UNIVERSAL DESIGN CURVES FOR VARACTOR VOLTAGE-TUNED CIRCUITS

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ABSTRACT

Voltage tuning of filters and oscillators with varactors is becoming widely used in communications equipment, to avoid problems of mechanical tuning and to decrease size and weight. The successful use of varactor tuning over fairly wide bands requires that adequate change in capacitance can be achieved within the tuning range, that the signal developed not be so large as to drive the varactor out of its nominal range, and that the d-c voltage developed for tuning purposes be held within fairly precise limits over its range of variation. Variations in power supply voltage on the tuning system can conceivably cause serious mistuning. Sensitivity of tuning to power supply variation can be modified by padding the tuning varactor with a fixed capacitance.

A set of universal design curves is given from which key design values can be obtained for most design problems involving varactor tuning. From these curves it is possible to determine sensitivity to power supply voltage changes and to tuning voltage.

The use of these design curves is illustrated by an application to a differential oscillator design, where the sensitivity to supply voltage changes can be decreased substantially with the aid of the curves.

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1 On leave from the Electrical Engineering Department, University of Kansas, Lawrence, Kansas.

1.0 <u>Fundamentals</u> <u>of</u> <u>Varactor</u> <u>Tuning.</u>[1] A varactor is a semiconductor p-n junction device, characterized by a high Q and a voltage-variable capacitance with reverse bias voltage. In a voltage-tuning application, the varactor is normally part of a tuned circuit which, in turn, is part of a filter or voltage-controlled oscillator (VCO). As the d-c bias on the varactor is varied its capacitance changes, causing the filter to track or the VCO to change frequency.

Varactors are of two main types, abrupt-junction and graded-junction. For voltage-tuning applications, the abrupt-junction type is preferred because a greater percentage change in capacitance can be obtained for a given tuning voltage range, due to the greater curvature in the capacitance-voltage characteristic..

For the abrupt-junction varactor type, to which all the following results pertain, the capacitance vs voltage law for reverse bias is

$$C = C_{min}\left[\frac{V_B + \phi}{V + \phi}\right]^{0.5} \tag{1}$$

where C_{min} is the capacitance at the breakdown voltage V_B, ϕ is very nearly the contact potential of the diode, and V is the tuning voltage magnitude.

Equation (1) can be used to derive various other forms of expressing the C-V relation. In particular, if V_0 is the maximum value of tuning voltage in an application, the corresponding value of capacitance would be

$$C_0 = C_{min}\left[\frac{V_B + \phi}{V_0 + \phi}\right]^{0.5} \tag{2}$$

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[1] Penfield, P. and R. P. Rafuse: <u>Varactor</u> <u>Applications</u>, The M.I.T. Press, 1962, pp. 63-73.

and one can then write

$$C = C_0 \left[\frac{V_0 + \phi}{V + \phi} \right]^{0.5} \tag{3}$$

which is the form to be used for obtaining universal design curves, with C_0 the minimum value of capacitance in the tuning range.

Equation (3) can be simplified for most tuning applications, since ordinarily the tuning voltage will be large compared to ϕ. When $V \gg \phi$

$$C \approx \frac{C_0}{(V/V_0)^{0.5}} \left[1 + (\phi/2) \frac{V - V_0}{VV_0} \right] \tag{4}$$

Normally the second term in this approximation is small enough that the expression

$$C = \frac{C_0}{(V/V_0)^{0.5}} \tag{5}$$

is adequate for the applications. In particular, this equation can be used with good accuracy when the varactor is applied as illustrated in Fig. 1, with the tuning range extending from V_1 to V_0, where $V_1 \gg \phi$. For example, if $V_0 = 50$ volts and $V_1 = 10$ volts, with $\phi = 0.5$ volt, the maximum error incurred by using Eq. (5) instead of Eq. (3) is about 2%. Ordinarily, with production devices, the exponent will differ from 0.5 by more than 2%, suggesting that the theoretical equation is a sufficiently inexact representation of the varactor that Eq. (5) might as well be used as Eq. (3), as long as they differ by only a few percent. Therefore, in the remainder of this paper, Eq. (5) will be used to represent the performance of a varactor in tuning applications.

We shall find it convenient to define $V/V_0 = x$ as the voltage-tuning variable. Thus for the varactor, the <u>tuning law</u> used is

$$C = C_0 x^{-0.5} \tag{6}$$

2.0 <u>Capacitance vs Voltage for a Series-Padded Varactor</u>. In series padding, a fixed capacitor C_s is placed in series with the varactor. This gives a composite capacitance C_T which has a different tuning law. We have

$$C_T = \frac{C_s C}{C_s + C} = \frac{C_s}{1 + (C_s/C_0)x^{0.5}} \tag{7}$$

We shall find it convenient to let $C_s/C_0 = \alpha$ and we refer to α as the parameter of Eq. (7). Thus we write

$$C_T = \frac{C_s}{1 + \alpha x^{0.5}} \tag{8}$$

as the <u>tuning law</u> for the series-padded varactor. When $\alpha = \infty$, Eq. (8) reduces to Eq. (6), since $\alpha = \infty$ means no padding.

3.0 <u>Capacitance vs Voltage for a Parallel-Padded Varactor</u>. In parallel padding, a fixed capacitor C_p is placed in parallel with the varactor. This gives a composite capacitance C_t which has a different tuning law. We have

$$C_t = C + C_p = C_p \left[1 + (C_0/C_p)x^{-0.5} \right] \tag{9}$$

We shall find it convenient to let $C_0/C_p = \theta$, and we refer to θ as the parameter of Eq. (9). Thus we write

$$C_t = C_p(1 + \theta x^{-0.5}) \tag{10}$$

as the tuning law for the parallel-padded varactor. When $\Theta = \infty$, Eq. (10) reduces to Eq. (7), since $\Theta = \infty$ means no padding.

4.0 Center Frequency vs. Tuning Voltage. Now the relations between center frequency of a parallel tuned circuit and tuning voltage will be derived for the unpadded and padded varactor.

4.0.1 The Unpadded Varactor. For the unpadded varactor serving as the capacitance in a parallel L-C circuit, the resonance frequency is

$$f_0 = \frac{1}{2\pi (LC)^{0.5}} \qquad (11)$$

and the maximum value of f_0 corresponds to the minimum capacitance, thus

$$(f_0)_{max} = \frac{1}{2\pi (LC_0)^{0.5}} \qquad (12)$$

Then, with Eq. (6) as the tuning law,

$$f_0 = (f_0)_{max} \, x^{0.25} \qquad (13)$$

The minimum frequency corresponds to $x = x_1 = V_1/V_0$, where V_1 is the minimum tuning voltage, thus

$$(f_0)_{min} = (f_0)_{max} \, x_1^{0.25} \qquad (14)$$

The tuning range of frequency is

$$F = (f_0)_{max} - (f_0)_{min} \qquad (15)$$

$$= (f_0)_{max}(1 - x_1^{0.25}) \qquad (16)$$

Equation 16 may be solved for x_1 in terms of a desired maximum resonance frequency and a desired tuning range, giving

$$x_1 = \left[1 - \frac{F}{(f_0)_{max}}\right]^4 \qquad (17)$$

Equation (16) may be used as the basis for a design chart, shown in Fig. 2. The lower portion of the vertical scale is expanded for ease of reading.

4.0.2 The Series-Padded Varactor. For a parallel-tuned circuit with a series-padded varactor as the capacitive element and $\alpha = C_s/C_0$, the center frequency can be written in the form

$$f_s = (f_s)_{max} \left[\frac{1 + \alpha x^{0.5}}{1 + \alpha} \right]^{0.5} \tag{18}$$

where

$$(f_s)_{max} = \frac{1}{2\pi \left[LC_s(1 + \alpha)^{-1} \right]^{0.5}} \tag{19}$$

From these equations we can solve for the voltage tuning range parameter x_1, where $x_1 = V_1/V_0$, in terms of the frequency tuning range F and the maximum resonance frequency $(f_s)_{max}$. We obtain

$$x_1 = \left\{ \frac{\left[1 - F/(f_s)_{max} \right]^2 (1 + \alpha) - 1}{\alpha} \right\}^2 \tag{20}$$

A family of curves can be plotted from Eq. (20) with α as a parameter. Such a family is shown in Fig. 3.

4.0.3 The Parallel-Padded Varactor. For the resonant circuit containing a parallel-padded varactor with $\theta = C_0/C_p$, the center frequency is

$$f_p = (f_p)_{max} \left[\frac{1 + \theta}{1 + \theta x^{-0.5}} \right]^{0.5} \tag{21}$$

where

$$(f_p)_{max} = \frac{1}{2\pi \left[LC_p(1 + \theta) \right]^{0.5}} \tag{22}$$

Solving for the tuning range parameter in terms of the frequency range F and the maximum center frequency, we have

$$x_1 = \theta^2 \left\{ \frac{1 + \theta}{\left[1 - F/(f_p)_{max}\right]^2} - 1 \right\}^{-2} \tag{23}$$

A family of curves can be plotted from Eq. (23) with θ as a parameter. Such a family is shown in Fig. 4.

5.0 Center Frequency Sensitivity to Voltage . The sensitivity of the resonant frequency of a tuned circuit to voltage is defined as the derivative of the center frequency with respect to voltage. This sensitivity will vary with the operating point. Also the sensitivity to tuning voltage will differ from the sensitivity to tuning supply voltage. It is assumed that the tuning voltage is obtained by tapping the tuning supply voltage. We designate the sensitivity as S_y^z where y identifies the particular center frequency being considered (f_0, f_s, or f_p) and z is the particular voltage involved (tuning voltage V or tuning supply voltage E). Whenever specific values are to be discussed, we use units of cycles/second per millivolt.

5.0.1 Unpadded Varactor. For the unpadded varactor, we can find $S_{f_0}^V$, the sensitivity of the center frequency to tuning voltage, by differentiating Eq. (13). This gives

$$S_{f_0}^V = \frac{df_0}{dx} \frac{dx}{dV} = \frac{(f_0)_{max}}{4V_0} (V/V_0)^{-0.75} \tag{24}$$

$$= (S_{f_0}^V)_{min} x^{-0.75} \tag{25}$$

where $(S_{f_0}^V)_{min} = (f_0)_{max}/4V_0$. The sensitivity of the center frequency with respect to changes in supply voltage E is given by

$$S_{f_0}^E = \frac{df_0}{dE} = \frac{df_0}{dx}\frac{dx}{dV}\frac{dV}{dE} \tag{26}$$

If we assume that $E = V_0$, then $V = xE$ and $dV/dE = x$, hence

$$S_{f_0}^E = (S_{f_0}^V)_{min}\, x^{0.25} \tag{27}$$

$$= (S_{f_0}^E)_{max}\, x^{0.25} \tag{28}$$

Note that the sensitivity of the center frequency to changes in supply voltage is a maximum at the point where the sensitivity to change in tuning voltage is a minimum, namely at the point where varactor capacitance is a minimum ($x = 1$).

5.0.2 Series-Padded Varactor. For the series-padded varactor, we find for the sensitivity to tuning voltage

$$S_{f_s}^V = (S_{f_s}^V)_{min}\left[\frac{1 + \alpha}{x(1 + \alpha x^{0.5})}\right]^{0.5} \tag{29}$$

where

$$(S_{f_s}^V)_{min} = \frac{\alpha\,(f_s)_{max}}{4V_0(1 + \alpha)} \tag{30}$$

The maximum sensitivity to tuning voltage occurs at $x = x_1$, i.e. at the lowest tuning voltage.

The sensitivity to supply voltage E, assuming $E = V_0$, is

$$S_{f_s}^E = (S_{f_s}^E)_{max}\left[\frac{x(1 + \alpha)}{(1 + \alpha x^{0.5})}\right]^{0.5} \tag{31}$$

where

$$(S_{f_s}^E)_{max} = (S_{f_s}^V)_{min} \tag{32}$$

As for the unpadded varactor, the maximum sensitivity to supply voltage is the same as the minimum sensitivity to tuning voltage, and occurs at $x = 1$.

5.0.3 <u>Parallel-Padded Varactor.</u> For the parallel-padded varactor the sensitivity to <u>tuning voltage</u> is

$$S_{f_p}^V = (S_{f_p}^V)_{min} \left[\frac{1 + \Theta}{x(1 + \Theta\, x^{-0.5})} \right]^{1.5} \tag{33}$$

where

$$(S_{f_p}^V)_{min} = \frac{\Theta(f_p)_{max}}{4V_0(1 + \Theta)} \tag{34}$$

The sensitivity is a maximum when $x = x_1$.

The sensitivity to change in <u>supply voltage</u> is, assuming $E = V_0$,

$$S_{f_p}^E = (S_{f_p}^V)_{min} \left[\frac{1 + \Theta}{x^{1/3} + \Theta\, x^{-1/6}} \right]^{1.5} \tag{35}$$

This sensitivity varies with x in a manner quite different from the other sensitivities considered. All others studied vary monotonically across the voltage tuning range. However this one, under some circumstances, will have a maximum at some point inside the limits of the tuning range. In particular, differentiating the sensitivity and setting it equal to zero, we find that this maximum occurs at $x = \Theta^2/4$. In order for this maximum to lie inside the tuning range, it is necessary that $\Theta < 2$.

Figure 5 shows curves of sensitivity of center frequency to supply voltage. The case $\Theta = 2$ is the maximally flat curve, and corresponds to holding the sensitivity to supply voltage relatively constant over a substantial portion of the tuning range. Figure 5 shows, for comparison, sensitivities for the unpadded varactor and for the series padded varactor. Also a parallel-padded varactor with $\Theta = 2^{1/2}$ is considered.

6.0 <u>Design Applications.</u>

The design charts presented can be used in various ways as aids in design. A summary of applications follows.

6.0.1 <u>Figure 2.</u> Figure 2 applies to a varactor of the abrupt-junction type without padding. When the maximum frequency and the tuning range are known, the required voltage range can be read directly from the chart. For example, if the frequency tuning range is to be 30% of the maximum frequency, it is seen that the ratio of minimum to maximum tuning voltage must be 0.24. These data apply to any varactor of the abrupt-junction type. To select a varactor, one takes the data from these curves together with the known circuit information. Alternatively, one can see from the curves in Fig. 2 that if the tuning voltage is limited to a range from one half the maximum value to the maximum value, the frequency tuning range can be only 16% of the maximum frequency.

6.0.2 <u>Figure 3.</u> When the frequency tuning range is a fairly small percentage of the maximum frequency, it may be desired to use series padding. The use of series padding reduces the sensitivity of the circuit to d-c power supply variation, and improves the linearity of signal handling. Figure 3 shows what percentage of the maximum frequency can be tuned with a given amount of padding and a particular ratio of minimum to maximum tuning voltage. For example, if it is desired to tune over a range which is 6% of the maximum frequency, a tuning voltage range from about 42% of maximum voltage to maximum voltage may be used if $\alpha = 0.5$, which means that the value of series capacitance C_s is one half of the minimum varactor capacitance C_0.

6.0.3 _Figure 4._ Parallel padding may be deliberate or may be impossible to prevent due to the presence of strays which effectively pad a varactor. Such padding reduces the percentage of frequency tuning range which can be achieved with a given voltage range. Figure 4 shows the relations between frequency tuning range and voltage range for specific amounts of padding. For example, if it is desired to tune 6% of the maximum frequency, a voltage ranging from about 54% of maximum to maximum will suffice with $\Theta = 0.5$, which means that the parallel padding capacitor has $C_p = 2C_0$, a capacitance twice as great as the minimum varactor capacitance.
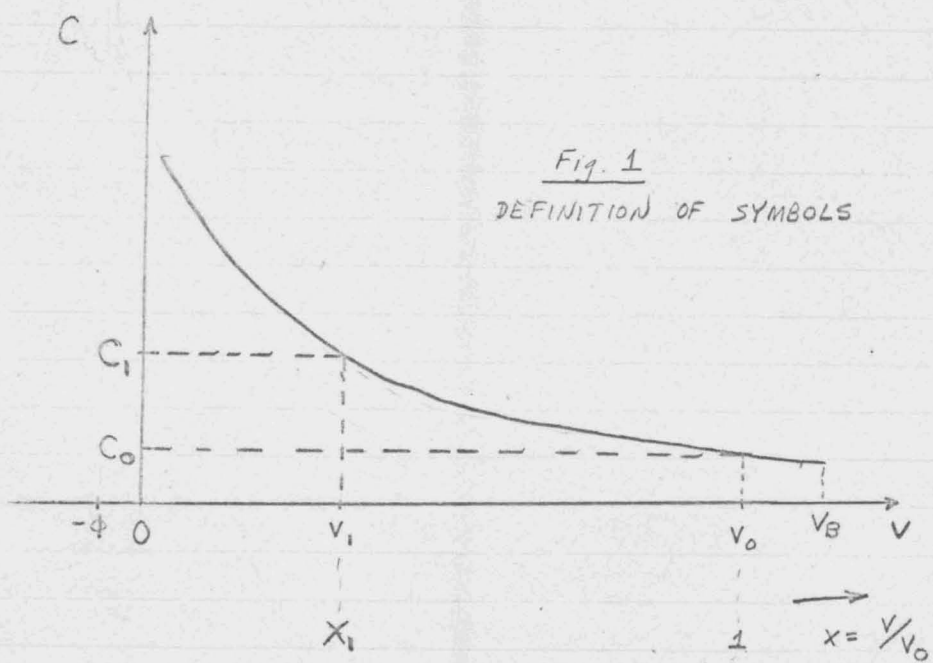
6.0.4 _Figure 5._ Figure 5 shows sensitivity of the resonance frequency to tuning voltage for an unpadded varactor and for series padding with $\alpha = 1$, and for parallel padding with $\Theta = 2$ and $2^{\frac{1}{2}}$. It is seen as mentioned earlier that the sensitivity can be maintained constant within a small percent over a wide tuning range for properly selected parallel padding.
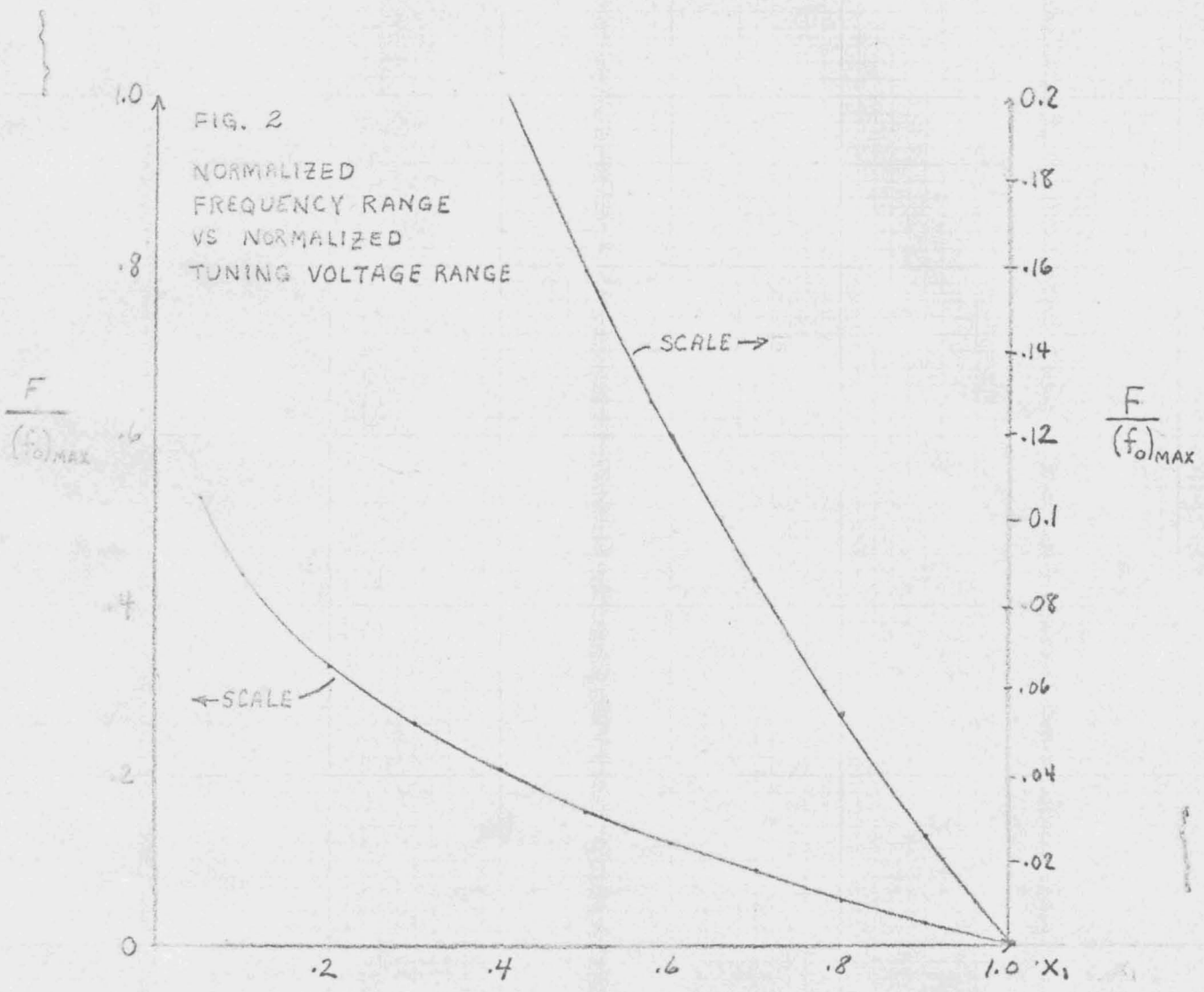
When the maximum frequency and the maximum tuning voltage are known, with the supply voltage equal to the maximum tuning voltage, these curves permit determination of sensitivity. For example, if the maximum frequency is 2 megacycles/sec and the maximum tuning voltage is 100 volts, at a tuning voltage of 60 volts the center frequency would change by 4.8 cps per millivolt for an unpadded varactor, by 3.3 cps per millivolt for a parallel-padded varactor with $\Theta = 2$, and by 2.1 cps per millivolt for a series-padded varactor with $\alpha = 1$.
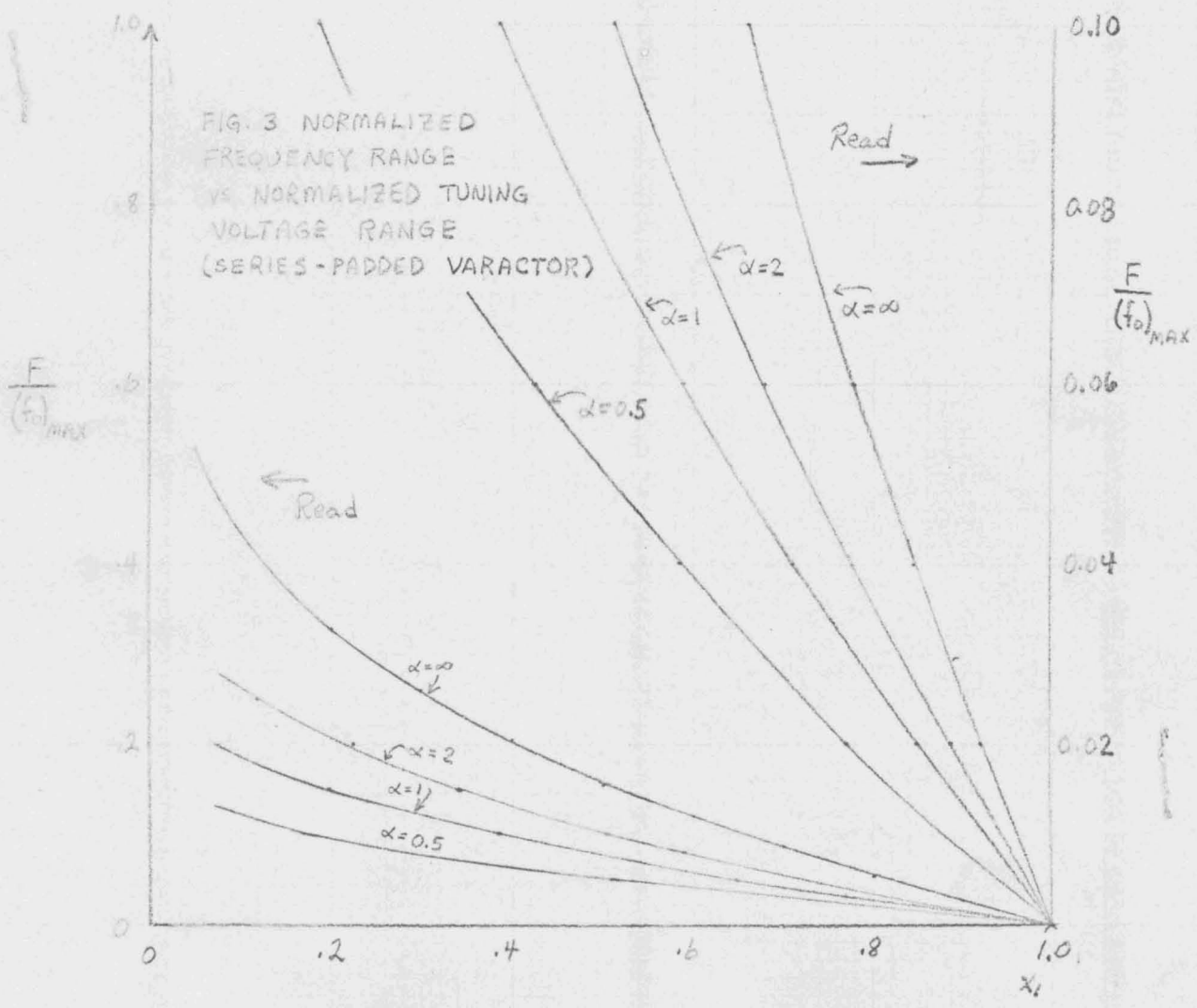
The flat characteristic for $\Theta = 2$ suggests a way of making a voltage-controlled source which is quite insensitive to supply-voltage variation. This can be done by making the output equal to the difference of two frequencies $f_g = f_1 - f_2$. The frequency f_1 can be varied by a tuning
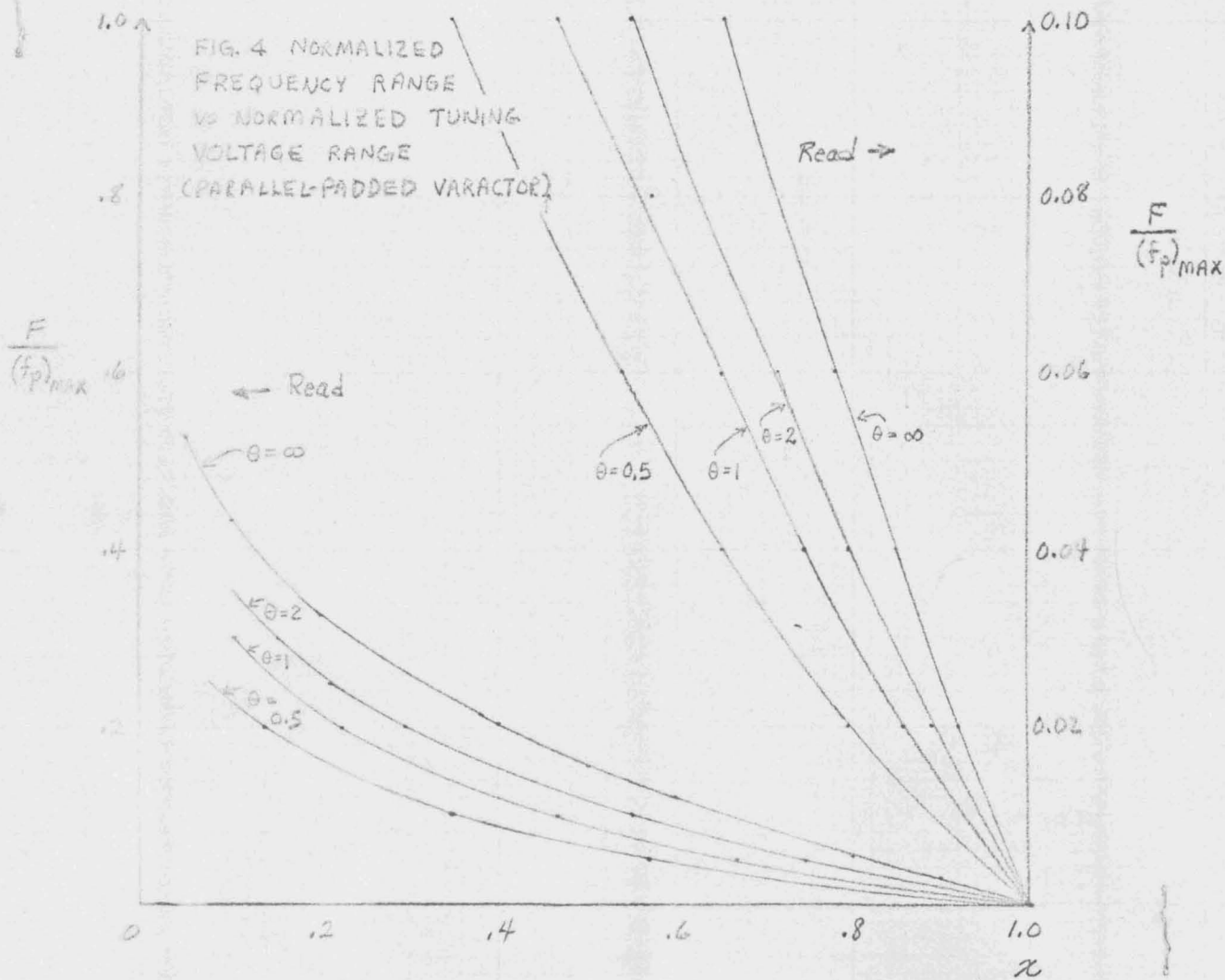
voltage, while the frequency f_2 remains constant. Thus f_g is effectively varied by the tuning voltage. Both f_1 and f_2 may be made to have the same sensitivity to supply voltage changes by including varactors in each individual oscillator. The varactor involved in producing f_1 has its voltage changed but proportional to the tuning supply voltage, while the varactor involved in producing f_2 does not have a changeable voltage except for changes in the tuning supply voltage, as its voltage is a constant proportion of the tuning supply voltage. This differential oscillator scheme substantially eliminates dependence of the output frequency on supply voltage variations, though it may still be sensitive to variations due to other causes.

7.0 <u>Summary</u>. Formulas and charts have been presented which speed and facilitate the design of voltage-tuned circuits using varactors as the variable tuning elements.

Fig. 1

DEFINITION OF SYMBOLS

FIG. 2

NORMALIZED
FREQUENCY RANGE
VS NORMALIZED
TUNING VOLTAGE RANGE

FIG. 3 NORMALIZED
FREQUENCY RANGE
VS NORMALIZED TUNING
VOLTAGE RANGE
(SERIES-PADDED VARACTOR)

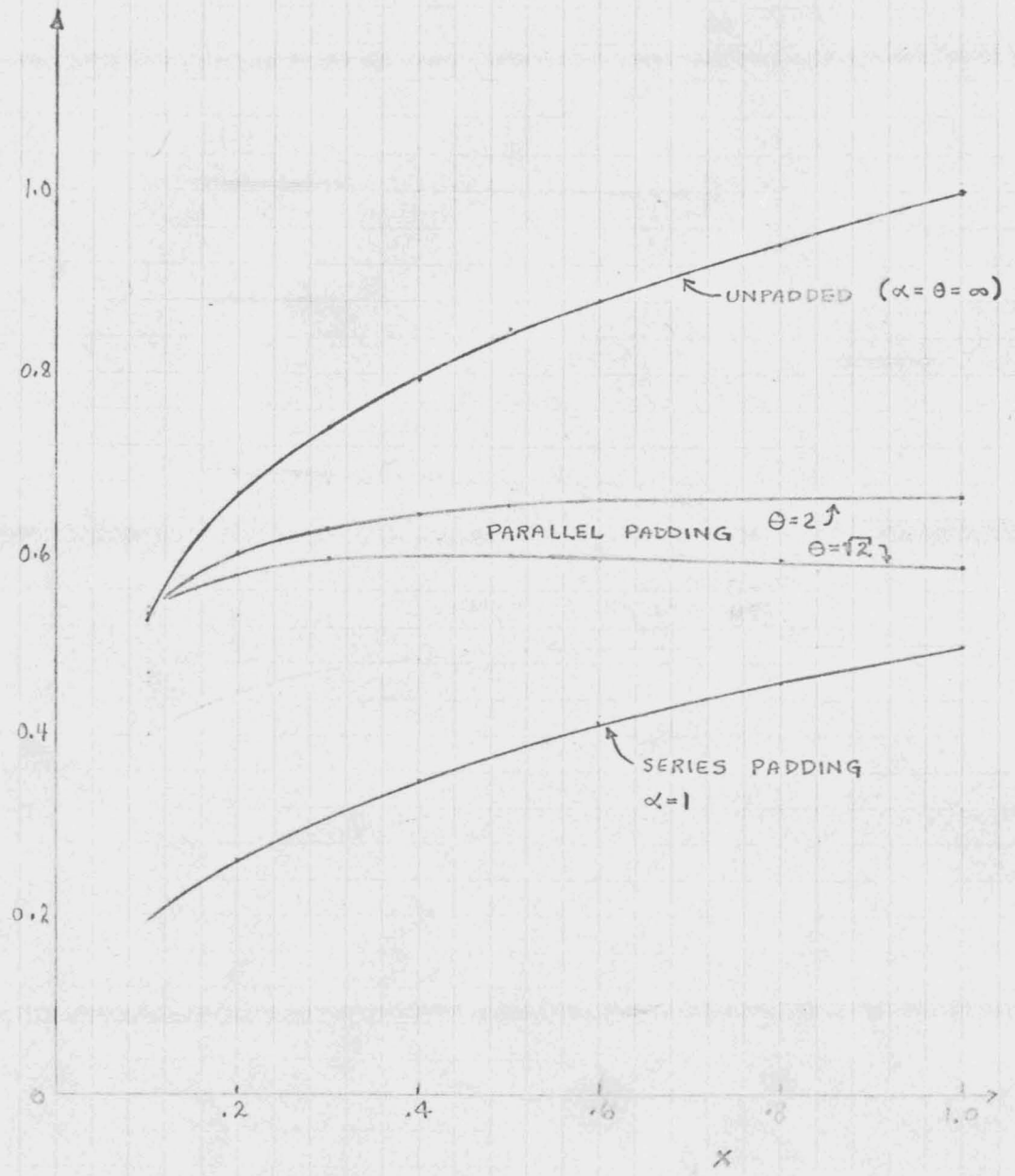FIG. 4 NORMALIZED FREQUENCY RANGE vs NORMALIZED TUNING VOLTAGE RANGE (PARALLEL-PADDED VARACTOR)

Fig. 5

Sensitivity of Center Frequency to Supply Voltage