

MODELING BEHAVIOR IN PUBLIC GOODS EXPERIMENTS

by

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Modeling Behavior in Public Goods Experiments

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## **DEDICATION**

This is dedicated to my loving wife Diana who has supported and encouraged me throughout this endeavor.

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## LIST OF ABBREVIATIONS

Constant Relative Risk Aversion .....	CRRA
Independent Identically Distributed.....	IID
Marginal Per Capita Return .....	MPCA

## **ABSTRACT**

### **MODELING BEHAVIOR IN PUBLIC GOODS EXPERIMENTS**

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This dissertation analyzes the behavior of participants in the class of economics experiments known as public goods games. It is well established that most participants in public goods games do not play the Nash equilibrium of zero contributed to the public good initially. By initially I mean either the first round of a repeated game or the only round in a one-shot game. The goal of this dissertation is to explain the observed behavior and to develop models that predict it.

In chapter 1, I describe a theory that models rational reciprocity. I develop a mathematical model based on expectation maximization that predicts that players will donate more than zero to the public good. The key factor is a player's level of belief that other players are like them. If I believe that other players are like me, I can expect other humans to arrive at the same decision with some probability. I show that if my belief level in the similarity of the other players to myself is high enough, I should logically play contribute-everything-to-the-public-good. The above theory, combined with

Bayesian belief adjustment, models several observed results: decay in cooperation; cheap talk and leadership increase contributions.

In chapter 2 I explore game ambiguity. I show that differences in what players perceive the game to be, explain a lot of the behavior seen in economics experiments. Most economics experiments define the payoffs, but rarely do they define what it means to win. This and other ambiguities in the definition of the experiment can lead to a wide variety of different behaviors. I argue that most participants in economics experiments perceive their situation as a game. In such situations altruism is an illogical strategy. They adopt a behavior appropriate for the known game which is most similar to the experiment rules. I tie this into Wicksteed's Commonsense of Political Economy (Wicksteed, Philip Henry 1935), arguing that he sees life as a collection of overlapping games. I follow Wicksteed's logic that people behave differently depending on what game they think they are playing and apply it to the behavior of people in economics experiments as well as to real life situations.

In chapter 3 I build on the literature on bet-hedging that invokes the Kelly criterion. Bet-hedging has been adopted by evolutionary biologists to show that evolutionary advantage belongs to those species or subspecies with the highest geometric mean number of offspring. I argue that humans see every decision in the context of long-term growth and that they instinctively apply the Kelly criterion to decide how much of their endowment to put at risk on any bet. Using this understanding I can explain why people in public goods games donate less than everything and more than nothing to the

public good. I show mathematically a direct relationship between the amount contributed to the public good and the level of belief they have that other people are like them.

## INTRODUCTION

“Perhaps the most fundamental question in experimental economics is whether findings from the lab are likely to provide reliable inferences outside of the laboratory.”

Steven D. Levitt, and John A. List. “What Do Laboratory Experiments Measuring Social Preferences Reveal about the Real World?”(Levitt and List 2007)

The following three chapters are an attempt to explain the observed behavior in public goods games. I am specifically trying to explain the behavior of participants in the experimental world as opposed to the real world. The results I obtain may be incidentally illuminating for behavior in the real world and obviously, the ultimate aim of the research is to improve our knowledge of the real world, but for the purposes of this paper, I will confine my analysis to the experimental world and only incidentally imply results that are applicable to the real world.

Although my examination of public goods games was initially driven by a desire to understand how moral preferences and approbation influenced the formation of wealth enhancing institutions, this work does not seek or claim to shed any light on this topic. Instead it takes as its objective the analysis and modeling of the observed results in a large body of literature on public goods experiments. Many of these experiments set out to illuminate behavior in specific real-world situations. The experimenters display considerable ingenuity in creating experiments that parallel these real world situations. They carefully explain how the conditions in the experiment are similar to the situation

they are trying to illuminate and at the end of the paper they draw conclusions about what to expect in real world situations. Many of these papers do not spend a lot of effort questioning whether the ingenious experiments are really useful in modeling the targeted situation.

Several papers have speculated that participants in the experiments can be influenced by their understanding of what the experimenter is looking for. There have even been experiments specifically designed to measure this effect (Cipani and Waite 1980). Other papers have looked into framing effects and have found some evidence that the way the context of the experiment is explained can have a small influence on how people behave (Rege and Telle 2004; Robison, Shupp, and Myers 2010; Schlüter and Vollan 2015). Many authors have speculated that people in experiments find it difficult to separate their behavior in the experiment from the world outside the experiment (Gintis 2009). They act within the experiment as if the experiment will be repeated indefinitely even if it is a one shot game with very specific rules.

A common theme in almost all of these papers is that large numbers of participants do not play the Nash equilibrium. Game theory is very clear that the Nash equilibrium is the expected play and game theory cannot explain human behavior that does not play the Nash equilibrium strategy. The consistency of the failure to play the Nash equilibrium has led to a whole series of games to explore the limits of this inconsistent behavior.

The following three chapters offer specific models for how human behavior could rationally and predictably lead to the observed results. Although the motivation for these

chapters is to understand how people behave in experimental situations and in some cases narrowly explain only behavior in such artificial environments, the analysis has pointed to a couple of insights into general behavior outside of the experimental environment. However, the main focus of this work should be understood as an investigation into the specific factors that affect people's behaviors when they are participating in economics experiments.

In their paper "What Do Laboratory Experiments Measuring Social Preferences Reveal about the Real World?" Steven D. Levitt, , and John A. List. (Levitt and List 2007) put forward a number of reasons why we should be careful when extrapolating experimental results into the real world. In their conclusions they argue, "that the choices that individuals make depend not just on financial implications, but also on the nature and degree of others' scrutiny, the particular context in which a decision is embedded, and the manner in which participants are selected to participate." Later they state, "by adopting experimental designs that recognize the potential weaknesses of the lab, the usefulness of lab studies can be enhanced." Their general approach is to study the ways in which the experimental world is significantly different from the real world. My approach is subtly different. Rather than comparing the experimental world to the real world, I start by trying to explain the behavior of participants as an isolated problem. Inevitably, the results point to ways in which this behavior is different from the real world, but my first objective is to explain the observed behavior in the experimental world. One of the consequences of this approach is that I come to conclusions about what is influencing



behavior in the experimental world that have no overlap with the three key differences listed in their conclusions.

## **CHAPTER ONE: RATIONAL RECIPROCITY: MODELLING OTHERS' BEHAVIOR**

### **Abstract**

People participating in public goods experiments do not consistently play the Nash equilibrium. I describe a theory that if I make a decision based on the material facts, I can expect other humans to arrive at the same decision with some probability. I show that if my belief level in the similarity of the other players to myself is high enough, I should logically play contribute-everything-to-the-public-good. The above theory, combined with Bayesian belief adjustment, models several observed results: decay in cooperation; cheap talk and leadership increase contributions.

Were it possible that a human creature could grow up to manhood in some solitary place, without any communication with his own species, he could no more think of his own character, of the propriety or demerit of his own sentiments and conduct, of the beauty or deformity of his own mind, than of the beauty or deformity of his own face. All these are objects which he cannot easily see, which naturally he does not look at, and with regard to which he is provided with no mirror which can present them to his view. Bring him into society, and he is immediately provided with the mirror which he wanted before. It is placed in the countenance and behaviour of those he lives with, which always mark when they enter into, and when they disapprove of his sentiments; and it is here that

he first views the propriety and impropriety of his own passions, the beauty and deformity of his own mind.

Adam Smith (1982a [1759]): The Theory of Moral Sentiments III.i.3.110

## **Introduction**

This paper presents a line of reasoning that provides a rational thought process that leads people to choose a strategy other than playing the Nash equilibrium.

Game theorists interpret the Nash equilibrium as the only rational play when the appropriate conditions apply. Ample experimental evidence shows that the majority of humans do not play the Nash equilibrium in a variety of experimental contexts. To explain this anomaly, economists invoke a concept which they label strong reciprocity. The advocates of strong reciprocity do not dispute the claim of the game theorists that the Nash equilibrium is the rational play. Instead, they claim that culture-gene coevolution predisposes modern man to cooperate in situations where there is no apparent advantage.

The conflict between game theory and strong reciprocity parallels the conflicting streams of thought among economists who analyze public choice. On the one hand there are a number of economists who claim that economic models should assume that Homo Economicus is a purely self-interested agent who always acts without consideration for others. On the other hand there are economists who claim that if there is a clear benefit to society, agents will find some way of realizing it. Most recent papers recognize both options and attempt to use a blend of them to explain behavior. The result is mostly a

fuzzy mess. These contrasting views have been around for a long time and economists continue to debate them today.

As Robert Kurzban observes (Kurzban, Burton-Chellew, and West 2015a) “in PGGs, in which not cooperating at all maximizes each individual's financial gain, numerous researchers have observed that individuals do indeed voluntarily contribute. This result has been taken to imply that humans have prosocial preferences, and based on that inference, a huge literature has emerged.” Here is a partial list of references that include a summary of the history of these ideas. Most of them cite Mancur Olsen’s book *The Logic of Collective Action* (Olson 2009) as the key item that refuted the previously widely held assumption that if there was a gain to be had, humans would find a way to realize it, but several of them trace the argument back to Hobbes and Kant.

(Falk and Fischbacher 2006)

(R. W. Rosenthal 1989)

(Dufwenberg and Kirchsteiger 2004)

(V. L. Smith 2003)

(Ambrus, A., & Pathak, P. (2009), n.d.)

(Gunnthorsdottir, Houser, and McCabe 2007)

(Joffily et al. 2014)

(Fehr and Gächter 2000)

(Rosenthal 1981)

(Rabin 1993)

(Masclet et al. 2003)

(Dannenberg 2016)  
(Rapoport and Chammah 1965a)  
(Isaac, Mark, McCue, and Plott 1985)  
(Gintis 2009)  
(Rapoport and Chammah 1965b)  
(Liebrand, 1984)  
(Kurzban, Burton-Chellew, and West 2015b)  
(Axelrod 1984)  
(Sober, Elliott 1998)  
(Henrich 2007)  
(Gintis 2005)

A number of academics from a variety of disciplines have produced models of culture-gene coevolution (Henrich 2007). These models assert that human civilization has evolved culturally and biologically in ways that are interdependent. Humans evolved culturally because biologically they are equipped to do so. W.E. Hamilton (Hamilton 1964) uses a model to determine the condition under which and altruistic behavior will survive. The condition is that  $rb > c$ , where  $c$  is the cost,  $b$  is the benefit to another person, and  $r$  is the probability that the two individuals have the same allele through descent from a common ancestor. Subsequent economists have built more complicated models to determine the conditions under which reciprocity is sustained. Henrich and Henrich (Henrich 2007) point out that although reciprocity appears to have evolved in humans

there is very little evidence that any such trait has appeared in any other animal including the other primates.

A key element of almost all these models is the conflation of reciprocity and altruism. This stems from the understanding that the Nash equilibrium is the rational play. Any choice that deviates from the Nash equilibrium must therefore be motivated by altruism and altruism is a quality we have inherited because of our evolutionary history.

As stated above, experiments have demonstrated repeatedly that people invited to participate in Public Goods games have a strong tendency to contribute more than zero to the “Public Good”. This basic fact has been established since 1978 (Marwell and Ames 1980; Marwell and Ames 1981; Marwell and Ames 1979; M. R. Isaac, McCue, and Plott 1985). Since then Economists have embarked on a research effort to examine this phenomenon in more detail. Experimenters studied repeated games where the same subjects played the same game several times (Chaudhuri 2010). It was found that under some circumstances (for example when the multiplier was low enough) players migrated towards the Nash equilibrium and under other conditions (for example when the multiplier was high enough) they migrated towards an equilibrium where most players contributed everything to the Public Good. This latter condition is described as the socially optimal outcome. One set of experimenters then explored punishment mechanisms (Fehr and Gächter 2000; Brandt, Hauert, and Sigmund 2003; Choi and Ahn 2013; Burnham 2014; Fehr and Fischbacher 2004; Raihani and Bshary 2011; Masclet, Noussair, and Villeval 2013; Masclet et al. 2003), while another set explored the effect of making the players’ choices public; at least to the other players in the game (Rege and

Telle 2004). Other researchers have examined the impact of communication (cheap talk) before the play and the role of leadership (Levy et al. 2011; Houser et al. 2014). The main thrust of the research has moved away from “why do players not play the Nash equilibrium?” to “what does it take to sustain the socially optimal play?”

In these papers various theories are proposed to explain players’ behavior. The main contenders are now recognized to be “altruism”, “fairness”, and “strong reciprocity”, with some economists arguing that strong reciprocity incorporates both altruism and fairness.

Altruism is usually defined as having utility for other people’s happiness/well-being and is represented theoretically by a utility function of the form:

$$U_i = f(R_i) + \sum_{j \neq i} g_j(R_j) \quad (1)$$

where  $i, j$  are the player ids,  $U$  is utility, and  $R$  is the reward or payout. The function  $f$  is the subjective response of the player  $i$  to a financial reward; it is almost certainly not linear and is only known to player  $i$ . The functions  $g$  are the utilities player  $i$  gets from the rewards to the other players. The usual assumption is that player  $i$  is indifferent to which of the other players receives the reward; hence the functions  $g$  are the same for each of them. In almost all cases, experimenters are careful to ensure that the players do not know each other and even if they are physically in the same room, they cannot

identify which people correspond to which players in the game. Since the experimenters are invariably trying to illuminate how strangers behave towards each other, this is reasonable.

Fairness is an intuitive concept that can either manifest itself as a tendency not to take more than the other player(s) unless there is some recognition that the player “deserves” to be preferred in some way, or as revenge when a player, who has been offered less than what they consider fair, punishes the other player. Dufwenberg and Kirchsteiger (2004) argue that fairness can be used to explain reciprocity. They construct a model that shows how a sense of fairness can explain the decay in cooperation in repeated public goods games, but have no explanation for the initial level of cooperation in the first round.

Strong Reciprocity is defined as “the predisposition to cooperate even when there is no apparent benefit in doing so” (“Strong Reciprocity” 2015). The implication is that it is illogical to play anything other than the Nash equilibrium. Hence the phrase “no apparent benefit”. While some economists may disagree with this characterization, the mainstream of thought is that the only logical play is the Nash equilibrium. For this reason strong reciprocity is usually seen as a form of altruism.

Strong reciprocity is not so much a cause of cooperation as a description of an effect. The effect is that humans behave irrationally in that they reciprocate a benevolent act, or an anticipated benevolent act by other players, by choosing to play the benevolent act of donating more than zero to the public good. Various explanations have been put forward to explain this benevolent act. Most of them boil down to some evolutionary



bias: because of the evolutionary path of human society, human beings are conditioned to “cooperate” (Kurzban, Burton-Chellew, and West 2015b; Zywicki 1999). Biologists have sought to establish ways that the early ancestors of man could have been selected for this trait (Axelrod, Robert M. 1984; Sober, Elliott 1998). The results are less than convincing in the context of public goods experiments. We are left with the conundrum that there is “no apparent benefit”.

Imagine a chess player being altruistic to his opponent. One might under teaching circumstances deliberately lose to one’s son or daughter in order to encourage them, but it is inconceivable that one would do so in any competition. When playing chess, one adopts a role and that role is defined by the rules of chess. One’s focus is entirely on winning and one exerts all one’s attention and ability towards that goal. One immerses oneself in the world defined by the rules of the game of chess. The same is true of participants in economics experiments. They are presented with a set of rules that define a virtual reality and told to “play a role” in the game. Their task is to play the game to the best of their ability based on the rules of the game as presented. There is no room, or incentive, to practice altruism.

### **A theory of rational reciprocity**

Economists have become so accustomed to the use of game theory and in particular the Nash equilibrium as the predictor of how players should act if they are rational, that they have been blinded to other potential methods of analysis.

Consider the typical public goods game. Each of four players is given an endowment of \$10 and told that they can keep any part of it for their own and contribute the rest to a public good. All contributions to the public good will be pooled and the pool multiplied by 2. The resulting augmented pool will be divided equally between the players. They take home the sum of the part of their endowment they kept and their share of the augmented pool.

The Nash equilibrium is defined as follows: “If each player has chosen a strategy and no player can benefit by changing strategies while the other players keep theirs unchanged, then the current set of strategy choices and the corresponding payoffs constitutes a Nash equilibrium.” It can be shown very easily that the unique Nash equilibrium for this game is for everyone to keep everything and take home \$10. However, it is obvious to any intelligent person that they could easily do better if everyone could be persuaded to contribute everything to the public good. If this could be achieved, each player would go home with \$20. This is clearly a superior outcome for each individual regardless of any value placed on the welfare of the other players. The problem is that if everyone else follows this cooperative strategy and one player keeps everything, he can go home with \$25. But this option is open to each player. Hence the Nash equilibrium. It appears inevitable that if everyone thinks the same way and understands the Nash equilibrium, each will go home with \$10. It is also obvious that they could do much better.

In the abstract to his paper, A Bounded-Rationality Approach to the Study of Noncooperative Games, Rosenthal (1981) states “The thesis of this paper is that finite,

noncooperative games possessing both complete and perfect information ought to be treated like one-player decision problems. That is, players ought to assign at every move subjective probabilities to every subsequent choice in the game and ought to make decisions via backward induction. This view is in contrast with the game theoretic approach of Nash equilibrium.”

This idea was explored during the following decades (Binmore 1987; McKelvey and Palfrey 1992) together with the concept of bounded rationality (R. W. Rosenthal 1989; Aumann 1995), but no coherent explanation for the behavior of participants in economic experiments emerged. Although the principle of treating each play as a one player decision problem was not disputed, no satisfactory method of arriving at the probabilities was proposed. The theory propounded below provides a rationale for calculating the probabilities that Rosenthal proposes.

Building on the concept of bounded rationality, Vernon Smith (V. L. Smith 2003) has argued for a model based on the concepts of ecological rationality and constructivist rationality. Ecological rationality drives behavior in recognized situations. In this mode people do not analyze all possible outcomes and actions in a particular problem, but use rules-of-thumb derived from past experience. In situations that do not immediately map to a recognized situation, people use constructivist logic, possibly bounded, to arrive at the chosen action. Occasionally constructivist logic is applied to recognized situations to see if a better rule-of-thumb can be achieved. The assumption that people use ecological rationality in no way invalidates the use of constructivist rationality to analyze a model. As Vernon Smith says, “Notice that our argument is in the form of a constructivist theory

that need not characterize the subjects' reasoning, even if it has predictive accuracy; i.e., constructive rationality may predict emergent ecologically rational outcomes. . .” The theory I present below is developed entirely using constructivist logic.

In their paper A Theory of Reciprocity Falk and Fischbacher (Falk and Fischbacher 2006), propose a model based on beliefs and beliefs about beliefs of what “type of player” the other players are. They postulate that the type of player can be derived from not only the actions of the other players, but also from the intentions revealed by those actions. While this model gives good predictions for the evolution of repeated games, it does not provide a basis for the behavior of players in one shot games or the first round of a repeated game.

My theory is based on the idea that each player models the behavior of other players based on their own experience and the level of belief they have that they are a typical member of society.

Building on Adam Smith's idea that we use society as a mirror to judge our own behavior, we can expect others to do the same. Our egos may tell us that we are unique and different, but logic dictates that the default assumption when encountering another individual that we have never encountered before, is that that individual is like us. We will quickly modify that opinion based on appearance, voice, choice of words, and tone, but in the absence of these cues our first assumption must be that they are like us.

Applying this principle to a public goods game, if I as a player in the game believe the other players are like me in both logical capabilities and preferences, I must conclude that if I choose to play the Nash equilibrium, they will do the same. It is

therefore extremely unlikely that I will be able to pull off the coup and go home with \$25. If I accept this reality, the best alternative for me is for everyone to go home with \$20. Is there any logical process that leads me to believe that this outcome is possible? Given that I believe other people are like me, if I can persuade myself that it is rational to donate everything to the public good, I can also persuade myself that the other players will do the same. As with most beliefs, the belief that I am like other people and that other people are like me is held subject to some probability. So the above statement has to be modified to read “I can also persuade myself that some of the other players will do the same . . .”, where the “some” is mathematically equivalent to the probability I assign to the belief that other players are like me.

Consider the following calculations:

Assume  $n$  players per game, an initial endowment of \$10, and the public good multiplier of  $\beta$ . For simplicity, start by constraining players to contribute all or nothing. This makes the mathematics more tractable. If the probability that other humans behave as I do is  $p$ , my expected payout is:

$$R = k_i + \frac{\beta}{n}(d_i + (n - 1)pd_i) \quad (2)$$

where  $k_i$  is the amount I keep,  $n$  is the number of players in my game, and  $d_i$  is the amount I donate. Note. I do not know what the other players are donating, but I expect

them to donate the same as I do with probability  $p$ , so the expected amount they donate is  $pd_i$ .

Since I have constrained the choice to all or nothing, I can write this as 2 equations (3) and (4), one in which I donate everything and the other in which I donate nothing.

$$R_{all} = 0 + \frac{\beta}{n}(10 + (n-1)p10) \quad (3)$$

$$R_0 = 10 + \frac{\beta}{n}(0 + (n-1)(1-p)10) \quad (4)$$

If  $R_{all} > R_0$  I should play donate everything to the public good. If  $R_{all} < R_0$  I should play donate nothing to the public good. If  $R_{all} = R_0$  it makes no difference.

$$R_{all} > R_0 \Leftrightarrow 0 + \frac{\beta}{n}(10 + (n-1)p10) > 10 + \frac{\beta}{n}(0 + (n-1)(1-p)10)$$

$$\beta(1 + (n-1)p) > n + \beta((n-1)(1-p))$$

$$\beta + \beta np - \beta p > n + \beta n - \beta np - \beta + \beta p$$

$$2\beta np - 2\beta p > n + \beta n - 2\beta$$

$$p > \frac{n + \beta n - 2\beta}{2\beta(n-1)} \quad (5)$$

For  $n=4$  and  $\beta=2$ :

$$p > \frac{4 + 8 - 4}{4(3)} = \frac{2}{3} = 0.667$$

Table 1 gives the values of  $p$  required for various values of  $n$  and  $\beta$

**Table 1 - values of  $p$  required for various values of  $n$  and  $\beta$**

	Values of $n$ – the number of players per game									
		2	3	4	5	6	7	8	9	10
Values of $\beta$ - the amount the donated sums are multiplied by	1	1	1	1	1	1	1	1	1	1
	1.5	0.6667	0.7500	0.7778	0.7917	0.8000	0.8056	0.8095	0.8125	0.8148
	2	0.5000	0.6250	0.6667	0.6875	0.7000	0.7083	0.7143	0.7188	0.7222
	2.5	0.4000	0.5500	0.6000	0.6250	0.6400	0.6500	0.6571	0.6625	0.6667
	3	0.3333	0.5000	0.5556	0.5833	0.6000	0.6111	0.6190	0.6250	0.6296
	3.5	0.2857	0.4643	0.5238	0.5536	0.5714	0.5833	0.5918	0.5982	0.6032
	4	0.2500	0.4375	0.5000	0.5313	0.5500	0.5625	0.5714	0.5781	0.5833
	4.5	0.2222	0.4167	0.4815	0.5139	0.5333	0.5463	0.5556	0.5625	0.5679
	5	0.2000	0.4000	0.4667	0.5000	0.5200	0.5333	0.5429	0.5500	0.5556
	5.5	0.1818	0.3864	0.4545	0.4886	0.5091	0.5227	0.5325	0.5398	0.5455
	6	0.1667	0.3750	0.4444	0.4792	0.5000	0.5139	0.5238	0.5313	0.5370

In Table 1, those cells where  $\beta$  is greater than  $n$  have values of  $p$  less than 0.5.

While it is extremely likely that a person believes others are like him with probability greater than 0.5, these cells are irrelevant because when  $\beta$  is greater than  $n$  the Nash equilibrium leads to the same action.

In the limit for large  $n$  and  $\beta = 2$ :

$$p > \frac{n + \beta n - 2\beta}{2\beta(n - 1)} \rightarrow \frac{1 + \beta}{2\beta} = \frac{3}{4} = 0.75$$

The value of  $p$  here is the belief level a person has to have that other players are sufficiently like him, to justify donating everything. In a 4 player game, if a person

believes that other people are likely to come to the same conclusion as he does with probability greater than  $p$ , it is logical to donate everything to the public good i.e. whenever  $p > 0.667$ . Notice that he comes to this conclusion before he makes his decision about how much to donate. In fact, he makes his decision about how much to donate based on the expected payout for each alternative.

I must also point out that his evaluation of  $p$  is a function of how certain he is about the rationality of the decision. To illustrate this point consider a game with  $n = 2$  and  $\beta = 2$ . In this game the payouts are given in Table 2:

**Table 2 - payouts for a game with  $n = 2$  and  $\beta = 2$**

Combination #	Player 1 donates	Player 2 donates	Payout for player 1
1	10	10	20
2	10	0	10
3	0	10	15
4	0	0	10

The payout to Player 1 when she donates 10 is greater than or equal to the payout when she donates 0 for all values of Player 2 donation. Therefore, donate 10 dominates donate 0 and is in fact the Nash equilibrium for this game. A player can therefore be very confident that the other players will come to the same decision as he does. For  $n > 2$  or  $\beta < 2$ , his confidence that the other players will reach the same decision as he does falls off.



I have just provided a logical reason for a rational being to donate everything if she believes the other members of the game are like her and will reason in the same way, in spite of the Nash equilibrium being to donate 0.

Why would an agent not consider this argument and then play keep everything? Clearly his decision to keep everything is not going to change the other players' decisions. So he can do better by keeping everything. The thought experiment that prevents this is rather like fencing with yourself in a mirror. As soon as you make a feint, your reflection counters with the speed of light. If you cannot convince yourself to play donate everything, your belief that others are like you results in the Nash equilibrium. Only if you can convince yourself to actually donate everything can this logic lead you to the conclusion that others will do likewise.

Adam Smith (Smith 1982b TMS III .I.3) uses the analogy of a mirror when explaining how an individual determines that his own behavior is socially acceptable. He explains that observing others' behavior is like looking at a mirror and seeing your own behavior potentially reflected in what other people do. By exercising judgment on other people's behavior the individual gains insight into how his own behavior is perceived. In my arguments the reverse process is taking place. Having learned what behavior is appropriate by looking into this mirror, he can now infer that others have done the same. It is therefore reasonable for him to model the world as if other people will behave in ways similar to the way that he behaves. This becomes the default position when encountering a stranger about whom he knows nothing, although he may form an initial judgment based on the appearance of the stranger. This is particularly true in

experimental situations as the experimenters are careful to avoid letting the participants identify each other.

A player can now construct a model of how the other players in the game will respond and apply the results of that model to make his choice, recognizing that not only is he modeling other players' behavior, but that he himself is an agent in the model. Having constructed and analyzed the model, the only conclusion he can logically come to is the one described above: if he believes he is a typical member of the human race, with probability greater than  $p$ , he must choose to donate everything. This is not really a violation of the Nash equilibrium, because the Nash equilibrium definition starts out with "If each player has chosen a strategy . . .". The problem is that each player has not chosen a strategy. They are in the process of choosing a strategy, and using the model he is constructing, they will arrive at the same decision that he does if and only if they are sufficiently like him.

A key to this argument is the intelligence of the other players. If for some reason, he considers other players inferior in reasoning to himself, he will choose the Nash equilibrium, either because he considers the other players incapable of following the same argument and he therefore expects them to keep everything, or because he considers them so stupid that they will play donate everything even though he chooses to keep everything. The only conditions under which it makes sense to donate everything is when you have a strong belief that others are like you and they are intelligent enough to see this possibility.

Public goods experiments using only economics graduate students as participants result in much lower contributions to the public good than games with randomly chosen participants (Marwell and Ames 1981). The standard explanation for this is that these graduate students are more intelligent than the randomly chosen participants. This would undermine my theory except for the fact that these students have been taught to consider the Nash equilibrium is the optimal and only rational play. Gintis (2005) suggests that the teaching of game theory to economics students has corrupted their normal and instinctive preferences. It is quite possible that this distorted view of human behavior has destroyed the fragile equilibrium implied by my theory. The power of an authoritatively presented theory like the Nash equilibrium swamps any instinctive idea that we are alike . . . and the mirror is broken.

In their book “the vanity of the philosopher” Peart and Levy (Peart, Sandra and Levy 2005) conclude that any model that assumes that there are inherent differences between people of different races or creeds risks leading to morally and economically inferior outcomes. They call this concept analytical egalitarianism. The theory I have expounded above leads to a very similar conclusion: on a personal level, any model I construct of how other people behave that does not assume that other people are inherently similar to me leads to an inferior outcome both for them and for me. One could summarize my theory as analytical egalitarianism applied to game theory.

This theory is arrived at from purely rational deductions, assuming that an intelligent rational agent optimizes her take based on the assumption that she is a typical member of the human race. There is no need for evolutionary bias towards strong

reciprocity. There is no need to invoke altruism. All agents act in a purely rational manner based on the information available to them. This argument does not mean that altruism and fairness play no part in economics experiments. Many experiments have shown that a sense of fairness and the revenge it inspires have been factors in the way play evolves in repeated games scenarios. What it does mean is that many of the scenarios which have previously been explained using strong reciprocity can be explained without any appeal to altruism or some evolutionary bias.

From this theory a number of previously puzzling results follow very naturally.

- “Why does cooperation fall off in repeated games?” If players are rational agents, they will modify their beliefs according to Bayes rule. If a player initially believes the other players are like him with a probability high enough to merit donating all, if after the first round, the result is not as expected, he will lower his probability in a Bayesian manner and over time his belief probability will decay until it reaches the point where it leads him to donate nothing. Fairness is another factor that comes into play in repeated games, but the effect is independent of the one I am modeling.
- Cheap talk has been shown to increase the initial level of donation as well as to sustain levels nearer to the social optimum (Levy et al. 2011). Cheap talk has been characterized as worthless because there is no enforcement attached. Using this model, it becomes apparent that cheap talk is simply a means of increasing the beliefs of the players that the other players think as they do, both regarding their awareness of the rules and the outcomes and their intention to play donate everything. Because

cheap talk effectively increases a player's confidence that the other players are like him, he is more likely to donate to the public good.

- Leadership has been shown to be effective in elevating the level of donations (Levy et al. 2011). Using this model, it is clear the effect of leadership is to increase the players' beliefs that the other players will do as they do if they follow the leader's recommendation. One of the findings of David Levy's paper is that computer generated leadership is ineffective. My model explains this by recognizing that the path by which the leader enhances the outcome is not by direction, but by enhancing belief in the similarities of understanding and of the motives of the group. Another finding is that an elected leader is more effective than an appointed one. The same argument illuminates this result.
- The model could also shed some light on the finding that average group IQ is more important for good institutions than individual IQ (Jones 2016). The analysis required to assess the level of probability needed to justify donating to the public good is not trivial and in fact it is unlikely that any players in any of the experiments have consciously performed it, but it is well established that the subconscious mind frequently performs economic optimization without any knowledge of the economic theories we use to analyze actions. If people with higher IQs are more likely to play according to this model than people with lower IQs, any repeated "game" in which a large percentage of the population is unable to arrive at this decision will cause the "cooperation" to decay rapidly.

- Group size has been studied extensively (Marwell and Ames 1979; R. M. Isaac, Walker, and Williams 1994; R. M. Isaac and Walker 1988; Szolnoki and Perc 2011). Isaac and Walker report some results that tie in well with my theory. They find empirically that as group size increases holding constant the marginal per capita return (MPCA), participation in the provision of the public good actually increases. They express surprise at this and point out that it is contrary to mainstream view of what should happen at the time. They also point out that if the MPCA is allowed to diminish naturally i.e. there is no increase in the multiplier as the group size increases, they get the expected result of increased free riding. Applying my theory to the first scenario, I need the belief level to meet the criteria derived above:

$$p > \frac{n+\beta n-2\beta}{2\beta(n-1)} \quad (6)$$

Holding MPCA constant implies  $\frac{\beta}{n} = k$ , where  $k$  is some constant. We can rewrite this as  $\beta = kn$ . This gives

$$p > \frac{n+kn^2-2kn}{2kn(n-1)} = \frac{1+kn-2k}{2k(n-1)} \quad (7)$$

By examination (or calculus) this can be shown to be a decreasing function of  $n$  for  $0 < k < 1$  and  $n > 2$ . Thus the required level of belief in the likeness of others to oneself decreases with group size provided the MPCA is held constant. When the MPCA is allowed to diminish in proportion to the group size, we get the results given in Table 1. Taking the example of  $\beta = 2$ , we see that  $p$  increases from 0.667 to 0.75 as  $n$  increases from 4 to infinity. So both of these results predicted by my theory are entirely consistent with the results of Isaac and Walker. The other papers cited are not inconsistent with the results of Isaac and Walker.

## **Conclusions**

It is both incorrect and unnecessary to invoke altruism to explain the observed behavior in public goods games. It is similarly incorrect and unnecessary to invoke evolutionary bias to explain strong reciprocity in public goods games. A belief in the similarity of other players to myself is sufficient to cause me to play the socially optimal play as opposed to the Nash equilibrium based on purely rational arguments.

## **CHAPTER TWO: WHAT IS THIS GAME AND HOW DO I WIN IT?**

### **Abstract**

I explore game ambiguity. I show that differences in what players perceive the game to be, explain a lot of the behavior seen in economics experiments. Most economics experiments define the payoffs, but rarely do they define what it means to win. This and other ambiguities in the definition of the experiment can lead to a wide variety of different behaviors. I argue that most participants in economics experiments perceive their situation as a game. In such situations altruism is an illogical strategy. They adopt a behavior appropriate for the known game which is most similar to the experiment rules. I tie this into Wicksteed's Commonsense of Political Economy, arguing that he sees life as a collection of overlapping games. I follow Wicksteed's logic that people behave differently depending on what game they think they are playing and apply it to the behavior of people in economics experiments as well as to real life situations.



The distinction that we have drawn between the selfish motive, which considers me alone, and the economic motive, which may consider any one but you, is well illustrated by the case of trustees. Trustees who have no personal interest whatever in the administration of the estates to which they give time and thought will often drive harder bargains—that is to say, will more rigidly exclude all thought or consideration of the advantage of the person with whom they are dealing—in their capacity as trustees than they would do in their private capacity. Thus we see that the very reason why a man feels absolutely precluded from in any way considering the interests of the person with whom he is transacting business may be precisely the fact that his motive in doing business at all is absolutely and entirely unselfish. The reason why, in this instance, there is no room for "you" in my consideration is just because "I" am myself already excluded from my own consideration. If I counted myself I should find room for you just so far as "I" take an interest in "you," but if I do not admit myself I cannot bring in your interests as part of my own programme. The "others" for whom I act are others than you, more completely and irrevocably other than I myself should be; for though I might myself adopt as mine some of your purposes, I cannot affiliate those purposes of yours upon these "others" for whom I am acting. The transaction then becomes more rigidly "economic," just because my motive in entering upon it is altruistic.

Philip Henry Wicksteed ( 1935): The Commonsense of Political Economy V.116

## **Introduction**

This chapter explores game ambiguity. Many economic experiments do not define what it means to win. Each participant may interpret winning differently. This leads to very different strategies when participating in the experiment. People in the real world treat various situations as if they were games. This leads to the idea that people use games as ecologically rational models to handle every-day situations. Many such situations occur frequently in a person's life and each person stores up a library of games

they use regularly. When people are confronted with a situation that is unfamiliar, they reach for the game in their library that appears to match the current situation most closely. With this insight it is possible to interpret how different people react in experiments based on their life experience and the games in their library. In the other direction we can construct a framework that allows us to interpret how behavior in experiments can be used to predict behavior in real life.

### **Game ambiguity - the nature of winning - different perceived objectives**

In Chapter 1, I mentioned the unreasonableness of expecting altruism in a game of chess. I contend that this is true for almost all the games people play. The whole concept of a game is to restrict action to a subset of life defined by the rules. We teach our children to play games from an early age and we teach them to abide by the rules. This is an important educational activity. It encourages the child to abstract their role from real life and act as though they were living in another reality. When we play a game, we are expected to immerse ourselves in the virtual reality defined by the rules of that game and to play the game to the best of our ability. This is an important skill. It enables us to see the world as others see it.

One part of the rules of most games is to define what constitutes winning. This is a key difference between games and real life. In most games there are clearly defined criteria for winning and there is no credit for achieving any other self-defined goal. A successful game can only be played when all players play by the same rules. Humans

seem to be very capable of projecting themselves into such a virtual reality and striving to win.

Many games that we play are constructed as zero sum games. Only one person or team wins. The others come in second or lose. This is less true in modern role playing internet games, but there is still an element of scoring and competition. We are by nature competitive and deliberately seek out ways to demonstrate that we are better than our peers by challenging them to competitive games.

Most economics experiments are set up as virtual reality games with defined rules and defined payouts. The people we invite to “play” these games almost certainly recognize the situation as a game. We can expect them to immerse themselves in the virtual world we provide them with and play to win as best they can. However, economics experiments are different from most traditional games. In spite of the fact that we carefully define the rules and the payouts, most experiments do not define what constitutes a win. In a typical public goods game, players might conclude that winning was achieved in at least 3 possible ways:

- Take home the largest possible amount of money
- Take home more money than any of the other players
- Take as much money as possible from the game host

Others can be imagined, but just these three lead to very different behaviors and strategies from the players. For objective 2, the Nash equilibrium play is very likely to be the chosen strategy. For objective 1, the argument outlined in chapter 1 leads to a strategy based on the probability that other players are like me. For objective 3, the clear

best strategy is to donate everything to the public good regardless of any other considerations. So depending on the player's understanding of the objective of the game, they would employ completely different strategies even given the same understanding of the rules.

A very telling illustration of this effect is found in Palacios-Huerta and Volij (Palacios-Huerta and Volij 2009). Using the centipede game (see below) they show that the best chess players tend to play the way Game Theory predicts. All the Grand Masters in their experiment played Take on the first opportunity. Their conclusion is that Grand Masters are better at working out the implications of their actions (whether by formal application of Game Theory or by general logical deduction) than the average student. Clearly they were assuming the optimal play for intelligent people capable of reasoning out the rule of the game would be the Nash equilibrium. I would suggest an alternative or complementary interpretation: rather than or as well as being better logicians, the chess players interpreted winning as beating their opponent, or taking home more money than their opponent. This would be natural to people who spend a considerable amount of time playing a game in which defeating your opponent is the primary objective. Chess players place more value on winning than on the material payouts – particularly when the payouts are relatively small. It is more important to them to “win” than to get \$100.

Another interesting result they report is that ordinary students when told they are playing against a chess expert, play “Take” on the first round in substantially and statistically significantly higher numbers than when they are playing against other “ordinary” students. Both of the theories propounded so far could contribute to this

effect. First, ordinary students may well think that chess players are different and therefore not sufficiently like themselves to induce Rational Reciprocity. Second, the fact that they are told they are playing against a chess expert, may cause them to adopt a different game as their model for this situation.

People not constantly involved in intense competition would interpret winning as taking home as much money as possible. A substantial number of people do in fact refuse to “take” until the very end of the game. But if the game master suggests that this is somehow like a game of chess, this option could be suppressed.

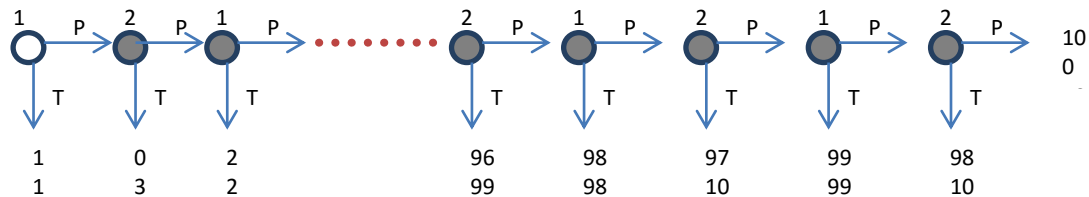
### **The centipede game**

The centipede game (Binmore 1996; Rosenthal 1981) is designed to test the validity of the Nash equilibrium as a guide to human behavior. The game is depicted in Figure 1. The rules of the game are as follows. Player 1 starts and has two possible actions (a) Take (T) – in which case the payouts are 1 for player 1 and 1 for player 2 and the game ends or (b) Pass (P) – in which case the pot is passed to player 2, one unit of payout is added to the pot and player 2 has two possible actions (a) Take – in which case the payouts are 0 for player 1 and 3 for player 2 or (b) Pass - in which case the pot is passed to the other player, one unit is added to it and the game repeats with the pot 2 units bigger than it was at the start. The game continues until the 198th node where player 2 has the choice to Take – with payouts of 101 for player 2 and 98 for player 1 or Pass – in which case the payout is 100 for each player. The choices for turn  $n$  ( $n < 198$ ) can be defined as

For  $n$  odd, player 1 can (a) Take and the payouts will be  $(n+1)/2$  for each player or  
 (b) Pass

For  $n$  even player 2 can (a) Take and the payouts will be  $(n/2 + 2)$  for player 2 and  
 $(n/2 - 1)$  for player 1 or (b) Pass

**Figure 1 The centipede game**



The classic treatment of this game is to use backwards induction. Starting from the last node, player 2 is going to choose Take to get 101 rather than the 100 he would get if he chose Pass. Looking at the previous move, player 1 will correctly conclude that Player 2 will Take in the final round and so she will choose Take on the last-but-one round because, knowing that player 2 is rational, she would prefer to get 99 to 98 which she expects to happen if she passes. This reasoning is continued until player 1 concludes that the only way to get anything is to Take at the first node. While this logic is apparently sound, experimental results are consistently different.

At this point I apply the theory of reciprocity expounded above. Although the players do not play simultaneously, the same logic applies. If I believe the other player is like me with a sufficiently high probability I have to assume that if I can persuade myself

to pass on this turn she will be able to persuade herself to pass on the next turn. If the other player is like me I can expect them to come to the same conclusion I do. For Player 2, the decision to Pass on the final turn may seem illogical, but unless he can convince himself that he will do this, he must entertain the possibility that Player 1 will fail to convince herself that she should Pass on the previous round. It is illogical for player 2 to set out on the game with a strategy of Take on the final round for to do so would imply that player 1 should Take on round 1. By Passing on earlier rounds, each player is complicit in an implied contract to Pass all the way to the end. If player 2 should play opportunistically on the final round, that is his prerogative and Player 1 has already decided that she does not care. Even if there is zero chance that player 2 will play Pass on the final round, it is in player 1's best interests to assume there is some chance and therefore play Pass on the last but one round.

Various Game Theorists have tried to explain the discrepancy formally. Nagel and Tang (1998) suggest that altruism can explain it. If you have utility for the other player's wellbeing, the overall utility of Pass at every stage of the game is greater than the given payoffs. Rosenthal (1981) argues that if one has reason to believe that the other player is going to Pass on the next turn, then it is advantageous to Pass at Turn 1, but he does not give any reason why one should have this belief. Using my theory of reciprocity all of these elaborate theories become unnecessary.

The results of experiments in the centipede game shows that many players pass in the initial rounds but decide to take at or around \$66. The combination of my theory of reciprocity and ambiguity as to what it means to win, can explain this behavior. As the

game progresses the ratio of the potential gain, to the potential loss if the other player decides to take on the next round, decreases. The probability that I attach to my belief that the other players are like me has to increase to compensate. If I add in some utility for beating the other player it is possible to construct the utility function that peaks at any value I choose to name.

Vernon Smith (2008) has argued that there are different types of people. One of the types is a cooperator and the other is self-interested. The implication is that this type is a characteristic of the person alone. In the above example I in no way consider chess players as more self-interested than the general population. The behavior is simply the result of identifying the economics experiment with the game of chess. By making this identification they exclude cooperation as an alternative and focus on defeating their opponent. Some of the chess players may be the most generous and kindest people in the world in other contexts, but because they have identified this situation as equivalent to a game of chess, these aspects of their personalities do not come into play.

The model has now become much more complex. We cannot define people as cooperator or self-interested. We must categorize them in terms of their library of games and their preference for identifying a given situation with one of those games. Since each person's library of games is a product of his experience which is at least partly a product of his preferences, it would appear at first sight that there are as many types as there are people but it is clear that serious chess players can be recognized by the way they play the centipede game. One could conclude that there is a niche occupied by serious chess



players. It is beyond the scope of this paper to determine whether other such niches exist and how they can be identified.

### **What then can economic experiments tell us about economic behavior?**

So far I have made three arguments. The first is that participants in economics experiments do not exercise altruism in the context of the experiment even if they are the most generous people on the planet. The second is that strong reciprocity is the logical result of believing that other people are like you. The third is that people will adopt different strategies depending on what they perceive the objective of the game to be. Do these arguments imply that economic experiments cannot tell us anything about the real world? I do not think that is the case.

There is a lot of literature based on experimental games. Economists devise a game that represents, more or less realistically, a real life situation that is of economic interest and then invite a group of people to play the game. Results can be very illuminating and this approach has resulted in significant insight into among other things bubbles on financial markets.

Real life is not simple. Each person decides for themselves what constitutes success for them. Winning at some narrowly defined game is, for some, the apex of success. For others, it may be having a happy family life. There are potentially as many different definitions of success as there are people in the world. Aristotle defined the ultimate goal in life as happiness, and philosophers have had a hard time refuting that. As economists we believe that prosperity is a facilitator of happiness. Although most

would agree that money does not necessarily lead to happiness, most people would agree that it is easier to be happy when you have at least some money or prosperity. As Dickens (2011) had Mr Micawber say:

"Annual income twenty pounds, annual expenditure nineteen [pounds] nineteen [shillings] and six [pence], result happiness. Annual income twenty pounds, annual expenditure twenty pounds ought and six, result misery."

Suffice it to say, the definition of winning in life is not a simple matter.

Modern internet games are somewhere between traditional games and real life. Many encourage the player to adopt a role that is quite different from their real persona. Many of these games do not have a specific definition of what it means to win the game, but almost all have some scoring mechanism and there is an implication that having a higher score than another player is an indication that you are better at the game than they are.

What then can we say about economics experiments? Are they games, or are they really representative of life? If they are games, what can they tell us about real life? I believe that economics experiments can and do tell us a lot about how people behave in real life. In this section I will argue that real-life is made up of overlapping games and that people use their models of how other people behave to achieve their goals in a given situation.

The central illustration Wicksteed (Wicksteed, Philip Henry 1935) uses in his analysis of the market is a housewife. He talks about the housewife going to the market to buy supplies for the family, how she gathers information on prices at various vendors,

and then uses those prices to purchase the optimal basket of goods for the family based on the budget and her assessment of the needs of the family. Interwoven with this analysis is a clearly separate activity that she undertakes as the distributor of the goods to meet the various needs of members of the family. He points out that she performs these two roles according to very different rules. He also points out that when in the market place she displays no altruism. Wicksteed agrees with Adam Smith (1982a WN I.ii.2.26) in thinking, “It is not from the benevolence of the butcher, the brewer, or the baker, that we expect our dinner, but from their regard to their own interest. We address ourselves, not to their humanity but to their self-love, and never talk to them of our own necessities but of their advantages.” On the other hand, when it comes to running the household, the rules are very different. She gives out the contents of her pantry according to rules that only she understands and does this so as to optimize the happiness of the family as a whole. Does she keep the choicest pieces for herself? Does she give preferential treatment to her husband or a favorite child? Does she perhaps go without in hard times to ensure the children get enough nourishment to grow up healthy and strong? Does she perhaps fear the wrath of her husband if he perceives that she is not keeping the family satisfied? All of these are possible and any or all of them may constitute the basis for the rules of the game as she understands them. In this, she could be considered as behaving altruistically, but it could be that she is simply fulfilling her side of an implied contract with the members of her family. Either way, she adopts the role of the mother of the house and plays that role to the best of her ability.

In another place, Wicksteed points out that trustees are much more likely to drive a harder bargain when acting on behalf of the trust than when acting on their own behalf. This is a natural consequence of adopting a different role and playing according to the rules associated with that role in the game that goes with it. When acting as a trustee, the agent is duty bound to obtain the best possible outcome for the trust. There can be no consideration of other interests. He must not consider giving better terms on a deal in return for support in an application to membership of a country club or other personal favor. When acting on his own behalf though, the agent can and does allow for a much wider set of possible and intangible benefits to affect the deal. Again, altruism is not necessary to the explanation. All the differences can be accounted for in the rules of the different games and the roles being played.

In certain games a person may define winning as achieving approbation. In others winning may be defined in terms of the success of a group the person belongs to. Certain rich people clearly derive satisfaction from creating charitable trusts. It seems callous to reduce all generous acts to the utility gained by the actor, but logically it seems quite consistent to do so. Adam Smith reveals in the wealth of nations that his goal in life is to be beloved: “If the chief part of human happiness arises from the consciousness of being beloved, as I believe it does . . . “ (Smith 1982b TMS I.ii.v.2.41). This clearly affects how he defines winning in the various games that make up life.

When people play games, they create a mental world which may have no relationship to real life. For example, when playing chess, the rules are completely artificial. They may have some parallel with real life, but during the game, the players

take no thought for the fact that what they are indulging in is effectively a war or that the pieces that they are “capturing” represent killing someone on the battlefield.

When countries go to war, we redefine “acceptable behavior” or we define a new set of game rules. Most people are able to adopt the new rules and perform acts that would and do appall them when the war is over or they are not part of it. People participate in computer games that call for killing their opponents in virtual reality. Experiments have been done in which people are told to play the game of torture. They have been persuaded to give electric shocks to the other players that would seriously hurt them as part of a game situation. Humans have a capacity to divorce themselves from the real world and live “as if” some other social order or even physical laws applied.

When it comes to institutions, we see a similar effect. The “players” on Wall Street are expected to behave very much as if they are playing a game. Their job is to win at the game of financial markets. The rules are reasonably well defined and within those rules they play to win. They will happily make a substantial gain in the market from a friend and have dinner with them the next day. The “game” is in some sense not part of real life. In some sense institutions are the rules of games we construct to make some aspects of life more manageable. Good institutions are perceived to be fair and serve to channel self-interest into societal gain.

We have to be very careful that we do not replace social norms with institutional rules that allow our normal consideration for others to be overridden by our competitive instincts. Bowles (2004) gives an example that illustrates this point. A day care center was exasperated by parents coming late to pick up their children. They tried setting up a

system of fines for parents who were late. The result was that lateness increased. The rules of the game had changed. Instead of it being rude for parents to be late, it had become a fee. The day care center had put a price on the inconvenience and it was no longer an insult to impose on them; it was using a service with a price. The day care center then removed the fine system, but the behavior did not return to what it had been. The parents now knew how much they were costing the daycare center and were willing to impose on them for that much. The “game” had changed from one governed by the rules of polite social behavior to one governed by the rules of commerce. The parents were now faced with a pure choice: do I value my time at more than  $x$  dollars? If I do, I can guiltlessly trade being late for that amount. Before the fine, the rules were those of social interaction and the trade-off was: do I value my reputation as a civilized member of society more than the inconvenience of being on time?”

For Wicksteed, a person’s life is made up of a collection of overlapping games, each with its own set of rules and in each game they adopt a role that they play to the best of their ability. They expect the other players in the game to do the same. They create a model of the game space that represents this particular aspect of life, based on their understanding of the rules and objectives. They use this model to predict the other players’ actions and optimize their outcomes. In a very real way this is how we reduce the complexity of the problem of life to a manageable level. This is a very plausible model for how we behave. What then does this mean for experimental economics?

The games we use to model reality come in all shapes and sizes. Some have very precise rules embodied in institutions whose laws are dictated by other people. Some

have very vague rules made up privately by an individual to govern behavior in a social context. Most of these games we can choose to play or not play depending on our goals in life. Some we are obliged to play in order to exist in modern society. If we choose to play a particular game, that game may or may not have clearly defined criteria for winning.

It is relatively easy to create economics experiments for those parts of life where the rules are well defined. Experimental economics has achieved some significant successes in these areas. It is much more difficult to create economics experiments for those parts of life where the rules are ambiguous and the goals ill-defined. It is perhaps most difficult to design experiments for those parts of life where the payouts are uncertain. Many aspects of economic policymaking fall into this last category.

Unfortunately most of the economics experiments related to this area ignore this complexity and specify quite precisely what the payouts for the game are. For example to model the behavior of governments trying to achieve consensus on how to deal with global warming, it is not very informative to conduct a game with very precise payouts. What governs the behavior of governments in this scenario could be more driven by different interpretations of the possible repercussions of continued carbon dioxide increases, than by whether or not they are willing to cooperate in a public goods game.

I see no reason why experiments cannot be constructed to assist in our understanding of games in which the rules are less well-defined, or the outcomes not known, but the construction of such games must be done with an awareness that what we are modeling is an area of life with very complex and ill-defined rules.

## **Conclusions**

Different views of what it means to win lead to different strategies. The ambiguity in the goal of many economics experiments leads players to adopt different strategies based on the goal they think they are striving for.

For the real world I have argued that we treat different situations as if they were different games. Each situation has its own set of rules, some very precise and complete, others vague and open-ended. I suggest that economic games are very useful in modeling behavior in those situations in life where the rules are precise and complete but less useful in those situations where the rules are vague and open-ended. It is particularly challenging to devise games or experiments to measure altruism (by altruism I mean altruism in its purest form of having utility for the well-being of others).

We use games as an educational device that we teach our children to use from an early age. As the child grows older they become aware of the rules of life. Some of these are propounded by their parents or guardian. We learn to play games to enable us to live in harmony. To some extent it is our ability to project ourselves into roles defined by rules that makes civilization possible.



## CHAPTER THREE: MAXIMIZING GROWTH

### Abstract

I build on the literature on bet-hedging that invokes the Kelly criterion. Bet-hedging has been adopted by evolutionary biologists to show that evolutionary advantage belongs to those species or subspecies with the highest geometric mean number of offspring. I argue that humans see every decision in the context of long-term growth and that they instinctively apply the Kelly criterion to decide how much of their endowment to put at risk on any bet. Using this understanding I can explain why people in public goods games donate less than everything and more than nothing to the public good. I show mathematically a direct relationship between the amount contributed to the public good and the level of belief they have that other people are like them.

If you can make one heap of all your winnings  
And risk it on one turn of pitch-and-toss,  
And lose, and start again at your beginnings  
And never breathe a word about your loss;

Rudyard Kipling: "If" (Kipling 1932)

### **An inconvenient result**

In chapter 1, Rational Reciprocity, I concluded that if I believe sufficiently strongly that the other players are sufficiently like me to make the same decision as I do, I should donate everything to the public good. For simplicity, I constrained the players to donate everything or nothing to the public good. This constraint simplified the mathematics considerably and provided a useful result. Removing this constraint makes the mathematics more difficult. It not only increases the options available for the player making the choice, it also introduces the possibility that even though the other players are like me they may choose to donate an amount different from the one I choose.

I will show below that even when I remove these assumptions and introduce probability distributions for the other players' donations, the model continues to predict that I should donate everything or nothing to the public good. This model is incapable of predicting a partial donation to the public good. Aliveness may come in different degrees. To study this fully I need to hypothesize a probability distribution for the amount donated by the other players.

In chapter 1, I made the rather extreme assumption that in the case where the player under consideration donates nothing to the public good, those players who are not like this player donate everything to the public good. I also pointed out in chapter 1 that the probability, or equivalently my level of belief, that other people are like me for a particular decision depends on how obvious it is that that decision is right. The example I use is for a two player game with the multiplier of 2. In this situation it is very easy to believe that everyone else will come to the same conclusion you do. The Nash equilibrium coincides with the decision that maximizes expected reward and is also the

same as the decision that delivers the social optimum. In the case where the player under consideration chooses to donate nothing to the public good, I concluded in chapter 1 that he needs to consider the other people are like him with a very low probability. This could arise either because he considers himself so much more intelligent than other people that they would be unable to grasp the subtleties of his reasoning, or because he believes himself to be much more self-centered than people in general and therefore that he can expect people to contribute to the public good even when he cannot convince himself that he should.

Marwell and Ames (1981) in their paper “Economists free ride does anyone else?” conclude that trained economists constitute the vast majority of participants in public goods games who contribute nothing to the public good on the first round of a repeated game or in a one-shot game. Economists have been taught that the Nash equilibrium is the rational strategy and when faced with a public goods game will carefully analyze it and determined that the only rational play is donate nothing. This is not because they believe themselves to be different from the rest of the world but because they believe everyone will come to the same conclusion. Because they have been trained to think that the Nash equilibrium is the proper treatment for this game, they naturally conclude that the optimal play is the Nash equilibrium regardless of what the other players may choose to donate.

My conclusion is that the majority of those players who choose to donate nothing are doing so not primarily because they believe that everybody else is different from them, but because they believe that that choice is the only rational one and they probably

expect the other players to do the same as they do. Based on this logic I believe it is reasonable to modify equations (3) through (5) in chapter 1 to reflect that the expected reward for player donating nothing to the public good does not include anything donated by the other players. That is the expected reward is given by:

$$R_0 = 10$$

So my condition for deciding whether to play donate everything or donate nothing becomes:

$$R_{all} > R_0 \Leftrightarrow 0 + \frac{\beta}{n}(10 + (n-1)p10) > 10 \quad (8)$$

$$\beta(1 + (n-1)p) > n$$

$$\beta + \beta np - \beta p > n$$

$$\beta np - \beta p > n - \beta$$

$$p > \frac{n - \beta}{\beta(n - 1)} \quad (9)$$

For n=4 and  $\beta=2$ :

$$p > \frac{4 - 2}{2(3)} = \frac{1}{3} = 0.333$$

As I pointed out in chapter 1 the assumption used there is the worst-case scenario. Under that assumption I need to believe that other people are like me with a probability of greater than or equal to two thirds in order to conclude that I should donate everything to the public good. With the argument above I have reduced that need to one third. This is clearly the best case scenario, but the argument to justify it is plausible. These two extreme assumptions both predict that a substantial number of people should donate everything to the public good. Reality may be somewhere between the two but I would argue that there is good reason to believe this assumption is closer to reality than the one in chapter 1. Later in this chapter I explore a model that predicts how much of my endowment I should donate to the public good. I remove the constraint that I donate all or nothing. To keep that model tractable I will assume that when a player chooses to give nothing to the public good, he does so believing that nobody else will contribute anything to the public good either.

I will now relax the constraint that each player contributes all or nothing to the public good. Because the player under consideration has no way of distinguishing who the other players are, we can assume he has the same belief probability distribution for each of the other players and that the decision of each player is independent. In fact the amounts donated by the other players are IID. Since I believe they are like me with probability  $p$ , I can make the following conclusions:

The mean of the distribution must be  $f$  (the fraction of the endowment that I am going to contribute when I have completed my optimization calculations) and the sum of

all the probabilities of non-zero contributions must equal  $p$  (the probability that they are like me). For continuous distributions this can be written:

$$\int_0^1 \rho(s) ds = p$$

$$\int_0^1 s \rho(s) ds = f$$

Since the distributions are IID, the expected value of the sum of  $n$  such distributions is simply  $n$  times the expected value of one of them or in this case  $nf$ . I can now write an expression for the expected reward in the PGG if I donate a fraction  $f$  of my endowment to the public good. For simplicity, I am normalizing the size of the endowment to 1 and assuming infinitely divisible money. There is no loss of generality in these assumptions.

$$E(R_f) = (1 - f) + \frac{\beta}{n}(f + (n - 1)pf) \tag{10}$$

This is a linear function of  $f$  and I can show that for the conditions that give  $E(R_f) > E(R_0)$ , this is an increasing function of  $f$ . The conclusion of this exercise is that for any belief probability distribution the player may have about how alike the other players are to himself, if he maximizes his expected reward and the probability is high enough to direct him to donate anything, it will direct him to donate everything.

This leaves me with a major problem. In the literature reporting on public goods games, the majority of players contribute something less than \$10 when they do not contribute zero. There are indeed some of them who contribute everything, but many

more contribute something less. To try and resolve this dilemma, I turn to investment theory.

### **The Kelly criterion**

In 1956, Kelly (Kelly Jr 1956) working with Shannon and Thorpe at Bell Labs, derived a formula for the optimal size of data packets to transmit over a noisy data line. They were also working on betting strategies and investment strategies and they came to realize the same formula could be applied to all three situations. The core of the theory is the realization that for optimal growth in repeated bets, you need to adopt a strategy that maximizes the geometric mean of your outcomes. Edward Thorpe explains it very succinctly in his paper understanding the Kelly criterion (Thorp 2010):

The Kelly Criterion is simple: bet or invest so as to maximize (after each bet) the expected growth rate of capital, which is equivalent to maximizing the expected value of the logarithm of wealth.

The mathematical foundation of this concept is fairly simple. Consider an investment that returns  $X_i$  in period  $i$ . Over  $n$  periods the value of this investment is given by:

$$Y_n = Y_0 \prod_{i=1}^n (1 + X_i) \tag{11}$$

Taking logs we get:

(12)

$$\ln \frac{Y_n}{Y_0} = \sum_{i=1}^n \ln(1 + X_i)$$

Now compare this to the standard growth rate formula. For a continuously compounded investment with growth rate  $r$  and time  $n$ , we get the standard formula:

$$Y_n = Y_0 e^{rn}$$

Or

$$r = \frac{1}{n} \ln \frac{Y_n}{Y_0}$$

So the growth rate of the investment above is given by:

(13)

$$r = \frac{1}{n} \ln \frac{Y_n}{Y_0} = \frac{1}{n} \sum_{i=1}^n \ln(1 + X_i)$$

If we now consider the return on this investment  $X_i$  as a random variable, it appears that to maximize growth, we must maximize the expected value of  $\ln(1 + X_i)$ . This is equivalent to maximizing the geometric mean of the random variable  $X$ .



Kelly points out in his original paper that the fraction to bet is always zero if there is no “advantage” to the bet. That is, unless the expected outcome is positive, you should not bet on the event. In mathematical terms this means that unless the expected (arithmetic mean) outcome is greater than zero, the Kelly criterion always dictates to bet nothing. This provides a useful, easy to calculate, lower limit on when to bet, or in public goods games language when to contribute something more than zero to the public good. The difference between optimizing strategies using arithmetic mean and geometric mean is that optimizing arithmetic mean dictates to bet nothing when the expectation is less than zero and bet everything when the expectation is greater than zero (even when it is infinitesimally small), while optimizing geometric mean dictates bet nothing when the expectation is negative and bet the Kelly fraction when the expectation is positive. In fact a commonly used summary for the Kelly criterion is bet “edge over odds”.

This idea of optimizing the geometric mean has been enthusiastically adopted by evolutionary biologists. There is a large body of literature on biological evolution and bet hedging (Beaumont et al. 2009; Bergstrom 2014; Carja, Liberman, and Feldman 2013; Graham, Smith, and Simons 2014; Heininger 2015; Holman 2016; Schreiber 2015; Seco-Hidalgo, Osuna, and De Pablos 2015; Tan 2015; Zhang, Brennan, and Lo 2014a; Zhang, Brennan, and Lo 2014b, and many more). In summary the literature is based on the following premise. If a species has two variants one of which produces 10 offspring in good years and zero in bad years, while the other variant produces 7 offspring in good years and 2 in bad years, over a long enough time frame, the first variant will die out and the second will thrive. In general, variants which have the highest geometric mean

number of offspring weighted by frequency of conditions, will have the highest growth rate. This is somewhat counterintuitive. At first sight one would expect the variant with the highest arithmetic mean to be most successful, but it can be shown that growth is maximized for the variant with the highest geometric mean.

The Rotando and Thorp (Rotando and Thorp 1992) show how this concept can be applied to investing in the stock market. They start by showing that an investment that delivers a return of  $b$  with probability  $p$  and which loses its principle with probability  $q$  has a growth rate which is a function of  $f$ , the fraction of my capital that I invest in each period, given by:

$$G(f) = E \ln \frac{Y_n}{Y_0} = p \ln(1 + bf) + q \ln(1 - f) \quad (14)$$

Differentiating with respect to  $f$ , we get:

$$G'(f) = \frac{pb}{1 + bf} - \frac{q}{1 - f}$$

Setting this equal to zero gives the first-order condition

$$\frac{pb}{1 + bf} - \frac{q}{1 - f} = 0$$

Or

$$f^* = \frac{(pb - q)}{b} \quad (15)$$

This is just one example of the principle worked out for a somewhat artificial game. In the book *fortunes formula* (Poundstone 2007), Poundstone describes how several of the theorists who explored this mathematical formula went on to establish successful hedge funds. Later in this chapter I will develop the mathematics for applying this principle to predicting how much of my endowment I should donate to the public good. And will show that under some circumstances the model predicts that I should donate an intermediate amount. But first I must argue why a player in a one-off public goods game should use a strategy that maximizes growth over multiple periods and multiple games.

Many papers on public goods games and other economics experiments have questioned whether the results can be partially explained by the hypothesis that humans are unable to isolate an individual bet from the continuum of life. They argue that because people see this game as part of a series of games that go on for an indefinite period, the folk theorem can be invoked. Using the folk theorem it is possible to show that for an indefinitely repeated prisoner's dilemma game, any outcome is an equilibrium. Many economics experiments including public goods games are equivalent to multiplayer prisoner's dilemma games. By this argument it is possible to explain people's behavior in public goods games. The theory is unsatisfactory because it fails to predict. It simply tells us that any outcome is possible.

Pérez-Marco (Pérez-Marco 2014) and Baker and McHale (Baker and McHale 2013) have shown that for investments with varying returns and varying probabilities, the optimum fraction of your endowment to invest on any turn is something less than the

Kelly factor. Life consists of a series of events with varying returns and varying probabilities. We can conclude that for optimal growth over a lifetime, the best strategy is to bet something less than the Kelly fraction on each event. The financial literature frequently recommends such a strategy, one of the most popular being half-Kelly. In the financial literature, this is arrived at somewhat arbitrarily by attributing this to a risk aversion preference, the two papers quoted above, provide a mathematical explanation of how this reduced Kelly fraction could feed into the evolutionary process that contributed to the most common utility functions. Since none of the literature provides any indication of the precise size of the reduction in the Kelly fraction predicted, I will proceed to work as if the Kelly fraction is the optimum and modify my predictions at the end.

If we take the hypothesis that humans are unable to isolate an individual bet from the continuum of life and further hypothesize that humans behave in a manner that maximizes growth over an indefinite period of time, we can apply the Kelly criterion to predict the fraction of any endowment the player will commit in a public goods game. As we have seen evolutionary biologists have embraced the maximum growth hypothesis enthusiastically. If we can conclude that species that maximize growth by choosing strategies with the highest geometric mean are those that survive in evolutionary time, it is reasonable to conclude that mankind could have evolved preferences consistent with this strategy following the traditional gene-culture evolutionary model. While it is clearly unreasonable to argue that human beings consciously calculate optimal strategy to achieve maximum growth, it is quite plausible to infer that subconscious mechanisms

guide us to actions consistent with this growth model by a process of natural selection. Such a behavioral bias would ignore the fact that a situation was a one-off bet, but would tend to treat every situation as embedded in the continuum of life.

Economists have long recognized that utility for wealth is not linear. In the late 1960s supporters of the Kelly criterion conducted a heated argument with Paul Samuelson and his supporters about whether pursuing an investment strategy based on maximizing growth of capital was valid. Paul Samuelson (Samuelson 1971) argued that we should be maximizing utility rather than capital growth. He seems to have inferred that E.O. Thorpe's hypothesis was a result of using logarithmic utility. Logarithmic utility is a utility function commonly used to model risk aversion, but Paul Samuelson argued that utility of  $-1/x$  was a better model of observed utility behavior. Both of these utility functions have constant relative risk aversion (CRRA). Empirical utility measures have been studied extensively (Hyperbolic Absolute Risk Aversion 2015; Tan 2015; Zhang, Brennan, and Lo 2014b; Ziemba 2015; Isoelastic Utility 2016) and new and very sophisticated utility functions of been developed. Most of them are still pretty close to logarithmic utility. Interestingly almost all the deviations from the logarithmic function would lead to people betting a slightly smaller fraction the capital than the Kelly criterion predicts. This is consistent with observed behavior and also with the results of Pérez-Marco (Pérez-Marco 2014) and Baker and McHale (Baker and McHale 2013)'s study quoted above.

Considering the attractiveness of geometric mean optimization to the evolutionary biologists and the relationship between empirically observed utility functions and

geometric mean optimization, it is a distinct possibility that the natural superiority of geometric mean optimization for survival scenarios is a direct cause of logarithmic-like utility functions in humans. This is argued very effectively by Zhang, Brennan, and Lo (Zhang, Brennan, and Lo 2014b).

In chapter 2, I postulated that humans behave in any given situation in a manner drawn from a library of games. As a natural extension of this theory it is reasonable to hypothesize that they play each game so as to maximize growth over the long term. It is therefore reasonable to expect players in a public goods experiment who have a belief that others are like them with above the threshold probability for that game, to commit a fraction of their endowment corresponding to the Kelly criterion to the public good, even in a one-shot game.

In a public goods game in which a player has a belief that the other players are like him with probability  $p$  we can construct a growth function and apply the Kelly criterion. For a 4-player game, in its most general form, the one period growth function for an investment of one unit of currency is:

$$G(f) = E \ln \frac{Y_1}{1} = \iiint_0^1 \ln \left( 1 - f + \frac{\beta}{4}f + \frac{\beta}{4}s + \frac{\beta}{4}t + \frac{\beta}{4}u \right) \rho(s)ds\rho(t)dt\rho(u)du$$

Where  $\beta$  is the multiplier for donations to the public good,  $f$  is the fraction of my endowment I should bet on this game, and  $\rho$  is the probability distribution function for the amount donated by the other three players. The distribution function  $\rho$  is the same for each of the other players because it is the subjective belief of the player we are considering and he does not know anything about the other players. The formula is

further complicated by the fact that the mean of the distribution function has to be constrained to be equal to  $f$  and the total probability must add up to  $p$ . ( $q = 1 - p$  is the probability that the other player is not like me and therefore contributes nothing). That is:

$$\begin{aligned}\int_0^1 \rho(s) ds &= p \\ \int_0^1 s\rho(s) ds &= f\end{aligned}\tag{16}$$

In this case, I cannot use the trick used above for the sum of the expected values, because the expected value of the logarithm of the sum of the variables is not the same as the logarithm of the sum of the expected values. So for mathematical tractability I'm going to dispense with the generic probability distribution and work on the assumption that if I choose to donate fraction  $f$  of my endowment, the other players who choose to donate will also donate the same fraction  $f$ . I will also assume that those who are not like me donate nothing. With these assumptions I proceed to a mathematical model.

Let  $p$  be the probability that others are like me.

Let  $q$  be the probability that others are not like me  $= 1 - p$

In a four player Public Goods Game in which I donate fraction  $f$  of my endowment to the public good, there are 4 possible outcomes corresponding to games in

which no other players contribute, one other player contributes, two other players contribute, and three other players contribute:

Outcome 1 :  $R = 1 - f + \frac{\beta}{4}f$  occurs with probability  $q^3$

Outcome 2 :  $R = 1 - f + \frac{2\beta}{4}f$  occurs with probability  $3q^2p$

Outcome 3 :  $R = 1 - f + \frac{3\beta}{4}f$  occurs with probability  $3qp^2$

Outcome 4 :  $R = 1 - f + \frac{4\beta}{4}f$  occurs with probability  $p^3$

For maximum growth, I need to maximize

$$G(f) = E\left(\ln\left(\frac{Y_n}{Y_0}\right)\right) \quad (17)$$

$$\begin{aligned} &= q^3 \ln\left(1 - f + \frac{\beta}{4}f\right) + 3q^2p \ln\left(1 - f + \frac{2\beta}{4}f\right) \\ &\quad + 3qp^2 \ln\left(1 - f + \frac{3\beta}{4}f\right) + p^3 \ln\left(1 - f + \frac{4\beta}{4}f\right) \\ &= q^3 \ln\left(1 + f\left(\frac{\beta}{4} - 1\right)\right) + 3q^2p \ln\left(1 + f\left(\frac{2\beta}{4} - 1\right)\right) \\ &\quad + 3qp^2 \ln\left(1 + f\left(\frac{3\beta}{4} - 1\right)\right) + p^3 \ln\left(1 + f\left(\frac{4\beta}{4} - 1\right)\right) \end{aligned}$$

$$G'(f) = \frac{q^3\left(\frac{\beta}{4} - 1\right)}{1 + f\left(\frac{\beta}{4} - 1\right)} + \frac{3q^2p\left(\frac{2\beta}{4} - 1\right)}{1 + f\left(\frac{2\beta}{4} - 1\right)} + \frac{3qp^2\left(\frac{3\beta}{4} - 1\right)}{1 + f\left(\frac{3\beta}{4} - 1\right)} + \frac{p^3\left(\frac{4\beta}{4} - 1\right)}{1 + f\left(\frac{4\beta}{4} - 1\right)} \quad (18)$$

Setting  $\beta = 2$ , this simplifies to:



$$\begin{aligned} & \frac{-\frac{1}{2}q^3}{1-\frac{1}{2}f} + \frac{0}{1} + \frac{\frac{1}{2}3qp^2}{1+\frac{1}{2}f} + \frac{p^3}{1+f} \\ &= \frac{-\frac{1}{2}q^3\left(1+\frac{1}{2}f\right)(1+f) + \frac{1}{2}3qp^2\left(1-\frac{1}{2}f\right)(1+f) + p^3\left(1-\frac{1}{2}f\right)\left(1+\frac{1}{2}f\right)}{\left(1-\frac{1}{2}f\right)\left(1+\frac{1}{2}f\right)(1+f)} \end{aligned}$$

For maximum growth this must be zero. Or:

$$\begin{aligned} & -\frac{1}{2}q^3\left(1+\frac{1}{2}f\right)(1+f) + \frac{3}{2}qp^2\left(1-\frac{1}{2}f\right)(1+f) + p^3\left(1-\frac{1}{2}f\right)\left(1+\frac{1}{2}f\right) = 0 \\ &= -\frac{1}{2}q^3 - \frac{1}{2} \cdot \frac{3}{2}q^3f - \frac{1}{2}q^3 \cdot \frac{1}{2}f^2 + \frac{3}{2}qp^2 + \frac{3}{2}qp^2 + \frac{3}{2}qp^2 \cdot \frac{1}{2}f - \frac{3}{2}qp^2 \cdot \frac{1}{2}f^2 + p^3 \\ & \quad - p^3 \cdot \frac{1}{4}f^2 \\ &= f^2\left(-\frac{1}{4}q^3 - \frac{3}{4}qp^2 - \frac{1}{4}p^3\right) + f\left(-\frac{3}{4}q^3 + \frac{3}{4}qp^2\right) + \left(-\frac{1}{2}q^3 + \frac{3}{2}qp^2 + p^3\right) \end{aligned}$$

Setting  $q = 1 - p$

$$\begin{aligned} &= f^2\left(-\frac{1}{4}(1-3p+3p^2-p^3) - \frac{3}{4}(p^2-p^3) - \frac{1}{4}p^3\right) \\ & \quad + f\left(-\frac{3}{4}(1-3p+3p^2-p^3) + \frac{3}{4}p^2 - \frac{3}{4}p^3\right) \\ & \quad + \left(-\frac{1}{2}(1-3p+3p^2-p^3) + \frac{3}{2}p^2 - \frac{3}{2}p^3 + p^3\right) \\ &= f^2\left(-\frac{1}{4} + \frac{3}{4}p - \frac{3}{2}p^2 + \frac{3}{4}p^3\right) + f\left(-\frac{3}{4} + \frac{9}{4}p - \frac{3}{4}p^2\right) + \left(-\frac{1}{2} + \frac{3}{2}p\right) = 0 \end{aligned} \tag{19}$$

I used Excel with the quadratic formula to calculate  $f$  for a selection of values of  $p$ . The results are in the Table 3 below.

**Table 3** Kelly fraction  $f$  for levels of belief  $p$

$p$	$f$
0.05	-0.99955
0.1	-0.99538
0.15	-0.97752
0.2	-0.90909
0.25	-0.68695
0.3	-0.29666
0.35	0.149643
0.4	0.580122
0.45	0.957197
0.5	1.264911
0.55	1.502297
0.6	1.677047
0.65	1.800394
0.7	1.883863
0.75	1.937686
0.8	1.970271
0.85	1.988242
0.9	1.996711
0.95	1.999609
1	2

Those values of  $f$  in the table which are greater than 0 (we are not interested in negative values of  $f$ ), must be the maximum of  $G(f)$  for the given values of  $p$ , because when  $f = 0$ ,  $G(f) = 0$ , and as  $f$  approaches 2 from below,  $G(f)$  approaches minus infinity, and there is exactly 1 stationary point between. So when  $G'(f) = 0$ , with  $0 < f < 2$  it must be a maximum.

For values of  $p$  between approximately one third and approximately 0.46, the theory predicts players will donate greater than zero but less than their entire endowment to the public good. For values of  $p$  above approximately 0.46, it predicts that if players were allowed to borrow at 0% interest, so that they could donate more than their endowment to the public good, they would.

Now that I have worked the problem and found the Kelly factor, I can look at what happens when I apply the modified Kelly fraction as discussed earlier. The general conclusion of these ideas was that to maximize growth with variable probability and variable reward, one should modify downwards the fraction of your endowment to bet. Using the fraction popular with the financial community of one half Kelly, we get the values in Table 4:

**Table 4** Half-Kelly fraction for levels of belief  $p$

$p$	$f/2$
0.05	-0.49978

0.1	-0.49769
0.15	-0.48876
0.2	-0.45455
0.25	-0.34347
0.3	-0.14833
0.35	0.074821
0.4	0.290061
0.45	0.478599
0.5	0.632456
0.55	0.751148
0.6	0.838524
0.65	0.900197
0.7	0.941932
0.75	0.968843
0.8	0.985135
0.85	0.994121
0.9	0.998356
0.95	0.999804
1	1

The threshold is still one third, but the fraction to bet rises more slowly so that there is now no level of  $p$  that predicts that I would want to borrow to contribute more

even if I could. Considering that the average amount contributed to the public good in most one-shot experiments and the first round of repeated Public Goods Game experiments is observed to be between 6 and 8, we can conclude from this that people believe that other people are like them with a probability of between 0.5 and 0.6.

At the beginning of this chapter, I showed that by changing the assumption about what people who are not like me would contribute when I contribute nothing, the breakeven probability for belief that others are like me is reduced from  $2/3$  to  $1/3$ . The analysis using the Kelly criterion in this part of the paper, is equivalent to the assumption that people who are not like me will contribute nothing even when I contribute nothing. Hence the threshold at  $p$  equals one third, below which a player should contribute nothing. Changing this assumption could raise that threshold to as high as two thirds, corresponding to the initial analysis in chapter 1. I would estimate that similar analysis based on this assumption would provide a range of fractions starting at zero corresponding to  $p$  equals two thirds and rising to one when  $p = 1$ . A reasonable estimate of the level of belief required for the observed contribution levels under this assumption would be 0.7 to 0.8.

## **Conclusion**

In chapter 1, I created a model that predicted nonzero contributions to the public good based on perfectly rational logic. The model in chapter 1 was unable to explain why players would ever donate anything other than zero or everything to the public good. In

this chapter I have improved that model using the idea that humans optimize growth over an extended period of time. Using some fairly severe assumptions to simplify calculations, I have shown that it is possible to predict the fraction of the endowment the player would contribute from the probability with which he believes other people are like him. Similarly, it is possible to deduce from the fraction of their endowment that players contribute to the public good the degree to which they believe others to be like them.

## CHAPTER FOUR: CONCLUSIONS

The model derived in chapters 1 and 3 based respectively on (i) maximizing expected reward under the assumption that other players will be sufficiently like me to make the same decision as I do and (ii) that humans are conditioned to act so as to maximize long term growth, has considerable explanatory power to predict the results reported in numerous papers in the published literature. Superficially the model is inconsistent with the standard game theory Nash equilibrium and certainly predicts different outcomes. However, on careful examination, the theories do not conflict with each other, because the condition in the definition of the Nash equilibrium “If each player has chosen a strategy . . .” is not met. Also, game theory typically does not concern itself with growth over the long term.

The idea that participants in economics experiments interpret the situation as a game, leads to two very strong predictions. The first is that pure altruism should never be displayed in any experiment that the participants can identify as such and the second is that unless the experimenter specifies precisely what the objective of the game is, there will be potentially a wide variety of strategies employed that are the result of how the participants perceive the game is won. One particular trap that experimenters can fall into is to think they can model real world situations in which the payouts are controversial. I have not come across any experiment that successfully simulates a

situation where people have different beliefs about the merits or value of the outcomes. Similarly, it is difficult if not futile to try to measure altruism in an experimental situation. The standard “game” for measuring altruism is the ultimatum game. I contend that this is not measuring altruism at all. All the reported results can be explained by expectation maximizing. The offeror is simply calculating what she thinks is the minimum amount she can get away with offering i.e. how much does she need to offer for the responder’s greed to overcome his anger at not being offered more?

Extrapolating the idea that participants’ strategies vary depending on what they perceive to be the objective of the game to the real world, opens up possibilities for better understanding how people behave. If they do indeed act in various situations according to rules they have evolved by acting in similar situations in the past, we must reexamine our economic models to take this into account. In particular, we must no longer assume that agents consistently belong to a certain type. The type displayed in any situation depends not only on the character of the agent, but also on the role in the game that she perceives herself to be playing.

### **Other factors affecting game outcomes**

In no sense can the above theories be considered complete descriptions of how people behave in economics experiments. There are clearly other factors that must be worked into the model but which would make it intractable. Many of them have already been explored in the published literature. Some of these factors are: a sense of what is fair, revenge and anger, utility for approbation, and possibly more.



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