways that large-scale research, which lacks context, so often does.

In collecting and analyzing data for this dissertation study, I discovered that this qualitative research offers a critical perspective on hyper-acceleration as a selective process which is assumed to be advantageous to its participants. This discovery motivated my decision to write smaller articles for publication within this dissertation. I first determined that my theoretical framework was the basis for advocacy for additional research on hyper-acceleration. Students' stories, as shared with different narrative analysis techniques in multiple smaller papers, challenge emerging assumptions being success in mathematics is measured and demonstrated by speed of content exposure. The three publication-ready manuscripts within this dissertation both argue for and share findings from this narrative inquiry. They are a conduit for knowledge which can lead to greater understanding of a larger social and mathematical phenomenon.

The National Council of Teachers of Mathematics ([NCTM], 2018) has called for critical conversations which address access to appropriately rigorous mathematics experiences and opportunities to build productive mathematics identities within our high schools. These conversations are often framed from a deficit perspective and focus on closing gaps in achievement for students who are not experiencing success (predicated on grades and standardized achievement) in mathematics. Without evidence of situative perspectives of opportunity to learn (OTL) within hyper-accelerated contexts, we will continue to condone an emergent form of tracking in our middle and high schools which both narrowly identifies and reifies smartness in mathematics. As more students are positioned on hyper-accelerated secondary pathways, we may also change the very nature of mathematics that is taught to those who are left behind. By understanding how mathematics is experienced in these contexts, we will be better positioned to have critical conversations about how mathematics can and should be experienced. Each and every student deserves the opportunity to engage in problem solving and productive struggle in collaborative and creative ways, and the value of hyper-acceleration should be questioned if it does not facilitate teaching and learning in these important ways.

Overview of Dissertation and Research Questions

This dissertation study builds a research-based argument for the situative examination of hyper-acceleration of Algebra I and develops a sequential narrative analysis protocol to describe the lived mathematical experiences of 15 recent graduates from one mid-Atlantic high school. Chapter 1 offers a statement of the problem in the form of a research commentary for the *Journal of Research in Mathematics Education* (Galanti, under review). I map the emergent phenomenon of hyper-acceleration of Algebra I back to broader acceleration policies of Algebra I to Grade 8. I further argue that national data set research from a content perspective of OTL (Reeves, Carnoy, & Addy, 2013; Tate, 2004) which reports positive outcomes with increased access to middle school algebra removes the historical distinction of Grade 8 Algebra I as a marker of mathematics success. Students thereby seek to hyper-accelerate their study of Algebra I to Grade 7 as they seek additional educational advantage (Lucas, 2001) in an increasingly competitive elite college admissions culture (Simzar et al., 2016). Yet research shows that an increasing number of students are studying one or more years of calculus in high school without constructing the deeper conceptual foundations which are necessary for persistence in advanced mathematics (Bressoud, 2017; Carlson, Madison, & West, 2015; Oehrtman, Carlson, & Thompson, 2008; Sadler & Sonnert, 2018). These contradictory implications for OTL in terms of meaningful, connected learning drive the need for research on the phenomenon of hyper-acceleration from a sociocultural perspective. By reimagining OTL within hyper-accelerated contexts as a situative trajectory of learning with a past, a present, and a future (Greeno & Gresalfi, 2008), we characterize the quality of learning within the phenomenon in the stories of individual students. While the historically narrow processes for identifying mathematically promising students (NCTM, 2016) must be acknowledged in situative research on hyper-acceleration, the voices of its current beneficiaries can capture the essence of OTL on these trajectories. Student experiences lay the foundation for broader attention to issues of privilege and access within this emergent track in mathematics education and have implications for students who are not participating in this track. Examination of the implications of hyper-acceleration for the development of productive mathematics identities within a growing "race to calculus" (Bressoud et al., 2012; NCTM, 2018) can and should inform acceleration policy and practice.

In Chapter 2, I provide an extensive literature review for conceptualizing OTL in hyper-accelerated Algebra I context from a situative perspective. Using the organizing categories of content exposure and coverage, content emphasis, and quality of instructional delivery (Tate, 2004), I synthesize the empirical history of acceleration of Algebra I and contemporary research on mathematical readiness for college calculus. By connecting potential implications for persistence in STEM course-taking and conceptual readiness for advanced mathematics to the more general policy context of Algebra I acceleration to earlier grades, I position hyper-acceleration as an emergent educational structure must be explored in contextual ways which relate the learner to the learning environment (Gee, 2008). This literature synthesis thus informs my design of a conceptual framework describing students' OTL in hyper-accelerated contexts from a situative/sociocultural perspective (Greeno & Gresalfi, 2008; Moss, 2008). This framework connects college students' retrospective narratives of hyper-accelerated Algebra I to their mathematics achievement and evolving identities of smartness in mathematics. Connecting achievement and identity is necessary to describe the quality of learning within classrooms and schools.

In Chapter 3, I explain my selection of a narrative inquiry methodology within a hermeneutic phenomenological paradigm (Heidegger, 1998; Kim, 2016) to interpret the lived experiences of recent high school graduates in one mid-Atlantic suburban school community who hyper-accelerated their study of Algebra I to Grade 7. This qualitative study of evolving identities as a trajectory of learning (Greeno & Gresalfi, 2008) within a shared figured mathematical world (Boaler & Greeno, 2000; Holland, Lachicotte, Skinner, & Cain, 1998; Horn, 2008) describes the complex intertwining of mathematical success and societal advancement which underlying the phenomenon of hyper-acceleration of Algebra I. I employ a sequence of three narrative analysis techniques (Knight & Sweeney, 2007; Riessman, 2008; Webster & Mertova, 2007) to explicate distinct meanings within focus group transcripts, across focus group transcripts, and within individual interview transcripts.

In Chapter 4, I describe an iterative narrative analysis of the data across all 15 participants using the first two of the three techniques described in Chapter 3. I first report the findings from critical events analysis (Webster & Mertova, 2007) within each of four focus group transcripts. This first stage of analysis yielded an initial set of thematic categories and identity claims (Carspecken, 1996; Dennis, 2018). Further thematic analysis of the resequenced focus group transcripts and tabular organization of extracted identity claims allowed me to build a cohesive story of hyper-acceleration of Algebra I at West Valley High School for an article in the *Journal of Mathematical Behavior* (Galanti, under review). Participant descriptions of their shared mathematical figured worlds became a story of the what, why, and how of mathematics learning across the lived experiences of the 15 students. A status of smartness in mathematics was conferred by their selection for gifted services in mathematics during elementary school, and Algebra I in Grade 7 became a necessity to preserve their social positioning within a middle school mathematical hierarchy. By constructing a shared narrative of successes and challenges over the subsequent six years of mathematics learning, I describe a changing relationship between OTL and status as students evolve in their understanding of what it means to be “good at mathematics”. This empirical examination provides a novel window on acceleration policies, equitable access to meaningful mathematics learning, and persistence on rigorous mathematical pathways toward STEM undergraduate study.

The critical events thematic analyses as reported in Chapter 4 yielded gendered implications for hyper-acceleration. In Chapter 5, I draw upon Mendick’s (2005) theorization of gendered binary oppositions to explore the tensions that hyper-accelerated young women face in mathematics as a masculine discipline. Using the third stage of narrative analysis as described in Chapter 3, I constructed syllogisms and enthymemes within the stories of three female participants which become rhetorical arguments (Knight & Sweeney, 2007) for the gendered experiences of hyper-acceleration. In this research article (Galanti, in preparation), I argue that the explicit identification of girls at the ages of 12 or 13 as “smart in mathematics” with hyper-acceleration has the potential to either encourage or detract from their interest and persistence in STEM. The emergence of mathematical resilience (Lee & Johnston-Wilder, 2017) as a theme in their stories of deceleration and subsequent mathematics course taking informs the field in novel ways about how high-achieving young women see themselves in competitive mathematics cultures.

Finally, in Chapter 6, I infer from the literature and from the findings of previous chapters to offer implications for stakeholders within other hyper-accelerated contexts. While prior research has documented the advantages of accelerating Algebra I to Grade 8 (Gamoran & Hannigan, 2000; Stein, Kaufman, Sherman, & Hillen, 2011; Smith, 1996;

Spielhagen, 2006) and more recently the negative consequences of universal Algebra I policies (Clotfelter, Ladd, & Vigdor, 2015; Domina, McEachin, Penner, & Penner, 2015; Remillard, Baker, Steele, Hoe, & Traynor, 2017), I have found no prior research examining the contextual consequences of hyper-accelerating study of Algebra I to Grade 7 or earlier for high-performing students. This research is imperative as school divisions respond to heightened expectations within their communities to offer the presumed educational advantage of Algebra I in Grade 7 or earlier. By advocating for this research and by interpreting student narratives which challenge unfounded beliefs that hyper-acceleration of Algebra I is the optimal pathway for every mathematically promising (NCTM, 2016) student, the three peer-reviewed publications in this dissertation begin to build a research base that can inform stakeholder decision-making. The academic outreach to wider audiences will extend perspectives that currently sit within informal web-based conversations on appropriate acceleration of Algebra I. Furthermore, the findings of this study support my advocacy for ensuring that opportunities to engage with mathematics in meaningful and substantive ways remain at the forefront of acceleration decisions within secondary mathematics.

Chapter One – Reimagining Opportunity to Learn with Hyper-acceleration of Algebra I

Abstract

An increasing number of students are *hyper-accelerating* their study of formal Algebra I in Grade 7 or earlier to distinguish themselves in mathematics. This phenomenon is occurring without evidence that these high-performing students are constructing conceptual foundations that are necessary for persistence in advanced mathematics. The author maps this phenomenon back to broader acceleration of Algebra I to Grade 8 and advocates for research on opportunities to learn (OTL) in this emergent context from a sociocultural perspective. OTL with hyper-acceleration of Algebra I is often presumed because these students are deemed successful by traditional metrics. However, students' pursuit of hyper-accelerated mathematics as a marker of distinction may negatively impact their longitudinal identity formation as productive doers of mathematics.

Introduction

The acceleration of formal algebra instruction to Grade 8 has expanded access to high school calculus with the presumed affordance of improving readiness for STEM undergraduate study (Clotfelter, Ladd, & Vigdor, 2015; Domina, McEachin, Penner, & Penner, 2015; Gamoran & Hannigan, 2000; Rickles, 2013). However, an unintended and controversial trend toward *hyper-acceleration* has emerged in parallel with this increased access to formal algebra in middle school. School divisions are now offering high school Algebra I courses in as early as Grade 6 to an increasing number of students who seek to distinguish themselves from their peers and to increase their competitiveness for admission to top universities (Loveless, 2013; Simzar, Domina, & Tran, 2016). Domina, Hanselman, Hwang, and McEachin (2016) described the "tracking up" which accompanied expanded enrollment in eighth-grade Algebra I in California public schools. Between 2003 and 2013 the percentage of students enrolled in eighth-grade Geometry in California more than tripled from 2% to 7%. In a nationally representative survey of students enrolled in first-semester STEM calculus courses in 2010 (Mathematics Association of America [MAA], 2017), 12.8% of respondents reported completing both Geometry and Algebra II by the end of Grade 9, and 88% of hyper-accelerated freshmen students had completed one or more years of calculus in high school.

Our current understandings of hyper-acceleration are based on the personal anecdotes and blog posts of school leaders, mathematics education specialists, teachers, students, and parents who are experiencing or challenging this phenomenon (e.g., Chou, 2018; Pemantle, 2016; Picciotto, 2014). Research on acceleration of Algebra I to Grade 8 is comprehensive, yet my thorough examination of the literature has not yielded any research on how hyper-accelerated students engage with mathematics content and persist in mathematics beyond courses required for high school graduation.

Therefore, this commentary synthesizes extant literature on the acceleration of formal algebra and the transition from high school to college calculus. This integrated base of literature informs the need for empirical examination of hyper-acceleration as an emergent phenomenon which can have unintended negative consequences. I will further argue that the study of hyper-acceleration responds to the recent National Council of Teachers of Mathematics (NCTM) Research Committee (2017) call for empirical work which will "contextualize mathematics education research within systemic barriers that continue to perpetuate and reinforce inequities in mathematics teaching and learning" (p. 382).

The Path to Hyper-acceleration

The push for hyper-acceleration is both a predictable consequence of algebra for all policies and an emergent form of curricular differentiation (Domina et al., 2016). Algebra I is often referred to as the gatekeeper for high school graduation and for access to advanced mathematics study (Moses, 1995). At the end of the 20th century, Robert Moses challenged preeminent mathematicians and education researchers at the Algebra Initiative Colloquium to view algebra as the "new civil right" (Lacampagne et al., 1995, p. 3). Lacampagne (1995) summarized the concerns that colloquium participants expressed about increasing access to rigorous mathematics for an increasingly diverse community of learners.

> How do we ensure that "algebra for all" is not "dumbing down" algebra? The mathematical community as well as parents of college-bound students will and should demand sound preparation in algebra for the college bound. We will be faced with building an algebra curriculum and pedagogy that will support the needs of all students. (LaCampagne, 1995, p. 2)

The implication that increased access could threaten the role of formal mathematics in preparing the elite students of the future foreshadowed the elevation of appropriate acceleration of Algebra I to the center of 21st century education policy. According to data published by the National Center for Education Statistics (NCES), the percentage of students enrolled in Algebra 1 or higher in eighth grade or earlier grew from 16% in 1990 to 42% of students in 2015 (Loveless, 2016). Prior research using national data sets has simultaneously endorsed and challenged this expansion in enrollment (Stein, Kaufman, Sherman, & Hillen, 2011). Students who study Algebra I in Grade 8 or earlier are more successful in advanced mathematics and more likely to attend college (Gamoran & Hannigan, 2000; Rickles, 2013; Smith, 1996; Spielhagen, 2006), yet broad acceleration of Algebra I to Grade 8 can have negative effects for students with lower prerequisite achievement (Clotfelter, Ladd, & Vigdor, 2015; Domina, McEachin, Penner, & Penner, 2015; Finkelstein, Fong, Tiffany-Morales, Shields, & Huang, 2012; Simzar, Domina, & Tran, 2016). The recent work of Schmidt and colleagues (2015) on disparate outcomes within U.S. schools suggests that the move toward increased access to eighth-grade algebra has created a new track called seventh-grade algebra. They also found that acceleration resulted in an overrepresentation of students from lower SES backgrounds in ninth-grade algebra and reduced opportunities to learn (OTL) for students across the spectrum of mathematical readiness. Thus, as hyper-acceleration proliferates, the implications for equity in mathematics education must be addressed.

Hyper-acceleration of Algebra I as a Matter of Equity

Hyper-acceleration as a social phenomenon exists without evidence that students are constructing the deeper conceptual foundations which are necessary for persistence in advanced mathematics (Carlson, Madison, & West, 2015; Bressoud, Camp, & Teague, 2012; Oehrtman, Carlson, & Thompson, 2008). Furthermore, these increasingly advanced course pathways can detract from the important acceleration policy objective of creating more rigorous and heterogenous mathematics experiences for more students.

Gutiérrez (2012) theorized a *dominant* axis of equity with its dimensions of access and achievement as separate from a *critical* axis with its dimensions of identity and power. She argued that tensions between these two axes orient the research community toward a broader agenda of social justice in which the mathematics education evolves to effect change. This tension is deeply embedded in individual and policy motivations to further accelerate the study of Algebra I. Hyper-accelerated learners have cultural capital by virtue of course selection and placement. In other words, these students sit at the pinnacle of the dominant axis of Gutiérrez's framework. Hyper-acceleration thus creates increasingly stratified structures of secondary mathematics courses and introduces critical questions about who has access to this cultural capital and how student identities define participation in these courses. Hyper-acceleration should motivate and empower mathematically promising students (NCTM, 2016); yet it introduces the potential to reify existing societal power structures and to devalue meaningful mathematics experiences in a potentially misguided and growing "race to calculus" (NCTM, 2018).

As a former electrical engineer, veteran high school mathematics teacher, and educational researcher, I am deeply concerned about this growing trend in secondary mathematics education without an empirical basis. Hyper-acceleration may be most prevalent in highly racially and/or socioeconomically stratified school divisions as particular students and parents seek to maintain an academic advantage from an effectively maintained inequality (EMI) perspective (Lucas, 2001) in the pursuit of optimal learning experiences.

There are serious equity issues that lie at the heart of hyper-acceleration, and these issues that deserve their own exploration. By virtue of arguably narrow processes for identifying students for this emergent track in mathematics education, hyper-acceleration can become a structural barrier to learning not only for those who struggle within these new tracks but also for those who operate outside of these tracks. In laying the foundation for this future work, this commentary will draw upon what is currently known about accelerated Algebra I to advocate for research which takes a sociocultural perspective on opportunities to learn (Gee, 2008; Greeno & Gresalfi, 2008; Moss, 2008) within hyper-accelerated mathematics contexts. Situated research can then inform broader policy conversations about this emergent phenomenon.

The Need for Contextual Research on Hyper-acceleration of Algebra I

As we acknowledge the social forces that drive hyper-acceleration, we also need to examine the impacts on curriculum and instruction. Narrow constructions of mathematics ability and achievement (Louie, 2017) that often characterize students' experiences in advanced secondary courses can compromise opportunities to engage with mathematics in meaningful ways (Boaler, 1997, 2015; Boaler & Greeno, 2000; Gutiérrez, 2002, 2012; Horn, 2006). Thus, hyper-acceleration itself becomes a potentially flawed narrative of mathematical competence and success. Three past NCTM presidents (Gojak, 2013; Larson, 2017; Seeley, 2005) have encouraged school divisions to emphasize powerful middle school mathematics curricula with algebraic thinking and proportional reasoning in lieu of acceleration. The MAA and NCTM jointly cautioned educators, parents, and students that the increasing acceleration of traditional secondary mathematics courses is not only "ineffective" but "counterproductive" (Bressoud et al., 2012, p. 2). The Achieve Working Group devised guidelines for acceleration within Common Core State Standards for Mathematics (CCSS-M), stating that curricula should not compacted before Grade 7 and that "placing students into tracks too early should be avoided at all costs" (p. 81). Additionally, Sheffield (2017) argued that the appropriateness of acceleration of secondary mathematics by more than one year for gifted students was a "dangerous myth" (p. 21) and "not beneficial for a majority of top students" (p. 22). Emergent stratification within middle school Algebra I courses can create an environment in which course enrollment is a "sign of superior intelligence" (Seeley, 2005) and introduces the potential to lower self-efficacy for accelerated students who struggle. Despite all of these challenges to hyper-acceleration by professional organizations and researchers, the pervasive beliefs that Advanced Placement courses and associated college credit are an advantage "make it unlikely that the push for mathematics acceleration among students, parents, and even some mathematics teachers, will lessen anytime soon" (NCTM, 2018, p. 20).

With increasing numbers of students choosing to hyper-accelerate secondary courses, there is a need for phenomenologically anchored research which describes student, parent, teacher, and administrator experiences in these emergent contexts. Furthermore, the field needs to consider the implications for OTL in terms of conceptual understanding, persistence in advanced mathematics, and evolving mathematical identities. Aguirre, Mayfield-Ingram, and Martin (2013) defined mathematical identity as "dispositions and deeply-held beliefs that students develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways across the contexts of their lives" (p. 14). This definition brings productive disposition, context, and achievement together in building mathematical competence (Berry, 2018). OTL with hyper-acceleration of Algebra I is often taken for granted because these students are deemed successful by traditional metrics of standardized tests and college enrollment. However, students' pursuit of hyper-accelerated mathematics as a marker of distinction may negatively impact their longitudinal identity formation as productive doers of mathematics.

Early Algebra I Acceleration and Paths to Advanced Mathematics

Until the early 1990's, enrollment in eighth-grade Algebra I had been reserved for a small percentage of students who had demonstrated strong aptitude in pre-algebra or met certain testing requirements. Prior research on the acceleration of Algebra I to middle school has focused on increasing access to appropriately challenging mathematics for traditionally underrepresented students with grades and standardized test scores as measures of successful outcomes (Clotfelter, Ladd, & Vignor, 2015; Remillard, Baker, Steele, Hoe, & Traynor, 2017; Ma, 2005a, 2005b; Rickles, 2013; Spielhagen, 2006; Stein et al., 2011). With the emergence of hyper-acceleration, it is especially important to consider the longitudinal implications for course taking and calculus readiness. Finkelstein and colleagues (2012) examined the mathematics course-taking sequences of 24,279 California students from Grades 7 through 12 using transcript data and standardized Algebra I, Geometry, and Algebra II test scores from 2004-2009 as proxies for achievement. The explanatory study quantified persistence on secondary STEM trajectories; discussion focused on negative effects for accelerated students who needed to repeat Algebra I to demonstrate mathematics proficiency for high school graduation. The researchers concluded that *when* students take Algebra I is less important than their readiness to access course content. My additional descriptive analysis of the data showed that 45% of the students who studied Algebra I in seventh grade repeated Algebra I in eighth grade. This statistic suggests that hyper-acceleration may not be fostering the mathematics understanding, motivation, and confidence that students need to continue on highly accelerated mathematics pathways.

Researchers have also begun to examine the consequences for undergraduate STEM readiness with broader acceleration of Algebra I to middle school and calculus instruction to high school (Bressoud, 2015a; Sadler & Sonnert, 2018; Wade, Sonnert, Sadler, Hazari, & Watson, 2016). Despite increased access to eighth-grade Algebra I, ACT (2017) reported that only 21% of the 2017 high school graduating class met their readiness benchmark for first-year STEM college courses. Accelerated students with interest in STEM undergraduate study may not be building necessary precalculus conceptual understandings, thereby experiencing delays in progress toward degrees or exiting the STEM pipeline (Carlson et al., 2015). Many students struggle in college-level mathematics because of "incomplete or insecure understandings" of algebraic topics situated within secondary mathematics curricula (Stewart & Reeder, 2017, p. 4).

The MAA study of Characteristics of Successful Program in College Calculus (CSPCC) surveyed 14,000 students at 213 colleges and universities enrolled in an entry level calculus course required for science, mathematics, and engineering majors. Over 67% of the surveyed students had studied calculus prior to starting at the college or university, and 33% of surveyed students failed to complete the course (Bressoud, Mesa, & Rasmussen, 2015). The researchers explained these findings as a "rush toward accumulation of problem-solving abilities in calculus while short-changing the broader preparation needed for success beyond calculus" (Bressoud et al., 2015, p. vi). In their July 2018 publication in JRME, Sadler and Sonnert reported that end-of-course grades in precalculus math courses along with SAT/ACT scores explained more than twice the variability in college achievement than grades in high school calculus courses. Accelerated students have also placed into non-credit bearing mathematics courses at the college level despite having completed calculus in high school (Bressoud, 2015b; Larnell, 2016).

Taken together, these findings speak to a need to understand how students are selected for hyper-acceleration of Algebra I and how they learn mathematics on this pathway. Potential deficits in algebraic understandings that are foundational for success in college calculus have not been explicated in large-scale research relying upon standardized test score outcomes as evidence of opportunities to learn with acceleration of Algebra I. Additional research that captures the contextual factors of Algebra I acceleration is needed to connect teaching and learning to mathematical dispositions and conceptual readiness for advanced mathematics.

Early Algebra vs. Algebra I Early

Research has shown that elementary and middle school students have a strong potential for abstract thinking through early algebra (Carraher, Schliemann, Brizuela, & Earnest, 2006). While most middle school students do not take Algebra I as early as Grade 7, algebraic thinking defined as early algebra is prevalent in middle school curricula and even in elementary mathematics curricula. Standards for algebraic thinking (National Governors Association Center for Best Practices, & Council of Chief State School Officers, 2010) and patterns, functions, and algebra (Virginia Department of Education, 2016) inform kindergarten curricula. Early algebra leverages algebraic context in problems without the structures of formal algebraic notation (Carraher, Schliemann, & Schwartz, 2008; Mason, 2017). These experiences are fundamental to the future abstraction of algebra once variables and symbolic notation are introduced (Driscoll, 1999). Yet educators' focus on creating rich early algebra experiences in the elementary and middle grades might be adversely impacted in schools where formal algebra instruction is increasingly available to younger students and perceived to be a better curricular choice.

Without new research that differentiates early algebra and algebra early explicitly for parents, teachers, and administrators, we will likely see a growing belief that hyper-acceleration of Algebra I is an optimal marker of mathematical success. The need to make these distinctions for teaching and learning becomes part of a "reimagining" of OTL in secondary mathematics education beyond the name of a course and the year in which it is completed. OTL in hyper-accelerated contexts should reflect a commitment to provide the time and resources to engage with content in meaningful and foundational ways (Larson, 2017). Unless the design and implementation of hyper-acceleration is grounded in empirical work that addresses expectations for teaching and learning, it has the potential to become a disadvantage for mathematically promising students. Accelerated exposure to formal algebra and calculus can weaken conceptual mathematical foundations and lessen productive dispositions toward mathematics for those who begin to struggle in mathematics during their high school years.

Reimagining Opportunity to Learn from a Situative Perspective

Despite these concerns, the demand for hyper-accelerated Algebra I courses as a marker of mathematical distinction is likely to persist in the absence of research which characterizes teaching and learning contexts. With this in mind, I reimagine OTL for hyper-accelerated students as a construct which characterizes meaningful learning experiences. OTL was originally defined by Husen (1967, as cited in Burstein, 1993, p. xviii) as "whether or not students have the opportunity to study a particular topic or learn how to solve a particular type of problem presented by the test" (pp. 162 - 163). This construct emerged out of the necessity to explain results of international assessments of mathematics and science learning and has been used in analysis of resource allocation and curriculum design. OTL has also been used as a policy instrument (McDonnell,

1995) to reframe the mathematics achievement gap in terms of inequitable access to experienced and qualified teachers, high expectations for success, appropriate challenging curricula, and per pupil funding (Flores, 2007; Schmidt, Cogan, Hoang, & McKnight, 2011) for traditionally marginalized groups.

Yet previous conceptualizations of OTL as a series of discrete constructs of content coverage, content exposure, content emphasis, and instructional delivery (Ellis, Kelton, & Rasmussen, 2014; Reeves, Carnoy, & Addy, 2013; Tate, 2004) are challenging to separate in psychometric research (Wang, 1998) and even more challenging to separate from individual student characteristics (McDonnell, 1995; Gee, 2008). As McDonnell (1995) noted, "Knowing that most ninth graders take algebra does not provide adequate information about their actual opportunity to learn algebra content. Enrollment statistics enumerated simply by course title convey little information about content or how that content is presented" (p. 310). By traditional measures of OTL, high-achieving mathematics students are presumed to have access to advanced content and quality instruction (Larson, 2017) and are assumed to be proficient in the mathematics topics in the curriculum of the courses. These traditional measures cannot capture the unintended consequences of hyper-acceleration in in terms of evolving mathematics identities and persistence in mathematics study.

Greeno and Gresalfi (2008) offered a *situative* perspective on OTL in which an individual's learning is conceptualized as a "trajectory of that person's participation in the community – a path with past and present, shaping possibilities for future participation" (p. 170). The selection for Algebra I in seventh grade or earlier, whether in the form of a sufficiently high score on test of mathematics ability, a teacher recommendation, or self-selection, becomes a presumption of competence in mathematics that is subject to challenge as the student moves through the next six years of mathematics study in middle school and high school. By reimagining OTL from a situative perspective, researchers can study the intersection of achievement and identity as a window on the meaning of success in these emerging contexts of accelerated Algebra I. Within my OTL framing, I define mathematics achievement as persistence in the advanced mathematics pipeline and gained conceptual understandings beyond procedural recollection and replication. I define mathematics identity as narratives of success and competence created by the context and culture of hyper-acceleration. Darragh (2016) proposed that identity as a construct can zoom in on individual experiences and zoom out on the sociopolitical discourses in which the individual is positioned, while Wenger (2010) affirmed identity as a learning trajectory which is "not an object, but a constant becoming….our identities incorporate the past and the future in the very process of negotiating the present" (pp. 133-4).

In hyper-accelerated contexts, teachers may not challenge students to think deeply, to justify their reasoning to others, or to make connections when students are otherwise successful by standardized measures. Students who operate as actors in these socially and culturally constructed worlds (Holland, Lachicotte, Skinner, & Cain, 1998) may experience didactic instruction and procedural routines that often characterize advanced secondary mathematics classrooms (Boaler, 1997; Boaler & Greeno, 2000; Schoenfeld, 1988) and limit opportunities for meaningful sense-making of foundational concepts. Beyond the classroom, student willingness to seek support, student interest in STEM careers, and student disposition to continue mathematics study become intertwined with changing beliefs about themselves as doers and beneficiaries of hyper-accelerated mathematics study.

Future Directions for Research

The reality of politics and the perception that "faster is better" is likely to continue to drive the political discourse in high-achieving school districts in the absence of new knowledge about the unintended consequences of acceleration. Questions about hyper-acceleration cannot be answered by large data set research in isolation; the voices of students, teachers, parents and administrators as they reflect on their experiences with mathematics learning trajectories that begin in seventh-grade Algebra I can describe the breadth and depth of this emergent phenomenon. This research is imperative as we strive to ensure that the academic and socioemotional needs of these young mathematicians and those who support them are not being compromised in a growing course completion race.

I propose multiple entry points to the empirical study of hyper-acceleration using a situated perspective on OTL. These entry points are organized by individual stakeholders and together lay the foundation for further study of hyper-acceleration from an equity perspective.

1. At the student level, research on hyper-acceleration can explicate the ways in which individual students make sense of mathematics content and how their understandings relate to their evolving identities. Previous studies have explored student identities as they position themselves or are positioned by

others as being “good at mathematics” (Bishop, 2012; Darragh, 2015; Mendick, 2005; Radovic, Black, Salas, & Williams, 2017). Examination of content understandings and narratives of evolving identities become foundational evidence of OTL in hyper-accelerated contexts.

2. At the teacher and department level, research can inform both the design of professional development experiences for teachers of Grade 7 Algebra I students and the development of course curricula which improves the quality of learning for mathematically promising students.
3. At the parent and administrator level, evidence of hyper-accelerated course trajectories, selection criteria, and student persistence can inform policy decisions which more equitably identify and support mathematically promising students.

Conclusion

Stinson (2004) specifically challenged mathematics educators to look at OTL beyond political structures such as tracking (Oakes, 2005) as he offered a critical perspective on the societal gatekeeping role that mathematics often plays. He asked, “How might mathematics educators ensure that gatekeeping mathematics becomes an inclusive instrument for empowerment rather than an exclusive instrument for stratification?” (p. 8). Stinson’s question is equally appropriate in considering the “tracking up” of middle school mathematics (Domina et al., 2016) which has accompanied broader access to Algebra I. The offering of more advanced mathematics courses in middle school is grounded in the desire of students, parents, and teachers to maintain a competitive edge for college admissions. OTL can be made explicit in the experiences of hyper-accelerated students whose notions of smartness evolve in parallel with participation in advanced courses on multi-year accelerated mathematics pathways. The intersection of achievement as persistence and conceptual understanding together with identities of competence can create compelling individual stories of OTL from a situative perspective within this emergent phenomenon of hyper-acceleration of Algebra I.

Reimagining OTL in the context of hyper-acceleration will offer a novel empirical basis for our shared pursuit of equitable educational structures across all mathematics education contexts. Acceleration of algebra to middle school is often framed as access to rigorous and relevant mathematics at the entrance to the secondary mathematics pipeline; however, it cannot be separated from its equally important role of building a foundation for continued course taking and productive dispositions in mathematics. Experiences with college-level mathematics content prior to the transition from high school to college calculus should prepare students for STEM success, yet research on insufficient mastery of algebraic skills and understanding of concept of function at the undergraduate level (Bressoud et al., 2015; Carlson et al., 2015; Sadler & Sonnert, 2018; Stewart & Reeder, 2017) challenges the quality of this preparation. Meaningful mathematics experiences should be available at all levels of readiness (NCTM, 2018), and further acceleration of Algebra I should not be at the expense of the development of positive mathematics identities and strong foundations for future learning.

Chapter Two – Literature Synthesis and Conceptual Framework

Chapter Overview

In this chapter, I offer a brief synthesis of prior research on the acceleration of Algebra I with a purposeful focus on high school mathematics achievement, course taking, and readiness for college calculus. Acceleration of Algebra I to Grade 8 has been a policy instrument intended to improve OTL in secondary mathematics (Domina et al., 2015; Loveless, 2013; Remillard et al., 2017; Schmidt et al., 2011), yet accompanying presumptions that additional acceleration improves student outcomes (Domina et al., 2016; Larson, 2017; Picciotto, 2014; Schmidt, 2009; Smith, 1996) are worthy of contemporary study. Relevant research for this literature synthesis is thus organized using traditional categorizations of OTL from a content perspective. These categorizations provide an empirical basis for expanding perspectives on OTL within the emergent phenomenon of hyper-acceleration of Algebra I.

I then elaborate on more recent conceptualizations of OTL which extend beyond traditional assessments of content knowledge and instructional delivery. A situative perspective on OTL (Gee, 2008; Greeno & Gresalfi, 2008) informs my design of a theoretical framework which integrates content and sociocultural experiences of OTL for students who hyper-accelerate their study of Algebra I and subsequent courses in secondary mathematics. In building this framework, I draw upon Moss' (2008) six criteria for evaluating OTL to frame the qualitative study of hyper-accelerated students' learning trajectories (Greeno & Gresalfi, 2008) within figured mathematical worlds (Boaler & Greeno, 2000; Holland et al., 1998; Horn, 2008). Finally, I juxtapose the operationalization of identity as narrative (Sfard & Prusak, 2005) with the recent theoretical works of scholars in the field of mathematics identity research (Darragh, 2016; Radovic, Black, Williams, & Salas, 2018) to describe OTL as a retrospective narrative of achievement and identity in hyper-accelerated contexts.

I chose to highlight the content perspectives on secondary mathematics outcomes in the literature to justify the need for phenomenological research of hyper-acceleration of Algebra I as an educational decision with longitudinal implications for mathematics learning. I began with "eighth grade", "middle school", "algebra", "acceleration", and "calculus readiness" as search terms in EBSCO host, Science Direct and PsychNet databases to compile relevant peer-reviewed publications. It is important to note the term "hyper-acceleration" as defined for this study was not used in as a search term for peer-reviewed publications; any empirical discussions of acceleration of Algebra I to Grade 7 or earlier are found within the broader context of research on Algebra I in Grade 8. After building a literature base of Algebra I acceleration, I widened my search to include "mathematics education", "opportunity to learn", "figured worlds", "narrative", and "identity". I sought to understand OTL from both content and sociocultural perspectives as an integrated conceptual lens for the study of hyper-acceleration of Algebra I. I used references within this literature to review additional research until a "point of saturation" (Randolph, 2009) was reached. Because hyper-acceleration is an emergent phenomenon lacking an empirical basis, I also relied upon relevant non-peer reviewed documents published on national mathematics organization websites including NCTM, MAA, and National Council of Supervisors of Mathematics (NCSM).

Prior Research on Algebra I Acceleration and Secondary Mathematics Outcomes

The majority of the research examining the consequences of increased early access to Algebra I has relied upon large-scale data sets and standardized test scores. Since Robert Moses' call for greater access to Algebra I in the late 20th century, researchers have explored the affordances and challenges of acceleration of Algebra I to Grade 8 or earlier (Gamoran & Hannigan, 2000; Ma, 2005a, 2005b; Rickles, 2013; Smith, 1996; Spielhagen, 2006; Walston & McCarroll, 2010). Empirical research on the effects of Algebra I acceleration can be broadly categorized in terms of effects on Grade 12 mathematics achievement, high-school course taking patterns, and classroom instructional rigor. Below, I organize prior research on the acceleration of Algebra I using the three categories of Tate's (2004) OTL framework. These categories were adapted from Stevens' (1993) variables related to instructional practice and student learning and elaborated on within the OTL research of Wang (1998) and more recently Reeves, Carnoy, and Addy (2013). Content exposure and coverage is defined by time and depth of experience with topics within a particular grade level or discipline. Empirical studies of the acceleration of Algebra I have quantified content exposure and coverage using traditional achievement test results. Content emphasis is defined as topic sequencing and the selection of students for low-order (memorization) or high-order (problem-solving) instruction. Content emphasis has been examined through studies of secondary mathematics course trajectories. Quality of instructional delivery aligns pedagogy with student achievement and has also been measured by traditional achievement test results.

Each of the studies selected for this literature review represents an underlying ontological commitment to understanding Algebra I acceleration as a policy which increases or decreases OTL in secondary mathematics. By categorizing prior research on the acceleration of Algebra I to Grade 8 or earlier using traditional OTL categories, implications for persistence in STEM course-taking and conceptual readiness for advanced mathematics within the more general policy context of Algebra I acceleration to earlier grades can be theorized.

Middle school Algebra I: OTL as content exposure and coverage. The four quantitative studies highlighted below report gains in student achievement through Grade 12 with access to Algebra I curriculum in Grade 8 or earlier. The methodological designs of these studies were consistent with traditional views of OTL as mathematics content taught and allocation of educational resources (Pullin & Haertel, 2008), yet each of these studies presented implications that affirmed the need for a more contextual examination of OTL which relates the hyper-accelerated Algebra I learner to the environment (Gee, 2008).

Smith (1996) conducted a correlational study of longitudinal transcript data from the High School and Beyond (HS&B) study of 5818 students who studied Algebra I in ninth grade and 1076 students who had studied Algebra I in eighth grade or earlier. She asked, “Does early entry into the mathematics pipeline give students an added advantage, either in their access to advanced coursework or their mathematics achievement, by the end of high school?" (p. 142). Smith acknowledged a "web of influences" (p. 143) on OTL by including socioeconomic status, tenth grade factors (achievement, curriculum track, and vocational aspirations), and number of mathematics courses as covariates. Smith concluded that "early access to algebra has a sustained positive effect on students, leading to more exposure to advanced mathematics curriculum and, in turn, higher mathematics performance by the end of high school" (p. 148).

Because her stated research goal was to find evidence for the advantages of broader access to Algebra I in middle school as predictive of mathematics achievement, Smith (1996) did not elaborate on descriptive statistics in her study indicating that over one-third of students who had early access to Algebra I were no longer on the more competitive "academic" mathematics track by Grade 10 (p. 145). In addition, only 25% of the students in the study who studied Algebra I in eighth grade took calculus in high school, thereby challenging the presumption that early access to advanced mathematics necessarily leads to persistence in advanced coursework. This dip in course taking could also suggest that accelerated access to abstract mathematics content negatively impacted the confidence or enjoyment of students who had demonstrated high mathematics ability in earlier grades. However, it is important to note that Smith did not provide data on the availability of calculus courses at the students' high schools nor mathematics graduation requirements. Both of these contextual conditions could also account for the small number of calculus takers. Finally, the study's achievement measure of discrete and algebraic mathematics understandings used in Grade 12 was the same as the measure used in Grade 10. This measure would not have captured the deeper conceptual understandings that students on a calculus preparatory track should have developed in their advanced secondary coursework.

Rickles (2013) elaborated on the challenge of controlling for the "web of influences" that Smith (1996) described by accounting for selection bias which systematically favors higher-achieving students. Using a nationally representative sample of eighth-grade students from the National Education Longitudinal Study of 1988 (NELS:88), Rickles modeled differential factors of placement and outcomes using regression analyses with inverse propensity score weighting for student, family and school characteristics. Although Rickles concluded that Grade 8 Algebra I had a positive effect on Grade 12 mathematics achievement test scores, pre-existing differences such as parents' education, gifted status, and prior grades in mathematics courses between algebra and non-algebra students accounted for at least half of the difference (p. 263). Rickles also noted the limitation of assuming that one student's course placement did not affect another's outcome. He speculated that there could be a potential watering down of course rigor with broader acceleration and that future research should examine how "classroom contextual effects mediate the effects of assignment to algebra" (p. 264).

Ma (2005a, 2005b) used hierarchical linear modeling with data collected annually for cohorts of 60 students from Grades 7 through Grade 12 in 51 middle and high school pairs within the Longitudinal Study of American Youth (LSAY) data set. This study compared the amount of mathematical growth and the degree of stability of growth across four mathematical areas (basic skills, algebra, geometry, and quantitative literacy) for accelerated and non-accelerated students. Mathematics achievement was measured yearly using National Assessment of Educational Progress (NAEP) test items, and additional levels of analysis accounted for student and student school characteristics. Like Smith (1996), Ma concluded that his analyses made the argument for broader access to Algebra I with appropriate investment in resources. Ma argued that with proper "school contextual properties" (2005b, p. 125), more regular students could be successful in accelerated mathematics.

More importantly, his findings with respect to gifted and honors students have interesting implications for the perceived affordances of hyper-accelerated study of mathematics content. Achievement growth was nearly equivalent for accelerated and non-accelerated gifted students and only slightly greater for accelerated honors in comparison with non-accelerated honors students. Ma hypothesized that lack of longitudinal difference in growth for gifted students was because early formal algebra coursework was not likely to offer a cognitive challenge nor elicit a greater motivation to learn. More importantly, school factors, including suburban school attendance and strong staff cooperation, explained 72% of the variance in overall achievement gains for accelerated gifted students. These findings are especially telling in light of recommendations for appropriate acceleration in NCTM's recent publication *Catalyzing Change in High School Mathematics: Initiating Critical Conversations* (2018). Mathematically promising students should have opportunities to pursue interests and engage in deep learning. Hyper-acceleration as a "race to calculus" may detract from these important opportunities, ultimately resulting in decreased persistence in STEM pathways.

All three researchers (Ma, 2005a, 2005b; Rickles, 2013; Smith, 1996) elaborated on the inherent limitations of standardized measures of achievement in characterizing the contexts of students' mathematics experiences. Ma (2005a) acknowledged that his study could not capture how well formal algebra was taught and that the small sample size limited his ability to isolate the years in which the greatest growth occurred.

The limitations of these studies are even more pronounced when considering the more recent phenomenon of hyper-acceleration. Both the HS&B data collections in 1980 and 1982 and LSAY cohort data collection between 1987 and 1993 occurred prior to the broad increase in Algebra I acceleration and the emergence of hyper-acceleration. The overall percentages of accelerated students [17% in Smith's (1996) study and 12.4% in Ma's (2005a) study] were well below the 42% of US students in recent NAEP data (Loveless, 2016). Rickles (2013) limited his sample to 941 schools with at least one student who studied algebra in eighth grade and one who did not study algebra in eighth grade. As a result, 41% of the students were accelerated in algebra. This percentage is more consistent with the most recent NAEP percentages, but Rickles could not have accounted for students who had studied algebra prior to eighth grade because his categorization only depended on the response to a question of whether the student took an algebra course during eighth grade. This distinction of degree of acceleration must inform future research on the timing of Algebra I.

Recent studies have reported on the unintended negative consequences in districts and states which adopted "algebra for all" policies in the 20 years since Smith's (1996) study (Clotfelter, Vigdor, & Ladd, 2015; Domina, McEachin, Penner, & Penner, 2015, Nomi, 2012). Longitudinal studies in four North Carolina school districts used quasi-experimental research with timing of Algebra I as the treatment variable and end-of course test results as the dependent variable. Researchers found "statistically significant harmful effects of acceleration" (Clotfelter et al., 2015, p.180) up to at least the 60th percentile of mathematics readiness as defined by sixth and seventh grade mathematics achievement. Over 80% of moderately performing students and 100% of low-performing students who had taken eighth-grade Algebra I failed to progress to calculus (Clotfelter et al., 2015). While no prior research has focused on hyper-acceleration as a separate phenomenon, these studies taken together raise questions about ways in which well-intended acceleration polices can have negative consequences with respect to content coverage and exposure for both mathematically promising students and those who are left behind in highly tracked middle schools. Broader access to Algebra I in Grade 8 could have the effect of moving the "gatekeeping" to Grade 7 or introducing ability grouping within Grade 8 Algebra I (Smith, 1996, p. 150), thus reducing or eliminating the affordances of acceleration. The contextual factors described by all three researchers underlie the need for qualitative examination of acceleration beyond quantitative achievement outcomes.

Middle school Algebra I: OTL as content emphasis. The above described studies of high school mathematics achievement for accelerated students (Ma, 2005a, 2005b; Rickles, 2013; Smith, 1996) did not explicitly address the subsequent secondary course pathways of students who study Algebra I in middle school. Course trajectories, or more specifically, decisions to retake Algebra I or to take courses over the summer, have implications for the quality of opportunities for deep understanding of pre-algebra, algebra, and precalculus content. Finkelstein and colleagues (2012) examined the mathematics course-taking sequences of 24,279 California students from Grades 7 through 12 using transcript data and standardized Algebra I, Geometry, and Algebra 2 test scores from 2004-2009 as proxies for achievement. The goal of this explanatory study was to quantify persistence on secondary STEM trajectories; their findings and discussion focused on negative effects for accelerated students who needed to repeat Algebra I to demonstrate mathematics proficiency for high school graduation. The researchers concluded that when students take Algebra I is less important than their readiness to access the content of the course.

While this statewide study focused on questions of broader access, I analyzed the published data to explicate assertions that broader acceleration of Algebra I to Grade 8 fuels hyper-acceleration of Algebra I to Grade 7 or earlier (Picciotto, 2014; Simzar et al., 2016). Fifty-six percent of students studied Algebra I in Grade 8 or earlier, yet only 35% of students continued on an accelerated pathway to calculus by Grade 12. Additional analysis of the published results indicated that nice percent of students achieved proficiency on the Geometry end-of-year exam in Grade 8; therefore, I inferred that nine percent of students successfully completed Algebra I in Grade 7 or earlier.

The 20 most common Grade 7-12 pathways in the overall data set accounted for 31.2% of the students. Four of the reported pathways began with Grade 7 Algebra, accounting for 4.97% of the total student sample. These pathways and percentages are described in Table 1. Further descriptive analysis of this data showed that 45% of the students who studied Algebra I in seventh grade repeated Algebra I in eighth grade and that 20% of them did not study calculus in high school. These high percentages suggest that hyper-acceleration may not be fostering the content emphasis that students need to persist on highly accelerated STEM pathways.

Table 1

Common Math Course Taking Patterns Beginning with Grade 7 Algebra

Grade 7	Grade 8	Grade 9	Grade 10	Grade 11	Grade 12	Percentage of Students
Algebra 1	Geometry	Algebra 2	Precalculus	Calculus	Statistics	1.43
Algebra 1	Algebra 1	Geometry	Algebra 2	Precalculus	Calculus	1.34
Algebra 1	Geometry	Algebra 2	Precalculus	Calculus	Calculus	1.28
Algebra 1	Algebra 1	Geometry	Algebra 2	Precalculus	Statistics	0.92

Note. Percentages calculated from dataset in “College Bound in Middle School and High School? How Math Course Sequences Matter” by N. Finkelstein, A. Fong, J. Tiffany-Morales, P. Shields, & M. Huang, (2012). Sacramento, CA: The Center for the Future of Teaching and Learning at WestEd.

The team acknowledged the limitations of large data set analysis of course trajectories because of the absence of insight about contextual inputs (Finkelstein et al., 2012, p. ix). They recommended that individual school districts or divisions replicate this type of analysis with respect to content and instruction and the transition from middle school to high school. Their study affirms the need for research which explores content emphasis in accelerated contexts beyond the simple reports of high school course completion. This additional analysis could be especially fruitful in understanding achievement and persistence on varying hyper-accelerated course trajectories.

Middle school Algebra I: OTL as quality of instructional delivery. Middle school teachers may have had less advanced coursework in algebra and function content in their preparation programs than high school teachers (Schmidt et al., 2007) and these differences yield statistically significant differences in mathematics knowledge for teaching (Hill, 2007). At the same time, there is a lack of research on how to prepare teachers of algebra with a balance of advanced mathematics content and "practice-based mathematics" (McCrory, Floden, Ferrini-Mundy, Reckase, & Senk, 2012, p. 587). Increasing access to Algebra I in earlier grades introduces a risk of instructional modification to focus on standardized test outcomes for students with lower measured aptitude or readiness. This "trimming" of content (McCrory et al., 2012, p. 605) to meet the anticipated needs of students in accelerated settings should not discount the more advanced mathematics ideas which the students will later encounter. With the potential for instructional emphasis on minimal competence as measured by standardized testing and the potential for less teacher fluency in algebra and function at the middle school level, OTL for mathematically promising students who are hyper-accelerated may be negatively impacted.

Research has also shown that learning opportunities may also be impacted in increasingly heterogenous Algebra I classrooms. In 1997, Chicago Public Schools implemented an algebra-for-all policy at the ninth-grade level. Allensworth, Nomi, Montgomery, and Lee (2009) concluded that low- and average-ability students experienced slightly higher subsequent math course failure rates with the new policy, and Nomi (2012) extended the analysis to identify the impact on students who would have taken Algebra I in ninth grade prior to the change in policy. She hypothesized that mathematics achievement could be adversely impacted by the resulting change in classroom composition. She used a short-interrupted time series quantitative design with adjustments for the change in the incoming skills of the students and to new retention policies that had been implemented the year prior to the broader access to Algebra. Nomi concluded that as peer ability levels declined by one standard deviation, high-skill student test scores would decline by 0.25 standard deviations (p. 500). Nomi also reported that the ninth-grade algebra-for-all policy had not been accompanied by professional development to support teachers in more heterogeneous classroom settings. Although the context of this study was universal access to Algebra I at the high school level, these findings may explain the decisions of some students and parents to hyper-accelerate the study of Algebra I to Grade 7 or earlier in pursuit of instructional rigor and classrooms with similarly achieving peers.

While the goal of increased access to Algebra I is often framed in the context of greater opportunity for challenging mathematics for all students, Stein and colleagues (2011) synthesized empirical evidence of universal and selective Algebra I acceleration policies and suggested that students in a non-tracked setting who would have been excluded from algebra under the “old rules” might be grouped together and taught a less rigorous version of algebra (p. 484). These “different algebras” are a reality as many middle schools with open-enrollment policies have created honors and non-honors algebra courses. The complexities of this relationship between OTL and student readiness for Algebra I acceleration create the need for “a research base that systematically covers all three: policy, instruction, and outcomes” (Stein et al., 2011, p. 486)

Stein and colleagues (2011) further called for a distinction between received and intended curricula in terms of courses offered, content covered, and pedagogical approaches. They argued that positive outcomes in contexts where students are selected for Algebra I do not have generalizable implications for OTL. When students are selected for acceleration, it is uncertain whether the algebra course taking is the true predictor for these positive outcomes or if other characteristics associated with the selection process or the conditions under which students took the course contribute to the positive outcomes.

This base of research literature examining the relationships between middle school algebra and high school mathematics outcomes, secondary mathematics course taking, and instructional rigor offers historical and methodological perspectives that should inform the design of new theoretical frameworks for research on the emergent phenomenon of hyper-acceleration.

Prior Research on Algebra I Acceleration and College Calculus Readiness

Because individual and policy motivations for the acceleration of Algebra I often include increased readiness for advanced STEM study, consideration of the preceding literature on secondary mathematics outcomes alongside literature on undergraduate calculus readiness more fully characterizes OTL in this emergent context. While the following studies were conducted to improve the quality of undergraduate calculus teaching, their findings offer profound implications for the conceptual foundations that must be built in the earliest algebra courses at the secondary level.

ACT (2017) used descriptive research to report STEM readiness for each test-taking student. In 2013, ACT defined a STEM benchmark combining typical grades from first-year college STEM courses, including calculus, in a single course success model to determine the ACT math and science scores associated with a 50% chance of earning a "B" or higher and a 75% chance of earning a "C" or higher (ACT, 2017, p. 3). ACT reported that only 21% of the 2017 ACT-tested high school graduating class met this benchmark designed to measure readiness for first-year STEM college courses despite broad access to Grade 8 Algebra.

Acceleration of Algebra I may shorten the window for students to build important foundational understandings for calculus, including proportional reasoning (Carlson et al., 2015; Sheffield, 2017); concept of function (Bardini, Pierce, Vincent, & King, 2014; Oehrtman, Carlson, & Thompson, 2008); and covariational reasoning (Bressoud et al., 2012; Smith & Thompson, 2007). Research on college calculus readiness (Sonnert, Sadler, Sadler, & Bressoud, 2015; Wade, Sonnert, Sadler, Hazari, & Watson, 2016) has the potential to enrich empirical understandings of Algebra I acceleration outcomes as an additional measure of OTL The traditional categories of OTL (Stevens, 1993; Tate 2004; Wang, 1998) of content exposure and coverage, content emphasis, and instructional delivery taken together with college calculus readiness contribute to a re-imagining of OTL which is more aligned with hyper-accelerated Algebra I contexts. Readiness for college calculus transcends traditional achievement measures of secondary mathematics mastery because of the need for connected conceptual understandings of precalculus mathematics. Thus, the content perspective on OTL in with hyper-acceleration of

Algebra I should encompass an instructional emphasis on conceptual understandings in preparation for the college calculus.

Preparation for college calculus: OTL as content coverage and exposure. Data on high school course trajectories and college calculus readiness is available from two national studies of first-year college students conducted within the last ten years. While these studies were not designed to examine secondary mathematics content and coverage, the results have implications for OTL with acceleration of Algebra I. In response to persistent high failure rates in entry-level STEM calculus courses, the MAA CSPCC study surveyed over 14,000 students at 213 colleges and universities enrolled in an entry-level calculus course required for science, mathematics, and engineering majors (Bressoud, 2015a). In his 2017 presentation at the NCSM annual conference, Bressoud estimated that only 150,000 of nearly 800,000 high school graduates who had previously studied calculus enrolled in Calculus II or higher in their first year of college. Over 67% of the CSPCC surveyed students had studied calculus in high school, and 33% of students failed to complete the college calculus course (Bressoud et al., 2015). Nearly 30% of students taking Calculus I in college also took precalculus in college even though they had done well in precalculus or even calculus in high school. While analysis of reported AP Calculus test scores could correlate examination scores with enrollment, there were no survey items on student reasoning for repeating calculus in college despite earning qualifying AP scores. Additional MAA surveys of high school course taking showed 13.5% of students with high school calculus credit took remedial (non-credit) math courses (Bressoud, 2015b).

High school course pathways were not a focus of the CSPCC study, yet my analysis of descriptive statistics from the survey dataset (MAA, 2017) again offers new justification for the study of hyper-acceleration as an emergent phenomenon. 10.5% of initial respondents reported having completed Geometry and Algebra 2 by the end of Grade 9, meaning that they likely completed Algebra I in Grade 7. In a continuing accelerated math course progression, these students would have studied precalculus or trigonometry during sophomore year followed by one to two years of AP, IB, or dual enrollment courses during junior and senior year. However, 26% of these hyper-accelerated students took another mathematics course in Grade 10. Because the study only reported on completion of Algebra 2, precalculus, calculus, and statistics, it is not clear what other course choices these students may have made.

Large-scale studies of high school course progression (Finklestein et al., 2012) and first-semester calculus experiences (Bressoud, 2015a) taken together raise questions about the development of conceptual understanding in preparation for STEM undergraduate study. When considered alongside recently published findings from the Factors Influencing College Success in STEM study (Sadler & Sonnert, 2018) the need for research on hyper-acceleration is clearer. End-of-course grades in precalculus math courses along with SAT/ACT scores explained more than twice the variability in college achievement than grades in high school calculus courses. This empirical evidence substantiates recommendations from professional organizations within the mathematics education community about appropriate acceleration (Bressoud et al., 2012; Larson, 2017, 2018; NCTM, 2018); the greater OTL may be associated with more time spent on algebra and precalculus topics and less with early access to calculus.

The students in the CSPCC study also responded to survey questions about their high school preparation at the beginning and at the end of Calculus I. At the beginning of the course, 81% of students felt prepared and 16% of students felt somewhat prepared. At the end of the course, only 56% of students felt that their high school mathematics experiences prepared them for calculus, while 31% felt that they had been somewhat prepared. The difference is more pronounced than this simple change in percentage; the end-of-course survey was only answered by the 67% of surveyed students who completed the course. The responses of students who did not complete the course would likely have increased the percentage of students who did not believe that they were well prepared for college calculus.

High school course selections and grades after acceleration of Algebra I depend upon a complex interaction of influences at the student level (prior achievement, confidence, interest, and motivation), the teacher level (recommendations, instructional emphasis); school level (structures of tracking, peer expectations); and outside of school level (parent expectations, college admissions guidance). An expanded sociocultural lens on OTL in relation to acceleration of Algebra I can supplement the findings of the previously described large-scale quantitative analyses and can offer important insight about demonstrated readiness and perceptions of algebraic foundations as constructed in the timing and trajectory of high school courses.

Preparation for college calculus: OTL as content emphasis. The importance of algebraic mastery, particularly with respect to functions, has guided significant mathematics research on college calculus readiness (Carlson et al., 2015, Ferrini-Mundy & Gaudard, 1992; Oehrtman et al., 2008; Stewart & Reeder, 2017; Thompson, 2013). These studies have examined the mathematical understandings as expected and as observed in post-secondary gatekeeper calculus courses. The findings have implications for secondary content emphasis as higher-order problem solving as an aspect of OTL (Wang, 1998). Concerns about lack of algebraic proficiency, particularly in relation to concept of function, have prompted this research. Mathematics education experts have long advocated for deeper conceptual understandings of algebra and function (Bardini et al., 2014; Carlson et al., 2015; Common Core Standards Writing Team, 2013; Eisenberg, 1992; Harel, 2007; Thompson, 1994; 2013) in preparation for calculus.

Problem solving foundations, specifically with respect to concept of function and quantities changing together, are built in secondary algebra coursework. Students with high school calculus experience have been shown to perform significantly better in college calculus than students without high school calculus experience, particularly on procedural items can measure memorization (Ferrini-Mundy & Gaudard, 1992). However, these outcome differences disappeared by the end of second semester calculus. The researchers concluded that instead of completing calculus, secondary mathematics experiences should focus on a “solid grounding in the understanding of functions and the conceptual underpinnings of calculus” (p. 68).

The mathematics education research community has a history of recognizing that functional understandings can offer either a window on sense-making in advanced mathematics or a barrier to learning. ”The idea of function as a static “thing”, which when introduced too early, is doomed to remain beyond the conception of many students” (Sfard, 1992, p. 77). Carlson (1998) further suggested that students may exit the undergraduate mathematics pipeline prematurely when they feel that the pace of instruction substitutes memorization for understanding.

Preparation for college calculus: OTL as instructional delivery. Preparing accelerated students to succeed in college calculus classrooms requires emphasis on conceptual understanding and pedagogical practices which engage students in quantitative reasoning and attend to mathematical meaning (Thompson, 2013). Wade et al. (2016) contrasted the perspectives of high school calculus teachers and college calculus professors on preparing students for success in transitioning from high school to post-secondary mathematics. The researchers compared responses of 84 high school pre-calculus and calculus teachers and 185 college calculus professors nationally representative survey data from the FICSMath project in 2009-2010. Their perspectives were coded and sorted by frequency; the results were organized in a cone of similarity as shown in Figure 1. While 24% of the statements made by professors addressed algebra content or pedagogy, only 4% of the statements by high school teachers addressed algebra. One professor stated, “Our students who struggle as they progress through the calculus sequence are generally not struggling with calculus but with their limited algebra skills. The students with the weakest algebra skills are the students to most likely drop out of the calculus sequence" (p. 11). The professors emphasized the need for work on families of functions, composition of functions, and function notation (Wade, 2011) and argued that a deeper focus on functions was a better preparation for tertiary calculus than secondary calculus.

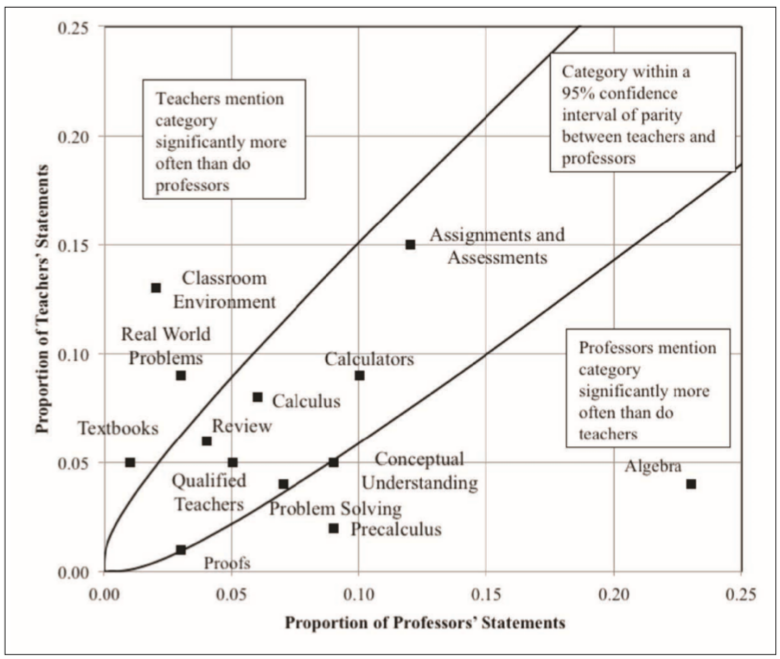

Figure 1. Cone of Similarity - created from the proportion of statements made by the teachers and professors. Retrieved from "A Comparison of Mathematics Teachers' and Professors' Views on Secondary Preparation for Tertiary Calculus" by Wade et al., 2016.

Further evidence of the differing orientations toward preparation for calculus was characterized by descriptions of Wade and colleagues of pedagogical expectations.

> Whereas the teachers had a stronger focus on the teaching process, the paramount concern for the professors was what students knew (or did not know) …the main area of disparity was that professors opined that there should be more focus on

algebra and precalculus without teaching secondary calculus, while teachers thought that calculus provided a means of reviewing algebra and precalculus concepts. (Wade et al., 2016, p. 13)

NCTM (2018) recently convened a High School Mathematics Task Force to identify the challenges of making Grade 9-12 mathematics work effectively for each and every student. They cautioned against acceleration as a "race to calculus" (p. 19) at the expense of building foundations in algebra and concept of function. Yet they acknowledged that the perceived value of completion of high school calculus as an advantage in competing for college admissions and earning college credits will continue to motivate students, parents and teachers to push for early Algebra I.

The assumed advantage which underlies the drive toward early calculus drives the need for research that connects hyper-acceleration as an emergent educational structure to OTL which relates the learner to the learning environment (Gee, 2008). The reality of politics and the perception that "faster is better" is likely to continue to drive the political discourse in high-achieving school districts in the absence of new knowledge. Questions about appropriate acceleration and college calculus readiness cannot be answered by large data set research in isolation; the stories of those who experience Algebra I in Grade 7 offer a novel entry point for research on OTL which reveals the breadth and depth this emergent phenomenon.

Reimagining OTL from a Situative Perspective

The earliest conceptualizations of OTL were research constructs assessing "whether or not students have had the opportunity to study a particular topic or learn how to solve a particular type of problem presented by the test" (Husen, 1967 as cited in Burstein, 1993, p. xviii). OTL was defined traditionally in terms of content taught, adequacy and allocation of educational resources, and classroom processes and practices (Pullin & Haertel, 2008, p, 17) as they relate to interpretations and uses of test scores.

OTL has also been used as a guiding idea in educational policy (McDonnell, 1995) to reframe the mathematics achievement gap in terms of inequitable access to experienced and qualified teachers, high expectations for success, appropriate challenging curricula, and per pupil funding (Flores, 2007; Schmidt et al., 2011) for traditionally disadvantaged groups as defined by race, socioeconomic status, or disability. Yet OTL as a series of discrete constructs of content coverage, content exposure, content emphasis, and instructional delivery are challenging to separate in psychometric research (Wang, 1998) and even more challenging to separate from individual student characteristics (Gee, 2008; McDonnell, 1995).

By traditional measures of OTL, high-achieving students are presumed to have access to advanced content and quality instruction (Larson, 2017) and are assumed to be proficient in the mathematics topics in the curriculum of the courses. Gee (2008) argued for a "mutually informing discussion" (p. 76) between researchers who employ psychometrics and researchers who employ sociocultural theory in the examination of learning. He further identified three complexities associated with the use of the content exposure and coverage measures of OTL: 1) they do not account for prior knowledge; 2) they do not necessarily distinguish between representations that are memorized and representations that are understood 3) and they do not distinguish between "input and intake" (p. 79) in the forms of content as taught and content as learned. Gee conceptualized "affordance/effectivity" pairs (p. 81) which distinguish between the resources or activities that are available to a learner and the capacity of the learner to take advantage of the available affordances. Within hyper-accelerated contexts, these affordances should be described as opportunities to engage meaningfully with content, to build problem-solving acumen, and to instill a desire to persist in advanced mathematics study.

Greeno and Gresalfi (2008) offered a situative perspective on OTL in which an individual's learning is conceptualized as a "trajectory of that person's participation in the community - a path with past and present, shaping possibilities for future participation (p. 170). Selection for Algebra I in Grade 7 or earlier, whether in the form of a sufficiently high score on test of mathematics ability, a teacher recommendation, parent selection, or a combination of the three, becomes a marker of success in mathematics. Yet this marker is subject to challenge as students move through the next six years of mathematics study in middle school and high school. Their experiences with mathematics are not fully described by traditional measures of mathematics achievement where content coverage and exposure are driven by standardized testing formats (Pullin & Haertel, 2008). Because hyper-accelerated students are often positioned as successful in mathematics in highly resourced schools which offer AP and IB calculus courses, OTL as traditionally conceptualized will not capture the important individual experiences with content and instruction that define this phenomenon.

Stinson (2004) specifically challenged mathematics educators to look at OTL beyond political structures such as tracking (Oakes, 2005) as he considered the societal gatekeeping role that mathematics often plays. He asked, "How might mathematics educators ensure that gatekeeping mathematics becomes an inclusive instrument for empowerment rather than an exclusive instrument for stratification?" (p. 8). While this question was posed with a critical stance on gatekeeping practices that limit the opportunities for students tracked into lower-level courses, Stinson's question is equally appropriate in considering the "tracking up" of middle school mathematics (Domina et al., 2016) which has accompanied broader access to Algebra I. The offering of more advanced mathematics courses in middle school is grounded in the desire of students, parents and teachers to maintain a competitive edge for college admissions for some students, but it also becomes a new gate that redefines opportunities for college admission and economic success.

The current lack of research on hyper-acceleration drives the need to probe students' individual yet shared longitudinal experiences. Students may begin to struggle to make conceptual sense of advanced material along multi-year trajectories and begin to question their previously constructed sense of competence and success in mathematics. Research in hyper-accelerated contexts must also be conducted with specific attention to conceptual readiness for college calculus combined with emergent beliefs about success as doers of mathematics. A situative perspective of OTL (Greeno & Gresalfi, 2008) can explicate the cognitive and pedagogical challenges of building a foundation for college calculus and evolving mathematical identities as framed by the context and culture of the classroom and the school. Prior research has shown that students may not necessarily be challenged to think deeply, to justify their reasoning to others, or to make connections they operate in figured mathematical worlds of didactic instruction and procedural routines that often characterize advanced mathematics classrooms (Boaler, 1997; Boaler & Greeno, 2000; Brahier, 1995; Schoenfeld, 1988). Examining hyper-acceleration as a mathematical figured world (Holland et al., 1998, Horn, 2008) may explicate important differences in and influences of student, parent, teacher, counselor and administrator beliefs about mathematical competence and success.

Hyper-accelerated contexts as figured mathematical worlds. Figured worlds in secondary mathematics have been defined at the classroom level by Boaler and Greeno (2000) and at the school level by Horn (2008); they are especially appropriate for the study of the hyper-acceleration of Algebra I as a sociocultural phenomenon. Boaler and Greeno (2000) and Horn (2008) drew on the work of Holland et al. (1998) as they examined identity formation from both narrative and positioning perspectives. Holland and colleagues introduced figured worlds as selective contexts in which positional identities are formed through “socially produced, culturally constituted activities” (pp. 40-41) which evolve with the actions of the participants as mediated by perceptions of “power, deference, entitlement, social affiliation and distance” (Holland et al., 1998, pp. 127–128). Horn (2008) elaborated on these perspectives on identity and agency as “self-understandings students develop about their relationships to the subject of mathematics and the understandings that are assigned to them through their position and encounters in the social world (p. 204). Within these figured worlds, “people are not only regarded as mathematics learners, assuming the cognitive order of the discipline, but people are negotiating a sense of self" (Boaler & Greeno, p. 189). In their interviews of 48 students from eight California high schools with high numbers of college bound students, Boaler and Greeno (2000) regarded all interview subjects as "successful students of mathematics" because they were enrolled in a fourth year of advanced mathematics (p. 175) and were appropriately positioned to continue the study of mathematics at the college level. Students from didactic classroom environments described a ritual of procedural demonstration and repetition with an expectation of individual work and results. The teachers positioned themselves as sole authorities who presented mathematics as a "closed, rules-bound subject" (p. 180) and positioned students as producers of correct answers. The informational resources of one-dimensional, formulaic learning and the absence of interpersonal resources in the didactic environment positioned students as received knowers in which passive learning was separated from active thinking. In contrast, students from discussion-based classrooms shared experiences of agency and collaboration in a community of shared learning. The interpersonal resources in the discussion-based environment positioned students as active, connected knowers. When teachers position themselves as facilitators of the construction of knowledge, cooperative mathematical discourse became shared thinking and learning which fostered conceptual understanding. Received knowing versus connected knowing as defined in this study parallels Gee's (2008) characterization of content input versus content intake that can define OTL advanced secondary classrooms.

High school mathematics identities are not only constructed in within classroom contexts but also across classrooms and schools. Horn (2008) conceptualized broader figured worlds in mathematics created by “course sequences that define the four-year mathematics curricula as they are lived, represented, enacted and negotiated by the students, teacher, and others who inhabit them” (p. 204). Horn studied the lived experiences of seven mathematics students from two high schools who had shown unexpected mathematics engagement in their first college preparatory courses. These “turnaround” students had differing access to resources beyond the classroom that directly impacted their abilities to position themselves as competent doers of mathematics and to continue to progress in a college preparatory curriculum. Students in an environment with teachers as collaborators who share a belief in the potential of students who demonstrate competence beyond the traditional definitions of mathematical success were more persistent and experienced more positive outcomes. Students also benefited from adapted curricula without “stigmatized spaces” of separated instruction in the form of remediation (p. 222).

While this study focused on the creation of more equitable college preparatory course sequences for traditionally underserved learners, Horn’s (2008) sociocultural framework of figured worlds is uniquely relevant for examining the experiences of hyper-accelerated learners who begin to struggle with abstract mathematics thinking and begin to question their continuing participation in advanced mathematics in their early high school years. Numerous studies have explored student identities as they position themselves or are positioned by others as “good at mathematics” (Bishop, 2012; Darragh, 2015; Mendick, 2005; Radovic et al., 2017). The decision to pursue a hyper-accelerated trajectory is a very public marker of mathematical competence, or a measure of intellect and capacity. Students' unwillingness to acknowledge any threats to this marker, in such equally public forms as repeating a course, moving to a non-honors course, or asking for assistance from a peer or a teacher, can redefine or end their continuing participation in advanced mathematics. "Student identities are constructed in particular contexts that define what it means to do math, what it means to be successful, and how such success might be positioned in interaction and over time" (Horn, 2008, p. 204). Advanced students may be at greater risk of constructing identities of competence as they enroll in Algebra I earlier than their peers but later struggle to master content in advanced courses and are advised to change their course progressions. These experiences may create newly "stigmatized spaces" (Horn, 2008) for students who are still on strong college preparatory trajectories.

Actors within hyper-accelerated mathematics figured worlds can encourage students with high prerequisite achievement to develop the agency that they need to persist in the mathematics pipeline and encourage them to take risks in the pursuit of deeper understandings which transcend grades. "If their opportunities for learning are limited to acquiring procedures, then their understanding and perception of mathematics can easily be limited to aspects of mathematical thinking in which the human agent is relatively passive" (Boaler & Greeno, 2000, p. 195). Emphasizing symbolic procedural competence in hyper-accelerated contexts may become an unintended barrier to more creative and collaborative application of mathematics in later courses. Student beliefs about memorization and correctness in mathematics may restrict their capacity to invest in opportunities for deeper conceptual understandings.

Research on the joint accomplishment of identity between individuals and their participation in the mathematics pipeline (Hand & Gresalfi, 2015) is especially important for accelerated students as they negotiate contextual ways of knowing and social constructions of success in mathematics. Perceptions of what it means to be successful in mathematics become part of a situative perspective on OTL which can explain persistence in the mathematics pipeline and conceptual readiness for STEM study.

Potential constructs for conceptualizing situative OTL. Prior research on tracking (Oakes, 2005) has focused on resulting disadvantages for low-achieving students, and there is an assumption that high-performing students necessarily benefit from access to accelerated courses (Larson, 2017). Boaler (1997) hypothesized that instead of facilitating challenges that promote deeper learning and understanding for these students, top set (most advanced) courses have negative effects on their identities as doers of mathematics with a stronger effect on girls. Using a three-year longitudinal case study in the United Kingdom, Boaler characterized individual anxiety, enjoyment, and achievement using classroom observations, questionnaires, and achievement tests for students in the top set courses of study in a school with eight tracks of instruction. Students were assigned to tracks based on entrance exams and achievement in prerequisite courses. Boaler observed that the increased pace of instruction and expectation of correct answers in the top set courses communicated to students that speed and memorization defined success in mathematics. "Students were often confused by mathematics lessons, sometimes managing to learn how to use a method, without any understanding of what it meant or how to use it; at other times not even managing this." (p. 170). The teachers articulated and demonstrated heightened expectations for these students that contributed to the competitive classroom environment. Boaler concluded that instructional pace and pressure to perform resulted in disaffection toward mathematics. Students were not confident that they would be able to use their learning in the future.

This 20-year-old contextual analysis offers a timely empirical challenge to the contemporary assumption that a more meaningful OTL is an affordance of hyper-acceleration. Boaler (2015) continues to advocate for meaningful detracked mathematics experiences in which students "ask good questions, map out pathways, reason about complex solutions, set up models and communicate in different forms", yet there has been no research on the contextual learning opportunities for hyper-accelerated students.

Students' perceptions of themselves as doers of mathematics has been encapsulated within the construct of productive disposition. The National Research Council (NRC, 2001) defined productive disposition, one of the five strands of mathematics proficiency, as "the habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence, and one's own efficacy." (p. 116). Students should be challenged to think deeply, to justify their reasoning to others, to make connections, to understand abstraction, and to apply their knowledge in rich problem-solving contexts. Former NCTM President Linda Gojak (2013) challenged the privileged view of middle school Algebra I as a status symbol (p. 2) and advocated for meaningful and relevant mathematics experiences with pre-algebra content. She also warned of the potential negative consequences for mathematics disposition if a student's mathematical struggle in accelerated coursework becomes unproductive. Stein et al. (2011) reported on two studies of mathematics disposition of students taking Algebra I in eighth grade (Brahier, 1995; McCoy, 2005) showing negative effects on confidence, engagement, and attitude. Brahier (1995) studied the mathematics disposition of students enrolled in eighth-grade algebra in 19 diverse Catholic schools. He defined mathematics disposition using the five components of the NCTM evaluation standards: interest and curiosity, perseverance, confidence, flexibility, and valuing the application of mathematics. Most students were motivated by a "desire to outperform" (Brahier, 1995, p. 5) their peers, and students and their parents felt that algebra was only important "because it was a prerequisite for other classes" (p. 7). His commentary offers a historical perspective about the emergence of hyper-acceleration as a marker of success in mathematics and the accompanying challenges of building meaningful mathematical experiences for young algebra learners.

> Since most of the lessons followed a traditional path of checking homework, "showing" new sample problems, and allowing students to begin their homework, there was little opportunity for students to demonstrate positive dispositions. Though very infrequent in my observations, the classroom experiences that involved teamwork, calculators, and "real-life" problems appeared to evoke positive disposition. The only way that a student can get out of this cycle and develop a positive disposition, therefore, is for the teacher to instruct in a way that

> would assist the student in appreciating that algebra is worth knowing in and of itself. (pp. 6-7)

Productive disposition and identities of competence in mathematics may be overlapping constructs, and more recent literature from scholars who take a sociocultural perspective on mathematics teaching and learning situates productive disposition within conceptualizations of mathematical identity. Aguirre, Mayfield-Ingram, and Martin (2013) defined mathematical identity as “dispositions and deeply-held beliefs that students develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways across the contexts of their lives” (p. 14). This definition reflects the current emphasis in mathematics education on meeting the needs of each and every learner along with the necessity of considering productive disposition, context, and achievement together in building mathematical competence (Berry, 2018). For hyper-accelerated learners, OTL as measured by achievement alone is taken for granted because they are successful by traditional metrics. However, students’ pursuit of accelerated mathematics as a marker of distinction may negatively impact their identity formation as productive doers and users of mathematics. Expressed identities, including productive dispositions, thus becomes an important accompanying characterization of OTL.

Conceptualizing OTL at the intersection of achievement and identity. Theoretical frameworks examining the co-construction of identity and mathematics knowledge have become increasingly prominent in mathematics education research (Bishop, 2012; Boaler & Greeno, 2000; Cobb, Gresalfi, & Hodge, 2009; Darragh, 2016;

Hand & Gresalfi, 2015; Horn, 2008; Larnell, 2016; Radovic et al., 2017; Sfard & Prusak, 2005). Hand and Gresalfi (2015) conceptualized identity as "one's participation in and across activities and the sense one makes of oneself in relation to these activities" (p. 191). Classroom context creates interpersonal and informational resources of participation that inform student beliefs about mathematics and about themselves (Hand & Gresalfi, 2015). Darragh (2016) proposed that identity as a construct can zoom in on individual experiences and zoom out on the sociopolitical discourses in which the individual is positioned. Wenger (2010) affirmed identity as a learning trajectory which is "not an object, but a constant becoming….our identities incorporate the past and the future in the very process of negotiating the present" (pp. 133-4). Bishop (2012) defined a situated yet malleable identity as "the ideas, often tacit, one has about who he or she is with respect to the subject of mathematics and its corresponding activities." (p. 39)

With the sociocultural turn (Lerman, 2000, Martin, 2013) and the sociopolitical turn (Gutiérrez, 2013) in mathematics education, researchers have decried the absence of theoretical consistency in conceptualizations of identity. Darragh (2016) described a dichotomy within the identity literature of mathematics education as she contrasted the psychological view of identity as acquired and the sociological view of identity as action. Identity as action serves the important purpose of "widening the lens" of identity by addressing the roles of social groupings and power (p. 29). Darragh analyzed 188 articles from 85 peer-reviewed journals and categorized research on mathematics identity as participative, narrative, discursive, psychoanalytic, or performative as constructed by social contexts. Radovic and colleagues (2018) challenged the theoretical overlap of

Darragh's (2016) five categorizations by drawing a distinction between conceptualization of identity and operationalization of identity. They offered three dimensions as a framework for conceptualizing identity (social/subjective, enacted/representational and change/stability) and five operationalizations of identity as individual attribute, narrative, relationship with specific practice, ways of acting, and the result of affordances/constraints of local practice. They also reported that enacted conceptualizations of identity were more prevalent in research in primary education, while representational and subjective conceptualizations were more prevalent in the secondary and post-secondary education, with narrative methodologies being the most common.

Research in secondary mathematics classrooms (Boaler & Greeno, 2000; Cobb et al., 2009; Darragh, 2016; Gresalfi & Cobb, 2011), and within secondary mathematics departments (Boaler & Staples, 2008; Horn, 2008) has offered empirical evidence of the complex connectedness of content experiences and mathematics identity. A situative perspective on OTL in hyper-accelerated contexts has the potential to explain potential challenges to identities of smartness in mathematics. Students may become more self-aware of threats to their evolving mathematical competence or may feel less connected to mathematical understanding. The affordance of access to more advanced mathematics content must be paired with effectivities, or capacities to act on these affordances (Gee, 2008) as participants in hyper-accelerated mathematics worlds. When there is a disconnect, the presumption of additional OTL is flawed.

The work of Boaler and Greeno (2000) and Horn (2008) together with the recent conceptualizations of identity in the theoretical work of Darragh (2016) and Radovic et al. (2018) inform a theoretical framework (Figure 2) for understanding OTL at the intersection of achievement and identity within figured worlds of hyper-acceleration. A situative perspective foregrounds the positioning of competence in mathematical interactions (Greeno & Gresalfi, 2008) as an evolving identity along a trajectory of experiences with advanced mathematics. OTL can thus be understood through retrospective narrative that describes the fluidity (Hall, Towers, & Martin, 2018) of individual's identities as they navigate hyper-accelerated contexts. Consistent with traditional conceptualization of OTL as a "relationship between schools and learning" (Floden, 2002), this framework acknowledges Gee's (2008) three complexities of the content perspective and values student stories in the evaluation of the quality of learning.

Within this situative framework, OTL is at the intersection of achievement and identity within hyper-accelerated contexts. I define achievement as persistence in the advanced mathematics pipeline and gained conceptual understandings beyond procedural recollection and replication which prepares students for success in college calculus. I define mathematics identity as characterizations of success and competence as constructed by structures and stakeholders in within and beyond hyper-accelerated classrooms.

Informed by Gee, Greeno, and Gresalfi and their sociocultural/situative perspectives on OTL, Moss (2008) tailored Engeström's (2001) questions for a theory of learning to propose criteria for evaluating OTL. I further adapted her questions (Table 2) to provide a structure for data collection and analysis that attends specifically to achievement and identity narratives in hyper-accelerated contexts. The italicized text is taken directly from her framework, and additional criteria were added during the study design and during data analysis.

Table 2

Narrating OTL in Hyper-accelerated Contexts

What mathematics did you learn?
• *Engagement with* conceptually rigorous algebra and functions content *in preparation for* college calculus • Valuation of meaning behind formulas and procedures • *Promotion of identities as learners who acquire habits of mind that sustain* advanced mathematics *learning*
How have you learned mathematics (within the classroom)?
• *Instructional tasks and activities with high cognitive demand* • *Explicit instruction characterized by models of* mathematical *expertise and problem solving, including use of* algebraic *reasoning, reasoning, and representation* • *Participation in activities which permit meaningful contributions to the group's work and foster interest in and motivation for continued* STEM *learning*
How have you learned mathematics (within the school)?
• *Social scaffolds to support learning* • *Strong teachers who are capable of teaching* conceptually rigorous content • Supportive relationships with peers • Competitive relationships with peers
Why did you learn mathematics?
• *Experiencing* mathematics *learning as* challenging, *meaningful and useful* • *Formulating and addressing problems* • *Questioning and evaluating conclusions* • Competing for elite college admissions • Aspiring to enter a STEM career • Maintaining identities of smartness

Note: Adapted from Moss (2008) in Criteria for Evaluating the Quality of Learning and Opportunity to Learn (p. 236) in *Assessment, Equity, and Opportunity to Learn.*

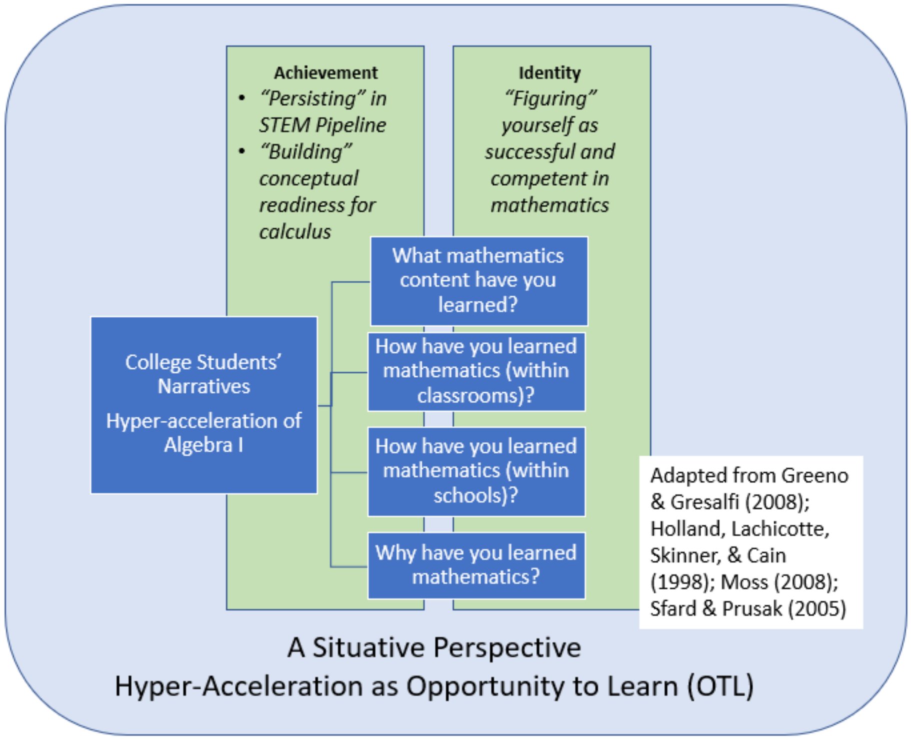

Figure 2. Theoretical framework: Hyper-acceleration as opportunity to learn (OTL)

It is important to emphasize that each of the aforementioned scholars are motivated by a more equitable measurement of OTL that challenges traditional forms of assessment. They pursue a more democratic schooling process that fosters a culture of lifelong learning for all students. Hyper-accelerated students have privileged access to resources and curriculum by virtue of their placement in their courses, the amplification of their voices in characterizing OTL in their experiences of hyper-acceleration will enhance empirical understandings of mathematical activity within the broader context of Algebra I learning trajectories within and across schools.

OTL as achievement and identity narratives. OTL in hyper-accelerated contexts can be both conceptualized and operationalized as the achievement and identity narratives of students in hyper-accelerated Algebra I contexts. Sfard and Prusak (2005) described identity as the missing link for researching the "complex didactic" between learning and sociocultural context (p. 5). Identity as narrative is "reifying, endorsable, and significant" (p. 17), and narrative is a triangulation between the identified person, the author, and the recipient which evolves over time (Hall et al., 2018; Radovic et al., 2018).

Sfard and Prusak (2005) foregrounded their operationalization of identity as narrative by stating that stories are not expressions of identity but are the identities themselves. Therefore, identities become a valuable tool for educational research which acknowledges the roles which humans play in shaping learning outcomes. By extension, operationalization of achievement and identity together as narrative in hyper-accelerated figured worlds becomes a research lens on characterizes OTL in meaningful ways beyond traditional modes of content assessment.

In the research of Sfard and Prusak (2005), the stories of Leah, an eleventh-grade advanced mathematics Israeli student, and Sonya, her immigrant classmate, were not descriptions of their identities. They were their identities, and arguably these identities both described and affected the quality of their learning. Leah's learning was ritualized to adhere to the rules of a social game with the goal of university acceptance and thus was less responsive to problem solving in unexpected contexts, while Sonya's learning was substantial because mathematics learning was part of being a complete human being and to support her professional aspirations. The 1377 citations of this article offer evidence of the operationalization of identity as narrative in the sociocultural turn (Lerman, 2000) in educational research. Their figured worlds became “horizons of meaning against which incidents, acts and individuals are interpreted” (Urrieta, 2007, p. 109). Learning opportunities for Leah and Sonya were thus constructed in the form of their narrated identities.

Larnell (2016) drew on Sfard and Prusak (2005) in his use of identity as narrative to examine the lived experiences of Black students in non-credit bearing remedial (NCBR) mathematics courses. Their contexts were defined by disproportionate enrollment of African American and Latinx students in NCBR courses and the lack of opportunity to build mathematics proficiency and productive disposition because of narrowly focused procedural curricula. His theoretical framework described the construction of mathematical identities in specific socialized educational spaces as narratives. He interpreted the words of his participants in terms of motivation to attain mathematical knowledge, opportunities and strategies for learning, constraints and barriers to participation, and individual capacity to perform (p. 240).

Larnell’s (2016) work illustrates the potential for employing Sfard and Prusak’s (2005) identity as narrative to make explicit both achievement and identity in constructing OTL from a situated perspective. One of his four longitudinal case studies focused on the lived experiences of Cedric, a high school salutatorian and first-generation college student. Cedric successfully completed differential calculus as a high school junior, did not study mathematics during his senior year, and planned to major in psychology. He described his high school mathematics experiences as challenging and worthwhile. “Cedric identified as a learner who appreciated productive struggle while doing mathematics and identified strongly with friends who were also top students” (Larnell, 2016, p. 255). Despite his previous success in mathematics and his recall of being described as a “natural” by high school calculus teacher, Cedric acted counter to his identity during his online mathematics placement. He recalled that he had not studied mathematics during his senior year, had rushed through the test, and had responded “I don’t know” to many of the questions.

Larnell (2016) described Cedric’s act of agency in placing himself in remedial mathematics as curious and theorized that Cedric believed that this level of mathematics was a sufficient starting place to continue to be successful. During his progress in the NCBR course, Cedric described a racialized identity threat as he questioned the meaning and significance of calculus classes without Black students and NCBR classes with primarily black students. While Larnell’s (2016) identity-oriented research was intended to inform policy and practice in NCBR courses, it also points toward potential value of operationalizing achievement and identity together as narrative in describing OTL for highly accelerated students. Students who do not study mathematics during senior year in high school are at greater risk for placement in remediated mathematics in college (Finkelstein et al., 2012), yet mathematics knowledge should not have an expiration date and meaningful learning is not simply forgotten. Cedric’s achievement narratives of success in accelerated mathematics classes contradicted his evolving identity as a mathematics student. Taken together, these stories explicate OTL as identity in accelerated secondary mathematics education.

Hyper-acceleration as a Matter of Access and Equity

OTL can be made explicit in the narratives of hyper-accelerated students whose individual notions of smartness and success evolve in parallel with participation in advanced courses on multi-year mathematics pathways. The intersection of achievement (persistence and conceptual understanding from a content perspective) together with figured identities of success and competence from a situative perspective can create compelling individual stories of OTL within the phenomenon of hyper-acceleration of Algebra I.

Acceleration of algebra to middle school is often framed as an issue of access to rigorous and relevant mathematics at the entrance to the secondary mathematics pipeline, however, it cannot be separated from its equally important role of building a foundation for persistence and productive dispositions in mathematics. Experiences with college-level mathematics content prior to the transition from high school to college calculus should prepare students for STEM success, yet research on insufficient mastery of algebraic skills and understanding of concept of function at the undergraduate level (Carlson et al., 2015; Bressoud et al., 2015, Sadler & Sonnert, 2018; Stewart & Reeder, 2017) challenges the quality of this preparation. Meaningful mathematics experiences should be available at all levels of readiness (NCTM, 2018), and hyper-accelerated mathematics in high school should be more than simply cultural capital in a game of dominant mathematics (Bourdieu, 1986; Gutiérrez, 2002) and a ticket to elite college admissions. It should instead open doors to STEM study for increasing numbers of interested students.

Over 70% of identity literature reviewed by Radovic and colleagues (2018) related to questions of equity, and prior research on OTL is motivated almost exclusively by psychometric questions of fairness in testing or questions of equity. In contrast, the phenomenon of hyper-acceleration is likely motivated by stakeholder presumptions of access and power. Questions of equity in mathematics education as a whole are deeply entrenched in schools where hyper-acceleration becomes a form of "tracking up" (Domina et al., 2016) , yet I must acknowledge that they are not made explicit in my OTL framework. As we come to understand the unintended consequences of hyper-accelerated mathematics study, we may find evidence of the ways in which these emergent course pathways drive or are driven by the mathematics of non-accelerated pathways. Advocates for meaningful secondary mathematics experiences which prepare students for postsecondary success must seek to publish research that offers a necessary counter narrative to the push to accelerate mathematics study. Capturing individual experiences from content and situated perspectives within the emergent phenomenon of hyper-acceleration in secondary mathematics may offer important evidence that can ultimately yield more equitable OTL for each and every student.

Conclusion

A situative perspective on OTL which integrates experiences with content and evolving identity is vital to understanding hyper-acceleration as promoting or restricting access to rich mathematics learning. Greeno and Gresalfi (2008) proposed that OTL describes the "affordances for student participation that support trajectories toward stronger valued capabilities and dispositions" (p. 193). In hyper-accelerated contexts, these affordances should include problem-solving opportunities which are high is cognitive demand. These learning experiences can build strong precalculus conceptual understandings, promote meaningful social interaction, and encourage participative structures which elicit conceptual agency. Yet these affordances (Gee, 2008) can only be realized with purposeful actions by teachers and students. Culturally situated beliefs about smartness and success in mathematics can either enable or impede progress in these learning environments. The effectivities (Gee, 2008), or capacities to benefit from these affordances, are mutually defined by each of the actors in figured worlds (Holland et al., 1998) of hyper-acceleration. With a situative perspective of OTL, hyper-acceleration becomes more than a faster exposure to mathematics information that is measured by grades and standardized test scores; it instead focuses on the identity development of the student as a doer of mathematics and achievement as persistence and pursuit of deeper conceptual understandings.

Chapter Three – Methodology

Chapter Overview

The research domain of mathematics education has its foundations in the positivist traditions of mathematics and experimental psychology (Ernest, 1999), yet a growing attentiveness to sociocultural and sociopolitical influences on teaching and learning (Gutiérrez, 2013; Lerman, 2000) has elicited new research paradigms. Schoenfeld (2004) described an unavoidable intertwining of mathematics and society and a perpetual struggle between equity and excellence that encapsulates the phenomenon of hyper-acceleration of Algebra I.

> According to popular belief at least, the facts of mathematics are universally true, its procedures universally correct, and both completely independent of culture…the counterpoint is that knowledge of any type, especially mathematics knowledge, is a powerful vehicle for social access and social mobility. Who gets to learn mathematics, and the nature of mathematics learned, are matters of consequence. (p. 255)

Research on OTL in hyper-accelerated contexts often relies on objective, standardized measures of mathematics content exposure and emphasis (Tate, 2004; Wang, 2000). Yet findings from these studies can be corroborated or challenged by the very subjective views that students, teachers, parents, and administrators in different contexts construct about avenues to success in mathematics. Prior research on acceleration of Algebra I has offered quantitative evidence of both positive consequences (Rickles, 2013; Smith, 1996; Stein et al., 2011) and unintended negative consequences (Bressoud et al., 2015; Clotfelter, Ladd, & Vigdor, 2015; Domina et al., 2015; Simzar et al., 2016). These studies rely on standardized test scores, course-taking and grades as measures of acquired mathematical understanding, yet prior researchers have acknowledged that Algebra I curriculum and instruction can vary from one context to another (Rickles, 2013; Schmidt, 2009; Stein et al., 2011). Such studies, informed by positivist and post-positivist ontologies, are veiled in a cloak of generalizability which cannot explicate the deeper implications of hyper-acceleration of Algebra I. Qualitative research which leverages students' retrospective stories of the phenomenon of hyper-acceleration of Algebra I can offer a contextual lens on OTL that is missing in large-scale quantitative studies of student outcomes.

For this dissertation study, I conducted five focus groups with recent high school graduates who studied Algebra I in Grade 7 and described their OTL from a situative perspective (Gee, 2008, Greeno & Gresalfi, 2008; Moss, 2008). A sequence of three narrative analysis techniques (Knight & Sweeney, 2007; Riessman, 2008; Webster & Mertova, 2007) using focus group and follow-on individual interview data for 15 participants supported my submission of two empirical research journal articles. The first manuscript, submitted to *Mathematical Teaching and Learning*, answered the following research question: How do recent graduates from one International Baccalaureate (IB) high school within a socioeconomically diverse suburban school narrate their OTL in mathematics after hyper-acceleration of Algebra I? The second article, submitted to *Educational Studies in Mathematics*, answered the following research question: How do high-achieving girls' rhetorical arguments of deceleration after Grade 7 Algebra I reveal gendered binary norms of participation in secondary and post-secondary mathematics?

My research questions reflect my desire to understand the lived realities of hyper-accelerated students as they strive to build mathematical proficiencies which are necessary to persist and succeed in advanced mathematics. Hyper-acceleration of Algebra I has the potential to introduce barriers to meaningful experiences with mathematics which are not necessarily reflected in research that measures larger trends in data such as completed coursework and standardized test scores. Within this qualitative study, achievement and identity converge to become a contextual lens on OTL in accelerated mathematics teaching and learning from a situative perspective (Gee, 2008; Greeno & Gresalfi, 2008; Moss, 2008). This OTL theoretical framework, as described in Chapter 2, informed my selection of a hermeneutical phenomenological philosophy and a narrative inquiry methodology to uncover the essence of student experiences with hyper-acceleration of Algebra I.

By situating my research within an interpretative phenomenological paradigm (Kim, 2016) for this project, I seek meanings in context (Mishler, 1995) by eliciting achievement and identity as narratives of college students from one IB high school who together reflect upon their shared experiences with hyper-acceleration of Algebra I. I bring my depth of knowledge as a former electrical engineer and as a high school mathematics educator to the search for profound understandings of ways in which “individual belief and action intersect with culture” (Denzin & Lincoln, 2011, p. 2) in focus groups and individual follow-on interviews.

In the subsequent sections in this chapter, I explain the selection of hermeneutic techniques of narrative inquiry to build empirical evidence of hyper-accelerated students’ experiences. Hermeneutic techniques must be understood with the broader realm of phenomenology as both a philosophy and a research movement (Eddles-Hirsch, 2015; Kim, 2016). I then describe the unique curriculum and assessment structures of two-year IB mathematics courses to argue for the selection of one IB high school as a rich context for the characterization of learning trajectories of hyper-accelerated students. I further argue that the narratives of college students from one IB high school provide a novel retrospective shared reflection on hyper-acceleration by which all of its affordances and challenges can be made explicit. I also describe my positionality as a veteran mathematics teacher at the same IB high school who brings her own deep understandings of this phenomenon to the data collection and analysis process. My role as a participant and a research instrument offer a more meaningful interpretation and construction of the multiplicities of truth which describe OTL with hyper-acceleration of Algebra I .

After describing the context of the study and its participants, I discuss the protocols for recruiting participants and collecting data in the from a series of focus groups and one-on-one interviews. I then present an intentional and iterative sequence of critical events analysis (Webster & Mertova, 2007), thematic analysis (Riessman, 2008), and structural analysis (Knight & Sweeney, 2007) which deconstructs and constructs students' lived experiences for each of the two articles in Chapters 3 and 4. I conclude by establishing the trustworthiness in this study and its limitations.

Phenomenology as a Theoretical Foundation for Narrative Inquiry

Meaningful reality is socially constructed (Crotty, 1998, p. 63). This ontological perspective underlies phenomenological research methodologies in which the perspective of the observer is "intertwined with the phenomenon" and is not expected to have objective characteristics (Mishler, 1979, p. 10). Within phenomenological paradigms, multiple truths can be revealed with a focus on the rich descriptions of participants' mathematical figured worlds (Boaler & Greeno, 2000; Holland et al., 1998; Horn, 2008). Qualitative investigations can capture the "essences and meanings of human experience" (Moustakas, 1994, p. 105) in which perceptions become the primary source of knowledge.

Eddles-Hirsch (2015) contrasted the philosophical postures of three different phenomenological approaches as attributed to Husserl, Heidegger, Sartre, and Merleau-Ponty in her resource for postgraduate educational researchers. In his transcendental phenomenology, Husserl (1999) theorized an alternative to the scientific method that captures a singularity of human consciousness and the world (Moustakas, 1994; as cited in Eddles-Hirsch, 2015). The researcher uses a bracketing technique to separate preconceptions, or natural attitude, from participant descriptions to describe the experiences under study. Husserl acknowledged the intentionality of the researcher in the examination of a particular phenomenon to uncover the underlying structures of noema, or objective experience of what is perceived, and noesis, or subjective experience of perceiving. Intuition and essence were necessarily separate, but Husserl acknowledged the "never-ending process" (Kim, 2016, p. 56) of bracketing one's own experiences because a researcher is continually building and revisiting personal experiences.

Heidegger, Husserl's student, argued that the researcher's intrinsic awareness could not be bracketed because of the impossibility of setting aside personal judgments. According to Heidegger, it is instead necessary to interpret human experience to fully understand the essence of a phenomenon. Heidegger's (1988) hermeneutic phenomenology extended the descriptive emphasis of Husserl's philosophy to interpretation of meanings deduced from the participant's life stories.

Existential phenomenology, most often associated with the writing of Sartre and Merleau-Ponty, also challenges empiricism and rejects the intentional separation of the consciousness and self as beyond the realm of possible (Eddles-Hirsch, 2015). Existentialism represents a shift from the distant perspective of describing a lived world to an interpretation of the consciousness of being in the world.

Of the three described philosophies, hermeneutic phenomenology is most aligned with the research goals for this study. It is an interpretivist inquiry which is consistent with my ontological commitment to OTL in hyper-accelerated mathematics as multi-faceted with differing contextual implications for each student. The specific tenets of hermeneutic phenomenology are especially appropriate for understanding the information-rich realities of hyper-acceleration. The participants are positioned as the knowers, and the essence of the phenomenon under study represents what is to be known through interpretation. The researcher interprets life worlds through storied experiences

(Langdridge, 2007) and makes her own beliefs and assumptions explicit in the interpretation. This research paradigm is aimed at "producing rich textual descriptions of the experiencing of selected phenomena in the life world of individuals that are able to connect with the experience of all of us collectively" (Smith, 1997, p. 80). Knowledge is produced using an iterative cycle of reading, reflective writing, and interpreting (Laverty, 2003). Hermeneutic phenomenological research explicates "unique, idiosyncratic meanings" (Cho & Trent, 2006, p. 328) and provides a contextual lens that is missing in quantitative studies of student outcomes. In contrast to the pursuit of variability within quantitative research paradigms, "variability in experience is generated within individuals...for the purpose of understanding the cognitive processes that lead to different (perceptual, cognitive) experiences" (Ercikan & Roth, 2009, p. 240).

The conceptual framework which informs this investigation, as presented in Chapter 2, is adapted from the combined works of Gee (2008), Greeno and Gresalfi (2008), and Moss (2008). It opens a space to examine the subjective juxtaposition of mathematics achievement and mathematics identity at the intersection of lived experiences of hyper-acceleration. van Manen's (1990) phenomenological vision affirms a narrative inquiry research design that captures individual truths of Algebra I hyper-acceleration which have been largely unexplored in mathematics education research.

> A good [phenomenological] description that constitutes the essence of something is construed so that the structure of a lived experience is revealed to us in such a fashion that we are now able to grasp the nature and significance of this experience in a hitherto unseen way. (p. 39)

Hyper-accelerated study of Algebra I is arguably a phenomenon with both societal and mathematical consequence. Although research has shown that accelerated study of Algebra I is advantageous for students with respect to advanced course-taking and college readiness college (Gamoran & Hannigan, 2000; Rickles, 2013; Smith, 1996; Spielhagen, 2006), individual stories about opportunities to engage with mathematics in meaningful ways and to build foundational understandings for success in calculus in hyper-accelerated contexts are subjective. Because my implicit assumptions about OTL with hyper-acceleration will be made explicit in my construction of student stories, the deeper essence of the phenomenon can be revealed.

Research Context

Because hyper-accelerated students typically complete two years of the traditional four-year high school mathematics sequence by the end of middle school, it is important to consider their subsequent opportunities to study advanced mathematics in high school as part of examining their learning trajectories. Many high schools in the United States offer advanced instruction with opportunities to earn college credit by examination in calculus or statistics in Advanced Placement (AP) and IB programs. High school students can also earn dual-enrollment mathematics credit with a college or university. In contrast with the single subject design of AP courses, the IB program emphasizes inter-disciplinary learning across six subject groups. Students may enroll in individual IB subject courses, and they may also pursue an IB diploma with a requirement for demonstrated mastery by disciplinary examinations, community service, and a culminating in-depth inquiry on a chosen topic.

Comparison of AP and IB curricula. The IB mathematics courses differ from AP courses because they are not specifically designed for college credit by examination. Precalculus and calculus topics are integrated within each of three IB mathematics two-year courses. While AP offers two single-year courses (AB Calculus and BC Calculus) with content aligned to typical college courses, the IB program offers two two-year mathematics course sequences (Table 3) which include substantive calculus and support different trajectories of mathematics achievement and college aspirations.

The IB program emphasize the role of mathematics in a larger educational landscape with the following aims for students:

• enjoy mathematics, and develop an appreciation of the elegance and power of mathematics

• develop an understanding of the principles and nature of mathematics

• communicate clearly and confidently in a variety of contexts

• develop logical, critical and creative thinking, and patience and persistence in problem-solving employ and refine their powers of abstraction and generalization

• apply and transfer skills to alternative situations, to other areas of knowledge and to future developments

• appreciate how developments in technology and mathematics have influenced each other (International Baccalaureate Organization [IBO], 2014a)

Within the IB program, students have more options for advanced mathematics course selection which are aligned with their changing goals for undergraduate study and their evolving identities as competent doers of mathematics. Students who study Algebra I in Grade 7 or earlier have three or more years in which to complete the IB sequence.

Table 3

IB Mathematics Courses

Course	Content Coverage	Sample Examination Questions
IB Mathematics Standard Level (SL)	Algebra Functions Trigonometry Vectors Statistics and Probability Calculus	**Question 1** A data set has a mean of 20 and a standard deviation of 6. Each value in the data set has 10 added to it. Write down the value of the new mean the new standard deviation Each value in the original data set is multiplied by 10. Write down the value of the new mean Find the value of the new variance. **Question 2** Given that $f(x) = 1/x$, answer the following. Find the first four derivatives of f (x). Write an expression for *f(n)* in terms of *x* and *n*
IB Higher Level (HL) Mathematics	Algebra Functions Trigonometry Vectors Statistics and Probability Calculus	**Question 1** The vectors a, b, c satisfy the equation $a + b + c = 0$. Show that $a \times b = b \times c = c \times a$. **Question 2** Consider the following system of equations: $x + y + z = 1$ $2x + 3y + z = 3$ $x + 3y - z = \lambda$ where $\lambda \in R$. Show that this system does not have a unique solution for any value of λ. Determine the value of λ for which the system is consistent. For this value of λ, find the general solution of the system.

Note: Retrieved from https://www.ibo.org/globalassets/publications/recognition/5_mathhl.pdf; https://www.ibo.org/globalassets/publications/recognition/5_mathsl.pdf;

For students who do not choose to stay on the most accelerated pathway toward multivariable calculus during Grade 12, the three levels of rigor within IB become a unique window on students' evolving beliefs about persistence, quality of precalculus conceptual understanding, and success. The anticipated variability of course choices over the a six-year mathematics learning trajectory creates a multi-dimensionality of context which will increase the depth of the phenomenological data.

While all IB mathematics examinations include calculus topics, the curriculum is specifically designed to prepare students for university study with integrated mathematics understandings and problem-solving proficiencies. The nature of IB examinations of mathematics proficiency contribute to the richness of this study because the questions elicit justification and proof in unfamiliar contexts. First, IB examination papers are administered at the end of senior year of high school regardless of the timing of course completion. All IB examinations specifically assess precalculus topics (algebra, functions, and trigonometry) in addition to calculus topics. Hyper-accelerated students may begin the IB course sequence as early as freshman year, but they must wait until the spring of senior year to take the IB mathematics examination. Thus, the examination is both a measure and an expectation of longitudinal understanding across the high school experience. Third, these examinations are 100% free response, and markschemes for scoring are specifically delineated with points for method and for accuracy. IB students must learn to organize and justify their mathematics thinking because all of their work is evaluated for both process and correctness. Finally, the IB end-of-course examination design offers the “most granular level of differentiation in student achievement by grade” of seven international mathematics curricula (Alcántara, 2016, p. 11).

The IB curriculum and examination structures are especially appropriate for describing OTL in hyper-accelerated contexts because of the curricular and assessment emphasis on connected understandings across multiple topics and the multiple pathways that students can choose. While this study did not use IB examination scores to triangulate student expression of quality of learning, the examination design had a significant impact on enacted curriculum, pedagogical choices, and student expectations. An informal survey of IB curricular offerings revealed that many U.S. high schools have tailored current their curricula to create modified one-year and two-year courses with context-specific prerequisites. Thus, IB program implementations can look very different across U.S. high schools. To maintain the fidelity of common high school course offerings as a context for longitudinal narrative collection and analysis, one IB high school was selected for recruitment of participants.

Setting. West Valley High School is a mid-Atlantic suburban public school with a socioeconomically and ethnically diverse population of over 2500 students and broad Algebra I acceleration policies. It offers both standard level and higher level mathematics courses along with dual-enrollment courses in multivariable calculus and linear algebra for which students can earn undergraduate mathematics credits from a nearby university.

IB programs are often placed in mixed-income or disadvantaged neighborhoods with a vision of breaking down barriers to rigorous educational opportunities for traditionally underrepresented students. Although West Valley High School has an open enrollment policy, the IB mathematics students are typically from White and Asian families. These students are intrinsically and extrinsically motivated to build social and mathematical capital in the form of accelerated coursework which will improve their chances for college admissions at selective institutions. Most students are accelerated by one to three years with respect to a traditional high school mathematics course sequence beginning with Algebra I in Grade 9.

Hyper-accelerated students at West Valley High School typically study Algebra II Honors in Grade 9. If they remain on the most rigorous course trajectory, they study Multivariable Calculus during senior year as a dual-enrollment course with a local university as shown in the rightmost course pathway in Figure 3. However, many hyper-accelerated students at West Valley High School choose alternate pathways, often taking both Standard Level (SL) mathematics and Higher Level (HL) mathematics courses to increase their time with precalculus course content. Others choose not to take a mathematics course during their senior year.

Many families grapple with difficult choices when their accelerated students began to struggle with mathematics. Their concerns about grades which are not As or Bs often lead to dilemmas about switching to non-honors classes. Students and parents often complain of an unacceptable gap between the non-honors courses and the honors courses. West Valley High School was an appropriate context for the study of OTL because it offers highly advanced mathematics course pathways with an expectation of connected mathematics understandings across multiple years.

Study Participants

In this interpretative research study, I am a study participant along with 21 West Valley High School graduates. Our shared understandings of the phenomenon of hyper-acceleration created an environment of trust in which deeper meanings emerged.

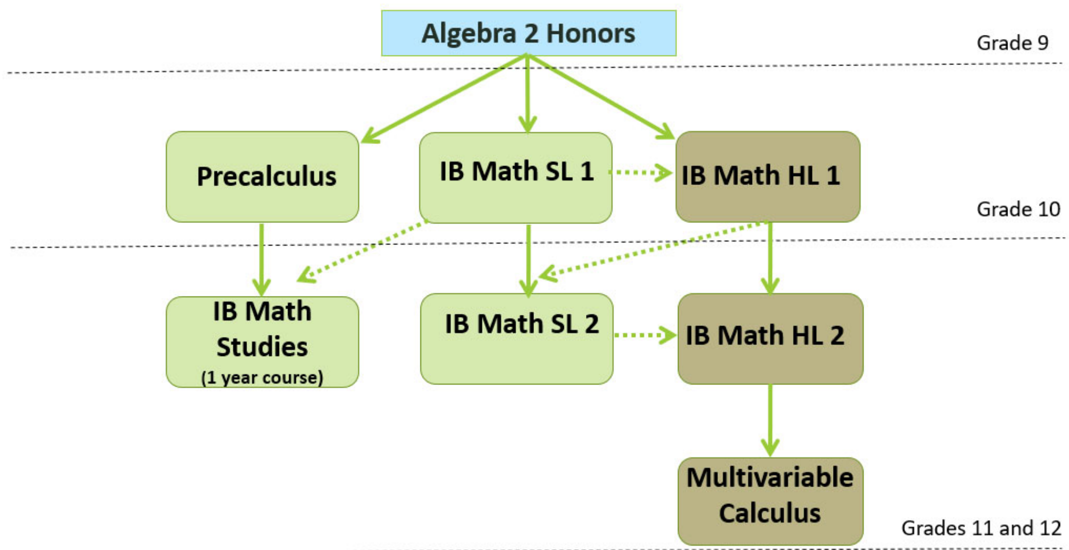

Figure 3. Advanced Mathematics Course Pathways at West Valley High School

Researcher positionality. I leveraged my professional understandings of IB curriculum and my relationships with the community to construct a rich data set of lived experiences from the stories of hyper-accelerated students. My worldview of OTL in at West Valley High School is informed by over thirty years of experiences as an electrical engineer, parent, high school mathematics teacher, and private tutor. In my pursuit of a bachelor's degree and a master's degree in electrical engineering, I relied heavily upon a solid and connected foundation of algebraic and trigonometric reasoning. I recognized the importance of productive struggle and opportunities to think about mathematics beyond correct answers on tests. My later parenting experiences reflected a range of tensions, choices, and regrets as I encouraged each of my four children to either speed up or slow down their mathematics studies when I questioned whether they were sufficiently or insufficiently challenged. I grappled with these choices and their implications for college admissions.

My current perspectives are informed by my continued witnessing of a lack of algebraic readiness in my role as an IB mathematics teacher at West Valley High School. My colleagues and I have often discussed the two schools within our school. A decision to study honors or IB mathematics is often a reflection of peer and parent expectations, ability to complete homework consistently, and financial access to private tutoring. These decisions do not necessarily reflect an interest in mathematics or a readiness for challenge at the honor level. At the same time, the mathematics experiences of many non-honors students at West Valley High School have been constrained by the lower expectations of engagement and achievement in classrooms that are disproportionately populated by black, Latin@/x, and special education students. I have counseled a number of parents whose children were struggling in honors mathematics but hesitated to request a course change because they didn’t want their children in a class with those kids. Grades and courses completed, not learning, have been of primary concern to many parents. Students at all levels of mathematics have resisted opportunities to discover and explore and were quick to say they were not good at mathematics.

My closeness to the phenomenon of as a teacher at West Valley High School, along with the relationships that I have built within surrounding community, positioned me uniquely to access, interpret, and share students' stories as empirical evidence which can inform hyper-acceleration policies. I brought my extensive doctoral study of precalculus conceptual understandings, my multi-year investment in educating hyper-accelerated students, and my commitment to creating meaningful OTL for each and every child as a crystalline lens (Denzin & Lincoln, 2011) on the multi-faceted essence of hyper-acceleration. My subjectivist worldview of the multidimensionality of hyper-acceleration informed a constructivist epistemology in which I placed all understandings, "scientific and non-scientific alike, on the very same footing" (Crotty, 1998, p. 16). This worldview recognized OTL as an important synthesis of the pursuit of algebraic fluency necessary for success in college calculus and evolving student perspectives as competent and successful doers of accelerated mathematics.

Recent high school graduates. The other study participants were graduates of West Valley High School between the years of 2014 and 2018 who studied Algebra I in Grade 7 and took an IB mathematics examination. Each of the participants was 18 years of age or older, and each reflected on a lifetime of experiences with school mathematics to explore the meaning of Grade 7 Algebra I. Although the phenomenon in this research study was rooted in enrollment in hyper-acceleration of Algebra I in Grade 7, the deeper implications for persistence, conceptual understanding, and mathematics identity formation were fully realized and articulated in retrospect. In their analysis of research on mathematical identity, Radovic and colleagues (2018) elaborated on Smith's (2010) examination of student success and persistence in advanced mathematics. They argued the importance of capturing decision-making at the critical transition from secondary education to postsecondary education to "engage students in the generation of narratives about their lives which can give meaning to their present and future choices". (p. 35) Additionally, the relationship of hyper-acceleration to selection of undergraduate major and institution was of critical importance in understanding the phenomenon of hyper-acceleration as motivated by long-term goals.

The collective imaginings of adults who were home from college and reflecting upon their common experiences in elementary school, middle school, and high school as informed by their recent experiences at the college level offered a depth and in breadth of description. As actors within the phenomenon, my participants and I will pursue a mutual construction of hyper-acceleration as an educational phenomenon, and the production of their narratives became a novel longitudinal lens on its implications.

Data Collection

I recruited participants by email invitation (Appendix B) distributed to West Valley High School IB students who had graduated within the past four years for focus groups and follow-on interviews. Because I am a teacher at West Valley High School, I specifically acknowledged my positionality and my motivation for conducting the study in the text of the email. The invitations were sent from the IB Coordinator's email address in Spring 2018 and Winter 2018 focus groups with appropriate permissions from local and school division authorities. I solicited the inputs of recent graduates to help teachers, parents, and counselors as they advise and support future students considering hyper-acceleration. Potential participants were offered two 90-minute focus group sessions as part of the dissertation pilot study in March 2019 and five 90-minute focus group sessions in December 2018-January 2019. I only had access to potential participants' email addresses if they expressed interest in participating. I also encouraged potential participants who responded to the original emails to invite fellow West Valley High School graduates who met the study selection criteria but who might not have received the recruitment email. Finally, I shared study details on social media outlets to increase community awareness of the study and to find qualified participants.

The use of an email database as my primary recruitment strategy increased the likelihood that participants represented the broad range of experiences with hyper-acceleration at West Valley High School. These recruitment strategies yielded seven study participants for the Spring 2018 data collection and 14 participants for Winter 2018 data collection; 15 of the 21 participants were my former students.

Focus group design. The use of focus groups as a qualitative data collection technique has have gained prominence in social science research (Sagoe, 2012; Williams & Katz, 2001). Focus groups were appropriate for this study because they built "knowledge, ideas, story-telling, self-presentation, and linguistic exchanges within a given cultural context" (Barbour & Kitzinger, 1998). The unique affordances of focus groups were the influence of a trusted moderator who can facilitate productive discussions; a high level of participant contribution with a social affirmation of shared experiences; dynamic data collection that responds to participant contributions; non-verbal cues as an additional research input; and interaction among participants (Sagoe, 2012).

The seven focus group sessions were conducted during the traditional winter break window for colleges and universities. The Spring 2018 sessions were conducted outside of school hours in a classroom at West Valley High School, and the Winter 2018 sessions and were conducted in a public library conference room three miles from West Valley High School. At the beginning of each focus group session, I asked participants to sign a consent to research form (Appendix A). I also asked them to complete a Google Form survey to enter their current undergraduate major (as applicable) along with the mathematics courses taken in Grades 7 – 12 and willingness to be contacted for follow-on interviews.

A re-envisioning the relationship between the researcher and the researched is most important aspect of narrative inquiry (Pinnegar & Daynes, 2007), and focus groups can be an “empowering, action-oriented” form of educational research (Williams & Katz, 2001). At the beginning of each focus group session, I shared my personal motivations the study of hyper-acceleration as growing phenomenon with a limited empirical basis. I encouraged them to share their mathematics experiences to help us to better understand how Grade 7 Algebra I formed their beliefs about success in mathematics and their interest in continuing in STEM study beyond high school. By positioning the focus group participants as researchers who were working with me to construct the essence of this phenomenon in ways that could help future students and teachers, they were more invested in a more purposeful exploration of their experiences.

During the remainder of the focus group session, I encouraged shared discourse and extended responses (Mishler, 1986; Polkinghorne, 1988) using a semi-structured interview protocol (Appendix C) aligned with the first research questions to "create the very events" (Denzin, 2000) that the narrative reflects upon. I wrote researcher commentaries immediately after each of the focus group , and I increased my closeness to the verbalized data (Sagoe, 2012) by annotated transcriptions within one week of recording to improve the overall quality of the study (Sagoe, 2012). These annotations took the form of interviewer comments clarifying participant's words about the unique aspects of West Valley High School, observer comments to capture actions, gestures, reactions during the focus groups, and dated researcher notes capturing reactions to the creation or rereading of the transcript (Carspecken, 1996; Dennis, 2018).

Because three of the seven scheduled focus group sessions had fewer than three participants, I used the data collected from these sessions to triangulate the findings from the remaining four focus groups. One participant was not able to attend any of the sessions, so I conducted a phone interview with her and used her contributions as further triangulation of data sources in later analysis. The 15 participants whose stories were formally analyzed as part of their contributions in focus groups with three or more participants are listed in Table 4.

Follow-on semi-structured interviews. I conducted follow-on 30-60-minute semi-structured interviews with a subset of individual participants. This discussion w were more informal than the focus group protocols and were tailored to elaborate their focus group conversations related to the three research sub-questions on persistence, conceptual understanding, and identity. I extracted specific transcript elements in a chronological sequence for each participant (Appendix D) and emailed them in advance to each of the participants. Participants were assured that these individual quotations did not represent the entirety of their hyper-acceleration stories; instead these transcript

Table 4

Focus Group Participant Characteristics

Student Name	Gender	Highest Math Course Completed	Undergraduate Major(s)
Focus Group 1 – Spring 2018			
Kevin	Male	Multivariable Calculus	Computer Science, Cybersecurity
Matthew	Male	IB Standard Level	Biochemistry
Stephen	Male	Multivariable Calculus	Mathematics, Computer Science
Liam	Male	Multivariable Calculus	Mathematics
Collin	Male	Multivariable Calculus	Engineering
Focus Groups 2-4 – Winter 2018			
Nadine	Female	Multivariable Calculus	Multivariable Calculus
Elizabeth	Female	IB Standard Level	Behavioral Neuroscience
Jonathan	Male	IB Higher Level	Communications
Mindy	Female	Multivariable Calculus	Mathematics
Robert	Male	IB Higher Level	Neuroscience
Ashley	Female	IB Standard Level	Environmental Engineering
Claire	Female	IB Standard Level	Communications
Jessica	Female	IB Higher Level	Integrative Studies
Samantha	Female	IB Higher Level	Public Policy
Michael	Male	Multivariable Calculus	Computer Science

excerpts became the anchors for further probing of meaning and intention. These interviews (in person or via Skype) were conducted six weeks after the Winter 2018 focus group sessions as a member-check for first stage of narrative analysis of this focus group data. The six weeks of time between the focus groups and the follow-on interviews allowed the participants to reflect upon and deepen their previous contributions (Polkinghorne, 2007). Questions were framed from an emic perspective, meaning that I sought to elicit and represent individual beliefs about culture while acknowledging that I cannot separate by own beliefs from data collection.

Reflective commentaries. I wrote reflective commentaries (Shenton, 2012) after each of the focus groups to position my perspectives within the findings. I revisited and updated these memos throughout the data analysis to consider how the study has changed my perspectives and how my perspectives were part of the emerging interpretations. These researcher commentaries were critical in the developing additional research questions to support the specific analyses in Chapters 4 and 5.

Data Analysis

An iterative sequence of critical events analysis (Webster & Mertova, 2007), thematic analysis (Riessman, 2008), and structural analysis (Knight & Sweeney, 2007) became a hermeneutic cycle of moving between transcriptions and the emerging description of the phenomenon (Polkinghorne, 1988, p. 177) which contributed to the reliability of the analysis. Each stage of interpretation (Figure 4) was continually informed by my years of witnessing of mathematics success and struggle as a West

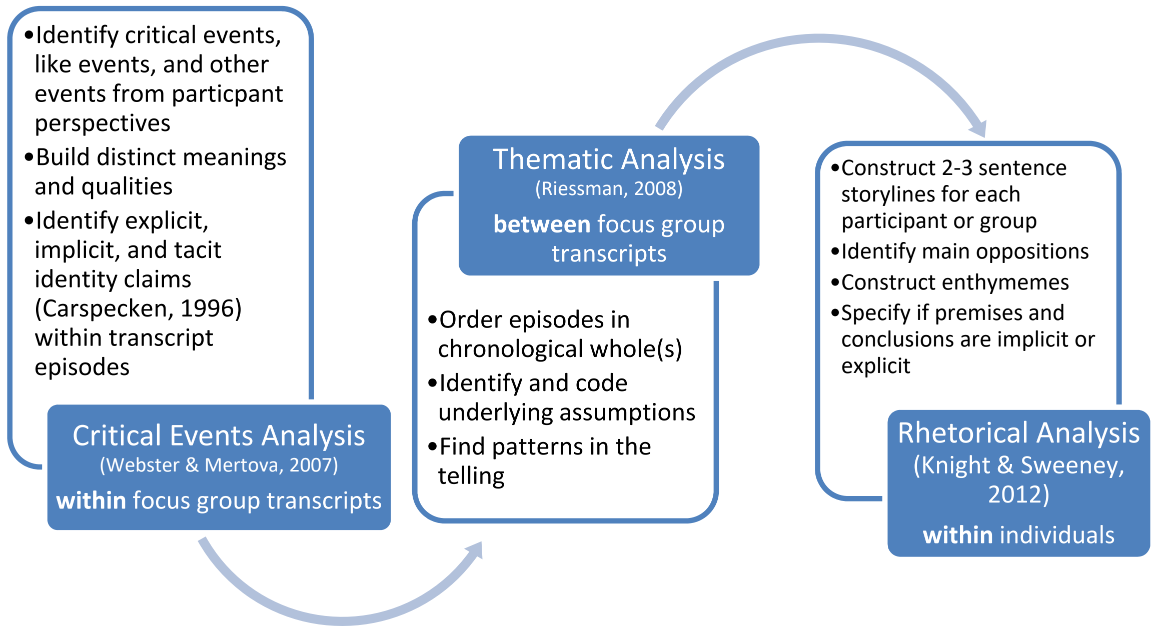

Figure 4. Three Stages of Narrative Analysis

Valley High School teacher. In the first stage of analysis, I identified critical, like, and other events (Webster & Mertova, 2007) and implicit, explicit, and tacit identity claims (Carspecken, 1996; Dennis, 2018) in each of the four focus group transcripts. I synthesizes these sets of codes to build an initial set of distinct meanings and categories describing OTL along hyper-accelerated learning trajectories as informed by the central questions in the theoretical framework in Chapter 2. In the second stage of analysis, I deconstructed and merged excerpts from the four transcripts to create cohesive chronological wholes (Riessman, 2008) defining temporal episodes spanning elementary school, middle school, high school, and college. This analysis across the focus groups confirmed and refined critical events analysis categories, revealing a contextual interplay between OTL and status over time. The first two stages of analysis provided the methodological and analytical basis for the article in Chapter 4.

For the third stage of analysis, I extracted transcript data for three individual students with a shared identity. This selection was motivated by the emergence of gender as an integral aspect of OTL in hyper-accelerated contexts. I used rhetorical techniques (Knight & Sweeney, 2007) along with the transcript analysis of their individual follow-on interviews to find the implicit meanings and logical narratives in these subsets of lived experiences. Analysis of these young women's stories of deceleration after Grade 7 Algebra I, along with literature on gender and mathematical subjectivities (Hottinger, 2016; Levya, 2017; Mendick, 2005), formed the theoretical and methodological basis for Chapter 5.

Stage 1: Critical events analysis within focus group transcripts. Webster and Mertova (2007) built upon Woods' (1993, p. 102) definition of critical events as the "right mix of ingredients at the right time in the right context" which reveal a change in a storyteller's worldview (p. 73). They also referenced Strauss' (1959) focus on critical events as retrospective construction of identity. Individuals realize the full meaning of past critical events "in a new light through reassessment and selective recollection" (Webster and Mertova, 2007, p. 77). Using the narrative analysis strategies outlined by Webster and Mertova (2007) and refined by Dennis (2018) in her doctoral course for narrative research, I first coded the focus group transcripts for critical, like, and other events from the perspective of the participants. Like events are similar in context to the critical events and serve to illustrate, confirm or repeat the experience of the critical event. Other events occur at the same time as the critical and like events and are interwoven in the analysis of the critical and like events.

I organized the critical, like, and other events using a set of distinct meanings and qualities which emerged in the critical events analysis. While acknowledging that student mathematical identities do not necessarily evolve in a linear fashion, (Dennis, 2018), the chronological organization of related excerpts of participant narratives within the focus group stories provided a lens on how smartness was explicitly identified by placement in Grade 7 Algebra I and was maintained, re-identified, or challenged by subsequent mathematics experiences.

I then coded participant identity claims as explicit, implicit and tacit (Carspecken, 1996). Explicit identity claims are clearly stated in the form of “I am such and such a person” (Dennis, 2018), while implicit identity claims are not directly stated but clear in the words of the participant. Tacit identity claims are implied or inferred. I developed an initial list of categories for each of the four transcripts using the event codes and identity analysis. These categories organized identity claims related to hyper-acceleration and motivation, beliefs about tracking, grades, competition, teachers, challenge, and applications of mathematics.

Stage 2: Thematic analysis across focus group transcripts. Thematic analysis of the focus group transcripts draws upon Riessman’s (2008) description of Williams’ (1984) thematic methodology in his narrative reconstruction. I combined the shared storytelling from the four focus groups as cohesive chronological wholes organized around identity claims and common course-taking trajectories. The uniqueness of individual experiences over time were made more explicit because they were juxtaposed within re-ordered narratives portraying the passage of time in related mathematics experiences. I retained the alignment of the participant's identity claims and transcript excerpt by line number to maintain insight into the individual tellings as part of the greater whole. Consistent with Williams' "full-blown account of the narrators" (Riessman, 2008, p. 58), I only carried over my comments and questions from the first stage of transcript analysis when necessary to clarify the account. By maintaining the focus group identifiers and transcript line numbers from the first stage of analysis, I could revisit the analysis in the original transcript as needed. I then coded underlying assumptions within each student's story to interpret the ways in which OTL and status can be conflated within and across student stories over time.

I had employed this sequence of critical events analysis and thematic analysis during the first two focus groups in Spring 2018 pilot study with five male undergraduate STEM majors from West Valley High School. Changing beliefs about smartness in mathematics emerged in their telling of their middle school, high school and college mathematical worlds. The selection for Algebra I in Grade 7 was the critical event that established identities of smartness in mathematics for middle school students. Smartness was challenged at different gates in high school mathematics, and smartness was redefined in college mathematics with a view of struggle as productive. These research findings were analogous to "apriori coding" (Saldaña, 2016) and informed my coding of underlying assumptions in my analysis of data from the three remaining focus groups. Riessman (2008) described Williams' (1984) commitment to constructing themes which are mediated by the researcher perspectives; the pilot study analysis informed but did not constrain my interpretation of the reordered transcripts. The additional analysis revealed identities of smartness are constructed over time with more complexity as an interaction of OTL and social status.

More specifically, the four questions within the OTL theoretical framework in Chapter 2 served as organizing categories for my presentation and discussion of findings from both focus groups in Chapter 4. These questions were 1) What mathematics do you learn? 2) How do you learn mathematics within the classroom? 3) How do you learn mathematics within the school? and 4) Why do you learn mathematics?

Stage 3: Structural analysis within and across individual interview transcripts. The goal of rhetorical analysis is to identify the "hidden elements of logical inference within narratives" (Knight & Sweeney, 2007, p. 226). Because the original focus group transcripts and the reordered transcripts encompassed the individual stories of 15 hyper-accelerated mathematics students, I will build upon the temporal sequencing from the thematic analysis in this third stage of analysis to capture the specific stories of a specific groups of participants. Drawing upon the strategies of Knight and Sweeney (2007), I used rhetorical analysis to explore implicit meanings and logical narratives in the form of syllogisms and enthymemes. Syllogisms are deductive arguments with major premises, minor premises, and conclusion, while enthymemes are syllogisms "in which one or more premises (or a conclusion) is missing, and needs to be filled in, because it has not been explicitly expressed" (Knight & Sweeney, 2007, p. 228).

This form of narrative analysis is especially powerful in this study context because adult college students are reflecting on choices to hyper-accelerate mathematics study that were made as many as 10 years earlier. These enthymemes become a form of syllogistic reasoning through which the reader can deduce the implicit meanings within the "naturalistic world of the participant" (Knight & Sweeney, 2007, p. 232) which may not be explicitly expressed.

The first two stages of narrative analysis revealed that mathematics is experienced differently by hyper-accelerated girls. I extracted sequential narratives and identity claims and descriptions of OTL as experienced for three of the female participants who had decelerated their high school mathematics trajectory in differing ways. I further identified gendered binary oppositions (Mendick, 2005) as subsets of potential arguments. Finally, I built syllogisms and enthymemes from their storylines and oppositions to describe how hyper-acceleration formed their beliefs about being good at mathematics.

The following oppositions, as identified in the Spring 2018 pilot study data with 5 male participants who are currently majoring in STEM, provided an important contrasting discourse on OTL which informed my analysis of girls' experiences of hyper-acceleration as constructed in the mixed gender Winter 2019 focus groups.

- coasting vs crashing (hitting the wall)
- speeding up (acceleration in middle school, fastest course trajectories in high school) vs slowing down (repeating or dropping courses in college)
- figuring it out on my own vs paying attention in class

- easy mathematics (memorizing, knowing numbers and formulas) vs. hard mathematics (explaining)
- competitive identity (not help-seeking) vs coping identity (help-seeking)

As an iterative yet cohesive whole, these three stages of narrative analysis provided an iterative restructuring and resequencing of the data in which each successive stage of analysis built both complexity and clarity in revealing the essence of the phenomenon of hyper-acceleration. The unanticipated triangulation between gendered focus group enactments also added to the richness of the analysis.

Trustworthiness

In narrative inquiry, it is "the readers who make the judgment about the plausibility of a knowledge claim based on the evidence and argument for the claim reported by the researcher" (Polkinghorne, 2007, p. 484). Schwandt (2007) offered a commentary on Lincoln's and Guba's (1985) widely cited operationalization of trustworthiness in interpretative naturalistic inquiry. He argued that any act of generating evidence itself is an interpretation because "the investigator cannot help but always be situated relative to (and cannot escape) social circumstances such as a web of beliefs, practices, standpoints" (p. 11). Lincoln and Guba (1985) proposed their four criteria for trustworthiness as "complementary" constructs (p. 131) to quantitative research counterparts as listed parenthetically – credibility (versus internal validity), transferability (versus external validity), dependability (versus reliability), and confirmability (vs objectivity). Together, these criteria provide a structure for critiquing the proposed data collection and analysis strategies used within this study and building a "convincing case" (Shenton, 2012, p. 73) for quality and rigor.

Because narrative inquiry and analysis is the reconstruction of student stories of hyper-acceleration for an intended audience which does not seek a singular truth, I also attended to Polkinghorne's (2007) assertion that validity is simultaneously "a function of intersubjective judgements" and "the consensus of a community" (p. 474). Loh (2013) acknowledged that there are narrative researchers who disagree with the Lincoln's and Guba's (1985) four trustworthiness criteria (Hammersley & Atkinson, 2007, Silverman and Marvasti, 2008), but he claimed a consensus within the larger qualitative research community about the influence of these criteria. I align myself with Loh (2013) as a narrative researcher who will "adhere to the trustworthiness criteria found in the broader qualitative field" (p. 3) in parallel with my own selection of criteria which align with my subjectivist ontology. As I acknowledge a co-existence of these perspectives on validity, I layered van Manen's (1990) four criteria for the quality of writing (orientation, strength, richness. and depth) in a dialogic relationship between myself and my readers while I maintained my attentiveness to the four criteria for trustworthiness offered by Lincoln & Guba (1985).

Credibility. Within interpretivist narrative inquiry, credibility depends not upon seeking truth in the reporting of events but instead capturing meaning and significance for the participants in storied descriptions (Polkinghorne, 2007). The following discussion of researcher background and qualifications, sampling strategy, peer scrutiny, triangulation of focus group data, member checking, and researcher reflective commentary promotes confidence (Shenton, 2012) that the multiple realities of hyper-accelerated math study are historical truths (Polkinghorne, 2007) from the perspectives of the participants.

Background, qualifications, and experiences of the researcher. I have seven years of teaching experience in the research context and four years of doctoral research on Algebra I acceleration and student and teacher beliefs about success in mathematics. I am uniquely positioned to interpret individual and shared student stories of hyper-acceleration at West Valley High School.

Purposive sampling from a uniquely representative population. The selection of West Valley High School as a research site combined my unique contextual knowledge with the stories of students who have a set of situative experiences in a mathematics curriculum with a high expectation of connected understandings across topics. The multiple voices within the focus groups both affirmed, encouraged, and challenged one another to explore the depth of the phenomenon as they exhibited "characteristics of similarity, dissimilarity, redundancy, and variety" (Stake, 1994, as cited in Shenton, 2012). At the same time, the richness and strength of the resulting interpretation is dependent upon finding participants who are reflective about their mathematical experiences. By recruiting from a pool of high school graduates during their college or university breaks and asking them to support future students in their own community, they are more likely to be personally invested as they collaborate to observe, question, and challenge the consequences of Algebra I acceleration. Multiple participants elected to participate because of their previous relationship with me or because of my reputation in the community. The years of common experiences that the participants could draw upon in these focus group settings increased participant engagement in conversations. They built upon one another's contributions. There connected recollections extended back to Grade 3.

Sample size. There is a lack of consensus on sample size and saturation for qualitative research. Theoretical saturation in qualitative studies is achieved when "no new properties, dimensions, or relationships emerge during analysis" (Strauss & Corbin, 1998, p. 143). Creswell (1998) recommended five to 25 participants in phenomenological research, while Morse (1994, as cited in Mason, 2010) recommended at least six participants. Mason (2010) analyzed qualitative studies in a database of graduate theses from Great Britain and Ireland and reported a range of 7-89 participants with a median number of 20 participants in phenomenological studies.

Participant attrition after initial expressions of interest resulted in fewer than the recommended four to six participants (Brown, 1999) in two of the four analyzed focus groups. However, use of data from focus groups attended by one or two students became supplemental interview transcripts which mitigated the impact of a smaller group sizes. The follow-on interviews by Skype together with my multi-year history of experiences at West Valley High School directly or indirectly with the participants contributed to "prolonged engagement" (Lincoln & Guba, 1985, p. 301) and "persistent observation" (p. 307) which further characterized culture and detects distortions in initial interpretations of focus group data.

Peer scrutiny. As a check of my interpretation of participant stories of OTL and construction of mathematics experiences in hyper-accelerated contexts, I relied on peer scrutiny (Shenton, 2012) from two university experts with relevant experience in secondary mathematics education. Their specific knowledge supported me in crafting an initial analysis that captured the intentionality and voice of the participants while encouraging a deeper study of how OTL may differ among participants.

The first expert was fellow doctoral student, Sara Birkhead, who was a middle school mathematics teacher on the shared campus of West Valley High School. Her common experiences with accelerated mathematics as socially constructed on our campus and her qualitative research expertise positioned her to singularly understand and to challenge my methods and analysis. She reviewed the distinct meaning and identity claims analysis from each of the four focus groups prior to the individual interviews to ensure that each of the participant's lived experiences were represented. She also supported my uncovering of embedded meanings of OTL and status that were later captured in the thematic codes created in the second state of analysis.

The second expert was my advisor and dissertation chair, Dr. Toya Jones Frank, who is a former secondary mathematics teacher and a scholar who focuses on issues of equity and identity in mathematics education in her research. Bi-weekly debriefing sessions with Dr. Frank assisted me in probing the influences of my prior engagement with teaching and parenting at West Valley High School and in testing emerging ideas and representations of OTL from a situative and from a gendered perspective. Written notes of each of these peer debriefings were maintained for the research audit trail (Lincoln & Guba, 1985).

Triangulation. In contrast with triangulation which “verifies and checks specific facts collected across resources” in truth-seeking research (Cho & Trent, 2006, p. 328), I relied upon a holistic triangulation as thick description across four separate focus groups with three stages of narrative analysis while simultaneously acknowledging the impossibility of affirming a truth (p. 333). The additional triangulation of focus group data with follow-on interview data and the three focus groups which did not meet the size criterion indicated both convergence and divergence of the constructed realities (Johnson & Christensen, 2014) across the four focus groups. Alternative representations of OTL which affirm or challenge the first and second stages of narrative analysis within this phenomenon informed my discussion and implications in Chapters 4 and 5 and increased the credibility of the study.

Member checking. Use of recursive member checking ensured intentionality and will contribute to the verisimilitude, or believability, of the study for its consumers (Loh, 2013). I conducted follow-on interviews with 8 of the 15 participants with transcripts of the follow-on interviews to allow them to confirm that their words had captured their intended meanings and to elaborate on initial findings across focus groups.

Transferability. The establishment of transferability, which is analogous to external validity in a quantitative study, is impossible in naturalistic research (Schwandt, Lincoln, & Guba, 2007). The selection of West Valley High School presumes that the phenomenon of hyper-acceleration is socioculturally manifested in high-achieving communities and thus improves transferability to similar contexts. Thick, rich descriptions combined with contextual definitions and detailed descriptions of course and learning trajectories increased the applicability of the study to other hyper-accelerated contexts. The research design of capturing retrospective adult perspectives on decisions from middle and high school years should encourage other researchers to examine this phenomenon in other contexts because of the easier access to participants and the mature retrospective lens that they bring to their consideration of the phenomenon.

Dependability and confirmability. A dependable research design may be viewed as a "prototype model" (Shenton, 2004, p. 71) which allows future researchers to build upon the work. Lincoln and Guba (1985) argued that dependability and confirmability are simultaneously met by careful, organized documentation. Acceptability of the documented process establishes the dependability of the process, while the product establishes the confirmability of the study (p. 318). In-depth methodological descriptions along with my transparency about changes made during data collection and analysis will allow the study to be evaluated, scrutinized, and repeated. I built a reflective commentary (Shenton, 2012) to record my impressions and decisions at the end of each of the data collection episode and analysis activity and to maintain an ongoing record of my own changing constructions of participant realities. These reflective commentaries, along with a careful cataloging of all focus group transcripts, discrete meanings documents, identity claim analysis documents, thematic categorizations, interview transcripts, and peer debriefing memos constitute an "audit trail" (Shenton, 2012, p. 72) providing traceability of research decisions and analyses. Specific details offered for each of the three narrative analysis methods also increases the transparency and therefore the quality of the contextual research (Demerath, 2006).

Limitations

Polkinghorne (2007) identified the following four threats to the validity in narrative research as a "disjunction" between experienced meanings and storied descriptions:

1) the limits of language to capture the complexity and depth of experienced meaning
2) the limits of reflection to bring notice to the layers of meaning that are present outside of awareness
3) the resistance of people because of social desirability to reveal fully the entire complexities of the felt meanings of which they are aware
4) the complexity caused by the fact that texts are often a cocreation of the interviewer and participant (p. 480)

The focus group design of this study can combat these threats to validity because experiences were articulated and constructed in concert by fifteen participants spanning multiple focus groups spanning five graduation years. Yet the focus group format with peers as the only participants can also exacerbate the hesitancy of individuals to reveal their personal experiences in depth because their stories are more public. Further studies with other stakeholders (parents, teachers, counselors, and administrators) would deepen our understanding of the phenomenon.

As I considered a potential audience for this research, I was motivated by the absence of context in prior quantitative studies of acceleration and the accompanying presumption of generalizability. The focus group design of this study cannot be fully extrapolated to other high school contexts, and many might challenge its research value for that reason alone. The study's interpretivist epistemological foundations may be challenged by some as yielding anecdotal and agenda-driven findings. However, this research study is grounded in a subjectivist epistemological view about the construction and valuation of mathematical understandings, Thus, my efforts to co-create understandings with my participants elevates their voices in illuminating the unintended realities of a culture of "faster as better" in mathematics teaching and learning. My overarching goal is to begin empirical conversations about hyper-acceleration and its contextual premises. I have also strived to develop and present a rigorous narrative inquiry methodology that encourages other researchers to replicate this research and to open new windows on OTL in hyper-accelerated contexts.

After defining their initial set of four criteria for trustworthiness, Lincoln and Guba (1986) subsequently offered a fifth criterion, authenticity, with a decidedly relativist view in terms of fairness in representing construction and empowering participant views in the pursuit of social agendas. Perhaps this methodology will also help us to answer even broader questions about the phenomenon from the perspectives of additional stakeholders in education. Who has access to hyper-accelerated mathematics and how these students are appropriately supported and encouraged?

Conclusion

Narrative inquiry grounded in a hermeneutic phenomenological paradigm constructed 15 hyper-accelerated students' stories of achievement and identity, thus describing OTL in secondary mathematics contexts in novel and important ways. Our sociopolitical realities, driven by a culture of high achievement and curricular differentiation, are redefining the ways in which our students build and use mathematics as a resource.

This study's methodology offers an empirical response to Larson's (2017) assertion that "it is plausible that students in the privileged top third experience mathematics instruction that cultivates their mathematical identities, conceptual understanding, and critical problem solving and thinking skills" (para 8). Such statements contribute to presumptions that hyper-acceleration of Algebra I offers the optimal pathway for mathematically promising students and increases the risk of rushing through the careful development of strong pre-algebra foundations. The design of this contextual exploration adds to the largely quantitative basis of literature on acceleration of Algebra I and can inform decision-making by parents, teachers, counselors, and administrators who select, design, and implement these course trajectories.

Chapter Four - "Just Solving for X": Retrospective Narratives of Opportunities of Learn on Hyper-accelerated Algebra I Pathways

Abstract

The presumption that the greatest opportunity to learn (OTL) comes with accelerated exposure to formal algebra motivates students and parents to pursue pathways which become markers of smartness in competition for elite college admissions. In the United States, students who *hyper-accelerate* their study of Algebra I to Grade 7 or earlier are often deemed successful from a content perspective of OTL which uses standardized test scores and secondary mathematics course taking as metrics. Yet retrospective narratives of students' lived experiences with accelerated algebra and calculus create compelling individual descriptions of OTL from a situative perspective. This phenomenological study of hyper-acceleration of Algebra I layers two narrative analysis techniques to interpret OTL in the stories of 15 recent International Baccalaureate program graduates from one U.S. high school. By explaining the what, why, and how of their mathematics learning, these students help us to better understand how hyper-acceleration may augment or diminish OTL. The students articulate a complex relationship between quality of learning and status as they evolve in their understanding of what it means to be "good at mathematics". This narrative analysis offers a novel empirical examination of acceleration policies, equitable access to advanced mathematics learning, and persistence on rigorous mathematical pathways.

Introduction

With fascinating precision,
mathematics is used as a preeminent resource for creating or evaluating strongly explanatory stories about our physical and sociopolitical realities. Based on the status and authority that those stories engender, mathematics is also employed as a gatekeeper in school contexts and actualized as a kind of intellectual property—a subject through which students must pass to endorse their academic merit.
(Larnell, 2016)

With the sociocultural turn (Lerman, 2000; Martin, 2013) and the sociopolitical turn (Gutiérrez, 2013) in mathematics education research, there is increasing attention on the inseparability of identity and mathematics teaching and learning. Research within secondary mathematics classrooms (Boaler, 1997; Boaler & Greeno, 2000; Darragh, 2016) and secondary mathematics departments (Boaler & Staples, 2008; Horn, 2008) has illuminated the complex connectedness of content experiences and social positioning (Gresalfi & Hand, 2019; Shah, 2017) in describing mathematics achievement and identity. Larnell's (2016) description of mathematics as intellectual property, or as a socially constructed marker of smartness (Dunleavy, 2018), is a call to examine systems and structures which perpetuate this powerful reality in secondary education. Narrow conceptualizations of mathematical ability and achievement (Louie, 2017) which often characterize students' experiences in advanced secondary courses can compromise opportunities for students at all levels of readiness to engage with mathematics in meaningful ways (Boaler & Greeno, 2000; Gutiérrez, 2002; Horn, 2006).

In a growing race for admissions to competitive colleges and universities in the United States, socially constructed markers of excellence in secondary mathematics are often course names and the years in which they are completed. While acceleration of

Algebra I to Grade 8 is broadly associated with more equitable access to rigorous mathematics (Ma, 2005; Rickles, 2013; Stein, Kaufman, Sherman, & Hillen, 2011), the *hyper-acceleration* of Algebra I to Grade 7 or earlier has become synonymous with academic merit. This emergent pathway motivates faster completion of secondary courses in the race to calculus (Bressoud, Camp, & Teague, 2012; National Council of Teachers of Mathematics [NCTM], 2018). Robin Pemantle (2016), a mathematician at an Ivy League university, summarized the false choices that high-achieving students face when timing of courses becomes the primary indicator of mathematical accomplishment.

> Many students do not have the option of choosing deep learning over acceleration. Therefore, acceleration is taken as a proxy for how mathematically apt a student is and for whether the student has sufficiently “challenged him/herself”, a favorite buzzword in college admissions.

This narrow framing of mathematical success is the subject of this study. The presumption that the greatest opportunities to learn (OTL) are associated with the most accelerated pathways frame the identities of students at all levels of achievement about what it means to be “good” at mathematics (Boaler, 1997; 2015; Darragh, 2015; Gutiérrez, 2002; Horn, 2006). These limited views of mathematical success can be affirmed or challenged by their experiences in classrooms and schools.

This interpretative phenomenological study narrates the retrospective mathematics stories of 15 recent International Baccalaureate (IB) program graduates from one racially and socioeconomically diverse U.S. high school. Each student had completed both Algebra I and Geometry in one of two middle schools prior to high school. By explaining the what, why, and how of their mathematics learning from a situated perspective (Greeno & Gresalfi, 2008; Moss, 2008), these students provide a novel research lens on the role of hyper-acceleration of Algebra I in augmenting or diminishing opportunities for meaningful mathematics learning.

It is important to acknowledge that aforementioned scholars are committed to more equitable construction of mathematics identities and more democratic schooling processes that foster a culture of lifelong learning for all students. Hyper-accelerated students have privileged access to resources, curriculum, and identities of competence by virtue of their positioning on this trajectory. Their identities of smartness in mathematics are conferred by our educational system. Yet there is a lack of research on the quality of learning within these emergent contexts because these students are deemed successful by traditional metrics. The narration of these students' experiences becomes empirical evidence of the individual and systemic effects of treating mathematics as intellectual property which in turn becomes a beacon of achievement and status. The presumption that all students who select or are selected for hyper-acceleration are well-served motivates this study. The resulting window on these students' evolving identities contributes to a more critical consideration of secondary mathematics course-taking trajectories within and across schools.

Algebra I as a "Moving" Gatekeeper

The gatekeeping role of Algebra I in the United States has been at the forefront of educational policy and evaluation for over 30 years. In 1997 the U.S Department of Education released a white paper titled "Mathematics Equals Opportunity" arguing that the key to understanding mathematics was studying Algebra I in middle school. This drive toward a more rigorous middle school mathematics curriculum would procure "an important advantage" (p. 2) in readiness for advanced mathematics and science coursework. Students who did not have gateway would be "behind" (p. 3) in appropriate course selection on the road to college.

In response to concerns about mathematical competitiveness in the global marketplace and about equitable access for students from traditionally underserved populations, many school divisions have enacted policies that encourage broader participation in formal algebra courses in middle school. According to data published by the National Center for Education Statistics, the percentage of students enrolled in Algebra 1 or higher in Grade 8 or earlier in the U.S. grew from 16% in 1990 to 42% of students in 2015 (Loveless, 2016). This increased access to Grade 8 Algebra has been accompanied by a greater interest in further acceleration of Algebra I as curricular intensification (Domina & Saldana, 2012). Between 2003 and 2013 the percentage of students enrolled in Grade 7 Algebra I in California more than tripled from 2% to 7% (Domina, Hanselman, Hwang, & McEachin, 2016) in parallel with universal Grade 8 Algebra I policies. In a nationally representative survey of students enrolled in first-semester STEM calculus courses in 2010 (Mathematical Association of America [MAA], 2017), 12.8% of respondents reported high school mathematics course sequences which would have been preceded by Algebra I and Geometry in middle school.

The emergence of this course-taking track called Grade 7 Algebra I in parallel with broader participation in Grade 8 Algebra I is consistent with the tenets of effectively maintained inequality (EMI) as theorized by Lucas (2001). "Advantaged actors secure for themselves and their children some degree of advantage wherever advantages are commonly possible" (p. 1652). In the context of acceleration of secondary mathematics, increased access to Grade 8 Algebra for students with a greater range of prerequisite achievement creates a quantitative leveling in Algebra I course taking. This phenomenon drives a need for a qualitative distinctions in the form of Grade 7 Algebra I, which becomes cultural capital, or increased access and achievement, along the dominant axis of equity in mathematics education (Gutiérrez, 2002). Early study of formal algebra may reify mathematical success in terms of memorization, speed, and correctness (Boaler, 1997) and may increase the risk of insufficient attentiveness to foundational conceptual understandings necessary for calculus (Bressoud et al., 2012). Experts in the teaching of algebraic reasoning in elementary and middle school distinguish algebra studied early from acceleration of coursework in that it leverages algebraic context in problems without the structures of formal algebraic notation (Carraher, Schliemann, & Schwartz, 2008; Mason, 2017). Hyper-accelerated experiences with algebra and geometry may also deconstruct productive dispositions toward mathematics for those who fail to maintain advanced course taking trajectories (Finkelstein, Fong, Tiffany-Morales, Shields, & Huang, 2012). Finally, hyper-acceleration as an emergent track in secondary mathematics may impede a more equitable examination of potential to learn beyond prior course achievement (Horn, 2006).

Theoretical Framework – Narrating OTL from a Situative Perspective

The hyper-acceleration of Algebra I to Grade 7 or earlier, often based on performance in elementary mathematics and scores on algebra readiness assessments, becomes a narrative of smartness which defines a student's position as a participant in advanced mathematics. Hyper-acceleration creates newly figured mathematical worlds (Boaler & Greeno, 2000; Horn 2008) because it builds selective contexts in which positional identities are formed through “socially produced, culturally constituted activities” (Holland, Lachicotte, Skinner, & Cain, 1998, pp. 40-41) and actions mediated by perceptions of “power, deference, entitlement, social affiliation and distance” (Holland et al., 1998, pp. 127–128). Identities of smartness in mathematics are thus socially constructed by interactions with peers, teachers, counselors, and administrators (Hatt, 2012) in the pursuit of educational advantage (Lucas, 2001). These identities may represent narrow views of mathematical competence (Gresalfi & Hand, 2019; Louie, 2017) which are affirmed or challenged as students advance through middle and high school.

Wenger (2010) further conceptualized identity as a learning trajectory which is “not an object, but a constant becoming….our identities incorporate the past and the future in the very process of negotiating the present” (pp. 133-4). Thus, the hyper-acceleration of Algebra I as enacted and experienced can best be understood by describing longitudinal formations of mathematical identities through middle school, high school, and college, thereby extending traditional discourses on OTL as access to content and resources.

Reconceptualizing OTL from a sociocultural perspective. The earliest conceptualizations of OTL were research constructs which assess whether students have had the opportunity to study a particular topic or learn how to solve a particular type of problem (Husen, 1967). OTL was thus described in terms of content taught, adequacy and allocation of educational resources, and classroom processes and practices (Pullin & Haertel, 2008, p, 17) to support interpretations of international test scores. OTL has also been used as a guiding idea in educational policy (McDonnell, 1995) in a move toward explaining the mathematics achievement gap as inequitable access to experienced and qualified teachers, high expectations for success, appropriate challenging curricula, and per pupil funding (Flores, 2007; Schmidt, Cogan, Huang, & McKnight, 2011)

Research on OTL in accelerated mathematics contexts often relies on objective, standardized measures of mathematics content exposure and emphasis (e.g., Stein et al., 2011; Tate, 2004; Wang, 2000). By traditional content perspectives of OTL, high-achieving students have access to advanced content and quality instruction and are deemed proficient by standardized measures of competence. A situative perspective of OTL instead foregrounds the positionings of competence in mathematical interactions (Greeno & Gresalfi, 2008) as an evolving identity along a trajectory of learning. In hyper-accelerated contexts, OTL can thus be described as the intersection of achievement and identity. Students become more self-aware of their evolving interest and confidence in advanced mathematics. Their changing views about the value of conceptual understanding along with their own places within overarching narratives of who is successful in mathematics describe the quality of learning. A situative perspective of OTL pairs the presumed affordances of early access to more advanced mathematics content with effectivities (Gee, 2008), or capacities to act on these affordances as participants in these newly figured mathematical worlds. When there are disconnects between access as an educational advantage consistent with EMI theories (Lucas, 2001) and effectivities as meaningful opportunities to engage with rigorous mathematics, the presumption of additional OTL with hyper-acceleration becomes flawed.

Narrating OTL at the intersection of achievement and identity. Building upon more recent conceptualizations of OTL as a "relationship between schools and learning" (Floden, 2002), the framework for this study broadens historical content perspectives of OTL and uses retrospective college students' stories to describe the quality of learning on hyper-accelerated secondary mathematics trajectories. Within this situative framework, achievement is persistence in the mathematics pipeline and valuation of conceptual understandings beyond procedural recollection and replication. Identities of smartness reflect beliefs about competence and are constructed by stakeholders within and beyond hyper-accelerated classrooms.

The combined works of Gee (2008), Greeno and Gresalfi (2008), and Moss (2008) on situative evaluation of OTL as a matter of equity also informed this framework design. The subjective juxtaposition of mathematics achievement and mathematics identity in hyper-accelerated contexts is revealed by students' articulations of the what, how, and why of their mathematics learning. OTL in hyper-accelerated Algebra I contexts is thus both conceptualized and operationalized within the achievement and identity narratives of students. Sfard and Prusak (2005) described identity as narrative for researching the "complex didactic" between learning and sociocultural context (p. 5) which acknowledges the roles which humans play in shaping learning outcomes over time (Darragh, 2016; Radovic, Black, Williams, & Salas; 2018). By extension, operationalization of achievement and identity together as retrospective narratives of hyper-accelerated figured worlds can describe both augmented and diminished opportunities to engage with mathematics in meaningful ways. While acceleration alone in mathematics is often presumed to increase OTL (Larson, 2017), a situative perspective on OTL introduces a higher expectation of engagement, struggle, and construction of deeper meanings. An augmented OTL in this sense becomes a more productive engagement of mathematics that transcends the ability to perform or replicate mathematics well, while a diminished OTL might communicate competent students should not have to struggle.

Methods

The tenets of hermeneutic phenomenology (Heidegger, 1988) are especially appropriate for interpreting the information-rich realities within narratives of the hyper-acceleration of formal algebra. Hermeneutical phenomenological research methods are grounded in an ontological perspective that meaningful reality is socially constructed (Crotty, 1998, p. 63). The perspective of the observer is "intertwined with the phenomenon" and is not expected to have objective characteristics (Mishler, 1979, p. 10). Multiple truths can be revealed with a focus on the participants' descriptions of learning in their figured mathematical worlds. This narrative inquiry will answer the following research question: How do recent graduates from one International Baccalaureate (IB) high school within a socioeconomically diverse suburban school describe their OTL in mathematics after hyper-acceleration of Algebra I?

Participants and setting. Because the decision to study formal algebra early is a marker of mathematics excellence valued in competitive college admissions, I suggest the phenomenon of hyper-acceleration is most richly constructed by college students who can together reflect upon their shared middle and high school experiences. Recent graduates of West Valley High School, a mid-Atlantic suburban IB public school with a socioeconomically and ethnically diverse population of over 2500 students and broad Algebra I acceleration policies, were recruited to participate in focus groups and follow-on interviews. The participants had studied Algebra I Honors in Grade 7 between 2009 and 2014 at one of two middle schools in the community and had completed Grades 9 through 12 at West Valley High School.

An IB high school provides an especially rich context for the study of hyper-acceleration. While Advanced Placement (AP) programs offer single-year courses (AB Calculus, BC Calculus, AP Statistics) with content aligned to typical college courses, IB programs offer two-year mathematics course sequences with different rigor which cover a wide range of precalculus, calculus, and statistics topics. The curriculum has an explicit expectation of connected mathematics understandings across multiple years of instruction and problem solving in unfamiliar contexts. Furthermore, IB programs are also often placed in mixed-income or disadvantaged neighborhoods with a vision of breaking down barriers to rigorous educational opportunities for traditionally underrepresented students.

Although West Valley High School has an open enrollment policy, the honors and IB mathematics students are typically from White or Asian families. Most students are accelerated by one to three years with respect to the traditional mathematics course sequence of Algebra I in Grade 9, yet West Valley High School is consistently scrutinized because of low standardized test scores for non-accelerated students.

When hyper-accelerated students at West Valley High School remain on the most rigorous course trajectory, they study multivariable calculus during senior year as a dual-enrollment course with a local university. Participant high school course pathways after Grade 9 Algebra II Honors are described in Table 5. Shaded cells represent a "deceleration" from the more rigorous of two IB mathematics course sequences. This deceleration resulted in taking three years instead of two to complete one IB course. Blank cells represent the choice not to enroll in a mathematics course during Grade 12.

Table 5

High School Mathematics Course Pathways after Algebra 2 in Grade 9

Participants	**Grade 10**	**Grade 11**	**Grade 12**
Claire, Matthew	Precalculus	IB Standard Precalculus	IB Standard Calculus
Ashley	IB Standard Precalculus	IB Higher Precalculus	IB Standard Calculus
Elizabeth	IB Standard Precalculus	IB Standard Calculus	
Jessica	IB Higher Precalculus	IB Standard Calculus	IB Higher Calculus
Samantha and Jonathan	IB Standard Precalculus	IB Higher Precalculus	IB Higher Calculus
Robert	IB Higher Precalculus	IB Higher Calculus	
Collin, Mindy, Liam, Nadine, Michael, Stephen, Kevin	IB Higher Precalculus	IB Higher Calculus	Multivariable Calculus

Consistent with an EMI framework which postulates the pursuit of both quantitative and qualitative advantages in tracked educational systems, many hyper-accelerated students and their parents grapple with difficult choices when students begin to struggle in advanced mathematics. Their concerns about grades which are not As or Bs often lead to dilemmas about switching to non-honors classes with less content coverage.

Students and parents often complain of an unacceptable gap between honors courses and non-honors courses.

Data collection and analysis. Each participant contributed to one of four 90-minute focus groups. The unique affordances of focus groups for narrative construction were the influence of a trusted moderator who could facilitate productive discussions; a high level of participant contribution with a social affirmation of shared experiences; dynamic data collection that responded to participant contributions; non-verbal cues as an additional research input; and interaction among participants (Segoe, 2012). A subsequent iterative sequence of critical events analysis (Webster & Mertova, 2007) and thematic analysis (Riessman, 2008) became a rich hermeneutic cycle of moving between transcriptions and the emerging description of the phenomenon which contributed to the trustworthiness of the analysis. In the first stage of analysis, focus group transcripts were coded for critical events and related events which revealed or substantiated fuller meanings in shared recollections. Further coding for implicit, explicit, and tacit identity claims (Carspecken, 1996) revealed the essence of evolving notions of mathematics success and competence. In the second stage of analysis, identity claims and their associated transcript excerpts were deconstructed and merged to create cohesive chronological episodes across all four focus groups. These temporal episodes spanned elementary school, middle school, high school, and college mathematics experiences. This analysis confirmed and refined distinct meanings (Dennis, 2018) within the critical events analysis categories. The resequenced transcripts were then coded for what, how, and why mathematics was learned with categorizations for OTL and status.

Follow-on individual interviews were conducted with a subset of the participants for member checking and deeper interpretation. Peer scrutiny, individual interviews with three graduates who were not able to attend the focus groups, and researcher reflective commentaries promote confidence (Shenton, 2012) that the multiple realities of hyper-acceleration described herein are historical truths (Polkinghorne, 2007) from the perspectives of the participants.

Researcher positionality. As a veteran teacher at West Valley High School during the time of the participants' attendance and as a doctoral candidate in mathematics education, I held a position of power in this interpretative research project. My participants articulated their evolving identities within an educational structure that encouraged them to challenge themselves in rigorous mathematics courses and to gain admission to prestigious colleges. I brought a different vantage point to our conversations. I sought to understand why too many of the hyper-accelerated students at West Valley High School struggle unproductively with content in IB courses. Because of my depth of experience with structures of acceleration, I was uniquely positioned to interpret participants' celebrations and frustrations in their evolving beliefs about success in mathematics.

While I followed a common semi-structured interview protocol for each of the four participant focus groups, I adjusted the sequencing of questions in real-time based on the direction that the participants took the conversations. I leveraged my knowledge of their teachers, their peers, and requisite mathematics content to probe topics more thoroughly. During individual follow-up interviews, I shared emergent findings with my participants and asked for confirmation of my interpretation of their words, actions, and reactions. They understood my commitment to careful construction of their multiplicities of truth which describe OTL with hyper-acceleration of formal algebra. The richness of this data and the depth of the findings reflect the authenticity and the transparency that only an insider's perspective could bring to this research.

Results and Discussion

Evolving beliefs about smartness in mathematics emerged in the participants' telling of their elementary, secondary, and college figured mathematical worlds. Their descriptions of elementary school selection processes for Algebra I in Grade 7 revealed a funneling effect in their journeys as high-achieving mathematics students. The Iowa Algebra Readiness Assessment (IARA) in Grade 6 was a common critical event that confirmed identities of smartness in mathematics which had been previously established by selection for gifted services in elementary school. Each of the participants vividly recalled the sorting function of the IARA and its' powerful significance as a percentile score which would qualify them to maintain their status among their peers as they transitioned to middle school. While teacher recommendations and prior achievement in mathematics may also have been part of the selection process, the participants framed placement simply and completely as a test score. Failure to qualify for Algebra I in Grade 7 was an ostensible threat to their identities as members of the most challenging course mathematics courses that their schools offered. Their shared discourse on the power of Grade 7 Algebra I as an academic gatekeeper and an instrument of stratification

(Stinson, 2004) is all the more impactful in juxtaposition of transcript excerpts across the four focus groups.

Jonathan	Well, I guess there's the Iowa Test, right? In sixth grade there was a whole lot of hype around that. Like what grade you got on that. I guess it goes even further back. I mean there the whole gifted program and all these kids were like, it seemed pretty prematurely competitive concerning academia or whatever. And so I remember going into the IOWA, it was quite a thing. I remember I barely got in, I got a 94th percentile and you had to be a 93rd percentile.
Kevin	I remember sort of the mentality just like going through elementary and middle school and it started when I was in the gifted program. It was like this expectation that if you don't get like if you're not in Algebra I than you are a dumbass.
Ashley	My sister told me she got 88%, so she didn't make Algebra I, and she was really devastated. She was like, "All my friends are in Algebra I. I only made 88%."
Michael	There was one kid who was already taking Algebra I at the middle school (as a 6th grader), and so it always felt very high pressure and competitive. Looking back it's kind of crazy that seventh graders were all crazy stressed about it.
Matthew	If you weren't going into Algebra I you would just be reviewing what you did in elementary school. I think almost a 1/3 of our class got 95 or higher percentile for that. A lot of people got 99.
Elizabeth	I just remember it was a very, if you don't get this on that then you are looked down upon kind of.
Jessica	Because I had always done higher math, I was like, "Oh, I am good at math. That's my thing". It seemed like the normal next step to do Algebra I in 7th Grade.
Robert	I don't remember ever thinking, "Oh, do I want to?" it was more like, "I really hope I did well on this test". I don't think I ever specifically liked math; it was more like I do like okay in math. I didn't want to be the one person who doesn't get to take this.

Claire My best friend, Mary, wasn't going into Algebra I. She was waiting until eighth grade, and I really wanted to stay in math with her. And my parents were like, "No, you got almost a perfect score on this test. You have to go into Algebra 1. It's not an option."

The pursuit of this marker of excellence began in Grade 3 when students' identities as gifted in mathematics were first socially and physically constructed. Participants recalled lunches and recesses at different times of the day than students who were not gifted in mathematics and friendships formed only with other gifted students. Yet as adults they openly wondered about the fairness of a system of tracking (Oakes, 2005) which communicates to other children that they are not good at mathematics. Their keen recollections of percentiles earned on the IARA became public metrics of competence articulated by their peers, their parents, and even their teachers. The presumption that the greatest OTL was associated with the course titled "Grade 7 Algebra I Honors" was evident in their articulated need to be in a course which provided a curricular intensification and perpetuated social stratification (Domina & Saldana, 2012). Failure to qualify for the course would have been analogous to failing in mathematics.

Quality of learning after hyper-acceleration of Algebra I. The narratives of secondary mathematics experiences which followed this critical event of selection for Algebra I in Grade 7 are organized by three situative criteria for evaluating OTL (Moss, 2008). Drawing on Engeström's (2001) questions for a theory of learning, Moss integrated the sociocultural and situated perspectives on the relationships between learners and their learning environments to probe the what, the how, and the why of mathematics learning. While these categorizations are distinct for purposes of analysis and presentation, these questions were not posed directly to the participants. Instead, the responses represent my deconstruction and resequencing of transcripts of shared participant stories of mathematics over six or more years as taught, learned, and experienced.

WHAT mathematics do I learn in the classroom? Recollections of middle school mathematics as highly competitive were vivid alongside a conspicuous absence of memories of mathematical learning which would be classified by researchers as meaningful. Smartness was measured by speed and by test scores. Liam, who is now a mathematics major, recalled feeling as if he was not smart when the results of end-of course Algebra I standardized tests were released and shared publicly among peers. “Me and Adam Miller we were the only ones who didn’t get a 500 in our class, and I got ripped a lot.” Their stories of Grade 7 Algebra I and Grade 8 Geometry classes did not reflect an augmented opportunity to engage meaningfully with mathematics. Instead, participants shared memories of Algebra I as an endeavor to “solve for x” using reliable and predictable procedures modeled by their teachers. Fond memories were descriptions of puzzles and contests. Several participants expressed their disdain for Geometry as different and disconnected from the Algebra I they had been comfortable doing.

Jessica (Grade 7) My teacher had these games associated with math concepts. For learning quadrants, and I still remember these to this day, she called it karate math, and to learn the quadrants we knew it was like one, two, three, four, and still sometimes I think about that. And then quadratic formula, we would sing it to like, "X equals negative b plus"…

Claire (Grade 7) My math teacher did like 30-minute PowerPoints where she went through everything. She had one of those projector things where she wrote on it and it appeared on the board. So I did really well in that class because she actually went through and did all the problems with us before she let us go.

Nadine (Grade 7) We had a Pi day, we had a pie competition, and my teacher brought pie and I loved math.

Michael (Grade 8) My eighth-grade Geometry teacher, she would give us these little puzzle sheets, which were applications of geometry. One was to try to transcribe a proof in Chinese using our knowledge of geometry or something.

Claire (Grade 8) Anytime I asked my Geometry teacher to re-explain something, she's like, "It's logic, it's not that hard." I was like, "No, it's not just like one specific answer." Geometry was like the proofs.

Jessica (Grade 8) The proof thing, I totally had forgotten about, and I was like, "I don't understand what they're telling me to do, I'm showing you the steps. Why do I have to write in English what I'm doing while I'm doing it?" It was just a lot of new things, and I didn't really get it.

These stories of games, competitions, and lack of understanding of the relevance of deductive reasoning revealed a diminished OTL consistent with Schoenfeld’s (1988) description of well-run Geometry classrooms where successful students were passive consumers of other’s mathematics and rapid problem solvers. Students did not engage with mathematics in ways that valued struggle as an avenue toward deeper learning. Their mathematical recollections instead associated enjoyment, entertainment, and right answers with positive experiences with mathematics. They did not describe learning that failure could be productive (Kapur & Bielaczyz, 2011).

Valuing conceptual understanding over right answers. With their retrospective consideration of the foundational content which was necessary for later courses, participant valuation of conceptual understanding over earlier descriptions of speed, competition, and efficiency emerged. Their beliefs about pursuing right answers evolved with the recognition that problem solving in Algebra I and Geometry could have been a more creative endeavor with space for multiple approaches and deeper learning. The rigor of their IB courses made many of the participants question the quality of their earlier mathematical preparation.

Liam: I just remember feeling very much like discouraged in math. Just like through middle school in general. I don’t think I really started enjoying math again probably towards til the end of high school when I was just doing bad on all of those exams but enjoying the process of learning. I just remember it was so much like if you didn’t get it you just didn’t think you were ever going to get it.

Ashley I came from feeling like there had to be one right way all the time, until like maybe halfway through high school. Probably once I started IB and it wasn't working. It made such a big difference for me to finally be like, oh, I can try different things, 'cause I was so set. I was like there's one right way to do everything. I was crazy about it, and it drives you insane to think, oh, I'm not getting it, I'm not getting it.

Mindy I realize more and more how important it is to cement certain things early on, like what vectors really are or why, like especially in calculus, like what these functions are doing. I think learning conceptually what that is, I think that that's really important. I think I struggled with that because I just wanted to get the right answer and then I started caring kind of late.

Samantha I had a lot of friends in my classes that would get so wrapped up with the anxiety of trying to get the 100 that they ended up just getting into really bad habits, or the pressure was too much, and they were like, "I don't deserve to be in this class." And it's not a matter of that, it's just a matter of shifting the perspective of what you were trying to achieve. Not the 100, but full understanding, and then if you have full understanding you will probably get the 100.

Kevin Someone with better logic skills could coast through something without understanding it all because they just like to write formulas from other stuff and then they’ll hit a wall later on.

Collin Yeah, does that mean they are performing well, so they are good at math at that moment and set up not to be good at math later?

Kevin At the end of the day, even if you are getting problems right, does that necessarily you are good at math if you don’t understand the concepts themselves?

The acquisition of a valuation of understanding concepts over the pursuit of status in the form of grades and course names was articulated across all four focus groups as participants together considered the advice that they would give to today's hyper-accelerated students. This divergence from early status-seeking goals of test scores and grades characterized a shift from beliefs in mathematics as performance to be recognized by peers toward a more productive view of mathematics (Boaler, 2015) in which struggle could be viewed as positive.

The participants' evolution from believing that success in mathematics was one right answer to be quickly grasped and performed signified an augmented OTL in advanced high school and college mathematics courses. For some participants, this shift in perspective created a social acceptability in their high school mathematical worlds with choosing a pathway which did not end with multivariable calculus. A choice to "decelerate" from the fastest trajectory was not perceived as diminishing status in retrospect; instead, it became its own form of persistence (Horn, 2008) as an interaction between the students and the mathematical worlds in which they operated. For some of the participants, deceleration allowed them to take more time with precalculus concepts to improve foundational understandings. Thus, deceleration could foster a more substantive and lasting participation in the learning of mathematics. Some who remained on the path to multivariable calculus openly wondered if they should have decelerated.

Applications of mathematics in personal projects. This later valuation of conceptual understanding was often narrated as applications of mathematics or problem solving which transcended the replication of knowledge they had received in classroom instruction. As a criterion for completion of their IB mathematics courses, they submitted a 12-page individual exploration of a mathematics topic. They valued this assessment of competence because it was a topic of personal choice and the assessment was rubric based. They appreciated struggle and challenge within their own mathematics investigations. Three students described their projects as their greatest successes in mathematics because they discovered avenues to achieve and to enjoy the subject in new ways.

Robert (Grade 11) I wanted to do like mathematical modeling of disease and it was the time when Ebola was like a big problem. So the model I first wanted to look at he said it would be a little bit complicated so I used like a simpler one but then I made it more complicated by modeling how many people would get sick, and therefore how much equipment they would need at different places.

Claire (Grade 12) I chose to do a card trick for my IB paper, the 21-card trick. I ended up accidentally finding that there were 12 different ways that you could do the 21-card trick and still have it come out correctly, I think that turned my brain on. I think that's part of the reason I decided to take the Game Theory class in college, because I had to put myself into this situation where I had to really think about this card trick, and I was so excited about having to figure out how to do this in so many different ways. So I think that definitely triggered my want to keep taking math.

Samantha: (Grade 12) I was like, "Oh my gosh, me playing with numbers can result in these really cool things." And I had the support to help me, too, so it wasn't really being competitive with myself, but it was seeing that I could get lost in this kind of study, whereas I've always identified as more of a liberal arts mind. Well, in some ways. And so that was really exciting for me, and also just the fact that I could exercise my brain in new ways without being told, "Okay, this is the proof you have to do." But I was creating new things myself. And so for me, that was a huge accomplishment, to see that I could take the reins in an area that I'd always been confident in, but I had always been told what to do in.

The IB mathematics exploration assignment represented augmented OTL for students who had not previously articulated a love of mathematics. Problem solving became more than attaining a correct answer (Schoenfeld, 1988) in their later mathematics journeys. The IB curriculum as exploration and reasoning in unfamiliar contexts itself represented an augmented OTL. Their memories of worksheets and word puzzles from earlier courses were replaced by mathematical solving of a deeper character in IB examination questions and explorations.

These descriptions of what mathematics they were learning in high school conveyed an important divergence from earlier extrinsic valuations of Grade 7 Algebra I. Their views of acceleration as a conferral of academic status became more intrinsic valuations of learning conceptually and applying the subject of mathematics as they became older. Yet this divergence was not equally expressed by all participants. For two participants who are majoring in mathematics, the enjoyment of the subject transcended teacher and course throughout middle school and high school; they created their own OTL. For others, OTL was narrated in the enjoyment and confidence that they discovered when striving for deeper understandings in rigorous and relevant mathematics.

HOW do I learn mathematics in the classroom? Identities of smartness which were conferred in the selection for Grade 7 Algebra I were challenged at different times as the participants navigated varying classroom environments in their subsequent courses. They articulated a need to maintain their status of smartness among their peers and believed that making mistakes compromised this status. The fear of appearing different from their peers in coupled with the fear of asking for help were both palpable threats to an augmented OTL on advanced mathematics trajectories.

They were supposed to be the students that were good at mathematics, and they avoided threats to these identities in their classrooms. If they did not understand something quickly, they felt that they were failing. Yet at the same time, they did not necessarily want to draw attention to themselves as being too good at math. The following resequenced narrative captures these tensions in their classroom relationships as the students managed points of struggle in their high school mathematics courses.

Robert: I remember there was maybe five of us who were freshmen in Algebra II and everyone else were sophomores. We all sat in this one clump, kind of towards the back.

Kevin: If you're coasting and you hit a wall, you don't really want to ask around. You don't really want to ask people. It's hard to ask for help.

Matthew: Especially when you're in this group like you don't need to study like you don't need help.

Liam: When you think everyone around you is getting an easy "A" and you are just sort of like uh it's embarrassing.

Jessica: I felt the expectations to have ... Like, because I'm in this class, I should just know, and I should pay attention, and therefore, and the whole notion of going afterwards to ask questions and be vulnerable to need to ask for help was always hard for me.

Michael: That was something I was always terrified of, was if I didn't get something immediately, I would freak out and be like, "Oh my God, I'm failing," or "I'm doing terribly," and being willing to just take a step back, and being like, "Okay, this one little thing is not gonna derail my entire existence," and then just being willing to ask questions, sit down, talk to your friends and work through things.

Collin: Yeah. That was part of growing up for me. Learning to ask for help, Learning that you can't shoulder everything.

These described tensions reflected their earliest notions that they should not need to ask for help along with a growing recognition that vulnerability and support seeking would lead to greater learning. A teacher's responsiveness to questions was critical in fostering a willingness in the classroom to risk being wrong and a belief that failure and struggle can be productive (Boaler, 1997; Boaler & Greeno, 2000). A teacher who created an environment of approachability was important in diffusing the oppositions that students felt in the form of seeking help as compromising their social status. Teachers who communicated that failure was acceptable in the process of learning mathematics more meaningfully also contributed to augmented OTL. Participants described classrooms in which learning was strongly related to being able to fail without threatening their identities.

Elizabeth — My Algebra II teacher always there to help, and never condescending. If I got something wrong, she was always there, she took every question, even if it may have seemed stupid, so it wasn't like she was putting anyone down. Which I feel is something that happens a lot in math classes, they're like "I don't even want to explain that to you."

Robert — My IB HL teacher definitely made me feel like he cared about how I was doing and even when I was having trouble he wasn't like, "Oh, you're just bad at math", it was very much like I'm willing to sit down with you and figure this out.

Samantha — If a teacher is really hard on a student that doesn't do well, or is like, "Ooh, some people failed this test," you know, I don't want to be that person who failed the test that everyone's looking at now. So that culture just was wiped out by the teacher never pointing fingers or being uncomfortable around the concept of failing, if that makes sense. So that ties into the whole encouragement thing, and encouraging students who aren't understanding to ask questions, and that it's okay to not understand a concept and fail in that regard, fail in understanding.

Their evolving beliefs that struggle could be productive were supported by test retake policies and by expectations for participation in classroom problem solving. Many students described an augmented OTL because they learned more when they prepared to retake tests. The inability to master content in the time allotted in a fast-paced curriculum did not need to represent failure. However, some participants decried test retakes as unfair. These contrasting views captured the inherent contradictions in grades as markers of smartness and grades as a measure of conceptual mastery. Students who articulated a strong competitive orientation resented retakes as a structure which diffused their own status as top performers in mathematics, while students who did not see themselves as competitors appreciated the additional time and opportunity to demonstrate mastery. The teacher's purposeful articulation of retakes as a structure which enabled more meaningful learning contributed to evolving student beliefs that conceptual understanding was more important than grades. Students also valued participatory structures in their classrooms which held them accountable to themselves and to one another through shared struggle in solving unfamiliar problems. Collaboration with peers became an augmented OTL which instilled confidence that they could work in mathematics at higher levels.

Hyper-acceleration alone did not inspire confidence, enjoyment, and interest in mathematics; instead it was the support of the teacher, the freedom to approach problems creatively and collaboratively, the acceptability of being less than perfect, the encouragement to seek help, and the application of mathematics in unfamiliar contexts that ultimately augmented learning opportunities.

WHY do I learn mathematics in the classroom? Within their expressions of an overarching extrinsic need to maintain status among their peers as they entered middle school, the focus groups participants offered divergent articulations of purpose in the subsequent pursuit of advanced mathematics in high school. Mindy and Stephen, now mathematics majors in college, described a lifelong love of math. Unlike other participants, maintaining status was not a primary motivator in their continuing pursuit of the highest level of mathematics throughout middle school and high school. Instead, they shared that their intrinsic valuation of mathematics propelled their trajectories toward multivariable calculus.

Mindy: I made the choice because I just wanted to take the hardest and math was easy for me and it was a no-brainer I think. I mean the gifted thing, being with people within the competitive environment, I guess, it kind of maybe egged me on a little bit, but I have always just loved math. My love of math was heightened and I was challenged to push myself.

Stephen: I guess I'm just oblivious to that kind of thing. I guess I just ignored other people and their opinions and I like math and I did it and so that's what I did from kindergarten to now.

The multiple IB course pathways at the high school level allowed other participants to make course-taking decisions that did not threaten their status; their choices became more personal and less public. These course selections reflected their individualized conceptualizations of challenge, enjoyment, and confidence, but admission to elite colleges and the expectation of college credit remained foregrounded in their retrospective reflections on their decisions. Their teachers played a prominent role in these choices; the passion and energy of the classroom teacher who articulates the importance of depth of learning has an undeniable power to inspire persistence in mathematics as a productive and enjoyable endeavor.

Two contrasting individual stories of students who sought challenge as a motivation for being in Grade 7 Algebra I reveal the power that teachers wield in discouraging or encouraging persistence in advanced mathematics.

Jonathan (Grade 6) I'm pretty sure I had to have a conversation with my teacher after the IOWA saying "Oh, you barely got it. Are you sure you want to do this?" And I was like, "Of course I do", because I felt like math was the most challenging and interesting subject to me at the time.

Elizabeth: (Grade 6) Yeah, I think part of it was, like I don't like being bored in class and I like being challenged by class, so Algebra I is the most challenging thing you can do as a seventh grader.

Jonathan and Elizabeth later described contrasting Grade 9 mathematics experiences, each of which reveals how the responsiveness of teachers and schools to the mathematical challenges that that children seek can impact their longitudinal beliefs about the why of studying mathematics. While Jonathan's Algebra II experiences did not provide the challenge that he sought during high school, Elizabeth discovered in Algebra II how she could have experienced mathematics in her earlier classes.

Jonathan (Grade 9) I can't even remember who I had as a teacher in Algebra II which is bizarre because I remember all my teachers. I feel like all of the kids who were a year or two above us who were in our class, math was definitely just another subject for them. With the whole bar kind of being brought down a bit, I definitely grew a little disenchanted with math, I guess. So I grew a little bit disinterested. I guess I wasn't as driven, so I was like "Yeah, I will just do standard level".

Elizabeth (Grade 9) I think there was a higher expectation of understanding (in Algebra II). I know that there were people in the class who didn't care as much as me, but that didn't make me think that the expectation was lower. I never really liked math, and I was never really confident. Then I came to high school and my teacher was so passionate about explaining why things are the way they are, and that made me enjoy math so much more, to get deeper into it rather than just be "Here's the formula", but to derive the formula and explain why it makes sense, and explain where it comes from, and to do a bunch of examples that were actually relevant. It was so changing.

While Elizabeth and Jonathan narrated very different experiences with Grade 9 Algebra II, they both decided to “decelerate” in Grade 10 and study IB mathematics at the standard level. Each of these students had the aptitude to continue on the trajectory toward multivariable calculus; Jonathan took pride in his perfect SAT mathematics score and Elizabeth scored in the top 10% internationally on her IB examination. However, they did not find value in pursuing the most rigorous mathematics within the broader landscape of their high school course choices.

Jonathan’s diminished OTL in Algebra II contributed to his decision to pursue other IB subjects in greater depth. In his follow-on interview, Jonathan elaborated on the importance of engaging students in mathematics and striking an appropriate balance between anxiety and boredom. His personal willingness to work toward understanding content was driven by his level of engagement. He expressed admiration for teachers who could provide broader perspectives about mathematics than simply teaching him how to complete a problem. Despite Elizabeth’s augmented OTL in Algebra II, she still did not feel confident enough to continue on the most accelerated pathway. She stated that more challenging middle school experiences would have built her confidence. She knew that she would need to take calculus for her neuroscience major in college and was content to wait. Their decisions point toward the important role that both interest and confidence can play in the continuing pursuit of advanced mathematics beyond the desire to maintain the status of being the most accelerated.

The why of learning mathematics was more about the why of taking mathematics for many of the focus group participants.. They secured the educational advantages and institutional cultural capital (Bourdieu, 1986; Gutiérrez, 2013; Lucas, 2001) that they needed to excel academically. All of the participants completed a two-year IB sequence in mathematics by the end of high school and attended prestigious universities across the United States. Their reflective articulations of whether they needed to be on mathematical pathways toward multivariable calculus in high school capture an ever-present tension between status and opportunities to engage in mathematics with enjoyment and confidence.

For many of the participants, hyper-acceleration in Grade 7 was not a choice driven by interest in mathematics. Instead, they were meeting social expectations and striving to outperform their peers. The following pairing of Mindy's expectation of a mathematical purpose and Jessica's articulation of her own lack of agency describe the extremes of a spectrum of identity claims across participants about why we study advanced mathematics. They questioned either their own places or the places of others in this figured mathematical world.

Mindy: I feel like it's important to maybe consider the value of doing this accelerated path. A lot of you aren't going to be doing a lot of math in your life, maybe applied math in your major, if it's a science major, but what was the value of doing it if you're not going to use it? Is it worth doing it, was it to get into college, was it to just learn math for the sake of learning math? Might you eventually use it, is it something like problem-solving skills in general? I'm not saying you shouldn't cause stress to yourself in high school by doing this accelerated path, but I'm wondering the reasons for people doing it, if they're not going to see themselves being a mathematician.

Jessica: I think I always had a little bit of imposter syndrome in that I always didn't quite felt like I belonged there. I was like, "I'm here, and I'm trying my best, but everybody else seems just a bit smarter," because you're with the people who are gonna go on to be mathematicians, so I think I was kind of comparing myself to them, and I was like, "I'm not them. Why am I here?"

> But it wasn't a huge anxiety, but just a little subtle back-of-my-mind voice was like, "Why are you here? I don't know, I just ... They told me to."

Mindy later offered a caveat to her statement about hyper-acceleration and meeting expectations. She acknowledged that twelve-year-old children should not be expected to know their career pathways when they choose to hyper-accelerate. Yet their early experiences with formal algebra did not foster openness to mistakes and productive struggle as a path toward learning mathematics well. OTL as meaningful engagement with mathematics was conspicuously absent in participant narratives of middle school mathematics. Algebra I in Grade 7 was unchallenging for many of the participants, and their recollections of high levels of competitiveness or overt recognition of smartness were devoid of specific memories of the importance of studying mathematics. Geometry, dismissed by some of the focus group participants as frustratingly different than the affirming repetition of “solving for x”, should have provided a critical foundation for calculus and deductive reasoning. However, but it was not valued as such by the participants as middle school students.

Quality of learning and mathematical competence. This examination of hyper-acceleration as a multi-year trajectory builds from Horn’s (2008) work on identities of competence as constructed across four years of high school mathematics curricula. Horn described remedial curricular tracks as figured worlds which reified narrow beliefs about students’ mathematical potential, and she argued for structural change within and across classrooms which would challenge socially constructed identities of competence. As an emergent track in today’s mathematical figured worlds, hyper-acceleration also assigns identities of competence and creates curricular structures through which rigorous mathematics courses become the “province of some students to the exclusion of others” (p. 208).

The identities of the students in this study have been shaped by political and cultural figured worlds in which they participate. Achievement is explicitly defined by grades earned and courses taken, while their identity claims are inextricably linked to *what* mathematics content was valuable, *how* mathematics was experienced in their classrooms, and *why* mathematics was a challenge worth seeking or not seeking. Louie (2017) elaborated on these ideas of identity and status by criticizing a culture of exclusion which “limits all students’ access to rich and meaningful mathematics learning experiences and further limits many students’ opportunities to develop identities as mathematically capable learners and thinkers.” (p. 489). The situative perspective on OTL (Greeno & Gresalfi, 2008) within hyper-acceleration thus becomes a critical lens which we can better understand and challenge this culture.

Implications

The interpretative layering and resequencing of the narratives of the college students in this study provided a novel window on the relationship between identity and persistence (Horn, 2008) and how success in mathematics is positioned over time for hyper-accelerated Algebra I students. These student stories rewrite the overarching narrative of hyper-acceleration as an educational advantage by motivating further conversations about the intrinsic value a pathway which should mean more than the continuing accrual of status. The students’ evolving valuations of conceptual understanding over grades earned should also elicit profound questions about the impetus of these realizations. Their appreciation of learning more than grades changed from middle school to college, and additional research is needed to understand if this change might be explained by the passage of time, the acquisition of more mature perspective, the nature of mathematics within IB curricula, or perhaps exposure to more abstract mathematics content which demanded more than competency-based instruction.

Expressions of augmented OTL were concentrated in IB courses in the later years of the secondary mathematics trajectory, and expressions of diminished OTL were more prevalent in the telling of Algebra I, Geometry, and Algebra II stories. These findings should inform curricular design and pedagogical choices in the critical middle school mathematics years. Students need early opportunities to engage in collaborative productive struggle through which failure becomes an avenue to deeper understandings. As long as the pursuit of hyper-acceleration is not informed by empirical evidence of the why, what, and how of mathematics learning in context, Grade 7 Algebra I risks being reduced to a sought-after prize consistent with an EMI perspective on education. Samantha characterized hyper-acceleration as a commodity that families seek, and Michael described a competitive mathematical hierarchy within West Valley High School. Neither of these descriptors emphasize the quality of mathematics learning within these emergent curricular structures. OTL as narrative in this study should help teachers and administrators to better meet the academic and social needs of students on these pathways and to help them to more fully realize their mathematical potential.

It is also important to acknowledge the potential goes undiscovered when tracks of smartness in mathematics are so explicitly presented at such young ages in diverse school communities. Identities of smartness are as important to those upon whom they are bestowed as to those upon whom they are not bestowed, and narrative inquiry compels us to interrogate the divisions of race and class which surround the phenomenon of hyper-acceleration. The intersection of achievement and identity along mathematics learning trajectories, starting in as early as Grade 3, can create unmalleable beliefs about who should be on these pathways. Elizabeth's uncomfortable reflection that most of the students in her Grade 7 Algebra I class were White despite the strong diversity of her middle school community drives the question that goes unasked in the West Valley community and in too many others like it. Who are the true beneficiaries of hyper-accelerated pathways, and how does the existence of these pathways impact broader access to rigorous mathematics learning within all secondary mathematics classrooms? The emergent valuation of conceptual understanding across the participant narratives in this study suggests that rich, unpredictable experiences with grade-level content in middle school in lieu of hyper-acceleration could better foster identities of competence and confidence for more students and inspire a lasting passion for mathematics. These experiences might also productively challenge our high-performing students' early social orientations toward competition and perfection in mathematics classrooms.

Conclusion

Perhaps the greatest significance in these narratives of the phenomenon of hyper-acceleration of Algebra I lies in the stories that remain untold. The voices of students who hyper-accelerated Algebra I and had painful mathematics experiences or did not persist in IB courses are largely absent in this resequenced narrative. Ashley's story in particular suggests that there is more to be understood about negative experiences with hyper-acceleration. She described Grade 7 Algebra I as “scarring” because she skipped a year of mathematics and did not learn fractions well. Her subsequent deceleration and her personal battle to successfully complete an IB mathematics sequence despite her early struggles became the ironic substance of her college admissions essay to a top-tier college.

Many parents grapple deeply with the decision to hyper-accelerate their children in mathematics because they do value the quality of learning and construction of solid mathematics foundations. Their stories live in the blog communities and social media exchanges which question policy and practice in schools like West Valley. Parents and teachers alike are looking for answers which can better meet the needs our children as individuals. Additional research offering a situative characterization of longitudinal OTL in other school communities will inform not only policies of selection for hyper-acceleration and but also practices which redefine success in our most advanced K-12 mathematics offerings. This research will also provide insight into classroom structures and teacher dispositions which appropriately support and challenge each of our young algebra learners.

Chapter Five - But You Are "Good at Mathematics": Women Who Decelerate After Algebra I in Grade 7

Abstract

Identities of smartness in mathematics are explicitly assigned to students in the United States when they are selected to hyper-accelerate their study of Algebra I to Grade 7 or earlier. Yet these identities evolve as hyper-accelerated students navigate secondary and post-secondary courses and reevaluate what it means to be "good at mathematics." This study narrates the identities of three hyper-accelerated girls who decelerated their mathematics course-taking trajectories on their pathways toward majoring in STEM fields. Their stories of opportunities to learn (OTL) from a situative perspective are constructed using syllogisms as a form of persuasion. Retrospective narratives of hyper-acceleration describe reveal a sociocultural interplay of acceleration as an educational advantage and mathematics as a masculine discipline. Their rhetorical arguments inform both policy and practice of Algebra I acceleration in secondary mathematics. Implications for high-achieving girls' participation in the STEM pipeline are presented.

Keywords: Identity, "good at mathematics", gender, secondary/postsecondary mathematics, acceleration, algebra

Introduction

In the United States, acceleration of Algebra I to 8th grade or earlier for students with demonstrated readiness as measured by elementary school classroom performance or standardized test scores has been shown to proffer an educational advantage with respect to grades, advanced mathematics course taking, and college attendance (Gamoran & Hannigan, 2000; Rickles, 2013; Spielhagen, 2006). As the number of students studying Algebra I in the middle schools increased from 16% in 1990 to 47% in 2015 (Loveless, 2016), there has been an accompanying increase in the number of students studying Algebra I in Grade 7 or earlier (Chou, 2018; Domina, Hanselman, Hwang & McEachin, 2016; Loveless, 2016; Picciotto, 2014). These students often seek to differentiate their academic performance in pursuit of elite college admissions.

Researchers have noted that increased access to hyper-accelerated mathematics yields different experiences for girls when compared to boys, generally. For example, prior research on “top set” learners, i.e., those in what would be considered hyper-accelerated classes in the United Kingdom (Boaler, 1997; Boaler, Wiliam, & Brown, 2000), described differential negative effects for girls who were placed in the most accelerated tracks of mathematics instruction between ages 10 and 13. These classrooms were highly procedural with an emphasis on didactic instruction. Girls expressed their frustrations and anxiety about the fast pace of courses in which speed, effortlessness, memorization, and right answers were valued over opportunities to understand mathematics well. Research of this nature highlighted how the well-intentioned actions of teachers, parents, and administrators to identify mathematically promising girls in late elementary and middle school for hyper-acceleration may introduce unintended negative consequences from at OTL perspective.

There is a growing international interest in narrating identities of high-achieving girls and their persistence in advanced mathematics. Recent publications describing gendered lived experiences of ability grouping in Norway and Australia (Foyn, Solomon, & Braathe, 2018; Wolfe, 2019) suggest that there is a commensurate need to examine how acceleration of mathematics may be differently experienced by girls. By emphasizing a sociocultural conceptualization of girls' development of positive mathematics identities (Radovic, Black, Salas, & Williams, 2017), this emergent direction in research recognizes gender as a social construct (Levya, 2017) which can expand traditional discourses of underperformance in women in STEM. The sociocultural turn in mathematics education research (Lerman, 2000) leverages identity as a situated narrative (Sfard & Prusak, 2005) of what means to be "good in mathematics" (Bishop, 2012; Darragh, 2015; Hall, Towers, & Martin, 2018; Radovic, Black, Wiliam, & Salas, 2018). The storied experiences of young women can describe ways in which opportunities to learn (OTL) are gendered with acceleration of secondary mathematics courses.

This study builds upon contemporary literature on gender and mathematics identity in secondary classrooms by narrating the subjective experiences of three young women who studied Algebra I in Grade 7 at one mid-Atlantic, socioeconomically and racially diverse International Baccalaureate (IB) high school. Hyper-acceleration is a growing phenomenon which confers a status of smartness in mathematics, and there is a need for research on the participation of girls in these competitive mathematics contexts at a critical time in their mathematics identity formation.

It is worthy of note that the cultural constructions of gender and mathematics (Hottinger, 2016) in this study describe OTL in a highly resourced community where hyper-acceleration carries a presumption of mathematical competence. The women who elected to participate in this study identify as White or Asian, and in the field, much research is written as to presume that "girls" is synonymous with White girls. Thus, it is also important to acknowledge a body of significant research (e.g., Esmonde, Brodie, Dookie, & Takeuchi, 2009; Gholson, 2016; Joseph, Hailu, & Boston, 2017) which addresses how gender intersects with other oppressed and privileged identities such as race, ethnicity, and class in assigning competence in mathematics education. This intersectionality often creates additional challenges and complexities for girls of color to construct positive and productive mathematics identities.

Theoretical Framework

A sociocultural perspective on OTL (Moss, 2008) examines the dynamic relationship between learners and their learning environments (Gee, 2008) and offers a situative lens on the OTL of hyper-accelerated students who would otherwise be deemed successful by grades and standardized test scores. Classroom and school contexts create interpersonal and informational resources of participation which define student' beliefs about mathematics and themselves (Hand & Gresalfi, 2015) in figured worlds of secondary mathematics (Boaler & Greeno, 2000; Holland, Lachicotte, Skinner, & Cain, 1998; Horn, 2008). According to Holland and colleagues, figured worlds are "socially constructed, culturally constituted activities" (pp. 40-41). Horn (2008) argued that mathematics course sequences become figured worlds which are "lived, represented, enacted, and negotiated by the students, teachers, and others who inhabit them" (p. 204), Persistence in advanced mathematics is an enactment of student identity, and retrospective stories of OTL in these hyper-accelerated Algebra I figured worlds become identity as narrative which is "reifying, endorsable, and significant" (Sfard & Prusak, 2005, p. 17). Drawing upon Bishop's (2012) definition of identity as "the ideas, often tacit, one has about who he or she is with respect to the subject of mathematics and its corresponding activities" (p. 39), I argue that these identities are made explicit in researchers' interpretations of stories of hyper-accelerated students whose individual notions of smartness and success evolve as they continue on advanced mathematics pathways. Contextual experiences as manifested in individuals' narrated identities are vital to understanding hyper-acceleration as a phenomenon which promotes or restricts access to meaningful mathematics learning and persistence on STEM trajectories.

These narrative identities are not independent of gender identity. Mendick's (2005) research on tensions that high school girls face in mathematics as a masculine discipline yielded a set of theorized gendered binary oppositions which can explain how girls navigate hyper-accelerated contexts. These oppositions provide an additional layer for examination of young women's longitudinal narratives of OTL in hyper-accelerated contexts. They may experience being "good at mathematics" differently than their male peers. These gendered oppositions include fast/slow, competitive/collaborative, active/passive, dynamic/static, naturally able/hardworking, real understanding/rote learning, rule-based/creative and really good at mathematics/good at mathematics. A higher masculine value is associated with the first descriptor, while a lesser feminine value is associated with the second descriptor. Mendick's (2005) assertion that "it is more difficult for girls and women to feel talented at and comfortable with mathematics and so to choose it and to do well at it" (p. 217) is worthy of scrutiny after girls are explicitly identified as smart by virtue of placement in Grade 7 Algebra. Levya (2017) called for research on gender in mathematics education which simultaneously considers institutional, interpersonal and cultural stereotypes, and as such it is impossible to ignore the very masculine underpinnings of hyper-acceleration as educational policy.

The further speeding up of mathematics teaching and learning beyond Algebra I in Grade 8 may become a socially constructed competition with differential gendered effects among students who have attained the highest mathematical status in their schools. Cultural discourses which construct mathematics achievement and ability to reason as masculine traits (Hottinger, 2016; Levya, 2017) can make it more difficult for girls in middle and high school to see themselves as productive knowers and doers of mathematics. By examining where high-achieving college women who are majoring in STEM located themselves within a common six-year mathematics trajectory launched by selection for Grade 7 Algebra I, we can gain new insight about the potentially gendered ways in which girls experience this phenomenon.

This study analyzes the stories of women who "decelerate" within or beyond this secondary mathematics trajectory by repeating courses, choosing less rigorous courses, or choosing not to continue in advanced mathematics. It answers the following research question: How do high-achieving girls' rhetorical arguments of deceleration after Grade 7 Algebra I reveal gendered binary norms of participation in secondary and post-secondary mathematics?

Methodology

This interpretative narrative study extends a larger empirical examination of 15 college students' retrospective stories of hyper-acceleration at an International Baccalaureate (IB) high school in the mid-Atlantic region of the United States. In contrast with Advanced Placement (AP) Calculus programs offered in many U.S high schools for college credit, the IB program emphasizes inter-disciplinary learning across six subject groups in preparation for post-secondary study. The Higher Level (HL) two-year course focuses on formal justification and proof with rigorous connectedness among precalculus and calculus topics presented in familiar and unfamiliar contexts. The Standard Level (SL) course approaches the same topics with less rigor and more contextual problem solving. The IB program emphasizes the role of mathematics in a larger educational landscape with the aims to develop critical thinking skills and persistence in problem solving (International Baccalaureate Organization, 2017).

Participants. West Valley High School is a racially and socioeconomically diverse suburban community of over 2000 students. IB classrooms at West Valley High School typically do not present behavioral or work completion challenges to teachers, but student anxiety about grades and achievement is pervasive. The participants completed Algebra II as freshmen, and hyper-acceleration afforded them the opportunity to complete a two-year Higher-Level IB mathematics sequence in Grades 10 and 11 as a prerequisite for dual-enrollment multivariable calculus and matrix algebra during Grade 12. In addition to attending the same high school, each of the fifteen original study participants had also completed Algebra I in Grade 7 and Geometry in Grade 8 at one of two middle schools between 2008 and 2012.

In the original study, five of the eight male participants maintained the fastest course taking trajectory as they studied multivariable calculus during Grade 12 and are pursuing an engineering or mathematics major in college. By contrast, only two of the seven female participants persisted on this most accelerated secondary trajectory. Thematic analysis in the broader study revealed the what, why, and how of mathematics learning (Author, under review) within the phenomenon of hyper-acceleration, but the questioning protocols were not specifically written to elicit gendered experiences. The emergence of themes of anxiety, frustration, lack of confidence, and imposter syndrome in the larger study motivated this finer grain analysis of narrated experiences of women who persisted in STEM after hyper-acceleration. The narratives of three female participants (Table 6) within the broader focus group study were selected for further interpretation in this study. They had decelerated their mathematics course taking in differing ways during high school and college, yet they were majoring in STEM fields.

Interpretative mode of inquiry. The specific tenets of hermeneutic phenomenology (Heidegger, 1988; Kim, 2016) as a mode of inquiry are especially appropriate for understanding the information-rich realities of hyper-acceleration. The participants are positioned as the knowers, and the researcher makes her own beliefs and assumptions explicit in the interpretation of participants' truths. In contrast to the pursuit of variability within quantitative research paradigms, “variability in experience is generated within individuals...for the purpose of understanding the cognitive processes that lead to different experiences” (Ercikan & Roth, 2009, p. 240). Hermeneutic phenomenological research explicates “unique, idiosyncratic meanings” (Cho & Trent, 2006, p. 328) and provides a contextual lens that is often missing in quantitative studies of student outcomes in accelerated secondary mathematics.

Table 6

Participant High School Mathematics Course Pathways after Algebra 2 in Grade 9

Participants	Grade 10	Grade 11	Grade 12
Elizabeth	IB Standard Level Year 1	IB Standard Level Year 2	None
Ashley	IB Standard Level Year 1	IB Higher Level Year 1	IB Standard Level Year 2
Nadine	IB Higher Level Year 1	IB Higher Level Year 2	Multivariable Calculus

Data collection. Participants were recruited for the larger focus group study by a Spring 2018 email invitation distributed to IB students who had graduated within the past four years. Their shared stories were constructed within and across four focus groups and individual follow-on interviews which probed participant characterizations of OTL from a situative perspective (Greeno & Gresalfi, 2008). Shared discourse and extended responses (Mishler, 1986; Polkinghorne, 1988) were encouraged in the focus group using a semi-structured interview protocol to “create the very events” (Denzin, 2000) that the narratives reflect upon. Follow-on interviews were conducted eight weeks after the focus groups and were designed to encourage participants to elaborate on specific transcript elements.

Data analysis. Using structural analysis techniques (Knight & Sweeney, 2007), I constructed syllogisms and enthymemes from resequenced focus group and individual transcripts for the three participants. Because all dialogue is a form of persuasion, narratives become logical arguments which unveil inferences which may be hidden within the text of the participant words. Syllogisms typically take the form of a major premise, a minor premise, and a conclusion, and they align closely with the participant's original explicit, implicit, and tacit identity claims (Carspecken, 1996). Enthymemes are arguments "in which one or more premises (or a conclusion) is missing, and needs to be filled in, because it has not been explicitly expressed" (Knight & Sweeney, 2007, p. 228). Because uncertainty is inherent in the construction of enthymemes, the researcher must clearly identify the basis for the implicit understandings.

Triangulation of findings from identity claim analysis documents in the broader study, researcher reflective memoing, member checking, and peer debriefing meetings improve the trustworthiness (Lincoln & Guba, 1985) of the study and promote confidence that the constructed arguments of gendered enactments of deceleration after hyper-acceleration of Algebra I are historical truths (Polkinghorne, 2007) from the perspective of the participants.

Researcher Positionality

As a woman engineer with a graduate degree from Stanford University and as a veteran mathematics teacher, I have a personal and professional understanding of the challenges that young women face in navigating STEM majors and careers. I am deeply committed to opening doors in STEM for more girls. Yet I theorize that the increasing acceleration of girls in middle school Algebra I may instead detract from the very opportunities that they need to reason about mathematics and to make critical connections across topics. I taught the three participants in this study during their Grade 9 mathematics courses. The depth of my knowledge of their classroom engagement, school context, and pedagogy and applications of advanced mathematics positioned me to infer the deeper meanings in their narratives of years of subsequent mathamatics study.

Findings and Discussion

Each of the three participants in this study made unique choices to decelerate their mathematics trajectories after Algebra I in Grade 7. These decisions to decelerate reflect a gendered questioning of their mathematics abilities, and deceleration itself is an enactment of Mendick's *fast vs slow* opposition and evidence of evolving beliefs about themselves within a competitive mathematics culture.

My naming of the three stories is intended to capture the gendered contradictions in the stories of these young women. They had garnered explicit recognition of their perceived natural mathematics ability with selection for Algebra I in Grade 7, but each made ostensibly lesser, or arguably feminine, choices about their persistence and performance in mathematics which challenge the presumed affordances of hyper-acceleration.

Elizabeth's story – The "naturally-able" leaver. Elizabeth identifies as a self-assured, academically driven White woman who was classified as a gifted learner in

Grade 3. She is now studying behavioral neuroscience at a top-tier state university after making two explicit choices to decelerate her mathematics beyond Grade 7 Algebra I. First, she enrolled in IB SL Year 1 during Grade 10, and she then chose to complete the second year of the SL course during Grade 11 in lieu of pursuing the more rigorous two-year HL course. Although she did not take a mathematics course during her senior year, Elizabeth scored in the top 10% internationally on her IB SL examination prior to graduation. She proudly described this accomplishment as her greatest achievement in mathematics and recalled her surprise when receiving her score.

> I literally screamed and fell on the floor because I literally thought I had failed. I was like “Colleges are going to think I’m crazy because I failed my IB exam and I am not taking math my senior year.”

In her follow-up interview, Elizabeth elaborated on the intensity of this reaction. The admissions counselor at her first-choice college had told her that she would not be admitted if she did not take a mathematics course during her senior year despite having completed a two-year IB course sequence. Elizabeth was frustrated by this narrow view of her mathematical proficiency, and despite the counselor’s admonition, Elizabeth chose to take an elective in lieu of mathematics to expand in her high school learning experiences. She expected to resume her study of calculus in college.

The following enthymemes are Elizabeth’s persuasive arguments about hyper-acceleration as both an academic necessity in a competitive culture and a trajectory upon which she did not build the confidence that she needed persist in the most rigorous courses. The underlined phrases within the enthymemes represent the implicit meanings about hyper-acceleration and OTL that I inferred from her narrative.

Enthymeme #1 – I had to take Algebra I in Grade 7 to keep being smart.

Everyone in a gifted and talented class in elementary school is smart.
(Major premise)
Everyone who is smart is selected for Algebra I in Grade 7.
(Minor premise)
Therefore, if you not selected for Algebra I in 7th grade, then you are not smart.
(Conclusion)

During our focus group conversations, Elizabeth shared distinct memories of being separated from her friends in Grade 3 after she was placed in the gifted center at her elementary school.

> I didn't go to recess with the people who were in general education or lunch with the people who were in general education. It was really intense. You were only with like the smart people who came from Center Ridge (another elementary school). It's very separated and from there it's kind of like competitive and has been ever since. I just remember that I always disliked math. I wasn't awful at it or anything, I just really didn't like it.

For Elizabeth, studying Algebra I in Grade 7 was not a purposeful decision driven by enjoyment of interest in mathematics; instead it was a necessary next step in preserving her positionality as smart in mathematics. She did not like being bored in class, and she took pride in taking advantage of the "opportunity to take a more difficult class than you are supposed to". Yet in retrospect, she voiced the tensions between status and OTL that hyper-acceleration may present for students who feel that they must be on this pathway to compete with their peers. The most accelerated pathway became a diminished opportunity to engage in mathematics.

> It was a big deal which math class you were in, and then I mean if you weren't taking Algebra 1 in seventh grade, then you weren't ... It wasn't even an option to not take Algebra 1 in seventh grade. There are definitely people that did it, because that's just what they have to do. That's what their parents are telling them they have to do, because they got high enough on the Iowa (algebraic readiness test), but maybe they didn't ever want to. There's tons of people who are just in

> classes, because that's the expectation, but it's not where they should be. And then, they end up hating learning.
>
> There's a lot of people who did do Algebra 1 in seventh grade, who probably it was not the right decision for them. I think it kind of destroyed their confidence in math.

Elizabeth's retrospective characterization of hyper-acceleration as a necessity with questionable learning value reflects an *active vs passive* gendered opposition in her mathematical figured world. The pursuit of hyper-accelerated mathematics as action to secure educational advantage is the more highly valued choice in the West Valley High School community. This pursuit mirrors discourses on male mathematicians as fast, competitive, confident, and independent (Mendick, 2005). Yet the passivity that Elizabeth describes as needing her teachers to challenging her in ways that build her confidence can have negative longitudinal implications for STEM motivation and persistence. It also raises the questions about the ways in which teacher expectations in early accelerated mathematics experiences may or may not foster a love of mathematics.

> ***Enthymeme #2 – Hyper-acceleration did not give me the confidence I needed to pursue the most advanced mathematics.***
>
> *In mathematics I want to be challenged <u>in a way that builds my confidence</u>.*
> (Major premise)
>
> *Middle school Algebra I and Geometry did not challenge me <u>in a way that built my confidence</u>.*
> (Minor premise)
>
> *Therefore, I did not want to continue on the highest level of mathematics in high school.*
> (Conclusion)

Elizabeth recalled that her Algebra I experience did not make her passionate about mathematics. She loved being challenged in her classes, and she felt that the material in Grade 7 Algebra I was very easy. In her Grade 8 Geometry classroom, she felt that there was too much of a disparity between expectations in some cases to simply replicate what they had been taught and expectations in other cases to do things they had never seen before. Elizabeth smiled as she described watching YouTube videos where students were asking questions that she would have wanted to ask in her own Geometry classroom. Elizabeth's beliefs about appropriate challenge become an argument for an important obligation for teachers in hyper-accelerated contexts to foster connected understandings across mathematics topics.

> I guess in the sense that it's the highest thing they offer, then supposedly you're going to be doing math that's difficult, and it's going to really make you think about the way math works, and how it should work, and just take what you've learned and make it further. Go further with it. Because I think something I didn't even realize until you get to calculus really, is that math is all connected. If somebody takes the time to tell you why it exists, and how it makes sense, then it's so much better than having to just remember, I don't know, the Pythagorean Theorem, with no concept of why it is that way.

Elizabeth offered a further contrast in OTL between her Algebra I and Algebra II experiences with respect to expectations for use of formulas.

> Algebra II was not, "Here's a formula. Here's the questions you're going to get, and then this is how you solve it using that formula." It was, "Here's how this formula works. Here's a question that might include tons of different topics, and you could solve it different ways, but you have to figure out how to get to the right answer." So, that's a higher expectation of understanding, than geometry where she (the teacher) apparently did have that higher expectation of understanding but didn't teach according to it.

She argued that a mathematics teacher has a responsibility to challenge her to build connected understandings that promote confidence in problem solving. She associated this additional challenge in high school with teachers who cared about her deeper learning.

In her further reflection on her choices to decelerate her mathematics, Elizabeth wondered how her trajectory might have changed had she had experienced appropriately challenging mathematics prior to high school. The importance of productive struggle and affirmation of teachers along every phase of a hyper-accelerated trajectory is captured in Elizabeth's speculation about the mathematics path she might have taken with appropriate challenge as OTL. She may have built the requisite confidence and arguably the passion that she needed to pursue the most rigorous mathematics pathway with more rigorous Algebra I and Geometry experiences. Yet at the same time, her subsequent description of students who "just get it" aligns with Mendick's (2005) gendered binary opposition of *naturally able vs hard-working*. Her lack of confidence, often characterized as a feminine trait, was a manifestation of her belief that she did not understand her middle school mathematics well enough. Her emerging belief that her mathematics ability came with her hard work in challenging courses also became a personal barrier to the highest mathematical trajectory.

> Higher Level mathematics was for the people who were geniuses, like they were so good, and they just got it. Everything I heard about Higher Level was like, "It's the worst IB class in the world. Don't take it. It's so scary." Then, I knew if I just started out with Higher Level, by senior year I'd probably be taking multi-var, and that's really scary. I think at that point in freshman year, I had gotten a lot more confident in how I understood math, but I think it just started too late. It was ninth grade. Maybe if I had been learning that way for the past three years, then I would have been like, "Oh, I can handle it." But, I didn't feel that way.

Elizabeth's inability to see herself in the highest level of mathematics reflects a masculinized view of who is really good at mathematics at West Valley High School. This view is consistent with prior research about high-ability girls and maladaptive motivational patterns (Dweck, 1986), wherein girls may avoid challenging mathematics courses because of fear of experiencing failure. These maladaptive patterns are countered (Boaler, 1997) in classroom settings where students are encouraged to take risks with an emphasis on time and support to build understandings with less emphasis on competition (Radovic et al., 2017; Solomon, 2007).

Elizabeth’s enthymemes about Grade 7 Algebra I as a necessary placement in her mathematics course taking and her later choices not to stay on the highest mathematics trajectory despite a consistent record of high achievement in mathematics become a logical argument about hyper-acceleration as an educational structure. An emphasis on status over meaningful learning in hyper-accelerated classroom can diminish OTL. Elizabeth’s perceptions of competition and a focus on right answers in her middle school mathematics classes did not foster love of mathematics nor a desire to pursue her study at the highest level.

Ashley’s story – The resilient “competitor”. Ashley identifies as a White collegiate athlete and environmental engineering major who aspires to serve in the military. Her struggles in mathematics began when she was identified for gifted services in mathematics after Grade 4. She remembered the well-intentioned decision of her teacher to recommend that she join the track which positioned students to qualify for Algebra 1 in Grade 7. Yet her belief that she was behind her peers created an unsustainable tension between her grades as a metric of her learning and her ability to engage in productive struggle which would lead to meaningful learning.

> "Oh, she's special, so we're going to recommend her for advanced placement in fifth grade." From fourth to fifth grade, I went from general education to gifted and talented. I guess I missed a grade of math somewhere in there, because gifted and talented was ahead in math.

> I was set up from the second I started in seventh grade to just be like well, I'm not good at this, I need to get a good grade, I'm not getting good grades in math, I have to get a good grade. I saw it with a lot of my friends, too. Everybody was always freaking out, and then half the time you're not really absorbing and thinking about it, you're just like oh, this is what I have to do, and then you can't do it right. I think that mindset is also what led you toward not going towards the multiple approaches to a problem. When you're like oh, I have to get this right, you see what the teacher does and you're like that's what I have to do to get an A, and you're just going to try and do that even if it doesn't really work for you.

Ashley attributed the subsequent challenges she faced in her middle and high school mathematics courses to both her own lack of ability and to the lost opportunity to build a foundation in fractions. She described multiple deceleration points in her high school and college mathematics trajectory. She chose IB SL mathematics during Grade 10 in anticipation of returning to the two-year HL trajectory during Grades 11 and 12, believing that additional time to experience precalculus content would allow her to be more successful. However, Ashley again decided to decelerate after completing one year of the HL sequence by returning to complete the SL course in Grade 12.

Ashley's story of anxiety and trauma in her mathematics courses between Grades 7 and 10 offers becomes a syllogism about the risks of accelerating students too early in mathematics. Students need opportunities build strong understandings of ratio and proportion (Gojak, 2013; Seeley, 2015) and algebraic thinking (Stewart & Reeder, 2017) to realize their full potential in calculus courses (Bressoud, Camp, & Teague, 2012). Emily described this lost opportunity explicitly in her narrative.

Syllogism #3 – Hyper-acceleration diminished my ability to build a strong mathematical foundation.

Students need to understand fractions to succeed in algebra and precalculus.
(Major premise)

Students who accelerate mathematics in elementary school as a prerequisite for Grade 7 Algebra I spend less time studying fractions.

(Minor premise)

Therefore, students who accelerate mathematics in elementary school may not understand fractions well enough to succeed in advanced mathematics.

(Conclusion)

Ashley recounted failing multiple tests in Grade 7 Algebra I and her fear that she would “get pulled from the program” if she did not start to earn passing grades. She recalled crying, “I hate math, I hate math!” when the Algebra I teacher called her parents to express concern about her progress. Her father helped her to learn fractions and she persisted in the course, but she summarized the experience as “scarring”. Her struggles in Algebra I created a diminished OTL in subsequent courses because asking questions was a sign of weakness.

Ashley’s association of fractions with her difficulties in mathematics came to a height during a mathematics test during the first year of her IB SL course.

> That day when I had a complete mental breakdown in math class. That was crazy. I was looking at the paper and the fractions. I guess fractions have always been the bane of my existence, 'cause they were literally swimming off the paper. I was sobbing in the back of the math class, and my teacher pulled me outside. Oh my god, that was crazy, 'cause I wasn't sleeping and I was a total mess.

Ashley’s persistence in IB mathematics and her continuation to a major in environmental engineering in college despite these earlier threats to belief in herself as competent in mathematics are a reflection of an acquired mathematical resilience (Lee & Johnston Wilder, 2017). Her longitudinal narrative counters perceived feminine traits of anxiety, avoidance, and helplessness and at the same time counters masculine traits of speed and natural ability. In her enthymeme justifying her deceleration, Ashley further credited her decisions to slow down in her mathematics progression as the reason that she could finally identify as "good at mathematics."

Enthymeme #4 – Deceleration helped me to learn that math was more than just right answers.

Students who accelerate before they are ready struggle in mathematics.
(Major premise)
Students who struggle <u>*need more time with topics*</u> *to understand that mathematics is more than just right answers.*
(Minor premise)
Therefore, students who accelerate may <u>*need more time with topics*</u> *to learn that mathematics is more than just right answers.*
(Conclusion)

The underlined phrases within this enthymeme represent my interpretation of Ashley's argument for deceleration as advantageous. While Ashley attributed her subsequent success in mathematics to taking the same courses over and over again, other gendered aspects of her development as a productive doer of mathematics can be inferred from her voicing of her deceleration decisions. She argued that her fear that she was "not good enough" at mathematics as measured by test grades restrained her from being willing to approach problems in multiple ways. Her aversion to risk taking in mathematics created a "mental block" when she wasn't getting problems right. Although she told her peers in the focus group that deceleration was a "path of least resistance" which would allow her to avoid the extreme challenges of mathematics, her later reflections during our one-on-one interview revealed that her own evolution in thinking about what it means to engage in mathematics. Her experiences in IB mathematics helped her to move beyond the more masculine constructions of success in mathematics as *ordered* and *rule based* (Mendick, 2005) and permitted her to move away from a traditionally feminine expectation of rote learning.

> I came from feeling like there had to be one right way all the time, until like maybe halfway through high school. It made such a big difference for me to finally be like, oh, I can try different things, 'cause I was so set. I was like there's one right way to do everything. I was crazy about it, and it drives you insane to think, oh, I'm not getting it, I'm not getting it.
>
> …the nature of IB probably, it's a lot less computational. We did more real-world sort of application problems, and my teacher would always stress that there wasn't one right way, I think. When I started doing it and doing different ways and being able to get the right answer while my peer was getting the right answer a different way, that's when I started to learn.

With the freedom to do mathematics in more than one way in her IB SL classes, Ashley's discovery of creativity as an attribute in mathematics made her more confident in herself as a doer of mathematics. Her later experiences in her college mathematics modeling course where the instructor focused on setting up approaches to problems which teams of students did not necessarily complete further bolstered her confidence.

`Ashley's final reflection on hyper-acceleration elicited her regret that she had studied Algebra I in Grade 7. She attributed her positioning on that pathway to the academic competition within the school and community.

> Honestly, with the accelerated math, I wish I hadn't done it. I think I would have gotten to the same place at the end of high school not doing accelerated math. I could have just done a normal trajectory. It was kind of pointless for me in the end, 'cause I missed so much, like something got messed up in the beginning there, even in Algebra 1 probably and geometry. In our area, it just gets so competitive and you get caught up in it. I didn't realize that there was another way to go.

Ashley's retrospective view on hyper-acceleration, a decision that by Elizabeth's reasoning was not optional, explains unexpected tensions for girls who are positioned to compete in a masculine pursuit of one right answer in mathematics. The gendered binary opposition *fast vs slow* is implicit within the phenomenon of hyper-acceleration, and girls who perceive that a slowing down may help them to learn content may also believe that they are thus less capable of competing on STEM pathways. Deceleration instead opened a door to deeper learning and persistence in STEM for Ashley.

Ashley's later college placement represented an additional deceleration as she did not demonstrate proficiency in the calculus content she had completed in high school. In her post-secondary mathematical modeling and engineering mathematics courses, she discovered that she was "good at mathematics" and spoke with pride about tutoring her peers in calculus. She maintained her belief that being successful in math was measured by grades, but she attributed her success in college to knowing how to make use of her resources and the problem-solving acumen she had acquired in her IB courses.

> Here, it's the only place I've ever gotten straight A's in math, and I think it's because in high school, I didn't fully grasp the idea of like oh, I'm knowing where to go back to, and knowing how to do problems is what matters, and not just getting the right answer, until I came here. When I say get an A, to be good at math, that's just I think the pressure cooker mindset that we have. That's why I didn't consider myself good until now. I was also so results oriented until recently, that's I think that's why I couldn't get an A. I don't know what's going on, but it's working. I'm trying to keep riding the wave.

Ashley's articulation of augmented OTL in her less-competitive college mathematics courses became her construction of a less masculine definition of success in mathematics. She had been convinced that her inability to earn an A in mathematics in her middle and high school courses was a reflection of her lesser competence in mathematics as rules-based computational performance. As she engaged in more collaborative learning and problem solving in her college course, she built more confidence as she could connect her mathematical thinking to her prior knowledge.

Nadine's story – The "logical" persister. Nadine is a candid, self-disciplined student who identifies as Asian. She is studying neuroscience at a top tier private university in southern United States. She was one of two women in the original study who maintained her hyper-accelerated trajectory from Algebra I in Grade 7 to dual enrollment multivariable calculus in Grade 12. Like Ashley and Elizabeth, she did not perceive hyper-acceleration of Algebra I as a choice in Grade 7. Her motivation for staying on the highest mathematics course trajectory to multivariable calculus as "logical" was consistent with a masculine view of accelerated mathematics. Despite having earned the status of being in HL mathematics and multivariable calculus, Nadine did not enjoy mathematics in Grades 11 and 12. She described her positioning in multivariable calculus as a "stigma" because her Type A peers who were not on the fastest trajectory assumed that she understood IB mathematics well and that she would be able to help them. Instead, she felt that the pace of teaching in multivariable calculus was too fast for her and that she was not understanding the content well. She described retaking every test in multivariable calculus to earn a "B", and she was afraid that her peers would discover this incongruence in her high school mathematics status. Her logical, higher-valued choice of hyper-acceleration suddenly became a diminished OTL when she began to question her own abilities. Nadine elaborated on a very feminine positioning as a rote learner when she recalled her mathematics participation in Grade 12.

> Abby and I sat in the back, this little back corner, and our teacher would occasionally call on us to answer questions but he knew that we wouldn't really know anything because we would probably learn it right before the exam. Abby and I didn't want to help (other people) because we didn't know enough and then it would come out that kind of by our senior year we were taking this level math but that doesn't necessarily mean that we were understanding what we were

doing. We were doing well in the class. We understood it enough to test but we didn't understand it enough to teach other people more of the basics.

Nadine's naming of her position in the mathematical hierarchy at her school as a "stigma" is consistent with Mendick's (2005) gendered binary opposition of being *good at mathematics vs being really good at mathematics*. This opposition is inherent in her choice of the word "stigma" and represents a conflicted identity within her hyper-accelerated figured world. While her peers recognized her as good at mathematics because she was enrolled in multivariable calculus and sought her help with prerequisite course work, Nadine instead measured her competence relative to three boys in her class who were performing mathematics with speed and demonstrated competence that she could not herself achieve. Nadine and Abby embraced a more feminine role in an *active vs passive* opposition in their multivariable calculus classroom as they physically positioned themselves to avoid being called upon by their teacher.

Nadine's choice not to engage in the competitive norms of her multivariable classroom where the boys raced to board to grab markers and to be the first to put answers on the board because they "knew what was up" or "read ahead" represented an implicit deceleration in her mathematics trajectory. This deceleration remains hidden behind the name of her mathematics course and her earned "B". She had already gained admission to her first-choice college, and her conversations with her college admissions counselors confirmed her belief that she only needed to pass multivariable calculus because the grade would not transfer to her college. Thus, Nadine chose not to work as hard at understanding multivariable calculus as she could have and stopped enjoying mathematics. She did not continue with advanced mathematics course taking in college and instead chose a course in logic to fulfill her mathematics graduation requirements.

Nadine's narration of her college mathematics experiences in her follow-up interview presented new questions about Grade 7 Algebra I as an augmented OTL in more advanced mathematics. The inherent contradictions in decisions to hyper-accelerate are presented in the following pair of syllogisms.

> ***Syllogism #5 – Hyper-acceleration is a disadvantage in college calculus readiness.***
>
> *College administrators believe that students who take calculus early in high school will forget what they have learned. They will require these students to retake Calculus I before they take more advanced college courses.*
> (Major premise)
> *Students on hyper-accelerated pathways take calculus early in high school.*
> (Minor premise)
> *Therefore, students who hyper-accelerate their study of Algebra I to Grade 7 will have to retake Calculus I in college.*
> (Conclusion)

Completion of one or more years of calculus in high school is widely recognized as a proxy for smartness in competitive college admissions (Bressoud et al., 2012) and hyper-accelerated students have space in their schedules for multiple years of calculus instruction. Nadine's retelling of her conversation with the dean of her college and his unwillingness to allow her to begin in Calculus II despite her earned dual-enrollment credit for multivariable calculus was an affront to her identity as "good at mathematics".

> You don't just forget things no matter how early you learn them. I think starting early hurt me a little bit in terms of wanting to pursue math. Plus I associated the stress of junior-senior year with hard math and I didn't want to continue with it. Southern University was going to make me start in Calc I. I didn't want to start in Calc I. Based on that conversation, if I had not accelerated everything, then he would have said, "You can skip Calc I because you just took Calc I last year." I could have majored in math at Southern University.

This forced deceleration was beyond her control and caused her to question the value of hyper-acceleration. Her experiences with multivariable calculus as culturally constructed at West Valley High School created a diminished OTL within the course, and this devaluing was perpetuated by a college administrator who told her the “logical” choice she had made in her competitive mathematics figured world had actually prepared her less well to persist in mathematics.

Nadine’s elaboration on her college mathematics experiences elicited an accompanying syllogism which adds further irony to the game of dominant mathematics (Gutiérrez, 2002) which hyper-accelerated students play.

> ***Syllogism #6 – Hyper-acceleration is an advantage in college calculus readiness.***
>
> *Students who do not take calculus in high school drown in their first college math courses.*
> (Major premise)
> *Hyper-accelerated students take two or three years of calculus in high school.*
> (Minor premise)
> *Therefore, students who hyper-accelerate their study of Algebra I to Grade 7 won’t drown in their college math courses.*
> (Conclusion)

Nadine valued the calculus knowledge that she had built during her IB courses in high school as important social and mathematical capital in her transition to college. She described the struggle of her peers in their first-year mathematics courses at her undergraduate institution and her appreciation of the opportunity to build a foundation in differential and integral calculus that would have allowed her to persist in mathematics had she chosen to.

> If I was thrown into a calc class, Calc I, II, III, then I would float. I wouldn't excel, but I'd float, versus if I hadn't taken it then I would've drowned. So I think

> for me, I feel good knowing that I know it. I want to hate that I took it (Algebra I) early, but at the same time I can't. I love the knowledge that I have.

Nadine's completion of multivariable calculus in high school should have positioned her in her cultural context as *really good at mathematics* within Mendick's gendered binary oppositions, but her struggles to understand the content and her college administrators questioning of her knowledge allowed her to easily take on the more feminine identity of just being *good at mathematics*, enough so that she could float in a college mathematics class.

Convergent Stories of Girls in Hyper-accelerated Figured Worlds

Hyper-acceleration as an educational structure communicated very strong messages to Elizabeth, Nadine, and Ashley about mathematics as a socially constructed discipline. It created an arguably masculine figured world of belonging or not belonging to a select and competitive group in which success in mathematics was manifested as natural ability, speed, and precision more than as creativity and collaboration. Elizabeth believed that she could not succeed at the highest level of mathematics despite a strong record of prior achievement because she didn't "just get it". Ashley hated mathematics because she was expected to learn it too quickly without understanding fractions, while Nadine elected not to participate in her highest-level classes so her struggles would remain hidden from view. Their unique enactments of deceleration became a feminine questioning of their places in a competitive mathematics culture at different points along a hyper-accelerated trajectory.

Yet Elizabeth, Ashley, and Nadine did not succumb to a stereotypical "not good enough assemblage" (Wolfe, 2019, p. 27). They clearly articulated their continuing interest in STEM and their desire to compete for elite college admissions. These motivators guaranteed their persistence in accelerated mathematics courses despite their doubts about the quality of their learning in their early courses or their own abilities to succeed in their later courses. Their choices to decelerate, while often aligned with prior research on gendered norms and potentially detrimental to increasing female participation in the STEM pipeline, also represented an emergent mathematical resilience (Lee & Johnston-Wilder, 2017). They voiced a critical recognition that struggle was necessary in order to succeed. Their descriptions of loving and hating math at various points in their mathematics journeys represented a feminine negotiation of their places as smart girls. Their unique deceleration choices can thus be viewed as productive or even necessary actions consistent with their knowledge that mathematics is personally and culturally valuable. Through deceleration, they were able to find the support they needed to persist in mathematics, whether it was enrolling in a less rigorous course, repeating challenging content, or seeking help from their teachers outside of class. Deceleration, when framed as a reflective, positive feminine choice, thus has the potential to help girls who question their places in hyper-accelerated figured worlds to discover their mathematical acumen and to see themselves in STEM career fields.

Implications and Directions for Future Research

The syllogisms and enthymemes in these young women's stories are not generalizable and do not suggest that deceleration is an appropriate choice for every girl who is considering a STEM career. In fact, there is a very real danger that framing deceleration as a positive choice simplistically affirms societal messages that girls are not knowers and practitioners of mathematics (Hottinger, 2016). If deceleration is a gendered decision, whether in the form of Ashley's and Elizabeth's IB Standard Level course selections or Nadine's classroom disengagement within in multivariable calculus, then it may have a related negative effect in reducing the presence of girls on the highest-level trajectories. An increasing gender ratio imbalance of meaningful participation in the most advanced courses can make it increasingly difficult for girls to see themselves in these classrooms despite their identification as gifted in mathematics (Kerr & Huffman, 2018). Furthermore, a greater gender imbalance in classrooms has been shown to have negative effects on gifted girls' academic self-concept (Preckel, Goetz, Pekrun, & Klein, 2008). Additional research is needed to understand how hyper-acceleration itself may contradict efforts to promote broader female participation in advanced secondary mathematics.

It also worth noting the challenges that Elizabeth, Ashley, and Nadine's rhetorical arguments present to Mendick's (2005) opposition of real understanding as masculine vs rote learning as feminine. These young women, arguably by virtue of their participation in an IB mathematics curriculum which demands that students makes abstractions and generalizations in creative ways, valued mathematics differently by the end of high school. The IB program aims were reflected in the young women's narratives; the affordances of the IB curriculum in promoting creative problems solving and in fostering resilience in advanced mathematics should be further examined.

Finally, rhetorical analysis of individual stories of hyper-acceleration through a research lens of gendered binary oppositions provides important empirical evidence of how explicit identification of girls at the age of 12 or 13 in smart in mathematics may either encourage or detract from their interest and persistence in STEM. The deductive reasoning in Elizabeth's, Ashley's and Nadine's stories, both explicit and implicit, provides contemporary insight into girls' evolving identities in secondary mathematics. Further use of a situative perspective on OTL can move us toward Mendick's (2005) vision of "classrooms in which teachers and learners are sensitive to the variety of identity work that learners are engaged on, in and through mathematics" (p. 218). If girls are impacted differently by hyper-acceleration, and its structure is rooted within masculine views of mathematics, we need to understand if we are further disadvantaging young women who are selected for these pathways. Guided by Boaler and Sengupta-Irving (2006), we should view gender not as an explanation for lesser participation but instead pursue research on "gender as response" to educational environments which may not be promoting deeper mathematical learning. When Algebra I in Grade 7 becomes more about intellectual status and confirming smartness than about providing meaningful learning opportunities on pathways to STEM, then we may be failing in a more critical objective of developing a love of mathematics in each of our mathematically promising students.

Chapter Six – Conclusion

It's time to begin the courageous work needed to intentionally and systematically remove tracking's barriers and the associated instructional practices. Instead, we must move toward creating pathways for success in mathematics for each and every student.
(Robert Q. Berry, III, President of National Council of Teachers of Mathematics)

Hyper-acceleration of Algebra I to Grade 7 is an unstudied dimension of recent exhortations to examine identities of competence and tracking in secondary mathematics classrooms. A student who studies Algebra I in Grade 7 is positioned as a privileged beneficiary of better resources and an ostensibly more certain pathway toward successful mathematics performance. Yet this dissertation study reveals that even with these presumed benefits of hyper-acceleration, there can be drawbacks with lasting mathematical consequences. The research commentary in Chapter 1 encourages the mathematics education community to challenge the presumed affordances of hyper-acceleration using contextual qualitative studies which operationalize OTL from a situative perspective. With a new focus on identity in a sociocultural turn in mathematics education research, OTL is measured by more than courses completed, grades earned, and competence as assigned by standardized proficiency tests. Instead, OTL from a situative perspective describes meaningful engagement in problem solving in the classroom, persistence in the secondary mathematics pipeline, and gained conceptual understandings beyond procedural recollection and replication. In the absence of such research in hyper-accelerated figured worlds, the current realities of mathematics course completion as a metric for intelligence and accomplishment will persist. The perception that “faster is better” in mathematics will continue to drive parent discourse and administrative decision-making. The potential barriers to meaningful learning that tracking of Algebra I in Grade 7 can create, both for students within the track and outside of the track, will remain.

With a growing socioeconomic divide in many communities across the United States, I theorize that the pursuit of hyper-acceleration of Algebra I as an educational advantage will grow in the absence of new knowledge about its potential ramifications for individual students, classrooms, and schools. The scholarly examination of OTL for hyper-accelerated students from a situative perspective can begin to counter these societal assumptions and contribute to new knowledge of meaningful secondary mathematical experiences on the pathway to calculus.

Significance of Research

In the literature review for Chapter 2, my juxtaposition of evidence from prior studies of acceleration of Algebra I to middle school alongside evidence of limited mathematical readiness for postsecondary calculus revealed a void in the literature. There are empirical contradictions between studies which relate improved secondary outcomes to increasing acceleration of Algebra I and studies which challenges acceleration to the detriment of building strong precalculus understandings. This contradiction works against positioning more students to pursue and persist along mathematically intense STEM trajectories. This literature review clarified the need for research on the contextual effects of Algebra I acceleration and the development of productive mathematics identities to help us to better understand these contradictions.

The courageous work of challenging and dismantling tracking (Berry, 2017; Oakes, 1985) cannot be accomplished with large-scale quantitative research which fails to attend to the sociocultural contexts which drive further tracking as a curricular advantage (Domina & Simzar, 2012; Lucas, 2001). Chapter 2 begins to challenge the presumption that hyper-acceleration offers a better mathematical pathway. It provides a conceptual framework for looking beyond the traditional ways by which we measure mathematical success. High-achieving students will always be deemed successful when success is defined as course completion, grades, and college attendance. Yet a deeper examination of the experience of learning mathematics in hyper-accelerated contexts as an opportunity to engage with conceptually challenging mathematics can tell us a different story. Examination of OTL from a situative perspective considers both achievement and evolving identities as manifestations of the quality of learning within secondary mathematics classrooms and schools.

Chapter 3 offers an ontological justification for the phenomenological study of hyper-acceleration using an interpretative philosophical paradigm. I argued the importance of my researcher positioning as a woman engineer and a veteran mathematics teacher with deep content and contextual understandings. Critical conversations about hyper-acceleration currently reside in social media, blog posts, teacher workrooms and conference networking sessions. My unique ability to explicate the tacit meanings in hyper-accelerated students' narratives at West Valley High School is the foundation of a methodology that begins to build the empirical essence of hyper-acceleration as a structure of tracking within a socioeconomically diverse setting. I further argued the affordances of situating this study within a singular context at West Valley High School because of the unique OTL associated with IB curriculum expectations for proof, justification, and connections in two-year course sequences. This situative study of OTL through retrospective student stories from the high school where I teach also promoted a contextual consideration of the evolving perceptions about what it means to be at good mathematics in Grades 7 through 12. The methodological design of a sequential layering of critical events, thematic, and structural analysis techniques (Knight & Sweeney, 2007; Riessman, 2008; Webster & Mertova, 2007) improved the quality of the generated narratives and clarified the shared experiences within phenomenon of hyper-acceleration at West Valley High School.

The what, how, and why of mathematics learning within the retrospective narratives of the 15 recent graduates from West Valley High School in Chapter 4 expands our empirical understandings effects of acceleration of Algebra I beyond quantitative analysis of district and state level achievement data. These student stories rewrite the overarching narrative of hyper-acceleration as an educational advantage and become the foundation for future conversations about the intrinsic value of hyper-acceleration as a pathway which means more than the continuing accrual of status. The pathway should also be built upon intrinsic values of challenge seeking, motivation, and interest in mathematics. The explicit associations of smartness with selection for the highest track in middle school mathematics in Grade 7 happened at a crucial time in each student's mathematical identity formation. The critical event of simply sitting in a Grade 7 Algebra I classroom became a catalyst which framed their mathematics identities in subsequent math courses. Their early descriptions of being “good at mathematics” as a performance, often characterized by speed, correctness, and competition, created longitudinal oppositions in their evolving identities as participants in IB mathematics discourses.

The use of sequential layering of narrative methodologies as described in Chapter 3 and enacted in Chapters 4 and 5 yielded identity claims within and across focus group transcripts which characterized OTL. The multiple restructurings of these identity claims elicited several oppositions in terms of quality of learning which warrant further study: coasting in earlier courses versus crashing in advanced courses; compromising status by needing help vs learning to seek help; replicating mathematics in Grade 7 Algebra I versus creating mathematics in IB courses; questioning proofs in Grade 8 Geometry versus valuing proof and derivation in IB courses; and answering questions correctly in middle school versus solving problems in high school. These articulations of changing positions in their secondary mathematical worlds as they were no longer “just solving for x” reflected the inherent tensions between OTL and status with hyper-acceleration. Consistent with the nearly 20-year-old research findings on high-ability students in tracked mathematical figured worlds (Boaler, 1997), hyper-acceleration continues to be a trajectory which presumes readiness and rewards performance with a faster exposure to content. However, it also introduces the risk of communicating to students that they do not belong in the most advanced mathematics courses. The participants in this study also questioned the value of these pathways for students who did not like math or did not understand what they were learning.

The emergence of common themes across the Chapter 4 stories of girls who hyper-accelerated their study of Algebra I at West Valley High School motivated my use of Mendick's (2005) gendered binary norms to further interpret their stories in Chapter 5. The development of enthymemes, or deductive arguments in which premises or conclusions are unstated but interpreted from contextual descriptions of participants, provided new insight about how girls navigate hyper-accelerated trajectories differently. Decelerating, or leaving the most advanced mathematical track in a high school, may promote more positive mathematical identities (Radovic et al., 2018) for girls who begin to question their abilities in performance-oriented classrooms. These rhetorical arguments, constructed from the storied experiences of three high-achieving girls in the Chapter 4 focus group study, provide a novel research lens on policy and practice of acceleration as it relates to encouragement of higher female participation in the STEM pipeline. Hyper-acceleration, an arguably masculine structure which communicates that faster is better in the competitive study of mathematics, may detract from the important work of encouraging broader participation. Subjective perceptions of ability to participate in advanced mathematics, despite the assigned status of smartness in Grade 7 Algebra I, cannot be separated from gendered norms. The enthymemes of participation and deceleration in Chapter 5 communicated a changing valuation of productive struggle as a means to deeper understanding along with a social acceptability repeating content to make connections that would support future STEM study. Furthermore, the resilience in the stories of the three young women, along with retrospective questioning of the value of hyper-acceleration, suggests that each of them may have achieved at even higher levels in mathematics with more meaningful mathematics experiences in middle school in lieu of earlier exposure to advanced content.

Chapters 4 and 5 describe OTL within the hyper-accelerated context of West Valley High School from a situative perspective and offer novel empirical evidence which begins the fill the gap in literature as outlined in Chapter 2. Stories of hyper-acceleration can reconcile the contradiction of positive outcomes with acceleration of Algebra I and lesser motivation, readiness, or persistence in STEM undergraduate study. As I argued in Chapter 1, use of this situative perspective motivates a reimagining of identity as a construct which defines OTL, and by extension, appropriate acceleration of secondary mathematics. Qualitative research which foregrounds the identity construction of students becomes a conceptualization of OTL beyond demographic information, test scores, and grades. These situative narratives help us to better understand how hyper-acceleration impacts culture and climate within classrooms and schools. They also explain student choices about persistence in STEM.

Implications for Stakeholders

Findings from the interpretative narrative research in this dissertation study have implications for policy and practice at multiple stakeholder levels. These findings can help us to better support students who are on these hyper-accelerated pathways. Furthermore, the findings should motivate further study of structures of tracking, gendered participation in advanced mathematics, and the presumed affordances of acceleration.

Students. Retrospective narratives, whether from aspiring mathematicians or from students who decelerated their high school courses on non-STEM pathways, affirm recommendations in the literature about the importance of building strong concepts of algebra and function in middle and high school mathematics. Nadine's contradictory syllogisms about the value of hyper-acceleration in the high school to college transition are consistent with Dr. Robin Pemantle's (2016) commentary on the false choices that today's high-achieving students face. Her perception of initial achievement advantages of earlier exposure to calculus are affirmed by research on the mathematics transition from high school to college (Bressoud, 2015a; Sadler & Sonnert, 2018), but her beliefs that a less accelerated pathway may have afforded her more opportunities in post-secondary mathematics are worthy of additional consideration. The many West Valley High School stories reveal that the pathway toward broader STEM participation, particularly for women, may not come with increasing acceleration of Algebra I. Mathematically promising students are deserving of research which more clearly defines appropriate acceleration.

This study also foregrounds questions about the *what* and *how* of mathematics learning in a situative OTL framework in relation to the repetition of advanced mathematics content. Data from the study of secondary mathematics progressions in California showing the high percentage of students who repeated Algebra I in Grade 8 alongside stories of repetition of content by the West Valley High School students represent a personal negotiation of achievement and identity. The interplay of learning mathematics well and maintaining status of smartness is best understood with the individual student experience as the unit of analysis. Yet future empirical examinations should be expanded to include more than described experiences; longitudinal analysis of how individual students make sense of specific concepts of algebra and function can more fully characterize their achievement. The use of clinical interviews to reveal a student's mathematical thinking alongside their storied experiences can create a richer description of conceptual understanding as part of a situative perspective on OTL.

We also need to understand more about the *why* of study of Algebra I in Grade 7 or earlier from students' perspectives. Hyper-acceleration as a competitive pursuit of educational advantage does not necessarily foster a longitudinal interest in mathematics. Hyper-acceleration as a mathematical advantage should provide a strong conceptual foundation for advanced calculus study. If hyper-acceleration does not foster both interest in mathematics and deeper understandings, then it is time to question our political investment in this privileged pathway. This assertion is substantiated by prior non-contextual research showing that high school understanding of algebra and geometry was nearly equivalent for accelerated and non-accelerated gifted students (Ma, 2005b) and that precalculus grades explained more twice the difference in college calculus achievement than high school calculus grades (Sadler & Sonnert, 2018). The "race to calculus" can thus become solely an educational advantage without attention to quality of learning. Securing a mathematical advantage along with an educational advantage in hyper-accelerated contexts can only be accomplished in classrooms facilitated by teachers who understand the difference between these advantages. Students then come to know the "why" of hyper-acceleration as an opportunity to engage in advanced mathematics in more meaningful ways aligned with their interest and readiness.

Teachers and curriculum. Hyper-acceleration will not be more than an educational advantage without specific attention the curricular design of secondary mathematics courses and how students are selected for participation. When teachers and mathematics departments structure curriculum which present formulas without attention to their genesis or encourage memorization of procedures to answer specific questions, they diminish OTL. Mathematically promising students deserve opportunities delve more deeply into content and to value mathematics as a problem-solving endeavor beyond producing correct answers. The retrospective descriptions of many of the West Valley High School students did not elicit evidence of truly challenging mathematics until they reached their IB courses.

Because more rigorous and relevant mathematics is an appropriate aim for each and every mathematics student, these narratives offer a novel lens on recent recommendations from NCTM (2018) about building essential understandings in early secondary mathematics courses. While this study was not designed to evaluate the quality of teaching of hyper-accelerated students, participant descriptions of OTL on this emergent course trajectory demand further research on curriculum and instruction in hyper-accelerated middle school mathematics courses. Grade 7 Algebra and Grade 8 Geometry must challenge students beyond preparing them to answer questions for competency-based standardized tests and multiple-choice examinations. If hyper-accelerated students are going to be viewed as the "smart ones" who teachers don't have to worry about, then we must again question the actual mathematical value of this trajectory. Teachers should pursue pedagogies which foster productive struggle and inspire students to make conceptual connections in preparation for more advanced courses.

College students' retrospective narratives can also motivate critical conversations about whether productive struggle and challenging mathematics experiences can perhaps be better facilitated in more heterogeneous classrooms where competition and status may be less intense and multiple approaches are celebrated. Teachers have a crucial role to play in helping their students to see that success in mathematics does not have equate to status; they can encourage their students at every level of mathematics to build identities of mathematical competence which transcend course names.

The participants' curricular choices at the IB level – whether a two-year SL course, two-year HL course, or a hybrid course over three years – reflected the messages they had received about who was good at mathematics from course descriptions, teachers, and classroom discourse. While each IB course encouraged students to build a mathematical toolbox to engage in creative problem solving, the prevailing belief at West Valley High School was that the HL course was for the "smartest" people who did not need to work hard at mathematics. Yet, unlike their perceived lack of choice about being in Algebra I in Grade 7, several of the hyper-accelerated students were willing to leave the most accelerated pathway when they feared that they were not smart enough to do well or when they wanted to take a safer path. The IB HL mathematics course description emphasizes formal justification, proof, and structure, stating that it is appropriate for students who are “intellectually equipped to appreciate the links between concepts in different concept areas” (IBO, 2014a). In contrast, the SL mathematics course description claims to “introduce concepts in a comprehensible and coherent way, rather than insisting on the mathematical rigour required for mathematics HL” (IBO, 2014b). These course descriptions of SL mathematics as less demanding and HL mathematics as based on fixed mathematical ability may help to explain why several of the participants could not or chose not to see themselves within HL mathematics. Curricular choices and identity development cannot be separated, and teachers have a powerful role to play in encouraging students to see themselves within the most advanced curricular offerings and to support them in persisting in these courses.

The variety of pathways chosen by the participants in this study should also encourage teachers at all grade levels to question the value of early acceleration when many students choose not to pursue the most rigorous courses at a later point on their secondary mathematics trajectories. The different narratives in this study can help teachers envision ways in which more rigorous experiences, especially in Algebra I and Geometry, could encourage more students to engage deeply in mathematics. Hyper-accelerated students should feel prepared and motivated to persist in mathematics in high school and beyond.

Additionally, these questions of persistence in advanced courses and appropriate rigor in middle school courses should challenge teachers to think more creatively about who is selected for these pathways. Standardized test scores and grades as metrics of readiness for hyper-acceleration constitute a narrow set of selection criteria that may fail to identify students equitably. Students who are not selected for hyper-acceleration may then have a lesser belief in their ability to participate in advanced courses.

Because hyper-acceleration is considered a marker of expected smartness in mathematics, it can also solidify student and teacher expectations that its participants shouldn't struggle. After all, as one of the study participants recalled her Grade 7 Algebra I teacher stating, "You are the smart ones." Yet struggle as a productive endeavor is exactly what prepares students to persist in STEM undergraduate study and careers. Struggle can further foster participation in mathematics as a collaborative and creative problem-solving experience in which each student has something to contribute. Students who are selected for a hyper-accelerated pathway should be able to describe and celebrate struggle as an avenue toward deeper learning.

Parents and administrators. Hyper-accelerated students navigate mathematical figured worlds which are created by adult stakeholders in their school communities. Parents and administrators should demand more educational research about the longitudinal impacts of the early study of Algebra I. The educational advantage that highly educated parents seek for their children may have unforeseen long-term negative consequences for the quality of their mathematics learning. The stories of anxiety, imposter syndrome, and lack of inspiration from boys and girls alike at West Valley High School should lead us to ask broader policy questions about what we value in K-12 mathematics study. If trusted adults create mathematical figured worlds for children in which the highest achievement is finishing mathematics content more quickly than peers or earning as much college credit as possible, then they fail to focus a quality of learning which inspires students to pursue and persist in advanced mathematics. In addition, the retrospective stories of the students at West Valley High School elicited their own adult questions of fairness along with descriptors of mathematics as a hierarchy and a commodity. The words of these successful graduates should give pause to all of us who claim to strive for more equitable access to mathematics education.

Several students in this study appreciated the opportunity to decelerate in Grades 10 through 12 and spend more time on precalculus topics. Yet these stories were from students who chose to attend a focus group about their mathematics journeys, completed IB courses, and took pride in their college outcomes. While deceleration was not framed as a regret by the participants in this study, it can become a marker of lost smartness for students who do not persist in advanced mathematics, and it may detract from the development of student identities as productive doers of mathematics. Students in this study did not repeat Algebra I in Grade 8. Yet my contextual knowledge and the findings of Finkelstein and colleagues (2012) affirms that many students make this choice when they struggle in Grade 7 Algebra. Parent, counselor, or administrator messaging that taking a course a second time is a readily available back-up plan when a student begins to struggle on an accelerated trajectory drives hyper-acceleration. This messaging is potentially damaging to students.

Further explorations of hyper-acceleration, as informed by students' narratives, can also foster critical conversations at the administrator level about tracking teachers as an inequitable practice (NCTM, 2018). When students and parents perceive that the best teachers are in the most advanced classes, they are more likely to conflate the greatest OTL with the most accelerated pathways independent of student interest or readiness. A reimagining of OTL as engagement with meaningful mathematics at every level of readiness can counter presumptions that more advanced mathematics is necessarily taught better.

Future Directions for Research

While I have personally questioned the wisdom of accelerating formal algebra instruction for over ten years, my goal in this interpretative, phenomenological dissertation study was not to make generalizable claims about hyper-acceleration as an emergent form of tracking. Hyper-acceleration will look different in different contexts, and the findings in this study cannot be interpreted to suggest that a "randomly selected" student will be served well or served poorly by selecting this mathematics trajectory. Yet my decision to conduct this study within a local context where I am a known and trusted researcher affirmed my drive to examine this phenomenon more broadly in the future. Hyper-acceleration of Algebra I privileges certain racial and socioeconomic backgrounds and increases exposure to mathematics content without necessarily improving the quality of learning. While structures of hyper-acceleration are often justified to meet the academic needs of students whose mathematical competence is assigned by traditionally narrow metrics, it is time to move the education research community forward in countering these claims. Hyper-acceleration of formal algebra is not simply a phenomenon worthy of study to describe quality of learning at the individual level; it is part of an inequitable educational system which should be challenged with new qualitative and quantitative evidence of implications for each of its stakeholders.

Students as stakeholders: Conceptual understandings of algebra and function. Validated instruments of algebraic reasoning and concept of function such as the Precalculus Concept Assessment (PCA) have been used to describe the content understandings of high school students, college precalculus students, and mathematics majors (Carlson, Oehrtman, & Engleke, 2010). Such instruments could be administered over multiple years to a representative sample of hyper-accelerated students to measure their longitudinal growth in foundational mathematics concepts for undergraduate STEM study. Follow-on individual interviews with categorical function tasks (Vrabel, 2014) could help to characterize the ways in which hyper-accelerated students make sense of foundational concepts of algebra and function. Research designs which foreground the acquired conceptual mathematical understandings of hyper-accelerated students can build a richer characterization of achievement which in turn deepens our understanding of the quality of learning.

Students of color as stakeholders: Selection and participation. Each of the study participants from West Valley High School was White or Asian. Hyper-acceleration creates a mathematical hierarchy which exacerbates stereotypes about who can do mathematics. My rhetorical analysis of girls' stories of hyperacceleration revealed gendered experiences with implications for broader participation. We also need to understand the unique ways in which students of color may traverse this phenomenon. Prior research using critical race theory and intersectionality theory to examine the lived experiences of high achieving secondary mathematics students (Berry, 2008; Davis, 2014; Jett, 2012; Joseph et al, 2017; Kirkland, 2012) can inform future research projects which address racialized structures of selection and participation in hyper-accelerated contexts.

Students and teachers as stakeholders: Relational identities in classrooms. The field of mathematics education is embracing a sociocultural turn in research as evidenced by four international reviews of research on identity within the last three years (Darragh, 2016, Graven & Heyd-Metzuyanim, 2019; Langer-Osuma & Esmonde, 2017; Radovic et al., 2017). The preponderance of identity literature focuses on sociohistorical narratives for students from minoritized backgrounds (Gresalfi & Hand, 2019), but I argue that the examination of students' evolving identities in hyper-accelerated contexts also has a place in the increased production of research on mathematical identity. Stories of hyper-acceleration, specifically in the articulation of ways in which this phenomenon both promotes and challenges mathematical competence, become another entry point by which the field can challenge inequitable educational structures.

We need to specifically examine how student and teacher mathematics identities relate to each other (Graven & Heyd-Metzuyanim, 2019) in the pursuit of more productive identity development in middle school Algebra I and Geometry classrooms. Student and teacher voices should be at the center of sociocultural research designs that explicitly acknowledge and address the social markers of accelerated mathematics course taking and the roles that peers, curriculum, parents, and administrators play in construction of competence. The tenets of an IB curriculum design at West Valley High

School mitigated unproductive beliefs that mathematics problems should be recognizable and solved with recalled procedures. Teacher beliefs about the potential of challenging curricula with problem solving and connections and how these beliefs interact with student beliefs about success in mathematics should be studied with the classroom as the unit of analysis. The characterization of norms, practices and interactions (Gresalfi & Hand, 2019; Horn, 2008) in hyper-accelerated classrooms can shed light on what it means to succeed in secondary mathematics classrooms and schools.

Colleges and universities as stakeholders: IB, AP, and college readiness. Statewide and nationwide examination of hyper-acceleration as policy and practice with implications for college readiness is a necessary next step in better understanding of the phenomenon in question. Research with representative samples of students and teachers, schools, and school divisions can be framed using my earlier definitions of achievement as persistence in advanced mathematics and readiness for college calculus and identity as beliefs about success and competence in mathematics. Correlational research on the timing of Algebra I and longitudinal mathematical outcomes can be modeled after prior transcript studies on secondary mathematics course progressions (Finkelstein et al., 2012) and self-reported student surveys of high school mathematics preparation, interest, and perceived competence in first-semester STEM calculus courses (Bressoud et al., 2015).

Although the IB curriculum at West Valley High School provided a rich context for study of OTL because of high expectations for connected understandings over multiple years of study, AP courses are much more accessible to today's secondary students than IB courses. The IB curriculum is offered in 947 high schools in the United

States (IBO, 2019) while AP Calculus courses are offered at over 14,000 schools (The College Board, 2018). Because AP courses are specifically designed to prepare students for examinations for college credit, OTL as narrated by students in AP courses will likely communicate a distinct culture and context with its own implications for hyper-acceleration. Our empirical understandings of this phenomenon will be stronger with an expansion of secondary contexts to include AP schools and research questions which elicit postsecondary perspectives on hyper-acceleration.

Conclusion

The narrative research design of this study challenges presumptions of both social and mathematical advantages with hyper-acceleration of Algebra I and asserts a broader educational purpose within and beyond this phenomenon. This pathway should provide meaningful and rigorous mathematics experiences and promote greater interest and participation in mathematics as a valued discipline. Findings related to diminished OTL and gendered experiences should motivate research on policies of hyper-acceleration and its implication for more equitable access to secondary mathematics for mathematically promising students.

It is impossible to ignore that the perpetuation of inequities in mathematics education at large are deeply intertwined in the phenomenon of hyper-acceleration as a pursuit of educational advantage from an EMI perspective (Lucas, 2001). These inequities are not made explicit in the theoretical OTL framework in this dissertation. Articulation of the lived experiences of the students at West Valley High School becomes but one of many new research windows that we need to create to better understand hyper-acceleration and its implications at both the individual level and the structural level.

The design of this interpretative narrative inquiry, with its layering of multiple analysis techniques, provides assurances not only to the scholarly community but to the participants from West Valley High School that their truths are being shared in ways which justify and motivate additional research. More specifically, the ways in which several West Valley High School female students positioned themselves in mathematics throughout high school and college reflect gendered perceptions of competition as unimportant or uncomfortable. Their early beliefs that mathematics was not a creative endeavor led them to question their places in hyper-accelerated figured worlds. The gendered undertones of hyper-acceleration as a static assignment of smartness and ability in Grade 7 should be made explicit in any future research framing.

In the absence of additional research, I fear that we will see future policy actions which purport to make hyper-accelerated pathways more accessible to more students of color as a positive movement in the name of equity. Analogous to achievement gap discourses (Gutiérrez, 2008) which encourage deficit thinking and negative discourse, efforts to broaden participation in hyper-accelerated trajectories introduce a real risk of further disadvantaging students who are uniquely positioned to challenge long-standing narratives of mathematics participation.

As we come to understand the unintended consequences of hyper-accelerated mathematics study in terms of evolving identities, patterns of selection and participation, and content as taught and learned, we will begin to fill the void of much-needed empirical evidence of the ways in which these emergent course pathways are accessed and navigated. Selection of and for hyper-accelerated pathways is inextricably linked to the mathematics of non-accelerated pathways. Future qualitative and quantitative research on achievement and identity in hyper-accelerated figured worlds can challenge its inherent privileged and resourced participation. We must recognize that being hyper-accelerated is in and of itself a narrative. This recognition in turn allows us to begin to question an achievement culture that that too often transcends a more critical focus on the quality of mathematical learning.

Appendix A

GMU IRB Approval Letter

GEORGE MASON UNIVERSITY

Office of Research Development, Integrity, and Assurance

Research Hall, 4400 University Drive, MS 6D5, Fairfax, Virginia 22030
Phone: 703-993-5445; Fax: 703-993-9590

DATE:	September 28, 2018
TO:	Meagan Call-Cummings
FROM:	George Mason University IRB
Project Title:	[1250936-2] College Student Experiences with Hyper-Acceleration of Algebra I
SUBMISSION TYPE:	Amendment/Modification
ACTION:	APPROVED
APPROVAL DATE:	September 28, 2018
EXPIRATION DATE:	June 13, 2019
REVIEW TYPE:	Expedited Review

Thank you for your submission of Amendment/Modification materials for this project. The George Mason University IRB has APPROVED your submission. This submission has received Expedited Review based on applicable federal regulations.

Please remember that all research must be conducted as described in the submitted materials.

Please remember that informed consent is a process beginning with a description of the project and insurance of participant understanding followed by a signed consent form unless the IRB has waived the requirement for a signature on the consent form or has waived the requirement for a consent process. Informed consent must continue throughout the project via a dialogue between the researcher and research participant. Federal regulations require that each participant receives a copy of the consent document.

Please note that any revision to previously approved materials must be approved by the IRB prior to initiation. Please use the appropriate revision forms for this procedure.

All UNANTICIPATED PROBLEMS involving risks to subjects or others and SERIOUS and UNEXPECTED adverse events must be reported promptly to the IRB office. Please use the appropriate reporting forms for this procedure. All FDA and sponsor reporting requirements should also be followed (if applicable).

All NON-COMPLIANCE issues or COMPLAINTS regarding this project must be reported promptly to the IRB.

The anniversary date of this study is June 13, 2019. This project requires continuing review by this committee on an annual basis. You may not collect data beyond this date without prior IRB approval. A continuing review form must be completed and submitted to the IRB at least 30 days prior to the anniversary date or upon completion of this project. Prior to the anniversary date, IRBNet will send you a reminder regarding continuing review procedures.

Please note that all research records must be retained for a minimum of five years, or as described in your submission, after the completion of the project.

Please note that department or other approvals may be required to conduct your research in addition to IRB approval.

If you have any questions, please contact Katie Brooks at (703) 993-4121 or kbrook14@gmu.edu. Please include your project title and reference number in all correspondence with this committee.

GMU IRB Standard Operating Procedures can be found here: http://oria.gmu.edu/1031-2/?_ga=1.12722615.1443740248.1411130601

Appendix B

Recruitment Letter

Dear Graduates,

Mrs. Terrie Galanti, IB Mathematics Teacher at West Valley High School, is conducting narrative research on the lived experiences in **mathematics for students who accelerated their study of Algebra I to 7th grade or earlier**. She would like to explore your stories of this growing phenomenon. You can offer such a unique and invaluable perspective as you reflect on your high school course selections, your learning experiences, your mathematics understandings, and your readiness for college mathematics.

Your voices can help us better support and advise future WVHS IB students.

Please join us for one of four Winter Break focus groups. She has planned two 90-minute sessions at the **Newton Regional Library in Meeting Room 1** on the following days and times:

1. Monday, December 17 at 3:00pm
2. Thursday, December 20 at 7:00pm
3. Thursday, December 27 at 7:00pm
4. Thursday, January 3 at 4:30pm

Please respond to Mrs. Galanti at tgalanti@fcps.edu if you plan to attend so that she can have plenty of refreshments at the ready! Last-minute attendees are ALWAYS welcome.

Since this is an external research project for her dissertation at George Mason University, she will ask you to sign a consent form. The product of the focus group conversations will be narrative stories that capture your perspectives on early Algebra I but maintain your anonymity.

If these times don't work for you but you are interested in a one-on-one conversation, please respond to Mrs. Galanti for alternate scheduling.

Thank you SO much for considering this invitation, and congratulations on all you are accomplishing as wonderful WVHS graduates!

IRB Net ID 1250936-2 (PI – Dr. Call-Cummings)
09/2018

Appendix C

Informed Consent Form

College Students' Experiences with Hyper-acceleration of Algebra I

RESEARCH PROCEDURES

This research is being conducted to understand the mathematics experiences of students who studied Algebra I in 7th grade or earlier. Focus group participation will require 90 minutes or less, and follow-on 30-minute individual interviews with the researcher will be at your discretion. If you agree to participate in the study, your perspectives will be captured in peer-reviewed publications of the experiences of highly accelerated mathematics students. You must be 18 years or older to participate in this study. If you are not 18, please indicate that you will not participate in the study and return the consent form.

RISKS

There are no foreseeable risks for participating in this research.

BENEFITS

There are no benefits to you as a participant other than to contribute to research in the hyper-acceleration of Algebra I.

CONFIDENTIALITY

The data in this study will be confidential. No identifying information about you will be included in any presentation or paper from this study. Audio recordings will be stored on a password-protected personal computer, and only the researchers will have access to the recordings and accompanying transcriptions. Transcriptions of the recordings will replace your names with a pseudonyms; audio recordings will be deleted within one month of transcription. Although focus group participants will be asked to keep the contents of the discussion confidential, due to the nature of a focus group, the researcher cannot control what participants might say outside of the research setting. Those who participate via Skype may review Microsoft's website for information about their privacy statement. https://privacy.microsoft.com/en-US/privacystatement/. While it is understood that no computer transmission can be perfectly secure, reasonable efforts will be made to protect the confidentiality of your transmission.

PARTICIPATION

You must have studied Algebra I during 7th grade or earlier to participate in this study. Your participation is voluntary, and you may withdraw from the study at any time and for any reason. If you decide not to participate or if you withdraw from the study, there is no penalty or loss of benefits to which you are otherwise entitled. There are no costs to you or any other party.

CONTACT

This research is being conducted by Ms. Terrie Galanti, a doctoral candidate at George Mason University. Ms. Galanti may be reached at 703-380-2614 or tgalanti@gmu.edu with questions. The Principal Investigator for this study is Dr. Meagan Call Cummings. She may be reached at 703-993-1718 or mcallcum@gmu.edu. You may contact the George Mason University Institutional Review Board office at 703-993-4121 if you have questions or comments regarding your rights as a participant in the research. This research has been reviewed according to George Mason University procedures governing your participation in this research

CONSENT

I have read this form and all of my questions have been answered (check one)

___ I agree to have my focus group and interview responses included in the analysis of data for researching purposes. I understand that I may be audio taped.

___ I **do not** wish to have my responses included in the analysis of data for research purposes.

____
Printed Name

____
Signature

Appendix D

Focus Group Semi-structured Interview Protocol

Focus Domain: College Student Experiences with Hyper-Accelerated Mathematics

Lead-Off Questions
How did you feel about mathematics as a 7th grader?
How do you feel about mathematics today?

Possible Follow-Up Questions:

1. Tell us about your greatest success in mathematics.

2. Tell us about your greatest struggle in mathematics.

3. What does it mean to be a "good" mathematics student?

4. What does it mean to be "good" at teaching math?
    Further probe: Tell us about the best math teacher you have ever seen/had.

5. How did your middle school mathematics experiences prepare you for high school?
    Further probe: What motivated you to study Algebra in 7th grade or to take a math course in the summer?

6. How did your high school mathematics experiences prepare you for college?

7. Tell us about a time when you were challenged to think deeply about mathematics.

8. How did you feel about retakes during high school? How do you feel about retakes now?

9. My students are very concerned about the "right way" to answer a question. What does the phase "right way" mean to you as a mathematics student?

10. What advice would you give a rising freshman in Algebra 2 Honors?

11. What advice would you give today's IB students about mathematics in college?

Appendix E

Example – Individual Follow-Up Interview Protocol

My dissertation study focuses on “opportunities to learn” mathematics well for students who study Algebra I in 7th grade.
I would like to understand more about the “what”, “why”, and “how” of your many experiences in mathematics.
In our follow-up conversation, I will ask you to tell more about the transcript excerpts below.

(**On GT center**) *It was really intense so I didn't get to see a bunch of my friends after third grade, so and you were only with the smart people who came from Evergreen. Yeah, it's very separated and then from there it's competitive, and it has been ever since.* (Lines 61-65)

Q: What does it mean for you to “compete” in mathematics?

(**On selection for Grade 7 Algebra**) *Oh right, oh yeah, that was a whole big thing. I just remember that I always disliked math. I wasn't awful at it or anything, I just didn't really like it, so I don’t know I always did okay in it or well enough and then I guessed I passed the IOWA because I did it in seventh grade. I honestly don't really remember. I just remember it was a very, if you don't get this on that then you are looked down upon kind of* (Lines 12, 33-39)

I don't like being bored in class and I like being challenged by class. Algebra I is the most challenging thing you can do as a seventh grader. So I was like, "Of course I'm going to do that", and it's an opportunity to take a more difficult class than you are supposed to so. (Lines 88-92)

Q: What did you expect in Algebra I?
Q: What does it mean to be “challenged” in mathematics?

(**On Algebra I in Grade 7**) . *If we turned in a language assignment for homework she would check it off, like people would do that, but I would always do my work because I can't not. My class was really easy and the math, she really taught it well. It was I don't know. It just didn't make me passionate about math in any way.* (Lines 100-104)

(**On Geometry in Grade 8**) *I actually was taught by like YouTube. So that was my geometry experience.* (Lines 182-183)

Q: How did you learn mathematics on You Tube?

(**On Algebra II**) *I think there was a higher expectation of understanding. I know that there were people in the class who didn't care as much as me, but that didn't make me think that the expectation was lower.* (Lines 263-265)

If I got something wrong, she was always there, she took every question, even if it may have seemed stupid, she didn't take it as a stupid question, so it wasn't like she was putting anyone down. Which I feel is something that happens a lot in math classes, they're like "I don't even want to explain that to you." But she would never do that. (Lines 221-226, 230-234)

Q: What was the higher expectation of understanding?

(On selecting math classes) *I definitely wasn't confident enough to take HL, and SL seemed like the right choice and didn't bother me that it wasn't like the highest choice to do.* (Lines 396-398)

Being a year or two ahead, or however far ahead we were in math, just gave me the flexibility to do other things I was interested in. (Lines 403-406)

Q: Describe your level of interest in math.

(On succeeding in math) *in the summer I found out I got a 7* (on the IB SL Exam) *and I literally like screamed and fell on the ground. And that was like definitely my greatest success by far because I literally thought I failed. I was like "Colleges are going to think I'm crazy because I failed my IB exam and I'm not taking math in my senior year.* (Lines 464-468)

Q: What made you think you had failed?

(On being good at math)
So you could be good at math, but if you're not confident then you're never going to really get to an answer and be like, "This is the answer", you're still going to be like "I'm not really sure." I think it's like, when I see someone who's really good at math, they just aren't afraid of it. (Lines 558-561)

If you can look at a problem that they haven't necessarily gone over the same type of problem before, but you can still apply the skills and get to an answer, because you know skillsets that allow you to get there. (Lines 631-634)

Q: Are you good at math?
Q: How would you have talked about being good at math as a 7th grader?

(Advice to future students) *Just don't stress so much about what percentage grade you got. Think about how much you understand what you're learning, and put effort in.* (Lines 789-792)

Q: Tell me about how you came to this.

Closing Q: What other thoughts do you have about accelerated mathematics?

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Biography

Terrie McLaughlin Galanti was the first woman in history to be ranked first in the order of merit when she graduated from the United States Air Force Academy in 1986 with a Bachelor of Science in Electrical Engineering. She earned her master's degree in Electrical Engineering from Stanford University in 1987 as a National Science Foundation Graduate Fellow. After serving as a military officer and systems engineer, she was a high school mathematics teacher and a Dean's Scholar in the College of Education and Human Development at George Mason University. She is specifically interested in how early study of Algebra I is related to opportunities to learn for mathematically promising students. She studies hyper-acceleration from a sociocultural perspective because this new track is socially constructed by parents, teachers, and administrators but should be mathematically advantageous for students. She also researches collaborative problem solving in model-eliciting activities as a vehicle for STEM integration in K-6 classrooms.