THE EMERGENCE OF SELF-GOVERNANCE INSTITUTIONS: AGENT-BASED SIMULATION OF GAME THEORETIC MODELS OF DEMOCRATIZATION

by

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A Dissertation
Submitted to the
Graduate Faculty
of
George Mason University
in Partial Fulfillment of
The Requirements for the Degree
of
Doctor of Philosophy
Computational Social Science

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Dedication

To Tim Reed, and my three wonderful children Maureen, Patrick and Peyton. You are loved.

Acknowledgements

This work would not have been possible without Dr. Robert Axtell's support, insight, focus and encouragement. I am also deeply grateful to the other members of my committee, Dr. Richard Wagner and Dr. Andrew Crooks. Thank you for your guidance and support. Karen Underwood in the Computational Social Science department was invaluable in keeping me calm and on track through the dissertation process.

Thanks to my husband Tim who was my rock and became chief griller, bottle washer, and take-out provider throughout this journey. Finally, I never would have been in the position to do this work were it not for the MITRE Corporation and their Educational Assistance program.

Table of Contents

List of Tables

List of Figures

Abstract

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Institutions are created by people interacting in complex ways with others in their socio-economic environment. A study of institutions should therefore study the people and interactions that create them. Acemoglu and Robinson (hereafter A&R) developed a game-theoretic framework for analyzing how economic incentives influence the way social groups shape institutions to allocate political and economic power. The A&R models assume groups or classes of people are completely rational and identical intra-group. Analytical difficulties impede A&R from exploring more realistic interactions. This dissertation utilizes an agent-based computational methodology to represent a subset of the A&R models. The computational model permits agents to be heterogeneous, which can affect outcomes at the group and aggregate levels. Specifically, with intra-group homogeneity the agent-based model reproduces the group-level threshold conditions affecting institutional choices found by A&R. I show that these results are robust to

parameter changes within the ranges defined by A&R. However, I then relax the intra-group homogeneity assumption, allowing exploration of the roles of income distributions and bounded intelligence on the larger outcomes for all groups. The population structure with heterogeneity can include a more realistic middle class. Modeling a middle class by using agent-based models with heterogeneous agents finds that the effect of a middle class is non-linear. This dissertation demonstrates the usefulness of agent-based modeling as a viable alternative quantitative methodology for studying complex institutions.

Chapter 1. Introduction

1.1 Motivation

The goal of this dissertation is to explain how socio-economic institutional decisions are chosen. There is a growing awareness of humankind's role as "stewards" of the planet. This interest in "sustainability" extends beyond the management of natural resources. We also study the sustainability of our socio-economic institutions of organization. The socio-economic system is considered sustainable if the institutions are renewable and stable over time in a way that allows humans to live lives they deem meaningful (Dumas & Urpelainen, 2014). Institutions tend to be more stable in nations with higher economic output than in poor nations. The median number of years before violent regime change in poor countries is seven years compared with 60 years for rich countries (Durant & Weintraub, 2014). There is considerable interest in identifying the causal factors of both stable governments and economic growth to better understand their interrelationship. It is argued that much of the economic growth in the last 200 years is due to institutional reforms introduced in Europe and North America between 1770 and 1830 (Acemoglu & Robinson, 2015; Fors & Olsson, 2007). As stewards, we want to understand the mechanisms and tipping points of environmental factors and human incentives that impact the economic growth and lead to stable, sustainable governance.

The large body of research on economic growth of countries has been focused on the macro level. The studies attempt to define relationships between macro-level inputs leading to economic growth. Many study the effects of education (e.g., Glaeser, 2004; Lipset, 1959), corruption (e.g., Mauro, 1995), natural resources (e.g., Sachs, 2005), geography (e.g., Krugman, 1991), fiscal policy (e.g., Krugman & Layard, 2012), and technological progress (e.g., Solow, 1956) on economic growth without exploring the human mechanisms by which the factors are assumed to cause growth. Institutions arise through interactions at multiple scales. At the micro-scale, individuals interacting in society make decisions based on what is likely to provide them more value. Governance institutions emerge from the individual decisions of citizens amidst the socio-economic factors influence those individual decisions. This dissertation explores the interactions between individuals at the micro-level that result in the macro-level emergence of institutional regimes, in order to better understand the conditions that will lead towards more longer lasting and economically healthy institutions.

A polity is defined as an organized society such as a nation, state, or city. The government is a complex system of people, laws, and officials that define and control the rules, regulations, and interactions of the people who live in a polity. North (1991) defines institutions as "the humanly devised constraints that structure political, economic and social interaction". North (1991) includes informal cultural norms and formal laws as institutions, but the key point is that institutions are created by people. A study of institutions should therefore study the people and interactions that create them.

Acemoglu, Johnson and Robinson (2005), find that democratic political institutions perform better than non-democratic institution in shaping the economic institutions of a country in ways that result in sustained economic growth. Acemoglu and Robinson (hereafter A&R), developed a ground-breaking game-theoretic framework (2006) for analyzing how economic incentives influence the way social groups shape institutions to allocate political and economic power. In contrast to the macro-level studies, their work describes micro level decisions and stresses that individual economic incentives determine political attitudes (2006, p. xii). Their work focuses on the conflict between the rich and poor over distribution policies of the political and economic institutions. A&R analysis of their game-theoretic models allowed them to construct a theory on the creation and consolidation of democracy (Acemoglu & Robinson, 2006).

An agent-based model (ABM) is a good methodology to gain insight into the incentives that influence the way social groups shape institutions. An ABM enables the researcher to model the individual decisions of people living within a society without the artificial a separation between society and government. Assuming a "representative agent" buries the interactions between parts of the economy and society by focusing on aggregates (Wagner, 2014b). Thus, a systems approach that exposes the interactions, rather than an equilibrium approach is needed to fully understand the economic analysis.

An ABM is more suitable than mathematical modeling because the mathematics needed to investigate the recursive nature of endogenous activity with heterogeneous players gets too difficult (Smaldino & Lubell, 2014). An ABM can more realistically capture the recursive dynamics and emerging macro-behavior between the variables attributed to the heterogeneous actors in a way that cannot be solved with strictly mathematical computation (Axtell, 2000). Although agent-based models are still rare compared to game-theoretic models, there is growing acceptance of agent-based models as a way of exploring human-environment interactions (Crooks, Castle, & Batty, 2008).

1.1.1 Research Question

This dissertation develops an agent-based model to explore Acemoglu & Robinson's (A&R's) theory concerning the Creation and Consolidation of Democracy. The dissertation attempts to answer the following research questions:

i) Can an agent-based model be developed that reproduces the results of Acemoglu & Robinson's game-theoretic models on the creation and consolidation of democracy?

ii) How do the more realistic assumptions of intra-group heterogeneity and limited rationality bring additional insights and understanding on the factors that affect the creation and consolidation of democracy?

iii) How does the presence of a middle class effect the economic determinants of the transition from authoritarian to democratic rule?

1.1.2 Research Contribution

The principle contribution of this work is to expand the understanding of the creation and consolidation of democracy through an agent-based model that makes explicit the interaction between the decisions of individuals and the emerging group level regime choice. Traditional research in economic growth of countries have focused on aggregate level indicators. In contrast, this dissertation builds upon the game-theoretic approach of individual incentives that Acemoglu and Robinson originated. It is the first agent-based model of Acemoglu & Robinson's theory of the creation and consolidation of democracy. Lastly, it offers additional insight beyond that achieved with the A&R game-theoretic models by addressing the limitations of that modeling technique.

This dissertation will demonstrate the usefulness of agent-based modeling as a viable alternative quantitative methodology for studying complex institutions. Using an agent-based model, I have modeled the more realistic assumptions of intra-group heterogeneity, and bounded rationality in order explore the roles of middle class income distributions and leadership on the larger outcomes for all groups.

I believe this work will advance the study of the emergence of political institutions by providing a complement to formal theory. Institutions are created by humans and, like physical infrastructure, must be maintained (Ostrom, 1990, p207-216). This effort may provide insight into maintenance priorities needed to continue sustainable and resilient governments. The use of agent-based modeling as a methodology enables us to build future institutions that are robust to variety of environmental changes (Jones-Rooy & Page, 2012).

1.2 Agentization as a Methodological Tool

1.2.1 Agent-Based Modeling as a Tool to Study Emergent Phenomenon

In this dissertation, I present several related agent-based models as a method of analyzing the relationships between an environment characterized by social and economic phenomena and the self-governing institution chosen by individuals who live in that environment.

An agent-based model is a computer simulation that models the responses of humans to a variety of experiences within their social and economic environment (Epstein & Axtell, 1996; Crooks & Heppenstall, 2012). To get a better handle on what a simulation is, consider video role playing games or, as espoused by Hanson (2001), the movies The Matrix, and the Truman Show. Computers execute the instructions that create the environments, bad guys, and good guys, and assign you attributes such as wealth, weapons, fighting skills, or dance moves that you use to get where or what you want. Imagine if you created the video game, how would you design it? What instructions would you have to include? You would probably have to figure out how many people would be in the game, what each of the people are trying to do, what type of behavior they are allowed to do to get to their objective, how much do they know about other people, and obstacles, how the game measures whether or not an agent, or group of agents, succeeded, and what happens when people meet other people or become aware of their environments. This is the same way you would specify an agent-based model. An agent-based model is a program that specifies the goal-oriented agents and their individual characteristics, the environments, and rules (e.g., behavior when interacting with other agents or the environment). Running such a model is as easy as pressing two buttons. First, pressing setup button which creates the virtual agents, like characters in a video game. Second, pressing a go button which causes the agents to interact, update their environment and influence other agents. The researcher then observes and monitors what happens (Axtell, 2000).

Agent-based modeling is an alternative to the more widely used game theory modeling. Game theory studies interactions between two or more participants where they choose their next action in order to maximize their payout or utility. Player decisions depend on the decisions and conditions of the other players in addition to their own. Interaction in game theory is generally indirect: the interactions occur through a representative player or signaling economic variables rather than a direct meeting of individuals. In political science, game-theoretic models are used to study conflict (e.g. Powell, 2006), negotiation (e.g. Brams, 1990), elections (e.g. Coughlin & Nitzan, 1981). In economics, game-theoretic models are used to investigate monetary policy (Barro & Gordon, 1983), corruption and lack of growth (Macrae, 1982), and institutions and growth (Matthews, 1986; Acemoglu et al., 2014).

Game-theoretic models are predicated on the assumption that players choose their next action by calculating: i) how different incentives will influence the behavior of other agents, and ii) the optimal strategy to reach a goal given the expected changes to the environment and other players. This is called "rationality". Rationality is critical because it leads to equilibria: a condition when all players play their best strategy assuming all other players are playing their best strategy with the same knowledge. By definition, in the state of equilibrium, no player can benefit from changing strategies because such a change would result in outcomes that are not optimal. This set of strategies and corresponding payoffs are called a Nash Equilibrium. An assumption of rationality allows the game theory modeler to solve the game by determining what decisions will offer the best payoffs. A rationality assumption implies that the players are conducting the same complex mathematical analysis themselves and taking actions to implement that strategy.

Critics of game theory focus their attention on the set of assumptions inherent to game theory: rationality, homogeneity, a lack of realistic interaction, and equilibrium. These game theory assumptions constitute the neoclassical "sweet spot" (Axtell, 2007). When the game theory assumptions are held together, game theory can be applied, but once one of these postulations are removed, the game theory model becomes an ineffective means of explaining behavior. The assumption that garners the most heated criticism stems from the broadly held recognition that humans don't have the capacity to figure out the best response for themselves much less for all other humans affecting their environment (Simon, 1978). "Bounded rationality" is used to describe the more realistic assumption that both the knowledge and computational power of the decision maker are severely limited (Simon, 1986, pp. 210-211). Homogeneity is an assumption in most economic models even while most economic modelers understand that a lack of that very same homogeneity in people's preferences is what gives rise to interactions of exchange in the first place. Which leads us to the third assumption: non-interaction. In game theory, the agents make decisions while considering: i) what the other agents will do, and ii) the change in aggregate metrics and the implication on their welfare. But there is no mechanism for "interacting" with other agents. Realistic human behavior is influenced by others through their social networks which can introduce positive and negative feedbacks into the system. Lastly, game theory models assume the output to be analyzed is an equilibrium decision. If one were to consider boundedly rational agents, then some of these heterogeneous (by necessity) agents may be making decisions that are out of alignment with the equilibrium decision. The interplay and interaction of these more varied responses is not accounted for in game theory.

These four assumptions are inherent in game theory because they are needed to a) make the math doable (Axtell, 2000) and b) allow deduction to synthesize results into a theory (Axelrod, 1997). Choosing the best game strategy often involves solving complex mathematical equations. And exploring these decisions and outcomes while relaxing the four assumptions of the previous paragraphs make for really complex relationships that may be resistant to direct mathematical analysis. Therefore, a key reason for choosing an agent-based model is that the emerging macro-behavior is not possible to solve with more traditional mathematical modeling means (Axtell, 2000). The interaction of agents with their friends or associates has impact upon the decisions that the agents make. This feedback is complex, non-linear and recursive, making the macro level consequences of these adaptive agents hard to deduce (Axelrod, 1997). These gravitational qualities point to computational methods like agent-based modeling in order to explain the net outcome.

While complex systems such as institutions and society itself are difficult to predict, they are not random. Traditional econometric models that assume a "representative agent" bury the human interactions between parts of the economy and society (Wagner, 2014b). In contrast, agent-based modeling is a simulation methodology that explores systems of diverse, adaptive, and boundedly rational agents by exposing their interactions (Axtell & Epstein, 1996). Agent-based models build up the complex system by modeling the many individual decisions of the agents. These decisions affect other agents and the world around them and provide feedback. The cumulative interactions of humans in their environment lead to patterns at the aggregate level that are not apparent in the individual decisions (Epstein & Axtell, 1996, p. 35). By programming the agents with valid micro-behaviors, the agent-based model is able to explain the emergence of collective behavior based on the individual behaviors of the interactive agents (Cioffi-Revilla, 2014, p. 212).

Agent-based models (ABM)s bring a set of unique capabilities not found in more established modeling methods. As discussed above, ABMs can help us offset some of the weaknesses inherent in other modeling methods and allows the modeler to explore the interrelationships between the agent behavior and the environment without making heroic assumptions (Chang & Harrington, 2005). In addition, they are well suited to studying complex adaptive systems and emergence. The formal notion of emergence means that the system exhibits characteristics and processes that are not present at the underlying component level (Gilbert & Troitzsch, 2005, p. 11). In this dissertation, I will be exploring the interplay of individual behaviors and choices resulting in the emergence of political equilibrium.

Classic examples of ABMs include Schelling's (1971) model that demonstrates how agents with mild preferences for their neighbors can lead to the emergence of segregated societies. Epstein and Axtell (1996), in one of the best examples of agent-based modeling, demonstrated that it is possible to build complex artificial societies complete with the transmission of culture, the rise of conflicts, the spread of a disease, the diffusion of price information, and migration - all based on simple participating agents.

While agent-based models have been shown to provide added value, it is still up to the researcher to determine what added value their specific agent-based model is providing over established methods. This dissertation will use the concept of “stylized facts” (Boland, 1987/1994) to compare the explanatory capability of my agent-based model with the more established A&R game-theoretic model. I am taking an “agentization” approach and replicating the stylized facts of A&R as well as providing some additional insights. Using agent-based models has allowed me to construct an artificial world and empower the agents living within. It enabled me to explore with more imagination the world that I am analyzing and allowed me to arrive at insights inscrutable to mathematical methods.

1.2.2 Agentization

Acemoglu & Robinson (2006) created a framework of mathematical game-theoretic models to model the effects of inequality between citizens and elite on a society’s selection of political organization. A&R identified and parameterized the equilibrium actions taken by two classes of people: rich and poor. This analysis led to their development of a theory concerning the consolidation and creation of democracy. In this dissertation, I explore A&R’s theory concerning the emergence of institutions by “agentizing” the A&R model of the Creation and Consolidation of Democracy as published in Economic Origins of Dictatorship and Democracy (2006, ch7 sec 4), in order to better understand the interplay of the individual behaviors and resulting regime choices. Specifically, I examine how the decision-making of citizens to choose revolution or status quo and the resulting equilibrium of institutional decisions.

Agentization is the process of creating an agent-based model to reproduce the results of a neoclassical model (Guerrero & Axtell 2011). This usage of an agent-based model is as a complement to formal theory (Axtell, 2000). The process of agentization is described by Guerrero & Axtell (2011) and consists of the following steps.

First, a computational model is built using restrictive assumptions of the traditional model. These assumptions include (1) rationality, that is, omniscient, fully rational agents with unlimited capacity to obtain information and compute relative benefits based on complex relationships. (2) Intra-group homogeneity. Although an integral aspect of A&R's model in inequality of the inter-group income between rich and poor, all agents of the same group or class have the same income. (3) Non-interactiveness. The agents in the A&M model interact with each other according to the Median Voter Theorem (MVT). The Medium Voter Theorem states that conflicting preferences among a diverse population is aggregated by the selected preferences of the median voter. It assumes that democratic decision processes lead to majorities in favor of a policy or decision (Acemoglu & Robinson, 2006, p. 93). In addition, it is recognized that even leaders of non-democratic regimes need to maintain the support of a group of people (elites) and their majority decisions may be represented by the median voter of their class (Acemoglu & Robinson, 2006, p. 94). This is not really an interaction since, in the model, all poor people have the same income, and there are more poor people than rich, the median voter will always be a poor person. Therefore, in the game-theoretic model, the decisions are determined by a poor person without regard to an "interaction".

Second, the agent-based model is implemented and shown to reproduce the results at various parameter settings. This is called "docking" (Axtell, 2000). Chapter 3 of this dissertation describes the initial agent-based model that includes the restrictive assumptions of the A&R models.

Third and last, once shown to reproduce the analytical results, the agent-based model is extended to include more realistic assumptions. I can now relax assumptions and see under what conditions the mathematical results still hold. The assumption of heterogeneity of income is relaxed first in section 4.1. Rather than all poor agents having income (y^p) and all rich agents having income (y^r), a more realistic heterogeneous income distribution is assigned to the agents by the model. However, the interaction is still MVT, and the agents are still fully rational. Section 4.2 describes the stylized facts of this model and section 4.3 describes verification, validation and results. The assumption of rationality is relaxed next in section 4.4. In this model, only some agents (leaders) have the capacity to calculate a best response. This is similar to the way retirement decisions have been modeled (Axtell, 2006). In Axtell's study of the timing of retirement, a small number of agents are rational and a majority are imitators. In this model, the majority of agents will follow one of the leaders. Leaders solve the collective action problem by helping their followers believe that it is in each person's own best interest to adopt the preference or position of the leader. Relaxing the assumption of rationality also changes the non-interaction assumption. It is no longer MVT. Some agents who are following a leader with different income level than their own will in essence be “adopting” his regime preferences. So, an agent’s preference is no longer determined by their income, but also by the influence of the representative income (ideology) of the leader. A&R emphasize the importance of ideology in solving the collective action problem (Acemoglu & Robinson, 2006, p. 126). The model with heterogeneous agents, can now be used to extend the boundaries of the neo-classical model and test the model in situations free from the limitations of mathematical tractability (Axtell, 2000).

1.3 Organization of the Dissertation

This dissertation is organized as follows. Chapter 2 provides background materials on institutions and research on the relationships between institutions and economic growth. Acemoglu & Robinson’s theory of the creation and consolidation of democracy is specifically examined.

Chapter 3 presents two ABM models using the restrictive game-theoretic assumptions. The first model reproduces the results of a subset of A&R’s game-theoretic models found in their book The Economic Origins of Dictatorship and Democracy (2006). The second, is a derivative of the first, and it reproduces the “Democratization in a Picture” diagrams that visually depict the decision space for elites based on inequality and costs of repression.

Next, I present in Chapter 4, two additional agent-based models which relax the restrictive assumptions of the game-theoretic models. In the first, the agents are endowed with intra-group heterogeneous incomes which simulate the impact of a middle class. In the second, the agents are endowed with limited rationality and follow the preferences of the few leaders who are fully rational.

Finally, Chapter 5 concludes the dissertation. I summarize the results of the previous chapters, and discuss their broader implications. With these findings in hand, I can more concretely describe how this work forwards the broader comparative analysis and agent-based modeling research areas, and propose extensions and experiments which can further build on the work presented here.

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2 Economic Origins of Political Institutions

2.1 Self-organizing Institutions

Societies organize in order to solve various problems that are solved more efficiently through collective action: e.g., irrigation, clean water, or the provision of defense. Some ways of organizing societies incent people to take risks, innovate and make incremental improvements to current processes. Many of these organization processes emerge through spontaneous "self-organization" rather than through formalized planning. Self-organized activities are those where each individual finds it in their own best interest to follow a certain "norm" (Ostrom, 2014).

The emergence of self-organization by individuals in a community is called "spontaneous order" by Hayek (1976, p. 33). One example is a practice of collecting driftwood in a fishing village on the Yorkshire coast. The first to that section of the beach could collect wood without interference and reserve it in a pile with a rock on top. If it wasn't collected within 2 high tides, then it was open for others (Walmsley, 1932 as described in Sugden, 1989). Other forms of resource allocation, such as assigning certain days or certain stretches of beaches, would require some central control. The fishing community found a way to self-coordinate the gathering of a collective resource.

Self-organization includes finding efficient ways to provide goods that benefit all members while ensuring all are contributing (Acemoglu et al., 2005, Ostrom, 1990, p. 26). As these norms become widely accepted, they become part of the more formally organized societal planning processes. The evolution of organizational rules is driven by individuals' experimentation to find solutions for social problems and turn conflict into peaceful and productive cooperation (Boettke & Candela, 2014). Societies ranging from informal associations to the highly structured governments of empires construct rules for acceptable behavior. Self-organized communities develop ways to monitor and enforce their rules (McGinnis & Walker, 2010). Through self-governance they craft institutional arrangements that enable them to effectively cope with economic and social incentives that would otherwise reduce the benefits to the community (McGinnis & Walker, 2010). Government rules emerge through designed changes to laws and regulations as well as through social norms of human action (Wagner & Runst, 2011).

A growing body of research is finding that the way a society organizes itself impacts its economic performance. The research on how government institutions affect growth extends an economics perspective on the study of social and legal norms and rules. This inquiry into the impact of institutions on economic growth began with Coase (1960) and is coined "New Institutional Economics". The Coase Theorem in "The Problem of Social Cost" (1960) postulates that there are always transaction costs and a goal of efficient governments is to compare alternative institutional arrangements in order to reduce transaction costs.

North (1994) has called institutions the “rules of the game,” referring to both the formal rules and norms for acceptable social behavior. North advocated the use of game theory to describe how institutional conditions evolve to reduce transaction costs. North concludes that individuals find conditions conducive to cooperation when they anticipate having to deal with the same people in the future and when they have complete information about other people in the transaction. Governments, on the other hand, tend to be made up of impersonal exchanges. North recognized that creating institutions that alter the benefits of these impersonal exchanges towards cooperation and mutually beneficial outcomes is a very complex process. He states that the work to create these economic institutions are successful only if the economic institutions are supported by appropriate political institutions (North, 1994).

When examining institutional change from an economic perspective, support for or opposition to democracy is based on the underlying economic factors rather than ideological preferences for democratic versus autocratic government. The field of Public Choice has been described as “the application of economics to political science” (Mueller, 2003, p. 1). All policy choices, whether democratic or non-democratic, have distributional implications. Citizens have preferences over distributional outcomes and evaluate different options according to their assessments of economic and social consequences. Individuals have preferences regarding economic institutions because of the allocation of resources that these institutions induce. Resources don’t allocate themselves. People allocate resources as a result of their interactions with other people and the governing intuitional frameworks (Wagner, 2014a). Individuals pay collectively for their institutions. If the institutions fail to supply adequate personal, social and property protection, individuals may have to provide it themselves at a cost or consider organizing collective action to change the institution. Different economic institutions will benefit different groups. Therefore, the groups of individuals with more political power will be able to shape economic institutions in their favor. People, including benevolent bureaucrats, act in their own self-interest (Tullock, 1976). Therefore, for governing institutions to be robust and sustainable, they need to be effective when people are at their worse (Boettke & Leeson, 2004, Leeson, 2007, Ostrom, 1999).

2.2 Institutions Cause Growth

While some research is focused on how individuals make institutional choices based on their individual gain, other research is focused on what effect the institutional choice has on economic growth of the society as a whole e.g., Alesina, 1994; Acemoglu & Johnson, 2003; Acemoglu, et al., 2005; Barro, 1991, 1996; Mauro, 1995; Easterly, 2001. This is of interest because countries with higher economic output tend to have longer lasting institutions and longer periods of peace (Durant & Weintraub, 2014).

There have been many studies examining the effects of violence and social unrest on economic growth. Most of the empirical literature have used cross sections of country level data. Barro (1991, 1996), Mauro (1995), and Alesina et al. (1996), have found that political instability negatively effects economic growth. Political instability is found to affect growth because such instability negatively affects investment and savings (Venieris & Gupta, 1986; Alesina & Perotti, 1996).

Haggard & Tiede (2011) explore how institutions that comprise "Rule of Law" impact economic growth. They investigated relationships between components of "Rule of Law" in a sample of 74 developing and transition economies in the timeframe 1985-2004. They find that the measures for a) restraint on government, b) property rights and c) corruption are correlated differently for developed countries than for developing countries. Haggard & Tiede find that level of violence was one of the most important characteristics explaining dissimilar economic growth among groups of otherwise similar countries. In addition, they find that "private capture" is as damaging to economic growth as would be unchecked predatory government (Haggard & Tiede, 2011). Their study finds that civil war leaves country 15% poorer than it would have been. In addition, decreasing homicide rate by 10% increases per capita GDP by .7 - 2.9% over the subsequent 5 years even when controlling for a variety of other determinants.

Giuliano, Mishra, & Spilimbergo (2012) use panel and factor analysis to study the effect of democracy on the adoption of economic reforms. They conclude that democratization has a positive effect on all economic reforms with significant effect on finance, agriculture, and trade.

Regressions on the measures show that long-run economic growth, investment rates and financial development are correlated with both contracting institutions and property rights institutions (Acemoglu & Johnson, 2003). Acemoglu and Robinson assert that the growth generated by extractive institutions is not as sustainable as growth generated by inclusive institutions. Extractive institutions tend to run out of steam because the individual incentives needed to continue the rate of growth lose strength over time (Acemoglu & Robinson, 2012, p. 150).

Throughout their extensive research, Acemoglu, Johnson and Robinson (2005) have found that sustained economic growth emerges when we observe the following:

1. Political institutions are used to increase and enforce personal and property rights for the broader population.
2. When political institutions effectively constrain predation, corruption and rent seeking.
3. When there are relatively few rents to be captured through government aegis.

Institutions that enforce personal and property rights for a broader population are found associated with democratic institutions rather than non-democratic. Acemoglu, Naidu, Restrepo and Robinson (2014) provide evidence that democratic institutions cause growth. They use regression to determine that transitions to democracy raise the GDP by 15% at 25 to 30 years after democratization. Their results suggest that democracy increases future GDP by encouraging investment, increasing schooling, inducing economic reforms, improving public good provision and reducing social unrest (Acemoglu et al., 2014). One concern is that the average duration of a regime is only seven years for poor countries (Durant & Weintraub, 2014), so the stability of such developing nations does not seem long enough to realize these benefits.

Democracy has been found to encourage growth through several mechanisms because democracy broadens access to political power (Acemoglu et al., 2014). First, democracy encourages economic reforms that shift the economic institutions to favor broad groups of people rather than the elite. Second, democracy is correlated with increased taxation and other government fees used to efficiently provide defense, keep the peace, and provide public services. Third, democracy increases investment through establishing an environment of credible commitments (Acemoglu et al., 2014).

Analysis has shown that while democracy may lead to economic growth, economic growth doesn't necessarily lead to democracy (Acemoglu & Robinson, 2012). And, while liberalization of economic institutions leads to economic growth, economic reforms do not inevitably lead to democracy (Giuliano et al., 2012). It may be possible in the short run for economic growth to be achieved through liberalization of the economic institutions without liberalization of the political institutions (Giuliano et al., 2012; Acemoglu & Robinson, 2012).

Acemoglu and Robinson have developed a framework to describe the creation and consolidation of democracy. The framework starts with the understanding of two foundations of political power, de jure, and de facto. De jure power is power formally granted by the governing institutions. De facto power is power through the ability to mobilize, motivate and influence change either through threat of violence or other collective action. Those with du jure power can use that power to change economic institutions and redistribute wealth. In order to avoid being overthrown, the ruling elite has to supply the minimum amount of public goods (defense, police, infrastructure) to keep populace satisfied. Change in political institutions alters de jure power. Political institutions can reform property rights institutions and reform financial institutions. Both reforms involve elites making a trade-off. The trade-off is between lowering rents from selling privileges and the increased equity and growth that results from increased productivity in the overall economy. Provided the du jure reforms do not radically alter the political structure of society, the identity of the elites, or the source of economic rents for elites, du jure reforms may change political power, but the loss will be offset by changes in de facto power (through bribery, lobbying, capture of political parties) (Acemoglu et al., 2005). A democracy grants a large amount of du jure power to citizens (versus elites), therefore, the elites will invest more in their de facto political power in a democracy than in a non-democracy. Elites invest to prevent future costs of wealth redistribution and loss of rents. The goal of this investment leads to a "captured democracy" where a democratic regime chooses economic institutions that favor the elite (Acemoglu et al., 2005).

2.3 Micro-Foundations of Acemoglu & Robinson Models

In their book, Economic Origins of Dictatorship and Democracy, A&R present a series of game-theoretic models to illustrate the dynamics of the creation and consolidation of democracy. They focus on the conflict between the rich and poor over the distributional policies of the political and economic institutions. They model the alternative tax and redistribution policies associated with democratic and non-democratic regimes. It is the distribution of power in the society that determines which economic policy gets enacted. In their models, elites are in power in non-democratic regimes. In democratic regimes, the citizens (poor and rich) have an equal vote, but because there are many more poor than rich, the median voter will be poor for this reason, the poor in democratic regimes are able to shape institutions in a way favorable to them. The elites can change a democratic regime to a non-democratic regime by mounting a coup. The elites can change a non-democratic regime to a democratic regime by democratization. The poor can influence the elites to democratize by threatening to have a revolution. And if a democracy is not redistributive enough, the poor can have a revolution.

These basic developmental tendencies mean that the elites are inclined to maintain an autocratic regime. The poor want more redistribution of income. If the poor can mount a credible threat of revolution, they can influence the elites to offer redistribution through taxes or to create a democracy. The poor know that concessions given by the elites is temporary and will last only as long as the poor can use their de facto power to maintain a credible threat of revolution. Therefore, the poor prefer a democracy. A democracy will give them du jure power they can use to effect more stable economic institutions in favor of the poor citizen. The key point of A&R's game-theoretic framework is that a transition to democracy shifts future power away from the elites to the citizens. The change in political institutions provides the credible commitment towards continued longer lasting policies favoring the poor citizen.

Elites do not prefer a democracy, but a revolution is always worse. A&R model revolution by reducing elite income to zero after redistributing all their income to the poor. The elites can also choose a third option: to repress or oppress the poor and prevent a successful uprising. Oppression has costs as well and, if these costs are higher than democratization, then the elites will choose democratization.

In summary, if inequality is high enough and the poor can organize a threat of revolution, then the elites can avoid a revolution by: i) changing policies (e.g., tax rates) to affect more redistribution, ii) changing institutional rules (e.g., democratization) to provide a commitment to future redistribution, and iii) repression.

The poor receive increasing benefits from increasing tax rates from 0 to their preferred rate. After that, the benefit decreases as tax rates continue to increase because of the increasing redistribution costs. Therefore, in a democracy, the poor know their preferred tax rate. If the elites can mount a threat of a coup then then poor will take action to prevent a coup by lowering taxes to appease the rich. However, if inequality is high enough, the elites will find it worth the cost of a coup to re-instate a non-democratic regime and take back political control. That is because, even though the poor are offering the lowest tax rate of 0, all parties know that this is temporary and based only on the elites' de facto power to mount a coup. If the elites mount a coup and install a non-democracy, they now have du jure power which lasts longer than a zero tax-rate. After the coup, the du jure power gives the elites the ability to change the economic institutions in their favor.

The next sections describe different components of the A&R model.

2.3.1 Basic Approach - Acemoglu & Robinson Theory of Democratization

According to A&R's theory, democracy consolidates when the elites are not incented to overthrow it. Consolidation depends on the i) the individual benefits of one regime versus another given the redistributional qualities of the regime, ii) the ability of the citizens to solve their collective action problems, iii) the ability of the institutions to solve the credible commitment problem, and iv) the level of economic inequality.

The A&R models utilize game theory to model preferences for democracy or non-democracy and regime transitions. First, they define two classes of people in a population with a mean income of $\bar{y}$. Minority elites are rich people, and their preference would be a non-democratic regime since it favors elites. The poor make up the majority of the population. A&R assign just two income levels and two types of individuals. It is assumed that the population is 1. There is a fraction $\boldsymbol{\delta}$, ($\boldsymbol{\delta} < ½$) of agents who are rich, each having income (y^r). There is a fraction (1-$\boldsymbol{\delta}$) of agents who are poor, each having income (y^p). The intra-group incomes are homogenous. The inter-group incomes are related to each by a factor value $\boldsymbol{\theta}$ which represents the share of income accruing to the rich. For example, the description of the relative inequality may be expressed as 5% of the people have 40% of the wealth. The income of the two classes of citizens is set up as:

$$y^r = (\boldsymbol{\theta} * \bar{y}) / \boldsymbol{\delta} \qquad (1)$$

$$y^p = (1 - \boldsymbol{\theta}) * \bar{y} / (1 - \boldsymbol{\delta}) \qquad (2)$$

2.3.2 Democracy vs Non-democracy

With a pretax income of y, a tax rate of τ and a distribution cost $C(\tau)$, in a population of mean income $\bar{y}$, an individual's preference for a political regime are defined by an individual's post-tax income $\hat{y}$ (Acemoglu & Robinson, 2006, p. 104)

$$\hat{y} = (1 - \tau)\, y + (\tau - C(\tau))\, \bar{y} = y + \tau(\bar{y} - y) - C(\tau)\, \bar{y} \qquad (3)$$

In a non-democracy with no threat of revolution, the tax rate is set to zero. There is no redistribution, so each person receives their income (y^r) or (y^p). If there is a credible threat of revolution, the rich may offer a concession tax or they may repress. Note that the more equal the society, the more likely the elites will offer a concession tax. The cost of the concession tax to the elites is low since not much redistribution is needed. At high levels of inequality, the poor will not find even the highest possible concession tax acceptable because a revolution will be more advantageous. To prevent a revolution, the elites will either democratize or repress. High levels of inequality make the prospect of a highly redistribution democracy very costly to the elites. Economic development that lowers inequality will have the effect of making a transition to democracy less costly to the elites since the resulting democracy will be less redistributive.

If the elites repress, both the rich and the poor lose a portion κ of their income and their payoffs are now $(1- \kappa)\ y^p$ and $(1- \kappa\ )\ y^r$ (Acemoglu & Robinson, 2006, p. 187). If the elites repress, it is always successful (Acemoglu & Robinson, 2006, p. 186). If the elites think that they cannot successfully repress, or that the cost of successful repression is too high, then the elites will form a democracy. A democracy will give the elites the benefits of their post-tax income (equation 3). In democracy, the taxes are expected to be more redistributive since the poor citizens are in power and will need to lower taxes only when the rich can mount a threat of coup.

If there is a revolution, the rich receive nothing in payoffs. The poor distribute all the income equally among themselves (Acemoglu & Robinson, 2006, p. 154).

$$V^p(R,\ \mu^H) = \frac{1-\mu}{(1-\delta)(1-\beta)}\ \bar{y} \qquad (4)$$

2.3.3 Collective Action Problems

The collective action problem refers to a condition in which a group would benefit from a certain action, but the costs are too great to be borne by one person. The solution is to share the costs of action but there are problems convincing all individuals to pay up initially and problems preventing free-riders. Olsen (1965) applies economic analysis to the study of group behavior and identifies a number of solutions to the collective action problem. Elinor Ostrom proposes a framework of design principles for creating successful institutions for self-governing solutions to the collective action problem (Ostrom, 1990, p. 90). Two of Ostrom's design principles concern monitoring and sanctions for persons who break the rules established for the institution. Weingast (1997) has developed a game-theoretic model demonstrating that it must be in the politician's best interest to respect political and economic rights of citizens by honoring term limits and respecting the outcome of an election. Weingast's (1997) work suggests that enforcing the rules of a democracy is a collective action problem among the citizens. This problem requires that the citizens agree on the limited powers of the state. It is a difficult problem because citizens tend to be in disagreement for key policy areas and yet must nonetheless coordinate their response to violations of democratic rights in order to bring the right incentives to the elites so that the elites follow the rules (Weingast, 1997). Tullock (1971) applies the research of collective action to revolutions.

The citizens have to solve the collective action problem in order to increase their power and change political and economic institutions. The citizens have to motivate others to participate in a revolution that may be costly to the individual but beneficial for the whole group. More than short term participation – citizens must motivate others to keep the credible threat of revolution active over a much longer term and under damaging circumstances. A revolution is modeled as the citizens taking the income of the elites and sharing it among each other. Acemoglu &Robinson (2006, pp. 122-125) show that the benefit of revolution to a citizen is greater than no revolution when the inequality $\boldsymbol{\theta}$ is greater than the fraction of the economy lost to revolution $\boldsymbol{\mu}$. A&R also represent that, when collective action is difficult, the cost of revolution (which includes the cost of organizing and motivating), will be higher. The necessity of solving the collective action problem for revolutions also means that there may be times when collection action problems are not overcome, and there is no threat of revolution. Although the citizens will fight for beneficial policies and redistribution in a non-democracy, they will fight harder for a switch to democracy. In a non-democracy, agreements on pro-citizen policies are temporary and will diminish as the citizens continue failing to solve the collective action problem. Without a credible threat of revolution, the elites are not incented to adopt some pro-citizen policies. These pro-citizen policies are modeled as re-distributional taxes. Similarly, the elites must solve the collective action problem in order for elites to mount a coup.

A&R examine several case studies of revolutions including Vietnam, Zimbabwe, and Rwanda with particular focus on how the collective action problem was solved. The solutions included direct benefits and the expectations of personal gain. Several scholars have noted the transitory nature of the collective action solution (e.g., Lichbach, 1995, p. 17; Tarrow, 1991; Ross & Gurr, 1989; Hardin, 1997). The transitory nature of the collective action problem is modeled by the changing values of the cost of coup and the cost of revolution (Acemoglu & Robinson, 2006, pp. 122-125).

Once a democracy is in place, the citizens must maintain solutions to the collective action problem so that the elites are incented to keep their promises (Weingast, 1997). The many volunteer, social, and civic organizations, as well as organized interest groups, may provide the platform and tools for coordination. Here the model assumes that the democratic institutions are sufficient to make the elites follow the rules of the government. Freedom of the press (rather than a captured press) may be an important component of effective democratic institutions. The A&R model demonstrates that if inequality is high and the democracy is not sufficiently redistributive, and if the citizens can overcome the collective action problem, then they may conduct a revolution to force a redistribution (Acemoglu & Robinson, 2006, p. 123).

2.3.4 Inequality

Many scholars have argued that a higher degree of inequality between the citizens and the elites increased the probability of revolution (e.g., Alesina & Perotti, 1996; Midlarsky, 1982; Lichbach, 1989; Acemoglu & Robinson, 2000). Much current research on economic relationships underpinning inequality does not take into account the role of institutions and political factors in the creation of inequality (Acemoglu & Robinson, 2015; Piketty & Saez, 2014). Easterly (2007) finds inequality prevents high quality democratic institutions from forming. He finds inequality lowers per capita income through mechanisms associated with schooling and democratic institutions (Easterly, 2007).

In addition, inequality factors into the decision for the elites to mount a coup and de-democratize. In a society with high inequality, the elites have greater incentives for bearing the cost of the coup and taking back control with an authoritarian regime in order to stop future redistributions. Tilly (2003) found that increases in inequality between groups of people in a democracy destabilizes the regime and increases the likelihood of reversion to undemocratic regime. Many analysts, including Aristotle, have thought inequality threatens democratic institutions. More recently, scholars have analyzed the role of inequality in the frequent occurrences of functioning democracies being overthrown and replaced with authoritarian regimes (e.g., Arat, 1991; Diamond, 1999; O'Donnell, 1999; Cheibub, Przeworski et al., 1996; Przeworski et al., 2000). Tilly (2003) references reversals that occurred in the last century in Greece, Italy, Germany, Spain, Portugal, Vichy France, Argentina, Bolivia, Brazil, Chile, Ecuador, Guatemala, Honduras, Panama, Peru and Uruguay.

The A&R model uses a measure of inequality constructed as the top $\boldsymbol{\delta}$ percent of citizens receive $\boldsymbol{\theta}$ percent of the total income. This measure is readily available in World Bank datasets provided by PovcalNet (2016). For a cross-country specification, see Table 43(PovcalNet, 2016) in Appendix A. Table 43 ranks 153 countries by the income share of each country's top 10% using World Bank data (PovcalNet, 2016).

The A&R model represents the relationship between inequality and revolution by express examination of the incentives for revolution. Large inter-group inequality means that a successful revolution would produce a bigger prize to divide up among the poor. A bigger prize represents a larger incentive for the poor to have a revolution or at least present a threat of a revolution. However, the A&R model also includes increased incentives for the elites to prevent democracy. As inequality increases, the amount of redistribution demanded by the citizens in a democracy will increase. Therefore, the rich will be less willing to democratize and more willing to pay the costs of repression. The way these two incentive forces play out is dependent upon other economic variables such as the costs of repression and the likelihood of the poor to maintain a threat of revolution. Glaeser (2005) found that inequality impacts the incentives of both the elites and the poor and can increase, decrease or not impact democratization at all.

In A&R's model, as long as there is a threat of a coup, the democracy is not stable, and is called either unconsolidated or semi-consolidated. The democracy is consolidated if there is no threat of a coup because either inequality is low (driving low redistribution), or the cost of coup is high enough to make this option not attractive. In the A&R model, once a coup is not an option, and a democratic regime has consolidated power, it never reverts to an autocratic regime. The A&R assumption is that the democratic institution controls inequality through taxation.

In contrast, Tilly (2003) contends that durable inequality threatens even consolidated democracies because the democratic institutions may not have kept pace with technology and are not exercising democratic control to adequately redistribute the value generated by financial capital, information, access networks, media, and scientific-technical knowledge. He asserts that government agents and other elite minority groups have incentives to help divert state power to their own advantage (Tilly, 2003).

2.3.5 Credible Commitment Problem

The credible commitment problem is the condition that elites can promise redistribution for this current period of time, but in an autocracy, where there are few restraints on the executive discretion, such promises for the future are not credible. A&R model the credible commitment problem by the way the players calculate their benefits. Each person calculates their benefit of the current promise as the benefit for that time period, plus the discounted value of what might happen in the unlimited view of the future (Acemoglu & Robinson, 2006, p. 155). In the future, the current policies may continue, or they may change.

It would be in the best interest of the rich to defuse a temporary crisis with the promise of redistribution to the poor with more pro-citizen policies. The poor are not stupid though, and there are many historic accounts of kings and other elites reneging on their promises as soon as the poor have returned to their villages and the ink is dry on the deal. A&R describe the peasants' revolt of 1381 in great detail (Acemoglu & Robinson, 2006, p. 137). King Richard II was 14 at the time. According to the accounts of Hilton (2003), Dobson (1983), and Dyer (1987), the peasants demanded freedom from some feudal labor restrictions, regulations and taxes. The peasants killed the Archbishop of Canterbury and the King's Treasurer. The peasants demanded that the Church's lands be confiscated and distributed to the citizens. The King agreed to all demands including a demand for pardons and non-retaliation. However, after the peasants went home, the King revoked the pardons, executed the rebel leaders, and claimed his promises were made under threat and not valid according to the law.

In another example, according to the account of Safford & Palacios (2002), the Spanish Empire in 1781 attempted to raise taxes in New Granada (now Columbia), South America. The government had a monopoly on the sale of cane liquor and tobacco. After the government raised prices, riots followed. Food shortages exacerbated the riots. As the riots gained support, the Spanish Governor fled and the Archbishop was left in control. He agreed to the rebel demands, appeased the rebels and persuaded them to go home. Then reinforcement troops from Cartagena arrived. The rebels were found and publicly executed, and the agreement was revoked.

The Glorious Revolution in England, 1688, provides an example of a revolt with institutional change that led to a different result. The monarchy and large landowners held the distribution of power in England until 1215 when King John was forced to sign the Magna Carta. This gave authority to assign and collect taxes to his council, which eventually became parliament. Before the Glorious Revolution 1688, the de jure power was still mostly in the hands of the king and his landowner supporters. Parliament tried to control spending and taxes. The Crown expropriated wealth through redefinition of rights. The expropriation of citizen's wealth led to civil war, a failed political institution, the return of the monarchy in 1660, which failed resulting in the Glorious Revolution and its fundamental redesign of the fiscal and governmental institutions (North & Weingast, 1989). The king's powers were severely curtailed. As parliament's interests shifted from land rents and local monopoly rents to the manufacturing and trade sectors, they used their political power to modify the economic institutions to their benefit. Parliament reduced taxes in the manufacturing sector and increased taxes in land. The Glorious Revolution and introduction of certain rights, changed institutions and constrained the king by making it infeasible or undesirable for the king to renege on commitments. (Pincus & Robinson, 2011).

Restrictions on government elites are believable only if there exists the ability for citizens to take action against the elites if the elites renege (Weingast, 1997). In this way, the citizens' ability to organize make it in the elites' best interest to keep the pro-citizen policies even if the incumbent elites change. Governments have incentives to renege on inconvenient commitments. Therefore, rule of law, property rights and contract enforcement cannot be credible unless there are effective limits on executive discretion. Olson (1965) showed that by credibly committing to restraints on executive discretion, even autocrats can benefit as a result of more robust economic activity and higher tax revenues. Autocrats will likewise be willing to make institutional, legal, and policy concessions to guard against the threat of citizen uprisings (Haggard & Tiede, 2011).

Autocratic regimes tend not to include effective limits on autocrats' power. Democratic institutions have limits on autocrats' power. Acemoglu &Robinson (2006, p. 155) argue that the democratic limits on autocrats' power provides the way to institutionalize the "credible commitment" to pro-citizen policies and redistribution.

With new democracies, there are difficulties conceding enough to the elites to influence them to create the democracy, while expecting elites to support the democratic institutions of executive constraints, term limits and elections. Unfortunately, new leaders are often incentivized to civil war to maintain their control in the new democracy rather than concede an election. Durant & Weintraub (2014) analyze the shortcomings of electoral institutions and develop a mathematical model of a new "turn taking" electoral institution that solves the design dilemma between 1) elite pacts (allocation of rents) needed to implement democracy with credible commitments and 2) the flexibility to ensure stability and peace. They argue that turn-taking institution makes it harder for elites to deploy and enforce exclusive ethnically favorable policies and makes it easier to coordinate on socially productive policy.

Acemoglu & Robinson (2006, p. 155) modeled the credible commitment problem by creating utility functions that look at the value of a decision in terms of what will happen today, as well as what is expected to happen tomorrow. Bellman equations are named after Richard Bellman (1957) who used the technique to write the value of a decision problem in terms of the payoff from the initial choices and the value of the remaining decision problem that results after the first choice is made. This breaks a complicated problem into more manageable parts (Bellman, 1957).

An example of a Bellman equation in shown in equation 5. The utility of the rich in a non-democracy with low threat of revolution is written as $V^r (N, \boldsymbol{\mu}^L)$. The value of that utility to a rich individual is equivalent to their current income (no taxes since there is a low threat of revolution) this year plus the value of what may happen in the future. The value of the future is multiplied by the discount factor to get the present value. With probability q in the future, there is a high threat of revolution and the elites democratize.

After establishing an initial democracy, there is a low threat of coup so that value is represented by $V^r(D,\boldsymbol{\varphi}^L)$. Also in the future, with probability (1-q), the revolution threat state stays low: $V^r(N, \boldsymbol{\mu}^L)$. Thus, it is a recursive equation:

$$V^r(N, \boldsymbol{\mu}^L) = y^r + \boldsymbol{\beta}[q\, V^r(D,\boldsymbol{\varphi}^L) + (1-q)V^r(N, \boldsymbol{\mu}^L) \qquad (5)$$

- (Acemoglu & Robinson, 2006, p. 235)

The equation is solved by finding values for the sub-components and solving two equations with two unknowns.

2.3.6 Model Assumptions of Acemoglu & Robinson's Dynamic Model of the Creation and Consolidation of Democracy

I closely reviewed Chapter 7, section 4. p. 231-245, which describes the Acemoglu & Robinson game-theoretic Dynamic Model of the Creation and Consolidation of Democracy. I documented the assumptions in Table 1 following the approach of Guerrero and Axtell (2011). These are the assumptions that have to be represented in the agent-based model for the agent-based model to replicate the results of A&R's game-theoretic model. The assumptions are numbered in column 1 of Table 1. Column 2 describes the assumption. And column 3 identifies the importance of the assumption.

Guerrero and Axtell (2011) describe three categories of assumptions. First-order assumptions are those assumptions inherent to game-theoretic models that must be confronted by the modeler when implementing an agent-based model. For example, the game-theoretic model assumes one agent, a fraction of which is rich and a fraction, poor.

In an agent-based model, a population of agents is explicitly instantiated. There is no way to implement a fraction of an agent. Non-interactiveness is another example. Changes between time period 1 and time period 2 result solely from the agents interacting with their environment and/or other agents. Second-order assumptions are the remaining three assumptions in the "neo-classical sweet spot: rationality, agent homogeneity and equilibrium. Modifying these assumptions provide the value-add of agent-based models to experiment with more realistic characteristics of the agents and environment. Third-order assumptions are specific to the theory being modeled, in this case, Acemoglu & Robinsons theory of the Creation and Consolidation of Democracy. The assumption modifications needed for the initial agentization are described in section 3.1.

Table 1: Microeconomic assumptions of model of the creation and consolidation of democracy

Number	Assumption	Order
1	Population consists of a population of size 1	1st
2	Population size is divided into two classes with fraction $\boldsymbol{\delta}$ of elites and (1-$\boldsymbol{\delta}$) poor citizens. $\boldsymbol{\delta} < ½$	1st
3	Rich agents have income y^r and poor agents have income y^p $y^r = (\boldsymbol{\theta} * \bar{y}) / \boldsymbol{\delta}$ $y^p = (1 - \boldsymbol{\theta}) * \bar{y} / (1 - \boldsymbol{\delta})$ (intra-group homogeneity)	2nd
4	All agents are aware of the cost of coup $\boldsymbol{\varphi}$ and the cost of revolution $\boldsymbol{\mu}$. (rationality)	2nd
5	Each person has preferences on regime decision. The preferences of the median voter determine the regime decisions enacted at the society level. The equilibrium of the game is for both elites and citizens to propose the ideal point of the median voter. (non-interactiveness)	1st
7	Infinite horizon model G^{∞}. Bellman equations are used to analyze the utilities consisting of the current year and looking into the future. (rationality)	2rd

Number	Assumption	Order
8	Agents expected utility at time t=0 is $U^1= E_0\sum_{t=0}^{\infty} \boldsymbol{\beta} \hat{y}_t^i$.	3rd
9	Citizens can mount a revolution by collective action technology. In a revolution, a portion $\boldsymbol{\mu}$ of the economy is destroyed. All rich agents lose all income, and remaining income is divided equally among the poor. $\boldsymbol{\mu}$ changes over time, perhaps due to changes in ability to organize, and is used to model the collective action problem.	3rd
10	In a democracy, elites can attempt a coup. After a coup, all agents lose a fraction $\boldsymbol{\varphi}$ of their income. $\boldsymbol{\varphi}$ changes over time and is and is used to model the limited ability of the citizens to commit to future tax concessions.	3rd
11	The game is either in a high or low threat state for revolution. In the low threat state, the cost $\boldsymbol{\mu}^L=1$. The probability of a high threat state $\boldsymbol{\mu}^L= \boldsymbol{\mu}$ is $q < 1/2$	3rd
12	The game is either in a high or low threat state for coup. In the low threat state for coup, the cost $\boldsymbol{\varphi}^L=1$. The probability of a high threat state $\boldsymbol{\varphi}^L= \boldsymbol{\varphi}$ is $s < 1/2$	3rd
13	If a coup is mounted, in the next time period $\boldsymbol{\mu}_t=\boldsymbol{\mu}^L=1$ so there is no immediate revolution	3rd
14	If a coup is mounted, in the next time period $\boldsymbol{\varphi}_t= \boldsymbol{\varphi}^L =1$ so there is no immediate coup	3rd
15	In each non-democratic period, the elites decide whether to democratize. If they do, the median voter, a citizen sets the tax rate.	3rd
16	When the elite mount a coup, the median elite, a rich person, sets the tax rate of 0.	3rd

2.3.7 Democratization in a Picture

The Acemoglu & Robinson theory of the creation and consolidation of democracy provides an empirical picture for the conditions that lead a society to become and stay a democracy. They map their analysis of inter-group inequality into critical values that determine what the decision space is and how those decisions are likely to be made. They discussed four paths of political development due to the underlying economic conditions. These four paths are numbered (1), (2), (3), and (4) in

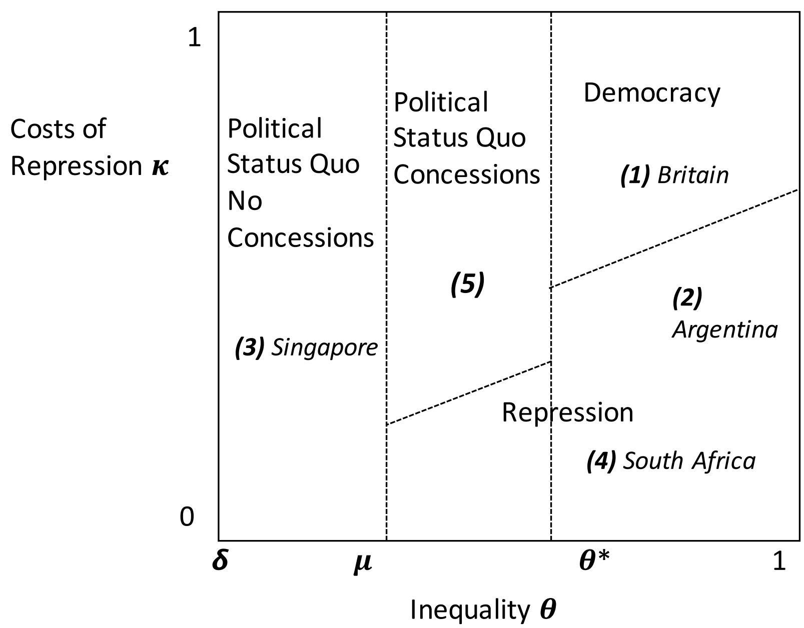
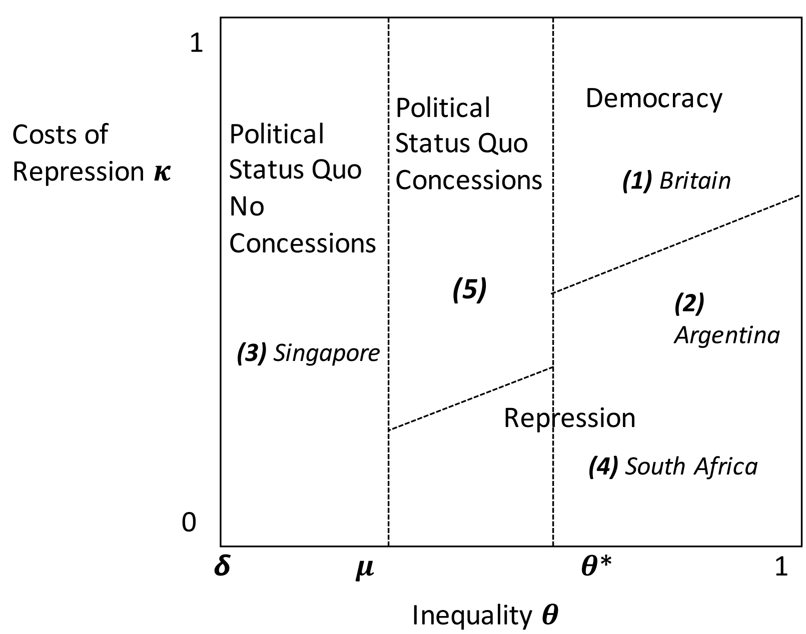


Figure 1. Later in the book, they discuss a fifth path associated with non-democracies offering redistribution to avoid revolution.

In the first path, in Britain, repressing the Glorious Rebellion was too costly and the elites installed a fully consolidated democracy. In this economic situation, the cost of a coup is also too great and the nation has remained a democracy. The second path is exemplified by Argentina. The cost of revolution was low which means the threat of revolution is high. The elites found it advantageous to create a democracy rather than repress. However, due to the high inequality, the cost of coups and revolutions are low relative to the benefit. The democracy is considered unconsolidated because the likelihood of a coup and subsequent revolutions are high. An unconsolidated democracy can be expected to undergo frequent regime changes between non-democracy, to democracy and back to non-democracy again. The third path is that of Singapore. The inequality is low enough in Singapore that the status quo can be maintained at relatively low cost and without much repression. The fourth path is that of South Africa. In South Africa, there was high inequality and relatively low cost of repression, so repression was used rather than democratization. There is a fifth path in which the inequality may be high but not enough to guarantee a revolution. In this space, the elites don't need to democratize to prevent a revolution. They have the choice of either offering additional redistribution, or preventing revolution through repression, depending on the cost of



repression.

Figure 1 shows the graphic reproduction of the Acemoglu & Robinson "Democratization in a Picture" showing the five paths.

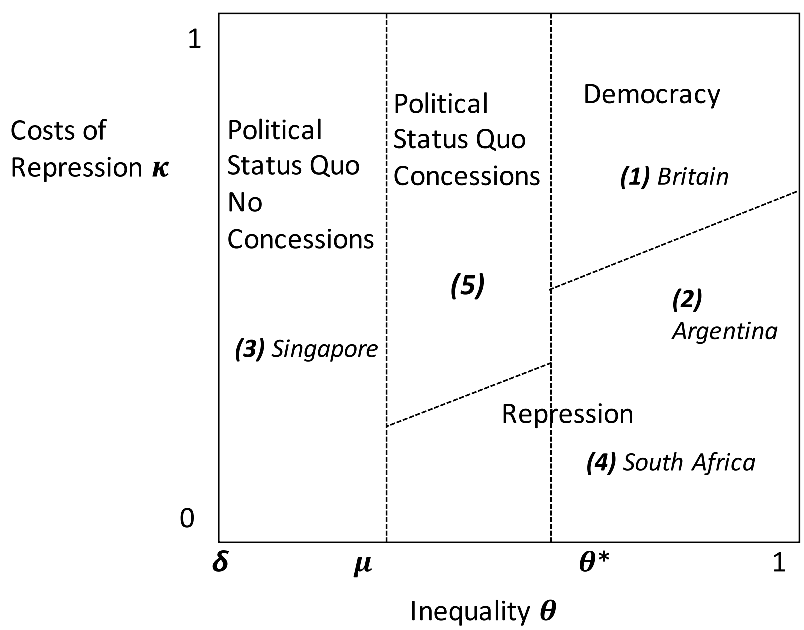


Figure 1: "Democratization in a Picture" (adapted from: Acemoglu & Robinson, 2006, pp. 44, 214)

The underlying economic conditions around inter-group income inequality and the costs and likelihood of coups, revolutions and repression, will determine the basic size and shape of the 5 paths. The horizontal axis is inequality. When inequality is low, there is no threat of revolution and thus no pressure for the elites to democratize. This area to the left of the vertical line marked with $\boldsymbol{\mu}$, has lower values $\boldsymbol{\theta}$ and over all values of repression $\boldsymbol{\kappa}$, the elites maintain the status quo without repression or policy concessions. This is similar to the path currently taken by Singapore.

The top area to the right of that line ($\boldsymbol{\theta} = \boldsymbol{\mu}$) is characterized by non-democratic regimes staying in power by making redistribution concessions to avoid revolution. Acemoglu & Robinson's Bellman equations and analysis of the utility of concessions versus democracy, show that decision to be independent of the cost of repression. It is therefore represented by another vertical line at $\boldsymbol{\theta}^*$. $\boldsymbol{\theta}^*$ represents the critical threshold of inequality below which a nation can avoid revolution through redistribution concessions. Although depending on the economics, they may still opt to use repression instead. To the right of $\boldsymbol{\theta}^*$ is a region where inequality is so high the regime must either democratize or use repression depending on the cost of repression. Acemoglu & Robinson assert that the decision to create a democracy is an economic one resulting from inter-group inequality and other economic conditions (Acemoglu & Robinson, 2006, p. xii).

2.4 Results of the Dynamic Model of the Creation and Consolidation of Democracy

Acemoglu and Robinson present their theory of the creation and consolidation of democracy in a series of propositions. Many of them are cumulative

Figure 2 depicts Proposition 7.2. and establishes the results of the Acemoglu & Robinson game-theoretic model of the Creation and Consolidation of Democracy. This proposition asserts that based on the underlying economic conditions of $\boldsymbol{\beta}$, $\boldsymbol{\varphi}$, $\boldsymbol{\mu}$, and $\boldsymbol{\kappa}$, and the political state P (democratic or non-democratic), the elites and the citizens will choose strategies which are best responses to each other. These strategies were analyzed in the model via Bellmen equations (Bellman, 1957) and assess the utility of regime choices as the current year's utility and the utility in the future discounted to the current year. Looking into the future, both the poor and rich understand that any current promises of redistribution or reduced taxation are valuable predicated on how likely they are to continue, which is a function of the underlying economic conditions. The main point of the Acemoglu & Robinson model is that the elites cannot commit to continued redistribution through taxation while they remain in power. Thus, democracy is a way for them to "institutionalize" the commitment to pro-citizen policies (Acemoglu & Robinson, 2006, p. 242).

Proposition 7.2: *there is a unique Markov perfect equilibrium in the game* $G^{\infty}(\boldsymbol{\beta})$. *Let* $\hat{\boldsymbol{\varphi}}$, $\boldsymbol{\varphi}^*$, $\boldsymbol{\kappa}^*$, $\bar{\boldsymbol{\kappa}}$, $\boldsymbol{\kappa}(\boldsymbol{\varphi})$, *and* $\breve{\boldsymbol{\kappa}}$, *be as defined previously. Then in this equilibrium:*

- *If* $\boldsymbol{\mu} \geq \boldsymbol{\mu}^*$, *the society remains nondemocratic. When* $\boldsymbol{\mu}_t = \boldsymbol{\mu}^L$, $\boldsymbol{\tau}^N = \boldsymbol{\tau}^r$ *and there is no redistribution. If* $\boldsymbol{\kappa} < \boldsymbol{\kappa}^*$, *then when* $\boldsymbol{\mu}_t = \boldsymbol{\mu}^H$, *the rich use repression. If* $\boldsymbol{\kappa} \geq \boldsymbol{\kappa}^*$, *then when* $\boldsymbol{\mu}_t = \boldsymbol{\mu}^H$, $\boldsymbol{\tau}^N = \hat{\boldsymbol{\tau}}$, *such that* $V^p(N, \boldsymbol{\mu}^H, \boldsymbol{\tau}^N = \hat{\boldsymbol{\tau}}) = V^p(R, \boldsymbol{\mu}^H,)$.
- *If* $\boldsymbol{\mu} < \boldsymbol{\mu}^*$, *then:*
  (1) *If* $\boldsymbol{\varphi} \geq \hat{\boldsymbol{\varphi}}$ *and* $\boldsymbol{\kappa} \geq \bar{\boldsymbol{\kappa}}$, *we are in a fully consolidated democracy. The society switches to democracy the first time* $\boldsymbol{\mu}_t = \boldsymbol{\mu}^H$ *and remains democratic thereafter, and taxes are always given by* $\boldsymbol{\tau}^D = \boldsymbol{\tau}^p$.
  (2) *If* $\boldsymbol{\varphi}^* \leq \boldsymbol{\varphi} < \hat{\boldsymbol{\varphi}}$ *and* $\boldsymbol{\kappa} \geq \boldsymbol{\kappa}(\boldsymbol{\varphi})$, *we are in a semo-consolidated democracy. The society switches to democracy the first time* $\boldsymbol{\mu}_t = \boldsymbol{\mu}^H$ *and remains democratic thereafter. When* $\boldsymbol{\varphi} = \boldsymbol{\varphi}^L$, $\boldsymbol{\tau}^D = \boldsymbol{\tau}^p$. *When* $\boldsymbol{\varphi}_t = \boldsymbol{\varphi}^H$, $\boldsymbol{\tau}^D = \boldsymbol{\tau}^p$, *democracy sets the tax rate* $\boldsymbol{\tau}^D = \tilde{\boldsymbol{\tau}} < \boldsymbol{\tau}^p$ *such that* $V^R(N, \boldsymbol{\mu}^L) - \boldsymbol{\varphi}\bar{y} = V^R(\boldsymbol{\varphi}^H, D, \boldsymbol{\tau}^D = \tilde{\boldsymbol{\tau}})$.
  (3) *If* $\boldsymbol{\varphi} < \boldsymbol{\varphi}^*$ *and* $\boldsymbol{\kappa} \geq \breve{\boldsymbol{\kappa}}$, *we are in an unconsolidated democracy. The society continuously switches regimes. In a nondemocratic regime, when* $\boldsymbol{\mu}_t = \boldsymbol{\mu}^L$, *the elites set* $\boldsymbol{\tau}^N = \boldsymbol{\tau}^r$; *when* $\boldsymbol{\mu}_t = \boldsymbol{\mu}^H$, *they democratize. In a democracy, when* $\boldsymbol{\varphi}_t = \boldsymbol{\varphi}^L$, $\boldsymbol{\tau}^D = \boldsymbol{\tau}^p$; *when* $\boldsymbol{\varphi}_t = \boldsymbol{\varphi}^H$, *there is a coup.*
  (4) *If* $\boldsymbol{\varphi} \geq \hat{\boldsymbol{\varphi}}$ *and* $\boldsymbol{\kappa} < \bar{\boldsymbol{\kappa}}$, *or* $\boldsymbol{\varphi}^* \leq \boldsymbol{\varphi} < \hat{\boldsymbol{\varphi}}$ *and* $\boldsymbol{\kappa} < \boldsymbol{\kappa}(\boldsymbol{\varphi})$, *or if* $\boldsymbol{\varphi} < \boldsymbol{\varphi}^*$ *and* $\boldsymbol{\kappa} < \breve{\boldsymbol{\kappa}}$, *when* $\boldsymbol{\mu}_t = \boldsymbol{\mu}^L$, $\boldsymbol{\tau}^N = \boldsymbol{\tau}^r$, *and there is no redistribution and when* $\boldsymbol{\mu}_t = \boldsymbol{\mu}^H$, *the elites use repression to stay in power.*

Figure 2: Proposition 7.2 (adapted from Acemoglu & Robinson, 2006, p. 242)

Acemoglu & Robinson assert in Proposition 7.2 that the equilibrium regime decisions depend on critical values for the cost of revolution $\boldsymbol{\mu}$, cost of coup $\boldsymbol{\varphi}$ and cost of repression $\boldsymbol{\kappa}$. Acemoglu & Robinson determined equations for these critical values. $\boldsymbol{\mu}^*$, $\hat{\boldsymbol{\varphi}}$, $\boldsymbol{\varphi}^*$, $\boldsymbol{\kappa}^*$, $\bar{\boldsymbol{\kappa}}$, $\breve{\boldsymbol{\kappa}}$, by analyzing the Bellman Equations from the game-theoretic model. Table 2 defines the critical values and their equations.

Table 2: Table of critical values for the creation and consolidation of democracy

Critical Value	Definition	Equation in dynamic game
$\boldsymbol{\mu}^*$	If cost of revolution is low such that $\boldsymbol{\mu} < \boldsymbol{\mu}^*$ even if the elites maximize the redistribution to the citizens preferred tax rate, it is not enough to stop a revolution.	$\boldsymbol{\mu}^* = \boldsymbol{\theta}$-(1-$\boldsymbol{\beta}$(1-q)) * ($\boldsymbol{\tau}^p(\boldsymbol{\theta}-\boldsymbol{\delta})$-(1- $\boldsymbol{\delta}$)$C(\boldsymbol{\tau}^p)$) 7.22
$\widehat{\boldsymbol{\varphi}}$	If cost of coup $\boldsymbol{\varphi} > \widehat{\boldsymbol{\varphi}}$, the value for the elites of staying in a democracy even if the citizens tax at their preferred tax rate $\boldsymbol{\tau}^D = \boldsymbol{\tau}^p$, will be greater than the value of a coup.	$\widehat{\boldsymbol{\varphi}} = \frac{1}{\boldsymbol{\theta}}\left(\frac{\boldsymbol{\delta} C(\boldsymbol{\tau}^p) - \boldsymbol{\tau}^p\ (\boldsymbol{\delta}-\boldsymbol{\theta})}{1-\boldsymbol{\beta}\ (1-\text{q})}\right)$ 7.16
$\boldsymbol{\varphi}^*$	If cost of coup $\boldsymbol{\varphi} < \boldsymbol{\varphi}^*$ the value for the elites of staying in a democracy even if the citizens reduce taxes to 0 during times of high threat of coup, is not enough to prevent a coup.	$\boldsymbol{\varphi}^* = \frac{1}{\boldsymbol{\theta}}\left(\frac{\boldsymbol{\beta}(\ q+s-1)(\boldsymbol{\tau}^p\ (\boldsymbol{\delta}-\boldsymbol{\theta}) - \boldsymbol{\delta} C(\boldsymbol{\tau}^p)}{1-\boldsymbol{\beta}\ (1-\text{q})}\right)$ 7.17
$\boldsymbol{\kappa}^*$	If there exists a concession tax that could prevent revolution $\boldsymbol{\mu} > \boldsymbol{\mu}^*$, then if the cost of repression is low $\boldsymbol{\kappa} < \boldsymbol{\kappa}^*$, then the elites will choose repression over offering a concession tax.	$\boldsymbol{\kappa}^* = \frac{1}{\theta}(\boldsymbol{\delta} C(\hat{\boldsymbol{\tau}}) - \hat{\boldsymbol{\tau}}\ (\boldsymbol{\delta}-\boldsymbol{\theta}))$ 6.20
$\overline{\boldsymbol{\kappa}}$	If cost of repression is low enough $\boldsymbol{\kappa} < \overline{\boldsymbol{\kappa}}$, then the elites will choose repression over a fully consolidated democracy. At $\overline{\boldsymbol{\kappa}}$, they are indifferent between democratization with high cost of a coup $\boldsymbol{\varphi} > \widehat{\boldsymbol{\varphi}}$) and repression.	$\overline{\boldsymbol{\kappa}} = \frac{\boldsymbol{\delta} C(\boldsymbol{\tau}^p) - \boldsymbol{\tau}^p\ (\boldsymbol{\delta}-\boldsymbol{\theta})}{\theta(1-\boldsymbol{\beta}\ (1-\text{q}))}$ $\quad \overline{\boldsymbol{\kappa}} > \boldsymbol{\kappa}^*$ 6.21
$\boldsymbol{\kappa}(\boldsymbol{\varphi})$	If cost of repression is low enough $\boldsymbol{\kappa} < \boldsymbol{\kappa}(\boldsymbol{\varphi})$, then the elites will choose repression over a semi-consolidated democracy.	The value of indifference increases with $\boldsymbol{\varphi}$.
$\breve{\boldsymbol{\kappa}}$	If the cost of repression $\boldsymbol{\kappa} < \breve{\boldsymbol{\kappa}}$ in a state with low $\boldsymbol{\varphi} < \boldsymbol{\varphi}^*$, the elites will choose repression over creating an unconsolidated democracy.	$\overline{\boldsymbol{\kappa}} > \boldsymbol{\kappa}(\boldsymbol{\varphi}) > \breve{\boldsymbol{\kappa}}$

3 Chapter Three: Initial Agentization

This chapter is structured in the following way. Section 3.1 presents an agentized model reproducing the Acemoglu & Robinson dynamic model of the creation and consolidation of democracy. In section 3.2 I examine the response of the model to a wide range of inputs and verify the results reproduce those of the A&R models. I provide evidence of the validity of the model in section 3.3.2 by reproducing the institutional arrangements that are found in several countries. I conclude by validating that the agent-based model does reproduce the stylized fact that inter-group inequality affects the institutional decisions made. I summarize how the model was validated over a wide range of parameters including high and low inequality extremes. This work "docks" the model with the Acemoglu and Robinson game-theoretic models in the sense of Axtell (2000).

In chapter, 4, I will extend this model to intra-group inequality to add value by introducing some additional explanatory power not present in the Acemoglu and Robinson models. The value-added by agent-based models over established methods is in being able to offer additional insights that better explain the phenomenon

3.1 Initial Agentization Model Design

This agent-based model implements the game-theoretic models of Acemoglu & Robinson's "Creation and Consolidation of Democracy". The models are described in chapter 7, section 4 of Economic Origins of Dictatorship and Democracy.

3.1.1 Initial Agentization Model Description

I developed the agent-based model in NetLogo 5.0.3 (Wilensky, 1999). NetLogo is an open source multi-agent programmable modeling and simulation environment developed at the Center for Connected Learning affiliated with Northwestern University.

3.1.2 Motivation

The purpose of this initial agentization is to replicate the insight provided by Acemoglu & Robinson's ground-breaking work developing a game-theoretic framework of the creation and consolidation of democracy. Their model provides insight into how economic incentives influence why some societies are democratic and some are not. Individuals evaluate different options and choose between strategies according to which strategy is anticipated to result in the best outcome for that individual. The model will address

- What effect does changing the inequality of the country have on the decisions of the elite in a non-democracy?
- What effect does changing the inequality of the country have on the decisions of the elite in a democracy?
- What effect does changing the likelihood of being in a high threat state for coups or revolution have on the decisions of the elite or the citizens?

3.1.3 Initial Agentization Approach

This dissertation presents two agent-based models. The first one, discussed in this chapter, keeps all the assumptions of the original game-theoretic set of models. The second model, discussed in chapter 4, modifies assumptions to make them more realistic following the example of Guerrero & Axtell (2011). These more realistic assumptions would have rendered the game-theoretic models mathematically intractable. However, they are possible to implement in an agent-based model and can provide additional insights not found in the original models. Table 3, lists the microeconomic assumptions of the game-theoretic models used in the Creation and Consolidation of Democracy. The next sections explain how these assumptions are instantiated in the agent-based model.

Table 3: Agent-based implementation of the microeconomic assumptions of A&R game-theoretic model

Number	Assumption of A&R model	Order	Agent-based implementation
1	Population consists of a population of size 1	1st	**MODIFIED**: N-agents
2	Population size is divided into two classes with fraction $\boldsymbol{\delta}$ of elites and (1-$\boldsymbol{\delta}$) poor citizens. $\boldsymbol{\delta} < ½$	1st	**MODIFIED**: Discreet agents. Elites = integer ($\boldsymbol{\delta}$ * N)
3	Rich agents have income y^r and poor agents have income y^p $y^r = (\boldsymbol{\theta} * \bar{y}) / \boldsymbol{\delta}$ $y^p = (1 - \boldsymbol{\theta}) * \bar{y} / (1 - \boldsymbol{\delta})$ (intra-group homogeneity)	2nd	Two classes with Homogenous Intra-group incomes
4	All agents are aware of the cost of coup $\boldsymbol{\varphi}$ and the cost of revolution $\boldsymbol{\mu}$. (rationality)	2nd	Global Variables

Number	Assumption of A&R model	Order	Agent-based implementation
5	Each person has preferences on regime decision. The preferences of the median voter determine the regime decisions enacted at the society level. The equilibrium of the game is for both elites and citizens to propose the ideal point of the median voter. (non-interactiveness)	1st	The medium voter agent, a poor agent determines the regime changes that are relevant to the poor. The medium voter elite, a rich agent, determines the regime changes relevant to the rich.
7	Infinite horizon model G^{∞}. Bellman equations are used to analyze the utilities consisting of the current year and looking into the future. (rationality)	2rd	Implements game-theoretic equations
8	Agents expected utility at time t=0 is $U^{1}= E_0\sum_{t=0}^{\infty} \boldsymbol{\beta} \hat{y}_t^i$.	3rd	Implements game-theoretic equations
9	Citizens can mount a revolution by collective action technology. In a revolution, a portion $\boldsymbol{\mu}$ of the economy is destroyed. All rich agents lose all income, and remaining income is divided equally among the poor. $\boldsymbol{\mu}$ changes over time, perhaps due to changes in ability to organize, and is used to model the collective action problem	3rd	Implements game-theoretic equations
10	In a democracy, elites can attempt a coup. After a coup, all agents lose a fraction $\boldsymbol{\varphi}$ of their income. $\boldsymbol{\varphi}$ changes over time and is and is used to model the limited ability of the citizens to commit to future tax concessions.	3rd	Implements game-theoretic equations
11	The game is either in a high or low threat state for revolution. In the low threat state, the cost $\boldsymbol{\mu}^{L}=1$. The probability of a high threat state $\boldsymbol{\mu}^{L}= \boldsymbol{\mu}$ is $q < 1/2$	3rd	Global variables
12	The game is either in a high or low threat state for coup. In the low threat state for coup, the cost $\boldsymbol{\varphi}^{L}=1$. The probability of a high threat state $\boldsymbol{\varphi}^{L}= \boldsymbol{\varphi}$ is $s < 1/2$	3rd	Global variables

Number	Assumption of A&R model	Order	Agent-based implementation
13	If a coup is mounted, in the next time period $\boldsymbol{\mu}_t=\boldsymbol{\mu}^L=1$ so there is no immediate revolution	3rd	Sets value after coup rather than random generation
14	If a coup is mounted, in the next time period $\boldsymbol{\varphi}_t= \boldsymbol{\varphi}^L=1$ so there is no immediate coup	3rd	Sets value after coup rather than random generation
15	In each non-democratic period, the elites decide whether to democratize. If they do, the median voter, a citizen sets the tax rate.	3rd	Tax rate is found by line-search algorithm.
16	When the elite mount a coup, the median elite, a rich person, sets the tax rate of 0.	3rd	Imbedded in game-theoretic equations

3.1.4 Agents

The original model assumes a population of 1, and precise fractions of rich and poor. This cannot be implemented in an agent-based model since the agents are discreet. There is a fixed population of agents divided into two classes: rich agents and poor agents. The rich agents comprise a percentage $\boldsymbol{\delta}$ of the population. $\boldsymbol{\delta}$ is less than 50%. Each agent, rich or poor, has an income. The average income of the rich agents, (y^r) is greater than the average income of the poor agents, (y^p). In the "docked" version of the model, all rich agents have the same income (y^r) and all poor agents have the income (y^p). In chapter 4, this assumption will be relaxed so that while the average income remains the same for each class of agent, there is a more realistic distribution of incomes.

3.1.4.1 Intra-group Homogenous Agent Incomes

In Acemoglu & Robinson's two-group model of redistributive politics, they define a simple model in which there are just two income levels and two types of individuals. It is assumed that the population is 1. There is a fraction $\boldsymbol{\delta}$ ($\boldsymbol{\delta} < ½$) of agents are rich each have income (y^r). The fraction (1-$\boldsymbol{\delta}$) of agents are poor and each have income y^p. The inter-group incomes are related to each by a factor value $\boldsymbol{\theta}$ which represents the share of income accruing to the rich. For example, the description of the relative inequality may be expressed as 5% of the people have 40% of the wealth. In the agent-based model, the population is defined by four values: i) given number of agents, ii) a selected fraction $\boldsymbol{\delta}$ of rich agents, iii) the mean income $\bar{y}$ of the population and iv) a value $\boldsymbol{\theta}$ which represents the share of income accruing to the rich. Agents are discreet; the actual number of rich agents is defined as the integer component of $\boldsymbol{\delta}$ * number of agents. The income of the two classes of citizens is set up as:

$$y^r = (\boldsymbol{\theta} * \bar{y}) / \boldsymbol{\delta} \qquad (6)$$

$$y^p = (1 - \boldsymbol{\theta}) * \bar{y} / (1 - \boldsymbol{\delta}) \qquad (7)$$

The mean of the poor class is less than the mean of the whole population which is less than the mean of the rich class: $y^p < \bar{y} < y^r$. The fraction of income that the rich control much be greater than their fraction of the population: $\boldsymbol{\theta} > \boldsymbol{\delta}$.

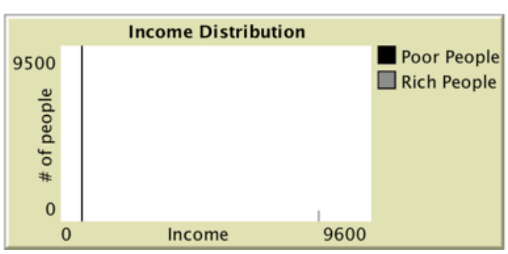


Figure 3: Homogeneous within group income

Figure 3 shows an income distribution example for 10,000 agents with mean income of the entire population of \$1000, and $\boldsymbol{\theta}$ is 40%. Thus 5% (500) agents are rich with mean income $y^r =$ \$8000 and 9500 agents are poor with mean income $y^p =$ \$631.

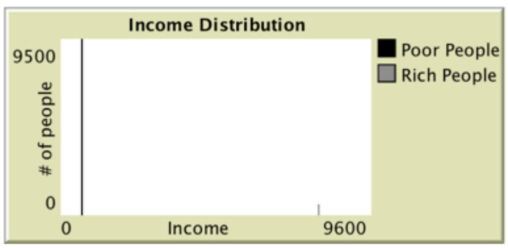


Figure 3: Homogeneous within group income

3.1.4.2 Knowledge

Each agent is cognizant of their own income, and the income of the other class. $\boldsymbol{\theta}$ represents the percent of total income (GDP) accruing to the rich. For example, if $\boldsymbol{\delta}$ is 5%, and the elite are characterized as "5% of the population have 25% of the income", then $\boldsymbol{\theta}$ in this case in 25%. Each agent is aware of the cost of a coup $\boldsymbol{\varphi}$ in terms of percent of GDP. Each agent is aware of the cost of revolution $\boldsymbol{\mu}$ in terms of percent of GDP. Each agent is aware of the future probability s of being in a high threat state of coup. Similarly, each agent is aware of the future probability q of being in a high threat state of revolution. Finally, they use this knowledge of income disparities between their class and the other, and the future threat of coups and revolutions, and what action they believe the other class is likely to take, to determine which decision is most likely to produce the best outcome for them. Table 4 lists the information used and decisions made by each agent class. The next section will explain in detail how those decisions are made.

Table 4: Decisions by agent class

Agent Class	Global Knowledge Used	Decisions Made
Rich (Elite)	$\bar{y}$ - mean income of the population y^r - mean income of the rich $\boldsymbol{\delta}$ – the percent of population considered rich $\boldsymbol{\theta}$ – the percent of total income shared among the rich $\boldsymbol{\varphi}$ – The cost of coup as percent of GDP $\boldsymbol{\mu}$ – the cost of revolution as percent of GDP $\boldsymbol{\beta}$ – discount factor **s** – the likelihood that there will be a high threat of coup **q** - the likelihood that there will be a high threat of revolution	**Polity is Non-Democratic** S – status quo. No redistribution of wealth T - offer concession tax to redistribute wealth and prevent revolution. D – democratize O – oppress (repress) **Polity is Democratic** P – pay tax set by democracy C – mount a coup and install a non-democracy
Poor	$\bar{y}$ - mean income of the population y^p - mean income of the poor $\boldsymbol{\delta}$ – the percent of population considered rich $\boldsymbol{\theta}$ – the percent of total income shared among the rich $\boldsymbol{\varphi}$ – The cost of coup as percent of GDP $\boldsymbol{\mu}$ – the cost of revolution as percent of GDP $\boldsymbol{\beta}$ – discount factor **s** – the likehood that there will be a high threat of coup **q** - the likehood that there will be a high threat of revolution	**Polity is Non-Democratic** S – accept status quo. No redistribution of wealth R – conduct a revolution **Polity is Democratic** P – pay tax set by democracy R – conduct a revolution to force more redistribution.

3.1.5 Rules

3.1.5.1 Median Voter Theorem

In the original A&R model, the regime decisions were made using the median voter theorem. In a democracy, all citizens rich and poor share in the political power. However, since there are more poor citizens than rich citizens, and since all poor citizens have the same intra-group income, the median voter is a poor citizen with income y^p. In a non-democracy, only the elite have political power. The median voter making decisions in a non-democracy is an elite agent. This assumption is maintained in this model.

3.1.5.2 Poor Preferred Tax Rate

A&R's dynamic model of the creation and consolidation of democracy builds on the assumptions defined in the earlier models in the book. Starting with the two-group model of redistributive politics, the political system determines an income tax rate $\boldsymbol{\tau} \geq 0$. There is a cost of the taxation system C ($\boldsymbol{\tau}$) and the remaining proceeds from the income tax are distributed equally to all citizens. The utility or after-tax incomes of our two classes of agents are:

$$V(y^r \mid \boldsymbol{\tau}) = y^r + \tau(\bar{y} - y^r) - C(\tau)\bar{y} \quad (8)$$

$$V(y^p \mid \boldsymbol{\tau}) = y^p + \tau(\bar{y} - y^p) - C(\tau)\bar{y} \quad (9)$$

Acemoglu & Robinson makes the following assumptions on the cost function $C(\tau)$ (Acemoglu & Robinson, 2006, p. 100,101):

i) the cost at zero income is zero, $C(0) = 0$

ii) the costs are increasing with the level of taxation $C'(.) > 0$

iii) the costs increase faster as tax rates increase $C''(.) > 0$

iv) $C'(0) = 0$

v) $C''(1) = 1$

This agent-based model implements the following cost function to meet these conditions.

$$C(\boldsymbol{\tau}) = \tfrac{1}{2}\boldsymbol{\tau}^2 \qquad (10)$$

Acemoglu & Robinson use the median voter concept to assert that in a democracy, the median voter determines the tax rate that is set. There are more poor agents than rich agents, therefore, the median voter is a poor agent. All poor agents have the same income. All poor agents will have the same preferred tax rate which is defined as the tax rate that will maximize their after-tax income given the mean income of the population, the income disparities between the two income groups, and cost of taxation.

The preferred tax rate for the poor is found by calculating the utility of the poor agent $V(y^p \mid \boldsymbol{\tau})$ (see equation 4) over tax rates $\boldsymbol{\tau}$ starting with 0 and incrementally increasing until the maximum tax rate of 1. My model implements a line search algorithm to find the maximum within the upper and lower bounds. The algorithm returns the tax rate $\boldsymbol{\tau}$ which provides the maximum utility. The preferred tax rate is hearafter referred to as $\boldsymbol{\tau}^P$.

3.1.5.3 Concession Tax to Prevent Coups

If the polity is a democracy, the poor would prefer to tax at their preferred rate $\boldsymbol{\tau}^P$. However, they base their decision on what they anticipate the elites to do. The elites may decide to mount of coup if they feel the benefits of taking over control with lower tax rates outweigh the cost the of coup. The poor consider this in their tax rate decision, and will offer a concession tax of $\boldsymbol{\tau}^D = \tilde{\tau}$. The poor evaluate what tax rate will offer the elites enough benefit so that they will decide not to mount a coup. The benefit to the elite of mounting the coup is calculated as the resulting benefit of being in a non-democracy minus the cost of the coup. This is described in equation 11(Acemoglu & Robinson, 2006, p. 233).

$$V^R(C, \boldsymbol{\varphi}) = V^r(N, \mu^L) - \boldsymbol{\varphi y^r} \qquad (11)$$

The decision for the poor is to set the tax rate low enough such that the benefit to the rich of staying in the democracy is greater than that of mounting a coup as described in equation 12 (Acemoglu & Robinson, 2006, p. 233).

$$V^r(D, \varphi^H, \tau^D = \tilde{\tau}) > V(N, \mu^L) - \boldsymbol{\varphi y^r} \qquad (12)$$

By analyzing the Bellman equations, Acemoglu & Robinson, (2006, p. 238) determine that the benefit to the rich of the non-democracy is as shown in equation 13.

$$V^r(N, \mu^L) = \frac{(1 - \beta(1-q))\, y^r + \beta^2 qs(\tilde{\tau}\,(\bar{y} - y^r) - C(\tilde{\tau})\,\bar{y}) + \beta q\,(1 - \beta s)\,(\tau^p(\bar{y} - y^r) - C(\tau^p)\,\bar{y})}{(1 - \beta)(1 - \beta(1-q))} \quad (13)$$

In addition, Acemoglu & Robinson, (2006, p. 237) determine that the benefit to the rich of staying in a democracy is as shown in equation 14.

$$V^r(D, \varphi^H, \tau^D = \tilde{\tau}) = \frac{y^r + (1 - \beta(1-s))\,(\tilde{\tau}(\bar{y} - y^r) - C(\tilde{\tau})\,\bar{y}) + \beta(1-s)\,(\tau^P(\bar{y} - y^r) - C(\tau^P)\,\bar{y})}{(1 - \beta)} \quad (14)$$

In the Acemoglu & Robinson model, tax rates are always greater than 0. Tax credits to enable greater redistribution are not considered. In addition, since τ^p provides the maximum utility to the poor, any tax $\tilde{\tau}$, would be less than or equal to τ^p. So the poor calculate utilities for the rich at tax rates varying incrementally from 0 to τ^p. The poor will first try a tax rate of zero which is the preferred tax rate of the rich. If the rich still prefer a coup over a democracy with a tax rate of 0, then there is no tax rate $\tilde{\tau}$ that will prevent a coup. If the rich prefer democracy over a tax rate of 0, the program will test tax rates $\tilde{\tau}$ that increase incrementally until the rich prefer a coup. The program returns the highest tax rate $\tilde{\tau} < \tau^p$. for which the rich will prefer not to have a coup.

3.1.5.4 Concession Tax to Prevent Revolution

If the polity is a non-democracy, the rich would prefer no redistribution and a tax rate of τ^N=0. In some economic conditions (high cost of revolution, high likelihood of coup), even with levels of high inequality, the poor would lose more by revolution than if they just kept the status quo with the non-democracy. For example, if there is a high likelihood of coup, then a democracy would be short lived and would be of less value to the poor. The rich however, want to avoid revolution at all cost. If there is a threat of revolution, one way the rich may be able to avoid revolution is by offering redistribution through increased tax rate $\hat{\tau}$. The rich will assess whether the poor can be offered increased redistribution through taxation such that their utility from staying in the non-democracy is greater than their utility from revolution. The rich will assess the utility of the poor for both revolution and staying in non-democracy with tax rate $\hat{\tau}$ at varying levels of $\hat{\tau}$ in order to find the lowest $\hat{\tau}$ that will prevent revolution.

In a revolution, the poor seize all income and divide it among themselves $(1 - \delta)$ individuals. But during revolution, a fraction μ of the economy $\bar{y}$ is destroyed. The utility of the poor from revolution discounted to the present is defined as shown in equation 15 (Acemoglu & Robinson, 2006, p. 238).

$$V^p(R,\ \mu^H) = \frac{1-\mu}{(1-\delta)(1-\beta)}\ \bar{y} \qquad (15)$$

The utility of the poor in a non-democracy depends on the likelihood q that they can pose a revolution threat and obtain a concession tax. Their utility is defined as shown in equation 16 (Acemoglu & Robinson, 2006, p. 239). When calculating the utility of the poor staying in the non-democracy, the rich assume they will offer concession tax of $\hat{\tau}$ when there is a threat of revolution, and no tax when there is no threat.

$$V^p(N,\ \mu^H,\ \tau^N = \hat{\tau}) = \frac{y^p + (1-\beta(1-q))\,(\hat{\tau}\,(\bar{y} - y^p) - C(\hat{\tau})\,\bar{y})}{1-\beta} \qquad (16)$$

The utility of revolution does not change with $\hat{\tau}$. The program calculates the utility of non-democracy at the maximum benefit to the poor $\hat{\tau} = \tau^p$. Next it incrementally decreases $\hat{\tau}$ until the utility of accepting the tax concession is equal to the utility of revolution. The program returns lowest value of $\hat{\tau}$ for which the poor will not have a revolution if such a value exists. Otherwise it returns τ^p.

3.1.5.5 Decision to Repress or Democratize

If the polity is non-democratic, and if the highest tax rate τ^p is not enough to stop a revolution, the elites will either democratize to prevent a revolution, or they will repress, depending on which is cheaper. When the elites evaluate their utility in a democracy they consider that their value depends on the likelihood of being in a democracy with a high coup threat. If there is a threat of coup, the citizens will offer concession tax $\tilde{\tau}$ that is lower than their preferred rate in order to avoid a coup. If there is no threat of coup, the citizens will keep their preferred tax τ^p and the democracy is considered "consolidated".

To make this decision of democratize or repress, the elites assess their utility of repression as the discounted sum of their income (no taxes) minus the cost of repression. The utility of repression takes into account that they will incur the cost of repression only during periods of high threat of revolution. The likelihood of high threat of revolution is q. This value function is defined as in equation 17 (Acemoglu & Robinson, 2006, p. 198).

$$V^r(O, \mu^H | \kappa) = \frac{y^r - (1 - \beta(1 - q)\kappa y^r}{(1 - \beta)} \qquad (17)$$

The elites compare this value to their utility if they democratize. If they democratize when there is little likelihood of mounting a threat of a coup, then the democracy is considered fully consolidated. The program identifies this case by checking if the cost of a coup $\boldsymbol{\varphi}$ is greater than the lowest cost of coup $\hat{\boldsymbol{\varphi}}$ experienced which did not result in a coup even at the highest tax rate τ^p. The logic is, if $\hat{\boldsymbol{\varphi}}$ did not result in a coup, at the highest tax rate τ^p, then a cost of $\boldsymbol{\varphi}$ which is greater than $\hat{\boldsymbol{\varphi}}$ will not result in a coup. The utility of a consolidated democracy assumes the poor will tax at their preferred tax rate for the foreseeable future because there is no likelihood of a high threat of coup. This utility is defined as shown in equation 18(Acemoglu & Robinson, 2006, p. 234).

$$V^r(D, \varphi^H, \tau^D = \tau^p) = \frac{y^r + \tau^p(\bar{y} - y^r) - C(\tau^p)\,\bar{y}}{(1 - \beta)} \qquad (18)$$

If the elites democratize when there is likelihood of a threat of a coup, then the democracy is considered either semi-consolidated or unconsolidated. It is considered semi consolidated in the situation that there exists a concession $\tilde{\tau} < \tau^p$ that could prevent a coup. It is considered unconsolidated when a concession tax $\tilde{\tau}$ that would prevent a coup does not exist and it is likely that the democratization will be followed by a coup. In both cases, the utility of the rich considers that citizens will alternate between setting tax rate τ^p when there is no threat and tax rate $\tilde{\tau}$ to prevent coup when there is a threat. The utility is defined as in equation 14 and is constructed by the democracy setting tax rate $\tilde{\tau}$ the first year (Acemoglu & Robinson, 2006, p. 238).

If the utility of democratization (equation 18 or 14) is greater than the utility of repression (equation 17), then the elites decide to democratize.

3.1.5.6 Decision to Repress or Offer Concession Tax

If the polity is non-democratic, and if there is a threat of revolution and the highest tax rate τ^p is enough to stop a revolution, the elites will either offer redistribution through a concession tax $\hat{\tau}$ to prevent a revolution, or they will repress, depending on which is cheaper.

To make this decision of redistribute or repress, the elites assess their utility of repression as described above with equation 17. The utility of repression takes into account that they will incur the cost of repression only during periods of high threat of revolution (Acemoglu & Robinson, 2006, p. 198).

The elites compare this value to their utility of making policy concessions and promising redistribution at a tax rate $\hat{\tau}$ less than τ^P but high enough to prevent revolution. The agent-based model finds this value as described in section 3.1.5.4. The utility to the rich of a non-democracy with a concession tax $\hat{\tau}$ applied for the foreseeable future results from similar analysis of elites being in a democracy and paying taxes of τ^P for the foreseeable future. That value is given in equation 18 above. Replacing τ^P with $\hat{\tau}$ provides the return to elites of always remaining in a non-democracy with tax rate $\hat{\tau}$. This utility is shown in equation 19.

$$V^r(N, \tau^N = \hat{\tau}) = \frac{y^r + \hat{\tau}(\bar{y} - y^r) - C(\hat{\tau})\,\bar{y}}{(1 - \beta)} \qquad (19)$$

If the utility of offering a concession tax $\hat{\tau}$ (equation 19) is greater than the utility of repression (equation 17), then the elites decide to offer a concession tax.

3.1.5.7 Decision to Mount a Coup

If the polity is democratic, the elites compare their utility of mounting a coup to the value of staying in the democracy and paying the concession tax offered from the citizens. The utility of mounting a coup is the value of being in a non-democracy with no threat of revolution minus the cost of the coup as defined in equation 20 (Acemoglu & Robinson, 2006, p. 233).

$$V^r(C) = V^r(N, \mu^L) - \varphi y^r \qquad (20)$$

The value of being in a non-democracy with no threat of revolution is shown in equation 13. The value of staying in a democracy where they pay τ^p during times of low threat of coup and they pay a concession tax $\tilde{\tau}$ during times of high threat of coup is shown in equation 14.

In the agent-based model, the elites compare the utility of mounting a coup (equation 20), to the utility of staying in the democracy (equation 14). If the utility of a coup is greater than that of staying in democracy, then the elites decide a have a coup. Following a coup, the cost of revolution is set to 1 (low threat of revolution) for one time period following the coup so that there is no immediate revolution (Acemoglu & Robinson, 2006, p. 231).

3.1.5.8 Decision to have a Revolution

The poor compare the utility of the current or offered democracy to the utility of revolutions. This occurs in two ways. First, if the maximum concession tax $\tilde{\tau}$ that the elites can offer in a non-democracy, is not redistributive enough, the elites may decide to democratize. Even if the elites democratize, the citizens may decide that the resulting democracy is still not redistributive enough and that a revolution is a better option for them. Second, if the democracy is not new, the citizens may decide after they make the concession tax $\tilde{\tau}$ promises to the elites, that they can get more value from a revolution. In both cases, the poor compare the utility of a revolution as defined in equation 15 to the utility of staying in the current democracy.

The poor consider that only in times of low threat of coup can they get a tax rate equal to their preferred rate τ^P. They also consider that in times of high threat of coup, they may be able to offer a concession tax, but there may also be a coup, and following that coup, and economic losses due to the coup, the tax rate in the non-democracy will depend on the poor's ability to pose a credible threat of revolution. The utility to the poor of staying in a democracy is conditioned on the future likelihood of coups and revolutions. The Bellman equations are defined as shown in equation 21 and 22 (Acemoglu & Robinson, 2006, p. 241). It describes the utility of a democracy to the poor as being the income from the current year plus the discounted value of the future. The income of the current year assumes a low threat of coup for the current year so the tax rate is maximized at τ^P. In equation 21, the value of the future considers that with probability s there will be a coup and the value to the poor is that of the economic loss of the coup $\boldsymbol{\varphi y^p}$ and the resulting non-democracy with low threat of revolution. The value of the future also considers that with probability (1-s) the citizens will keep the current democracy with value $V^p(D,\boldsymbol{\varphi}^L)$.

$$V^p(D,\boldsymbol{\varphi}^L) = y^p + \tau^P(\bar{y} - y^p) - C(\tau^P)\,\bar{y} \\ + \boldsymbol{\beta}[s\,(V^p(N,\mu^L) - \boldsymbol{\varphi y^p}) + (1\text{-}s)\,V^p(D,\boldsymbol{\varphi}^L) \qquad (21)$$

The value of the poor in a non-democracy is determined to be the value of the current year's income (no taxes) and the discounted value of what may happen in the future. As shown in equation 22, in the future, a revolution is expected with probability q and with probability (1-q) the citizens would remain in the non-democracy.

$$V^p(N, \mu^L) = y^p + \boldsymbol{\beta}[q\, V^p(D, \boldsymbol{\varphi}^L) + (1-q)\, V^p(N, \mu^L)] \qquad (22)$$

Acemoglu & Robinson solve 21 and 22 as two equations with two unknowns. Solving for $V^p(D, \boldsymbol{\varphi}^L)$ they find the utility as shown in equation 23 (Acemoglu & Robinson, 2006, p. 241).

$$V^p(D, \varphi^L) = \frac{y^p\,((1 - \varphi\beta s)(1 - \beta(1-q)) + \beta s) + (1 - \beta(1-q))\ (\tau^p(\bar{y} - y^p) - C(\tau^p)\,\bar{y})}{(1 - \beta(1-s))(1 - \beta(1-q)) - \beta^2 sq} \qquad (23)$$

In the agent-based model, the poor compare their utility of having a revolution (equation 15) to the value of staying in the democracy (equation 23). If the value of revolution is greater than staying in current democracy, then they have a revolution.

3.1.6 Implementation

3.1.6.1 Model Interface

The model user-interface as shown in Figure 4, allows the user to change the global knowledge variables and through a set of sliders. On the left, the user selects whether the initial polity is Democratic or Non-democratic. In addition, the user sets the population size (NumAgents), the percent of the population considered rich, $\boldsymbol{\delta}$ (FractionRichAgentstoNumAgents), the share of income accruing to the rich $\boldsymbol{\theta}$, the discount factor $\boldsymbol{\beta}$, the average income $\bar{\mathbf{y}}$**,** the likelihood of threat of revolution **q,** and the likelihood of threat of coup **s**.

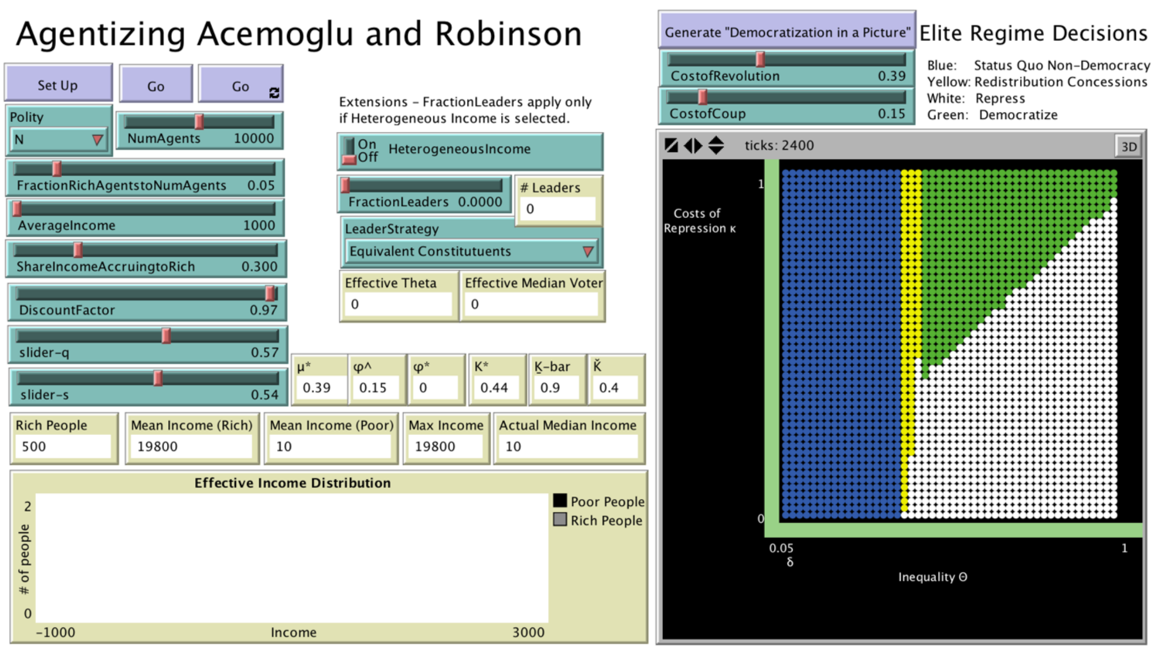


Figure 4: Model user interface

The middle section marked "Extension" consists of switches and parameters used to relax the game-theoretic assumptions of homogeneity and rationality. These parameters will be discussed in Chapter 4.

The model can be run in two modes. Section 3.1.6.2 describes the subset of A&R models defined as "A Dynamic Model of the Creation and Consolidation of Democracy" (Acemoglu & Robinson, 2006, p. 231). Section 3.4.describes the "Democratization in a Picture" mode which represents the empirical implications about intra-group inequality (Acemoglu & Robinson, 2006, p. 214).

3.1.6.2 Creation and Consolidation of Democracy Dynamic Model Mode

In the Creation and Consolidation of Democracy Dynamic Model Mode, inequality $\boldsymbol{\theta}$ is set by the user and is static throughout the simulation as the agents respond to different economic situations consisting of the cost of coup and cost of revolutions. The cost of coups and revolutions are dynamic and drawn randomly from a uniform distribution of discreet values in increments of .01 [0, 1]. This setup differs from the "Democratization in a Picture" mode where the economic conditions of the cost of coup and revolutions are held static and the model draws the decision space based values of repression $\boldsymbol{\kappa}$ and inequality $\boldsymbol{\theta}$.

In Creation and Consolidation of Democracy Dynamic Model mode, the user selects values on the left-hand side of the user interface. These include the parameters identified in Table 5 as "user-entered". Then the user selects the "Set Up" button which initiates the income distribution. Then the user selects "Go", either the single instance or the repeated instance "Go" button. The cost of coup and the cost of revolution are random variables that the simulation assigns at the beginning of each time step.

Table 5: Parameters for creation and consolidation of democracy

Parameter	Range	Change
Number of Agents	**1 – 10,000**	**Static: User-Entered**
Polity	**N, D**	**Dynamic: User-Initialized**
$\bar{y}$ **- mean income of the population**	**0 – $100,000**	**Static: User-Entered**
$\boldsymbol{\delta}$ **– the percent of population considered rich**	**(0, 0.25)**	**Static: User-Entered**
$\boldsymbol{\theta}$ **– the percent of total income shared among the rich**	**Range (0,1) Discreet values in increments of .01**	**Static: User-Entered**
$\boldsymbol{\varphi}$ **– The cost of coup as percent of GDP**	**Range [0, 1] Discreet values in increments of .01**	**Dynamic: uniform discreet distribution**
$\boldsymbol{\mu}$ **– the cost of revolution as percent of GDP**	**Range [0,1] Discreet values in increments of .01**	**Dynamic: uniform discreet distribution**
$\boldsymbol{\beta}$ **– discount factor**	**Range [0,1] Discreet values in increments of .01**	**Static: User-Entered**
s – **the likelihood that there will be a high threat of coup**	**Range [0,1] Discreet values in increments of .01**	**Static: User-Entered**
q - **the likelihood that there will be a high threat of revolution**	**Range [0,1] Discreet values in increments of .01**	**Static: User-Entered**

The polity variable is initialized by the user as either democratic or non-democratic. It is dynamic in that it may change during the simulation. By the end of each time step, the agents have made decisions which determine whether the polity is democratic or non-democratic for the next time step. In the repeated instance “Go”, the model continues to the new time step with the updated polity variable D - Democracy or N-Non-democracy, and new random variables are chosen for the cost of coup and the cost of revolution.

3.1.6.3 Model Flow

This section describes the model flow. Figure 5 illustrates the flow of activities and processing for the agent-based model to reproduce the game-theoretic model of the Creation and Consolidation of Democracy.

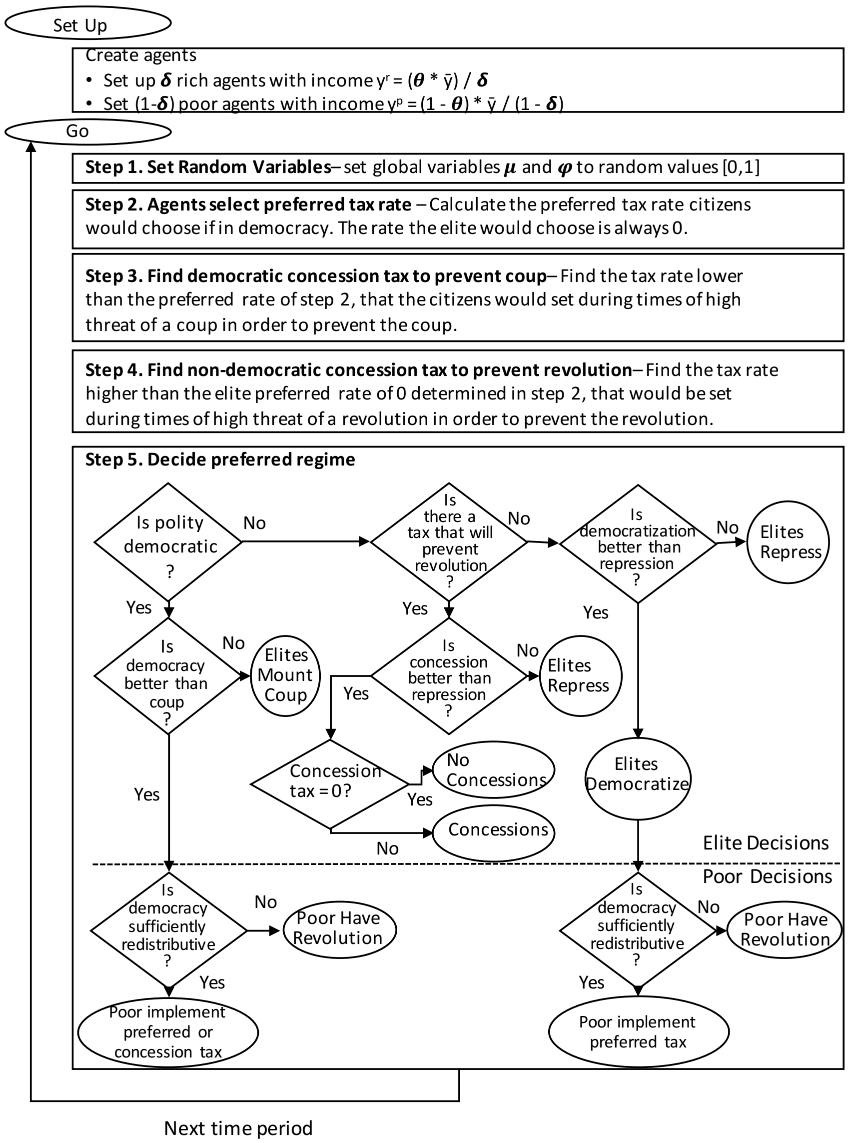


Figure 5: Model flow of the creation and consolidation of democracy

<u>SETUP</u> The "Setup" button on the user interface sets up the community of agents as described in the process flow of Figure 5. The "Setup" button executes the following steps:

1. *Setup constants with initial values*

2. *Set the average income for each class y^r and y^p*
3. *Set up Agents*

Set up $\boldsymbol{\delta}$ *rich agents with income* $y^r = (\boldsymbol{\theta} * \bar{y}) / \boldsymbol{\delta}$

Set (1-$\boldsymbol{\delta}$*) poor agents with income* $y^p = (1 - \boldsymbol{\theta}) * \bar{y} / (1 - \boldsymbol{\delta}$

GO The user next selects either the "Go" button or the "Go Forever" button to run the simulation. The Go button executes the commands in steps 1 through 5 (Figure 5) just once. The "Go Forever" button executes the commands in steps 1 through 5 continuously. When the "Go Forever" button is selected, the equilibrium decisions may change the polity and the new polity variable is carried over to the next time step.

Step 1. Set Random Variables

The A&R model states that the variables $\boldsymbol{\mu}$ and $\boldsymbol{\varphi}$ are exogenous and are known to all players / citizens. The model assigns a random number between (0,1] to $\boldsymbol{\mu}$ and a separate random number between (0,1] to $\boldsymbol{\varphi}$.

Step 2. Agents select preferred tax rate

The agent-based model calls a routine that assigns a value to $\boldsymbol{\tau}$, the tax rate that maximizes the utility of the median voter. Since the median voter is a poor agent, this is called the poor preferred tax rate and is identified as $\boldsymbol{\tau}^p$. The model performs processing as described in section 3.1.5.2 Poor Preferred Tax Rate.

Step 3. Find democratic concession tax to prevent coup

The agent-based model calls a routine that assigns a value to $\tilde{\boldsymbol{\tau}}$**,** the concession tax set by the poor in a democracy. $\tilde{\boldsymbol{\tau}}$ is set as high as possible to provide the most value to the citizens and as low as necessary to provide enough value to the elites in order to prevent a coup. The model performs processing as described in section 3.1.5.3 Concession Tax to Prevent Coups. If the elites would prefer a coup even over a tax rate of 0 in a democracy, then the coup flag is set.

Step 4. Find non-democratic concession tax to prevent revolution

The agent-based model calls a routine that assigns a value to $\hat{\tau}$, the concession tax set by the rich in a non-democracy as necessary to prevent a revolution. The model performs processing as described in section 3.1.5.4 Concession Tax to Prevent Revolution. This value of $\hat{\tau}$, is used by both the elites and the poor in determining their utilities in various regimes. If a value of $\hat{\tau}$, is not found then the flag "revolution imminent" is set. This flag will be checked by the elites, and if set means that a concession tax will not be sufficient to prevent a revolution. Therefore, the Elites will have to either repress or create a democracy when there is a threat of revolution.

Step 5. Decide preferred regime

The agent-based model executes the process flow as defined in step 5 of Figure 5. Step 5 results in the equilibrium policy and regime decisions being made by the rich and poor agents. If the polity is non-democratic, the elites determine if there is a tax rate that will prevent revolution by checking whether the flag "revolution imminent" was set during step 4. If a tax can't prevent a revolution, then the elites decide between democratization or repression by following the process described in section 3.1.5.5. If the elites repress, it is assumed to be successful (Acemoglu & Robinson, 2006, p. 186). If the elites democratize, the poor will evaluate their utility under the new democracy and assess whether it is redistributive enough or if they would be better off with a revolution following the process described in section 3.1.5.8. If they have a revolution, a fraction of the economy is destroyed and the poor share the remaining income. The cost of a coup, $\boldsymbol{\varphi}$, is set to 1 so that democracy has a chance (Acemoglu & Robinson, 2006, p. 231). If they don't have a revolution, the poor preferred tax rate $\boldsymbol{\tau}^p$ identified in section 3.1.5.2 is set by the median voter.

If the flag "revolution imminent" was not set in step 4, then the elites know they can set a concession tax appropriately to prevent a revolution. However, the elites may choose to repress rather than offer a concession tax following the processing in section 3.1.5.6. If the elites choose to repress, then it is assumed to be successful (Acemoglu & Robinson, 2006, p. 186). If they choose to offer the concession tax to prevent revolution, then the tax found in step 4 following the processing of in section 3.1.5.4 would be implemented by the elites and is assumed to be successful in preventing revolution. Perhaps revolution is so costly to the poor that no concession tax is needed. If no concession tax is needed, then a tax of 0 would have been found in step 4 and that would be implemented. This state is considered "Political Status Quo No Concessions" (Acemoglu & Robinson, 2006, p. 214).

Returning to the decision diamond in the top left of step 5, if the polity is democratic, the elites decide whether to have a coup following the process of section 3.1.5.7. If they mount a coup, the cost of revolution $\boldsymbol{\mu}$ is set to 1 so that it is not immediately followed by revolution in the same time period (Acemoglu & Robinson, 2006, p. 231). If they do not have a coup, the poor will evaluate their utility assess whether the democracy is redistributive enough or if they would be better off with a revolution following the process described in section 3.1.5.8. If they have a revolution, a fraction of the economy is destroyed and the poor share the remaining income. The cost of a coup, $\boldsymbol{\varphi}$, is set to 1 so that the new democracy has a chance (Acemoglu & Robinson, 2006, p. 231). If they don't have a revolution, the concession tax $\tilde{\tau}$ as described in section 3.1.5.3 is set by the median voter. It may be that the concession tax as high as τ^{p} is acceptable and implemented.

3.2 Stylized Facts: Identification of Critical Values

Acemoglu & Robinson assert in Proposition 7.2 (see section 2.4) that the equilibrium regime decisions depend on critical values for the cost of revolution $\boldsymbol{\mu}$, cost of coup $\boldsymbol{\varphi}$ and cost of repression $\boldsymbol{\kappa}$. These critical values $\boldsymbol{\mu}^*$, $\hat{\boldsymbol{\varphi}}$, $\boldsymbol{\varphi}^*$, $\boldsymbol{\kappa}^*$, $\bar{\boldsymbol{\kappa}}$, $\breve{\boldsymbol{\kappa}}$ are defined in Table 6. Rather than calculating them directly, the agent-based model finds these critical values over time by observing the choices made. Table 6 summarizes how the Agent-based model finds these critical values.

Acemoglu & Robinson use these critical values in order to explain the phenomenon that democratization is a way for the elites to commit to future redistribution. Phenomenon that is explained by a model are called stylized facts. In addition, stylized facts provide the point of reference that can be used to compare different types of models (Boland, 1987/1994).

Table 6: Table of ABM implementation of critical values

Critical Value	<u>**Agent-based method**</u> For a given economic environment consisting of inequality $\boldsymbol{\theta}$, a discount factor $\boldsymbol{\beta}$, and the likelihood of threat of revolution q, and likelihood of threat of coup s:	<u>**Equation in dynamic game**</u>
$\boldsymbol{\mu}^*$	Over time, the elites will find the highest cost of revolution $\boldsymbol{\mu}^*$ for which the poor will still have a revolution even with an offer of the maximum redistribution, $\boldsymbol{\tau}^p$.	$\boldsymbol{\mu}^* = \boldsymbol{\theta}-(1-\boldsymbol{\beta}(1-q))$ * $(\boldsymbol{\tau}^p(\boldsymbol{\theta}-\boldsymbol{\delta})-(1-\boldsymbol{\delta})C(\boldsymbol{\tau}^p))$ (A&R, 2006, eq. 7.22)
$\widehat{\boldsymbol{\varphi}}$	The agents assess the lowest $\boldsymbol{\varphi}$ experienced which did not result in a coup, even at the highest tax rate τ^p.	$\widehat{\boldsymbol{\varphi}} = \frac{1}{\boldsymbol{\theta}}\left(\frac{\boldsymbol{\delta}C(\boldsymbol{\tau}^p)-\boldsymbol{\tau}^p(\boldsymbol{\delta}-\boldsymbol{\theta})}{1-\boldsymbol{\beta}(1-q)}\right)$ (A&R, 2006, eq. 7.16)
$\boldsymbol{\varphi}^*$	The agents assess the highest $\boldsymbol{\varphi}$ experienced which resulted in a coup even at the lowest tax rate of 0.	$\boldsymbol{\varphi}^* = \frac{1}{\boldsymbol{\theta}}\left(\frac{\boldsymbol{\beta}(q+s-1)(\boldsymbol{\tau}^p(\boldsymbol{\delta}-\boldsymbol{\theta})-\boldsymbol{\delta}C(\boldsymbol{\tau}^p)}{1-\boldsymbol{\beta}(1-q)}\right)$ (A&R, 2006, eq. 7.17)
$\boldsymbol{\kappa}^*$	Agents assess the highest $\boldsymbol{\kappa}$ experienced with which the elites still preferred repression over concession.	$\boldsymbol{\kappa}^* = \frac{1}{\boldsymbol{\theta}}(\boldsymbol{\delta}C(\hat{\boldsymbol{\tau}}) - \hat{\boldsymbol{\tau}}(\boldsymbol{\delta}-\boldsymbol{\theta}))$ (A&R, 2006, eq. 6.20)
$\breve{\boldsymbol{\kappa}}$	Agents assess the highest $\boldsymbol{\kappa}$ experienced with which the elites will choose repression over creating an unconsolidated democracy.	$\overline{\boldsymbol{\kappa}} > \boldsymbol{\kappa}(\boldsymbol{\varphi}) > \breve{\boldsymbol{\kappa}}$ (A&R, 2006, p. 243)
$\overline{\boldsymbol{\kappa}}$	Agents assess the highest $\boldsymbol{\kappa}$ experienced with which the then the elites will choose repression over a fully consolidated democracy. At $\overline{\boldsymbol{\kappa}}$, they are indifferent between democratization with high cost of a coup $\boldsymbol{\varphi} > \hat{\varphi}$ and repression.	$\overline{\boldsymbol{\kappa}} = \frac{\boldsymbol{\delta}C(\boldsymbol{\tau}^p)-\boldsymbol{\tau}^p(\boldsymbol{\delta}-\boldsymbol{\theta})}{\theta(1-\boldsymbol{\beta}(1-q))}$ $\overline{\boldsymbol{\kappa}} > \boldsymbol{\kappa}^*$ (A&R, 2006, eq. 6.21)

The observed phenomenon is described in the paragraph below. The following subsections describes the agent-based modeling process to find the critical values in more detail.

As long as the poor can exert pressure for redistribution in a non-democracy, the poor will get redistributive concessions (where inequality is low and repression costs are high enough). As inequality increases, the amount of redistribution can't be met by concessions, because the poor know that the redistribution may stop next time period, if they can't continue to maintain the pressure. Given this uncertainty, they are more likely to have a revolution. Then the elites are facing the unproductive costs of repression, or they will democratize. In effect, democratization is a way for the elites to make a continued promise of redistribution in order to avoid the costs of revolution (Acemoglu & Robinson, 2006, p. 155).

3.2.1 Cost of Revolution

If the polity is currently non-democratic, and if the cost of revolution $\boldsymbol{\mu}$, is below the critical threshold $\boldsymbol{\mu}^*$, then even the maximum amount of redistribution $\tau^N = \tau^p$, will result in a revolution. The elites, though, will lose everything in a revolution, and therefore, the only option for them is to democratize. Acemoglu & Robinson have found an equation for the value of $\boldsymbol{\mu}^*$(see Table 6). In the agent-based simulation, the elites don't know the value of $\boldsymbol{\mu}^*$. For the current economic conditions of inequality, likelihood of revolution and discount factor, the poor calculate the utility of poor for the choice of revolution and the choice of staying in the non-democracy with a concession of redistribution. Over time, the elites will find the highest cost of revolution $\boldsymbol{\mu}^*$ for which the poor will still have a revolution even with an offer of the maximum redistribution.

3.2.2 Cost of Coup

If the polity is currently a democracy, and if the rich will not prefer a coup at the maximum tax rate τ^{p}, then the democracy is called "fully consolidated". There will be a cost of coup $\widehat{\varphi}$, that as long as $\boldsymbol{\varphi} < \widehat{\varphi}$, a coup is never beneficial for the elites even if the poor tax at $\tau^{D} = \tau^{p}$. Acemoglu & Robinson have calculated the value $\widehat{\varphi}$ by finding the value of $\boldsymbol{\varphi}$ for which the rich are indifferent between having a coup, or living forever in a democracy with $\tau^{D} = \tau^{p}$ (see Table 6). In the agent-based simulation, the poor agents don't know the value of $\widehat{\varphi}$. For the current economic conditions of inequality, likelihood of revolution and discount factor, the poor calculate the utility of rich for the choice of coup and the choice of staying in the democracy. The poor will discover that as long as the cost of coup remain above a critical threshold $\widehat{\varphi}$, the rich will never have a coup, and the poor can tax at their highest preferred rate.

For a given set of economic conditions, when the cost of coup is below the critical threshold $\boldsymbol{\varphi}^*$, the democracy is considered "unconsolidated" and even a strategy of setting $\tau^{D} = 0$ will not prevent a coup. Acemoglu and Robinson have determined an equation for $\boldsymbol{\varphi}^*$ (see Table 6). In the simulation, the poor will find over time that as $\boldsymbol{\varphi}$ changes, sometimes there is coup even when $\tau^{D} = 0$ and sometimes there isn't. The poor will discover highest cost of coup $\boldsymbol{\varphi}$ seen so far, for which there is always a coup regardless of the concession tax offered.

If a polity is non-democratic, then the values of $\boldsymbol{\varphi}^*$ and $\widehat{\varphi}$ tell you whether, if the elites were to form a democracy, and the fundamental economic conditions remained the same, would the democracy be subject to coups (unconsolidated) or consolidated?

3.2.3 Cost of Repression

If the polity is currently a non-democracy, and if there is no threat of revolution, then the elites continue the status quo with no policy changes or tax concessions. However, if there is a threat of revolution, then the elites must decide between repression, redistribution, and democratization. If inequality is low enough, then there exists a tax rate that will provide the same utility to the poor as a revolution. In these economic conditions, a concession tax is sufficient to prevent a revolution. However, if the cost of repression $\boldsymbol{\kappa}$ is low enough $\boldsymbol{\kappa} < \boldsymbol{\kappa}^*$ then the elites will prefer repression over redistribution. The critical value $\boldsymbol{\kappa}^*$ represents the cost of repression where the elites are indifferent between repression and redistribution through a concession tax. The equation Acemoglu and Robinson have found for $\boldsymbol{\kappa}^*$ is shown in Table 6.

If inequality is over a certain point, then even if the elites offer the maximum redistribution through a tax rate equal to the poor preferred tax rate, it is not enough to stop a revolution. The elites must either democratize, or repress. That decision depends on the current cost of repression $\boldsymbol{\kappa}$ and the likelihood of coups in the future which would return the polity to a non-democracy and stop the redistribution. If the cost of coup is high $\boldsymbol{\varphi} > \hat{\boldsymbol{\varphi}}$, then a coup is never beneficial to the rich even if the poor set their preferred tax rate τ^p. Once the democracy is created, it never returns to a non-democracy and the elites will expect to pay the preferred redistribution tax τ^p for the foreseeable future. This situation is called a "consolidated democracy". The critical value $\bar{\boldsymbol{\kappa}}$ represents the cost of repression where the elites are indifferent between repression and democratization to a consolidated democracy. The equation Acemoglu and Robinson have found for $\bar{\boldsymbol{\kappa}}$ is shown in Table 6.

If the cost of coup is low $\boldsymbol{\varphi} < \boldsymbol{\varphi}^*$, then the democratic regime created is expected to be short-lived and a coup is likely to occur in the future. This situation is called an "unconsolidated democracy". The critical value $\breve{\boldsymbol{\kappa}}$ represents the cost of repression where the elites are indifferent between repression and democratization to a unconsolidated democracy. The equation Acemoglu and Robinson have found for $\breve{\boldsymbol{\kappa}}$ is shown in Table 6.

An unconsolidated democracy is preferred by the elites over a consolidated democracy, so indifference value of the cost of repression is higher in the environmental conditions that would create a consolidated democracy. $\bar{\boldsymbol{\kappa}} > \breve{\boldsymbol{\kappa}}$ indicates that the elites are willing to pay more in repression costs to avoid a consolidated democracy.

3.3 Model Verification and Validation

Verification evaluates whether the model was built correctly and validation evaluates how well it represents the real world (North & Macal, 2007). An agent-based model simulates an outcome and hence must be run multiple times while the set of potential outcomes are studied with statistical tools. An ABM inherently recognizes that there is always a little bit of chance involved in life's processes. Even when an agent-based model with run with the same parameters, there will be run-to-run variations due to some of the random components of the model. These random components include the timing and sequencing of when agent because active, sense their world, take an action, and monitor effects.

3.3.1 Verification

The model was tested for the accuracy of its logic. Errors in the coding could create erroneous results or leave unusual behavior in specific circumstances. The code was examined for errors. Outputs at interim stages were captured and evaluated to verify the processing logic was producing the correct interim results and the correct final results.

Several methodological approaches were used to analyze the model outputs and verify the model is working as designed. Many of these tests begin with a “baseline” set of parameters of a typical simulation run. Then those parameters are modified in different ways producing different results which can be analyzed to gain a better understanding of the complex interactions being modeled.

For this agent-based model, the baseline case is defined in Table 7. Two of these parameters concern how to describe the income distribution of the population. In the A&R models, they describe the income distribution by specifying the fraction of population considered rich, and the share of income they receive. For example, description of the U.S. income distribution can be made many ways as shown in Table 8 (Williams, 2016). This table used data from the Internal Revenue Service (IRS). One can say 1% of the population earns 19% of the income. Or one can say 5% of the population earns 34.4% of the income. Or one can say the top 50% of the population earns 88.5% of the income. In this agent-based model, for verification and validation, I used baseline values of $\boldsymbol{\delta}$ = 5% and $\boldsymbol{\theta}$ = 30%.

Table 7: Parameters for baseline case

Parameter	Range	Baseline Value	Change
Number of Agents	1 – 10,000	10,000	Static: User-Entered
Polity	N, D	N	Dynamic: User-Initialized
$\bar{y}$ - mean income of the population	0 – $100,000	$1,000	Static: User-Entered
δ – the percent of population considered rich	(0, 0.25)	.05	Static: User-Entered
θ – the percent of total income shared among the rich	Range (0,1) Discreet values in increments of .01	.30	Static: User-Entered
φ – The cost of coup as percent of GDP	Range [0, 1] Discreet values in increments of .01	Uniform on (0,1]	Dynamic: uniform discreet distribution
μ – the cost of revolution as percent of GDP	Range [0,1] Discreet values in increments of .01	Uniform on (0,1]	Dynamic: uniform discreet distribution
β – discount factor	Range [0,1] Discreet values in increments of .01	.97	Static: User-Entered
s – the likelihood that there will be a high threat of coup	Range [0,1] Discreet values in increments of .01	.25	Dynamic: uniform discreet distribution
q - the likelihood that there will be a high threat of revolution	Range [0,1] Discreet values in increments of .01	.25	Dynamic: uniform discreet distribution

Table 8: Income distribution in the United States (2013). (Williams, 2016).

Income Category	Percent of All U.S. Income
Top 1%	19%
Top 5%	34.4%
Top 10%	45.9%
Top 25%	68.1%
Top 50%	88.5%

There are other methodologies used to describe share of income and it depends on what types of income such a labor, and capital gains, are included. For a cross-country specification, see Table 43 in Appendix A. This table lists the country ranking of the income share of the top 10%. In this World Bank methodology, the United States, where the top 10% control 30.19% of the income, is ranked 72 out of 153 countries. The first, Namibia had the highest inequality with the top 10% accruing 51.84% of the income. Azerbaijan had the lowest, with the top 10% accruing just 17.41% of the income. However, Piketty and Saez (2014) used IRS data in their analysis on the growth of inequality in the United States and reports that the top 10% earned between 45% and 50% of the income, in corroboration with Williams (2016). It appears that both methodologies are in use. For cross-country comparisons in this dissertation, I will use the data reported by PovcalNet research on income inequality and poverty survey of 153 countries using World Bank data (PovcalNet, 2016).

For comparison to the United States (where the top 10% control 30.19% of the income), in South America, Colombia has the highest income share: 10% of the population hold 41.9% of the income and Argentina has the lowest income share: 10% of the population hold 30.56% of the income (PovcalNet, 2016). A high-inequality example is South Africa, which ranks second, where the top 10% hold 51.26% of the income. A low-inequality example is Norway (rank 151) where the top 10% earn 20.90% of the income.

The following methodology was used for verification. Each methodology and associated tables of results are discussed in the succeeding sections.

Baseline Convergence – I run the model with the baseline parameters identified in Table 7 to see how long it takes for the critical values to converge to a steady value. This length of time is used to evaluate the sensitivity analysis. It takes time for the agent based model to converge to a steady value because the agents do not have enough experiences. The critical values were recorded at increasing time steps to understand how quickly enough experiences were found to determine the critical values. The A&R models do not have a concept of convergence, because their agents are able to calculate the best solution in advance.

Sensitivity Analysis - Sensitivity analysis in an agent-based model (Gilbert and Troitzch, 2005, p. 24) is performed by instantiating the model multiple times, each time varying one of the parameters. To perform the sensitivity analysis, I varied each input parameter while keeping the other parameters the same as the baseline case. Thirty baseline runs were performed with the baseline values in Table 7. In addition, the model was run with each user selected parameter modified over 3 additional values. The model was run 30 times with each combination of values in order to verify the results make sense and the model is working as intended.

Relative Value Testing – This activity consists of examining relationships between inputs and outputs. Each input value was modified during the sensitivity analysis, and the resulting critical values were recorded. The impact that the change in input value had on the critical values was recorded and compared to what was expected.

3.3.1.1 Baseline Convergence

The baseline case was run and critical values recorded at four time steps: 10 ticks, 100 ticks, 1000 ticks and 10,000. Table 9 shows the mean and standard deviation for the critical values at those time steps. There were 30 trials at t = 10, and I found a large amount of variance in the values. To understand this, consider that when the elites make the decision to mount a coup even when offered a tax rate of 0, they check to see if they had made that decision at any higher values of $\boldsymbol{\varphi}$, if not they have found the value of $\boldsymbol{\varphi}$* which is the maximum value of $\boldsymbol{\varphi}$ for which they will always have a coup regardless of the tax rate offered. It takes several hundreds of iterations of assessing the conditions and making decisions for the critical values to converge on a number with little variance. Therefore, in the beginning of the agent-based simulation, the values of $\boldsymbol{\mu}$*, $\hat{\boldsymbol{\varphi}}$, $\boldsymbol{\varphi}$*, $\boldsymbol{\kappa}$*, $\breve{\boldsymbol{\kappa}}$, and $\bar{\boldsymbol{\kappa}}$ will change until there is enough experience to build the steady state value. At t=10,000, $\boldsymbol{\kappa}$* was the only value that had any variance and it was very slight.

Table 9: Baseline case convergence

	At t=10		At t=100		At t=1000		At t=10,000	
	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation
$\boldsymbol{\mu}^*$	0.196	0.082	0.282	0.011	0.290	0.000	0.290	0.000
$\boldsymbol{\varphi}^*$	0.308	0.086	0.395	0.009	0.400	0.000	0.400	0.000
$\hat{\boldsymbol{\varphi}}$	0.824	0.081	0.739	0.012	0.730	0.000	0.730	0.000
$\boldsymbol{\kappa}^*$	0.217	0.200	0.027	0.023	0.010	0.002	0.011	0.004
$\breve{\boldsymbol{\kappa}}$	0.864	0.120	0.699	0.087	0.618	0.012	0.610	0.000
$\bar{\boldsymbol{\kappa}}$	0.986	0.040	0.948	0.062	0.850	0.021	0.830	0.000

3.3.1.2 Sensitivity Analysis and Relative Value Testing

In Scenario 1, I modified the mean income $\bar{y}$. Table 10 shows the results of the parameter sweeps when the baseline values are held constant while the mean income $\bar{y}$ of the population is modified from 100 to 100,000. 30 trials were run at each value of $\bar{y}$. The critical values were recorded at time step 10,000 to ensure variance is not due to the agents not having enough experiences.

Table 10: Scenario 1 - modifying mean income

	At $\bar{y}$=100		Baseline Value At $\bar{y}$=1,000		At $\bar{y}$=10,000		At $\bar{y}$=100,000	
	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation
$\boldsymbol{\mu}$*	0.290	0.000	0.290	0.000	0.290	0.000	0.290	0.000
$\boldsymbol{\varphi}$*	0.410	0.000	0.400	0.000	0.400	0.000	0.400	0.000
$\hat{\boldsymbol{\varphi}}$	0.740	0.000	0.730	0.000	0.730	0.000	0.730	0.000
$\boldsymbol{\kappa}$*	0.033	0.065	0.011	0.004	0.010	0.000	0.010	0.000
$\breve{\boldsymbol{\kappa}}$	0.620	0.000	0.610	0.000	0.610	0.000	0.610	0.000
$\bar{\boldsymbol{\kappa}}$	0.850	0.000	0.830	0.000	0.830	0.000	0.830	0.000

As the mean income increases from 100 to 1,000 there are very slight changes in the critical values. $\boldsymbol{\mu}$* doesn't change. $\breve{\boldsymbol{\kappa}}$ decreases from .85 to .83. $\bar{\boldsymbol{\kappa}}$ decreases from .62 to .61. Only the critical value for $\boldsymbol{\kappa}^*$ has a non-zero variance at the end of 10,000 steps. The insensitivity of the critical values to changes in income is as expected because it is the relative values of the incomes of the rich class and the poor class rather than the nominal value of the income level that affects regime decisions.

For scenario 2, Table 11 shows that as the number of agents in the simulation changes the critical values change very little in this agent-based model with homogeneous agents. As the number of agents increase from 100 to 20,000 there are slight changes in the critical values. But none of the parameters for number of agents resulted in critical values outside 1/2 standard deviation from the baseline value. This is as expected since because all the agents are homogenous, and the population is defined by its relative values of $\boldsymbol{\delta}$ and $\boldsymbol{\theta}$.

Table 11: Scenario 2 - modifying number of agents

	At **N=100**		At N**=1,000**		Baseline Value At N**=10,000**		At **N=20,000**	
	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation
$\boldsymbol{\mu}$*	0.290	0.000	0.290	0.000	0.290	0.000	0.290	0.000
$\boldsymbol{\varphi}$*	0.400	0.000	0.400	0.000	0.400	0.000	0.400	0.000
$\hat{\boldsymbol{\varphi}}$	0.730	0.000	0.730	0.000	0.730	0.000	0.730	0.000
$\boldsymbol{\kappa}$*	0.010	0.002	0.011	0.004	0.011	0.004	0.010	0.002
$\breve{\boldsymbol{\kappa}}$	0.619	0.013	0.616	0.008	0.610	0.000	0.618	0.010
$\bar{\boldsymbol{\kappa}}$	0.844	0.016	0.841	0.013	0.830	0.000	0.843	0.015

In Scenario 3, I modified the discount factor $\boldsymbol{\beta}$. Table 12 shows the results of the sensitivity analysis when the baseline values are held constant while the discount factor $\boldsymbol{\beta}$ of the economy is decreased from .99 to .80. All values are recorded at time step 10,000.

Table 12: Scenario 3 - modifying discount factor

	At β =.99		Baseline Value At β =.97		At β =.90		At β =.80	
	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation
μ^*	0.290	0.000	0.290	0.000	0.290	0.000	0.280	0.000
φ^*	0.430	0.000	0.400	0.000	0.310	0.000	0.220	0.000
$\hat{\varphi}$	0.770	0.000	0.730	0.000	0.610	0.000	0.490	0.000
κ^*	0.012	0.006	0.011	0.004	0.010	0.000	0.015	0.009
$\breve{\kappa}$	0.660	0.000	0.610	0.000	0.470	0.002	0.342	0.005
$\bar{\kappa}$	0.880	0.000	0.830	0.000	0.700	0.002	0.570	0.000

The discount factor is the rate by which you multiply a benefit in the future to get the net present value. It represents the time value of money and the uncertainty risk. The lower the discount factor, the higher the risk of receiving that benefit in the future. Countries with high inflation have a lower discount factor. Table 13 defines what happens to the critical values as the discount factor is modified.

Table 13: Relative value analysis of discount factor

Critical Value	Change
μ^*	As the discount factor decreases, μ^* changes very little.
φ^*	As the discount factor decreases, φ^* decreases.
$\hat{\varphi}$	As the discount factor decreases, $\hat{\varphi}$ decreases.
κ^*	As the discount factor decreases, κ^* stays at a very low value.
$\breve{\kappa}$	As the discount factor decreases, $\breve{\kappa}$ decreases.
$\bar{\kappa}$	As the discount factor decreases, $\bar{\kappa}$ decreases.

We can interpret these results by looking at Figure 6. The value of $\boldsymbol{\theta}$ that corresponds to $\boldsymbol{\mu}^*$ is called $\boldsymbol{\theta}^*$ and demarks the vertical line between whether, in a non-democracy with high cost of repression, the elites choose to offer concessions or democratize. This vertical line is insensitive to changes in the discount factor $\boldsymbol{\beta}$. However, the indifference between repression and democratization is affected by changes in $\boldsymbol{\beta}$. As the future value decreases, the elites will be more likely to democratize (unconsolidated) than use repression. As the future value decreases, the elites will be more likely to democratize (consolidated) than use repression. Thus, democratizations to both consolidated and unconsolidated democracies become more likely when the discount factor is lower. This tracks well with the South American countries with high fiscal volatility that create democracies rather readily even though they may be short-lived.

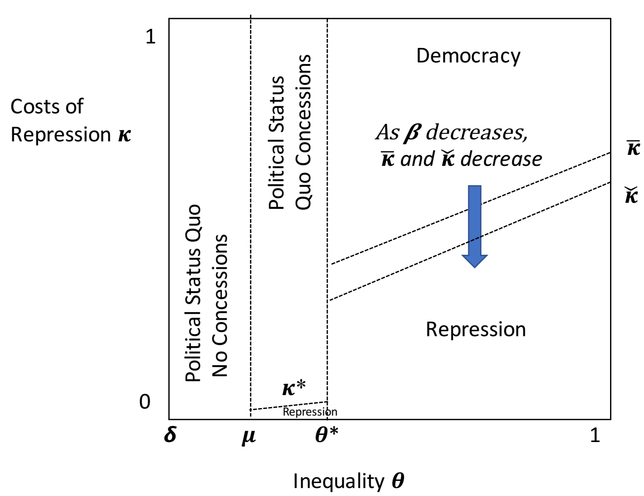


Figure 6: Effects of changes in $\boldsymbol{\beta}$ on "Democratization in a Picture"

In Scenario 4, I modify the percent $\boldsymbol{\delta}$ of population that is rich. Table 14 shows the percent $\boldsymbol{\delta}$ of rich agents is modified from .01 to .10. All values are recorded at time step – 10,000. The baseline value of $\boldsymbol{\theta}$ is .30 so all values in the parameter sweep meet the condition of the A&R models that $\boldsymbol{\delta} < \boldsymbol{\theta}$.

Table 14: Scenario 4 – modifying percent $\boldsymbol{\delta}$ of population that is rich

	At $\boldsymbol{\delta}$ =.01		At $\boldsymbol{\delta}$ =.02		Baseline Value At $\boldsymbol{\delta}$ =.05		At $\boldsymbol{\delta}$ =.20	
	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation
$\boldsymbol{\mu}^*$	0.280	0.000	0.280	0.000	0.290	0.000	0.290	0.000
$\boldsymbol{\varphi}^*$	0.500	0.000	0.470	0.000	0.400	0.000	0.080	0.000
$\hat{\boldsymbol{\varphi}}$	0.880	0.000	0.840	0.000	0.730	0.000	0.160	0.000
$\boldsymbol{\kappa}^*$	0.037	0.053	0.028	0.038	0.011	0.004	0.010	0.000
$\breve{\boldsymbol{\kappa}}$	0.770	0.000	0.720	0.000	0.610	0.000	0.160	0.020
$\bar{\boldsymbol{\kappa}}$	1.000	0.000	0.991	0.003	0.830	0.000	0.181	0.003

Table 15 defines what happens to the critical values as the percent $\boldsymbol{\delta}$ of the population that is rich changes. The critical values in general depend on the difference between $\boldsymbol{\delta}$ and the income share accruing to the rich $\boldsymbol{\theta}$. Consider that when $\boldsymbol{\theta}$ is held constant, increasing $\boldsymbol{\delta}$ means decreasing inequality. As inequality decreases, $\boldsymbol{\varphi}^*$ and $\hat{\boldsymbol{\varphi}}$ decrease. This means a consolidated democracy is much more likely, and an unconsolidated democracy becomes less likely. This makes sense since with low inequality, the elites have less incentive to mount a coup, so the democracy will be more stable.

Table 15: Relative value analysis percent $\boldsymbol{\delta}$ rich agents

Critical Value	Change
$\boldsymbol{\mu}^*$	As the percent $\boldsymbol{\delta}$ of rich agents increases, $\boldsymbol{\mu}^*$ decreases slightly.
$\boldsymbol{\varphi}^*$	As the percent $\boldsymbol{\delta}$ of rich agents increases, $\boldsymbol{\varphi}^*$ decreases
$\hat{\boldsymbol{\varphi}}$	As the percent $\boldsymbol{\delta}$ of rich agents increases, $\hat{\boldsymbol{\varphi}}$ decreases
$\boldsymbol{\kappa}^*$	As the percent $\boldsymbol{\delta}$ of rich agents increases, $\boldsymbol{\kappa}^*$ remains low and decreases
$\breve{\boldsymbol{\kappa}}$	As the percent $\boldsymbol{\delta}$ of rich agents increases, $\breve{\boldsymbol{\kappa}}$ decreases
$\bar{\boldsymbol{\kappa}}$	As the percent $\boldsymbol{\delta}$ of rich agents increases, $\bar{\boldsymbol{\kappa}}$ decreases

Again, we can interpret these results by looking at Figure 7. The value of $\boldsymbol{\theta}$ that corresponds to $\boldsymbol{\mu}^*$ is called $\boldsymbol{\theta}^*$ and demarks the vertical line between whether, in a non-democracy with high cost of repression, the elites choose to offer concessions or democratize. This vertical line is only slightly sensitive to changes in $\boldsymbol{\delta}$ while keeping the same baseline $\boldsymbol{\theta}$. However, both $\bar{\boldsymbol{\kappa}}$ and $\breve{\boldsymbol{\kappa}}$ decrease as the percent $\boldsymbol{\delta}$ increases and gets closer to $\boldsymbol{\theta}$ (.30). When inequality decreases, the democracy is expected to be less redistributive. When the democracy is less redistributive, it is less threatening to the elites, $\bar{\boldsymbol{\kappa}}$ and $\breve{\boldsymbol{\kappa}}$ are shifted down meaning that repression becomes optimal only for lower repression costs, and the area of democratization gets expanded. The effect of less redistributive democracies being less threatening to elites is discussed in the book Economic Origins of Dictatorship and Democracy (Acemoglu & Robinson, 2006, p. 215. figure 6.5).

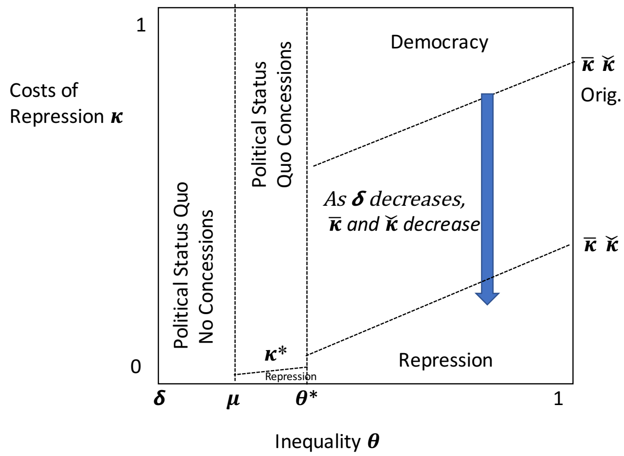


Figure 7: Effects of changes in $\boldsymbol{\delta}$ on "Democratization in a Picture"

In Scenario 5, I modify the percent of income accruing to the rich $\boldsymbol{\theta}$. Table 16 shows the results of the parameter sweeps when the baseline values are held constant while the percent of income accruing to the rich $\boldsymbol{\theta}$ is modified from .08 to .80. The baseline value of $\boldsymbol{\delta}$ is .05, so the minimum value of $\boldsymbol{\theta}$ tested was .08 so all values in the parameter sweep meet the condition of the A&R models that $\boldsymbol{\delta} < \boldsymbol{\theta}$. All values are recorded at time step – 10,000.

Table 16: Scenario 5 - modifying percent of income accruing to the rich $\boldsymbol{\theta}$

	At $\boldsymbol{\theta}$ =.08		At $\boldsymbol{\theta}$ =.2		Baseline Value At $\boldsymbol{\theta}$ =.30		At $\boldsymbol{\theta}$ =.80	
	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation
$\boldsymbol{\mu}^*$	0.080	0.000	0.190	0.000	0.290	0.000	0.710	0.000
$\boldsymbol{\varphi}^*$	0.020	0.000	0.210	0.000	0.400	0.000	1.000	0.000
$\widehat{\boldsymbol{\varphi}}$	0.030	0.000	0.370	0.000	0.730	0.000	1.000	0.000
$\boldsymbol{\kappa}^*$	0.010	0.000	0.010	0.000	0.011	0.004	0.233	0.196
$\breve{\boldsymbol{\kappa}}$	0.052	0.007	0.330	0.002	0.610	0.000	1.000	0.000
$\overline{\boldsymbol{\kappa}}$	0.052	0.007	0.450	0.000	0.830	0.000	1.000	0.000

Table 17 defines what happens to the critical values as the share of income accruing to the rich $\boldsymbol{\theta}$ changes. The critical values in general depend on the difference between $\boldsymbol{\delta}$ and the income share accruing to the rich $\boldsymbol{\theta}$. Consider that when $\boldsymbol{\delta}$ is held constant, increasing $\boldsymbol{\theta}$ means increasing inequality. As inequality increases, $\boldsymbol{\varphi}^*$ and $\widehat{\boldsymbol{\varphi}}$ increase. This means a consolidated democracy is much less likely, and an unconsolidated democracy becomes more likely. This makes sense since with high inequality, the elites have more incentive to mount a coup to avoid redistribution, so the democracy, if created, is more likely to be unconsolidated and less stable. This tracks well with countries in South America with high inequality and which tend to switch relatively frequently between democratic and non-democratic regimes.

Table 17: Relative value analysis of changes to percent of income accruing to the rich ($\boldsymbol{\theta}$)

Critical Value	Change
$\boldsymbol{\mu}$*	As the percent $\boldsymbol{\theta}$ increases, $\boldsymbol{\mu}$* increases
$\boldsymbol{\varphi}$*	As the percent $\boldsymbol{\theta}$ increases, $\boldsymbol{\varphi}$* increases
$\widehat{\boldsymbol{\varphi}}$	As the percent $\boldsymbol{\theta}$ increases $\widehat{\boldsymbol{\varphi}}$ increases
$\boldsymbol{\kappa}$*	As the percent $\boldsymbol{\theta}$ increases, $\boldsymbol{\kappa}$* increases
$\breve{\boldsymbol{\kappa}}$	As the percent $\boldsymbol{\theta}$ increases, $\breve{\boldsymbol{\kappa}}$ increases
$\overline{\boldsymbol{\kappa}}$	As the percent $\boldsymbol{\theta}$ increases, $\overline{\boldsymbol{\kappa}}$ increases

Figure 8 shows the results of the changes in the critical values on "Democratization in a Picture". The value of $\boldsymbol{\theta}$ that corresponds to $\boldsymbol{\mu}$* is called $\boldsymbol{\theta}$* and demarks the vertical line between whether, in a non-democracy with high cost of repression, the elites choose to offer concessions or democratize. As inequality increases, and keeping all other parameters the same, the point where concession prevents revolution, $\boldsymbol{\mu}$*, increases. This makes sense since as inequality increases, the elites have more incentive to maintain the non-democracy rather than to democratize. Table 17 shows that $\boldsymbol{\kappa}$*, $\overline{\boldsymbol{\kappa}}$ and $\breve{\boldsymbol{\kappa}}$ increases as $\boldsymbol{\theta}$ increases. The increase in $\boldsymbol{\kappa}$* means that the elites are more likely to use repression rather than concession as inequality increases. The increase in $\overline{\boldsymbol{\kappa}}$ and $\breve{\boldsymbol{\kappa}}$ means that democratizations to both consolidated and unconsolidated democracies become less likely when inequality is increased and repression becomes more likely. These influences are shown all "Democratization in a Picture" diagrams by the upward slanting lines between repression and concession/democratization. As inequality increases, repression becomes more likely.

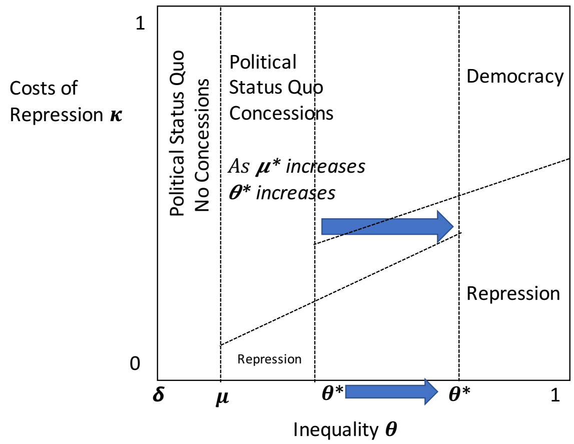


Figure 8: Effects of changes in θ on "Democratization in a Picture"

In Scenario 6, I modify the likelihood of threat of revolution q. Table 18 shows the results of the parameter sweeps when the baseline values are held constant while the likelihood of threat of revolution q increases from .05 to .35. All values are recorded at time step – 10,000.

Table 18: Scenario 6 - modifying likelihood of threat of revolution q

	At q =.05		At q =.15		Baseline Value At q = .25		At q =.35	
	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation
μ^*	0.290	0.000	0.290	0.000	0.290	0.000	0.280	0.000
φ^*	1.000	0.000	0.740	0.000	0.400	0.000	0.230	0.000
$\hat{\varphi}$	1.000	0.000	1.000	0.000	0.730	0.000	0.520	0.000
κ^*	0.037	0.052	0.014	0.014	0.011	0.004	0.028	0.040
$\breve{\kappa}$	1.000	0.000	0.941	0.003	0.610	0.000	0.482	0.041
$\bar{\kappa}$	1.000	0.000	1.000	0.000	0.830	0.000	0.631	0.018

Table 19 defines what happens to the critical values as the likelihood of threat of revolution q changes. As the likelihood of a high threat state of revolution increases, $\boldsymbol{\varphi}^*$ and $\widehat{\boldsymbol{\varphi}}$ decrease. This means a consolidated democracy is much more likely, and an unconsolidated democracy becomes less likely. This makes sense since as long as the poor can mount a credible threat, the elites have more incentive to keep the democracy created rather than mount a coup in the future. If there is little likelihood of a high threat state, then even if the rich are convinced to create a democracy, it is more likely to be an unconsolidated democracy and may switch more readily to an autocratic regime in the future.

Table 19: Relative value analysis of likelihood of threat of revolution q

Critical Value	<u>Change</u>
$\boldsymbol{\mu}^*$	As the likelihood of threat of revolution q increases, $\boldsymbol{\mu}^*$ is not affected
$\boldsymbol{\varphi}^*$	As the likelihood of threat of revolution q increases, $\boldsymbol{\varphi}^*$ decreases
$\widehat{\boldsymbol{\varphi}}$	As the likelihood of threat of revolution q increases, $\widehat{\boldsymbol{\varphi}}$ decreases
$\boldsymbol{\kappa}^*$	As the likelihood of threat of revolution q increases, $\boldsymbol{\kappa}^*$ remains low
$\breve{\boldsymbol{\kappa}}$	As the likelihood of threat of revolution q increases, $\breve{\boldsymbol{\kappa}}$ decreases
$\bar{\boldsymbol{\kappa}}$	As the likelihood of threat of revolution q increases, $\bar{\boldsymbol{\kappa}}$ decreases

Again, we can interpret these results by looking at Figure 9. The value of $\boldsymbol{\theta}$ that corresponds to $\boldsymbol{\mu}^*$ is called $\boldsymbol{\theta}^*$ and demarks the vertical line between whether, in a non-democracy with high cost of repression, the elites choose to offer concessions or democratize. This vertical line is not sensitive to changes in **q** while keeping the same baseline $\boldsymbol{\theta}$. $\boldsymbol{\kappa}$* remains low. However, both $\bar{\boldsymbol{\kappa}}$ and $\breve{\boldsymbol{\kappa}}$ decrease as **q** increases. Thus, democratizations to both consolidated and unconsolidated democracies become more likely when the threat of revolution remains high. And repression becomes less likely. This is seen in the world when dictators, even of countries with low inequality like Belarus, reduce rights such as free speech, protests, and congregation, in order to make it difficult to make a credible threat of revolution.

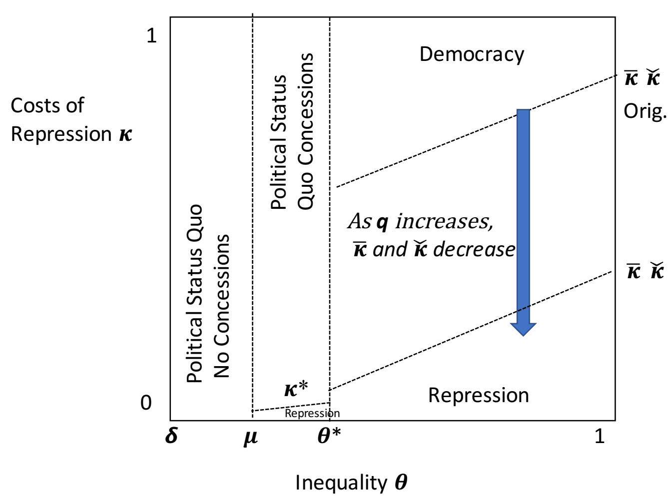


Figure 9: Effects of changes in q on "Democratization in a Picture"

In Scenario 7, I modify the likelihood of threat of coup **s**. Table 20 shows the results of the parameter sweeps when the baseline values are held constant while the likelihood of threat of coup **s** increases from .05 to .35. All values are recorded at time step – 10,000.

Table 20: Scenario 7 - modifying likelihood of threat of coup s

	At s =.05		At s =.15		Baseline Value At s =.25		At s =.35	
	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation
$\boldsymbol{\mu}$*	0.290	0.000	0.290	0.000	0.290	0.000	0.290	0.000
$\boldsymbol{\varphi}$*	0.560	0.000	0.480	0.000	0.400	0.000	0.320	0.000
$\hat{\boldsymbol{\varphi}}$	0.770	0.000	0.750	0.000	0.730	0.000	0.710	0.000
$\boldsymbol{\kappa}$*	0.010	0.000	0.011	0.003	0.011	0.004	0.011	0.004
$\breve{\boldsymbol{\kappa}}$	0.770	0.000	0.690	0.000	0.610	0.000	0.531	0.003
$\overline{\boldsymbol{\kappa}}$	0.831	0.003	0.831	0.003	0.830	0.000	0.831	0.003

Table 21 defines what happens to the critical values as the likelihood of threat of coup ***s*** changes. As the likelihood of a high threat state of coup increases, $\boldsymbol{\varphi}$* decreases, which means an unconsolidated democracy becomes less likely. It is interesting that $\hat{\boldsymbol{\varphi}}$ decreases very slowly. This means decreasing the likelihood of coup does not change much the likelihood of creating a consolidated democracy. $\overline{\boldsymbol{\kappa}}$ which is the indifference between creating an unconsolidated democracy or repression is also insensitive to changes in q.

Table 21: Relative value analysis of likelihood of threat of coup s

Critical Value	Change
$\boldsymbol{\mu}^*$	As the likelihood of threat of coup s increases, $\boldsymbol{\mu}^*$ is not affected
$\boldsymbol{\varphi}^*$	As the likelihood of threat of coup s increases, $\boldsymbol{\varphi}^*$ decreases
$\widehat{\boldsymbol{\varphi}}$	As the likelihood of threat of coup s increases, $\widehat{\boldsymbol{\varphi}}$ decreases slightly
$\boldsymbol{\kappa}^*$	As the likelihood of threat of coup s increases, $\boldsymbol{\kappa}^*$ remains low
$\breve{\boldsymbol{\kappa}}$	As the likelihood of threat of coup s increases, $\breve{\boldsymbol{\kappa}}$ decreases
$\overline{\boldsymbol{\kappa}}$	As the likelihood of threat of coup s increases, $\overline{\boldsymbol{\kappa}}$ remains high

Again, we can interpret these results by looking at Figure 10. The value of $\boldsymbol{\theta}$ that corresponds to $\boldsymbol{\mu}^*$ is called $\boldsymbol{\theta}^*$ and demarks the vertical line between whether, in a non-democracy with high cost of repression, the elites choose to offer concessions or democratize. This vertical line is not sensitive to changes in **s** while keeping the same baseline $\boldsymbol{\theta}$. $\boldsymbol{\kappa}^*$, the indifference line between repression and a concession tax to the left of $\boldsymbol{\theta}^*$, remains low. $\overline{\boldsymbol{\kappa}}$ remains high which means the indifference between creating a consolidated democracy or repression is high and is not very sensitive to changes in q. However, $\breve{\boldsymbol{\kappa}}$ decreases as **q** increases. Thus, democratizations to unconsolidated democracies become relatively more likely while repression becomes relatively less likely when the threat of coup remains high. This is seen in the world when elites, such as in Africa are willing to create the democracy, because they retain a large amount of de facto power, and know that as long as they can provide a credible threat of a coup, they can keep taxes and redistribution low.

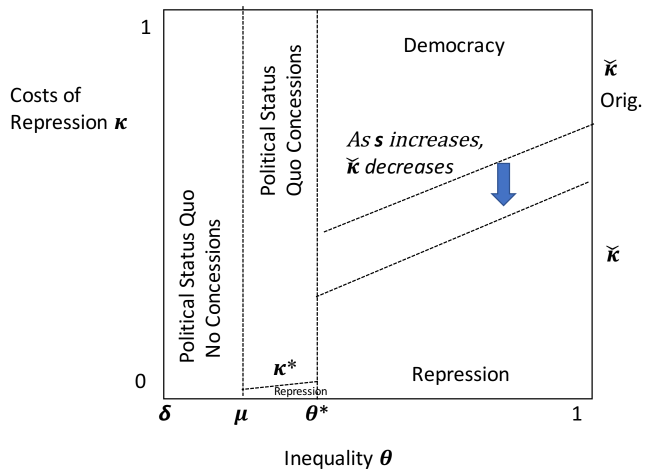


Figure 10: Democratization in a Picture (adapted from: Acemoglu & Robinson, 2006, pp. 44, 214)

3.3.2 Validation and Results

Validation determines how well the model represents the real world (North & Macal, 2007). Both the A&R game-theoretic model and this agent-based model are considered abstract models. Abstract models are very useful in describing the complex relationships between multiple influencers. In general, abstract models are validated by qualitatively comparing how the inputs respond to the outputs with results seen in the real world. Acemoglu and Robinson's book Economic Origins of Dictatorship and Democracy include many validating case studies of historical regime decisions that they use for their validation. Acemoglu & Robinson maps their theoretical results from their models to these case studies. Since my agent-based model is also an exploratory model, the model is validated by ascertaining that the results appear consistent with real life.

Validation of agent-based models is challenging and depends its purpose (Crooks, Castle, & Batty, 2008). In this model, the outputs are the regime decisions made by the agents. There is still a lot unknown about how a society chooses their governing institutions. Acemoglu and Robinson have a series of models that propose a decision-making process based on individuals evaluating the relative value of different options. They find that a group of critical values based on the underlying economic conditions determine what regime decisions the individual selects from the ones that are available. The agent-based model produces these same critical values. When the underlying economic conditions are instantiated with real values, the game-theoretic solutions to the critical values can be evaluated and numerical results can be found. Similarly, empirical data is generated by the agent-based model that can be compared against empirical data generated by the game-theoretic equations. The availability of empirical data allows historical or "explanatory" models to be validated with statistical methods (Crooks, Castle, & Batty, 2008).

Cross-Validation verifies outputs correlate with another model that has already been validated. The model is quantitatively validated comparing the critical values with the critical values calculated by the Acemoglu and Robinson Game-theoretic model of the creation and consolidation of democracy (Acemoglu & Robinson, 2006, pp. 231-246). This validation "docks" the model in the sense of Axtell et al., (1996). Two areas of cross-validation are performed. The following validation methodologies are used.

- Cross Validation of Critical Values – The game-theoretic model and the agent-based model are instantiated with the same underlying economic conditions. Each model then produces a set of critical value outputs $\boldsymbol{\mu}^*$, $\hat{\boldsymbol{\varphi}}$, $\boldsymbol{\varphi}^*$, $\boldsymbol{\kappa}^*$, $\breve{\boldsymbol{\kappa}}$, and $\bar{\boldsymbol{\kappa}}$ that determine how the individual make decisions on institutional preferences.
- Macro-level validation – The results of the Agent-Based model is compared to three instances of real-world countries in order to assess whether the aggregate patterns correspond to real-world? The effect of the change in inputs on the change in critical values and resulting institutional decisions is compared against real-world examples of a high inequality, low inequality and medium inequality country along with country specific data on the discount factor.
- Cross Validation of Democratization in a Picture - The agent-based model produces an output called "democratization in a picture" and is compared against the graphics used by Acemoglu and Robinson

3.3.2.1 Cross-Validation of Critical Values

In this section, I compare the results of agent-based model to the game-theoretic results. The comparisons are made for three scenarios that tests how agent-based model performs on extreme values.

(1) Using the baseline values of Table 7 where 5% of the population has 30% of the income.

(2) A very very low inequality scenario where 5% of the population has 8% of the income.

(3) A very high inequality scenario where 1% of the population has 60% of the income.

In the game-theoretic model, the critical values are calculated by solving the Bellman equations of utility and finding the point of indifference relative to the specific critical value being solved. Conversely, in the agent-based model, the critical values are not calculated by the agents. Their experience builds up on when certain decisions makes the most sense. In the beginning, the values of $\boldsymbol{\mu}^*$, $\hat{\boldsymbol{\varphi}}$, $\boldsymbol{\varphi}^*$, $\boldsymbol{\kappa}^*$, $\breve{\boldsymbol{\kappa}}$, and $\bar{\boldsymbol{\kappa}}$ will change until there is enough experience to build the steady state value.

The calculated values require solving for the preferred tax rate $\boldsymbol{\tau}^p$ and for the tax rate $\hat{\boldsymbol{\tau}}$ if it exists that will prevent a revolution. I found $\boldsymbol{\tau}^p$ by evaluating the utility of the median income person at various tax rates. Intra-group incomes are homogenous, and there are more poor people than rich. Therefore, the median income person will have the same income as a poor person. I varied the tax rate τ^p in equation 9 and recorded outcomes in order to find the maximum value.

The costs of the taxation system are again given by equation 10. See section 3.1.5.2 for more information. To calculate these values from the game-theoretic results I used a spreadsheet rather than the agent-based model.

To find the tax rate $\hat{\boldsymbol{\tau}}$ that will prevent a revolution, if such a tax rate exists, I used Acemoglu and Robinson's equations for calculating the utility of revolutions and non-democracy. The utility of revolutions is given by equation 15 and the utility of the poor in a non-democracy is given by equation 16 (Acemoglu & Robinson, 2006 p. 239). Research on conflict has found that civil war on average make a country 15% poorer than it would have been without civil war (Haggard & Tiede, 2011). Therefore, I used to the value of .15 for μ.

The utility of the poor in a non-democracy depends on the likelihood q that they can pose a revolution threat and obtain a concession tax. When calculating the utility of the poor staying in the non-democracy, the rich assume they will offer concession tax of $\hat{\boldsymbol{\tau}}$ when there is a threat of revolution, and no tax when there is no threat (equation 16).

To find $\hat{\boldsymbol{\tau}}$ for the game-theoretic values, I constructed a spreadsheet that evaluated equations 15 and 16 at different tax rates $\hat{\boldsymbol{\tau}}$ and selected the lowest rate for which the utility of non-democracy is greater than that of revolution $V^p(N, \mu^H, \tau^N = \hat{\boldsymbol{\tau}}) > V^p(R, \mu^H)$.

Table 22:Simulation results compared to calculated game-theoretic results

	Baseline Values $\boldsymbol{\delta}$ =10%, $\boldsymbol{\theta}$ = 30% $\boldsymbol{\tau}^p$ = .263 $\hat{\boldsymbol{\tau}}$ = N/A μ. = .15			Low Inequality Scenario $\boldsymbol{\delta}$ =5%, $\boldsymbol{\theta}$ = 8% $\boldsymbol{\tau}^p$ = 0 $\hat{\boldsymbol{\tau}}$ = 0 μ. = .15			High Inequality Scenario $\boldsymbol{\delta}$ =1%, $\boldsymbol{\theta}$ = 60% $\boldsymbol{\tau}^p$ = .596 $\hat{\boldsymbol{\tau}}$ =N/A μ. = .15		
	Game-theoretic Result	ABM Average of 30 trials	% Var	Game-theoretic Result	ABM Average of 30 trials	% Var	Game-theoretic Result	ABM Average of 30 trials	
$\boldsymbol{\mu}$*	0.291	0.290	0.3%	0.080	0.080	0.0%	0.552	0.550	0.4%
$\boldsymbol{\varphi}$*	0.393	0.400	-1.8%	0.000	0.020	N/A	*2.162	1.000	N/A
$\hat{\boldsymbol{\varphi}}$	0.825	0.730	11.5%	0.000	0.030	N/A	*1.046	1.000	N/A
$\boldsymbol{\kappa}$*	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
$\breve{\boldsymbol{\kappa}}$	$\bar{\boldsymbol{\kappa}} \geq \breve{\boldsymbol{\kappa}}$	0.610	true	$\bar{\boldsymbol{\kappa}} \geq \breve{\boldsymbol{\kappa}}$	0.059	true	$\bar{\boldsymbol{\kappa}} \geq \breve{\boldsymbol{\kappa}}$	1.000	true
$\bar{\boldsymbol{\kappa}}$	0.825	0.830	0.83%	0.000	0.059	N/A	*2.162	1.000	

* The high inequality scenario when applied to the game-theoretic equations for $\boldsymbol{\varphi}$*, $\hat{\boldsymbol{\varphi}}$, $and\ \bar{\boldsymbol{\kappa}}$, yielded values greater than 1. However, they should be limited to [0,1]. The agent-based model correctly reported the maximum allowable value of 1.

In Table 22, the calculated values from the game-theoretic model are compared to my agent-based model results for the baseline case and extreme cases of low and high inequality to validate that the model can reproduce A&R game-theoretic results. The table has three areas, one for each scenario: i) baseline, ii) low inequality, and iii) high inequality. In each area, there are three columns: the game-theoretic result, the ABM result and the % variance between the ABM and game-theoretic results. These values are found for the 6 critical values: $\boldsymbol{\mu}^*$, $\hat{\boldsymbol{\varphi}}$, $\boldsymbol{\varphi}^*$, $\boldsymbol{\kappa}^*$, $\breve{\boldsymbol{\kappa}}$, and $\bar{\boldsymbol{\kappa}}$**.** Acemoglu and Robinson do not actually provide an equation for $\breve{\boldsymbol{\kappa}}$. However, they find that $\bar{\boldsymbol{\kappa}}>\breve{\boldsymbol{\kappa}}$. Therefore, the validation checks whether $\bar{\boldsymbol{\kappa}}$ found by the agent-based model is greater than the results for $\breve{\boldsymbol{\kappa}}$ found by the agent-based model. In the case of the low inequality scenario, note that the preferred tax rate is 0. This is because at low inequality, the loss due to the redistribution system costs more than the benefits.

The game-theoretic results for the critical values $\hat{\boldsymbol{\varphi}}$, $\boldsymbol{\varphi}^*$ and $\bar{\boldsymbol{\kappa}}$ are exactly equal to zero, because the poor preferred tax rate is zero, and the concession tax rate is zero. In the agent-based model however, the critical values are found with some very small value above zero. The model does not directly calculate the critical value. There may be some cases in the agents experience where certain decisions are made that contribute towards the non-zero value. The variances for these cases is marked by N/A.

The values for $\boldsymbol{\kappa}^*$ are not applicable for all three scenarios because the definition of $\boldsymbol{\kappa}^*$ is the elite indifference between repression and offering a concession tax to prevent revolution. This then depends on there being a concession tax $\hat{\boldsymbol{\tau}}$ that would prevent a revolution. In all three cases, such a concession tax does not exist. In the baseline case and the High inequality case, the poor would have a revolution regardless of the tax. And in the Low Inequality scenario, a concession tax is not needed to prevent revolution. The poor do not find that an attractive alternative.

In all three scenarios, including the extreme values of very high inequality, and very low inequality, the agent-based model found that $\boldsymbol{\varphi}^* < \hat{\boldsymbol{\varphi}}$. (Acemoglu & Robinson, 2006, p. 237). In all three scenarios, including the extreme values of very high inequality, and very low inequality, the agent-based model found that $\bar{\boldsymbol{\kappa}} > \breve{\boldsymbol{\kappa}}$ and $\bar{\boldsymbol{\kappa}} > \boldsymbol{\kappa}^*$ (Acemoglu & Robinson, 2006, pp 199, 243). In addition, the agent-based model reproduced critical values of the game-theoretic model in ten out of the twelve cases where critical values existed. Of the 6 critical value percent variances calculated in Table 22, I find that 4 were within 1%, another 1 was within 2% and 1 within 12%. In three cases, when the critical value was determined to be zero by the game-theoretic model, the agent-based model found a small non-zero value and did not reproduce the critical value of 0. Lastly, in three cases, solving the game-theoretic equations returned a critical value greater than 1 even though A&R had defined the critical value to be between 0 and 1. The agent-based model correctly returned the maximum value of 1.

3.3.2.2 Validation of Critical Values for sample countries

In this section, I compare how well the model performs for parameters seen currently in the world. I examine results using parameters that are similar to those found in a non-democratic country near the median in terms of inequality, one with very high inequality and one with very low inequality compared to other countries. The data was obtained from the PovcalNet research on income inequality and poverty survey of 153 countries (PovcalNet, 2016).

Most dictatorships do not make their country's income distribution known. Belarus, and Swaziland are two dictatorships that are near extremes of income inequality and included in the report. Lao PDR, the communist People's Democratic Republic, is a dictatorship near the median level of inequality.

Belarus, is a presidential republic in name, but called a dictatorship by the CIA Factbook (2017) and scholarly reports (for example, see McFaul, 2002). The PovcalNet research program reports that the top 10% in Belarus garnered just 21.54% of the income (2016). That a very low rate of inequality, in fact, the eighth lowest in the report. At the beginning of the regime change from communist rule in 1990, analysts hoped that practical leaders could guide the country along an evolutionary path to democracy. But the president of Belarus, Lukashenko, found autocracy more convenient. The old hard liners from the communist regime quickly worked with him to consolidate his authoritarian rule (McFaul, M. (2002). In comparing Belarus to Acemoglu & Robinson's paths to democracy, the low inequality may place it in a similar path to Singapore, where there is an un-challenged non-democracy. But based on reports of repression (McFaul, M. (2002) it is most like in the section where repression is used instead of offering concessions. Acemoglu & Robinson do not identify a country in this region of the "Democratization in a Picture".

Table 23: Belarus had steady low inequality. (Source: PovcalNet, 2016)

Belarus Low Inequality – Top 10% Share of Income	
Year	Share of Income
1993	19.40
1995	22.55
1998	24.44
1999	24.86
2000	23.54
2001	24.52
2002	23.92
2003	23.10
2004	21.47
2005	21.95
2006	22.32
2007	23.01
2008	21.95
2009	22.58
2010	22.34
2011	21.47
2012	21.54

Laos PDR, is communist dictatorship in mainland Southeast Asia (CIA Factbook (2017). Its inequality is slightly higher than the median level with 10% of the population earning 30.84% of the income. However, Laos consistently ranks among the least free in the world according to Freedom House (Gwartney, et al., 2015). Lack of freedom of speech, freedom of association and no culture or organization focused on human rights, make it one of the most repressive regimes.

Swaziland is an absolute monarchy style dictatorship in Africa surrounded by South Africa on three sides and Mozambique on the fourth. Its inequality value is the 14th highest, with 10% of the people taking 40% of the income. For the years for which data is available, inequality appears to be going down. However, that does not tell the whole story. This is more likely a case where the leader is taking his whole country down into more and more poverty. Its estimated GDP real growth rate has been declining over the last three years: 3.6% in 2014, 1.1% in 2015 and -.4% in 2016 (CIA Factbook, 2017).

Table 24: Swaziland high inequality. (Source: PovcalNet, 2016)

Swaziland High Inequality – Top 10% Share of Income	
Year	Share Income
1994	49.53
2000	44.13
2009	39.98

Table 25 compares the agent-based model resulting critical values to those found by the game-theoretic calculations for three cases of non-democracy: A low inequality scenario like Belarus, a median level inequality like Laos, and a high inequality scenario like that found in Swaziland. The values for $\boldsymbol{\kappa}^*$ are not applicable for all three scenarios because the definition of $\boldsymbol{\kappa}^*$ is the elite indifference between repression and offering a concession tax to prevent revolution. This then depends on there being a concession tax $\hat{\boldsymbol{\tau}}$ that would prevent a revolution. In all three cases, such a concession tax does not exist, the poor would have a revolution regardless of the tax. The reason they don't (according to Acemoglu & Robinson's theory), is that repression is less costly to the elites than democratization and therefore, the elites are repressing. The table validates that the agent-based model can reproduce the critical criteria results of Acemoglu and Robinson for three different real-world cases. Of the 18 critical value percent variances calculated in Table 22 and Table 25, I find that 7 were within 1%, another 6 were within 5% and 4 within 10%.

Table 25:Agent-based model results compared to calculated game-theoretic results

	Low Inequality Scenario (Belarus) $\boldsymbol{\delta}$ =10%, $\boldsymbol{\theta}$ = 22%, $\boldsymbol{\beta}$=.995 $\boldsymbol{\tau}^{p}$ = .132 $\hat{\boldsymbol{\tau}}$ =N/A			Median Inequality Scenario (Lao PDR) $\boldsymbol{\delta}$ =10%, $\boldsymbol{\theta}$ = 31%, $\boldsymbol{\beta}$=.877 $\boldsymbol{\tau}^{p}$ = .233 $\hat{\boldsymbol{\tau}}$ =N/A			High Inequality Scenario (Swaziland) $\boldsymbol{\delta}$ =10%, $\boldsymbol{\theta}$ = 40%, $\boldsymbol{\beta}$=.962 $\boldsymbol{\tau}^{p}$ = .334 $\hat{\boldsymbol{\tau}}$ =N/A		
	Game-theoretic Result	ABM: Average of 30 trials	%var	Game-theoretic Result	ABM: Average of 30 trials	%var	Game-theoretic Result	ABM: Average of 30 trials	%var
$\boldsymbol{\mu}^*$	0.218	0.210	3.7%	0.302	0.300	0.7%	0.386	0.380	1.6%
$\boldsymbol{\varphi}^*$	0.142	0.150	-5.6%	0.205	0.210	-2.4%	0.441	0.450	-2.0%
$\hat{\boldsymbol{\varphi}}$	0.299	0.270	9.7%	0.487	0.450	7.6%	0.950	0.900	5.3%
$\boldsymbol{\kappa}^*$	N/A	N/A		N/A	N/A		N/A	N/A	
$\breve{\boldsymbol{\kappa}}$	$\bar{\boldsymbol{\kappa}} \geq \breve{\boldsymbol{\kappa}}$	0.243	true	$\bar{\boldsymbol{\kappa}} \geq \breve{\boldsymbol{\kappa}}$	0.333	true	$\bar{\boldsymbol{\kappa}} \geq \breve{\boldsymbol{\kappa}}$	0.690	true
$\bar{\boldsymbol{\kappa}}$	0.299	0.310	-3.7%	0.487	0.490	-0.6%	0.950	0.952	-0.2%

3.4 Variation of Initial ABM - "Democratization in a Picture"

3.4.1 Model Description

I begin with the model described earlier in this chapter. This model extends the initial model in Netlogo 5.0.3 (Wilensky, 1999).

3.4.2 Motivation

This is a variation of the basic model. It reproduces the "Democratization in a Picture" as described in Acemoglu and Robinson (2006, p 43, 214) for further validation that the agent-based model reproduces the A&R results.

3.4.3 "Democratization in a Picture" Mode

The model can be run to draw a point in time decision picture based on the current economic conditions of the discount factor, costs of coup and costs of revolution. Acemoglu & Robinson describe this as "democratization in a picture" because it shows when the elites decide to democratize, repress, or offer a concession tax based on the value of $\boldsymbol{\theta}$ – the percent of total income shared among the rich – and the cost of repression $\boldsymbol{\kappa}$. To generate the "Democratization in a Picture" the user selects the static variables for the cost of revolution $\boldsymbol{\mu}$ and cost of coup $\boldsymbol{\varphi}$ and the simulation runs a parameter sweep over theta $\boldsymbol{\theta}$ and $\boldsymbol{\kappa}$ to generate the graph on the right of the user interface (Figure 11). This setup differs from the Creation and Consolidation of Democracy mode where inequality is held constant and the agents respond to the changing economic conditions of the cost of coup and cost of revolution. The vertical axis of the graph shows the values of repression $\boldsymbol{\kappa}$. As the simulation runs, critical values of $\boldsymbol{\theta}^*$, $\boldsymbol{\kappa}^*$, $\bar{\boldsymbol{\kappa}}$, $\breve{\boldsymbol{\kappa}}$ that affect the decision to democratize become known and are updated on the appropriate axis of the graph.

3.4.4 Agents

The agents remain the same as in the initial agentization.

3.4.5 Rules

The rules remain the same as in the initial agentization.

3.4.6 Implementation

3.4.6.1 Model Interface

The Netlogo user-interface (see Figure 11) is the same as in the initial model of the Creation and Consolidation of Democracy. On the left, the user sets the population size, the percent of the population considered rich $\boldsymbol{\delta}$, the discount factor $\boldsymbol{\beta}$, the average income $\bar{\mathbf{y}}$**,** the likelihood of threat of revolution **q,** and the likelihood of thread of coup **s**. The share of income accruing to the rich $\boldsymbol{\theta}$, is not used because that value is varied between $\boldsymbol{\delta}$ and 1 in order to generate the picture. The user needs to select two additional parameters that are held static in order to generate the picture: the cost of coup $\boldsymbol{\varphi}$ and the cost of revolution $\boldsymbol{\mu}$.

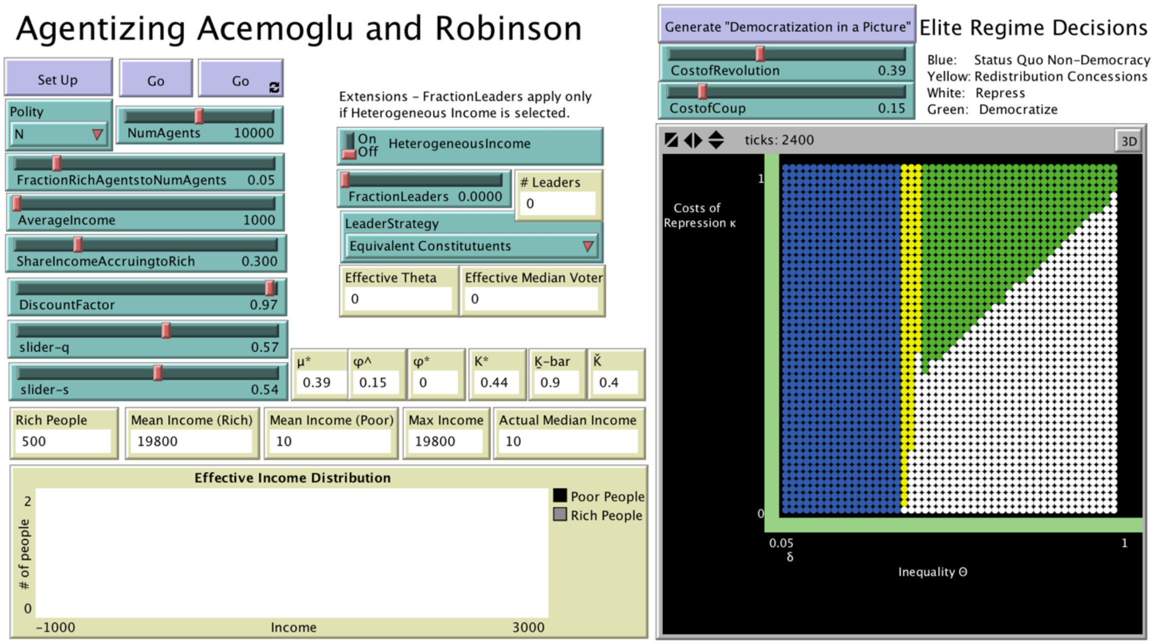


Figure 11: Model user interface

3.4.6.2 Model Flow

This section describes the model flow as shown in Figure 12: Model flow of "Democratization in a Picture to reproduce the "Democratization in a Picture" graph. The initial set up processing is different for Democratization in a Picture mode than in the previous model. However, steps 2 through 5 are exactly the same. After Step 5 is called, there is a new Step 6 in which the elite decision is graphed on the picture, the polity is reset to N, $\boldsymbol{\kappa}$ is incremented by .02, and the simulation calls the routine to execute commands in steps 2 through 6 again. This occurs for all values of $\boldsymbol{\kappa}$ from 0 to 1 in increments of .02. and steps 2 through 6 is called for all values of $\boldsymbol{\theta}$ from a minimum of $\boldsymbol{\delta}$ to 1.

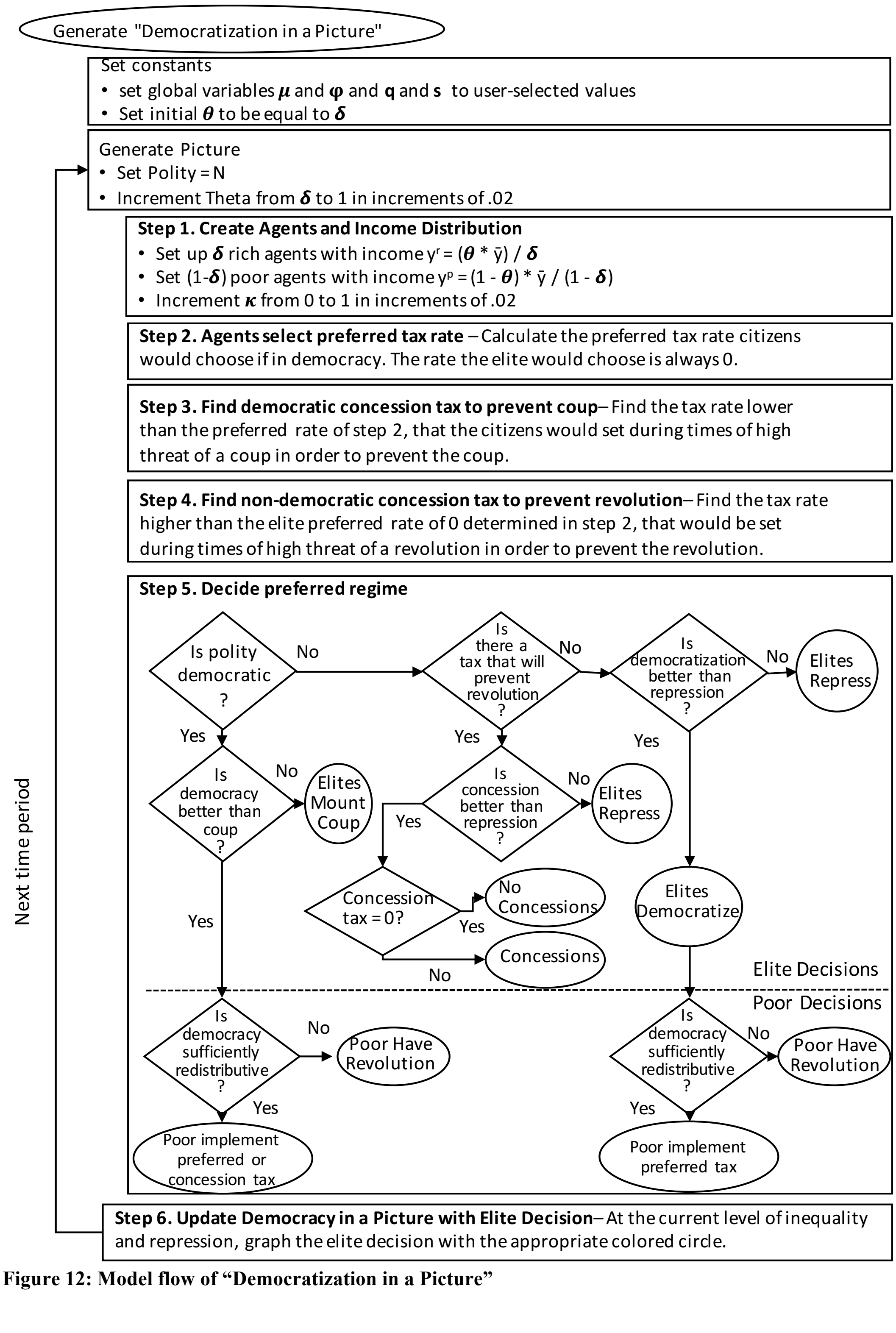


Figure 12: Model flow of "Democratization in a Picture"

GENERATE POLITCAL OUTCOME PICTURE The user selects the button GENERATE POLITCAL OUTCOME PICTURE. The "Setup" button on the user interface is not used. The Generate Political Outcome Picture first calls a special set up routine to set up the community of agents as described in the process flow of Figure 12. The "Setup" routine executes the following steps:

1. Setup constants with initial values including $\boldsymbol{\mu}$ and $\boldsymbol{\varphi}$ but not $\boldsymbol{\theta}$
2. Set initial $\boldsymbol{\theta}$ to be equal to $\boldsymbol{\delta}$

Generate Picture Next the simulation sets up the repeating loops that increment $\boldsymbol{\theta}$ and $\boldsymbol{\kappa}$. For the outer loop, it resets Polity to be N "non-democracy" at the beginning of every run. Then it executes Step 1. "create agents" that sets up the new income levels based on the incrementally changed inequality and starts the inner loop of incrementing $\boldsymbol{\kappa}$.

Step 1. Create Agents

1. Set up $\boldsymbol{\delta}$ rich agents with income $y^r = (\boldsymbol{\theta} * \bar{y}) / \boldsymbol{\delta}$
2. Set (1-$\boldsymbol{\delta}$) poor agents with income $y^p = (1 - \boldsymbol{\theta}) * \bar{y} / (1 - \boldsymbol{\delta}$
3. Set Initial $\boldsymbol{\kappa}$ to be zero

Next, the simulation calls the routine in to execute the commands in steps 2 through 6 (Figure 12). Steps 2 through 5 are the same as in the initial model and described in section 3.1.6.3. After step 5, **Step 6** is executed in which the elite decision is graphed on the picture, the Polity is reset to N, $\boldsymbol{\kappa}$ is incremented by .02, and the simulation calls the routine to execute commands in steps 2 through 6 again. This occurs for all values of $\boldsymbol{\kappa}$ from 0 to 1 in increments of .02.

After $\boldsymbol{\kappa}$ reaches 1 and the loop processing is complete, the polity is reset to N, and processing returns to **GENERATE PICTURE** where $\boldsymbol{\theta}$ is incremented by .02, and Create agents is called to reset the agents with new incomes based on the new inequality $\boldsymbol{\theta}$. Then for each new value of $\boldsymbol{\theta}$, $\boldsymbol{\kappa}$ is incremented by .02

3.5 Model Verification and Validation

3.5.1 Verification

The model was verified similarly to the initial agentization (section 3.3.1) with code walkthroughs and testing of extreme values to make sure a picture was still created correctly. Figure 13 shows the graph created with baseline values as identified in Table 28. This graph was created by the simulation model and presented on the user interface while the simulation was running.

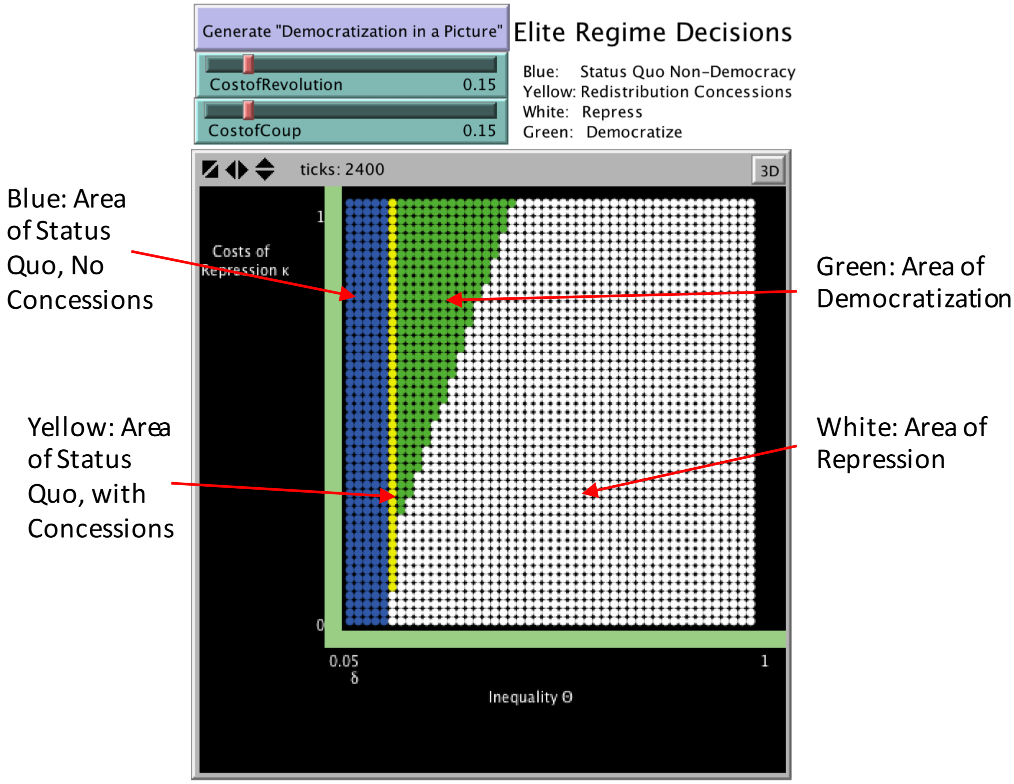


Figure 13: "Democratization in a Picture" with baseline values

The blue area is the area in the decision space where with low inequality, the elites keep the non-democracy, and there is no credible threat of revolution, so there are no concessions, and no repression is needed. The yellow area is the decision space where there is a credible threat of revolution, and if repression costs are high enough, the elites will grant more redistribution through taxation. The white area under the yellow shows where, if the repression costs are very low, the elites will choose to repress. The green area is the decision space where the inequality is too high to solve by redistribution through taxation. And, if the repression costs are high, the elites will choose to democratize, either to a consolidated or non-consolidated democracy. If the repression costs are low, either because the citizens can't mobilize or organize an opposition, then the elites will choose to repress.

Table 26: Parameters for baseline case

Parameter	Value
Number of Agents	**10,000**
Polity	**N**
$\bar{y}$ - **mean income of the population**	**1,000**
$\boldsymbol{\delta}$ **–** the percent of population considered rich	**.05**
$\boldsymbol{\varphi}$ **– The cost of coup as percent of GDP**	**.15**
$\boldsymbol{\mu}$ **– the cost of revolution as percent of GDP**	**.15**
$\boldsymbol{\beta}$ **– discount factor**	**.97**
s – the likelihood that there will be a high threat of coup	**.25**
q - **the likelihood that there will be a high threat of revolution**	**.25**

To test the model with extreme values, I choose to lower the likelihood of a high threat state of revolution to q= .01 while keeping the other parameters the same. The image on the left of

Figure 14 shows the "Democratization in a Picture" with a low likelihood of high threat state for revolution. The image on the right in

Figure 14 shows the "Democratization in a Picture" with a high likelihood (q=.49) of high threat state for revolution. The image shows that the elites will be more likely to democratize rather than repress if there is a high likelihood of a high threat state of revolution in the future. The same result was found in the first model of the Creation and consolidation of Democracy (see Figure 9). However, this view shows that this tendency exists over a range of medium and higher inequality, not just high inequality countries. This supports what is seen in the world today by dictatorships with wide ranges on inequality such as Belarus and Lao PDR reducing freedom of expression and freedom of association in order to reduce the likelihood of a high threat state.

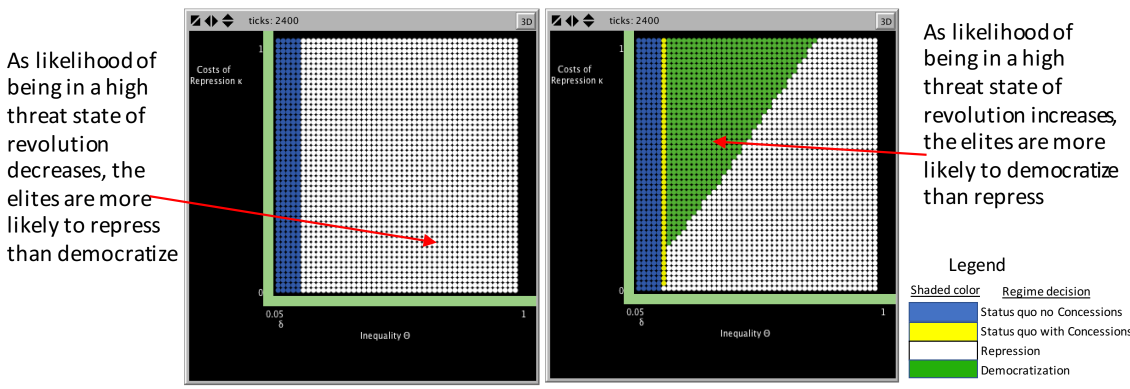
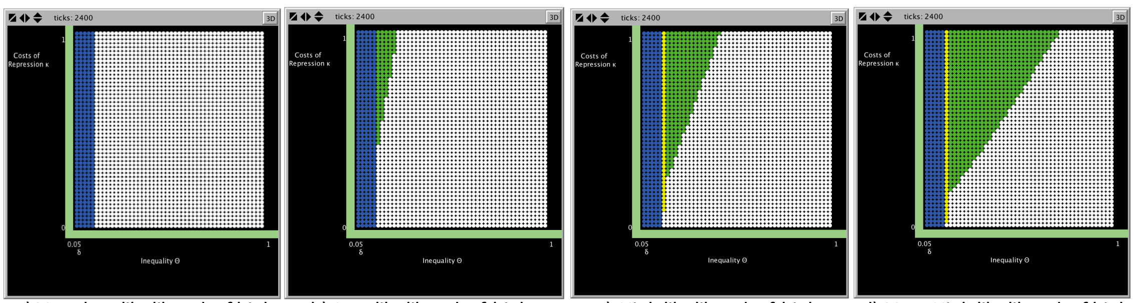


Figure 14: Comparison of low (left) and high (right) values of likelihood of high threat of revolution

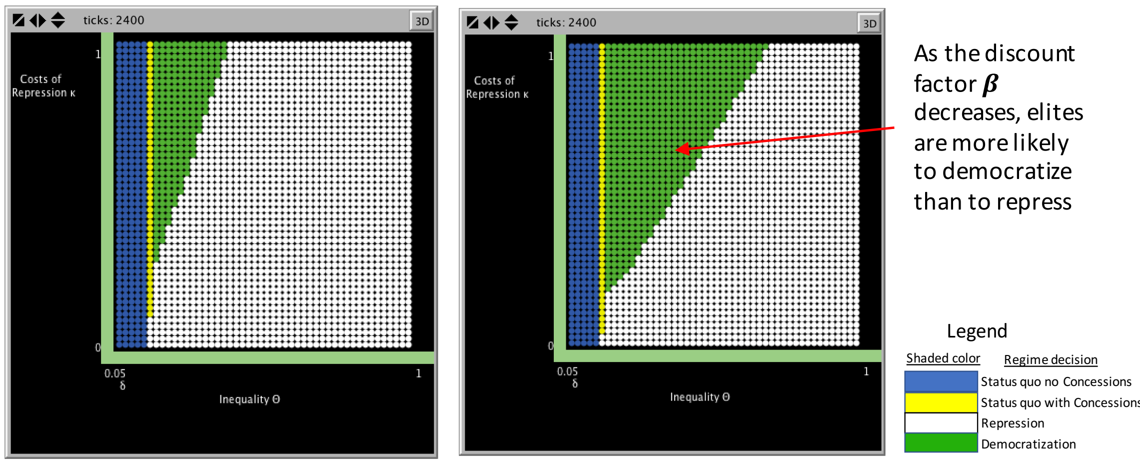
Acemoglu and Robinson theorized that democratizations would occur in countries with medium inequality and described an inverted U relationship between inequality and democratization (Acemoglu &Robinson, 2006, p. 80). In low inequality, there would not be enough incentive for the poor to have a revolution. In high inequality, the elites would be incented to repress. However, this appears not to hold in an environment with low likelihood of being in a state of high threat of revolution. Acemoglu and Robinson discuss that the high threat state is a rare occurrence. Although the authors don’t give a numerical value to “rare” it seems reasonable that rare would be 10% or less. Figure 15 shows that if being in a high threat state is a rare event (10% or less). Democratizations are unlikely to happen at any level of inequality. At a likelihood of high threat state (q), at 10%, the cost of repression would need to be well over 50% and inequality very low, for democratization to occur.



a) Very low likelihood of high threat of revolution q=.01 | b) Low likelihood of high threat of revolution q=.10 | c) High likelihood of high threat of revolution q=.25 | d) Very High likelihood of high threat of revolution q=.49

Figure 15: Democratizations unlikely unless there is a high likelihood of high threat state.

Next, I tested the model with a low discount factor of .80 and a high discount factor of .99 while keeping the other parameters the same. The image on the left of Figure 16 depicts the decision space when the discount factor is high at $\boldsymbol{\beta}$=.99. The image on the right shows how that changes when the discount factor decreases to .8. As the discount factor decreases, the elites are more likely to democratize than to repress. This same effect was found in the verification of the Creation and Consolidation of Democracy model (see Figure 6). However, now I find that the relationship holds true for a wide range of inequality $\boldsymbol{\theta}$. The effect that decreasing the discount factor $\boldsymbol{\beta}$ has on lowering the critical values $\boldsymbol{\kappa}$ is stronger as inequality increases. This is shown in the picture by the drop in the line of indifference between repression (white) and democratization (green) is more pronounced to the right of the picture.

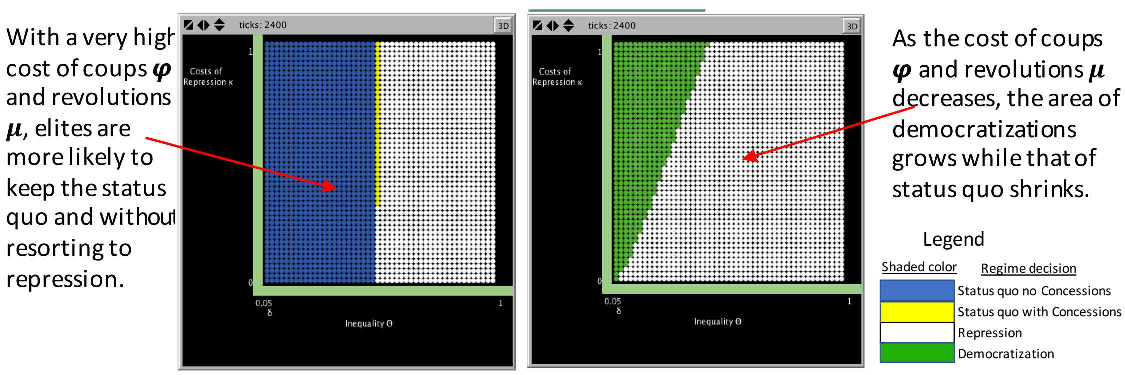


(a) High discount factor $\boldsymbol{\beta}$=.99 (b) Low discount factor $\boldsymbol{\beta}$=.80

Figure 16: Comparison of low (left) and high (right) discount factors

Last, I tested the model with high costs for coups $\boldsymbol{\varphi}$ and revolutions $\boldsymbol{\mu}$ and with low costs for coups and revolutions. The image on the left of Figure 17 depicts the decision space when revolutions and coups are very costly. The cost of coup $\boldsymbol{\varphi}$ and the cost of revolution $\boldsymbol{\mu}$ was set to =.50. The picture shows that the area of Status Quo (blue) has been extended to the right. This matches real world situations in areas like Singapore where even though there is high inequality, there is high growth and the citizens enjoy a high standard of living. Thus, it is expected that the cost of revolution to the citizens is very high and makes it easier for the elites to maintain the status quo with little or no repression.

The image on the right shows how that changes when the cost of coup $\boldsymbol{\varphi}$ and the cost of revolution $\boldsymbol{\mu}$ was set to =.02. As the cost of coup $\boldsymbol{\varphi}$ and the cost of revolution $\boldsymbol{\mu}$ decreases, the area of democratizations grows while that of status quo shrinks. Since in the Creation and Consolidation of Democracy model, the cost of coup $\boldsymbol{\varphi}$ and the cost of revolution $\boldsymbol{\mu}$, change throughout the simulation, that model could not bring insight into this effect. But keeping $\boldsymbol{\varphi}$ and $\boldsymbol{\mu}$ static while varying inequality and the cost of repression, brings greater focus on the effect of the cost of regime changes.



(a) High cost of coup $\boldsymbol{\varphi}$ and cost of revolution $\boldsymbol{\mu}$ =.50

(b) Low cost of coup $\boldsymbol{\varphi}$ and cost of revolution $\boldsymbol{\mu}$ =.02

Figure 17: Comparison of low (left) and high (right) costs of coups and revolutions

Acemoglu and Robinson's theory that democratizations occur at medium levels of inequality seems to apply at medium levels of cost of coup and cost of revolution. If the cost of coup and revolution is very low, then democratizations will occur at low levels of inequality. If the cost of coup and revolution is high, then democratizations aren't likely to occur.

3.5.2 Validation of Democratization in a Picture

Validation of an abstract model is generally accomplished by observing how the model outputs change with changes in the inputs and comparing the relationships in the model to the relationships found in the real world. In addition, validation is often performed by Cross-validation. Cross Validation verifies that the outputs correlate with another model that has already been validated with the real world. The "Democratization in a picture" model is qualitatively validated by comparing the model output with the Democratization in a picture analysis of Acemoglu and Robinson's Game-theoretic model (Acemoglu & Robinson, 2006, p. 214). This validation "docks" the model in the sense of Axtell et al., (1996).

"Democratization in Picture" illustrates the different regime decisions that get made based on the economic circumstances. In Figure 18, the diagram to the left shows the diagram as presented by Acemoglu and Robinson to describe the Creation and Consolidation of Democracy (Acemoglu & Robinson, 2006, p. 214. I have modified the picture to add the empirical examples discussed by Acemoglu and Robinson in an earlier section of the book, to the picture. This diagram is fully described in section 2.3.7. The diagram to the right shows the picture as drawn by the Agent-based model. It has the same five sections as in the Acemoglu and Robinson theoretical diagram. The diagram drawn by the agent-based model is made by color coding the agents' decisions graphed according to the vertical axis of Cost of repression $\boldsymbol{\kappa}$ and the horizontal axis of inequality $\boldsymbol{\theta}$. The agent decisions are based on the underlying economic conditions.

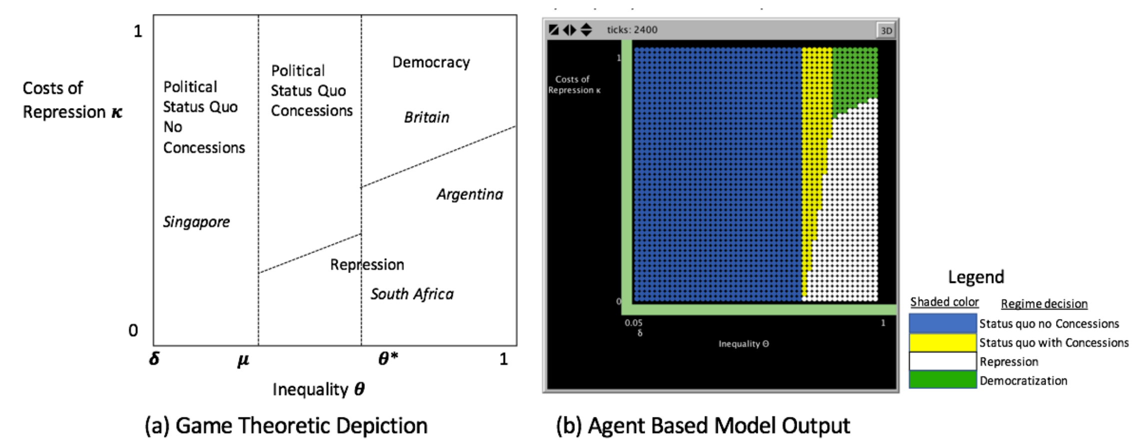


Figure 18: Comparison of "Democratization in a Picture" (left) (adapted from: Acemoglu & Robinson 2006, pp. 44, 214), with agent-based model results (right).

3.6 Conclusion

This Chapter has described two agent-based models. The first is an agent-based model for simulating the creation and consolidation of democracy. The agent-based model agentizes a subset of the game-theoretic models of Acemoglu & Robinson (2006, ch 7, section 4). Specifically, it agentizes the model titled "A Dynamic Model of the Creation and Consolidation of Democracy". The second model agentizes the same game-theoretic models as the first, but varies the parameters differently in order to draw what Acemoglu & Robinson call "Democratization in a Picture" (2006, p. 43) and

“Democratization in a Picture (2006, p. 214). The picture illustrates the paths that Acemoglu & Robinson predict will lead to democracy following their theory of democratizations. Both models instantiate two classes of agents: rich, and poor. The classes of agents are identified by their income level relative to each other. The rich have a higher income than the poor. The level of inequality is defined as the share of income accruing to the rich.

The agent-based model reproduces the assumptions as closely as possible of the game-theoretic models. One necessary change is that, instead of one agent, a population of agents are create. There is an integer number of rich agents rather than a fraction. Although there are two classes, and inter-group income is heterogeneous as characterized by the inequality, intra-group income is homogenous. Keeping this intra-group homogeneity assumption, the agent-based model reproduces the group-level threshold conditions affecting institutional choices found by Acemoglu & Robinson. My validation methods show that these results are robust to parameter changes within the ranges defined by Acemoglu & Robinson.

I found that the empirical values arrived at by the agent-based model through the agents interacting with each other reproduce the Acemoglu & Robinson game-theoretic results in the following ways:

1. In all scenarios, including the extreme values of very high inequality, and very low inequality, the agent-based model found that $\boldsymbol{\varphi}^* < \hat{\boldsymbol{\varphi}}$. $\bar{\boldsymbol{\kappa}} > \check{\boldsymbol{\kappa}}$ and $\bar{\boldsymbol{\kappa}} > \boldsymbol{\kappa}^*$ as specified by Acemoglu & Robinson (2006, p. 199, 237, 243).

2. The agent-based model reproduced critical values of the game-theoretic model as the input parameters were varied. Half of the critical values were within 2% of the game-theoretic results. Of the 18 critical value percent variances calculated in Table 22 and Table 25, I find that 7 were within 1%, another 6 were within 5% and 4 within 10%.
3. The agent-based model reproduced case studies of three non-democratic regimes. In the three instances of low inequality (Belarus), medium inequality (Lao PDR) and high inequality (Swaziland), the model found that the critical values would indicate that the elites were using repression. Repression was mentioned in articles about each country's political and economic condition.
4. The "Democratization in a Picture" results of the agent-based model reproduce pictures with the same 5 areas as Acemoglu & Robinson theoretical pictures and was able to bring additional insight into the effect of the cost of coup and revolutions.

In the A&R game-theoretic model the critical values of $\boldsymbol{\mu}^*$, $\hat{\boldsymbol{\varphi}}$, $\boldsymbol{\varphi}^*$, $\boldsymbol{\kappa}^*$, $\breve{\boldsymbol{\kappa}}$, and $\bar{\boldsymbol{\kappa}}$ were analyzed mathematically and calculated directly. Conversely, in the agent-based model, the critical values are not calculated by the agents. Their experience builds up when certain decisions are made. In the beginning, the values of $\boldsymbol{\mu}^*$, $\hat{\boldsymbol{\varphi}}$, $\boldsymbol{\varphi}^*$, $\boldsymbol{\kappa}^*$, $\breve{\boldsymbol{\kappa}}$, and $\bar{\boldsymbol{\kappa}}$ will change until there is enough experience to build the steady state value.

In the game-theoretic model, Acemoglu and Robinson conduct relative value analysis on how changes in the underlying economic conditions affect the critical values.

In this chapter I conducted relative value analysis on the results from the agent-based model. It seems easier to do the analysis on the agent-based model because it's so easy to change parameters and see the effects in the change of the Democratization Picture. The relative value analysis reveals that when the likelihood of being in a state of high threat of revolution is low (which A&R assert is the norm) their inverted U shaped relationship between inequality and democratizations doesn't hold. The agent-based model finds that the likelihood of being in a state of high threat of revolution must be at least around .25 in order for the inverted U shape relationship to appear. In addition, the agent-based model finds that at medium levels of cost of coup and cost of revolution, (around .15), their U-shaped prediction holds. However, these results are brittle when you move away from average costs. At low costs of coups and revolutions, democratizations occur easiest at the lowest levels of inequality. At high costs of coups and revolutions, democratizations never occur with democratizations being unnecessary at low levels of inequality and with the elites choosing repression at high levels. The agent-based model is a better laboratory with built in visualization of "what if" experiments than complex and unwieldy equations.

4 Chapter Four: Using Agent-Based Models to Relax Assumptions

This model relaxes the intra-group homogenous income assumption of the A&R game-theoretic models.

4.1 Model Design - Heterogeneity of Income Distribution

4.1.1 Model Description

This model extends the initial model in Netlogo 5.0.3 (Wilensky, 1999).

4.1.2 Motivation

I begin with the initial agentization model described in chapter 3. That model has two classes, rich and poor. The intra-group incomes are homogenous. The purpose of this model is to relax the intra-group homogeneity assumption of the initial agentization model to gain additional insights into how a more realistic distribution of income would affect outcomes at the individual and class level. Acemoglu & Robinson suggest that a large middle class will make democracy more likely, and more likely to persist if democracy does emerge. Easterly (2001) has shown the importance of a middle class in democratizations. His cross-country studies have found more democracies in societies with a larger share of income going to the middle class.

In the validation and verification of this model, the homogenous model is used to represent a two-class society. The intra-group heterogeneous income model is used to represent a society with a middle class.

4.1.3 Agentization Approach

This model modifies assumptions to make them more realistic following the example of Guerrero and Axtell (2011). These more realistic assumptions would have rendered the game-theoretic models mathematically intractable. However, they are possible to implement in an agent-based model and can provide additional insights not found in the original models. Table 27 lists the microeconomic assumptions of the game-theoretic models used in the Creation and Consolidation of Democracy. The next section explains how assumption # 3 is modified and implemented in the agent-based model.

Table 27: Agent-based implementation of the microeconomic assumptions

Number	**Assumption**	**Order**	**Agent-based implementation**
1	Population consists of a population of size 1	1st	N-agents
2	Population size is divided into two classes with fraction $\boldsymbol{\delta}$ of elites and (1- $\boldsymbol{\delta}$) poor citizens. $\boldsymbol{\delta} < ½$	1st	Discreet agents. Elites = integer ($\boldsymbol{\delta}$ * N)
3	Rich agents have income y^r and poor agents have income y^p $y^r = (\boldsymbol{\theta} * \bar{y}) / \boldsymbol{\delta}$ $y^p = (1 - \boldsymbol{\theta}) * \bar{y} / (1 - \boldsymbol{\delta})$ (intra-group homogeneity)	2nd	**MODIFIED:** Two classes with heterogeneous Intra-group incomes still keeping these relationships $y^r = (\boldsymbol{\theta} * \bar{y}) / \boldsymbol{\delta}$ $y^p = (1 - \boldsymbol{\theta}) * \bar{y} / (1 - \boldsymbol{\delta})$

Number	Assumption	Order	Agent-based implementation
4	All agents are aware of the cost of coup $\boldsymbol{\varphi}$ and the cost of revolution $\boldsymbol{\mu}$. (rationality)	2nd	Global Variables
5	Each person has preferences on regime decision. The preferences of the median voter determine the regime decisions enacted at the society level. The equilibrium of the game is for both elites and citizens to propose the ideal point of the median voter. (non-interactiveness)	1st	The medium voter agent, a poor agent determines the regime changes that are relevant to the poor. The medium voter elite, a rich agent, determines the regime changes relevant to the rich.
7	Infinite horizon model G^{∞}. Bellman equations are used to analyze the utilities consisting of the current year and looking into the future. (rationality)	2rd	Implements game-theoretic equations
8	Agents expected utility at time t=0 is $U^{1}= E_0\sum_{t=0}^{\infty}\boldsymbol{\beta}\hat{y}_t^i$.	3rd	Implements game-theoretic equations
9	Citizens can mount a revolution by collective action technology. In a revolution, a portion $\boldsymbol{\mu}$ of the economy is destroyed. All rich agents lose all income, and remaining income is divided equally among the poor. $\boldsymbol{\mu}$ changes over time, perhaps due to changes in ability to organize, and is used to model the changes in ability to organize, and is used to model the collective action problem	3rd	Implements game-theoretic equations
10	In a democracy, elites can attempt a coup. After a coup, all agents lose a fraction $\boldsymbol{\varphi}$ of their income. $\boldsymbol{\varphi}$ changes over time and is and is used to model the limited ability of the citizens to commit to future tax concessions.	3rd	Implements game-theoretic equations
11	The game is either in a high or low threat state for revolution. In the low threat state, the cost $\boldsymbol{\mu}^{L}=1$. The probability of a high threat state $\boldsymbol{\mu}^{L}=\boldsymbol{\mu}$ is $q < 1/2$	3rd	Global variables

Number	Assumption	Order	Agent-based implementation
12	The game is either in a high or low threat state for coup. In the low threat state for coup, the cost $\boldsymbol{\varphi}^L=1$. The probability of a high threat state $\boldsymbol{\varphi}^L= \boldsymbol{\varphi}$ is s < 1/2	3rd	Global variables
13	If a coup is mounted, in the next time period $\boldsymbol{\mu}_t=\boldsymbol{\mu}^L=1$ so there is no immediate revolution	3rd	Sets value after coup rather than random generation
14	If a coup is mounted, in the next time period $\boldsymbol{\varphi}_t= \boldsymbol{\varphi}^L =1$ so there is no immediate coup	3rd	Sets value after coup rather than random generation
15	In each non-democratic period, the elites decide whether to democratize. If they do, the median voter, a citizen sets the tax rate.	3rd	Tax rate is found by line-search algorithm.
16	When the elite mount a coup, the median elite, a rich person, sets the tax rate of 0.	3rd	Imbedded in game-theoretic equations

4.1.4 Agents

Similar to the initial agentization, there are two types of agents: rich agents and poor agents. In this model, I relax the homogeneity assumption of A&R's simple two-group model of redistributive politics. Instead of just two types of individuals with just two income levels, one for poor agents and one for rich, this model instantiates a more realistic distribution of incomes among rich and poor agents. This model keeps the group level characteristics defined as in the simple model in order to allow comparison of results. There are still two classes of individuals. The fraction $\boldsymbol{\delta}$ ($\boldsymbol{\delta}$ < ½) of agents are rich and the mean income of the rich is y^r. The fraction (1-$\boldsymbol{\delta}$) of agents are poor and the mean income of the poor is y^p. The population is defined by four values: i) given number of agents, ii) a selected fraction $\boldsymbol{\delta}$ of rich agents, iii) the mean income $\bar{y}$ of the population and iv) a value θ which represents the share of income accruing to the rich. Similar to the simple model, the description of the relative inequality may be expressed, for example, as 5% of the people have 40% of the wealth. Then the mean income of the two classes of citizens is set up as:

$y^r = (\theta * \bar{y}) / \delta$

$y^p = (1 - \theta) * \bar{y} / (1 - \delta)$

And again, similar to the simple model, because $y^p < \bar{y} < y^r$, then θ must be greater than δ. This can also be expressed by stating that the fraction of income that the rich control much be greater than their fraction of the population.

While the group level characteristics are the same with the simple model, the individual attributes are not. Each agent is endowed with a heterogeneous income in a way that maintains the group level aggregates. The model does this by setting up a distribution of income among the agents, and selecting the δ * N agents with the highest incomes and calling them rich agents. The other agents are called poor agents.

In order to instantiate a graceful distribution for a wide range of inequality θ values, the model initiates three different distributions. The distribution for the low inequality case begins with uses a normal distribution. The one for middle inequality uses a combination of normal and exponential distributions and the one for high inequality just uses exponential. The first variable in the random-normal function is the mean, and the second is the standard deviation. The random-exponential function has a variable for the mean. The following section describes the Netlogo code for the three distributions.

Low Inequality starting distribution

set income int (random-normal (y-bar / 1.1) (y-bar / 3))

Medium Inequality starting distribution

set income int $\frac{\text{(random-normal (y-bar/3) (y-bar /1.1) +random-exponential (y-bar/2.5))}}{2}$

High Inequality starting distribution

set income int random-exponential (y-bar)

The incomes of individual rich agents are modified until the mean income of the rich agents is equal to the mean, (y^r). Incomes of individual poor agents are modified until the mean income of the poor agents is equal to the mean, (y^r). This maintains the group level characteristics specified by the inequality metric $\boldsymbol{\theta}$.

As a comparative example, Figure 19 shows the distribution of income among two groups, rich and poor, with homogenous intra-group distributions. There were 10,000 agents with mean income of the entire population of \$1000, and $\boldsymbol{\theta}$ is 40%. Thus 5% (500) agents are rich with mean income y^r = \$8000 and 9500 agents are poor with mean income y^p = \$631. Figure 20 shows the heterogeneous income distribution. The distribution has the same group level characteristics. There are 1000 agents with mean income $\bar{y}$ = \$1000. However, the distribution is more realistic. Again, 5% of the agents are configured to have 40% of the income. The mean group level incomes are the same: y^r = \$8000 and y^p = \$631. In the figures, the distribution of the poor agents is shown in the dark black color, while the distribution of the rich agents is shown in grey. The heterogeneous distribution has a very long tail which extends outside the range of the graph in Figure 20. The user interface displays the max income as $589,000, almost 600 times the population mean income. When inequality θ is increased to .80, the distribution is even more dramatic (see Figure 21).

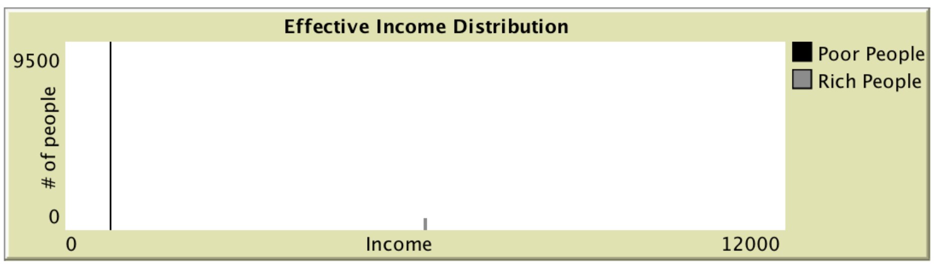


Figure 19: Homogeneous within group income distribution with $\delta = .05$, $\theta = .30$

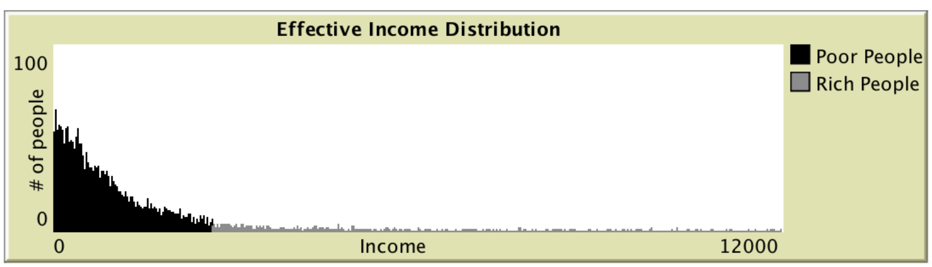


Figure 20: Heterogeneous within group income distribution with $\delta = .05$, $\theta = .30$

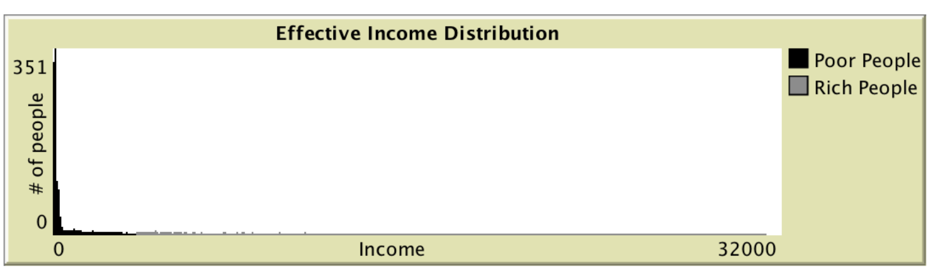


Figure 21: Heterogeneous within group income distribution with $\delta = .05$, $\theta = .80$

Not all distributions are as skewed, when inequality is low, the share accruing to the rich will be lower, and the distribution may be more rounded, with possibly a larger middle class. Figure 22 displays one example distribution when 5% of the population has 15% of the income. Again, the population mean income $\bar{y}$ is still \$1000. However, the distribution looks very different. With the change in inequality, the group level mean incomes, y^r and y^p, are now = \$3000 and \$894 respectively. The maximum income attributed to an agent is \$20,986 (and is outside the range of the graph shown in Figure 22).

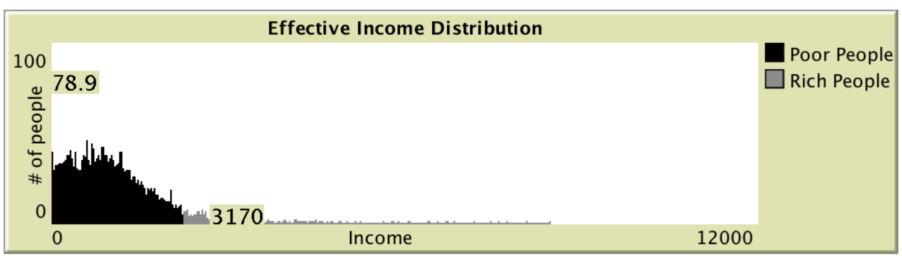


Figure 22: Heterogeneous income distribution with $\boldsymbol{\delta} = .05$, $\boldsymbol{\theta} = .15$.

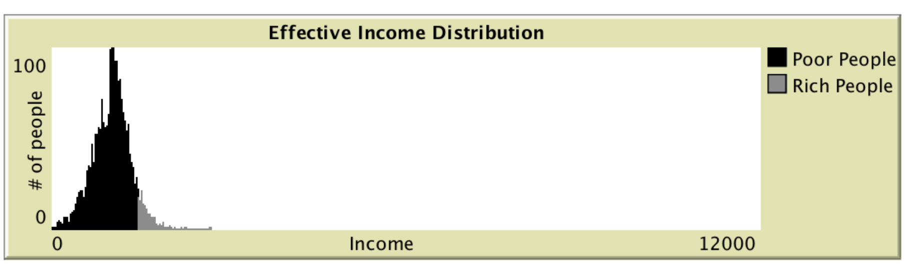


Figure 23: Heterogeneous income distribution with $\boldsymbol{\delta} = .05$, $\boldsymbol{\theta} = .08$

The agent-based model implements the income distributions using a combination of Netlogo random-normal and random-exponential distribution functions. While the initial agentization model could support the wide range of values for inequality $\boldsymbol{\theta}$, there are limitations with heterogeneous model. A&R's game-theoretic model includes the constraint that $\boldsymbol{\theta} > \boldsymbol{\delta}$ in order to solve the requirement that $y^p < \bar{y} < y^r$ (Acemoglu & Robinson, 2006, p. 104). The closer $\boldsymbol{\theta}$ is to $\boldsymbol{\delta}$, the closer y^p is to y^r. In a heterogeneous distribution, to have y^p be only incrementally less than y^r, would result in a distribution much like the homogeneous distribution with two frequency bars: one at y^p, and one at $y^r = y^p + \boldsymbol{\varepsilon}$, where $\boldsymbol{\varepsilon}$ is a very small number. The agent-based model with heterogeneous incomes adds a restriction that $\boldsymbol{\theta} > (1.5 * \boldsymbol{\delta})$ in order to achieve a varied distribution of income.

While Figure 20 through Figure 23 shows different distributions depending on the values of $\bar{y}$, $\boldsymbol{\delta}$ and $\boldsymbol{\theta}$, they are only one instance of a multitude of distributions that may be created by the model from the same $\bar{y}$, $\boldsymbol{\delta}$ and $\boldsymbol{\theta}$. Figure 24 shows three slightly different income distributions generated with $\bar{y} = 1000$, $\boldsymbol{\delta} = .05$, and $\boldsymbol{\theta} = .20$. In Figure 24 the heavy right tail of the high-income earners is truncated in order to present graphs with similar scale.

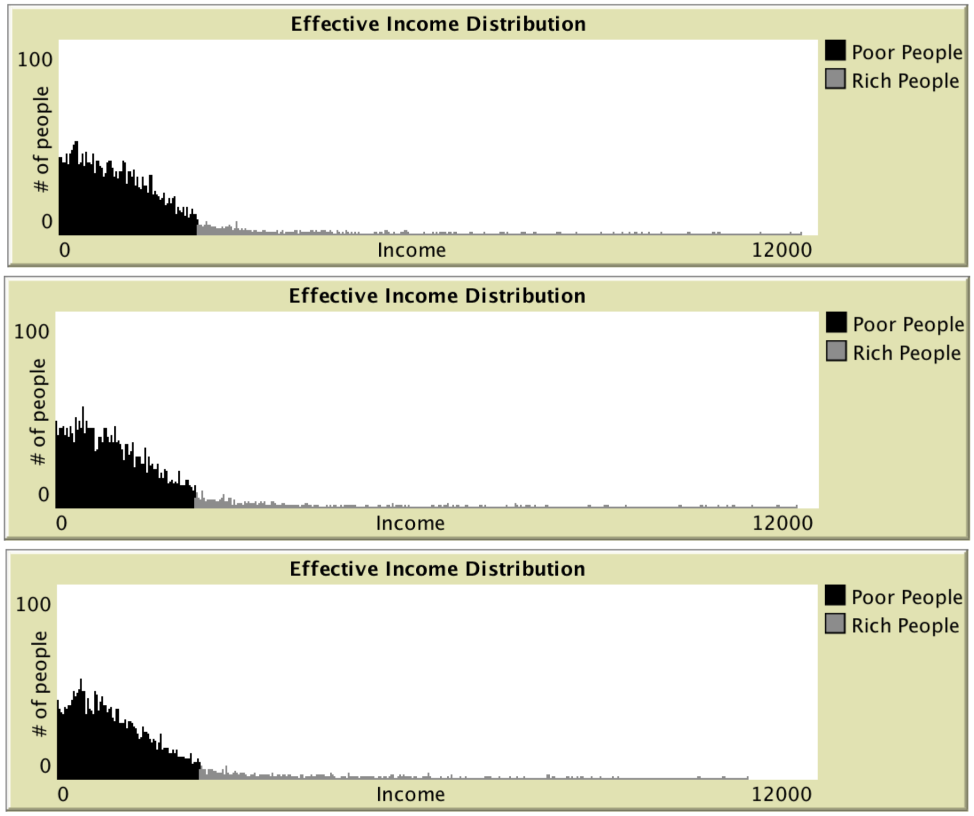


Figure 24: Three slightly different income distributions with δ = .05, θ = .20

The following Netlogo plot setup commands are included in the histogram on the user interface. The first command says that if the mode is on generate "Democratization in a Picture" then don't graph the income distributions since they will change with every change in inequality θ along the horizontal axis. The second command cuts off the long right tail so that the middle class can be seen more clearly. The third one adjusts the pen interval to scale to the average income so that the graph shape is not affected by the average income.

```
if GeneratePoliticalOutcomesPicture? [stop]
```

```
set-plot-x-range 0 int (2 * mean [income] of richpeople)

set-plot-pen-interval (AverageIncome * 5) / 1000
```

4.1.5 Rules

The rules remain the same as in the initial agentization.

4.1.6 Implementation

The Implementation is the same as in the initial agentization.

4.2 Stylized Facts: Identification of Critical Values

The outcome measures and critical values are the same as in the initial agentization. See Table 6.

4.3 Model Verification and Validation

4.3.1 Verification

The model was verified similarly to the initial agentization. A code walkthrough was conducted. In addition, parameter sweeps were performed to verify reasonableness of results. The baseline parameters were kept the same as in the initial agentization in order to make meaningful comparisons. The table of baseline parameters is included below as Table 28 for convenience to the reader.

Table 28: Parameters for baseline case with heterogeneous intra-group income

Parameter	Range	Baseline Value	Change
Number of Agents (N)	**1 – 10,000**	**2,000**	**Static: User-Entered**
Polity	**N, D**	**N**	**Dynamic: User-Initialized**
$\bar{y}$ - mean income of the population	**0 – $100,000**	**$1,000**	**Static: User-Entered**

δ – the percent of population considered rich	**(0, 0.25)**	**.05**	**Static: User-Entered**
θ – the percent of total income shared among the rich	**Range [1.5 * δ, 1] Discreet values in increments of .01**	**.30**	**Static: User-Entered**
φ – The cost of coup as percent of GDP	**Range [0, 1] Discreet values in increments of .01**	**Uniform on (0,1]**	**Dynamic: uniform discreet distribution**
μ – the cost of revolution as percent of GDP	**Range [0,1] Discreet values in increments of .01**	**Uniform on (0,1]**	**Dynamic: uniform discreet distribution**
β – discount factor	**Range [0,1] Discreet values in increments of .01**	**.97**	**Static: User-Entered**
s – **the likelihood that there will be a high threat of coup**	**Range [0,1] Discreet values in increments of .01**	**.25**	**Static: User-Entered**
q - **the likelihood that there will be a high threat of revolution**	**Range [0,1] Discreet values in increments of .01**	**.25**	**Static: User-Entered**

4.3.1.1 Baseline Convergence

The baseline case was run and critical values recorded at four time steps: 10 ticks, 100 ticks, 1000 ticks and 10,000. Table 29 shows the mean and standard deviation for the critical values at those time steps. There were 30 trials at t = 10, and like with the initial agentization, I found a large amount of variance in the values. At t = 10,000 there was still more variance in the heterogeneous model than the initial agentization. This because the core models depend upon the median voter theorem. In the initial agentization, there are more poor people than rich people and all poor people have the same income, the median voter is one of the poor people. All poor people have the same mean income. Therefore, the median voter has the same income as the mean poor voter. However, in this model with heterogeneous intra-group incomes, the distribution is calculated again for every one of the 30 trials of each time duration scenario. For example, the 30 runs at time step t=10 were each initialized with slightly different income distributions. The group level means in the heterogeneous experiment were the same as for the homogenous agent-based model. The mean poor income was the same for all 30 trials, and the mean rich income was the same for all 30 trials. However, the actual incomes of each agent differ slightly because the distributions are randomly generated. The actual income of each agent is different; therefore, the median voter will be slightly different in each run. It is expected then that if the median voter is different, the preferred tax rate will vary more between runs. Likewise, the concession tax to prevent revolutions or prevent coups is expected to vary. Since those tax rates are the building blocks of evaluating utility, the utilities of the agents are expected to vary and the critical values which are dependent on the analysis of those utilities is also expected to vary.

Table 29: Baseline case convergence for intra-group heterogeneity

	At t=10		At t=100		At t=1000		At t=10,000	
	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation
$\boldsymbol{\mu}^*$	0.214	0.069	0.283	0.009	0.290	0.000	0.290	0.000
$\boldsymbol{\varphi}^*$	0.530	0.053	0.577	0.026	0.582	0.023	0.572	0.019
$\hat{\boldsymbol{\varphi}}$	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000
$\boldsymbol{\kappa}^*$	0.104	0.081	0.025	0.019	0.010	0.000	0.010	0.000
$\breve{\boldsymbol{\kappa}}$	0.990	0.034	0.929	0.055	0.889	0.037	0.871	0.029
$\bar{\boldsymbol{\kappa}}$	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000

Table 30 compares the critical values between a population with intra-group homogenous income (no middle class) and a population with intra-group heterogeneous income (with a middle class). At the baseline parameters for the underlying economic conditions, the presence of a middle class increased the coup constraint $\hat{\boldsymbol{\varphi}}$ by 37% from .730 to .1. An increasing coup constraint means an increasing likelihood that the new democracy will not be consolidated. The coup constraint is 1,which means that if democratizations do happen, they will be semi-consolidated or unconsolidated. A semi-consolidated democracy means that the citizens will not be able to enact the maximum taxation. The citizens will have to offer tax concessions in order to prevent a coup. $\boldsymbol{\varphi}^*$ was .4 in the homogenous population. With a middle class, and keeping $\boldsymbol{\theta}$ at the baseline value of .3, $\boldsymbol{\varphi}^*$ was 0.613. When the cost of coup is less than $\boldsymbol{\varphi}^*$, democratizations will result in an unconsolidated democracy. An unconsolidated democracy means that no amount of tax concessions will prevent a coup. For $\boldsymbol{\varphi}^*$ to increase means an increasing likelihood that the new democracy will be unconsolidated rather than consolidated. If unconsolidated, the next time the elites are able to overcome the collective action problem and threaten a coup, they will succeed. However, both $\bar{\boldsymbol{\kappa}}$ and $\breve{\boldsymbol{\kappa}}$ increase with a middle class. Thus, repression is more likely with a middle class and democratizations to both consolidated and unconsolidated democracies become less likely. In summary, with 5% of the populace accruing 30% of the income, a population with more middle class rather than distinct rich and poor classes is more likely to be repressed, and in cases where it does form a democracy, the democracy is less likely to be consolidated.

Table 30: Baseline case convergence comparison between homogeneous and heterogeneous incomes

	At t=10,000	
	Homogeneous income distribution	Heterogeneous income distribution
$\boldsymbol{\mu}*$	0.290	0.290
$\boldsymbol{\varphi}*$	0.400	0.572
$\widehat{\boldsymbol{\varphi}}$	0.730	1.000
$\boldsymbol{\kappa}*$	0.010	0.010
$\breve{\boldsymbol{\kappa}}$	0.610	0.871
$\overline{\boldsymbol{\kappa}}$	0.830	1.000

4.3.1.2 Sensitivity Analysis and Relative Value Testing

Table 31 shows that as the number of agents in the simulation changes the critical values change very little in this agent-based model with heterogeneous agents. However, the variation in critical values due to the integer rounding of number of agents decreases as the population of agents grows to 20,000.

Table 31: Modifying number of agents – heterogeneous incomes

	At N=100		At N=1,000		Baseline Value At N=10,000		At N=20,000	
	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation
$\boldsymbol{\mu}*$	0.291	0.003	0.290	0.000	0.290	0.000	0.290	0.000
$\boldsymbol{\varphi}*$	0.580	0.132	0.563	0.064	0.572	0.019	0.582	0.019
$\widehat{\boldsymbol{\varphi}}$	0.956	0.103	0.987	0.043	1.000	0.000	1.000	0.000
$\boldsymbol{\kappa}*$	0.010	0.002	0.011	0.003	0.010	0.000	0.011	0.004
$\breve{\boldsymbol{\kappa}}$	0.848	0.152	0.856	0.094	0.871	0.029	0.885	0.028
$\overline{\boldsymbol{\kappa}}$	0.967	0.079	0.997	0.015	1.000	0.000	1.000	0.000

The critical mechanism that I want to investigate is how inequality affects democratizations. Table 32 shows the results when the baseline values are held constant while the percent of income accruing to the rich $\boldsymbol{\theta}$ is modified from .08 to .80. The baseline value of $\boldsymbol{\delta}$ is .05. The minimum value of $\boldsymbol{\theta}$ tested was .08 so all values in the parameter sweep meet the condition of the A&R models that $\boldsymbol{\delta} < \boldsymbol{\theta}$. All values are recorded at time step – 10,000.

Table 33 defines what happens to the critical values as the share of income accruing to the rich $\boldsymbol{\theta}$ changes. Similar to the case with homogenous intra-group incomes, as inequality increases, $\boldsymbol{\varphi}^*$ and $\widehat{\boldsymbol{\varphi}}$ increase. This means a consolidated democracy is much less likely, and an unconsolidated democracy becomes more likely. This makes sense since with high inequality, the elites have more incentive to mount a coup to avoid redistribution, so the democracy, if created, is more likely to be unconsolidated and less stable.

Table 32: Modifying percent of income accruing to the rich ($\boldsymbol{\theta}$)- heterogeneous income

	At θ =.08		At θ =.2		Baseline Value At θ =.30		At θ =.80	
	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation
μ*	0.080	0.000	0.190	0.000	0.290	0.000	0.720	0.000
φ*	0.000	0.000	0.261	0.008	0.572	0.019	1.000	0.000
$\hat{\varphi}$	0.010	0.000	0.434	0.059	1.000	0.000	1.000	0.000
κ*	0.216	0.278	0.010	0.000	0.010	0.000	0.308	0.284
$\breve{\kappa}$	0.513	0.325	0.402	0.014	0.871	0.029	1.000	0.000
$\bar{\kappa}$	0.513	0.325	0.551	0.018	1.000	0.000	1.000	0.000

Table 33: Relative value analysis of percent of income accruing to the rich ($\boldsymbol{\theta}$)

Critical Value	Change
μ*	As the percent θ increases, μ* increases
φ*	As the percent θ increases, φ* decreases
$\hat{\varphi}$	As the percent θ increases $\hat{\varphi}$ stays high until very high inequality and then the coup constraint drops quickly.
κ*	As the percent θ increases, κ* stays low
$\breve{\kappa}$	As the percent θ increases, $\breve{\kappa}$ stays high until very high inequality and then the coup constraint drops quickly.
$\bar{\kappa}$	As the percent θ increases, $\bar{\kappa}$ stays high until very high inequality and then the coup constraint drops quickly.

I found in the baseline case (inequality θ = .3) that heterogeneous incomes (middle class) increased the coup constraint $\hat{\varphi}$ which makes consolidated democracies less likely (see Table 30). A consolidated democracy is one in which there is no threat of coup. In Table 34 I compare the critical values found with the initial ABM with homogeneous incomes (Table 16) to the critical values for the ABM with heterogeneous incomes for the other inequality θ values of .08, .20, and .80. From the comparison in

Table 34, I find the effect of a middle class on the critical values to be non-linear with respect to the inequality measure $\boldsymbol{\theta}$:

- At very low inequality ($\boldsymbol{\theta}=0.08$), the coup constraint $\hat{\boldsymbol{\varphi}}$ and $\boldsymbol{\varphi}$* decreases with a middle class. This means, where a democracy does form, the democracy is more likely to be consolidated. K* (the line of indifference between concession and repression) is much higher with a middle class meaning that the elites are more like to repress than offer concessions.
- At low and medium inequality ($\boldsymbol{\theta}=0.2$ and $\boldsymbol{\theta}=0.3$) the coup constraint increases with a middle class. This means, where a democracy does form, the democracy is less likely to be consolidated.
- $\overline{\boldsymbol{\kappa}}$ and $\breve{\boldsymbol{\kappa}}$ also increase with a middle class for populations with low to medium high inequality. Thus, at low to medium-high income inequality, repression becomes more likely with a middle class and democratization less likely.
- At very high inequality ($\boldsymbol{\theta}$ =.80), the middle class does not make much of a difference.

Table 34: Effect of middle-class on low inequality and high inequality populations

	At θ =.08		At θ =.2		At θ =.3		At θ =.80	
	Homogeneous income distribution	Heterogeneous income distribution	Homogeneous income distribution	Heterogeneous income distribution	Homogeneous income distribution	Heterogeneous income distribution	Homogeneous income distribution	Heterogeneous income distribution
μ^*	0.080	0.080	0.190	0.190	0.290	0.290	0.710	0.720
φ^*	0.020	0.000	0.210	0.261	0.400	0.572	1.000	1.000
$\hat{\varphi}$	0.030	0.010	0.370	0.434	0.730	1.000	1.000	1.000
κ^*	0.010	0.216	0.010	0.010	0.010	0.010	0.233	0.308
$\breve{\kappa}$	0.052	0.513	0.330	0.402	0.610	0.871	1.000	1.000
$\bar{\kappa}$	0.052	0.513	0.450	0.551	0.830	1.000	1.000	1.000

4.3.2 Validation and Results

An exploratory model is validated by comparing the trends with the real-world. Table 35 compares the critical values found for the heterogeneous model to the critical values found by the game-theoretic calculations for A&R's intragroup homogeneous model. These comparisons are made for the inequality and discount parameters of three non-democratic countries. Also note that $\boldsymbol{\delta}$ has changed from the baseline values. For example, in the baseline case 5% of the population accrued 30% of the income. This is different from the Lao PDR case where 10% of the population accrues 31% of the income.

Table 35: Analysis of effect of middle class on regime decisions in 3 non-democratic countries.

	Low Inequality Scenario (Belarus) $\boldsymbol{\delta}$ =10%, $\boldsymbol{\theta}$ = 22%, $\boldsymbol{\beta}$=.995 $\boldsymbol{\tau}^{p}$ = .132 $\hat{\boldsymbol{\tau}}$ =N/A			Median Inequality Scenario (Lao PDR) $\boldsymbol{\delta}$ =10%, $\boldsymbol{\theta}$ = 31%, $\boldsymbol{\beta}$=.877 $\boldsymbol{\tau}^{p}$ = .233 $\hat{\boldsymbol{\tau}}$ =N/A			High Inequality Scenario (Swaziland) $\boldsymbol{\delta}$ =10%, $\boldsymbol{\theta}$ = 40%, $\boldsymbol{\beta}$=.962 $\boldsymbol{\tau}^{p}$ = .334 $\hat{\boldsymbol{\tau}}$ =N/A		
	Game-theoretic homogenous Result	ABM: Average of 30 trials		Game Theoretic homogenous Result	ABM: Average of 30 trials		Game-theoretic homogenous Result	ABM: Average of 30 trials	
$\boldsymbol{\mu}^*$	0.218	0.210		0.302	0.300	+	0.386	0.380	+
$\boldsymbol{\varphi}^*$	0.142	0.071		0.205	0.188	+	0.441	0.542	+
$\hat{\boldsymbol{\varphi}}$	0.299	0.081	-	0.487	0.431	-	0.950	0.992	-
$\boldsymbol{\kappa}^*$	N/A	N/A	-	N/A	N/A	-	N/A	N/A	-
$\breve{\boldsymbol{\kappa}}$	$\bar{\boldsymbol{\kappa}} \geq \breve{\boldsymbol{\kappa}}$	0.139	+	$\bar{\boldsymbol{\kappa}} \geq \breve{\boldsymbol{\kappa}}$	0.296	+	$\bar{\boldsymbol{\kappa}} \geq \breve{\boldsymbol{\kappa}}$	0.825	+
$\bar{\boldsymbol{\kappa}}$	0.299	0.161	+	0.487	0.446	+	0.950	1.000	+

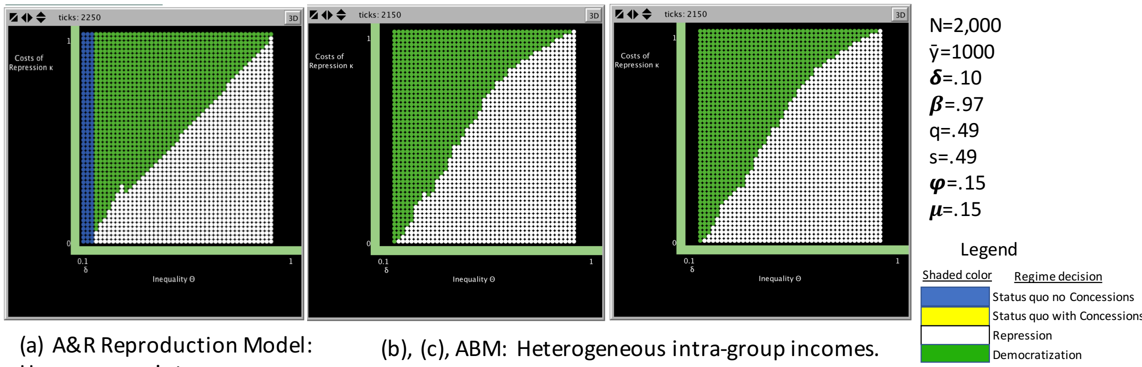
With this comparison, I find:

- $\boldsymbol{\mu}^*$, The level of inequality that makes the benefit of revolutions to be greater than the costs, does not seem to be sensitive to whether there is a middle class.
- Having a middle class makes $\boldsymbol{\varphi}^*$ which determines whether a democracy will be unconsolidated, and $\hat{\boldsymbol{\varphi}}$ which determined whether a democracy will be consolidated, higher in a country like Swaziland with medium high inequality and a medium discount factor. This means, in Swaziland, if a democracy were formed, it is less likely to be consolidated if there is a middle class.

- The opposite is true for lower inequality countries. Having a middle class lowers $\boldsymbol{\varphi}^*$ and $\widehat{\boldsymbol{\varphi}}$ for countries like Belarus and Lao which increases the stability of a democracy if it were to be created. This shows that the effect of a middle class on the stability of a democracy is non-linear.
- The coup constraint $\widehat{\boldsymbol{\varphi}}$ is very low for a country like Belarus with a middle class than it would be for a country with similarly low inequality and high discount factor but without a middle class. Both the A&R homogenous intra-group model and the agent-based heterogeneous model find that if a democracy were to be created, it is more likely to be consolidated in Belarus than in Lao or Swaziland. The agent based heterogeneous model found a lower $\widehat{\boldsymbol{\varphi}}$ (higher likelihood of a consolidated democracy) than the homogenous game-theoretic model for both Belarus and Lao.

4.3.3 "Democratization in a Picture" mode.

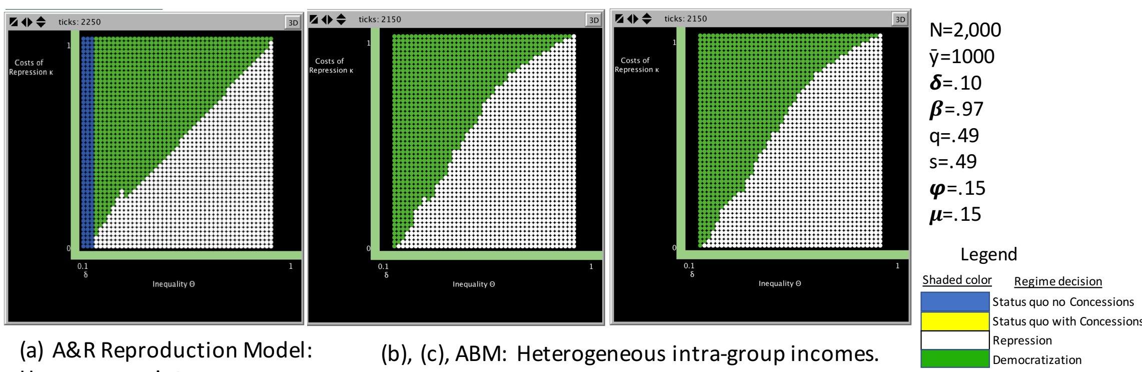
Democratization in a Picture mode works the same for heterogeneous incomes as it does for homogeneous incomes. See section 3.4. This functionality has the same model flow as described in Figure 12: Model flow of "Democratization in a Picture. The only difference is that the income distributions are heterogeneous rather than homogeneous. "Democratization in Picture" illustrates the different regime decisions that get made based on the economic circumstances.



(a) A&R Reproduction Model: Homogenous intra-group incomes.

(b), (c), ABM: Heterogeneous intra-group incomes.

Figure 25: Effect of changes in q for homogeneous vs heterogeneous



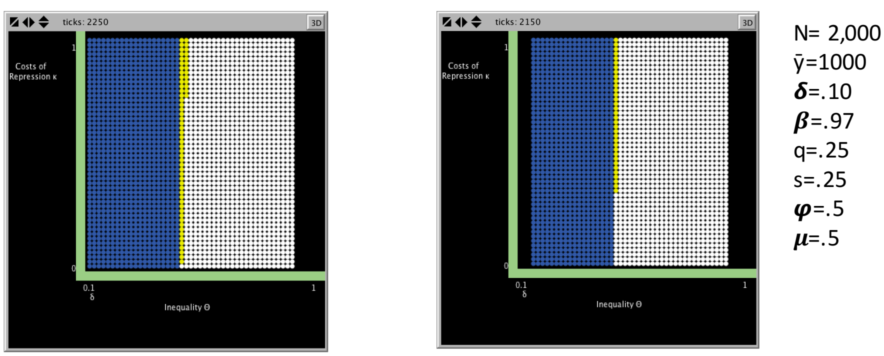
(a) A&R Reproduction Model: Homogenous intra-group incomes.

(b), (c), ABM: Heterogeneous intra-group incomes.

Figure 25 shows a scenario with the parameters for likelihood of revolution and coups set higher than baseline. In this thought experiment, it might be because the technology has made organization much easier. Picture (a) was drawn by the A&R reproduction model of homogenous intra-group agents. Figures (b) and (c) are two different runs of the heterogeneous model with the same parameters. The heterogeneous model has slight variations. Even though the poor mean income and the rich mean income is the same for both homogenous and heterogeneous models, the income distributions are different, leading to slightly different pictures. At lower levels of inequality, the middle class (heterogeneous model) makes democratization is more likely and status quo non-democracy less likely. However, that is not the case at medium and higher levels of inequality. Also note that the line of indifference between creating a democracy and repression is curved with a middle class. The effect of the middle class is non-linear with respect to inequality. This supports the results of the analysis of critical values. Countries with a high inequality like Swaziland are in a high point of the curve and are less likely to democratize with a middle class than without. Acemoglu and Robinson argue that the middle class acts as a buffer between the rich and poor, and limiting redistribution, and making democratization less costly to the elites (Acemoglu & Robinson, 2006, p. 258). This analysis show that when there is a high likelihood of high threat of revolution, the presence of a middle class does not make democratizations more likely except at extremely low levels of inequality. The assertion that lowering inequality may tip a country into democratization doesn't seem to hold for high values of likelihood of revolution and medium to high levels of inequality.

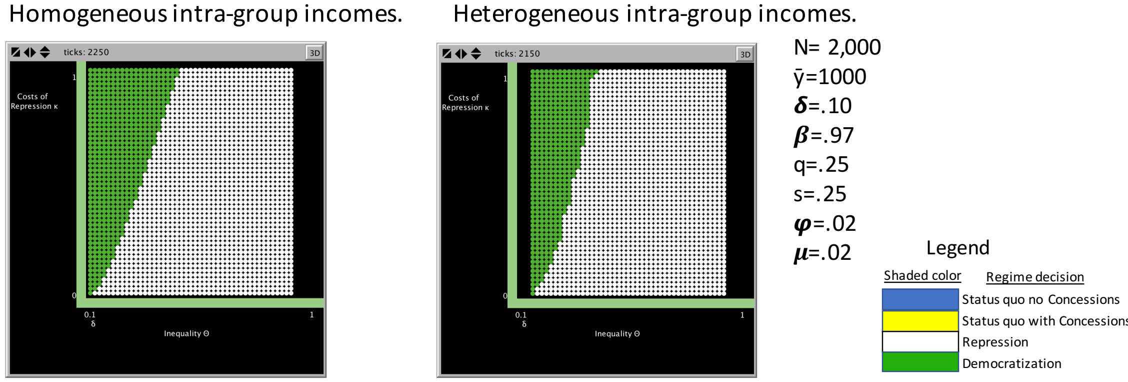
In Figure 26 the diagrams on the left is made by the A&R replicate homogeneous agent model and the diagrams on the right are made by the heterogeneous agent model showing the effects of the middle class. Pictures (a) and (b) shows that with high cost of coup and revolutions, the middle class does not have much of an effect. With a high cost of coups and revolutions, such as we see with high GDP countries like Singapore, the elites tend to keep the status quo at lower levels of equality and to repress more as inequality increases. Pictures (c) and (d) show the difference with a low cost of revolutions and coups. A low cost of revolution makes it more likely that a democracy will be formed compared to a high cost of revolution. However, in figure (d), it appears that the presence of a middle class makes does not make democratizations more likely at low inequality but does make democratization less likely at higher inequality as repression becomes more advantageous. Again, with a middle class, the line of indifference between democratization and repression is curved.

A&R Model Results **ABM Model with Middle Class**



(a) High cost of coup and revolutions. Homogeneous intra-group incomes.

(b) High cost of coup and revolutions. Heterogeneous intra-group incomes.



(c) Low cost of coup and revolutions. Homogeneous intra-group incomes.

(d) Low cost of coup and revolutions. Heterogeneous intra-group incomes.

Figure 26: "Democratization in a Picture" - homogeneous compared to heterogeneous income

4.4 Model Design: Bounded Rationality and Heterogeneous Interactions

4.4.1 Model Description

One of the most unrealistic assumptions of game theoretic models is that the agents are omniscient. They know everything. This model extends the heterogeneous-income model of section 4.1. In this model, only some small fraction, the leaders, has the capacity and desire to do the analysis and determine the most advantageous course of action. The majority of agents are followers of one of the leaders. This model contains heterogeneous distributions and cross-scale human-environment interactions between local decisions, and sub-group influencers and group level outcome. These characteristics have been found to be more likely to lead to complex outcomes (Miller & Page, 2007 p. 231).

4.4.2 Motivation

The purpose of this model is to utilize the heterogeneous-income model to relax the rationality and non-interaction assumption. The original A&R game-theoretic models assumed each agent calculated the benefits of alternative decisions with unlimited look-ahead to the future and being fully cognizant of the underlying economic characteristics and the trade-off decisions of the other class. A more realistic assumption is that a relatively small number of agents are fully rational and the rest of the population adopts the preferences of others. Changing this assumption necessitates a change in the non-interaction assumption. The A&R models stylized the "interaction" among the population with the medium voter theorem (MVT). This agent-based model will modify that stylized interaction to include representational leadership. This will provide additional insights into how a more realistic decision based on multiple stakeholders with different preferences would affect outcomes at the individual and class level.

4.4.3 Agentization Approach

This model modifies assumptions to make them more realistic following the example of Guerrero and Axtell (2011). These more realistic assumptions would have rendered the game-theoretic models mathematically intractable. However, they are possible to implement in an agent-based model and can provide additional insights not found in the original models. Table 36 lists the microeconomic assumptions of the game-theoretic models used in the Creation and Consolidation of Democracy. The next section explains how assumption # 4 and # 5 are modified and implemented in the agent-based model. The boundedly rational agents know only their individual desires and decisions.

Table 36: Agent-based implementation of the microeconomic assumptions

Number	Assumption	Order	Agent-based implementation
1	Population consists of a population of size 1	1st	N-agents
2	Population size is divided into two classes with fraction $\boldsymbol{\delta}$ of elites and (1-$\boldsymbol{\delta}$) poor citizens. $\boldsymbol{\delta} < ½$	1st	Discreet agents. Elites = integer ($\boldsymbol{\delta}$ * N)

Number	Assumption	Order	Agent-based implementation
3	Rich agents have income y^r and poor agents have income y^p $y^r = (\boldsymbol{\theta} * \bar{y}) / \boldsymbol{\delta}$ $y^p = (1 - \boldsymbol{\theta}) * \bar{y} / (1 - \boldsymbol{\delta})$ (intra-group homogeneity)	2nd	**MODIFIED:** Two classes with heterogeneous Intra-group incomes still keeping these relationships $y^r = (\boldsymbol{\theta} * \bar{y}) / \boldsymbol{\delta}$ $y^p = (1 - \boldsymbol{\theta}) * \bar{y} / (1 - \boldsymbol{\delta})$
4	All agents are aware of the cost of coup $\boldsymbol{\varphi}$ and the cost of revolution $\boldsymbol{\mu}$. (rationality)	2nd	**MODIFIED** Only leaders are aware of cost of coup $\boldsymbol{\varphi}$ and cost of revolution $\boldsymbol{\mu}$ and take this into account when determining their preferred tax rate.
5	Each person has preferences on regime decision. The preferences of the median voter determine the regime decisions enacted at the society level. The equilibrium of the game is for both elites and citizens to propose the ideal point of the median voter. (non-interactiveness)	1st	**MODIFIED** Rather than taking the incomes of the entire population into account when determining the preferred tax rate, the majority of the population will imitate the preference of the leader whose income is closest to their own and thus "represents" them.
7	Infinite horizon model G^{∞}. Bellman equations are used to analyze the utilities consisting of the current year and looking into the future. (rationality)	2rd	Implements game-theoretic equations
8	Agents expected utility at time t=0 is $U^1 = E_0 \sum_{t=0}^{\infty} \boldsymbol{\beta} \hat{y}_t^i$.	3rd	Implements game-theoretic equations

Number	Assumption	Order	Agent-based implementation
9	Citizens can mount a revolution by collective action technology. In a revolution, a portion $\boldsymbol{\mu}$ of the economy is destroyed. All rich agents lose all income, and remaining income is divided equally among the poor. $\boldsymbol{\mu}$ changes over time, perhaps due to changes in ability to organize, and is used to model the limited ability of the elites to commit to future redistribution.	3rd	Implements game-theoretic equations
10	In a democracy, elites can attempt a coup. After a coup, all agents lose a fraction $\boldsymbol{\varphi}$ of their income. $\boldsymbol{\varphi}$ changes over time and is and is used to model the limited ability of the citizens to commit to future tax concessions.	3rd	Implements game-theoretic equations
11	The game is either in a high or low threat state for revolution. In the low threat state, the cost $\boldsymbol{\mu}^L=1$. The probability of a high threat state $\boldsymbol{\mu}^L= \boldsymbol{\mu}$ is $q < 1/2$	3rd	Global variables
12	The game is either in a high or low threat state for coup. In the low threat state for coup, the cost $\boldsymbol{\varphi}^L=1$. The probability of a high threat state $\boldsymbol{\varphi}^L= \boldsymbol{\varphi}$ is $s < 1/2$	3rd	Global variables
13	If a coup is mounted, in the next time period $\boldsymbol{\mu}_t=\boldsymbol{\mu}^L=1$ so there is no immediate revolution	3rd	Sets value after coup rather than random generation
14	If a coup is mounted, in the next time period $\boldsymbol{\varphi}_t= \boldsymbol{\varphi}^L =1$ so there is no immediate coup	3rd	Sets value after coup rather than random generation
15	In each non-democratic period, the elites decide whether to democratize. If they do, the median voter, a citizen sets the tax rate.	3rd	Tax rate is found by line-search algorithm.
16	When the elite mount a coup, the median elite, a rich person, sets the tax rate of 0.	3rd	Imbedded in game-theoretic equations

4.4.4 Agents

This model extends the heterogeneous income distribution model described in section 4.1. Again, there are two types of agents: rich agents and poor agents. In this model, I relax the homogeneity assumption of A&R's simple two-group model of redistributive politics the same way as done in section 4.1. The model instantiates a more realistic distribution of incomes among rich and poor agents. The model keeps the group level characteristics defined as in the simple model in order to allow comparison of results.

The extension for this model is that now, only a fraction $\boldsymbol{\alpha}$ of the population are fully rational. The rest are imitators and follow the decisions of their leader that most closely represents their income characteristics. If the leader's strategy is random, then the leaders arise at random and advocate for their own income level. Each agent in the population selects a leader whose constituent income reflects a value closest to their own. Leaders select themselves as leaders. In each simulation, income distributions may be slightly different. The constituent incomes of the leaders are randomly chosen among the distribution, therefore, some leaders will have more followers than others.

If the leaders' strategy is "equivalent constituents" then the leaders strategical position themselves in relation to the rest of the leaders trying to get the most support. In this mode, the model assumes the leaders are able to divide the support equally among themselves. The income of the leader represents the income of the population that the leader is advocating for. This doesn't necessarily mean that the agent actually has that income level. The "income" parameter for leaders represents the "constituent" income level. It means that the leader is advocating a decision that best serves agents with an income level near the leaders assigned level.

In this model, the population is defined by six values:

i) given number of agents,

ii) a selected fraction $\boldsymbol{\delta}$ of rich agents,

iii) the mean income $\bar{y}$ of the population,

iv) a value $\boldsymbol{\theta}$ which represents the share of income accruing to the rich,

v) a fraction $\boldsymbol{\alpha}$ of leaders

4.4.5 Rules

The rules remain the same as in the initial agentization with the following modifications.

4.4.5.1 Minimum Voter Theorem

In the original A&R model, the regime decisions were made using the medium voter theorem. In a democracy, all citizens rich and poor share in the political power. However, since there are more poor citizens than rich citizens, and since all poor citizens have the same intra-group income, the medium voter is a poor citizen with income (y^p).

In a non-democracy, only the elite have political power. The medium voter making decisions in a non-democracy is an elite agent. And since they are all homogenous, it doesn't matter which one. The median voter is an elite citizen with income (y^r).

In this agent based model, the medium voter concept is modified slightly. Each agent is either a leader or an imitator. If she is a leader, then she represents a constituency of agents who want an outcome most closely aligned with her represented income. This is implemented in the model by having all the non-leaders look at their income, and find the leader with the income closest to their own, and adjusting their preferences to match that of the representational leader. In this way, the agents "support" is for the leader whose income is closest to their own.

The model support two strategies for leaders: i) Random, and ii) Equivalent Constituency. If the "Randomsets" strategy is selected, then leaders arise randomly advocate for their actual income. If "Equivalent Constituency" is selected, then the leaders adopt a representational income such that each leader will have an approximately equivalent share of support from the population. If there are 3 leaders, approximately 1/3 of the population will adopt the income each leader. It is approximate because sometimes there are multiple agents with the same income. If this block of agents lies on a border supporting one agent, the actual cut off may be in the middle of the block but all agents in that block will follow the same leader. Thus, some leaders may have slightly more agents than others.

4.4.6 *Implementation*

The Implementation is the same as in the initial agentization.

4.4.6.1 *Model Interface*

The Netlogo user-interface (see Figure 27 allows the user to change the global knowledge variables and through a set of sliders. Two new global variables a fraction ***α*** of leaders, and the leader strategy, have been added to the user interface at the top center.

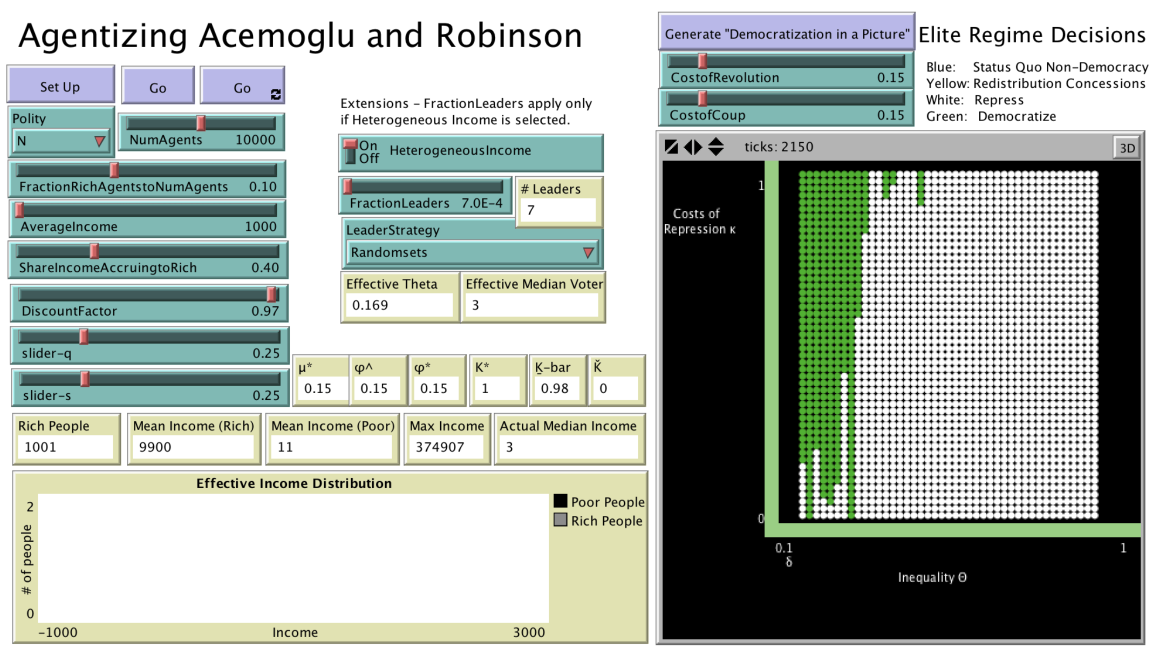


Figure 27: Model user interface

The model can still be run in two modes. The dynamic mode runs the Dynamic Model of the Creation and Consolidation of Democracy (Acemoglu & Robinson, 2006, p.

231). And the "Democratization in a Picture" mode represents the empirical implications about intra-group inequality. It illustrates the circumstances under which different regimes are chosen.

4.4.6.2 Model Flow

The Model Flow is the same as described in section 3.1.6.3 with the modification to the Setup routine to establish leaders and adjust agent incomes to follow leaders. Figure 28 illustrates the flow of activities and processing for the agent-based model of the Creation and Consolidation of Democracy with limited rationality. The change in the Setup routine is detailed below.

SETUP The "Setup" button on the user interface sets up the community of agents as described in the process flow of Figure 5. The "Setup" button executes the following steps:

1. *Setup constants with initial values*
2. *Set the average income for each class y^r and y^p*
3. *Set up Agents*

- *Set up $\boldsymbol{\delta}$ rich agents with income $y^r = (\boldsymbol{\theta} * \bar{y}) / \boldsymbol{\delta}$*
- *Set (1-$\boldsymbol{\delta}$) poor agents with income $y^p = (1 - \boldsymbol{\theta}) * \bar{y} / (1 - \boldsymbol{\delta})$*
- *Set up Heterogeneous Income Distributions*
- *Randomly select ($\boldsymbol{\alpha}$*Numagents) agents for leaderset. If less than one, randomly select one agent as leader.*
- *If strategy = "Randomset" then use list of leaders' incomes.*
- *Else strategy = "Strategic Constituents". Determine the interval length and create a list of incomes that are the median incomes for each equally divided interval. Assign each income to a different leader.*
- *Each agent updates income with one of the set of leaders incomes that is closest to their income.*

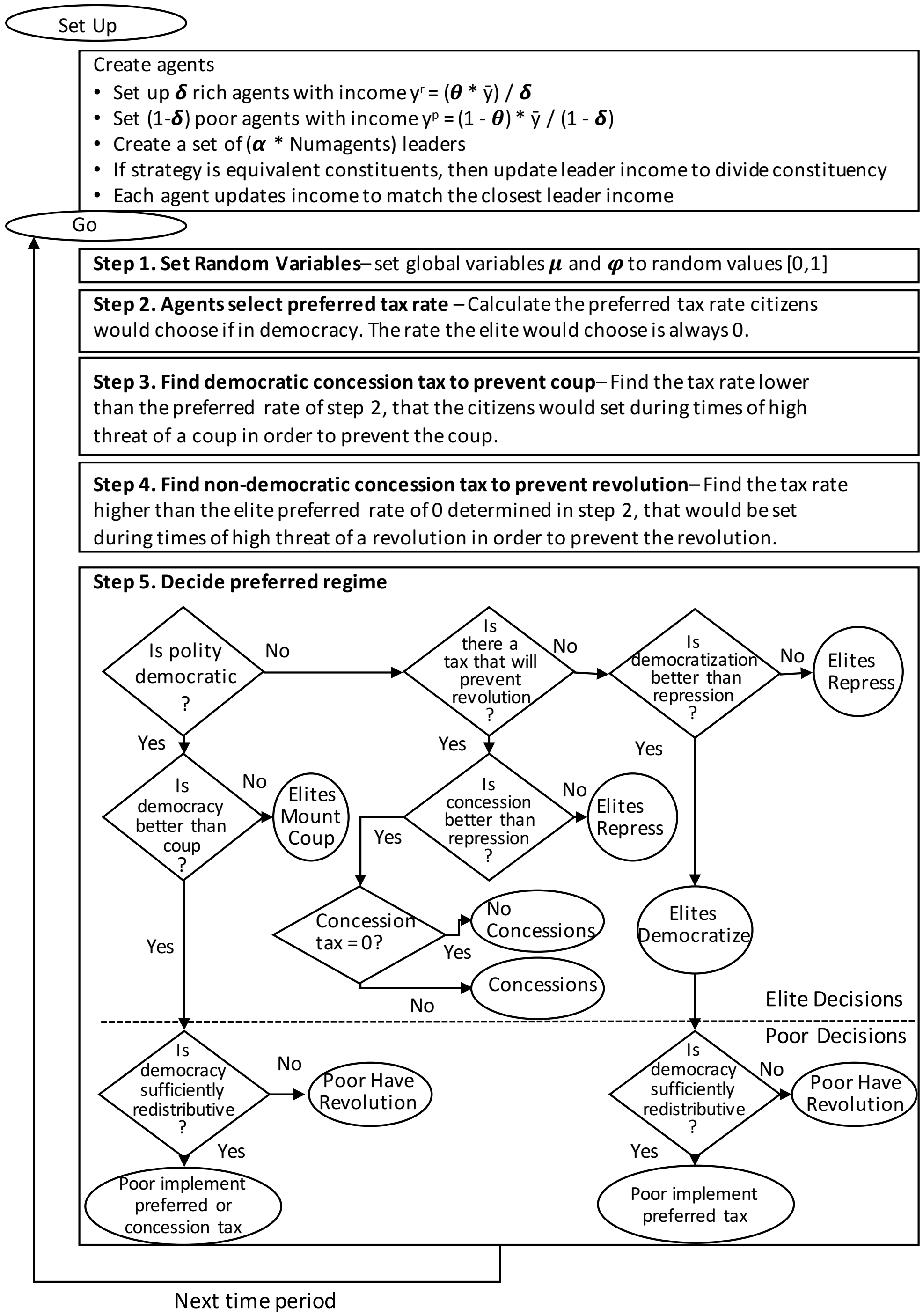


Figure 28: Model flow of the creation and consolidation of democracy- limited rationality

4.5 Model Verification and Validation

4.5.1 Verification

The model was verified similarly to the initial agentization. A code walkthrough was conducted. In addition, parameter sweeps were performed to verify reasonableness of results. Outputs at interim stages were captured and evaluated to verify the processing logic was producing the correct interim results and the correct final results.

The baseline parameters remain the same as in the heterogeneous income model with the addition of the fraction of population considered a leader $\boldsymbol{\alpha}$, and the strategy of random or equivalent constituents.

Table 37: Parameters for baseline case with heterogeneous intra-group income

Parameter	Range	Baseline Value	Change
Number of Agents (N)	**1 – 10,000**	**2,000**	**Static: User-Entered**
Polity	**N, D**	**N**	**Dynamic: User-Initialized**
ȳ - mean income of the population	**0 – $100,000**	**$1,000**	**Static: User-Entered**
$\boldsymbol{\delta}$ – the percent of population considered rich	**(0, 0.25)**	**.05**	**Static: User-Entered**
$\boldsymbol{\theta}$ – the percent of total income shared among the rich	**Range (0,1) Discreet values in increments of .01**	**.30**	**Static: User-Entered**
$\boldsymbol{\varphi}$ – The cost of coup as percent of GDP	**Range [0, 1] Discreet values in increments of .01**	**Uniform on (0,1]**	**Dynamic: uniform discreet distribution**
$\boldsymbol{\mu}$ – the cost of revolution as percent of GDP	**Range [0,1] Discreet values in increments of .01**	**Uniform on (0,1]**	**Dynamic: uniform discreet distribution**

β – discount factor	**Range [0,1] Discreet values in increments of .01**	**.97**	**Static: User-Entered**
s – **the likelihood that there will be a high threat of coup**	**Range [0,1] Discreet values in increments of .01**	**.25**	**Static: User-Entered**
q - **the likelihood that there will be a high threat of revolution**	**Range [0,1] Discreet values in increments of .01**	**.25**	**Static: User-Entered**
α - **fractionleaders**	**Range [0, 0.1] Discreet values in increments of .0001**	**.0005**	**Static: User-Entered**
Leader Strategy	Random Equivalent Constituents	Random	**Static: User-Entered**

4.5.1.1 Baseline Convergence

The baseline test was run and critical values recorded at timesteps 10, 100, 1000, and 10,000 (Table 38). While the heterogeneous model showed more run to run variation than the homogeneous model, this boundedly rational model with leadership strategy = Randomsets showed more run-to-run variation than the heterogeneous model with fully rational agents. Even at t=10,000, the variance was considerable. This is as expected. The leaders are selected randomly, therefore, there is a wider variance around the median income of the population.

Table 38: Baseline case convergence for intra-group heterogeneity and leader strategy = "Randomset"

	At t=10		At t=100		At t=1000		At t=10,000	
	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation
$\boldsymbol{\mu}^*$	0.226	0.072	0.292	0.014	0.293	0.006	0.294	0.010
$\boldsymbol{\varphi}^*$	0.505	0.275	0.642	0.233	0.627	0.233	0.592	0.243
$\hat{\boldsymbol{\varphi}}$	0.891	0.221	0.927	0.143	0.908	0.221	0.881	0.246
$\boldsymbol{\kappa}^*$	0.274	0.297	0.018	0.013	0.044	0.181	0.010	0.002
$\breve{\boldsymbol{\kappa}}$	0.910	0.204	0.902	0.115	0.853	0.191	0.799	0.237
$\bar{\boldsymbol{\kappa}}$	0.972	0.155	0.969	0.080	0.938	0.160	0.907	0.220

Table 39 shows the baseline convergence cases for the leadership strategy = Equivalent Constituency. Variance of the critical values are similar to that of the heterogeneous model with fully rational agents.

Table 39: Baseline case convergence for intra-group heterogeneity and leader strategy ="Equivalent Constituency"

	At t=10		At t=100		At t=1000		At t=10,000	
	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation
$\boldsymbol{\mu}^*$	0.229	0.065	0.285	0.008	0.290	0.000	0.290	0.000
$\boldsymbol{\varphi}^*$	0.485	0.115	0.573	0.022	0.583	0.023	0.582	0.021
$\hat{\boldsymbol{\varphi}}$	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000
$\boldsymbol{\kappa}^*$	0.123	0.102	0.020	0.014	0.011	0.003	0.011	0.003
$\breve{\boldsymbol{\kappa}}$	0.997	0.016	0.942	0.037	0.889	0.035	0.884	0.031
$\bar{\boldsymbol{\kappa}}$	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000

Table 40 compares the baseline values of the four variations of the model: i) Homogenous intra-group incomes, ii) Heterogeneous intra-group incomes, iii) Limited Rationality (Equivalent Constituency), and iv) Limited Rationality (Random Leadership. The presence of a middle classes increases the coup constraint by 37%, which means an increasing likelihood that the new democracy will not be consolidated. Leadership with equivalent constituency has similar critical values as the full rationality model. At the baseline values, leadership with random leaders, reduces the impact of the middle class.

Table 40: Baseline case convergence comparison

	At t=10,000			
	Homogeneous intra-group incomes	Heterogeneous intra-group incomes	Limited Rationality (Equivalent Constituency)	Limited Rationality (Random Leadership
$\boldsymbol{\mu}^*$	0.290	0.290	0.290	0.294
$\boldsymbol{\varphi}^*$	0.400	0.572	0.582	0.592
$\hat{\boldsymbol{\varphi}}$	0.730	1.000	1.000	0.881
$\boldsymbol{\kappa}^*$	0.010	0.010	0.011	0.010
$\breve{\boldsymbol{\kappa}}$	0.610	0.871	0.884	0.799
$\bar{\boldsymbol{\kappa}}$	0.830	1.000	1.000	0.907

4.5.1.2 Sensitivity Analysis and Relative Value Testing

Sensitivity testing was performed on the new variable $\boldsymbol{\alpha}$. Table 41 displays the results of the sensitivity analysis on the percent of leaders $\boldsymbol{\alpha}$ using the option of random leaders.

Table 41: Modify percent of leaders $\boldsymbol{\alpha}$

	At α =.0002		Baseline Value At α =.0005		At α =.001		At α =.01	
	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation	ABM Average of 30 trials	Standard Deviation
μ^*	0.305	0.017	0.294	0.010	0.290	0.002	0.290	0.000
φ^*	0.615	0.406	0.592	0.243	0.614	0.136	0.578	0.025
$\hat{\varphi}$	0.720	0.415	0.881	0.246	0.968	0.121	1.000	0.000
κ^*	0.122	0.292	0.010	0.002	0.011	0.004	0.011	0.004
$\breve{\kappa}$	0.786	0.316	0.799	0.237	0.884	0.147	0.878	0.037
$\bar{\kappa}$	0.818	0.299	0.907	0.220	0.974	0.101	1.000	0.000

Table 42 defines what happens to the critical values as the fraction of leaders α change. As the fraction of leaders increase, the coup constraint $\hat{\varphi}$, increases. This means that a consolidated democracy will be less likely. $\bar{\kappa}$ and $\breve{\kappa}$ increases which means that at the baseline values, democratization is less likely and repression more likely at lower numbers of leaders.

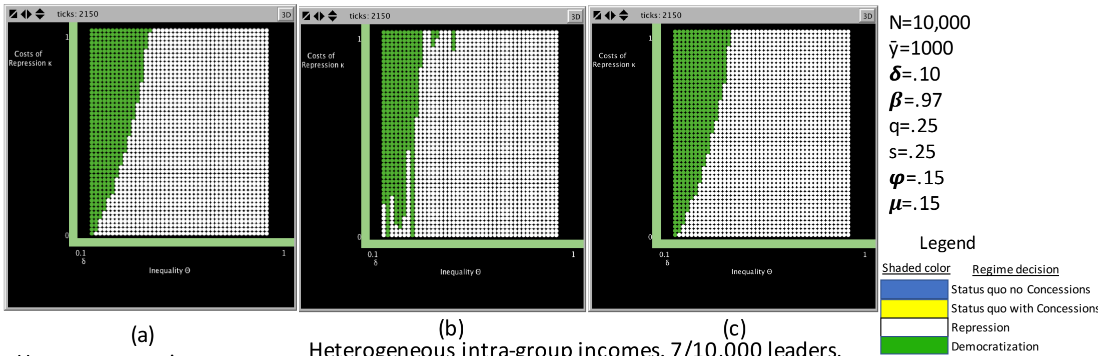
Table 42: Relative value analysis

Critical Value	Change
μ^*	As the fraction of leaders α increases, μ^* stays the same
φ^*	As the fraction of leaders α increases, φ^* increases
$\hat{\varphi}$	As the fraction of leaders α increases, $\hat{\varphi}$ increases
κ^*	As the fraction of leaders α increases, κ^* decreases
$\breve{\kappa}$	As the fraction of leaders α increases, $\breve{\kappa}$ increases
$\bar{\kappa}$	As the fraction of leaders α increases, $\bar{\kappa}$ increases

4.5.2 Validation

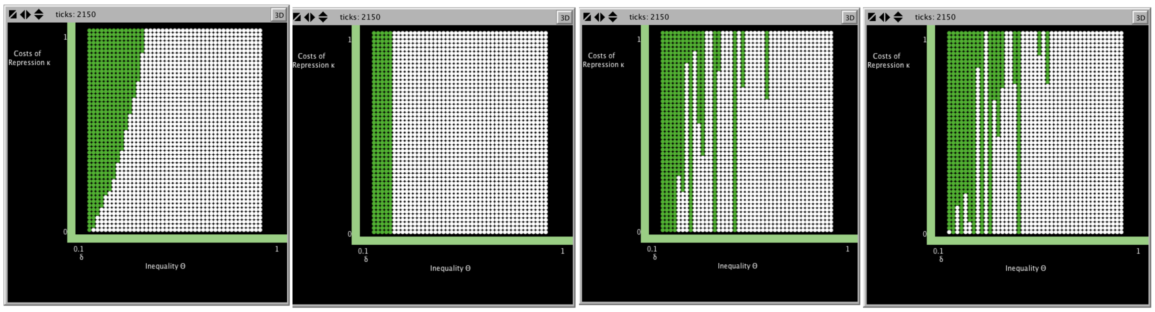
The purpose of this model is to explore what might happen if citizens didn't rationally evaluate their choices. In this model while some small number of citizens are rational, most citizens will choose a leader a and adopt the preferences of their leader. It is difficult to compare this thought experiment to real world data. Therefore, the model will be validated by assessing the validity of "Democratization in a picture" results.

Figure 29 shows three sample trials of "Democratization in a Picture". The heterogeneous model instantiates a new distribution at every new level of $\boldsymbol{\theta}$ leading to slight run to run variations. Picture (a) shows heterogeneous intra-group income with rational agents. Parameters are set as indicated on the right. Picture (b) shows the model results with 7 leaders in the population of 10,000 and with random leader strategy. The median income changes from trial to trial because the leaders are randomly selected and represent the incomes closest to their own. Picture (c) shows the results with 7 leaders in the population of 10,000 but with equivalent constituent strategy. "Equivalent Constituency" models the case when the leaders position themselves to get the most support given all the other leaders.



(a) Heterogeneous intra-group incomes. All Rational.

(b) (c) Heterogeneous intra-group incomes. 7/10,000 leaders. (b) Random leaders and (c) equivalent constituency



(d) 6/10,000 leaders. (e) 2/10,000 leaders. Heterogeneous intra-group incomes. Equivalent constituency.

(f) 2/10,000 leaders. (g) 2/10,000 leaders. Heterogeneous intra-group incomes. Random leaders

Figure 29: "Democratization in a Picture" – relaxing rationality

The next three pictures show additional leadership variations at the same baseline parameter setting. Picture (d) shows the results with 6 leaders in the population and with equivalent constituent strategy. This is similar to the case with 7 leaders. Picture (e) shows the results with 2 leaders in the population and with equivalent constituent strategy. Since there is an even number of leaders who have each strategically represented half of the vote, half the population is aligned with one leader and half with the other. This dramatically changes the effective median voter and reduces the area of democratization in low inequality settings. Pictures (f) and (g) shows the results with 2 leaders in the population and with random strategy. There are only two leaders which randomly may both represent an income higher than the median income or both represent an income lower than the median. This dramatically changes the effective median voter. One conclusion drawn from pictures b), f) and g) is that limiting rationality results in more variability in the ability for underlying economic conditions to trigger a democratization.

By looking at the display monitors I could tell areas of democratization occurred when the effective median voter was higher than the actual median voter. This expands the possibility of democratizations depending on the political leaders at the time and the constituency they are representing. The main conclusion drawn is that the distribution of the leaders' representative constituency affects the determination of the effective median voter and may thus affect the requested tax rate and regime decisions made by considering that tax rate.

4.6 Conclusion of Using Agent-Based Models to Relax Assumptions

Chapter 4 has described two extensions of the initial agent-based model of the Creation and Consolidation of Democracy. Section 4.1 describes the model where intra-group incomes are heterogeneous with a varying distribution while still keeping the group level mean incomes and the inequality measure $\boldsymbol{\theta}$ the same. Section 4.4 describes the model where i) agents' incomes are heterogeneous, and ii) not all agents have unlimited rationality. A small percentage of agents are analytical and have unlimited rationality.

The remainder of the agents choose a leader and select the leader's preferences as their own.

One limitation of the heterogeneous model is that inequality $\boldsymbol{\theta}$ values can't be close to $\boldsymbol{\delta}$. A & R specify that $\boldsymbol{\theta}$ must be greater than $\boldsymbol{\delta}$. Intuitively, a population doesn't have two distinguished classes if the 5% of the population with the highest income, accrue 5% of the income. In an agent based implementation as $\boldsymbol{\theta}$ becomes close to $\boldsymbol{\delta}$, the intra-group population becomes homogenous. I added the limitation that that $\boldsymbol{\theta} > (1.5 * \boldsymbol{\delta})$ in order to achieve a varied distribution of income. By running the baseline model, I found that the heterogeneous agents had slightly different income distributions each trial which resulting in larger variances in the critical values.

The heterogeneous model was verified by running sensitivity analysis on the critical values by varying the input parameters. It was validated by comparing the critical values to those determined by A&R. I found that when a population has a middle class, it has the following effect on the critical values:

1. At the baseline case, when a middle class is present, the coup constraint $\widehat{\boldsymbol{\varphi}}$ increased. An increasing coup constraint means an increasing likelihood that the new democracy will not be consolidated. Thus, in the new democracy, the citizens will not be able to enact their maximum preferred taxation. The critical value $\boldsymbol{\varphi}^*$ increased as well which means increasing likelihood that the democracy will be unconsolidated, that is, subject to coups.
2. At very low inequality ($\boldsymbol{\theta}$=0.08), the coup constraint $\widehat{\boldsymbol{\varphi}}$ and $\boldsymbol{\varphi}^*$ decreases with a middle class. This means, where a democracy does form, the democracy is

more likely to be consolidated. K* (the line of indifference between concession and repression) is much higher with a middle class meaning that the elites are more like to repress than offer concessions.

3. At low and medium inequality ($\boldsymbol{\theta}$=0.2 and $\boldsymbol{\theta}$=0.3) the coup constraint increases with a middle class. This means, where a democracy does form, the democracy is less likely to be consolidated. This finding was supported by the analysis of specific economic conditions in Belarus and Lao.
4. $\bar{\boldsymbol{\kappa}}$ and $\breve{\boldsymbol{\kappa}}$ also increase with a middle class for populations with low to medium high inequality. Thus, at low to medium-high income inequality, repression becomes more likely with a middle class and democratization less likely.
5. Having a middle class makes $\boldsymbol{\varphi}$* which determines whether a democracy will be unconsolidated, and $\hat{\boldsymbol{\varphi}}$ which determined whether a democracy will be consolidated, higher in a country like Swaziland with high inequality and a medium discount factor. This means, in Swaziland, if a democracy were formed, it is less likely to be consolidated if there is a middle class.

The "Democratization in a Picture" model showed that

1. While the middle class may make democratizations more likely at the baseline parameters, it does not hold for all levels of underlying economic values. In particular:
    a. At high probability of credible threat of revolution, democratizations seem less likely except at very low inequality
    b.) At low cost of revolution, democratizations seem less likely

c. At high cost of revolution democratizations don’t occur and are insensitive to a middle class.

2. In economic environments with low inequality, the middle class does not make a difference between elites being incented to create a democracy and keeping a non-democratic or repressive regime.
3. In an environment without a middle class, the line of indifference for the elites between democracy and repression is linear. Contrast that with an environment with a middle class, I find the line of indifference for the elites between democracy and repression to be non-linear.

The boundedly rational model was verified by running sensitivity analysis on the critical values by varying the input parameters. It was validated by comparing the critical values to those determined by A&R. I found that when a population has limited rationality, it has the following effect on the critical values:

1. When the Leadership strategy of Equivalent constituents is selected, the model results are very similar to those of the fully rational heterogeneous model. This is because when the voters are evenly split among candidates, the effective median stays the same.
2. When leaders arise randomly, then an increase in democratizations depends on whether the effective representation has a higher or lower median income. The leaders’ representative constituency affects the determination of the median voter and may thus affect the requested

tax rate and regime decisions made by considering that tax rate. Examination of the "Democratization in a picture" model yielded the following insight: If the representative leaders represent a constituent with a higher than median income, their demands are less threatening to the elites, resulting in a higher chance of democratization.

3. Limiting rationality results in more variability in the ability for underlying economic conditions to trigger a democratization.

5 Chapter Five: Summary

5.1 Summary

This dissertation presents an agent based political economy model to study the determinants of the transition to democratic regimes from the bottom up. Much of current literature is focused on the macro-economic characteristics. Societal decisions however, are not just an outcome of macro-level characteristics. Social groups reflect and shape the operations of institutions controlling decision-making and power allocation. As a complement to the existing macro-level understanding, this dissertation focuses on the micro-economic foundations that lead self-interested agents to shape the institutions that allocate political and economic power. This context provides the roles for political institutions and may lead to design of political institutions that increase economic growth and stability.

The principle contribution of this work is to expand the understanding of the creation and consolidation of democracy through an agent-based model that makes explicit the interaction between the decisions of individuals and the emerging group level regime choice. This dissertation builds upon the game-theoretic approach that Acemoglu and Robinson originated. Their game-theoretic model analyzes democratization in a context of the trade-offs between democratization and other types of concessions and repression can be evaluated. Their model is limited by assumptions that all agents are fully rational and capable of infinite horizon analysis and that all agents with a class are identical.

Using an ABM allows me to relax these assumptions inherent in the A&R game-theoretic model: homogeneity, rationality, and non-interactiveness so that the impact of a middle class can be more realistically represented and their affects understood. The agent based model and analysis in this dissertation makes it more explicit than the A&R game-theoretic model, how certain factors help or hinder democracy.

Using the methodology of agentization (Guerrero & Axtell, 2011), I developed a model of the creation and consolidation of democracy keeping the principle assumptions of A&R game-theoretic models. There were a few necessary changes. For example, in the game-theoretic model, A&R assume there is one agent in the model. A fraction $\boldsymbol{\delta}$ are rich and (1- $\boldsymbol{\delta}$) are poor (Acemoglu & Robinson, 2006, p. 104). To convert that to an agent-based model, I allow the number of agents, **N,** to be selected by the user. The fraction $\boldsymbol{\delta}$ of rich agents is also specified by the ABM user. The ABM approximation for the number of rich people ($\boldsymbol{\delta}$ x N in the A&R model) is by necessity, the closest integer value to the product $\boldsymbol{\delta}$ x N. The number of poor agents then is also an integer. In A&R's model, the values for the income of the rich and the poor depend upon: i) how many of the agents are rich $\boldsymbol{\delta}$, and ii) the percentage of income accruing to the rich. The ABM approximation for the number of rich people, int ($\boldsymbol{\delta}$ x N), may result in slight variations of the values for the rich person's income (y^r), and the poor person's income (y^p).

In this dissertation, I show that the agent-based model reproduces the group-level threshold conditions affecting institutional choices found by A&R. I validate the ABM model by showing that these results are robust to parameter changes within the ranges defined by A&R with slight variances in part caused by the integer approximation of the number of rich and poor agents. This type of validation is called “docking” (Axtell, 2000). Docking is necessary to remove the criticism of arbitrariness from the agent-based model: however, by itself it does not create added value over the more established game-theoretic methodology. Therefore, sensitivity analysis was performed to investigate how robust the A&R results were to changes in underlying economic conditions.

Principally, added value is provided by the ABM when the restrictive assumptions are relaxed. First, I created a variation of the software model that assigns each agent their own heterogeneous income. Now, instead of a class of rich agents with homogenous income (y^r), and a class of poor agents with homogenous income (y^p), the model includes agents with income spanning a wide spectrum of incomes. In effect, the ABM models a more realistic “middle class”. I run the model keeping the other A&R assumptions the same. I found that having a middle class does affect the macro-level regime outcomes in non-linear ways.

Next, I limit agent rationality. Instead of omniscient agents, with tremendous capacity to calculate their infinite horizon payoff’s, I create a population of agents where a small percentage of them act rationally and calculate the utility of their alternatives. However, the majority of agents choose a leader and follow their leader. They don’t think beyond knowing who their leader is and what preferences their leader holds. Limitations in cognition or knowledge is referred to as “bounded rationality”.

This dissertation demonstrates the usefulness of agent-based modeling as a viable alternative quantitative methodology for studying complex institutions. Using an agent-based model, I have modeled the more realistic assumptions of intra-group heterogeneity, and bounded rationality in order explore the roles of middle class income distributions and leadership on the larger outcomes for all groups.

While agent-based modeling addresses many of the challenges of Game-theoretic models, ABM models are faced with their own challenges. The structure of my dissertation addresses the 7 challenges of agent-based models described in Crooks, Castle, & Batty (2008).

Purpose: The purpose of the agent-based model is to explore Acemoglu & Robinson's theory of the Creation and Consolidation of Democracy in order to answer the research questions in this dissertation Section 1.1.1. The value added over the A&R model is that this ABM can be used to answer the question "what is the effect of a middle class on the democratization processes?" It is reasonable for the agent-based model to be an abstract model because the A&R model is an abstract model. There is insufficient data on the underlying economic variables such as cost of coup, cost of revolution, and the high threat states of coups and revolutions to build a consolidative model with high fidelity to the real world. This ABM is an exploratory model. Exploratory modeling provides computational experiments to reveal how the world would behave in different "what if" circumstances of varying assumptions and hypotheses (Bankes, 1993).

Theory: I translated the Acemoglu & Robinson theory of the creation and consolidation of democracy into a form where the restrictive assumptions about homogeneity and perfect rationality can be relaxed following the methodology of agentization (Guerrero & Axtell, 2011). The process of agentization makes explicit the assumptions about the model.

Replication and Experiment: The confidence in this model is derived from the validation that this model reproduces results from the Acemoglu & Robinson theory and game-theoretic models. Such a validation technique is called "docking" (Axtell et al., 1996).

Verification, Calibration, and Validation: Code walkthrough were conducted, and interim results monitored and verified. I conducted sensitivity analysis by parameter sweeps to verify the models functioned as expected even with extreme values. The model was validated against the existing A&R model results. Empirical results of the agent-based model were compared to empirical results of the A&R game-theoretic model.

Agent Representation, Aggregation and Dynamics: The agent representations are clearly defined and rules and relationship between individual agent decisions, the class (rich and poor) decisions and the macro-level regime decisions are clearly described. In this ABM model, following Acemoglu & Robinson, the agents are not spatial and do not move. Spatial and regional influences may be an interested extension.

Operational Modeling: I developed the model in Netlogo and made use of its built-in agent handling routines, GUI interface and visualization capabilities. This allows a consistency of results.

Sharing and Dissemination: The model presents the economic influences on political decisions in a very clear way that is easy to test and experiment with. Although I have not planned a specific sharing and dissemination method, the ability to imbed the model in a web page means that it may have applications as educational material. There are many programs starting to make "hands on" models available to students around the world. This agent-based model of the creation and consolidation of democracy would be a valuable addition to a student's curriculum.

I believe this work will advance the study of the emergence of political institutions by providing a complement to formal theory. Institutions are created by humans and, like physical infrastructure, must be maintained (Ostrom, 1990, p207-216). This effort may provide insight into maintenance priorities needed to continue sustainable and resilient governments. The use of agent-based modeling as a methodology enables us to build future institutions that are robust to variety of environmental changes (Jones-Rooy & Page, 2012).

5.2 Conclusion

This work uses an agent-based computational methodology to build a much richer political economy model to study the creation and consolidation of democracy without the limitations of the game theoretic techniques used by Acemoglu and Robinson in their book Economic Origins of Dictatorship and Democracy (2006). Acemoglu & Robinson developed a formal game-theoretic framework for analyzing how economic incentives influence the way social groups shape institutions to allocate political and economic power. The A&R models assume groups or classes of people are completely rational and identical intra-group. My work expands the understanding of the creation and consolidation of democracy through an agent-based model that makes explicit the interaction between the decisions of individuals and the emerging group level regime choice. Using an ABM allows me to relax the homogeneity and rationality assumptions inherent in the A&R game-theoretic model, so that characteristics of a middle class, can be more realistically represented and understood. A&R predict that democratization would occur at a medium level of inequality. Too little inequality, and the majority have little incentive to engage in revolution. Too much inequality and the rich have too much to lose, and therefore will respond with repression. They describe this as an inverted U-shaped relationship between inequality and democratization (A&R, 2006, p. 80). Because of this, they predict that that a decrease in inequality in highly unequal societies would make democratization more likely (A&R, 2006, p. 82).

Acemoglu and Robinson saw the advantage of a model which included the middle class but the limitations of their formal infinite-horizon game theoretic model of the creation and consolidation of democracy prevented that analysis. Instead, they offer some additional analysis in terms of a single time period static game with limited modeling of the commitment problem. The resulting analysis is less formal and does not include specifications of the strategies (Acemoglu & Robinson, 2006, p. 255). A&R argue that the middle class acts as a buffer between the rich and poor, and limiting redistribution, and making democratization less costly to the elites (Acemoglu & Robinson, 2006, p. 258).

5.3 Results

Using the methodology of agentization (Guerrero & Axtell, 2011), I developed a model of the creation and consolidation of democracy keeping the principle assumptions of A&R game-theoretic models. There was one necessary change. I implemented a discreet number of agents in the model, rather than a single agent as in A&R's game-theoretic model. The ABM approximation for the number of rich people resulted in slight variations of the values for the rich person's income (y^r), and the poor person's income (y^p) but these differences diminish as the number of agents increase. In this dissertation, I show that the agent-based model reproduces the group-level threshold conditions affecting institutional choices found by Acemoglu & Robinson. I validate the ABM model by showing that these results are robust to parameter changes within the ranges defined by Acemoglu & Robinson with slight variances in part caused by the integer approximation of the number of rich and poor agents. This type of validation is called "docking" (Axtell 2000). Docking is necessary to remove the criticism of arbitrariness from the agent-based model. However, by itself "docking" does not create added value over the more established game-theoretic methodology. My initial agentization of A&R's game-theoretic models adds value by making it easier to do the analysis on the agent-based model. The built-in GUI interface and visualizations make it easy to change parameters and see the effects of the change in the Democratization Picture. The agent-based model is a better laboratory with built in visualization of "what if" experiments than complex and unwieldy equations.

The Agent-based model reproduces Acemoglu & Robinsons stylized facts. For example, it shows that as the discount factor $\boldsymbol{\beta}$ decreases, democratizations are more likely. This tracks well with the South American countries with high fiscal volatility that create democracies rather readily even though they may be short-lived. The "docked" ABM reproduces the finding that democratizations are more likely to occur with a medium level of inequality for the baseline economic parameters. In addition, the "Democratization in a Picture" shows the inverted U-shape relationship between inequality and democratization. However, this relationship doesn't exist for some economic conditions.

1. At low probability of credible threat of revolution, democracy does not occur even at the medium levels of inequality.
2. At very low costs of revolutions or coup, there is no area of "status quo" and democratization occur easiest at lowest levels of inequality.
3. At very high costs of revolutions or coups, democratizations never occur. Elites choose "status quo" at medium levels of inequality.

Adding intra-group heterogeneity reveals that having a middle class affects the macro-level regime outcomes in the following ways:

- At the baseline case, when a middle class is present, there is an increasing likelihood that the new democracy will not be consolidated. Thus, in the new democracy, the citizens will not be able to enact their maximum preferred taxation. In addition, there is increasing likelihood that the democracy will be unconsolidated, that is, subject to coups.

- Because the line of indifference between repression and democratization, is curved, in some economic conditions, the middle class makes democratizations more likely. This is consistent with Acemoglu & Robinson's findings that a middle class acts as a buffer between the rich and poor and limits redistribution (Acemoglu & Robinson, 2006, p. 258). Economic development that lowers inequality will have the effect of making a transition to democracy less costly to the elites since the resulting democracy will be less redistributive. The rich have less to lose by democratization (compared to the cost of repression) when there is a middle class. However, for societies with low and medium inequality, this dissertation finds that democratization become less likely, a departure from Acemoglu & Robinson's theory.
- By analyzing the "Democratization in a Picture" we find the middle class does not increase the likelihood of democratizations in the following economic conditions:
    - At high probability of credible threat of revolution, democratizations seem less likely except at very low inequality
    - At low cost of revolution, democratizations seem less likely
    - At high cost of revolution democratizations don't occur and are insensitive to a middle class.

The second assumption that I relax is agent rationality. Instead of omniscient agents (with tremendous capacity to calculate their infinite horizon payoffs), I create a population of agents where a small percentage of them act rationally and calculate the utility of their alternatives. However, the majority of agents choose a leader and adopt the preferences of their leader. Limitations in cognition or knowledge is referred to as "bounded rationality".

Results from this model show that the leaders' representative constituency affects the determination of the median voter and may thus affect the requested tax rate and regime decisions made by considering that tax rate. Examination of the "Democratization in a picture" model yielded the following insight. Limiting rationality results in more variability in ability for underlying economic conditions to trigger a democratization. If the representative leaders represent a constituency with a higher than median income, their demands are less threatening to the elites, resulting in a higher chance of democratization.

This dissertation demonstrates the usefulness of agent-based modeling as a viable alternative quantitative methodology for studying complex institutions. Using an agent-based model, I have modeled the more realistic assumptions of intra-group heterogeneity, and bounded rationality in order explore the roles of middle class income distributions and leadership on the larger outcomes for all groups.

5.4 Future Work

The agent based model is built using a standard platform and, with future work, can be incorporated into online learning websites. It provides a sandbox for students of political economy to conduct experiments with variations of the underlying economic conditions and the ability to receive instant visualizations of the effect on regime decisions. There are additional areas for continued research.

The heterogeneous model selected a distribution that satisfied the macro-level characteristics of average income, percent of rich agents, and the percent of income that they hold. However, there could be multiple variations of distributions that address those conditions. Some would have a middle-class closer to the rich and some would have a middle-class closer to the poor. Some would have a large number of people in the middle class and in others, the middle class would be sparse. The research would investigate how these differences in distributions affect the regime decisions and whether there are tipping points of the distributions. The expectations of mobility between the classes is another area for research.

This work investigated the effects on "Democratization in a picture". This work could be extended to look into the effects on "Consolidation in a picture". This view would show the decision space of the elites and the poor in democratic regimes around coups and concessions based on inequality and cost of coup.

There may be differences in regime choice among how closely knit the society is to nearby societies. For example, isolated countries have different decisions compared to globally connected countries.

Lastly, this model took a very simple approach to limited rationality. Future research would link more complex mechanisms. The ABM approach is more flexible than the game-theoretic approach in addressing other mechanisms discussed in Acemoglu & Robinson's book: globalization, political parties and political identity, the role of the military, variations in democratic institutions, mechanisms of collective action for revolution. These studies would benefit from the inherent capability of agent-based models to address limited rationality and heterogeneity.

Appendix A: Income Share Held by Highest 10% - Country Ranking

The following tables depicts the country ranking of income share. Data is from the most recent year available for that country. "Income share held by highest 10% - Country Ranking" PovcalNet (2016).

Table 43: Income Share Held by Top 10%- country ranking (Source: World Bank, Development Research Group)

Rank	Country	Income Share	Year	Rank	Country	Income Share	Year
1	Namibia	51.84	2009	16	Lesotho	40.88	2010
2	South Africa	51.26	2011	17	Swaziland	39.98	2009
3	Botswana	49.61	2009	18	Panama	39.96	2013
4	Haiti	48.21	2012	19	Mexico	38.86	2012
5	Comoros	48.10	2004	20	Kenya	38.83	2005
6	Central African Republic	46.22	2008	21	Paraguay	37.56	2013
7	Zambia	45.23	2010	22	Malawi	37.53	2010
8	Rwanda	44.34	2010	23	Dominican Republic	37.41	2013
9	Belize	42.43	1999	24	Costa Rica	37.28	2013
10	Guinea-Bissau	42.00	2010	25	Cabo Verde	37.14	2007
11	Colombia	41.94	2013	26	The Gambia	36.85	2003
12	Guatemala	41.83	2011	27	Mozambique	36.78	2008
13	Brazil	41.82	2013	28	Ecuador	36.53	2013
14	Honduras	41.48	2013	29	Jamaica	35.76	2004
15	Chile	41.47	2013	30	Solomon Islands	35.75	2005

Rank	Country	Income Share	Year	Rank	Country	Income Share	Year
31	Bolivia	35.55	2013	60	Dem. Rep. Congo	32.00	2012
32	Nicaragua	35.19	2009	61	Sri Lanka	31.80	2012
33	Samoa	34.70	2008	62	Turkmenistan	31.73	1998
34	Fiji	34.66	2008	63	Israel	31.27	2010
35	Malaysia	34.59	2009	64	Senegal	31.05	2011
36	Togo	34.53	2011	65	Tanzania	30.95	2011
37	Benin	34.48	2011	65	Uruguay	30.95	2013
38	Djibouti	34.39	2012	67	Lao PDR	30.84	2012
39	El Salvador	34.35	2013	68	Bhutan	30.63	2012
40	Macedonia	34.16	2008	69	Argentina	30.56	2013
41	Guyana	34.11	1998	70	Turkey	30.50	2012
42	Venezuela	34.07	2006	71	Thailand	30.43	2012
43	Uganda	33.90	2012	72	United States	30.19	2013
44	Seychelles	33.75	2006	73	Vietnam	30.05	2012
45	Philippines	33.40	2012	74	Yemen	29.99	2005
46	Peru	33.39	2013	75	India	29.98	2011
47	Madagascar	33.16	2010	75	China	29.98	2010
48	Gabon	33.15	2005	77	Trinidad and Tobago	29.89	1992
49	Morocco	33.10	2007	78	Georgia	29.86	2013
50	Cameroon	33.04	2007	79	Congo	29.85	2011
51	Nigeria	32.73	2009	80	Tonga	29.39	2009
52	Ghana	32.69	2005	81	Iran	29.08	2013
53	Papua New Guinea	32.68	2009	82	Mauritius	29.04	2012
54	Côte d'Ivoire	32.64	2008	83	Vanuatu	28.92	2010
55	St. Lucia	32.48	1995	84	Syrian Arab Republic	28.68	2004
56	Chad	32.44	2011	85	Kiribati	28.50	2006
57	Angola	32.31	2008	86	Uzbekistan	28.33	2003
58	Russia	32.22	2012	87	Liberia	28.29	2007
59	Burkina Faso	32.05	2009	88	Indonesia	28.18	2010

Rank	Country	Income Share	Year	Rank	Country	Income Share	Year
89	Cyprus	28.04	2012	117	Poland	25.63	2012
90	Burundi	27.97	2006	118	Pakistan	25.62	2010
91	Mauritania	27.86	2008	119	Armenia	25.56	2013
92	Portugal	27.77	2012	120	Ireland	25.55	2012
93	Jordan	27.45	2010	121	Estonia	25.25	2012
94	Ethiopia	27.40	2010	122	Cambodia	25.23	2012
95	Bulgaria	27.38	2012	123	Bosnia and Herzegovina	25.09	2007
96	Tunisia	27.03	2010	124	Montenegro	25.05	2013
97	Algeria	26.89	1995	125	Switzerland	25.02	2012
98	Sierra Leone	26.88	2011	126	Japan	24.77	2008
99	Lithuania	26.84	2012	127	United Kingdom	24.66	2012
100	France	26.84	2012	128	São Tomé and Principe	24.18	2010
101	Bangladesh	26.81	2010	129	Tajikistan	24.13	2009
102	Greece	26.74	2012	130	Hungary	23.93	201
103	Sudan	26.68	2009	131	Iraq	23.70	2012
104	Timor-Leste	26.66	2007	132	Germany	23.69	2011
105	Latvia	26.63	2012	133	Croatia	23.63	2011
106	Egypt	26.57	2008	134	Denmark	23.47	2012
107	Australia	26.53	2010	135	Austria	23.45	2012
108	Luxembourg	26.47	2012	136	Moldova	23.26	2013
109	Niger	26.43	2011	137	Serbia	23.24	2010
110	Guinea	26.41	2012	138	Kyrgyz Republic	23.14	2012
111	Nepal	26.32	2010	139	Albania	22.93	2012
112	Italy	26.30	2012	140	Netherlands	22.57	2012
113	Mongolia	26.19	2012	141	Finland	22.29	2012
114	Spain	25.90	2012	142	Czech Republic	22.17	2012
115	Canada	25.74	2010	143	Iceland	22.11	2012
116	Mali	25.68	2009	144	Kazakhstan	22.00	2013

Rank	Country	Income Share	Year	Rank	Country	Income Share	Year
145	Belgium	21.96	2012				
146	Belarus	21.54	2012				
147	Sweden	21.50	2012				
148	Romania	21.46	2012				
149	Slovenia	21.07	2012				
149	Ukraine	21.07	2013				
151	Norway	20.90	2012				
152	Slovak Republic	20.50	2012				

Appendix B: Agent Based Model of Emergence of Self-governing Institutions

The Netlogo Code for the agent based model is included.

```
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;
;;  Elaine Reed
;;  Department of Computational Social Science, George Mason University
;;  Dissertation Project
;;  This program is written in NetLogo 5.3.1
;;  This Netlogo model reproduces the Acemoglu & Robinson (A&R)
;;  dynamic model of the creation and consolidation of democracy.
;;  (Acemoglu and Robinson, 2006, ch 7 section 4). This is an
;;  agentization of the A&R game theoretic models. The A&R model
;;  provides insight into how economic incentives influence why
;;  some societies are democratic and some are not.
;;  Individuals evaluate different options and choose
;;  between strategies according to which strategy is
;;  anticipated to result in the best outcome for that individual.
;;  This agent-based model allows the user to relax the homogeneity
;;  assumption and set up a more realistic distribution of income.
;;  In addition, this agent based model allows the user to relax the
;;  rationality assumption such that some agents are rational and
;;  others are immitators and adopt the preferences of the leader.
;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;


globals [
  y-bar          ; average income of all agents (GDP)
  y-poor          ; mean income of poor agents
  y-rich         ; mean income of rich agents
  theta          ; inequality
  beta           ; discount factor - this gives the value of the future income, it is not discount
rate
```

delta ; fraction of rich agents, there are (1 - delta) * N poor agents
tau ; current tax rate
poorpreferredtau ; poor preferred tax rate,
richpreferredtau ; rich preferred tax rate is 0.
tautilde ; concession tax set by poor to prevent coup
tauhat ; concession tax set by rich to prevent revolution
mu ; Cost of revolution (fraction of economy that is destroyed)
phi ; cost of coup
mulist ; list of mu for memorylength years
philist ; list of phi for memorylength years
elitedecisionlist; list of elitedecisions for memorylength years
poordecisionlist ; list of elitedecisions for memorylength years
incomelist ; list of income values used to find median
leaderlist ; list of (representative) income values of the leaders
peopleset ; agentset of rich and poor agents
leaderset ; agentset of leaders
richincome ; lowest income of richagents
medianincome ; median income of all rich and poor
taulist ; list of taxrate with resulting utility used to find preferred tax rate of poor people
phi-star ; if cost of coup < phi-star, reducing taxes to zero will not prevent a coup
mu-star ; revolution constraint: if cost of revolution is < mu-star, maximizing redistribution will not prevent revolution.
phi-hat ; coup constraint: if cost of coup > phi-hat, rich will not have coup even in situation of maximimal redistribution
coup? ; indicates that a coup is prefereble to staying in democracy for the elites.
revolutionimminent? ; indicates that no tax rate can be found to give utility greater than that of revolution
democratization? ; indicates that a democracy is preferable to repression for the elites.
revolution? ; indicates that a revolution is prefereable to the planned democracy
k ; cost of repression
k-bar ; line of indifference between repression and creating a consolidated democracy
k-check ; line of indifference between repression and creating a unconsolidated democracy
k-star ;
q ; probability of a state with High threat of revolution H arises. (1-q) is probability of a state with Low threat of revolution L arises.
s ; probability of a state with high threat of coup H arises. (1-s) is probability of a state with low threat of coup L arises. Both q and s are less than ½ p. 231
graphpatches ; patches needed to make the "democratization in a picture"
generatepoliticaloutcomespicture?; flag to indicate what mode to run the model
effectivetheta ; with representational leadershiip, the effective theta changes
effectivey-rich ;mean income of richpeople considering representational leadership

```
  effectivey-poor  ;mean income of poorpeople considering representational leadership
  effectivey-bar   ; effective mean income
  effectivemedianvoter ; effective medianvoter considering representational leadership
  maxincomeofrichpeople; maximum income
  threatsofrevolutionandcoup       ; in future extension, determines how the likelihood of
high threat states for revolutions and coups are determined
  memorylength                   ; in future extension, may be used to determine likelihood of
high threat state
  changesincostofrevolutionandcoup ; in future extension, may be used to have more
realistic changes in cost of revolution and coup than random uniform.
  debug?          ; turn debugging on
  marktheta        ; identify a level of inequality on the democratization in a picture
]

breed [richpeople richperson]  ; there are delta x Numagents number of rich people
breed [poorpeople poorperson]  ; the rest are poor people
breed [people person]          ; agents are set up as people, before income is assigned and
they become either rich or poor.
breed [GraphPoints GraphPoint] ; these "agents" just hold the colored points needed to
make the "democratization in a picture"

people-own [
  payoff        ; not used. in future extension, individual agents may track their individual
payoffs
  income
  leader?
]

poorpeople-own [
  payoff       ; not used. in future extension, individual agents may track their individual
payoffs
  income
  leader?
]

richpeople-own [
  payoff       ; not used. in future extension, individual agents may track their individual
payoffs
  income
  leader?
]

GraphPoints-own [
]
```

```
to setup
  clear-all
  set generatepoliticaloutcomespicture? false
  set debug? false
  set theta ShareIncomeAccruingtoRich ; assign shorter name
  set marktheta theta
  set threatsofrevolutionandcoup "Slider variables q and s"
  ; in the future, threats-of-revolution-and-coup could have the folowing values:
  ;"Slider variables q and s" - default, q and s are set by the user via sliders.
  ;"Based on history of cost of revolution and coup" - q and s are based on history of
likelihood of revolution and coup
  ;"Random uniform distribution" - q and s are randomly generated
  set changesincostofrevolutionandcoup "Random uniform distribution"
  ; in future, changesincostofrevolutionandcoup may be based on incremental changes
from the current cost rather than random uniform distribution these would be the options
  ;"Random uniform distribution"
  ;"Incremental change from random starting point"
  setupconstants         ; set up global constants
  initializemu           ; provide agents with single random memory of threat of revolution -
list of memorylength mu values
  initializephi          ; provide agents with random memory of threat of coup -  list of
memorylength phi values
  setup-graph            ; colors graph patches for elite decisions in a view
  updateaverageincomes    ; updates y-poor, and y-rich based on theta
  setup-people           ; set up rich and poor breeds with either group-level homogenous or
heterogeneous income
  reset-ticks
end

to setupconstants
  set mulist []             ; past memory list of probability of a state with High threat of
revolution arises
  set philist []            ; past memory list of probability of a state with High threat of coup
arises
  set elitedecisionlist []
  set poordecisionlist []
  set memorylength 25        ; in future, memory length could be used to evaluate
likelihood of coup and revolution
  set k-bar 1               ; initial value before experience is used to update this
  set k-check 1              ; initial value before experience is used to update this
  set k-star 1              ; initial value before experience is used to update this
```

```
  set delta FractionRichAgentstoNumAgents ; assign shorter name
  set beta DiscountFactor               ; assign shorter name
  set y-bar averageincome               ; assign shorter name
  set incomelist []
  set taulist [] ; set of taxrate with resulting utility used to find preferred tax rate of poor
people
  set phi-star 0             ; initial value before experience is used to update this
  set phi-hat 1              ; initial value before experience is used to update this
  set mu-star 0              ; initial value before experience is used to update this
  set coup? false
  set revolutionimminent? false
  set democratization? false
  set revolution? false
end

to setup-graph
  ;; create the axis
  set graphpatches patches with [(pycor >= -9 and pxcor = -9) or (pxcor >=  -9 and pycor
= -9)]
  ask graphpatches [ set pcolor green + 2 ]

  ;; create the annotated axis labels with patch labels
  ask patches with [pxcor = -8 and pycor = -10]
  [
    set plabel delta
  ]
  ask patches with [pxcor = -8 and pycor = -11]
  [
    set plabel "δ"
  ]
  ;let one 10; (((1 - delta) * 20) - 8)
  let one (((1 - delta) * 25) - 8)
  print one
  ask patches with [pxcor = int one and pycor = -10]
  [
    set plabel 1
  ]
  ask patches with [pxcor = int (((1 - delta) * 12.5) - 8) and pycor = -10]
  [
    set plabel precision (delta + (1 - delta) / 2) 2
  ]
  ask patches with [pxcor = int (((1 - delta) * 6.25) - 8) and pycor = -10]
  [
    set plabel precision (delta + (1 - delta) / 4) 2
```

```
  ]
  ask patches with [pxcor = int (((1 - delta) * 18.75) - 8) and pycor = -10]
  [
    set plabel precision (delta + 3 * (1 - delta) / 4) 2
  ]
  ask patches with [pxcor = int (delta + (marktheta * (1 - delta) * 25) - 8) and pycor = -11]
  [
    set plabel "*"
  ]
  ask patches with [pxcor = 4 and pycor = -12]
  [
    set plabel "Inequality Θ"
  ]
  ask patches with [pxcor = -13 and pycor = 13]
  [
    set plabel "Costs of"
  ]
  ask patches with [pxcor = -12 and pycor = 12]
  [
    set plabel "Repression κ"
  ]
  ask patches with [pxcor = -10 and pycor = 15]
  [
    set plabel "1"
  ]
  ask patches with [pxcor = -10 and pycor = -8]
  [
    set plabel "0"
  ]

end

to updateaverageincomes
  set y-poor  int (((1 - theta) *  y-bar) / (1 - delta))       ; mean income of poor agent - this
is true for heterogeneous or homogenous income
  set y-rich  int ((theta *  y-bar) / delta )                 ; mean income of rich agent - this is
true for heterogeneous or homogenous income
end

to setup-people

  ifelse count richpeople = 0     ; if no agents, then create Numagents
  [
    create-people (NumAgents)
```

```
    [
      set income 0
      set leader? false
      hide-turtle
    ]
  ]
  [ ; else we are in "generating picture mode" and people already exist
    ask richpeople
    [
      set breed people
      set leader? false
      set income 0
    ]
    ask poorpeople
    [
      set breed people
      set leader? false
      set income 0
    ]
  ]
  ifelse HeterogeneousIncome
  [

    SetupIncomeDistributions     ; Call the routine to create heterogeneous income
distribution
    set peopleset (turtle-set turtles with [breed = poorpeople]  turtles with [breed =
richpeople] )
    set incomelist sort [income] of peopleset ; set income list before agents adjust their
income to follow the leader in order to get true median
                                        ;
                                        ; The following code is executed only if there are some
designated leaders and the rest of the population are followers.
                                        ;
    if FractionLeaders != 0      ; assign some agents to be leaders
    [
      set leaderset n-of (FractionLeaders * NumAgents) peopleset ; choose leaders
randomly
      If not any? leaderset [set leaderset n-of 1 peopleset]     ; make sure there is at least one
leader
      let numleaders count leaderset
      ifelse LeaderStrategy = "Randomsets"  ; if leaders are randomly picked - keep their
income as the representative income
      [
```

```
        set leaderlist [income] of leaderset  ; get a list of the incomes belonging to the
leaders
      ]
      [  ; else leaderstrategy is   "Equivalent Constituents" and change income to evenly
distribute voters among them.
        let interval int (numagents / numleaders)  ; the approx number of voters for each
candidate
        ask leaderset
        [
          set income 0   ; initialize their incomes to zero so you can track who has been
updated.
        ]
        ; to evenly divide the constituency, each leader gets (numagents / numleaders)
constituents including herself
        ; so each leader will have a representative income and agents with incomes closest to
that income will follow them.
        ; thus we just need to find the representational incomes that are in the middle of each
evenly divided subset of the population
        ifelse int interval mod 2 = 0
          [   ; if even, find the average of the two incomes in the middle
              ;type "even "
            set leaderlist []
            let itemfirstincome round ((interval + 1) / 2) - 1  ; list starts with item 0 so find
midpoint and subtract 1, this is the lower item
            let i 0
            ;print itemfirstincome
            repeat numleaders
              [
                ; find the two incomes in the middle of the interval and average them, then add
to the list of leaders incomes.
                set leaderlist fput round (   ((item ((i * interval) +  itemfirstincome) incomelist)
+ (item ((i * interval) + itemfirstincome + 1) incomelist) ) / 2)   leaderlist
                set i (i + 1)
              ]
            ;print leaderlist
          ]
          [  ; if odd, find the midpoint and subtract one
            set leaderlist []
            let itemfirstincome ((interval + 1) / 2) - 1  ; list starts with item 0 so find midpoint
and subtract 1
            let i 0
            ;print itemfirstincome
            repeat numleaders
              [
```

```
          set leaderlist fput (item ((i * interval) +  itemfirstincome) incomelist) leaderlist
          set i (i + 1)
         ]
        print leaderlist
       ]
      foreach leaderlist
       [
        ask one-of leaderset with [income = 0]
        [
          set income ?               ; replace zeroed-out income with one of strategically
selected incomes.
        ]

       ]
     ]


     ;type "before assigning " print [income] of peopleset
     ask peopleset              ; update income to match that of leader - people adopt their
leader's position on tax policy.
     [

       ;type income type " "
       let differencelist map [ abs (? - income) ] leaderlist
       ; set income corresponding to leader with income closest to agent's own income
       set income item 1 first sort-by [item 0 ?1 < item 0 ?2] (map list differencelist
leaderlist)

     ]
     ;type "after assigning " print [income] of peopleset
     ; note that y-bar is set by the user and y-rich and y-poor is a function of y-bar and
theta
     ; so equations based on y-rich and y-poor do not change, just the preferred tau
changes when voters follow their leaders
     set effectivey-rich mean [income] of richpeople
     set effectivey-poor mean [income] of poorpeople
     set effectivey-bar mean [income] of peopleset
     set effectivetheta (effectivey-rich * delta) / effectivey-bar
     set effectivemedianvoter median [income] of peopleset
    ]  ; end if leaders
      ; this ends the code unique to having leaders


  ]   ; end if heterogeneous income
```

```
  [    ; else homogenous income so set up rich and poor people based on delta
    ask n-of (delta * NumAgents) people
    [
      set breed richpeople
      set income int y-rich
    ]
    ask people with [income = 0]    ; everyone who is not rich, is set as poor
    [
      set breed poorpeople
      set income int y-poor
    ]
    let poorlist [income] of poorpeople
    let richlist [income] of richpeople
    set incomelist sentence poorlist richlist ; incomelist is used to find median income,
which is always y-poor for homogeneous
  ]
  set maxincomeofrichpeople max incomelist
  set medianincome median incomelist ; find the median income whether heterogeneous
or homogeneous, however if there are leaders then
                                    ; the effective median income rather than median income is used
to find tau
end


to-report incomedifference [i lst]           ; for future use if q and s are determined by
history of threat state of coups and revolutions
  report length filter [? = i] lst
end

to initializemu
  repeat memorylength [
    set mulist fput (((random 100) + 1) / 100) mulist ; initialize mulist with random
numbers between (0, 1]
  ]
end

to initializephi
  repeat memorylength [
    set philist fput (((random 100) + 1) / 100) philist ; initialize philist with random
numbers between (0, 1]
  ]
end

to initializek
```

```
  set k ((random 100) + 1) / 100 ; initialize k
end

to generate              ; this mode generates the "democracy in a picture" and calls the
necessary setup and go routines.
  clear-all
  set generatepoliticaloutcomespicture? true
  set debug? false
  set memorylength 25
  set threatsofrevolutionandcoup "Slider variables q and s"
  set changesincostofrevolutionandcoup "Random uniform distribution"
  setupconstants
  set phi CostofCoup          ; assign shorter name to GUI variable
  set mu CostofRevolution     ; assign shorter name to GUI variable
  set q slider-q              ; assign shorter name to GUI variable
  set s slider-s              ; assign shorter name to GUI variable
  set theta FractionRichAgentstoNumAgents
  if heterogeneousIncome [set theta precision (1.5 * FractionRichAgentstoNumAgents) 3
]
  setup-graph                 ; colors graph patches for elite decisions in a view
  reset-ticks
  sweepthetaandk              ; run routine incrementing inequality theta and cost of
repression K, to draw "democracy in a picture"
end

to sweepthetaandk
  set Polity "N"
  while [theta < 1]   ; increment theta from delta to <1 by .02
  [
    type " theta " print theta
    updateaverageincomes ; updates y-poor, and y-rich based on theta
                    ;print "setup-people"
    setup-people    ; set up rich and poor breeds with either group-level homogenous or
heterogeneous income
    set k 0
    while [k < 1]    ; increment k from 0 to .98
    [
      go           ; run the go routine
      set Polity "N" ; reset Polity to non-democratic
      set k k + .02
    ]
    set theta theta + .02
  ]
end
```

```
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;                                                                             ;
; The "Go" routine determines the regime decisions based on the economic conditions   ;
;                                                                             ;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
to go

  if not generatepoliticaloutcomespicture?
  [ ; when generating the "Democratization in Picture" mu and phi are user entered do not
change randomly.
    updatemuandphi ; randomly assign cost of revolution mu and cost of coup phi
    updateqands    ; determine q and s either from slider variables, randomly, or from
history of cost of revolution and coups depending on user selection
  ]

  set revolutionimminent? false
  if debug? [type " phi " type phi type " mu " type mu type " q " type q type " s " type s
type " k " type k type " theta " print theta]
  findpoorpreferredtaxrate ; this sets tau
  finddemconcessiontax     ; if democratic, the poor may lower taxes to tautilde in order to
prevent a coup. this sets tautilde
  findnondemconcessiontax  ; if non-democratic the rich may lower taxes to tauhat to
prevent a revolution. this sets tauhat
  ifelse Polity = "N"      ; if non-democracy,
  [
    ifelse revolutionimminent? ; The elite check to see if there is a tax rate that might
prevent revolution,
    [
      repressordemocratize ; if not, the elites decide whether to avoid revolution by
repression or democratization
                           ; even if they democratize, the poor may still have a revolution if the
democracy is not redistributive enough.
    ]
    [ ; else, there is a concession tax that will prevent revolution. thus the elites will
concede or repress.
      repressorconcessiontax
    ]
  ]
  [ ; if democracy,
    couporpaytax   ; the elites decide whether to have a coup or pay taxes set by the
democracy. if they don't have a coup, the poor may still
                   ; have a revolution if the democracy is not redistributive enough.
```

```
  ]
  if debug? [type "mu " type mu type " phi " type phi type " phi-hat " type phi-hat type "
phi-star " type phi-star type " k " type k type " k-star " type k-star type " k-bar " type k-
bar type " k-check " print k-check]

  tick
end

to updateqands     ; in this model, q and s are always set by the user with sliders. There
may be extensions in the future
  ifelse threatsofrevolutionandcoup = "Based on history of cost of revolution and coup"
  [
    set q 1 - frequency 1 mulist / length mulist       ; the percentage frequency that mu is
less than 1
    set s 1 - frequency 1 philist / length philist     ; the percentage frequency that phi is less
than 1
    if debug? [type "q and s " type q type " " print s]
  ]
  [
    ifelse threatsofrevolutionandcoup = "Random uniform distribution"
    [
      set q (((random 100) + 1) / 100)
      set s (((random 100) + 1) / 100)
      if debug? [type "q and s " type q type " " print s]
    ]
    [
      set q slider-q      ; this is the default mode and the only active mode in this current
model
      set s slider-s      ; this is the default mode and the only active mode in this current
model
    ]
  ]
end

to-report frequency [i lst]           ; for future use if q and s are determined by history of
threat state of coups and revolutions
  report length filter [? = i] lst
end

to updatemuandphi
  ifelse coup? ; if there is a coup, mu is set to 1 so that there is no immediate revolution.
A&R p. 231
  [
    set mulist fput 1 mulist
```

```
    set coup? false           ; reset flag
  ]
  [
    set mulist fput (((random 100) + 1) / 100) mulist
  ]
  set mulist remove-item memorylength mulist
  ifelse (democratization? or revolution?) ; if there is a democratization, phi is set to 1 to
give democracy a chance. A&R p. 231
  [
    set philist fput 1 philist
    set democratization? false   ; reset flags
    set revolution? false        ; reset flags
  ]
  [
    set philist fput (((random 100) + 1) / 100) philist
  ]
  set philist remove-item memorylength philist
  set phi first philist
  set mu first mulist
  initializek         ; call routine to set random cost of repression
  if debug? [type "next timestep phi, mu and k " type phi type " " type mu type " " print k]
end

;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
; findpoorpreferredtaxrate finds the preferred  ;
; rate of the citizens (median voter) and       ;
; assigns it to the variable tau.               ;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

to findpoorpreferredtaxrate
  ; find agent with median income (from both rich and poor)
  ;set medianincome median incomelist ;
  ;type "medianincome " print medianincome
  if debug? [type "medianincome " print medianincome]
  let up-b 1 ; upper bound
  let low-b 0 ; lower bound
  let x 10    ; interval
  ifelse effectivemedianvoter = 0
  [
    set taulist LineSearchUtility2 medianincome low-b up-b   ; if homogenous or
heterogeneous without leaders use the medianincome
  ]
  [
```

```
    set taulist LineSearchUtility2 effectivemedianvoter low-b up-b  ;heterogeneous with
leaders use the effectivemedianincome
  ]
  set tau item 0 taulist    ; tau is the preferred tax rate
  if debug? [type "tau " print tau]
end

to finddemconcessiontax
  ; citizens evaluate the threat of coup and set the tax rate taudem =  =< tau.
  ; this is as high as possible but less than than or equal to the preferred tax rate,
  ; and low enough to prevent a coup.
  ; If the coup threat is low (phi = 1) then they keep tau
  ; if the coup threat  is high they set the tax rate tautilde such that the utility
  ; the rich get from a coup minus the losses from a coup, is less than the utility
  ; the rich would get from staying in democracy and paying tautilde.
  ; To decide whether to mount a coup, the rich compare their value in a non-democracy
  ; minus the cost of a coup to their utility of staying in a democracy. If q is high
  ; (high probability of a state with high threat of revolution),
  ; the value of a coup is reduced. If s is high, high probability of a coup means
  ; the elites pay relatively low taxes, so democracy is less costly to them
  ; and value of democracy increases

  ifelse testtautildezero? ; see if there will be a coup even if tautilde is set to 0
  [
    if debug? [print "If democracy, there will be a coup even if tautilde is set to 0 "]
    if Polity = "D" [set coup? true]  ; if true, then nothing will prevent a coup
    if phi > phi-star
    [
      set phi-star phi
      if debug? [type "will always have coup when phi is less than " print phi-star]
    ]
  ]
  [
    if debug? [print "find tautilde the highest concession tax less than tau that will prevent
a coup "]
    findtautilde ; if the elites won't have a coup at tautilde = 0 find the highest concession
tax less than tau that will prevent a coup.
  ]
  if debug? [type "final tautilde " print tautilde]
end


to-report testtautildezero?       ; see if there will be a coup even if tautilde is set to 0
  set tautilde 0
```

```
  let couputility (CalcVrichNmulow q s tau tautilde) - (phi * y-rich)
  let richpaytaxutility CalcVrichDtautildezero s tau tautilde
  if debug? [type "rich utility from coup " print couputility]
  if debug? [type "rich utility from paying concession tax 0 " print richpaytaxutility]
  ifelse couputility > richpaytaxutility
  [
    report true
  ]
  [
    report false
  ]
end

to findtautilde  ; the poor may lower taxes to prevent a coup. this routine finds the
concession tax
  let trytautilde 0
  while [trytautilde < tau]   ; find the highest concession tax less than the poor preferred
tax rate
  [
    let couputility (CalcVrichNmulow q s tau trytautilde) - (phi * y-rich)
    let richpaytaxutility CalcVrichDtautildezero s tau trytautilde
    if (couputility < richpaytaxutility)
    [
      set tautilde trytautilde
    ]
    set trytautilde trytautilde + .1
  ]
  let maxtry min list tau (tautilde + .1)
  set trytautilde tautilde + .01
  if trytautilde > tau
    [
      set trytautilde tau ; the concession tax is always less than or equal to the poor
preferred tax.
      set tautilde tau
      if phi < phi-hat    ; find the lowest phi-hat for which there is never a coup, even when
taxrate = tau (poor preferred rate)
        [
          set phi-hat phi
          if debug? [type "will never have coup when phi is greater than " print phi-hat]
        ]

    ]
  while [trytautilde < maxtry]   ; find the highest concession tax less than the poor
preferred tax rate
```

```
  [
    let couputility (CalcVrichNmulow q s tau trytautilde) - (phi * y-rich)
    let richpaytaxutility CalcVrichDtautildezero s tau trytautilde
    if (couputility < richpaytaxutility)
    [
      set tautilde trytautilde
    ]
    set trytautilde trytautilde + .01
    if trytautilde > tau
      [
        set trytautilde tau ; the concession tax is always less than or equal to the poor
preferred tax.
        set tautilde tau
        if phi < phi-hat    ; find the lowest phi-hat for which there is never a coup, even
when taxrate = tau (poor preferred rate)
          [
            set phi-hat phi
            if debug? [type "will never have coup when phi is greater than " print phi-hat]
          ]

      ]

  ]

end

to findnondemconcessiontax
  ; If there is a revolution, elites lose everything. They want to prevent this. They have
three ways:
  ; democratize, repress, set a concession tax rate.
  ; If they democratize, the party that comes to power decides whether to keep the taxrate
set earlier stage 2
  ; or to set a new tax rate. s = probability of a state with high threat of coup H arises.
  ; (1-s) is probability of a state with low threat of coup L arises.
  ; q = probability of a state with high threat of revolution H arises.
  ; (1-q) is probability of a state with low threat of revolution L arises.

  ; The elites try to stop a revlution by raising taxes - that is, by increasing redistribution
  ; the elite evaluate the threat of revolution and set the tax rate taunondem
  ; as low as possible but less than than or equal to the preferred tax rate,
  ; and high enough to prevent a coup.
  ; If the revolution threat is low (mu = 1) then they keep a 0 tax rate.
  ; if the revolution threat  is high they set the tax rate tauhat such that the utility
```

```
  ; the poor get from a revolution minus the losses from a revolution, is less than the
utility
  ; the poor would get from staying in the non-democracy and receiving income
distribution based on tau-hat.


  ifelse testtauhattau?  ; test even if tauhat is set as high as tau, will the poor still have a
revolution
  [
    if debug? [print "even if tau-hat is set as high as tau, the poor still prefer revolution "]
    set revolutionimminent? true   ; even if tauhat is set as high as tau, the poor still prefer
revolution
    if mu > mu-star    ; so at this value of mu, even if touhat is set as high as tau, the poor
still prefer revolution
    [
      set mu-star mu    ; find the highest mu-star at which the poor will always prefer
revolution
      if debug? [type "poor will always prefer revolution when mu is less than " print mu-
star]
    ]
  ]
  [
    if debug? [print "find the lowest concession tax less than tau that will prevent a
revolution "]
    findtauhat
    ; if the poor won't have a revolution at tauhat = tau, find the lowest concession tax less
than tau that will prevent a revolution.
    ; find benefit to poor of non-democracy and raise taxes until it equals revolution.
  ]
  if debug? [type "final tauhat " print tauhat]
end


to-report testtauhattau?  ; find the maximum utility that can be given to the citizens
without democratizing and see if it's enough to stop a revolution
  let revolutionutility CalcVpoorrevolution
  let poorgettaxutility CalcVpoornondemmaxtax tau  ; the maximum utility will be at their
preferred tax rate tau
  if debug? [type "poor utility from revolution " print revolutionutility]
  if debug? [type "poor utility from getting concession tax as high as tau " print
CalcVpoornondemmaxtax tau]
  ifelse revolutionutility > poorgettaxutility
  [
    set tauhat tau
```

```
    report true ; even if tauhat is set as high as tau, the poor still prefer revolution
  ]
  [
    report false  ; there exists a tauhat less than tau that gives more utility to the poor than
revolution
  ]
end

to findtauhat    ; find max benefit to poor of non-democracy and lower taxes until it
equals revolution.
  let trytauhat tau
  let revolutionutility CalcVpoorrevolution ; utility of revolution is independent of tax rate
  let poorgettaxutility CalcVpoornondemmaxtax trytauhat
  while [poorgettaxutility >= revolutionutility and trytauhat > 0]   ; find the lowest
concession tax that will prevent a revolution
  [ ; coarse search
    set trytauhat trytauhat - .1
    set revolutionutility CalcVpoorrevolution
    set poorgettaxutility CalcVpoornondemmaxtax trytauhat
  ] ; at this point, tax will be slightly too low so perform a finer search going up towards
tau
  if trytauhat < 0 ; for high levels of mu, the poor may not prefer revolution even if the tax
rate is 0.
  [
    set trytauhat 0   ; the lowest allowable tax rate is 0
    set revolutionutility CalcVpoorrevolution
    set poorgettaxutility CalcVpoornondemmaxtax trytauhat
    set tauhat trytauhat  ; starting point
                        ; type "trytauhat " print trytauhat
  ]
  if debug? [type "end coarse trytauhat " type trytauhat type " concession tax " type
poorgettaxutility type " revolution " print revolutionutility]
  while [poorgettaxutility < revolutionutility ]   ; raise taxes until you will prevent a
revolution
  [  ; fine search
    set trytauhat trytauhat + .01
    set revolutionutility CalcVpoorrevolution
    set poorgettaxutility CalcVpoornondemmaxtax trytauhat
  ] ; at this point trytauhat is less than .01 higher than tax needed to prevent a revolution
  set trytauhat trytauhat - .01
  set poorgettaxutility CalcVpoornondemmaxtax trytauhat
  if trytauhat < 0 [set trytauhat 0]
  if debug? [type "end fine search trytauhat " type trytauhat type " concession tax " type
poorgettaxutility type " revolution " print revolutionutility]
```

```
  while [poorgettaxutility < revolutionutility]   ; raise taxes until you will prevent a
revolution
  [  ; finest search
    set trytauhat trytauhat + .001
    set revolutionutility CalcVpoorrevolution
    set poorgettaxutility CalcVpoornondemmaxtax trytauhat
  ] ; at this point trytauhat is less than .001 higher than tax needed to prevent a revolution
  set tauhat trytauhat
  if debug? [type "end finest search trytauhat " type trytauhat type " concession tax " type
poorgettaxutility type " revolution " print revolutionutility]
  if debug? [type "poor utility from rich concession tax tauhat " type tauhat type " " print
CalcVpoornondemmaxtax tauhat]
end

to repressordemocratize
  ;If the highest tax rate tau is not enough to stop a revolution, the Elites will either
democratize to prevent a revolution
  ;or they will repress, depending on which is cheaper. In democracy VR has different
values depending on the cost of a coup
  ; which determines what tax rate the citizens would set.
  let repressutility CalcVrichrepress
  let democratizeutility CalcVrichdemocratize s tau tautilde
  if debug? [type "repress utility " type repressutility type " democratize utility " print
democratizeutility]
  ifelse (repressutility > democratizeutility)
  [
    if debug? [print "Elites Oppress"]
    set elitedecisionlist fput "O" elitedecisionlist
    Create-GraphPoints 1       ; create the point for Repression / Opression
    [
      set shape "circle"
      set size .5
      set color white
      setxy (((theta - delta) * 24) - 8) ((k * 24) - 8)
      ;set label k
    ]
    if length elitedecisionlist > memorylength
    [
      set elitedecisionlist remove-item memorylength elitedecisionlist
    ]
    if debug? [type "Elite decisions " print elitedecisionlist]
    ifelse phi >= phi-hat ; where you would expect the democracy to be fully consolidated
- no concession tax needed because a coup is too costly.
    [
```

```
    if k > k-bar
    [
      set k-bar k ; find the highest k, for which the elites will repress rather than
democratize
      if k-check > k-bar [set k-check k-bar] ; k-check is always equal to or lower than k-
bar
      if debug? [type "Elites Oppress and will always oppress rather than democratize at
tau when k is less than k-bar " print k-bar]
    ]
  ]
  [  ; else phi < phi-hat
    if phi <= phi-star ; - where you would expect the democracy to be unconsolidated,
coups are easy, but revolutions are easy as well.
    [
      if k > k-check
      [
        set k-check k ; find the highest k, for which the elites will repress rather than
democratize to an unconsolidated democracy
        if debug? [type "Elites Oppress and will always oppress rather than democratize at
a concession tax when k is less than k-check " print k-check]
      ]
    ] ; end if phi < phi-star
  ]

  if debug? [print "Poor are oppressed"]
  set poordecisionlist fput "O" poordecisionlist
  if length poordecisionlist > memorylength
  [
    set poordecisionlist remove-item memorylength poordecisionlist
  ]
  if debug? [type "poor decisions " print poordecisionlist]
 ]
 [ ; else democratize
  if debug? [print "Elites Democratize"]
  set polity "D"
  set democratization? true
  set elitedecisionlist fput "D" elitedecisionlist
  Create-GraphPoints 1
  [
    set shape "circle"     ; create the point for democratization
    set size .5
    set color green
    setxy (((theta - delta) * 24) - 8) ((k * 24) - 8)
    ;set label k
```

```
   ]
   if length elitedecisionlist > memorylength
   [
     set elitedecisionlist remove-item memorylength elitedecisionlist
   ]
   ifelse phi >= phi-hat ; where you would expect the democracy to be fully consolidated
- no concession tax needed because a coup is too costly.
   [
     if k < k-bar
     [
       set k-bar k ; find the lowest k, for which the elites will democratize rather than
repress
       if k-check > k-bar [set k-check k-bar] ; k-check is always equal to or lower than k-
bar
       if debug? [type "Elites democratize at tau and will always democratize rather than
repress when k is greater than k-bar " print k-bar]
     ]
   ]
   [  ; else phi < phi-hat
     if phi <= phi-star ; - where you would expect the democracy to be unconsolidated,
coups are easy, but revolutions are easy as well.
     [
       if k < k-check
       [
         set k-check k ; find the lowest k, for which the elites will democratize to an
unconsolidated democracy rather than repress
       ]
     ] ; end if phi < phi-star
   ]
   if debug? [type "elite decisions "print elitedecisionlist]
   if debug? [print "Poor decide whether to have revolution "] ; if the democracy is not
sufficiently redistributive, the poor may still have a revolution.
   revolutionornot
  ]
end

to repressorconcessiontax
  let repressutility CalcVrichrepress
  let concessiontaxutility CalcVrichconcession
  if debug? [type "repress utility " type repressutility type " rich concession utility at
tauhat " type tauhat type " " print concessiontaxutility]
  ifelse (repressutility > concessiontaxutility)
  [
    if debug? [print "Elites Oppress"]
```

```
    if k > k-star
    [
      set k-star k ;
      if debug? [ type "Elites Oppress and will always oppress rather than concede taxes
when k is less than k-star" print k-star]
    ]
    set elitedecisionlist fput "O" elitedecisionlist
    Create-GraphPoints 1   ; create the point for repression / oppression
    [
      set shape "circle"
      set size .5
      set color white
      setxy (((theta - delta) * 24) - 8) ((k * 24) - 8)
      ;set label k
    ]
    if length elitedecisionlist > memorylength
    [
      set elitedecisionlist remove-item memorylength elitedecisionlist
    ]
    if debug? [type "elite decisions " print elitedecisionlist]
    if debug? [print "Poor are oppressed" ] ; by definition, the amount of repression and
cost are sufficient to prevent a revolution
    set poordecisionlist fput "O" poordecisionlist

    if length poordecisionlist > memorylength
    [
      set poordecisionlist remove-item memorylength poordecisionlist
    ]
    if debug? [type "poor decisions " print poordecisionlist]
  ]
  [
    if debug? [print "Elites Impose Concession Tax"]
    if k < k-star
    [
      set k-star k ;
      if debug? [ type "Elites concede redistribution and will always concede rather than
oppress when k is greater than k-star" print k-star]
    ]
    ifelse tauhat > 0
    [
      set elitedecisionlist fput "T" elitedecisionlist
      Create-GraphPoints 1    ; create the point for offer concessions
      [
        set shape "circle"
```

```
      set size .5
      set color Yellow
      setxy (((theta - delta) * 24) - 8) ((k * 24) - 8)
      ;set label k
     ]
    ]
    [
     set elitedecisionlist fput "S" elitedecisionlist
     Create-GraphPoints 1      ; create the point for non-democratic status quo
     [
      set shape "circle"
      set size .5
      set color blue
      setxy (((theta - delta) * 24) - 8) ((k * 24) - 8)
      ;set label k
     ]
    ]
    if length elitedecisionlist > memorylength
    [
     set elitedecisionlist remove-item memorylength elitedecisionlist
    ]
    if debug? [type "elite decisions "print elitedecisionlist]
    if debug? [print "Poor gain redistribution taxes"]
    set poordecisionlist fput "T" poordecisionlist
    if length poordecisionlist > memorylength
    [
     set poordecisionlist remove-item memorylength poordecisionlist
    ]
    if debug? [type "poor decisions " print poordecisionlist]
  ]
end

to couporpaytax
  ifelse coup?
  [
    if debug? [print "Elites Mount Coup "]
    set polity "N"
    set elitedecisionlist fput "C" elitedecisionlist
    if length elitedecisionlist > memorylength
    [
     set elitedecisionlist remove-item memorylength elitedecisionlist
    ]
    if debug? [type "elite decisions " print elitedecisionlist]
```

```
    if debug? [print "Poor set concession tax"]
    set poordecisionlist fput "T" poordecisionlist
    if length poordecisionlist > memorylength
    [
      set poordecisionlist remove-item memorylength poordecisionlist
    ]
    if debug? [type "poor decisions " print poordecisionlist]
  ]
  [
    if debug? [print "Democratic Elites Pay Tax"]
    set elitedecisionlist fput "P" elitedecisionlist
    if length elitedecisionlist > memorylength
    [
      set elitedecisionlist remove-item memorylength elitedecisionlist
    ]
    if debug? [type "elite decisions "print elitedecisionlist]
    if debug? [print "Poor decide whether to have revolution "] ; if the democracy is not
sufficiently redistributive, the poor may still have a revolution.
    revolutionornot
  ]
end

to revolutionornot
  let revolutionutility CalcVpoorrevolution
  let acceptdemocracy CalcVpoorunconsolidateddemocracy
  if debug? [type "revolution utility " type revolutionutility type "
acceptunconsolidateddemocracy " print acceptdemocracy ]
  ifelse (revolutionutility > acceptdemocracy)
    [
      if debug? [print "Poor have revolution "]
      set polity "D"
      set revolution? true
      set poordecisionlist fput "R" poordecisionlist
      if length poordecisionlist > memorylength
      [
        set poordecisionlist remove-item memorylength poordecisionlist
      ]
      if debug? [type "poor decisions "print poordecisionlist]
    ]
    [  ; else accept democracy
      if debug? [print "Poor keep democracy "]
      set poordecisionlist fput "T" poordecisionlist
      if length poordecisionlist > memorylength
        [
```

```
        set poordecisionlist remove-item memorylength poordecisionlist
       ]
     if debug? [type "poor decisions "print poordecisionlist]
   ]
end


to-report CalcVrichNmulow [tempq temps temptau temptautilde]; used for utility of rich
mounting a coup
  report ((((1 - beta * (1 - tempq)) * y-rich) + (beta ^ 2) * tempq * temps * (temptautilde *
(y-bar - y-rich) - (distributioncost temptautilde) * y-bar)
    + (beta * tempq * (1 - (beta * temps)) * (temptau * (y-bar - y-rich) - ((distributioncost
temptau) * y-bar ))) )
      / ((1 - beta) * ( 1 - beta * (1 - tempq)  )) )
end

to-report CalcVrichDtautildezero [ temps temptau temptautilde]
  report (y-rich + (1 - (beta * (1 - temps))) * (temptautilde * (y-bar - y-rich) -
((distributioncost temptautilde) * y-bar))
    + beta * (1 - temps) * (temptau * (y-bar - y-rich) - ((distributioncost temptau) * y-bar ))
)
  / (1 - beta)
end

to-report CalcVpoorrevolution
  report (y-bar * (1 - mu)) / ((1 - delta) * (1 - beta))
end

to-report CalcVpoornondemmaxtax [temptauhat]
  report ((y-poor + ((1 - (beta * (1 - q))) *  (temptauhat * (y-bar - y-poor) -
((distributioncost temptauhat) * y-bar )) ))
    / (1 - beta))
end

to-report CalcVrichrepress
  report (y-rich - ((1 - (beta * (1 - q))) * k * y-rich))  / (1 - beta)
end

to-report CalcVrichdemocratize [ temps temptau temptautilde]
  Ifelse phi > phi-hat ; no chance of coup, democracy is fully consolidated
  [
    report (y-rich + (temptau * (y-bar - y-rich)) - ((distributioncost temptau) * y-bar ) )
    / (1 - beta)
  ]
```

```
 [
   ifelse phi <= phi-star
     [
       if debug? [type "unconsolidated "]
       if debug? [type "temptautilde " print temptautilde]
       report (y-rich + (1 - (beta * (1 - temps))) * (temptautilde * (y-bar - y-rich) -
((distributioncost temptautilde) * y-bar))
         + beta * (1 - temps) * (temptau * (y-bar - y-rich) - ((distributioncost temptau) * y-
bar )) )
       / (1 - beta)
     ]
     [   ; else phi-star < phi < phi-hat, democracy is partially consolidated
       if debug? [type "semi-consolidated "]
       report (y-rich + (1 - (beta * (1 - temps))) * (temptautilde * (y-bar - y-rich) -
((distributioncost temptautilde) * y-bar))
         + beta * (1 - temps) * (temptau * (y-bar - y-rich) - ((distributioncost temptau) * y-
bar )) )
       / (1 - beta)
     ]
   ;    [ ; this used a value for Vr (D, phiLow) which assumes the first year is at tau rather
than the concession tax tautilde
   ;      if debug? [type "unconsolidated "]
   ;      if debug? [type "tautilde " print tautilde]
   ;      report (y-rich + (beta * s * ((tautilde * (y-bar - y-rich)) - ((distributioncost tautilde)
* y-bar)))
   ;        + (1 - (beta * s)) * (tau * (y-bar - y-rich) - ((distributioncost tau) * y-bar )))
   ;      / (1 - beta)
   ;    ]
   ;    [   ; else phi-star < phi < phi-hat, democracy is partially consolidated
   ;      type "semi-consolidated "
   ;      report (y-rich + (beta * s * ((tautilde * (y-bar - y-rich)) - ((distributioncost tautilde)
* y-bar)))
   ;        + (1 - (beta * s)) * (tau * (y-bar - y-rich) - ((distributioncost tau) * y-bar )))
   ;      / (1 - beta)
   ;    ]
 ]
end

to-report CalcVrichconcession ; this is coded as the rich expecting to always have to tax
at tau-hat, rather than a temporary tax
  report (y-rich +  (tauhat * (y-bar - y-rich) - ((distributioncost tauhat) * y-bar)) )
  / (1 - beta)
end
```

```
to-report CalcVpoorunconsolidateddemocracy
  report (y-poor * ((1 - (phi * beta * s))  * ( 1 - beta * (1 - q)) + (beta * s))
    +  (1 - (beta * (1 - q) )) * (tau * (y-bar - y-poor) - ((distributioncost tau) * y-bar ))
)
  / ((1 - (beta *(1 - s))) * ( 1 - beta * (1 - q)) - (beta * beta * s * q ) )
end



to-report distributioncost [taxrate]      ; this routine calculates the cost of redistribution
  report  .5 * (taxrate ^ 2)              ; based on assumptions in A&R p 100, 101
end

;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;
; SETUPINCOMEDISTRIBUTIONS
; this routine sets up heterogeneous income distributions that keeps the
; inter-group relationships of mean income, theta, mean poor income and mean rich
income
; these are arbitrary distributions assgned to show the diference that having a
; more realistic distribution of income would have compared to homogenous intra-group
; incomes where all poor people have income = y-poor and all rich people have income
; equal to y-rich.
; initial distributions are set up and then the incomes adjusted in order to make sure the
; group level statistics stay the same.
;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;


to SetupIncomeDistributions
  Ifelse  (((theta - delta) / theta) + delta ) < .85
  [
    Ifelse (((theta - delta) / theta) + (2 / 3) * delta ) < .65 ;< .3
    [ ;very low inequality - use a distribution that works for very low inequality

      ask people
      [
        set income int (    (random-normal (y-bar / 1.1) ( y-bar / 3 )) + 0 *  random-
exponential (y-bar / 2.5) / 2  )
        while [income < 0 ] [ set income int (   (random-normal (y-bar / 1.1) ( y-bar / 3 )) + 0
*  random-exponential (y-bar / 2.5) / 2)  ]
      ]
```

```
    ] ; endif very low inequality
    [ ; else midlin inequality - start with a distribution between the low and the high.
      ask people
      [
        set income int (    (random-normal (y-bar / 3) ( y-bar / 1.1 )) +  random-exponential
(y-bar / 2.5) / 2  )
        while [income < 0 ] [ set income int (   (random-normal (y-bar / 3) ( y-bar / 1.1 )) +
random-exponential (y-bar / 2.5) / 2 )  ]
      ]
    ]
  ]; end low inequality
  [ ; else high inequality

    ask people
    [
      Ifelse (((theta - delta) / theta) + (2 / 3) * delta ) < .98 ;< .3
        [
          set income int random-exponential (y-bar / 1)
          set income int random-exponential ((1 + theta) / 1.5 * y-bar )
        ]
        [
          set income int random-exponential ( y-bar / 1)
          set income int random-exponential ((1 + theta) / 1.5 * y-bar )
        ]
    ]
  ]
  set incomelist sort [income] of people
  ;set medianincome median incomelist  ; set medianincome for heterogeneous
distribution - but this is before we adjust for theta, need to find median later
  let index int ((1 - delta) * count (people))
  set richincome item index incomelist            ; Determine who's rich and who's poor
based on delta
  ask people
  [
    ifelse income >= richincome
    [
      set breed richpeople             ; assign rich people
    ]
    [
      set breed poorpeople             ; assign poor people
    ]
  ]
  ; the rest of the routing adjusts the incomes of individual agents in order to keep the
desired relationship theta
```

```
  ; between the mean income of the rich and the mean income of the poor

  let tempmean mean [income] of richpeople


  ifelse mean [income] of richpeople > y-rich ; if we overshot the mark
    [
      while [ mean [income] of richpeople > y-rich]
      [
        ask one-of richpeople with-max [income]  ; lower income
        [
          let tempincome income

          set income int ((y-rich * count richpeople) - sum [income] of other richpeople)

          if income < richincome [  set income int (tempincome - ( (tempincome -
richincome) / 2)) ]
        ]
      ]
    ];end if we overshot the mark


    [ ; else mean [income] of richpeople < y-rich and we need to increase mean

      while [ mean [income] of richpeople < y-rich]
      [
        ask one-of richpeople with [income > richincome]
        [
          set income int (income + (income - richincome) * 2)   ; raise income
        ]
      ]
      if mean [income] of richpeople > y-rich ; if we overshot the mark
      [

        ask one-of richpeople with-max [income]
        [
          let tempincome income
          set income int ((y-rich * count richpeople ) - sum [income] of other richpeople)    ;
lower income
          if income < richincome [set income int (tempincome - (( tempincome - richincome)
/ 2 ))]
        ]
      ] ;end overshot the mark
    ] ; end increase mean of richpeople
```

```
  let maxpoor max [income] of poorpeople
  let targetincome ((richincome - y-poor) / 2)

  while [ mean [income] of poorpeople < y-poor]       ; need to raise mean income
  [

    ask one-of poorpeople with [income <= y-poor]
    [

      set income int (random-normal (y-poor / 1) ( y-poor / 5) )  ; raise income
      while [income >= richincome or income < 0] [set income int (random-normal (y-poor
/ 1) ( y-poor / 5) ) ]

    ]
  ]

  while [ mean [income] of poorpeople > y-poor]   ; need to low mean income
  [

    ask one-of poorpeople with [income > (y-poor / 2)]
    [

      set income int (random-normal (y-poor / 3) ( y-poor / 2) )    ; lower income
      while [income >= richincome or income < 0] [set income int (random-normal (y-poor
/ 3) ( y-poor / 2) ) ]

    ]
  ]

end

; This function tries varying taxrates and returns the taxrate associated with the most
utility.
; This function works only if the utility is single peaked. could easily be updated to start
with
; 10 % ranges and then finding the peak within each range and then finding the best of the
peaks

to-report LineSearchUtility [tempmedianincome lb ub repeatcount]
  ;  let t 0

  let midrate  (lb + ub) / 2
```

```
  let resultlist (list (list lb calcutility lb tempmedianincome (distributioncost lb) )
    (list midrate calcutility midrate tempmedianincome (distributioncost midrate))
    (list ub calcutility ub tempmedianincome (distributioncost ub)))

  set resultlist sort-by [item 1 ?1 < item 1 ?2] resultList
  ;type "resultlist " show resultlist

  repeat repeatcount
  [
    set resultlist but-first resultlist
    set midrate (item 0 item 0 resultlist + item 0 item 1 resultList) / 2
    set midrate (list midrate calcutility midrate tempmedianincome (distributioncost
midrate))
    set resultlist fput midrate resultlist
    set resultlist sort-by [item 1 ?1 < item 1 ?2] resultlist
    ; type "resultlist " show resultlist
  ]
  ;show aList
  report last resultList
end

to-report LineSearchUtility2 [tempmedianincome lb ub]  ; if not single peaked divides
into intervals and runs linesearch on each interval.

  let interval (ub - lb) / 10
  let mid1 lb + interval
  let mid2 mid1 + interval
  let mid3 mid2 + interval
  let mid4 mid3 + interval
  let mid5 mid4 + interval
  let mid6 mid5 + interval
  let mid7 mid6 + interval
  let mid8 mid7 + interval
  let mid9 mid8 + interval

  let y 10

  let midlist1 LineSearchUtility tempmedianincome lb mid1 y
  let midlist2 LineSearchUtility tempmedianincome mid1 mid2 y
  let midlist3 LineSearchUtility tempmedianincome mid2 mid3 y
  let midlist4 LineSearchUtility tempmedianincome mid3 mid4 y
  let midlist5 LineSearchUtility tempmedianincome mid4 mid5 y
  let midlist6 LineSearchUtility tempmedianincome mid5 mid6 y
  let midlist7 LineSearchUtility tempmedianincome mid6 mid7 y
```

```
  let midlist8 LineSearchUtility tempmedianincome mid7 mid8 y
  let midlist9 LineSearchUtility tempmedianincome mid8 mid9 y
  let midlist10 LineSearchUtility tempmedianincome mid9 ub y


  let thislist (list midlist1 midlist2 midlist3 midlist4 midlist5 midlist6 midlist7 midlist8
midlist9 midlist10)
  if debug? [print thislist]
  ;
  ;
  set thislist sort-by [item 1 ?1 < item 1 ?2] thisList
  if debug? [type "thislist " print thislist]
  report last thisList


end

to-report calcutility [temptau tempincome temptaxcost]          ; this is the agents utility in
a single year based on the tax rates.
  report (tempincome + temptau * (y-bar - tempincome) - (temptaxcost * y-bar))
end
```

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Biography

Elaine Reed received a Bachelor of Science Degree in Electrical Engineering from Virginia Tech, and a Master of Science Degree in Electrical Engineering from the George Washington University. She is a certified Project Manager Professional (PMP). She has led global businesses and government organizations to achieve transformative capabilities in information technology over the past 30 years. She has worked at MITRE for the last 10 years encouraging, exploring and enabling change.