

PRIME LABELINGS OF ALL CUBIC BIPARTITE GRAPHS WITH AT LEAST 10 VERTICES AND AT MOST 18 VERTICES

S. SCHLUCHTER AND T. WILSON

ABSTRACT. This document contains prime labelings of all cubic bipartite graphs with at least 10 and at most 18 vertices.

1. INTRODUCTION

A graph G with n vertices has a *prime labeling* if there is a bijection $\phi: V(G) \rightarrow \{1, 2, 3, \dots, n\}$ such that $\{v_i, v_j\} \in E(G)$ implies that the greatest common divisor of $\phi(v_i)$ and $\phi(v_j)$ is 1. If a graph has a prime labeling, then the graph is *prime*.

This article contains some of the prime labeling data referenced in [2]. The data was generated using [2, Construction 3.3], and the data appears in Section 3. The graphs were generated using [1].

2. HOW TO READ THIS DOCUMENT

Each cubic bipartite appearing in Section 3 and its corresponding prime labeling are represented with a specially constructed block matrix.

Consider the block matrix appearing in Figure 1, and let B be the lower-right block, which denotes an adjacency matrix for a cubic bipartite graph G . The upper-right and lower-left blocks contain the prime labeling. If the leftmost entry in row i is labeled x , the uppermost entry in column j is labeled y and $B_{ij} = 1$, then the vertices labeled x and y are adjacent; if $B_{ij} = 0$, then the vertices labeled x and y are not adjacent. The upper-left block contains the symbol M_i^j , which signifies that this is the i^{th} labeled cubic bipartite graph with j vertices. The prime labeling is also drawn graphically in Figure 1 for the reader's convenience.

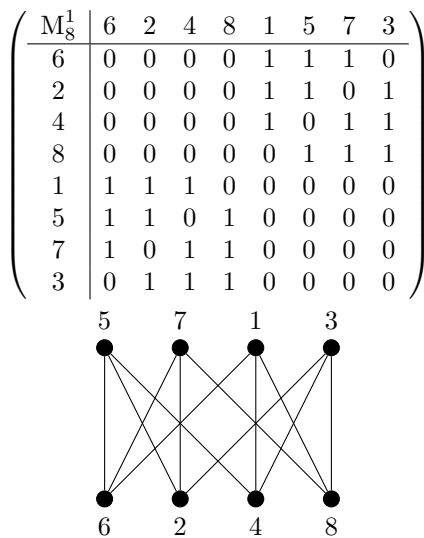


FIGURE 1. A sample prime labeling of a cubic bipartite graph and a block-matrix representation of the graph and the labeling.

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Key words and phrases. Graph labeling; prime graph.

M_{18}^{156}	6	12	2	18	10	14	4	8	16	5	7	1	11	13	17	3	9	15
6	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	1	0	0	1	1	0	0	0	0
2	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	1
18	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	0	0	0
10	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1	0
14	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	1	0
4	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	1
8	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1
5	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
11	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0
13	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0
15	0	0	1	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0

M_{18}^{157}	6	12	10	18	2	14	4	8	16	7	5	1	11	13	17	3	15	9
6	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	1	0	0	1	1	0	0	0	0
10	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	1
18	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	0	0	0
2	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	1	0
14	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	0
4	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1
8	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1
7	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
11	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0
13	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0
3	0	0	1	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0
9	0	0	1	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0

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DEPARTMENT OF MATHEMATICAL SCIENCES, GEORGE MASON UNIVERSITY, 4400 UNIVERSITY DRIVE, MS: 3F2, FAIRFAX, VA 22030, UNITED STATES OF AMERICA

E-mail address: sschluch@gmu.edu

GEORGE MASON UNIVERSITY, 4400 UNIVERSITY DRIVE, MS: 3F2, FAIRFAX, VA 22030, UNITED STATES OF AMERICA

E-mail address: twilso19@gmu.edu