EMERGENCE AND SUSTAINABILITY OF TEACHERS' PRODUCTIVE DISPOSITION THROUGH PROFESSIONAL DEVELOPMENT EXPERIENCES VIEWED THROUGH AN ANDRAGOGICAL LENS

By

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Dedication

This is dedicated to my husband, Jamey, who is the reason I am able to accomplish my goals; my daughter, Jasmine, who models for me what it means to work hard academically; and my son, Richard, who proves challenges can be conquered every day; my son, David, who reminds me to take a breath and laugh – especially at myself.

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I thank my family, colleagues and friends who have provided the support that enabled me to finish my dissertation. Jamey, my husband, has been my rock, and to his credit balanced support with reminders to finish. Whether taking the kids, ensuring the study was an environment in which I could be productive, or reviving crashed technology – he was always there.

My children, Jasmine, Richard, and David, thank you for your continued understanding and for being an inspiration – each in your own way.

Thank you to my parents who have always encouraged me and provided enough support for me to find my own way.

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Thank you to all the students and teachers who have helped me be a better learner and teacher of mathematics.

Finally, thank you to Dr. Suh for believing in me again, and again, and again. Your, and Dr. Hjalmarson's mentorship and guidance are appreciated more than words can say. And Dr. Galuzzo, you were right – just start writing.

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List of Abbreviations

Association of Mathematics Teacher Educators……... AMTE
Cognitively Guided Instruction.....CGI
Department of Defense Education Activity..... DoDEA
International Baccalaureate.....IB
Interstate New Teacher Assessment and Support Consortium..... …INTASC
Mathematics for Teaching Knowledge.....MTK
Mathematics Professional Development Institutes.....MPDI
National Council for Accreditation of Teacher Educators ..... NCATE
National Council of Supervisors of Mathematics.....NCSM
National Council of Teachers of Mathematics .....NCTM
National Research Council ..... NRC
National Security Agency.....NSA

Abstract

EMERGENCE AND SUSTAINABILITY OF TEACHERS' PRODUCTIVE DISPOSITION THROUGH PROFESSIONAL DEVELOPMENT EXPERIENCES VIEWED THROUGH AN ANDRAGOGICAL LENS

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This dissertation addresses the development or sustainability of teachers' productive dispositions through professional development and related professional experiences when viewed from an andragogical perspective. The study identified experiences which both promoted and hindered productive disposition, and then considered these events using the andragogical framework (Knowles, Holton, & Swanson, 2011). This dissertation study includes a literature review of relevant topics including andragogy, professional development, teacher change and productive disposition. Teachers who participated in a grant funded professional development opportunity were interviewed, and an account taken of the math experiences throughout their lives, to include K- 12 mathematics instruction. This dissertation is meant to help identify professional development opportunities and professional experiences teachers have which promote a change or pathway toward productive disposition. The research considered, from an andragogical perspective, the professional development opportunities and conditions which will better

promote the emergence or sustainability of a teacher's productive disposition. The research also identified circumstances or events which hinder or curb a productive disposition, again, from an andragogical perspective.

Chapter One

Teacher affect is critical, but there is little empirical evidence regarding how teacher affect develops or is maintained for practicing teachers (Philipp, 2007). An identified key component of teacher affect is a productive disposition (Jacobson & Kilpatrick, 2015). There are at least a few reasons for this lack of empirical evidence and research on productive disposition: there are broad and changing definitions; it represents one piece of the inter-related components of affect, identifying and developing reliable and valid tools to measure productive disposition is difficult, and there is the need to account for numerous variables which may influence productive disposition. Finally, productive disposition appears to be a construct which changes over time throughout a teacher's career.

One possible origin for the definition of productive disposition is in the work of the National Council of Teachers of Mathematics' process standards (NCTM; 1989). The organization noted it was an important attribute for students of mathematics. Time and further research led practitioners to believe it was an important construct to conceptualize for teachers of mathematics. Researchers like McIntosh (1997), and Royster, Harris, and Schoeps (1999) also defined the term in relation to students. By 2001, researchers (Freeman, 2007; McClain & Cobb, 2001; National Research Council, 2001) and organizations such as The National Council for the Accreditation of Teacher Education

(NCATE, 2001) and the Interstate New Teacher Assessment and Support Consortium (INTASC; as cited in Hampton, 2008) discussed productive disposition as a critical component for teaching mathematics. Each of these organizations, however, defined the term differently and made it part of different larger constructs.

In some cases, productive disposition was seen as a key component or attribute which defined a good teacher. The National Reseach Council (NRC; 2001) identified it as one of five interwoven strands which defined proficiency in mathematics. Freeman (2007) noted productive disposition was part of a larger construct called attributes which helped determine effective teaching. NCATE viewed disposition as one of three components effective teachers had, and renamed and redefined it in 2007, referring to it as professional disposition. While other researchers acknowledged productive disposition was part of a larger construct, they also noted aspects of productive disposition, such as identifying mathematically reasonable and convincing arguments should be delineated and measured (McClain & Cobb, 2001). It is increasingly clear in the literature that productive disposition is complex, and thus without a clear definition is difficult to measure.

The research also tends to present productive disposition within a specific context, such as a prospective teacher's course of study (Charalambous, 2015; Jong & Hodges, 2015; Philipp, 2007) or a professional development opportunity for practicing teachers (Jacobson & Izsak, 2015; Lewis, Fischman, & Riggs, 2015). Although some of this research took place over the course of a semester or a multi-year project, and in some cases accounted for early experiences as teachers (Jong & Hodges, 2015), there are many

more variables which may impact a teachers' disposition. These variables include experiences as learners (K- 12), experiences in preparatory programs (if they were in a traditional program), student teaching experiences, mentors, professional development experiences, department collaboration, and administrative support.

The purpose of this research was to further explore how this aspect of teacher affect, called productive disposition for teaching mathematics (Jacobson & Kilpatrick, 2015), might be developed or sustained through professional development and other professional experiences. The following paragraphs more fully explain the rationale for choosing this topic, provide the operational definition used, and present the research questions.

Significance of the Research

One of the most contentious issues in education reform and policy is preparation of prospective teachers (Ingersoll, Merrill, & May, 2012). The Association of Mathematics Teacher Educators (AMTE) proposed a guide for a well-prepared beginning mathematics teacher. However, AMTE (2017) acknowledged that, "the development of teachers' content and teaching knowledge, skills, and dispositions occurs over a career-long trajectory" (p. 3). The continuum presented by the organization is represented in Figure 1. Moving from one stage of the continuum to the next, and therefore attrition, is dependent on preparation, continued development and support, and productive disposition. A teacher's experiences in each of these sections of the continuum is critical.

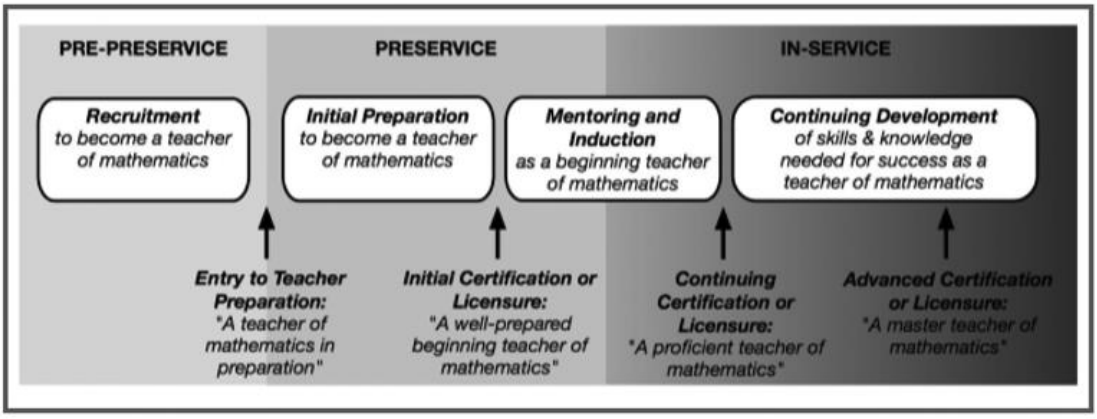

Figure 1. The teacher development curriculum. Association of Mathematics Teacher Educators. (2017). *Standards of Preparing Teachers of Mathematics*. Retrieved from amte.net/standards

Individuals who decide to go into teaching may do so because they want to help others, to inspire learning, or to share something about which they are passionate like mathematics (Curtis, 2012). These sentiments inherently seem to convey an initial positive disposition or affect toward students or toward content. Unfortunately, the United States has a shortage of highly-qualified teachers. Furthermore, in fields like mathematics, the shortage is considered hard-to-staff (Feng & Sass, 2018; Harrell, Thompson, & Brooks, 2019; Ingersoll & May, 2012; Ingersoll & Perda, 2010; McConnell III, 2017; Zumwalt, Natriello, Randi, Rutter, & Sawyer, 2017). This shortage is due to two primary factors: many teachers are retiring or leaving the profession, and the decrease in college graduates majoring in mathematics seeking a career in education.

Retention of qualified math teachers is one of the most concerning and challenging issues in education today (LaTurner, 2002; McConnell, 2017; Rinke &

Mawhinney, 2017). Several researchers reported that anywhere from about 20-50% leave the profession in the first 5 years (Amick, Martinez, & Taylor, 2017; Curtis, 2012; Ingersoll et al., 2012; McConnell, 2017). Ingersoll et al. found in 2004- 2015 data that 14.5% of math teachers left after the first year. Furthermore, losing teachers who have appropriate training and experience and continue to grow professionally has implications for student performance (Ronfeldt, Loeb, & Wyckoff, 2013). Rinke and Mawhinney (2017) agreedwith the AMTE that teacher expertise grows and develops over the length of their career, and further note there is a negative impact on student achievement when teachers with more experience leave the profession. Winters, Dixon, and Greene (2011) provided some quantitative evidence that higher performing teachers are more likely to seek other career pathways.Part of addressing the attrition rate is recruiting math teachers, as noted in the AMTE continuum (2017). LaTurner (2002) noted the contributions of recent college graduates is important to mathematics education, especially given the number of teachers retiring or leaving the profession; however, there are serious concerns given the projections for future math educators.

The President's Council of Advisors on Science and Technology stated there would be a shortage of over 1 million college graduates in related fields over the next 10 years (as cited by McConnell, 2017, p. 1). Add to this statistic the acknowledgement that math educators are in what is considered a high-need area, and the situation becomes more critical. Further complicating the implications for future mathematics education is that pedagogical training is important to retention. Future math educators must experience rigorous preparation/instruction in content and how to teach the content for it

to be more likely they will stay in the classroom. Ingersoll et al. (2012) found, referencing data from 2004- 2005, that 24.6% of math teachers with little or no pedagogical training left the profession, while only 9.8% of those with comprehensive training left the profession. However, what may be considered a traditional path (degree in mathematics or math education with content and methods courses), or programs which help ensure comprehensive training are not the current standard.

The need for mathematics teachers, and the projections that the situation will only become more desperate, has led many states to consider alternate routes to certification (Boyd et al., 2012; Zumwalt et al., 2017). It is estimated by Ingersoll et al. (2012) that more than 40% of the nation's teachers enter the field through alternative routes. Due to these alternatives, the related statistics in mathematics education are as disconcerting. For example, LaTurner (2002) found that 46.4% of new math and science teachers in Grades 6- 12 in her sample were only minimally qualified. Zumwalt et al. noted that many of the alternate route programs end up embedded in a college/university, so teachers earn a degree eventually; it is these teachers who serve our most at-risk students. However, the development and training of teachers takes time, which means many students are not receiving instruction from qualified staff. Zumwalt et al. further stated that support through programs, courses, and professional development are critical to ensuring the teachers are well-prepared. Although the attrition of teachers who are not qualified may be a preferable outcome, LaTurner (2002) argued considering retention of all groups is critical given the increased need for math and science teachers.

This research, in summary, notes the attrition of mathematics teachers, lack of current undergraduates in the field, loss of especially effective or experienced teachers, and weaknesses of alternative paths to teacher certification. These deficits create an increasingly critical demand for effective math educators who will stay committed to the profession. Given these realities, the model and continuum must be reconceptualized, or at least more flexible in its implementation to account for the preparation of those for whom math education was not a first choice, and thus do not follow a traditional path to teaching. The critical components of the continuum remain professional development and disposition. In the first phase of their research, Amick et al., (2017) were able to identify two categories of experiences which increased teachers' dispositions, which they described as enthusiasm for teaching mathematics. The first category involved the teachers' engagement with someone who had an, "infectious, enthusiastic attitude about teaching" (p. 458). The second was the teachers' observations of student thinking and success after implementing new strategies (Amick et al., 2017). In summary, interventions to retain teachers involved various types of professional development and disposition.

The professional development of teachers must consider many variables, including the teachers' paths to education, math background and content knowledge, experience (teaching and non-teaching), and various supports which may impact them in the workplace. The goal is to increase teachers' content and pedagogical knowledge in such a way that the knowledge is translated into practice (Desimone, Porter, Garet, Yoon, & Birman, 2002). Furthermore, the goal of the implementation of this knowledge should

lead to increased student performance (Wei, Darling-Hammond, Andree, Richardson, & Orphanos, 2009).

Professional development opportunities are varied and therefore their effectiveness is dependent on a number of attributes, only some of which can be controlled by the facilitator. For example, professional development which inherently creates a disconnect between the professional development (by setting or content) and teachers' classroom practices and experiences will minimize the effectiveness of the professional development (Cobb, McClain, de Silva Lamberg, & Dean, 2003). Given the needs of the teachers and the backgrounds of many coming from non-traditional paths, there is an increased focus to ensure the professional development opportunities offer pedagogical and subject content (Battey & Franke, 2008). In addition to aspects professional development facilitators can control, there are many over which they may not have influence or control: support from the teachers' supervising administrators, alignment of professional development with division/school goals, teachers' access to teacher leaders or knowledgeable others, supportive collaborative teams for the teachers, time, and the ability of the teacher to sustain their own growth (Cobb et al., 2003; Fullan, 2001; Loucks-Horsley, Stiles, Mundry, Love, & Hewson, 2010; Mewborn, 2010; Reeves, 2010; & Wei et al., 2009). As noted by Rinke and Mawhinney (2017) the "push and pull" of these variables directly influence a teacher's trajectory. It is arguable that a mitigating factor is how the "pushing and pulling" influences their disposition towards mathematics and teaching mathematics.

In 2015, an entire volume of the *Journal of Mathematics Teacher Education* was devoted to productive disposition in teachers. In the opening article, Jacobson and Kilpatrick (2015) argued that productive disposition is an important, and often overlooked, aspect of effective teaching. Several researchers have noted that the importance of teacher affect lies in its influence on instruction and learning (Cross, 2009; Pajares, 1992; Philipp, 2007). Charalambous (2015) noted a specific case in Yin's (2009) work to argue that regardless of how difficult it may be to promote changes in a teachers' belief and knowledge, "the price of not taking any action may be too high for student learning" (p. 443). Thompson (1992) noted that focusing either on math and pedagogical knowledge or beliefs as separate entities, results in, "an incomplete practice" (p. 131). However, the connections between these pieces are elusive because the relationship is likely based on an interactive process. Jacobson and Kilpatrick said, "The central assumption of strong, nonlinear relationships between productive disposition for teaching mathematics, teacher knowledge, and teaching practice implies an overarching hypothesis that change in one aspect is inextricably linked with change in the others" (Thompson, 1992, p. 402). The authors noted because their statement was a hypothesis due to a lack of empirical data, and a nonlinear relationship is likely, more coordinated research was needed to determine aspects of the relationship, such as "teacher affect in mathematics teacher education" (p. 402).

I have worked in several positions in which the importance of a teacher's productive disposition has been clear to me. My experiences began with my own math background and using my math content knowledge and skills to support teachers when I

served as a practicum supervisor for Iowa State University. Since then I have worked as a high school math teacher, division math coordinator, undergraduate and graduate instructor/facilitator, and currently as a high school principal. The difference the teacher's affect, and specifically productive disposition has for their students' development in and appreciation for mathematics is evidenced in so many ways. I can recount many examples while serving in each of these roles in which I observed, productive disposition both thrive and wane. And yet, despite years as a practitioner who has worked with teachers my entire career, I have little understanding of the emergence and sustainability of productive disposition. The antecedent, mitigating factors, and ultimate result of what I observed would serve to confirm my observations and help me better understand how to facilitate productive disposition for more of the educators with whom I interact in these various roles.

The data collected, and the results of this study will contribute to a growing body of research on identifying observable attributes of productive disposition. The study will also strive to identify its emergence or growth in teachers, and those factors which contribute to or hinder the sustainability of a teacher's productive disposition. This information will serve to inform educators of mathematics teachers, those who offer professional development, and professionals who guide or coach mathematics teachers.

Defining Productive Disposition

In an effort to best understand teachers' productive disposition, it was important to look at its origins in math literature. Specifically, the role of productive disposition in students' mathematical proficiency. The NRC (2001) identified five strands which were

necessary for students to develop mathematical proficiency. These strands are conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. The first four of these components are related to a student's knowledge – skills and ways of thinking or reasoning within the area of mathematics – and the final strand is an affective attribute. The authors noted it was important to think of these strands as interwoven and dependent on each other for a student to achieve mathematical proficiency. The authors make a comparison to proficient teaching of mathematics, stating, "Just as mathematical proficiency itself involves interwoven strands, teaching for mathematical proficiency requires similarly related components" (NRC, p. 380). The authors acknowledged productive disposition for teachers is a critical component of teaching as it directly influences their development as mathematicians and math educators, and therefore directly impacts their students' mathematical development. Since this seminal work, the definition of productive disposition has evolved, especially as it relates to teaching. Jacobson and Kilpatrick's (2015) definition is the most comprehensive and will therefore be used for this study. Their definition is as follows: "Productive disposition for teaching mathematics is mathematics teachers' malleable orientation toward – and concomitant beliefs, attitudes and emotions about – their own professional growth, the subject of mathematics, and its teaching and learning that influences their own and their students' successful mathematics learning" (p. 402).

Research Questions

The purpose of this dissertation is to provide insight on teachers' behaviors and thoughts, in relation to professional development, which provide evidence of a productive

disposition toward mathematics and mathematics instruction. The professional development provided will place teachers in purely educational settings as learners and teachers over a sustained period of time, focus on pedagogy within the context of relevant content, and consider support provided by the instructional team and the teacher's supervisors and colleagues. Given this, the following questions will be researched:

1. How do teachers describe the changes in their beliefs, attitudes and emotions about (a) their own professional growth, (b) the subject of mathematics, and (c) mathematics teaching and learning over their teaching careers?
2. How can the andragogical framework be used to identify and explain events which serve as catalysts which promote or hinder teachers' productive dispositions?
3. What are indicators, of which professional development designers should be aware, which will further promote or hinder productive disposition? Is there evidence regarding how these indicators should be addressed to maximize growth?

Chapter Two

This chapter presents a review of the literature on andragogy, professional development, and productive disposition. The review begins by discussing andragogy, a theoretical framework for working with adult learners. *Andragogy* (art and science of helping adults learn) was originally based on four assumptions which differentiate it from pedagogy (Knowles, 1970). These original assumptions and the related work which followed have direct implications for professional development for practitioners. The next part of the chapter discusses several of these implications and considers various forms of professional development. The section on professional development also helps delineate which forms address the assumptions of andragogy and where they fall short. Finally, whether an experience like professional development meets the needs of an adult learner has implications for the adult learner's disposition. Therefore, the next section of this chapter explores productive disposition. The definition is shown to be one that has evolved, with perspectives of various professional organizations and researchers contributing to the definition. To help better understand both the application of the definition and its components as it relates to teachers, I will summarize several studies. The studies explore productive disposition of pre-service teachers in education programs and in-service teachers engaged in a professional development opportunity.

Andragogy

Adult learners have accumulated various personal and professional experiences, such as their experience as a learner in Grades K-12, interactions with their teachers/mentors, pre-practitioner/student teaching experiences, and teaching experiences, all of which serve to frame and inform how their ideas about math and teaching are defined and perceived. Adult learners' experiences at a given point of time are quite varied. Knowles (1970) theoretical framework, andragogy, accounts for these variations. As stated by Knowles et al. (2011), andragogy accounts for "the lack of homogeneity among learners and learning situations and illustrates that the learning transaction is a multifaceted activity" (p. 146). Andragogy also accounts for the goals and learning environment which will influence the outcomes of a learning experience for an individual adult learner. Mezirow (1981) summarized andragogy as "an organized and sustained effort to assist adults to learn in a way that enhances their capacity to function as self-directed learners" (p. 21).

Andragogy is a derivation of the Greek root –*agogus*—which means leading. Man is translated from the Greek word *aner*. Thus, andragogy is the art and science of helping adults learn (Knowles, 1970, p. 38). Andragogy is based on six critical assumptions accepted as truths in relation to adult learners. A re-crafted version of Taylor and Kroth's (2009, p. 46), interpretation of these assumptions is as follows:

1. Self-concept. As children, people are often directed in terms of what they should do, when and why—and thus dependent on others. As adults, our self-concept changes to one of independence. Adults avoid or resist situations in which decisions are made for them.

2. Experience. Adults have accumulated a wealth of experiences which not only guide our decision-making but also become a resource from which we draw as we learn. In fact, it is argued by Knowles (1970) prior experience is our richest resource.
3. Readiness to learn. Adults' readiness to learn is directly related to the roles adults play in their lives. There is a direct correlation between adults' readiness to learn and the perceived relevancy of the information.
4. Orientation to learn. Children often are placated with the knowledge that what they are learning will be applicable some day. Adults, however, determine the immediate application of the information in efforts to problem-solve current life events. If an adult does not perceive the information will be immediately applicable, then the adult loses interest and their focus is directed elsewhere.
5. Motivation to learn. Adults are primarily driven to learn by internal factors: goals, self-esteem, increased knowledge/understanding, etc.
6. The need to know. Adults must understand and appreciate the need to have the information. When adults believe the information valuable, they take ownership of learning and of the information.

A later model created by Knowles et al. (2011) identified three dimensions of andragogy. The first dimension includes the goals and purposes for learning. The second dimension incorporates individual and situational differences. The third dimension is the foundation for andragogy – the original six core learning principles: a learner's need to

know, self-concept of the learner, prior experience of the learner, readiness to learn, orientation to learning, and motivation to learn (Knowles et al., 2011).

The first-dimension (Knowles et al., 2011) addresses whether the participants' reason for learning is for the good of society – societal; to promote productivity – institutional; or for personal growth - individual (Knowles et al., 2011). A common perspective among researchers and practitioners is to consider an individual's growth. Institutional growth refers to an organization's efforts to improve, a common goal fostered by a common development experience for all. There are also societal based motivations, such as becoming knowledgeable about a teaching process/strategy. Examples of this dimension are lesson study, project-based learning, inquiry-based lessons, or constructivism. Although andragogy addresses a framework for an individual adult's learning, it acknowledges that learning happens in varied contexts. "Individual learning may occur for the purpose of advancing individual, institution, or societal growth" (Knowles et al., 2011, p. 150).

The second dimension, individual and situational differences, considers subject-matter differences, situational differences, and individual differences (Knowles et al., 2011). Subject-matter differences refer to the idea that different content requires the use of different learning strategies, and thusdifferent instructional strategies. The determining factors may be the nature of the content itself, or whether the content is new for the learner. Situational differences include learning in a familiar environment versus a new environment, or an independent learning environment versus a group learning environment. These settings also have implications for learning and instructional

strategies. Individual differences account for how the knowledge and experiences of each individual affect the individual's learning. Individual differences may also include cognitive, personality and prior knowledge differences, such as those noted by Jonassen and Grabowski (1993, as cited by Knowles et al., 2011). Although andragogy first emerged to differentiate adult learners from other types of learners, it must now be "further tailored to fit the uniqueness among adults" (Knowles et al., 2011, p. 152).

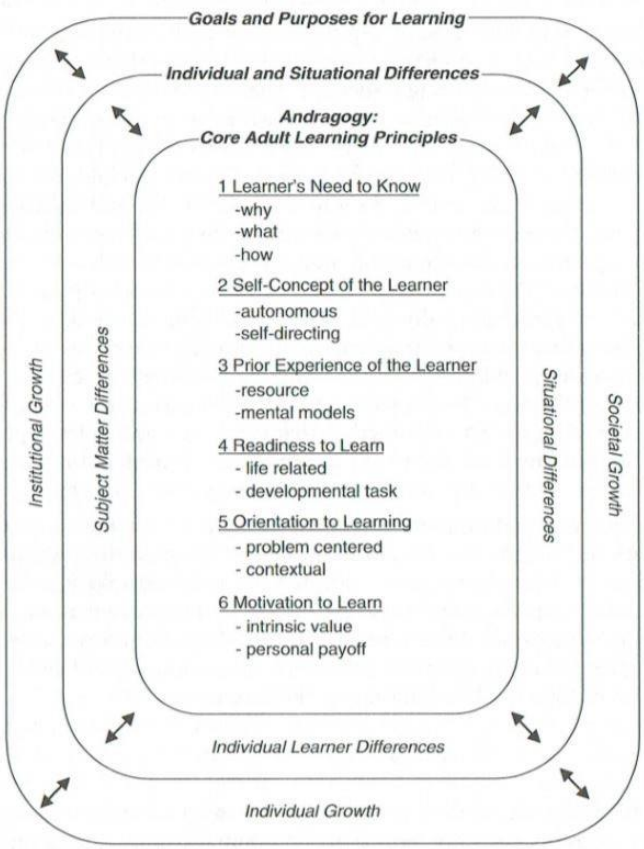

Figure 2. Andragogy in practice. Knowles, M.S., Holton, E.F., & Swanson, R.A. (2011). *The adult learner: The definitive classic in adult education and human resource development* (7th ed.). Burlington, MA: Elsevier, Inc.

The final dimension, the core adult learning principles, have not changed much since their inception. The six core principles are: (1) learner's need to know, (2) self-concept of the learner, (3) prior experience of the learner, 4) readiness to learn, 5) orientation to learning, and 6) motivation to learn. Each of these has specific attributes as noted in Figure 2.

An implication of andragogy, specifically because a learner's experiences ground their beliefs and perspectives of mathematics and teaching mathematics to an unmalleable state, is that individual, institutional and societal calls for change can be difficult. Change is a process which often entails experiences which create cognitive dissonance, causing the adult learner to question their beliefs or perspective.

> [Teacher change] requires investments in those things that allow teachers, as change-agents, to grapple with transformations of ideas and behavior: time for learning about, looking at, discussing, struggling with, trying out, constructing, and reconstructing new ways of thinking and teaching. (Darling-Hammond, 1990, p. 240).

And, a change in, "teachers' knowledge, beliefs, and instructional strategies is necessary for professional growth" (Sowder, 2007, p. 161). Therefore, professional development aimed to promote professional growth must provide opportunities for teachers which challenge their beliefs and perspectives, while offering support and understanding from an andragogical perspective.

Professional Development

The purpose of staff development is to ensure that teachers' content and pedagogical knowledge are improved and translated to practice (Desimone et al., 2002), with an ultimate goal of improving students' mathematical understanding and development. Relevancy to the teacher is the primary impetus which makes even the chance of transference to the classroom and improvement in student performance a reality (Battey & Franke, 2008). These ideas relate directly to Knowles (2011) core principles. Relevance in this paper refers to anything related to the day-to-day constructs with which the teacher must function: curriculum, pedagogy, time, school and division goals, support, and implications for student achievement. All these pieces from teacher knowledge to educational environment ultimately depend on teacher affect, a key component of which is productive disposition.

The professional development of teachers in education has long been of concern to both researchers and practitioners. In an era of accountability and reform the issues and pressures surrounding effective professional development are even greater. The issue of accountability, including high-stakes testing, has put in writing what educators have always known: they are responsible for ensuring every student meets a standard. Educators must reach more students and a more diverse population of students. Effective professional development ultimately promotes "improvements in teachers' knowledge and instructional practice, as well as improved student learning outcomes" (Wei et al., 2009, p. 2). There has been a lot of research done and theories put forward about professional development—the forms it may take, critical components, some research on sustainability, and various paradigms/theories through which to frame/view professional

development. Most importantly, the goal of professional development must be to ensure a relevancy for teachers which incorporates pedagogy for teachers, the "sense-making" of mathematics for teachers and students, and efficacy, which translates to productive disposition for the teacher as a learner and a teacher.

Prior to the current century, there was little research regarding how or what caused professional development to translate into practice (be effective) according to Wilson and Berne (1999). However, Stein, Silver, and Smith (1999) identified emerging and more powerful structures for professional development which were different from the ineffective one-day opportunity (Wei et al., 2009). This section will highlight key strategy clusters of professional development identified by Loucks-Horsley et al. (2010). Two of the strategy clusters will be presented as critical components of professional development: increasing teachers' mathematical content knowledge and teachers' pedagogical content knowledge. The third cluster will be incorporated within the first cluster as the teacher's knowledge of the alignment and implementation of the written curriculum is inherent in the teacher's content knowledge. Next the section will explore some of the inherent concerns and critical issues in professional development. Various professional contexts or structures, which is the fourth strategy cluster identified by the authors, will be introduced and accompanied by research and literature that helps identify how the knowledge issues and critical concerns are currently being addressed.

Mathematical and pedagogical content knowledge. While there are a variety of components that researchers cite as critical to producing effective staff development, two of the most commonly mentioned components/goals are increasing teachers'

mathematical content knowledge and pedagogical content knowledge (Loucks-Horsley et al., 2010; Mewborn, 2003; National Council of Supervisors of Mathematics, 2009; Reeves, 2010; Wei et al., 2009). Teachers' content knowledge, in this case, refers to teachers' knowledge of mathematics as well as of the mathematics (or curriculum) they teach. Pedagogical content knowledge refers to understanding how students learn and make sense of mathematics and how that informs using purposeful instructional strategies, and of course, having some proficiency as a teacher with those strategies.

Teachers of secondary mathematics, specifically at the high school level, are often assumed to have the content knowledge necessary to teach mathematics simply because they have a mathematics degree. The importance of a teacher's content knowledge is evident in everything from states' requirements for teachers to pass content-based assessments before they are awarded a professional license to the ever-growing popularity of programs that award K – 8 teachers advanced specialist degrees in mathematics. And yet, research has shown that teachers do not grasp the conceptual underpinnings of mathematics (Mewborn, 2003). Adding to the complexity of this grave concern is that content knowledge is not just about what teachers know, whether conceptually or procedurally. Loucks-Horsley et al. (2010) discussed in various ways that teachers of mathematics must have a deep understanding of the content, of the ways in which students come to understand the content, and the alternate and often incorrect conceptions students have or assumptions they make about the content and why.

Acquiring content knowledge in this sense means that teachers may first need to relearn mathematics. Specifically, they must learn mathematics in a way that develops

conceptual understanding while grounded in their own efforts as they struggle with rich mathematical tasks (Wei et al., 2009). Teachers need to learn to "argue" mathematics and develop ideas about what constitutes proof. They need to have opportunities to build on prior knowledge, recognize patterns and relationships and how to ask the next best question to deepen their understanding of mathematics. In other words, "teachers need to experience learning the way they will implement it in the classroom and experiment with new behavior and gain new understandings" (Loucks-Horsley et al., 2010, p. 43).

It is at this point of working with the mathematics in rich contextual constructs that teachers will inevitably come to moments of cognitive dissonance and begin to truly grow in their own mathematical understanding. It is also critical at this time that the facilitator of the professional development models instruction for the teacher as a learner that can be implemented in the classroom. Teachers need to have experiences as learners that model how they can work with their students (Loucks-Horsley et al., 2010).

Teachers will also develop content knowledge by studying student work to identify patterns in student thinking—those thoughts that are mathematically sound and those that are not (Kazemi & Franke, 2004). Given their own experiences with the same mathematical content, teachers will reflect on how misconceptions develop. Due to purposeful facilitation teachers are able to identify instructional strategies to combat those misconceptions. In other words, teachers identify instructional strategies to add to their repertoire that coincides with their developing understanding of mathematics. Inherent in this repertoire are ideas about new forms of assessment, ways in which teachers can determine what students really understand. Without this broad conception of

content knowledge, teachers' pedagogical content knowledge is limited (Loucks-Horsley et al., 2010).

Pedagogical content knowledge, defined as teachers' awareness of how pieces of content knowledge interact with one another and how they can influence purposeful and effective instruction, is as important as mathematical content knowledge (Shulman, 1986). The National Council of Supervisors of Mathematics (NCSM; 2008) discusses this in its fourth principle in leadership: assessment leadership. In terms of classroom practice, NCSM states that leaders (teachers) must use formative assessments to inform instruction and student learning. Teachers must develop active listening skills that enable them to make sense of students' knowledge base and mathematical reasoning, and teachers must have the ability to analyze students' ideas. As aforementioned, the ability to enact this knowledge is most influenced by participating in professional development in which the teacher experiences this as a learner. Additionally, they must have the opportunity to develop these skills within professional development and receive constructive feedback from facilitators and colleagues (Mewborn, 2003). All these experiences in which the teacher becomes the student lend themselves to another critical attribute of staff development: the social aspect of learning or collaboration.

A safe and collegial environment provides much more opportunity for teachers to dialogue, take risks, reflect on student work, implement teaching practices which lead to precise mathematical and pedagogical language, and to identify sound teaching practices (Loucks-Horsley et al., 2010; Wei et al., 2009). While this may not necessarily change teachers' practices, it is more likely to challenge their status quo thinking by creating

further dissonance in both their mathematical and pedagogical content knowledge. And without a reason to believe that what they are doing could be done better or some uneasiness, practice is unlikely to change. However, there are other aspects of professional development which may inhibit a teacher's growth/development.

Historical concerns. Research which helps identify concerns about professional development is prevalent. There are four primary concerns which repeatedly are present in the literature. The first, and likely most prevalent problem is the disconnect teachers often experience between the setting of professional development and their practice. This is a primary reason that staff development efforts do not' translate to implementation in the classroom argued Cobb et al. (2003). This disconnect is amplified by simple environment; teachers complete the staff development in an environment completely different from where they practice. Cobb et al. stated the best location for staff development is within an/the educational setting. The researchers concluded efforts to translate professional development to practice worked best when the staff development occurred in the teachers' educational setting; the professional development had to navigate many of the same constraints as part of the educational process that the teacher does.

Second, despite efforts to promote a change in the structure of professional development, Wei et al. (2009) stated at least 90% of teachers reported attending a variety of professional learning opportunities. However, most of these opportunities were still isolated conferences or workshops, with less than 16 hr of contact time. Professional development must take place over a prolonged period, either by design or because there

are opportunities for the participants to follow-up through collaboration, observation, or analysis. For the professional development ideas to be sustained and impact teachers' practice, teachers must have the opportunity to continue the dialogue and reflect on various aspects of the educational environment and student thinking. This may happen with the support of the professional development team, a school-based teacher team, or their local administration.

Third, there are also pedagogical and subject content hurdles. Teachers must be given the opportunity to increase both their pedagogical content and subject content knowledge related to the concepts which they teach. Battey and Franke (2008) noted the factors which impact teachers' choice regarding which staff development opportunities would translate into practice require staff development be restructured. "We must work to provide opportunities for teachers to work together . . . in ways that allow teachers to make sense of their knowledge, skills, and identities in relation to norms in both professional development and classroom practice" (p. 147). The professional development opportunities were primarily subject-based, and there was no follow through or support once the contact time ended. The report also made it clear there has been little change in the content of professional development.

Professional development must align with school/division/regional reform efforts or initiatives (Garet, Porter, Desimon, Birman, & Yoon, 2001; Penuel, Fishman, Yamaguchi, & Gallagher, 2007; Supovitz, Mayer, & Kahle, 2000). In order to increase relevancy and therefore change, teachers must make connections between what they learn in professional development and what is expected of them in their schools or divisions.

Contextual concerns. There are several inherent concerns regarding the consistency and integrity with which staff development practices are enacted in the classroom. This section will briefly discuss a few of those most cited in the literature: support from administrative leaders, the development of teacher leaders, collaboration, and time, all of which lend themselves to the over-riding concern of sustainability. These also align with Knowles' et al. (2011) outer two dimensions of andragogy.

Support from administrative leaders. The support for the professional development may come, or not come, from a variety of places. Some of these include support of the teacher's supervising administrator/leader, division/regional specialists and colleagues. Of these, the support of the supervising administrator/leader has proven to be one of the most influential in the enactment of professional development practices (Heller, 2004; Loucks-Horsley et al., 2010). A quick reflection provides enough evidence regarding the validity of this concern.

Administrative leaders who are school based can ensure that teachers have the resources needed to be successful in enacting professional development. They can schedule time in the day to ensure teachers have time to look at student work, collaborate, plan, and design purposeful instruction. Administrators also have control over financial resources that pay for the materials and tools needed by teachers to aid instruction. The financial resources can also be used to develop school-based leadership capacity specifically in mathematics education. Administrators can demonstrate support by attending professional development with the teachers or by helping to educate and communicate with parents regarding effective mathematics instruction. Ultimately,

leaders hold the teachers accountable through supervision and evaluation practices and so, have a critical role in providing effective and growth-producing feedback to teachers in line with the goals of the professional development (Reeves, 2010).

Division/Regional specialists also play a critical role in the enactment of knowledge gained in professional development. These specialists serve as resources with whom teachers can follow up with questions, seek further guidance, or ask for observational feedback. Specialists can observe teachers and help teachers analyze student work and help them engage in reflective practices. Assuming the specialists are in part responsible for the facilitation of the original staff development they can offer specific guidance and insights related to the intended outcomes of the staff development on classroom practices. Finally, mathematics specialists are content experts, or should be, both in mathematical and pedagogical content. They therefore have a responsibility to offer insights to teachers and to help them continue to develop, and more importantly to help develop school-based teacher leaders.

Development of teacher leaders. "Sustaining change means sustaining leadership and spreading it widely throughout the system" (in Loucks-Horsley et al., 2010, p. 40). Districts must utilize central office staff personnel and principals to develop site-based teacher leaders. These leaders serve as true content experts and are more readily available to teachers. They can better perform two crucial tasks: provide more coaching in the classroom and support job-embedded learning for all of the teachers with whom they work. Ensuring the knowledge acquired in staff development transfers to the context of

teachers' jobs in the classroom is critical to claiming effective staff development has occurred (Loucks-Horsley et al., 2010; Mewborn, 2010; Wei et al., 2009).

Collegial collaboration. A third, and integral part of support for teachers, is that of collaboration with colleagues, which is situated in the teacher's instructional setting (Cobb et al., 2003; Gilles, Willis, & Elias, 2010; Knight, 2002; Loucks-Horsley et al., 2010; Mewborn, 2003; Wei et al., 2009). The opportunities to both attend the professional development and work collaboratively after the training with colleagues is crucial in enabling the teacher to implement strategies and to continue to develop knowledge from the staff development. Colleagues serve as each other's eyes, ears and sounding board. Just as students have different learning trajectories and make sense of mathematics in a variety of ways, teachers also have different learning trajectories and make sense of the two knowledge constructs in a variety of ways. When teachers share these perspectives with each other, they perpetuate their own continued growth and development. "As research deepens our understanding of how teachers learn, many scholars have begun to place greater emphasis on job-embedded and collaborative teacher learning" (Wei et al., 2009, p. 8). However, participating in structured staff development, observing each other, and collaborating all take time.

Time. To affect teachers' instructional practices, it is not just the aforementioned aspects that are time-intensive. Systemically changing practice requires that staff development take place over time (Fullan, 2001; Loucks-Horsley et al., 2010; Reeves, 2010; Wei et al., 2009). Research and work related to staff development and the change process demonstrates that it can take 3- 5 years for systemic change to take place. Any

new innovation, even exposure to research-based strategies will face resistance, go through an implementation dip (Fullan, 2001) and not become fully implemented until either the teachers see a need for change or see that changes are increasing student achievement. For staff development to have this kind of longevity, it must have all of the leadership supports mentioned above, and the resources (including financial) to support the efforts and a commitment to the purpose of the efforts by all stakeholders so that they see the value in carrying on. In other words, it must have all of the attributes necessary to be sustainable.

Sustainability. Without all the components delineated above, the sustainability of staff development efforts would not be possible. Sustainability, according to Fullan (2001) requires systemic thinking implemented in practice. It requires full leadership supports throughout a division and ownership on the part of the teachers. Loucks-Horsley et al. (2010) also argued that for staff development initiatives to be sustainable they must be systemic in the philosophical alignment with other initiatives.

Fogleman, Fishman and Krajcik (2006) worked with the Detroit public school system to create systemic leadership that would create staff development opportunities for teachers in line with the district's philosophies. They met with about 80 identified teachers each month for 14 months in work circles. Specific concerns were identified by the lead teachers and participating teachers and then lessons or instructional strategies were developed by the lead teachers to address these concerns. The authors found that while it was an effective form of professional development overall, the participating teachers often relied on the work created by the lead teachers as "the answer." In other

words, they did not consult with the lead teachers nor tweak the ideas/lessons on their own to meet the needs of their own students and therefore did not continue to grow as professionals themselves, thus ending its sustainability at a collegial and individual level.

Franke, Carpenter, Fennema, Ansell and Behrend (1998) conducted a qualitative study over the course of 4 years with three teachers using Cognitively Guided Instruction (CGI) staff development practices to determine if self-sustaining generative change would occur. The CGI staff development sessions focused on children's mathematical thinking to promote teachers' conversations about inquiry and their understanding of students' thinking. It was hypothesized that this would promote sustainable change in the teachers' practices. They found that while the three teachers had very different learning and implementation trajectories, CGI did in fact change the teachers' instructional practices and lead to self-sustaining growth. However, they questioned how sustainability is gained without time-intensive staff development or in staff development with a completely different structure.

Sustainability of any professional development initiative is obviously context-embedded and time dependent. Therefore, each medium through which staff development is delivered offers its own challenges and questions related to sustainability. Complicating matters is that while staff development can and should be socially situated, a teacher's learning and implementation in practice, as confirmed by Franke et al. (1998), is very individualized. The implication for a large division or region as it strives to meet and address the needs of hundreds of teachers in the area of mathematics is that the professional development will likely be very complex. The selection of the staff

development structure will greatly impact the effectiveness of the staff development and therefore how it is implemented in the classroom.

Professional development structures. Professional development is not a one-size-fits all scenario. The structure picked will depend on the purpose and goals of the staff development. While there is not one right structure, there are some that may not be appropriate based on the goals of the professional development, and there is one structure which has proven primarily ineffective regardless of the goals. The ineffective structure is the one-day workshop and so, will not be addressed in this review. Three structures will receive attention: workshop/institute professional development, online staff development and school-embedded staff development. The first two fall into the category of formal professional development opportunities and the last into job-embedded professional development as defined by Wei et al. (2009).

Formal professional development opportunities. Formal professional development opportunities as defined by Wei et al. (2009) include workshops, conferences, and training during or outside of school contract hr. The authors reported that 92% of teachers participated in workshops, conferences or other training sessions in 2003-2004. In general, these opportunities did not meet the criteria identified in the research as best practice; they fell short both in rigor and in duration.

Workshop/institute professional development opportunities. Workshop or institute professional development opportunities may constitute a week or more of training, usually over the summer, in which teachers explore mathematical and pedagogical content knowledge. This training may or may not then lead to a form of

follow-up by the facilitators. In some cases, site-based teachers are left to their own devices regarding implementation and follow through. In other cases, facilitators or mathematics education leaders conduct follow-up meetings, on-going collaboration, or workshops with teachers.

Friel (1996 cited in Loucks-Horsley et al., 2010) conducted a study focused on work with teachers in Grades 1- 6 to help teachers better understand statistics and how to teach statistical concepts. The project involved nine of the ten Mathematics and Science Education Network centers in North Carolina. The first year of the project was spent planning the staff development. During the first year of implementation, 57 teachers attended a 3-week summer institute. The facilitators spent the following year visiting each regional teacher team exploring the teaching of statistics and how to integrate statistics across the curriculums. In the second year, seven of the teachers from the first year helped plan and implement the staff development for 24 new teachers each. They were able to share their experiences, were initiated into the world of facilitating staff development and developed themselves as coaches and mentors. In the third year, 84 of the teachers involved were selected to serve as statistics educators throughout the state and provide professional development for other teachers. Before facilitating staff development, they participated in a 1-week institute about staff development. Additionally, the site educators at the nine centers each held a 2-week institute for 24 new teachers. What they found was that professional development that modeled exciting learning environments translated to classrooms and helped to create more teacher leaders.

Hill and Loewenberg-Ball (2004) wrote an article about California's Mathematics Professional Development Institutes (MPDI). Their goal was to determine if teachers' mathematical knowledge for teaching increased because of MPDI. For the article, mathematical knowledge for teaching was defined as both content knowledge and pedagogical content knowledge. The study included several hundred teachers, and thus gave them an opportunity to pilot content knowledge test items on a large scale. Teachers attended summer institutes of between 40-120 hr taught by mathematicians and mathematics educators. The teachers also participated in up to 80 hr of staff development during the following school year. The authors' goal was to research knowledge gained by 2300 teachers that participated in a 2001 workshop, although the MPDI program served over 23,000 teachers over its 3-year implementation. They were only able to gather pre- and post-assessment data on 398 of the participants. The authors found statistically significant gains in teacher content knowledge. However, because of a variety of variables (e.g. reliability, validity) there was no control group and that data was collected on only 2 points in time. The researchers stated that their findings were not definitive and pointed out that a benefit of this approach was that the effectiveness of the program was not dependent on teachers' perceptions, but rather empirical data. While this study did not cover classroom implications, the authors were satisfied that it addressed knowledge needed to teach mathematics and teachers' growth in that knowledge (Hill & Loewenberg-Ball, 2004).

Online staff development. The exponential increase of technological advances has not only contributed to the use of technology in classrooms but the use of technology

as a delivery system of staff development for teachers. New technological capabilities have enabled teachers to do everything from participate in webinars set up by various education related agencies to enroll in online courses individually. Technology has also given divisions and area education regencies the opportunity to request that specific staff development sessions be created to meet the specific needs of their teachers, immediate and poignant staff development that can be saved and easily maintained for reuse (M. Lynch, personal communication, February 13, 2010).

Chen, Chen and Tsai (2009) conducted a research study to determine if the use of synchronous online discussion served as an effective web-based professional development tool. Six discussion strings were transcribed and analyzed, and interviews were conducted with 10 of the 61 participating teachers. While the authors did find that this system served well as a learning tool and was used for support purposes and to connect socially, it was determined that this system was not effective for some participating teachers. Additionally, the authors state that the online discussion format had little advantage over face-to-face discussions.

In 2009, Koc, Peker and Osmanoglu reported positive results when they used case study videos as a tool for promoting content and pedagogical discussions as a model for online professional development. The authors studied 26 teachers including pre-service teachers, in-service teachers and the veteran teacher, who was video-taped. The results of their study indicated that teachers were able to make connections between theory and practice. The authors pointed out that the short duration of the staff development, the small sample size, and the fact that the video-taped teacher was one of the researchers

may have created some bias, limiting them from making a more substantial argument for the effectiveness of this form of professional development.

Russell, Kleiman, Carey and Douglas (2009) compared a self-paced and cohort based online course for middle school algebra teachers. The self-paced group did not have any additional supports, but the online group had an online instructor, an online facilitator and the ability to communicate synchronously with classmates. The researchers found no significant differences between the two groups in the areas of teachers' mathematical understanding, pedagogical beliefs or instructional practices based on the pre- and post-test data. They did note, however, that all of the teachers self-selected to take the course online, which may limit generalizability. Additionally, the results reported were derived from half of the number of original participants (231) due to attrition or failure to take the pre- or post-test.

Researchers Herrington, Herrington, Hoban and Reid (2009) sought to determine if online professional development transferred to teachers' classroom practice. They had 170 teachers participate in a self-paced modular staff development program that was designed to increase teachers' skills in using information technology applications in their classrooms (Herrington et al., 2009). The researchers used a constant comparative method to determine major themes in teacher and classroom observations, teacher and student interviews and additional artifacts. There were mixed results. The quality and usefulness of the modules themselves was an issue for some teachers. In some cases, there were either technological problems (with the module, school computer system, or teacher's access to technology), or the teachers did not have the time to get comfortable

enough with the technology to use it in class. In cases where more than one teacher in a building was participating in the program, it was more likely to be learned, used, and found to be of value. Many teachers believed the acquisition of new computer/technological skills was a positive outcome in terms of their professional development. There was evidence that some teachers incorporated new content and strategies based on the modules and that their own learning trajectory was stimulated. The authors concluded that when site-based resources supported the incorporation of technology, the benefits to student learning were profound. In these cases, teachers were more likely to be determined to implement what they had learned. In the cases where the environment was not conducive to implementation, teachers gave up and reported the experience to be a waste of time.

School embedded staff development. In 2003-2004, teacher responses indicated that in relation to job-embedded professional development, 39.8% had participated in individual or collaborative research, 70.4% regularly collaborated with teachers regarding instruction, 63% participated in peer observations, and 45.7% served as a mentor or coach (Wei et al., 2009, p. 40). Regular collaboration and peer observations may have been the most common because they are completed with releative ease due to the availability of colleagues and aligned schedules. These results may also be due to the emerging constructs in the United States, which are referred to as professional learning communities and lesson study.

DuFour and Eaker (1998), who built on the work of Lave and Wagner (1991), provided guidance on developing productive professional learning communities. DuFour

and Faker described a professional learning community as a teaching environment in which teachers (or schools or divisions) have a shared vision, inquiry promotes growth, educators work in collaborative teams, educators are action-oriented, educators strive for continuous improvement (for the sake of educators and students), and success is based on results not intentions. Knight (2002) made a similar argument stating that professional development policies and practices should continue to expect learning communities to build a collective pedagogical repertoire, purposefully choosing from the repertoire to meet the learning needs of the students and auditing. He defined auditing as the discussion teachers need to have about which teacher covers a particular concept at a specific time, and why. For example, the teacher who purposefully chooses to go over the new geometry vocabulary before diving into the related unit verses the teacher who dives in to the content covering vocabulary as the vocabulary is needed. Teachers need to be reflective about these practices to determine their effects on students' understanding of the content and subsequent performance. Related to these constructs, Cobb et al. (2003) tried to bridge the perceived dichotomy between educational constructs (hierarchies, structures, etc.) and the purpose of systemic professional development. Their argument was contextualized in the school and district as one large professional learning community.

Lesson study is another prominent form of job-embedded professional development. In lesson study, an algebra teacher team, for example, designs a lesson with student learning goals in mind. One of the teachers implements the lesson, and the other team members observe the lesson, noting student conversation, modeling, and

questioning. After the lesson, the teachers discuss the lesson from the perspective of the students' learning/sense-making. The lesson may then be modified or changed completely and is then taught by another member of the group while the others observe, again taking notes, which will further guide instructional choices. This cycle may go through several iterations as determined by the lesson study group. This process is ongoing with the goals of increasing mathematical content knowledge and pedagogical content knowledge, creating opportunities to observe students' mathematical reasoning, reaping the benefits of collegial networks, identifying the connections between daily practice and long-term goals, increasing motivation and sense of efficacy in students, and creating and making available quality lesson plans (Lewis, Perry & Hurd, 2004).

In 2002, Fernandez published an article describing lesson study and cited a study that identified the challenges of implementing lesson study in the United States. The findings were based on two lesson study groups, one conducted with 14 K- 8 teachers in a diverse urban district and the second with 19 middle school lead teachers and staff developers from a community school district in New York City. The researchers found that teachers had great difficulty attending to children's mathematical thinking. Researchers did find that emerging teacher leaders demonstrated growth in this area and concluded attending to students' mathematical thinking can be learned. They specifically suggested that professional development focus on attending to, interpreting and deciding how to respond to students' thoughts.

Lewis, Perry, Hurd and O'Connell (2006) wrote an article about the development of lesson study at Highlands Elementary School (400-500 K-5 students) in California.

They initially conducted two lesson study cycles in 2000-2001 with a small group of teachers and upon the group's presentation to the faculty that spring, more teachers asked to participate. In 2006 they were in their sixth year of implementation and all faculty members were participating. Each lesson study group consisted of three to six teachers and each group shared information with the faculty at regular intervals. For each lesson study cycle a common research theme was chosen as a focus, such as closing the achievement gap, so that there was a common dialogue among lesson study groups. During its implementation at Highlands, lesson study went through four primary changes that took faculty members from the superficial aspects of lesson study to those that change practice:

> Lesson study is about teacher learning, not just about lessons; effective lesson study hinges on skillful observations and subsequent discussions; lesson study is enhanced by turning to outside sources of knowledge; and the phases of the lesson study cycle are balanced and integrated. (Lewis et al., 2006, pp. 274-275).

At the end of the staff-wide implementation, Highlands had more job-embedded opportunities for teachers' professional development, and student achievement had improved.

Based on various research, there are several aspects of professional development which must be carefully considered prior to implementation. And yet, careful consideration and attention to these components may still not lead to a teacher implementing change where needed; teachers need to believe a change on their part will yield positive outcomes for students. The interactions and "order" of these components –

professional development, teacher change, student learning outcomes, and teachers' affect has also been a focus of research.

Teacher change and teacher beliefs and attitudes

Studies conducted in the late 1980s and early 1990s noted related changes which needed to occur as teachers shifted to a constructivist or developmental view of students' learning of mathematics. The four changes identified by Carpenter, Fennema, Peterson, and Carey (1988) and Wood, Cobb, and Yackel (1991), were (1) seeing students as learners who can pose questions, explore solutions, and construct knowledge; (2) realizing instruction could be based on students' thinking and developmental progression; (3) recognizing class discourse as an intellectual authority; and (4) realizing reasoning efforts of the students and the teacher can produce and confirm mathematical knowledge (Nelson, 1997). Although the changes in belief were documented, Nelson pointed out the experiences needed and the relationship between the changes in teachers' beliefs and practices was undetermined.

About this same time, Guskey (1986) proposed a model identifying a sequence or process connecting professional development to teachers' beliefs and attitudes. In the model, Guskey conveyed that a change in a teacher's beliefs and attitudes is an end result. In other words, the staff development promotes a change in the teacher's practices. Once those changes are implemented, and the teacher notes a change in the students' performance, the result will be a change in the teacher's beliefs and attitudes. In other words, the staff development promotes a change in the teacher's practices. Once those changes are implemented, and the teacher notes a change in the students' performance,

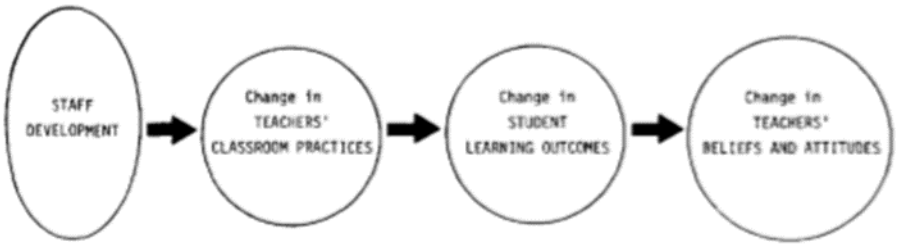

Figure 3. Guskey's model of the process of teacher change. Guskey, T. R. (1986). Staff development and the process of teacher change. *Educational Researcher, 15(5)*, p. 7. doi:10.2307/1174780.

the result will be a change in the teacher's beliefs and attitudes. Guskey noted the ideas must be further researched given the complexity of teacher change. He also stated that being attentive to the order could have implications for the impact of staff development on a teacher's change and the sustainability of that change (Guskey, 1986). Not all researchers model these ideas as such a linear progression, with beliefs and attitudes the end-result.

Clarke and Hollingsworth (2002) proposed a model representing a more interconnected relationship using similar components. They also accounted for various domains (e.g., professional development, classroom practice, and outcomes), which may impact beliefs and attitudes directly. The model indicates there is a direct interconnected relationship between professional development and aspects of a teacher's personal domain, which includes beliefs and attitudes. Based on this model, these attributes can,

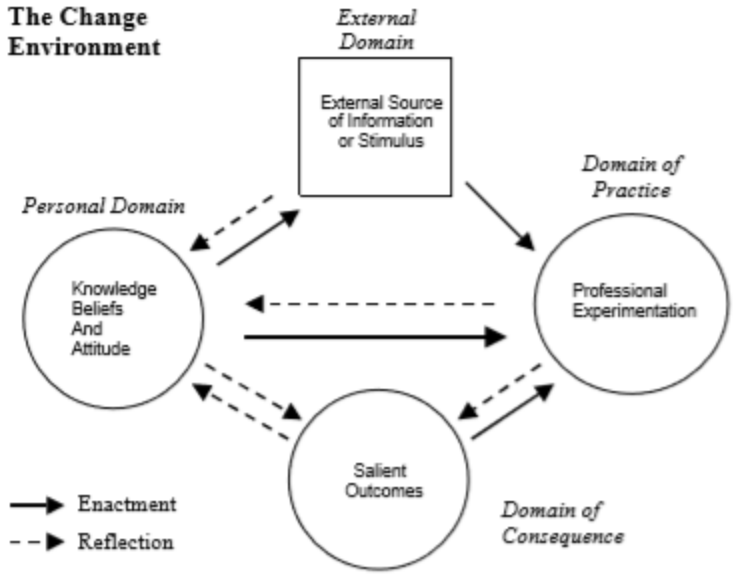

Figure 4. The interconnected model of professional growth. Clarke, D., & Hollingsworth, H. (2002). Elaborating a model of teacher professional growth. *Teaching and Teacher Education, 18,* p. 951. doi:10.1016/s0742-051x(02)00053-7.

in turn, influence classroom practice. The model allows for teachers' attitudes and beliefs to both influence, and be influenced by, other critical factors; it clearly represents a construct in which the personal domain is not the end result, but part of a fluid and interconnected structure that strives to explain teacher growth. Adhering to this model means accepting that teacher affect, and for the purpose of this study, productive disposition specifically, is a critical component of professional growth. A teacher's professional growth can easily be swayed, both positively and negatively, due to various components which contribute to the potential for teacher change. A key component of this interconnected web is productive disposition.

Productive Disposition

Productive disposition is a key component of the National Council of Teachers of Mathematics' (NCTM) process standards. The idea has been highlighted and explored by several researchers. The concept of productive disposition and its meaning have gone through several iterations. Productive disposition is also often considered to be part of a larger construct, such as affect. As another specific example, Lewis et al. (2015) noted proclivities for teaching mathematics, which constitute a form of teacher reasoning. Among these proclivities are four domains, one of which is productive disposition. A historical development of the definition of productive disposition is necessary to best understand what productive disposition encompasses.

Development of a definition. Disposition was initially defined and researched in terms of a students' disposition. Mathematics disposition of students was defined by the NCTM as "not simply attitudes but a tendency to think and to act in positive ways" in the *Curriculum and Evaluation Standards for School Mathematics* (1989, p. 233). This definition does not specifically speak to positivity as it relates to math so is open to interpretation. Transferring this definition to the teaching of mathematics was not done, nor would it have been useful.

In 1997, McIntosh defined productive disposition as the students "usual mood; temperament, a habitual inclination, tendency" (p. 93). McIntosh identified various components of productive disposition including: attitudes, beliefs, confidence, cooperative skills, confidence, and the rejection of mathematics stereotypes. This definition still left much to be interpreted. This definition was also not translated nor were parallels drawn to teaching mathematics.

Fostering a Good Mathematical Disposition was published by NCTM in 1999. The organization's then president posed a series of questions which would prove important in determining a student's disposition. The questions addressed a student's willingness to persevere when presented with more challenging tasks. About this same time, Royster, et al. (1999), using previous definitions, noted confidence, interest, and perseverance as specific indicators of a student's disposition. Although with these ideas the parallels to teaching were becoming clearer, there were still not any direct correlations made.

Much more headway was made regarding an operational definition of productive disposition and specific parallels drawn between learning mathematics and teaching mathematics. McClain and Cobb (2001) noted that a critical component of a productive disposition in mathematics was being able to identify mathematically reasonable and persuasive arguments. Another definition came out of a model for thinking about mathematical proficiency of students – attributes needed to develop a quality math student. Kirkpatrick et al. (2001) identified five strands as critical components of a student's mathematical proficiency development. The first four of these strands – conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning – can be summarized as content knowledge and procedural skill. The fifth strand identified by the NRC was productive disposition. The authors defined productive disposition as a "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence in one's own efficacy" (2001, p. 116). The authors used the strand/string representation to further clarify their point that these

attributes must be woven together, and not considered in isolation, to be most effective. Although the authors' intent was to shed light on students' mathematical development, the authors also made a definitive comparison to the teaching of mathematics, noting teaching for mathematical proficiency requires similar interwoven components. They delineated that a teacher must have a, "productive disposition toward mathematics, teaching, learning, and the improvement of practice" (Kirkpatrick et al., 2001, p. 380). These insights and definitions served to further inform teaching organizations regarding teacher quality and attributes.

Freeman (2007) noted that through about 2002 teacher quality was defined by teacher's content knowledge, pedagogical skills, and attributes. After 2002 various organizations strived to further delineate aspects of a quality teacher. The National Council for Accreditation of Teacher Educators (NCATE) stated effective teachers must possess content knowledge, pedagogical skills, and dispositions. NCATE's (2007) reference to dispositions and the definition changed in 2007. The organization switched from calling it dispositions to professional dispositions. The definition adopted by NCATE for professional disposition was the "professional attitudes, values, and beliefs demonstrated through both verbal and non-verbal behaviors as educators interact with students, families, colleagues, and communities. These positive behaviors support learning and development" (NCATE, 2007, p. 30).

A second influential organization, The Interstate New Teacher Assessment and Support Consortium (INTASC, 2008) also provided guidance regarding teacher disposition. INTASC identified core strands which include teacher knowledge (what

teachers know), skills (what teachers can do), and dispositions (what teachers should be like) to be responsible practitioners (as cited in Hampton, 2008). The organization noted that a teacher's dispositions should include believing children can learn at high levels, taking responsibility for a positive classroom climate, valuing ongoing assessment to promote student learning, establishing respectful and productive relationships with parents and guardians, and willing collaboration with colleagues to improve students' learning environment (as cited in Hampton, 2010).

Since the initial identification and conception of disposition, several researchers have worked to more clearly name, define and measure productive disposition. Terms including disposition, professional disposition, productive disposition and mathematical disposition have all been used to identify this idea. Research has also spanned a variety of fields and age groups or levels of education. In a concerted effort to inform the mathematical education community, a focused group of related research appeared in the *Journal of Research in Mathematics Education* in 2015. Among these research articles were papers by Lewis et al. (2015) and Jacobson and Kilpatrick (2015).

Lewis et al. (2015) researched proclivities for teaching mathematics. The four domains of proclivities, as defined by the authors are productive disposition, openness to new ideas, thought processes, and organization of mathematical thought. The focus of their research was productive disposition. They defined productive disposition using the definition set by the NRC (2001): Productive disposition refers to the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer

of mathematics. They used the four components identified by this definition and added a fifth: sense-making of mathematics, seeing mathematics as useful and worthwhile, persisting, self-efficacy as a learner and doer, and seeking and using resources (Lewis et al., 2015).

In this same journal, Jacobson and Kilpatrick (2015) expounded on their own definition of productive definition. This definition will be used for this study:

> Productive disposition for teaching mathematics is mathematics teachers' malleable orientation toward – and concomitant beliefs, attitudes and emotions about – their own professional growth, the subject of mathematics, and its teaching and learning that influences their own and their students' successful mathematics learning. (Jacobson & Kilpatrick, 2015, p. 402).

Productive disposition towards mathematics and the teaching of mathematics. At an early age, students see mathematics as interesting, worthwhile, and as something at which they can be successful. Therefore, students are willing to work at mathematics (NRC, 2001). Again, the authors draw a parallel to teaching. Teachers must have a productive disposition about their own mathematical knowledge, instructional practice, and learning. Teachers must see mathematics as worthwhile, believe students can be successful and that they can be successful teachers of mathematics.

Moldavan and Mullis (1998) studied autobiographies of prospective teachers. Although there were a couple exceptions, the overriding findings were that teachers' dispositions towards mathematics were directly related to their experiences as math students. Teachers who fostered perseverance, were accepting of mistakes, and promoted

creativity tended to develop students with a productive disposition towards mathematics. In contrast, teachers who seemed unwelcoming, intolerant of mistakes, and taught math as a set of rules, developed students who continued to dislike math and not view math as a worthwhile pursuit.

These ideas align with an indicator for teachers established by the Association of Mathematics Teacher Educators (AMTE). The indicator begins with a definition closely tied to Kilpatrick et al.(2015), such as sensible, useful, and worthwhile for all, and this overview relates it directly to a mindset for beginning teachers. The organization cites the National Research Council (NRC), stating that new mathematics teachers know success is dependent on a productive disposition toward mathematics and hard work (2017, p. 9).

> [Well prepared beginning teachers] believe that requisite characteristics of high-quality teaching of mathematics include a commitment to sense making in mathematical thinking, teaching, and learning and to developing habits of mind, including curiosity, imagination, inventiveness, risk-taking, and persistence. For example, when faced with a challenge to common practice or to their current understandings or beliefs, well-prepared beginning teachers have the intellectual courage and mathematical disposition to not reject the challenge but to investigate the proposed idea, applying their own critical thinking and using all available resources. (AMTE, 2017, p. 9)

Given the number of researchers and organizations which note productive disposition as a key component to effective teaching and improved student performance, it is critical to look more closely at the development of productive disposition and its maintenance.

Developing a productive disposition. Goldsmith and Schifter (1997) noted the ability for a teacher to persevere is influenced by a teacher's initial or developing motivation and disposition. They argued there is no thorough explanation regarding why teachers persevere when faced with frustration as they relearn or redefine teaching and are struggling with the ideas such as the importance of being attentive to students' thinking. The authors believed that having or developing a positive disposition is part of the explanation. They cited examples, such as the ambiguity of open-ended tasks/questions used in instruction, allowing students to struggle as a part of learning, and addressing issues with students, parents, and administrators as examples of a needed productive disposition. It is important to note that some situations may serve as a catalyst for teachers' productive dispositions and a wall for other teachers. Therefore, working to identify situations or strategies which will promote productive disposition is important.

The NRC (2001) noted that for students to attain a productive disposition and maintain it, students must see their success as a result of the work and perseverance they put in to understanding the subject and not a result of innate ability. Students who believe learning math is dependent on ability show less interest and so, often do not perform as well. Likewise, teachers must believe that mathematics, their ability to make sense of students' thinking, and the instructional practices they use are interwoven. To develop and maintain a productive disposition, teachers must also believe their ability to learn about these interwoven strands is dependent on their work and effort, and not innate abilities they or their students possess. This study will therefore use the definition presented by Jacobson and Kilpatrick's (2015):

> Productive disposition for teaching mathematics is mathematics teachers' malleable orientation toward – and concomitant beliefs, attitudes and emotions about – their own professional growth, the subject of mathematics, and its teaching and learning that influences their own and their students' successful mathematics learning. (p. 402)

Efforts to assess productive disposition. Approaches to assessing productive disposition have varied, likely because it is hard to measure (Jacobson & Kilpatrick, 2015). Researchers have attempted to measure this concept from a holistic standpoint and as a specific part of a concept based on the operational definition used in the research. Qualitative and quantitative data have been used. The AMTE (2017) made recommendations regarding the assessment of disposition. They note that effective assessments provide data on various aspects of positive disposition as it relates to teaching, including engaging in mathematics, identity as a teacher and learner, and dedication/perseverance in supporting the mathematics learning of every individual student. AMTE further suggests that assessment may start with a self-assessment but must include assessments by colleagues, particularly those who work closely with the teacher as a practitioner. Finally, they suggest that teachers' dispositions and beliefs can be further assessed based on their reflection on supporting students and their mathematical development or growth.

Charalambous (2015) reported assessing knowledge and beliefs was primarily done by assessing one of these domains in isolation. Although research on both was conducted, and thus the body of research grew for both; research which sought to

understand a relationship between the two was relatively non-existent. Recently, researchers have looked at the two simultaneously. Researchers have either explored knowledge and beliefs in parallel or examined ways to understand or explain their relationship (Charalambous, 2015). The first approach has produced primarily quantitative data collected after participating in a teacher education program, which Charalambous argued leads to many questions. Charalambous expressed concern over unanswered questions such as how the two domains interact and if one domain can compensate or hinder the other. He then argued that working at the intersection of the two domains using qualitative methods provided initial insights which helped to understand their relationship.

Charalambous (2015) considered several research efforts which explored productive disposition and how it was assessed in conjunction with knowledge. He noted two primary concerns regarding the assessments: the first was that the studies, save one, assessed the domains in practicing teachers at one moment in time. The other study explored the changes of pre-student teachers' beliefs and knowledge, but all of the data was self-reported. Charalambous then presented a multiple-case study conducted by Yin (2009) which addressed both of these concerns. Yin utilized observations in a simulated environment, a multiple-choice content test, and a PowerPoint showing various stages of a lesson on which participants were asked to comment, throughout their enrollment at a university in content and methods courses. He did note, however, that given the simulated environment, the study likely spoke more to the pre-student teachers' potential to participate in the practices studied. Charalambous concluded that productive

disposition/beliefs cannot be assessed in isolation. He cited Yin's work to argue that the relationship between knowledge and beliefs, and assessing them and their relationship is "dynamic and complex" (Charalambous, 2015, p. 443).

Jacobson and Kilpatrick (2015) argued that to assess productive disposition, there needs to be more understood about the specificity of the items being posed and the instrument level itself. They also noted that in order to address the malleable part of the definition of productive disposition, the contextual factors which help productive disposition manifest in practice must be considered. The teacher must be viewed as a teacher and a learner to assess productive disposition. "Future studies, especially those focused on practicing teachers, should expect and attend to interactions between context and nonmathematical affect, on one hand, and the relationships between mathematics-related affect and mathematics instruction and learning, on the other" (Jacobson & Kilpatrick, 2015, p. 405). The authors subscribed to Charalambous' claim that to meet these goals in assessing productive disposition, rather than use large scale measurements, common measures can be used to provide supplemental data which is acquired through ethnographic and case-study based approaches to generalize findings.

Related studies. A 2015 volume of the *Journal of Mathematics Teacher Education* focused on productive disposition. The opening article (Jacobson & Kilpatrick, 2015) focused on a definition, highlighted the need to consider the cognitive and affective (productive disposition) domains as interwoven components, made recommendations for future research, and provided an overview of the following five articles which proposed ways to, "conceptualize, measure, and apply constructs of productive disposition for

teaching through research on teacher education" (p. 404). These five articles are summarized in the following paragraphs.

In the first article, Jong and Hodges (2015) explored the development of pre-service elementary teachers' attitudes towards mathematics in relation to their experiences as learners in kindergarten through twelfth grade and within a teacher education program. The researchers used both the Rasch Rating Scale Model and parametric analysis methods on 146 pre-service teachers as they moved through their mathematics methods course and student teaching experience. The authors noted that although the participants' attitudes as learners and students were very different from their attitudes as teachers, the two domains were connected. The results of the study indicated significant changes in attitude occurred, especially for those teachers who entered with a negative attitude and experienced quality math instruction during their student teaching. The authors noted two key thoughts at the conclusion of their study. The first was whether or not the changes would be sustainable into their teaching experiences. The second was to note although the data provided evidence of a connection between math knowledge and productive disposition, more information was needed to determine more precisely the connection between the two (Jong & Hodges, 2015).

In the next article, Charalambous (2015) explored how a teacher's knowledge and beliefs are related to the decisions a teacher makes and the actions they take. He researched three prospective teachers in a large midwestern university who were participating in a 1- year master's program. He noted the most notable difference between the three was their math education/background. Charalambous' research, specifically,

was aimed at helping to determine if deficits in either the knowledge/cognitive domain and the affective/productive disposition domain can be negated by strength in the other domain. Charalambous collected math teaching knowledge assessments, surveys which investigated beliefs and images of teaching, and a simulated teaching environment to enable respondents to participate in different pedagogical experiences and reflect on the experiences. The data was collected both at the beginning of the 1-year course and at the end; the course spanned approximately 7 months. Charalambous determined that strength in one domain cannot compensate for weaknesses in the other. Moreover, he found that the productive impact of one domain can be mediated by weaknesses in the other. Charalambous concluded, in an effort to inform practice, that both teacher education programs and in-service teacher professional development must strive to promote changes in both domains simultaneously.

The purpose of Lewis et al. (2015) study was to directly study proclivities for teaching mathematics as having an impact on improving mathematics instruction. One of the proclivity domains is productive disposition. The researchers viewed this as an opportunity to develop ways to measure the proclivities. The participants were 63 teachers who worked with Grades 4- 8 and were part of a National Science Foundation funded professional development program which ran annually for 5 years. The professional development curricula focused on algebraic thinking and reasoning in Grades 1-8. The teachers participated in a 2-week summer institute which focused on algebraic thinking and reasoning. Teachers participated in a 10-day lesson study component, monthly seminars, and met in small groups once a month. The researchers

developed a 27-item assessment on proclivities, which was administered the third, fourth, and fifth years of the program. The validity of the items was determined by conducting interviews with teachers not in the program to identify discrepancies regarding intent and interpretation. The results of the study were, first, the identification of three clusters of teachers. Cluster 1 was the group of teachers least likely to challenge students and use varied approaches and Cluster 3 consisted of teaches most likely to engage in these practices. The researchers noted teachers did move from Cluster 1 to Cluster 2 and Cluster 2 to Cluster 3 over the course of the study. Lewis et al. also noted "a correlation between the measure of proclivities for teaching mathematics and the measures of mathematical quality in observed instruction" (2015, p. 461). They recommended several things including refinement of the instrument, collecting data on more participants, collecting data on teachers who are not part of a professional development program, and to add dimensions to the proclivities to address the three levels of proclivities they identified – doing mathematics, thinking about how students do mathematics, and helping students develop habits of mind in and for doing mathematics.

Jacobson and Izsak (2015) researched the connections between teachers' opportunities to experience drawn models as learners and their knowledge and motivation to use models in instruction. A three-part survey was administered to 990 middle school teachers from across the nations. The three parts of the survey were a knowledge assessment, a professional history and teaching experience questionnaire, and a motivation questionnaire. The history and teaching questionnaire asked teachers about what opportunities they had to learn about drawn models. The motivation questionnaire

included if teachers had anxiety about the use of models in instruction, their self-concept regarding their ability to use them, and if teachers valued the use of models. The researchers found that there was a strong relationship between "teachers' opportunities to learn, their knowledge and motivation, and self-reports of instructional practices with respect to drawn models" (Jacobson & Izsac, 2015, p. 482). This was especially true in comparison to general pre-service teacher experiences. They also determined teachers must have the opportunity to experience the use of drawn models and their own growth as learners using drawn models. "It is unlikely that teachers would independently construct relevant knowledge and instructional practices without such opportunities, especially if they did not observe or experience instruction that involved drawn models . . . themselves" (Jacobson & Izsak, 2015, p. 482). The most important finding was that teachers' motivation to use drawn models was completely responsible for the relationship between mathematical knowledge and use of models in instruction. They concluded, in the case of drawn models, teachers must experience purposeful instruction on both the uses of drawn models to represent mathematical thinking and on motivations for using mathematical modeling – again, the intersection of the knowledge and affect domains was noted critical. The researchers called for more research, using this as a foundation, to better understand how experiences of teachers translate to their practice (Jacobson & Izsak, 2015).

In the final article, Philipp and Siegfried (2015) presented their summary of productive disposition as a developing construct. They began by looking at the definition of productive disposition and the relationships between its appearance in the mathematics

proficiency strands presented by the NRC (2001) and implications for teacher educators. These recommendations delineated the three approaches taken in the above noted studies: "connecting existing research to productive disposition . . . using productive disposition through which to view an area of interest and . . . studying productive disposition directly" (Philipp & Siegried, 2015, p. 491-493). Philipp and Siegfried then presented some considerations when studying productive disposition. The first consideration described was a clear definition for productive disposition as it relates to the researcher's study, noting that each of the authors in the above articles used different definitions. The second consideration was whether it is a local or global attribute; in other words, is it only present in an individual when they work with algebra or is the presence of productive disposition context independent. The third consideration identified by the authors is whether there is more than one kind of productive disposition. For example, a teacher asked to engage in a rich task may engage a productive disposition as a learner, but research must also consider if the teacher will also engage a productive disposition as a teacher to think through how students may approach the task and how they can help all students access the task. The fourth is the researcher must be mindful of the importance of the teacher in conceptualizing productive disposition. They noted that although teachers may not know the term, they will know the construct when they see or experience productive disposition. Finally, the authors provided some suggestions regarding methods. They stated that a variety of approaches are needed to inform the field, such as "in-depth case studies to examine deeply a small number of subjects,

studies like those published in this issue to develop instruments, and even informal attempts to grapple with the construct" (Philipp & Siegfried, 2015, p. 499).

Chapter Three

This purpose of this study was to investigate the emergence and sustainability of teachers' productive dispositions. Specifically, teachers' professional development experiences and the influence of those experiences on the teachers' disposition from an andragogical perspective was researched. The research explored behaviors and thoughts of teachers, in relation to professional development, which provided evidence of a productive disposition toward mathematics and mathematics instruction. The research was guided by Jacobson and Kilpatrick's (2015) definition which is the most comprehensive and was therefore used for this study. Their definition is as follows:

> Productive disposition for teaching mathematics is mathematics teachers' malleable orientation toward – and concomitant beliefs, attitudes and emotions about – their own professional growth, the subject of mathematics, and its teaching and learning that influences their own and their students' successful mathematics learning. (Jacobson & Kilpatrick, 2015, p. 402)

From this definition, and using the andragogical framework, the following research questions were derived:

1. How do teachers describe the changes in their beliefs, attitudes and emotions about (a) their own professional growth, (b) the subject of mathematics, and (c) mathematics teaching and learning over their teaching careers?

2. How can the andragogical framework be used to identify and explain events which serve as catalysts which promote or hinder teachers' productive dispositions?
3. What are indicators, of which professional development designers should be aware, which will further promote or hinder productive disposition? Is there evidence regarding how these indicators should be addressed to maximize growth?

Conceptual Framework

In order to identify paradigms/frameworks which would guide the research, several factors were considered. The first was my own knowledge about and experience with the professional development opportunity used to select candidates as well as my possible relationship with the respondents. After some exploration, it was determined interpretivism and narrative approaches would help analyze the data in this study.

Researcher's introspective considerations. It was clear from the onset of this research that my thinking/ideas and my experiences guided the decisions I made in determining this was an important topic to explore and how I would conduct the research. I have been interested in andragogy as a construct for a while. So, as I began to reflect on my experiences and why and when things have been important to me, I realized my thoughts could be framed in andragogy's three dimensions: core adult learning principles, individual and situational differences, and goals and purposes of learning (Knowles et al., 2011). I also quickly realized as I went through various iterations of the research questions, my journey was based in social contexts and interpretations of those social

contexts. It was clear my research may take a similar journey, bringing me to Maxwell's (2005) description of qualitative research when he noted, "Any component of the design may need to be reconsidered or modified during the study in response to new developments or to changes on some other component" (p. 2). Maxwell went on to explain that the activities of qualitative research happen simultaneously and iteratively, each aspect of the research impacting the others. Hammersley and Atkinson (1995) noted that qualitative research "should be a reflexive process operating through every stage of the project" (p. 24). This does not mean, however, there was no conceptual framework or purposeful design, for without these the research would likely fail. Therefore, the following paragraphs cover the research analysis approaches for my methodology and how these informed various components of the research.

The reality of who my participants might be made it impossible to believe I would be a distant observer or researcher in the strictest sense of the word. I also acknowledged that my involvement with the possible participants, and knowledge of the professional development courses from which the participants were chosen, may be helpful in conducting this research. Ethnography encourages that researchers take a more active role to better understand the culture being studied; however, the practice readily acknowledges that the relationships established, possible bias, and intrusion in others' lives are real concerns (Wolcott, 2008). A framework which would enable me to acknowledge my possible role/influence on the participants and understand or analyze this component was critical. Maxwell (2005) stated, "Separating your research from other aspects of your life cuts you off from a major source of insights, hypotheses, and validity

checks" (p. 38). The framework also needed to account for how I tend to make sense of the social contexts and interactions with the participants. As a researcher, my goal was to not only conduct my study and learn from the participants but to also reflect with them on mathematical knowledge and the practice of teaching mathematics.

I arrived at three conclusions. My analysis needed to account for participants' changes over time and for realities and ideas which were socially constructed, in which it was possible I was a part of the social construct. The research was designed to consider the participants' experiences over time, from elementary experiences to teaching experiences. The approach selected to consider a historical timeline and teachers' reflections was a narrative approach. The second approach chosen to use as part of my analysis was interpretivism. This approach enabled me to account for the social construct of the research and my involvement or possible relationships with the participants, and the perceptions of those social constructs. Finally, I considered all of this within the andragogical framework. The goal was to account for the importance or supportive elements of various factors of each participants' professional lives which impact their productive disposition as explained by the three dimensions of andragogy.

Research Design

This study employed primarily qualitative methodology which combined the complementary analytical approaches of narrative and interpretivism within the andragogical framework. In the following paragraphs, I discuss my research design methods, including the design of the study, the context from which participants were chosen, the selection of participants, the participants, and the data collection and analysis.

A collective case study approach was chosen due to the varying and complex factors which might influence a teacher's productive disposition. A case study is research based in a bounded system – bound by time, place, or a specific event. In the case of this research, the bound was participation in a specific form of professional development. The professional development experience was a course combining mathematics content with lesson study offered at an east coast university, with variations, each year over the course of 6 years. The case part of the definition is also met as those initially identified to participate in the research had all completed the content and lesson study portions of one of the courses.

The goal of case study is to use the intricacies of a case, or in this research, several cases, to determine connections between phenomenon. In this research, the goal was to determine and describe a relationship between professional development experiences and productive disposition. The relationships was considered from an andragogical perspective. The case study method enabled opportunities for more profound interviews, conversation, and analysis (Glesne, 2011, p. 22). Because more than one participant was included, it was considered a collective case study. This allowed for the research of a phenomenon,in this case, the development and sustainability of productive disposition through professional development opportunities. As the study required a look at the development of the teacher over time, a narrative and interpretive analysis approach was used.

In accordance with Emerson, Fretz, and Shaw (1995), a narrative approach was taken to more fully consider the timelines and stories/reflections presented by the

respondents. Specifically, to think of the shared experiences as "fieldnote tales, which recount interactions, incidents, and events as dynamic actions in an unfolding scene" (Emerson et al., 1995, p. 85). The authors noted the tales are interpretations of the researcher based on an extended account of related or interconnected ideas over a day's time. However, a narrative approach implies the researcher/ethnographer identifies either events which are similar or involve the same people, or note an event means, "actions progress, develop over time, and sometimes lead to immediate outcomes" (Emerson, et al., 1995, p. 89). The authors state this strategy may lead the researcher to identify or presume connections which do not exist as it can become more interpretive. Given this concern, careful consideration was given to the methods and purposeful efforts to help mitigate erroneous findings of connectivity.

Interpretivism was developed by idealists who believed that,"the world cannot exist independently of the mind – or of ideas" (Glesne, 2011, p. 8). Glesne stated the goal of interpretivism is to understand a person's' ideas, actions, and interactions in specific situations or in relation to the more global culture. Interpretivism, according to Maxwell (2005), is understanding the meaning of the events in a person's life and the meaning of their actions from the person's perspective. Ontologically, the parallel is that meaning and realities are socially constructed, dependent on numerous variables, and change over time. In the case of this research, it required interacting with people in the social context or in a related context and understanding their perceptions. The approach itself allowed for a number of realities inherent in the research.

Glesne (2011) identified several components of this approach which served as critical aspects for consideration (p. 9). The approach is based in the assumptions that meaning and reality are socially constructed, taking into account many immeasurable and complex variables. The research approach implication is that the researcher is the instrument. Cognitively, as the instrument, the researcher must take an inductive approach, searching for patterns while identifying complexity. Finally, the researcher may have personal involvement in the event or action, and thus may have empathetic understanding for what the participants experienced, and the meaning derived.

Professional Development Setting

The participants were all enrolled in two or three summer graduate courses which occurred over the course of 6 years. The specific courses/summers in which the participants of this study were enrolled are not reported to protect the anonymity of all the participants. The summer graduate courses were grant-sponsored courses conducted by researchers from both the Math Education and Math Departments at an east coast university. The courses were facilitated by the researchers and their graduate students. The researchers worked with local division leaders to encourage instructional teams to participate. Participants received both a stipend and a course credit.

Each course had a few critical components. Teachers were brought together for a week during the summer and it was requested they come in school or division teams or at least with a colleague. During this week, the participants were immersed in content for which they were responsible as a teacher. The courses were designed to have a specific focus such as algebraic reasoning, proportional reasoning, or identifying and creating rich

tasks. The tasks/problems given to the participants aligned with the state standards the participants were to teach; however, tasks were differentiated to allow for a variety of approaches and thus aligned with and were appropriate for multiple grade levels. Participants were asked to explore and engage in math tasks. They were encouraged to model tasks in a variety of ways and dialogue with others regarding their reasoning and models. As each person/group developed confidence in their understanding, the facilitators encouraged whole class discussions. The facilitators guided discussionso the participants could make connections between models, reasoning, and in some cases, different interpretations which led to different answers which were still mathematically valid (e.g., a different answer based on how the whole was defined). The goal of the facilitators was to model the instructional strategies and techniques it was hoped the participants would then use in their own classrooms. The participants were then asked to implement elements of their week-long experience into a lesson study cycle.

Prior to implementation of the lesson study, the participants were introduced to the lesson study process, provided instruction and watched an exemplar lesson study via video. The participants had discussions about lesson study – design, purpose, goals, and implications for practice. They discussed how each of these were apparent in the video, and they discussed their observations regarding the roles of the lesson study team members. Participants then determined a self-selected group to complete a lesson study cycle. Their chosen team was assigned a graduate student to guide them through the lesson study cycle to be based in the content of the summer graduate course. Again, the content of these courses was designed to align with the curriculum for which the teachers

were responsible. One of the teachers was selected or volunteered to implement the lesson in their classroom during the school year. The rest of the team, including the graduate student, observed the lesson as part of the lesson study team. Observers were asked to focus on student thinking, representations, and questions posed by students to each other and during whole class discussions. Student work was collected at the end of the lesson. The graduate student facilitated a discussion after the lesson, the purpose of which was to explore student thinking. The discussion was based on the models created by the students and the student conversations the observers scribed. The graduate student also led a debrief on the lesson study process to that point: experiencing the math themselves, purposeful selection of a task, design of the lesson, initial implementation of the lesson, and possible implications of student thinking on the lesson implementation and future design. The team then identified next steps for the teacher who conducted the lesson and determined how each member might alter the lesson before implementing it in their own classroom. The rest of the team then implemented the lesson independently and wrote a reflection on the lesson, which focused on an analysis of student thinking, implications of student thinking, and next steps. The teams then met to discuss the various observations and summarize their experiences and findings. Each team presented their experiences and findings to the entire class at a conference held in the spring.

My Role in the Professional Development

My advisor was one of the lead investigators of the grant funded professional development course. I worked with the courses in a few different capacities. I served as a co-facilitator of the course work. This involved selecting or designing rich mathematical

tasks in which to engage teachers. As a co-facilitator, I challenged them to model their thinking and to model problems in more than one way. Facilitators also strived to support and encourage those that appeared less comfortable with the math or the modeling. We asked participants, both those thriving and struggling, to walk us through their thinking or explain their model. Facilitators collaborated as we observed participants working on tasks. This enabled the facilitators to identify similar strategies, misconceptions, different interpretations, and connections which were made. This information steered our efforts to purposefully guide discussions to further the participants' learning. I also served as a lesson study knowledgeable other which involved helping the team develop their lesson, observing the lesson, and leading the discussion and debrief after the lesson was implemented. I also attended the team presentations, sometimes as a facilitator or knowledgeable other who had worked with the teams and sometimes as an invited graduate student observer. I was also brought in to work as a mentor to a new knowledgeable other who was leading a lesson study team. Finally, for a couple of years, I held a leadership position in a local division's math office and helped promote the program to teachers, teams, and administrators.

Selection of Participants

Purposeful selection of participants for this case study was critical; a justifiable selection strategy was identified (Glesne, 2011, p. 44). Given my role and relationship with possible participants, I was mindful of those who may feel they had insight into the purpose of my research and my thoughts about the interview questions. As a goal was to also account for variations from an andragogical perspective, and the data was considered

using an interpretivist approach, I chose to use maximum variation sampling. The goal was to select individuals who represented a range in teaching experience, professional development experiences, and personal accounts of productive disposition. The hope was to search for some common patterns across despite, these variations (Glesne, 2011). Narrowing down participants for the case study who had disparate attributes, but who established common patterns, lent the findings of this research to generalizability (Cresswell, 2008).

In an effort to narrow down potential participants, existing quantitative and qualitative data from the grant research, which may be helpful was identified. Existing quantitative data included pre- and post-assessments in which modeling was utilized (number of problems in which modeling was used, number of models used, scores, etc.), surveys given during the graduate courses, the number of various types of professional development candidates attended, etc. The existing qualitative data included surveys, interviews, and written responses to course tasks. One of the primary researchers reviewed existing data and shared recommendations for possible participants. The data for those recommended was then discussed and ultimately 12 possible participants were identified. The participants also met the goal of maximum variation sampling. Although this research was conducted during a busy time of the year for educators, it was believed at least three or four would be willing and able to participate. The primary researcher of the grant project emailed each of the 12 possible participants individually, introducing me and letting them know I would be contacting them about my dissertation research. I then sent individual emails to each of the 12 individuals, asking for their assistance, and

directing them to the survey (to be discussed later). Of the 12 people contacted, 11 agreed to participate and completed the survey. The twelfth participant also contacted me and was willing, but the final chapters had already been written. At the recommendation of my committee, I tried to have as many participate as possible. Due to timing (e.g., end of the school year, vacations, times of day the respondents were available, and the geographical spread of the respondents), this research used data collected from nine participants.

Participants

Nine teachers/coaches participated in this study: Sam, Paula, Edward, Amanda, Ricki, Jamie, Christine, Faye, and Kim. Pseudonyms are used for all participants to protect the anonymity of the individuals. All the participants were involved in two or three of the summer courses. A couple of the course participants, in time, served as lesson study facilitators. They work in several adjacent school divisions in a metropolitan area on the East Coast. Variations in their teaching experiences include teaching from 3 to over 30 years; working in one to three different divisions (one taught in another country); teaching in one to seven different schools; working in the private and public sector of education; teaching grades K- 12; and working as a teacher, mentor, and coach. Most of the participants had teaching experience with Grades K-8, had 11- 20 years of experience, and taught in the public education setting.

Table 1

Participant demographics

Name	Years of experience	Divisions	Schools	Grade bands in which each taught/coached
Sam	3	1	1	K- 5
Paula	5	1	2	9-12
Edward	10	1	1	K- 5, 6- 8
Amanda	11 (2 as a coach)	1	1	K- 5
Ricki	11	3	7	K- 5, 6- 8
Jamie	13	2	4	K- 5, 6- 8, 9- 12
Christine	17 (4 as a coach)	1	2	6-8
Faye	30	2	5	K- 12
Kim	30 +	3	7	K- 5, 6- 8

Data Sources

The courses were grant-based endeavors, a basis for which was the collection of several types of data. The data collected addressed research questions posed by both the faculty researchers and by graduate students. Researchers and graduate students collected specific data outside of the course structure material and observations. This data included survey data related to teaching and class interviews. This data was archived and therefore available for this research. Assessments, reflections, interviews, videos, and survey data were considered to inform the research design, guide semi-structured interview questions, identify possible research participants, and help triangulate data.

Survey. Once possible participants were identified using existing data, a request asking they participate, and a survey link was sent to each individually. The survey was

based on a 7-point Likert scale survey from Yin's (2009) work cited by Charalambous (2015) to "tap into participants' beliefs about what it means to do, teach, and learn mathematics" (p. 433). It was designed to establish a self-report about how these beliefs may have changed over time (see Appendix A). There was an initial concern that the adaptation may imply to the participant a change was expected. However, it was determined the interview and follow-up questions would inform both if participants had these beliefs and provide an opportunity to validate information shared in the survey, to include any change in their beliefs.

Interview and follow-up questions. A semi-structured interview (see Appendix B) was conducted with each participant of the case study. In accordance with the design, the semi-structured interview was chosen to allow for the social context, my involvement in the coursework, and the narrative and interpretive analysis of the data (Glesne, 2011, p. 9). The interview allowed for follow-up questions and explorations which happened authentically. The purpose of the semi-structured interview was to gain an understanding of the mathematical history and mathematical development of the participant, both as a student of mathematics and as a teacher of mathematics. The questions included the types/forms of professional development in which they participated throughout their career. These answers to these questions provided information related to the andragogical aspects of the research framework. It is important to note the interviews, again based on time and logistical constraints, happened in person and via video and phone calls. Following the first interview, and after initial reviews of the existing and new data, follow-up questions via email and phone were posed to the participants to clarify shared

information or address conflicting information. Again, phone and email were used to address convenience concerns for the participants and given the timeline of the study.

Data Collection

Data collection has five interrelated steps, beginning with participant selection (Cresswell, 2008,). Participant selection has already been discussed. This section will provide details on three of following steps: types of data to collect; locating, modifying or developing instruments; and the actual collection.

Existing data. As aforementioned, data collection began by reviewing existing quantitative and qualitative data. The quantitative data included data from a mat beliefs survey, performance data from the courses, and in some cases, scores on Hill and Loewenberg-Ball's Mathematics for Teaching Knowledge (MKT) assessment (Hill, Loewenberg-Ball, & Schilling, 2008). The qualitative data included interviews, surveys, lesson study notes and presentations, and assessments. This data was used to identify potential participants for the study. However, it was not analyzed for the purpose of informing the results of this study.

Survey question data. An open-ended survey (see Appendix A) was used to provide initial consent for the research, demographic data, self-reported beliefs on changes in the teacher's views about mathematics and teaching and learning mathematics, and to provide initial data for the research questions. The questionnaire focused on teachers' ideas and feelings about various learning and teaching components when they first began teaching and how they feel about those same components now. The survey is an adaptation of a 7-point Likert scale survey from Yin's work (2009) cited by

Charalambous (2015) to, “tap into participants’ beliefs about what it means to do, teach, and learn mathematics” (p. 433). The format of the survey was changed from a 7-point Likert-scale to open-ended sentence starters. Again, there was a concern the respondents would feel they needed to report a change. In an effort to help mitigate this possible perception of the respondents, it was decided the following interview would be used to help confirm teachers’ reports of change (or in some cases, little change).

I also relied on my experiences as a math educator and professional development provider in modifying these ideas so that they addressed common topics and themes which I discussed, experienced, or observed working with teachers. This included my role and participation in the grant-funded courses and the ideas which emerged in the guidance and support I needed to provide, and the discussions I had with the teachers enrolled in the course. Therefore, some ideas/statements were removed completely. The goal was to create an opportunity for the responder to interpret meaning which might inform aspects of the conceptual framework. The questions were also designed to provide insight into the participants as adult learners. The responses were to, it was hoped, shed light on changes the participants identify in themselves as doers and teachers of mathematics at two times in their adult lives, thus providing some andragogical insight.

Interview questions. The interview questions (Appendix B) served to guide a semi-structured interview for the case study participants. Questions 1- 4 provided information about the teachers’ math education, training, and experiences. There was an interpretive nature to Question 3 as it was meant to reveal which activities they considered contributors to their professional growth. The question enabled me to ask

about other professional development activities in which they participated but did not mention as or consider growth producing. Questions 5- 8 were designed to more directly address Research Questions 2 and 3. The goal was to identify activities and experiences which participants believed changed their thinking, challenged them, helped them grow professionally, or hindered their growth. They also had the opportunity to share how these activities/experiences made them feel and what followed that may or may not have supported them as they worked through those experiences.

Follow-up questions and participant additions. These questions, which were few, were specific to each case study participant. The questions were asked with two purposes in mind: to determine validity of my interpretations of their responses and help eliminate the possibility of my own bias, or to gather more information to clarify a response or resolve any discrepancies within the first two instruments. In a few cases the participants contacted me offering additional information or something they wanted to clarify. This information was also added to the data.

Data Analysis

The surveys, interviews and data analysis will occur occurred between late-May 2019 and the end of June 2019. Although an advantage to this was an ability to capture and follow up on memories or thoughts of the participant while they were still fresh in their minds (Weiss, 1994, p. 57), the timeline was primarily dictated by program completion deadlines. The surveys were sent to each of the initial candidates within a span of a few hr. However, the timing of the interviews and follow-up questions varied

based on researcher and participant availability, and timing of initial review of the survey and interview data.

The interviews were recorded by two devices. The better recording of the two was transcribed. Transcriptions were made using a professional transcription service. Those transcripts were reviewed for accuracy by listening to the tapes while reading the transcript. Notes were made regarding any missing information in the transcript which could be discerned by the researcher on the tape, or evident in the notes taken during the interview. The researcher also took notes during the interviews, noting key ideas and possible relevant observations about mood, inflection, or non-verbal behaviors observed during the interview in an effort to capture every nuance possible (Weiss, 1994).

Due to the time constraints, I did not wait for the all the interviews to be transcribed and reviewed as described above before determining follow-up questions for each participant. Fortunately, the turn-around for each transcription was very quick (within 24 hr) so I was able to simultaneously consider each participant's recording, transcription, and related notes in a timely manner. I listened and looked for themes/patterns, considering relevant word usage which emerged in each participant's interview and, in some cases, between the respondents (Maxwell, 2005). These interviews were also compared to the participants' survey responses to determine common ideas and discrepancies for which I needed clarification or more information.

Once all interviews were transcribed, I attempted an open-coding process to help identify possible categories for the individual respondents (Creswell, 2008). I felt I had some direction regarding the participants ideas or beliefs which may be categorized

(Maxwell, 2005). I used an electronic computer-based system, Dedoose (2018), to identify key words or ideas which I believed may constitute substantive categories. I believed this would be more easily done as the program enabled me to track the frequency with which a key word or idea occured. I found I had many, and there did no't seem to be an emerging pattern or big themes or ideas after completing the coding for the first four participants. I went back to various notes I had made throughout the design process. I realized although the terms and ideas varied, if I used more broad ideas/themes the data fit an aspect of the dimensions of andragogy or pieces of the working definition of productive disposition. At this realization, I started over and changed my strategy.

I created coding names which aligned with the individual's background (informing the andragogical perspective of the participant) and with the dimensions of andragogy. I also created coding names which represented various aspects of the working definition: evidence of malleable orientation/concomitant beliefs, professional growth, subject of math, and math teaching and learning. I considered Kilpatrick's (2001) definition, which noted ideas of sensibility, that math is useful and worthwhile, and diligence. I added these codes to more specifically inform aspects of the definition, as well as codes for the pieces of the definition from Lewis et al. (2015), efficacy as a learner and doer and seeking and using resources. In other words, I switched from open-coding to a form of a priori coding. I used these codes to analyze the surveys, interviews and follow-up information for each respondent. This process was then completed for the other participants, so that each case was considered independently first (Strauss & Corbin, 1998). I worked to identify big ideas within each respondents' answers, and

aspects which might inform possible concept maps. I then moved on to the subsequent participants, again considering learning and developmental trajectories which may determine or inform concept maps. The need to consider various concept maps was inherent in the claim that concepts within knowledge for teaching and productive disposition are interactive (Jacobsen & Kilpatrick, 2015). This strategy used for the individual respondents helped me meet my goal to use narrative and interpretive analysis approaches within an andragogical framework. I also gained insight for a possible concept map to better help address the research questions.

Although I believed I had enough information to conduct some cross-case analysis, I needed a way to connect my initial open-coding strategy with a priori coding. I decided to take the current coding (a priori) for the respondents' as a set, group the information by code name, and then review data within each code name to identify key ideas or themes. After this review (which was essentially a second coding) was completed, my initial summary ideas regarding the cross-case analysis and participants' direct quotes were considered in relation to the proposed concept map and the research questions.

Validity

The use of multiple sources to initially identify potential candidates, the survey, the interview with observation notes, and the follow-up questions within my research design helped to provide validation for my findings. Additionally, these efforts mitigated validity threats (Maxwell, 2005). Therefore, there was construct validity. By using

existing data and following up with respondents regarding inconsistencies, seeking alternate explanations, or to confirm my interpretations, I established internal validity.

The primary data sources for this research were the survey, interview, and follow-up questions. Because I was concerned about my possible influence on respondents, especially those I knew (two of them), and my involvement in the professional development used as a starting place to identify participants, I worked purposefully to mitigate the impact of these components. I asked open-ended questions in a semi-structured interview format. I avoided leading questions and often asked for clarification on responses to ensure I had specific answers including the how and why (Maxwell, 2005). When participants asked for more guidance on an open-ended question, I provided several possibilities for interpretation. I also circled back during the interview to see if they provided different information or clarified their initial response. I also used follow-up questions after the first interview if there were still inconsistencies or I felt more detail was needed.

It is important to note, again, that all data is from the participants' self-reports. Save two of the participants, I did not have any background knowledge of the participants. Each of the 11 who completed the survey were willing to fully participate. Fortunately, with the nine that I interviewed, there was enough communication prior to the interview that they started the interview willing to engage and relaxed. For the few who seemed a bit nervous, I sensed by the second or third question they were at ease, so I found ways to circle back to the initial questions to see if they had more information to provide. I provided ample wait time, making a note if I had a question or follow-up idea,

so they could share their thoughts and thinking process completely. The interviews were conducted in methods of their choosing: their home, store/coffee shop near their home, video conference, and two by phone.

As a researcher, and to better understand my own development in the andragogical framework (and at the recommendation of my committee), I completed both my survey and had a fellow graduate student interview me using the interview protocol. This served two purposes. First, it was a personal reminder of the various events which informed and shaped me as a mathematician, teacher of mathematics, and teacher of teachers. Second, it helped me identify where my personal biases may lie. Although I believe all of my experiences (learner, teacher, professional development and curriculum specialist, college course facilitator, and school administrator) have been invaluable, they are also responsible for shaping my ideas and beliefs. I therefore respected my participants and their insights from the perspective of each of these roles.

Limitations

The narratives presented on each of these individuals in this study present enough variables to make them unique. Although not generalizable, it is believed this collective case study provides some ideas and themes which may serve to inform various stakeholders and therefore be further explored. Also, the final analysis was based solely on self-report. There was no data gathered on the teachers or coaches in their work environments—working with students, colleagues, or teams. All the participants provided specific examples at various points in time from their childhood to present, which adds credibility to their recollection and reflection on their journey. In several instances, it

appeared genuine connections were made by them for the first time. Although in some cases, their connection may have been due to a connection I had made, so I asked a relevant question to see if they made the same connection. This also added credibility to the implications for the connections between an individual's experiences as a learner and their beliefs as a teacher, again, tying back to an andragogical framework.

Chapter Four

The results are presented in four sections. The first section describes each participant's journey: a narrative of the teachers' mathematics backgrounds and development as a learner and a teacher. Connections are then drawn between the individual's journey and their placement within the andragogical framework. This placement is important as it identifies the needs and differences for each participant which would need to be acknowledged or addressed to promote productive disposition. Next, the evidence of productive disposition and possible factors related to its emergence or sustainability are presented for each participant. Although the focus is on the productive disposition of the participants as practitioners, it is important to acknowledge similar attributes of the participants in their formative years, so examples from their formative years are used when appropriate.

The second section presents summary findings regarding the andragogical framework, specifically the ways in which professional development met their needs as adult learners. This section provides evidence that when andragogical needs are met, the participant was more likely to express ideas aligned with the definition of productive disposition. The third section presents similarities of the participants in relation to productive disposition. Specifics of their professional experiences which either supported

or hampered their productive disposition and how these experiences relate to andragogy are discussed.

The final section presents summary findings. The findings present connections between the andragogical framework and the definition for productive disposition. It is clear in these findings there are occasions of both overlap between these two ideas and instances in which the andragogical framework provides information for stakeholders in promoting and sustaining productive disposition.

Participant Narratives

The following section provides a brief narrative of each of the participants' journeys in mathematics as a learner and a practitioner. Each person's math background, which includes their kindergarten through 12th grade experiences, preparation to teach math, professional development experiences as a practitioner, and changes to their practice, is presented. Following this narrative is an exploration of the pieces of the andragogical framework with which each participant arrives to adult learning experiences. Finally, evidence of their productive disposition is presented. Again, the evidence presented is meant to align with the definition by Kilpatrick et al. (2001), to include the attributes (learner and doer, diligence, useful and worthwhile, and seeks and uses resources), which have been used by other researchers to further delineate the definition.

Sam. Sam did not begin her college years intending to be a teacher. By circumstance after her college years, she decided teaching was a calling. She then earned a master's degree in education. Sam is a teacher with 3 years of experience. She has

taught in one division and one school throughout the 3 years. During her tenure she has taught a middle grade but taught advanced mathematics curriculum for the grade level.

Mathematics background and development as a learner and a teacher. Sam's experiences throughout her K-12 and college tenure may be considered very traditional. She was taught using direct instruction and noted only a couple positive experiences throughout her math courses. She also provided evidence of her persistence. Sam ultimately grew into math, which is explained in the following paragraphs. Her first really rich experience with math was not until she took an elementary math methods course as part of her master's in education program.

K- 12 math experiences. Sam recalled she was a poor math student. She remembered struggling and noted examples like not memorizing her multiplication tables until she was in fourth-grade, and it was a third-grade expectation. The instruction she received she described as procedural. She thought part of what caused her to struggle was there was "one way" to work the problems. Sam said she was good at memorizing but didn't really understand. She felt the way to be successful was to "parrot" back whatever procedure was demonstrated. Sam was selected, although she does not recall how, to be involved in a group that worked on logic problems with a math specialist. She enjoyed this setting, saying it was more like problem solving, reasoning, or working on puzzles. Overall, she said her elementary experience gave her a poor foundation for future math courses. However, she said she somehow stayed or was put on an accelerated track.

Sam was placed in Pre-Algebra in 7th grade and noted she again did not have a very good experience. Her overarching memories are that her teacher was awful; she

spent the year confused and not feeling she understood any of the math. Regardless, she was promoted to Algebra I in 8th grade. She was very aware she was a year ahead of the general population. Her algebra teacher remains one of the best parts of her school career. He was "such a wonderful person." She recalled his patience and offers to provide extra help. In hindsight, she knows she was not ready for Algebra, but she still worked very hard and appreciated all he did to try and help her. Ultimately, she didn't do well and repeated the course the following summer. Sam did much better the second time in the course.

Sam's high school math courses brought more struggle. Geometry she remembered was among the courses in which she was successful – success defined by a B or B+. She then took Algebra II and Pre-Calculus in the International Baccalaureate (IB) program. She said despite lots of hard work and tutors her sophomore and junior years, she did not do well. Her teacher her junior year, noting her hard work and outside assistance, recommended her for the next math course in the IB program. However, she said, "I finally got smart and was like, nope." She took Statistics her senior year. Her tone and inflection implied she did not consider this a real math class, but she did note she got an "A."

Preparation to teach math. Sam earned an undergraduate degree in biology. She had always been interested in the life sciences, so this was a natural pursuit for her. During her college years, she said she had come around with math. She took Calculus and did fine. She said it still took a lot of hard work on her part but that "Things started clicking." She felt her attitude towards math changed due to the personal success after a

lot of hard work and struggle along the way. Given her biology degree, her mother-in-law (a teacher) invited Sam to present lessons to her fifth-grade class. She loved being in the classes, seeing the kids' excitement and decided she wanted to pursue teaching.

She applied and was accepted into a graduate education program. Her first math course was an elementary math methods course. She said the course got her excited about math. The instructor provided resources and ideas about how to teach math. Sam said these resources and ideas are what excited her. She felt she was given a way to reach students and make the math accessible. She noted this was exciting because she felt this was part of the art of teaching, knowing the resources and your students and picking what will be best based on your students' needs.

Professional development experiences as a practitioner. Sam has participated in a variety of professional development opportunities. She said her grade-level professional learning team is strong but not in terms of discussing math specifically. As she taught the advanced math section for her grade, she and the teachers from the upper grade did share resources and ideas, and they demonstrated respect for each other. Sam attended a lot of courses provided by her division – she estimated three or four each semester. She said the summer courses created even more excitement for her than the methods course. She again noted the relevancy of the resources provided. Sam believed the more resources she has the better so she can meet students' needs – and again, that is exciting for her. She also felt she benefitted from being engaged with mathematics and the lesson study cycles.

Sam was very aware the tasks given to them during the summer week aligned with the standards they were to teach. She also noted because she had "come-around"

with math, she now loves a challenge. She now appreciates and takes advantage of opportunities to stretch herself. She noted the use of models as a specific example. The summer courses were viewed by Sam as an opportunity to try and try again in a safe, non-threatening place – no judgement. Sam said sometimes it works, and sometimes it doesn't – and that's okay. She also recalled a specific geometry problem they were asked to model. Everyone found the problem challenging, and although she didn't get it at first, Sam said she kept working on it and slowly started to make sense of the problem. She said she is probably more excited by the challenge given, her continued struggles, and ultimate perceived success in mathematics. She added she wasn't afraid of math, which is how she used to be in high school. Her mindset is

> Okay, I'm going to try different things and I'm going to talk to different people. And maybe it won't work the first time, but I'm going to eventually figure this out even if I don't figure it out on my own. Collectively, in the end, I'll get it. (Sam)

In the lesson study cycles she completed, she noted they were "fascinating" and that she learned a lot. She had the opportunity to both be an observer and be the teacher. She appreciated the opportunity to work with teachers as they modified the same lesson to address different grade level objectives and was surprised and interested in what the students were doing with the tasks. Most enlightening for her were the observations her team made of her students' thinking. She said it was invaluable to hear what the observers gleaned from the other students. She reflected on all of the "great math stuff" which may be happening in your classroom and you, as the teacher, aren't right there to observe it. She said it helped her better appreciate rich task work.

Sam credits the summer classes for proving to her she really can "do" math. The courses have forced her to understand the math and reasoning behind the procedures. She acknowledges there are plenty of mathematical procedures she can complete that she doesn't understand, but she also believes she could figure them out if she took time to explore them. She also knows she can, for example, model division of fractions and understand why she is doing, what she is doing, and explain it to an adult. Sam noted this has been "huge" for her. She said this realization is huge because it also means she can help students understand math and, "why it matters."

Sam attended the summer courses in two situations: one in which she knew a few people and another in which she didn't. She appreciated the support and camaraderie experienced when attending with people she knew but said meeting new people was also interesting as they offered a different perspective. She also acknowledged regardless, she had the support of her administrator and her grade level and upper level math teams if she wanted to try something new, so both experiences had important benefits. Sam noted, however, that the administrative support may be because she teaches the advanced math course. She has observed there seems to be less autonomy for her grade level math peers. She also implied there is some parent influence on her grade level peers. Sam stated she would struggle if she was expected to be in "lock-step" with what other math teachers were doing. Sam stated, and it was clear observationally, this was an internal conflict for her. She also said although they share resources, she does not believe others use most of what she shares. Sam said unless they experience what she has experienced, they are unlikely to change their instruction.

Changes in practice. Sam said she implemented strategies, methodology and resources immediately. She said this happened in part because the lesson study teams were implementing them as part of the course. She noted, however, she implemented others right away, too. She used the proportional reasoning book by Lamon (2005) as an example of a resource she used right away. Sam admitted that sometimes using ideas from this text meant telling students that she had something new for them, and "'We're going to see how it goes."

Sam also noted she was insistent this year that students use models. She began her year with a focus on modeling (e.g., hands-on manipulatives, diagrams, or pictures). Students did not get it initially, but by the end of the year she reported the students loved modeling problems. She noted at the end of the year when she gave math tasks to work groups, they would immediately engage in modeling the problem, even if she hadn't asked them to use a model. Interestingly, although she wasn't sure it helped them on the state assessment, she knew the experiences they had as mathematicians were more important and overall, more powerful. Sam also said she felt the students were much more flexible in their thinking than she; so, she loved seeing what students, especially those who didn't think themselves the best math student, would do mathematically. She hoped their confidence increased through these experiences.

There are several beliefs for Sam which have changed in the 3 years. These include, again by self-report, changes such as viewing math as a set of procedures and rules to believing math, "requires accuracy, but is fluid, flexible, and open-ended." Although she used to think sharing incorrect mathematical reasoning was bad and would

confuse students, she now believes it puts the focus on the process of mathematics rather than the product, helps bring attention to misconceptions, and can help build a sense of community by establishing trust and support.

Sam's journey within the andragogical framework. Sam's adult learning experiences were likely most determined by the core principle self-concept as a learner. She did not, in her early adulthood, demonstrate self-directed learning or an ability to identify or access helpful resources. Her references to not being ready for topics like algebra and her report that her hard work finally paid off in college and things started to "click" are clear indicators of individual learner differences. As noted previously, she grew into math in her college years. Once she experienced some success and determined what she wanted to do as a career, her motivation to learn, another core principle, was met. She saw the payoff of her hard work and that success certainly provided some positive intrinsic motivation.

Sam's continued growth from the time she decided to be a teacher then moved fluidly through the various dimensions of andragogy. The initial core principles were evident. She needed to know how to best support students. Her perception of herself as a learner of mathematics had improved, especially through her methods course. She was able to understand the lack of supportive experiences she had in her formative years and identify resources, or at least acknowledge that resources were available, which would help her understand mathematics. And again, given her chosen career path, she had a readiness, orientation, and motivation to learn.

In relation to individual and situational differences, Sam's account of her adult life addressed primarily individual learner and situational differences. She readily recognized how she learns in comparison to others, which also served to help her better understand her formative years of math education. She also spoke consistently to situations or environment created in the math methods course and the summer courses. She described safe and supportive environments in which exploration and mistakes were okay. Sam spoke to relevance of the content and the related resources and strategies which were shared. She also shared the positive learning experiences which arose in the completion of the lesson study cycles. These attributes which resulted in rich learning experiences and that she deemed important were only true in these specific professional development situations/settings.

The only area of the goals and purposes for learning dimension, based on Sam's responses, for which there is evidence is individual growth. Sam's participation in professional development opportunities, including the division sessions, she did primarily on her own. She had the support of her administration, but the administration was not asking her to attend. Sam clearly seeked opportunities to advance her own learning and professional growth.

Based on the information, Sam's needs for a productive development experience centered on providing her with engaging and relative tasks and providing her opportunities to rely on her relatively new self-concept to make sense of the mathematics. Given this, she likely made the connection to both her need to know and her orientation needs. She also clearly noted she needs a safe environment – one which allows her time

to account for her individual learning differences which are grounded in her willingness to be diligent.

Sam and productive disposition. An aspect of productive disposition which is evident throughout Sam's life is diligence. Through struggle and frustration, she continued to work at math. Although she had a few positive experiences in her formative years and was provided extra help and a tutor in high school, she did not feel successful. And yet, she continued to work at math. Her initial success in college was not noted by her as a change agent, rather she just summed it up by saying, "Things started clicking." The first real change agent for her was the math methods course. Her reflections provide evidence that she realized that math should make sense and there were resources available to help her make sense of mathematics. As she continued to be given new tasks through her methods course and the summer courses, her earlier diligence paid off. She spoke to this when she discussed facing challenging tasks, trying several approaches because initial attempts don't work, and knowing that ultimately, she will figure the math out. Through the methods course and summer courses she clearly saw herself as a learner and doer of math.

Given her formative year experiences, she also quickly saw what her own education was lacking and that she could influence students' success in math using the very strategies and tasks she had experienced. Her continued pursuit of new resources and ideas speaks to both her orientation towards professional growth and toward math teaching and learning. As previously noted, she now views math as a fluid and flexible, both in how it is taught and learned. She specifically spoke to her belief she can impact

students' success in mathematics when she talked about why acquiring resources was so valuable. She stated that the art of instruction is finding the resources which will best meet students' needs.

Although the usefulness and worthwhileness of pure math was only expressed by Sam in one instance, she spoke often to the usefulness and worthwhileness of her professional development experiences. Whether referencing the resources, strategies, or collegiality, Sam noted their value to her as a learner and teacher of mathematics.

Sam's productive disposition was centered in her belief in diligence. She learned perseverance was rewarded at a young age but as an adult still remembered the struggle this presented. The early methods course changed her experiences as a learner. Within the andragogical framework, this provided her new ideas from which to draw about learning and teaching mathematics. These types of continued experiences fueled her productive disposition, as did the observed success of her students (andragogical motivation to learn).

Paula. Paula went to college top become a math educator. She felt this was primarily due to her father's influence. He was an engineer, so math was prioritized in their household. Paula said her father was very supportive and, at a critical time for Paula, told her to work where she was in math – work with what was in front of her. She credits her father for creating a positive math experience for her and instilling a desire to pursue math. Paula has taught for 5 years. Although she has only worked in one division, she's worked in two different high schools.

Mathematics background and development as a learner and a teacher. Paula's overall math education background was traditional. She described her math courses K-12 as using the textbook curriculum and the teachers' use of direct instruction. The only exception to this were hands-on experiences in first-grade with tools like base-10 blocks. She also had profound test anxiety which caused some difficult situations during her education. The anxiety was caused by a pressure she put on herself to do well, combined with self-doubt. Her dad's continued support was a primary factor in contributing to her more positive math identity. An experience late in her college career also helped her finally see herself as a learner and doer of mathematics. This was followed by positive experiences in the summer graduate courses.

K- 12 math experiences. Paula recalled a lot about her first-grade math experience. She remembered using manipulatives, specifically hundreds, tens and one's blocks. She remembered using these to represent and count out ideas about place value. After this experience in first-grade she said it became a lot of seatwork and long worksheets. Everything was taught through a textbook. Paula remembered developing text anxiety, which became critical in fifth-grade. She was in an honors class and began performing poorly on exams. Her teacher noticed her anxiety and told her the advanced class was not worth the stress and anxiety it was causing. So, Paul was moved to the regular fifth-grade math class. The stress was alleviated, but she though it also meant maybe she wasn't good at math; she didn't belong with the advanced students. Paula remembers it was then, in retrospect, that her Dad's support was so evident. He told her, "You just work wherever you are . . . learning whatever track you're in, and you can still

do math." Despite the understanding of her teacher and support from her dad, Paula described the whole situation as traumatizing.

Paula didn't remember much about her middle school math courses, save they were very procedural, and again, seemed to be textbook driven. High school was the same, save she decided she needed to prove to herself she could do math and be with the advanced students. She continued to struggle with test anxiety. However, she was determined get "caught back up" and take AP Calculus her senior year. So, she took a course over the summer prior to her senior year. Upon reflection, she said there was still a bit of a chip on her shoulder regarding her fifth-grade experience. Although Paula now acknowledges the AP Calculus is an advanced course for a senior, she said she felt better because at the time she considered AP Calculus to be on grade-level. She also remembered it meant having an intimidating teacher her senior year; someone who pushed them and made her less afraid of failure. She failed the first test, but also learned from the teacher it was about defending the answer she provided. His strategy to grill them "forwards and backwards" made a student both want to be prepared and helped make failure less scary to Paula.

Preparation to teach math. Paula began college in the secondary math education program. She took various calculus classes, a class on proofs, and specifically remembers a Geometry class she believed was just for secondary math teachers. Based on her memory of the content, it was both a Euclidean and non-Euclidean concepts-based course. It was very hard for her, primarily because she had to apply what she'd learned. She did not feel you could study for the assessments. She remembered feeling paralyzed

when given a test. She did not do well on the first test and failed the second. When the third test session ended, the teacher offered her a chance to stay and finish after she internally focused. Paula earned an "A" on the exam, and by the final learned how to apply the concepts "under pressure." The greatest benefit, according to Paula, was mastering the skill of applying mathematics. She also learned from this experience that, "although failure is uncomfortable, it is not a permanent state."

Paula also took a methods course in college. The instructor had the students engage in activities which directly corresponded to specific standards in the state. The teacher was a member of the education department at Paula's university and had been a secondary math teacher for 15-20 years. Paula moved after graduating, so she did not teach in the state where she received her degree.

Professional development experiences as a practitioner. Paula, although she acknowledged her division offers numerous professional development opportunities, said division classes are hard to attend during the school year. She implied life is just too busy, and there is always a lot going on (personally and professionally). She also worked with a professional learning team which met once a week but based on her thoughts did not see these as opportunities for her own professional growth. Paula noted the most influential and helpful have been the summer graduate courses.

She appreciated that the content part of the course was during the summer. Specifically, she noted the ability to focus on the mathematics content without distractions, which she found beneficial. She learned that all students could be provided rich opportunities in math and that a single task designed purposefully could engage all

learners. Learning how to differentiate—again that she didn't need a hundred different activities, just a well-designed task—was key to reaching all students. Paula learned this works because students can talk about what they see—patterns, make observations, and build their ideas and reasoning from that starting observation. She also said the lesson study provided her with a lot of ideas and relevant information. She learned, for example, how differently proportional reasoning could be taught in the elementary schools, especially in comparison to how she learned it. Paula also spoke to the accountability associated with the lesson study—knowing she had to implement the strategies and tasks in her classroom. The combination of the lesson study team support and accountability to implement were also beneficial aspects of the courses to Paula. It was also noted she described in detail the job-embedded nature of the lesson study cycle with appreciation – for the process and its relevance to her classroom. She had participated in the summer courses by herself and with a colleague. She said although she learned a lot when she attended by herself, she felt attending with more colleagues assured continued support at the school site.

Although Paula recalled various examples of test anxiety, she could not recall any specific math task or concept which challenged her or made he uncomfortable mathematically. She said in response she feels this is because she developed a mindset or belief that it is okay if she doesn't understand something right away. It's also okay if she has a different answer or a less direct way to get to the answer. She now knows she can approach any problem with confidence. She even said some students may understand a

problem or task before her because they see something she doesn't or see the problem from a different prospective – and all of this is okay.

Paula recalled times during the summer course when participants shared their strategies. She was interested in how elementary teachers approached various problems, especially when completing proportional reasoning problems. She noted that the models created by the elementary teachers often enabled them to get to the answer more easily than the equation she used to solve the same problem. These experiences led her to a couple realizations: elementary mathematics is just as rich as high school math and students can make great connections and that varied strategies used to approach a task can make the task accessible at several grade levels, to include elementary to high. She had similar experiences in her classroom.

Changes in practice. Paula had a student this year that often saw patterns no one else saw; it made his solutions very interesting. Ultimately, the class would look at what the student did and try to figure out why his approach worked. She cultivated this differentiated approach to problem solving in her English learner double-blocked classroom. She had a student whom she believed got bored with some of the more procedural-based tasks and associated skills, but modeled complex thinking when presented with a rich task.

Paula said it took her awhile to be truly comfortable with a new approach to teaching given her own K-12 experiences. She said she did change her teaching methods and learned that to learn to teach well and reach all students, experience is critical. Trying new things and using strategies shared in the courses and by her colleagues was a key to

getting these experiences. She was fortunate to have good co-workers who helped her with strategies to support students.

Paula's journey within the andragogical paradigm. Paula's experiences in math from kindergarten to college included several challenging situations. These events, and specific people's reactions to them, helped define her adult learning core principles. Her father made it clear it was about doing the math in front of her – working at that math, whatever it was. From this she gained a self-directing concept of herself as a learner. She took matters into her own hands by taking a course over the summer and experienced an intrinsic pay-off when she proved to herself she could be in Calculus. Her calculus teacher challenged her but also helped her build a sense of autonomy and payoff for her efforts. She felt good about herself knowing she had made it to Calculus and learned to defend her thinking. She learned it didn't matter so much whether you were right or wrong, her calculus teacher expected students to be able to explain their thinking.

Paula's college geometry teacher helped her further define herself as an adult learner. She encouraged Paula to use her resources (in this case, the fundamentals of the class), apply them (self-directed and autonomous thinking) and addressed her individual learner differences in this dimension of andragogy by allowing her to stay and work on the test. The geometry teacher met a personality need within the individual learner differences realm: Paula's test anxiety. Again, Paula was provided with an opportunity to prove to herself she could do the math. Paula's continued growth as a mathematician and math teacher has been for the purpose of addressing her individual growth. Although at

times she has worked with her team or attended professional opportunities with her team, she does a lot on her own in her effort to best meet students' needs.

Paula had established a strong self-concept as a learner. She worked independently and held specific expectations which were evidenced in her young adulthood prior to her senior year. Paula's two primary areas of need were prior experiences of the learner and individual learner differences. Her high school calculus teacher and her college geometry teacher provided experiences in which she learned she could draw from her knowledge and resources to defend her reasoning and apply mathematics. These teachers also helped her address an individual learning difference, specifically, her anxiety. Although these were initial steps, it is also clear these are needs Paula requires to have met in professional development opportunities.

Paula and productive disposition. It is not surprising given Paula's math background that she believes in diligence for herself and her students. She expressed several times that it is okay if she or her students can't come up with a strategy that helps or a solution right away. She believes they are all learners and doers of math, and it just takes time to find the approach that works for the individual. She's learned that all students, accelerated and struggling, need tasks which cater to all the different needs. Students can make observations, identify patterns, and then students can build on what makes sense. She noted she's also learned some students will find a good strategy before she does. "Maybe some students might even get it faster. They might see something I don't, so [I am] okay with different perspectives." No matter the ability level of the student she knows she can teach them how to think and give them strategies to problem

solve – give them opportunities to make sense of the math and be successful. She also said failing is a critical part of the learning process.

Paula made an interesting observation when discussing her history and what she sees in her classroom. She believes her anxiety stemmed from a need to be right; she didn't want to "lose face" by making a mistake or failing. As a teacher she learned this is, in general, true of girls. The boys, however, in her experience are not as concerned. They make careless mistakes and have true misconceptions but by making those mistakes, they seem to learn faster. Based on her observations, "The more you fail, the more you learn. Just as long as you pick yourself up and you just keep going and going, that's how everyone learns." So, from her perspective, to be a learner and doer of mathematics you must be okay with mistakes and be willing to fail. These ideas are part of the malleable process toward growth and successful learning of mathematics.

The experiences she expressed regarding teaching, which included providing varied strategies and opportunities for students to be successful, are evidence that her need to acquire new prior learning was met. Through professional development she has acquired new ideas which guide her decision making and from which she can draw as she and her students learn together. Her anxiety, which was rooted in a need to have everything correct, is also evident in her approach to individual learning differences – hers and her students. She reported she is okay with students understanding something first or doing a task differently; she now sees it as an opportunity to learn. Moreover, she wishes more students (especially girls) were willing to make mistakes as she believed more learning happens from mistakes. This is clear evidence that she had experiences

which helped her address the root cause of her anxiety, which again was an individual learning difference.

Edward. Edward considered himself a good math student throughout his K-12 experience. Although, he noted he just "muscled through" the curriculum. He earned an undergraduate degree in mathematics. However, he had no intention of teaching. He found out a school needed a temporary math teacher, so he applied and was hired. Edward said he immediately felt this was his niche and pursued a full-time teaching job. Edward taught for 5 years in one school, Grades 5-8 (fifth-grade math through Geometry). The school was a private Catholic school run by the diocese.

Mathematics background and development as a learner and a teacher. Edward's K- 12 learning experiences would be considered traditional. He remembered feeling the math classes moved too slowly early on and that ideas were just repeated throughout elementary school. In middle school, he realized some of his peers were learning things that were more advanced and in retrospect, that he made assumptions about how math worked and assumed he was right. He enjoyed high school overall, both the curriculum and some of his teachers. During his sophomore year of college, he had to pick a major, and he chose math because it had the fewest requirements. He walked into a temporary teaching job and decided based on this experience to get a master's degree in education and teach math.

K- 12 math experiences. Edward remembered what had to be one of his first experiences in math. The teacher asked the students to determine the number of jelly beans in a container. He was certain he was going to win. Although someone else won,

he thought it was cool at the time that a teacher presented them with such an open-ended question. After this initial experience, he found math tedious. Teachers asked him to repeat operations but use bigger numbers. Class did not move quickly enough for him. He also continued to wonder why he was not in the gifted program. His reprieve from the tedium came from his mother. She asked him to solve math problems while they were in the car. He said he taught himself how to do those problems and enjoyed doing it. His mother also had books he could work in, and he specifically remembered trying to teach himself double-digit multiplication by following the instructions. He remembered that at first it confused him but doesn't remember if he eventually figured it out.

In middle school, sixth-grade specifically, a friend asked Edward if he knew the value of 10 to the second power. Edward remembered saying, "20," and his friend said, "No, it's 100." He remembered he immediately wanted to know why but doesn't remember when he learned why. He remembered the content of his middle school classes, and part of his introduction to algebra in seventh-grade. The teacher asked them to combine like terms in a four- or five-term expression. He said he was cocky and decided he just needed to put everything together in one term. Edward remembered a female student, who he did not feel was very smart, got the correct answer. Again, he wondered why that answer was correct but again didn't remember learning how to combine like terms. He also got involved in math contests his seventh-grade year, sometimes outperforming the gifted students, although he attributed this to his ability to make better guesses. In eighth-grade he was placed in Algebra I with gifted students and

remembered that he was happy. The algebra teacher was a good teacher and Edward did pretty well, but his dominant memory was he was happy.

Edward liked his high school courses. In Geometry, he enjoyed making conjectures and coming up with theorems. His teacher gave him credit for making conjectures which Edward said, “was nice.” They built a bridge in Geometry, which Edward questioned at that time because he felt the project had more to do with physics, so he didn’t understand why they did the project in his Geometry class. Edward didn’t really study for his high school classes but still maintained “B” averages. In Calculus, the teacher gave problems of the week. Edward was determined to solve every one of these problems. He appreciated that the teacher offered something extra which was different from the day-to-day work.

Preparation to teach math. Edward earned a bachelor’s degree in mathematics. His sophomore year he chose math because the program had the least number of requirements. Edward also felt he could “muscle through it” as he was able to “muscle through” math before. He felt at the time either you understand math, or you don’t, regardless of effort. He continued to do well, despite a weaker work ethic because he understood it. After graduating he “walked into” a temporary math teaching position. Edward felt the first day, he may have found his career. He recalled he had an epiphany in the first few weeks when he realized he was “called to do this.” He enrolled in a master’s degree program in education and completed his degree while teaching. The program included a methods course, but he did not have any specific memories of the course.

Professional development experiences as a practitioner. Edward helped revise the standards in the curriculum. He also attended half-day in services offered by the diocese. He became a member of NCTM and attended a regional conference. Edward's only real take-away from all these experiences was the opportunity to meet other teachers and hear what they were doing. As the sole middle school math teacher in his school, collaboration at this event was important. Edward attended a session on reversibility – how to turn a question around to prompt student thinking. For example, rather than asking the perimeter of a rectangle with side lengths two and five, asking the possible dimensions of a rectangle if the perimeter is 14. He realized this strategy could be applied to any problem and used this right away in his instruction. There were a few other take-aways from the conference which he still uses.

Edward completed the summer graduate program courses also. Again, as the sole teacher in his school until this past year, he looked forward to the conversations with other math teachers. He wanted to know what worked for them, with what they were struggling, cool ideas they had, what technology they were using, etc. Just the chance to be collegial was a benefit. Edward said the last time teachers from various schools within the diocese were together was essentially a direct instruction session. He kept thinking, "I hate this. Just break us into groups, and it'd be so awesome." He found humor in the fact that unlike the summer graduate courses, the presenter was not modeling what he was asking the teachers to do. Edward acknowledged seeing and experiencing what a classroom should be like was important.

Edward remembered a task in the summer graduate program in which they were to model fraction operations; they were to try and model multiplication and division. At the time, he felt the exercise was a waste of time and just wanted to use the rule because he didn't know how to model it and thought it would take "forever." Edward now finds it funny that he recognizes this experience as why he's "sort of converted." He realized the modeling and sense-making of the concepts lent themselves to developing and understanding what he knew only as a procedure. This task and others like it, in addition to observations he made of his students, are what prompted him to implement these ideas into his practice.

Changes in practice. Edward knew not all students were understanding fraction operations. They didn't know what they were supposed to do, if they knew what to do, and they didn't really understand why. Through his experience in the summer graduate class, he realized modeling helped the process make sense. He used this strategy later with students, although believes it was a gradual introduction and process rather than purposefully implemented with complete immersion. Edward would rather now spend a month working with similar ideas because he saw it helped the students develop better number sense. He now thinks just memorizing steps is stupid and wants his students to develop a level of understanding where the step they should take is intuitive, but when asked they can explain why they are choosing the step. He also sees this kind of reasoning as a student's' opportunity to develop more efficient methods/procedures.

He used several other ideas from the summer graduate course, but in general, he said they are introduced gradually. He exposed students to manipulatives, often "just

winging it in the classroom." For example, he told a class they were going to use base 10 blocks to think about long division. He told them he knew it would work and that they would just figure it out together. Edward began making changes the first few months into teaching because he realized what he was doing was not working. He knew there had to be a better way and started trying new things and using the ideas he gleaned from the professional development activities. It was not necessary for him to be comfortable with the resource or strategy first; he just kept working to help students understand the mathematics. Therefore, he said the change has been gradual and continual.

Edward's journey within the andragogical framework. Edward's primary attribute within the core principles is that he was an autonomous and self-directed learner. He said a few times he did and could "muscle through." Although in his formative years he often wondered why or how in relation to math or a specific math problem, he did not follow up or remember learning the why or how. As an adult that same need to know emerged initially because he felt there was a better way to teach students math. It is likely the other principles happened simultaneously or quickly followed.

The situation in which Edward found himself, the only middle school teacher, prompted him to find a different situation in which he could learn more about teaching and get more resources to support students. He often mentioned the excitement he experienced meeting and collaborating with other teachers. He said the best part of the summer education course was there were other teachers from other schools in the diocese for which he worked. He was able to collaborate with them and see what they were doing with students. Edward also noted the recent addition of another middle school math

teacher at his school has greatly enhanced his professional situation. The two teachers worked with the manipulatives and made their own connections and realized why procedures work. Edward said they almost had more fun than the students.

Edward's growth has been primarily driven by his individual need to learn more to support students. He did attend the first summer course because it helped him meet a licensure requirement, but he went back because of what he learned and gained from the experience. Although other teachers whom he knew from his sister schools attended also, which may imply institutional growth, he did not know they planned to attend until he arrived for class the first day. His goal and purpose for attending and learning can be fully attributed to his desire to grow as an individual.

Edward, as with the first two participants, had a strong self-concept as a learner. The initial needs he had related to the core principles were readiness to learn and orientation to learning. He saw his students were struggling, which was a life-related situation, so he was ready to learn. Edward was looking for things which were immediately applicable (orientation). As noted, several times, he did not need to be comfortable with the tool/resource, he just needed to know it would work, and he would implement it and "muscle through" with the students. Finally, he had a situational difference need; he was an isolated teacher and sought collegiality and collaboration.

Edward and productive disposition. Although Edward said he muscled through in both his formative years and college courses, his description does not really speak to diligence. It did not appear he did this from some sense of determination to figure it out. In fact, despite memories of specifically wondering why something worked or why his

answer was incorrect, he did not remember working to figure it, or even when he figured out the related math. This expression more likely speaks to thinking of himself as a learner and doer of math. This continues to be evident in his descriptions of lessons implemented with his students.

Edward viewed his students as learners and doers, and as he expressed a few times, was willing to learn and do the math alongside them. He provided a couple specific examples in which he supplied a manipulative to work on a concept and said they (he and the class) would figure it out together. He did not approach the lesson with a specific plan about how it would work (e.g., the lesson with the base 10 blocks), but he was confident it was the right tool and that they would make sense of it together. This directly lends itself to his belief math should made sense to both himself and his students. He discussed his "conversion" catalyst – modeling operations with fractions. The experience provided him the opportunity to make sense of working with fractions and further helped him understand why the procedure works.

Edward often expressed dichotomous beliefs about students' abilities to be successful in mathematics. Several times he said students get it or they don't; this included when he reflected about his decision to major in math. He just "got it" for the most part and would push through what he did not. He observed a few years ago working with adults that, "Wow, some people [are] never going to get it. They're really just limited in how they understand." However, he also noted he had a female student who really struggled in math, and this past year was one of the top students in algebra. "I definitely see kids who've advanced so much. And so, the flip side, there are kids who

graduate . . . and I'm like, 'Wow, this kid's really owned it.'" Upon further exploration of this idea, he expressed all students can learn math. What he realized over time, and in part thanks to the use of some smart responders, was that students have holes in their learning. It is difficult to identify and address all the holes. So, he believes students can be successful in math, and there are tools to support their sense-making of math, but he feels it would take him having 1:1 time with each of them to identify their deficits/holes and how to address them. "It started with those responders, and realizing . . . there's something wrong, these kids just never got it. You have to focus more than just a day of teaching procedures before kids actually understand a concept." Edward now spends a lot of time asking students what something means, why their answer is correct, how they know, and telling them if they can't articulate their thinking, then they don't really understand the math. He pushes them toward explaining their thinking.

Overall, Edward had a very malleable orientation toward math and math teaching and learning. His confidence, or self-described cockiness, enabled him to take a resource with which he's less familiar, but intuitively knows is appropriate, and explore mathematics with his colleagues and students. Edward believed all students can learn mathematics but struggles with how to address the deficits individuals have by the time he has them in class at the upper-middle school level. So, he muscled through trying to figure out how best to support all students.

Edward's professional development experiences have met his andragogical needs of providing relevant and immediately applicable information. He noted his students' overall improved performance and a few individuals who made great strides – not to

mention his own growth mathematically. It is clear he embraced any opportunity to work with colleagues, thus meeting his situational difference need and further promoting a productive disposition. Edward noted the professional development must actually provide opportunities for colleagues to engage with each other – not doing so frustrated him. The needs Edward currently expressed were again based in readiness, orientation and situation. Edward continues to struggle with the dichotomous reality that he believes all students are learners and doers but also believes he does not have the time to identify and address all the gaps in each individual student's learning and get them to grade level.

Amanda. Amanda was a good math student from kindergarten through college and majored in mathematics. She did not have any real instances of struggling mathematically, save a few more difficult problems here and there which she was able to resolve with some help from her dad or a night's sleep. Her first exposure to manipulatives and varied strategies did not occur until her methods courses. However, they were still primarily theory based. Her greatest learning experiences occurred when she became a practitioner and realized more needed to be done to reach all learners.

Mathematics background and development as a learner and a teacher. Amanda's K-12 experiences were extremely traditional, and she has very few distinct memories regarding those years. She was a good math student and knew entering college she wanted to be a math teacher or math coach. College course work was her first real exposure to manipulatives and true sense-making of mathematics. Amanda has taught math for 11 years, the last 2 years she worked as a math coach. She taught upper level

elementary students and was a coach for Grades K-5 teachers. All her teaching experiences have been in one school.

K- 12 math experiences. Amanda recalled little about her formative year math experiences. Teachers modeled problems at the board – everything was procedure based. She brought a text book home and worked the assigned problems. She was certain there was some use of place value blocks and that she had picture representations of them. However, Amanda did not use concrete models or manipulatives, or not that she remembers. She progressed mathematically, completing Calculus prior to high school graduation.

Although Amanda saw her friends and classmates struggle throughout her school tenure, this was not the case for her. Sometimes a topic, and more likely the last few problems of an assignment, would be difficult for her. She took the work home, and her dad would help. This often led to frustration because her dad would solve it and show her what he did, but she would say, "I can't get it that way. That's not how my teacher showed me to do it." This led to further frustration and sometimes long nights and tears. But they worked together until she was able to solve the problem correctly. She was also aware her friends were not, in comparison, intuitive mathematically and hated math. She at least understood what she was supposed to do according to the teacher, and it was a matter of addressing just the few problems she did incorrectly. As an adult, she believes what the teacher demonstrated was THE way the teacher knew how to do the problem and that fear kept the teacher from student exploration; fear the teacher wouldn't

understand the student's approach or that the approach wouldn't work, and the teacher wouldn't be able to help the student.

Preparation to teach math. Amanda intended to be an elementary teacher or math coach. The elementary education program was a master's degree program which required the students get a bachelor's degree in a content area. She chose math because she had always done well, enjoyed math, and thought it would make her more marketable. So, Amanda took all the math courses required to earn the bachelor's degree. The college math courses were also easy for her. She said she had to buckle down sometimes and work hard to get through it. She also remembered going to sleep with a problem on her mind and a light bulb going off the next morning. College was the first time she experienced this kind of delayed understanding, but what she liked about math was that there was always a way to get to the answer. Her undergraduate program rolled right in to a master's in elementary education program.

Amanda was required to take a math education course and two math methods courses as part of the program. During the method courses, she was exposed to manipulatives and questioning. She specifically remembers discussing the work of Madeline Hunter. Her reaction to this was two-fold given that she was not exposed to these during her formative years. Sometimes she thought, "Ohhh, this is why some of that works." On the other hand, because math had always come easily to her and for the most part she understood the "why," she felt the work was extraneous and made it take a lot longer to get to the answer. "But at the same time, this is cool, how you can figure it out." However, the majority of her time was spent learning about various learning theories.

This was obviously not the same as seeing some manipulatives used in practice or how they are implemented and work for students; she also realized the videos modeling the theories with the perfect classroom with five kids did not prepare her for the realities of a classroom.

Professional development experiences as a practitioner. Amanda read a lot of blogs to get ideas, specifically looking for alternative methods to teach concepts. She participated in several opportunities through her division and took online classes. The opportunities she sought had a math focus or were aimed at how to better support a subgroup population (e.g., English language learners); she chose the sessions of the latter type because of the demographics at her school. Amanda also participated in the summer graduate courses.

Amanda found these experiences important because they kept her up to date by providing her with fresh information or important reminders. Sometimes she returned to her school with a different graphic organizer to use in an already planned lesson or a new strategy or manipulative to incorporate. The experiences also provided her with different approaches to the same task across grade levels. Because she worked at the elementary level, she appreciated the approaches of teachers who worked up through the high school level. She observed some teachers taking an algebraic approach, and because of her current placement and experience she did not approach it algebraically, although she understood the representation. She was also surprised to observe that elementary approaches, which used pictures and modeling to solve complex tasks, actually helped elementary teachers arrive at an answer prior to their high school counterparts.

Amanda left every professional development experience with something – a new skill, resource, strategy or a different way of thinking. Therefore, she did not feel there was one course or activity that changed her life. Most impactful to her teacher career were her teaching experiences. She had to "dig down to the base level" to help students be successful in math because they had not mastered content from 2 or 3 years prior. It was these experiences and working to help a variety of students that proved to have the greatest influence on her teaching style and day-to-day decisions.

Changes in practice. Amanda knew the changes in her practice were directly influenced by her experiences teaching and the professional development activities. She quickly learned she had to meet students where they were and then make the grade level objectives as accessible as possible. The professional development activities in which she participated provided her with resources and strategies to help make the math accessible. She knew what she was learning had to be implemented in her classroom immediately and purposefully in order to best help students.

Amanda constantly incorporated new ideas in her classroom and helped fill in missing math background. "Every experience is different. I taught three sections of math this year, and literally every class was different. And I had to prepare for every class in a different way, even when I'm teaching the exact same standard." For Amanda, this meant introducing different manipulatives and strategies for each class. To meet one class where they were, she spent a significant amount of time on denominators which was not a grade-level objective. The efforts paid off; she reflected, "We were able to do [grade

level] thinking through word problems using the like denominators ,and when we transitioned to unlike denominators, our life was a whole lot easier."

In a particularly weak class, Amanda and a co-teacher completely changed the environment and approach. They created learning centers, used every hands-on manipulative in their classroom, taught to small groups, used sentence frames, created visuals, provided scaffolded notes, and tried to decrease the cognitive load in other areas so the students could make progress in math. The co-teachers noted the students' willingness and that the students' confidence grew, which in turn fueled the teachers' efforts to make the math "as accessible as possible."

Amanda also noted the impact that varied strategies and differentiation can have for the students. There are tasks she gave her class that she previously believed her students would not grasp, especially without some basic algebraic knowledge. She observed that although higher-level gifted students struggled because they were looking to tables and doing the problem numerically, there was a weaker and less confident math student who solved the problem quickly using pictures. She asked all students to look at the thinking and make sense of what was in the picture. When the class confirmed the thinking and solution were valid, she reminded everyone that pictures were their number one tool.

Amanda's journey within the andragogical framework. As described, Amanda has always been a good math student who almost always understood the "why" behind the math. Although she wanted to be a math teacher or coach, initially it was not because she knew her peers in school struggled or because she thought it could be done

differently or better. She had a comfort level with math and a desire to help others learn math. Given this, as an adult learner, her need to know came about because her early experiences in teaching provided evidence for her that not all students understood math, and she wanted to "fix" the situation. Although she had experienced many successes, experience is still the driving force for her need to know. The other principles then follow closely behind, to include the motivation to learn which is an intrinsic reward when her students do well.

The situation Amanda was in, specifically, the demographics of her school, presented its own unique set of challenges. She worked with a high percentage of various populations who struggle academically. Amanda was fortunate as she worked with many teachers she considered "like-minded" and not "old school" because they supported the same kind of math thinking and instruction Amanda wanted for her students. Situationally, she was in an environment which forced her to think differently about instruction and seek resources but was also an environment that was supportive and collaborative.

Amanda's primary purpose for learning was her own individual growth, with an ultimate goal of improving her students' mathematical understanding. "You have to have some pride in [the students' thinking and engagement and perseverance] and satisfaction that you're helping them feel success or challenging them to think in a different way." Amanda recognized the process is a never-ending journey, and so there is always room for her to grow professionally. She expressed her learners' needs to know more. "[My students' recent success] provides validation for the strategies and skills and an approach

of teaching; things we did worked, but there's still room to grow. We didn't all make it, so what else could we have done for some of these other kids that didn't make it?" Amanda's observations of success for herself and her students feed her belief that all can be learners and doers of math.

Amanda presented herself as someone confident in both her knowledge as a math learner and teacher. Her enrollment in professional development was primarily to meet a readiness to learn need. She had life-related circumstances for which she sought relevant information; in this case different strategies to meet the needs of her diverse learners. She was also fortunate to have a supportive and collegial environment of "like-minded" teachers. Although her efforts met the needs of many learners, she acknowledged not all students did well. Therefore, her readiness to learn continued to be her primary andragogical need.

Amanda and productive disposition. It was evident in Amanda's history that she always considered herself a learner and doer of mathematics. She knew that some of her friends were not successful in mathematics and some hated the subject. Prior to teaching, her belief was students needed to be directed through procedures and have a lot of time to practice. "Repeated direct exposure to procedures and processes" were needed to learn math skills. Once she started teaching, her orientation toward learning and teaching changed. Amanda learned quickly that what students needed was a resource, a visual, or another approach. She actively sought and used a variety of resources to help all her students become learners and doers of mathematics. Amanda said, "[What motivates me] is something that looks at what everybody can be doing and how this can work for

everybody or can be tweaked for everybody." Her productive disposition is derived from a cycle of professional development which provided her with resources, which in turn helped her students be more successful but also helped her further identify deficits and misconceptions, which then took her back to professional development and seeking more resources.

Amanda said one of the most difficult challenges is working with students who are not ready yet and the accompanying curriculum expectations and state assessments.

> Sometimes kids aren't ready yet. You're not ready to multiply fractions, you're not there but I have to teach that and push this test down on you when I know we would be better served spending our time doing other things that you could build yourself up on. (Amanda)

As noted earlier (denominator work), Amanda did her best to use a manipulative to help students make sense of and learn the area of deficit, while addressing the given curriculum. Amanda also said that although her teacher team discusses objectives and appropriate activities, there isn't time to talk about student misconceptions. She felt fortunate because she can fairly quickly determine students' misconceptions or confusion, but she also felt teachers need more time to explore individual student's thinking and collaborate with colleagues about how to best address the deficits/misconceptions. It's discouraging to her that if teachers don't know what the student did and, "don't understand why they're doing that and just say [here's how you do it], it doesn't help a kid not make that same mistake again." Amanda viewed student thinking as another valuable resource to inform instruction.

Amanda's need to receive relevant information was consistently met by most of her professional development experiences. The information quickly translated to the other core principles for Amanda. She noted, specifically a motivation to continue on her growth trajectory based on her students' increased understanding and improved performance. Additionally, her professional development experiences were further supported by her school team. These experiences promoted a productive disposition for her toward professional development. She spoke directly to a cycle: readiness to learn need (andragogy), and professional development met need by providing relevant and immediately applicable resources. When she implemented resources, there was increased student understanding; however, some students still struggled so there was a new readiness to learn need. Thus, she was back at the beginning of the cycle.

Ricki. Ricki had several mathematics experiences which were difficult, but overall, she was a good mathematician. She did not get a degree in math or teaching initially. However, life circumstances provided an unconventional path to teaching math. She has now taught math or been a teacher leader of some sort for 11 years. Ricki worked for three divisions in seven different schools. Her professional experiences included working with students in Grades K-6 and teachers.

Mathematics background and development as a learner and a teacher. Ricki was a good math teacher and consistently in honors or advanced courses. However, there were a few key events in which she went to a less challenging honors course and these events caused her some frustration and doubt. Ricki earned a degree in history, due to life circumstances, and got in to teaching when she applied to an emergency teacher

credentialing program. Her initial methods and training courses in her first couple years of teaching did not provide much guidance and support. However, soon after, Ricki found herself in a series of rich experiences which shaped her as a teacher, instructional coach, and resource teacher.

K- 12 math experiences. Ricki considered herself a high learner in the area of mathematics in elementary school. Her favorite activity was the mad math minute because computation came easy to her. This was due to her mother more than her elementary school experiences. When Ricki's parents moved to the United States, her mother became a certified public accountant. Ricki's mother would often provide her with math problems involving coins; Ricki wanted the change her mother would get at the store, so she could go to the gumball machine or get things form the Hello Kitty store. Ricki would try to figure out how many of each coin was needed to purchase something. Her parents and other family members expressed fascination that she was so "flexible with numbers." She went into a private school with this "flexibility with numbers" and so often felt she was "above whatever was being instructed." Her dad, who was a high school counselor, noticed Ricki was getting the same homework as her peers and decided she needed to be in a gifted program, so they moved her to a public school.

In the fifth-grade gifted class, Ricki was placed in the lowest math group, and was shocked. The lowest group was doing sixth-grade math, but Ricki was devastated she was not placed in Math Seven, even though she was still in an advanced group. Her father was determined she be in Algebra as a seventh-grader, so she was one of two seventh-graders placed in Algebra with eighth-graders. She was not interested in math but in socializing

(it should be noted the school hosted Grades 7-9). She had a "C" or "D" by the end of the first six-week marking period. The teachers recommended she go to the Grade 7 honors class. She felt like she failed her parents. She did well in the class, and her peers thought she was amazing because she knew so much math. She was comfortable and relieved by the new placement. Algebra I was much easier for her as an eighth-grader. She was promoted to Geometry for her ninth-grade year.

Ricki's parents had similar expectations of Ricki's sister, so Ricki was in Geometry with her sister who was in eighth-grade. Ricki struggled in the course. She hated the text and remembered it was small, thick, and brown; she could clearly still picture the book. The book was full of theorems, proofs, words, and mathematician's' biographies. She remembered the teacher, too. She did not have a lot of geometric experiences prior to the class, really hated the class, and ended up not doing well in the class. Her sister received an "A," which further discouraged and depressed Ricki in terms of her math ability. Ricki recognized as an adult that she has weak spatial awareness skills and feels robbed that she was never exposed to manipulatives or nets to help her make sense of math. Instruction consisted of explanations and examples followed by independent work.

In high school, Ricki took Algebra II/Trigonometry in the International Baccalaureate (IB) Program as a 10th grader. Ricki grew up on the West Coast, and her teacher was from the East Coast; her accent, hair, lipstick and nails all fascinated Ricki. Her teacher was also her field hockey coach, and Ricki developed a good rapport with her and still feels the teacher was the most influential person in her math journey. The teacher

provided the fundamentals of a concept, expecting notes to be taken. She then expected students to dialogue about a related task. After exploration of the task, the students received problem sets. The students could talk and work together. Ricki did well in the class. The following year Ricki was placed with another teacher for IB Math Analysis who had a very different approach to teaching. The teacher did not develop a rapport with students, the students were in rows, she lectured, and students couldn't talk. Ricki's grade slid, and Ricki told her parents, counselor and principal she wanted to take Math Analysis with her 10th grade teacher. She is still surprised she said this to her parents and that they allowed her to change classes. Again, the change was a relief. Class was dialogical again and felt comfortable to Ricki. The next course given her track was IB Calculus. Without telling her parents, she decided not to take IB Calculus; in part because she did not like the AP Calculus teacher and figured she could take Calculus in college. She enrolled in IB Statistics and earned an "A."

Preparation to teach math. Ricki went to college intending to earn a degree in business economics (5-year program). She met her now husband in college, and her parents told her they could not get married until she had a degree. She earned a degree in history, even though it is her least favorite subject, because she could write well (a lot of blue book assessments), and it would only take 3 years to complete the degree. She didn't know what to do with the degree. Ricki heard about an emergency credentialing program for teachers. Her husband, who had taught high school, didn't think high school would be a good fit, so he encouraged her to get her elementary teaching license. She started teaching while earning her license.

Professional development experiences as a practitioner. Because Ricki's training really started as she was teaching, her preparation to teach math and professional experiences as a practitioner overlapped. She was required to take two methods courses as part of the credentialing program. Both classes were terrible. One was just terrible overall. The other was terrible because the manipulatives used in the course were not available at her school. The manipulatives were meant to support a specific program, which also was not used at her school. Ricki realized years later one of the manipulatives was a set of Cuisenaire rods. The teacher told them if you don't get anything else, get these. They did not have them at her school, and Ricki would not see them again for many years. At the conclusion of her program, she felt "completely ill prepared to teach math."

Ricki and her husband moved to the East Coast, and she taught in an English Language Learner program for a year. She realized she missed teaching math, so asked to do so. Again, she felt relief. However, five of the teachers decided to use flexible math grouping, and three, including Ricki, were left out. In the end, Ricki was glad she was not involved because she was able to work with her students in the way she felt was most helpful. Her students did well, if not better than some of the other students. After a couple of years, she decided to apply to be a math lead for the school. She was awarded the position and required to go through a lead teacher course.

The lead teacher course was the first real experience which provided her the knowledge and support she needed. The course covered both pedagogical and content knowledge that she "had been missing." Also, the manipulatives that were used were in

her classroom and available to others at her school. The course and materials, and the fact that the classes were after school, enabled her to dig deeper into the content and explore ideas she didn't have time to during the school day. Ricki took the course over the school year and then shared what she learned the following summer with the teachers. Two key things happened for her as a result: she was viewed as a leader at her school and she experienced this "complete other world" that she had been missing.

Ricki also had an opportunity through the National Security Agency (NSA). The NSA asked the teachers collaborate to write lesson plans to help teachers everywhere enact math. The lessons were posted online for teachers to access. The NSA also provided resources from respected publishers and the NCTM for the teachers to consider as they wrote the lessons and the teachers could keep them. For example, they received a set of decimal base 10 blocks. During this time, she also worked for the division creating state crosswalk maps (mapping previous learning targets within a curriculum framework to the new targets adopted by the state).

Ricki's family moved out of state and she did not teach for the next 10 years. When they relocated back to the area she'd last taught, she applied for a math resource position. The position was funded through a grant from the Department of Defense Education Activity (DoDEA). The grant provided a coach from Math Solutions to work with Ricki directly for 58 days. The direct guidance and support deepened both her pedagogical and content knowledge. One of the resources they explored was a text about linear models for fractions. The coach, regarding a lesson in the text, said they were going to model a problem using Cuisenaire rods. Afterwards, she and Ricki searched the

school building as the coach was sure the school had sets of the rods. They found at least 15 sets, unused. A critical piece of advice Ricki was given by the coach was to "be in the content," so Ricki immersed herself. The coach ended up working with Ricki for 4 years.

As an instructional coach and math resource teacher, Ricki also participated in professional development provided by the division and attended the NCTM conference. At the math conference, Ricki spoke to another resource teacher who recommended Ricki talk to a professor about a doctorate program. Ricki met with the professor and enrolled in the summer graduate courses and eventually became a knowledgeable other/lead for lesson study teams. In addition to the immersion in the content, Ricki stated the lesson study cycle was valuable for her and the teachers she guided. She said it was important the facilitators model the entire process for the teachers and ensure the teachers did the math themselves prior to the lesson. The conversations were rich and informative and helped the teachers be successful with their lesson study cycles.

One opportunity that challenged Ricki was a graduate Geometry course for math teachers. Again, she did not feel successful in her high school Geometry class so there was already some anxiety or trepidation. During one particular class, the instructor drew dotted lines in relation to a task on the board; what others clearly saw/inferred from the original information on the board was now clear to Ricki. When the instructor drew the broken lines, it felt to Ricki as if "this veil had been lifted . . . I could access the math."

Clearly, Ricki had multiple professional development experiences which contributed to her professional growth. Ricki most appreciates being engaged in the

content and identifying ideas, strategies or resources she can immediately use to help students or "build capacity in other teachers."

Changes in practice. Ricki's experiences as a practitioner led to changes in both her classroom and in the guidance and professional development she provided for teachers. As a teacher, Ricki used a variety of manipulatives and instructional strategies to help students access and understand the content. A specific memory she had was the use of the Rekenrek rack. The students were using them to explore a lesson and during the number talk that followed, a student said the strategy they used was the commutative property. She said, "This is a highly transient school, and kids are talking about the commutative property because they see it on a Rekenrek."

Ricki used the advice from her coach to immerse herself in the content with the teachers, too. She encouraged teachers to engage in the math with the manipulatives they should use with the students. She exposed staff to Cuisenaire rods and had them explore various tasks. The teachers were excited because there were concepts that made sense to them for the first time. The teachers continued to work at the math. She observed, "They saw themselves engaged in higher-level tasks and higher-level thinking, and we gave them time to explore the tools. They weren't skeptical, and then once they started engaging with these tasks, they were sold." At the end of the professional development, all the teachers wanted the rods for their students. Ricki worked with administration to get Cuisenaire rods for the teachers.

Ricki's journey within the andragogical framework. As an adult learner, and whether she was aware or not, Ricki was drawn to math both because she was a learner

and doer and because she realized she did not make sense of math the way most of her peers had during her formative years. Throughout the interview she specifically stated, or shared feelings and thoughts which informed, the state of her math identity at different times in her life. Within the core principles she seems primarily driven by her motivation to learn. This motivation was sometimes based in the intrinsic value of her efforts—proving to herself she could do the math or make a difference in the lives of students or teachers. Other times, as often expressed in her interview, her motivation stemmed from a need for a personal payoff. In Ricki's case, the personal payoff was to feel seen, valued, appreciated and recognized for her efforts. Ricki also believed her learning style was very self-directed (self-concept of learner). In the survey in relation to several questions which she interpreted as traditional teaching and learning strategies, she simply wrote that now she thinks specific topics were "not my style." This also clearly pointed to her recognition of her individual learning differences.

As an individual learner, she believed she approaches math differently than most individuals. As an adult, she recognized that in high school she needed math to be task and discussion based to do well. She still believes her weakness is geometry. She gave specific examples to confirm she is not spatially aware (e.g., packing items in Pyrex containers) and that she still does not understand fractions using Cuisenaire rods without a lot of work. She even gave reasons she felt the teachers with whom she worked made quick connections with the rods when she could not. Ricki also spoke to situational differences which matter to her. Growth happened for her when she had the opportunity

to explore and discuss with others whether in a classroom, professional development setting, or being coached.

Examples of both the individual differences and situational differences are the two times in her life when she was exposed to Cuisenaire rods. The first time was in her methods course which was part of her emergency credentialing program. The teacher taught them directly, and she didn't recall time to explore tasks using the rods nor time to discuss them with classmates. Although she stated the instructor emphasized their importance, given her background and struggles in geometry and with spatial reasoning, she was not open to learning about them. Finally, they were not something she had at her school. Fast forward to just under 20 years later. Ricki had experienced various approaches to teaching and learning and was more available to alternative approaches. She worked with a personal coach on Cuisenaire rods, modeling different tasks and discussing the use of this resource/tool. Although she admitted she still struggled with them due to her self-identified deficits, she saw the value of using them for other learners. She then sought them out in her building and encouraged a principal to buy them for the staff. The emotional place she was and her math self-concept the second time, coupled with a supportive and collaborative situation made a huge difference for her in terms of her own readiness and willingness to learn.

Ricki's goals and purposes for learning were primarily based in individual growth. In some cases, it related back to her self-concept based in proving that she could do the math and have a positive impact on others. Certainly, her division, as an institution, was looking for ways to grow and build capacity, but initially she sought out

those opportunities for her own professional growth. Over time, Ricki wanted to lead the institutional growth and expressed a desire to build capacity. Her purpose is now two-fold: individual growth and institutional growth.

Ricki demonstrated a need to have her individual, subject matter, and situational differences met as a learner. She clearly articulated examples in which these needs were met: high school teacher who created dialogical learning environment, one-to-one mentor who took time to model and experience the math with Ricki as a learner (e.g., with the Cuisenaire rods), and the geometry teacher who drew in what Ricki could not infer from the drawing on the board. Having these differences acknowledged, as well as providing feedback which contributes to her self-esteem (motivation to learn) are critical needs Ricki must have met to establish a productive disposition.

Ricki and productive disposition. Ricki often spoke to her strengths and weakness as a math learner and doer. She was very clear that as a learner and doer, she was successful in a variety of areas of mathematics. However, she continued to identify geometry, due to her deficits in spatial reasoning, as an area of deficit. Ricki's diligence is the reason for her continued growth and progress in geometry. Her experiences, self-concept, and math identity play out in her productive disposition. Ricki believed all students and teachers can learn and do math. Given her experiences, she said in a variety of ways that math is only accessible when the teacher understands the needs of the learners, and tools/strategies are used that enable students to make sense of mathematics. Even when she spoke about the teachers and their excitement about the Cuisenaire rods,

she said the excitement was because the concepts finally made sense to these adult learners because of their previous experiences.

Ricki's return to the field and continued efforts to increase and deepen her pedagogical and content knowledge speak to her diligence as an educator who wants to provide better support for her students and the teachers with whom she works. Her malleable orientation toward math and the ability of learners was best summarized when she said, "I used to think students [who were good in math] were mathematically gifted. Now I thinks students must ALL be given high quality mathematics," and, "I used to think students who struggle mathematically could not grasp concepts. Now I think students who struggle mathematically have not had opportunities for success." She believes all students can be learners and doers of math provided they have rich opportunities in which the teacher removes "whatever the barrier is" to meet the students' needs.

Ricki's frustration increased both when teachers didn't have time to collaborate or had the time but didn't collaborate effectively and when there was no support for teachers. It is important that teachers fully discuss the lesson, including which strategies they are going to useand engage themselves in the math the students will be doing. She expressed to her team, "This [collaborative team] time is like church to me. We have to start on time, we have to be meaningful and purposeful in what we're doing." Ricki added that follow through and continued support is also critical. In reference to a professional development session on number talks that was done at the school, Ricki remembered the teachers were introduced to the idea, told how it worked, observed key

aspects of the strategy, and then watched a video of a number talk – and that was it; then the teachers were supposed to implement number talks.

> If I have one experience, even if I have two experiences, even if I have three experiences . . . if you were not in my classroom showing me how this is going to work with my students, it's not going to work. I'm not going to do it. What support do I have? There was no support in the building. (Ricki)

Follow through, continued guidance or a coach who provides continued modeling and support similar to what Ricki received, are requirements in order for teachers to continue to make positive changes.

Ricki's experiences have, over all, led to a productive disposition. However, there were clearly times where her disposition has been challenged. These circumstances tie directly back to the needs identified for her within the andragogical framework. She was most productive when her learning style and subject matter needs were met. In some cases, meeting these needs required a different approach or resources, and in some cases, it required she received direct, individual instruction. Additionally, she not only struggled, but she was agitated when the situation did not provide direct and sustained , whether for her or for the teachers with whom she worked.

Jamie. Jamie grew up and went to college in another country. She intended to teach so she earned degrees in physics education and math education. When she moved to the United States, more coursework was required for her to teach. Jamie is a teacher with 13 years of experience, again both in and out of the United States. She taught in two

divisions and four schools. Jamie worked with students in Kindergarten through high school, teaching students in subjects through Algebra I.

Mathematics background and development as a learner and a teacher. Jamie had traditional math experiences in her formative years. Her experiences in college, as she earned education degrees in physics and math,were similar. Her memories of these experiences were few. Jamie was not pleased with her students' overall performance in her first teaching job, but she also did not know why some students were not performing well. She remembered the first course she took which challenged her ideas about math and math instruction. Specifically, the summer college courses she took taught her to rethink her learning about math and math instruction.

K- 12 math experiences. Jamie's initial math experiences were described as traditional. "You listen to the teacher, you do what the teacher says, and then we practice constantly repeating over and over like the teacher did it." They didn't have a middle school, just the primary grades and then high school. Her high school experience was similar. The teacher explained the process or procedure, which she had to understand/memorize and then solve the problems "that way." Jamie felt the other classes were about pure memorization, and they had to memorize fast. In math she had to understand the process the teacher explained and then repeat the process. She gravitated toward math because she hated pure memorization. Jamie was a good math student and found it was easy for her to understand. Her coursework in high school took her through trigonometry. She recognized in those early years that her friends were not having the same experience with math—that many struggled. She didn't give this much thought

beyond that they struggled. She didn't have any specific memories of her experiences in her formative years.

Preparation to teach math. Jamie earned an undergraduate degree in physics education. In this program, she took levels of calculus, statistics, and other traditional math degree courses. She then earned a masters in math education. This required more math courses. Ricki did not recall any specific methods or pedagogy classes. Jamie did not have any specific memories regarding her math courses, save math continued to be a subject at which she did well. Reflecting on her preparation, she said, "I feel like I got the content." She began teaching in her home country after completing these degrees. Jamie taught for a couple of years in her home country before moving to America. To teach in the United States, she had to complete a college level Geometry course. This was primarily a content course; it was required because the state licensing board said she "didn't have enough background in it."

Professional development experiences as a practitioner. While teaching in her home country, despite knowing the content well-enough to teach it, Jamie noticed even when she tutored her students, they were not performing to her expectations. She questioned what she was doing, or not doing, and the implications of her actions in relation to student performance. Jamie came to the United States and completed the required geometry content course, so she could teach. Jamie got a job, and although she was experiencing success similar to the success of her peers, she felt there was more to teaching and reaching all students. She learned the institution where she took the Geometry course offered a summer content and lesson study course, so enrolled. "I

started to realize [students weren't meeting my expectations] because it wasn't really authentic learning; maybe it was more of memorizing steps." This realization was a break-through for her in regard to her teaching. She considered all of the summer courses "a major factor" in her professional growth. The courses empowered her, and she felt the strategies would empower students and enable them to be mathematical thinkers. She recognized these strategies and approaches were completely different from anything she experienced in her formative years.

The summer graduate courses provided her ways to engage students. She remembered courses that explored strategies like using multiple representations and number talks. In the course which explored multiple representations, she initially felt it was a bit challenging. Soon, however, the process felt like it was a competition to see who could model it first and who could find the most representations – and explain them, of course. She liked that there weren't rules; it was about what made sense to each person. She was inspired to see her peers look at each other's models and want to know how each person had used the model to make sense of the problem. There was excitement throughout the room and people wanted to share their models with the whole group. This confirmed for her the power of using open strategies which enabled each person to make sense of the problem for themselves.

Jamie attended division professional development opportunities. These were also helpful and often presented in an approach which aligned with the summer graduate courses. Jamie was also fortunate to work with an effective team at her school. Last year, the group received an award for the gains the students made on state standardized tests.

She said this was due to their focus on meeting the new learning targets. They plan to focus next year on the most appropriate tools or strategies to help meet the learning targets. She also encouraged her teachers to attend the training with her, which a couple did.

Changes in practice. Jamie changed her practices the school year after her first summer graduate course. The students were empowered. Students who thought they did not know math felt like they could do things and make progress. They wanted to "do my own learning." She observed her students develop different formulas from their models and prove that the formulas were mathematically correct. Students were able to use strategies to make sense of concepts like percent change. More importantly, when the class had a discussion, students were able to articulate how their model showed that percent change was the change from the original price to the new price. During another lesson, students derived the formula for the surface areas of a pyramid. She still remembers the student who broke the surface area up into its faces and wrote the area for each piece and then worked to put them together (combine like terms). "That was meaningful to me." She continued to incorporate the various tools and strategies as appropriate and "[I saw] my kids do [math], and it's just amazing! It's amazing because you see authentic learning from your students."

Jamie's journey within the andragogical framework. Math was always fairly easy for Jamie to understand. She thought when people didn't understand math it was because they just weren't good at math. Because she always understood what the teacher

or professor said she should do procedurally, she always had a readiness to learn and orientation towards learning math.

The core principles which prompted learning were an orientation and a motivation to learn more about math and teaching math because not all her students were doing well. As a teacher, she needed to know why and how to help the students. Jamie recognized through the summer graduate courses that she had individual learner differences and that there were individual learner differences among her students that needed to be addressed. She also acknowledged situational differences; the direct instruction of her classroom in her formative years compared to the open, safe environment the adults experienced in the summer course. As with several of the others, her purpose for seeking learning opportunities was for individual growth. She tried to influence institutional growth by inviting her colleagues, and a couple have attended. This effort was based in a desire or need to change the situation at her school. Her colleagues were unsure of the strategies and tools she was suggesting, and she felt resistance, which was an uncomfortable situation. So, she encouraged them to attend the next class with her, so they had an opportunity for similar learning experiences. One did, and then wanted to continue attending, which changed her school situation as the teachers could support each other's efforts.

Jamie, throughout her life, had a strong self-concept as a learner. It was not until she taught and observed students not meeting her expectations that andragogical needs surfaced. Those were a life-related situation (readiness to learn) and a need for what she was learning to be applicable in addressing her students' needs (orientation to learning).

Because she noted a Geometry course she took met those needs and helped her improve student performance, she enrolled in related courses offered by the same institution. Because these experiences met her needs, she invited her peers to attend, also. This was also an attempt to improve a situational difference as she believed if the colleague experiences what Jamie had, the colleague would be less likely to be resistant to the instructional changes Jamie proposed. All of these experiences contributed to her productive disposition.

Jamie and productive disposition. Jamie changed from believing there were just people who are not good at math to believing students just need opportunities to make connections. Students who struggle mathematically have trouble finding a connection to the math concept and therefore don't know how to process the information. By helping students with various approaches, encouraging models and representations and establishing meaningful discourse, she helped students make sense of the math and see themselves as learners and doers. She focused her efforts on establishing meaningful discourse with peers and allowing them to experience a productive struggle. Students demonstrated their ability to her as learners and doers when they were able to develop a representation she'd not considered, developed their own algorithm, or when students were empowered to try things on their own. When provided opportunities to make sense of math, she saw her kids do math, and "it [was] just amazing." Math for her is no longer a subject at which you are good or not; for Jamie it is a subject that demands students experience authentic learning in order to be successful.

Jamie's own increased math and instructional knowledge, and the resultant improved performance of her students promoted her productive disposition. The success of her students furthered her belief that all students are learners and doers provided the appropriate supports. Jamie furthered her own ability to have a productive deposition by inviting her colleagues to join her in the professional development,
thus addressing a situational difference.

Christine. Christine had an interesting elementary and high school set of experiences due to the size of her school. She went to college to be an elementary teacher. The program required the students add an additional endorsement. She chose middle school math. She has taught for 17 years in two different schools within a division. She spent the last 4 years as an instructional coach; this is a general assignment in which she is asked to work with all curriculum areas, so she does still have opportunities to work with math teachers.

Mathematics background and development as a learner and a teacher. Christine's elementary experience, due to a small school, meant she was often in mixed grade-level classes. She was accelerated by middle school but began to struggle in Algebra I in eighth-grade. It was some tutoring experiences in college which convinced her to seek an endorsement in middle school math. She was exposed to new approaches to teaching, for example the use of manipulatives, in middle school. However, she didn't really have rich opportunities to inform her teaching until she was a practitioner.

K- 12 math experiences. Christine's elementary years (K- 6) consisted of sitting and doing work. She partly attributed this to the small school she attended. The teacher

had two grade levels in one classroom. This meant a lesson was taught to one grade level and they were given work to do, and then the teacher would teach a lesson to the other grade level, give them work and then returned to the first set of students. She remembered as a second grader she was a good math student and often would do the work assigned to the third graders. She could choose to extend and challenge herself. The lessons were all procedural; there were not multiple strategies or models – nothing with manipulatives. It was "just pretty much learning the basic facts." In elementary school, she enjoyed doing math and helping her friends who struggled.

Christine had been placed in the gifted program, so she was enrolled in accelerated classes. Therefore, she took Algebra a year early. She struggled with the way the material was presented so often brought her work home. Her dad helped her through the course. He helped her understand variables and "how they worked," and how different things went together. The teacher, she thought, just wanted to coach and was only there for a couple of years. He wasn't a good teacher, and Christine doubted he really understood the content. Christine did not attribute any of what she learned that year to her school environment: "The teacher was not the person that was going to get me through the class that year. It was my dad."

In high school Christine was still in a small rural school, so she only had exposure to two teachers. One teacher she had for 2 years. She vividly remembered the teacher sat at an overhead projector – the kind which had a roller to pull the laminate across the screen. He sat and wrote problems on the overhead, completing each step, and gave them homework which was due the next day. Most of the students copied work off students

(like Christine), who seemed to understand what was happening in the class. She had the teacher for Geometry. She didn't understand the proofs and couldn't make sense of how to put them together. She was very frustrated with her teacher and told the teacher she didn't think he was teaching them well and that she didn't understand what he was saying. He added to her frustration when she recalled his response was that if she just did a few more problems she would understand. In retrospect, she knew what she needed was something different—a different learning experience. As an adult, she believed he took for granted the level of knowledge he had about math, so he didn't do a good job of helping his students have that same level of understanding. Her Algebra II teacher was also her choir teacher. Therefore, Christine had a good relationship with her prior to becoming her math student. The year went okay, which Christine credited to the already established relationship with the teacher.

Preparation to teach math. Christine went to college to earn an elementary education degree. The university she attended mandated elementary education majors had to also choose a middle school content area, special education, or pre-K instruction. Christine was leaning toward middle school but could not decide on a content area. She tutored some middle school special education students as part of her college program, and it happened she was asked to help them in math. It reminded her how much she had loved math early in her school career. Christine realized that if she had had some different teachers she may have been more confident in math and pursued a different degree.

> But because I had those middle and high school experiences where I wasn't still encouraged to grow and have the growth mindset, I felt like you know what, if I teach the middle school level I can do this for other kids. (Christine)

So, she chose math and took college level math courses and methods courses focused on middle school math. This was the first time she was exposed to manipulatives and learned how to get students to explain their reasoning. She was able to continue these experiences through the summer graduate courses.

Professional development experiences as a practitioner. Christine was able to participate in a few of the summer graduate courses. She attended them with different teams of teachers from her division and feels she has "grown up" through the program. These experiences inspired her to apply for a coaching position in the county, so she had several professional development experiences working with other coaches and teachers in her division, too. Christine also found several ideas and sources on Twitter (2019), and it helped her keep up with math trends. She attended state and national conferences a couple of times during her career. Christine was invited to help write a test bank of math questions. This provided her an opportunity to get in to good discussions about what the learning objectives meant, the experiences students needed to understand the targets, and how the target should be assessed. She also said her collaborative teams when she was teaching were important. She met with two groups: teachers who taught the same math classes, and teachers who taught the same students. These were a critical means of support when she was teaching.

Upon reflection of the various forms of professional development in which she's participated, Christine said the summer graduate courses were the "gold standard." It immersed her in a week of content and then provided the guidance and support to complete a lesson study cycle. This was the most helpful, in terms of forms of professional development, because she was with teachers and they were being challenged in positive ways that she can then use to support teachers in her coaching role. She said,

> If we're at something that's really engaging, or getting the teachers to think, I can always come back to them and say, "Now remember when we were sitting there talking with so and so and they said this . . . Do you think that's something you want to try?" (Christine)

She went on to say that professional development has to mean something, not just how to write an objective or plan your lesson. It has to help you reach students and help students understand mathematics.

A professional development experience that challenged her early in her career was a push to do more open-ended tasks. She was concerned about the pacing of the curriculum and was sure there was not time to do these tasks. She also did not believe the tasks would have an impact or that students would want to do them. Christine was sure it was a fad, the next best thing and would soon be gone. After a lot of work and planning, she realized that a balance could be struck between doing what was good for students and staying on pace. She also realized that the tasks were rich tasks, and so there was a lot of value in having the students work through them and "giving the kids a chance to think and kind of learn for themselves." Adding to her belief in the power of these tasks was

the exploration of the Five Practices (Smith & Stein, 2011). She said, "Orchestrating a good classroom discussion [using a rich task] – I finally realized, oh, I see how this fits together now. Oh, I see how to make this work in a classroom." She also realized the reason they were rich tasks was because more content was covered in the task than in a standard lesson; the kids would have to apply things they did earlier in the year and incorporate those ideas in to the new content. Her awareness that her own math development may have been different had she had different supports, and experiencing alternative teaching strategies early in her career, inspired her to continually change/improve her practice.

Changes in practice. Based on her own account, Christine's goal as a teacher was to provide rich and supportive math experiences for students which she did not have in her formative years. Therefore, she sought ways to change practice, not just her practice but the practice of teaching and learning she had experienced. Christine implemented many different strategies, used manipulatives, and tried various approaches. As a teacher she was constantly trying to improve the learning experience for her students. As a coach, she worked to increase and improve the skills and knowledge of the teachers, so they can better support students' learning.

Christine's most rewarding experience and where she saw these efforts make an impact was in the Algebra class she taught for eighth graders. She was confident giving them a chance to struggle and battle with the math. She supported them as they worked through the math telling them "we" can get it. She used everything she'd learned to help them make sense of the math, so they would see and feel they were capable math

students. The models, multiple representations, patterns, discussions, questioning techniques – everything Christine had learned to do to help students was paying off. She felt like things had come full circle for her. Christine was now that middle school algebra teacher working with accelerated kids trying to be sure they understood the math.

Christine's journey within the andragogical framework. Christine, in her formative years, experienced periods of success and confidence, and periods of frustration and doubt. The turning point for her was when she tutored middle school and high school students. She was reminded she liked math, and specifically she wanted to teach middle school math. Christine had a readiness to learn that was life-related, a motivation to learn because she wanted to help students in way she felt was lacking in her own life and a need to know how to better support students who were struggling.

The experiences Christine had tutoring also informed differences she felt were critical. She understood she learned differently than some, and the strategies her dad used were what helped her. She also felt she had subject matter differences; after tutoring the high school students, she knew she would not be comfortable at high school, likely because of her geometry experience. The instructional experiences she needed to feel comfortable with high school math beginning with geometry had not yet happened, so she decided she would teach middle school math.

Christine chose to develop professionally and continue learning for her own growth. She did speak to institutional growth when she talked about supportive collaborative teams dedicated to improving learning for all students. She also experienced institutional growth when she worked as a coach. It is important to note that there were

also times when due to situational differences, growth was left to her individually. She talked about situational differences which, although they did not hinder her growth, also did not promote growth. Specifically, teams that lose focus or administrators which take a laissez-faire attitude. These circumstances made growth and productive disposition difficult.

Although Christine had a good self-concept as a learner, she recognized a need to have a different set of prior experiences she could use to both help herself better understand math and better support her students. The graduate courses provided this; in her own words, she "grew up through the program." She needed to be presented with more resources and ideas to guide her instructional decisions and draw from both as a learner and a teacher. The experiences which met theses needs changed her beliefs about math and math teaching and learning, contributing to her productive disposition.

Christine and productive disposition. Christine, perhaps based on her own experiences, spoke to the idea of diligence often. She wanted students to struggle, with the appropriate support, and realize if they don't understand it at first it is okay. She emphasized, "We can get it." In this sentiment, she conveyed that she saw the students and herself as a unit – a collective of learners and doers of mathematics. She noted a critical year for her was the first time she taught Algebra to middle school students. She said she had to relearn the material with them. Her success and the students' success undoubtedly helped her understand that a "we" approach in teaching can be a positive strategy. When a student did something she didn't understand, she used to believe their strategy was wrong and wouldn't work. Now Christine assumes there is mathematical

reasoning happening and that she should ask questions to understand so she knows best how to guide or support the student. She is also now convinced that when a student does not understand something, they have different needs so need a different strategy to be able to make sense of the mathematics. Her immediate response to this is to ask what resources or strategies they have that might help them understand the concept or help them figure out how to do the problem; this is the same sentiment she expressed when she talked about how she approaches a problem about which she is unsure. Christine used to think if she didn't know how to do a problem it was because she didn't have the right skills or know the procedure; now she knows she has resources and strategies which can help her make sense of the math. She clearly sees teaching and learning as a fluid and ever-changing process that is dependent on the individual needs of the learner.

As a professional educator, Christine is discouraged primarily by three things. The first is the pressure of standardized tests. However, the two that are most difficult are what could be described as mediocrity and lack of focus on what's important. The first refers to teachers' belief that if administration is "leaving the teacher alone" they must be doing alright. The problem is two-fold; the teachers don't feel a need to improve their practice, and although she acknowledged the administrators are often busy with struggling students or teachers, she also believed with more attention there are many teachers who could grow well beyond mediocrity. The second is working with a teaching team that complains or focuses on the latest division change or gossip rather than plan purposefully or talk about student thinking. The teachers' actions don't necessarily have a negative impact on her or take away the progress she's made, but the actions do negate an

opportunity for them to grow as a team and make purposeful decisions in the best interest of students.

Overall, Christine displayed a productive disposition toward math, math learning, and her professional growth. This is primarily due to opportunities she had which allowed her to redefine how math is learned and how it can be taught – a new set of prior experiences. She was fortunate to have a supportive supervisor and situations in which she had access to supportive colleagues. Again, there were times when this was not the case, but overall, she has been in supportive situations. Her productive disposition was further supported by the growth and success of her math students who were able to experience math in a way which she did not in her formative years as a reflection of her primary goal.

Faye. As a first-generation citizen, Faye had a passion for languages. She was also a good math student. A college recruiter suggested to her that math was a language, and that if she could teach math, there would always be a job for her. She earned a degree that allowed her to teach high school and did teach high school for many years. However, she switched to elementary school where she's spent most of her career. She's taught K-12 grade for 30 years, educating students in five different schools located in two different divisions.

Mathematics background and development as a learner and a teacher. Faye had a traditional math education in her early years. This changed a bit in high school where she had opportunities to engage in math differently. She majored in secondary math and was greatly influenced by her college mentor who provided direct and specific ideas

about how to reach struggling learners. As a professional educator, she constantly sought opportunities to increase her knowledge and implemented what she'd learned with enthusiasm. Her students' reactions to her methods further encouraged her to seek out more strategies and ideas.

K- 12 math experiences. Her parents enrolled her in a K-8 Catholic school. She remembered that the learning was very rote. In elementary school, she would repeat her addition facts, times tables, or whatever the teacher asked. The learning was very rote and based in memorization. There was nothing that was really hands-on or engaging. Faye was a good math student and so was able to advance to algebra in eighth-grade. The instruction was still very direct, but she was able to understand and apply whatever method or procedure was provided, so was able to follow and perform well.

Faye also attended a Catholic high school. The teachers at that level were a bit different because they did have them engage in some hands-on activities. Her first high school course was Geometry, and she found she "really loved geometry." She loved to write the proofs, which were not strictly two-column proofs. There was a boy in the class with whom she would always compare how many steps each had in their proofs. Faye continued to do well in math and for the most part found she understood what was presented by the teachers. She also remembered her dad doing various homework assignments with her as she worked at home. She recalled that she and her dad both liked to be creative, so there was likely some exploration happening when her dad worked on the math problems alongside her.

Faye's parents didn't know much about college but did know they wanted all of their kids to receive a higher education degree. Her school hosted a small college fair, so Faye attended. She knew she wanted to be a teacher and was pretty sure she wanted to teach language (she learned her mother's native language as a child and could read, write and speak in the language). A college recruiter told her if she taught math as a language, since she loved languages, she would always have a job. So, she went to college to become a math teacher.

Preparation to teach math. As part of her secondary math education program, Faye had to take a methods course. The instructor sat them at tables and talked to them. She did not remember him showing or modeling anything for them. Many students fell asleep. The professor had an intern who did all the grading and most of his work for him from Faye's perspective. The best part of the program was the mentor she was assigned. Her practicum and student teaching hr were at a middle school for students with special needs and a high school with many at-risk youths. Faye remembered a couple specific things about her mentor. The first was the mentor knew the students at these schools "really needed something, and she taught us to [reach and engage them]." The mentor would model and say, "This is the way you show the kids how to do it. And this is how you can make them repeat what you're doing, but in a fun way, using . . . crayons and colors."

There was another time when Faye's cooperating teacher gave her a lesson to teach with the lesson plan written out. Her mentor asked how she was going to do it based on what the cooperating teacher had suggested. Faye said she didn't know, that the

teacher had just given it to her and told her to teach it. Her mentor said, "No, no. He didn't go through the step by step procedure of it?" The teacher hadn't explained the lesson or strategies to her, so the mentor showed her the "A to Z" of implementing a lesson. She gave Faye what she thought was the best way to present it, how to model it on the board and use colors to clearly delineate specific parts. She encouraged Faye to let the students use crayons and focus on one piece or part at a time. The lesson was one that was usually an introduction to graphing. Faye said that to this day, it is one of her favorite lessons to teach and that she always thinks of that mentor when she teaches it.

Professional development experiences as a practitioner. Faye got a job teaching high school at an all boys' school. Early in the experience, she decided she wanted to know where students started mathematically and how they arrived (had developed) when they got to high school. So, she took some elementary school math methods courses. She also wanted to do creative things in the classroom but wasn't sure how to bring that out in a math classroom. She decided to try some of the hands-on and visual things she was learning in the elementary methods course with the high school boys. And her boys, "Oh my gosh, they absolutely loved it . . . They loved to build . . . I'd brought them back to . . . 'Oh, we haven't done this since we were in fifth grade.'" This experience encouraged her to seek out more opportunities to learn about various creative strategies and resources.

There was minimal support from the diocese regarding professional development. Faye was one of the only female teachers, and the rest were brothers who seemed to do their own thing. She decided to pursue a master's in elementary education. She noted the

strategies she'd learned in the elementary methods course or developed for the high school students were the exact strategies being promoted in the master's program. However, the focus seemed to be on reading and writing. So, as they looked at books Faye tried to figure out how the same books could be used to engage students in math or another subject area.

Faye moved to a division in another state and found several opportunities to grow professionally. She attended NCTM conferences. She also participated in many courses offered by her division. Because she had always liked language and taken language courses in college, she often took division courses that would help her figure out how to support English language learners. She knew from her own experiences, and her experiences helping her mom learn English, that there are still words she doesn't know in English. She felt she could put herself in their place and understand the struggle with vocabulary, especially given math vocabulary can be difficult, and figure out how to help them "link" to the vocabulary. In relation to this issue, she noted she learns from students. One student made a connection between matrices and compound words. Faye felt it was brilliant. "[The student] taught me how to do it this way."

Faye liked to learn by experiencing the strategy or lesson given what's worked in the classroom. She also wanted to be sure the why, how, and what do you think are covered in professional development because that's what she expects of her students. To address this, she sought out courses (mostly online) presented by professionals. She's also identified people in her division who present this way and will attend their professional development offerings. Faye took advantage of offers from specialists or

coaches to come and teach lessons in her classroom; she observed how the strategies they're recommending work for her students. Finally, she's attended the summer graduate courses. She appreciated the diversity of the teachers and their strategies. She said she often felt humbled by the approaches elementary teachers took when approaching a problem. Their models and reasoning generated different ways of thinking for her, and she wanted to try the methods of thinking and reasoning they were sharing. These experiences reminded her she tends to think a certain way, and these opened her mind to the variety of approaches students may use and to challenge herself to try different approaches.

Changes in practice. It is evident in Faye's history, she implemented many changes to her practice over the years. It started with re-introducing hands-on activities in the all boys' school to engage the students and promote learning. Faye provided example after example of things she's done in her classroom to improve student understanding. Faye observed that when students aren't empowered to find a link into how math works, or when no one empowers them to show they know something about the math, they shut down. And when students shut down, that's when not understanding math really happens because they are not given or taking advantage of opportunities to show what they know. To remedy this, one strategy she used was to show her own weakness in mathematics. "I'll look at the problem and say, 'You know, I really don't know what to do with this problem. Who can help me?' And in that moment, [students] are saying, 'Let's try this, or let's try this, or let's try this.'" And from this interaction, Faye gets all the students to

generate ideas and engage. She truly believed when you show kids your human and have weaknesses, too, they are willing to give so much more.

Faye's journey within the andragogical framework. Faye's dominant core adult principles were her need to know and her orientation to learning. Early in her career, she wanted to know how students develop mathematically. She sought to get this question answered, and in her desire to incorporate creativity observed how students make sense of mathematics. Throughout her career, she identified various problems; for example, she needed to help English language learners make links to math using language. Again, she signed up for related classes and listened to students' thinking to identify ways she could help students make connections.

Faye spoke most directly to the situational differences she's experienced as an adult learner. She struggled to find collaborative professional opportunities in the Catholic school in which she first taught and initially in the school she is now. Her current school has two math teachers. When she first arrived she and the other teacher taught different classes and did not collaborate often. That teacher departed, and a new teacher was hired. The new teacher and she share courses so that they are both teaching most of the classes. This change promoted a collaborative environment in which they share ideas and strategies to best support students. She also spoke about a situation in which each teacher was supposed to teach the same thing in the same way on the same day. She found this very limiting, specifically, "that it didn't make me progress and learn as much," and she struggled to operate in a paradigm with which she fundamentally

disagreed. Faye learned that the professional situation in which she finds herself has an impact on her ability to learn and grow within the environment.

All of Faye's pursuits have been for the sake of individual growth to benefit students. She wanted to increase her knowledge, so she could engage students, address deficits to help students make connections, or better understand how students develop mathematically. Regarding the summer graduate course, Faye said,

> I got to see what [other teachers] were doing, and be in their classrooms, and see other classrooms. I mean, I love to see how another classroom works because it just empowers me more, to use part of what they're doing in my own room. (Faye)

Regardless of the problem or situation she wanted to address, Faye's goal was to increase her own knowledge to meet her own expectations regarding personal growth.

Faye had a strong self-concept regarding math. She had a learner's need to know how students developed mathematically. She also needed to believe that information she received would help her meet the needs of male high school students (in her first years of teaching). To Faye's credit, she was adept at transferring knowledge to different contexts and could therefore determine the relevancy of the information herself (readiness to learn). This was evident in her recollection of the elementary methods course and the English strategies, both of which she used to identify strategies for her classroom. Continually gaining new insights, implementing those ideas, and observing the benefits to her students ultimately contributed to her productive disposition.

Faye and productive disposition. Faye's view of math as a language informed her belief that everyone can learn and do math, just as everyone can acquire a language. She believed that math, like language can be taught several different ways. There are several ways to get to an answer and to express the same thought. In line with these thoughts, she said that teaching math is multivariant and depends yearly on students' understanding. All these ideas are indicators of a malleable orientation towards teaching and learning. Clear evidence has also been provided that she sees students as learners and doers of mathematics. She makes specific and purposeful efforts to help students see themselves as learners and doers. When students are struggling with a math problem, she asks students to talk her through what they do know about the problem. She also owns her weaknesses and shares misconceptions and mistakes as a way to expand learning. In Faye's math world every situation is an opportunity for her, her students, or her colleagues to explore mathematics and discuss, question, and write about the how and why.

Faye is also a model for a malleable orientation toward professional growth. She has focused her growth both on overall knowledge and specific aspects of her professional career, usually determined by the population of students she was teaching. Faye also recognized that the strategies, resources, and approaches are transferable. Examples of this are her transferring what were presented as strategies for the English elementary classroom to math or those presented for the elementary math classroom to her high school class of boys. She understands a good strategy is a good strategy, regardless of the subject/content, when implemented purposefully.

Faye's productive disposition is fueled by her students continued increased understanding and her own ability to transfer knowledge to different contexts. Although professional development experiences may be credited for meeting her need to know and her orientation to learning, she was responsible for determining relevancy, especially when she participated in professional activities outside the math realm or the grade levels she teaches. These varied experiences contributed to her ability to engage students and promote their growth, all of which, in turn, promote and sustain her productive disposition.

Kim. Kim knew from an early age she wanted to be teacher. She went to college and enrolled in an education program. She moved often early in her career and had to meet additional certification requirements with two of the moves. Kim has taught for over 30 years. Most of her experience has been at the middle school level, but she has taught students in Grades K-8. Kim taught in seven different schools in three divisions.

Mathematics background and development as a learner and a teacher. Kim was an average math student. Her middle school experience was her best, although she doesn't remember why or what the teacher did in particular that made his classes such a good experience. She does know the teacher she had for seventh and eighth grade created a strong math foundation for Kim. His efforts helped her get through Algebra. However, Geometry did not go well, and it was the last class she took in high school.

K- 12 math experiences. Kim attended elementary school on a military base where her father was stationed. She doesn't remember a lot about elementary math classes but believes they were taught traditionally. She had one distinct memory. She was

putting a problem on the board, and the teacher said something positive to her. Kim knew in that moment she wanted to be a teacher. It may have had this effect because she needed validation, or the intrinsic feeling she experienced knowing she'd done it right, but she just knew she wanted to be a teacher (and it's never changed).

Kim had the same teacher in seventh and eighth grade. Although she doesn't remember how he taught or how she learned the material, she thought he was a pretty good teacher. Kim said her first year with the teacher, her dad was in Vietnam and she liked the teacher (older man in his 60s) and the content, so she found some comfort in his classroom. He helped Kim believe she could do what she needed to do to be successful. He expected students to show their work and be able to explain their thinking. She was certain he created enough of a foundation for her in math that it helped her through her first year of high school math. She couldn't give a "definitive moment or something that" he did specifically, but she was sure it is to his credit that she did well in Algebra. She did not have a positive experience in Geometry. The teacher was not very good, and Kim struggled to make sense of the geometric concepts. Her memories of the teacher were that he briefly explained how to do something and then gave them problems to complete. Kim did not enroll in any more math classes after her sophomore year because of her experience in Geometry.

Preparation to teach math. Kim went to college and earned a degree in elementary education. She took the CLEP (College Board, 2019) test and passed out of the math class required of elementary education majors. Kim took all of the other courses required including a methods course. The course did not introduce many manipulatives,

and Kim does not remember getting much from this course. Her one take-away was the usefulness of Cuisenaire rods. Kim's student-teaching supervisor, however, was flexible mathematically toward both teaching and learning. She encouraged that same flexibility when Kim worked with students. The supervising teacher let the practicum students try different strategies and worked with the students in a fluid manner. Kim's take away from this was that she could find students' "success spots" and use those to help the students get where they needed to be.

Professional development experiences as a practitioner. Kim moved often early in her career, and so she had to complete different licensing requirements. One division required she take a Geometry and Technology course, and another required she take a Statistics and General Education course. She remembered she liked the geometry teacher. However, the class was a struggle. She recognized it was because she didn't have the foundation she needed, and this class was full of proofs. She said she decided either people are proof people or they're not. She wrote down everything in the Statistic class,which she was quick to point out, was the content now expected of high school students.

Kim participated in various opportunities offered by the division. She said her first real eye-opening experience was a number sense class in which she enrolled which was part of a math specialist program coordinated by the division. The course was "beyond pivotal, and it's beyond powerful" because it determines everything she does every day for every student. The class was challenged to work in different bases to make sense of grouping and reminded that a number is just a symbol that represents a quantity.

Kim challenged herself to work in bases other than those done in class and constantly thought about the idea of quantity verses the idea of a symbol to represent that quantity. After completing that course, her goal was to look for activities, projects, and other ways for kids to learn and to share what she learned with colleagues. She attended local and state conferences, participated in division opportunities, and took part in the summer graduate courses.

A recent summer course she found beneficial was the course which focused on rich tasks. The teachers found rich tasks and supportive resources. They explored them and discussed what made them rich tasks. The participants were also asked to put together rich tasks and present them. Kim learned a few things from this experience. First, she learned the resources were not things you can just find on the internet; it takes a network and word of mouth to find the good, rich tasks. Also, she noted the participants had varied levels of comfort because the rich tasks often don't have just one answer. This is another aspect of these opportunities she appreciated: they're taught with respect to the teachers as adults, some coming in with no experience and some with a lot. The professional development experience meets a variety of learners and needs, and tasks are presented that are accessible and challenging to all. These are exactly the types of problems in which she wanted to engage in during professional development and offered to her students.

Changes in practice. Kim stated several times she was constantly looking for more effective strategies and tools to help students. Kim talked about the way she used the idea of quantity from the number sense class to help students with everything from

multiplication as area and the array model of multiplication which provides a visual of distribution, to helping students better understand fractions. The implementation of the various strategies was very purposeful because "Students need the opportunity to be guided through how to use manipulatives, multiple representations, taking quantities apart, discovering place values through rich tasks . . . There are so many ways to math – yes, math is a verb!"

Another big change for Kim has been to provide opportunities for students to experience "productive struggle." She had to work at being purposeful about implementing and supporting these types of lessons, which she now does a couple times a week. When she first started teaching, providing a set of well-defined steps or demonstrating problems for the students was just "the way math was taught." Now, she presents problems for them to solve. She reminded students they have the tools/resources to make sense of the problem and know the math inherent in the problem, but it is their problem to solve. Then, she "steps back." A further benefit, especially since she started using the rich, open-ended tasks, was that it promotes questioning and dialogue. Students consider each other's work and say things like, "How come he got the right answer, but his mathematical thinking [is different]. That's not this."

Another overriding mantra for her has been insisting her students use exact and purposeful mathematical language. She lets them say or write what they are thinking; however, it first comes to them because she wants them to share. However, when necessary, she also asked that they "resay" it. She does it in a supportive way but makes it clear that when communicating about mathematics, for the sake of all learners in the

room, it is important the language be precise and an accurate representation of their thinking. She even supported this idea when she gave students work. She gave them a lot of white space because it's her way of saying "Okay, let your mind go. Just go." When students understand the problem, there is still likely room to clearly convey their thinking.

Kim's journey within the andragogical framework. Kim expressed in several ways in relation to the core principles, she considered herself an autonomous and self-directed learner. Kim, because of the foundation given to her by her middle school teacher, in general felt she was "savvy enough" to be able to figure things out from a text, instruction or an example. She could walk into a math class and figure out the pieces, roll them together and "get it." She felt she could take any math and eventually figure it out. As a teacher ,her motivation to learn math and various approaches stems from the intrinsic rewards of doing so. The convergence of her self-concept and motivation are apparent when she expresses her individual learner differences.

Kim realized as an adult learner she experiences three different feelings as she learns. The first is saturation – when there is too much information for her to process or it is information regarding a subject in which she has less comfort. She gets to a point where she literally has to walk away and try to process what's already been put in front of her before she can consider more information. The second feeling is one in which the material is within her grasp and she knows she can figure it out, even when others seem uncomfortable with the math or that there may be more than one solution. She knows there's a solution she can figure out. The third is when she figures it out and as an

individual learner experiences jubilation. A specific example was when she explored different bases in a number sense course. As soon as she was comfortable with base five, and worked on the other couple bases recommended, she said she couldn't stop. The class had moved on, and she was considering other bases. She was so excited, she was making sense of grouping. Kim learned how to mediate these differences in her as a learner in the larger classroom or professional development session in part due to subject matter and situational differences.

Kim noted her willingness to persevere, both in relation to her middle school teacher role and the experiences she had in graduate classes which had a lot to do with feeling it was a safe space, and the facilitators were supportive. When these instructors observed she'd reached saturation, they addressed it and knew how to help her take a step back and think about what she knew. She also recognized her saturation points occur most often with geometry related topics, probably because proofs all looked the same to her, and she couldn't make sense of diagrams in high school. As previously noted she didn't do well. She also took college geometry courses, and although the instructors were good and helpful, she believed her foundational deficits made them more of a struggle. She had also decided that there were people who are good at proofs and those who are not – thus the one area in which she felt she couldn't be successful in math. Given this and that the courses were required, many of the core principles were not met, and thus these were not productive experiences for her.

Of some interest, she shared in the interview that in her twenties, she learned her eyes track differently. For example, one eye was up, and one was down and so they

focused on different spots. Obviously, she must have done a pretty good job compensating throughout her life to that point. When asked if she thought perhaps the tracking issue contributed to her difficulty with geometry given its visual nature (e.g., diagrams), she seemed to consider it for the first time and said, "Probably. Yeah, probably."

Kim, from very early on, had a good self-concept. She continued to express a belief she could do math, save perhaps geometry, which is likely due to a combination of individual and subject matter differences, and a lack of prior experience from which she can draw to help her make sense of geometry. She stated in her student-teaching experience she was encouraged to draw from various resources and mental models, furthering rich prior experiences for her as a learner. Therefore, her primary need within the framework was to have individual differences met, specifically understanding for when she is saturated, working toward a solution, or exploring a math concept or strategy further.

Kim and productive disposition. Kim clearly, and in many ways, spoke to her malleable orientation toward math and math teaching and learning; she considers herself and her students both learners and doers and notes that math requires diligence. Kim's efforts to vary instruction to better meet students' needs and her own willingness to explore math from the roles of teacher and learner are evidence of her malleable orientation. Interestingly though, she said although standards and students change, these tools, methods, and resources are a constant. They will always work. She must just match the strategy purposefully with the standard and with the student's in mind.

Kim engaged herself in mathematics and expected her students to do so also. Kim said several times she knew she could tackle any math put in front of her. She has a belief she will figure it out. Kim said she even wondered if she went back and took a foundational Geometry course if it would make sense now, acknowledging a new set of foundational prior experiences are needed in the subject. She was finally in a place where she believed it was possible she could be a learner and doer of geometry. In relation to her students, she appreciated an opportunity to help them see themselves as mathematicians. Kim said,

> What [that access to math] allows me to do in my classroom for my students, those kids who come in that have never passed a [state test], or they've decided, "'Okay, I'm just not a math person,'""this lets me show them yes, they're a math person. It doesn't have to look like this kid's work over here. It's theirs. (Kim)

Kim knew if she and her students continued to work at math they wouldfigure it out. It just requires diligence. When describing herself, diligence is described as thinking about ways to approach the problem, considering what others are thinking, trying things and then eventually rolling it together, and then she makes sense of the math. In relation to studentsm she offered white space, so they can let their minds go and try different things. She also spoke about ensuring students have opportunities to experience "productive struggle," which is also an opportunity to demonstrate diligence.

Kim's productive disposition was quite evident when you talked to her. It stemmed from her continued growth as a learner of mathematics as she made more sense of various aspects of math. Her experiences, in turn, motivated her to provide her

students with similar experiences so they could "math." (Again, she believed math was a verb.) Her students' success and growth further fueled her productive disposition. Kim was constantly looking for opportunities which would inform her own understanding of math and teaching math, and ultimately her students' understanding. This was evidence of Kim's productive disposition toward math, math teaching and learning and her own professional growth.

Andragogy

Several of the participants talked about meeting the students where they were by teaching grade level objectives while acknowledging their differences and addressing their deficits. The purpose of andragogy is to help identify what it means to meet adult learners where they are. It could be argued adult learners are just presented with many more differences based on a life time of experiences. As a core principle, the role of the learner's experiences means:

> Adults come in to an educational activity with both a greater volume and a different quality of experience from that of youths. By virtue of simply having lived longer, they have accumulated more experience than they had as youths. But they also have had a different kind of experience. This difference in quantity and quality of experience have several consequences for adult education. (Knowles et al, 2011, p. 64)

The preceding narratives presented andragogical attributes of the participants which needed to be met, providing evidence for the aspects of andragogy that should be or were addressed for them to have a productive learning experience. This section will

summarize findings regarding the aspects of the framework which needed to be or were met and how aspects of professional development met their adult learning needs.

The core adult learning principles. There was evidence of a need for all the core principles to be met among the participants. A few principles were more prevalent. Readiess to learn and learner's need to kow were the most prevalent. Also prevalent, and often intertwined, were orientation to learning and motivation to learn. These principles,

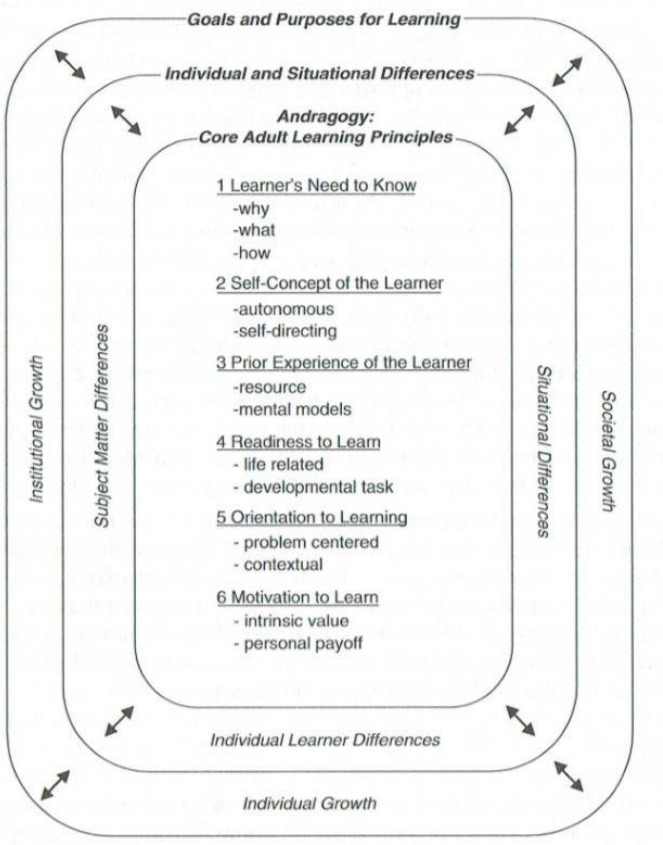

Figure 2. Andragogy in practice.Knowles, M.S., Holton, E.F., & Swanson, R.A. (2011). *The adult learner: The definitive classic in adult education and human resource development* (7th ed.). Burlington, MA: Elsevier, Inc.

how the needs were determined and whether it was met through professional development experiences is discussed in the following paragraphs.

Readiness to learn. "Adults become ready to learn those things they need to know and be able to do in order to cope with real life situations" (Knowles et al., 2011, p. 65). In other words, adult learners are ready to learn when they are faced with a situation they want to understand or be able to address more effectively. The participants in this group were in real-life situations or had experienced events which prompted questions with which they were grappling. These questions fell primarily into three categories: How could they ensure their students had a better experience than they had in their K-12 grade experience? Why do some students understand math and others do not, or why does one student understand this idea but not another idea? And, how do students make sense of math or develop mathematically? Examples of them expressing these ideas included:

> I would say what made me struggle was . . . having to do things one way. And I had no idea what I was doing. It was just memorize this. And I could memorize it for a bit, but I didn't really understand it. (Sam)

> But because I had those middle and high school experiences where I wasn't still encouraged to grow and have the growth mindset, I felt like you know what, if I teach middle school level I can do this for other kids. (Christine)

> I keep on wondering what's going on, why is it like that? Why I can't get these children to get perfect scores. (Jamie)

> But what I always tried to do, even a lot of county courses, to see how I could link the [English learner] population to math, because [I'm first generation] and 'there's still some things, vocabulary I 'don't know in English. So, I feel like I can put myself in their place and understand that when they are looking at all this vocabulary, and math vocabulary can be very difficult, but how to try and get them to link into it. And I think that probably was the drive for me, to learn more and more, and how to be more creative, and how to teach differently . . . So, I actually took some [elementary methods] classes as well. Because I was teaching high school, but I also wanted to know where everything came from. I mean, where [students] started, and where they got to. (Faye)

> There's got to be a better way of doing this math thing . . . And so it's almost that limitation of just realizing that like for my kids, not all the kids are going to be able to do this. Not all the kids are going to be able to see this concept from this perspective. (Edward)

> [There's a] fear that I don't understand what [the student's] are doing or I don't under why it works. (Amanda)

The participants therefore sought out experiences they believed would help them answer these questions. The teachers enrolled with a set of expectations or beliefs that the professional development experience would help them answer these questions and thus

help them better support their students. The professional development facilitators then had to convince the participants the information and experiences presented would both be of benefit to the teachers (learners need to know) and that specifically, the experiences would help them address their real-life situations (orientation to learn).

Learner's need to know and orientation to learning. The experiences that best met their need to know immersed them in the content they teach and modeled for them how to teach math; they experienced the content as a learner.

> Those classes, you're immersed, but you're not just set up by yourself. You've got a ton of people there to help you, but it's immersion, but it's a good immersion, if that makes sense. (Kim)

> I used to think sharing different models or explanations in the classroom was confusing to students . . . Now I think sharing different models or explanations in the classroom is helpful, since all students are seeing and understanding math differently. (Edward)

They experienced the benefits of the resources and the teaching strategies. Furthermore, these experiences ensured the orientation principle was met because they saw it as immediately applicable. Participants were able to approach the work from the perspective of the students with whom they work. They were also able to observe their peers applying various strategies – a representation of varied types of learners at multiple grade levels. This made what they were learning appear immediately applicable.

I think that my gut is to look for an equation, to look for how to solve [a problem] from there. My teaching background says that's not the best way per se. (Amanda)

It's really interesting how elementary school teachers approach a problem. There's been, specifically in the proportional reasoning class, it was interesting to see how high school and middle school teachers, primarily high school teachers, but approach a problem compared to the elementary, and . . . 'there's some proportion problems where if you drew a model, you could get it much more easily than trying to set up an equation and solve the problem. (Paula)

Yeah, actually, that's one of the reasons why I took the courses . . . Because there were so many different, I love the differentiation of the students there. And in seeing how some of those [elementary teachers] put their examples on the board. To me, I felt very humbled by it. But maybe it's because they thought of it differently. And many of them were in elementary school, and I thought, "Wow. That's so fantastic. How do they come up with that? I wouldn't have even gone there." It humbled me quite a bit, but I love the fact that they generated a different way of thinking from me. It really made me, because we tend to think in the same way most of the time, or how does it work, we tend to follow the same method of following through. But when someone opened-up my mind and was like, wow, that's a great way of doing it. I've got to try it. I've found myself trying it in their

way a couple of times, and it really expanded me so much. So, the humbleness is so good for me. (Faye)

What I liked about the . . . classes is that I'm one of the lower grade teachers there so I really like seeing how everybody through tenth grade content wise approaches a task, and some of that has really changed my thinking in terms of a problem I would have read and thought this isn't like an algebraic approach right off the bat. And sometimes the elementary teachers actually solve them faster because we use pictures and modeling versus trying to develop this complex algebraic equation that you can solve it with, but it takes longer. (Amanda)

Christina reflected after spending time reading a text and having it modeled in a course.

I think one of the big things that changed [my perception about making time for sound pedagogy] was reading the *Five Practices* book. So, orchestrating a good classroom discussion. That book was probably kind of a classroom changer for me; I think because I finally realized, oh, I see how this fits together now. Oh, I see how to make this work in a math classroom. (Christine)

Participants also identified several types of professional development which did not meet their need to know or orientation needs. One that was mentioned was curriculum work in which related methods to teach or resources were not also discussed. A second type was any development in which someone talked to them or at them about

how to teach but didn't model it or invite the teachers to engage in any way. Edward reflected on a professional development opportunity which he felt was a lost opportunity:

> I look forward to conversations I have with my fellow math teachers. And just, "What are you struggling with? What works for you? What are some cool ideas? What technology?" Or even its just the camaraderie. It's like everything. It's just there's so many things. And, it's just unfortunate, like the last time our diocese had all the middle school teachers who were teaching high school classes, like Algebra and Geometry . . . we didn't actually meet. We were just listening to people talk. And like, "I hate this. Just break us in to groups, and it'd be so awesome." Yeah, it's just sort of funny. It's just like model what you expect of us. (Edward)

Christine shared an experience in which several people, who had been out of the classroom for many years brought teachers together to discuss alignment as part of a professional development. The experience did not meet her orientation needs because the leaders of the program did not understand the contextual concerns of their classes or relate the alignment to critical building blocks. In each of these cases, the teachers' needs were not being met. They did not believe the experience met their needs nor directly helped them address any current real-life situations with any immediacy. There was no relevance to their practice, and without relevance it is clear the motivation to learn principle was not met.

Motivation to learn. The participants were primarily motivated by internal motivators, although one expressed a mix of internal and external motivators and another

expressed mostly external motivators. The rewards, whether intrinsic or extrinsic, sometimes occurred immediately. For example, some immediate motivation stemmed from figuring something out during the professional development, making connections to another's thinking, having a model validated, being asked to share their thinking because it was different, or identifying a strategy that would work for a student/learner. However, the more impactful motivators, those that ultimately changed the teachers' practice, were often delayed. There is a time lapse, for example, between attending a professional development and seeing the impact the implementation of the strategies has on student success.

> That has really solidified my belief in these practices, too. That you get these kids that didn't traditionally think of themselves as the smartest math kid, but they show you in class they can do so many amazing things. (Sam)

> And I thought, well, let me try some of the things, like the hands-on stuff. Let them visually do things. Let them cut things apart. Let them put them together. To see how they loved it. And my boys took to it like, oh my gosh, they absolutely loved it . . . All of the sudden they found a new-found reason to do something. (Faye)

According to Tough, as cited in Knowles et al. (2011) factors which hamper motivation are lack of support, time constraints, lack of resources, or situations which violate the core principles. These were also experienced by the participants.

But it wasn't about the tools, because the tools can sit in the closet. They've been sitting there for 15 years at the school I was at. They were there, no one touched them. No one used them because the teachers didn't know how to use them. They didn't have the support. (Ricki)

(In relation to Number Talks) The big absence was follow through. So, no support, no follow through. If I've had one experience, even if I have two experiences, even if I have three experiences, if you were not in my classroom showing me how this is going to work with my students, it's not going to work. I'm not going to do it. What support do I have? Or, if you don't have a team of teachers that understand how. (Ricki)

We get stuck because we don't have, at least in elementary school, we have common planning time but not really. I need more. Everything's very CLT oriented, [but that doesn't leave] a lot of time to really dig into [the math]. (Amanda)

The manipulatives that we used in the math methods course were not available in my school. I don't ever remember . . . Then I remember we went through this math program . . . a terrible program. It was targeted at Title 1 schools and very low performing schools. So none of these manipulatives that [the program] had matched anything that I had. So I was completely ill prepared to teach math. (Ricki)

You look at your calendar and you feel like I don't have much time left. (Jamie)

Individual and situational differences. This dimension involves three possible differences: individual differences, subject matter differences, and situational differences. Most of what the participants shared falls in the individual and situational difference areas. However, there were a few who clearly identified their own subject matter differences and its impact on them as learners and practitioners.

Individual learner differences. Individual learner differences refer to the cognitive, personality and prior knowledge differences of the learner. Jonnasen and Grawbowski (1993) noted cognitive differences refer to mental abilities, cognitive controls, information gathering and organizing and learning styles (preferences). Personality differences refer to the adult's attention and engagement styles, and expectancy and incentive style attribute has become much more connected to adult psychology as the andragogy framework has developed (Knowles et al, 2011). Finally, prior knowledge is a part of individual learner differences. The participants in this study had several of their learner differences met.

Ricki said several times that she was not good at spatial reasoning, which is a cognitive difference. She shared she doesn't accurately determine how much of substance or how many items may fit in a container. She said she didn't see or perceive visually what many of her peers or classmates did when looking at a geometric figure. Ricki remembered an experience in a math specialist Geometry course. The teacher "showed me what visually I couldn't perceive, or I didn't have the spatial awareness, or the

perception, or the depth perception to be able to [make sense of what was on the board]." Ricki was so appreciative that the teacher compensated for Ricki's spatial/visual deficits – she said she felt "seen." In the survey, in response to traditional teaching strategies, Ricki often answered, "not my style." Direct instruction, memorization, algorithms, pointing out a specific error – all of them were not her style. Other participants elaborated on these ideas whereas Ricki seemed to be expressing she had a unique learning style in which none of these had any benefit. By contrast. a few others said they were very spatial and classes like Geometry were exciting for them. Kim noted the individual differences when the teachers collaborated:

> With anything I've ever done in math, especially where there is a collaborative effort, you have people that see that right away, "Oh, here, here, here, and here." Then there are other people like, "I don't see it that way," or they'll go to the algorithm where nine times out of ten, I'll draw a picture . . . then other people say they didn't see it that way. (Kim)

Paula had a specific personality difference throughout her life – anxiety related to performance in mathematics. This was addressed by a college instructor by providing her time to take back locus of control – another aspect of personality differences. Kim talked about the open-ended problems and that she appreciated there was more than one correct answer. However, she noted some teacher's discomfort with this ambiguity. "I'm comfortable with it. Some people aren't. [There's] not one answer. It's not one answer for this problem." Sam said, regarding taking risks, that the summer graduate courses

specifically provided an "opportunity to challenge myself in a very safe place where I don't feel threatened [or worried about what others may think or say]."

Prior knowledge, specifically the lack of foundation in a subject area had a profound impact on a couple of the participants. In college, Christine tutored middle school students and high school students. The high school students were in Geometry – the course which ended her high school math track. Through those experiences, she decided high school was not where she wanted to be. Christine believed if she'd had a different experience, such as more supportive teachers in her formative years, she may have entered college and pursued engineering. However, even now, despite doing well in Geometry Education courses she took while a practitioner, she still does not believe high school would be a good fit. Kim expressed similar concerns. Geometry was difficult for her as a high school student. Despite two practitioner experiences in Geometry classes, with good and supportive teachers, she becomes anxious and "saturated." In one class she said the way it was presented was just not how she made sense of things. Kim said of the other practitioner experience, "I also knew that [the instructor] would be supportive. I could ask her a question any time. So that was really important . . . In retrospect, I didn't have as much foundational geometry as maybe I should have." She attributed this to her experience in high school. She wondered if she retook a true high school or Introductory Geometry class if she would feel differently toward the subject. Ricki is another participant who said her lack of prior knowledge in geometry had an impact on her journey. "The way geometry is instructed now, I feel robbed because I feel like if I would have had those experiences I would have understood."

Again, a couple of these participants talked about having good and supportive geometry experiences in their adult and practitioner lives. They had instructors who clearly knew the material and were patient. However, these participants still don't feel they have overcome the negative impact of their lack of prior knowledge or low achievement in this subject area.

Situational differences. Andragogy acknowledges the social, logistical, and setting differences within a learning situation. These are all part of situational differences. To address this aspect of the dimension, participants were asked about professional development and site-based settings.

The participants shared the differences of attending professional development sessions with a team or as an individual. In general, those who experienced impactful professional development with a colleague and alone felt there were equivalent benefits to both. Their thoughts regarding the two situations were that it is supportive and nice to have people that you know with you. The relationship is already established so you already have someone with whom you can work and reflect on what you are learning. There was also an advantage to attending professional development with people you 'didn't know because you got a different perspective. In the case of the summer graduate course and lesson study, another benefit was seeing other schools and educational environments. Sam found it "really surprising and interesting" to see different teachers implementing the same lesson, modifying it by grade level, and experience varied outcomes. Faye said,

> I got to work with people that were in other school systems, and I got to see what they were doing, and be in their rooms, and see other classrooms. I mean, I love to see how another classroom works because it just empowers me more, to use part of what they're doing in my own room.

The more critical piece was the support the teachers had at their site when they wanted to implement change. Fellow teachers, school practices/circumstances, and administrators had an impact on their success and continued growth.

The participants provided aspects of their site-based support which contributed to their continued learning and growth and those that did not. Participants who attended with a colleague had an easier time implementing and sustaining instructional change because there was a peer with whom they could continue to collaborate. However, those who attended professional development without a peer often found their colleagues were hesitant to implement what was being suggested. Their teams were supportive and shared resources but did not want to join the participant in trying something new or growth was hampered because the team did not stay focused on students and helping them learn.

> They're just wonderful and amazing, and I think we have a really, really good team where we're all contributing. We're all thinking. We all have a really great bank of resources that we pull from. But I think because they haven't had an opportunity to personally learn about it and experience it themselves, I think they haven't drunk the Kool-Aid yet in that regard. (Sam)

> It's a little bit of a challenge [to implement new strategies]. There is some kind of resistance that I felt. So, what I did is encourage them to go to the class with me so that they will see firsthand. and I have my go-teacher go with me last summer and now she's been like "Let's go to [that class] again." (Jamie)

> Sometimes some of the colleagues that I've had have maybe held me back a little bit or maybe not held me back but not helped me grow. Because while we did always have the content time, it wasn't always productive. A lot of times, it's really hard to get adults all on the same pace to focus your time and use your time wisely when you have that content time because I've worked with several teachers that would bring all their other drama in their life, if they're going through a divorce, if they're going through this with their parents, or this with their kids and that's very draining when you're teaching. (Christine)

Edward, contrarily, reported excitement from a co-worker who didn't attend the professional development. He said he and his co-teacher actually have more fun than the students using the resources and figuring out how and why things worked. Faye had a similar experience. However, she attributed this to a change in the courses she and her counterpart teach. Her prior fellow department member split classes with her so they didn't have any classes in common. The new teacher and she share almost every class. This gives them shared content and experiences to consider and the new teacher wants to collaborate and share strategies.

Others struggled situationally because they worked in a school where the expectation was they all teach at the same pace and in the same way or they are isolated. In these cases, learning/growth is minimized because the social aspect of the process feels scripted and impersonal or is essentially non-existent.

> I get the sense that our gen ed teachers [are] monitored more closely than what I do . . . Just knowing and feeling the way that I do about math, I would have a very, very hard time having to be lock-step with what everyone else is doing. (Sam)

So what makes me very sad, and still does at times when I see it happening, or I hear it . . .

> . is when everybody in the same school, in the same course, has to teach the same thing in the same way, with the same testing window that there is, at the same time, so that everything's uniform. And I've been in that situation. For me, I found personally, I became like a robot. And I wasn't even thinking what I had to do. I'm just like, this is what I have to do the next day, this is what I have to test. And I found that it didn't make me progress and learn as much. (Faye)

> I think just the bond between co-workers, that's a make or break situation. I've been in situations where I've gotten isolated, and it's very hard to teach well if you're isolated. (Paula)

A final situational difference the participants addressed was the support of their administration. In general, their administrators were supportive, although one noted she

thinks it's only because she teaches advanced classes. However, a couple of them pointed out supportive really meant the teachers could "do their thing," and so the teachers were "left alone." This seeming indifference carried its own message, which also minimized opportunities for growth. One said administrators can also be prescriptive, which doesn't allow for growth.

> I know there have been a couple of principals that I've worked for that I don't feel like have done anything to help me grow as a teacher. They're just sort of . . . you could just keep doing what you're doing when . . . No teacher wants to hear that. We all want to grow and have some different experiences. I think one of the saddest things about the profession of education is that sometimes teachers say, "Well, admin leaves me alone, so I must be doing things okay." And I mean I get it because administrators are usually on teachers that aren't doing what they're supposed to, but there's a lot of kind of mediocre teachers there that can grow more if they had positive encouragement to do a little something different. (Christine)

> Or, also the administration was very like rigid in terms of this is what you need to teach, this is how you need to teach it. (Edward)

Identifying and being attentive to the individual and situational differences of each learner provide a unique set of challenges for professional development specialists, school teams, and administrators. However, there is still one more consideration regarding differences – subject matter differences.

Subject matter differences. Subject matter differences refer to the simply stated, but complex reality that "not all subject matter can be taught or learned in the same way" (Knowles et al., 2011, p. 151). All of the participants shared examples of using different approaches, and noted their colleagues made sense of problems in different ways. And as already mentioned, there are a couple who believed if they had been taught a subject (e.g., geometry) the way they believed students are experiencing it now, then perhaps they would have a better foundation in various areas of math.

The individual and situational differences dimension/ring of the andragogical framework presents many insights and challenges in working with an adult learner. A final component which serves to inform are the individual's goals and purposes for learning.

Goals and purposes for learning. The goals and purposes of learning provide information about the expected developmental outcomes. There was some evidence of participants' awareness of growth due to societal and institutional factors. They recognized the discussion and research within the larger context of society which is framing math instruction and how teachers should be trained. For some of the participants, there were division or school efforts to improve instruction through change. However, as noted in the discussion regarding the participants' readiness to learn, the goal and purpose was for the sake of individual growth, so they could best support their students.

Individual growth. Participants' experiences prior to becoming educators or their early practitioner experiences prompted beliefs and questions. These beliefs and

questions led them to the conclusion that there must be a better way to engage students in mathematics to promote understanding. They each sought professional development opportunities (courses, rich team discussions, on-line blog recommendations, coaches/mentors, etc.) which they felt would provide them the knowledge and tools to help them confirm what they believed.

Meeting these teachers where they were and addressing their adult learning needs through the andragogical framework was complex. The evidence provided thus far seems to indicate that meeting their needs as adult learners happened in a variety of ways. The evidence has also helped reveal some clear aspects of their professional lives which were hampering, or at least not promoting, continued learning. This may have implications for the emergence, and more importantly, the sustainability of the teachers' productive disposition.

Productive Disposition

This section presents information regarding the participants' productive disposition.

> Productive disposition for teaching mathematics is mathematics teachers' malleable orientation toward – and concomitant beliefs, attitudes and emotions about – their own professional growth, the subject of mathematics, and its teaching and learning that influences their own and their students' successful mathematics learning. (Jacobsen & Kilpatrick, 2015, p. 402)

Included within this definition to better present teachers' beliefs are their references to learners and doers of mathematics, belief in diligence, usefulness and worthwhileness of

mathematics, and their efforts to seek and use resources. It is importation to point out the participants rarely spoke to the usefulness and worthwhileness of mathematics itself, rather the usefulness and worthwhileness of their professional experiences.

Professional growth. The teachers shared stories and beliefs which provided evidence of professional growth. They did this by expressing their views of themselves as learners and doers of mathematics, views about innate ability verses diligence, efforts to seek and participate in useful and worthwhile tasks and professional development opportunities, search for and use of resources, and thoughts about instructional strategies. A few examples of the presentation of these ideas for a handful of the participants which best represents the group follows.

> [The first year I taught middle school algebra] was a challenge because I was having to relearn math with them. I used to think when I struggled with a math problem it was because I didn't know the skill I needed to solve it. Now I think what resources do I have to be able to learn and understand this problem? I used to think sharing different models or explanations in the classroom was not worth the time. Pick the strategy most kids will be able to use and force everyone to do it. Now I think it is important to the mathematical growth of the students. (Christine)

> I used to think teaching math involved directing students through procedures and giving them ample time to practice. Now I think it is engaging student in various strategies to tackle problems and using their knowledge to find solutions. I used to

think students needed repeated exposure to procedures and processes to learn math skills. Now I think students need repeated exposures to models and mathematical ideas that they investigate and work on with groups. Sorting, error analysis, various strategies, etc. I think [professional development is about keeping yourself fresh. That's why I do it. Anything that keeps you on top of your game. (Amanda)

I'm saying because [the summer graduate courses] made me a better teacher, and they continue to make me a better teacher. They're not making me a better teacher because that's what I have to do, they're making me a better teacher because of what they cover, how they presented it, how I got to practice it, and how I get to bring it into my classroom without exception. That is just is huge. I used to think students who struggle mathematically either just did not do the work (practice) or that [they were] pushed through the curriculum, leaving many children with gaps in their understanding. Now I think students need the opportunity to and be guided through how to use manipulatives, multiple representations, taking quantities apart, discovering place value through rich tasks. (Kim)

I used to think math teaching was unengaging, straightforward, and boring. Now I think teaching is engaging, challenging, and exciting. I used to think teaching math was challenging because students wouldn't pay attention and I wouldn't be able to control them. Now I think it is easy because students naturally want to learn and understand . . . and those that don't will still do the work if I am

> respectful to them and get to know them. Just trying to teach them to remember rules, especially with fractions, kids just weren't understanding. So, I feel like it doesn't matter. Some kids are getting it and some kids aren't, and it's just like this isn't education. And so, it's like rather than just drill and kill, let's actually try to create some extra experiences that involves visuals and hands-on, and just basic things that they can come back to. (Edward)

The teachers' productive disposition toward their professional growth stemmed from both positive experiences they had in professional development and their students' success which they attributed to the change in their teaching practices based on the professional development.

> I just love seeing . . . Especially because even though I'm more comfortable with math, I still don't have that flexibility I think that these kids do at least at this age. So, I just love seeing what they do. That has really solidified my belief in these practices, too. That you get these kids that didn't traditionally think of themselves as the smartest math kid, but they show you in class they can do so many amazing things. (Sam)

> I'm really loving [multiple representations]. Like I've seen my kids do it and it's just amazing . . . You will see authentic learning from your students . . . My students were able to come up with various formulas and prove that it's right, and they compared. And I mean these students . . . I have seen [the students] who

> really didn't have any idea, so they were just really absorbing what their classmates are doing. (Jamie)

These positive changes in their students' learning encouraged the teachers to seek out more similar professional development experiences. The courses or opportunities most often mentioned were the summer graduate courses, math specialist related courses, and for a couple of the participants, it was the opportunity to work one-one with a mentor. For Ricky, the multiple opportunities to work one-one with a coach were invaluable. Finally, one of the participants had a lot of direct guidance and modeling from her college practicum supervisor.

Subject of math. In general, participants' views regarding the subject of math changed from math being about rules, procedures and memorization to one which could be learned through exploration and sense-making. The participants who had always been good at math because they understood the rules and procedures quickly acknowledged not all of their peers did. They recognized those same friends often hated math, and Faye said she saw it as a practitioner. Students who viewed math as rule-based believed they are either good at math or they are not. Faye said she noticed this happens around fourth to sixth grade.

The participants' statements regarding how they viewed math previously included: a set of procedures and rules; was not a language-was only practice; a gateway to success; about practice, practice, practice – the more practice the better you would be at doing it; a rote subject; a series of procedures applied to different contexts; all about learning the different algorithms; and linear and structured-once you finish a concept you

move on. By comparison, following professional development the participants in corresponding order, said: math requires accuracy, but is fluid, flexible, and open-ended; is a language, and can be taught several ways; is a civil right; is more about learning how to think and analyze, while still needing the numerical understanding of how things relate; as a language to communicate patterns around us; is a flexible way of thinking in that there are many ways to approach a problem and achieve a solution; is about learning how to use the learned mathematical concepts in solving a mathematical problem where various approaches and representations in arriving to the solution of the problem is acceptable; and is fluid allowing for concepts to be continually played with and enjoyed.

Throughout their interviews the participants, both those who felt they did well in math and those that struggled, also conveyed math was about rules and procedures. Sam specifically remembered fractions and trying to perform and understand the rules. She felt like she had no idea what she was doing. As an adult, in a professional development opportunity, she was working with models for the division of fractions. She remembered the feeling she had when she was confident in what she was doing and was able to explain it to another adult. She described it as a huge moment for her. Amanda and Edward both provided examples of using manipulatives and experiencing excitement when they realized why a procedure worked.

The participants' understanding of math and appreciation for the realization that math should make sense was a lengthy journey for most of them. Their interviews revealed this was not what most of them thought through high school; it was still procedural. The realization, whether through a methods course or professional

development activity, contributed to their malleable orientation toward math as a subject. These experiences also contributed to their belief in and commitment toward a different teaching and learning paradigm.

Math teaching and learning. Some of the participants had a few examples of what they considered rich teaching and learning experiences prior to college. Ricki, for example, shared her first experience in which math was learned within a social context – the teacher set the classroom up to be dialogical. A couple had memories of using things like base 10 blocks or models. However, the use of these strategies was isolated, so did not become a tool they regularly used or something that was, as Ricki said, "compressed" into their brain. She talked about developing a level of comfort with something, thus it was compressed into your brain, so that it became second nature or a go-to strategy or resource. A few of the teachers talked about the idea of comfort from a teaching perspective before implementing a tool or strategy, also.

Ricki clearly sees the power of using Cuisenaire rods, and had conducted professional development with them and encouraged principals to buy them. However, she noted she was still not comfortable using them (which she attributed to the spatial nature of the tool). Edward expressed a willingness to "just try" things with his students. However, he also acknowledged that it takes time to fully understand the tool and implement change purposefully in the classroom. In reference to himself and his co-working with some manipulatives, he recalled how much fun they had, that they were making sense of math and making great connections, but, he also said:

> And so, I think it takes a year of [exploring with the tools or strategies]. I feel like with a lot of teachers it's like the teachers need to first see it, and usually it's by teaching that you realize that, "Oh. yeah." And then a year or two later, [teachers] can actually integrate it into instruction so that it's a smooth experience for the kids. (Edward)

Edward is, however, willing to "muscle through" with the students and figure the math out using manipulatives with the students. He implemented a lesson using base 10 blocks and acknowledged "winging it." He told the class he knew they could be used to make sense of long division, saying, "I know it can work. Let's just figure it out." Other teachers expressed examples of learning with the students or empowering them to direct their own learning.

The participants expressed an idea of "we" in the classroom; the teachers and students experienced math learning together. This happened because they made efforts to empower the students. Jamie liked the summer graduate courses because "The approach to teaching is so different, and it empowers students to be a thinker . . . some students have gone through, 'I don't know math.' However, in that approach, they feel like, 'Hey, I can do some things here and I can do my own learning.'" Faye felt empowering students can make all the difference in their mathematical development. She said if they aren't empowered to believe and experience that they can be learners of mathematics, then they shut down. If that happens they certainly aren't going to understand math or even believe that math can make sense.

Participants also shared thoughts which suggested that both they and the students are both teachers and learners in the classroom as opposed to operating as if teachers teach and students learn. Many acknowledged that when students did something with math that they didn't understand, they assumed the students were wrong, or in a couple cases that the student was smarter than them. The student was therefore either redirected or ignored, respectively (often because the teachers were afraid to appear as if they didn't know/understand). The participants now see situations like these as opportunities to better understand the students thinking, which informs them as a learner and teacher of mathematics. Specifically, the participants said when they don't understand a student's work: I need to listen to THEIR method, I ask questions until I do, I love it! I think there is so much value in the different ways people think, I need to talk to the student and have him or her explain the process, it opens up an opportunity for the student to enter the discussion, and I want to understand it because (1) I want to know how their brain works and (2) I might want to steal their strategy to include in my repertoire of math teaching tricks.

The participants' productive disposition was established and maintained through an iterative cycle. The participants learned math differently and learned it could be taught differently. Some immediately and some over time implemented the different strategies they'd experienced. Upon seeing their student's success, the participants sought more ways to improve their practices and the cycle began again. They also saw everyone in the classroom as a teacher and learner in that the process of teaching and learning is fluid and flexible. They understood that each class and each student is different, so they continued

to search for new ways to make math accessible and understandable and prepare for the new things they must learn to help students and,in turn, learn from students.

Summary Findings

The following paragraphs consider the relationship, based on what the participants shared, between the andragogical framework and productive disposition. In some cases, there appeared to be an overlap between the two ideas. In other cases, it can be argued that aspects of the andragogical framework (e.g., the participants' needs) must be met in order to help ensure a productive disposition. The participants' attitudes and beliefs toward math, math teaching and learning, and their own professional growth were all influenced by their experiences.

Each of the participants demonstrated at least a good self-concept as a learner of mathematics. They believed themselves to have the skills, in at least most areas of math and certainly those areas for which they were responsible as educators, to be autonomous learners. They expressed a desire to be treated as adults and to have minimal limitations put on them in professional development settings. This included their teaching environment (did not want to teach "lock-step" with other teachers). However, they all also acknowledged a time when a push or directive was needed to model division of fractions, create an additional representation, explore tasks as rich experiences in that they promoted dialogue and covered more than the identified objective, etc. Whether the belief formed in their formative years, or later in their early adult lives, it is important to note that they all viewed themselves as independent math learners within the andragogical framework. It is equally important to recognize the overlap this principle

has with viewing themselves as learners and doers of mathematics within the definition of productive disposition. They all believed themselves capable of doing the math put in front of them. However, they also acknowledged that not all learners had the tools which made math accessible.

Many of the participants approached staff development opportunities with two aspects of the andragogical framework which needed to be addressed: prior experience of the learner and individual learner differences. The prior experiences of the learner need stemmed from two situations: the participant did not have a rich experience in their formative years or their students were struggling or not meeting their performance expectations. Regardless of the reason, the participants wanted to better meet the needs of learners. They knew they did not have the prior knowledge as a learner or teacher from which to draw to meet the students' needs. Therefore, in line with the definition of productive disposition, they sought resources and strategies to build their instructional repertoire, thus creating "newer" prior experiences from which they could draw. The participants then used these new experiences appropriately in the classroom. It is important to note that many were aware there was a right type of professional development activity depending on the need. Richer instructional needs were met through opportunities such as the graduate courses. However, if a new graphic organizer was needed to support sense-making, then a blog or other on-line site was just as effective, where effectiveness was determined by the success of their students.

The individual learner differences need was based in several differences ranging from how they best learned math to issues with anxiety. Although clearly differences in

how they learn translated to their students, the participants spoke to other various individual needs they had in professional development experiences which could also translate to working with students. A few participants said they needed a safe environment in which math could be explored, and they felt safe to take risks and make mistakes. Others talked about the accessibility of the tasks with which they were presented so whether they were elementary or secondary-minded, they could approach the problem based on their individual strengths or knowledge. In this case, there is little overlap between the andragogical need and the definition for productive disposition. In the cases where individual differences (and related subject matter differences) were not met, the participants' expressed frustration or that the experience did not translate to their own growth or the math classroom (e.g., Ricki and her first experience with Cuisenaire rods, Kim and practitioner geometry experiences, etc.).

The next two most prominent andragogical aspects present were the participants' readiness to learn and their orientation to learning. In the case of their readiness to learn, the participants needed the information to be relevant and to address a life-related situation they were experiencing. They also needed to believe the information was immediately applicable (orientation). Many had situations in which students were struggling, and they wanted interventions or strategies to help the students. Again, there is little overlap with the definition of productive disposition. In this case there is evidence a professional development experience provided them something which they thought might improve learning for their students, they implemented the change, and then their attitude and concomitant beliefs regarding teaching and learning were changed when

students' understanding increased. It is important to note that two participants pointed to the accountability of implementing the instructional change through lesson study, and many talked about the necessity of support with the same resources available at their school or guidance.The participants expressed seeing their students as learners and doers of mathematics because of these experiences which then also promoted productive disposition. This trend then likely also changed their beliefs about effective professional development and their own growth.

Situational differences created a need for a few of the participants. Edward needed opportunities to collaborate and work with other teachers on similar student deficits. Therefore, he attended professional opportunities and sought opportunities to meet this need. He also noted he hated professional development sessions which could provide opportunities for collaboration and did not. A couple of participants talked about support at their school, which they addressed or tried to address by inviting colleagues to attend the professional development with them. Two teachers noted individual mentors or coaches which were paramount in their professional development, both of whom are now coaches attempting to do this for other teachers. One of them, Ricki, said in many ways, although she believed instruction could be better, this individualized attention was critical to her math content knowledge and to her belief in the ability of all to learn provided they have appropriate supports/resources. This area does not overlap with the definition of productive disposition, save seeking supportive situations as a resource. It is arguable that creating situational supports for teachers based on their needs contributes to their productive disposition.

The final prominent aspect of andragogy was the participants' desire to grow professionally as individuals. Although a few became involved in larger institutional or societal constructs for growth, they sought those opportunities to further their individual growth. Each of them was able to articulate ways in which they grew professionally, both in their content and their pedagogical knowledge. They also noted experiences which did not help them grow (e.g., writing goals, curriculum work in one case, non-productive team conversations, teaching lock-step, being isolated, and a laissez-faire approach by administrators). It is arguable that participants avoided or stopped attending types or structures of professional development that they believed did not help them grow. However, as they experienced growth their attitudes toward and beliefs about specific forms of professional development also changed. The culmination of these experiences, therefore, contributed to their overall productive disposition toward their own professional growth.

Chapter Five

The purpose of this dissertation is to provide insight on teachers' behaviors and thoughts in relation to professional development and productive disposition toward mathematics and mathematics instruction. The professional development provided placed teachers in purely educational settings as learners and teachers over a sustained period of time, with a focus on pedagogy within the context of relevant content, and considered support provided by the instructional team and the teacher's supervisors and colleagues. Given this, the following questions were researched:

1. How do teachers describe the changes in their beliefs, attitudes and emotions about (a) their own professional growth, (b) the subject of mathematics, and (c) mathematics teaching and learning over their teaching careers?
2. How can the andragogical framework be used to identify and explain events which serve as catalysts which promote or hinder teachers' productive dispositions?
3. What are indicators, of which professional development designers should be aware, which will further promote or hinder productive disposition? Is there evidence regarding how these indicators should be addressed to maximize growth?

Nine teachers who participated in at least two summer graduate courses were identified by the primary researcher of the program as professionals who demonstrated growth as mathematics teachers and learners. The participants then completed a survey and an interview. They also responded to follow up questions which were posed to confirm or clarify information. Two of the respondents also sent follow-up emails after their interview. The purpose of the survey and interview was to identify in what ways they demonstrated productive disposition and the professional experiences which influenced their productive disposition. Included in the professional experiences were methods courses; professional development opportunities; and support provided by instructors, professional development facilitators, colleagues and administrators. The support they received was viewed through an andragogical framework.

Research Question 1

Teachers described changes in their beliefs, attitudes and emotions about their own professional growth, the subject of mathematics, and mathematics teaching and learning as an iterative and continual process. Although they described some specific learning experiences as catalysts or "conversion" moments, it was iterative because the teachers acquired new knowledge, implemented what they had learned, identified more needs or questions, and sought more learning experiences. Each new professional learning experience – professional development session, classroom experience, team collaboration, debrief with administrator, etc. – provided them with information which they then used to make purposeful decisions regarding instruction. When students' understanding increases, teachers are encouraged to seek more opportunities which will

inform their instruction. Therefore, the change in their practice is best represented by Clarke and Hollingsworth's (2002) interconnected model of professional growth. The teachers also referred to this as a process or journey. They articulated clear examples of improved practice and changes in their beliefs, but also stated there is much more to learn and room for growth as practitioners.

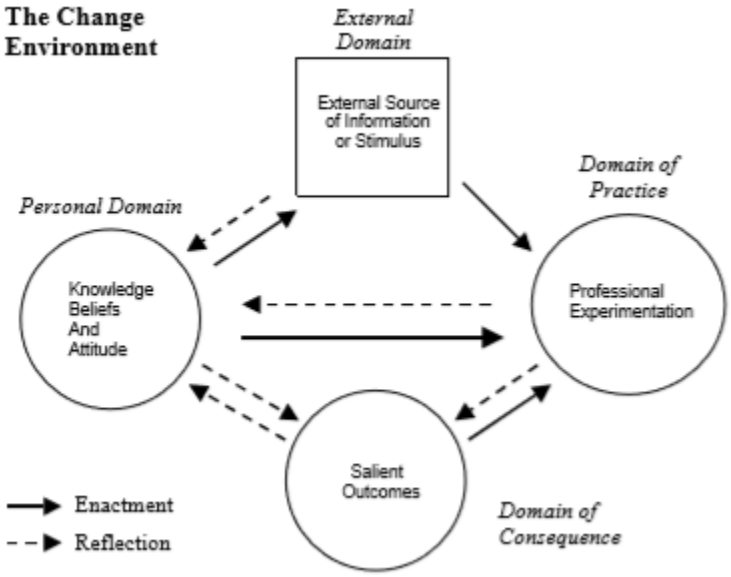

Figure 6. The interconnected model of professional growth. Clarke, D. & Hollingsworth, H. (2002). Elaborating a model of teacher professional growth. *Teaching and Teacher Education, 18,* p. 951. doi:10.1016/s0742-051x(02)00053-7.

Changes related to their professional growth were attributed to three types of experiences. The first were experiences which asked that they explore the content they teach but required that they approach the math differently or specifically from the perspective of their students. Manipulatives, models, pattern recognition, rich tasks, and

discussion were all used as tools to support their "re-learning" of the content. The second set of experiences were revelations they had when they saw their students as learners and doers. Many were amazed by the thinking which their students were capable of when given the opportunity. This prompted the teachers to gain more pedagogical and content knowledge so the teachers could further promote student thinking. A third experience which contributed to their beliefs about professional growth was the lesson study experience. They appreciated the opportunity to visit other classrooms and see how teachers instructed and students engaged in mathematics instruction. A few of them identified the lesson study process as a powerful professional growth experience. It helped them establish clear beliefs about expectations for any collaborative learning team when the goals are professional growth and improved student performance.

The teachers' changes in their beliefs, attitudes and emotions about the subject of mathematics derived from their own experiences and those of their students. Whether they considered themselves good at math or not, experiences in their adult lives provided them with evidence that math was not a rote and procedural subject. Given opportunities to explore math the participants learned they could "make sense of" math. The teachers expressed they no longer thought of math as an either you understand it, or you don't subject; it was a subject which can be figured out with the right resources and perseverance. Teachers with and without math degrees also expressed their fascination with the approaches taken by their colleagues. High school (math degreed) teachers were humbled by the approaches by their elementary counterparts, and elementary teachers noted their strategies sometimes enabled them to make sense of tasks before their high

school colleagues, confirming for both groups that math can be accessible to all, and one approach does not work for all learners. As teachers enacted these practices in their classroom, they observed similar phenomenon among their students. This included the gifted student who struggled with tasks and the struggling student who modeled the pattern and convinced others their thinking was mathematically sound. These combined experiences changed their beliefs about math. As Kim said, "There are lots of ways to math. Yes, math is a verb!"

The teachers' changes in their beliefs, attitudes and emotions about teaching and learning derived from rich professional development experiences but more so from their experiences with students. As discussed previously, the observations they made, and the knowledge acquired in rich professional development experiences helped them see that math could be taught differently. However, even those who attended these sessions wanting to know how to teach better were sometimes hesitant about how these strategies would work with students. They talked about being resistant, feeling they didn't have time, and unsure of their own comfort level with the ideas that were shared. It was implementing these strategies in their own classroom and with students in a realistic setting that convinced them teaching and learning could be different and powerful. One participant said that was the best part of the lesson study cycle; it was a measure of accountability to implement the tasks and strategies. Without the expectation to complete the cycle implementing what had been modeled for them, it is likely many would have waited. Once the teacher observed the student engagement and the thinking of her own students, she was more willing to implement other ideas.

Research Question 2

The catalysts which promote a teacher's productive disposition are based in rich and productive experiences. It is important to note the participants in this study believed or had hope that math teaching and learning could be better facilitated, save Faye who wanted to know how students developed mathematically. Therefore, they had a need that required attention. It can then be inferred they were open to thinking about teaching and learning differently and just needed the right set of circumstances. For most of the participants, the catalyst or catalysts were learning experiences they had with their adult peers. It may have been a particular task, a week in which they focused on a strategy (e.g., modeling or mathematical discussions), or a teacher who provided new or interesting strategies throughout a course or participating in a lesson study cycle. What was key was that their needs (from an andragogical perspective) were met. Different approaches were modeled, thinking shared, individual learning differences appreciated, situational differences acknowledged, and the participants felt they were in a safe place to explore math teaching and learning. A couple of participants spoke to one event as a catalyst, but the theme was that it was a culmination of rich experiences. These experiences/catalysts also included what happened in their classrooms.

Believing the strategies would help students and experiencing their benefit with peers is very different from observing the benefit of the strategies with their own students. The participants all spoke to the inspirational and motivating aspects of observing their students' successes. Two of them recalled a situation with a particular student that confirmed for them as teachers they were on the right path toward improving

instruction for students. These experiences met their motivation to learn core principle need. There was an intrinsic reward for them in both their students' mathematical progress and in their perceived instructional growth.

The circumstances which hindered productive disposition, or which did not promote it were all examples of violations of the andragogical core principles or dimensions/rings (individual and situational differences or goals and purposes for learning). It is again acknowledged that there is some overlap between the components of andragogy, so it could be argued any of these best fits with a different component, but the point is the circumstances fall within the andragogical framework.

A few of the teachers talked about their prior knowledge (or lack thereof) or that as an adult they realized they learn differently. Their lack of prior knowledge created frustrating situations when the instructor or facilitator either made assumptions about what they knew in relation to the course content or did not scaffold to address deficits. In the case of these participants, this confirmed their belief they were not good in this particular subject (geometry). All but one retains this feeling today, although I feel if they kept trying or participated in an Introductory Geometry course, they could learn geometry. Ricki, although she feels she was robbed in her formative years, did not retain the belief she was bad at geometry. Fortunately for her, she had an experience in which an instructor drew in a diagram the aspects of the diagram other students were able to infer. Ricki described feeling "being seen" and how much she appreciated that the teacher attended to individual learning differences.

Most of the participants expressed situational differences which impacted productive disposition. The situations all involved a lack of support. Teachers attended professional development in which manipulatives or a software tool was part of the instruction. However, these tools were not provided to the teachers by the facilitator, nor available or purchased at their school site. Some teachers returned to teams which were resistant to the strategies they suggested, or to situations of isolation. Others worked in schools in which the expectation was every teacher do the same thing on the same day and give the same assessment. This was discouraging for all who experienced or observed this in their school setting. A final situational difference was one of perceived administrative indifference. In these cases, the teachers were disheartened because they were considered good teachers and were "left alone." So, although it didn't hamper the teachers' growth or productive disposition, it also didn't promote a productive disposition.

Two of the teachers talked briefly about standardized testing; they expressed it in such a way that it pointed to a conflict between societal growth and situational differences (or students' individual learner differences). From a societal perspective they understood the need for accountability. However, the pressure to complete the prescribed curriculum often conflicts with their beliefs about what is instructionally sound for their students. The pacing, especially as they near the time for the state standardized test, limits their ability to allow students to make sense of the mathematics. Both teachers have found an approach which works for them; one has learned to balance the two throughout

the year and the other works to thoroughly cover most of the curriculum and works with her vertical team regarding any deficits.

Although the participants identified several circumstances which were discouraging or frustrating, there were fortunately none that have been constant experiences throughout their career, save the standardized testing. These experiences and situations have informed them about what is professionally helpful to them and what they hope to avoid in any instructional setting. Although not explicitly asked or stated, there is some evidence these unproductive experiences informed how they work with students, too.

The andragogical framework sometimes overlaps with productive disposition as noted in Figure 7: andragogical framework (AF) ~ professional development (PD). For example, the andragogical self-concept with which each participant arrived to professional development aligns with them seeing themselves as learners and doers of mathematics within the definition of productive disposition. There is also some overlap between prior knowledge of the learner and (andragogy) and seeking and using resources (productive disposition). The data collected regarding other aspects of andragogy which were prominent, including readiness to learn, orientation to learn, motivation to learn, individual differences, situational differences, subject matter differences and individual growth, seem to indicate a directional relationship between andragogy and productive disposition which is also noted in Figure 7: andragogy framework (AF) → productive disposition (PD). In other words, teachers have needs within the andragogical framework they must have met. If overall, these needs are not met for an individual teacher, it is

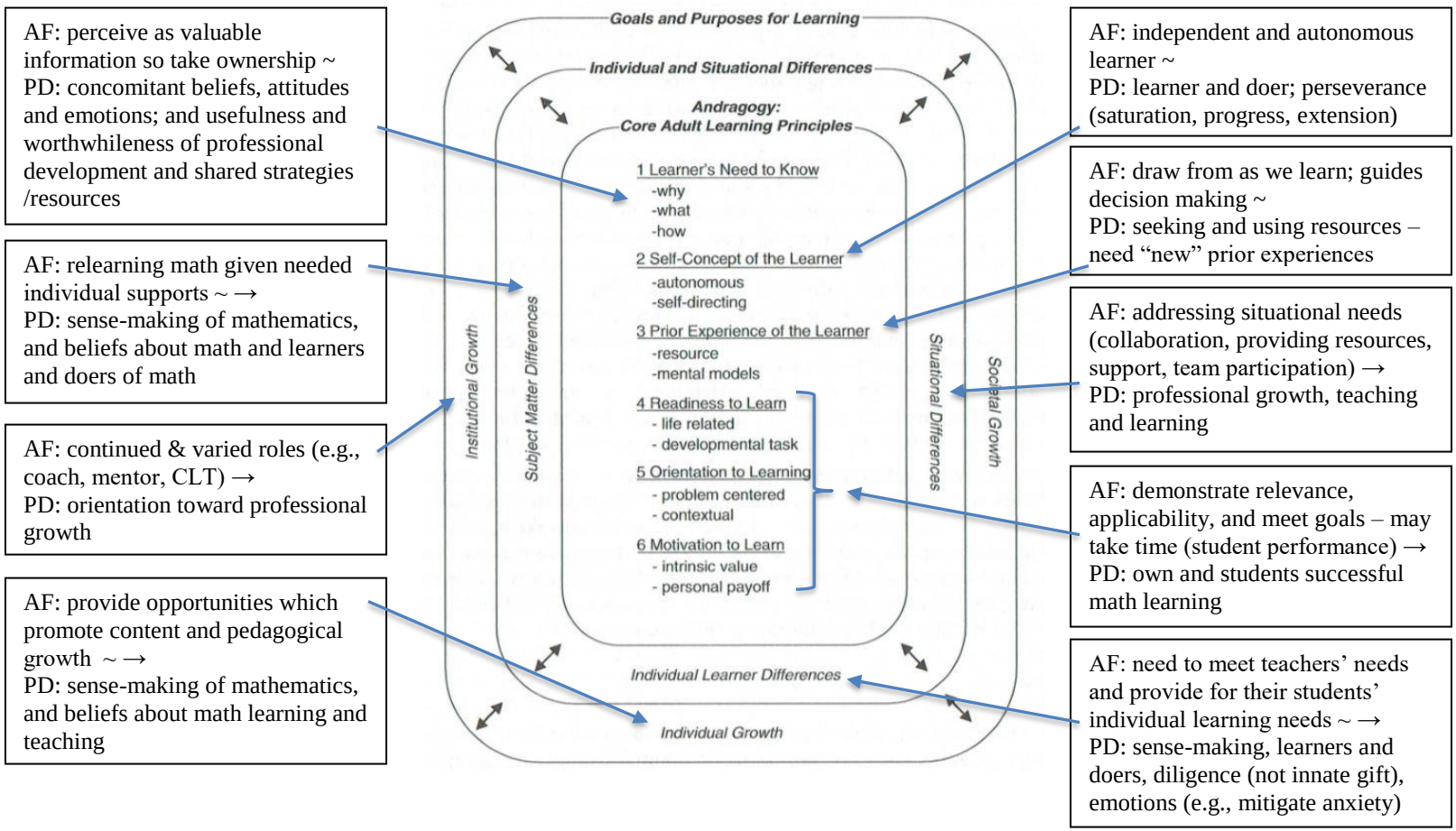

Figure 7. Andragogy and productive disposition. Modified from: Knowles, M.S., Holton, E.F., and Swanson, R.A. (2011). *The adult learner: The definitive classic in adult education and human resource development* (7th ed.). Burlington, MA: Elsevier, Inc.

difficult for the teacher to develop or maintain a productive disposition. In the cases of these participants, most of their andragogical needs were regularly met, thus they developed and maintained a productive disposition toward math, teaching and learning, and their own professional growth.

Research Question 3

Given the evidence in this research, professional development designers need to attend to the teachers from an andragogical perspective and with an understanding of the interactive play of the components of Clarke and Hollingsworth's (2002) interconnected model of professional growth. The participants in the study, when asked about aspects of their professional career which were difficult, all reported examples of their needs not being met as an adult learner.

Professional development specialists who can attend to teachers as adult learners and meet their needs as noted in the andragogical framework will best promote productive disposition. There were life-events which shaped the participants of this study, provided information about their needs and perceived insecurities, and helped in the identification of when and how their needs were met. As it is unlikely specialists could gather an equivalent background prior to professional development, they should consider the following: administer an opening survey to gather some initial information on the participants which help reveal their beliefs and feelings about mathematics; ensure the lessons are differentiated so they are accessible to all learners; attend to the teachers and note when they are frustrated, excited, overwhelmed so support and validation can be provided; create a safe environment in which risk-taking, questioning, and sharing ideas

and misconceptions are encouraged; and provide learning and teaching environments for them which are realistic.

Reality dictates that some of the aspects of the andragogical framework are out of professional development specialists' purview. For example, they cannot control the team support or administrative guidance for the teacher. They can however, take steps to mitigate these factors. Coordinators of professional development should provide resources (e.g., manipulatives) and support (e.g., follow-up observations, planning sessions, or data analysis). They should invite colleagues and administrators to conduct observations with them or attend part of the training. At a minimum they should share the goals of the professional development with the supervising administrators and what the administrators should expect to see as a result of the training or make suggestions about how the administrator can support the teacher.

Implications for Stakeholders

The implications for the professional development facilitators are delineated above. However, there are also implications for administrators and teachers. Teachers need the support and continued guidance of their supervising administrators. It is important the administrators work toward an understanding of what the teachers experienced, the teacher's plan for implementation, and the intended student learning outcomes. There is often a bit of discomfort with the implementation of new strategies or tools, so the administrator should be attentive to the teacher and guide them through this, ensuring the conversation and efforts are focused on what is best for students. The administrator should also provide continued support through observations, reflections and

by discussing student performance with the teachers. Finally, the administrator must be aware of the andragogical needs of the teacher. For example, an administrator should note a teacher's discomfort if they feel they need to be doing exactly what another teacher is doing and encourage the teacher to meet the needs of their students. Or, the administrated should note when a teacher is uncomfortable with a new teaching assignment, determine why, and do what is necessary to provide the support so the teacher becomes comfortable with the assignment. A final example would be noting the support of the teacher's team. It is important the focus of team conversations be on student thinking and promoting student growth in mathematics by identifying appropriate resources and strategies.

Teachers need to be mindful of their own needs as learners and teachers. Although it sounds simple, it is often hard for adults to advocate for themselves. For the purpose of professional growth and productive disposition, adult learners need to, respectfully and professionally, advocate for themselves when their needs are not being met. They should be prepared to identify what can be done to meet their need(s) or to discuss options when they are unsure how the need can be addressed. They must be receptive to the truth that meeting some needs may take time (e.g., school team support and collaboration regarding the implementation of new strategies or tools). They must also be ready to act to support their needs. In the case of these participants, one invited colleagues to attend professional development with them, another sought out collaborative environments as he was an isolated teacher, and another asked herself how information from professional development experiences outside the realm of high school

math could be applied to teaching high school math. In each of these cases, the teachers addressed situational concerns on their own.

Recommendations for Future Research

There are two specific recommendations for future research. The first is a longitudinal study with a similar number of participants. Although existing data from the past few years was considered for this research, there would be benefit in having first-hand knowledge and first-person observations over time of the participants in various situations besides the professional development. Lesson, team collaboration meetings, and meetings the teachers have with their administrators could also be observed. This would better inform the researcher regarding if and how various needs of the adult learner within the andragogical framework are being met. A more pragmatic approach could be considered regarding the relationship between the andragogical framework and productive disposition. This would also enable the researcher to determine if there is specific catalyst which promotes productive disposition or if it emerges gradually over time due to a culmination of experiences. Finally, it would mitigate the dependence on the participants self-reports.

A second recommendation is researchers should consider teachers who are not reported to exemplify a productive disposition. As noted in the participant selection for this research, participants who were perceived to have a productive disposition and for whom there was data to support this perception, were chosen for the study. As Christine clearly stated, there are many mediocre teachers who do not seem motivated to change their practice, are resistant, or are fine being "left alone" by administration. Teachers who

have not had rich professional development and classroom experiences as described by these participants would offer a different perspective on productive disposition and how their adult learning needs are met. Furthermore, research which considers teachers such as these participants alongside teachers who do not appear to have a productive disposition may help determine if an event/circumstance prevented/discouraged the teachers from seeking opportunities which support productive disposition, or if there are steps which can be taken, or catalysts which will help teachers take advantage of opportunities that promote a productive disposition.

Closing Thoughts

This research considered the emergence and sustainability of the productive disposition of teachers. As the research was conducted with the adult learner in mind, the andragogical framework was applied. Each participant's mathematical development, training, and professional development experiences was considered. This information was used to help identify each participant's adult learning placement and needs within the andragogical framework. The research then provided evidence of the teacher's productive disposition. Once each participant was discussed, a comparison was done of the group in relation to both andragogy and productive disposition in an effort to identify common themes or ideas.

In general, courses or professional development experiences which immersed the teachers in the content they teach using varied strategies and manipulatives. The combined experiences in their classrooms confirmed the validity of these practices, highlighted student thinking, and promoted and sustained the teachers' productive

dispositions. Although there were circumstances or events which threatened each participant's productive disposition, all of which were examples of their needs not being met from an andragogical perspective, the circumstances and events were mitigated because an overwhelming majority of their needs were being met. Additionally, in the two encompassing experiences which seem most critical in promoting and sustaining professional development – rich learning and teaching experiences – the adult learners' needs were being met.

Appendix A

Survey

1. I read the consent form and agree to be a part of this study.
2. How many years have you taught mathematics?
3. In how many school systems/divisions have you taught?
4. In how many different schools have you taught?
5. Which grade levels have you taught?
6. Create a timeline by listing events which have had an impact on your teaching career. You need not present a lot of detail, just list the events and include a sentence or two about why the event had an impact.
7. Math
   I used to think math…
   Now I think math…
8. Teaching math
   I used to think teaching math…
   Now I think teaching math…
9. Students (in relation to doing math)
   I used to think students (in relation to doing math)…
   Now I think students (in relation to doing math)…
10. Students who struggle mathematically
    I used to think students who struggled mathematically…
    Now I think students who struggle mathematically…
11. When I struggled with a math problem
    I used to think when I struggled with a math problem…
    Now I think when I struggled with a math problem…
12. Sharing different models or explanations
    I used to think sharing different models or explanations…

Appendix A (Continued)

Now I think sharing different models or explanations…

13. Sharing incorrect mathematical reasoning
    I used to think sharing incorrect mathematical reasoning…
    Now I think sharing incorrect mathematical reasoning…
14. Clear and well-defined algorithm/set of steps
    I used to think explaining a clear and well-defined algorithm/set of steps to solve a problem was…
    Now I think explaining a clear and well-defined algorithm/set of steps to solve a problem is…
15. Pointing out a student's error
    I used to think specifically pointing out a student's error or telling them exactly how to do a problem was…
    Now I think specifically pointing out a student's error or telling them exactly how to do a problem is…
16. Memorization
    I used to think the role of memorization in mathematics was…
    Now I think the role of memorization in mathematics is…
17. Student strategy
    I used to think when a student used a strategy I didn't understand…
    Now I think when a student uses a strategy I don't understand…
18. Student math proficiency
    I used to think the best indicator of a student's math proficiency was…
    Now I think the best indicator of a student's math proficiency is…
19. Lesson closure
    I used to think closure to a lesson meant…
    Now I think closure to a lesson means…
20. I am willing to be a part of the next phase of this study which includes 1 – 2 interviews. Therefore, I am providing my contact information: Name, email address, and phone number.

Survey modified from: Charalambous, C. Y. (2015). Working at the intersection of teacher knowledge, teacher beliefs, and teaching practice: a multiple-case study. *Journal of Mathematics Teacher Education, 18,* 427-445. doi:10.1007/s10857-015-9318-7

Appendix B

Semi-Structured Interview Protocol

The purpose of the semi-structured interview was to gain an understanding of the mathematical development and educational history of the participant. This included the types/forms of professional development in which they had participated throughout their career. This interview provided information related to the andragogical aspects of the research framework. From this interview and the survey, follow-up questions were developed where necessary to confirm interpretations or clarify information shared.

Interview: Semi-Structured Questions

1. What do you remember about your math learning experiences in elementary school? Middle school? High school?
2. In which mathematics courses or experiences did you participate to prepare you to teach? College courses – math and methods?
3. In which activities do you participate which contribute to your professional growth?
4. What types of professional development have you participated in since you started teaching? How many of each? How often?

Appendix B (Continued)

5. What aspects of professional development do you find the most helpful and why?
6. What was an experience you had during a professional development activity/task which was difficult, made you uncomfortable mathematically, or challenged your thinking? How did you feel about it at the time? How do you feel about it now?
7. What aspects of your career, including teaching environment, have you found to be the most supportive/helpful?
8. What experiences in your professional career stand out to you and why?

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Biography

Katherine J. Bolluyt-Meints graduated from South Hamilton High School, Jewell, Iowa in 1988. She received a Bachelor of Science degree in mathematics and secondary education from Iowa State University in 1994, and a Master of Science degree in professional studies in higher education from Iowa State University in 1996. She was employed as a teacher at Interstate 35 High School in Truro, Iowa for 2 years before moving to Virginia. She taught at E. C. Glass High School in Lynchburg for 2 years and Forest Park High School in Woodbridge, Virginia for 6 years. During that time, she earned a Master of Science degree in education administration, completing the degree in 2005. She then worked as the secondary math coordinator for Prince William County Schools from 2006-2009. Katherine J. Bolluyt-Meints became an assistant principal for Brentsville District High School in 2009 and was promoted to principal in 2011.